Phased Array Damage Detection and Damage Classification in Guided Wave Structural Health Monitoring

Daewon Kim

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Michael K. Philen, Chair
Rakesh K. Kapania
Mayuresh J. Patil
Daniel J. Inman

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(ABSTRACT)

Although nondestructive evaluation techniques have been implemented in many industry fields and proved to be useful, they are generally expensive, time consuming, and the results may not always be reliable. To overcome these drawbacks, structural health monitoring (SHM) systems have received significant attention in the past two decades. As structural systems are becoming more complicated and new materials are being developed, new methodologies, theories, and approaches in SHM have been developed for damage detection, diagnosis, and prognosis.

Among the methods developed, the guided Lamb wave based SHM can be a promising technique for damage evaluation since it provides reliable damage information through signals propagating over large distance with little loss of amplitude. While this method is effective for damage assessment, the guided Lamb wave contains complicated mode characteristics, i.e. an infinite number of wave modes exist and these modes are generally dispersive. For this reason, a minimum number of wave modes and various signal processing algorithms are implemented to obtain better signal interpretations.

Phased array beamsteering is an effective means for damage detection in guided Lamb wave SHM systems. Using this method, the wave energy can be focused at localized directions or areas by controlled excitation time delay of each array element. In this research, two types of transducers are utilized as phased array elements to compare beamsteering characteristics. Monolithic piezoceramic (PZT) transducers are investigated for beamsteering by assuming omnidirectional point sources for each actuator. MacroFiber Composite (MFC) transducers with anisotropic actuation are also studied, considering the wave main lobe width, main lobe magnitude, and side lobe levels. Analysis results demonstrate that the MFC phased arrays perform better than the PZT phased arrays for a range of beamsteering angles and have reduced
main lobe width and side lobe levels. Experiments using the PZT and MFC phased arrays on an 
aluminum plate are also performed and compared to the analysis results.

A time-frequency signal processing algorithm coupled with a machine learning method can 
form a robust damage diagnostic system. Four types of such algorithms, i.e. short time Fourier 
transform, Wigner-Ville distribution, wavelet transform, and matching pursuit, are investigated 
to select an appropriate algorithm for damage classification, and a spectrogram based on short 
time Fourier transform is adopted for its suitability. A machine learning algorithm called 
Adaboost is chosen due to its effectiveness and high accuracy performance. The classification is 
preformed using spectrograms and Adaboost for crack and corrosion damages. Artificial cracks 
and corrosions are created in Abaqus® to obtain the training samples consist of spectrograms. 
Several beam experiments in laboratory and additional simulations are also performed to get the 
testing samples for Adaboost. The analysis results show that not only correct damage 
classification is possible, but the confidence levels of each sample are acquired.
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Table of Contents

1. Introduction ........................................................................................................................... 1
   1.1 Structural Health Monitoring ........................................................................................... 2
   1.2 Structural Health Monitoring Approaches ......................................................................... 3
   1.2.1 Vibration Based Method ........................................................................................... 3
   1.2.2 Impedance Based Method .......................................................................................... 3
   1.2.3 Guided Wave Based Method ....................................................................................... 4
   1.3 Literature Review ............................................................................................................. 7
   1.3.1 Damage Detection ..................................................................................................... 8
   1.3.2 Damage Diagnosis ................................................................................................... 31
   1.4 Damage Types for Classification ..................................................................................... 37
   1.5 Problem Statement and Research Objectives ................................................................... 39
   1.6 Dissertation Outline ......................................................................................................... 39

2. Guided Wave Field Expressions ............................................................................................. 41
   2.1 Mathematical Displacement Formulations ........................................................................ 41
   2.1.1 Rectangular Piezoceramics ....................................................................................... 47
   2.1.2 MacroFiber Composites ............................................................................................ 50
   2.2 Sensor Response Formulations ....................................................................................... 51
   2.3 Conclusion ...................................................................................................................... 54

3. Beamsteering Analysis and Discussion ................................................................................ 55
   3.1 Limitations of Omni-Directional Beamsteering Algorithms for MFC Phased Arrays ..... 55
   3.2 Analysis and Discussion of Beamsteering with PZT and MFC Phased Arrays ............... 59
   3.3 Conclusion ...................................................................................................................... 62

4. Experimental Validations ....................................................................................................... 63
   4.1 Sensor Voltage Response ............................................................................................... 63
   4.2 Phased Array Beamsteering .......................................................................................... 66
   4.3 Conclusion ...................................................................................................................... 71

5. Damage and Signal Analysis ................................................................................................. 72
   5.1 Damage Forms ............................................................................................................... 72
List of Figures

Figure 1.1. Phase velocity dispersion curves for aluminum (E=73.1 GPa, ν=0.33, ρ=2780 kg/m³) ......................................................................................................................................................... 6
Figure 1.2. Group velocity dispersion curves for aluminum (E=73.1 GPa, ν=0.33, ρ=2780 kg/m³) ......................................................................................................................................................... 7
Figure 2.1. Isotropic plate structure with the PZT and MFC actuators ........................................ 42
Figure 2.2. Particle directions for symmetric and anti-symmetric modes .................................. 44
Figure 2.3. Illustration of the (a) MacroFiber Composite actuator and (b) actuator package [102] ....................................................................................................................................................... 50
Figure 3.1. Simulated wave propagations (a) and wave pattern (b) for one PZT ......................... 56
Figure 3.2. Simulated wave propagations (a) and wave pattern (b) for one MFC ....................... 56
Figure 3.3. Phased array response of PZT and MFC using generalized beamsteering expressions ....................................................................................................................................................... 57
Figure 3.4. (a) Beamforming to 60° for PZT and MFC, (b) degree lines at 60° and 74° for MFC investigation, and (c) amplitude using two MFC sources for 60° and 74° degree lines using Eq. (3.1) ....................................................................................................................................................... 58
Figure 3.5. Relative time delays for each actuator for different angles using (a) 2 actuators, (b) 4 actuators, and (c) 6 actuators. ....................................................................................................................................................... 60
Figure 3.6. Phased array response with optimal time delays using 6 actuators (a) 85° and (b) 65° ....................................................................................................................................................... 60
Figure 3.7. Main lobe and side lobe areas for different angles using (a) 2 actuators, (b) 4 actuators, and (c) 6 actuators ....................................................................................................................................................... 61
Figure 3.8. Area ratios for different angles using (a) 2 actuators, (b) 4 actuators, and (c) 6 actuators ....................................................................................................................................................... 62
Figure 4.1. Sensor response amplitudes for S0 and A0 modes (PSI-5H4E) .................................. 64
Figure 4.2. Comparison between theory and experimental results for A0 mode ......................... 65
Figure 4.3. Experiment overview (a) and multi-channel amplifier module (b) ....................... 66
Figure 4.4. Experimental setup ............................................................................................. 68
Figure 4.5. Data cleansing of measured data ................................................................................ 68
Figure 4.6. Normalized beamforming results from experiment (left) and theory (right) for target angle of 60° and $r_t=10D$ (in meter) ....................................................................................................................................................... 69
Figure 4.7. Normalized amplitudes for PZT (a) and MFC (b) arrays for target angle of 60° measured at $r_t=2D$. Main and side lobe areas for PZT (c) and MFC (d) for the 1st quadrant .... 71
Figure 5.1. Induced aluminum corrosion using saline solution (a) top and side views, (b) cross-section of undamaged region, and (c) cross-section in the middle of corrosion ................................. 73
Figure 5.2. Excitation signals with (a) 1-cycle sine, (b) 1-cycle Hanning windowed sine, (c) 2.5-cycles sine, and (d) 2.5-cycles Hanning windowed sine ....................................................................... 75
Figure 5.3. Fourier transform of (a) 1-cycle sine, (b) 1-cycle Hanning windowed sine, (c) 2.5-cycles sine, and (d) 2.5-cycles Hanning windowed sine ............................................................. 76
Figure 5.4. Wave propagation at 250 μs with various inputs (a) 1-cycle sine, (b) 1-cycle Hanning windowed sine, (c) 2.5-cycles sine, and (d) 2.5-cycles Hanning windowed sine ......................... 77
Figure 5.5. Time signals at 20 cm from actuators for (a) 1-cycle sine, (b) 1-cycle Hanning windowed sine, (c) 2.5-cycles sine, and (d) 2.5-cycles Hanning windowed sine ......................... 78
Figure 5.6. Simulation of dispersive A0 mode; (a) space-time map, (b) original signal, and (c) 1 m traveled signal ........................................................................................................................................ 79
Figure 5.7. Simulation of non-dispersive S0 mode; (a) space-time map, (b) original signal, and (c) 1 m traveled signal .................................................................................................................. 80
Figure 5.8. Dispersion removal of A0 mode (20 kHz and 0.8 m distance); (a) original signal, (b) dispersed signal, and (c) after dispersion removal ................................................................................. 82
Figure 5.9. Cross-correlation and Hilbert transform; (a) original signal, (b) traveled signal with noise, and (c) processed signal ...................................................................................................... 83
Figure 5.10. Normalized simulation results of crack response at 60° (a) w/o threshold (b) w/ threshold ........................................................................................................................................ 84
Figure 6.1. Spectrogram of sensed signal (a) test setup, (b) sensed signal, (c) sensed signal from damage, and (d) spectrogram of damage signal ........................................................................ 89
Figure 6.2. Time-frequency representation of damage signal (a) WVD and (b) Choi-Williams ................................................................................................................................. 91
Figure 6.3. Scalogram of damage signal ............................................................................................................................................................................................................. 93
Figure 6.4. Time-frequency representation of damage signal (a) Gabor MP and (b) chirplet MP .............................................................................................................................. 94
Figure 6.5. WVD analysis (a) combined signal, (b) WVD TFR, and (c) Choi-Williams TFR .... 96
Figure 6.6. Chirplet MP decomposition of damage signal ........................................................................................................................................................................................................ 97
Figure 6.7. Damage signal and time-frequency plots (a) whole signal, (b) TFR of chirplet MP decomposition, (c) signal from crack damage, and (d) spectrogram of crack signal .......... 99
Figure 6.8. Atom separations using chirplet MP decomposition ........................................................................................................................................................................................................ 100
Figure 6.9. Original signal and reconstructed signal using 10 atoms ........................................ 102
Figure 7.1. Adaboost iteration process at the beginning ........................................................................................................................................................................................................ 106
Figure 7.2. Adaboost iteration process ................................................................................................. 107
Figure 7.3. Final hypothesis decision .................................................................................................. 108
Figure 7.4. Classification trees ............................................................................................................. 109
Figure 7.5. 2-dimensional Abaqus® models (a) crack and (b) corrosion ........................................ 111
Figure 7.6. STFT of crack with D=15cm and αT=2T/8 (a) whole signal (b) damage signal .... 112
Figure 7.7. Spectrograms for cracks with (left) 15-cm distance and (right) 20-cm distance .... 112
Figure 7.8. Reflection study (a) spectrograms of cracks with D=15cm, (b) whole time signal of cracks with D=10cm, (c) normalized crack signals with D=10cm, and (d) crack reflection

Figure 7.9. Wave propagation of (a) A0 at 50 kHz, (b) S0 at 50 kHz, (c) A0 at 150 kHz, and (d) S0 at 150 kHz ........................................................................................................................ 114

Figure 7.10. Wave structures of 50 kHz, A0 mode showing in-plane and out-of-plane displacement profiles across the plate thickness for one wavelength λ.............. 115

Figure 7.11. Spectrograms for corruptions with 15-cm distance (left) and 20-cm distance (right); ............................................................................................................................................. 116

Figure 7.12. Additional training samples following the trained patterns. Examples of (a) crack 15 cm and (b) corrosion 15 cm, 5 cm width, and 2T/8 depth .............................................. 117

Figure 7.13. Experiments performed to get testing samples (a) crack at 13-cm with 2T/8 depth and (b) corrosion at 13-cm having 6-cm width and 2T/8 depth ................................................. 118

Figure 7.14. Testing samples with 2T/8 depths: (a) experiment 13 cm crack, (b) experiment 13 cm corrosion with 6 cm width, (c) Abaqus® 10 cm crack, (d) Abaqus® 17 cm pitting corrosion with 4 cm width, (e) Abaqus® 18 cm crack, and (f) Abaqus® 10 cm corrosion with 5 cm width ......................................................................................................................... 119

Figure 7.15. Cross-section of aluminum corrosion (a) and Abaqus® pitting corrosion model (b) ............................................................................................................................................. 120
List of Tables

Table 6.1. Signal information for each atom ................................................................. 101
Table 7.1. Confidence levels of testing samples ......................................................... 120
Table 7.2. Confidence levels of training samples ....................................................... 121
Table A.1. PZT parameters for 2-D Abaqus® model ................................................. 146
Chapter 1

1. Introduction

Manufacturing, constructions, transportation systems, and new materials have been rapidly developed as the desire for convenience has been increased in recent decades. Although the quality of lifestyle has been significantly improved, the brisk developments have sometimes resulted in unexpected blemishes, such as building collapses, machinery and transportation failures, causing severe damages or even loss of lives. These incidents remind us not only the importance of accurate planning, analysis, and manufacturing, but also the significance of the appropriate maintenance and safety measurements throughout the life cycle of the product we are making. The latter part may be more difficult to control since there are too many unknown factors that can influence its life cycle. In an endeavor to surmount these difficulties, a number of methods that could extend products’ service life have been developed, so called nondestructive testing (NDT) or nondestructive evaluation (NDE) methods.

Nondestructive implies that there is no harm or destruction on the tested object during damage evaluation. NDT is the term generally used for detecting the presence of damage, but without the capability of damage characterization or further analysis. NDE is an extension of NDT with additional quantitative features recommending further determinations with the results [1]. NDE is often used interchangeably with NDT in the field. The most common nondestructive test methods are visual inspection, penetrant test, eddy current test, ultrasonic test, and acoustic emission test. The advantages and limitations of these methods can be found in Thomas [2]. The NDE methods are primarily performed with a prior knowledge of damage locations or in the local areas where cyclic or severe loadings may occur [3, 4]. Although these techniques have been performed in many industry fields and proved to be valuable methods, they are generally expensive and time consuming since the tested components may have to be stopped or disassembled during the test. Since all inspections or measurements rely on practitioners’
judgment based on their experiences, knowledge, and support equipments, the results may not always be reliable. Therefore, some of these NDE tests are transformed into structural health monitoring (SHM) systems by integrating transducers on or into the structures [5]. The birth of SHM originates from the pursuit of an impeccable evaluation system over the previous NDE methods. The reliable SHM system is being challenged because of more complicated structural components and constant materials developments. In this circumstance, great deals of research on SHM have been done and are under investigation utilizing various approach directions and methodologies.

1.1 Structural Health Monitoring

The objective of SHM technology is to employ autonomous damage detection strategies to monitor the integrity of a structure in real time [5]. An SHM system can be further advanced by adding prognosis capability or usage monitoring systems [6].

The process to monitor damages that adversely affect the structural system integrity has been well described in previous works [1, 3, 7]. Although this process might be explained differently in each of these papers, they are essentially describing the same steps. The SHM procedure can be macroscopically divided into three fundamental steps.

Step 1. Detection: damage existence and location evaluation
Step 2. Diagnosis: damage assessment with classification and extent estimation
Step 3. Prognosis: prediction of remaining useful life and further management

The first step is damage detection. The SHM system should be able to find the existence and locations of damage in a given material or structural engineering system. Damage diagnosis is the second step, which can provide information about damage types and possibly extent of damage. The final step is damage prognosis that is related to estimating remained useful life of a system. Usually, SHM system includes a network of integrated sensors and actuators with advanced data acquisition, computation, and signal interpretation. Due to the benefits of SHM, such as reducing maintenance costs, preventing catastrophic failure, and improving safety and reliability, it has received a considerable amount of attention from both industrial sectors and government agencies encompassing many engineering fields, such as bridge monitoring [8], pipeline inspection [9], aging aircraft [10, 11], and space operation vehicles [12, 13].
1.2 Structural Health Monitoring Approaches

Different methods have been developed for SHM applications over the last two decades. Three methods that have been mostly investigated are described in the following. The vibration based method was extensively studied in the 80’s and 90’s [4] and still being investigated. Both the impedance based and guided wave based methods are relatively new and have been intensely studied as SHM systems for more than a decade.

1.2.1 Vibration Based Method

Most of NDE methods are designed to inspect local areas. The need to detect damages in structures globally and continuously led the development of a vibration approach that can use structural dynamics responses [14, 15]. The basic premise of the global vibration based method is that the changes in modal parameters of structures are directly related to the physical parameters [16, 17]. Review of this method is well organized by Doebling et al. [4] and overview of different approaches used in the method is given by Fritzen [18]. The basis of the method is that formation or expansion of damages alters the physical properties, such as mass and stiffness, of the structural system, which in turn changes the system dynamical responses. The differences of the dynamic behavior of the system enable it to locate damage or inform the nature of damage types. The most significant drawback of this method is the lack of sensitivity. Early stage damages may not have significant influences on the system with lower order modes that are generally used in this method.

1.2.2 Impedance Based Method

The impedance based method by utilizing sensor-actuator network was first developed by Sun et al. [19]. The impedance based method utilizes small piezoceramic (PZT) patches bonded to a structure to monitor any changes in structural impedance to detect damage. By measuring changes in the electrical impedance of the PZT, any changes in the mechanical impedance can be detected due to the electromechanical coupling of the PZT [20, 21]. Unlike the vibration based method, this method typically utilizes relatively high frequencies (above 30 kHz) through the collocated PZT transducer and measures the changes in impedance. Due to the higher frequencies, the system is more sensitive to minor changes than the vibration method.
Advantages of the method include low power requirements for excitation (less than 1V), the possibility to be applied to complex structures, and no dependence on an analytical model for implementation. In addition, since the electrical impedance of the PZT is directly related to the structural impedance of a host structure, the number of PZT requirements and data processing procedures can also be minimized [20, 22-24]. The drawbacks of this method are related to damage location and sensitivity. The method does not give information of damage location and the sensitivity decreases notably as the distance between a transducer and damage increases.

1.2.3 Guided Wave Based Method

While the impedance based approach is more appropriate for near field damage detection, the guided wave propagation method can be applicable for near and far field damage detection and has received increasing attention in recent years [10, 25-29]. Unlike many commonly known localized nondestructive approaches, the guided waves have the advantage of wave propagation over large distances with little loss of amplitude in most materials [30, 31]. Thus, a significant advantage over the above mentioned nondestructive methods is that the sensors do not need to be located in the vicinity of damage.

Guided waves can be defined as elastic waves that propagate along the confined geometric boundaries of the structure. They can be divided into two types depending upon the boundary conditions. The first type is surface acoustic waves that usually propagate along the surfaces of structures. Rayleigh, Stoneley, and Love waves are the examples of the surface acoustic waves. They usually propagate a long distance and the amplitude of the waves attenuates with the depth into the structure. Another type of waves is bounded plate waves so called guided Lamb waves. The guided Lamb waves (guided waves hereafter) have infinite number of modes and are mostly dispersive, meaning the wave velocity depends on the product of excitation frequency and structural thickness. Although the guided wave has complicated characteristics, it has been widely used as means for SHM systems due to its advantage of long travel distance and straightforward signal interpretations.

In the guided wave SHM, a pulsed excitation signal is typically generated by an actuator. If damage exists along the propagation path of the signal, some portion of the signal reflects back to the original position or propagates through the damage and recorded by sensors. The former
approach is called the pulse-echo method that both actuator and sensor are usually collocated. The latter is called the pitch-catch method that the transmitted signal is saved by the sensor located at some distance from the actuator [32]. The sensed signal obtained from either of these methods is generally analyzed by comparing to the base signal of the pristine structure.

**Guided Wave Dispersion Curves**

Understanding the characteristics of wave propagation is the most fundamental part in guided wave SHM. For both active and passive inspections, a pulsed wave signal is typically generated from actuators and external sources, respectively. This wave signal is basically divided into two modes, namely symmetric and antisymmetric wave modes. Each mode contains infinite number of modes and they are generally dispersive, i.e. wave velocity depends on the frequency. Not only does the guided wave have two modes and is dispersive, there also exist two wave velocities: phase velocity and group velocity. The phase velocity is the velocity of the phase of one frequency. The group velocity is the velocity of a wave packet, which indicates the energy propagation velocity. The group velocity component is predominantly used to analyze the sensed signals to characterize the presence and types of damages in guided wave SHM. Figure 1.1 shows the phase velocity dispersion curves for an aluminum plate with parameters used in the parenthesis, and a corresponding group velocity for the same aluminum plate is shown in Figure 1.2.

Under a normal dispersion case, i.e. the phase velocity being faster than the group velocity, a new wave originates from the back of a wave packet, then travels to the front in the packet and vanishes. If the phase and group velocities are the same, there is no dispersion. If the phase velocity is slower than the group velocity, the opposite phenomenon occurs, i.e. a new wave originates from front of the packet and disappears to the rear [33].
Figure 1.1. Phase velocity dispersion curves for aluminum (E=73.1 GPa, ν=0.33, ρ=2780 kg/m³)

As mentioned, the group velocity dispersion curves are very important in understanding the guided wave characteristics and utilizing the appropriate wave mode for SHM applications. The general toneburst excitation signal, whether in symmetric or antisymmetric mode, contains broad band frequency components. The waves corresponding to the different frequencies travel at different velocities that the excited toneburst signal elongates as the wave travels further. Therefore, the signal with no or little dispersions range in Figure 1.2 can be a preferred choice, such as ones with the fundamental symmetric mode (S0) less than 0.5 MHz-mm or ones in the fundamental antisymmetric mode (A0) over 1 MHz-mm for this aluminum case. Another important consideration is the existence of at least two modes at any frequency. Only two fundamental modes exist until at a certain frequency-thickness component in Figure 1.2. As the component increases, more modes are generated so that the interpretation of the sensed signal becomes more complicated. Although other high frequency wave modes can also be selected, two fundamental wave modes, i.e. A0 and S0, are more preferred in order to ease the sensed signal interpretations (signal range under 1.5 MHz-mm for an aluminum plate).
1.3 Literature Review

The principle theory of the guided wave for structural damage detection has more than a hundred years of history. The theoretical expression for the infinite plate waves were first developed by two eminent researchers, Rayleigh and Lamb, before the 20\textsuperscript{th} century [34-36] with further examinations during World War I [37]. Instead of adopting their theories for damage detection, a few physicists including Sokoloff [38] had performed x-ray experiments to find metal flaws before World War II began. The War brought more and rapid attention on machine developments and also provided the basis for the nondestructive evaluation.

During the era of rapid growth in nondestructive techniques, one of the brilliant engineers was Firestone who employed the theoretical plate wave expressions to the nondestructive evaluation of sheet metals [39-42]. His invention, the supersonic rectroscope, utilized a single transducer for damage detection utilizing the pulse-echo method. After the War, the veiled knowledge of structural damage detections methods was transferred to industry resulting in gradual development on various kinds of nondestructive evaluation techniques. Worlton [43, 44] was one among those investigating the use of guided wave and might be the first one who confirmed Lamb’s original theory [45].
In 1960s and 1970s, the guided wave theory was fully documented by Viktorov [46] and Graff [47]. Viktorov investigated nondestructive testing of an isotropic plate using ultrasonic Rayleigh and Lamb waves for surface defects and boundary reflections. Graff compiled comprehensive work on elastic wave motion in solids including strings and rods. The number of publications of nondestructive testing using the guided wave started to increase around this time due to the increased computing capability and industrial growth. The transition of term used for damage evaluation was observed in the beginning of 1990s, changing from nondestructive testing to structural health monitoring (SHM) due to material developments with advanced technology. General introduction and review of SHM were previously summarized [3, 48, 49] and a comprehensive review of guided wave SHM can be found in Chimenti [50], Raghavan and Cesnik [51], and Rose [30].

1.3.1 Damage Detection

Damage can be defined as a change of system that can negatively affect its designed performance [3]. There are different types of damage incurred in various structural applications used in aerospace, civil, and mechanical systems. The formed damages can vary depending upon the materials used, structural loading patterns or severities, environmental conditions, and so on. These damages can be detected, directly or indirectly, using different types of sensors that are based on the laws of physical principles. The sensors can be densely distributed or assembled in one place to cover the designated inspection area. The sensed signals are then analyzed by utilizing signal processing algorithms to get more information about damages.

Although there are some issues regarding damage detection considering accessibility, economic feasibility, and inspection capability, most of damages can be detected by applying some advanced inspection methods developed from different application fields. In general, the most frequent damage types incurred in metallic and composite structures are cracks and delaminations, respectively [52]. In metallic structures, the second most frequent damage incurred is corrosion according to the previous survey [53]. Some of investigations studied by other researchers on these metallic and composite damages are described in the following section.
Damage Types and Interactions

Crack detection has been widely investigated using different approaches by many researchers [54, 55]. Pena-Macias et al. [56] and Giurgiutiu et al. [57] investigated crack growth images generated by the piezoceramic phased arrays in metallic structures. Lu et al. [58] studied the reflected and transmitted wave energies for through-the-thickness cracks by varying their lengths and orientations. Qing et al. [59] investigated a built-in piezoceramic actuator/sensor network to monitor crack growth in a rocket engine pipe and concluded that the A₀ mode was more sensitive for crack growth than the S₀ mode. Ihn and Chang [60] developed a built-in piezoceramic diagnostic system to monitor fatigue crack growth in metallic structures. Their system was tested with a notched aluminum plate and showed a good correlation of actual crack growth and visual inspection.

For corrosion damages, Thomas et al. [61] used embedded piezoelectric wafer active sensor (PWAS) to examine corrosion damages. The mechanically formed corrosion damages were placed on an aluminum plate with a grid of PWAS sensors and the amplitude with phase changes were observed as corrosion depth were varied. Simmers et al. [21] studied the presence of corrosion damage using the impedance based SHM method. Multiple light corrosions are identified from an aluminum beam and progressions of damages were adequately tracked. Fasel et al. [62] investigated corrosion damages in an aluminum plate using an array of piezoceramic patches. Electrolytic corrosion was evaluated through the spatio-temporal algorithm for its existence, location, and extent. Michaels and Michaels [63] presented a method to determine the location and quantification of corroded area using an acoustic wave-field imaging technique.

The damages in composite materials have been extensively studied in recent years due to the rapid usage growth in different industries. Firstly, the characteristics of wave propagation on layered laminates have been continually developed by many researchers [64-71]. Seale et al. [31] studied fatigue and thermal damages incurred in composite plates. The presence of damage could alter material parameters in a plate, such as stiffness, density, and thickness. These changes could be related to the guided wave velocities so that some useful damage information was extracted. Guo and Cawley [72] studied detection of delaminations in composite laminates using the S₀ guided wave mode. The delaminated location was varied through the thickness, placing it between different layers. Both numerical and experimental analyses were performed.
and the results showed that signal reflection from delaminations was highly dependent upon their locations.

Some researchers [73, 74] studied a feasibility to detect delaminations in narrow composite beams. Some PZT transducers were surface-bonded on top of the beams and a dominant A_0 mode toneburst signal was generated from a signal actuator. Valdes and Soutis [73] used a Kevlar/epoxy beam and calculated the magnitude of wavelet coefficients on the sensed signals. The measured damages were closely located to the predicted regions with less than 10% errors. In Ip and Mai’s research [74], the area of delaminations was increased to observe signal responses. They also looked at the results obtained from a finite element method for further investigations.

Sohn et al. [75] developed an online monitoring system for delaminations detection in quasi-isotropic composite plates. This active sensing system, consisted of 16 PZT patches, was operated in a round-robin fashion examining 66 different path combinations. The damage features were extracted from the sensed signals that processed through wavelet transform. The delaminated plate was further examined with local temperature variation and clamped boundary conditions.

Diamanti et al. [76] studied the use of a linear PZT actuator array to detect delaminations type damages in quasi-isotropic composite laminates. The array elements were excited in same phase with low frequency A_0 mode. Stiffened composite panels were also examined to study damage detectability. Different damages were characterized varying the locations and impact energies on the panels. The results showed that damages on web or cap could not be determined with their testing setup.

Kudela et al. [77] proposed an algorithm based on a circular pattern sensor array for damage detection in composite plates. Simulation studies were carried out based on a spectral element method using the A_0 wave mode. A 2-D damage map was drawn after applying the algorithm on the plate containing two crack damages. They concluded that a circular sensor array would be easier to monitor damages in a composite plate than using a grid of sensors due to directional velocity dispersions in composite materials.

Staszewski et al. [78] developed two different methods to detect impact damage on composite materials. The first method was an active approach utilizing a 3-D laser vibrometer.
The location and severity of delaminations in composite plate was measured from the scanned results that separated in-plane and out-of-plane displacement components. The other method was a passive approach that used a triangulation procedure and the genetic algorithm to find impact damage in a composite wing section. Benefits of each method were described and combining the two methods was suggested for further examinations.

Quaeghebeur et al. [79] examined the relation between damage detectability and piezoceramic actuator configurations, excitation frequencies, and wave modes. The relation was determined from the theoretical analysis that performed for 32-ply composite laminate that contained a small Teflon tape between layers. Applying a pitch-catch method, the sensor signals were captured along the damaged and undamaged paths for two different frequencies. The results showed that A₀, A₁, and S₁ modes were sensitive for the delaminations while S₀ was not affected much.

Salas and Cesnik [80] investigated wave propagation characteristics in several multi-layered composite laminates. The CLOVER transducer and a PZT actuator were used to send out wave signals through the laminates. The peak-to-peak signal amplitudes and in-plane wave speed were measured from both a laser vibrometer and a finite element package. The beamsteering results showed that the wavepackets eventually traveled perpendicular to the group velocity slowness curves, not to the direction they were originally excited.

There are not only cracks, corrosion, and delaminations types of damage, but many other damage types can also be formed, such as loose bolts, disbonding, and fiber breakages. None of these damages should be disdained since any damage, if exists, could lead to a catastrophic system failure. Since it may not be easy to distinguish different types of damages, the interactions to the damages with the transmitted guided waves to get the better sense of the interpreted signals should also be considered. Some of previous researches about the signal interaction with damages are presented.

Rokhlin [81, 82] used the Wiener-Hopf method and the method of multiple diffraction in order to study wave scatterings caused by a symmetric crack that was formed parallel to the structure surface. The solutions came out identical using these two different methods. Alleyne and Cawley [83] numerically and experimentally studied the interaction of guided waves with notches in plate structures. The results showed that the sensitivity of guided wave signal to a
notch depended on various parameters such as frequency-thickness product, incoming mode type, mode order, and the geometry of the notch.

Cho et al. [84, 85] studied the reflected and transmitted wave scatterings due to damages using a hybrid boundary element method. Different types of damages/parts, such as crack, step discontinuity, and tapered damage, were analytically and experimentally examined for various excitation frequencies and damage depths in plate structures. A mode conversion due to abrupt geometric change was also investigated. Castings et al. [86] used a modal decomposition method to analyze wave interactions due to symmetric internal cracks and open cracks. The reflection and transmission coefficients were obtained as functions of crack depths and the mode conversions were also studied.

Lowe et al. [87, 88] analyzed the reflections due to rectangular notches using a finite element method. The width and depth of the notches were varied under a range of frequencies for each of the fundamental guided wave modes. The results showed that the reflection coefficients were functions of axial stresses and shear stresses from the propagated wave. Gunawan and Hirose [89] developed a hybrid mode-exciting method to solve wave interactions due to internal cracks and edge reflections.

Flores-Lopez and Gregory [90] studied wave scatterings due to a thin surface crack using a projection method based on energy conservation and reciprocal principle. The results were compared well with other published results based on a finite element method. Terrien et al. [91, 92] applied a combined finite element and modal decomposition method to investigate wave interactions with damaged regions. The former was implemented to model the damage region and the latter was used to get the reflection and transmission coefficients and to see mode conversions. Benmeddour et al. [93, 94] and Ramadas et al. [95, 96] also numerically and experimentally studied the wave interactions due to the symmetric and anti-symmetric notches formed in aluminum plates and delaminations in composite laminates, respectively.

**Transducer Developments**

The important factor in the transition from NDE to SHM was the use of smart materials that enabled the autonomous monitoring with their permanent surface attachments or embedded configurations. In many SHM applications, various actuator and sensor transducers, such as piezoelectric wedge, comb transducer, piezoelectric ceramics, fiber composites, electromagnetic
acoustic transducers, magnetostrictive materials, metal-core, fiber optics, comparative vacuum monitoring, micro-electro-mechanical system, and even lasers have been utilized to monitor the integrity of structures and to collect damage information.

Selection of the appropriate actuators and sensors is not only dependent upon the damage characteristics but also upon applicability on the structural components, installation simplicity, and the cost of complete setup and maintenance. Some of the above mentioned transducers, such as piezoelectric wedge, are not in much use these days or being used for specific applications due to spatial limitations. Although some transducers are no longer being used, new actuators and sensors for SHM are constantly being developed such as embeddable piezoelectric acoustic sensor array [97] and carbon nanotubes [98-100].

Piezoceramic Based Transducers

Among the above mentioned transducers, piezoelectric based transducers, especially piezoelectric ceramics (piezoceramics) have been explored and utilized extensively due to their broad bandwidth, easy structural integration or attachment, reliability, and their electromechanical coupling [25, 101, 102]. After the discovery of piezoelectric effect by the Curie brothers in 1880, piezoceramic transducers become the most common devices for SHM applications [32]. The most common type being used in guided wave SHM is the piezoceramic wafer transducer that is very thin (~0.2 mm), making it easy to unobtrusively integrate in the structure.

The important characteristic of the piezoceramics is its converse effects. The direct effect relates that the mechanical strain produces electrical charge on the surface of piezoceramics. This is the principle effect used in the piezoceramic sensors such as piezoelectric accelerometers or pressure sensors. Conversely, mechanical strain is generated when piezoceramic is subjected to an electric field. This converse effect is used for actuators, thus piezoceramics are excellent materials that can be used as both actuators and sensors. The most common types of transducer materials are PZT (lead zirconium titanate) and PVDF (polyvinylidene fluoride). The former piezoceramic is quite brittle and usually not recommended to be used in curved surface. The latter piezopolymer is flexible and has large compliances. Although the piezopolymers are easier to fabricate and can be handled with less care than piezoceramics, they are less sensitive and produce more noise than piezoceramics [103].
Crawley and de Luis [104] would be the first ones who looked at the possibility of using piezoelectric actuators as the elements of intelligent structures. Piezoelectric actuators are investigated as practical means for dynamic vibration and shape controller. The attached and embedded actuators on and within three beam models were tested and their results were well compared to the analytical predictions. A shear lag effect caused by bonding layer between sensor and structure was investigated for PZT [104, 105] and PVDF [106]. Sirohi and Chopra investigated piezoelectric elements as strain sensors. The advantages and limitations of PZT and PVDF sensors were discussed.

One early study using built-in piezoelectric transducers for guided wave SHM was done by Keilers and Chang [107]. In their work, four piezoceramic transducers were mounted on the surface of composite laminates to detect the size and location of delaminations. Frequency responses of the baseline and delaminated structure were compared to identify damage characteristics. Chang was a precursor of employing built-in piezoceramics for guided wave SHM in 1990s [108, 109] and his works were probably the cornerstone for the significant increase in SHM related research in 2000s.

Among the eminent researchers in the field of guided wave SHM, Giurgiutiu has done exceptional works investigating from sensor analysis to damage prognosis using piezoelectric wafer active sensors [10, 24, 110]. One of his work [101] showed comprehensive analysis of the guided Lamb wave interaction with a structure containing defects. A shear lag mechanism between monolithic piezoelectric wafer active sensors (PWAS) and a plate structure was studied varying thickness of the bonding layer. Based on this result, a pin force was assumed along the perimeter of the PWAS actuator. The in-plane strain formulation was derived from a 2-D plane to find out the tuning mechanism of wave mode. The theoretical and experimental results confirmed that it was possible to actuate the PWAS with one predominant wave mode with much less contribution from the other mode.

Raghavan and Cesnik [111, 112] has also investigated guided wave SHM. One notable work was the theoretical modeling of finite dimensional piezoceramics [25]. He developed mathematical formulations of the guided wave field for an isotropic plate with an arbitrary shape actuator attached on and excited with a certain narrow band signal. This formulation was based on the 3-D linear elasticity, advancing the previously known 2-D expressions. Numerical and
experimental results were compared for rectangular and ring shape actuators. In addition, the output voltage response of a piezoceramic sensor that was surface bonded on the plate was derived and experiments were performed to verify the analytical results.

Lanzara et al. [113] studied the influence of interface de-bonding on the performance of piezoceramic actuator using numerical simulations and experimental tests. Various bonding conditions were applied to see the response of the guided wave propagation under wide frequency range. Numerical simulation was done using a 2-D spectral element method. The results showed that partial interface de-bonding could influence not only the signal amplitude reduction but it could also incur phase delays. A peripheral or outermost interface de-bonding seemed to be the worst among those investigated causing significant amplitude drops.

To overcome its brittleness of the piezoceramic based transducer, the fiber composite type transducers were developed. The most common piezoelectric fiber composite transducers are the AFC (active fiber composite) and MFC™ (MacroFiber composite) actuators. Both of these fiber composites use extruded unidirectional piezoceramic fibers embedded in thermosetting polymer matrix. The interdigitated electrodes are etched in a polymer film and placed next to the top and bottom of the piezoceramics. The actuation principles and the shapes of packages for both transducers are similar as explained in Brunner et al. [114].

The AFC was first developed by Bent and Hagood in the middle of the 1990’s [115], while the MFC was developed at NASA Langley Research Center during the late 90’s by Wilkie et al. [102]. Both fiber composites transducers possess the in-plane and directional actuation/sensing abilities along the piezoelectric 3-3 mode (unidirectional piezoceramic fibers direction). Although both fiber composites are quite similar, AFC may take more manufacturing difficulties and require a bit higher voltages to actuate due to the round shape of piezoelectric fibers and more dielectric matrix materials.

The MFC uses the square piezoelectric fibers resulting less dielectric matrix material and relatively lower voltage requirements. These two types of piezoelectric fiber composites have been widely investigated for vibration and noise control, structural health monitoring, morphing, energy harvesting, and many other applications [116, 117]. Di Scalea et al. [118, 119] studied the use of MFCs for damage detection in composite materials. Disbonding between composite wing skins to spar was studied by examining the transmitted signal strength through the defected
bonds. The MFCs were also formed in rosettes to locate damage spots using the directional characteristics of MFC and time-of-flight signal information. Birchmeier et al. [120] investigated AFCs as guided wave actuators. After an AFC actuator that was attached on an aluminum plate was excited with various frequencies, the in-plane and out-of-plane structural responses for $S_0$ and $A_0$ wave modes were measured using a laser interferometer. The results showed that AFCs were suitable actuators for guided wave generation for SHM.

Although not intensely investigated, the metal-core piezoelectric fiber can also be used for damage detection with guided waves. The transducer seems to have some advantages over aforementioned transducers except fabrication difficulties, such as no need of interdigitated electrodes, high S/N ratio, and good durability [121, 122]. One recent study using this sensor was done by Liu et al. [123]. They examined the metal-core piezoelectric fiber sensor that has good directional properties for guided wave sensing in aluminum plate. Damage localization was done through a metal-core rosette utilizing guided wave propagation.

Piezoelectric paint patch was also developed to overcome the brittleness of PZT. Egusa and Iwasawa [124] applied the piezoelectric paint for SHM applications. The paint was coated on aluminum beam and its sensitivity was measured for both low frequency vibration and high frequency acoustic emission, varying the coating thicknesses. The results showed that the piezoelectric paint had potential to be used as a vibration sensor. Recently the piezoelectric paint sensor patches were attached onto metallic structures to examine damages. Surface crack on steel beam was detected by a surface attached paint sensor [125]. More advanced analysis was performed using a piezoelectric paint phased array to locate damages in aluminum plate using a Hilbert-Hwang transform [126].

**Electromagnetic Acoustic Transducers**

Although not as sensitive as piezoceramic based transducers, an electromagnetic acoustic transducer (EMAT) based on non-contacting coupling mechanism can transmit and receive ultrasonic signals by coupling magnetic fields to conductive materials [127]. There are basically two types of EMATs. In a magnetostrictive EMAT, ferromagnetic materials undergo magnetostriction that they stretch under a magnetic field and become normal when the field is removed. The other type, a Lorentz EMAT, is based on the induction of electromagnetic force due to a current conductor under a magnetic field [128]. These EMATs can generate
longitudinal, shear horizontal, or guided waves in conductive materials. One of the early investigations using electromagnetic effects for damage detection in guided waves was done in 1970’s [129] and its mechanism was well explained in several papers [32, 130, 131].

Murayama et al. [132, 133] used a magnetostrictive EMAT to generate different guided wave modes and frequency signals by manipulating meander line coils in thin steel plates. Both guided wave and non-dispersive shear horizontal plate wave were generated and their sensitivity on rough surface damage was compared. Similarly, Kwun and Kim [134] also generated and sensed different wave modes using a channel shape magnetostrictive sensor. Li et al. [135] used a Lorentz EMAT to generate single fundamental guided wave mode in steel plate and various thickness pipes. Experiments were performed to detect a damage formed in a steel pipe using the $A_0$ guided wave mode and the results showed clear differences between undamaged and damaged structures.

Ho et al. [136] applied a continuous wavelet transform for the damage signals obtained using Lorentz EMATs. Three types of artificial defects were formed in aluminum plates and sensed by the EMATs. Tomographic images of the damages were constructed from the signal amplitudes for some of guided wave modes.

Lee et al. [137, 138] developed an orientation-adjustable magnetostrictive patch EMAT that was capable of sending the signal waves in certain directions using the shear-horizontal guided waves. Unlike conventional piezoceramic phased arrays, wave direction was controlled by rotating solenoid arrays inside of the patch. Due to the use of non-dispersive shear-horizontal waves, post signal processing might not be required.

Dixon et al. [139] used a pulsed Nd:YAG laser to generate the guided wave modes and EMAT sensors to detect in-plane displacements of metallic plates. A stress-corrosion crack was generated in the plates and the laser was scanned across the plate surface and the sensors were measured a sudden change with the laser that was placed near or on the crack.

Fiber Optic Transducers

Other than the piezoelectric and magnetostrictive based transducers that have been utilized for guided wave SHM, one frequently being employed is the fiber optic sensors for static and dynamic strain sensing. Although there are some drawbacks using the fiber optic sensors, such
as difficult integration, unfeasible replacement, dependence on alignment, and so on, some attractive advantages over the previous discussed transducers exist. The fiber optic sensor has a very high operational frequency bandwidth (up to 25 MHz), high sensitivity, flexibility, and is immune to electromagnetic interferences [140-142].

Gachagan et al. [143] presented the work on damage detection in composite laminates using fiber optic Michelson and Mach-Zehnder interferometer systems. Different types of damages with surface bonded and embedded fiber optic sensors were also examined for the carbon fiber and the glass fiber reinforced composite laminates. Culshaw et al. [144] studied a hybrid system of embedded ultrasonic source array and fiber optic sensor array to detect centimeter scale delaminations, holes, and impact damages. Qing et al. [145] also developed a hybrid diagnostic system using PZT and fiber optic sensors. The PZTs were used to excite the structures and the fiber optic sensors were employed to capture structural responses.

The fiber optics were not only used as sensors but also used as actuators [146]. Firstly, five fiber optic acoustic actuators were distributed along a single optical fiber to generate acoustic signals through an aluminum plate and the signals were measured by a PZT sensor. Additionally, the combined model of fiber optic acoustic actuators and sensors were tested and their results were compared to the PZT results.

The fiber Bragg grating (FBG) sensor constructed with very short reflective grating segments has also been investigated. Takeda et al. [147] developed a high-speed optical wavelength interrogation system to detect delaminations of composite laminates. It was found that the gage length of the FBG sensor should be shorter than 1/7 of the guided wave wavelength for a better signal detection.

Betz et al. [148, 149] also investigated the FBG sensors in guided wave SHM. The ultrasonic sensing system using the FGB was introduced with numerical and experimental validations. A damaged aluminum plate with various sizes of holes was examined to determine the severities of damages. Rajic et al. [150] used an array of FGB sensor package for guided wave SHM applications. They concluded that a signal modal decomposition through a single fiber sensor with distributed Bragg gratings could be beneficial since it provided better diagnostic assessment.
The light was not only used in the fiber optic sensors, but the laser pulse also was utilized as excitation sources. Murray et al. [151] used an array of 10 laser pulse sources to generate ultrasonic waves in aluminum specimen. A directional scan was achieved by the spatially and temporally modulated laser pulses and the propagated wave signal was detected using the FBG sensors. Recently, Yashiro et al. [152] developed a scanning method to show the ultrasonic wave propagation using a pulsed laser and a fixed transducer. The method was used to examine a steel plate with the slit that was located in the center of the plate and an artificial defect placed at an elbow pipe.

Other Types of Transducers

Among many different kinds of transducers used for guided wave SHM, new technologies to assemble and efficiently intercommunicate between the transducers have been materialized. Instead of separately managing or controlling the actuating and sensing elements, the integrated network solution packages have been developed allowing intelligent and rapid signal analysis. Among the integrated solutions, some of the recognized packages are SMART Layer® from Acellent, M.E.T.I.-disk from Metis Design, DNP from Advanced Structure Monitoring, and HELP Layer® from ONERA.

SMART Layer® (Stanford Multi-Actuator Receiver Transduction) is a network of distributed piezoelectric actuators and sensor embedded in a thin dielectric film to evaluate the structural conditions [153]. The SMART Layer® can be surfaced bonded or embedded into any structures and it does not degrade the material characteristics. Multiple actuator and sensor nodes are covered in thin film layers and the pitch-catch method is used between the actuator-sensor nodes.

Yang and Chang [154] studied of detecting bracket loose from carbon-carbon thermal protection panels. A PZT embedded sensor washer with the aid of SMART Layers® was implemented to see the degradation of the brackets connecting two panels. Signal attenuation-based diagnostic was done by identifying failure modes and loosing brackets. Experimental tests were performed under cyclic loading with the panel of ten brackets and the results showed that the system was reliable within a designed operational range.

Qing et al. [59] also used SMART Layers® for crack growth detection in a rocket engine pipe. Small sizes of crack damages were detected using the system. The results showed that a crack
growing away from the actuator-sensor path was more sensitive than one growing in and also A\textsubscript{0} mode was found to be more sensitive than S\textsubscript{0} mode for crack growth detection.

Kessler and Shim [155] developed an SHM package called a monitoring and evaluation technology integration (M.E.T.I.) disk that enclosed piezoceramic actuator, sensor, integrated circuit, and connectors. Different configurations of actuators and sensors layers have been developed to locate damages [156]. The HELP Layer\textsuperscript{®} (Hybrid Electromagnetic Performing Layer) is composed of two elements. One element is a circuit layer inducing eddy currents and measuring the electric field, and the other element is a system that generates the excitation signals [5, 157].

Although not an integrated solution, a micro-electro-mechanical system (MEMS) sensor can be another state-of-the-art technology that can be used in guided wave SHM. Neumann et al. [158] manufactured both piezoresistive and capacitive MEMS sensors for detecting ultrasonic signals and described the advantages of piezoresistive sensors in terms of size and sensitivity. Pattnaik et al. [159] examined the optical MEMS pressure sensor and compared with the piezoresistive and capacitive sensors. The optical MEMS sensor was found to be more sensitive due to the cumulative nature of the sensor and be more noise resistant for mechanical/thermal variations.

**Detection Methodologies**

Based on the physical principles on each of these transducers previously mentioned, the guided wave signals can be generated or sensed through the data acquisition systems. An initial alert that usually can be noticed during the acquisition would be the signals that contain unwanted noises. This may be inevitable since they can be caused by electromechanical interferences or environmental disturbances, but the noise signal is undesired since it hinders from extracting the accurate signal information. In order to extract the clear signals, various procedures can be taken, such as increasing actuation voltages, averaging out the signals through repeated measurements, or employing some signal de-noising algorithms using discrete wavelet filters. Once the signals can be separable from the noises or if the signal to noise ratio is high enough, some signal processing methods can be utilized to get more information about the status of the structures.
The signals that contain damage information can be measured by adopting different detection methodologies. For instance, multiple actuators and sensors can be distributed over a certain area or embedded in structural components. Instead of this distributed network, a set of geometrically concentrated transducers can be used to cover the same area, such as phased array systems. Furthermore, the transducers may not even be needed to detect the propagated guided waves if a laser vibrometer can be used. No matter how the signals are detected, the propagated guided wave should be present in the structures.

As an endeavor to get better signal information, signal frequency tuning has been investigated by researchers. Rose et al. [160] examined pipe structures for damage detection using an eight elements circumferential array. The signals for two different damage types were measured with various frequency values. The results showed that one damage type was more sensitive at 275 kHz while the other was best detected from 300 to 350 kHz frequency range signals. Santoni et al. [161] studied of guided wave frequency mode tuning using a phased array system. With an aluminum plate with crack damage, the voltage responses of each wave mode were firstly measured for different frequencies. The frequencies with the highest single mode measurement and the larger $S_0/A_0$ voltage ratio were taken to examine the crack on the plate. The results showed that the damage was better detected using the frequency with the highest voltage response, not the voltage ratio.

In the damage detection realm using the guided waves, two basic approaches generally exist, i.e. pitch-catch and pulse-echo. The pitch-catch method employing separate transmitter and receiver was first used to detect a flaw in solids in 1931, which was patented by Muhlhauser [162]. This approach has been widely practiced to find damages in various structural components since the sensed signal from another location can possibly indicate the existence of discontinuities. On the other hand, the pulse-echo method, which was introduced by Firestone in Chapter 1, uses a combinatorial transducer concept such that a transmitted signal can be detected by a collocated sensor if the signal comes back from discontinuities. This method has also attracted researchers since a dense network of transducers may not be required to cover a large area if transducer elements are properly aligned. Other than these two approaches using physical transducers, there are also laser-based actuators and sensors. Some of the aforementioned methods that have broadly been investigated are presented in the following sections.
Pitch-catch Based Damage Detections

Among many damage detection strategies using various transducers, one particular method that must be analyzed with the pitch-catch approach is the time-reversal method. This damage detection method was developed by Fink et al. [163-166] using a time-reversal mirror process that enabled baseline-free damage detection. The time-reversal process is based on the assumption that an original input signal can be reconstructed at the excitation point if the signal is reemitted time-reversely from another point. If there are defects along the wave propagation path, the original signal cannot be reconstructed. Therefore, the baseline signals may be unnecessary since this technique only compares the original and reconstructed signals.

Wang et al. [167] studied a time-reversal imaging method using a distributed actuator and sensor network to find damage in aluminum plate. The developed method was able to locate the damage and somewhat estimated its size in low image resolution obtained from four PZT sensors. Sohn et al. [168-171] has extensively studied the time-reversal method for different applications in SHM. Not only crack damage in a metal plate was evaluated using this baseline free method, but delamination types of damage incurred in composite materials were also tested with aid of signal processing algorithms. Xu and Giurgiutiu [172] presented theoretical model of guided wave time-reversal and studied the algorithm with single and two wave modes. The results showed that the guided wave input signal was fully time-reversible if a single mode was excited, which could be done by frequency tuning.

Similar to the time-reversal imaging method, the tomographic reconstruction maps for damaged structures were obtained employing the pitch-catch method to inspect a large section of structure. Malyarenko and Hinders [173] constructed a quantitative guided wave velocity map to visualize different damages in isotropic plates. The map was created using the data collected over the perimeter of the plates considering the relation between the wave velocity and plate thickness. Zhao et al. [11] used sparse PZT transducers in a circular pattern and applied a probabilistic based correlation algorithm to process crack and corrosion damage signals on aircraft wing. Similar tomographic damage maps were constructed using eight piezoceramic sensors and the signal difference coefficients to locate the damages and to monitor their growth.

Belanger and Cawley [174] performed the feasibility study of a straight ray tomography using guided waves. In order to reconstruct a corrosion patch profile in a metallic plate, the
time-of-flight straight ray tomography based on guided wave dispersion was applied providing a
table depth shape of damage. Both finite elements simulation and experimental results showed that the straight ray theory might be inaccurate predicting damage features so that a
diffraction tomography was instead recommended with a low frequency system.

**Phased Array with Pulse-echo Method**

Damages mostly need to be located in the proximity of sensors or on the wave propagation
paths in order for damage features to be obtained. If damages are positioned away from sensors
or not on the wave paths, an SHM system such as the pitch-catch method may not be appropriate
or correctly diagnose the existing damages. Another efficient way to monitor structures is based
on the pulse-echo method and one method that can be effectively applied is the phased array
beamsteering method.

The phased array techniques utilize a group of distributed transducers that mostly surface-
bonded on structures. There are two ways to construct phased array based SHM. Beamforming
from actuators arrays can be usually done by exciting the elements with different times or
frequency variations so that the wave energy can be focused at localized directions or areas.
Beamsensing from sensor arrays is done similarly by processing the measured signals through
appropriate time or frequency algorithms. A phased array based SHM can be meant by applying
one of these two methods or both for the system analyzed.

For an isotropic or quasi-isotropic structure, the general assumption is that wave propagation
through the structure is uniformly circular from excitation sources. In beamforming case, while
the propagating wave from the planar arrays can be intensified into a desired direction, it suffers
from small side lobes propagating other directions that could cause difficulty in signal
interpretations. Increasing the number of actuators in the array can be one solution but this may
not be an attractive due to physical constraints and the complicated control electronics. Exciting
the actuators with differential voltages/weightings, such as binomial or Dolph-Tschebyscheff
[175] antenna methods, can be another way but they generally widen the width of the main wave
beam.

In the historical part in the phased array systems, ultrasonography would be the first field that
the beamsteering using the phased array technique was implemented. The B-scan ultrasound
imaging system was only capable to visualize static targets within the body until the early 70’s
Since then, medical imaging techniques have been dramatically improved enabling real-time 3-D scanning images and even 4-D moving animations. The phased array systems for SHM have been developed as computation capability becomes faster and larger.

The feasibility of using phased arrays for SHM was attempted by some pioneers. Joshi [177] studied the potential of phased array concept for SHM system and provided some analytic results and insights. Murray et al. [151] used an array of laser sources to generate surface waves in an aluminum plate. Deutsch et al. [178] used a simple self-focusing technique using a linear phased array. A single transducer at the center of the array was excited and the reflected signals from a defect were measured with all the elements in the array. The time of flight data were adjusted to excite all elements with the recalculated time delays to focus the combined signal to the defect. The sensed signals were then timely readjusted to maximize the amplitude.

Wooh et al. [179-181] studied a beamsteering characteristic of linear phased array using numerical methods. Various transducer parameters including number of elements, spacing between the elements, steering angle, and element size were investigated. It was found that increasing number of elements in the array enhanced beam directivity but it also increased the near blind region where beamsteering could not be used. The optimal transducer design was also discussed but no experiments were performed.

In 2000s, Wilcox [27] brilliantly developed omni-directional circular pattern array transducers using EMATs and implemented a phased addition algorithm to steer the beam directions. Two prototype models were investigated, i.e. Type-I with densely placed transducer elements in a circular pattern and Type-II with elements in a single ring pattern, to get B-scan images of a tested plate. The dispersion compensation algorithm that eliminated a wave packet spreading as the wave propagated along the structure and the deconvolution algorithm were applied to get better beamsteering results. Experiments were performed on a steel plate containing various defects, such as holes and pitting corrosions, and the results showed good agreement with the finite element predictions.

Purekar et al. [182] investigated phased array filters for damage detection in isotropic plates. Beamsteering was done by applying different weightings on each array element and the transient sensor responses were obtained to provide damage information. A linear PZT array and a clamp-fixed damage were experimentally studied to validate the phased array filter in an aluminum
plate. Sundaraman et al. [183] examined sparse phased arrays with spatio-temporal adaptive filters for SHM applications. The filtered signals were processed to provide information regarding damage detection, localization, and quantification. The sparse array system was tested with both steel and composite plates to verify analysis results.

Rajagopalan et al. [184] used a piezoceramic single transducer and multiple receivers to detect damages in composite plates. Their algorithm based on a phased addition algorithm was able to construct damage images for the tested composite plates. The results were obtained based on the assumption that phase and group velocity directions were being equal. Olsen et al. [185] discussed a phased array beamsteering algorithm and presented analytical results for some desired target angles. Numerical and experimental results as well as analytical results were compared for the PZT arrays attached on an aluminum plate.

Recently, Yu and Giurgiutiu [186, 187] studied the beamforming characteristics using 1-D and 2-D phased array transducers by applying the embedded ultrasonic structural radar (EUSR) algorithm. The genetic beamforming principle was developed and the signal processing techniques to improve the scanned image were implemented to the sensed signals, through wavelet transform, cross-correlation, and the Hilbert transform. Different number of transducers and the element spacing, as well as various array configurations, were investigated to optimize the beamforming results. Experiments for 1-D and 2-D phased arrays were performed for some inclined cracks and pin holes. The scanned images of these defects through the EUSR algorithm came out similar to the real defects.

Pena-Macias et al. [56] investigated crack growth under a composite bonded repair in a metallic structure with a phased array SHM system. The system was able to indicate the size of the crack under fatigue loading, which followed actual crack size. Another study was done to evaluate the system performance in detecting damage with composite laminates. For this, a small crack was cut parallel to the longitudinal array axis at a certain distance away from the array center. Although the processed image was more indistinct than the one from the metallic structure, it was possible to observe the crack image from the system. Vishnuvardhan et al. [188] examined damages in a composite plate using a single transducer and multiple receiver circular array. A phased addition algorithm in accordance with appropriate dispersion velocity data was implemented to reconstruct the plate image with delaminations.
Purekar and Pines [189] used a PZT sensor array to detect delaminations in a composite plate. Two approaches were examined based on classical laminate plate theory and finite element model. It was concluded that phased array algorithm accompanied with a sensor array was necessary in order to detect damage region. Yoo et al. [126] investigated a capability of 2-D phased sensor arrays made of piezoelectric paint patches for possible SHM applications. Three different array configurations were studied to examine the directivity of signals. A spiral sensor array was experimentally investigated further to detect damages located in an aluminum plate. Signal processing based on the empirical mode decomposition and the Hilbert-Huang transform was applied to enhance the analysis results.

Higuti et al. [190] applied the PZT linear arrays and guided waves to obtain the high resolution images for various damage types in an isotropic plate structures. A synthetic aperture method was employed to attain the maps for these damages utilizing the apodization techniques and polarity scopes. In experiments, non-dispersive $S_0$ wave mode and two linear PZT arrays located in perpendicular to improve the damage visibilities were used to find eight defects that were sparsely distributed in 1-mm aluminum plates. The results came out quite good with some defect images showing titled damage angles. Engholm and Stepinski [191] presented a minimum variance distortion-less response approach using multiple sensor arrays and a signal transmitter. Signal dispersions were compressed taking the theoretical dispersion curves into consideration in the algorithm. The processed results were compared with the conventional delay and sum results.

**Numerical Simulation Methods**

There are many numerical simulation methods implemented for guided wave SHM [26], such as finite difference method (FDM), finite element method (FEM), boundary element method (BEM), finite strip element method (FSEM), spectral element method (SEM), mass spring lattice method (MSLM), and local interaction simulation approach (LISA). The basis of FDM was developed by Yee in 1966 [192] and has mainly been used to model seismic wave propagations. Although somewhat unstable, the results can be obtained fast with less memory use compared to the FEM so that the FDM has been also used in guided wave SHM analysis for isotropic metals [193, 194], composites [195], and even for lead-rubber bearings [196].

The FEM has been a most popular structural analysis method due to its diversity and flexibility since its development at early 20th century. Alleyne and Cawley [83] used the FEM
for analysis of guided wave interactions with different defects. The results showed the signal sensitivity for various notch parameters, especially inclined and width-varying notch configurations. Moulin et al. [197] studied guided wave SHM for composite materials using the FEM coupled with the normal modes expansion method.

Bartoli et al. [198] examined inclined flaws in railroad tracks using a commercial finite element package. Reflection coefficients for the cracks with different sizes and orientations are measured and compared for a range of frequency values. Srivastava and Lanza di Scalea [199] investigated a quantitative method for extracting damage information from isotropic and layered anisotropic materials. A semi-analytical finite element method was utilized to analyze the reflected and transmitted wave energies for the notch shape and delaminations shape damages.

The BEM is a more targeted analysis tool that spends less time and memory than the FEM by attempting to use the given boundary values [200]. The method may be more suitable for surface analysis than volume analysis. Cho and Rose [201] used the BEM to study the guided wave mode conversion due to the edge of a circular hole in a steel plate. Although the mentioned FDM, FEM, and BEM are fairly accurate, these can generally be time consuming and use relatively large memory storages compared to the FSEM, which is based on a lower level of discretization [202]. Liu et al. [203] studied wave scattering due to a crack using the FSEM for the submerged steel and composite laminates. The numerical results showed that the influence of fluid addition was much less for the submerged steel but there was significant amplitude reduction on the displacement response for the composite laminates.

There are two types of SEMs, i.e. FFT-based SEM proposed by Doyle [204] and orthogonal polynomial-based SEM proposed by Patera [205]. The former is more suited for simple 1-D problems and the latter becomes more versatile with complex structural geometry [206]. A 3-D SEM analysis was investigated by Peng et al. [207] for a crack damage detection in an aluminum plate. The results were compared with the 2-D SEM analysis and concluded that the 3-D SEM was able to model the structures more precisely since the out-of-place displacements could be obtained. The MSLM developed by Harumi [208] is similar to the FDM. Yim and Sohn [209] used the MSLM for 2-D crack analysis and compared the results with the FDM analysis. The reliability of the results was checked with the analytical results that were based on the Taylor series and von Neumann analysis methods.
The LISA was developed by Delsanto et al. for 1-D [210], 2-D [211], and 3-D [212] wave propagations, which the method was similar to the FDM but simulated the wave propagation heuristically. Lee and Staszewski [26, 213, 214] have implemented the LISA for the guided wave propagation and damage detection in metallic structures. They demonstrated that the LISA could be efficiently applied to model sharp interferences and discontinuities in structures. One investigation showed the relation between the damage size, such as length and width of crack, and the transmitted/reflected signal amplitudes. This numerical approach was also employed for sensor location studies. The optimal sensor locations were determined where the amplitude was largely increased with a presented damage. Both of these studies were validated with experiments and the results were matched well to the analysis. Ruzzene et al. [215] used a scanning laser doppler vibrometer (SLDV) to visualize propagated wave in an aluminum plate. Numerical work was performed using the LISA for the plate with a longitudinal slit and the experiments with the SLDV were also preformed to verify the numerical results. Both results show similar patterns for the presence of damage.

Other Detection Methods and Applications

There are many advanced ways to detect damages in structural systems and the application fields using guided wave SHM for damage detection can be innumerable. The optimization study in terms of damage detection techniques and sensor placements using guided waves can increase detectability. Alleyne and Cawley [216] studied a way of optimizing guided wave inspection technique and discussed of appropriate mode and frequency selections. Signal processing of a time-domain signal and two dimensional frequency-domains were investigated. Worden and Burrows [217] used an artificial neural network for fault detection in a cantilever plate. The genetic algorithm coupled with the simulated annealing was applied to determine optimal sensor locations. The results showed that the simulated annealing proved to be a useful method to find the best sensor distribution.

Bland and Kapania [218] investigated of using a genetic algorithm to optimize actuator and sensor locations for a damaged plate. The LISA was employed for wave propagation in the plate and the obtained sensor data was used to evaluate a sensor performance metric that indicated damage states. The results showed that the sensor placed at optimized locations performed much better than evenly distributed sensors. Das et al. [219] presented the analytical and experimental
work for the detection of delaminations in composite materials. The presence of delaminations in composites was evaluated by an optimized sensor network. The optimization was performed considering the sensor overlapping areas, number of sensors, and some actuation parameters and material properties. Flynn and Todd [220] introduced a new and novel Bayesian approach for optimal transducer placement using the global detection and false alarm rates. The objective function of the system was aimed to maximize the probability detection rate or minimize the false alarms with minimum overall cost.

Other physical theories or physics based methods have also been investigated. Lin and Yuan [221] used Mindlin plate theory to analytically model a transient guided wave in an aluminum plate. The analysis results were experimentally verified by sending a pure $A_0$ wave mode from a pair of circular PZT actuators. Furthermore, the derivation of sensor output voltage was obtained in terms of given excitation signals and the results were experimentally validated. Ghoshal et al. [28] developed a physics-based simulation model based on classical plate theory for an anti-symmetric guided wave mode. Acoustic wave generation and propagation was developed with a plate model that could reduce a computational cost. This model could be useful for reducing the number of channels and for optimizing sensory systems.

The laser vibrometry can be an excellent tool to observe the insights of wave propagation through structural systems. Staszewski who co-investigated the LISA is one of vigorous researchers in the area of SHM [222-224]. His investigation of fatigue crack detection in metals using 3-D laser vibrometry was particularly interesting since a full guided wave field could be graphically presented. It was experimentally observed that the out-of-plane displacement attenuated significantly while the in-plane displacement attenuated much less with the propagated waves. Considering the dispersion characteristics of metal structures, these observations were true since more dispersive $A_0$ mode was dominated by out-of-plane vibration and relatively non-dispersive $S_0$ mode was dominated by in-plane vibration.

Olson et al. [225] studied guided wave propagations in aluminum and composite plates using the FEM, 1-D, and 3-D laser vibrometers. The propagated results for the fundamental wave modes were compared and showed a good agreement for a thin aluminum plate; however, there were some noticeable differences between the results for a thick composite plate.
The guided wave SHM has also been investigated for structures with complex geometries and configurations. Ryu and Wang [226] examined the effectiveness of piezoceramic actuators on curved beams. Different types and configurations of surface-bonded actuators are studied both analytically and numerically. Experiments were carried out to verify the numerical results and the results matched well with the numerical predictions. Yang and Qiao [227] studied three different engineering materials, i.e. aluminum, composite laminates, and composite sandwich panels for guided wave damage detection. Theoretical, numerical, and experimental works were presented utilizing piezoceramic transducers and the pulse-echo method. They concluded that the pulse-echo method combined with piezoceramics could be an effective way to detect damages.

Song et al. [228] investigated the guided wave signal detection using PZT transducers through an aluminum honeycomb sandwich panel. The numerical and experimental results were compared and the energy attenuation due to the different geometric core cell parameters was briefly discussed. Chen et al. [229] examined guided wave propagation in a submerged aluminum structure. A fluid-solid coupled medium was investigated varying the thickness of the fluid layer, both with numerically and experimentally. The pulse-echo technique and the fundamental A$_0$ mode were used due to its better sensitivity to damage on this study. It was found that the wave energy dropped significantly as fluid layer increased causing possible erroneous interpretations. After appropriately compensating the signals and applying a probabilistic imaging technique, corrosion damage on the plate was successfully identified.

One of the accelerated areas in guided wave SHM is to study temperature effects for both structures and transducers. Konstantinidis et al. [230] investigated the long term stability of an SHM system under temperature variation. The baseline subtraction method was utilized to find a defect in an isotropic plate. The temperature variation was found to cause significant errors using this method due to the thermal deformation of structures and the changed dispersion curves. Raghavan and Cesnik [231] investigated guided wave sensor responses with elaborated temperature conditions. An appropriate bonding agent was first chosen among high temperature-resistant epoxies and wave propagation experiments were performed in an autoclave to compare signal responses under varied structural temperatures. It was found that the modulus of elasticity was the most sensitive parameter under temperature variation. In addition, they found that the sensitivity dropped significantly beyond 80°C for the aluminum plate for the frequency values investigated.
Lanza Di Scalea and Salamone [232] examined temperature effects on the change of signal amplitudes in an aluminum plate. The analytical models were derived to model the effects and the pitch-catch experiments were performed to verify the theoretical results for the temperature range from -40°C to 60°C. The results showed that the signal amplitude became the maximum at 20°C but decreased as the temperature either increased or decreased from 20°C. Salamone et al. [233] continued this study with composite laminates and obtained similar results. Clarke et al. [234] studied a temperature compensation method by combining the optimal baseline subtraction and stretch methods to reduce the number of necessary baseline signals. The compensation method was found to be highly dependent upon signal complexity and mode purity.

1.3.2 Damage Diagnosis

Among the consecutive damage evaluation steps in SHM systems, a damage diagnosis step can be very important since it is the direct factor that influences the estimation of the remaining useful life of structures. The prognosis results can be significantly different because damage development phases for different damages, such as crack and corrosion, have dissimilar patterns. In order to correctly diagnose and possibly classify the damage types, adequate signal processing algorithms should be applied to the sensed signals.

The importance of signal processing was mentioned by Meyer [235] with four objectives, i.e. to analyze accurately, code efficiently, transmit rapidly, and reconstruct carefully, as all of the signal information is effectively presented and also hidden in the complicated graphical representations. Among the signal processing tools currently being employed, it cannot be determined that one tool works best for guided wave SHM since the selection of an appropriate signal processing tool should be dependent upon many factors that require careful considerations. One of the prominent tools that can use all of the sensed signal information, such as time, amplitude, energy, and frequency, is the time-frequency method, and it is discussed in the next section.

Time-Frequency Representation

A traditional signal processing method for a sensed signal is analyzing with either time or frequency signal information. Each analysis has major drawback, i.e. the time-domain analysis cannot be applied complex signals and the frequency-domain analysis only provides global
information. The time-frequency analysis was introduced to vanquish these drawbacks. Some of the time-frequency methods proven to be efficient are short time Fourier Transform (STFT), Wigner-Ville distribution (WVD), wavelet transform (WT), and matching pursuit (MP).

Chin et al. [236] compared the Wigner distribution and the STFT as non-destructive evaluation tools for simulated ultrasonic signals. The analysis was done for the reflected flaw echoes and the back scattered signals from inhomogeneous materials. The results showed the outperformance of STFT owing to non-presence of cross terms in Wigner distribution. Grondel et al. [237] used the guided wave and acoustic emission techniques to monitor crack initiation and its growth around rivets in metallic structures. A spectrogram was used to see the relation between physical crack emergence and time-frequency variations.

Ihn and Chang [60] used a spectrogram to extract energy distributions for the baseline and crack damaged structures while monitoring crack growth. The obtained energies were used to calculate damage index values which were used to distinguish different crack sizes. Hong et al. [238] developed a dispersion based STFT that time-frequency tiles in a spectrogram were depending upon the dispersion characteristics of the guided waves.

Prosser et al. [239] employed a pseudo-WVD to analyze acoustic signals and demonstrated the generation of dispersion curves from the measured signals. The technique was applied to the signals obtained from aluminum plates and unidirectional composite laminates. The dispersion curve results obtained from the technique were closely agreed with the theoretical curves.

The WT has been widely investigated as means for structural damage detection using an impact force or arbitrary excitation signals [240-244]. The WT based on the guided wave was applied for delaminations detection in composite materials. Lemistre and Balageas [245] used several surfaced bonded thin PZTs to find impact delaminations in a 16-ply composite laminate. The mode conversion due to the delaminations, damage localization, and size estimation were studied.

Paget et al. [246] presented a wavelet coefficients damage detection method to decompose the guided wave signals detected by PZT sensors. This wavelet coefficients method was applied to assess damage extent made by five different impact loadings on a cross-ply composite laminate. Lestari and Qiao [247] evaluated three different damages, i.e. delaminations, saw cut, and impact damage, with wavelets employing the guided wave propagation in composite beams.
Park et al. [248] used both impedance and guided wave based methods to detect cracks and loosen bolts in steel bridge components.

A continuous WT and a pattern recognition technique were employed to improve the damage detectability. Grabowska et al. [249] investigated distinguishing different states of damages on long rod and beam models. A spectral element method was used for numerical modeling and the WT was used for signal processing. Experimental results showed good correlation comparing energy levels and wavelet coefficients. Lu et al. [250] presented a signal processing method that could reduce the effects of environmental and operational signal anomalies, which was based on an adaptive harmonic WT and the principal component analysis. This combined method was examined by comparing signal detectability and sensitivity of damaged and undamaged structures. Li et al. [251] studied to select optimal mother wavelets using a Shannon entropy of wavelet coefficients. Various mother wavelets were applied to the sensed signals obtained from composite delaminations and the results showed that the optimally selected mother wavelets provided most accurate damage localization.

Hong et al. [252, 253] investigated signal processing techniques with MP in guided wave SHM. The noisy and dispersive crack damage signals in a circular cylinder were processed through Gabor and chirplet functions in the algorithm. Several damages were assessed and comparison of these two was also discussed. Das et al. [254] developed a faster signal decomposition algorithm using MP and a Monte Carlo algorithm. Experiments were carried out to study the efficiency of the algorithm using healthy and damaged composite laminates. The simulations showed that the algorithm resulted faster decomposition compare to the ordinary matching pursuit without decreasing the accuracy. Some practical uses of matching pursuit were also found.

Raghavan and Cesnik [111] discussed modal separations of overlapped sensed signals and presented an algorithm based on a chirplet matching pursuit. Numerical simulations and experiments were performed with a clamped aluminum plate to show the validity of the proposed algorithm. Chakraborty et al. [255] investigated classifying loosened fastener damages using the MP TFR for the sensor signals and Bayesian sensor fusion approach. An aluminum plate fastened at corners and surface mounted PZT sensors were used for damage detection. The general and modified MP decompositions were performed for the PZT sensor signals using
Gaussian functions and the TFR results were processed through the Bayesian fusion approach. The results showed that this method could successfully classify five different damage classes.

Although the MP decomposition can provide high resolution TFRs, calculating the best represented functions out of the abundant dictionary may be timely expensive. A signal processing method called Hilbert-Huang transform (HHT) in conjunction with empirical mode decomposition was developed [256, 257], decomposing non-stationary signals into the intrinsic mode functions and obtaining instantaneous frequency components.

Quek et al. [258] investigated the practicality of the HHT as a signal processing tool for localized and different forms of damages. Three forms of damages, i.e. aluminum crack, delaminations in sandwich panel, and damage in reinforced concrete, and one input signal distortions were investigated. The results showed that the HHT was able to identify these damages but needed more caution when analyzing the distorted signals.

Pines and Salvino [259] briefly discussed a few well known signal processing methods and used the empirical model decomposition coupled with the HHT for structural damage detection. The magnitude, phase, and damping information were obtained by processing the sensed signals through this coupled algorithm. Damage localization and estimation were performed by analyzing the phase properties.

**Machine Learning Methods**

There are two main categories in the machine learning algorithms, i.e. supervised and unsupervised learning. The former requires a pair of input objects and output values to form a postulated relationship. Through the learning process, their connections can be adjusted, learned, and corrected to improve the performance of the algorithm. Classification and regression problems are generally considered as supervised learning [7]. The latter concerns of characterizing and organizing measured data sets, such as density estimation and clustering.

**Support Vector Machines**

One well regarded classification method that belongs to the supervised machine learning methods is support vector machines (SVM). The SVM maps the extracted feature vectors of input space into a high dimensional hyper-plane to maximize margin. One of the early use of SVM for SHM was performed by Mita and Taniguchi [260] who applied the SVM to
characterize damage features in aluminum plates. Yuan and Chu [261] presented a multi-class SVM classification algorithm for damage detection. This one-to-others algorithm was based on a binary tree classifier made of several ordinary two-class classifiers. The algorithm was tested on a turbo pump rotor test bed and its classification results were compared with a few known algorithms. The results showed that this algorithm was faster and provided a highest precision ratio among the tested ones.

Chattopadhyay et al. [262] used one-class SVM to classify damage in bolted joints and also different damage types in composite laminates. A Gaussian kernel was chosen to map the input patterns into a high dimensional feature space. The experiment results showed the capability of this algorithm for distinguishing different torque states, crack size, and damage classification.

Park et al. [263] applied two-step SVMs for railroad track damage identification. Both the impedance and guided wave based SHM were applied to evaluate a hole and a long cut damage in railroad track. The first step SVM was used for damage detection and the second was employed for damage classification, establishing the optimal separating planes on the feature map. Isa and Rajkumar [264] used a RBF kernel based SVM to detect damages in pipeline. PZT sensors and the guided wave propagation principle were employed to obtain the sensed signals that processed through the discrete wavelet transform to extract damage features. Other types of kernel functions were examined for this application but the RBF kernel worked best among the examined.

Artificial Neural Networks

Artificial Neural Networks (ANNs) was also utilized for damage diagnosis and classification using different approaches [265-268]. The ANNs is a machine learning tool that captures the complex relation between inputs and outputs, as the neurons in human brain. Challis et al. [269] employed the ANNs to classify obtained data signals from a T-joint. A combined guided wave signal was excited from the actuator located on one side of the joint and it propagated towards the bond and through the other part. The frequency spectrum was obtained for the sensed signals and these were used as the inputs to feed-forward networks to classify the data.

Additional work was performed by Todd and Challis [270] using the same approach. Legendre et al. [271] applied wavelet transform and ANNs for an aluminum weld quality classification. The guided wave generated from EMATs was reflected from the weld and the
WT was used for feature extraction. A neural classifier was able to interpret the weld quality class resulting more than 90% of correct classifications. Su and Ye [272] identified composite delaminations through an intelligent signal processing and pattern recognition package consisted of various signals processing techniques. Factitious delaminations were inserted in composite laminates that were modeled with a finite element package. The sensed signals were received using the guided wave and processed with the WT, digital damage fingerprints, and a multilayer feed-forward ANNs. The results showed that this algorithm package closely identified the location of the delaminations.

An additional study using digital damage fingerprints and multilayer ANNs was done by them [273, 274]. A similar study of finding damage location using ANNs were performed by Mustapha et al. [275]. Cau et al. [276] used ANNs to diagnose damages formed in inaccessible pipeline structures. The artificial signal sets were obtained from finite element methods and processed through a principal component analysis to reduce data dimensions. The processed data were inputted to a multilayer feed-forward ANNs to obtain the diagnosed damage information. Moll et al. [277] developed a new formulation to localize multiple damages in anisotropic plate-like structures. A multiple PZT actuator/sensor network was used for sensing damage signals and a local ANNs was applied to model the nonlinear dispersion curves. Damage localization was done utilizing a probabilistic density function and an expectation-maximization algorithm.

**Other Diagnosis Approaches**

Damage classification in guided wave SHM was also performed using Hidden Markov Models (HMMs) that can be used to predict underlying dynamic system characteristics. Chakraborty et al. [278] combined the MP decomposition with HMMs for damage classification in a lug joint. Different noise levels were added to the damage signals obtained from the FEM for a 6 mm crack. The classification results showed over 90% probability even with high noise levels. Zhou et al. [279] also presented a signal processing algorithm based on HMMs coupled with the MP decomposition. The algorithm integrated into a Bayesian decision framework was implemented to classify various types of composite laminate damages, such as delaminations, impact damage, and progressive tensile damage. A sensor fusion process combining each local classification results was also applied to get better overall classification.
The damage identifications were carried out employing other unique and novel approaches. Su and Ye [280] presented a technique for damage identification through active sensor network called a signal processing package. This package contained various analysis tools including wavelet based signal filtration, signal extraction using principal component analysis, feature vector construction, information mapping, and a pattern recognition.

Zhang et al. [281] investigated the characteristics of damages through the scattering index matrices using a finite element modeling procedure. The developed model was clearly able to distinguish circular holes and crack shape damages. The size, shape, and orientation of crack-like damages were experimentally determined by a total focusing method obtained with a linear ultrasonic array. Unless some prior knowledge regarding to the damages was known, they concluded that it would be impossible to deduce the exact damage geometry from the inverse of scattering matrices.

Ng et al. [282] used a two-stage probabilistic optimization technique and the spectral element method for damage identification. The global optimum of an objective function was found by a simulated annealing and the probability density function was maximized utilizing a simplex search method. The results showed correct identification of all damage parameters, such as location, length, and depth. The method not only characterized the different damage factors, but was also able to quantify the uncertainties of predicted values.

1.4 Damage Types for Classification

As mentioned in section 1.3.1, the most frequent damage types incurred in metallic structures are cracks and corrosions. Crack in the aerospace industry is a well known type of damage. This can be formed by various causes, such as fatigue, stress, temperature, impact, incomplete repair, manufacturing anomaly, and so on. Fatigue cracks are formed in structures mainly from repeated loadings. For instance, fuselage can be under cyclic loadings due to the change of cabin pressures. A wing flutter with periodic motion during flight and cyclic deployments of landing gears are also some of the examples that can cause crack damages. Cracks due to stresses can be formed when components in a system are consistently over or close to their structural loading capacities. Constant change of environmental conditions like latitude and altitude during flights also can incur crack damages. Once an initial crack grows over the critical size, it can lead to
catastrophic system failures. While cracks are generally formed in long and narrow configurations, corrosions can be caused through various processes in transportation systems.

Corrosion can be formed from a deterioration process caused by electrochemical reaction in material, usually metal, with its environment. This is in general a natural phenomenon moving from an unstable high energy state to a stable low energy state. The reaction process results in a loss of material and changes the material properties, such as strength or ductility, of the original material part. Corrosion formation in aircraft varies depending upon materials used, application environments, and/or the depth of attack. There are many forms of corrosions that can be caused in metallic structures.

Corrosion generally results in thinning of the metal, and it is usually not uniform but varies depending upon the intense of chemical reaction. Perforation can even occur on thin materials when the reaction is severe. The galvanic corrosion can occur on negatively charged metal surfaces and it is more severe at the interface between the dissimilar metals or electrically conductive non-metals. It occurs in much more rapid rate than the general corrosion. The pitting corrosion occurs when a portion of metal surfaces undergo rapid chemical attack leaving pits and craters. Various size and shape of pits can be formed when this portion of metal surface is severely affected. Aluminum and magnesium alloys are susceptible to the pitting corrosions. The intergranular corrosion occurs due to the chemical attack on grain boundaries of metals. The grain boundaries are chemically differ from the grains and may react differently from electrochemical process. Exfoliation is a form of this corrosion that results the separation of the outer layer from the main metal body. This corrosion is common on aluminum and carbon steel. There are additional forms of corrosions but mentioned above are better known to the aerospace industries.

According to the Department of Defense (DoD) annual cost of corrosion report published in July 2009 [283], the total annual corrosion cost of DoD infrastructure and equipment is $16.9 billion. Among this, the costs for Air Force aircraft and missile equipment, Navy and Marine Corps aviation, and Army aviation and missiles are $5.4 billion, $3.0 billion, and $1.6 billion, respectively. This corrosion cost consists of man-hours, such as inspection and repair, materials usage, corrosion facilities, training, and research and development. Unexpectedly though, the 2nd and 3rd highest cost components in a top 20 corrosion cost ranking chart are the inspection and
special inspection costs, with the combined total of $1.528 billion. This study implies that considerable amount of expense can be saved by utilizing autonomous damage monitoring systems instead of employing man-based inspection systems.

1.5 Problem Statement and Research Objectives

There has been considerable amount of research in guided wave SHM pertaining to damage detection and diagnosis in material and structural engineering systems. Much of these investigations generally consider distributed actuator and sensor networks. Such network systems might be effective if the damage is located in the vicinity of sensors or along the line of propagating wave between actuators and sensors. In a plate structure, however, it is usually difficult to predict potential locations for damage occurrences. An effective way to detect damage contents can be considered by scanning a certain area by sending out wave beams into specific directions or steering them for the desired range of angles. Beamsteering using a phased array system is therefore an adequate approach for damage detection of plate structures. SHM systems should also be capable of diagnosing the damage, i.e. classifying damage types and estimating severities. Damage classification is an important step since all the following processes, such as severity assessment and estimation of remained useful life of a system, are highly based on this classified results. The overall goal of this research is first to develop an improved damage detection system based on guided wave beamsteering using anisotropic transducers. The second goal for the research is to correctly classify different types of structural damages through advanced signal processing algorithms and a machine learning method.

1.6 Dissertation Outline

The dissertation consists of eight chapters that are organized as follows.

The first chapter introduces some background information and motivation for the current research. A comprehensive review of damage detection and diagnosis in guided wave SHM systems are included and the research objectives are stated.

The second chapter explains the derivations and results of analytical displacement formulations for the guided wave field in an isotropic plate structure. The voltage response of a piezoceramic sensor due to in-plane strains is also presented.
The phased array responses for piezoceramic and Macrofiber composite actuators are compared for harmonic wave propagation and the limitations of the general beamsteering formula are investigated in the third chapter.

The fourth chapter discusses the experimental results of piezoceramic sensor response and beamsteering results using both PZT and MFC linear phased array systems. The analytical results are compared with experiments showing close match one another.

Some essential elements for a robust damage detection and localization system are presented in the fifth chapter. A beamforming and beamsensing algorithm and some signal processing algorithms are applied to the signals obtained from a cracked plate.

The sixth chapter presents four well known time-frequency representation methods for damage diagnosis analysis. Each method is examined in detail to select a suitable method for damage classification.

A machine learning algorithm called Adaboost is utilized to classify crack and corrosion damages in the seventh chapter. Both numerical and experimental analyses are preformed to obtain the training and testing samples. The results show correct classification and provide the confidence levels for each sample.

Finally, the eighth chapter summarizes the research and discusses some future recommendations.
Chapter 2

2. Guided Wave Field Expressions

General displacement formulations are derived for the guided wave field in an isotropic plate structure. These expressions are obtained assuming that an arbitrary thin transducer is perfectly bonded on an infinite isotropic plate structure. The displacement expressions using a rectangular piezoceramic actuator and Macrofiber composites actuator are obtained. The expressions for the voltage response of a PZT sensor due to in-plane strains are presented.

2.1 Mathematical Displacement Formulations

The characteristics of an infinite medium with a propagated wave due to an external force can be generally expressed with particle displacement formulations. The first step of getting these expressions is to consider a dynamic equilibrium of a point in the medium. In a global sense, the conservation of momentum and Gauss theorem lead to the following equation of motion in an elastic medium [284].

\[ \tau_{ij,j} + \rho f_i = \rho \ddot{u}_i \]  
\[(2.1)\]

where the variables \(\tau_{ij}\), \(\rho\), \(f_i\), and \(u_i\) are the stress tensor at a point, the mass density of structure, body force vector, and the displacement vector, respectively.

Assuming homogeneity and isotropy of the material structure, only two independent constants termed Lamé constants are sufficient to express the linear elastic isotropic behavior. The stress-strain relation is succinctly written as,

\[ \tau_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} ; \quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \]  
\[(2.2)\]

where \(\lambda\) and \(\mu\) are known as the Lamé constants.
The Navier-Lamé elasticity equation is derived from substituting Eq. (2.2) into Eq. (2.1) for an isotropic and linear elastic materials, which can be written as [285],

\[
(\lambda + \mu)u_{j,i} + \mu u_{i,j} + \rho f_i = \rho \ddot{u}_i \tag{2.3}
\]

where the displacement vector \( u_i \) is in the \( x_1, x_2, \) and \( x_3 \) directions in Figure 2.1.

The host structure is assumed to be an infinite isotropic plate with thickness \( 2b \) where the top and bottom surfaces are \( +b \) and \( -b \), respectively (Figure 2.1). The external force is applied by a rectangular actuator, where \( a_1 \) is half the width of the actuator in the \( x_1 \) direction and \( a_2 \) is half the width in the \( x_2 \) direction. Similar displacement formulations are developed and presented for the symmetric Lamb wave modes in Raghavan et al. [25, 286].

![Figure 2.1. Isotropic plate structure with the PZT and MFC actuators](image)

Setting the body forces \( f_i = 0 \), the general solutions of displacement can be obtained by Helmholtz’s decomposition, where the vector that is piecewise continuous and differentiable in the finite closed region can be resolved into gradient and curl functions. Using a scalar potential \( \phi \) and a vector potential \( \psi \), Lamé gives the following displacement vector field equation.

\[
\nabla \phi + \nabla \times \psi = u \tag{2.4}
\]

Substituting Eq. (2.4) into Eq. (2.3) and regrouping yields the following uncoupled wave equations.

\[
\nabla^2 \phi - \frac{\ddot{\phi}}{c_d^2} = 0 \ ; \ \nabla^2 \psi - \frac{\ddot{\psi}}{c_r^2} = 0 \tag{2.5}
\]
where the dilatational wave velocity, $c_d$, is $\sqrt{(\lambda + 2\mu)/\rho}$ and the rotational wave velocity, $c_r$, is $\sqrt{\mu/\rho}$. Eq. (2.5) shows that the scalar potential $\phi$ propagates with the velocity $c_d$ and the vector potential $\psi$ with the velocity $c_r$.

The completeness theorem [287] also adds one more condition in the vector field, imposing the divergence-free condition (zero net-flux) on $\psi$.

$$\nabla \cdot \psi = 0$$  \hspace{1cm} (2.6)

The solutions representing harmonic waves travelling in the $x_1$ and $x_2$ directions and standing waves in the $x_3$ direction can be assumed in the form,

$$\phi = \hat{\phi}(x_3) e^{-i(\xi_1 x_1 + \xi_2 x_2 - \omega t)}$$

$$\psi = \hat{\psi}(x_3) e^{-i(\xi_1 x_1 + \xi_2 x_2 - \omega t)}$$  \hspace{1cm} (2.7)

where $\xi_i$ and $\omega$ are the wave number in $x_i$ directions and angular frequency, respectively.

After substituting these assumed solutions into Eq.(2.5), the following equations are obtained.

$$\frac{d^2 \hat{\phi}}{dx_3^2} + p^2 \hat{\phi} = 0; \quad p^2 = \left( \frac{\omega^2}{c_d^2} - \xi_1^2 - \xi_2^2 \right)$$

$$\frac{d^2 \hat{\psi}}{dx_3^2} + q^2 \hat{\psi} = 0; \quad q^2 = \left( \frac{\omega^2}{c_r^2} - \xi_1^2 - \xi_2^2 \right)$$  \hspace{1cm} (2.8)

The most convenient solutions of the above equations with a scalar potential $\hat{\phi}$ and a vector potential $\hat{\psi}$ are obtained below,

$$\hat{\phi}(x_3) = A \sin\left(px_3\right) + B \cos\left(px_3\right)$$

$$\hat{\psi}(x_3) = C_i \sin\left(qx_3\right) + C_j \cos\left(qx_3\right)$$  \hspace{1cm} (2.9)

Substituting Eq. (2.9) into Eq. (2.7), the following scalar and vector potential equations are obtained.
\[
\varphi = \left[ A \sin(p x_3) + B \cos(p x_3) \right] e^{-i(\xi_1 x_1 + \xi_2 x_2 - \omega t)}
\]
\[
\psi_1 = \left[ C_1 \sin(q x_3) + C_2 \cos(q x_3) \right] e^{-i(\xi_1 x_1 + \xi_2 x_2 - \omega t)}
\]
\[
\psi_2 = \left[ C_3 \sin(q x_3) + C_4 \cos(q x_3) \right] e^{-i(\xi_1 x_1 + \xi_2 x_2 - \omega t)}
\]
\[
\psi_3 = \left[ C_5 \sin(q x_3) + C_6 \cos(q x_3) \right] e^{-i(\xi_1 x_1 + \xi_2 x_2 - \omega t)}
\]

(2.10)

Substituting Eq. (2.10) into Eq. (2.4) yields three displacement equations that each can be divided into the sine and cosine functions.

\[
u_1 = \left[ A \sin(p x_3)(-i \xi_1) + C_4 \sin(q x_3)(-i \xi_2) + q C_4 \sin(q x_3) \right] e^{-i(\xi_1 x_1 + \xi_2 x_2 - \omega t)}
\]
\[
u_2 = \left[ A \sin(p x_3)(-i \xi_2) - q C_2 \sin(q x_3) - C_5 \sin(q x_3)(-i \xi_1) \right] e^{-i(\xi_1 x_1 + \xi_2 x_2 - \omega t)}
\]
\[
u_3 = \left[ -p B \sin(p x_3) + C_3 \sin(q x_3)(-i \xi_1) - C_1 \sin(q x_3)(-i \xi_2) \right] e^{-i(\xi_1 x_1 + \xi_2 x_2 - \omega t)}
\]

(2.11)

Figure 2.2 illustrates the symmetric and antisymmetric wave modes with particle moving directions corresponding to each mode. In the symmetric mode, the in-plane displacements move in a cosine pattern, while these displacements in the antisymmetric modes move with a sine pattern. The out-of-plane displacements move in the opposite way. Considering the wave direction and particle movements in an isotropic plate along with three displacement equations obtained, it can be shown that \(A, C_2, C_4, \) and \(C_5\) are associated with the antisymmetric Lamb wave modes, while \(B, C_1, C_3, \) and \(C_6\) are associated with the symmetric Lamb wave modes.
The displacements in the symmetric mode are then obtained below from Eq. (2.11).

\[ u_1|_S = \left[ B \cos(px) \left( -i \xi_1 \right) + C_6 \cos(qx) \left( -i \xi_2 \right) - qC_3 \cos(qx) \right] e^{-i(\xi_1 x + \xi_2 y - \omega t)} \]

\[ u_2|_S = \left[ B \cos(px) \left( -i \xi_2 \right) + C_1 \cos(qx) - C_6 \cos(qx) \left( -i \xi_1 \right) \right] e^{-i(\xi_1 x + \xi_2 y - \omega t)} \tag{2.12} \]

\[ u_3|_S = \left[ -pB \sin(px) + C_3 \sin(qx) \left( -i \xi_1 \right) - C_1 \sin(qx) \left( -i \xi_2 \right) \right] e^{-i(\xi_1 x + \xi_2 y - \omega t)} \]

To the contrary, the displacements in the antisymmetric mode are obtained below.

\[ u_1|_A = \left[ A \sin(px) \left( -i \xi_1 \right) + C_5 \sin(qx) \left( -i \xi_2 \right) + qC_4 \sin(qx) \right] e^{-i(\xi_1 x + \xi_2 y - \omega t)} \]

\[ u_2|_A = \left[ A \sin(px) \left( -i \xi_2 \right) - qC_2 \sin(qx) - C_5 \sin(qx) \left( -i \xi_1 \right) \right] e^{-i(\xi_1 x + \xi_2 y - \omega t)} \tag{2.13} \]

\[ u_3|_A = \left[ pA \cos(px) + C_4 \cos(qx) \left( -i \xi_1 \right) - C_2 \cos(qx) \left( -i \xi_2 \right) \right] e^{-i(\xi_1 x + \xi_2 y - \omega t)} \]

Using Eqs. (2.2), (2.4), and (2.10), the stress components on the top free surface of the plate can be obtained. The double Fourier transform is applied to the stresses to obtain the final displacements solutions. Then the stresses components in the wave number domain become,

Symmetric mode:

\[ \hat{\varepsilon}_{31} \left( \xi_1, \xi_2 \right) |^S = \mu \left( B \left( 2i \xi_1 p \right) \sin(pb) + C_1 \left( \xi_1 \xi_2 \right) \sin(qb) + C_3 \left( q^2 - \xi_1^2 \right) \sin(qb) \right) \]

\[ \hat{\varepsilon}_{32} \left( \xi_1, \xi_2 \right) |^S = \mu \left( B \left( 2i \xi_2 p \right) \sin(pb) + C_1 \left( \xi_2^2 - q^2 \right) \sin(qb) - C_3 \xi_1 \xi_2 \sin(qb) \right) \]

\[ \hat{\varepsilon}_{33} \left( \xi_1, \xi_2 \right) |^S = \mu \left( B \left( \xi_1^2 + \xi_2^2 - q^2 \right) \cos(pb) + C_1 \left( 2i \xi_2 \right) \cos(qb) - C_3 \left( 2i \xi_1 \right) \cos(qb) \right) \tag{2.14} \]

Antisymmetric mode:

\[ \hat{\varepsilon}_{31} \left( \xi_1, \xi_2 \right) |_A = \mu \left( A \left( -2i \xi_1 p \right) \cos(pb) + C_2 \left( \xi_1 \xi_2 \right) \cos(qb) + C_4 \left( q^2 - \xi_1^2 \right) \cos(qb) \right) \]

\[ \hat{\varepsilon}_{32} \left( \xi_1, \xi_2 \right) |_A = \mu \left( A \left( -2i \xi_2 p \right) \cos(pb) + C_2 \left( \xi_2^2 - q^2 \right) \cos(qb) - C_4 \xi_1 \xi_2 \cos(qb) \right) \]

\[ \hat{\varepsilon}_{33} \left( \xi_1, \xi_2 \right) |_A = \mu \left( A \left( \xi_1^2 + \xi_2^2 - q^2 \right) \sin(pb) - C_2 \left( 2i \xi_2 \right) \sin(qb) + C_4 \left( 2i \xi_1 \right) \sin(qb) \right) \tag{2.15} \]

The constants in the above equations can be obtained applying boundary conditions around the actuator attached on the free surface of the plate. The interfacial shear stresses in the bond
layer between the actuator and the structure are investigated, corresponding to shear lag parameter [104]. In an ideal bonding condition, i.e. very thin bond layer, it is a reasonable assumption that the interfacial shear stresses are concentrated along the edges of the actuator. Then the stress components on the top free surface due to an external traction force, \( \tau_0 \), and their wave number domain components through the Fourier transform can be expressed as,

\[
\tau_{31}(x_1,x_2) = \tau_0 E_1(x_1,x_2); \quad \tau_{32}(x_1,x_2) = \tau_0 E_2(x_1,x_2); \quad \tau_{33}(x_1,x_2) = 0
\]

\[
\tilde{\tau}_{31}(\xi_1,\xi_2) = \tau_0 \tilde{E}_1(\xi_1,\xi_2); \quad \tilde{\tau}_{32}(\xi_1,\xi_2) = \tau_0 \tilde{E}_2(\xi_1,\xi_2); \quad \tilde{\tau}_{33}(\xi_1,\xi_2) = 0 \quad (2.16)
\]

where \( E_{1,2} \) are spatially defined edges in which the traction force acts upon around the actuator.

From Eq.(2.6), we get additional fourth equations to solve the unknown constants for both modes.

\[
-C_i i\xi_1 \sin(qb) - C_i i\xi_2 \sin(qb) - C_i q \sin(qb) = 0
\]

\[
-C_i i\xi_1 \cos(qb) - C_i i\xi_2 \cos(qb) + C_i q \cos(qb) = 0 \quad (2.17)
\]

The unknown constants can be evaluated on the top free surface, \( x_3 = b \), using Eqs. (2.15), (2.16), and (2.17).

**Symmetric mode:**

\[
\begin{bmatrix}
(2i\xi_1 p)\sin(pb) & (\xi_1 \xi_2)\sin(qb) & (q^2 - \xi_1^2)\sin(qb) & (i\xi_2 q)\sin(qb) \\
(2i\xi_2 p)\sin(pb) & (\xi_1^2 - q^2)\sin(qb) & (-\xi_1 \xi_2)\sin(qb) & -(i\xi_1 q)\sin(qb) \\
(\xi_1^2 + \xi_2^2 - q^2)\cos(pb) & (2iq\xi_2)\cos(qb) & -(2iq\xi_1)\cos(qb) & 0 \\
0 & -i\xi_2\sin(qb) & -i\xi_2\sin(qb) & -q\sin(qb)
\end{bmatrix} \begin{bmatrix} B \\ C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} \tau_0 \\ \tilde{E}_1(\xi_1,\xi_2) \\ \tilde{E}_2(\xi_1,\xi_2) \end{bmatrix} \quad (2.18)
\]

**Antisymmetric mode:**

\[
\begin{bmatrix}
(-2i\xi_1 p)\cos(pb) & (\xi_1 \xi_2)\cos(qb) & (q^2 - \xi_1^2)\cos(qb) & (i\xi_2 q)\cos(qb) \\
(-2i\xi_2 p)\cos(pb) & (\xi_1^2 - q^2)\cos(qb) & (-\xi_1 \xi_2)\cos(qb) & (i\xi_1 q)\cos(qb) \\
(\xi_1^2 + \xi_2^2 - q^2)\sin(pb) & (-2iq\xi_2)\sin(qb) & (2iq\xi_1)\sin(qb) & 0 \\
0 & -i\xi_1\cos(qb) & -i\xi_2\cos(qb) & q\cos(qb)
\end{bmatrix} \begin{bmatrix} A \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} \tau_0 \\ \tilde{E}_1(\xi_1,\xi_2) \\ \tilde{E}_2(\xi_1,\xi_2) \end{bmatrix} \quad (2.19)
\]

The following three equations show the transformed displacement components for both the symmetric and antisymmetric modes on the free surface. The symmetric displacement
components are obtained following the top lines of Eqs. (2.20) to (2.22), while the bottom lines give the antisymmetric displacement components.

\[
\begin{align*}
q_{11}^S &= \frac{\cot(qb)}{\tan(qb)} \int_{-\infty}^{\infty} \frac{e^{-i(\xi_1 + \xi_2 - ax)}}{4\pi^2 \mu q} D_{11}(\xi) d\xi d\xi_2 (2.20) \\
n_{21}^S &= \frac{\cot(qb)}{\tan(qb)} \int_{-\infty}^{\infty} \frac{e^{-i(\xi_1 + \xi_2 - ax)}}{4\pi^2 \mu q} D_{11}(\xi) d\xi d\xi_2 (2.21) \\
n_{31}^S &= \frac{\tau_0}{4\pi^2 \mu} \int_{-\infty}^{\infty} \frac{e^{-i(\xi_1 + \xi_2 - ax)}}{4\pi^2 \mu q} D_{11}(\xi) d\xi d\xi_2 (2.22)
\end{align*}
\]

where \( \xi_1^2 = \xi_1^2 + \xi_2^2 \) and the denominator \( D_{11}(\xi) = \left(\xi_1^2 - q^2\right)^2 \left(\xi_1 - \xi_2\right) \left(\xi_1 + \xi_2\right) \left(2\xi_1^2 + \xi_2^2\right) \left(2\xi_1^2 - \xi_2^2\right) \). The products of sine and cosine are denoted as \( A_p = \sin(qb) \cos(pb) \) and \( B_p = \sin(pb) \cos(qb) \).

Three equations from Eq. (2.20) to Eq. (2.22) are the general analytic displacement components for the symmetric and antisymmetric mode responses of the system shown in Figure 2.1.

2.1.1 Rectangular Piezoceramics

For a rectangular PZT attached to the plate, the forcing directional functions in Eq. (2.16) can be described using the Dirac delta and Heaviside functions. The wavenumber domain expressions for the PZT actuator using double Fourier transforms are shown in Eq. (2.23).

\[
\begin{align*}
E_1 &= \left(\delta(x_1 - a_1) - \delta(x_1 + a_1)\right) \left(H(x_2 + a_2) - H(x_2 - a_2)\right) \\
E_2 &= \left(H(x_1 + a_1) - H(x_1 - a_1)\right) \left(\delta(x_2 - a_2) - \delta(x_2 + a_2)\right)
\end{align*}
\]

The out-of-plane displacement is considered in the subsequent analysis since the amplitudes of the propagating wave can be measured with a laser vibrometer in the experiments.
Substitution of Eq. (2.23) into Eq. (2.22) results in the following equation for the out-of-plane displacement for the system.

Symmetric mode:

\[
 u_{1|s} = \frac{\tau_0}{\pi^2 \mu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} q \xi \sin(\xi_1 a_1) \sin(\xi_2 a_2) \frac{T_s(\xi)}{\xi} e^{-i(\xi_1 x_1 + \xi_2 x_2 - o\xi)} d\xi_1 d\xi_2
\]

\[
 u_{2|s} = \frac{\tau_0}{\pi^2 \mu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} q \xi \sin(\xi_1 a_1) \sin(\xi_2 a_2) \frac{T_s(\xi)}{\xi} e^{-i(\xi_1 x_1 + \xi_2 x_2 - o\xi)} d\xi_1 d\xi_2
\]

\[
 u_{3|s} = \frac{\tau_0}{\pi^2 \mu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} q \xi \sin(\xi_1 a_1) \sin(\xi_2 a_2) \frac{T_s(\xi)}{\xi} e^{-i(\xi_1 x_1 + \xi_2 x_2 - o\xi)} d\xi_1 d\xi_2
\]

(2.24)

\[
 T_s(\xi) = (\xi^2 + q^2) \cos(pb) \cos(qb)
\]

where \( T_{s3}(\xi) = \xi^2 \left[ (\xi^2 - q^2) \sin(qb) \cos(pb) + 2pq \sin(pb) \cos(qb) \right] \)

\[
 D_s(\xi) = (\xi^2 - q^2)^2 \sin(pb) \cos(pb) + 4\xi^2 pq \sin(pb) \cos(qb)
\]

Antisymmetric mode:

\[
 u_{1|a} = \frac{\tau_0}{\pi^2 \mu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} q \xi \sin(\xi_1 a_1) \sin(\xi_2 a_2) \frac{T_a(\xi)}{\xi} e^{-i(\xi_1 x_1 + \xi_2 x_2 - o\xi)} d\xi_1 d\xi_2
\]

\[
 u_{2|a} = \frac{\tau_0}{\pi^2 \mu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} q \xi \sin(\xi_1 a_1) \sin(\xi_2 a_2) \frac{T_a(\xi)}{\xi} e^{-i(\xi_1 x_1 + \xi_2 x_2 - o\xi)} d\xi_1 d\xi_2
\]

\[
 u_{3|a} = \frac{\tau_0}{\pi^2 \mu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} q \xi \sin(\xi_1 a_1) \sin(\xi_2 a_2) \frac{T_a(\xi)}{\xi} e^{-i(\xi_1 x_1 + \xi_2 x_2 - o\xi)} d\xi_1 d\xi_2
\]

(2.25)

\[
 T_a(\xi) = (\xi^2 + q^2) \sin(pb) \sin(qb)
\]

where \( T_{a3}(\xi) = \xi^2 \left[ (\xi^2 - q^2) \sin(pb) \cos(qb) + 2pq \sin(qb) \cos(pb) \right] \)

\[
 D_a(\xi) = (\xi^2 - q^2)^2 \sin(pb) \cos(pb) + 4\xi^2 pq \sin(qb) \cos(pb)
\]

Introducing a change of variables, the above equation is converted into polar coordinates yielding more convenient formation, as shown below for the antisymmetric out-of-plane displacement.

\[
 u_{3|a} = \frac{\tau_0}{\pi^2 \mu} \int_{0}^{2\pi} \int_{0}^{\infty} \sin(\xi a_1 \cos\gamma) \sin(\xi a_2 \sin\gamma) \frac{T_{a3}(\xi)}{\xi \cos\gamma \sin\gamma} e^{-i(\xi x_1 \cos\gamma + \xi x_2 \sin\gamma - o\xi)} d\gamma d\xi
\]

(2.26)
The evaluation of the integral with singular points where the roots of \( D_A = 0 \) can be solved by the residue theorem [288] shown in Eq. (2.27). Using the theorem, other displacement equations in polar coordinate like Eq. (2.26) can be further simplified by eliminating the infinite integral.

\[
\left. \int_{-\infty}^{\infty} \frac{\pi}{\xi} \sum_{\xi} \text{Res}\left(I\left(\tilde{\xi}\right)\right) \right|_{\xi \to \infty} = 2\pi i \sum_{\xi} \text{Res}\left(I\left(\tilde{\xi}\right)\right)
\]

where \( I \) is the integrand and \( \tilde{\xi} \) is the poles of the function \( I \).

Note that the second term in Eq. (2.27) in the residue theorem approaches zero as \( \xi \) approaches infinity for the system represented by Eqs. (2.24) and (2.25) since the degree of the polynomial in the denominator is two orders higher than the numerator. Since the function is also even, Eq. (2.27) can be rewritten as Eq. (2.28).

\[
\int_{-\infty}^{\infty} \frac{\pi}{\xi} \sum_{\xi} \text{Res}\left(I\left(\tilde{\xi}\right)\right) = 2\pi i \sum_{\xi} \text{Res}\left(I\left(\tilde{\xi}\right)\right)
\]

where, for example, the antisymmetric out-of-plane integrand is shown below.

\[
I = \frac{2\pi}{\xi} \sum_{\xi} \text{Res}\left(I\left(\tilde{\xi}\right)\right)
\]

Using Eq. (2.28), the meromorphic function, Eq. (2.26), can be now expressed as,

\[
|t_3|_{\bar{A}} = \frac{2\pi i}{\xi} \sum_{\xi} \text{Res}\left(I\left(\tilde{\xi}\right)\right)
\]

The integral type of Eq. (2.29) can be approximately solved with either the method of stationary phase or the method of steepest descent for large \( \eta \) [285] as shown in Eq.(2.30). This type of integral commonly appears in waveguide problems and the above two methods are regularly used for deriving asymptotic expansions. Both methods result in the same approximation as shown below.

\[
\int_{a}^{b} f(\zeta)e^{i\eta h(\zeta)}d\zeta \approx \sqrt{\frac{2\pi}{\eta h''(\zeta_0)}} f(\zeta_0) e^{i[\eta h(\zeta_0) + (\pi/4)\text{sgn} h'(\zeta_0)]}
\]
where \( sgn \) indicates the sign of the quantity following it.

Comparing Eqs. (2.29) and (2.30), all the displacement expressions of the PZT are finally obtained as below,

\[
\begin{align*}
    u_1^S|_A & = \sum_{\zeta} \frac{q\tau_{0}}{\pi\mu} \left( \frac{D^S_{A\zeta}(\tilde{\xi})}{D^S_{A\zeta}(\tilde{\xi})} \right) \sqrt{2\pi} \frac{\sin(\tilde{\xi}a_1 \cos \theta)\sin(\tilde{\xi}a_2 \sin \theta)}{\xi r} e^{\left[ \omega t - \xi r + (\pi/4) \right]} \\
    u_2^S|_A & = \sum_{\zeta} \frac{q\tau_{0}}{\pi\mu} \left( \frac{D^S_{A\zeta}(\tilde{\xi})}{D^S_{A\zeta}(\tilde{\xi})} \right) \sqrt{2\pi} \frac{\sin(\tilde{\xi}a_1 \cos \theta)\sin(\tilde{\xi}a_2 \sin \theta)}{\xi r} e^{\left[ \omega t - \xi r + (\pi/4) \right]} \\
    u_3^S|_A & = \sum_{\zeta} \frac{\tau_{0}i}{\pi\mu} \left( \frac{D^S_{A\zeta}(\tilde{\xi})}{D^S_{A\zeta}(\tilde{\xi})} \right) \sqrt{2\pi} \frac{\sin(\tilde{\xi}a_1 \cos \theta)\sin(\tilde{\xi}a_2 \sin \theta)}{\xi r} e^{\left[ \omega t - \xi r + (\pi/4) \right]},
\end{align*}
\]

where \( \theta = \tan^{-1}(x_2/x_1) \) and \( r = \sqrt{x_1^2 + x_2^2} \).

2.1.2 MacroFiber Composites

As briefly discussed in section 1.3.1, the MacroFiber Composite (MFC) possesses anisotropic actuation by using interdigitated electrodes and unidirectional piezoceramic fibers embedded in a thermosetting polymer matrix [102]. Since the poling and alignment of the fibers are in the same direction (Figure 2.3), the actuation authority of MFC is large in the fiber direction and minimal normal to the fibers. Therefore, we can assume that the traction force in this direction normal to the fibers is approximately zero [25].

Figure 2.3. Illustration of the (a) MacroFiber Composite actuator and (b) actuator package [102]
Considering the directional authority of the actuator, the forcing functions for the MFC can be expressed in Eq. (2.32) using the Dirac delta and the Heaviside function. Note that direction 1 is the orthogonal direction to the fibers.

\[
E_1 = 0; \quad \tilde{E}_1 = 0 \\
E_2 = (H(x_1 + a_1) - H(x_1 - a_1))(\delta(x_2 - a_2) - \delta(x_2 + a_2)); \quad \tilde{E}_2 = -4i\sin(\xi_1a_1)\sin(\xi_2a_2)/\xi_1
\] (2.32)

Following the same steps outlined in the previous section for the rectangular PZT, the out-of-plane displacements in polar coordinates are expressed as,

\[
u_3|_A^S = \frac{\tau_0}{\pi^2 \mu} \int_0^{2\pi} \tan \gamma \sin(\xi_1 a_1 \cos \gamma) \sin(\xi_2 a_2 \sin \gamma) \frac{T_{A3}^{S3}(\xi)}{\xi^D_A(\xi)} e^{-i(\xi_1 \cos \gamma + x_2 \sin \gamma - \omega t)} d\gamma d\xi
\] (2.33)

By applications of the residue theorem and the method of stationary phase, the out-of-plane displacement of the system with an attached MFC actuator is expressed below.

\[
u_3|_A^S = \sum_{\xi} \frac{\tau_0}{\pi^3 \mu} \frac{T_{A3}^{S3}(\xi)}{\xi^D_A(\xi)} \sqrt{\frac{2\pi}{\xi_1 r}} \tan \theta \sin(\xi_1 a_1 \cos \theta) \sin(\xi_2 a_2 \sin \theta) e^{i[\omega t - \xi r + (\pi/4)]}
\] (2.34)

In the experimental analysis, excitation signals from the actuators are accompanied with a relatively lower frequency of around 20 kHz. It is theoretically and experimentally determined that, with an actuator attached on one side of an aluminum plate, the amplitudes of symmetric wave mode due to an external force are much weaker than the ones for the antisymmetric wave mode at this frequency. Due to its amplitude ascendancy of the antisymmetric modes over the symmetric modes at lower frequencies, the antisymmetric modes are considered for the subsequent analysis.

### 2.2 Sensor Response Formulations

In general, the piezoceramic sensors are embedded or attached on the surface of the host structure. These thin piezoceramic sensors convert in-plane strains to electric voltage by the converse piezoelectric effect. Since the displacement equations are already obtained in section 2.1, it is feasible to obtain the voltage outputs from the piezoceramic sensors [25, 101, 106, 289]
The general constitutive equations for a linear piezoelectric sensor can be written below [290].

\[ D_i = d_{ik} \sigma_k + \varepsilon_{ij} E_j \]  

(2.35)

where \( D_i \), \( d_{ik} \), \( \sigma_k \), \( \varepsilon_{ij} \), and \( E_j \) are electric displacement, piezoelectric constant, stress component, dielectric constant, and electric field component, respectively. For a sensor, the applied external electric field, \( E_j \), becomes zero. Then Eq. (2.35) yields the following electric displacement vector equations [106, 291].

\[
\begin{align*}
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix} &= \begin{bmatrix}
0 & 0 & 0 & d_{15} & 0 \\
0 & 0 & 0 & d_{15} & 0 \\
d_{31} & d_{31} & d_{33} & 0 & 0
\end{bmatrix} \begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{31}
\end{bmatrix} \\
\end{align*}
\]  

(2.36)

It is a reasonable assumption that the electric displacements in 1- and 2-directions (\( D_1 \) and \( D_2 \)) and \( \sigma_{33} \) are zeros since the PZT sensor is thin and poled in 3-direction. The electric displacement in 3-direction is then notated below.

\[ D_3 = d_{31} \left( \frac{E_c}{1 - \nu_c} \varepsilon_{ii} \right) \]  

(2.37)

where \( E_c \), \( \nu_c \), and \( \varepsilon_{ii} \) are Young’s modulus, Poisson ratio, and strain of the piezoceramic sensor, respectively.

Under constant stress, the inter-relation between piezoelectric constants can also be written as below [290].

\[ d_{31} = \varepsilon_{33} g_{31}; \quad \varepsilon_{33} = k_c \varepsilon_0 \]  

(2.38)

Substituting Eq. (2.38) into Eq. (2.37), the following electric displacement is obtained.

\[ D_3 = \frac{k_c \varepsilon_0 g_{31} E_c}{1 - \nu_c} (\varepsilon_{ii}) \]  

(2.39)
where \( \varepsilon_{33}, g_{31}, k_c, \) and \( \varepsilon_0 \) are permittivity, piezoelectric constant, relative permittivity, and vacuum permittivity (8.854 x 10\(^{-12}\) Farad/meter), respectively.

The sensor voltage, produced between the electrodes of the PZT, has a relation with the capacitance of the sensor and the charge generated. The total electric charge of the sensor is the product of electric displacements and the area of the sensor shown in Eq. (2.40).

\[
Q_c = \int_{\text{Area}} D_3 dA = \frac{k_c \varepsilon_0 g_{31} E_c}{1 - v_c} \int_{\text{Area}} \varepsilon_0 dA
\]

(2.40)

The capacitance of the PZT sensor can be calculated if the geometry of the piezoceramics is known.

\[
C_c = \frac{k_c \varepsilon_0 A_c}{t_c}
\]

(2.41)

where \( A_c \) and \( t_c \) are the surface area and the thickness of the sensor. Since the sensor voltage response is the electric charge over the capacitance, it can be written as below.

\[
V_c = \frac{g_{31} E_c t_c}{A_c (1 - v_c)} \int_{\text{Area}} (\varepsilon_{11} + \varepsilon_{22}) dA
\]

Or

\[
V_c = \frac{g_{31} E_c t_c}{A_c (1 - v_c)} \int_{-s_1}^{+s_1} \int_{-s_2}^{+s_2} \left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} \right) dx_2 dx_1
\]

(2.42)

where \( s_1 \) and \( s_2 \) are the half lengths of the sensor in 1- and 2-directions.

The analogue equations for \( u_1 \) and \( u_2 \) from Eq. (2.29) can be written in which the symmetric mode is obtained following the top part of the equations, while the bottom parts give the antisymmetric displacement mode equations, as given in Eq. (2.43).
Substituting Eq. (2.43) into Eq. (2.42), the output voltages for both of the symmetric and antisymmetric modes of the PZT sensor are finally obtained as,

\[
V_{c}^{\xi} = \frac{4\tau_{0}g_{31}E_{t}^{c}}{\pi \mu s_{1} s_{2}(1-v_{c})} \frac{q T_{d}^{S}(\bar{\xi})}{(D_{d}^{S}(\bar{\xi}))'} \int_{0}^{2\pi} \csc(2\gamma) \sin(\bar{\xi} a_{1} \cos \gamma) \sin(\bar{\xi} a_{2} \sin \gamma) e^{-i(\bar{\xi}(x_{1} \cos \gamma + x_{2} \sin \gamma) - \omega t)} d\gamma (2.44)
\]

2.3 Conclusion

Displacement formulations for both symmetric and antisymmetric wave modes are derived for the guided wave field in an infinite isotropic plate structure. Plate displacements at a point excited by a rectangular PZT actuator and an MFC actuator are obtained assuming the perfect bonding between the actuators and the plate. Voltage outputs from the PZT sensor is also presented from the converse piezoelectric effect using the in-plane strains derived from the previously obtained displacement formulations.
Chapter 3

3. Beamsteering Analysis and Discussion

The first part of the chapter presents the limitations of the general beamsteering formula assuming omni-directional point excitations. This formula is found to be incapable of predicting the actual response for the phased array with anisotropic actuations. The genetic algorithm is then employed to determine correct beamsteering time delays. The phased array responses for PZT and MFC actuators are studied by comparing the main lobe and side lobe areas. The area ratio plots are also drawn to compare the performance of both phased arrays.

3.1 Limitations of Omni-Directional Beamsteering Algorithms for MFC Phased Arrays

For the first part of the analysis, the harmonic wave propagation of a single MFC and PZT actuator is compared. The two actuators are equal in size and are located at the center of a free isotropic plate. The PZT fibers in the MFC are aligned in the vertical direction, and thus the maximum actuation authority of the MFC is in the vertical direction. Both plots in Figure 3.1(a) and Figure 3.2(a) show the normalized wave propagation simulation results from the analytic solutions derived for the PZT and MFC, respectively (Eqs. (2.31) and (2.34)). Their wave patterns are shown on the same figures (right), where the radius indicates the magnitude of the wave and the angle indicates the direction of the wave propagations. The values in the plots have been also normalized for comparison purposes. Since the electromechanical coupling of the PZT is equal in the 1- and 2- directions and the size of the actuator is small with respect to the plate, the out of plane displacement is equal in all directions for the PZT. On the other hand, due to the anisotropic actuation, the wave pattern for MFC is maximum in the PZT fiber direction and minimum in the direction normal to the fibers. The reduced amplitude along the horizontal (normal to fiber) direction can be advantageous for certain applications requiring minimized wave propagation in specific directions.
Due to the anisotropic actuation, the beamforming algorithm using the omni-directional assumption may not be applicable for the MFC actuators since the magnitude of the wave propagation is dependent upon the PZT fiber orientation with respect to the host structure. To illustrate this, the general beamforming algorithm for omni-directional point excitations is applied to two equivalent systems, one having PZT actuators and the other having MFC actuators. The general formula is shown in Eq. (3.1) [186].

$$BF\left(\sigma_m, s_m, \theta_0\right) = \frac{1}{M} \sum_{m=0}^{M-1} \frac{\sigma_m}{\sqrt{r_m/r}} e^{i\omega \left[ \frac{r_m - r_m}{c} \delta_m(\theta_0) \right]}$$

(3.1)

where $\sigma_m$, $s_m$, $\theta_0$, $M$, $r$, $r_m$, $c$, and $\delta_m$ are the weighting factors, actuator location vectors, target direction, number of actuators, distance from origin, distance from actuator, wave velocity, and delays, respectively.

Shown in Figure 3.3(a) are the beamforming results for the PZT and MFC using Eq. (3.1) for the PZT (left plot) and the MFC (right plot). Each system has eight actuators with the time
delays ($\delta_m$) being equal for both the PZT and MFC systems. The rest of the beamforming conditions are identical to those used in Yu et al. [186], i.e. the spacing between the actuators is the half wavelength, distance from the origin to target is ten times of the array span and uniform weighting factors. As shown in this normalized plot, the side lobes are clearly lower for the MFC than the PZT for the $90^0$ angle and the beamsteering angles are exactly $90^0$ for both systems. However for angles other than $90^0$, the general beamforming equation in Eq. (3.1) is not applicable for the MFC actuators as shown in Figure 3.3(b), where there is about $5^0$ offset from desired angle for the $40^0$ beamforming direction. The generalized beamsteering formula is incapable of accurately predicting the actual response for the MFC phased array.

To understand the limitations associated with the use of Eq. (3.1) for the MFC phased array, a simple linear array having two MFC sources is examined for a desired angle of $60^0$. As shown in Figure 3.4(a), the largest amplitude of the PZT array is at $60^0$ as predicted by Eq. (3.1) while the maximum amplitude for the MFC array occurs at $74^0$ instead of the desired angle of $60^0$. In examining the error associated with the use of the equation for the MFC array, the amplitudes at two points residing on the solid radial lines of $60^0$ and $74^0$ are studied as indicated in Figure 3.4(b). The system response for each actuator along the $60^0$ and $74^0$ lines are plotted in Figure 3.4(c). The distance from the origin, located in the middle of the two sources, to the target points is set to be 10 times of the distance ($d$) of two sources.
In Figure 3.4(c), the x-axis is the distance along the 60° and 74° radial lines from 9d to 11d, and the y-axis are the actuator responses. The solid blue line (right plot) is the sum of two MFC sources in the 74° direction and the solid black line (left plot) is the sum in the 60° direction. As shown in these plots, the sum of the responses of the two actuators at 74° is greater than at 60°. The generalized beamsteering algorithm in Eq. (3.1) is derived such that the waves from each actuator are in-phase which ensures that the maximum amplitude is achieved in the desired direction with an omni-directional point source. Although the responses are in-phase at 60° as seen in Fig. 6(c), the maximum amplitude is greater at 74° where the response of each actuator are slightly out-of-phase. The larger amplitude occurring at 74° instead of 60° is a result of the anisotropic actuation of the MFC, where the amplitude of MFC is getting larger as the angle shifts toward the embedded PZT fiber direction, 90° in this case. Simply stated, the increase in amplitude of the waves slightly out-of-phase at larger angles is greater than the sum of two
smaller responses that are in-phase since the magnitude of the wave from each MFC actuator is maximum at 90°.

3.2 Analysis and Discussion of Beamsteering with PZT and MFC Phased Arrays

As discussed in the previous section, there is an error in the beamsteering direction with the application of Eq. (3.1) for the MFC phased array due to the omni-directional point assumption in the beamforming algorithm. As a result of the complexity of the analytical problem for the MFC phased array, an exact analytical solution for predicting the exact beamforming direction cannot be determined. Therefore the genetic algorithm (GA) is employed for determining the desired beamforming solutions for the MFC phased array based upon an objective function that considers the beamforming angle, the amplitude in the beamforming direction, and the side lobe magnitudes.

In the linear PZT and MFC phased array analysis, various numbers of PZT and MFC array elements (2, 4, and 6) are investigated. The independent variables in the analysis are the relative delay times between each actuator. In the optimization routine for determining the correct beamsteering delay times, the maximum main lobe amplitudes of the PZT and MFC arrays are equal to each other, are maximum in amplitude, and occur at the exact beamsteering angle. The distance between each actuator is half of the wavelength and the size of actuators are identical to each other. The frequency of the excitation signal is 20 kHz and its wave speed is 770 m/s through the aluminum plate structure. In the GA, using the roulette wheel selection, an initial population size of 2,000 with 50 children and 2 elitists are used, and the convergence tolerance is such that the maximum values of 20 generations remain constant within a tolerance of $1 \times 10^{-10}$.

The time delay results for the PZT and MFC phased arrays are shown in Figure 3.5, where the first actuator (left one of the array) is excited at 0 second. Note that the delay times for the PZT are determined using Eq. (3.1) since the amplitude in the beamsteering direction is maximum and the side lobes are minimum. For each discrete data point for the MFC, the GA is executed 5 times having randomly selected initial guesses. The largest change observed in the final value was never greater than 1% of the data point shown in the figure. As shown, the delay times for the MFC actuators are larger than the delay times for the PZT. Since the amplitude of the wave displacement for the MFC increases as the angle approaches 90° (fiber direction), longer delay times are required to ‘steer’ the beam in the correct direction. Interestingly, it is
also seen that the delay times for the MFC actuators approach the delay times for the PZTs as the number of actuators are increased.

The phased array responses for both systems are shown in Figure 3.6 for target angles at 85° and 65°. Six actuators are used in each array with the optimal time delays shown in Figure 3.5(c). Unlike the results in Figure 3.3(b) for the MFC actuator, the main lobes of the wave are correctly located to the desired target angles by using the optimal delay values.

The main lobe and side lobe areas for both the PZT and MFC phased arrays are shown in Figure 3.7 for increasing number of actuators. The main lobe area of the PZT system is greater than the MFC system for all steering angles and number of actuators. Since the magnitudes are equal for both systems, the larger main lobe area observed with the PZT phased array system is a result of an increase in the main lobe width. For each actuator case, the side lobe areas for the
PZT phased array are larger than the MFC system in the 90° direction as a result of the PZT fiber alignment in the MFC actuator. As the beamforming angle decreases from 90°, the side lobe areas of the MFC approaches and exceeds the PZT side lobes, and thus the improved performance of the MFC phased array over the PZT system decreases as the beamsteering angle decreases. However, increasing the number of actuators increases this performance range for the MFC actuator.

From Figure 3.7(c), a local minimum is observed around 70° for the PZT array having 6 actuators. This behavior can also be seen when the number of PZT actuators is further increased. As a result of the omni-directionality of the PZT actuator, the constructive and destructive side lobe interferences can be more significant, compared to the MFC actuators. At certain angles, the destructive interferences of the PZT side lobes result in a local minimum in the side lobe area.

The ratio of the main lobe to the side lobe areas in Figure 3.8 can be determined from the analysis results presented in Figure 3.7 for the considered beamforming angles and number of actuators. Similar to Figure 3.7 for the absolute main/side lobe areas, the results illustrate the improved performance of the MFC phased array in reference to the PZT phased array for a range of desired beamsteering angles. For the area ratios, the intercepting points of the PZT and MFC are 72°, 61°, and 52° for two, four, and six actuators, respectively. The area ratio plots (Figure 3.8) along with the absolute values (Figure 3.7) can provide essential information regarding the preferred choice of actuator type for the desired range of beamsteering angles.
3.3 Conclusion

The general beamsteering formula with omni-directional point excitations is incapable of predicting the actual response for the phased array with anisotropic actuations. The correct beamsteering time delays are obtained for PZT and MFC phased arrays using genetic algorithms and the array responses are compared for the main lobe and side lobe areas. The performance of both arrays is discussed based on the area ratio plots. The results show the improved performance of the MFC phased array in reference to the PZT phased array for a certain range beamsteering angles.
Chapter 4

4. Experimental Validations

Experiments are performed using the phased array technique with PZT and MFC actuators and the results are compared to the analysis presented in Chapter 3. Firstly, the sensor responses due to in-plane strains are studied for the A0 and S0 modes, utilizing monolithic piezoceramics. The second part presents the experimental results of the phased array beamsteering using the PZT and MFC linear arrays. Six actuators are used for each linear array system to steer the wave directions, by exciting each actuator with the calculated time delay. The beamsteering results of both systems are compared.

4.1 Sensor Voltage Response

The normalized voltage amplitude results of both the A0 and S0 modes for an aluminum plate is shown in Figure 4.1 using Eq. (2.44). The piezoceramic transducer used for the analysis is a PSI-5H4E (Piezo Systems, inc.) with dimensions of 10 mm x 10 mm x 0.267 mm. The structure is considered as an aluminum plate having a thickness of 3.18 mm. The analysis results show that the voltage response of each mode is significantly different, i.e. the A0 mode responses largely at relatively lower frequency range (max around 700 kHz), while the S0 mode increases steadily with the increasing frequency. Giurgiutiu [101] mentioned a ‘sweet spot’ where the S0 mode voltage response is predominant over the A0 mode response, as indicated in the figure. The sensor response at 20 kHz that is used for beamsteering analysis in previous chapter is also indicated on the same figure. The figure shows that the A0 mode voltage response at this frequency is much higher than the S0 mode response.
The voltage response of a piezoceramic sensor is experimentally validated for the A0 mode using a 1000 mm x 10 mm x 3.2 mm aluminum beam. Two PSI-5H4E piezoceramic actuators previously mentioned are attached on the middle length of the beam (500 mm). They are collocated but each one is attached on the opposite side of the aluminum beam surface to send out a pure A0 wave mode signal. A sensor, possessing the same type and dimensions as actuators, is attached at 70 mm apart from the actuators to measure the propagated A0 mode input signal. The close distance between the actuators and the sensor is chosen to get a better signal resolution as well as to reduce signal dispersions. Four-cycles Hanning windowed signal with an input voltage of ±5V is excited from both actuators. While one actuator sends out the original signal, the other actuator sends an inverted signal through an inverting amplifier to excite a pure A0 wave mode. The input signals are sent through a NI PXI-6120 system and the output signals are saved through a NI PXI-6133 system. The peak-to-peak voltage is measured 12 times at every 10 kHz to average out the signal. The normalized results for the A0 mode at frequencies up to 100 kHz are shown in Figure 4.2. The solid line is normalized voltage response of the A0 mode with the original transducer dimensions. The dotted line is the same response with the reduced transducer dimensions (85%) and the small rectangles are experimental results. The theoretical solid line in the figure is obtained assuming the perfect bonding layer, i.e. matrix between piezoceramics and the aluminum beam is extremely thin. However, the effective dimensions of the transducer could be smaller than the actual dimension due to the ‘shear lag’
effect [104]. Wang et al. [29] obtained good agreement between the analytical and the experimental results by using 15% smaller than the physical actuator dimensions considering the shear lag effect. Therefore, the effective actuator dimensions used in the analysis are reduced to 85% of the original dimensions as shown in Figure 4.2. The experimental results seem to follow the voltage response of the original transducer at lower frequency range, but the overall voltage response seems close to the reduced actuator responses.

![Figure 4.2. Comparison between theory and experimental results for A0 mode](image)

The voltage responses above 100 kHz in the experiments were not measured due to the sampling limitations of the NI PXI DAQ systems (2.5 MS/s). As the sampling step size decreases with higher frequencies, the discrete signal for the input and the quantized signal for the output tend to improperly represent the continuous input and the sensed signals. This discrepancy affects the overall wave propagation and results in unacceptable signal responses. The fundamental symmetric mode (S0) was not measured since the voltage responses for this mode were very weak for the frequencies less than 100 kHz, as shown in Figure 4.1. It was hard to correctly capture its reflected wave mode from the sensed signal.
4.2 Phased Array Beamsteering

The analytical beamsteering results of the PZT and MFC phased arrays are validated by performing experiments using an aluminum plate according to the schematic shown in Figure 4.3(a). Arrays of each type of actuator are attached on one side of a 2024 aluminum rectangular plate (910 mm x 910 mm x 3.18 mm). The active area of both PZT (American Piezo Ceramics, APC-850) and MFC (Smart Material, cropped from M4010P1) actuators are identical (10 mm x 10 mm). Each actuator is directly connected from the outputs of a multi-channel amplifier module built from six operational amplifiers (M.S. Kennedy, MSK 601) and appropriate resistors and capacitors (Figure 4.3(b)) to provide a large voltage gain (-20 V/V). The MSK 601 operational amplifier is especially useful for this application since it has high (220 V) peak-to-peak voltage swing at high slew rates and a wideband frequency range (max 1 MHz). Each input of the amplifier module is then connected to D/A channels of an 1103 dSPACE rapid control prototyping controller board. Agilent E3620a and Xantrex XHR 150-7 power supplies are used to provide the necessary power to the amplifier module. Once the actuators in the array are excited, a laser vibrometer measures the out-of-plane response of the plate. The measured data is then monitored in LabVIEW and saved for further analysis.

![Figure 4.3. Experiment overview (a) and multi-channel amplifier module (b)](image.png)

In the experiment, the actuators attached to the plate (Figure 4.4) are sequentially excited with a 4-cycle Hanning windowed sinusoidal toneburst. The peak-to-peak voltages applied for
the PZT and MFC actuators are 80 V and 200 V, respectively, which limits the maximum current to less than 0.2 A at the specified operating frequency. Two rubber straps are used to hang the plate on top, and the bottom of the plate is clamped to eliminate plate fluctuation during the test. The measured area of the plate is 0.2 m by 0.2 m square, where its bottom-left corner is located at the center of the actuator array. The center area of the plate is examined to avoid wave reflections caused by the open edges of the plate. In both actuator arrays, the distance between adjacent actuators is 19.3 mm resulting in 96.3 mm array width. Due to sampling limitations of the D/A output channels on the dSPACE control board, the maximum excitation frequency was restricted to a value less than 50 kHz. Using the results from an experiment conducted in the lab for a single actuator, it was determined that the fundamental antisymmetric mode (A0) is predominantly excited at a frequency of 20 kHz. Accordingly, a signal frequency of 20 kHz is chosen for demonstrating the beamforming performance of the phased arrays and the corresponding phase and group velocities for this test setup are 770 m/s and 1550 m/s, respectively. The time response of the displacement and velocity of the plate is recorded through the laser vibrometer. The laser vibrometer (Polytec OFV-505) is attached to two high performance Newport linear stages. The linear stages control the position of the vibrometer in both horizontal and vertical directions through MATLAB™ or LabVIEW™. A vibrometer controller (Polytec OFV-5000) with a high frequency velocity and displacement decoder (VD-05) and a high resolution decoder (DD-200) measure the response. Once the excitation and response of the plate are recorded, the laser vibrometer moves to the next point and repeats the procedure. The measured square area is scanned with 40 equally spaced grids in both directions resulting in 1600 measurement points having a grid space of 5.1 mm. The data is acquired for all grid points through a NI PXI-6133 DAQ system with a sampling rate of 2 MS/s.
The measurement is repeated four times for each grid point with sampling frequency of 1 Hz to average out the signal. Averaging the signals is a necessary step since the unwanted noises can be produced by electrical and mechanical interferences or environmental disturbances. Most of noises in the measured data are due to the amplifier module built for the experiment. Signal de-noising using the discrete Meyer wavelet filters with soft heuristicSURE thresholding is used to reduce high frequency white noise. Figure 4.5 shows an example of noisy measured data and its de-noised data curve employing the above mentioned wavelet filter. To compare the theoretical results, experiments are performed for every 5° starting at 60° and ending with 90° while assuming the target distance ($r_t$) is located ten times the array width ($D$).
The experimental and theoretical results for the target angle of 60° are shown in Figure 4.6 for both the PZT and MFC arrays. The total duration of the time captured for each point is 400 μs at 2 M/sec sampling rate. The simulation results for the PZT array are presented for time equal to 216 μs and the results for the MFC array are presented for time equal to 244 μs to demonstrate the timely propagation of a group wave as shown in the figure. Notice that there is close agreement in the plots between the experimental and theoretical simulation results. In each array, both experimental and theoretical results indicate that the group of waves propagates toward the desired direction. The main and side lobes are also developed in a similar pattern for both arrays. As shown in top plots, the PZT side lobes are clearly visible on both sides of the main lobe envelope, whereas the MFC side lobes plots in bottom are imperceptible for the direction perpendicular to the PZT fiber direction of the MFC actuator. The group of waves shown in the figure appears to have more than 5 cycles although the excited signal is a 4-cycle windowed toneburst due to the group velocity dispersion.

Figure 4.6. Normalized beamforming results from experiment (left) and theory (right) for target angle of 60° and \( r_t=10D \) (in meter)
Note that the above results are obtained using time delays determined under the assumption that the target points are located at $r_t=10D$ or 0.963 m from the center of the array. However in the experiments, the maximum target distance is bounded by the measured area, i.e. 0.2 m by 0.2 m. Expanding to a larger area (10$D \times 10D$) would have implied more than 2 weeks of continuous testing for both plates at each desired angle since each measurement point requires approximately half a minute to take measurements and position the laser at a new position. Thus, the results for the 2D x 2D area obtained in the experiment for the 1$^{\text{st}}$ quadrant only are compared with analysis. Note that the main lobe and side lobe areas theoretically obtained in Section V are calculated considering both quadrants above the actuator array line, i.e. 1$^{\text{st}}$ and 2$^{\text{nd}}$ quadrants.

The normalized amplitudes for the desired angle of 60° and at a radius of 2D ($r_t=2D$) are shown in Figure 4.7(a-b) for the PZT and MFC arrays. The experimental results match well with the theoretical results. Close observation at the angle for the maximum amplitude reveals that the maximum angle is indeed not on 60°, but around 58° for both of the actuators. This is due to the difference between the target distance used for the calculation ($r_t=10D$), and the one used to measure the amplitude ($r_t=2D$).

There is a definite difference between the theoretically obtained main and side lobe areas in Figure 3.7 and the areas in Figure 4.7(c-d). As mentioned above, the results in Chapter 3 are measured for the combined region of the 1$^{\text{st}}$ and 2$^{\text{nd}}$ quadrants; however, the results in Figure 4.7 are only measured for the 1$^{\text{st}}$ quadrant. Like the amplitude plot of (a) and (b), the experimental and theoretical results closely match one another, therefore validating the theory. Although it only shows the 1$^{\text{st}}$ quadrant, it is clearly observable that the MFC side lobe areas are smaller than the ones for the PZT for all angles measured.
4.3 Conclusion

Experimental results of the PZT sensor response and beamsteering results using both PZT and MFC phased arrays are shown in this chapter. The monolithic PZT sensor responses for the A0 and S0 modes are theoretically obtained and experiments show good agreements with the theoretical results. The phased array beamsteering using both actuators are also experimentally evaluated for the first quadrant for certain beamsteering angles. The experimental and theoretical results closely match one another and the results show that the sidelobe area of the MFC is smaller than the one for the PZT for all the angles examined.
Chapter 5

5. Damage and Signal Analysis

In previous chapters, the phased array damage detection systems based on beamforming in plate structures were presented using the linear phased array transducers. This chapter delivers additional damage detection and location estimation approaches by discussing different types of damages, a fundamental study about the excitation signals, and a few signal processing methods, such as dispersion removal, cross-correlation, and Hilbert transform. Using these signal processing methods, a beamforming and beamsensing algorithm is applied to an aluminum plate containing crack damage and the processed results in 2-dimensional space is presented.

5.1 Damage Forms

Damage can be defined as a factor that negatively influences the system performance and causes a potential malfunction of the system that sometimes results in catastrophic system failures. The most frequent damage types incurred in aerospace industries are cracks for metallic structures and delaminations for composite structures [52]. In metallic structures, the second most frequent damage incurred is corrosion [53]. Each type of these damages has been investigated by other researchers as mentioned in section 1.3.1. The goal of this and next chapters is to discuss the necessary processes in classifying crack and corrosion damages in metallic structures.

In order to distinguish different types of damages, there must be distinct differences between the damage types. A general difference between crack and corrosion damages is that a crack is usually initiated much narrower and sharper in one direction than the other directions, but noticeable corrosion has a wider affected area for all directions.

Cracks can be initially formed on one side of structure surface and developed further into through-thickness cracks as number of cyclic loadings or applied loadings increase. Since the
development process from the initial to the final through-thickness crack is relatively fast compared to other damage types, it is critical to detect the crack in its early stage. On the other hand, it is not easy to define a certain form for corrosion damage due to its diverse and nonuniform deterioration process. For the damage diagnosis in this research, a pitting corrosion that frequently forms in common metals is induced in an aluminum plate to analyze its damage features, as shown in Figure 5.1. In order to create corrosion on the plate, a saline solution filled fluid reservoir is first attached on the plate with the rim of the reservoir covered with a rubber sealant. A coiled 12 cm x 1 cm x 0.1 cm aluminum electrode is put into the solution and 0.8A current is forced through the plate and the electrode for about 30 minutes. This corrosion process is similar to the natural phenomenon since negatively charged electrons in the plate are reacted with the positively charged electrode in the solution elements. This results in the loss of plate material in the saline solution forming corrosion damage. Figure 5.1(a) shows the top and side views of the corrosion. The diameter of the induced corrosion is about 4 cm. The depth of the corrosion varies due to the current applied, processing time, electrode surface area, and size of the plate. Figure 5.1(b) and (c) show the magnified views of the cross sections for the undamaged and the corroded region of the plate, respectively.

Figure 5.1. Induced aluminum corrosion using saline solution (a) top and side views, (b) cross-section of undamaged region, and (c) cross-section in the middle of corrosion
5.2 Signal Analysis

The most fundamental element in guided wave SHM is the wave signals that are excited from the actuators, propagated through the structures, and reflected and/or transmitted from the damages. Although the region that the wave signal propagates is dependent upon materials, applied actuator voltages, and signal frequencies, it can be generally assumed that the damage signals can be detectable. In many cases, the potential sites or hot spots for damages can be estimated such that the damage monitoring systems can be set at near to the locations apt to be damaged. In the following analysis, an actuation and sensing monitoring system using PZT transducers is assumed to be placed close to these hot spots. Before discussing the methods for distinguishing two types of damages, a fundamental study of an excitation signal in guided wave SHM is carried out and presented in the next section.

5.2.1 Excitation Signal Selection

A first step in guided wave SHM would be a selection of an appropriate excitation signal. This may not only be dependent upon damage characteristics and the materials that the signals go through, but it can also be influenced by the excitation signal itself.

To investigate appropriate excitation signals, four different sinusoidal signals are examined for thin PZT actuators. Other signals such as an impulse or a long time-wise signal are not considered due to its wide frequency range and a longer signal spread due to dispersion, respectively. These four inputs are 1-cycle and 2.5-cycles with Hanning and without Hanning windowed sine signals, as shown in Figure 5.2. The signal frequencies are 50 kHz for all cases and the PZT actuators are excited with these signals at the beginning of the measurements and remained still after being excited.
The Fourier transform of these excitation signals are analyzed considering the whole time period of the signals, i.e. the excitation signal added with zero inputs for the whole time period. Each of the Fourier transformed results of these time signals is shown in Figure 5.3. One thing immediately noticed is that the widths of frequency envelopes are dependent upon the number of cycles of the excitation signal. The 1-cycle excitation signals have the wide main frequency envelopes compared to the 2.5-cycles signals. In addition, the smaller side-frequency envelopes are also formed for the signals without Hanning window (full sine). Although the Fourier transforms of the Hanning windowed signals do not have these smaller multiple side-frequency envelopes, their main frequency envelopes are much wider than the full sine signals. Lastly, the peak amplitudes from the transform are smaller with the Hanning windowed signals than the
ones with full sine, which means that stronger signals can be excited from these full sine signals compare to the Hanning windowed signals.

![Figure 5.3](image)

Figure 5.3. Fourier transform of (a) 1-cycle sine, (b) 1-cycle Hanning windowed sine, (c) 2.5-cycles sine, and (d) 2.5-cycles Hanning windowed sine

To observe how the Fourier transform results relate to the wave propagations, each excitation signal is generated from a pair of PZT actuators on a thin 2 m aluminum beam in Abaqus®. A pure A0 mode signal is generated by attaching one PZT on top of the beam and the other on the bottom, and transmitting a normal and an inverse excitation signals on each one, respectively. Four different signals mentioned above are excited and propagated through the beam. A symmetric condition is applied in the middle of the PZT to the left side of the beam. The wave propagation results at 250 μs are shown for each signal in Figure 5.4. The propagated results match with the frequency analysis as expected. A 1-cycle Hanning windowed signal, shown in Figure 5.4(b), has undesirable large signal spread due to the wide mainlobe frequency range as
shown in Figure 5.3(b). Although full sine signals have narrower mainlobes than the Hanning windowed signals with the same cycles, Figure 5.4(a) and (c) show that they also become dispersive due to the smaller frequency sidelobes shown in Figure 5.3(a) and (c).

In order to see the differences more clearly for these wave propagation results, the particle displacements at 20 cm from the PZT center are measured for the whole time period and the results are shown in Figure 5.5. The numbers of fluctuation cycles counted at this point are 6.5, 6.5, 8, and 5.5 for 1-cycle full, 1-cycle Hanning, 2.5-cycles full, and 2.5-cycles Hanning signals, respectively. The widths of the wave signals are also proportional to the number of cycles. The results show that the 2.5-cycles Hanning windowed sine signal can be the most appropriate signal to be used out of these four signals. It is previously mentioned that an impulse and a long sinusoidal signal are out of consideration because of their wide main frequency range and a long signal spread due to signal dispersion. It may also be preferred if .5-cycle is added for non-dispersive excitation signals since it provides an odd peak in the signal. Considering these conditions and the analysis results from this section, it can be determined that an appropriate excitation signal in the guided wave SHM would be a short-cycle Hanning windowed toneburst signals, maybe containing 2.5 to 4 signal cycles. Some of the Hanning windowed toneburst signals with these cycles are used for the remaining analysis.
5.2.2 Dispersion Effects and Removal

As discussed in section 1.2.3 and also shown in the previous section, the guided Lamb wave has a dispersive nature where the wave velocity is dependent upon frequency. Since it is observed that dispersions can be severe for some excitation signals at certain frequencies, the next step would be investigating more about the dispersion effects on the excitation signals.

Wilcox et al. [292] discusses two ways to predict the dispersive wave propagation using the Fourier decomposition and the group velocity dispersion curves. The Fourier decomposition has been applied for this study and the space-time map illustrating the dispersive propagation is shown in Figure 5.6. The spreading of the wave packet is due to the different wave velocity that
is frequency dependent. The peaks and troughs of the wave packet are shown in grayscale in the top figure, where the black lines indicate troughs of the wave packet and white lines in-between are peaks of the wave. The space-time maps for the A0 mode and the S0 mode are shown in Figure 5.6(a) and in Figure 5.7(a), respectively. The original signal at the initial time is a 4-cycles Hanning windowed toneburst with the center frequency of 20 kHz. The 4-cycles are used to readily show the effects of dispersion. The 3.18 mm aluminum plate with the same properties used in the previous chapter is studied for the wave packet propagating through the plate.

![Image](image.png)

Figure 5.6. Simulation of dispersive A0 mode; (a) space-time map, (b) original signal, and (c) 1 m traveled signal

As shown in the dispersion curves of Figure 1.2, the group velocity of the A0 mode is highly dispersive around 60 kHz-mm frequency-thickness (due to 20 kHz x 3.18 mm). Figure 5.6(b) shows the original signal at the initial time while Figure 5.6(c) shows the dispersed signal after
the wave packet traveled 1 meter through the plate. Two features can be drawn from these time-amplitude results. The A0 mode at low frequency is very dispersive that the duration of wave packet is increased as wave propagates. In addition, the amplitude of the wave is decreased as the dispersive wave packet propagates further. Both of these may be undesirable since they may hinder accurate interpretations of the sensed signals.

On the other hand, the S0 mode wave packet keeps its shape after it is traveled 1 meter as shown in Figure 5.7. This is expected since the dispersion curve is almost flat (little dispersion) at this frequency-thickness curve shown in Figure 1.2. Since the group velocity of the S0 mode wave is faster than the A0 mode at lower frequencies, the wave packet arrives after 1 meter in

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Figure 5.7. Simulation of non-dispersive S0 mode; (a) space-time map, (b) original signal, and (c) 1 m traveled signal
shorter time than A0 mode (Figure 5.7(c)). Since there is almost no dispersion, the original signal remains the same and amplitudes are not decreased. This may be the reason why the S0 mode has been a more favorable choice for SHM applications in the perspective of dispersion and amplitudes. The choice of wave mode, however, is more complicated. For instance, due to the particle movement in each mode, the S0 mode may be better for finding crack types of damage while the A0 mode can be better for delaminations types of damage. Another consideration for choosing the desired mode for damage detection is the wavelength of the mode. The A0 mode has a smaller wavelength than the S0 mode since the velocity of the S0 mode is greater than the A0 mode at the same frequency. This suggests that the A0 mode can be more sensitive to smaller size damage compared to the S0 mode wave [185].

As previously mentioned, the signal spreading due to the dispersive nature of the guided Lamb wave in the plate-like structures may sometimes be undesirable because this phenomenon reduces signal resolutions and hinders the signal interpretation due to the inclined signal overlapping. Different approaches to reduce or remove the dispersion effects have been attempted [293-297]. Among these approaches, the dispersion compensation method using a signal processing technique [296] and the dispersion removal method using a linear mapping technique [297] are the most intelligent and concise methods that can be applicable in current SHM systems. The former method uses a back-propagation concept that reverses the dispersion process by compressing wave packet to the zero time. The latter approach recovers the original shape of a wave packet by using the linear relation of wavenumbers. Xu et al. [298] compared these two methods and concluded that the dispersion removal method outperforms the dispersion compensation method and it also takes less computational time. The drawback of the dispersion removal method, though, is that a specific bandwidth range has to be chosen to get the valid results. For instance, one-third of the center frequency on each side of input signal has to be selected to remove dispersion effects. Figure 5.8 shows the results of the dispersion removal method that is applied to the A0 mode dispersion shown in Figure 5.6. The propagated distance of the signal is 0.8 meters under the same conditions. Figure 5.8(a) is the original signal, and (b) is the dispersed signal after 0.8 meters, and (c) shows the signal after applying the dispersion removal method. By using this method, the dispersion effects are almost fully removed from the signal that has traveled 0.8 m.
5.2.3 Additional Signal Analysis

After the dispersion is removed from the signal, one may apply additional signal processing methods to obtain more damage feature information. Among these methods, two primary methods that can be coupled well are the cross-correlation and the Hilbert transform. These algorithms can be utilized to the sensed signals in order to obtain the damage locations and extract damage characteristics. Cross-correlation is a signal processing technique that measures the similarity of two signals. One of the signals slides along the axis in time where two signals are located on and gives the integral of their product. The result reaches its maximum when the two signals match best. It also provides the signal delay of a certain system and enables reduction of unwanted signal noise. The obtained signal is then processed to the Hilbert transform. An envelope which is the outline of signal variation often contains important signal information. This envelope is constructed from a complex analytic signal where the real part is the original signal and the imaginary part is the Hilbert transform of the original signal. Accurate propagation distances or times can be obtained from the envelopes of the cross-correlation functions. An illustrative example is shown in Figure 5.9 where the original 4-cycles Hanning windowed signal is shown in (a) and the traveled signal with some unwanted noise is
shown in (b), and signal after cross-correlation and an envelope with Hilbert transform is shown in (c). The cross-correlated signal has its maximum at the value 500 since the original signal matches with the traveled signal after it moves to this value. An envelope in red dots is drawn after the signal is processed through Hilbert transform. This red line obviously provides time or distance information of the signal and also provides the overall shape of the traveled signal.

![Cross-correlation and Hilbert transform](image)

Figure 5.9. Cross-correlation and Hilbert transform; (a) original signal, (b) traveled signal with noise, and (c) processed signal

### 5.2.4 2-Dimensional Scanning Study

A beamforming and beamsensing algorithm using linear phased arrays discussed in the previous chapters is studied in this section, applying the signal processing techniques mentioned in the previous sections. A 3.18 mm aluminum plate, with a crack located at 60° and 20 cm away from the origin in the center of a phased array, is considered for illustrating the methodology of obtaining the 2-D image with the beamforming and beamsensing algorithms.

The crack is about 4 cm and oriented perpendicular to the direction of the wave. The linear PZT phased array with 6 actuators that is used in the previous investigation is implemented again, considering the dispersion effect in the aluminum plate. A 4-cycle Hanning windowed sinusoidal toneburst are sequentially excited to aim the mainlobe angle to go to 60° angle.
Beamsensing, using 6 PZT sensors that are collocated with the actuators, is also applied to measure the reflected signals from the crack. The signal processing techniques such as cross-correlation and Hilbert transform are applied to get the signal features out from the simulated results. The mode conversion due to the crack is not considered in this analysis. It is also assumed that the signal just before the crack and right after the reflection conversely change its phase with 60% reduced signal amplitudes. Figure 5.10(a) shows the simulation result of the crack response after applying the beamforming and beamsensing algorithm. Note that since this is a linear phased array, the same result can be obtained if the crack is located at 300° instead of 60°. The obtained response from the simulation appears larger than the size of the crack. This is due to the large wavelength of the signal that propagated with the relatively low 20 kHz frequency and also by the dispersion effects of the actuated wave packet. The thick blur lines shown at other angles are phantom images from the signal processing utilized for this analysis. A threshold of 80% of the maximum amplitude is applied in Figure 5.10(b) to extract the damage feature out. With this threshold level, the location and shape of the damage are clearly shown.

![Figure 5.10](image)

Figure 5.10. Normalized simulation results of crack response at 60° (a) w/o threshold (b) w/ threshold
5.3 Conclusion

In order to efficiently and correctly detect damages, this chapter discusses some essential elements for a robust damage detection system. A fundamental study of the excitation signal is firstly presented determining a preferred choice of the input signal. Dispersion effect, dispersion removal, and the cross-correlation combined with the Hilbert transform are applied to the signal to obtain damage features. The beamforming and beamsensing algorithm accompanied with these signal processing methods are applied to the signals obtained from a plate with crack damage in order to see the 2-dimensional scanning image of the damaged plate.

The damage detection and localization done by applying these processing tools is the first step in guided wave SHM that builds a foundation for the next step of damage diagnosis. More advanced signal processing algorithms are presented in next chapter for diagnosis/classification since this can be generally achieved utilizing more complicated algorithms.
Chapter 6

6. Time-Frequency Representations

For damage diagnosis, advanced signal processing algorithms are necessary to correctly perform the analysis. There are various time-frequency signal analysis methods that can be used for damage diagnosis in guided wave SHM. Among those, four well known signal processing algorithms, such as STFT, WVD, WT, and MP, are discussed in the chapter. Each method is briefly presented and its time-frequency representation for a signal with damage is investigated. In order to select an appropriate signal processing algorithm for damage classification, each one is also examined in detail at the end of the chapter. Finally, the spectrogram based on STFT is chosen due to its relatively clear feature boundaries.

6.1 Damage Diagnosis

The phased array beamsteering based on the beamforming/beamsensing algorithm can provide some of damage information, such as the location, the approximate size of the damage, and possibly the severity of the damage as discussed in Chapter 5. The damage location can be deduced from the time of flight information. The size of damage can be estimated by applying a pertinent threshold but the damage result is dependent upon the threshold level. The severity of damage can only be evaluated if multiple damages are detected and their signal amplitudes are compared from the normalized damage signals. Although this seems to provide sufficient damage information, one critical factor is in fact neglected, i.e. damage classification.

Among the consecutive damage evaluation steps in SHM systems, a damage diagnosis and classification step can be very important since it is the direct factor that influences the estimation of the remaining useful life of structures. The prognosis results can be significantly different because damage development phases for different damages, such as crack and corrosion, have dissimilar patterns. In order to correctly diagnose and classify the damage types, all sensed
signal information, such as time, amplitude, and frequency, should be processed through a robust 
signal processing algorithm. One such an algorithm that can process and provide the above 
signal information is a time-frequency analysis that is discussed in the next section.

6.2 Time-Frequency Representations

A conventional way of analyzing a sensed signal is to represent it in the time-domain or in 
the frequency-domain to gather useful information. The time-domain analysis is a simple 
process that allows detecting and possibly diagnosing anomaly signals of interest. Time-
synchronous averaging and autocorrelation are some of the techniques that can improve signal 
processing capabilities. The major drawback of time-domain analysis is its inapplicability to 
diagnose complex signals with multiple components [259]. The frequency-domain analysis 
based on Fourier transform has well been accepted as a main signal processing technique. This 
analysis enables to identify the hidden signal components that are unable to isolate from complex 
signals. Although widely accepted, one crucial shortcoming is that this frequency-domain 
analysis does not represent time-varying phenomenon but only provides global signal 
information due to its periodic nature. The combined time-frequency representation was 
therefore introduced to overcome these drawbacks. The general descriptions and reviews about 
the time-frequency analysis can be found in several references [224, 299-303]. In this analysis, a 
sensed signal from a damage is processed through the known time-frequency analysis methods, 
i.e. short time Fourier transform, Wigner-Ville distribution, wavelet transform, and matching 
pursuit.

6.2.1 Short Time Fourier Transform

Time-varying signals can be adequately characterized by jointly using time- and frequency-
domains such that more information can be captured from the localized time and frequency 
components. The most basic method among the time-frequency representations (TFR) is the 
short time Fourier transform (STFT). The STFT is done by dividing the time-domain signal into 
a number of short segments, multiplying with a window function \( g(t) \), and performing Fourier 
transform analysis on each of these segments.

The modified signal before applying Fourier transform can be described as,
\[ f_i(\tau) = f(\tau) g(\tau - t), \]

where \( t \) is the time point of interest and \( \tau \) is the running time. Then, the short time Fourier transform around the specific time point, \( S_f \), and the energy density spectrum, \( P_{sp} \), can be obtained as shown below [299, 303],

\[
S_f(t, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\tau) g(\tau - t) e^{-j\omega \tau} d\tau,
\]

\[
P_{sp}(t, \omega) = |S_f(t, \omega)|^2,
\]

where \( f(\tau) \) and \( g(\tau-t) \) are the signal and the window function, respectively.

The energy density spectrum as drawn in a time-frequency map is called spectrogram. Although STFT works quite well for most of signals, one drawback can be the time frequency tradeoff that shortening signal segment to improve time resolution can reduce frequency resolution and vice versa. In addition, an optimal length of the segment window may not be chosen for the signals that contain several different features.

A spectrogram is obtained for the signal obtained from a 2-D Abaqus® beam model, as shown in Figure 6.1(a). The length and thickness of the aluminum beam are 2 m and 3.175 mm and a pair of 1 cm width PZT actuators is attached in the middle of the beam, each one facing the opposite side of the beam. An artificial corrosion damage of 3 cm width with 45° ends and a quarter beam thickness is cut from the beam at 20 cm from the PZTs. A pure A0, 2.5 cycles Hanning windowed sine signal with 50 kHz center frequency is generated from the pair of PZT actuators such that the guided Lamb waves are propagated through the beam. The in-plane displacement is measured at the intersection point between the PZT on top and the upper surface of the beam and its results are shown in Figure 6.1(b) and (c). Considering that there must be only two wave modes, i.e. A0 and S0, reflected from the damage due to the excitation frequency and also taking the pattern of the damage signal into account, it can be determined that there are multiple A0 and S0 mode signals overlapped in this time period of interest.

Figure 6.1(d) shows the spectrogram for this sensed signal from the corrosion damage. A Gaussian window is used to get the spectrogram but other window functions, such as Hanning,
Hamming, and Blackman, are also produced similar result plots. It can be noticed from the spectrogram that most of signal energy is placed around the 50 kHz excitation frequency.

![Schematic diagram of test setup](image)

Figure 6.1. Spectrogram of sensed signal (a) test setup, (b) sensed signal, (c) sensed signal from damage, and (d) spectrogram of damage signal

### 6.2.2 Wigner-Ville Distribution

Other approaches have been developed to overcome some of the shortcomings of STFT. One of the most studied approaches is the Wigner-Ville distribution (WVD) [304, 305]. Although Wigner proposed and Ville further developed the distribution with two different...
approaches in the middle of 20th century, WVD was actually applied for signal analysis in 1980’s [300]. The WVD is achieved by taking Fourier transform of the instantaneous autocorrelation function, which compares the signal with itself rather than multiplying with a window function like STFT [306]. This method basically measures overlapping of the past time signal with future time signal. This also holds for the frequency domain since the forms are basically identical.

The WVD in terms of the signal, \( f(t) \), and its spectrum, \( F(\omega) \), are written as,

\[
WV(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f\left(t + \frac{\tau}{2}\right) f^\star\left(t - \frac{\tau}{2}\right) e^{-j\omega\tau} d\tau
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} F\left(\omega + \frac{\gamma}{2}\right) F^\star\left(\omega - \frac{\gamma}{2}\right) e^{j\gamma} d\gamma
\]

(6.4)

Since it always multiplies two portions of signals and there is nothing in time before and after a finite length of signal, WVD is zero at the starting and ending time of signal. There also can be interference or cross terms in which the signal is zero. If non-zero signals are located same time away towards positive and negative directions, their inner product is not zero adding a cross term in-between. Figure 6.2(a) is the TFR based on WVD and it shows an apparent cross term inside of the left elliptical outline. This ellipse is almost symmetric in both time and frequency domains such that the cross term may be more readily visible.

Although the WVD has a great advantage over the STFT producing much better time-frequency resolution map, it suffers from the presence of cross terms and poor noise properties [299]. One method to alleviate noise and suppress the cross terms, called pseudo-WVD, is done by multiplying the product with a window function. This helps reducing the size of the cross terms but also reduces the frequency or time resolutions on the map. Other types of distributions, such as smoothed WVD, Born-Jordan, and Choi-Williams, have been developed, but there are always compromises between the time-frequency resolution and interference terms [111]. Figure 6.2(b) shows the TFR from Choi-Wiliams. It produces a bit smoother plot for this signal than WVD meaning that noise level is reduced, but the cross term still exists inside of the left circle in the time-frequency plot.
Figure 6.2. Time-frequency representation of damage signal (a) WVD and (b) Choi-Williams

6.2.3 Wavelet Transform

Another approach is wavelet transform (WT) and the concept is well introduced and reviewed by many scholars [235, 307-310], specifically with the SHM topic [311]. While sinusoidal waves are used as the basis functions to decompose the received signals in Fourier transform based analysis such as STFT, the signal transform based on wavelets, WT, can be better employed for more complicated signals. Wavelet transforms are divided into continuous and discrete WTs. The continuous WT is in general good for time-frequency analysis and the discrete WT is better for signal decomposition and feature selection [224]. Wavelets are localized wavelike functions that can be dilated and translated during the analysis. The choice of best wavelet for a particular application depends on the nature of the signal or expected outcomes. Certain mathematical criteria must be satisfied for the wavelet and they are shortly discussed in a handbook [309]. The first requirement about the wavelet is that it must have finite energy.

\[
E = \int_{-\infty}^{+\infty} |\psi(t)|^2 \, dt < \infty \tag{6.5}
\]

The squared sum of wavelet magnitude must be finite. Secondly, it must have a zero average or mean value, as shown in Eq. (6.6). The \( C_g \) in Eq. (6.7) called admissibility constant ensures that the wavelet has no zero frequency components.

\[
\int_{-\infty}^{+\infty} \psi(t) \, dt = 0 \tag{6.6}
\]
\[ C_g = \int_0^\infty \frac{\left| \hat{\psi}(f) \right|^2}{f} df < \infty \quad \text{where} \quad \hat{\psi}(f) = \int_{-\infty}^{\infty} \psi(t) e^{-j\omega t} dt \quad (6.7) \]

A large number of wavelets are available, such as Haar, Morlet, Mexican hat, Daubechies, and Gaussian, and each one has its own wavelet properties that each WT in accordance with a specific wavelet may produce different result.

Since the continuous WT is good for time-frequency analysis, it is applied and discussed further. A continuous WT is described as a convolution of wavelet function to the signal analyzed. The WT and its time-frequency energy density, \( P_{\nu f} \), called scalogram are written as,

\[ WT(a,b) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{b}} \psi^* \left( \frac{t-a}{b} \right) dt \quad (6.8) \]

\[ P_{\nu f}(a,\omega) = \left| WT(a,b) \right|^2. \quad (6.9) \]

where \( \psi(t) \), \( a \), and \( b \) are the mother wavelet, the translation parameter, and the dilation or scale parameter, respectively. The translation parameter is related to time and the scale is analogous to frequency [311]. Figure 6.3 shows the scalogram of the sensed signal from the corrosion damage. A Gabor wavelet \( \psi(t) \) obtained from a Gaussian window is used as the mother wavelet.

The time-frequency plane in WT basically differs from the one in STFT. While the size of the division boxes in the time-frequency plane, called Heisenberg boxes, in STFT is uniform, the time and frequency widths of the Heisenberg boxes in WT differs due to the stretchable or dilatable wavelets. At low frequencies, the frequency resolution of the scalogram is finer than the spectrogram and vice versa for the time resolution. At high frequencies, the previous sentence becomes exactly the opposite. The scalogram in Figure 6.3 shows this well compared to the spectrogram of Figure 6.1(d), providing blurry images at high frequency range and, although hard to observe, slightly finer images at lower frequency range.
6.2.4 Matching Pursuit

Some of the limitations mentioned in previous signal processing techniques may be overcome by using the matching pursuit (MP) decomposition. The MP enables not only to remove the cross terms shown in WVD and yield high resolution TFR, but it also reduces poor noise properties due to its iterative nature. The MP decomposition algorithm was first introduced by Mallat [312] and also independently by Qian [313]. The MP decomposes the signal into piece by piece from a large dictionary of analyzing functions. It is an acquisitive algorithm that an analyzing function, which best represents part or all of the remaining signals, is selected at each decomposition stage [309, 314]. The iterative process allows the original signal to be represented as a weighted linear combination of analyzing functions or ‘atoms’. Some of additional studies related with wave-based signal processing and matching pursuit can be found in several papers [111, 252, 255, 315-318].

The first iteration of MP decomposition is done by finding an analyzing function from a preselected dictionary, $D$, of functions. An analyzing vector, $\varphi_{p_0}$, is chosen such that it gives the largest inner product with the signal $f$,

$$f = \langle f, \varphi_{p_0} \rangle \varphi_{p_0} + Rf,$$  \hspace{1cm} (6.10)$$

where $Rf$ is the residual term after approximating the original signal and is orthogonal to $\varphi_{p_0}$. The iteration is repeated on the residues obtained previously and the $n$th order residue $R^n f$ is computed as,

$$R^n f = \langle R^n f, \varphi_{p_n} \rangle \varphi_{p_n} + R^{n+1} f.$$  \hspace{1cm} (6.11)$$
At any iteration stage, the original signal is divided into two components, i.e. the first reconstructed part with an inner product and the residual component $R^{n+1}f$. Then, the original signal is represented with the sum of all decomposed atoms and residues,

$$f = \sum_{n=0}^{N-1} \langle R^n f, \varphi_{p_n} \rangle \varphi_{p_n} + R^N f.$$  \hspace{1cm} (6.12)

The maximum iteration $N$ can be determined for each signal based on the mathematical stopping criteria or from the set of the minimum residual energy.

The Gabor and chirplet atoms are used to decompose the sensed signal and their MP results using Lastwave [319] are shown in Figure 6.4. Gabor atoms based on Gaussian envelopes are the most common analyzing functions in MP because of their simpler shape variation through translating, scaling, and modulating the atoms [320]. Chirplet atoms may be better if the analyzed signal is dispersed or antisymmetric that is frequently shown in guided wave SHM. The difference of these two MP is shown using 5 atoms that contain most of high energies compare to the rest of the remaining atoms in the figure. Clear differences are observed between the time-frequency plots by using these two dictionaries, as well as from other time-frequency analysis results. From MP decomposition, it may be possible to separate out the wave modes unlike the previously discussed TFRs. The decomposition using the chirplet dictionary can show the frequency variation inside of an atom compare to the Gabor decomposition. In less than 1 MHz-mm range of the group velocity dispersion curve for an aluminum plate, S0 and A0 wave modes can have positive and negative chirplets due to their negative and positive dispersion slopes, respectively. Considering this, one may have an insight of how the signal is reflected from the damage and measured at the sensor in a certain order.

Figure 6.4. Time-frequency representation of damage signal (a) Gabor MP and (b) chirplet MP
6.3 Time-Frequency Selection for Damage Classification

A time-frequency analysis of the analyzed signal can be a useful method for damage diagnosis and classification. Four signal processing algorithms have been discussed in the previous section. In this section, the algorithms are examined in great detail for selection of the appropriate method for damage diagnosis.

As briefly discussed in section 6.2.2, signal processing algorithms based on WVD may develop interference on the regions where there are zero signals. To examine the WVD in more detail and understand the limitation, the WVD is used to analyze a signal consisting of two sine components with frequencies of 6 Hz low frequency ($f_1$) and 60 Hz high frequency ($f_2$). The $N$ points signal is multiplied with a symmetric Hanning window as shown in Eq. (6.13).

$$S_w = \left[ \sin\left(2\pi f_1 t\right) + \sin\left(2\pi f_2 t\right) \right] \times \text{Hann}$$

$$= \left[ \sin\left(2\pi f_1 t\right) + \sin\left(2\pi f_2 t\right) \right] \times \frac{1}{2} \left( 1 - \cos\left(2\pi \frac{n}{N}\right) \right), \quad 0 \leq n \leq N$$

The signal is recorded for one second as shown in Figure 6.5(a). The TFRs based on WVD and Choi-Williams are shown in Figure 6.5(b) and (c), respectively. In these time-frequency plots, the dominant frequency components, 6 Hz and 60 Hz, are clearly visible and the cross terms are also formed between these two frequency lines. The frequency signals get stronger in Choi-Williams but the cross terms still exist in large and low resolution vertical lines. Therefore, correct damage information may not be obtained if multiple wave mode signals, such as A0 and S0 modes, are overlapped by using the algorithms based on WVD.
Figure 6.5. WVD analysis (a) combined signal, (b) WVD TFR, and (c) Choi-Williams TFR

The MP is an appealing signal processing algorithm for guided wave SHM as it effectively demonstrates the wave mode decomposition in section 6.2.4. From Figure 6.4(b), it can be noticed that most of atoms except the smallest atom on the TFR are more separable by using the chirplet based MP decomposition than the Gabor based. Each of these atoms is more examined in Figure 6.6. The four signal characteristics of each atom are tabulated within a colored time-frequency map in the figure. In the table, the time and frequency values of each atom are obtained from the highest amplitude of each elliptical atom. The chirp rate, \( c \), is the rate of change of the instantaneous frequency [321] that can be calculated from Eq. (6.14). The signal energy, or atomic energy in the table, is the area under the squared atomic signal.

\[
c = \frac{d}{dt} \left( \frac{1}{2\pi} \frac{d\phi(t)}{dt} \right) = \frac{1}{2\pi} \frac{d^2\phi(t)}{dt^2}, \quad \phi(t) = \text{phase} \tag{6.14}
\]

From the colors of elliptical atoms, it is obvious that the second atom from the left has the highest signal energy and the fourth atom has the lowest signal energy, as also indicated in the table of the figure. The center frequency values of each atom are not consistent considering that the excitation signal has 50 kHz center frequency, and the chirp rates for the atom signals also differ in numbers as well as in their signs. One possible explanation is previously mentioned
regarding the slope of dispersion curves. From the group dispersion curve shown in Figure 1.2, the slopes of S0 and A0 wave modes in less than 1 MHz-mm range have negative and positive values. The signs of these slopes are correlated with the signs of the chirp rates. For instance, since all toneburst excitation signals have certain ranges of input frequencies that are similar to Figure 5.3, the signals with higher frequencies propagates faster than the ones with lower frequencies for the A0 mode. If a sensor were located in a certain distance from an actuator, the sensor would record the high frequency signals first and then the lower frequency signals would follow. This sensed signal would form an ellipse in a time-frequency map that the sign of the major axis of the ellipse is negative. This is a chirplet atom with a negative sign as shown in the third and the last atoms in Figure 6.6. Similarly, the atoms with positive chirp rates would be related with the S0 wave mode signals. This can give the modal information about each atom.

As discussed so far, the chirplet based MP decomposition can provide useful information regarding the damage signal. Since time information is given, one can estimate the location of damage converting time to distance with a known wave speed. The chirp rate of each atom can provide wave mode information that mode reflection from damage can be evaluated. The signal energy can give information about where most energy is placed along the signal and which wave mode is more reflected from damage.

The time-frequency analysis based on the chirplet MP seems to be an attractive method for damage classification considering the abundant signal information obtainable from the time-frequency map. However, there are a couple of important drawbacks of this method for damage
classification. The intelligent algorithm of chirplet MP sometimes has a weakness due to its calculation process when applied to the guided wave SHM. The MP is a greedy algorithm that seeks an analyzing function, which best represent the remaining signals. In some cases, if multiple mode signals are connected continuously, the best represented function in the algorithm may be calculated for the whole signal period instead of separating functions for each mode.

An example is presented below for crack damage in an aluminum beam. A test setup is quite similar to Figure 6.1(a). Instead of having a corrosion-like damage, a crack-like narrow notch with 0.5 mm width and 0.8 mm depth is located 20 cm from the PZT actuators. Other test conditions are the same as the corrosion test and the in-plane displacement result is shown in Figure 6.7(a). Since a 50 kHz pure A0 mode is excited, it is assumed that only two wave modes, i.e. S0 and A0, are reflected from the crack damage and are also pointed out on the signal based on their wave speeds.

The chirplet based MP decomposition is performed for the signal and the results of largest 10 atoms (signal energy over 0.01) are shown in Figure 6.7(b). The obtained TFR image for the crack damage is unexpected. For the signal time period for the crack, three long and horizontal atoms are shown in lieu of two atoms per each mode. The same crack signal shown in Figure 6.7(c) is also processed through STFT and its spectrogram is shown in Figure 6.7(d). The TFR results from chirplet MP and STFT seem to be different where the spectrogram seems to be correlated with the crack signal while the TFR based on chirplet MP displays an odd image pattern.
Figure 6.7. Damage signal and time-frequency plots (a) whole signal, (b) TFR of chirplet MP decomposition, (c) signal from crack damage, and (d) spectrogram of crack signal.

In order to see the iteration process for decomposition of Figure 6.7(b), the atom signals per iteration are separated from the remaining signal as shown in Figure 6.8. In the figure, the atom numbers are in order of signal energy, i.e. the lower the atom number, the higher its signal energy.
As expected, the largest signal energy is due to the excitation signal and the second and third largest signals are also come from the excitation as shown in the figure. For the signal from the crack damage, three long analyzing functions, i.e. atom 4, atom 6 and atom 9, are selected to represent this portion instead of two separate atoms for each wave mode. This does not happen for all continuous signals. In many cases, the MP algorithm can successfully separate different
wave modes from the overlapped or continuous signals in guided wave SHM, as shown in the corrosion example. Similar signal information in Figure 6.6 for each atom is shown in Table 6.1. Unlike the table shown in Figure 6.6, not much useful crack information can be extracted from this table since there are no clear distinctions between the atoms except the atoms for the crack damage have small positive chirp rates. These rates may not provide helpful information since the atoms do not present adequate wave modes.

<table>
<thead>
<tr>
<th>Atom number</th>
<th>Time (s)</th>
<th>Frequency (kHz)</th>
<th>Chirp rate (kHz/ms)</th>
<th>Signal energy (coeff$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.2e-5</td>
<td>47.37</td>
<td>-92.75</td>
<td>8.3241</td>
</tr>
<tr>
<td>2</td>
<td>1.6e-5</td>
<td>31.25</td>
<td>0</td>
<td>0.8307</td>
</tr>
<tr>
<td>3</td>
<td>4.0e-5</td>
<td>31.25</td>
<td>0</td>
<td>0.2476</td>
</tr>
<tr>
<td>4</td>
<td>19.2e-5</td>
<td>54.73</td>
<td>8.66</td>
<td>0.1475</td>
</tr>
<tr>
<td>5</td>
<td>2.4e-5</td>
<td>44.10</td>
<td>-1520.96</td>
<td>0.1226</td>
</tr>
<tr>
<td>6</td>
<td>19.2e-5</td>
<td>38.78</td>
<td>5.18</td>
<td>0.0707</td>
</tr>
<tr>
<td>7</td>
<td>4.8e-5</td>
<td>31.25</td>
<td>0</td>
<td>0.0446</td>
</tr>
<tr>
<td>8</td>
<td>3.2e-5</td>
<td>62.50</td>
<td>0</td>
<td>0.0235</td>
</tr>
<tr>
<td>9</td>
<td>19.2e-5</td>
<td>72.05</td>
<td>6.30</td>
<td>0.0231</td>
</tr>
<tr>
<td>10</td>
<td>1.6e-5</td>
<td>31.25</td>
<td>0</td>
<td>0.0176</td>
</tr>
</tbody>
</table>

Figure 6.9 shows the original signal and the reconstructed signal using above 10 atoms in solid and dotted lines, respectively. The reconstructed signal is built by simply adding the atoms and it matches well to the original signal for the most of the part. The signal for the crack damage part is magnified in the figure to closely look at the differences between the two. The sum of three horizontal atoms for the crack damage is not identical to the original signal but it is closely comparable with small differences only at the end of the signal. It can be expected that the next atom would compensate for this difference.
Although the chirplet based MP decomposition can be a very effective algorithm to be used for guided wave SHM, it sometimes provides faulty time-frequency information. Like the WVD, correct damage information may not be obtained using this method. As discussed in section 6.2.3, the TFR image of a signal using WT can be blurry at high frequency range and this image may be unfavorable for damage diagnosis since it does not provide clear feature boundaries. Due to these limitations, the STFT is selected for damage diagnosis, based upon the TFR results for crack and corrosion damage types.

6.4 Conclusion

This chapter discusses various time-frequency analysis methods using the sensed signals from crack and corrosion damages. Four different signal processing algorithms, such as STFT, WVD, WT, and MP, are discussed and their TFR examples are shown. The purpose of this chapter is to select the appropriate algorithm to be used for damage diagnosis and possibly classification in guided wave SHM. After examining these algorithms in more detail, the spectrogram based on STFT is chosen as a suitable signal processing method for damage diagnosis and classification.
Chapter 7

7. Damage Classification

Damage classification is performed using a machine learning algorithm, Adaboost, for crack and corrosion damages in aluminum structures. Artificial crack and corrosion damages are created in FEM for obtaining damage signals and the signals are processed through a signal processing algorithm. Once the spectrograms for both damage signals are obtained, they are used as the training samples for the Adaboost algorithm. Beam experiments in laboratory and additional simulations are also performed to get the testing samples. The results show that it is feasible to classify different damage types and obtain the confidence levels of each sample utilizing Adaboost with time-frequency representations.

7.1 Machine Learning

The damage information in SHM can be obtained by analyzing the signals acquired from various detection methods. These methods can be considered as the tools that are aimed to assist accurate damage diagnosis and proper prognosis. Some tools may be faster in obtaining the outcomes or may provide more accurate results, but the nature of the sensed damage signals does not change much although the methodology used for the detection varies. Therefore, advanced signal processing techniques are typically employed to extract more damage features out of the sensed signals. This can add additional damage information onto a previously obtained damage database and can eventually result in abundant data sets for the different damage signals. If a large database of information is available, then damage diagnosis and further prognosis may yield more accurate results. With this abundant database though, there are more chances that the information boundaries for the different damages would overlap each other and even the same ones can sometimes be unmatched. Therefore, to get more confidence in the diagnosis results, the accuracy for the damage diagnosis should be based on the probabilistic decision analysis.
Damage characteristics underlying the probabilistic analysis can be estimated and may be assessed by using some machine learning methods.

Through the previous two chapters, two most frequency damage types, cracks and corrosions, incurred in metallic structures have been studied. The probabilities of these two types of damage can be assessed using a machine learning method. Although some feature extraction methods for damage diagnosis, such as dispersion removal or compensation [298], principal component analysis [250], and time-frequency analysis in the previous chapter, have been utilized for SHM applications, a damage classification method combined with some of these feature extraction methods would significantly enhance damage diagnosis and further prognosis decisions.

One of the well regarded classification methods in SHM applications at the present time is support vector machines (SVM). SVM is a machine learning technique that maps the extracted feature vectors of input space into a high dimensional space by using the kernel functions. An optimal separating hyper-plane is then constructed maximizing the margin between the class supporting vectors by solving a quadratic optimization problem. However, the classification method being investigated in this research is an adaptive boosting algorithm called Adaboost. Adaboost, one of the top 10 data mining algorithms [322], is also a machine learning algorithm that produces highly accurate classification results. An accurate classification is achieved by combining a set of weak hypotheses that each emphasizes more on misclassified feature vectors [323]. While Adaboost refers to a linear programming, SVM corresponds to a quadratic programming that can be more computationally expensive than the linear one [323-325]. In addition, the kernel matrix of SVM quadratically grows that training of the algorithm generally requires large sets of data and high algorithmic complexity. SVMs also depend upon the choice of a kernel function and its parameters. In contrast, small data sets can be used to train Adaboost since data can be re-sampled during iteration process [326].

SVM has been utilized for damage classification by a few researchers. One-class SVM is used to classify damage in bolted joints with fatigue cracks and composite laminates with different damage types [262]. SVM is also used for railroad track damage identification [263]. Adaboost has not been fully adopted as damage classification method in SHM applications yet. In areas other than SHM, Adaboost and SVM have been compared by their accuracies or performances, and Adaboost has been shown to outperform SVM for certain types of problems.
For this reason, Adaboost is chosen as the classifier for crack and corrosion damages in this research. More information about Adaboost is presented and discussed in the next section.

### 7.2 Adaboost Algorithm

Adaboost is one of the most powerful machine learning methods introduced in 1990s by Freund and Schapire [323, 329]. The key idea of the algorithm is that it adds more weights on the misclassified feature vectors but reduces weights for the vectors that are correctly classified. The pseudo code for Adaboost algorithm is given below.

<table>
<thead>
<tr>
<th>Adaboost algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given:</strong> $(x_1, y_1), ..., (x_m, y_m)$ where $x_i \in X, y_i \in Y = {-1, +1}$</td>
</tr>
<tr>
<td>Initialize $D_1(i) = 1/m$</td>
</tr>
<tr>
<td>For $t = 1, ..., T$:</td>
</tr>
<tr>
<td>- Train weak learner using distribution $D_t$.</td>
</tr>
<tr>
<td>- Get weak hypothesis $h_t: X \rightarrow {-1, +1}$ with error.</td>
</tr>
<tr>
<td>$\epsilon_t = \Pr_{i \sim D_t}[h_t(x_i) \neq y_i]$</td>
</tr>
<tr>
<td>- Choose $\alpha_t = \frac{1}{2} \ln \left( \frac{1-\epsilon_t}{\epsilon_t} \right)$.</td>
</tr>
<tr>
<td>- Update: $D_{t+1}(i) = \frac{D_t(i)}{Z_t} \times \begin{cases} e^{-\alpha_t} &amp; \text{if } h_t(x_i) = y_i \ e^{\alpha_t} &amp; \text{if } h_t(x_i) \neq y_i \end{cases}$</td>
</tr>
<tr>
<td>where $Z_t$ is a normalization factor (chosen so that $D_{t+1}$ will be a distribution).</td>
</tr>
<tr>
<td>Output the final hypothesis: $H(x) = sign \left( \sum_{t=1}^{T} \alpha_t h_t(x) \right)$</td>
</tr>
</tbody>
</table>

The algorithm takes the input training set $(x_1, y_1), ..., (x_m, y_m)$ where each label $x_i$ is in some domain $X$ and $y_i$ is in some label set $Y$, assuming here with $Y = \{-1, +1\}$. To better explain the algorithm, an example is shown for the binary classification of 20 samples in which one class (+1) is represented with 10 red crosses and the other class (-1) with 10 blue circles, as shown in Figure 7.1. The variable $m$ is 20 for the 20 elements of the set $X$. 
The one main idea of the algorithm is an iterative weight distribution. Initial weight distribution on the training sample $i$ is equally set as $D_1(i)=1/m$. At the first iteration, since all weights are evenly distributed and $m$ is 20, the weight distribution of each sample becomes 0.05.

In each iteration step, the weights are increased for the incorrectly classified samples and the error is measured by summing these weights.

$$
\varepsilon_t = \Pr_{i \sim D_t}[h_t(x_i) \neq y_i] = \sum_{i: h_t(x_i) \neq y_i} D_t(i) \quad (7.1)
$$

Once the weak hypothesis $h_t$ has been determined with the least error, a parameter $\alpha_t$ is calculated according to the equation shown in the algorithm. This parameter has an important relation with the error, $\varepsilon_t$, such that it gets larger as the error gets smaller and vice versa.

Figure 7.1 shows the early iteration steps for 20 training samples. A threshold line, weak learner $h_1$, is drawn in which there is a minimum error in the first iteration. At the first step, there are 5 samples that are incorrectly classified as shown in the dotted boxes of Figure 7.1(b). Since $D_1(i)=0.05$, the probability of error samples, $\varepsilon_1$, becomes 0.25 thus yielding the parameter $\alpha_1$ as 0.55.

Using the calculated error and the parameter, the distribution, $D_t$, can be updated in the next iteration step using the below relation.
• Update:
\[
D_{t+1}(i) = \frac{D_t(i)}{Z_t} \times \begin{cases} 
 e^{-\alpha_t} & \text{if } h_t(x_i) = y_i \\
 e^{\alpha_t} & \text{if } h_t(x_i) \neq y_i 
\end{cases}
\]
\[
= \frac{D_t(i) \exp(-\alpha_t h_t(x_i))}{Z_t}
\]
(7.2)

where \(Z_t\) is a normalization factor (chosen so that \(D_{t+1}\) will be a distribution).

The distribution rule defines that the weights of correctly classified examples decrease and the ones with incorrectly classified gain more weights in the next step. Figure 7.1(c) shows that the incorrectly classified circles in the dotted boxes get larger, \(D_2(i)\)s as 0.087, while the rest of correctly classified examples get smaller, \(D_2(i)\)s as 0.029, compare to the equally distributed weights in the first iteration.

![Figure 7.2. Adaboost iteration process](image)

The next iterations are similar to the previous one with a little bit more calculations. Once the weak learner with least error, \(h_2\), is selected at the next step, the incorrectly classified
samples are again determined as shown in Figure 7.2(a). The probabilistic error, \( \varepsilon_2 \), for the incorrectly classified samples in the box is obtained from the previous \( D_2(i) \) values as 0.145 and its corresponding parameter \( \alpha_2 \) is computed as 0.89. A new weight distribution set, \( D_3(i) \), is then updated from these values following the algorithm. As noticed in Figure 7.2(b), (d), and (f), the sample weights that correctly classified in all steps get very small.

The final hypothesis \( H \) is a weighted majority vote of the \( T \) weak hypothesis that is obtained from the sign of the sum of \( a_t \) assigned to weak hypothesis \( h_t \) as shown in below equation.

Output the final hypothesis:

\[
H(x) = \text{sign} \left( \sum_{t=1}^{T} a_t h_t(x) \right)
\]  

(7.3)

In Figure 7.3, the parameter \( a_t \) at each step is multiplied with the weak hypothesis \( h_t \), which divides one class (+1) in the gray region to the other class (-1) in the white region. After summing all of these multiplications, the final hypothesis is drawn from the sample signs as shown in the figure. It should be noted that the summation values for the samples located far from the class boundary, the thick line in the middle, are larger than those near the boundary. These samples are the ones that their weights get smaller as iteration increases.

Figure 7.3. Final hypothesis decision

With the above example, it is mentioned that the weak learner is determined to yield the least error with respect to the distribution at each iteration step. Due to its flexibility of the Adaboost algorithm, various methods can be combined with the algorithm to find the weak learners, such
as neural networks [330] and SVMs [331]. Although these methods can produce good performance results with Adaboost, they may accompany with the complex overfitting and the appropriate setting of kernel width problems [331]. The method chosen to find the weak learner in this research is the classification tree. The classification tree is one of the top off-the-shelf classification methods and it performs well across a wide range of situations [332]. The algorithm is readily understandable and can be applied in most of the classification problems.

A brief explanation of the classification tree is described. In general classification problems, any case in a sample belongs to a specific class. Consider $X$ to be the domain that consists of possible variables of $\mathbf{x} = (x_1, \ldots, x_k)$. In a classification tree, the tree-structured classifiers are constructed by continuously splitting the domain $X$ based on the predetermined set of rules, such as a minimal error. Subsets of $X$ from the split are subsequently formed building a hierarchical structure. Figure 7.4 shows an example of a classification tree construction. A recursive partitioning, shown in the left figure, denotes the independent variables by $x_1$ and $x_2$ and the class variables by $Y_1$ and $Y_2$. The recursive partitioning divides the independent variable space $X$ into the non-overlapping rectangles. The first split divides the domain $X$ into $\{x_2 \leq 0.65\}$ and $\{x_2 > 0.65\}$. Additional splits can be astutely chosen to increase the purity of the resulting rectangles, as selected here as $\{x_2 \leq 0.65, x_1 \leq 0.2 | Y_1\}$, $\{x_2 \leq 0.65, x_1 > 0.2 | Y_2\}$, $\{x_2 > 0.65, x_1 \leq 0.5 | Y_1\}$, and $\{x_2 > 0.65, x_1 > 0.5 | Y_2\}$. This recursive partitioning is called a classification tree since a split from one node can produce two successor nodes as shown in the right figure.
In this research for damage classification of crack and corrosion with time-frequency representations, the GML Adaboost Matlab toolbox [333] is used with a certain number of maximum tree splits. A slightly modified Adaboost algorithm called Gentle Adaboost from the original Real Adaboost algorithm is utilized since it provides more robust and stable results [334].

7.3 Adaboost Training and Testing Samples

For most of machine learning algorithms, many empirical training samples are generally required to educate the algorithm and to extract some useful information. Acquiring training or testing samples for damages in guided wave SHM may not be an easy task since a reliable and useful damage database does not exist at the present time. In order to obtain two different sets of damage samples for crack and corrosion, a finite element analysis tool, Abaqus®, is employed instead of performing numerous experiments to get the samples. Although the guided wave mostly propagates through a 3-dimensional structural space, the wave interaction from damage and its sensing of the reflected signals can be regarded as a 2-dimensional interaction following the path of the main wave signal. Therefore in the following study, the coplanar analysis using Abaqus® is carried out to examine the wave interactions for the crack and corrosion damage cases.

7.3.1 Training Samples using Finite Element Analysis

To obtain the training samples in 2-dimensional Abaqus® models, the crack is represented as a thin notch and the corrosion is modeled with a trapezoidal channel shape with 45° cut at both ends as shown in Figure 7.5. The longer direction of a narrow crack is assumed to be oriented perpendicular to the propagated wave direction and it is located at D from a pair of thin 1 cm width PZT actuators as shown in Figure 7.5(a). Considering the wavelength of the excitation frequency, two different distances for D are chosen as 15 cm and 20 cm. The crack has 0.5 mm width, Wk, and its depth, αT, varies from 1/8 to 1/2 of the aluminum beam thickness T (3.175 mm). On the other hand, since corrosion damage is usually wider as shown in Figure 5.1, its cross-section is assumed to be a trapezoidal channel having 3 cm and 5 cm width, Wc, like shown in Figure 7.5(b). The other corrosion parameters for D and αT are the same with crack cases.
As in the previous time-frequency analysis, a pure A0 mode, 2.5 cycles Hanning windowed toneburst signal with the 50 kHz center frequency is generated from the pair of PZT actuators. The wavelength of the signal based on this center frequency is 2.4 cm thus the total length of the excitation signal is approximately 6 cm. Since A0 mode around this frequency is dispersive, the total length of the signal is more spread as the wave propagates further along its path.

Once the signal is excited from the actuators, some portion of the propagating wave signal is reflected back from the damage and the rest of the signal gets through the damage. Although a PZT sensor is not constructed in the Abaqus® model, the in-plane displacements of the reflected signals are measured at the point on the upper surface of the beam and near to the PZT actuators. The in-plane displacements are taken since the wafer type PZTs are only capable of detecting the in-plane movements as previously discussed.

As presented in the previous chapter, a spectrogram based on STFT is obtained for the sensed signals for damage classification. An example is shown for the crack located at 15 cm from the PZT actuators (D=15 cm) with 0.8 mm depth (αT=2T/8) in Figure 7.6. The total time signal and its spectrogram are shown in Figure 7.6(a) and the signal for the crack, inside of the dotted line, and its spectrogram are also shown in Figure 7.6(b). Like Figure 6.1, the excitation signal is first sensed and the reflected signal from the crack is sensed around at 100 μs. The crack signal length is longer than the excitation signal since it contains both A0 and S0 wave modes. A spectrogram shown in the bottom of Figure 7.6(a) indicates that the maximum signal energy is due to the large amplitude excitation signal. Since our interest lies in for the damage signal, only
crack signal is extracted from the whole signal and its spectrogram (50 to 400 μs in the time domain and 0 to 150 kHz in the frequency domain) is shown in Figure 7.6(b).

![Figure 7.6. STFT of crack with D=15cm and αT=2T/8](image)

(a) whole signal (b) damage signal

Training samples for the crack damage are obtained following the above procedure varying the length D and the depth αT. The width of the crack, Wk, is maintained as 0.5 mm. Total of 8 time-frequency plots for the training crack samples are obtained for now using two D lengths (15 cm and 20 cm) and four αT depths (T/8, 2T/8, 3T/8, and 4T/8) as shown in Figure 7.7.

![Figure 7.7. Spectrograms for cracks with (left) 15-cm distance and (right) 20-cm distance](image)
The time-frequency plots in the figure look similar for all 15 cm cracks and all 20 cm cracks in the black and white images. The locations for the highest signal amplitude are in fact changing as the crack depth increases in both plot images as shown in Figure 7.8(a). Since S0 wave should be detected earlier than A0 wave due to their wave speeds, one can see that the amplitude of S0 mode reflection gets larger as the crack depth increases for this 15 cm distance case. In order to clearly observe this phenomenon, some cracks with different crack depths, located at 10 cm from the PZT actuators, are examined and their measured time signals are shown in Figure 7.8(b). It is obvious to see that there are overall larger signal reflections with the increased crack depth, as shown in the box of the figure.

Figure 7.8. Reflection study (a) spectrograms of cracks with D=15cm, (b) whole time signal of cracks with D=10cm, (c) normalized crack signals with D=10cm, and (d) crack reflection coefficients for A0 incident mode with fT=1MHz-mm [90] (reprinted with permission from M. A. Flores-Lopez and R. D. Gregory, Journal of the Acoustical Society of America, vol. 119, pp. 2041-2049, 2006)
Besides the overall increasing signal reflections due to the deeper cracks, one can observe
that the lower reflection of S0 mode for shallower cracks gets larger as the crack depth increases
as shown in Figure 7.8(c). The figure is drawn by normalizing the maximum A0 amplitudes for
all depths of crack signals. At the 4T/8 crack depth, the S0 mode reflection becomes even
slightly larger than the A0 mode reflection. This feature is also shown in Flores-Lopez [90] for
an elastic plate under the 1 MHz-mm frequency-thickness condition and their A0 crack reflection
results based on the projection method is shown in Figure 7.8(d). The figure shows that the
reflection coefficients for these two modes are changing as the crack depth increases.

As discussed, the magnitudes of wave reflections are not proportional to the crack depth for
both A0 and S0 wave modes. This may be explained by the propagated wave patterns of the
excitation signal mode. Figure 7.9 shows four wave propagation shapes for both fundamental
wave modes at 50 kHz and 150 kHz. It is obvious to see that the wavelength at 50 kHz is longer
than the one with 150 kHz.

Figure 7.9. Wave propagation of (a) A0 at 50 kHz, (b) S0 at 50 kHz, (c) A0 at 150 kHz,
and (d) S0 at 150 kHz
From the middle regions between the maximum and minimum amplitude lines in Figure 7.9(a), it can be noticed that the material particles move in both in-plane and out-of-plane directions. In other words, the antisymmetric mode is not entirely consisted with the out-of-plane displacement components and similarly the symmetric mode cannot be thought of as a mode with only in-plane displacements. The A0 wave mode with 50 kHz in Figure 7.9(a) is more examined for one wavelength period in Figure 7.10 to discuss that the wave reflection magnitudes are not proportional to the crack depth. These small plots can be thought as the mode shapes at the points that divides the wavelength, \( \lambda \) shown in Figure 7.9(a), into eight equal lengths. The parameter \( A \) in the equation above each plot is an arbitrary distance and \( \lambda \) is the wavelength about 2.4 cm with 50 kHz frequency. The vertical axis of each plot is the thickness of the plate structure. The first plot on the top-left shows no in-plane \((u_1)\) displacements but large negative out-of-plane \((u_3)\) displacements. This is the identical shape to the beginning point of the \( \lambda \) in Figure 7.9(a) where it shows the downward only particle movements. At the middle point of \( \lambda \) in Figure 7.9(a), particles only move upward and this is the same for the bottom-left plot in Figure 7.10. In addition, there are no \( u_3 \) movements but only \( u_1 \) displacements for \((2/8)\lambda \) and \((6/8)\lambda \) points. These profiles show the insight of guided wave propagation. Since the displacement profiles vary as the wavelength changes, the wave reflections due to different crack depth are disproportionate. When these \( u_1 \) and \( u_3 \) displacements are added together, wave propagation patterns are produced like in Figure 7.9.

![Figure 7.10. Wave structures of 50 kHz, A0 mode showing in-plane and out-of-plane displacement profiles across the plate thickness for one wavelength \( \lambda \).](image-url)
Returning to the wave reflections with various crack depths, the mode shapes are not linearly proportional to the plate thickness. With this, one also needs to consider the boundary refractions. Once the wave signals are reflected from a crack, these scattered signal particles interact with each other, diverge, and also refract from the plate boundaries. This complex process can produce other wave modes and may not yield the directly proportional reflections for various crack depths.

![Figure 7.11. Spectrograms for corrosions with 15-cm distance (left) and 20-cm distance (right); 3-cm corrosion width (top 4 plots) and 5-cm corrosion width (bottom 4 plots)](image)

The training samples for the corrosion damage are also obtained from Abaqus® with varying the length D, the depth αT, and the width Wc, of Figure 7.5(b). Total of 16 time-frequency plots for the training corrosion samples are obtained using the same crack sample dimensions for the
D and $\alpha T$, with 3 cm and 5 cm corrosion width, $W_c$, values. The time-frequency results for the corrosion samples are shown in Figure 7.11.

Unlike the time-frequency plots for the crack damages, it is difficult to find the regular patterns out of these corrosion damage plots. One should be cautious to conclude that the corrosion features in the spectrograms are longer in the time domain because of the longer corrosion width compared to the crack width. Under certain circumstances, this may not be correct. The crack and corrosion damage features in the time-frequency domain are not directly proportional to their widths. They are more dependent upon the excitation signal, frequency bandwidth, center frequency, and the distance between the actuators and damage. For the Abaqus® corrosion damages, the thickness variations at both ends of the damage can cause wave scatterings and mode conversions. The reflected scattering signals are also refracted from the plate boundaries and propagates back from the damage. The sensed corrosion signals with various depths and widths can differ in many degrees as shown in the figure.

Due to the extensive time required for creating finite elements models for each damage type, locations, and dimensions, there are only 8 crack and 16 corrosion training samples for the machine learning algorithm. The classification accuracy can be increased by having more distinctly classified training samples. In order to increase the accuracy of the classified results, more training samples are generated by moving the damage features in time-frequency plots through the time-domain as shown in Figure 7.12. Transferring the damage features with only time-domain direction is considered acceptable since it is unlikely that the frequency of the damage signal would change significantly from the excitation frequency.

![Trained pattern](image1)

![Trained pattern](image2)

Figure 7.12. Additional training samples following the trained patterns. Examples of (a) crack 15 cm and (b) corrosion 15 cm, 5 cm width, and 2T/8 depth

Four time-frequency damage features from each damage type are used to get additional training samples. The utilized four crack damages come from the damage results with 15 cm and 20 cm distances, each having $T/8$ and $3T/8$ depths. The additional corrosion samples are
obtained using the same distances but all with 2T/8 depths, each with 3 cm and 5 cm widths. The total numbers of training samples for crack and corrosion are increased from 8 and 16 to 42 and 40, respectively.

7.3.2 Testing Samples using Experimental Beam Specimens for Crack and Corrosion

The testing samples for classification for each damage type are obtained from the beam experiments and Abaqus® results. For the experimental results, a pair of thin 1 cm width PZT actuators is attached on each side of the beam with 1.9 m x 1 cm x 3.175 mm dimensions. These actuators are located on the middle of the beam and a PZT sensor with the same dimension is attached at right next to the actuator on the top surface as shown in Figure 7.13. A crack with 2T/8 depth is cut from the beam using a hacksaw at 13 cm distance from the actuators. A corrosion with 6 cm width and 2T/8 depth is also made using a milling machine at the same distance for the corrosion testing sample.

![Figure 7.13. Experiments performed to get testing samples (a) crack at 13-cm with 2T/8 depth and (b) corrosion at 13-cm having 6-cm width and 2T/8 depth](image)

Like the simulation models, a pure A0 mode, 2.5 cycles Hanning windowed toneburst signal with 50 kHz center frequency is generated from the thin PZT actuators. The peak-to-peak voltage of the input signal is 17 V. The input signal is generated from LabVIEW and transmitted through a NI PXI-6120 system and the sensed signal is obtained through a NI PXI-6133 system with a sampling rate of 1 MS/s. Total 30 measurements are taken and the signals are averaged to reduce the noise levels. The spectrograms of these averaged signals for each damage are presented in Figure 7.14(a) and (b) for crack and corrosion, respectively.
Four additional time-frequency testing samples, two of each, are acquired from Abaqus® models. One particular corrosion model is drawn based upon the cross-section of the aluminum corrosion experiment in Chapter 5. A chemical attack on the aluminum surface can cause tiny pits as shown in Figure 7.15(a). A similar pitting corrosion is modeled to represent the actual pitting corrosions with Abaqus® as shown in Figure 7.15(b). The actual pits have various sizes and irregular patterns but these are uniformly modeled in Abaqus®. The distance from the PZT actuators to the corrosion damage is 17 cm and its width and depth are 4 cm and 2T/8, respectively. The other Abaqus® corrosion sample is obtained from 5 cm width corrosion placed at 10 cm from the actuators. Two Abaqus® crack models are also formed that a 2T/8 crack is placed at 10 cm for one case and the other case with 18 cm, under the same testing conditions.

The time-frequency plots for all of the received signals from the Abaqus® models are shown in Figure 7.14(c) through (f). Figures (c) and (e) are the results from the Abaqus® crack models while (d) and (f) are the corrosion models results. These testing time-frequency result plots have the same time and frequency domain boundaries as with the previous training samples.
7.4 Classified Results

The Adaboost algorithm code was run on an AMD Athlon 2.61 GHz dual core processor with 1.93 GB of RAM and each run took about 4.5 minutes with 50 iterations. The classification results of the tested damage samples came out correctly, i.e. tested cracks resulted as ‘cracks’ and tested corrosions resulted as ‘corrosions’. The algorithm was not only used for damage classification, but it also could be used for obtaining the confidence levels that estimate the probability of their predictions.

The confidence level can be defined as the extent of sample location from the classification boundary. It was previously mentioned that the confidence level or the summation in the final hypothesis would be larger if a sample is located far from the class boundary. To obtain the confidence levels for the testing samples, the references are also obtained from the maximum summation quantities. The results of the confidence levels for all the correctly classified testing samples are shown in Table 7.1. Since all confidence levels are more than 50% in both damage classes, they are correctly classified indicating the highest confidence of 88.6% for the Abaqus® crack located at 10 cm and the lowest confidence of 59.5% for the experiment corrosion sample.

<table>
<thead>
<tr>
<th>Crack</th>
<th>%</th>
<th>Corrosion</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment 13 cm</td>
<td>74.8%</td>
<td>Experiment 13 cm with 6 cm width</td>
<td>59.5%</td>
</tr>
<tr>
<td>Abaqus® 10 cm</td>
<td>88.6%</td>
<td>Abaqus® 17 cm with 4 cm width (pitting corrosion)</td>
<td>62.2%</td>
</tr>
<tr>
<td>Abaqus® 18 cm</td>
<td>63.1%</td>
<td>Abaqus® 10 cm with 5 cm width</td>
<td>68.1%</td>
</tr>
</tbody>
</table>
In order to examine how well the training samples are affiliated in their class categories or how much they contribute to their class families, each training sample is separately tested through the Adabost algorithm. Note that there have been total of 42 and 40 training samples for crack and corrosion damages, respectively, including those additional samples that are trained through the time domains. For this analysis, only the actual training samples obtained from the Abaqus® simulation models are tested, excluding those additional time-adjusted samples.

In the algorithm, one training sample becomes a testing sample and the rest of the samples in each class are used as the training samples. The confidence levels of each of the training samples for both damages are shown in Table 7.2.

<table>
<thead>
<tr>
<th>Crack</th>
<th>15 cm distance</th>
<th>20 cm distance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T/8</td>
<td>2T/8</td>
</tr>
<tr>
<td>15 cm distance</td>
<td>73.0%</td>
<td>85.3%</td>
</tr>
<tr>
<td>20 cm distance</td>
<td>79.8%</td>
<td>81.5%</td>
</tr>
</tbody>
</table>

As shown in the table, most of the crack training samples belong to the crack class with more than or close to 70% of the confidence levels except the crack located at 20 cm with 4T/8 depth. Since the confidence levels are closely related to the degree of class affiliation, the lower confidence level means that the sample may not effectively contribute to the class it belongs. The spectrogram of this crack in Figure 7.7 seems a bit different from others showing slightly weak amplitudes at the tail section in the feature. The corrosion training samples show similar confidence results but appear to be less consistent. Although most of them are accurately categorized, a few samples are incorrectly classified, such as the ones at 15 cm distance with 5 cm width (4T/8) and 20 cm distance with 5 cm width (3T/8). These outlier samples would have negative effects on correct classification. Although they may have counter influences, it is hard to consider that the spectrograms for these two corrosion samples in Figure 7.11 look similar to the crack training samples. They both have some strong peak amplitudes at front of the features.
but the overall images seem to be somewhat closer to the other corrosion samples. The irregular patterns of the corrosion samples may have yielded these odd confidence levels.

Generally the correct classification and confidence levels depend on the training sample configurations, number of training samples, number of dimensions, number of iterations, and so on. Although the training samples used in this analysis are much less than typically applied when using a machine learning algorithm such as Adaboost, the tested results from above tables show that damage classification based on time-frequency plots is feasible for damage classification.

7.5 Conclusion

Damage classification is demonstrated using spectrograms and a machine learning algorithm called Adaboost. The damage signals of two most frequent damage types, crack and corrosion, incurred in metal structures are simulated with a finite element tool, Abaqus®, to obtain the training samples consisted of spectrograms. Beam experiments and additional simulations are also performed to get the testing samples. From the results, not only the correct damage classification is possible but the confidence levels of each testing sample can also be acquired.
Chapter 8

8. Conclusions and Recommendations

The general steps of structural health monitoring system can be categorized into damage detection, diagnosis, and prognosis. Among these steps, this research intends to focus on the possible approaches to improve or advance the current detection and diagnosis methodologies. In damage detection fields, a phased array beamsteering using guided Lamb wave is investigated using the monolithic piezoceramic and MacroFiber composite transducers. This beamsteering method is attractive since the wave energy can be concentrated at localized directions or areas by controlled excitation time delays, enabling localized damage diagnosis. In damage diagnosis fields, a new diagnosis methodology is developed using a robust machine learning algorithm called Adaboost for damage classification. The algorithm employs the time-frequency damage information as the training and testing samples and correctly classifies the damages with the confidence levels.

8.1 Summary and Conclusions

For the phased array beamsteering, the general displacement formulations are derived for symmetric and antisymmetric wave modes in guided wave field. These formulations based on omni-directional point excitations show some limitations when predicting the actual response for the phased array with anisotropic actuation. The correct beamsteering time delays are obtained for the anisotropic phased array using the genetic algorithms and its array responses are compared with the piezoceramic phased array for the main lobe and side lobe areas. The results show the improved performance of the anisotropic phased array in reference to the piezoceramic phased array for a certain range beamsteering angles.

In addition, the theoretical investigations on the phased array responses are verified with experiments. Firstly the monolithic piezoceramic sensor responses due to in-plane strains are
experimentally examined for A0 and S0 modes and the results show good agreement with the analysis. The phased array beamsteering using both piezoceramic and MacroFiber composite actuators are also investigated for certain beamsteering angles. The experimental and theoretical results closely match to one another and the results show that the sidelobe area of the MacroFiber composite beamsteering is smaller than the one for the piezoceramic for all the angles examined.

For damage diagnosis, a fundamental study about the excitation signals and signal analysis methods are studied to determine the input signals and to enhance signal characteristics. The beamsteering algorithm accompanied with these signal analysis methods is also applied for 2-dimensional crack localization. The time-frequency signal processing algorithms, such as short time Fourier transform, Wigner-Ville distribution, wavelet transform, and matching pursuit, are investigated to select the appropriate algorithm for damage classification. Each method is assessed its suitability for the usage and a spectrogram based on STFT is chosen for damage classification.

Finally, damage classification is performed using spectrograms and a machine learning algorithm called Adaboost for crack and corrosion damages. A finite element analysis tool, Abaqus®, is implemented to simulate these damages with aluminum structures. The training samples for the algorithm are adopted from the damage spectrograms. Several beam experiments and additional simulations are preformed to get the testing samples. The analysis results show that not only the correct damage classification is possible, but the confidence levels of each sample can also be acquired using Adaboost.

8.2 Recommendations

The Adaboost algorithm used in this research is capable of a binary classification. Multi-class classification can be done using other Adaboost algorithms, such as Adaboost M1, M2, MH, or combining it with other machine learning algorithms, such as decision trees or support vector machines, to increase the confidence levels. To perform the multi-class classification though, abundant data sets for each damage class and careful selection of class categories are required. Instead of classifying two different damage types performed in this research, it can be further investigated by differentiating the size, extent, and location of each damage type.
Although the phased array beamsteering and damage classification are separately investigated in this research, a more robust algorithm can be achieved by combining these two. The former beamsteering can yield the exact locations and possibly extents of damages and the latter damage classification using a machine learning algorithm can provide accurate damage diagnosis information such as damage classes or severities. Once the beamsteering is used to scan a certain area of structures and find damages, the signal that propagates through the damage center can be evaluated with damage classification algorithms.

More advanced materials have been used in aerospace industries and one of the frontier materials being used is composites. In the composite materials, delaminations are the most frequently incurred damage type but it is not easy to detect because of its invisible nature. The beamsteering algorithm investigated in this research can also be used to obtain the damage information if the dispersion and slowness curves for the composite laminates are acquired. It may also be feasible to distinguish different damage types for composite laminates utilizing the time-frequency representations and Adaboost machine learning algorithm.

The time-reversal method combined with Adaboost or other machine learning algorithms can be another option. The key idea of time-reversal method is that the reconstructed signal after the time-reversal process may differ from the original input signal if damage exists between transducers. Signal information, such as time, amplitude, and frequency values, for these reconstructed signals may be different for different types or severities of damages. Those differences can be used for damage classification and assessment using machine learning algorithms associated with signal processing methods.
Bibliography


Appendices

Appendix A. 2-Dimensional Abaqus® PZT Model Properties

The 2-Dimensional PZT properties used for the Abaqus® models discussed in Chapter 6 and 7 are shown in Table A.1. The implemented PZT type is an APC 850 from American Piezo Ceramic International Ltd.

![Diagram of 2D PZT model](image)

Table A.1. PZT parameters for 2-D Abaqus® model

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>PZT</th>
<th>Attached surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>7500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic Properties (modulus: Pa)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E1</td>
<td>6.30e10</td>
<td>n12</td>
<td>0.532</td>
</tr>
<tr>
<td>G12</td>
<td></td>
<td></td>
<td>2.30e10</td>
</tr>
<tr>
<td>E2</td>
<td>5.40e10</td>
<td>n13</td>
<td>0.301</td>
</tr>
<tr>
<td>G13</td>
<td></td>
<td></td>
<td>2.35e10</td>
</tr>
<tr>
<td>E3</td>
<td>6.30e10</td>
<td>n23</td>
<td>0.532</td>
</tr>
<tr>
<td>G23</td>
<td></td>
<td></td>
<td>2.30e10</td>
</tr>
<tr>
<td>Piezoelectric Coupling Properties (m/V)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d1 11</td>
<td>0</td>
<td>d2 11</td>
<td>-1.75e-10</td>
</tr>
<tr>
<td>d1 22</td>
<td>0</td>
<td>d2 22</td>
<td>4.00e-10</td>
</tr>
<tr>
<td>d1 33</td>
<td>0</td>
<td>d2 33</td>
<td>-1.75e-10</td>
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<tr>
<td>d1 12</td>
<td>5.90e-10</td>
<td>d2 12</td>
<td>0</td>
</tr>
<tr>
<td>d1 13</td>
<td>0</td>
<td>d2 13</td>
<td>0</td>
</tr>
<tr>
<td>d1 23</td>
<td>0</td>
<td>d2 23</td>
<td>0</td>
</tr>
<tr>
<td>Dielectric Properties (Farad/m)</td>
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<td></td>
</tr>
<tr>
<td>D11</td>
<td>1.51e-8</td>
<td>D22</td>
<td>1.30e-8</td>
</tr>
<tr>
<td>D33</td>
<td></td>
<td></td>
<td>1.51e-8</td>
</tr>
</tbody>
</table>

Appendix B. Derivation of Guided Lamb Wave Equations

The following presents the mathematical derivations for 2-dimensional guided Lamb wave equations. For an isotropic and linear elastic material, the Navier-Lamé’s equation is given in Eq. (2.3) and also shown below in a different form.

\[
\left( \lambda + \mu \right) u_{j,j,i} + \mu u_{i,j,j} + \rho f_i = \rho \ddot{u}_i \\
(\lambda + 2\mu)\nabla(\nabla \cdot u) - \mu \nabla \times \nabla \times u = \rho \frac{\partial^2 u}{\partial t^2}
\]

The general solutions for the displacement can be obtained by Helmholtz’s decomposition, where the vector that is piecewise continuous and differentiable in the finite closed region can be resolved into gradient and curl functions. Using a scalar potential \( \phi \) and a vector potential \( \mathbf{\varphi} \), Lamé gives the following displacement vector field equation.

\[
\mathbf{u} = \nabla \phi + \nabla \times \mathbf{\varphi}
\]

The completeness theorem also adds one more condition in the vector field, imposing the divergence-free (zero net-flux) condition.

\[
\nabla \cdot \mathbf{\varphi} = 0
\]

Substituting Eq. (2) into the second equation of Eq. (1),

\[
(\lambda + 2\mu)\nabla(\nabla \cdot (\nabla \phi + \nabla \times \mathbf{\varphi})) - \mu \nabla \times \nabla \times (\nabla \phi + \nabla \times \mathbf{\varphi}) = \rho (\nabla \ddot{\phi} + \nabla \times \ddot{\mathbf{\varphi}})
\]

Or

\[
(\lambda + 2\mu)\nabla\left(\nabla^2 \phi + \nabla \cdot (\nabla \times \mathbf{\varphi})\right) - \mu \nabla \times \nabla \times (\nabla \phi + \nabla \times \mathbf{\varphi}) = \rho (\nabla \ddot{\phi} + \nabla \times \ddot{\mathbf{\varphi}})
\]

Using the below vector identities, the Eq. (4) can be more simplified.

\[
\nabla \cdot (\nabla \times \mathbf{\varphi}) = 0 \\
\nabla \times (\nabla \phi) = 0 \\
\nabla \times (\nabla \times \mathbf{\varphi}) = \nabla \cdot (\nabla \cdot \mathbf{\varphi}) - \nabla^2 \mathbf{\varphi}
\]

Again substituting Eqs. (3) and (5) into Eq. (4), we get

\[
(\lambda + 2\mu)\nabla\left(\nabla^2 \phi\right) - \mu \nabla \times (-\nabla^2 \mathbf{\varphi}) = \rho (\nabla \ddot{\phi} + \nabla \times \ddot{\mathbf{\varphi}})
\]

Or
\[
\n\nabla \left( (\lambda + 2\mu) \nabla^2 \phi - \rho \ddot{\phi} \right) + \nabla \times \left( \mu \nabla^2 \varphi - \rho \ddot{\varphi} \right) = 0
\]

To satisfy the above equation, both equations should be zeros, i.e.

\[
(\lambda + 2\mu) \nabla^2 \phi - \rho \ddot{\phi} = 0; \quad \mu \nabla^2 \varphi - \rho \ddot{\varphi} = 0
\]

Or

\[
\nabla^2 \phi - \frac{\rho}{(\lambda + 2\mu)} \ddot{\phi} = \nabla^2 \phi - \frac{1}{C_i^2} \ddot{\phi} = 0 \quad \Rightarrow \quad \nabla^2 \phi = \frac{1}{C_i^2} \ddot{\phi}
\]

\[
\nabla^2 \varphi - \frac{\mu}{\rho} \ddot{\varphi} = \nabla^2 \varphi - \frac{1}{C_i^2} \ddot{\varphi} = 0 \quad \Rightarrow \quad \nabla^2 \varphi = \frac{1}{C_i^2} \ddot{\varphi}
\]

where the longitudinal velocity \( C_i^2 = \frac{\lambda + 2\mu}{\rho} \) and the transverse velocity \( C_i^2 = \frac{\mu}{\rho} \).

Above figure shows the coordinates axes that extend infinitely along the \( x_1 \) and \( x_2 \) directions.

If the in-plane problem is defined in the \( x_1 x_3 \)-plane, \( u_1 \) and \( u_3 \) are nonzero and \( u_2 \) is equal to zero.

The displacement vector \( \mathbf{u} \) of a two dimensional plane is then,

\[
\mathbf{u} = (u_1, 0, u_3)
\]

From Eq. (6), the governing equations are

\[
\nabla^2 \phi - \frac{1}{C_i^2} \ddot{\phi} = \phi_{,11} + \phi_{,33} - \frac{1}{C_i^2} \ddot{\phi} = 0
\]

where \( \phi = \phi(x_1, x_3, t) \) (7)

\[
\nabla^2 \varphi - \frac{1}{C_i^2} \ddot{\varphi} = \phi_{,11} + \phi_{,33} - \frac{1}{C_i^2} \ddot{\varphi} = 0
\]

Since \( u_2 = 0 \), the values of \( \varphi_1 \) and \( \varphi_3 \) are either constant or zero as shown below.
\[\nabla \times \varphi = \left| \begin{array}{ccc} i_1 & i_2 & i_3 \\ \frac{\partial \varphi_1}{\partial x_1} & \frac{\partial \varphi_2}{\partial x_2} & \frac{\partial \varphi_3}{\partial x_3} \end{array} \right| = \hat{i}_1 \left( \frac{\partial \varphi_2}{\partial x_1} - \frac{\partial \varphi_1}{\partial x_3} \right) + \hat{i}_2 \left( \frac{\partial \varphi_3}{\partial x_1} - \frac{\partial \varphi_2}{\partial x_3} \right) + \hat{i}_3 \left( \frac{\partial \varphi_1}{\partial x_2} - \frac{\partial \varphi_3}{\partial x_2} \right)\]

Or

\[\nabla \times \varphi = ( -\varphi_3 ) \hat{i}_1 + ( \varphi_1 ) \hat{i}_3 \quad \text{where} \quad \varphi = \varphi_2 = \varphi(x_1, x_3, t)\]

Therefore from Eq. (2),

\[
\begin{align*}
  u_1 &= \phi_1 - \varphi_3 \\
  u_2 &= 0 \\
  u_3 &= \phi_3 + \varphi_1
\end{align*}
\]

(8)

We are seeking the harmonic solution of the governing Eq. (6) that represents a harmonic wave traveling in the direction \(x_1\). Assuming the solutions of Eq. (6) in the following forms,

\[
\begin{align*}
  \phi &= \Phi(x_3) e^{i(kx_1 - wt)} \\
  \varphi &= \Psi(x_3) e^{i(kx_1 - wt)}
\end{align*}
\]

(9)

These solutions represent the traveling waves in the \(x_1\) direction and the standing waves in the \(x_3\) direction. Substituting Eq. (9) into Eq. (7) yields,

\[-k^2 \Phi(x_3) + \Phi(x_3)_{,33} - \frac{1}{C_i^2} (-w^2 \Phi(x_3)) = \frac{\partial^2 \Phi(x_3)}{\partial x_3^2} + \left( \frac{w^2}{C_i^2} - k^2 \right) \Phi(x_3) = 0\]

Similarly,

\[
\frac{\partial^2 \Psi(x_1)}{\partial x_3^2} + \left( \frac{w^2}{C_i^2} - k^2 \right) \Psi(x_3) = 0
\]

(10)

where \(p^2 = \frac{w^2}{C_l^2} - k^2\), \(q^2 = \frac{w^2}{C_i^2} - k^2\), and \(k = \frac{w}{C_p}\).

If \(p^2\) and \(q^2\) are both positive, i.e. \(C_i < C_l < C_p\), the solutions of these equations become

\[
\begin{align*}
  \Phi(x_3) &= A_1 \sin(px_3) + A_2 \cos(px_3) \\
  \Psi(x_3) &= B_1 \sin(qx_3) + B_2 \cos(qx_3)
\end{align*}
\]
Hence,
\[
\phi = (A_1 \sin(px_3) + A_2 \cos(px_3)) e^{i(kx_3 - wt)} \\
\varphi = (B_1 \sin(qx_3) + B_2 \cos(qx_3)) e^{i(qx_3 - wt)}
\] (11)

Substituting Eq. (11) into Eq. (8) yields
\[
u_1 = \phi_3 - \varphi_3 = \left[ ik \left( A_1 \sin(px_3) + A_2 \cos(px_3) \right) - q \left( B_1 \cos(qx_3) - B_2 \sin(qx_3) \right) \right] e^{i(kx_3 - wt)} \\
\nu_3 = \phi_3 + \varphi_1 = \left[ p \left( A_1 \cos(px_3) - A_2 \sin(px_3) \right) + ik \left( B_1 \sin(qx_3) + B_2 \cos(qx_3) \right) \right] e^{i(kx_3 - wt)}
\]

Collecting the sine and cosine terms from the above equations
\[
u_1 = \left[ ikA_2 \cos(px_3) - qB_1 \cos(qx_3) \right] e^{i\alpha} + \left[ ikA_1 \sin(px_3) + qB_2 \sin(qx_3) \right] e^{i\alpha} \\
\nu_3 = \left[ -pA_2 \sin(px_3) + ikB_1 \sin(qx_3) \right] e^{i\alpha} + \left[ pA_1 \cos(px_3) + ikB_2 \cos(qx_3) \right] e^{i\alpha}
\] (12)

where \( \alpha = (kx_3 - wt) \) is the phase.

The figure above is the deformation and particle movement directions in 2-D case. From the symmetric mode in the figure, \( \nu_1 \) follows the cosine behavior while \( \nu_3 \) follows the sine behavior with respect to the mid plane. On the other hand, for the anti-symmetric mode, particles follow the sine behavior for \( \nu_1 \) direction and the cosine behavior for \( \nu_3 \) direction. Therefore, the particle displacement vectors in Eq. (12) can be divided into the symmetric and anti-symmetric modes; letting \( A_1 \) and \( B_2 \) equal zero yields the symmetric mode, whereas \( A_2 \) and \( B_1 \) equal zero yields the anti-symmetric mode.
Symmetric Mode

From Eq. (12), the symmetric motion with respect to the mid plane is

\[
\begin{align*}
  u_1 &= [ikA_2 \cos(px_3) - qB_1 \cos(qx_3)] e^{i(k_x-x_3 - wt)} \\
  u_3 &= [-pA_2 \sin(px_3) + ikB_1 \sin(qx_3)] e^{i(k_x-x_3 - wt)}
\end{align*}
\]  

(13)

The corresponding stresses can be written in terms of the potentials given below.

\[
\begin{align*}
  \sigma_{31} &= \mu \left( \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} \right) \\
  \sigma_{33} &= \lambda \left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \right) + 2\mu \frac{\partial u_3}{\partial x_3}
\end{align*}
\]  

(14)

where Lamé constants \( \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \) and \( \mu = \frac{E}{2(1+\nu)} \)

The derivatives of \( u_1 \) and \( u_3 \) are then,

\[
\begin{align*}
  \frac{\partial u_1}{\partial x_1} &= ik \left[ ikA_2 \cos(px_3) - qB_1 \cos(qx_3) \right] e^{ia} = \left[ -k^2 A_2 \cos(px_3) - ikqB_1 \cos(qx_3) \right] e^{ia} \\
  \frac{\partial u_1}{\partial x_3} &= \left[ -ikpA_2 \sin(px_3) + q^2 B_1 \sin(qx_3) \right] e^{ia} \\
  \frac{\partial u_3}{\partial x_1} &= ik \left[ -pA_2 \sin(px_3) + ikB_1 \sin(qx_3) \right] e^{ia} = \left[ -ikpA_2 \sin(px_3) - k^2 B_1 \sin(qx_3) \right] e^{ia} \\
  \frac{\partial u_3}{\partial x_3} &= \left[ -p^2 A_2 \cos(px_3) + ikqB_1 \cos(qx_3) \right] e^{ia}
\end{align*}
\]

Substituting these derivatives into Eq. (14) yields

\[
\begin{align*}
  \sigma_{31} &= \mu \left[ -2ikpA_2 \sin(px_3) + (q^2 - k^2)B_1 \sin(qx_3) \right] e^{ia} \\
  \sigma_{33} &= \left[ -\lambda (k^2 + p^2) A_2 \cos(px_3) - 2\mu \left( p^2 A_2 \cos(px_3) - ikqB_1 \cos(qx_3) \right) \right] e^{ia}
\end{align*}
\]

(15)

The constants \( A_2 \) and \( B_1 \) are determined from boundary conditions (stress free at the upper and lower surfaces), i.e.

\[
\sigma_{31} = \sigma_{33} = 0 \text{ at } x_3 = \pm h
\]
Applying the boundary conditions, Eq. (15) becomes

\[
2ikpA_2 \sin(ph) = (q^2 - k^2)B_1 \sin(qh)
\]
\[
(\lambda k^2 + \lambda p^2 + 2\mu p^2)A_2 \cos(ph) = 2\mu ikqB_1 \cos(qh)
\]

Or

\[
\frac{A_2}{B_1} = \frac{(q^2 - k^2) \sin(qh)}{2ikp \sin(ph)} = \frac{2\mu ikq \cos(qh)}{(\lambda k^2 + \lambda p^2 + 2\mu p^2) \cos(ph)}
\]

The denominator, \((\lambda k^2 + \lambda p^2 + 2\mu p^2)\), on the right hand side can be simplified to

\[
C_i^2 = \frac{\lambda + 2\mu}{\rho} \Rightarrow \lambda = C_i^2 \rho - 2\mu
\]
\[
C_i^2 = \frac{\mu}{\rho}, \quad \frac{w^2}{C_i^2} = p^2 + k^2, \quad \text{and} \quad \frac{w^2}{C_i^2} = q^2 + k^2
\]

\[
\lambda(k^2 + p^2) + 2\mu p^2 = (C_i^2 \rho - 2\mu)(k^2 + p^2) + 2\mu p^2
\]
\[
= \rho C_i^2 (k^2 + p^2) - 2\mu k^2
\]
\[
= \rho w^2 - 2\rho C_i^2 k^2 = \rho C_i^2 \left(\frac{w^2}{C_i^2} - 2k^2\right)
\]
\[
= \rho C_i^2 (q^2 - k^2) = \mu (q^2 - k^2)
\]

Then,

\[
\frac{A_2}{B_1} = \frac{(q^2 - k^2) \sin(qh)}{2ikp \sin(ph)} = \frac{2\mu ikq \cos(qh)}{\mu (q^2 - k^2) \cos(ph)}
\]

Or

\[
\frac{(q^2 - k^2) \tan(qh)}{2ikp \tan(ph)} = \frac{2ikq}{(q^2 - k^2)}
\]

(16)

Therefore, the dispersion equation for the symmetric mode is obtained as,

\[
\frac{\tan(qh)}{\tan(ph)} = \frac{4k^2 pq}{(q^2 - k^2)^2}
\]

(17)
**Anti-symmetric Mode**

From Eq. (12), the anti-symmetric motion with respect to the mid plane is

\[
\begin{align*}
    u_1 &= [ikA_i \sin(px_i) + qB_2 \sin(qx_i)] e^{i(kx_i-wt)} \\
    u_3 &= [pA_i \cos(px_i) + ikB_2 \cos(qx_i)] e^{i(kx_i-wt)}
\end{align*}
\]

The derivatives of \( u_1 \) and \( u_3 \) are

\[
\begin{align*}
    \frac{\partial u_i}{\partial x_i} &= ik[kA_i \sin(px_i) + qB_2 \sin(qx_i)] e^{i\alpha} = \left[-k^2 A_i \sin(px_i) + ikqB_2 \sin(qx_i)\right] e^{i\alpha} \\
    \frac{\partial u_i}{\partial x_3} &= ik[pA_i \cos(px_i) + q^2 B_2 \cos(qx_i)] e^{i\alpha} \\
    \frac{\partial u_3}{\partial x_i} &= ik[pA_i \cos(px_i) + ikB_2 \cos(qx_i)] e^{i\alpha} = \left[ikpA_i \cos(px_i) - k^2 B_2 \cos(qx_i)\right] e^{i\alpha} \\
    \frac{\partial u_3}{\partial x_3} &= \left[-p^2 A_i \sin(px_i) - ikqB_2 \sin(qx_i)\right] e^{i\alpha}
\end{align*}
\]

Substituting these derivatives into Eq. (14) yields

\[
\sigma_{31} = \mu \left[2ikpA_i \cos(px_i) + (q^2 - k^2)B_2 \cos(qx_i)\right] e^{i\alpha}
\]

\[
\sigma_{33} = \left[-\lambda(k^2 + p^2)A_i \sin(px_i) - 2\mu\left(p^2 A_i \sin(px_i) + ikqB_2 \sin(qx_i)\right)\right] e^{i\alpha}
\] (18)

The constants \( A_1 \) and \( B_2 \) are also determined from boundary conditions (stress free at upper and lower surface), i.e.

\[
\sigma_{31} = \sigma_{33} = 0 \text{ at } x_3 = \pm h
\]

Applying the boundary conditions into Eq. (18)

\[
\begin{align*}
    2ikpA_i \cos(ph) &= -(q^2 - k^2)B_2 \cos(qh) \\
    (\lambda k^2 + \lambda p^2 + 2\mu p^2)A_i \sin(ph) &= -2\mu ikqB_2 \sin(qh)
\end{align*}
\]

Or

\[
\frac{A_i}{B_2} = \frac{-(q^2 - k^2) \cos(qh)}{2ikp \cos(ph)} = \frac{-2\mu ikq \sin(qh)}{(\lambda k^2 + \lambda p^2 + 2\mu p^2) \sin(ph)}
\]
Then,

\[
\frac{A_1}{B_2} = \frac{-(q^2 - k^2) \cos(qh)}{2ikp \cos(ph)} = \frac{-2ikq \sin(qh)}{(q^2 - k^2) \sin(ph)}
\]

Or

\[
\frac{-2ikq \tan(qh)}{(q^2 - k^2) \tan(ph)} = \frac{-(q^2 - k^2)}{2ikp}
\]

Therefore the dispersion equation for the anti-symmetric mode is obtained as,

\[
\frac{\tan(qh)}{\tan(ph)} = -\frac{(q^2 - k^2)^2}{4k^2 pq}
\]

(20)

Finally, the Lamb wave equation is obtained as shown below

\[
\frac{\tan(qh)}{\tan(ph)} = -\left[ \frac{4k^2 pq}{(q^2 - k^2)^2} \right]^{\pm 1}
\]

(21)

where +1 applies for symmetric modes, whereas -1 applies for anti-symmetric modes.