ANALYSIS OF SURFACE PRESSURE AND VELOCITY FLUCTUATIONS IN THE FLOW OVER SURFACE-MOUNTED PRISMS

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(Abstract)

The full-scale value of the Reynolds number associated with wind loads on structures is of the order of $10^7$. This is further complicated by the high levels of turbulence fluctuations associated with strong winds. On the other hand, numerical and wind tunnel simulations are usually carried out at smaller values of $Re$. Consequently, the validation of these simulations should only be based on physical phenomena derived with tools capable of their identification. In this work, two physical aspects related to extreme wind loads on low-rise structures are examined. The first includes the statistical properties and prediction of pressure peaks. The second involves the identification of linear and nonlinear relations between pressure peaks and associated velocity fluctuations.

The first part of this thesis is concerned with the statistical properties of surface pressure time series and their variations under different incident flow conditions. Various statistical tools, including space-time correlation, conditional sampling, the probability plot and the probability plot correlation coefficient, are used to characterize pressure peaks measured on the top surface of a surface-mounted prism. The results show that the Gamma distribution provides generally the best statistical description for the pressure time series, and that the method of moments is sufficient for determining its parameters. Additionally, the shape parameter of the Gamma distribution can be directly related to the incident flow conditions. As for prediction of pressure peaks, the results show that the probability of non-exceedence can best be derived from the Gumbel distribution. Two approaches for peak prediction, based on analysis of the parent pressure time series and of observed peaks, are presented. The prediction based on the parent time series yields more conservative estimates of the probability of non-exceedence.
The second part of this thesis is concerned with determining the linear and nonlinear relations between pressure peaks and the velocity field. Validated by analytical test signals, the wavelet-based analysis is proven to be effective and accurate in detecting intermittent linear and nonlinear relations between the pressure and velocity fluctuations. In particular, intermittent linear and nonlinear velocity-pressure relations are observed over the nondimensional frequency range \( fH/U < 0.32 \). These results provide the basis for flow parameters and characteristics required in the simulation of the wind loads on structures.
Dedication

To my parents and my grandmother
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Chapter 1

Introduction

1.1 Background and motivation

Natural wind hazards, such as hurricanes, tornadoes and strong wind storms, cause loss of life and have negative effects on natural resources and economic development. Post-disaster investigations of low-rise structures, such as single-family dwellings and light industrial buildings, consistently show that most of the damage is usually associated with the failure of roof coverings. The failure of roof sheathing can cause failure of trusses and rafters. Additionally, roof failures leave large openings which result in severe water damage. Roof failures are usually the result of extreme suction pressures induced by flow separation. Consequently, the prediction of the suction pressure and the associated extreme loads in these regions is the first step in improving the design guidelines in terms of building code requirements and inspection procedures.

The design guidelines for wind loads in the current codes come mostly from wind tunnel simulations. Yet, over the past decade, full-scale data have become more available. Comparisons of pressure coefficients from full-scale measurements and wind tunnel experiments show a high level of variance in the magnitude of surface pressure peaks (Tieleman et al., 1992, [1] and 1996, [2]). This variance is primarily due to variations in the turbulence characteristics, such as integral length scales, energy levels, small-scale turbulence and turbulence intensities of different velocity components in the incident flow. With the advancements in computational methodologies and
computing power, it is expected that numerical simulations will be used more often for the prediction of wind loads on structures. The different sources of the pressure coefficient data along with the dependence of these data on the incident flow characteristics pose several important questions. First, what constitutes a peak pressure in terms of the simulation time and data acquisition? For instance, a full-scale measurement might constitute of a single record. Obviously, it will have one peak value. On the other hand, a wind tunnel or numerical simulation can be run over a long time and could consist of a large number of records. The question becomes how do we compare a peak from a large number of records to a peak from a single record? Moreover, would an additional record from wind tunnel or numerical simulation yield a higher peak? Second, on what basis should data from different experiments and simulations be compared? Finally, what procedures should be followed to ensure that the same physical phenomena, in terms of causes and characteristics, govern the full-scale and simulated wind loads? It has been established that the turbulence level in the incident flow affects the flow separation and the levels of measured pressure peaks. The question remains as to what turbulence scales affect the pressure peaks and would thus need to be simulated. For instance, do the scales that are half or third or tenth of the characteristic dimensions of the structure need to be simulated for accurate prediction of pressure peaks? The results presented in this thesis aim at answering all of the above questions.

Over the past two decades, studies on wind-induced loads on low-rise structures have been the subject of many research efforts. Full-scale experimental results have become available since 1992, when field data began to be obtained from the experimental building at Texas Tech University (Levitan, 1992a, [3] and 1992b, [4]). Special emphasis has also been placed on wind tunnel simulations. To reproduce field measurements in a wind tunnel, different sets of criteria have been proposed and discussed. In addition to careful scaling of the incident turbulence integral scale and the aerodynamic roughness length, the effect of small-scale turbulence on the development of the separated shear layers and associated vortices have been shown to be
important (Tieleman, 1996, [5]). This leads to the concept of duplicating Melbourne’s small-scale turbulence parameter (Melbourne, 1980, [6]). Different floor-roughness configurations were consequently made in the wind tunnel simulations conducted at Clemson University, to test the effects of different simulation criteria. Based on these simulations, some progresses and pitfalls have been presented by Tieleman et al. (1992, [1] and 1996, [2]). However, and because of the complexity of the problem, a commonly accepted simulation technique is still far from settled.

It has been established that the wind-induced pressure forces can be treated as random variables and can, thus, be described by probability distribution functions. If the process is Gaussian, all the key parameters and its peak distribution can be obtained theoretically (Davenport, 1964, [7]). However, unfortunately, recent studies have shown that the time series representing the external pressure forces are generally not normally distributed. Kasperski (2000, [8]) studied the distributions of the parent time series and their peak values for four different locations of the pressure taps: the stagnation region, the separation zone downstream of the leading edge, the region downstream from the reattachment point and downwind wall. Kasperski’s results showed that the Gumbel distribution (extreme value distribution type I) is suitable for the description of peaks in most cases. Sadek and Simiu (2002, [9]) adapted Grigoriu’s theory (Gioffre, 2000, [10]) and developed a translation model to estimate the peaks of internal forces induced by wind in low-rise building frames. By comparing the goodness of fit among several candidates, the Gamma distribution was found to provide the best fit for the parent time series.

For the validation of numerical and wind-tunnel simulations of wind loads on structures, physical characterization of the pressure-velocity relation is especially important. Sarkar et al. (1997, [11]) visualized the corner vortices on a full-scale experimental building for oblique incident winds, and measured the dimensions of the separation bubble that results from normal incident winds. Kramer et al. (1989, [12]) simulated the full-scale
low-rise building in a wind tunnel, and visualized the corner vortices that formed due to cornering winds. The corner pressure distribution on a model prism and its variation with wind angle were measured and presented by Lin et al. (1995, [13]), based on the wind tunnel experiments at the University of Western Ontario. These visualizations and direct measurements led to a better understanding of the flow phenomena. Tieleman (1992, [1]) pointed out the occurrence of the secondary separation, which would take place when the flow under the primary corner vortices moves toward the leading edge while encountering an increasing adverse pressure gradient. More recent studies, by Kawai et al. (1996, [14]), Kawai (1997, [15]) and Marwood et al. (1997, [16]), presented extensive results on the behavior and characteristics of corner vortices on low-rise buildings, which provides a solid basis for investigation of surface pressures.

To assess the effect of the flow field on surface pressures on low-rise buildings, Hajj et al. (1997, [17]) examined the relation between frequency components of the incident wind and the surface pressure fluctuations. Their results showed that linear and nonlinear coherence between the incident velocity and surface pressure fluctuations were very low. A subsequent work, by Tieleman et al. (1998, [18]) also using Fourier-domain analysis, reported that the significance of the far-field velocity contributions to the surface pressure fluctuations was limited to a very low frequency range. The limitations of Fourier-based analysis for the investigation of intermittent wind loads were also pointed out.

Jordan et al. (1997, [19]) studied the influence of the incident flow field on surface pressures using wavelet analysis, which ensured better performance in processing nonstationary and intermittent signals. In fact, this effort was primarily concentrated on the linear relations between the far-field velocity and pressure fluctuations. High levels of linear couplings between the incident velocity fluctuations and the surface pressure fluctuations were identified successfully. The discrete wavelet transform was also used, by Hajj et al. (1999, [20], 2000, [21]), to characterize intermittency
1.2 Objective

The objective of this thesis is two-fold. The first is to examine whether pressure time series and individual peaks measured in wind tunnel simulations under different flow configurations can be described statistically by probability density functions. The second is to determine the extent of the linear and nonlinear relations between the velocity fluctuations in the near-field and observed pressure peaks. The realization of this objective will allow for better reproduction of extreme wind loads by setting the parameters needed in terms of simulating the turbulence characteristics of the incident flow.

This thesis is divided into two major parts. In Chapter 2, the experimental setup is introduced. The statistical characterization of pressure time series and pressure peaks is presented in Chapter 3. Assessment of the capabilities of the use of wavelet-based higher-order spectral moments is presented in Chapter 4 through their applications to analytical signals. The identification of linear and nonlinear relations between the velocity fluctuations and the simultaneously observed pressure peaks by using wavelet-based higher-order spectral moments is presented and discussed in Chapter 5. Finally, conclusions are summarized in Chapter 6.
Chapter 2
Experimental setup

Two different sets of experiments were conducted for this work. The first set consisted of pressure measurements and was designed to statistically characterize the pressure peaks under different roughness configurations. The second set consisted of simultaneous measurements of surface pressure and velocity fluctuations in the near-field flow. All observations were made from experiments on a 1:50 scale model of the WERFL experimental building placed on the turntable of the boundary layer wind tunnel of the Wind Load Test Facility (WLTF) at Clemson University in Clemson, South Carolina. The wind tunnel is of the open-return variety with a 2m x 3m cross-section and a test section length of 16m from the entrance to the test section to the center of the turntable. The wind tunnel is powered by a pair of fans with a diameter of 1.8m and controlled by a VTL Series 3500 Adjustable Frequency drive manufactured by Danfoss Inc. In this facility, flow simulations can be varied by changing roughness configurations, and by placing spires, baffles and trips varying in shape, size and quantity at different upstream locations with respect to the model prism.

2.1 Pressure measurements

The pressure measurements were made by a system using eight Scanivalve Model 48JMG-48 port pressure switches each instrumented with a Setra Model 239 differential pressure transducer. This system allowed for the simultaneous observation of pressures at eight different pressure tap loca-
tions. The pressure at the static port of a pitot-static tube, mounted in the free stream at about 1.5m above the tunnel floor, was used as a reference for the transducers.

In Part I of this work, observations from two sets of pressure taps placed on the upper surface of the prism model are studied. The locations and arrangement of the taps are illustrated in figure 2.1 and Table 2.1. The pressure taps of set A were designed for studying the pressures under the separation bubble due to normal incident flow. The eight pressure taps in set A are located along a line normal to the leading edge at points that are equivalent to the positions of the WERFL taps 50900 through 50909. Pressure taps of set B are lined up along a ray originating from the roof corner and making an angle of $15^\circ$ with the leading roof edge. This set of pressure taps was designed for pressure measurements under one of the corner vortices that typically form as a result of the three-dimensional separation of the oblique incident flow.

![Figure 2.1: Arrangement of the pressure taps set A and set B on the surface of the model and the zoning of the roof; tap coordinates are given in Table 2.1.](image)

In ASCE-7 Standard (1995, [27]), boundaries of different roof zones are given for determining wind loads on structures as shown in figure 2.1. Based on this zoning, three of the pressure taps of set A are located in zone 2 along
the leading edge, and the remaining five taps are in zone 1. For set B which lines up along axis $S$, the first three taps are in the corner zone 3, while the following five taps fall in zone 2.

### 2.1.1 Floor-roughness configurations and flow parameters

Many studies (Tieleman et al., 1996, [5], 1992, [1] and 1996, [2]) have shown that the addition of small spires just upstream of the model location will result in an increase of small-scale turbulence content in the incident flow, and therefore better simulate the pressures in the separated shear layer. The pressure measurements analyzed in this work were obtained under different upstream floor-roughness configurations. These configurations were purposefully designed to generate varying turbulence intensities and other upwind conditions. The description of these configurations is as follows:

**Configuration#1**  The flow is created in a bare tunnel without any spires or surface roughness.

**Configuration#2**  consists of the bare tunnel but with three spires and a boundary layer trip placed at the tunnel entrance. The three spires that extend the height of the tunnel are 0.24m wide on the top and 0.41m wide at the floor.

**Configuration#3**  is equivalent to Config.2, with the addition of a row
of six small spires equally spaced across the tunnel at approximately 1.50m upstream of the model location. These spires measured 9cm wide at the base and 61cm high and had a thickness of 1.3cm.

**Configuration #4** consists of the spires at the tunnel entrance as presented under Config.2, followed by 2.2m bare floor, followed by four baffles made of one by six lumber spanning the tunnel and 91cm apart.

**Configuration #6** consists of four spires at the tunnel entrance measuring 7.5cm wide at the top and 23cm wide at the base with a 10cm high trip board. The first 1.22m from the spire is bare floor, followed in order by 2.44m of cubic roughness (size 3cm), 2.44m bare floor, 2.44m roughness (size 3cm) and 1.22m bare floor.

**Configuration #7** is identical to the previous configuration with the last sheet of roughness before the turntable removed and replaced with five small spires.

**Configuration #8** consists of six 11.5cm by 20cm spires at 50cm centers across the tunnel, spanning the entire height of the test section together with a 13cm high trip board all placed 6.4m upstream from the model location.

A summary of the pertinent flow parameters observed at the model roof height for each configuration of the incident flow is presented in Table 2.2. The turbulence intensities of the $u$, $v$ and $w$ velocity components are denoted by $S_u/U$, $S_v/U$ and $S_w/U$, respectively, where $U$ is the mean wind velocity at roof height $H=7.9$cm. $\alpha$ is used to denote the exponent of the power-law representation of the mean velocity distribution; $u_*$ denotes the friction velocity, and $z_0$ denotes the aerodynamic roughness length obtained from the mean velocity profile. $S$ denotes the parameter of the small scale turbulence, which is usually defined as

$$S = \left[ \frac{n S_u(n)}{U^2} \right] \times 10^6 \tag{2.1}$$

and should be evaluated at $n = 10U/H$. Here, $S_u(n)$ is the spectral density of the streamwise velocity fluctuations at frequency $n$. Also in Table 2.2,
Table 2.2: Flow parameters observed at the model location at prism height under different flow configurations

<table>
<thead>
<tr>
<th>Conf.</th>
<th>Su/U (%)</th>
<th>Sv/U (%)</th>
<th>Sw/U (%)</th>
<th>α</th>
<th>u∗ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.1</td>
<td>5.3</td>
<td>4.3</td>
<td>0.132</td>
<td>0.554</td>
</tr>
<tr>
<td>2</td>
<td>14.3</td>
<td>13.6</td>
<td>7.6</td>
<td>0.109</td>
<td>0.412</td>
</tr>
<tr>
<td>3</td>
<td>17.8</td>
<td>17.9</td>
<td>11.6</td>
<td>0.099</td>
<td>0.319</td>
</tr>
<tr>
<td>4</td>
<td>18.8</td>
<td>15.7</td>
<td>10.2</td>
<td>0.107</td>
<td>0.291</td>
</tr>
<tr>
<td>6</td>
<td>16.2</td>
<td>12.0</td>
<td>8.6</td>
<td>0.079</td>
<td>0.396</td>
</tr>
<tr>
<td>7</td>
<td>17.1</td>
<td>16.0</td>
<td>11.9</td>
<td>0.086</td>
<td>0.253</td>
</tr>
<tr>
<td>8</td>
<td>19.3</td>
<td>18.2</td>
<td>12.5</td>
<td>0.085</td>
<td>0.257</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conf.</th>
<th>z0 × 10^3 (mm)</th>
<th>S</th>
<th>Lux (cm)</th>
<th>Lvx (cm)</th>
<th>Lwx (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43</td>
<td>11</td>
<td>19</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>12</td>
<td>68</td>
<td>49</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>2.4</td>
<td>41</td>
<td>39</td>
<td>37</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>1.66</td>
<td>28</td>
<td>65</td>
<td>33</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>4.04</td>
<td>21</td>
<td>66</td>
<td>27</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>0.44</td>
<td>133</td>
<td>26</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>0.84</td>
<td>54</td>
<td>34</td>
<td>24</td>
<td>9</td>
</tr>
</tbody>
</table>

Lux, Lvx, Lwx denote the turbulence integral scales obtained from the auto-correlation of the u, v and w components, respectively (see also [22] for more detail). It is important to note that these parameters cannot be varied independently. For instance, increasing the small scale turbulence is usually accompanied by a decrease in the integral length scale.

2.1.2 Pressure coefficient and area load

In the course of the experiments, pressure signals were measured and compared with the static reference pressure that is measured upstream separately, which yielded the pressure coefficient Cp. The pressure coefficients at the eight locations of the taps were measured simultaneously. The total sampling time for a complete record was 120 seconds, with a sampling frequency 2000Hz. As proposed by Lieblein (1974, [23]), the variance of the peak pressure coefficients from individual records approaches its smallest value when the number of records is at least 16. So for this experiment, each identical measurement was conducted independently 16 times.

The area load is also a 120-second-long time series, but not a measured
Table 2.3: Weight coefficients for pressure taps of set A and set B

<table>
<thead>
<tr>
<th>Tap set A</th>
<th>50900</th>
<th>50901</th>
<th>50902</th>
<th>50903</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight coef.</td>
<td>0.0821</td>
<td>0.0821</td>
<td>0.09833</td>
<td>0.09833</td>
</tr>
<tr>
<td>Tap set A</td>
<td>50904</td>
<td>50905</td>
<td>50907</td>
<td>50909</td>
</tr>
<tr>
<td>weight coef.</td>
<td>0.09833</td>
<td>0.1475</td>
<td>0.19666</td>
<td>0.19666</td>
</tr>
<tr>
<td>Tap set B</td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
<td>S4</td>
</tr>
<tr>
<td>weight coef.</td>
<td>0.1713</td>
<td>0.1320</td>
<td>0.1421</td>
<td>0.0674</td>
</tr>
<tr>
<td>Tap set B</td>
<td>S5</td>
<td>S6</td>
<td>S7</td>
<td>S8</td>
</tr>
<tr>
<td>weight coef.</td>
<td>0.0899</td>
<td>0.1320</td>
<td>0.1303</td>
<td>0.1348</td>
</tr>
</tbody>
</table>

record. It was obtained by averaging all the instantaneous time series from the eight pressure taps. These eight measured time series were weighted by weight coefficients that vary with tap locations. The weight coefficients for set A and set B are given in Table 2.3.

In the following studies in Part I, codes with six digits are used to identify different records: the first digit denotes the configuration number, and the following three digits represent azimuths: 025 for 25° and 000 for 0°. The last two digits preceded by a hyphen represent the repeat number, 01-16. For example, 6025-10 means that the floor-roughness is Config.6, with azimuth 25° and repeat No.10.

2.2 Pressure and velocity measurements

In Part II of this thesis, the pressure-velocity relations are studied. For this purpose, pressures were measured with Setra model differential transducers connected to Scanivalve pressure switches. Pressure taps were set on the top surface of the model. The arrangement of pressure taps for the studies in Part II are shown in figure 2.2. For each record, three pressure signals and one velocity signal were recorded simultaneously. Tap locations were carefully selected for normal and oblique incidence cases. Velocity measurements were made with standard hot-wire anemometry. The location of the anemometer in each record would be over one of the pressure taps, varying with records. Detailed tap and anemometer locations are presented
in Table 2.4, and denoted as Hotsam records for convenience. All the Hotsam records can be classified into two groups: normal cases (with azimuth 180°) and oblique cases (with azimuth 225°). The upwind floor roughness for Hotsam measurements was Config.2 as described in Section 2.1.1. More details of the experimental setup and measurements for this set are given in Janajreh, 1998, [24].

Figure 2.2: The WERFL test building and tap locations: the 5-digit tap numbers are the same as those on the model; not all the taps are shown in this figure.
Table 2.4: Tap and anemometer locations on the model; HW: Hot-wire; $p_1$, $p_2$, $p_3$: pressure taps for simultaneous measurements; $z'$: the distance the probe is over the pressure tap; also see figure 2.2 for the locations of pressure taps.

<table>
<thead>
<tr>
<th>No.</th>
<th>anemometer</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>azimuth</th>
<th>Probe over</th>
<th>$z'$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hotsam8</td>
<td>HW</td>
<td>50904</td>
<td>50903</td>
<td>50902</td>
<td>180</td>
<td>50900</td>
<td>1.50</td>
</tr>
<tr>
<td>Hotsam9</td>
<td>HW</td>
<td>50904</td>
<td>50903</td>
<td>50902</td>
<td>180</td>
<td>50900</td>
<td>1.00</td>
</tr>
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<td>Hotsam10</td>
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<td>HW</td>
<td>50900</td>
<td>50905</td>
<td>50907</td>
<td>180</td>
<td>50900</td>
<td>0.50</td>
</tr>
<tr>
<td>Hotsam12</td>
<td>HW</td>
<td>50904</td>
<td>50903</td>
<td>50902</td>
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Part I

Pressure peak statistics and prediction
Chapter 3

Space-time correlation and statistical prediction of pressure peaks

In earlier studies (Tieleman et al., 1996, [5], 1999, [20], 2003, [22] and Tian, 1999, [25]), many characteristics of surface pressures on surface-mounted prisms were presented. Some of the most important characteristics that are relevant to the present work can be summarized as follows:

- Flow separation occurs at the leading roof edge for normal cases (azimuth 0 degrees) or at the roof corner for oblique cases (azimuth non-zero). This gives rise to extreme suction pressures in the separated regions, which is one of the major factors for building damages.

- The magnitudes of the pressure coefficients in different cases of oblique incidence are usually much higher than those measured in normal incidence cases.

- In both incidence cases, the magnitude of the peak pressure coefficients drops rapidly with increasing distance away from the leading edge or the roof corner.

- The turbulence intensities and small-scale turbulence are important to simulating extreme pressures in critical areas on the building roof.

- The mean pressure coefficients drop in magnitude with increasing turbulence intensities, while the fluctuating pressures (peak and RMS of the pressure coefficients) vary in the opposite trend.
• Studies of the 2-D joint probability density shows that large pressure peaks tend to have long durations.

In this chapter, space correlations between pressures are calculated to obtain an overview of the peak distribution on the model surface. Conditional sampling is carried out for some cases to show the connections between pressure peaks at different locations. Furthermore, the convection velocity of pressure peaks is also estimated with the space-time correlation. To study the pressure time series statistically, the Gamma distribution is used to match the probability distribution of the parent time series, and the Gumbel distribution is used to predict the distribution of pressure peaks. The goodness of fit of both distributions is also assessed quantitatively.

3.1 Correlation studies

3.1.1 Definition of correlation coefficient

Quantitatively, the correlation coefficient between two random variables $X$ and $Y$ is defined as

$$\rho_{XY} = \frac{\text{Cov}(X, Y)}{\sqrt{D(X)}\sqrt{D(Y)}}$$  \hspace{1cm} (3.1)

where $\text{Cov}(X, Y)$ is the covariance between $X$ and $Y$, and defined as $\text{Cov}(X, Y) = (X - \bar{X})(Y - \bar{Y})$. $D(X)$ denotes the variance of $X$.

The correlation coefficient $\rho_{XY}$ has two important properties. These are

• $|\rho_{XY}| \leq 1$

• $|\rho_{XY}| = 1 \iff$ There exist constants $a$ and $b$ that satisfy $P\{Y = aX + b\} = 1$

These properties imply that if and only if two time series are linearly dependent, the correlation coefficient will be equal to unity. Near-zero $|\rho_{XY}|$ means that the two time series $X$ and $Y$ are not linearly related. Values between 0 and 1 indicate partial linear coupling. The correlation coefficient can be negative, which indicates linear, but out-of-phase, coupling.
3.1.2 Space correlation

Correlation coefficient between pressure coefficients from different locations

Space correlations of the pressure coefficients as measured along tap sets A and B that are shown in figure 3.1 were calculated using equation 3.1. Figure 3.1 shows the space correlations between pressure coefficients at different locations for tap set B as measured in record 8025-04. Obviously, pressures at taps closer to tap S1 have higher levels of linear correlation with the reference pressure. However, the lowest correlation coefficient in set B is still over 0.85, which implies that all the eight pressures in set B are well correlated, even though some of the pressure taps in the set could be separated by a relatively large distance. It should be stressed that this is not usual, because such a high linear correlation is not an instantaneous relation, but a long-term one. This might be due to the corner vortex that sways over the pressure taps of set B. Physically, the pair of corner vortices are relatively steady and influential structures in the flow field, although in fact they might sway. Therefore, the surface pressures along axis S are largely influenced by the same flow structure, the corner vortex. As a result, the linear space correlations between pressures in set B have high levels.

Figure 3.2 shows the space correlations between pressure coefficients at different taps in set A as measured in record 3000-04. The pressures at tap 50903 is selected as the reference. In the region from tap 50900 to tap 50905, all pressures have a high correlation, over 0.9, with the pressures at tap 50903. Further downstream, the correlation coefficients drop sharply from 0.8 to below 0.3, as observed at taps 50907 and 50909. This shows that the surface pressures are highly correlated over a relatively large range from the leading edge to tap 50905. This level of high correlation drops significantly beyond tap 50907. This could result from the growing convections and flow reversals in the region of flow reattachment.
Correlation with conditional sampling

As an effective technique for turbulence analysis, conditional sampling has been widely used in numerous subjects. The purpose of conducting conditional sampling is usually to sort out meaningful events from turbulent signals by setting one or more criteria or thresholds. This technique is especially popular in the characterization of coherent structures in turbulent flows. In this work, the interest is in pressure peaks. Suction peaks in critical areas such as leading roof edges and roof corners are regarded as a major factor responsible for wind-induced damage to low-rise structures. Thus, in this section, conditional sampling is used to sort out local pressure peaks, in an effort to calculate space correlations for all-peak time series.

The conditional sampling is applied by performing the following steps: First, the mean values of all the relevant time series are removed. Second, one particular time series, denoted by $T_A$, is selected to be conditionally sampled. In the conditional sampling criteria applied here, pressure coefficients in $T_A$ that exceed three times the root-mean-square (RMS) of the entire time series are retained. Pressure coefficients that do not satisfy this
Figure 3.2: Correlation coefficients between pressures in set A; the pressure coefficient at tap 50903 is selected as the reference; Config.3; record 3000-04 criterion are eliminated. For other records, only the simultaneous pressure coefficients are retained, regardless of their values. The process can also be described as follows: if time series $T_A$ is chosen to be conditionally sampled, and $T_B$ is another simultaneous record, we let

$$
\begin{cases}
  T_{A_i}' = T_{A_i}, T_{B_i}' = T_{B_i} & T_{A_i} \geq 3 \cdot RMS(T_A) \\
  T_{A_i}' = T_{B_i}' = 0 & \text{otherwise}
\end{cases} 
\quad (i = 1, 2, \cdots, 240000) 
\tag{3.2}
$$

Then, with the two sampled time series $T_A$ and $T_B$ (still denoted as $T_A$ and $T_B$ instead of $T_A'$ and $T_B'$ for convenience), the correlation coefficient $R_{AB}$ is determined by the area definition as Saathoff (1989, [26]) proposed, i.e.

$$
R_{AB} = \frac{T_{AB}}{\sqrt{T_{AA}T_{BB}}} 
\tag{3.3}
$$

where $T_{AA}$, $T_{BB}$ and $T_{AB}$ are the areas under the time series $T_A \cdot T_A$, $T_B \cdot T_B$ and $T_A \cdot T_B$, respectively. In practice, however, another form is usually used for calculation:
As an example, data from record 8025-04 are examined again. The pressure time series at tap S1 is chosen for conditional sampling. The correlation coefficients between the pressure time series after sampling are calculated, using tap S1 as a reference. Figure 3.3 compares the correlation coefficients for both original and conditionally sampled time series. The results show clearly that the correlation coefficients become higher when conditional sampling is applied. All the correlation coefficients have values that are larger than 0.97. As another example, figure 3.4 shows the comparison of correlation coefficients between pressures measured in record 4025-01 with and without the conditional sampling. Extremely high correlation coefficients are obtained again when conditional sampling was applied. The lowest correlation coefficient, between taps S1 and S8, is above 0.97, manifesting a consistently perfect correlation between pressure peaks for set B. In contrast, the correlation coefficients are obviously lower without conditional sampling. This is a strong indication that there exist some common characteristics in pressure peaks as measured by the different taps.

### 3.1.3 Space-time correlation

Correlation between pressure peaks were also studied in cases where slight convection in pressure peaks was observed. Figure 3.5 shows data segments of the pressure time series measured by the different taps in Set B. The downstream convection of local pressure peaks can be observed near t=110s. In this case, the local pressure peak emerges earliest at pressure tap S1, followed by, in sequence, at tap S2 down to tap S8. This reveals clearly a slow convection of pressure peaks starting at the roof corner and moving downstream.
To quantify the convection velocity between two locations, the upstream time series was shifted by incremental time delays. Once the correlation coefficient between the shifted time series and the downstream one reaches the maximum, the corresponding time delay is considered as the time it takes for the pressure peak to be convected to the downstream location. The convection velocity $V_c$ is then determined by the relation

$$ V_c = \frac{D}{T} $$

(3.5)

where $D$ is the distance between the two locations, and $T$ is the time delay with respect to the maximum correlation coefficient.

Figure 3.6 shows the correlation coefficients between the pressure at tap S1 and the remaining pressures in the same set. The pressure at tap S1 was shifted by incremental time delays to determine the highest correlations with other pressure taps. The filled square dots in figure 3.6 show clearly the time delays for which the maximum correlation occurs. In fact, this time delay is exactly the period during which the pressure peak is convected from the upstream location to the adjacent downstream one.
Based on the time delays estimated by space-time correlations shown in figure 3.6, the convection velocity for the particular peak occurring near t=110s can be determined using equation 3.5. The process of estimating the peak’s convection velocity at the tap locations is presented in Table 3.1. Obviously, when the distance from tap S1 to the current tap location is large enough, i.e., beyond tap S5, the local convection velocity of the peak has a stable estimate. At taps S5, S6, S7 and S8, the convection velocity is approximately 5.5 m/s, which is nearly half of the mean wind speed at roof height.

Moreover, investigation of many cases indicates that the convection velocity of pressure peaks is not constant, even under identical upstream conditions. According to Tieleman et al. (2003, [22]), the downstream convection of pressure peaks in oblique incident flow is influenced by the status of the corner vortex that forms around the roof corner. Different spatial orientations and oscillations of the vortex axis result in different values of the
convection velocity. Kawai et al. (1996, [14]) found out that the axis of the corner vortex (or conical vortex) is not straight but spiral, and therefore, the convection velocities at locations on axis $S$ are not the same.

3.2 Distribution and prediction of pressure peaks

3.2.1 Probability distribution of pressure time series

In ASCE-7 Standard (1995, [27]), surface pressures are assumed to be normally distributed for convenience. However, this assumption does not apply to low-rise structures for which the time-varying surface pressures and loads along the roof leading edge and roof ridges are generally non-Gaussian (Tieleman et al., 2003, [28]). In this section, the probability distribution functions of pressure coefficients are determined.
Figure 3.6: Correlation coefficients between pressure time series of tap set B with incremental time delays; the pressure of tap S1 selected to be shifted; unit time delay 2.5ms; t=110.0s; record 8025-04

Table 3.1: Convection velocity of the pressure peak near t=110s as shown in figure 3.5

<table>
<thead>
<tr>
<th>Tap</th>
<th>Coordinate (x,y) (cm)</th>
<th>Distance to tap S1 (cm)</th>
<th>Observed time delay (ms)</th>
<th>Convection velocity (m/s)</th>
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</thead>
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<tr>
<td>S2</td>
<td>(0.97, 0.29)</td>
<td>0.383</td>
<td>0.0</td>
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<td>S3</td>
<td>(1.48, 0.45)</td>
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<td>S4</td>
<td>(1.97, 0.53)</td>
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<td>S5</td>
<td>(2.70, 0.79)</td>
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<td>S6</td>
<td>(3.43, 1.00)</td>
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<tr>
<td>S7</td>
<td>(4.17, 1.19)</td>
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<td>S8</td>
<td>(4.89, 1.38)</td>
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The histogram of the pressure coefficients of record 8025-04 is shown in figure 3.7. Obviously, the shape of the histogram is skewed to the left, indicating that the associated Probability Distribution Function (PDF) of the entire time series deviates from the symmetric normal distribution. In many earlier studies (Gioffre et al., 2000, [10] and Sadek et al., 2002, [9]), it has been recommended that the Gamma distribution be used to fit the PDF of the pressure time series. The goodness of the Gamma distribution has been validated for modeled internal forces in low-rise building frames.
The PDF of the three-parameter Gamma distribution is defined as:

\[ f(x) = \frac{(x-\mu)^{\gamma-1}Exp\left(-\frac{x-\mu}{\beta}\right)}{\beta \Gamma(\gamma)}, \quad (x \geq \mu; \gamma, \beta > 0) \]  

(3.6)

where the three parameters \( \mu, \beta \) and \( \gamma \) are the location, scale and shape parameters, respectively. \( \Gamma(\cdot) \) denotes the Gamma function defined as:

\[ \Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt \]  

(3.7)

Fitting a PDF with the Gamma distribution is obtained by finding the appropriate parameters \( \mu, \beta \) and \( \gamma \). The Gamma distribution is completely determined when the three parameters are estimated.

By assuming that the shape parameter is known, the probability plot can be used to determine the remaining two parameters: \( \mu \) and \( \beta \). According to [29], the probability plot is the type of graph that consists of:

- Vertical axis: Ordered values in the time series;
- Horizontal axis: Order statistic medians for the given distribution;
the statistic median is defined as:

\[ N(i) = G(U(i)) \]

where \( U(i) \) denotes the uniform order statistic medians and \( G \) is the percent point function for the desired distribution (more details are given in [29]).

The probability plot should be a straight line if the parent time series is strictly Gamma distributed. However, in reality, results from pressure time series show that they do not precisely follow a Gamma distribution. Therefore, small deviations from the fitting straight line are often unavoidable. The correlation coefficient between the probability plot and its fitting straight line is defined as the Probability Plot Correlation Coefficient (PPCC). Obviously, a perfectly straight probability plot will yield a unit value for the PPCC.

By the method of moments, the estimators for the shape, scale and location parameters of the Gamma distribution are given as:

\[
\begin{align*}
\gamma &= \frac{(2/S)^2}{2} \\
\beta &= \sigma S/2 \\
\mu &= \bar{X} - 2\sigma/S
\end{align*}
\]

(3.8)

where \( \bar{X} \), \( \sigma \) and \( S \) denote respectively the mean, the standard deviation and the skewness of the time series.

One may use the PPCC to optimize the parameters on the basis of the preliminary estimators as obtained from equation 3.8. The following is the procedure: using equation 3.8 one can obtain preliminary estimators of the three parameters. Then, by temporarily fixing \( \beta \) and \( \mu \), the shape parameter is varied in the vicinity of its estimator. Corresponding to each test shape parameter, a PPCC can be obtained. The highest PPCC should result from the optimal shape parameter. Once the shape parameter is determined, the other two parameters are optimized using the probability plot.
Figure 3.7 shows the histogram of the parent time series for record 8025-04 at tap S1 with its fitting Gamma PDF superimposed on it. Obviously, the Gamma PDF determined by the three preliminary estimators fits the histogram very well. For this plot, these estimators are: $\gamma=3.8897$, $\mu=0.5403$ and $\beta=1.0582$. Figure 3.8 illustrates the optimization of shape parameter by comparing PPCCs around the preliminary estimator. The range of test shape parameters were extended from 3.1 to 4.7, approximately centered at 3.8897. The PPCC that corresponds to each test shape parameter was then calculated. Using figure 3.8, the optimal shape parameter, as determined from the climax of the PPCC arc, is 4.0764. This value is slightly larger than the one obtained from the moment estimators, 3.8897. The PPCC associated with the optimal shape parameter has a value over 0.9996. After determining the shape parameter, the probability plot is produced as shown in figure 3.9, and then the other two parameters are modified. The optimal location parameter turns out to be 0.4437, and the optimal scale parameter is 1.0334. They both have slight excursions from values obtained with the moment estimators.

For a better comparison, Table 3.2 lists the mean, standard deviation, skewness and kurtosis of 17 load time series, for 7 different configurations, observed from tap set A with an azimuth of 0°. Common equations for calculating skewness and kurtosis can be found in [29]. These parameters are used to estimate the shape, location and scale parameters of the Gamma and normal distributions as described above. For the same 17 load time series, optimized Gamma parameters obtained by using probability plots are also determined. A comparison of the results is presented in Table 3.3. The PPCC for the normal and Gamma distributions are listed in columns No.5 and 9 of Table 3.3. It is noted that the PPCC values for Gamma distributions are always higher than those for normal distributions. This implies that the Gamma distribution provides a better fit for load time series for all configurations. On the other hand, by comparing the estimated and optimized Gamma parameters in Table 3.3, it is observed that, invariably, there is very little or no difference between the two sets of parameters. This
Figure 3.8: Varying shape parameters and their corresponding PPCCs; location and scale parameters estimated by the method of moments; record 8025-04, tap S1 indicates that for normal incidence cases, the fitting Gamma distribution determined by the moment estimators is suitable for the representation of wind loads on structures.

Table 3.4 lists the mean, standard deviation, skewness and kurtosis of 17 load time series for the case of oblique incidence, with an azimuth of 25°. Table 3.5 lists the moment estimators of the scale, location and shape parameters for the fitting Gamma distributions, and their optimized counterparts. The PPCC values, used to quantify the goodness of fit by both Gamma and normal distributions, are also listed in Table 3.5. Again, it is noted that, for the case of oblique incidence, the PPCC values for Gamma distributions are invariably higher than those for normal distributions. This is an indication of better suitability of the Gamma distribution as a fit to load time series for all configurations. One may also note again that there are only small discrepancies between the location, shape and scale parameters calculated from the moment estimators and their optimized counterparts.
Table 3.2: Statistics of time series; set A, load, azimuth 0 degrees

<table>
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<tr>
<th>Conf./repeat</th>
<th>Mean</th>
<th>Std</th>
<th>Skewness</th>
<th>Kurtosis</th>
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<td>0.284</td>
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<td>3.657</td>
</tr>
<tr>
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<td>1.257</td>
<td>0.385</td>
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<td>5.938</td>
</tr>
<tr>
<td>8/09</td>
<td>1.265</td>
<td>0.385</td>
<td>0.836</td>
<td>4.232</td>
</tr>
</tbody>
</table>

Table 3.3: Parameters for Gamma and normal distributions (estimates based on Table 3.2); \( \gamma \): shape parameter; \( \mu \): location parameter; \( \beta \): scale parameter; set A, load, azimuth 0 degrees

<table>
<thead>
<tr>
<th>Conf./repeat</th>
<th>( \gamma )</th>
<th>( \mu )</th>
<th>( \beta )</th>
<th>PPCC</th>
<th>( \gamma )</th>
<th>( \mu )</th>
<th>( \beta )</th>
<th>PPCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/01</td>
<td>49.70</td>
<td>0.022</td>
<td>0.0234</td>
<td>0.99992</td>
<td>56.99</td>
<td>0.0206</td>
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<td>0.99757</td>
</tr>
<tr>
<td>1/03</td>
<td>66.62</td>
<td>0.0193</td>
<td>-0.157</td>
<td>0.99996</td>
<td>54.18</td>
<td>0.0214</td>
<td>-0.032</td>
<td>0.99824</td>
</tr>
<tr>
<td>1/16</td>
<td>92.78</td>
<td>0.0163</td>
<td>-0.389</td>
<td>0.99987</td>
<td>86.59</td>
<td>0.0168</td>
<td>-0.337</td>
<td>0.99858</td>
</tr>
<tr>
<td>2/01</td>
<td>7.408</td>
<td>0.108</td>
<td>0.300</td>
<td>0.99983</td>
<td>7.605</td>
<td>0.107</td>
<td>0.289</td>
<td>0.98605</td>
</tr>
<tr>
<td>2/03</td>
<td>5.850</td>
<td>0.125</td>
<td>0.381</td>
<td>0.99979</td>
<td>6.240</td>
<td>0.121</td>
<td>0.358</td>
<td>0.98253</td>
</tr>
<tr>
<td>3/11</td>
<td>4.700</td>
<td>0.165</td>
<td>0.452</td>
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<td>5.139</td>
<td>0.158</td>
<td>0.417</td>
<td>0.97856</td>
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<tr>
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<td>0.125</td>
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<td>7.864</td>
<td>0.123</td>
<td>0.279</td>
<td>0.98636</td>
</tr>
<tr>
<td>4/01</td>
<td>6.381</td>
<td>0.171</td>
<td>0.106</td>
<td>0.99967</td>
<td>5.923</td>
<td>0.177</td>
<td>0.146</td>
<td>0.98142</td>
</tr>
<tr>
<td>4/09</td>
<td>4.426</td>
<td>0.200</td>
<td>0.255</td>
<td>0.99956</td>
<td>4.758</td>
<td>0.193</td>
<td>0.223</td>
<td>0.97648</td>
</tr>
<tr>
<td>6/02</td>
<td>11.25</td>
<td>0.080</td>
<td>0.205</td>
<td>0.99987</td>
<td>11.89</td>
<td>0.078</td>
<td>0.180</td>
<td>0.99067</td>
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<td>0.093</td>
<td>0.237</td>
<td>0.99982</td>
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<td>0.088</td>
<td>0.189</td>
<td>0.98876</td>
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<tr>
<td>7/07</td>
<td>8.084</td>
<td>0.117</td>
<td>0.371</td>
<td>0.99907</td>
<td>9.431</td>
<td>0.108</td>
<td>0.296</td>
<td>0.98790</td>
</tr>
<tr>
<td>7/09</td>
<td>21.56</td>
<td>0.066</td>
<td>-0.118</td>
<td>0.99929</td>
<td>24.36</td>
<td>0.062</td>
<td>-0.207</td>
<td>0.99487</td>
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<tr>
<td>8/01</td>
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<td>0.205</td>
<td>0.535</td>
<td>0.99792</td>
<td>4.620</td>
<td>0.179</td>
<td>0.431</td>
<td>0.97536</td>
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<tr>
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<td>0.161</td>
<td>0.345</td>
<td>0.99940</td>
<td>6.048</td>
<td>0.156</td>
<td>0.320</td>
<td>0.98214</td>
</tr>
</tbody>
</table>
In practice, searching for the optimal parameters for fitting Gamma distributions is quite inefficient and time-consuming. Since excellent closeness between the moment estimators and the optimal parameters is exhibited, as shown in Tables 3.3 and 3.5, for both normal and oblique cases, it can be stated that the moment estimators should be used to match pressure time series, and in most cases, the fit is satisfactory.

Having made the above conclusion about the suitability of the Gamma distribution for fitting wind loads, it must be pointed out that this might not be the case for some individual taps. For instance and for individual taps of Config.1 and Config.7, the normal distribution describes the parent time series better than the Gamma distribution (Tieleman et al., 2003, [28]). Figure 3.10 shows the histogram of the time series observed at tap 50909, record 1000-01. Obviously, the histogram is quite symmetric, which makes the normal distribution a natural candidate. The matching normal PDF is also shown against the histogram. For such a parent time series, a matching Gamma PDF does not exist. In fact, by the method of moments,
Table 3.4: Statistics of time series; set B, load, azimuth 25 degrees

<table>
<thead>
<tr>
<th>Conf./repeat</th>
<th>Mean</th>
<th>Std</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/01</td>
<td>4.010</td>
<td>0.567</td>
<td>0.086</td>
<td>2.80</td>
</tr>
<tr>
<td>1/02</td>
<td>3.960</td>
<td>0.551</td>
<td>0.153</td>
<td>2.89</td>
</tr>
<tr>
<td>1/03</td>
<td>4.013</td>
<td>0.586</td>
<td>0.260</td>
<td>2.97</td>
</tr>
<tr>
<td>2/01</td>
<td>3.244</td>
<td>1.075</td>
<td>0.957</td>
<td>4.55</td>
</tr>
<tr>
<td>2/04</td>
<td>3.328</td>
<td>1.068</td>
<td>0.711</td>
<td>3.55</td>
</tr>
<tr>
<td>3/04</td>
<td>3.320</td>
<td>1.151</td>
<td>0.714</td>
<td>3.66</td>
</tr>
<tr>
<td>3/15</td>
<td>3.319</td>
<td>1.138</td>
<td>1.003</td>
<td>5.17</td>
</tr>
<tr>
<td>4/08</td>
<td>3.210</td>
<td>1.252</td>
<td>0.985</td>
<td>4.32</td>
</tr>
<tr>
<td>4/09</td>
<td>3.213</td>
<td>1.240</td>
<td>0.890</td>
<td>3.97</td>
</tr>
<tr>
<td>4/13</td>
<td>3.300</td>
<td>1.307</td>
<td>0.924</td>
<td>3.98</td>
</tr>
<tr>
<td>6/05</td>
<td>3.126</td>
<td>0.882</td>
<td>0.663</td>
<td>3.50</td>
</tr>
<tr>
<td>6/06</td>
<td>3.057</td>
<td>0.849</td>
<td>0.696</td>
<td>3.63</td>
</tr>
<tr>
<td>7/04</td>
<td>3.646</td>
<td>0.894</td>
<td>0.609</td>
<td>3.81</td>
</tr>
<tr>
<td>7/10</td>
<td>3.685</td>
<td>0.916</td>
<td>0.771</td>
<td>4.33</td>
</tr>
<tr>
<td>8/05</td>
<td>3.356</td>
<td>1.315</td>
<td>1.118</td>
<td>5.23</td>
</tr>
<tr>
<td>8/14</td>
<td>3.383</td>
<td>1.245</td>
<td>0.944</td>
<td>4.89</td>
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</table>

Table 3.5: Parameters for Gamma and normal distributions (estimates based on Table 3.4); $\gamma$: shape parameter; $\mu$: location parameter; $\beta$: scale parameter; set B, load, azimuth 25 degrees

<table>
<thead>
<tr>
<th>Conf./repeat</th>
<th>Moment estimators</th>
<th>Gamma distribution</th>
<th>Normal distribution</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>$\gamma$</td>
<td>$\mu$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>1/01</td>
<td>546.5</td>
<td>0.024</td>
<td>-9.244</td>
</tr>
<tr>
<td>1/02</td>
<td>171.4</td>
<td>0.042</td>
<td>-3.245</td>
</tr>
<tr>
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<td>59.33</td>
<td>0.076</td>
<td>-0.503</td>
</tr>
<tr>
<td>2/01</td>
<td>4.36</td>
<td>0.515</td>
<td>0.999</td>
</tr>
<tr>
<td>2/04</td>
<td>7.91</td>
<td>0.380</td>
<td>0.323</td>
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<td>3/04</td>
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<td>0.411</td>
<td>0.952</td>
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<td>3/15</td>
<td>3.98</td>
<td>0.571</td>
<td>1.051</td>
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<tr>
<td>4/08</td>
<td>4.12</td>
<td>0.617</td>
<td>0.669</td>
</tr>
<tr>
<td>4/09</td>
<td>5.05</td>
<td>0.552</td>
<td>0.426</td>
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<tr>
<td>4/13</td>
<td>4.69</td>
<td>0.604</td>
<td>0.471</td>
</tr>
<tr>
<td>6/05</td>
<td>9.09</td>
<td>0.293</td>
<td>0.467</td>
</tr>
<tr>
<td>6/06</td>
<td>8.26</td>
<td>0.295</td>
<td>0.618</td>
</tr>
<tr>
<td>7/04</td>
<td>10.79</td>
<td>0.272</td>
<td>0.71</td>
</tr>
<tr>
<td>7/10</td>
<td>6.72</td>
<td>0.353</td>
<td>1.309</td>
</tr>
<tr>
<td>8/05</td>
<td>3.20</td>
<td>0.735</td>
<td>1.005</td>
</tr>
<tr>
<td>8/14</td>
<td>4.49</td>
<td>0.588</td>
<td>0.744</td>
</tr>
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</table>
the three parameters for a possible Gamma distribution are $\gamma=502.7650$, $\mu=-4.02$ and $\beta=0.0101$. The shape parameter $\gamma$ is so large that the corresponding term in equation 3.6 approaches infinity when the parameters are substituted into it. Consequently, a matching Gamma distribution cannot be found for such a parent time series.

Figure 3.10: Histogram with fitting normal PDF; tap 50909; record 1000-01

Figure 3.11 presents the histogram of the parent time series, fitted with both normal and Gamma PDFs for measurements at tap 50900 in record 1000-01. The parameters of both PDFs were estimated by the method of moments. It is obvious that the Gamma PDF provides a better fit. In this case, the three parameters for the fitting Gamma distribution are $\gamma=14.37$, $\mu=0.4336$ and $\beta=0.0454$. It should be noted that the shape parameter is not large, which assures the existence of the Gamma PDF. Consequently, the normal distribution is not the best for all measurements by all taps in Config.1.

Table 3.6 presents the skewness, kurtosis and the three parameters for the associated fitting Gamma distribution of the parent time series, record 1000, repeats 01-16. Results for two taps are compared here: tap 50900
and tap 50909. Obviously, the shape parameter of the time series at tap 50900 is quite normal, while for tap 50909, the corresponding shape parameters become very large. Hence, no Gamma distribution can be found to match the histograms for time series at tap 50909. It should be noted here that pressure tap 50900 is in the separation region, whereas tap 50909 is in the region of flow reattachment. Consequently, even for the same flow configuration, the pressure coefficients over taps in different flow regions exhibit different statistical characteristics and should be fitted by different probability distribution functions.

To avoid extreme cases as it is obvious by the fitting of records from different taps of Config.1, load time series are considered to determine the effect of the incident turbulence intensities on the skewness, kurtosis, and Gamma-distribution parameters. Table 3.7 lists the average (over 16 repeats) parameters of the time series, the fitting Gamma distribution against the incident turbulence intensities for both normal (set A) and oblique (set B) incident flow cases. The three parameters for the Gamma distribution, namely the shape, scale and location parameters, are estimated by
Table 3.6: Skewness, kurtosis and the three parameters for the fitting Gamma distribution: $\mu$, $\beta$ and $\gamma$; taps 50900 and 50909; record 1000, repeats 01-16

<table>
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<tr>
<th>Repeat</th>
<th>Tap</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>$\mu$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
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<td>-4.0200</td>
<td>0.0101</td>
<td>502.7650</td>
</tr>
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</table>
Table 3.7: Average parameters for time series, Gamma distribution, incident turbulence intensities and parameters of small-scale turbulence

<table>
<thead>
<tr>
<th>Conf.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Shape</th>
<th>Scale</th>
<th>Location</th>
<th>$S_u/U$ (%)</th>
<th>$S_v/U$ (%)</th>
<th>$S$</th>
</tr>
</thead>
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<tr>
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<td>0.371</td>
<td>19.3</td>
<td>18.2</td>
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Set B, load, azimuth 25 degrees

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<tr>
<th>Conf.</th>
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<th>Kurtosis</th>
<th>Shape</th>
<th>Scale</th>
<th>Location</th>
<th>$S_u/U$ (%)</th>
<th>$S_v/U$ (%)</th>
<th>$S$</th>
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<td>19.3</td>
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</tbody>
</table>

The method of moments based on the basic statistical parameters such as the skewness and kurtosis. It is obvious that for Config.1, whose u- and v- turbulence intensities are both significantly lower than those of other configurations, the skewness is much lower than that of other configurations. The low skewness results in a high shape parameter, which may lead to the failure of the Gamma distribution as a fit to the time series. Moreover, high values of skewness and kurtosis can be associated with the cases of higher turbulence intensities for both sets A and B. As for the Gamma-distribution parameters, higher turbulence intensities consistently lead to higher scale and location parameters and lower shape parameters.

3.2.2 Probability distribution of pressure peaks

Gumbel distribution

The extreme value type I distribution, also referred to as Gumbel distribution, is theoretically the distribution of the maximum values of a Gamma distributed time series ([29]). The Cumulative Distribution Function (CDF) of the Gumbel distribution (maximum) is commonly expressed as

$$F(x) = e^{-e^{-\frac{x-a}{b}}}$$ (3.9)
where $a$ is the location parameter and $b$ is the scale parameter. An example of the CDF curve is presented in figure 3.12. In many applications, it is convenient to transform the original expression into a linear relation by taking twice of negative natural logarithm to both sides. The transformed CDF is then given by

$$-\ln(-\ln(F(x))) = \frac{1}{b}x - \frac{a}{b} = Px - Q \quad (3.10)$$

Obviously, such a relation is linear. The new CDF $\tilde{F}(x) = -\ln(-\ln(F(x)))$ is determined by two coefficients, $P$ and $Q$, which come from the location and scale parameters of the original Gumbel distribution. It will be shown that this transformation results in significant simplification of the analysis. Thereafter, the scale produced by taking twice of negative natural logarithm to an originally linear scale is referred to as the Gumbel scale.

![Figure 3.12: Gumbel cumulative distribution function for the maximum case](image)

The PDF of the Gumbel distribution is commonly defined as

$$f(x) = \text{Exp}\left(\frac{x-a}{b}\right)\text{Exp}\left(-\text{Exp}\left(\frac{x-a}{b}\right)/b\right) \quad (3.11)$$
As a successful precedent, Kasperski (2000, [8]) used the Gumbel distribution to fit the peaks of bending moments in a low-rise structure. In this section, we will use the Gumbel distribution to see whether it can be used as an appropriate distribution to describe surface pressure peaks and wind loads under different flow conditions.

**Probability of non-exceedence**

Unlike the whole time series which includes a large number of data points, the number of pressure peaks from sixteen repeats is too small to yield reliable statistical results. To obtain a larger sample of pressure peaks, we split each of the sixteen 120-second records into four sub-records. Thus, we obtain 64 pressure peaks, each of which has the maximum magnitude in a 30-second sub-record.

Following the above process, the 64 peaks were sorted in an ascendant order. The probability of non-exceedence is defined according to the following procedure: the probability that a certain peak is likely not to exceed the smallest (or largest in absolute sense) peak is $1/64$. Similarly, a peak is likely not to exceed the second smallest peak with a probability of $2/64$, ... And finally the probability of which a peak is smaller than the second largest peak is $63/64$. Thus, in correspondence with the ascendant series of pressure peaks, its non-exceedence probability turns out to be a series of $\{1/64, 2/64, \ldots, 63/64\}$.

In fact, the probability of non-exceedence is largely equivalent to the CDF of a certain random variable. Therefore, the Gumbel scaled non-exceedence probability (or CDF) should bear a linear relation with the peak value $x$, provided that the Gumbel distribution constitutes an acceptable description of the peaks.

In figure 3.13, the probability of non-exceedence of the 64 observed pressure peaks of record 8025 (repeats 01 to 16) is plotted on the Gumbel scale.
As expected, all the resulting points approximately fall onto a straight line. Such a fitting straight line was determined by the least squares method. Obviously, this straight line matches the 64 points very well. This confirms the assumption that the Gumbel distribution can be used to give a reasonable description of the peak distribution. Furthermore, this process provides an efficient approach to determine the parameters of the Gumbel distribution. Making use of the 64 observed peak values, one can always find a fitting straight line by the least squares. Then on the Gumbel scale, it is not difficult to obtain the slope and intersect of the line. Hence, the expression of the scaled CDF of the Gumbel distribution becomes known. As a final step, one can take the inverse transform of the straight line equation to obtain the expression of the original Gumbel distribution. This resulting Gumbel distribution is exactly the distribution of the pressure peaks. In practice, however, there is no need to conduct this process, because the probability of non-exceedence on the Gumbel scale is sufficiently straightforward and informative in most applications.

![Figure 3.13: Probability of non-exceedence and the fitting straight line of the 64 observed pressure peaks on the Gumbel scale; tap S2; record 8025](image)

As another example for normal incidence cases, figure 3.14 shows the probability of non-exceedence of the 64 peaks observed from tap 50900,
record 2000. The roughness configuration is No.2, and the azimuth is 0°. Again, all 64 resulting points can be fitted along a straight line, which is also shown in the same figure. This is an indication of the effectiveness of the Gumbel distribution in describing surface pressure peaks for both normal and oblique cases.

Figure 3.14: Probability of non-exceedence and the fitting straight line of the 64 observed pressure peaks on the Gumbel scale; tap 50900; record 2000

For observations from different locations, the fitting straight line on the Gumbel scale has different slopes and intersects. This implies that the parameters of the Gumbel distribution are not constant for all cases. And in some cases, the resulting points on the Gumbel scale could also have a tilted tail at the end of large peak values, which forms a tendency to deviate from the fitting straight line. Such a situation is shown in figure 3.15, for record 1025 and tap S6. From the figure, it is observed that the circular points start to deviate from the straight line near the peak value of 4.5. Beyond this value, the straight line is definitely not an acceptable fit. In part, such a deviation results from the possibility that the Gumbel distribution might not be suitable for the pressure peaks at some taps of Config.1. This problem has been discussed in detail in Section 3.2.1.
Figure 3.15: Probability of non-exceedence and the fitting straight line of the 64 observed pressure peaks on the Gumbel scale; tap S6; record 1025

The deviation of peaks from the fitting straight line can also be due to the limitation of the measuring time. Typically a full-scale measurement lasts many hours, whereas for wind tunnel experiments, it is practically impossible to conduct measurements over long durations. This inability might partially account for the deviation in some cases. In this sense, the true reason for the deviation will become clear when the experiments are conducted over sufficiently long durations.

**Prediction with observed pressure peaks**

In most cases, when the parent time series follows the Gamma distribution, the peak values are theoretically Gumbel distributed. Based on the 64 observed pressure peaks, the fitting straight line can be used to estimate the probability of non-exceedence for any given peak value. This is quite an effective and straightforward approach.

The process is illustrated in figure 3.16. With a limited population of 64 pressure peaks, one can produce a fitting line by the least squares
method. Then, for a given peak value, the probability of non-exceedence is determined from the Gumbel scaled diagram, even if the given peak is not actually observed. However, in fact, due to the reasons presented earlier, the resulting Gumbel distribution may not precisely represent the real statistics of the peaks. Therefore further analysis is required to confirm the validity of this simple method.

![Graph showing probability of non-exceedence versus peak pressure coefficient]

**Figure 3.16:** Illustration of making predictions with the scaled Gumbel CFD line

**Prediction of pressure peaks with the parent time series**

Because 64 observed peaks may not provide a sufficiently large sample, another approach to predict pressure peaks is proposed. Following the discussion in Section 3.2.1, it is assumed that the Gamma distribution has already been established as a fair fit for the parent time series. By the method of moments again, estimators for the two parameters of the Gumbel distribution are given by:

\[
\begin{align*}
    b &= \sqrt{6} \sigma \\
    a &= \bar{X} - 0.5772b
\end{align*}
\]

where the two parameters \( a \) and \( b \) are respectively the location and scale parameters of the Gumbel distribution as defined in equation 3.11. \( \bar{X} \) de-
notes the mean of the time series, and $\sigma$ denotes the standard deviation.

In this approach, a mapping procedure is followed. A numerical CDF of the peak distribution is determined by mapping the corresponding points from the standard normal CDF curve onto the Gamma CDF curve of the parent time series. After taking derivative of the numerical CDF, one may find the numerical PDF of the extreme value distribution. This peak distribution should not have large discrepancy from the Gumbel PDF, since the parent time series is Gamma distributed. Using equation 3.12, one can determine the two parameters of the Gumbel distribution. Then finally, the numerical representation of the Gumbel PDF are smoothed by the analytical result. Hence, the only source needed for this procedure is the parent time series, which provides a large number of data for prediction. This procedure also shows that the peak information is in fact implied in the parent time series. The detailed instructions and discussions for this mapping procedure is presented by Sadek et al. (2002, [9]).

**Comparison of the two prediction approaches**

Figure 3.17 shows the results by the two aforementioned approaches for record 4025-01, load. There are 17 straight lines in the figure: the dashed line is based on the 64 observed pressure peaks, and the other 16 lines are produced by the mapping procedure for all the 16 repeats of the record. Apparently, the group of mapping results are primarily on the right of the fitting line. This indicates that for a given potential peak value, the mapping approach will give a lower probability of non-exceedence, while the approach based on observed peaks will give a higher probability of non-exceedence. Therefore, the mapping approach is systematically more conservative. Sadek et al. (2002, [9]) pointed out that the mapping approach is more stable and thus superior to the method that uses observed peaks. However, because of the discrepancy of the 16 straight lines, it seems logical to deduce that the duration of 120 seconds for a single record is far from being sufficient. To obtain results that are statistically stable, one should
expect a cluster of 16 converged lines, but not a group of scattered lines.

Figure 3.17: Comparison of the two peak predicting methods: hollow circular dots: observed 64 peaks; dash line: fitting straight line for the 64 peaks; solid lines: Gumbel scaled peak CDFs for 16 repeats predicted by the mapping procedure; record 4025; load

Results and discussion

Figure 3.18 shows the fitting Gamma and Gumbel PDFs for the parent load time series and the associated extreme values (pressure peaks) for record 7000-01. The Gamma distribution is determined by the method of moments introduced in Section 3.2.1, and the Gumbel distribution is established by the mapping procedure and moment estimators. It is noted that the PDF curve of the peaks is comparatively towering and steep, implying the narrower range of probable values. It is also observed that the occurrence of a peak of 5.5 in value is even not improbable, although the observed pressures in the time series are primarily bounded between 0.5 and 3.4. Another example is shown in figure 3.19, for a case of oblique incidence as measured in Config.4, repeat 01. Obviously, the range of peak values is much smaller than that of the parent time series, and a peak value of 17.0 is likely to occur with a non-zero probability.
From the estimated Gumbel distribution of the pressure peaks, mean peak load coefficients with 84%, 97.5% and 99% probability of non-exceedence can be calculated. The dependence of the mean peak load coefficients for the three probabilities of non-exceedence on the values of the shape parameter for different configurations is presented in figures 3.20 and 3.21, for pressure taps in both sets A and B. The observed peak load coefficients for the three probability of non-exceedence for different configurations are also included in the two figures. Both figures show a trend, whereby an increase in the shape parameter can be associated with a decrease in the mean peak value. This is true for all configurations except Config.7, where a high level of small-scale turbulence was introduced in the incident flow. Hence, the level of the pressure peaks can be associated with the shape parameter of the fitting Gamma distribution for the parent time series. Since by the method of moments, the shape parameter is inversely proportional to the skewness of the time series, a more skewed Gamma distribution curve (with higher skewness and thus lower shape parameter) leads to higher levels of pressure peaks. This property holds for both normal and oblique incidence.
Figure 3.19: Parent time series fitted by the Gamma distribution and its extreme values predicted by the Gumbel distribution; record 4025-01; load cases. Also shown in figures 3.20 and 3.21 are the mean observed peak values for the same three levels of probability of non-exceedence for different configurations. Obviously, these values are in the same trend as the estimated peak values for both normal and oblique incidence cases.

To assess the effect of the turbulence intensities in the incident flow, wind loads and peak distributions were fitted by Gamma and Gumbel distributions, respectively, for records 2000-01 and 8000-01. The two selected records represent high turbulence intensities (Config.8) and low turbulence intensities (Config.2), as shown in Table 3.7. The fitting PDF curves are presented in figure 3.22. By comparing the two curves, it is noted that the distribution of the peak values for Config.2 is largely over the range between 2.7 and 4.0, while the peak distribution for Config.8 is over a range between 4.0 and 5.0. Moreover, the PDF curve of the peak values for Config.8 is much flatter than that for Config.2. This reveals that higher turbulence intensities in the normal incident flow result in higher probable peak values and larger standard deviation for the peak distribution. For oblique incidence cases, the fitting Gamma and Gumbel distributions for the parent
Figure 3.20: Estimated and observed mean peak load coefficients with 84%, 97.5% and 99% probability of non-exceedence and the corresponding shape parameters of the fitting Gamma distribution for the parent time series; load; set A.

time series and the peak values of the load coefficient, are shown in figure 3.23. Two records, 2025-01 with low turbulence intensities, and 8025-01 with high turbulence intensities, are considered again. It is noted that, although the difference between the distributions of the parent time series is not significant, the probable peak values for record 8025-01 is generally higher than those for record 2025-01. Moreover, the PDF curve of the peak values for record 8025-01 is flatter than that for record 8025-01. Hence, the results imply that higher turbulence intensities in the oblique incident flow lead to higher probable peak values and larger standard deviation for the peak distribution.

The effect of the small-scale turbulence, as shown in Table 3.7, on the distribution of pressure peaks are studied, by examining the fitting Gamma and Gumbel PDFs for the parent time series and peak values for both normal and oblique cases. The fitting PDF curves for records 7000-01, with a high parameter of small-scale turbulence, and 8000-01, with a low pa-
Figure 3.21: Estimated and observed mean peak load coefficients with 84%, 97.5% and 99% probability of non-exceedence and the corresponding shape parameters of the fitting Gamma distribution for the parent time series; load; set B

Parameter of small-scale turbulence, are shown in figure 3.24 for the normal incidence case. The fitting PDF curves for records 7025-01 and 8025-01 are presented in figure 3.25, for the oblique incidence cases. It is noted, from the two figures, that the PDF curves of the parent time series for records 8000-01 and 8025-01 are more skewed than those for records 7000-01 and 7025-01, respectively. Furthermore, the probable peak values for records 8000-01 and 8025-01 are obviously higher and more scattered than those for records 7000-01 and 7025-01, respectively. Therefore, the observations imply that with a high content of small-scale turbulence, as for Config.7, the skewness of the distribution of the parent time series tends to be lower, which results in a higher shape parameter for the fitting Gamma PDF and a generally lower but more concentrated peak values. This is applicable to both normal and oblique incidence cases.
Figure 3.22: Pressure coefficient and pressure peak distributions fitted by Gamma and Gumbel distributions; Config.2 and Config.8; load; azimuth 0°; set A

Figure 3.23: Pressure coefficient and pressure peak distributions fitted by Gamma and Gumbel distributions; Config.2 and Config.8; load; azimuth 25°; set B
Figure 3.24: Pressure coefficient and pressure peak distributions fitted by Gamma and Gumbel distributions; Config.7 and Config.8; load; azimuth 0°; set A

Figure 3.25: Pressure coefficient and pressure peak distributions fitted by Gamma and Gumbel distributions; Config.7 and Config.8; load; azimuth 25°; set B
Part II

Pressure-velocity relations
Chapter 4

Detection of nonlinear phenomena in nonstationary signals

In relation to wind loads on structures, Fourier-domain analysis has been the main tool used to describe the incident wind, its turbulent characteristics, and their effects on surface pressures. The underlying process in Fourier decomposition is the representation of a signal onto a set of complex sinusoids. Such a representation leads to many shortcomings when applied to analyzing time variations of turbulent scales in wind velocity components and associated surface pressure coefficients. On the other hand, Hajj et al. (1998, [30]) and Jordan et al. (1997, [19]) showed that the wavelet representation is more advantageous in the analysis of full-scale wind characteristics and pressure peaks. The results of [30] only stressed the linear relation between far-field low-frequency mean flow unsteadiness and observed pressure peaks. One objective of this thesis is to study the linear and nonlinear near-field velocity and surface pressure fluctuations. Consequently, the need arises for the development of tools capable of detecting nonlinear couplings in intermittent events or nonstationary signals. In this chapter, the wavelet-based higher order spectral moments are presented and developed as a tool to identify nonlinear effects in nonstationary phenomena. Analysis and application of the proposed techniques to analytic signals show that the wavelet-based moments are suitable for detecting intermittent linear and nonlinear relations embedded in statistical noise.
4.1 Definition and implications of stationarity

A random process is considered stationary when the mean value is a constant and the various higher-order correlation functions are functions of the time difference only, i.e.

\[
E[x(t)] = \text{constant} \quad (4.1)
\]

\[
E[x(t)x^*(t - \tau)] = R(\tau) \quad (4.2)
\]

\[
E[x(t)x^*(t - \tau_1)x^*(t - \tau_2)] = R(\tau_1, \tau_2) \quad (4.3)
\]

\[
\vdots
\]

\[
E[x(t)x^*(t - \tau_1) \cdots x^*(t - \tau_n)] = R(\tau_1, \cdots, \tau_n) \quad (4.4)
\]

where \(E[\cdots]\) denotes ensemble average, and \(R(\cdots)\) denotes the correlation function.

In many applications, the interest is only in weakly stationary random processes, i.e., when only equations 4.1 and 4.2 are satisfied. The implications of weak stationarity is that if a random process is represented by a sum of complex sinusoids, their random complex amplitudes are uncorrelated. When the hierarchy of correlation functions presented above show time variations, the signals are termed nonstationary. One implication is that the statistics cannot be estimated over long periods. They can only be approximated by taking short data segments from the parent time series. Consequently, the representation of a nonstationary random process by a sum of complex sinusoids would not be appropriate.

4.2 Detection of nonlinearities with phase-coupling

Hajj et al. (1997, [31]) pointed out that in nonlinear systems, frequency components interact with each other to transfer energy to their sum or difference component. More importantly, during such a process, the phase of
the sum or difference component is related to the phases of the primary interacting modes. A schematic of a quadratic nonlinearity between three coupled complex frequency components is presented in figure 4.1. The figure shows three quadratically coupled frequency components at $f_i$, $f_j$ and their sum $f_i + f_j$. Their corresponding phase angles are denoted as $a_x(f_i)$, $a_x(f_j)$ and $a_x(f_i + f_j)$. The level of phase coupling between the three frequency components is represented by the time variations of the relation 

$$\Delta a = a_x(f_i + f_j) - (a_x(f_i) + a_x(f_j))$$

Perfect phase coupling is obtained when this phase relation is time-independent. As will be shown below, this phase relation is actually given by the phase of the third-order spectral moment known as the bispectrum (Kim and Powers, 1979, [32]).

Figure 4.1: Quadratic phase coupling between two frequency components and their sum component in the spectrum of signal $x(t)$; $\tilde{F}_x(f_i)$ and $\tilde{F}_x(f_j)$ denote two frequency components at respectively $f_i$ and $f_j$ in the spectrum of signal $x(t)$; $\tilde{F}_x(f_i + f_j)$ represents their sum component at frequency $f_i + f_j$; $a_x(f)$ is the phase angle for the frequency component $\tilde{F}_x(f)$.  

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4.3 Fourier-based spectral analysis

Fourier-domain analysis has been one of the most effective tools used for signal processing. Quantities derived from the Fourier transform, including Short Time Fourier Transform (STFT), Fourier-based spectra and higher-order spectral moments, have been widely used to detect nonstationary and nonlinear aspects in energy-limited signals. Fourier-based analysis tends to be the starting point for signal processing in many applications.

4.3.1 Definitions

The Fourier transform of a zero-mean signal $x(t)$, which is a real function satisfying the Dirichlet conditions ($x(t)$ bounded with only a finite number of maxima, minima and/or discontinuities over any one period), is defined as:

$$\hat{x}(f) = \int_{-\infty}^{+\infty} x(t)e^{-i2\pi ft} dt$$  \hspace{1cm} (4.5)

where $f$ is the frequency variable in the frequency domain, and $i = \sqrt{-1}$. The inverse Fourier transform is then given by

$$x(t) = \int_{-\infty}^{+\infty} \hat{x}(f)e^{i2\pi ft} df$$  \hspace{1cm} (4.6)

Using the Fourier transform, one can define many quantities relevant to the understanding of physical phenomena. The power spectrum, defined as the Fourier transform of the second-order covariance function, is estimated as:

$$P_{xx}(f) = \lim_{T \to \infty} \frac{1}{T} E[\hat{x}(f)\hat{x}^*(f)]$$  \hspace{1cm} (4.7)

where $T$ denotes the time duration of the signal $x(t)$. $E[\cdots]$ denotes a time average, and $(\cdots)^*$ represents the complex conjugate. The power spectrum yields an estimate of the distribution of energy density among frequency components of the fluctuations. The cross-power spectrum for two zero-mean signals $x(t)$ and $y(t)$ can be defined as

$$P_{xy}(f) = \lim_{T \to \infty} \frac{1}{T} E[\hat{x}(f)\hat{y}^*(f)]$$  \hspace{1cm} (4.8)
The cross-power spectrum is usually normalized to yield the linear coherence, which is given by

$$C_{xy}(f) = \frac{|P_{xy}(f)|^2}{P_{xx}(f)P_{yy}(f)}$$

(4.9)

Based on Schwarz inequality, the linear coherence assumes values between 0 and 1. A zero value indicates no coherence. A value of one indicates perfect coherence, and values between 0 and 1 indicate partial coherence.

Higher-order spectral analysis of fluctuations yields quantities relevant to the identification of nonlinear relations between frequency components. The bispectrum, defined as the double Fourier transform of the third-order covariance function, yields a measure of the phase relation between three components with frequencies that sum to zero (Kim and Powers, 1979, [32]). For three frequency components, \(f_i\), \(f_j\) and their sum \(f_i + f_j\), the auto-bispectrum is estimated as:

$$B_{xxx}(f_i, f_j) = \lim_{T \to \infty} \frac{1}{T} E[\hat{x}(f_i + f_j)\hat{x}^*(f_i)\hat{x}^*(f_j)]$$

(4.10)

Similarly, the cross-bispectrum between two frequency components at \(f_i\) and \(f_j\) in one signal \(x(t)\) and the frequency component at \(f_i + f_j\) in another signal \(y(t)\) is estimated as

$$B_{xyy}(f_i, f_j) = \lim_{T \to \infty} \frac{1}{T} E[\hat{x}(f_i + f_j)\hat{y}^*(f_i)\hat{y}^*(f_j)]$$

(4.11)

In both equations 4.10 and 4.11, \(f_i\) and \(f_j\) are arbitrary frequencies limited by the Nyquist frequency \(f_N\). If the three frequency components at \(f_i\), \(f_j\) and \(f_i + f_j\) are independent, i.e. each mode is characterized by a statistically independent random phase, the bispectrum will have a near-zero value after the statistical averaging is carried out. On the other hand, if the three modes are coupled through a quadratic interaction mechanism, a phase coherence will exist among them. Under these conditions, the averaging will lead to a large value for the bispectrum. The bispectrum, thus, preserves the phase information between coupled frequency components in
the form of a phase relation, which could not be obtained from the power spectrum. Because of this capability, the bispectrum could be used to identify quadratically coupled frequency components in one or more signals.

In order to remove the dependence of the bispectrum on the amplitudes of the three frequency components under consideration, the bispectrum is usually normalized to yield values bounded between 0 and 1. The normalized auto-bispectrum, referred to as auto-bicoherence, is defined as:

$$b_{xxx}^2(f_i, f_j) = \frac{|B_{xxx}(f_i, f_j)|^2}{E[|\hat{x}(f_i + f_j)|^2]E[|\hat{x}(f_i)\hat{x}(f_j)|^2]}$$

Similarly, the cross-bicoherence, is defined as

$$b_{xyy}^2(f_i, f_j) = \frac{|B_{xyy}(f_i, f_j)|^2}{E[|\hat{x}(f_i + f_j)|^2]E[|\hat{y}(f_i)\hat{y}(f_j)|^2]}$$

In both of the above expressions, the triplet of frequency components at frequencies $f_i$, $f_j$ and $f_i + f_j$ are quadratically coupled if $b^2(f_i, f_j) = 1$, not quadratically coupled if $b^2(f_i, f_j) = 0$, and partially coupled if $0 < b^2(f_i, f_j) < 1$.

The auto-bispectrum possesses important symmetry properties (Kim and Powers, 1979, [32]), which allows for significant reduction in its computation. Based on these properties, the auto-bispectrum or auto-bicoherence needs to be calculated only in the $S$ region in figure 4.2, while the cross-bispectrum needs to be calculated in both $S$ and $D$ regions of the same figure.

### 4.4 Wavelet-based analysis

The bispectral moments and associated bicoherence that are based on the Fourier transform do not preserve temporal information. Yet, in many applications, the signal might be nonstationary and/or the interest lies in specific intermittent events. Under these conditions, there would be a need to quantify the nonlinear interactions as a function of time. Obviously, and because of the averaging needed for their computations, the Fourier-based
bispectral moments are not adequate to determine phase couplings that take place over short durations. Because the wavelet transform has proven to be powerful and efficient in analysis of localized events, the wavelet-based bispectral analysis has been suggested as an efficient tool to detect short-term phase couplings (van Milligen, et al., 1995, [33]).

The application of wavelet analysis started in 1980’s, although the first mention of wavelets could be dated back to 1909, when Harr described the Harr wavelet in an appendix to his thesis (Meyer, 1993, [34]). To date, the wavelet analysis has been employed in a variety of disciplines (Addison, 2002, [35]). The fundamental advantage of the wavelet-based analysis is its ability to unfold signals into both time and scales, and thus, overcome the shortcomings of applying Fourier-based analysis to nonstationary signals. It should, however, be noted here that the interpretation of the wavelet-based analysis must be carefully conducted, because the resulting wavelet coefficients may contain redundant information. Compared with the short
time Fourier transform, which is also more effective than the Fourier analysis in processing nonstationary signals, the wavelet analysis yields better resolutions in both time and frequency/scale domains, since the wavelet analysis makes use of flexible window widths (Mallat, 1995, [36]).

### 4.4.1 Continuous wavelet transform

The Continuous Wavelet Transform (CWT) of a real signal \( x(t) \) is defined as:

\[
W_x(a, \tau) = \int_{-\infty}^{+\infty} x(t) \psi_{a,\tau}^*(t) dt \tag{4.14}
\]

where \( a \) and \( \tau \) are scale and time variables respectively, and \( \psi_{a,\tau} \) represents the wavelet family generated by continuous translations and dilations of the mother wavelet \( \psi(t) \), and \( W_x(a, \tau) \) are the resulting wavelet coefficients.

Typically, these translations and dilations are obtained by

\[
\psi_{a,\tau}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t - \tau}{a}\right) \tag{4.15}
\]

In this equation, the \( \frac{1}{\sqrt{a}} \) coefficient is usually chosen so that all wavelets will have the same energy for every scale. For computational purposes, it is more convenient to express the wavelet transform in the Fourier domain. The convolution in equation 4.14 is then simplified to a direct multiplication:

\[
W_x(a, \tau) = \int_{-\infty}^{+\infty} \hat{x}(f) \hat{\psi}_{a,\tau}^*(f) df \tag{4.16}
\]

Here, \( f \) is the frequency variable, and \( \hat{\psi}_{a,\tau}(f) \) is the Fourier transform of \( \psi_{a,\tau}(t) \). Based on this relation, the wavelet coefficients can be interpreted as filtered versions of \( x(t) \) that are band-pass filtered by \( \hat{\psi}_{a,\tau}(f) \). It is important to note that equation 4.16 holds for any family of wavelets (Addison, 2002, [35]).

The Morlet wavelet (Goupilland et al., 1984, [37]; Torrence et al., 1998, [38] and Addison, 2002, [35]) to be implemented in this work is defined as

\[
\psi(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2} \tag{4.17}
\]
In this definition, $\omega_0$ is set equal to 6.0 to approximately satisfy the wavelet admissibility condition. For the Morlet wavelet, the relation between the frequency and scale is given as (Torrence et al., 1998, [38])

$$f = \frac{c}{a} \text{ where } c = \frac{\omega_0 + \sqrt{2 + \omega_0^2}}{4\pi}$$  \hspace{1cm} (4.18)

Hence, when $\omega_0=6.0$, $f = \frac{0.9434}{a}$.

### 4.4.2 Wavelet spectral moments

Similarly to the Fourier-based spectral moments defined in Section 4.3, one can define a hierarchy of wavelet-based higher-order spectral moments. The wavelet power spectrum is defined as

$$P_{wx}(a) = \int_T W_x^*(a, \tau)W_x(a, \tau)d\tau$$  \hspace{1cm} (4.19)

which yields the energy distribution among all scales over the integration time $T$. The wavelet cross-spectrum, defined as

$$P_{wy}(a) = \int_T W_x^*(a, \tau)W_y(a, \tau)d\tau$$  \hspace{1cm} (4.20)

can be used to determine the level of coupling between equal scales (or frequencies) in two time series over the integration time length $T$. The normalized wavelet cross spectrum, namely, the wavelet linear coherence, is defined as

$$C_{xy}^w(a) = \frac{|P_{xy}(a)|^2}{P_{xx}(a)P_{yy}(a)}$$  \hspace{1cm} (4.21)

yields a measure of the linear coherence of the same scale in two time series over the integration time length $T$. Similarly to the Fourier-based linear coherence, $C_{xy}^w(a)$ is bounded by zero and one. A near-zero value indicates no coupling, while a near-one value indicates a perfect coupling. Values between zero and one indicate partial couplings.

Based on the definition of the auto-bispectrum in equation 4.10, one can define the wavelet auto-bispectrum as

$$B_{xxx}^w(a_1, a_2) = \int_T W_x^*(a, \tau)W_x(a_1, \tau)W_x(a_2, \tau)d\tau$$  \hspace{1cm} (4.22)
where $\frac{1}{a} = \frac{1}{a_1} + \frac{1}{a_2}$ is used to satisfy the frequency sum-rule $f = f_1 + f_2$. Additionally, the wavelet cross-bispectrum between two real time series $x(t)$ and $y(t)$ can be defined as

$$B^w_{yxx}(a_1, a_2) = \int_T W^*_y(a, \tau)W_x(a_1, \tau)W_x(a_2, \tau)d\tau \quad (4.23)$$

The wavelet auto- and cross-bispectrum can also be normalized to have values bounded between zero and one. They, respectively, yield the wavelet auto-bicoherence, defined as

$$\tilde{b}^2_{xxx}(a_1, a_2) = \frac{|B^w_{xxx}(a_1, a_2)|^2}{(\int_T |W_x(a_1, \tau)W_x(a_2, \tau)|^2d\tau)(\int_T |W_x(a, \tau)|^2d\tau)} \quad (4.24)$$

and the wavelet cross-bicoherence, defined as

$$\tilde{b}^2_{yxx}(a_1, a_2) = \frac{|B^w_{yxx}(a_1, a_2)|^2}{(\int_T |W_x(a_1, \tau)W_x(a_2, \tau)|^2d\tau)(\int_T |W_y(a, \tau)|^2d\tau)} \quad (4.25)$$

In addition to the wavelet-based bicoherence, one can define the summed-bicoherence as (Koronovski et al., 2002, [39])

$$b_\Sigma(a) = \sqrt{\frac{1}{N_s} \sum \left[ b^2(a_1, a_2) \right]^2} \quad (4.26)$$

This summation quantifies how the scale $a$ is affected by quadratic couplings of all scales $a_1$ and $a_2$ that satisfy the frequency sum-rule. Consequently, the summed-bicoherence can be used to determine which scale (or frequency component) contains the most quadratic couplings among all scales or frequencies that satisfy the sum-rule.

### 4.4.3 Statistical noise level

Van Milligen et al. (1997, [40] and 1995, [33]) pointed out that because the wavelets of the Morlet wavelet family are not orthogonal, the wavelet coefficients are not statistically independent. Due to the periodicity of the wavelets of scale $a$, two statistically independent estimates of the wavelet coefficients are separated by $a/2$, or by a number of samples $a \times F_s/2$, where $F_s$ is the sampling frequency. When the integration time contains $N$ data
points, the number of independent points or samples is actually \(N/(aF_s/2)\). Consequently, one can define a statistical noise level in the wavelet linear coherence as

\[
\epsilon[C_{xy}^w(a)] \approx 2\left[\frac{F_s}{f} \frac{1}{N}\right]^{1/2}
\] (4.27)

Obviously, at low frequencies, i.e. as \(f\) approaches zero, the noise level will mathematically approach infinity. Although there is no physical meaning for infinite noise, the noise level at finite low frequencies is expected to be relatively high.

For the wavelet bicoherence, van Milligen et al. (1995, [33]) estimates the noise level to be:

\[
\epsilon[\tilde{b}^2(f_1, f_2)] \approx \left[\frac{F_s/2}{\min(|f_1|, |f_2|, |f_1 + f_2|)} \frac{1}{N}\right]^{1/2}
\] (4.28)

Again, at low frequencies, the statistical noise level may dominate the bicoherence.

### 4.5 Fourier- and wavelet-based spectral analysis applied to analytical signals

The usefulness and limitations of the analyzing procedures presented in Sections 4.3 and 4.4 are demonstrated by applying them to two analytical data sets. Each of these sets consists of two signals, with 1500 data points in each signal. The sampling rate in both sets is assumed to be 1000Hz, which is equivalent to the sampling rate of the pressure and the velocity fluctuations to be analyzed in Chapter 5.

In both data sets, each signal consists of three time intervals: interval I covers the times between 0.601 and 0.900 seconds; interval II covers the times between 0.376 and 0.600 seconds and between 0.901 and 1.125 seconds; interval III covers the times between 0.001 and 0.375 seconds and between 1.126 and 1.500 seconds. Note that all three intervals are symmetric about \(t = 0.750\) seconds. In both data sets, different phase coupling
patterns are generated over different time intervals to test the ability of wavelet-based spectral moments in the detection of intermittent nonlinear couplings.

4.5.1 Data set No.1

In the first data set, the two signals, referred to as \(p\) and \(u\), are given by

\[
\begin{align*}
\{ &
p = p_0 + 3\left[\sin(2\pi16t) + \sin(2\pi40t) + \sin(2\pi56t)\right] + 2\text{Rand} \quad \text{(Interval I)} \\
&
u = u_0 + 3\left[\cos(2\pi8t) + \cos(2\pi20t) + \cos(2\pi28t)\right] + 2\text{Rand} \quad \text{(Interval I)} \\
&
p = p_0 + 3\left[\sin(2\pi40t) + \sin(2\pi56t)\right] + 2\text{Rand} \quad \text{(Interval II)} \\
&
u = u_0 + 2\text{Rand} \quad \text{(Interval II)} \\
&
p = p_0 + 2\text{Rand} \quad \text{(Interval III)} \\
&
u = u_0 + 2\text{Rand} \quad \text{(Interval III)} \\
\end{align*}
\]

(4.29)

The mean values, \(p_0\) and \(u_0\), were set equal to -1.2 and 9.8, respectively. The added normally distributed random noise, \(\text{Rand}\), has a zero mean and a standard deviation of one.

In interval I, both \(p\) and \(u\) signals have three characteristic frequency components. Additionally, the three characteristic frequencies of \(p\) are respectively the double of those of \(u\), and their phases are set to be zero. This yields a constant phase relation which simulates the self-interaction of frequency components in \(u\) to transfer energy to their sum components in \(p\). In interval II, \(u\) consists of a pure random noise, while \(p\) retains two of its original sinusoidal frequency components at 40Hz and 56Hz. Hence, the two signals become decoupled in this interval. In interval III, farther away from the center, the fluctuations in both signals consist of noise.

Figure 4.3 shows the \(p\) and \(u\) signals in data set No.1 and as formulated in equation 4.29. The full-length Fourier power spectra of the two signals are shown in figure 4.4. These spectra were obtained by averaging 5 segments of the signals, each consisting of 256 data points. Three local peaks are plainly visible near 16Hz, 40Hz and 56Hz in the spectrum of \(p\). The spectrum of \(u\) also exhibits three peaks near 8Hz, 20Hz and 28Hz. While the spectra show the peaks in both signals, it is important to note that the magnitudes of these peaks do not match with the magnitudes set in
equation 4.29. The issues of leakage and the frequency resolution, as well as the fact that only five segments were used in the estimation of the power spectrum, have contributed to the enhanced noise levels and amplitude variations as observed in the spectra. Figure 4.5 shows the Fourier-based linear coherence. Since the two signals contain different frequency components, it is expected to obtain low linear coherence at all frequencies. The high values for the linear coherence near 15Hz, 80Hz and 90Hz are most likely due to the small number of segments used in the averaging process.

Figure 4.3: Time series of $p(t)$ and $u(t)$ as defined in equation 4.29

Figure 4.6 shows the Fourier auto-bicoherence of the two signals, with contour levels 0.5, 0.7 and 0.9. In the bicoherence plot of $p$, the (40, 16), (56, -16) and (56, -40) pairs of frequency components show levels higher than 0.7. This is due to the relation among the frequency components of these pairs, and to the fact that they have a constant phase relation. The auto-bicoherence plot of the $u$ signal shows high bicoherence values for the frequency pairs (20, 8), (28, -8) and (28, -20), which manifest that the three major components in the $u$ signal are coupled. However, it should be noted that the high couplings cover a broad range of frequencies around these components. Again, the small number of segments used in the averaging
of the bispectral moments yields estimates for the spectral moments with high variance which also explains why some other frequency components have high levels of bicoherence.

As for the cross-bicoherence presented in Figure 4.7, high values are noted for the frequency pairs (8, 8), (20, 20) and (28, 28). This shows how the frequency components of the \( u \) signal are coupled with their sums that constitute the signal \( p \) in interval I. Yet, the levels of 0.5 and 0.7 do not reflect the perfect bicoherence set in interval I. Additionally, other high coupling levels are observed among other components. The presence of highly coupled modes that do not constitute a part of the \( p \) and \( u \) signals implies that estimating the bicoherence from five segments over the duration of 1.5 seconds fails to determine the phase couplings set in interval I. Consequently, the Fourier-based analysis is not effective in detecting intermittent phase coupling. In summary, it fails to separate the couplings happening in interval I from the decouplings in intervals II and III.

The failure of the Fourier-based analysis in detecting phase couplings
that take place over relatively short durations and the suitability of the wavelet-based analysis in characterizing nonstationary phenomena lead to the idea of implementing wavelet-based spectral analysis on data set No.1.

Figure 4.8 shows the wavelet power spectra for \( p \) and \( u \) signals defined in equation 4.29. The spectra are calculated over three different integration time lengths. Centered at \( t=0.75s \), the three integration times, 0.3s, 0.85s and 1.4s, cover intervals I, I+II and I+II+III respectively. In these spectra, the frequency resolution 0.667Hz is determined by the sampling frequency 1000Hz divided by the total number of data points in the time series, 1500. Over the time length of 0.3s, which covers interval I, both signals have perfect phase relations. The other two integration time lengths cover regions where noise is a part of the signals. In Figure 4.8, the three characteristic frequency components are identified by frequency ranges around the major frequencies in both \( p \) and \( u \) wavelet spectra. It can also be noted that for larger integration times, the peaks in the power spectra become more flattened. However, the three major frequency components can still be identified. The linear coherence between the two signals is presented in figure 4.9. The noise levels were estimated according to equation 4.27.
Figure 4.6: Fourier auto-bicoherence of the full-length (a): $p(t)$ and (b): $u(t)$ shown in figure 4.3; contour levels are set at 0.5, 0.7 and 0.9; frequency resolution $df=3.906\text{Hz}$

Obviously, the longer integration time lengths result in lower noise levels. The results clearly show that all coherence levels are lower than the noise levels. This is not unexpected, since the frequency components in the two signals do not have any linear relations.

Figures 4.10-4.12 show contours of the wavelet auto-bicoherence of both signals for the three integration time lengths, 0.3s, 0.85s and 1.40s. The first time length, 0.3s, covers interval I, over which the frequency components of the $p$ and $u$ signals have a constant phase relation. The other two time lengths extend to cover intervals where noise was added. In each of figures 4.10-4.12, the three straight lines parallel to the $f_i$ and $f_j$ axes and the ray that originates from the origin and makes an angle of $-45^\circ$ from the positive $f_i$ axis are contours of statistical noise estimated by equation 4.28. The equation shows that higher frequency components have lower noise levels. Moreover, when the number of data points in the segment increases, the noise level drops accordingly. Hence, taking figure 4.10 as an example, the lines that are farthest from the $f_i$ and $f_j$ axes and the ray described above represent the noise level of 0.7, and the contours that are closest to them have a noise level of 0.9. The contours in between represent noise level of 0.8. For easier comparison, the contour levels of the bicoherence are
Figure 4.7: Fourier cross-bicoherence between the full-length $p(t)$ and $u(t)$ shown in figure 4.3; contour levels are set at 0.5, 0.7 and 0.9; frequency resolution $df = 3.906$Hz

set to be the same as those of the statistical noise.

In figure 4.10, which shows the wavelet auto-bicoherence of both $p$ and $u$ signals over an integration time length of 0.3s, three coupling pairs in $p$, (40, 16), (56, -16) and (56, -40), are clearly observed. The high level of bicoherence (over 0.9) manifests the expected perfect quadratic self-coupling between the frequency components at 16Hz and 40Hz in $p$ in interval I. The self-coupling in $u$ between frequency components at 8Hz and 20Hz is also identified by the three frequency pairs (20, 8), (28, -20) and (28, -8) in the wavelet auto-bicoherence of $u$. Although the integration time length was only 0.3s, the results are not affected by noise, except over the low-frequency ranges. Figure 4.11 shows the wavelet auto-bicoherence of $p$ and $u$ obtained over an integration time of 0.85s, covering intervals I and II. The frequency components in both signals become decoupled in interval II. As a result, the auto-bicoherence levels of both $p$ and $u$ drop slightly to 0.5, 0.6 and 0.7. When the integration time length is increased to 1.4s, which covers interval III, figure 4.12 shows that the bicoherence levels at the coupled frequency pairs do not drop off much from their previous values. Their levels are still 0.5, 0.6 and 0.7. This is due to the fact that the duration of interval III is much shorter than the summed durations of intervals I and II.
Figure 4.8: Wavelet power spectra of $p(t)$ and $u(t)$ shown in figure 4.3 and estimated over three different integration time lengths: 0.30s, 0.85s and 1.40s; frequency resolution $df=0.667$Hz

Figure 4.13-4.15 show the wavelet cross-bicoherences between the $p$ and $u$ signals with the same three integration time lengths, namely 0.3s, 0.85s and 1.4s. The results exhibit high values at the frequency pairs of (8, 8), (20, 20) and (28, 28) in figure 4.13, which identifies the perfect preset frequency doublings in interval I. In figure 4.14, which shows the cross-bicoherence between $p$ and $u$ for the integration time of 0.85s, the levels of coupling among the three frequency pairs decreases. This decrease is significant at the higher frequency pairs (20, 20) and (28, 28). For the integration time length of 1.4s, figure 4.15 shows that the bicoherence levels do not decline as considerably as with the first extension of the integration time. The coupling between the frequency pair (8, 8) is still distinct. Again, this is due to the fact that the duration of interval III is much shorter than the summed durations of intervals I and II.

In summary, the analysis of data set No.1 shows that the wavelet bispectral analysis performs much better than the Fourier bispectral analysis
Figure 4.9: Wavelet linear coherence between $p(t)$ and $u(t)$ shown in figure 4.3 and estimated over three different integration time lengths: 0.30s, 0.85s and 1.40s; frequency resolution $df=0.667$Hz

when examining nonlinear couplings that take place over relatively short-time durations. It reduces statistical noise appreciably, and retains only useful information.

4.5.2 Data set No.2

For data set No.2, the $p$ and $u$ signals are given by

$$\begin{cases}
p = p_0 + 3[\sin(2\pi 16t) + \sin(2\pi 40t) + \sin(2\pi 56t)] + 2\text{Rand} & \text{(Interval I)} \\
u = u_0 + 3[\cos(2\pi 8t) + \cos(2\pi 20t) + \cos(2\pi 56t)] + 2\text{Rand} & \text{(Interval I)} \\
p = p_0 + 2\text{Rand} & \text{(Interval II)} \\
u = u_0 + 2\text{Rand} & \text{(Interval II)} \\
p = p_0 + 2\text{Rand} & \text{(Interval III)} \\
u = u_0 + 2\text{Rand} & \text{(Interval III)}
\end{cases}$$

(4.30)

In this data set, the $p$ and $u$ signals consist of random noise except in interval I. In interval I, the frequency components in $p$ at 16Hz and 40Hz are respectively coupled with the frequency components in $u$ at 8Hz and 20Hz. Additionally, both signals have a frequency component at 56Hz, which should result in a linear coupling between $p$ and $u$ near this frequency.
Figure 4.10: Contour plots of the wavelet auto-bicoherence of (a): $p(t)$ and (b): $u(t)$ shown in figure 4.3 along with the associated statistical noise; integration centered at $t=0.75s$; integration time length: 0.30s; contour levels for both auto-bicoherence and noise are set at 0.7, 0.8 and 0.9; frequency resolution $df=0.667$Hz

Plots of the $p$ and $u$ signals, as defined in equation 4.30, are presented in figure 4.16. The three characteristic frequency components at 16Hz, 40Hz and 56Hz in $p$ and at 8Hz, 20Hz and 56Hz in $u$ are properly identified in the power spectra presented in figure 4.17. The preset linear relation between $p$ and $u$ at 56Hz in interval I is accurately detected by the Fourier-based linear coherence presented in figure 4.18. Nevertheless, the linear coherence is also high near 20-30Hz and 43Hz, which is again the result of insufficient number of segments for averaging. Therefore, the Fourier-based linear coherence fails to isolate the coupled frequencies from erroneous ones, and, thus, does not provide reliable information. Figure 4.19 shows that the Fourier auto-bicoherence is contaminated by erroneous coupling patterns for the same reason presented before. Three coupling peaks in $p$ are discerned at (40, 16), (56, -16) and (56, -40), accompanied by other high-bicoherence pairs that are scattered over a wide range of frequencies. For the $u$ signal, all detected quadratic couplings in its auto-bicoherence plot are not physical, because the three characteristic frequencies present in interval I, namely, 8Hz, 20Hz and 56Hz, do not follow the frequency sum-rule. Similarly, in figure 4.20, which shows the Fourier cross-bicoherence between the two sig-
Figure 4.11: Contour plots of the wavelet auto-bicoherence of (a): $p(t)$ and (b): $u(t)$ shown in figure 4.3 along with the associated statistical noise; integration centered at $t=0.75$s; integration time length: 0.85s; contour levels for both auto-bicoherence and noise are set at 0.5, 0.6 and 0.7; frequency resolution $df=0.667\text{Hz}$

signals, the two expected coupling peaks are actually observed at $(8, 8)$ and $(20, 20)$, with bicoherence levels over 0.9. However there are other frequency pairs with bicoherence levels higher than 0.9, such as $(45, -5)$ and $(62, -20)$. This implies that the Fourier-based cross-bicoherence is also contaminated by excessive noise, and, thus, it is not suitable for identification of phase couplings that take place over short durations of a time series.

Using three different integration time lengths 0.3s, 0.85s and 1.4s, the wavelet power spectra of the $p$ and $u$ signals are presented in figure 4.21. The results clearly show that the wavelet power spectrum is capable of identifying all characteristic frequency components. The perfect linear relation at 56Hz in interval I is detected by a large peak (over 0.9) in the frequency range around 56Hz in the wavelet linear coherence, presented in figure 4.22. As the integration time length is increased, the wavelet linear coherence drops accordingly. This results from the additive random noise in intervals II and III. The extension of integration time lengths diminishes the perfect coupling pattern in interval I and yields lower levels of the linear coherence over the frequency range around 56Hz.
Figure 4.12: Contour plots of the wavelet auto-bicoherence of (a): $p(t)$ and (b): $u(t)$ shown in figure 4.3 along with the associated statistical noise; integration centered at $t=0.75$s; integration time length: 1.40s; contour levels for both auto-bicoherence and noise are set at 0.5, 0.6 and 0.7; frequency resolution $df=0.667$Hz

The wavelet auto-bicoherence of the $p$ and $u$ signals were calculated over the same three integration time lengths, namely, 0.3s, 0.85s and 1.4s. The results are shown in figures 4.23-4.25, along with their associated noise levels. In figure 4.23, the perfect self-coupling in $p$ over interval I is manifested by three frequency pairs $(40, 16)$, $(56, -16)$ and $(56, -40)$. Their bicoherence levels are all over 0.85, well overcoming their noise levels. As for the $u$ signal, no self-coupling is detected. This is correct, because the three frequency components in $u$ do not follow the frequency sum-rule. When the integration time length increases to 0.85s and 1.4s, the bicoherence levels of the three frequency pairs in $p$ drop accordingly, and still no coupling pattern is detected in $u$.

Figures 4.26-4.28 show the wavelet cross-bicoherence between $p$ and $u$ for the same integration time lengths. For this data set, as designed, there are two frequency doublings in interval I, which are properly identified by frequency pairs $(8, 8)$ and $(20, 20)$ in figure 4.26. The level of the cross-bicoherence is almost 0.9. Over the longer integration time lengths, the two
Figure 4.13: Contour plots of the wavelet cross-bicoherence between $p(t)$ and $u(t)$ shown in figure 4.3 along with the associated statistical noise; integration centered at $t=0.75s$; integration time length: 0.30s; contour levels for both cross-bicoherence and noise are set at 0.7, 0.8 and 0.9; frequency resolution $df=0.667Hz$

frequency pairs are still distinct in figures 4.27 and 4.28. Their values drop accordingly compared with figure 4.26.

In summary, the results of the two analytical data sets have clearly demonstrated that the wavelet-based analysis has significant advantages over the Fourier-based analysis, when investigating intermittent nonlinear events in nonstationary signals. They are especially effective when significant coupling patterns are embedded in background noise. Based on the analysis presented in this chapter, wavelet-based spectral moments will be used to identify linear and nonlinear relations between surface pressure and velocity fluctuations. The details are presented in Chapter 5.
Figure 4.14: Contour plots of the wavelet cross-bicoherence between $p(t)$ and $u(t)$ shown in figure 4.3 along with the associated statistical noise; integration centered at $t=0.75s$; integration time length: 0.85s; contour levels for both cross-bicoherence and noise are set at 0.5, 0.6 and 0.7; frequency resolution $df=0.667Hz$

Figure 4.15: Contour plots of the wavelet cross-bicoherence between $p(t)$ and $u(t)$ shown in figure 4.3 along with the associated statistical noise; integration centered at $t=0.75s$; integration time length: 1.40s; contour levels for both cross-bicoherence and noise are set at 0.5, 0.6 and 0.7; frequency resolution $df=0.667Hz$
Figure 4.16: Time series of $p(t)$ and $u(t)$ as defined in equation 4.30

Figure 4.17: Fourier power spectra of the full-length $p(t)$ and $u(t)$ shown in figure 4.16; frequency resolution $df=3.906\text{Hz}$
Figure 4.18: Fourier linear coherence between the full-length $p(t)$ and $u(t)$ shown in figure 4.16

Figure 4.19: Fourier auto-bicoherence of the full-length (a): $p(t)$ and (b): $u(t)$ shown in figure 4.16; contour levels are set at 0.5, 0.7 and 0.9; frequency resolution $df=3.906$Hz
Figure 4.20: Fourier cross-bicoherence between the full-length $p(t)$ and $u(t)$ shown in figure 4.16; contour levels are set at 0.5, 0.7 and 0.9; frequency resolution $df=3.906\text{Hz}$

Figure 4.21: Wavelet power spectra of $p(t)$ and $u(t)$ shown in figure 4.16 and estimated over three different integration time lengths: 0.30s, 0.85s and 1.40s; frequency resolution $df=0.667\text{Hz}$
Figure 4.22: Wavelet linear coherence between $p(t)$ and $u(t)$ shown in figure 4.16 and estimated over three different integration time lengths: 0.30s, 0.85s and 1.40s; frequency resolution $df=0.667\text{Hz}$

Figure 4.23: Contour plots of the wavelet auto-bicoherence of (a): $p(t)$ and (b): $u(t)$ shown in figure 4.16 along with the associated statistical noise; integration centered at $t=0.75\text{s}$; integration time length: 0.30s; contour levels for both auto-bicoherence and noise are set at 0.8, 0.85 and 0.95; frequency resolution $df=0.667\text{Hz}$
Figure 4.24: Contour plots of the wavelet auto-bicoherence of (a): $p(t)$ and (b): $u(t)$ shown in figure 4.16 along with the associated statistical noise; integration centered at $t=0.75s$; integration time length: 0.85s; contour levels for both auto-bicoherence and noise are set at 0.7, 0.8 and 0.9; frequency resolution $df=0.667Hz$

Figure 4.25: Contour plots of the wavelet auto-bicoherence of (a): $p(t)$ and (b): $u(t)$ shown in figure 4.16 along with the associated statistical noise; integration centered at $t=0.75s$; integration time length: 1.40s; contour levels for both auto-bicoherence and noise are set at 0.7, 0.8 and 0.9; frequency resolution $df=0.667Hz$
Figure 4.26: Contour plots of the wavelet cross-bicoherence between $p(t)$ and $u(t)$ shown in figure 4.16 along with the associated statistical noise; integration centered at $t=0.75s$; integration time length: 0.30s; contour levels for both cross-bicoherence and noise are set at 0.8, 0.85 and 0.95; frequency resolution $df=0.667Hz$

Figure 4.27: Contour plots of the wavelet cross-bicoherence between $p(t)$ and $u(t)$ shown in figure 4.16 along with the associated statistical noise; integration centered at $t=0.75s$; integration time length: 0.85s; contour levels for both cross-bicoherence and noise are set at 0.7, 0.8 and 0.9; frequency resolution $df=0.667Hz$
Figure 4.28: Contour plots of the wavelet cross-bicoherence between $p(t)$ and $u(t)$ shown in figure 4.16 along with the associated statistical noise; integration centered at $t=0.75s$; integration time length: 1.40s; contour levels for both cross-bicoherence and noise are set at 0.7, 0.8 and 0.9; frequency resolution $df=0.667\text{Hz}$
Chapter 5

Velocity-pressure relations

Validation of wind-tunnel and numerical simulations of wind loads on structures must be performed with tools capable of identifying flow dynamics responsible for peak loads. Many characteristics of the incident flow field, including turbulence intensities, the integral length scale and energy level of the small scale turbulence, affect the near flow field and the resulting characteristics of surface pressure peaks. To assess the roles played by different aspects of the incident flow and the flow adjacent to the surface of the structure on the characteristics of extreme wind loads, one needs to establish a relation between the velocity components and the surface pressure peaks. One such relation is represented by the quasi-steady approach, whereby the pressure fluctuations are related to the far field fluctuations in the velocity and the flow direction. However, this approach does not account for specific flow dynamics in the near-field that affect pressure peaks. A more physical relation is the one obtained by taking the divergence of the equation of motion and applying the continuity equation. The resulting equation relates the pressure at any point to the velocity fluctuations in the entire flow field.

In this chapter, we apply the techniques discussed in Chapter 4 to detect intermittent linear and nonlinear relations between the velocity fluctuations in the near-field and surface pressure fluctuations. In particular, the interest is in pressure peaks observed in the separated regions under normal and oblique incident flow cases. The objective is to determine what scales affect the surface pressures.
5.1 Aerodynamic forces: buffeting vs. interaction

The incident flow could affect aerodynamic forces on structures through two different mechanisms, namely buffeting and interaction (Bearman, 1978, [41]). In this sense, the flow field can be divided into two regions: the far-field and the near-field.

The buffeting forces result from low-frequency fluctuations with wavelengths that exceed the structure’s major dimensions. This exhibits the influence of the upstream conditions (the far-field) on the wind loads. Through buffeting, the surface pressures vary in accord with the unsteadiness of the mean wind speed and its direction. Although they provide acceptable predictions of aerodynamic forces in stagnation regions ($C_{p,mean} > 0$) (Tieleman, 2003, [42]), theories based on the buffeting effect alone cannot account for the flow properties and surface pressures in separated regions.

The interaction effect of the incident flow on the wind loads is a complex combination of the interactions between the incident flow and the flow separation, shear layer development, vortex generation and flow reattachment. It is logical to assume that all turbulence scales in the incident flow affect the level of pressure peaks in the separated region. The small scales are primarily responsible for the roll-up of the separated shear layer, while the large-scale turbulence brings the vortices to a full maturity (Saathoff et al., 1997, [43] and Li et al., 1995, [44]). Hence, an investigation of the near-field interactions become especially important, because peak suctions are observed directly beneath the separation bubble or the corner vortices (Marwood et al., 1997, [16]).

Because of the interest in intermittent pressure peaks and associated velocity events, the wavelet-based higher-order statistical moments, discussed
in Chapter 4, are used to examine the linear and nonlinear interactions between the pressure and velocity fluctuations in the near-field. The multiscale property of the wavelet analysis makes it possible to recognize separately the roles of the small and large scales. Additionally, the low noise level resulting from its application to short time durations allows for the identification of intermittent linear and nonlinear relations.

5.2 Overview of the velocity-pressure relation

One approach used to model the relation between the velocity fluctuations and surface pressure coefficients is the quasi-steady theory (Kawai, 1983, [45]). In this approach, the surface pressure fluctuations are proportional to the instantaneous far-field wind velocity and direction. Janajreh (1998, [24]) detailed the linear and nonlinear versions of the quasi-steady theory and the resulting frequency domain admittance functions that relate spectral components of the far-field velocity components and surface pressures. The analysis performed by Janajreh on measured far-field fluctuations and surface pressures showed that the quasi-steady theory in its different forms cannot be used to predict extreme pressure coefficients.

A more physical relation between the mean and fluctuating components of the flow field and the surface pressure fluctuations is governed by Poisson’s equation, whose solution is given as (Townsend, 1976, [46])

\[
p(\vec{r}') = \frac{1}{4\pi} \int \int \left[ 2 \frac{\partial U_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \frac{\partial^2}{\partial x_i \partial x_j} \left( u_i u_j - u_i \bar{u}_j \right) \right] \frac{dV(\vec{r})}{|\vec{r}' - \vec{r}|} \quad (5.1)
\]

Obviously, the pressure fluctuations, on the left hand side of equation 5.1, is a function of velocity gradients throughout the flow field. Based on equation 5.1, the local pressure fluctuations are influenced not only by the far-field incident flow, but also by the gradients of mean velocity and velocity fluctuations in the near-field.
The integrand in equation 5.1 consists of two terms. The first term, 
\[2 \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i},\]
is determined by the product of the gradients of the mean and fluctuating velocity components. This term represents the linear contribution of the velocity field to the pressure fluctuations. The second term, 
\[\frac{\partial^2}{\partial x_i \partial x_j} (u_i u_j - \bar{u}_i \bar{u}_j),\]
reflects the nonlinear (quadratic) contribution of the fluctuating velocities to the surface pressures. Considering that the spatial derivative is independent of the temporal derivative, we will investigate the velocity field directly instead of its gradient distribution (Tieleman et al., 1998, [18]). In the following discussion, we will apply a hierarchy of wavelet-based spectral moments to quantify the linear and quadratic relations between surface pressures and velocity fluctuations in the near-field.

5.3 Results and discussion

Four representative pressure peaks are isolated from records Hotsam10, Hotsam11 and Hotsam14 for the assessment of the velocity-pressure relation in the case of normal flow incidence. Four other pressure peaks, measured in records Hotsam47, Hotsam48 and Hotsam49, are investigated in the case of oblique flow incidence. The pressure taps and anemometer locations for the measurements performed in these records are illustrated in figure 5.1. The detailed coordinates of the measuring sensors are presented in Tables 2.1 and 2.4.

5.3.1 Normal incidence cases

\(\sqrt{\text{Pressure coefficients near } t=17.580s, \text{ from Hotsam10}}\)

Figure 5.2 shows three simultaneously measured surface pressure coefficients \(p_1, p_2, p_3\) at taps 50900, 50905 and 50907, respectively, and the \(u\)-component of the velocity fluctuations \(u\) over a period of 2 seconds. These measurements are a part of the Hotsam10 record. In nondimensional sense and denoting the height of the prism as \(H\), the three pressure taps, \(p_1, p_2, p_3\), are set, respectively, at \(0.035H, 0.37H\) and \(0.52H\) on the top surface of the
Figure 5.1: Pressure tap and anemometer locations for normal and oblique incidence cases; detailed coordinates of measuring sensors are presented in Tables 2.1 and 2.4.

prism. The u-component of the velocity fluctuations was measured at 0.3H above pressure tap 50900. This puts the velocity measurement on the outside edge of the shear layer. The pressure peaks in both $p_1$ and $p_2$ in figure 5.2 near $t=17.580s$ are identified for analysis. The associated pressure peak in $p_3$ appears to be delayed by about 0.1s. This downstream convection of pressure peaks has been studied by the space-time correlation presented in Section 3.1.3. On a large scale (slow variations) basis, the velocity and pressure data segments presented in figure 5.2 clearly show a trend, whereby an increase in the local speed is accompanied by a decrease in the pressure peak. Yet, the interest is in the small variations which cause the significant drop in the pressure as observed near $t=17.580s$.

Figure 5.3 shows the Fourier-based power spectrum of the velocity fluctuations, estimated by averaging 10 segments, each consisting of 4096 data points. Thus, each of the 10 segments was 4.096 seconds long, which yields a frequency resolution of 0.244Hz. By inspecting the velocity power spectrum, it is noted that the velocity signal exhibits the characteristics of turbulent
fluctuations. The spectrum can be nicely matched by the Kolmogoroff \(-5/3\)-law in the range between 2Hz \((fH/U=0.016)\) and 100Hz \((fH/U=0.8)\).

Figures 5.4-5.7 show, respectively, contour plots of the magnitudes of wavelet coefficients of the velocity and the three pressure time series, presented in figure 5.2. These plots give a measure of the energy contained in the different frequency ranges (or scales) as a function of time. The wavelet coefficients of the velocity fluctuations exhibit a large magnitude over a frequency range around 20Hz near \(t=17.5s\). Simultaneously, the wavelet coefficients of both \(p_1\) and \(p_2\) exhibit relatively large magnitudes over two frequency ranges near 10Hz and 20Hz with streaks that indicate enhanced energy levels of the smaller scales. The contour plot of the wavelet coefficient magnitude for \(p_3\), shown in figure 5.7, reveals different characteristics than the ones observed from \(p_1\) and \(p_2\). In particular, two ranges of energy-
containing scales centered around 25Hz and 40Hz are observed. The results of the plots of the magnitude of the wavelet coefficients clearly show that there is a relation between the pressure peaks in $p_1$ and $p_2$ near $t=17.5s$, measured by taps 50900 and 50905, and the velocity fluctuations as measured 0.3$H$ above tap 50900.

The power of a specified frequency band over a defined integration time can be determined from the wavelet power spectrum. Figure 5.8 shows the wavelet power spectra of the velocity fluctuations and the three pressure time series that are plotted in figure 5.2. To examine the effects of using different integration times in estimating the wavelet power spectrum, three integration time lengths, namely 0.44s, 1.0s and 2.0s, are chosen. The 0.44s time length corresponds to the observed duration of the pressure peak. For the time length of 0.44s, it is noted that high-energy density is concentrated over the frequency range near 20Hz, which corresponds to the high-level contours centered at 20Hz in the plot of the magnitude of the wavelet coefficients presented in figure 5.4. The power spectra of $p_1$ and $p_2$ are fairly similar; both of them show a high level of energy density over the frequency
Figure 5.4: Wavelet coefficient magnitudes of the velocity fluctuations presented in figure 5.2

range near 10Hz, 20Hz and 40Hz. In comparison, the power spectra of \( p_3 \) is quite different. A relatively high level of energy is noted over a wide range that extends between 25Hz and 45Hz. When the time length is elongated to 1.0s and subsequently, to 2.0s, it is noted that the high-energy spectral peaks are flattened. Consequently, it is logical to connect the high-energy spectral ranges observed over the 0.44s integration time with the pressure peak observed near \( t=17.580s \).

The wavelet linear coherence between each of the three surface pressure signals and the velocity fluctuations are estimated for the same three time lengths, namely, 0.44s, 1.0s and 2.0s, and presented in figure 5.9, along with the associated noise levels. The results show a high level linear coherence (over 0.9) between \( p_1 \) and \( u \) in the range of frequencies around 20Hz. As the time length is extended to 1.0s, and subsequently, to 2.0s, this coherence level, over the same frequency range, drops significantly to about 0.5. This implies that the linear coupling between \( p_1 \) and \( u \) over this frequency range is highly intermittent and is associated with the peak event. At the same time, the linear coherence between \( p_2 \) and \( u \) exhibits high levels for
the frequencies near 20Hz when the integration time length is set to 0.44s, which indicates that there also exists a linear relation between $p_2$ and $u$ over this range of frequencies. This linear relation is also intermittent and highly dependent on the peak event. The linear coherence between $p_3$ and $u$ shows no appreciable level of linear coherence, especially over the same range of frequencies in $p_1 - u$ and $p_2 - u$ that are highly coupled. In summary, the wavelet-based power spectral analysis and linear coherence results associate a frequency range near 20Hz with the pressure peaks simultaneously observed at taps 50900 and 50905 and relate this range to the same frequency range in the velocity fluctuations measured 0.3$H$ above tap 50900.

An assessment of the nonlinear relations between spectral peaks in the velocity and pressure fluctuations is best determined from cross-bicoherence estimates between the two signals. Yet, it is as useful to assess the extent of nonlinear couplings in the velocity and pressure fluctuations. The autobicoherence contour plots, presented in figures 5.10-5.12, show the nonlinearly coupled frequency components (or scales) in the velocity and pressure time series. The three figures correspond to three integration time lengths:
Figure 5.6: Wavelet coefficient magnitudes of the pressure record at tap 50905 presented in figure 5.2

0.44s, 1.0s and 2.0s. For the time length of 0.44s, a major coupling represented by the frequency pair (40, -20) is noted. This coupling shows that the two frequency components at 20Hz and 40Hz in the velocity fluctuations are quadratically coupled. Comparing the contours at (40, -20) in the velocity auto-bicoherence plots for three integration time lengths in figures 5.10-5.12, it is noted that the auto-bicoherence value for this coupling pattern decays significantly as the integration time is increased. Consequently, this coupling in the velocity fluctuations is highly intermittent and can be associated with the peak event. In figure 5.10, major coupling patterns over the frequency ranges near 20Hz and 40Hz are also identifiable in the auto-bicoherence plots of both $p_1$ and $p_2$. This high level of coupling is not existent in the auto-bicoherence plot of $p_3$, even for the shortest duration of 0.44s. As the integration time length is increased, the auto-bicoherence in both $p_1$ and $p_2$ signals over the frequency ranges between 20 and 40Hz decreases rapidly. This implies that the coupling over this range in both $p_1$ and $p_2$ is closely associated with the peak event.

Figures 5.13-5.15 show the wavelet-based cross-bicoherence of $p_1 - u$, 


$p_2 - u$ and $p_3 - u$ for the three integration time lengths, 0.44s, 1.0s and 2.0s. It is of interest to note, from figure 5.13, that cross-coupling over the frequency range centered around 20Hz and 40Hz is detected between $p_1$ and $u$, with a bicoherence level over 0.7. The same coupling pattern, but at a lower level, is also identified between $p_2$ and $u$. However, from the cross-bicoherence estimates of $p_3 - u$, no major coupling is observed. When the integration time length is extended, the bicoherence levels of both $p_1 - u$ and $p_2 - u$ over the same frequency range decrease considerably, indicating intermittency and specific nonlinear relations between the velocity and pressure fluctuations during the occurrence of the peak event.

The wavelet-based analysis and results yield the following conclusions regarding linear and nonlinear velocity-pressure relations when a surface pressure peak is formed. For the case of normal incidence as measured in record Hotsam10, the energy-containing scales in the frequency range near 20Hz in the velocity fluctuations plays a vital role in both linear and nonlinear pressure-velocity relations. This frequency range is coupled with the frequency range near 40Hz in the velocity fluctuations. The two frequency
Figure 5.8: Wavelet power spectra of (a): $u$; (b): $p_1$; (c): $p_2$; (d): $p_3$ for the records shown in figure 5.2. Three different integration time lengths, 0.44s, 1.0s and 2.0s, are used.

ranges near 20Hz and 40Hz are quadratically coupled with their difference components in the pressure fluctuations measured by $p_1$ and $p_2$. Additionally, the frequency range near 20Hz in the velocity fluctuations is linearly coupled with the frequency range near 20Hz in both $p_1$ and $p_2$. Thus, different routes, through which velocity fluctuations affect the development of the surface pressure peak, are identified. One route is a linear relation, the second is a nonlinear coupling in the velocity fluctuations that linearly affect the pressure fluctuations; and the third is a direct nonlinear relation between frequency ranges in the velocity fluctuations and their algebraic sum in the pressure fluctuations.
Figure 5.9: Wavelet linear coherence of (a): $p_1 - u$; (b): $p_2 - u$; (c): $p_3 - u$ for the records shown in figure 5.2. Three different integration time lengths, 0.44s, 1.0s and 2.0s, are used. The dotted lines indicate the associated noise levels.

√ Pressure coefficients near t=62.800s, from Hotsam10

To assess the reliability of the discussion and results presented above, the same analysis procedure is applied to another peak observed in the same record, Hotsam10. Two-second data segments of the u-component of the velocity fluctuations and the surface pressure coefficients, $p_1$, $p_2$ and $p_3$ at taps 50900, 50905 and 50907, respectively, over a period of two seconds are shown in figure 5.16. The interest is in the pressure peaks observed near t=62.8s in both $p_1$ and $p_2$. These peaks are associated with an increase in the magnitude of the simultaneously measured velocity fluctuations. Similarly to the case discussed above, there is no simultaneous pressure peak in the time series of $p_3$.

Contour plots of the magnitudes of wavelet coefficients of $u$, $p_1$, $p_2$ and

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Figure 5.10: Wavelet auto-bicoherence of (a): $u$; (b): $p_1$; (c): $p_2$; (d): $p_3$ and the associated statistical noise for the records shown in figure 5.2; integration centered at $t=17.580$ seconds; integration time length: 0.44s; contour levels for both auto-bicoherence and statistical noise: 0.3, 0.5 and 0.7.

$p_3$ records of figure 5.16 are shown in figures 5.17-5.20, respectively. Near the time $t=62.8s$, which corresponds to the occurrence time of the surface pressure peak, high contour levels are noted over the frequency ranges near 15Hz in the velocity fluctuations, and over the frequency range near 15Hz, 35Hz and 50Hz in the pressure fluctuations represented by $p_1$ and $p_2$. There is no clear energy-containing scale near $t=62.8s$ in figure 5.20, which shows the magnitude of the wavelet coefficients at tap 50907. In fact, figure 5.20 shows that the level of the magnitudes of wavelet coefficients for $p_3$ are generally lower than those of $p_1$ and $p_2$. 
Figure 5.11: Wavelet auto-bicoherence of (a): $u$; (b): $p_1$; (c): $p_2$; (d): $p_3$ and the associated statistical noise for the records shown in figure 5.2; integration centered at $t=17.580$ seconds; integration time length: 1.0s; contour levels for both auto-bicoherence and statistical noise: 0.3, 0.5 and 0.7

The wavelet power spectra of the velocity and the three pressure fluctuations are calculated for three integration time lengths, 0.35s, 1.0s and 2.0s, and presented in figure 5.21. The wavelet-based power spectra of $p_1$ and $p_2$ over the shortest time length show relatively higher energy levels in the frequency range around 15Hz and 35Hz. The power spectrum of $p_3$ shows a relatively low energy over the range of frequencies near 15Hz. This indicates again that $p_1$ and $p_2$ share similar characteristics, whereas $p_3$ is quite different from them. Consequently, the frequency range around 15Hz can be associated with the observed peak in $p_1$ and $p_2$. The linear coherence over the same three integration time lengths were estimated for
Figure 5.12: Wavelet auto-bicoherence of (a): $u$; (b): $p_1$; (c): $p_2$; (d): $p_3$ and the associated statistical noise for the records shown in figure 5.2; integration centered at $t=17.580$ seconds; integration time length: 2.0s; contour levels for both auto-bicoherence and statistical noise: 0.3, 0.5 and 0.7

$p_1 - u$, $p_2 - u$ and $p_3 - u$, and are presented in figure 5.22. The peaks in the linear coherence around 15Hz in both $p_1 - u$ and $p_2 - u$ are noted. Over longer integration time lengths, the levels of the local linear coherence decay considerably to below 0.4, implying that the two linear relations are highly intermittent and closely associated with the peak events in the respective pressure fluctuations. For the linear coherence between $p_3$ and $u$, all values are below the noise levels. Hence, no linear relation can be identified with reasonable confidence.

The wavelet auto-bicoherence of $u$, $p_1$, $p_2$ and $p_3$, used to assess the
nonlinear relations between the different scales in the velocity and pressure fluctuations, are presented in figures 5.23-5.25, respectively for the three integration time lengths: 0.35s, 1.0s and 2.0s. For the time length of 0.35s, which covers the pressure peaks in $p_1$ and $p_2$, a major coupling between the frequency ranges around 35Hz and 15Hz are noted in $u$, $p_1$ and $p_2$, but not in $p_3$. When the integration time is extended to 1.0s and 2.0s, the auto-bicoherence levels for the same frequency range decrease sensitively, which shows the intermittency of these couplings. To examine the nonlinear relations between the velocity and pressure fluctuations, the wavelet cross-bicoherence for $p_1 - u$, $p_2 - u$ and $p_3 - u$ were calculated and are plotted in
Figure 5.14: Wavelet cross-bicoherence of (a): $p_1 - u$; (b): $p_2 - u$; (c): $p_3 - u$ and the associated statistical noise for the records shown in figure 5.2; integration centered at $t=17.580$ seconds; integration time length: 1.0s; contour levels for both cross-bicoherence and statistical noise: 0.3, 0.5 and 0.7 figures 5.26-5.28, respectively for the three integration time lengths. From figure 5.26, a cross-coupling between $p_1$ and $u$ is detected in the frequency ranges around 35Hz and 15Hz, which is also observed between $p_2$ and $u$. However, the cross-bicoherence of $p_3 - u$ does not show the same relation. With the extension of the integration time length to 1.0s, and subsequently, 2.0s, all observed high cross-bicoherence levels drop significantly, indicating that all the major couplings are associated with the respective peak events.

The analysis and results presented above show that the energy-containing scales over the frequency ranges near 15Hz and 35Hz in the velocity fluctu-
Figure 5.15: Wavelet cross-bicoherence of (a): $p_1 - u$; (b): $p_2 - u$; (c): $p_3 - u$ and the associated statistical noise for the records shown in figure 5.2; integration centered at $t=17.580$ seconds; integration time length: 2.0s; contour levels for both cross-bicoherence and statistical noise: 0.3, 0.5 and 0.7.

The range of frequencies near 15Hz in the velocity fluctuations has a nearly perfect linear relation with the same frequency range in both $p_1$ and $p_2$. The auto-bicoherence of the velocity fluctuations reveals a high level of coupling between the frequency ranges near 15Hz and 35Hz. Nonlinear coupling between the two frequency ranges near 15Hz and 35Hz in the velocity fluctuations and their difference in both $p_1$ and $p_2$ is also observed. Hence, the velocity fluctuations have both linear and nonlinear relations with the pressure fluctuations at $p_1$ and $p_2$ over the duration when a peak was observed. On the other hand, there are no clear linear or nonlinear relations between
Figure 5.16: Two-second time series of the u-component of the velocity fluctuations and the surface pressures at taps 50900, 50905 and 50907 as measured in record Hotsam10

$p_3$ and $u$. The pressure fluctuations at $p_3$ did not exhibit a simultaneous peak.

✓ Pressure peaks from records Hotsam11 and Hotsam14

The results presented above are extended by examining the linear and non-linear relations between pressure peaks observed at the same pressure taps and velocity fluctuations measured at distances of 0.15$H$ (from record Hotsam11) and 0.075$H$ (from record Hotsam14) above tap 50900.

Pressure peak near $t=15.900s$, at tap 50900, from Hotsam11  Figure 5.29 shows the pressure signal at tap 50900 and the associated u-component of the velocity fluctuations at a distance of 0.15$H$ above the pressure tap over a period of 2.0 seconds, centered at $t=15.900s$. The plots
Figure 5.17: Wavelet coefficient magnitudes of the velocity fluctuations presented in figure 5.16 clearly show that the pressure peak near t=15.9s can be related to an increase in the magnitude of the velocity signal. It can also be estimated that the duration of the this pressure peak is approximately 0.5s.

Figure 5.30 shows the wavelet-based power spectra of the velocity and pressure time series, presented in figure 5.29, for three integration time lengths, 0.5s, 1.0s and 2.0s. For the shortest time length, it is noted that four frequency ranges, centered near 10Hz, 20Hz and 30Hz and 50Hz, contain relatively high levels of energy in the both velocity and pressure fluctuations. When the integration time length is extended to 1.0s and, subsequently, to 2.0s, the local energy peaks near 30Hz, 50Hz in the velocity fluctuations and near 20Hz, 50Hz in the pressure fluctuations become flattened, which distinguishes these four energy-containing scales as intermittent ones and relates them to the observed pressure peak. To examine the linear relation between the velocity and pressure fluctuations, the wavelet-based linear coherence for the three integration time lengths were estimated, and the results, along with the noise levels, are presented in figure 5.31. For the integration time length of 0.5s, high levels of linear coherence are observed.
Figure 5.18: Wavelet coefficient magnitudes of the pressure record at tap 50900 presented in figure 5.16

over the frequency ranges near 10Hz, 20Hz, 30Hz and 50Hz. Over the longer time lengths, the linear coherence levels in all the four frequency ranges drop to some extent. This shows that the linear relations between the velocity and pressure fluctuations are closely associated with the peak event occurring near t=15.900s.

The wavelet auto-bicoherence of the velocity and pressure fluctuations are shown in figure 5.32, for integration time length of 0.5s. High-level coupling pairs are observed near (30, -20) and (20, -10) in the auto-bicoherence plot of the velocity fluctuations, indicating couplings between two frequency ranges near 30Hz and 20Hz and between two frequency ranges near 20Hz and 10Hz. Similarly, in the pressure fluctuations, couplings are detected between the frequency ranges near 20Hz and 10Hz, and near 30Hz and 10Hz, and to a lower level near 50Hz and 20Hz. Over the longer integration time lengths, 1.0s and 2.0s, the bicoherence levels (figures not shown) for these coupled frequency ranges decrease considerably, which is an indication of the association of these couplings with the peak event.
To detect the nonlinear relations between the velocity and pressure fluctuations, a contour plot of the wavelet cross-bicoherence between the velocity and pressure fluctuations for the shortest integration time length, 0.5s, is presented in figure 5.33. Obviously, two nonlinear couplings can be observed from the cross-bicoherence plot: one is between the frequency ranges near 30Hz and 10Hz, and the other is between the frequency ranges near 20Hz and 10Hz. The cross-bicoherence levels over these two pairs of frequency ranges decrease significantly (figures not shown), when the integration time length is increased to 1.0s, and subsequently, to 2.0s. This again verifies the intermittence of the two nonlinear relations.

The analysis and results presented above reveal that the four energy-containing scales near 10Hz, 20Hz, 30Hz and 50Hz in the pressure fluctuations are related to the peak event. These scales are linearly and nonlinearly related to the scales in the velocity fluctuations.
Figure 5.20: Wavelet coefficient magnitudes of the pressure record at tap 50907 presented in figure 5.16

**Pressure peak near t=19.250s, at tap 50900, from Hotsam14** Figure 5.34 shows two-second velocity and pressure signals centered at t=19.250s as measured in record Hotsam14. It is noted again that the pressure peak near t=19.250s is accompanied by an increase in the local magnitude of the velocity. The duration of this peak event is approximately 0.3s.

From figure 5.35, which presents the wavelet power spectra of the velocity and pressure fluctuations for three incremental integration time lengths, 0.3s, 1.0s and 2.0s, it is noted that, in both the velocity and surface pressure fluctuations, there are two major energy-containing scales, represented by the frequency ranges near 15Hz and 30Hz. To assess the linear relation between the velocity and pressure fluctuations, the wavelet linear coherence for three integration time lengths were estimated and are presented in figure 5.36. Over the shortest integration time length, relatively high levels of linear coherence are observed near 15Hz, 30Hz, 60Hz and 90Hz. When the integration time length is elongated to 1.0s and then to 2.0s, the linear coherence level over the frequency range near 15Hz decays appreciably, implying the high intermittence of this linear relation and its relevance to the
Figure 5.21: Wavelet power spectra of (a): $u$; (b): $p_1$; (c): $p_2$; (d): $p_3$ for the records shown in figure 5.16. Three different integration time lengths, 0.35s, 1.0s and 2.0s are used.

pressure peak.

The wavelet auto-bicoherence for the three integration time lengths, 0.3s, 1.0s and 2.0s, was also estimated for the assessment of nonlinear relations between the velocity and pressure fluctuations. Figure 5.37 shows the wavelet auto-bicoherence of the velocity and pressure fluctuations for the shortest integration time length, 0.3s. In the velocity fluctuations, one coupling represented by a frequency pair near (30, -15) is noted, which indicates a nonlinear coupling between the frequency range near 30Hz and the frequency range near 15Hz. In the pressure fluctuations, there is also one major coupling over the frequency ranges centered near 25Hz and 10Hz, respectively.

Figure 5.38 shows the wavelet cross-bicoherence between the velocity
Figure 5.22: Wavelet linear coherence of (a): $p_1 - u$; (b): $p_2 - u$; (c): $p_3 - u$ for the records shown in figure 5.16. Three different integration time lengths, 0.35s, 1.0s and 2.0s, are used. The dotted lines indicate the associated noise levels.

and pressure fluctuations for the shortest integration time length, 0.3s. The plot exhibits one major coupling near (30, -15). This coupling represents nonlinear relation between the frequency ranges near 30Hz and 15Hz in the velocity signal and their difference in the pressure fluctuations. This high level of bicoherence drops when the integration time length is extended (figures not show), which reveals the association of this nonlinear relation with the peak event.

Pressure-velocity relations for different elevations in normal incidence cases  The results presented above show that the velocity fluctuations at all locations in the flow field influence the surface pressure fluctuations through various linear/nonlinear couplings. On the other hand, there are similar characteristics in the relations between different pairs of pres-
Figure 5.23: Wavelet auto-bicoherence of (a): $u$; (b): $p_1$; (c): $p_2$; (d): $p_3$ and the associated statistical noise for the records shown in figure 5.16; integration centered at $t=62.800$ seconds; integration time length: 0.35s; contour levels for both auto-bicoherence and statistical noise: 0.4, 0.6 and 0.8

sure and velocity fluctuations. Energy-containing scales consistently play a critical role in characterizing the pressure peaks.

It is also important to note that as the distance between the pressure tap and the anemometer is reduced, small scales become more actively involved in both linear and nonlinear relations. In terms of the relation between the velocity fluctuations and the pressure fluctuations measured at tap 50900, for the peak event near $t=19.250$ s from record Hotsam14, peaks in the linear coherence are observed in higher frequency ranges near 60Hz and 90Hz, as shown in figure 5.36. For the peak event near $t=15.900$ s from record Hot-
Figure 5.24: Wavelet auto-bicoherence of (a): $u$; (b): $p_1$; (c): $p_2$; (d): $p_3$ and the associated statistical noise for the records shown in figure 5.16; integration centered at $t=62.800$ seconds; integration time length: 1.0s; contour levels for both auto-bicoherence and statistical noise: 0.4, 0.6 and 0.8

sam11, high-level linear coupling can be found in the frequency range near 50Hz, as shown in figure 5.31. In comparison, however, for the peak events near $t=17.580$s and $t=62.800$s, the linearly coupled frequency components are largely limited to the range below 30Hz, as shown in figures 5.9 and 5.22. The same phenomenon can be also observed from nonlinear relations. For instance, for the peak event near $t=19.250$s from record Hotsam14, a wide range of frequencies, from 50Hz to 70Hz, becomes nonlinearly coupled, as shown in the wavelet cross-bicoherence plot presented in figure 5.38. This trend clearly shows that the velocity fluctuations at a closer distance tend to have a wider range of frequencies that are linearly and/or nonlinearly
Figure 5.25: Wavelet auto-bicoherence of (a): $u$; (b): $p_1$; (c): $p_2$; (d): $p_3$ and the associated statistical noise for the records shown in figure 5.16; integration centered at $t=62.800$ seconds; integration time length: 2.0s; contour levels for both auto-bicoherence and statistical noise: 0.4, 0.6 and 0.8
coupled with the pressure fluctuations during the occurrence of a peak.

5.3.2 Oblique incidence cases

In the oblique incidence case, the flow is at $45^\circ$ to two sides of the prism. Under these conditions, the flow separation, induced by the leading edges on top of the prism, contains two conical vortices, which resemble delta-wing vortices. As shown in figure 5.1, the tap located under one of the two vortices is tap 50209. In this section, the linear and nonlinear relations between pressure peaks observed at this tap and the velocity fluctuations
Figure 5.26: Wavelet cross-bicoherence of (a): \( p_1 - u \); (b): \( p_2 - u \); (c): \( p_3 - u \) and the associated statistical noise for the records shown in figure 5.16; integration centered at \( t=62.800 \) seconds; integration time length: 0.35s; contour levels for both cross-bicoherence and statistical noise: 0.4, 0.6 and 0.8 of the streamwise components, as measured above the taps, are determined.

\[ \sqrt{ \text{Pressure coefficients near } t=11.950 \text{s, from Hotsam49} } \]

Figure 5.39 shows two-second segments of the u-component of the velocity and pressure fluctuations measured in the Hotsam49 record. It is important to note that the occurrence of the pressure peak is associated with a rise in magnitude of the velocity. On the other hand, the interest is in the small variations which are a part of the peak near \( t=11.950 \)s. The long-time Fourier power spectrum of the velocity fluctuations was estimated using
Figure 5.27: Wavelet cross-bicoherence of (a): $p_1 - u$; (b): $p_2 - u$; (c): $p_3 - u$ and the associated statistical noise for the records shown in figure 5.16; integration centered at $t=62.800$ seconds; integration time length: 1.0s; contour levels for both cross-bicoherence and statistical noise: 0.4, 0.6 and 0.8

40960 data points that were split into 10 segments. The resulting power spectrum is shown in figure 5.40. The results clearly show that the flow field exhibits turbulence characteristics with the energy level of the fluctuations having the Kolmogoroff -5/3-law in the range of frequencies between 2Hz and 80Hz.

Figures 5.41 and 5.42 show respectively the magnitudes of the wavelet coefficients of the velocity and pressure fluctuations near the peak event. The plot in figure 5.41 does not show a clear high level of energy in the velocity spectrum that can be associated with the pressure peak. On the
Figure 5.28: Wavelet cross-bicoherence of (a): $p_1 - u$; (b): $p_2 - u$; (c): $p_3 - u$ and the associated statistical noise for the records shown in figure 5.16; integration centered at $t=62.800$ seconds; integration time length: 2.0s; contour levels for both cross-bicoherence and statistical noise: 0.4, 0.6 and 0.8.

On the other hand, figure 5.42, which shows the magnitude of the wavelet coefficients of the pressure fluctuations, exhibits a clear peak in the frequency range around 15Hz. From figure 5.43, which shows the wavelet power spectra of both velocity and pressure fluctuations obtained over three different integration time lengths, 0.4s, 1.0s and 2.0s, it is noted that in the velocity fluctuations, the frequency ranges near 10Hz, 15Hz and 25Hz contain relatively higher energy levels than their side-bands. The pressure fluctuations show two major energy-containing ranges near 10Hz and 15Hz. When the integration time length is increased, the aforementioned energy-containing scales are altered and/or shifted to some degree, which implies that the
Figure 5.29: Two-second time series of the u-component of the velocity fluctuations and the surface pressures at tap 50900 as measured in record Hotsam11.

Characteristic frequency ranges detected over the shortest integration time are intermittent and are associated with the peak event. By inspecting the wavelet linear coherence estimated over the same three integration time lengths and presented in figure 5.44, linear relations in the frequency ranges near 10Hz and 15Hz are noted. For extended integration ranges, the levels of linear coherence over both of these ranges decrease significantly. This clearly shows that there is a linear relation between the velocity and pressure fluctuations over the time of occurrence of the pressure peak.

Figures 5.45-5.47 show the wavelet auto-bicoherence for three different integration time lengths of the velocity and pressure fluctuations presented in figure 5.39. Over the shortest integration time length, figure 5.45(a) shows two nonlinearly coupled frequency regions in the auto-bicoherence plot of the velocity fluctuations. It is important to note that the couplings involve all three energy-containing frequency ranges identified from the velocity power spectrum. From the auto-bicoherence plot of the pressure fluctuations in figure 5.45(b), the highest coupled pair is detected between the
Figure 5.30: Wavelet power spectra of the velocity fluctuations ($P_{uu}$) and the pressure fluctuations ($P_{pp}$) for the records shown in figure 5.29. Three different integration time lengths, 0.5s, 1.0s and 2.0s, are used.

frequency ranges near 10Hz and 5Hz, which involves an interaction between the two energy-containing frequency ranges near 10Hz and 15Hz. Estimates of the auto-bicoherence over the longer integration time lengths presented in figures 5.46 and 5.47 show that all the high coupling levels observed in figure 5.45 decrease significantly, which implies that the detected nonlinear couplings are intermittent and associated with the peak event.
Figure 5.31: Wavelet linear coherence between the velocity and pressure fluctuations for the records shown in figure 5.29. Three different integration time lengths, 0.5s, 1.0s and 2.0s, are used. The dotted lines indicate the associated noise levels.

Figure 5.32: Wavelet auto-bicoherence of (a): velocity fluctuations; (b): pressure fluctuations and the associated statistical noise for the records shown in figure 5.29; integration centered at t=15.900 seconds; integration time length: 0.50s; contour levels for both auto-bicoherence and statistical noise: 0.4, 0.6 and 0.8
Figure 5.33: Wavelet cross-bicoherence between the velocity and pressure fluctuations and the associated statistical noise for the records shown in figure 5.29; integration centered at $t=15.900$ seconds; integration time length: 0.50s; contour levels for both cross-bicoherence and statistical noise: 0.4, 0.6 and 0.8

Figure 5.34: Two-second time series of the u-component of the velocity fluctuations and the surface pressures at tap 50900 as measured in record Hotsam14
Figure 5.35: Wavelet power spectra of the velocity fluctuations ($P_{uu}$) and the pressure fluctuations ($P_{pp}$) for the records shown in figure 5.34. Three different integration time lengths, 0.3s, 1.0s and 2.0s, are used.

Figure 5.36: Wavelet linear coherence between the velocity and pressure fluctuations for the records shown in figure 5.34. Three different integration time lengths, 0.3s, 1.0s and 2.0s, are used. The dotted lines indicate the associated noise levels.
Figure 5.37: Wavelet auto-bicoherence of (a): velocity fluctuations; (b): pressure fluctuations and the associated statistical noise for the records shown in figure 5.34; integration centered at $t=19.250$ seconds; integration time length: 0.3s; contour levels for both auto-bicoherence and statistical noise: 0.4, 0.6 and 0.8

Figure 5.38: Wavelet auto-bicoherence of (a): velocity fluctuations; (b): pressure fluctuations and the associated statistical noise for the records shown in figure 5.34; integration centered at $t=19.250$ seconds; integration time length: 0.3s; contour levels for both cross-bicoherence and statistical noise: 0.4, 0.6 and 0.8
Figure 5.39: Two-second time series of the u-component of the velocity fluctuations and the surface pressures at taps 50209 as measured in record Hotsam49

Figure 5.40: Fourier power spectrum of the velocity fluctuations ($P_{uu}$) of record Hotsam49
Figure 5.41: Wavelet coefficient magnitudes of the velocity fluctuations presented in figure 5.39

Figure 5.42: Wavelet coefficient magnitudes of the pressure record at tap 50209 presented in figure 5.39
Figure 5.43: Wavelet power spectra of the velocity fluctuations ($P_{uu}$) and the pressure fluctuations ($P_{pp}$) for the records shown in figure 5.39. Three different integration time lengths, 0.4s, 1.0s and 2.0s are used.

Figure 5.44: Wavelet linear coherence between the velocity and pressure fluctuations for the records shown in figure 5.39. Three different integration time lengths, 0.4s, 1.0s and 2.0s, are used. The dotted lines indicate the associated noise levels.
Figure 5.45: Wavelet auto-bicoherence of (a): velocity fluctuations; (b): pressure fluctuations and the associated statistical noise for the records shown in figure 5.39; integration centered at \(t=11.950\) seconds; integration time length: 0.40s; contour levels for both auto-bicoherence and statistical noise: 0.5, 0.7 and 0.9
Figure 5.46: Wavelet auto-bicoherence of (a): velocity fluctuations; (b): pressure fluctuations and the associated statistical noise for the records shown in figure 5.39; integration centered at t=11.950 seconds; integration time length: 1.0s; contour levels for both auto-bicoherence and statistical noise: 0.5, 0.7 and 0.9

Figure 5.48 presents contours of the cross-bicoherence between the pressure and velocity fluctuations over the shortest integration time length, 0.4s. Two evident couplings are observed from the cross-bicoherence contours, namely in the frequency range near 25Hz and 5Hz and near 15Hz and 10Hz. The cross-bicoherence level decreases significantly as the integration time length is increased to 1.0s and 2.0s, as shown in figures 5.49 and 5.50. This implies that the two nonlinear relations between the pressure and velocity fluctuations are highly intermittent and associated with the peak event.

√ Pressure coefficients near t=38.920s, from Hotsam49

To assess the reliability of the conclusions made above for one observed peak, another peak from the same record is analyzed. Figure 5.51 shows two-second data segments, from record Hotsam49, of the velocity and pressure fluctuations. Again, it is noted that the pressure peak observed at t=38.920s is associated with an increase in the magnitude of the local velocity. The duration of the this pressure peak is slightly less than 0.2s. The
magnitude of the wavelet coefficients of the both velocity and pressure fluctuations are presented in figures 5.52 and 5.53, respectively. Local peaks can be identified near $t=38.920s$ in the frequency range around 15Hz in the velocity fluctuations and around 15Hz and 35Hz in the pressure fluctuations.

Figure 5.54 shows the wavelet power spectra of the velocity and pressure fluctuations, as presented in figure 5.51, estimated over three different time lengths: 0.2s, 1.0s and 2.0s. The spectrum of the velocity fluctuations clearly shows relatively high levels of energy in the frequency ranges near 15Hz, 25Hz and 35Hz. The associated pressure spectrum shows two energy-containing frequency ranges near 15Hz and 35Hz. As the integration time length is extended, the energy-containing scales in both spectra become averaged out and disappear, which shows the high intermittency of these energy-containing scales and their association with the pressure peaks. From figure 5.55, which presents the wavelet linear coherence between the velocity and pressure fluctuations of figure 5.51 for three integration time lengths, high levels of linear coherence over the frequency ranges near 15Hz and 35Hz are noted. Over longer integration time lengths, the
linear coherence over both of these ranges decays rapidly, manifesting a strong association of this linear coherence with the peak event.

Nonlinear couplings in the velocity and pressure fluctuations of figure 5.51 are detected in the auto-bicoherence plots that are shown in figure 5.56 for the shortest integration time length. In the velocity fluctuations, two frequency ranges near (35, -10) and (25, -10) exhibit high levels of nonlinear coupling. From the pressure fluctuations, one coupling near (25, -10) is noted. Figures 5.57 and 5.58 show the auto-bicoherence of the velocity and pressure fluctuations over the longer integration time lengths, 1.0s and 2.0s. In comparison with figure 5.56, all the previously detected nonlinear coupling patterns disappear from the auto-bicoherence contours, which implies their close association with the peak event. Figure 5.59 shows the wavelet cross-bicoherence between the velocity and pressure fluctuations estimated over 0.3s. Two major nonlinear coupling frequency ranges near (35, -15) and near (35, -25) are noted. The coupling (35, -25) connects again the two major energy-containing scales in the velocity fluctuations with their difference components in the pressure fluctuations. Moreover, the two frequency
components also interact with their difference in the velocity fluctuations through the coupling (35, -10). This leads to a high linear coupling near 10Hz between the velocity and pressure fluctuations, as obtained from figure 5.55. The other nonlinear relation, (35, -15), represents the interaction between two major energy-containing scales in the velocity fluctuations and their difference frequency range near 20Hz in the pressure fluctuations. It is also remarkable that the two frequency components in the pressure fluctuations that are involved in the nonlinear relations are also coupled with each other through the coupling near (22, -10), observed in figure 5.56. Over the longer time lengths, as shown in figures 5.60 and 5.61, the two nonlinear relations become obviously averaged out. Therefore, the whole set of self- and mutual couplings reveal active interactions between energy-containing scales in both velocity and pressure fluctuations. These interactions are associated with the peak event, and are highly intermittent.
Figure 5.50: Wavelet cross-bicoherence between the velocity and pressure fluctuations and the associated statistical noise for the records shown in figure 5.39; integration centered at t=11.950 seconds; integration time length: 2.0s; contour levels for both cross-bicoherence and statistical noise: 0.5, 0.7 and 0.9

✓ Pressure peaks from records Hotsam47 and Hotsam48

Pressure peak near t=22.850s, at tap 50209, from Hotsam47  The results presented above are extended by examining the linear and nonlinear relations between peaks observed at the same pressure tap, namely 50209, and the velocity fluctuations measured at distances of 0.3\(H\) (from record Hotsam47) and 0.225\(H\) (from record Hotsam48). Figure 5.62 shows records of the simultaneously measured velocity and pressure fluctuations in the Hotsam47 record. The interest is in the pressure peak centered at t=22.850s, whose duration is estimated to be about 0.4s. It is observed again that the pressure peak is accompanied by a rise in the magnitude of the local velocity signal. Figure 5.63 shows the wavelet power spectra of the velocity and pressure fluctuations estimated over three integration time lengths, 0.4s, 1.0s and 2.0s. For the shortest time length, namely 0.4s, three energy-containing frequency ranges, near 10Hz, 15Hz and 25Hz, are identified in the velocity fluctuations. Similarly, there are also three energy-containing scales over the frequency ranges near 5Hz, 15Hz and 25Hz in the pressure fluctuations. The wavelet linear coherence, between the two signals presented in figure 5.64, shows peaks near the frequency ranges identified
To assess the nonlinear relations in the velocity and pressure fluctuations respectively and between them, wavelet auto- and cross-bicoherence were calculated for the three integration time lengths, 0.4s, 1.0s and 2.0s.

The contour plots of the auto- and cross-bicoherence for the shortest integration time length, namely 0.4s, are presented in figures 5.65 and 5.66, respectively. The auto-bicoherence contour plot of the velocity fluctuations show no dominant coupling patterns. In the contour plot of the auto-bicoherence of the pressure fluctuations, two major couplings are identified near (40, -15) and (25, -10), which represent couplings among the energetic spectral ranges identified in the wavelet power spectra. Over longer integration time lengths (figures not shown), the bicoherence level for these frequency ranges decrease rapidly, exhibiting the intermittence of the detected nonlinear couplings and their association with the peak event. The
Figure 5.52: Wavelet coefficient magnitudes of the velocity fluctuations presented in figure 5.51

contour plot of the cross-bicoherence between velocity and pressure fluctuations presented in figure 5.66 shows one distinct coupling over the frequency range near (25, -15). This high-level bicoherence indicates a major nonlinear coupling over the frequency ranges near 25Hz and 15Hz in the velocity fluctuations and the frequency range near their difference in the pressure fluctuations. This relation is also associated with the peak event, according to the contour plots of the cross-bicoherence obtained over longer integration time lengths (figures not shown).

Pressure peak near \( t = 1.430 \text{s}, \) at tap 50209, from Hotsam48  Figure 5.67 shows two-second time series of the simultaneously measured velocity and pressure signals in record Hotsam48. The interest is in the peak centered near \( t = 1.430 \text{s}, \) which is accompanied by an increase in the magnitude of the velocity. Figure 5.68 shows the wavelet power spectra of both velocity and pressure fluctuations of figure 5.67, estimated over three integration time lengths, 0.36s, 1.0s and 2.0s. Two frequency ranges, near 10Hz and 20Hz, are identified as energy-containing scales in the velocity
fluctuations, and two frequency ranges, near 10Hz and 25Hz, are identified as energy-containing scales in the pressure fluctuations. To detect linear relations between the velocity and pressure fluctuations, the wavelet linear coherence were estimated over three integration time lengths, 0.36s, 1.0s and 2.0s, and are presented in figure 5.69. For the shortest integration time length, 0.36s, the linear coherence is consistently low at all frequencies, except for a peak over the frequency ranges near 15Hz, 40Hz and 80Hz. The linear coherence at this peak drops significantly, when the integration time length is extended to 1.0s, and subsequently, 2.0s.

Figure 5.70 shows the wavelet auto-bicoherence of the velocity and pressure fluctuations of figure 5.67. For the shortest integration time length, 0.36s, high-valued contours concentrated in the frequency range near (20, -10) are observed in the wavelet auto-bicoherence plot of the velocity fluctuations, which represent a coupling in the velocity fluctuations over the frequency ranges near 20Hz and 10Hz. Similarly, high-valued contours centered at the frequency pair near (25, -15), are noted, in the pressure fluctuations over the frequency ranges near 25Hz and 15Hz. Over the longer
Figure 5.54: Wavelet power spectra of the velocity fluctuations \( (P_{uu}) \) and the pressure fluctuations \( (P_{pp}) \) for the records shown in figure 5.51. Three different integration time lengths, 0.2s, 1.0s and 2.0s are used.

integration time lengths, 1.0s and 2.0s, these two self-couplings disappear (figures not shown), revealing that these two nonlinear relations are intermittent and associated with the pressure peak. Figure 5.71 shows the contour plot of the wavelet cross-bicoherence of the velocity and pressure fluctuations, estimated over the three integration time lengths. For the shortest time, namely 0.36s, high level contours are noted at the frequency pair near \((20, -10)\). This represents a cross-coupling over the frequency ranges near 20Hz and 10Hz in the velocity fluctuations and the frequency range near their difference frequency, namely 10Hz, in the pressure fluctuations. Particularly, it is important to note that the same two frequency ranges, near 20Hz and 10Hz, in the velocity fluctuations are also self-coupled with their difference frequency range. Hence, the two difference scales near 10Hz in the both velocity and pressure fluctuations should be linearly coupled. This verifies the linear relation over the frequency range near 10Hz, as detected from figure 5.69.
Figure 5.55: Wavelet linear coherence between the velocity and pressure fluctuations for the records shown in figure 5.51. Three different integration time lengths, 0.2s, 1.0s and 2.0s, are used. The dotted lines indicate the associated noise levels.

**Pressure-velocity relations for different elevations in oblique incidence cases**  By comparing the results of the analyses performed on simultaneous velocity and surface pressure measurements from records Hot-sam47, Hotsam48 and Hotsam49, it is noted that the surface pressure fluctuations have different linear and nonlinear relations with the velocity fluctuations measured over the pressure tap with different distances. It clearly reveals that all points in the whole flow field (at least in the near field) have influence on the surface pressure fluctuations through linear and nonlinear interactions.

It is also important to point out, based on the results for both normal and oblique cases, that frequency ranges below 40Hz in velocity and pressure fluctuations were identified to be very active in linear and nonlinear couplings. Hence, the nondimensional frequency, \( fH/U \), that is equivalent to 40Hz is approximately 0.32, where \( H \) denotes the height of the prism, 8cm, and \( U \) denotes the approaching wind speed, around 10m/s. Tieleman et al. (1998, [18]) reported that the nonlinear contribution from the near-
field and far-field velocity fluctuations to the surface pressures on models is insignificant, when the nondimensional frequency $f_H/U$ is higher than 0.122. However, it is shown, in this work, that frequencies up to $f_H/U=0.32$ are critical to intermittent linear and nonlinear relations between pressure peaks and their associated velocity fluctuations in the near-field. Consequently, for wind tunnel and numerical simulations, the frequency range up to $f_H/U=0.32$ should be reproduced, in order to properly simulate the extreme wind loads on low-rise buildings in full-scale experiments.
Figure 5.57: Wavelet auto-bicoherence of (a): velocity fluctuations; (b): pressure fluctuations and the associated statistical noise for the records shown in figure 5.51; integration centered at t=38.920 seconds; integration time length: 1.0s; contour levels for both auto-bicoherence and statistical noise: 0.5, 0.7 and 0.9

Figure 5.58: Wavelet auto-bicoherence of (a): velocity fluctuations; (b): pressure fluctuations and the associated statistical noise for the records shown in figure 5.51; integration centered at t=38.920 seconds; integration time length: 2.0s; contour levels for both auto-bicoherence and statistical noise: 0.5, 0.7 and 0.9
Figure 5.59: Wavelet cross-bicoherence between the velocity and pressure fluctuations and the associated statistical noise for the records shown in figure 5.51; integration centered at $t=38.920$ seconds; integration time length: 0.20s; contour levels for both cross-bicoherence and statistical noise: 0.5, 0.7 and 0.9

Figure 5.60: Wavelet cross-bicoherence between the velocity and pressure fluctuations and the associated statistical noise for the records shown in figure 5.51; integration centered at $t=38.920$ seconds; integration time length: 1.0s; contour levels for both cross-bicoherence and statistical noise: 0.5, 0.7 and 0.9
Figure 5.61: Wavelet cross-bicoherence between the velocity and pressure fluctuations and the associated statistical noise for the records shown in figure 5.51; integration centered at $t=38.920$ seconds; integration time length: 2.0s; contour levels for both cross-bicoherence and statistical noise: 0.5, 0.7 and 0.9

Figure 5.62: Two-second time series of the $u$-component of the velocity fluctuations and the surface pressures at taps 50209 as measured in record Hotsam47
Figure 5.63: Wavelet power spectra of the velocity fluctuations ($P_{uu}$) and the pressure fluctuations ($P_{pp}$) for the records shown in figure 5.62. Three different integration time lengths, 0.4s, 1.0s and 2.0s are used.

Figure 5.64: Wavelet linear coherence between the velocity and pressure fluctuations for the records shown in figure 5.62. Three different integration time lengths, 0.4s, 1.0s and 2.0s, are used. The dotted lines indicate the associated noise levels.
Figure 5.65: Wavelet auto-bicoherence of (a): velocity fluctuations; (b): pressure fluctuations and the associated statistical noise for the records shown in figure 5.62; integration centered at $t=22.850$ seconds; integration time length: 0.40s; contour levels for both auto-bicoherence and statistical noise: 0.4, 0.6 and 0.8

Figure 5.66: Wavelet cross-bicoherence between the velocity and pressure fluctuations and the associated statistical noise for the records shown in figure 5.62; integration centered at $t=22.850$ seconds; integration time length: 0.40s; contour levels for both cross-bicoherence and statistical noise: 0.4, 0.6 and 0.8
Figure 5.67: Two-second time series of the u-component of the velocity fluctuations and the surface pressures at taps 50209 as measured in record Hotsam48

Figure 5.68: Wavelet power spectra of the velocity fluctuations ($P_{uu}$) and the pressure fluctuations ($P_{pp}$) for the records shown in figure 5.67. Three different integration time lengths, 0.36s, 1.0s and 2.0s are used.
Figure 5.69: Wavelet linear coherence between the velocity and pressure fluctuations for the records shown in figure 5.67. Three different integration time lengths, 0.36s, 1.0s and 2.0s, are used. The dotted lines indicate the associated noise levels.

Figure 5.70: Wavelet auto-bicoherence of (a): velocity fluctuations; (b): pressure fluctuations and the associated statistical noise for the records shown in figure 5.67; integration centered at t=1.430 seconds; integration time length: 0.36s; contour levels for both auto-bicoherence and statistical noise: 0.4, 0.6 and 0.8
Figure 5.71: Wavelet cross-bicoherence between the velocity and pressure fluctuations and the associated statistical noise for the records shown in figure 5.67; integration centered at t=1.430 seconds; integration time length: 0.36s; contour levels for both cross-bicoherence and statistical noise: 0.4, 0.6 and 0.8
Chapter 6

Conclusions

This thesis dealt with two aspects related to the characterization and simulation of wind loads on structures. The analysis performed aims at the development of tools that should be used in the validation of model tests and numerically simulated data against full-scale measurements.

The first part of this thesis is concerned with the statistical properties of surface pressure time series and their variations under different incident flow conditions. The results show that unless the levels of turbulence intensity in the incident flow are low (below 10%), the Gamma distribution provides the best fit for the PDF of the pressure time series. Two different approaches are presented for determining the parameters of the Gamma distribution. The first approach is based on the moments (mean, standard deviation and skewness) of the time series. The second approach is based on an optimization of procedure of the probability plot correlation coefficient. The results show that the method of moments is sufficient for determining the parameters of the Gamma distribution.

The analysis presented in the first part of this thesis also includes two approaches to predict the distribution of pressure peaks. One approach is a mapping procedure, which makes use of the whole parent time series, and the other approach is based on the observed pressure peaks from 64 independent records. Both approaches show that for the cases where the parent time series is described by the Gamma distribution, the Gumbel distribution is suitable for describing the distribution of the pressure peaks. The
results show that the prediction based on the parent time series yields more conservative estimates of the probability of non-exceedence. This is due to the fact that the number of pressure peaks, 64, might not be large enough to statistically characterize the pressure peaks.

The results presented in Part I can be summarized as follows: for both normal and oblique cases, the higher turbulence intensities in the incident flow generally result in higher pressure peaks. Moreover, the level of the pressure peak values can be related to the shape parameter of the fitting Gamma distribution for the parent time series. Lower shape parameters generally correspond to higher peak values, except for Config.7, which has a much larger parameter of small-scale turbulence than all other configurations. This result matches with the predictions obtained by fitting the pressure peaks with the Gumbel distribution.

In the second part of this thesis, the linear and nonlinear relations between the velocity fluctuations in the near flow field and simultaneously measured surface pressures are identified. Particular interest was taken into these relations during the occurrence of pressure peaks. This necessitated the use of a hierarchy of wavelet-based spectral moments. These moments are proven to be more suitable than their Fourier-based counterparts for the detection of intermittent linear and nonlinear relations. Through the evaluation of wavelet-based higher-order spectral moments over different integration time lengths, linear and nonlinear coupling patterns that are associated with the peak events are identified.

The results of the second part show that, for all cases, major linear and nonlinear couplings are enhanced by energy-containing scales. In the cases of normal incidence, observed pressure peaks near the leading edge are highly coupled with the velocity fluctuations as measured in the shear layer. Lesser coupling patterns are detected between these velocity fluctuations and the pressure fluctuations that are measured at taps located further downstream. It is also shown, for both normal and oblique incidence cases,
that as the distance between the measuring locations of the velocity and pressure fluctuations is decreased, higher frequency ranges become involved in the linear and nonlinear couplings. Consistently, it is found that the intermittent linear and nonlinear velocity-pressure relations are observed over the nondimensional frequency range $fH/U < 0.32$. This observation stresses the requirement that flow characteristics in this frequency range must be reproduced in wind tunnel and numerical simulations of wind loads on structures.
Bibliography


Vita

Zhongfu Ge was born on September 24th, 1974 in Shanghai, China, as the only child of his parents.

After graduating from high school, Zhongfu Ge joined the Department of Engineering Mechanics at Shanghai Jiao Tong University, Shanghai, China, for his higher education in 1993. Through four years of hard work, he earned his bachelor’s degree in engineering mechanics, and was admitted to the graduate school as one of the top five students of the department in 1997.

In graduate school at Shanghai Jiao Tong University, Zhongfu Ge chose fluid mechanics as his major, advised by Professor Qu-yuan Ye. In June 2000, he earned his MS degree. The thesis was on numerical simulation of high-speed gas flow field.

Zhongfu Ge came to the Department of Engineering Science and Mechanics at Virginia Polytechnic Institute and State University in August 2000 in pursuit of a Ph.D degree. His research has been focused on wind engineering, turbulence and signal processing, advised by Dr. Muhammad R. Hajj. In December 2004, he is going to defend his dissertation.