Modeling and Testing of Fast Response, Fiber-Optic Temperature Sensors

by

Michael James Tonks

Dissertation submitted to the Faculty of Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in

Aerospace Engineering

Approved by:
Dr. Joseph A. Schetz, Chair
Dr. Elaine P. Scott
Dr. Kent A. Murphy
Dr. Rakesh K. Kapania
Dr. William J. Devenport

January 2006
Blacksburg, Virginia

Keywords: Fiber-Optic Sensors, Temperature Sensors, Shock Tubes, Inverse Heat Transfer

© 2006, Michael James Tonks
Modeling and Testing of Fast Response, Fiber-Optic Temperature Sensors

Michael J. Tonks, Ph.D.
Virginia Polytechnic Institute and State University, 2006
Advisor: Dr. Joseph A. Schetz

ABSTRACT

The objective of this work was to design, analyze and test a fast response fiber-optic temperature probe and sensor. The sensor is intended for measuring rapid temperature changes such as produced by a blast wave formed by a detonation. This work was performed in coordination with Luna Innovations Incorporated, and the design is based on extensions of an existing fiber-optic temperature sensor developed by Luna. The sensor consists of a glass fiber with an optical wafer attached to the tip. A basic description of the principles behind the fiber-optic temperature sensor and an accompanying demodulation system is provided.

For experimental validation tests, shock tubes were used to simulate the blast wave experienced at a distance of 3.0 m from the detonation of 22.7 kg of TNT. The flow conditions were predicted using idealized shock tube theory. The temperature sensors were tested in three configurations, flush at the end of the shock tube, extended on a probe 2.54 cm into the flow and extended on a probe 12.7 cm into the flow. The total temperature was expected to change from 300 K to 1130 K for the flush wall experiments and from 300 K to 960 K for the probe experiments. During the initial 0.1 milliseconds of the data the temperature only changed 8 K when the sensors were flush in the end of the shock tube. The sensor temperature changed 36 K during the same time when mounted on a probe in the flow. Schlieren pictures were taken of the flow in the shock
tube to further understand the shock tube environment. Contrary to ideal shock tube theory, it was discovered that the flow did not remain stagnant in the end of the shock tube after the shock reflects from the end of the shock tube. Instead, the effects of turbulence were recorded with the fiber-optic sensors, and this turbulence was also captured in the schlieren photographs. A fast-response thermocouple was used to collect data for comparison with the fiber-optic sensor, and the fiber-optic sensor was proven to have a faster response time compared to the thermocouple. When the sensors were extended 12.7 cm into the flow, the fiber-optic sensors recorded a temperature change of 143 K compared to 38 K recorded by the thermocouple during the 0.5 millisecond test. This corresponds to 22% of the change of total temperature in the air recorded by the fiber-optic sensor and only 6% recorded by the thermocouple. Put another way, the fiber-optic sensor experience a rate of temperature change equal to 2.9x10^5 K/s and the thermocouple changed at a rate of 0.79x10^5 K/s. The data recorded from the fiber-optic sensor also contained much less noise than the thermocouple data.

An unsteady finite element thermal model was created using ANSYS to predict the temperature response of the sensor. Test cases with known analytical solutions were used to verify the ANSYS modeling procedures. The shock tube flow environment was also modeled with Fluent, a commercially available CFD code. Fluent was used to determine the heat transfer between the shock tube flow and the sensor. The convection film coefficient for the flow was predicted by Fluent to be 27,150 W/m²K for the front of the wafer and 13,385 W/m²K for the side. The Fluent results were used with the ANSYS model to predict the response of the fiber-optic sensor when exposed to the shock tube flow. The results from the Fluent/ANSYS model were compared to the fiber-optic measurements taken in the shock tube. It was seen that the heat flux to the sensor was slightly over-predicted by the model, and the heat losses from the wafer were also over-predicted. Since the prediction fell within the uncertainty of the measurement, it was found to be in good agreement with the measured values.

Inverse heat transfer methods were used to determine the total temperature of the flow from the measured data. Both the total temperature and the film coefficient were
determined simultaneously during this process. It was found that for short testing times, there were many possible solutions. In order to obtain ultimate success with this method, the uncertainty of the demodulation system must be improved and/or the simple analytical thermal model used to predict the response of the sensor needs to match the physical sensor. Whenever possible, longer testing times should be employed. Promising suggestions for extending this approach are provided.
Acknowledgements

First I would like to thank my family for their support and encouragement throughout my educational experiences. My parents always provided wisdom and advice; my children were my greatest source of joy and my wife provides all of the missing pieces to my soul.

I have been blessed to know many great teachers. Dr. Joseph A. Schetz has given much valued help and mentoring. I appreciate the masterful way in which theoretical knowledge was combined with practical application to teach valuable lessons. Thank you to all of my committee members for their help and patience. I also thank my fourth grade teacher, Mrs. Keller, for showing me the joy of learning.

Special thanks to Luna Innovations Inc. for providing me with this research opportunity. In particular Matthew Palmer helped in research and design discussions. He also joined with Matthew Davis and David Slusher in running the fiber-optic side of the experiments.

Without help from Bruce Stanger, James Lambert and Steve Edwards the experiments performed in this work would not have been possible.

Research on this project was supported in part by the United States Army, Engineering Research and Development Center, under Contract No. W912HZ-04-C-0006, awarded to Luna Innovations Incorporated. Such support does not constitute an endorsement by the Army or Luna of the views and opinions expressed herein.
# Table of Contents

Abstract .............................................................................................................. ii
Acknowledgements .......................................................................................... v
Table of Contents ............................................................................................ vi
List of Figures .................................................................................................. ix
List of Tables ................................................................................................... xiv
Nomenclature ................................................................................................... xv

1. Introduction .................................................................................................. 1
2. Review of Literature ................................................................................... 4 
   2.1. Deflagration / Detonation Measurements ........................................ 4 
   2.2. Fiber-Optic Temperature Sensors .................................................... 5 
   2.3. Inverse Heat Transfer Methods ......................................................... 6 
3. Fiber-Optic Temperature Sensors ............................................................... 7 
   3.1. Basic Temperature Sensor Design ................................................... 8 
   3.2. Fiber-Optic Signal Processing System ............................................. 9 
       3.2.1. Hyperscan System Specifications .......................................... 10 
4. Shock Tubes ................................................................................................. 12 
   4.1. Flow Properties in the Shock Tube .................................................... 14 
       4.1.1. Shock Tube Relations .............................................................. 15 
       4.1.2. Graphical Solution Method .................................................... 18 
       4.1.3. Nominal Testing Time ............................................................. 22 
   4.2. Prediction Results ............................................................................ 24 
   4.3. Real Gas Effects ............................................................................. 26 
5. Computational Modeling ............................................................................. 30
5.1. Modeling the Fiber-Optic Temperature Sensor ................................................. 30
  5.1.1. Finite Element Model ........................................................................ 33
  5.1.2. Constant Surface Heat Flux .............................................................. 37
  5.1.3. Constant Surface Temperature ........................................................... 39
5.2. Flowfield Modeling .................................................................................. 40
  5.2.1. Inviscid Solution .............................................................................. 43
  5.2.2. Viscous Solution .............................................................................. 45
5.3. Composite Computational Model for Flow and Sensor Response ......................... 48
6. Shock Tube Experiments ............................................................................ 50
    6.1.1. Shock Tube Operating Procedures ............................................. 51
    6.1.2. Preliminary Results ........................................................................ 53
  6.2. Schlieren Photography ........................................................................... 57
    6.2.1. Schlieren Setup ........................................................................... 57
    6.2.2. Schlieren Pictures ........................................................................ 61
  6.3. Sensors Extended into the Flow ............................................................. 63
    6.3.1. Short Probe .................................................................................. 70
    6.3.2. Longer Probe ................................................................................ 70
    6.3.3. Testing Conditions ......................................................................... 71
    6.3.4. Experimental Results ................................................................. 72
7. Data Analysis .............................................................................................. 77
  7.1. Inverse Heat Transfer Solution Method .............................................. 77
    7.1.1. Least Squares Parameter Estimation ........................................... 79
    7.1.2. Least Squares with Regularization ............................................... 80
7.2. Application of Inverse Heat Transfer Methods to the Fiber-Optic Temperature Sensors ............................................. 80
8. Computational Results and Comparison with Experiments ..... 94
9. Discussion .................................................................................................................. 99
  9.1. Future Work ........................................................................................................... 106
References .................................................................................................................. 108
Appendix ...................................................................................................................... 112
# List of Figures

1.1: U.S. Army Corp of Engineers Explosives Testing Facility .............. 1  
1.2: Picture of Fiber-Optic Temperature Sensor ................................. 2  
3.1: Extrinsic Fabry-Perot Interferometry ........................................ 7  
3.2: Temperature Sensor Design (not to scale) .................................. 8  
3.3: Picture of a Temperature Sensor ................................................ 8  
3.4: Single Wavelength Demodulation System ................................. 9  
3.5: Dual Wavelength Demodulation System ................................. 10  
4.1: Shock Tube Operation .......................................................... 12  
4.2: Reflected Shock Wave / Contact Surface Interaction .................... 13  
4.3: Shock Tube x-t Diagram ......................................................... 14  
4.4: Graphical Method – Step 1 ....................................................... 20  
4.5: Graphical Method – Step 2 ....................................................... 21  
4.6: Graphical Method – Step 3 ....................................................... 22  
4.7: Variation of the Flow Parameters in Air, Behind the Normal  
    Reflected Shock Wave with the incident Shock Wave Mach  
    Number .......................................................... 29  
5.1: Fiber-Optic Sensor at the Tip of a Probe ................................. 33  
5.2: ANSYS Heat Transfer Model Configuration .................................. 34  
5.3: Meshing for the ANSYS Model ................................................. 36  
5.4: Sensor with 100,000 W/m² applied to the Surface ....................... 38  
5.5: Temperature Distribution in Two Semi-Infinite Solids Initially at  
    Different Temperatures ....................................................... 40  
5.6: Temperature Distribution in the Sensor when in Contact with Hot,  
    Stagnant Air .......................................................... 41
5.7: Flowfield Model at the Sensor Tip ........................................ 42
5.8: Fluent Model Geometry .................................................... 42
5.9: Mach Number Contours – Inviscid Flow Solution (Mesh Size: 9 µm x 7 µm Near the Wafer) ........................................... 44
5.10: Inviscid Fluent Predictions of the Pressure Change at the Sensor 45
5.11: Fluent Model Grid Independence ........................................ 46
5.12: Mach Number Contours – Viscous Flow ............................. 47
5.13: Total Temperature Contours – Viscous Flow ....................... 47
6.1: Shock Tube Configuration .................................................. 50
6.2: Driver Pressure and Temperature Measurements .................... 52
6.3: Comparison of Medtherm and Fiber-Optic Sensor Responses with the Sensors Flush in the End of the Shock Tube – Run 1 .............. 54
6.4: Comparison of Medtherm and Fiber-Optic Sensor Responses with the Sensors Flush in the End of the Shock Tube – Run 2 .............. 55
6.5: Comparison of Medtherm and Fiber-Optic Sensor Responses with the Sensors Flush in the End of the Shock Tube – Run 3 .............. 55
6.6: Comparison of ANSYS Prediction and Fiber-Optic Sensor Response with the Sensors Flush in the End of Shock Tube – Run 3 ....................................................... 56
6.7: Design for Shock Tube Modifications .................................... 57
6.8: Spool Piece Mounted on End of Shock Tube ............................ 58
6.9: Schlieren Photography Setup ............................................... 59
6.10: Diaphragm Bursting Wire ................................................... 60
6.11: Burst Diaphragms ............................................................. 61
6.12: Schlieren Photograph of the Incident Wave Approaching the End of the Shock Tube ....................................................... 62
6.13: Schlieren Photograph of Stagnant Region Behind Reflected Shock Wave ........................................... 62
6.14: Schlieren Photograph of Turbulence Reaching the End of the Shock Tube ........................................... 63
6.15: Fiber-Optic and Medtherm Probes ........................................... 65
6.16: Fiber-Optic Probe Tip Design ........................................... 66
6.17: Thermocouple Probe Tip Design ........................................... 66
6.18: Schlieren Photograph of Incident Wave Approaching the Flat-Tip Probe ........................................... 68
6.19: Schlieren Photograph of Developing Flow over a Flat-Tip Probe ........................................... 68
6.20: Schlieren Photograph of Fully Developed Flow over a Flat-Tip Probe ........................................... 68
6.21: Schlieren Photograph of Incident Wave Passing over Pointed-Tip Probe ........................................... 69
6.22: Schlieren Photograph of Developed Flow over Pointed-Tip Probe ........................................... 69
6.23: Schlieren Photograph of Reflected Wave Approaching the End of the Pointed-Tip Probe ........................................... 69
6.24: Short Probe Design ........................................... 70
6.25: Longer Probe Design ........................................... 71
6.26: Conditions for Sensor Failure ........................................... 72
6.27: Fiber-Optic Data – Short Probe ........................................... 73
6.28: Comparison of Fiber-Optic and Thermocouple Data – Short Probe ........................................... 74
6.29: Fiber-Optic Data – Longer Probe ........................................... 75
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.30</td>
<td>Comparison of Fiber-Optic and Thermocouple Data – Longer Probe</td>
<td>76</td>
</tr>
<tr>
<td>7.1</td>
<td>Classification of Heat Conduction Problems</td>
<td>78</td>
</tr>
<tr>
<td>7.2</td>
<td>Regularization Scaling Factor Independence Study</td>
<td>82</td>
</tr>
<tr>
<td>7.3</td>
<td>Contour Plot of Objective Function (Eq. 7.4) Using Data from Five Inch Probe Shock Tube Experiments</td>
<td>83</td>
</tr>
<tr>
<td>7.4</td>
<td>Minima Shown at Bottom of Trough</td>
<td>84</td>
</tr>
<tr>
<td>7.5</td>
<td>Ideal Response of Fiber-Optic Temperature Sensor over the Duration of Shock Tube Tests</td>
<td>85</td>
</tr>
<tr>
<td>7.6</td>
<td>Ideal Response of Fiber-Optic Temperature Sensor – 10 Times Longer Duration than Shock Tube Tests</td>
<td>86</td>
</tr>
<tr>
<td>7.7</td>
<td>Effects on the Objective Function of Lengthening the Testing Time</td>
<td>86</td>
</tr>
<tr>
<td>7.8</td>
<td>Sensitivity Coefficients for the Inverse Process</td>
<td>88</td>
</tr>
<tr>
<td>7.9</td>
<td>Determining the Minimum Testing Time Required for use of the Inverse Heat Transfer Method</td>
<td>89</td>
</tr>
<tr>
<td>7.10</td>
<td>Comparison of how the Initial Estimate of Total Temperature Affects Required Testing Time</td>
<td>90</td>
</tr>
<tr>
<td>7.11</td>
<td>Effect of Sampling Frequency on Required Testing Time</td>
<td>91</td>
</tr>
<tr>
<td>7.12</td>
<td>Theoretical Model for the Inverse Method Compared to Measure Data Points</td>
<td>93</td>
</tr>
<tr>
<td>7.13</td>
<td>Theoretical Model for the Inverse Method Compared to Measure Data Points with Measurement Uncertainties</td>
<td>93</td>
</tr>
<tr>
<td>8.1</td>
<td>Temperature Contour Plot of the Fluent/ANSYS Model</td>
<td>94</td>
</tr>
<tr>
<td>8.2</td>
<td>Fluent/ANSYS Time History Results</td>
<td>95</td>
</tr>
</tbody>
</table>
8.3: Comparison of Fiber-Optic Data and Fluent/ANSYS Model
Results – Short Probe .................................................. 96

8.4: Comparison of Fiber-Optic Data and Fluent/ANSYS Model
Results – Short Probe with Uncertainty of the Hyperscan Shown 97

8.5: Comparison of Fiber-Optic Data and Fluent/ANSYS Model
Results – Longer Probe .................................................. 98

8.6: Comparison of Fiber-Optic Data and Fluent/ANSYS Model
Results – Longer Probe Uncertainty of the Measured Data Shown 98
List of Tables

4.1: Initial Shock Tube Conditions .............................................. 25
4.2: Calculated Properties for Gas States 2, 3, and 5 ................. 25
5.1: Keypoint Coordinates for the Ansys Model ......................... 35
6.1: Testing Conditions with the Sensors Flush in Shock Tube Wall .. 53
6.2: Shock Tube Testing Conditions ........................................... 72
7.1: Some Possible Solutions from the Inverse Heat Transfer Problem 92
Nomenclature

a  Speed of Sound
A_s  Surface Area
b  Unknown Optimization Parameter
c_1, c_2  Optimization Regularization Constants
C_p  Constant Pressure Specific Heat
c_v  Constant Volume Specific Heat
e  Internal Energy
\dot{E}  Energy Rate
h  Enthalpy
\dot{h}  Convection Film Coefficient
h  Planck’s Constant
\dot{h}^*  Convection Film Coefficient Initial Approximation
k  Conductivity
K  Stagnation Point Velocity Gradient
L  Length
M  Mach Number
P  Pressure
Pr  Prandtl Number
q''  Heat Flux
R  Gas Constant
S  Objective Function
T  Temperature
t  Time
T_{aw}  Adiabatic Wall Temperature
T_t  Total Temperature
T_t^*  Total Temperature Initial Approximation
u  Fluid Speed
U_e  Boundary Layer Edge Velocity
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>Shock Speed</td>
</tr>
<tr>
<td>( Y_i )</td>
<td>Measured Data Points</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Parameter for Simplifying Shock Tube Calculations</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Parameter for Simplifying Shock Tube Calculations</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Shock Thickness</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Specific Heat Ratio</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Boltzman’s Constant</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Viscosity</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Courant Number</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Time Constant</td>
</tr>
</tbody>
</table>
Chapter 1. Introduction

As many weapons contain highly volatile substances, concerns arise about the possibility of a mass detonation occurring when large stockpiles of weapons are present in concentrated areas, such as munitions depots or ammunition production facilities. A mass detonation occurs when an accidental explosion of a single round of ammunition causes a large quantity of the surrounding ammunition to also detonate. Safety guidelines for facilities that house these materials are developed by studying the deflagration process. In addition, as new warfare technologies are developed, one area of research involves the discovery of new energetic materials for use as propellants or explosives. One method for testing energetic materials involves the deflagration and/or detonation of said materials and measuring the heat generated during these processes.

The United States Army Corps of Engineers currently operates facilities where such tests occur. Two of these facilities are located at the White Sands Missile Range, NM, and at the Linchburg Mine, Magdalena, NM. At these facilities, the energetic material is burned or detonated within a concrete bunker. An exhaust pipe is fitted to the bunker to allow for pressure relief. Figure 1.1 gives a sketch of one of the bunkers used in these experiments. The bunker and exhaust pipe are instrumented at various locations to collect pressure, temperature and heat flux data. Due to the nature of the detonation tests, the instruments used to collect data must be extremely fast and robust.

![Figure 1.1 U.S. Army Corps of Engineers Explosives Testing Facility](image)
Luna Innovations, Inc. of Blacksburg, VA is currently developing a fast response fiber-optic temperature sensor for use in these and similar testing facilities. Fiber-optic sensors have some distinct advantages over electronic sensors. Such sensors are generally very small and highly sensitive. Thus, they can have a very rapid time response. Fiber-optics offer a wide bandwidth and high data rate without the interference associated with electrical components. When security is an issue, optical fibers are a more secure form of data transmission since it is extremely difficult to tap into the fibers without destroying the fiber and thus the data transmission path. While electric components are subject to electromagnetic interference from lightning or power switching, fiber optics are not. Also, since only light travels through the optical fiber, there is no possibility of an electrical spark in a fiber-optic sensor. This makes a fiber-optic sensor an ideal instrument for use in volatile environments. The temperature sensors studied here are based on the principle of extrinsic Fabry-Perot interferometry. They are constructed by attaching an optical wafer material to the end of a glass fiber (see Fig. 1.2). The temperature change of the sensor is determined by examining how the thickness and index of refraction of this wafer change.

![Figure 1.2 Picture of Fiber-Optic Temperature Sensor](image)

This work details our efforts in testing and modeling these fiber-optic temperature sensors. A brief explanation of the fiber-optic sensors and demodulation system is included.
Experiments were performed in shock tubes to test the response of the sensor. The sensors were tested in three different configurations:

1- mounted flush in the end of the shock tube,
2- protruding from the end of the shock tube 2.54 cm (1.0 inch) into the flow, and
3- protruding 12.7 cm (5.0 inches) from the end of the shock tube.

The response of the fiber-optic sensors was compared to the response of high frequency response thermocouples. To further the understanding of events occurring within the shock tube, schlieren pictures of the flow were taken. These pictures detail the flow as the shock passes over the sensor and is reflected from the end of the shock tube.

Efforts were undertaken to use the fiber-optic sensor measured data to predict the temperature change of the air in the shock tube experiments. This involves predicting a large temperature change in the air by observing a relatively small temperature change of the sensor. Inverse heat transfer methods were employed for this purpose.

A complex model was developed to provide a detailed description of the flow and the sensor behavior and to aid in understanding these processes. A finite element model of the sensors was generated with the use of ANSYS, a popular commercially available finite element software package. This model was used as a design tool for understanding the response of the sensor to various temperature inputs. The computational fluid dynamics program Fluent was used to model the flow environment inside the shock tube. The Fluent model was used in conjunction with the ANSYS model in an effort to predict the sensor response to the shock tube flow.
Chapter 2. Review of Literature

2.1. Deflagration / Detonation Measurements

Four reports were provided by the US Army Corp of Engineers detailing measurements that have been made in their facilities in the past. In every test thermocouples, thermal flux gages and pressure transducers were the instruments used to record the flow data. These reports helped to define the design space for the intended fiber-optic temperature sensors.

Knox [21] has documented the effects of detonating explosives in an enclosed environment. Peak pressures up to 400 psi were experienced along with heat flux values of 100,000 to 300,000 W/m². Tests were completed in part to help validate a computer simulation of the detonation process.

When the propellant burn test was performed by Knox [22], heat flux values of up to 400,000 to 800,000 W/m² were measured. The temperature changed about 600 K within about 16 seconds, but the pressure only increased by about 35 psi. The deflagration tests lasted much longer when compared to the detonation tests.

In the propellant burning experiments performed by Joachim [19], the measured heat flux reached up to 1,500,000 W/m². Some of the propellants during this test were burned, and some were detonated. The purpose of this test was to determine the conditions that would cause detonation.

Finally, Joachim [18] showed some correlations between measured heat flux values and differentiated temperature time histories. He concludes that for “slow” thermal phenomena a thermocouple can be calibrated for use as a crude heat flux sensor.

In general, many methods have been developed to measure what happens during an explosion. Due to the short duration and violent nature of explosions, the number of useful sensors to be used during experiments is severely limited. Baker [3] reported in
1973 that there were many pressure transducers, one density transducer, a limited number of techniques for measuring velocity and essentially no temperature sensors that were suitable for use in blast experiments. Technological advances in temperature measurement techniques have since produced a few methods that qualify for use in these experiments. Danehy and Alderfer [7] have reported that thermocouples are still not preferred for measuring such short duration events. In underwater shock experiments, the shock travels at about 1500 m/s, and the fastest available thermocouples have a response time of about 2 milliseconds in air and a few hundred milliseconds in water. Instead, Danehy details five methods capable of measuring these underwater shocks, including fiber-optic sensors, which respond faster than thermocouples.

2.2. Fiber-Optic Temperature Sensors

Recent efforts have been made to develop fiber-optic sensors for measuring temperature. Several methods have been proposed, but only a few will be discussed here.

Dils developed a temperature sensor constructed of sapphire optical fiber that is described in Ref. 9. This sensor is formed by depositing a thin metal film onto the end of the fiber into a cup shape. A sapphire coating is then fit over the metal film. This film acts as a black body and the radiation it emits is transmitted back along the fiber to a detector. The temperature of the film is determined by measuring the amount of radiation energy sent back through the fiber.

Zhang [42] explored the use of fiber-optic temperature sensors in harsh environments and developed a temperature sensor that transmitted the temperature information in the spectrum of the light signal. This design was based on splitting and polarizing the light beam. Because the polarizing crystal properties are dependent on temperature, when the two light signals are analyzed they will be out of phase with one another. The temperature of the polarizer can be determined by studying this phase shift.

Many fiber-optic temperature sensors can be found that are based on Fabry-Perot interferometry. Most commercially available temperature sensors of this type currently
have an air gap between either two fibers or a fiber and a reflective surface. The sensor studied here is also based on Fabry-Perot interferometry, but it is of different construction. Details are provided in chapter 3.

2.3. **Inverse Heat Transfer Methods**

Inverse heat transfer methods have been shown to be useful to solve many heat transfer problems [1, 4, and 27]. A few of these include:

1. estimate the thermophysical properties of materials [24, 37, 41]
2. estimate the boundary heat flux from internal temperature measurements in a fluid [31, 36]
3. determine the contact resistance or heat transfer at an interface [29, 35]
4. estimate the heat flux or heat transfer coefficient from a surface temperature measurement [8, 14, 25, 26, 39]

It is proposed here to use the inverse heat transfer methods to determine the fluid temperature and film coefficient defining convection heat transfer into a surface by monitoring the temperature change of that surface. Although instances have been found where either the temperature, or the film coefficient have been determined separately, no report was found detailing the possibility of finding them both at the same time as intended here.
Chapter 3. Fiber-Optic Temperature Sensors

Fiber-optic technology has been used to make sensors that measure displacement, pressure, vibrations, motion, velocity and temperature, to name a few. Fiber-optic sensors have some distinct advantages over electrical sensors. Fiber-optics offer a wide bandwidth and high data rate without the interference associated with electrical components. Fiber-optic sensors can also be smaller and have faster responses than electrical sensors. While electric components are subject to electromagnetic interference from lightning or power switching, fiber optics are not. There are no sparks produced by optical fibers, which makes them an ideal instrument for use in volatile environments. Also, optical fibers are a more secure form of data transmission, since it is extremely difficult to tap into the fibers without destroying the fiber.

The temperature sensors studied here were developed based on the principle of extrinsic Fabry-Perot interferometry (EFPI). In EFPI sensors, a light signal is sent along a fiber. Part of the light is reflected from the end of the fiber, and the remaining light travels through a low-finesse Fabry-Perot cavity to a reflective surface where it is reflected back into the fiber (see Fig. 3.1). The two reflections, $R_1$ and $R_2$, form an interferometric pattern, through which the distance between the two surfaces (gap) can be measured. The fringe pattern is developed by the interference of these two reflections with each other, and this fringe pattern changes as the gap changes. Sensors relying on extrinsic Fabry-Perot interferometry are designed so that as the measured quantity changes, the gap will also change in a predictable fashion. The change in the measured quantity is therefore determined by monitoring how the gap changes.

![Figure 3.1 Extrinsic Fabry-Perot Interferometry](image-url)
3.1. Basic Temperature Sensor Design

The basic temperature sensor was developed by Luna Innovations Inc. and consists of a fused silica (glass) fiber with a thin optical wafer attached to the tip as illustrated in Fig 3.2. An actual sensor can be seen in Fig. 3.3. The wafer acts as a Fabry-Perot cavity. Part of the light signal is reflected from the fiber/wafer junction, and part is reflected from the exposed end of the wafer. These two reflections form the interferometric pattern needed to measure the change in temperature. As the temperature of the wafer changes, the index of refraction and the thickness of the wafer (due to thermal expansion) change. The resulting interference pattern is interpreted, and the gap length can be determined within ±18 nm.

The overall diameter of the sensor is determined by the diameter of the fiber (currently 125 μm). However, the light signal travels only through the center of the fiber along a path with a diameter of about 5 μm. Also, since the thickness length of the wafer is utilized as the Fabry-Perot cavity, it is not possible to determine whether there are temperature variations within the wafer. Therefore, the recorded temperature is an average temperature of the wafer material that occupies the center 5 μm in diameter and
extends the entire thickness of the wafer. Wafer materials with high thermal conductivities are chosen in order to minimize temperature variations within the wafer material.

Fused silica fibers will work up to a fiber temperature of about 1100 K. If greater temperatures are required, the fiber can be replaced with a sapphire fiber that operates up to a fiber temperature of 2200 K. If the fiber is insulated radially, the silicon wafer will attain a much greater temperature than the fiber when large heat fluxes are applied. The sensors will be functional until the fiber reaches these temperature limits.

The sensors studied here had fused silica fibers 125 μm in diameter with optical wafers 35 μm thick. The wafers were square with a length 85 μm on a side and were centered on the end of the fiber. These sensors have been tested to withstand repeated loading up to a wafer temperature of 1400 K without failure and a wafer temperature of 1700 K was measured prior to sensor failure.

3.2. Fiber-Optic Signal Processing System

Figure 3.4 shows the conceptual layout of a single wavelength demodulation system. A single laser signal is sent along the fiber to the sensor. The two reflections are formed in the sensor and transmitted back along the same fiber to a detector. A measurement of the intensity of the light signal received by the detector shows a rapidly fluctuating sinusoidal field. Two main disadvantages of this system are the nonlinearity of the transfer function and directional ambiguity.

![Figure 3.4 Single Wavelength Demodulation System]
The directional ambiguity is a result of a sinusoidal signal from the sensor. The signal is a measure of the constructive and destructive interference of the two reflections in the sensor due to the change in length traversed by the light making up the second reflection, hence the name interferometer. If a change in the gap occurs at a peak or valley of the sinusoid, it will not be detected because the slope of the transfer function is zero at these points. Therefore, the sensitivity of the system approaches zero at these points.

For these reasons Luna Innovations Inc. has developed the Hyperscan data acquisition system. This system uses two laser signals that are 90 degrees out of phase with each other (Fig. 3.5). The sensor will produce two reflections for each laser, thereby two interferometric patterns are received by the detector. If the change in gap occurs at a peak or valley of one of the laser signals, it will occur in the linear portion of the other. By this means, the change in gap will always occur in a linear portion of one of the laser signals. By comparing the two laser signals, the gap change is unambiguously determined.

![Figure 3.5 Dual Wavelength Demodulation System](image)

### 3.2.1. Hyperscan System Specifications

The two lasers that produce the optical signal for the Hyperscan system are alternately switched on and off at a rate 3 times the fastest expected change in the sensing system. This is done to separate the two signals and prevent the two fringe patterns from interfering with one another. The Hyperscan system has an adjustable sample speed that is capable of measuring 5000 to one million samples per second. Data are collected via a
PC by utilizing an off-the-shelf Adlink 20MHz A/D card with PCI interface. The card clock is generated by the Hyperscan system for synchronization of the A/D conversion with the laser switching events. The data are captured and stored on the hard drive with Luna’s proprietary in-house software. The data are then post processed, again with Luna proprietary software, to convert the captured voltage to relative gap change.
Chapter 4. Shock Tubes

A shock tube is a device that can be used to produce very short duration flows of high-temperature, high-pressure gases. Details on shock tubes and their operation are given by Glass [12]. A simple shock tube consists of two sections of pipe with constant cross-sectional areas. Figure 4.1 illustrates what happens when a shock tube is operated. The numbers correspond to different gas states, and the arrows indicate the direction of travel for shock waves and expansion waves. The gases in these two pipe sections are initially at different pressures and are separated by a diaphragm (states 1 & 4). Rapidly removing the diaphragm generates a short duration flow from the high-pressure (driver) region to the low-pressure (driven) region. A shock wave travels into the low-pressure gas, while an expansion wave travels into the high-pressure region. The shock wave increases the pressure and temperature of the driven gas (2), while the expansion wave...
decreases the pressure and temperature of the driver gas (3). The flows generated by these two waves are separated by a contact surface. These flows (2 & 3) have the same pressure and velocity, but generally have different temperatures and densities. If the end of the driven section of the shock tube is capped off, the shock wave is reflected off the end. This reflected shock wave produces a region of stagnant gas at the end of the shock tube (5) with even greater temperature and pressure.

When the reflected shock wave encounters the contact surface separating flows 2 and 3, there are two possible outcomes. A shock will travel through the contact surface into region 3, and either an expansion wave or a shock wave will be reflected back toward the end of the tube. In either case, the newly formed regions 7 and 8 will be separated by a contact surface, as shown in Fig. 4.2. Just like regions 2 and 3, they will have the same pressure and velocity, but different temperatures and densities. Whether a shock wave or an expansion wave is reflected back to the end of the tube is determined by the internal energies of the gases in states 2 and 3. Therefore, the types of gas used for the driven and driver gases influence this reaction.

Finally, the wave generated from the contact surface - shock wave interaction reflects off the end of the shock tube. The end of the shock tube only experiences two events, the reflection of the first shock wave and the reflection of this second wave. These two events define the nominal testing time for a sensor that is placed flush in the end of the shock tube. A sensor in such a configuration will measure the properties of the gas in state 5. If the sensor is mounted on a probe and extended into the shock tube, it can
measure the properties of the gas in state 2. The nominal testing time for this configuration is determined to be from the time that the initial incident shock wave passes over the sensor until the time when the reflected shock wave passes over the sensor. This is illustrated in the above x-t diagram. The numbers in the x-t diagram represent the different gas states. The labels “E”, “S” and “C” are assigned to expansion waves, shock waves and contact surfaces respectively. The arrows following these labels show the direction in which the wave is propagating. The nominal testing times for the two sensor configurations discussed above are also shown.

4.1. Flow Properties in the Shock Tube

Two methods were studied to predict the states of the gases throughout the shock tube interactions. Both methods are derived from the conservation laws. They assume isentropic flow through expansion waves and that the shock waves travel through a calorically perfect gas. The caloric perfect gas assumption is valid for air temperatures less than 1000 K. For temperatures greater than 1000 K, the specific heat ratios are no
longer constant but instead are dependent upon the temperature. Therefore, a solution which takes into account this dependency needs to be used for higher temperatures.

4.1.1. Shock Tube Relations

The first method is based on equations for normal shock relations [2, 12]. This method seeks to predict the properties of the gas that result from the passing of the shock waves. This is done assuming that the initial conditions in regions 1 and 4 are known. In order to simplify calculations, two variables are introduced, $\alpha_i = \frac{\gamma_i + 1}{\gamma_i - 1}$ and $\beta_i = \frac{\gamma_i - 1}{2\gamma_i}$ where $\gamma_i$ is the value of the specific heat ratio of the gas at state i.

In the following equations, ratios were combined into a single variable when that ratio consisted of the same parameter at different states. For example, the ratio $P_4 / P_1$ becomes $P_{41}$. Once the initial conditions are known, these equations can be used to determine the properties in states 2 and 3. Equation 4.1 is solved by iterating the value of $P_2$ until the right hand side of the equation is equal to the known ratio $P_{41}$. The variables $a_1$ and $a_4$ are the speed of sound in gases 1 and 4. These are determined by the relation $a_i = \sqrt{\gamma_i R_i T_i}$, with $R_i$ being the gas constant of the fluid in state i.

$$P_{41} = P_{21} \cdot \left[ 1 - \frac{(\gamma_4 - 1) \cdot \frac{a_1}{a_4} \cdot (P_{21} - 1)}{\sqrt{2 \cdot \gamma_1 \cdot (2 \cdot \gamma_1 + (\gamma_1 + 1) \cdot (P_{21} - 1))}} \right]^{-\frac{2\gamma_4}{\gamma_4 - 1}}$$

Equations 4.2 through 4.5 are used in determining the rest of the properties in state 2. The normal shock equations relate temperature, density, and speed (u) to the pressure ratio $P_{21}$. The speed of sound of state 2 is determined once $T_2$ is known.

For testing the fiber-optic temperature sensors, shock tubes were used to create a step change in the temperature of the air in the driven section. The thickness of a shock wave
is described by the relation $\varepsilon = \frac{\mu}{\rho \Delta u}$ in Ref. [23]. The viscosity and density are calculated at the sonic condition. Using values for the shock speed and temperature change produced in shock tubes, the rate of temperature change in the air that a shock wave passes through can be determined. For a shock traveling at approximately 1000 m/s in atmospheric air, the shock thickness is on the order of $\varepsilon = 0.3e^{-6}$ m. Assuming that the temperature change across the shock is 1000 K, the rate of temperature change as the shock passes through the air is very large, $\frac{\Delta T}{\Delta t} = O(3.3e^{12})$. Therefore, a shock tube can be used to create a very effective temperature step change.

The sensors were then used to measure this change in temperature. Since stronger shock waves result in larger temperature changes, as seen in Eq. 4.2, increasing the pressure ratio $P_{21}$ will increase the change in temperature. From Eq. 4.1 it can be seen that $P_{21}$ will increase by either increasing the pressure ratio $P_{41}$, or by decreasing the ratio $a_1/a_4$. For safety reasons it is more desirable to increase $a_4$ than to increase the driver pressure. Since $a_4 = \sqrt{\gamma_4 \cdot R_4 \cdot T_4}$ this can be accomplished by heating the driver gas or choosing a different driver gas with large values for $\gamma_4$ and $R_4$. Helium is a popular choice for the driver gas, because the speed of sound in helium is larger than the speed of sound in air and it is inert. Helium was, therefore, chosen here as the driver gas to increase the shock strength.

$$T_{21} = P_{21} \left( \frac{\alpha_1 + P_{21}}{1 + \alpha_1 \cdot P_{21}} \right)$$

(4.2)

$$\rho_{21} = \frac{1 + \alpha_1 \cdot P_{21}}{\alpha_1 + P_{21}}$$

(4.3)
Contrary to shock waves, the expansion wave is assumed to be an isentropic process. Therefore, by knowing the ratio $P_{34}$, the temperature and density of the gas in region 3 can be found through isentropic relations. Since regions 3 and 2 are separated only by the contact surface, $P_3 = P_2$ and $u_3 = u_2$. Again, the speed of sound is found once the temperature is known. While the properties of the driven gas were used to determine state 2, the properties of the driver gas are used in the equations for state 3.

When the shock approaches the end of the shock tube, the air mass behind the shock is flowing at a velocity $u_2$. If the end of the shock tube is solid, it is impossible for the flow to travel through the end. To satisfy this non-penetration condition, the flow at the end of the tube must maintain zero velocity after the shock reflects from the end of the tube ($u_5 = 0$). In other words, the shock that is reflected from the end of the tube must be of sufficient strength to force the flow velocity in region 2 to change from a value of $u_2$ to zero. Equations 4.9 through 4.12 predict the properties in state 5.

$$u_2 = a_1 \cdot \frac{P_{21} - 1}{\gamma_1 \cdot \sqrt{\beta_1 \cdot (\alpha_1 \cdot P_{21} + 1)}}$$

(4.4)

$$a_2 = \sqrt{\gamma_1 \cdot R_1 \cdot T_2}$$

(4.5)

$$T_{34} = P_{34}^{\frac{2}{\gamma_4}}$$

(4.6)

$$\rho_{34} = P_{34}^{\frac{1}{\gamma_4}}$$

(4.7)

$$a_3 = \sqrt{\gamma_4 \cdot R_4 \cdot T_3}$$

(4.8)
\[ T_{52} = \frac{P_{52} \cdot (\alpha_1 + P_{52})}{1 + \alpha_1 \cdot P_{52}} \]  

(4.10)

\[ \rho_{52} = \frac{1 + \alpha_1 \cdot P_{52}}{\alpha_1 + P_{52}} \]  

(4.11)

\[ a_5 = \sqrt{\gamma_1 \cdot R_1 \cdot T_5} \]  

(4.12)

Utilizing these equations, it is relatively simple to determine the gas properties in states 2 through 5. The nominal testing time of a sensor mounted on a probe in order to measure the properties in state 2 can also be determined by using the calculated wave speeds and measuring the length of the driven section of pipe. However, in order to determine the nominal testing time for a sensor mounted flush with the end of the shock tube (measuring properties in state 5) one needs to know what kind of wave is formed by the interaction of the reflected shock wave with the contact surface. The testing time will be dependent on whether a shock wave or an expansion wave next travels toward the end of the shock tube. As mentioned previously, in order to determine which type of wave results from this interaction, the internal energies of the gases in states 2 and 3 are required. In order to avoid some complexity associated with this process, the graphical solution method was utilized.

### 4.1.2. Graphical Solution Method

The second method discussed here uses a u-P shock polar to determine the gas properties at the different states in the shock tube as described by both Shapiro [34] and Courant [6]. The graphical method simplifies determining which interaction occurs when the reflected shock encounters the contact surface. This method is based on the following equations that describe the relationship between the velocity, pressure and Mach number for the shocks.
In these equations, the subscript ‘i’ is used for the properties before the passing of either a shock or expansion wave, and the subscript ‘j’ is used for the properties after the passing of the wave. Shocks waves are described by Eqs. 4.13 and 4.14. For the right hand side of Eq. 4.13, a right-running wave is positive and a left-running wave is negative. Points for the u-P polar come from varying the value of $M_x$ and finding $u_j$ and $P_j$. The value of $M_x$ begins at 1 and is gradually increased. The resulting values of $u$ and $P$ then form points on the line showing the passing of the shock wave. Once the pressure and velocity are known, the temperature and density are found using the shock tube relations from the previous method.

$$\frac{u_j - u_i}{a_i} = \pm \frac{2}{\gamma + 1} \left( M_x - 1 \right)$$

(4.13)

$$\frac{P_j}{P_i} = 1 + \frac{2 \cdot \gamma}{\gamma + 1} \left( M_x^2 - 1 \right)$$

(4.14)

The expansion waves are modeled with the isentropic equation relating the pressure ratio with the velocity (Eq. 4.15). In the case of expansion waves, the points for the graph are generated by inserting different velocity values, $u_j$, and finding the corresponding pressures $P_j$. The line formed from these points indicates the passing of the expansion wave. The remaining properties are found using isentropic relations.

$$\frac{P_j}{P_i} = \left[ 1 - \frac{\gamma - 1}{2} \left( \frac{Abs(u_j - u_i)}{a_i} \right) \right]^{2\gamma/(\gamma - 1)}$$

(4.15)

The following procedure is shown in Fig. 4.4 through 4.6. The gas states indicated by the arrows and the numbers correspond to those in Fig. 4.3. Shocks are plotted as solid lines, while expansion waves are plotted as circles. The initial values of the gases in states 1 and 4 that were used to create these plots were taken from one of the experiments.
to be discussed later. The first shock from states 1 to 2 is modeled with Eqs. 4.13 (positive sign) and 4.14. This shock starts at state 1 and runs up and to the right. There is an expansion wave that starts at state 4 and is plotted down and to the right. The pressure and velocity of states 2 and 3 are found at the point where these two lines intersect. This is shown in Fig. 4.4.

![Graphical Method- Step 1](image)

Figure 4.4  Graphical Method- Step 1

Next, Eqs. 4.13 (negative sign) and 4.14 are used again until the velocity returns to zero, simulating the shock reflected from the end of the shock tube and bringing the driven gas to state 5. These same equations are used with the driver gas constants to simulate the passing of the shock through the contact surface. This shock wave changes the driver gas from state 3 to state 8. The point that represents the pressure and velocity of the gas in state 8 lies on this line. These are plotted on the graph in Fig. 4.5.
The pressure and velocity of the driven gas in state 7 must equal those of the driver gas in state 8, because these two gases are only separated by a contact surface (see Fig. 4.3). Since the velocity of the gas in state 5 is constrained to be zero (non-penetration condition at the end of the shock tube), the conditions at state 5 are now known. By comparing the line from state 2 to state 5 and the line from state 3 to 8, it can be determined whether an expansion wave or a shock wave is the result of the reflected wave / contact surface interaction. If the line from 3 to 8 crosses the y axis at a higher pressure than the line from 2 to 5, a shock will be reflected from the contact surface toward the end of the shock tube. This shock wave would raise the pressure in region 5 so that $P_7$ would equal $P_8$. When the line from 2 to 5 crosses the y axis at a greater pressure than the line from 3 to 8, as it does in this case, the pressure in state 5 will be reduced by an expansion wave in order for $P_7$ to equal $P_8$. Using Eq. 4.15, an expansion wave starts at state 5 runs down to the left until it intersects the line from state 3 to state 8. This intersection reveals the pressure and velocity of states 7 and 8. This last step is illustrated in Fig. 4.6.
4.1.3. Nominal Testing Time

By knowing the velocities $u_2$, $w_{1-2}$ (velocity of shock from state 1 to 2), $w_{2-5}$, and $w_{5-7}$ (velocity of expansion wave 5-7) the nominal testing time for the sensors can be determined. Equation 4.16 describes the speed of a shock wave $w_{1-2}$, and the Mach number of this wave is found by the relation $M_{1-2} = w_{1-2}/a_1$. The Mach number of the reflected wave is found by using Eq. 4.17, and the speed of this wave with respect to the lab space reference frame, $w_{2-5}$, is found with Eq. 4.18. Expansion waves travel at the speed of sound of the medium they are traveling into, therefore, $w_{5-7} = a_5$.

\[ w_{1-2} = a_1 \sqrt{\frac{\gamma + 1}{2\gamma} \left( \frac{P_2}{P_1} - 1 \right) + 1} \]  

(4.16)

\[ \frac{M_{2-5}^2}{M_{2-5}^2 - 1} = \frac{M_{1-2}^2}{M_{1-2}^2 - 1} \sqrt{1 + \frac{2(\gamma - 1)}{(\gamma + 1)^2} \cdot (M_{1-2}^2 - 1) \cdot \left( \gamma + \frac{1}{M_{1-2}^2} \right)} \]  

(4.17)

Figure 4.6   Graphical Method- Step 3
\[ w_{2-5} = a_2 \cdot M_{2-5} - u_2 \]

(4.18)

The following discussion describes the process for determining the nominal testing time for a sensor embedded flush with the end of the shock tube. The first time of interest, \( t_1 \), is the moment when the incident shock is reflected off the end of the shock tube. The time it takes for the reflected shock to travel from the end of the shock tube to the contact surface is \( t_2 \). The position of the contact surface when this interaction takes place is dependent on \( t_2 \) by the relationship: contact position = \( u_2 (t_1 + t_2) \). Since \( t_2 \) is also dependent on the position of the contact surface, this is a circular equation that must be solved iteratively. This is simplified and shown in Eq. 4.20. The third important time period, \( t_3 \), is the time it takes for the expansion wave formed from the reflected shock wave/contact surface interaction to travel from this contact point to the end of the tube. The nominal testing time is then the sum of \( t_2 \) and \( t_3 \).

\[
\begin{align*}
  t_1 &= \frac{L_{driven}}{W_{1-2}} \\
  t_2 &= \frac{L_{driven} - u_2 \cdot (t_1 + t_2)}{w_{2-5}} \\
  t_3 &= \frac{L_{driven} - u_2 \cdot (t_1 + t_2)}{W_{5-7}} \\
  t_{test} &= t_2 + t_3
\end{align*}
\]

(4.19) \hspace{1cm} (4.20) \hspace{1cm} (4.21) \hspace{1cm} (4.22)

A sensor on a probe extended into the flow from the end of the shock tube was used to measure the flow properties in state 2. In such a configuration, the first time of interest is defined as the time the incident wave takes to travel from the tip of the probe to the end of the shock tube, Eq. 4.23. As long as the contact surface separating gas states 2 and 3
does not reach the probe tip before the reflected wave, the end of the test occurs when the reflected wave passes back over the tip of the probe. Equation 4.24 shows the time the reflected wave takes to travel from the end of the shock tube to the probe tip, and Eq. 4.25 gives the testing time for a sensor on a probe.

\[ t_1 = \frac{L_{probe}}{W_{1-2}} \]  \hspace{1cm} (4.23)

\[ t_2 = \frac{L_{probe}}{W_{2-5}} \]  \hspace{1cm} (4.24)

\[ t_{test} = t_1 + t_2 \]  \hspace{1cm} (4.25)

### 4.2. Prediction Results

The following table contains the required initial conditions for determining the shock tube flow conditions. The values measured during experiments are shown in bold and all values were chosen to match the initial conditions of one of the experiments. The initial pressures and temperatures (state 1 and 4) were chosen to produce the desired shock strength. Once pressure and temperature are found, the density values are determined from the perfect gas law, \( P = \rho * R * T \). Two different shock tubes were used in the experiments. The results shown here are for a shock tube that has a driver section that is 2.44 m (8 ft) long and a driven section 6.34 m (20 ft, 9 in) long. The shock tube was designed and the initial conditions were chosen to produce a flow simulating the blast wave from an explosion of 22.7 kg of TNT at a distance of 3m.
Figure 4.6 was used to illustrate the u-P polar method. That figure contains the results obtained using the initial conditions in Table 1 with the graphical method. Table 4.2 shows the results of the shock tube equations using the initial conditions listed in Table 4.1. The results from both methods are identical. The main benefit of using the graphical method is that it easily shows that an expansion wave is formed and propagates toward the end of the shock tube when the reflected shock encounters the contact surface. As seen above, this is important to determine the nominal testing time for the experiments.

<table>
<thead>
<tr>
<th>Table 4.1 Initial Shock Tube Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1 (Air)</td>
</tr>
<tr>
<td>L_{driven} (length of driven section)</td>
</tr>
<tr>
<td>R1 (gas constant)</td>
</tr>
<tr>
<td>γ1 (specific heat ratio)</td>
</tr>
<tr>
<td>P1 (pressure)</td>
</tr>
<tr>
<td>T1 (temperature)</td>
</tr>
<tr>
<td>ρ1 (density)</td>
</tr>
<tr>
<td>a1 (speed of sound)</td>
</tr>
<tr>
<td>α1</td>
</tr>
<tr>
<td>β1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4.2 Calculated Properties for Gas States 2,3 and 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 2 (Air)</td>
</tr>
<tr>
<td>Pressure</td>
</tr>
<tr>
<td>Temperature</td>
</tr>
<tr>
<td>ρ (density)</td>
</tr>
<tr>
<td>U (flow speed)</td>
</tr>
<tr>
<td>a (speed of sound)</td>
</tr>
</tbody>
</table>
The results show that the stagnant air at the end of the shock tube experiences a temperature step change from 298 K to 1245 K. The air theoretically remains at this temperature for the duration of the nominal testing time, $t = 0.00178$ seconds (probe mounted flush in the end of the shock tube). Previous experimental results show that the actual testing time is approximately half of that predicted by the theory [12]. The discrepancy between the theory and the experimental results lies in the definition of the contact surface. The theory describes the contact surface as a boundary between the driver and driven gases that has no thickness. In actuality, the contact surface is a region of turbulent mixing. Experiments have shown that the contact surface arrives at a point 20 ft downstream of the diaphragm about fifty percent sooner than the theory predicts. This would imply that the actual testing time for this experiment is about 0.00089 seconds.

4.3. Real Gas Effects

The calorically perfect gas assumption is invalid in air above a temperature of 1000 K due to two phenomena. Below 1000 K, the majority of the molecular energy in the gas is manifested in either rotational or translational motions of the molecules. The equations that define the rotational and translational energy are linearly dependent on temperature. As the gas temperature reaches 600 K, some of the atoms within the molecules start to move relative to one another. The energy used in the movement of the atoms is defined as vibrational energy, and for shock tube analysis the amount of vibrational energy is considered significant when the temperature reaches 1000 K. As the temperature increases even further, dissociation of the molecules will occur, and the energy required for dissociation is called electronic energy. The total energy of a diatomic molecule measured above the zero point ($T = 0$ K) is defined by Eq. 4.26 where $h$ is Planck’s constant, $\kappa$ is the Boltzmann constant, and $v$ is the fundamental vibration frequency of the molecule.

Equation 4.26 combined with the definition of specific heat at constant volume, $c_v = (\partial e/\partial T)_v$, reveals that the specific heat will be constant at low temperatures. Once the temperature increases to a point where vibration is excited, the specific heat will be a
function of temperature. Since the temperature of the gas after the reflected shock is greater than 1000 K for the conditions of interest here, the specific heat ratios can no longer be assumed to be constant for the gas in state 5. The vibrational motion of the atoms uses some of the energy that would otherwise be used in either rotation or translation of the molecules. Therefore, the temperature of the gas will be lower than predicted by the normal shock relations.

\[
e_{tot} = e_{trans} + e_{rot} + e_{vib} + e_{el} \\
= \frac{3}{2} RT + RT + \frac{hv}{\kappa T} - 1 - RT + e_{el}
\]

(4.26)

Figure 4.7 is a chart taken from NAVORD Report 1488 (Vol. 6) Handbook of Supersonic Aerodynamics Section 18 Shock Tubes, by I. I. Glass and J. Gordon Hall [12]. This chart was used to correct for the vibration effects on the properties of the gas in state 5. In Fig. 4.7, the Mach number of the incident shock wave is plotted along the horizontal axis and labeled \( W_{11} \). The right hand axis gives values for ratios of temperature and density in state 5 compared to state 2 (for example: \( T_{52} = T_5/T_2 \)) and also the reflected wave speed \( W_{21} \). A star superscript indicates that the value has been corrected for vibrational effects. The earlier analysis showed the expected incident shock wave Mach number to be about 2.78. Applying the correction from Fig. 4.7, the expected temperature behind the reflected shock wave is adjusted from 1245 K to 1208 K. This also affects the speed of sound in region 5 and the speed of the reflected shock. The prediction of the nominal testing time is affected as well. The new predicted nominal testing time becomes \( t = 0.0019 \) seconds. Again, experiments have shown this time prediction to be about twice as long as what the value measured experimentally. The expected nominal testing time is actually 0.00094 seconds.

A sensor placed in the end of the shock tube will be affected by the temperature change the air in front of it. In this case, the sensor is expected to be affected by a gas
temperature step change from 298 to 1208 K. The duration of this step change is expected to be slightly less than 1 millisecond. A sensor mounted on a probe would experience a step change from 298 to 724 K. These step changes will be measured and used to characterize the fiber optic temperature sensor.
Figure 4.7 : Variation of the Flow Parameters in Air, Behind the Normal Reflected Shock Wave with the Incident Shock Wave Mach number (w11) (vibration excitation corrections) (Glass Fig. 2.4-2)
Chapter 5. Computational Modeling

Computational models are commonly used today in the design and analysis of engineering products. In this case, models were developed to be used as a tool to further understand the flow environment of the shock tube and to understand how the fiber-optic sensors respond to various inputs. A finite element model of the sensor was generated with ANSYS for analyzing the thermal response of the sensor and test cases were used to validate the modeling procedures. Due to the complex nature of the flow around the sensor in the shock tube, a model of the shock tube environment was created in Fluent. Information from the Fluent model about the heat flux from the flow was then used as an input for the ANSYS model in order to predict the temperature response of the sensor in the shock tube tests. In this chapter, the modeling of the fiber-optic sensor itself is discussed, starting with an idealized analytical treatment. Modeling of the flowfield is presented in section 5.2 and the process for combining the Fluent and ANSYS models is presented in section 5.3.

5.1. Modeling the Fiber-Optic Temperature Sensor

A simplified energy balance was first performed on the sensor wafer for design purposes. This analysis points out the important parameters for design and use of the sensors. Beginning with the statement for the conservation of energy and applying it to the wafer volume

\[ \dot{E}_{st} = (\dot{E}_{in} - \dot{E}_{out}) + \dot{E}_g. \]  

(5.1)

It was assumed that there is no heat generation \((\dot{E}_g \rightarrow 0)\) and also the heat that is lost from the wafer to the fiber can be ignored \((\dot{E}_{out} \rightarrow 0)\). The lumped capacitance approximation was assumed to apply to the wafer \((\dot{E}_{st} \rightarrow \rho V C_p \frac{dT_{wafer}}{dt})\), and the method of heat transfer into the wafer is convection in a supersonic flow \((\dot{E}_{in} \rightarrow hA_s(T_{in} - T_{wafer})\)). Equation 5.2 was used to govern the heat transfer to the sensor wafer in supersonic flow.
\[ \rho V C_p \frac{dT_{wafer}}{dt} + h A_s (T_{wafer} - T_{aw}) = 0 \]

By definition, the adiabatic wall temperature is

\[ T_{aw} = T_\infty + r \frac{U_e^2}{2C_{p-air}}, \]

where \( r = \Pr^{1/2} \) for laminar flow and \( \Pr^{1/3} \) for turbulent flow. Laminar flow is assumed in the shock tube and the sensor is located at the front of a probe. Substituting the definition for the adiabatic wall temperature into Eq. 5.2 yields the first-order system in Eq. 5.4. The definition for the adiabatic wall temperature involves the specific heat for the air or fluid and the lumped capacity assumption involves the specific heat for the wafer. These are labeled accordingly in Eq. 5.4.

\[ \rho C_{p-wafer} V \frac{dT_{wafer}}{dt} + h A_s \left( T_{wafer} - \left( T_\infty + \Pr^{1/2} \frac{U_e^2}{2C_{p-air}} \right) \right) = 0 \]

This first-order system can be solved to determine the temperature response of the sensor. Substituting for the time constant of the system, \( \tau = (\rho V C_{p-wafer})/h A_s \), and using the initial condition of \( T = T_\circ \) when \( t = 0 \) yields Eq. 5.5.

\[ T_{wafer}(t) = T_\infty + \frac{\sqrt{\Pr \cdot U_e^2}}{2C_{p-air}} + e^{-\frac{t}{\tau}} \left( T_\circ - T_\infty - \frac{\sqrt{\Pr \cdot U_e^2}}{2C_{p-air}} \right) \]

Since one of the main objectives in the design and use of these sensors is to have a very fast response, design decisions were made in order to minimize the time constant, \( \tau \).
Ideally, the design of the sensor housing needs to increase the film coefficient, $h$, and the V/A_s ratio of the wafer should be minimized. The wafer material and size are constrained due to manufacturing concerns. Therefore, the sensor housing design needed to have a large film coefficient.

Also, if the housing were designed in such a manner as to locate the wafer at the stagnation point, the solution would be further simplified. At the stagnation point, $U_e=0$, therefore the adiabatic wall temperature is the same as the total temperature of the flow, $T_t$. The solution for the wafer temperature given these conditions is shown in Eq. 5.6.

$$T_{\text{wafer}}(t) = T_t + e^{-rac{t}{\tau}}(T_o - T_t)$$

(5.6)

For the probes used in the shock tube experiments, the tips consisted of a stainless steel tube, 1.6 mm OD and 0.5 mm ID. The end of the tube contained a stainless steel capillary and a glass capillary. These were used to reduce the ID of the tube enough to hold the optical fiber. The tip was then ground to a 30 degree angle. These probes endeavored to place the fiber-optic sensors at the stagnation point of the probe. Figure 5.1 illustrates how the probe tips were constructed.

White [40] gives a relationship for the heat transfer to a blunt tipped cylinder in supersonic flow based on Newtonian impact theory. This relationship is found in Eq. 5.7 where the density, viscosity and enthalpy at both the wall and edge of the boundary layer are required. $K$ is the velocity gradient at the stagnation point. White gives an estimate for this value on a flat-nosed probe based on the diameter of the probe. Using Eq. 5.7

$$q_w = 0.763 \Pr^{-0.6} \left( \rho_e \mu_e K \right)^{1/2} \left( \frac{\rho_w \mu_w}{\rho_e \mu_e} \right)^{0.1} (h_e - h_w)$$

(5.7)
the heat flux at the stagnation point of the probe was estimated to be about $7 \times 10^6$ W/m$^2$. This estimate neglects the pointed tip of the probe, and therefore provides a lower boundary for the expected value.

5.1.1. Finite Element Model

With modern computational tools, a more accurate treatment without the assumptions of the lumped capacity analysis can be obtained. ANSYS is a popular finite element analysis software package developed by ANSYS, Inc. This FEA package is used to solve a wide variety of problems including static/dynamic structural analysis (including linear and nonlinear problems), heat transfer, fluid flow, acoustic and electromagnetic problems. A finite element model of the temperature sensor was developed using ANSYS to help understand the temperature response of the sensor resulting from various heat transfer conditions.

Still, the complete problem here is very complex. It is fully 3D and unsteady with complex geometry. In order to simplify the modeling of the temperature sensor
somewhat, the wafer on the tip of the optical fiber was assumed to cover the entire surface of the fiber. This assumption allowed for the sensor to be modeled axisymmetrically. Furthermore, the optical fiber was assumed to be insulated radially, allowing only for one dimensional heat transfer along the axis in the glass fiber. Within the wafer, however, multidimensional heat transfer was allowed depending on the boundary conditions applied to the wafer. The configuration of the sensor in the ANSYS model is represented in Fig. 5.2.

The modeling process in ANSYS includes defining the material properties and geometry, meshing the geometry, defining the contacts between materials, applying loads, obtaining the solution, and analyzing the solution. The material properties for the wafer material and the optical fiber were specified using data provided at www.matweb.com and are listed in the appendix. For creating a thermal model, the density, specific heat and thermal conductivity are required. If the structural effects are of interest, due to thermal expansion in this model, the thermal expansion coefficient, Poisson’s ratio and the Young’s modulus of elasticity are also required. The material
properties vary as the temperature changes. This variation is accounted for by creating a table of the material property values at different temperatures. When the temperature of the model is between values listed in the table, ANSYS linearly interpolates between the values and uses an approximated value.

ANSYS utilizes different element types for different kinds of problems. An element type that supports thermal analysis is required to examine the heat transfer throughout the fiber-optic sensor. The model was generated using Plane-13 elements, which are two dimensional, rectangular elements that support thermal and structural problems and can also be used in axisymmetric problems.

The geometry of the model was set up through the use of the keypoints listed in Table 5.1. All dimensions are given in meters. The coordinate system origin is based along the axis of symmetry, at the junction between the wafer and the optical fiber. The thickness of the wafer in the model was adjusted in order to match the volume of the wafer in the model with the volume of the physical wafer used for the temperature sensor. The physical sensor had a wafer that was 85 x 85 microns square and 35 microns thick. The wafer on the ANSYS model was created as a cylinder with a radius of 62.5 microns and a thickness of 20.6 microns. The area representing the wafer is formed with keypoints 1, 2, 3 and 4 at the corners. Keypoints 1, 2, 5 and 6 were used as the corners for the optical fiber portion of the model. Since keypoints 1 and 2 were used to form both the wafer and the fiber, the two regions were formed sharing the same line. This created a perfect thermal contact between the wafer and the fiber in the ANSYS model.

Table 5.1 Keypoint Coordinates for the ANSYS Model

<table>
<thead>
<tr>
<th>Keypoint Number</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>6.25 e-5</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>6.25 e-5</td>
<td>2.06 e-5</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>2.06 e-5</td>
</tr>
<tr>
<td>5</td>
<td>6.25 e-5</td>
<td>5 e-4</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>5 e-4</td>
</tr>
</tbody>
</table>
The length of the glass fiber in the model was defined by the Y coordinate for keypoints 5 and 6. This length was important for matching the boundary condition in the model with what physically occurred in the sensor. The only temperature known in the glass fiber was the initial temperature. If the fiber in the model was long enough, it could be considered a semi-infinite solid, and the boundary condition at the opposite end of the fiber could be defined. The temperature in the fiber far from the wafer could be assumed to remain at the initial temperature of the sensor. This condition was ensured by checking the temperature at this boundary upon completion of running the model for different loading conditions. If the boundary temperature changed, the model assumed that the boundary was insulated; therefore the resulting temperature values within the model were too large. The results were discarded, and the model fiber was lengthened before running again. In this way, the fiber was modeled as a semi-infinite solid. For the testing times and heat transfer conditions of interest a fiber length of 0.05 cm was sufficient.

Since the geometry of the model was very simple, a mapped mesh was created. The mesh in the vicinity of the wafer can be seen in Fig. 5.3. The wafer area is blue and the glass fiber is red. The mesh was refined in the wafer to attain accuracy in this region of the model and coarsened in the fiber to save computational time. The wafer area was
meshed with a 20 x 20 uniform grid. The glass fiber was meshed with a 20 x 50 grid
with the y dimension of each layer of elements further from the wafer increasing 20%
above the previous layer. The mapped mesh was formed by specifying the number of
element divisions along each line and then having the ANSYS meshing tool mesh the
interior areas. Grid independence was established by running the model with twice as
many elements in both the x and y dimensions, but the results did not change noticeably.

Two test cases with known analytical solutions were used for testing the modeling
procedures. Both cases involve semi-infinite solids where heat is added at the surface to
a material of infinite length. The exact solutions were solved for comparison with the
results from the ANSYS model. In order to simulate the semi-infinite solid, both the
wafer and fiber areas were modeled with the same material properties, creating a solid
cylinder of a single material. Again, the temperature at the far end of the fiber was
monitored to ensure it did not change.

5.1.2. Constant Surface Heat Flux

The first test case involves applying a constant heat flux to the surface of a semi-

infinite solid. Equation 5.8 describes the temperature distribution in the solid at any
depth, x, from the surface and at any time, t. The material properties of the wafer
material were used for the semi-infinite solid for this case. An initial temperature of 300
K and a heat flux of 1,000,000 W/m² were applied to the front surface for two seconds.
The heat flux of 1,000,000 W/m² is representative of the range of heat flux conditions in
the deflagration tests. The exact solution yields a surface temperature of 310.18 K after
2.0 seconds.

\[
T(x, t) - T_o = \frac{2 q_o'' (\alpha t / \pi)^{1/2}}{k} \exp \left( -\frac{x^2}{4 \alpha t} \right) - \frac{q_o'' x}{k} \text{erfc} \left( \frac{x}{2 \sqrt{\alpha t}} \right)
\]

(5.8)

The same boundary conditions were applied to the ANSYS model of the semi-infinite
solid and the model returned a surface temperature of 310.17 K after 2.0 seconds. The
The material properties of the model were then changed so that the ANSYS model simulated the fiber-optic sensor. Again, the initial temperature was 300 K and a heat flux of 1,000,000 W/m² was applied to the surface. The run time was reduced to only 0.02 seconds. Figure 5.4 shows the result of this calculation performed for the tip of the sensor. For plotting purposes, the model is displayed with a three quarter revolution about the axis of symmetry. Of particular interest is the temperature distribution within the wafer. Even though the surface temperature of the wafer changed approximately 86 K, the temperature difference between the front face of the wafer and the wafer/fiber junction was only 0.18 K. This supports the earlier assumption of applying the lumped
capacitance approximation to the wafer because the temperature variation in the wafer is so small. Also, of interest is the fact that the temperature at the surface of the wafer is much greater than in the semi-infinite solid calculations even though the exposure time was reduced by two orders of magnitude.

5.1.3. Constant Surface Temperature

A particularly interesting case occurs when two semi-infinite solids initially at uniform but different temperatures are brought together. Assuming there is a perfect thermal contact between the two solids; the temperature of the contact surfaces will experience a step change to an intermediate value. The contact temperature remains constant as long as the semi-infinite solid approximation is valid.

This case closely matches what is expected to occur at the end of a shock tube. The end of the shock tube and the air in contact with it are considered as the semi-infinite materials. The air at the end of the shock tube theoretically remains stagnant and experiences a step change in temperature as the shock is reflected. This assumption will hold as long as both materials maintain their respective initial temperatures far away from the contact surface. The air also must remain stagnant at the end of the shock tube for this approximation to apply. The temperature distribution for the materials in this case is shown in Fig. 5.5, and the expression for the temperature at the contact surface is Eq. 5.9. The subscripts A and B follow properties of materials A and B, respectively. The surface temperature is dependent on the initial temperatures of both materials A and B, as well as the thermal conductivity, density and specific heat of each material.

\[
T_s = \frac{(k \rho C_p)_A^{1/2} T_{A,i} + (k \rho C_p)_B^{1/2} T_{B,i}}{(k \rho C_p)_A^{1/2} + (k \rho C_p)_B^{1/2}}
\]

(5.9)

This loading condition was met in ANSYS by modeling a third area above the wafer and assigning it the material properties of air. The two areas representing the wafer and fiber were both given the material properties of the fiber for this analysis. The initial
The temperature of the air was set at 1245 K, and the initial temperature of the glass fiber was 300 K. These values were taken from the analysis in Ch. 4 for one of the shock tube experiments. The analytical solution yields a surface temperature of 321.9 K while the ANSYS model returned a result of 321.8 K for the surface temperature. Once again, the model matched well with the analytical solution.

The model was then altered so the wafer and fiber no longer had the same material properties. The temperature distribution in the sensor can be seen in Fig. 5.6. The colors and labels in Fig. 5.6 specify where the different materials within the model lie. The temperature of the wafer was not constant as time progressed because the semi-infinite solid approximation no longer applied. This model was used as a tool to study the response of the sensor when it was mounted flush in the end of the shock tube.

5.2. Flowfield Modeling

In order to develop a complete computational model of the temperature sensor response, a computational model of the flow is required to provide the unsteady heat flux input to the sensor. The flow is 3D and unsteady with complex geometry, so some
idealizations were used. Again, the wafer was taken as a cylindrical disk that covered the entire end of the optical fiber. This allowed for an axisymmetric model of the flow around the sensor. The tip of the sensor in the model is shown in Fig. 5.7. The stagnation point and the point where the sensor met the housing are labeled. These two points are of interest because they define the boundaries of the wafer in the model. The wafer surface was divided into two faces. The front face is the side of the wafer that is facing into the flow. The side face of the wafer is the assumed cylindrical wall of the sensor model which is everywhere perpendicular to the free stream flow in this axisymmetric model.

Two different probes were used for the experiments. The first probe extended the sensors 2.54 cm (1.0 in) from the end of the shock tube into the flow, while the second extended the sensors 12.7 cm (5.0 inches). However, the geometry for both probes was identical within 2.54 cm of the sensor tip. Since both probes were identical near the sensor, only the first 2.54 cm of the probes were modeled. The knowledge gained from this model could be applied to both probe configurations.
Fluent is a commercially available computational fluid dynamics software package. A model was developed using Fluent to simulate the flow inside of the shock tube. The sensors were mounted into a probe and extended into the shock tube flow. This was done to increase the film coefficient to the sensors and thereby reduce the time constant of the system. The Fluent model was used to determine the flow properties at the surface of the sensor. Specifically, an overall film coefficient, $h$, was determined.

The geometry and meshing of the model were accomplished through the use of Gambit, a mesh generation program. Gambit is also owned by Fluent and is recommended as the preferred preprocessor tool. Figure 5.8 displays the geometry of the Fluent model. The lines defining the boundary of the fluid flow were drawn and the boundary conditions defined. The axis of symmetry for the model extends from the stagnation point on the front of the wafer to the upstream boundary of the model. The boundary upstream of the sensor was set as a moving wall, creating a piston from this.
surface. According to shock tube theory, the contact surface between the driven and driver gases behaves the same as a piston. As this piston moved, the air in front of it was compressed and a shock wave was formed. This shock wave then became the incident shock. The strength of this shock was determined by the speed of the piston. With a piston velocity of 696 m/s the resulting shock wave traveled at 959 m/s. The flow behind this incident shock wave traveled at the same speed as the piston with a total temperature of 960 K. These conditions matched well with the experimental conditions in the shock tube.

The Fluent model was broken into five areas for meshing purposes. These areas are shown and labeled in Fig. 5.8 as well. Near the probe tip the elements were created small so as to have enough element layers within the boundary layer to ensure accuracy. The goal was to have at least 10 elements within the boundary layer thickness. If this element size were used throughout the model, there would be too many elements and the computational time would be too large. Therefore, the element size was increased further away from the probe tip.

It was determined that only 7.62 cm (3.0 in) of the flow upstream and one inch downstream of the probe tip needed to be modeled. The upstream length was determined from knowledge about the velocity of the shock and the contact surface. Enough time was allocated for the shock to travel past the wafer and reach the end of the computational space before the contact surface reached the probe tip.

5.2.1. Inviscid Solution

The inviscid flow problem takes less time to solve than the viscous problem. It was used to ensure that the mesh distribution was correct in the regions far from the probe tip. The model was solved with the 2nd Order Upwind Differencing Method for the fluid flow and the First-Order Implicit Method for the heat equation. The 2nd Order Upwind Method is stable if the Courant number is between zero and two, and the First-Order Implicit Method is unconditionally stable for the heat equation. The Courant number is
defined as $\nu = w \frac{\Delta t}{\Delta x}$, with $w$ as the wave speed. If the time step and mesh size are chosen incorrectly, the flow properties smear across the shock. This had the effect of widening the shock front. Since the mesh size is not uniform throughout the model, contour plots of the free stream flow were examined and the mesh size was adjusted to eliminate these errors in the free stream flow. This was an iterative process to find a combination of time step and mesh spacing that would suffice to reduce errors in the free stream flow upstream of the probe. Figure 5.9 is a contour plot of the flow Mach number. The free stream Mach number of the flow around the probe is 1.29. This figure also shows the formation of the bow shock after the incident shock wave has passed over the probe tip.

The flow was monitored at the stagnation point and the housing point (defined in Fig. 5.7). Figure 5.10 shows how the pressure at these two locations changes as the shock passes over the wafer. From Fig. 5.10 it is seen that the pressure in the vicinity around
the wafer reaches equilibrium about 1.5 e-5 seconds after the incident wave first reaches the probe tip. This time became the focus for determining the transient response to the passing of the shock of the flow around the sensor.

### 5.2.2. Viscous Solution

The final Fluent calculation involved solving the viscous flow problem. The mesh around the tip of the sensor was refined in an attempt to achieve at least 10 elements within the boundary layer. This was accomplished using a boundary layer tool built into Gambit. A grid independence study was performed and the results can be seen in Fig. 5.11. This plot shows the values of heat flux at the stagnation point and the housing point during the transient flow around the probe. The open data points are associated with a mesh distribution that had 9 points within the boundary layer at the stagnation point. The mesh was then refined and the filled data points correspond to a mesh with 19 points in the boundary layer at the stagnation point. All solutions presented here were obtained with the fine mesh.

![Inviscid Fluent Predictions of the Pressure Change at the Sensor](image)

**Figure 5.10** Inviscid Fluent Predictions of the Pressure Change at the Sensor
The Mach number profile around the tip of the probe can be seen in Fig. 5.12. This figure shows that the flow separated from the edge of the wafer as it traveled from the front face to the side face of the wafer. The separation of the flow here results in a larger thermal boundary on the side face of the wafer.

The thermal boundary layer for the flow around the tip of the probe is shown in the plot of total temperature in Fig. 5.13. The temperature contours across the side of the wafer are more spread out due to the separation of the flow in this area. The thermal boundary layer across the front face of the wafer is much smaller.

![Diagram of heat flux over time](image)

**Figure 5.11** Fluent Model Grid Independence
Figure 5.12  Mach number Contours – Viscous Flow

Figure 5.13  Total Temperature Contours – Viscous Flow
5.3. Composite Computational Model for Flow and Sensor Response

Once an adequate solution was found for the viscous flow, the required heat transfer parameters were obtained from the Fluent model. During the transient portion of the fluid flow, the heat flux from the air to the wafer changed dramatically. The thermal transient time was defined as the time from when the shock wave first passes the tip of the wafer to the time when the heat flux reached a steady value. The heat flux during this transient time was previously shown in Fig. 5.11 as a part of the grid independence study for the viscous solution. Conservatively, the flow was considered to be at thermal equilibrium around 1.5 to 2 microseconds after the shock wave first contacted the wafer. Since the expected testing time for the shock tube experiments was about 100 microseconds for the 2.54 cm probe and 500 microseconds for the 12.7 cm probe, the time when the flow was unsteady was deemed insignificant. Any effects on the sensor during this transient fluid response were assumed to be small when compared to the steady state effects.

The value of the heat flux from the flow to the wafer for each mesh point along the surface of the wafer in the Fluent model was recorded. These heat flux values were used to find the film coefficient for both the front and side faces of the wafer. The film coefficient values were determined from Newton’s law of cooling for supersonic flow,

\[ q'' = h(T_{\text{wafer}} - T_{\text{flow}}) \].

Again at the stagnation point, the adiabatic wall temperature was the same as the total temperature. The total temperature for the flow was 960 K and the wafer temperature was assumed to be 300 K since only a small change in temperature was predicted for the transient flow time (1.5 to 2 microseconds). An average film coefficient along a face of the wafer was found by dividing the heat flux at each point by this temperature difference and applying an area weighted average to the corresponding film coefficient values. For the modeled flow conditions, the front face of the wafer had a film coefficient of 27,149 W/m²K and the side face of the wafer had a film coefficient of 13,485 W/m²K.
Once an estimate was obtained for the film coefficient, the *ANSYS* model was used to predict the temperature response of the fiber-optic sensor to the flow conditions. The film coefficients were applied to their respective surfaces in the *ANSYS* model and the bulk fluid temperature was set at 960 K. The *ANSYS* model was then run to obtain the prediction. The results from this *Fluent/ANSYS* modeling were then compared to the measurements obtained from the shock tube experiments. The results will be presented later.
Chapter 6. Shock Tube Experiments

Testing of the fiber-optic temperature sensors was performed in shock tubes. The shock tubes were used to create a step change in the temperature of the air at the end of the driven section, and this step change was measured by the temperature sensors. Two different shock tubes were used. Both are located in room 33 Randolph Hall on the campus of Virginia Tech in Blacksburg, Virginia. The shorter shock tube has a driver length of 1.52 m (5 ft) and a driven length of 3.05 m (10 ft). This shock tube was used to test the sensors when they were mounted flush in the end of the shock tube. Later, tests were performed with the sensors extended on probes into the flow. These tests were performed in a longer shock tube with a driver length of 2.4 m (8 ft) and a driven length of 6.32 m (20 ft, 9 in). These shock tubes are constructed from two sections of pipe with constant cross sectional area. The two pipe sections are separated by a diaphragm consisting of layers of Mylar sheets, each 7 mils thick.

The testing configuration is illustrated in Fig. 6.1. As seen in Ch. 4, the temperature change produced by the shock tube can be predicted if the type of gases used and the initial temperatures and pressures of the gasses are known. Helium was used as the driver gas and air was used as the driven gas. A thermocouple and a pressure transducer
were placed in the driver end in order to monitor the initial conditions of the driver gas. The pressure in the driven section was assumed to be the atmospheric pressure in the room. A Medtherm coaxial thermocouple and the fiber-optic sensor were placed in the driven end of the shock tube. Medtherm thermocouples were chosen as a comparator because they were the fastest commercially available thermocouples. A comparison between the fiber-optic sensors and the fastest available thermocouple was desired to show the benefits of fiber-optic technology. In all cases, the Hyperscan system developed by Luna Innovations Inc. was used to record the output of the fiber-optic sensors. The output from the Medtherm thermocouple was recorded at a sampling frequency of 100 kHz using LabView. Because the testing procedures changed between tests, further details of how the shock tubes were operated will be presented as the individual experiments are described.


The first experiments were performed with the faces of the fiber-optic sensor and the end of the Medtherm thermocouple mounted flush in the end of the shock tube. The sensors mounted in this manner measured the temperature change resulting from the reflection of the incident shock wave off the end of the shock tube (see Ch. 4). In this configuration, the Medtherm thermocouple used had a diameter of 1/16 inch. These tests were performed to compare the fiber-optic sensors with the Medtherm thermocouple.

6.1.1. Shock Tube Operating Procedures

The first step in operating the shock tube was to evacuate the air from the driver section. This was done so that helium could be used as the driver gas. The vacuum pump was used to evacuate the air and then helium was used to fill the driver section. The valve between the shock tube and the helium tank was opened and the driver pressure increased until the diaphragms ruptured. These tests relied on the driver pressure to burst the diaphragms and start the shock tube. Due to small differences between the Mylar disks, the pressure and temperature of the driver gas were somewhat
different for each run. The data from the pressure transducer and the thermocouple in the driver section were used to determine what conditions existed in the shock tube. Figure 6.2 is a display of the pressure and temperature in the driver section for the first test in this configuration. The pressure scale is plotted on the left vertical axis and the temperature is on the right. The jump in both temperature and pressure at about 4 seconds marks the opening of the valve between the helium tank and the shock tube. The drop of both temperature and pressure at approximately 9.5 seconds on the graph is the point where the expansion wave contacts the end of the driver section. The values of temperature and pressure just prior to the expansion wave were taken as the temperature and pressure of the driver section.

Table 6.1 lists the testing conditions of the first experiments that were performed. These conditions are calculated following the procedure outlined in chapter 4. The testing conditions for runs 1 and 3 are similar with testing temperatures of about 1130 K. However, the conditions for run 2 are vastly different. The testing temperature for run 2 is expected to only reach about 904 K. This discrepancy is most likely due to a scratch or

![Figure 6.2 Driver Pressure and Temperature Measurements](image-url)
impurity of some kind in the Mylar disks used for the diaphragm in run 2 resulting in a lower driver pressure.

Table 6.1  Testing Conditions with the Sensors Flush in Shock Tube Wall

<table>
<thead>
<tr>
<th></th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driven Pressure (P₁), Pa</td>
<td>94,250</td>
<td>94,250</td>
<td>94,250</td>
</tr>
<tr>
<td>Driven Temperature (T₁), K</td>
<td>298.5</td>
<td>297.3</td>
<td>297.1</td>
</tr>
<tr>
<td>Driver Pressure (P₄), Pa</td>
<td>2,401,313</td>
<td>1,417,872</td>
<td>2,386,186</td>
</tr>
<tr>
<td>Driver Temperature (T₄), K</td>
<td>300.7</td>
<td>307.1</td>
<td>310.4</td>
</tr>
<tr>
<td>Total Temperature (T₅), K</td>
<td>1129.8</td>
<td>903.8</td>
<td>1133.0</td>
</tr>
<tr>
<td>Nominal Testing Time, s</td>
<td>0.00134</td>
<td>0.00177</td>
<td>0.00133</td>
</tr>
</tbody>
</table>

6.1.2. Preliminary Results

The experiments performed with the sensors mounted flush in the end of the shock tube were performed in a shock tube with a 3 m driven section. These tests were performed in June 2004. These were some of the first experiments performed trying to use the Hyperscan system with this type of sensor. They were considered intermediary experiments and were useful to help learn more about the fiber-optic temperature sensors and the Hyperscan data acquisition system. It was later learned that there were flaws in the demodulation algorithm that was used to process this data. The errors were fixed for later experiments, but the values for the fiber-optic temperature sensor may not be accurate for this configuration. However, these tests introduced suspicions that there were deficiencies in the ability to predict the flow conditions with shock tube theory. Because of these experiments, the schlieren photographs were taken detailing the turbulence phenomenon at the end of the shock tube. Even though no conclusions can be drawn from these flawed results as to actual numbers, the general trend of the data is correct. For this reason, the fiber-optic sensor data are shown as dashed curves for these experiments. The thermocouple results are correct.

Figure 6.3 shows a comparison of the fiber-optic sensor results with the output from the Medtherm thermocouple. It is apparent that the fiber-optic sensors have much less
noise than the thermocouple. The reduction in noise content is one of the great advantages of fiber-optic systems over electrical systems. Another observation can be made from comparing these two signals. The difference between the fiber-optic signal and the thermocouple signal is not very large through the first 0.65 ms. The sensors are embedded in a large thermal mass at the end of the shock tube and are greatly affected by the presence of this heat sink. The fiber-optic sensors had an overall temperature change of about 34 K while the air temperature was predicted to change about 830 K. The large difference between the temperature change of the sensor and the predicted temperature change of the air indicates that the heat transfer film coefficient, \( h \), was very small. A small value for the film coefficient is expected in this configuration because the heat transfer is due to stagnant air or free convection. A new configuration was planned that would increase the film coefficient by placing the sensors on a probe extended into the flow. This configuration would change the heat transfer mode from free convection to forced convection. The data from runs two and three with the sensors flush at the end of the shock tube are displayed in Figs. 6.4 and 6.5. The same trends can be seen in these two plots.

![Graph](image)

**Figure 6.3** Comparison of Medtherm and Fiber-Optic Sensor Responses with the Sensors Flush in the End of the Shock Tube – Run 1
Figure 6.4 Comparison of Medtherm and Fiber-Optic Sensor Responses with the Sensors Flush in the End of the Shock Tube – Run 2

Figure 6.5 Comparison of Medtherm and Fiber-Optic Sensor Responses with the Sensors Flush in the End of the Shock Tube – Run 3
The fiber-optic sensor measurement was also compared with a prediction from the ANSYS model. This comparison is shown in Fig. 6.6. There is a poor match between the ANSYS prediction and the actual measurement. Of particular interest is the change in slope of the measured data that occurs at about 0.0001 seconds. This anomaly occurred at approximately the same time in every test. The ANSYS model did not contain enough information about the flow in the shock tube to explain why this change occurred. It was decided to take schlieren photographs of the flow in order to better understand what was happening in the shock tube environment. The ANSYS model in this case assumed that the air at the end of the shock tube remained stagnant (as does normal shock theory, see Ch. 4), so any air movement at the end of the shock tube would cause discrepancies between the ANSYS model and the measurements.

Figure 6.6  Comparison of ANSYS Prediction and Fiber-Optic Sensor Response with the Sensors Flush in the End of Shock Tube – Run 3
6.2. Schlieren Photography

Schlieren photographs were used to further understand the flow phenomena that the sensors were exposed to in the shock tube. A schlieren photograph responds to the change in density gradients in transparent mediums and is particularly useful in supersonic flow to help one visualize what is happening in the flow. Detailed techniques for schlieren photography are found in Ref. [38].

6.2.1. Schlieren Setup

A spool piece was designed and built to allow for windows to be placed in the end of the shock tube. This design is seen in Fig. 6.7 and the spool piece can be seen attached to the end of the shock tube in Fig. 6.8. The windows were made of Plexiglas and were tested with the schlieren system to ensure that they were of good enough optical quality to be useful. These windows were 7.1 cm (2.8 in) long and 2.0 cm (0.8 in) wide. The

Figure 6.7 Design for Shock Tube Modifications
windows were considered “good enough” if when looked at with the schlieren system no visible deviations in the windows could be detected. A plug was inserted into the spool piece in order to bring the effective end of the shock tube to the edge of the windows. The temperature sensors were instrumented in this plug.

The schlieren system used for the experiments is shown in Fig. 6.9. The light source was a 12 volt light bulb with a straight filament. The light from this bulb was focused by use of a lens and shown through a parabolic mirror attached to the wall above the shock tube. This mirror is 15.2 cm (6 in) in diameter and has focal length of 1.22 m (48 in). The light then reflected from this mirror and traveled through the windows and the test section in the shock tube. Then, it was reflected from a flat mirror past a knife edge and into the camera. The amount of light that is blocked by the knife edge determines the sensitivity of the pictures. When more light is blocked, the schlieren effect will be more
sensitive to changes in the density gradient, resulting in more detailed pictures. However, if too much light is blocked, there will be insufficient light for the exposure of the picture.

The camera used in these experiments is manufactured by Hadland Photonics, currently owned by DRS Technologies. This camera is a digital camera with four charge-coupled devices (CCDs) that can be programmed separately as to exposure length, gain, and delay time. A microphone was attached to the shock tube close to the location where the diaphragms were located. When the diaphragms burst, the noise was detected by the microphone, which then sent an electrical signal to the camera. In this manner the camera received a trigger signal, and the process of taking the pictures was initiated. The exposure time for the pictures taken was 500 ns. The delay between the trigger and when exactly the pictures were taken had to be determined by experimentation because the microphone received the noise signal through a combination of the metal of the shock tube and the air.
The timing delay between the trigger signal and when the camera took the pictures became critical in order to capture a picture of the shock. The incident shocks captured by the photographs were traveling at a speed approximately 960 m/s and as mentioned earlier the windows were only 7.1 cm long. Since the goal of the schlieren system is to take pictures through the window when the shock is present, small deviations in loading pressure needed to be avoided. These would result in changes in the shock speed large enough to affect the picture timing.

To make the shock tube experiments more repeatable and the picture timing more stable, the method of how the diaphragms were burst was changed. Previously the driver section of the shock tube was pressurized until the diaphragms burst by themselves. The new method consisted of a thin Chromel wire (about 127 μm in diameter) was taken from a braided type K thermocouple and placed between the diaphragms. Figure 6.10 shows the device used to hold the chromel wires in place. It is a piece of PC board with a hole in the middle where the wire stretched across the shock tube. The copper was etched from the board leaving a strip to solder the wire to and provide electrical contact to the
wire. Four 7 mil Mylar disks were used as the diaphragm. Three were placed on the driver side of the wire and one on the driven side. These electrically insulated the wire and copper strips from the metal of the shock tube. With this wire in place, the driver section was pressurized and allowed to cool. A current was then sent through the wire, heating the wire and melting the diaphragms until they burst. A picture of the burst diaphragms is found in Fig. 6.11. In this manner the driver pressure of the shock tube was able to be specified previous to running the shock tube and controlled with much more accuracy than before. The shock tube relations could then be used to predict when the shock would be within the 7.1 cm length of the windows and this timing delay was programmed into the camera.

6.2.2. Schlieren Pictures

Figures 6.12 through 6.14 show details about what happens as the incident shock wave approaches and reflects from the end of the shock tube. In each picture, the end of the shock tube is on the right side of the picture and the arrows above the shock waves indicate which direction they are traveling. Figure 6.12 shows the incident wave approaching the end of the shock tube. The black specs near the end of the shock tube are dirt particles on the windows.
The reflected wave is shown in Fig. 6.13. The reflected wave is traveling slower than the incident wave from the lab space reference frame, therefore it appears thicker in the pictures. This picture was taken about 6 e-5 seconds after the shock wave was reflected. There is a region of stagnant gas in contact with the end of the shock tube as shock tube theory predicts, but also visible is the development of a turbulent region directly behind the reflected wave. The development of this turbulence is not predicted by the shock tube theory.

Figure 6.14 was taken approximately 0.00016 seconds after the shock is reflected. This figure shows that the turbulence behind the reflected wave has reached the end of the

![Figure 6.12 Schlieren Photograph of the Incident Wave Approaching the End of the Shock Tube](image)

![Figure 6.13 Schlieren Photograph of Stagnant Region behind Reflected Shock Wave](image)
There is no longer a region of stagnant gas in contact with the end of the shock tube.

These pictures explain why there was an anomaly in the data collected with the sensors mounted flush in the end of the shock tube. Immediately after the incident wave is reflected, the air at the end of the shock tube remains stagnant. The data and pictures reveal that about 0.0001 seconds after the shock wave is reflected, a turbulent flow comes into contact with the end of the shock tube. This turbulent flow increases the heat transfer to the sensors and a drastic change in the slope of the recorded temperatures is seen.

### 6.3. Sensors Extended into the Flow

In order to increase the film coefficient, $h$, for the heat transfer from the fluid, the sensors were placed on probes and extended into the flow. By extending the sensors into the flow, the experiment was designed to measure the temperature of the air after the passing of the incident shock wave. Once the reflected wave reached the sensors, the run was considered over. According to Chapter 4, the sensors on probes measured the temperature in gas state 2, whereas with the sensors flush in the end of the shock tube the change due to the increased film coefficient. Extending the sensors into the flow also allowed the problems with the turbulent flow to be avoided. Since these sensors are...
designed to measure the temperature after the passing of a blast wave, this was also a better simulation of the purpose for which these sensors were designed.

Two configurations were tested. In the first configuration the probes protruded 2.54 cm, and the second configuration extended the sensors 12.7 cm from the end of the shock tube. Both fiber optic sensors and Medtherm thermocouples were used in the experiments, and the results were compared. The Medtherm thermocouple used is a high frequency, coaxial thermocouple with only a 0.38 mm overall diameter. The small diameter of this thermocouple allowed for construction of probes for the thermocouples with a similar tip geometry to the fiber-optic probes.

The probes were designed to increase $h$ as much as possible. The outer support of the probe consisted of a stainless steel tube 1.6 mm OD and 0.5 mm ID. Within the end of the tube was located a stainless steel capillary and a glass capillary. These brought the overall ID of the probe to 165 microns. This was needed to hold the fiber-optic sensor in place. The tip of the tube was ground to a 30 degree angle. Grinding the tip of the probe had a great effect on increasing the heat transfer to the probe tip. The probes for the Medtherm thermocouples were also constructed of the same 1.6 mm OD tubing. This tip was ground to a 30 degree angle, and the thermocouple was inserted into the tube. The tip of the thermocouple was extended past the tubing so as to extend the surface of the thermocouple about 0.05 mm past the tubing. This position was to match the distance that the sensor wafer extended past the tubing. The probe tips for the Medtherm and fiber-optic sensors are shown in Fig. 6.15. The design details for these probes are found in Figs. 6.16 and 6.17.
Figure 6.15  Fiber-Optic and Medtherm Probes
Figure 6.16  Fiber-Optic Probe Tip Design

Figure 6.17  Thermocouple Probe Tip Design
Schlieren pictures were taken to compare the difference between a flat-tip probe and a pointed probe. These pictures are shown in Fig. 6.18 through Fig. 6.23. Figures 6.18, 6.19 and 6.20 show the flow development over a flat tipped probe. These pictures clearly show the bow shock development as the incident shock wave passes the probe tip. The shock standoff distance can also be detected in Fig 6.19 and Fig. 6.20.

When the pointed tipped probe was photographed it was more difficult to detect the bow shock. This is due to the fact that the bow shock is weaker around the pointed probe than around the flat-tip probe. The knife edge was adjusted to block more of the light beam, increasing the sensitivity of the schlieren effect. In Fig. 6.21 through Fig. 6.23 the sensitivity was increased to the point where turbulence is plainly visible upstream of the probe tip. This turbulence is introduced to the flow near the side wall of the shock tube when the windows are placed in the shock tube. Flat surfaced windows were fitted to a round interior surface of the shock tube. The two surfaces do not match perfectly, forming a small cavity around the edges of the windows. Curved surface windows were not formed due to difficulty in manufacturing and also optical complications. Even with the sensitivity of the schlieren system increased, the bow shock in front of the pointed probe was only slightly detectable. The shock standoff distance is smaller than with the flat probe; it cannot be seen in the photographs. The pointed probe will respond faster to changes in the free stream flow because the bow shock is weaker.
Figure 6.18 Schlieren Photograph of Incident Wave Approaching the Flat-Tip Probe

Figure 6.19 Schlieren Photograph of Developing Flow over a Flat-Tip Probe

Figure 6.20 Schlieren Photograph of Fully Developed Flow over a Flat-Tip Probe
Figure 6.21 Schlieren Photograph of Incident Wave Passing over Pointed-Tip Probe

Figure 6.22 Schlieren Photograph of Developed Flow over Pointed-Tip Probe

Figure 6.23 Schlieren Photograph of Reflected Wave Approaching the End of the Pointed-Tip Probe
6.3.1. Short Probe

The 2.54 cm probes were attached to a brass pipe plug with ½ inch NPT threading (shown in Fig. 6.24). The end of the shock tube was modified so the surface of the plug was flush with the inner surface of the shock tube when assembled.

With the probe sticking 2.54 cm from the shock tube surface, shock tube experiments were performed. The testing time in this case started when the incident wave contacted the tip of the probe. The end of the test was defined to be the instant when the reflected wave passed over the tip of the probe. The testing time was predicted to be about 0.0001 seconds in this configuration.

![Figure 6.24 Short Probe Design](image)

6.3.2. Longer Probe

The same basic probe tip design for the short (2.54 cm) probe was also used for the first part of the longer (12.7 cm) probe. After the first 2.54 cm the probe was built up in layers to support the longer probe. The diameter of the probe near the end of the shock
tube was 1.27 cm to stiffen the probe. The design for the longer probe can be seen in Fig. 6.25.

![Longer Probe Design](image)

**Figure 6.25 Longer Probe Design**

### 6.3.3. Testing Conditions

The following table lists the testing conditions for the experiments performed with the sensors extended into the flow. Each of these measurements was taken on different days, but this table shows that the shock tube experiments were very repeatable. The pressure of the driver section, P4, was the parameter that was controlled to determine the shock tube flow. This pressure was loaded within a gage reading of 0.69 kPa (0.1 psi) between runs. The pressure transducer was calibrated prior to use, but was only able to be calibrated up to 758 kPa (110 psi). Since the loading pressure for the experiments was about 3,075 kPa (446 psi), the uncertainty for this measurement was estimated to be about 34.5 kPa (5 psi). A jitter analysis was used to determine the uncertainty in the total temperature prediction. It was determined that the uncertainty associated with each of the total temperature values in Table 6.1 was ± 5.4 K, or about ± 0.8% of the expected temperature change.
Occasionally, a piece of the Mylar diaphragm would hurl through the shock tube and hit the fiber-optic sensor. Sometimes this debris would cause the sensor to fail, and sometimes the sensor would survive. Figure 6.26 shows what happened when one particularly large piece of diaphragm material struck the probe. This sensor did not survive.

### Table 6.2  Shock Tube Testing Conditions

<table>
<thead>
<tr>
<th></th>
<th>Medtherm 1”</th>
<th>Medtherm 5”</th>
<th>Fiber-Optic 1”</th>
<th>Fiber-Optic 5”</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 (mbar)</td>
<td>943.2</td>
<td>944.6</td>
<td>952.8</td>
<td>951.1</td>
</tr>
<tr>
<td>T1 (K)</td>
<td>295</td>
<td>295</td>
<td>295</td>
<td>296</td>
</tr>
<tr>
<td>P4 (psi)</td>
<td>446</td>
<td>446</td>
<td>449</td>
<td>449</td>
</tr>
<tr>
<td>T4 (K)</td>
<td>295</td>
<td>295</td>
<td>295</td>
<td>296</td>
</tr>
<tr>
<td>Mach number</td>
<td>1.295</td>
<td>1.294</td>
<td>1.294</td>
<td>1.294</td>
</tr>
<tr>
<td>Static Temp (K)</td>
<td>717.7</td>
<td>717.4</td>
<td>719.7</td>
<td>719.8</td>
</tr>
<tr>
<td>Total Temp (K)</td>
<td>958.3</td>
<td>957.8</td>
<td>960.8</td>
<td>961.0</td>
</tr>
</tbody>
</table>

**6.3.4. Experimental Results**

The results of the shock tube experiments are given below. The experiments with the sensors mounted on one inch and five inch probes were performed in July 2005 and November 2005 respectively. These tests were conducted in a shock tube with a driven section of 6.4 meters. These tests were performed with both the fiber-optic sensors and with Medtherm thermocouples. The following figures compare the thermocouple and fiber-optic data and then the fiber-optic data are compared with the model predictions.
Three sets of data collected with the sensors extended into the flow 2.54 cm are shown in Fig. 6.27. The data show good agreement between data sets. Runs 1 and 2 were sampled at a frequency of 200 kHz and run 3 was sampled at 1 MHz. The duration for each test was about 0.0001 seconds.

Figure 6.28 shows a comparison between run 1 of the fiber-optic data and a representative measurement made with the thermocouple probe extended 2.54 cm into the flow. The thermocouple data are represented by the filled data points, while the fiber-optic data are represented by the open data points. Figure 6.28 shows that the fiber-optic sensors responded much faster than the Medtherm high frequency thermocouples. The probe design for the fiber-optic sensors was not exactly the same as the probe design for the thermocouples, so this is not a direct comparison. However, the fiber optic probe was able to have a better design (smaller tip diameter) because the fiber-optic sensors are

Figure 6.27  Fiber-Optic Data – Short Probe
smaller than even the smallest available thermocouples (about 0.13 mm compared to 0.38 mm diameter).

When using the longer probe, only two reliable data sets were obtained. The measured data for the 12.7 cm probe are plotted in Fig. 6.29. As with the short probe, the data sets have fairly good agreement, although toward the end of the second run two the data sets for the longer probe start to diverge. It was observed that the signal from the fiber-optic sensor had changed from run 1 to run 2. After run 2, the fiber-optic sensor had failed. It is suspected that the sensor was starting to fail, i.e. the wafer was separating from the glass fiber, during run 2. For this reason, the data from run 1 is used throughout the rest of the analysis for the longer probe experiment. Figure 6.30 reaffirms that the fiber-optic sensors had a much faster response time when compared to the thermocouples. While the change in temperature of the fiber-optic sensor was almost 145 K or 22% of the change in total temperature of the air, the thermocouple only registered a change in temperature of 38 K or 6% of the air temperature change.
Figure 6.29  Fiber-Optic Data – Longer Probe
Figure 6.30  Comparison of Fiber-Optic and Thermocouple Data – Longer Probe
Chapter 7. Data Analysis

When an instrument is used to measure a physical quantity the measured value rarely matches perfectly with the true value of the physical quantity. This is especially true for dynamic measurements where the value of the physical quantity changes as time progresses. In fluid flow measurements, often the mere presence of the sensor will have an effect on the fluid property that the sensor is trying to measure. For example, when measuring the temperature in a fluid some of the energy from that fluid is used to heat the sensor, therefore the temperature of the fluid is lower than it would be without the presence of the sensor. Errors in the measurement system must be accounted for if the true value of the physical quantity is to be found.

Also, for most dynamic measurements there exists a settling time where it takes a sensor a certain amount of time to respond to a change in the input value. For such a measurement, the output of the sensor does not match the input value, but if enough information is known about the behavior of the sensor, the input value can be determined by analyzing the measured values. One of the difficult tasks that engineers face in designing experiments is to determine how to evaluate the output of a measurement system when the time history behavior of the input to the sensor is unknown. Although the Fluent/ANSYS model developed here is a useful tool for design and analysis of the sensors, it is not a practical tool for analyzing the measured data on a routine basis. For this study, an inverse heat conduction analysis method was explored as a way to deduce the temperature of the air flow from the measured temperature values over a short time. This chapter will detail some of the work completed in this area.

7.1. Inverse Heat Transfer Solution Method

In direct heat conduction problems, the temperature distribution within a solid is determined when the boundary conditions, initial conditions, heat generation, geometry, and thermal properties are all known. Inverse heat conduction problems involve measuring the temperature history of the solid and determining unknown information about the boundary conditions, initial conditions or heat generation. For design
For a problem to be well-posed, it must satisfy three conditions [4, 27, 37].

1. The solution must exist.
2. The solution must be unique.
3. The solution must be stable when small changes in input data exist.
Inverse heat transfer problems do not qualify as well-posed problems, because they are very sensitive to small changes in the input data, i.e. errors in the measured data or modeling errors. Also, for some problems uniqueness becomes an issue.

7.1.1. Least Squares Parameter Estimation

In order to successfully solve an inverse heat transfer problem, the problem is generally reformulated into a form that although may not be well-posed, it is “better”-posed [27]. Most methods for solving inverse heat transfer problems involve an optimization procedure to minimize an objective function. One of the simplest procedures for solving an inverse heat transfer problem is the least squares minimization approach [27, 37]. The objective function, $S$, for the least squares approach is found in Eq. 7.1. In this equation, $Y_i$ represents the measured values and $T_i(t)$ is the predicted values from a theoretical model. The variable $b$ is the unknown input to the problem. This is the quantity that the inverse method is determining.

$$S(b) = \sum_{i=1}^{M} (Y_i - T_i(b))^2$$

(7.1)

If the measured data contain any amount of uncertainty or small errors, and/or the theoretical model does not perfectly match the physical system the objective function cannot be driven to zero. The accuracy of this method depends upon the errors in the measured values and the quality of the theoretical model. The desired value of $b$ is the value that minimizes the objective functions, $S$. The problem to be solved is to find the value of $b$ so that $\frac{dS}{db} = 0$. When only one parameter, $b$, is desired, this is straightforward with one set of measurements yielding answers for one set of unknowns.

This method is not, however, limited to solving for only one unknown parameter. In the case of estimating multiple unknown parameters the unknown variables are represented by a matrix $b$. Where $b$ is made up of the $N$ unknowns $b_1, b_2, b_3, \ldots b_N$. The desired solution is the set of unknown parameters $b$ that minimize the objective function.
The objective function is minimized by requiring the partial derivative of $S$ with respect to each unknown parameter to equal zero ($\frac{\partial S}{\partial b_1} = \frac{\partial S}{\partial b_2} = \frac{\partial S}{\partial b_3} = \ldots = \frac{\partial S}{\partial b_N} = 0$).

### 7.1.2. Least Squares with Regularization

The least squares procedure can become unstable and experience large oscillations. In order to reduce the magnitude of these numerical errors and to smooth out the effects of small errors in the measured values, a “regularization” term is often included in the objective function [27, 37]. Equation 7.2 is an objective function used in the least squares with regularization procedure to solve for two unknown parameters. The first term is recognized as the residual between measured and predicted values. The regularizations terms follow. In the regularization terms, $c_i$ are weighting factors that apply a relative importance to each term. The terms $b_i^*$ are predetermined values or initial estimates for each unknown parameter “$i$”. As the weighting factor for a certain parameter is increased, the optimization procedure will force the final values of the unknown parameter towards the initial guess for that parameter. If a lot of confidence lies in the initial estimate value for a certain unknown parameter, then the weight factor for that parameter should be increased. Likewise, if there is little confidence in one of the initial guess values, that weight factor should be reduced.

$$S(b_1, b_2) = \sum_{i=1}^{M} (Y_i - T_i(b_1, b_2))^2 + c_1 (b_1 - b_1^*)^2 + c_2 (b_2 - b_2^*)^2$$  \hspace{1cm} (7.2)

### 7.2. Application of Inverse Heat Transfer Methods to the Fiber-Optic Temperature Sensors

A simplified energy balance was performed on the sensor for design purposes in Chapter 5. It was assumed that the lumped capacitance approximation was valid for the wafer and that no heat was lost from the wafer to the optical fiber. Also assumed was that the wafer was located at the stagnation point of the probe and that the sensor was in
laminar flow. Equation 7.3 was found to describe the temperature of the wafer under these conditions with the time constant of the system described by 

\[ \tau = \frac{\rho V C_{p-wafer}}{h A_s}. \]

\[ T_{wafer}(t) = T_i + e^{-\frac{t}{\tau}} \left( T_o - T_i \right) \]

(7.3)

This equation was used here as the theoretical model to give predicted solutions for use in the inverse heat transfer method. When making measurements with the temperature sensors, the exact flow environment is not known. Therefore, the film coefficient, \( h \), is one of the unknown parameters. The other unknown parameter in this equation is the gas total temperature, \( T_t \). Determining the total temperature of the gas flow is of primary concern, but unless the film coefficient is known beforehand, both will need to be found concurrently. In principle, one could use the more exact Fluent/ANSYS computational model here, but this is impractical due to the excessive computational burden.

The objective function in Eq. 7.2 was altered so that each term was normalized. This simplified determining any scaling factors. The normalized equation is shown below.

The initial estimate of the total temperature was taken as the last recorded temperature from the set of experimental data. The initial estimate for the film coefficient was set at 10,000 W/m²K. This is a conservative estimate from the values calculated with the Fluent model. This estimate is meant to just get the optimizer in the range of the actual solution.

\[ S(h, T_t) = \sum_{i=1}^{M} \left( 1 - \frac{Y_i}{T_i(h, T_t)} \right)^2 + c_1 \left( 1 - \frac{h^*}{h} \right)^2 + c_2 \left( 1 - \frac{T_t^*}{T_t} \right)^2 \]

(7.4)

In order to determine the values for the scaling factors \( c_1 \) and \( c_2 \), an independence study was performed. A set of “measured” data points, \( Y_i \), were simulated using Eq. 7.3 to generate the data points. Values of \( T_t = 1500 \) K and \( h = 250 \) W/m²K were used to generate this data set. If the inverse solution process were to work perfectly, these values
would be obtained as the solution values for $T_t$ and $h$. Random error with limits of ±5% of the temperature change was applied to introduce some imperfection to the simulated “measured” data. The Mathematica command “NMinimize” was employed as the optimizer. This command has the capability to determine the overall minimum of a function even with the presence of local minima. The value of $T_t^*$ was varied to determine the effect that changing $c_1$ and $c_2$ had on the optimization results for different initial estimates of total temperature. The effects of this study can be seen in Fig. 7.2 below. In this study, the values of $c_1$ and $c_2$ were constrained to be equal and are simply labeled as $c$.

![Graph showing the effect of regularization scaling factor independence study](image)

**Figure 7.2** Regularization Scaling Factor Independence Study

As can be seen in Fig. 7.2, regularization aids the optimizer in avoiding numerical error. As the scaling factors were decreased too much, the optimizer solutions began diverging from the solution of $T_t = 1500$ K. There was some motivation to have small values for the scaling factors. As the scaling factors decrease, the optimization results
become independent of the initial estimate of total temperature. The values of $c_1 = c_2 = 5.5e-6$ were chosen, because they were determined to be the smallest scaling factor values allowed before the optimizer diverged from the solution.

A typical contour plot of the objective function (Eq. 7.4) is seen in Fig. 7.3. The measured data points $Y_i$ were taken from a shock tube experiment with the sensor extended on the end of the longer probe. The value of the objective function is plotted on the vertical axis and the values of the film coefficient and total temperature are plotted on the two horizontal axes. This figure shows that there are multiple solutions to this problem. Instead of just one minimum within the objective function contour plot, a trough of possible solutions is present. A closer view of the bottom of the trough shows that the bottom of the trough is not smooth. In fact, there are many local minima throughout the domain as can be seen in Fig. 7.4.

![Contour Plot of Objective Function (Eq. 7.4) Using Data from Five Inch Probe Shock Tube Experiments](image)
Since the heat transfer to the sensor wafer is by means of convection, it was beneficial to look at the convection process during the short duration of the shock tube tests in more detail. Three combinations of film coefficient and total temperature were chosen from the bottom of the trough in Fig. 7.4. The values of the possible solution sets (total temperature $K$, film coefficient $W/m^2K$) chosen were: (600, 36600), (960, 14600) and (1200, 10400). These film coefficient and total temperature pairs were used with Eq. 7.3 to plot the “ideal” temperature response of the sensor to the possible combinations of film coefficient and total temperature. This plot is shown in Fig. 7.5.

Figure 7.5 shows that the even though total temperature values were chosen up to 600 K apart, the ideal responses for each case never deviate from one another more than 5 K within the time of interest for the shock tube tests. This is logical when the boundary condition for the wafer is thought of in terms of heat flux. Although the heat flux is dependent on both the film coefficient and the total temperature, at any one instant there are an infinite number of combinations of film coefficient and total temperature that can combine to give the required heat flux. These two parameters are coupled. In order to use the inverse heat transfer method to solve this problem, these two parameters need to be decoupled.
Figure 7.6 shows the ideal response of the sensor again, but allows for a much longer test time (10x). In this figure it can be seen that if the testing time were lengthened, the film coefficient and total temperature would decouple due to the availability of a longer time history to use in the inverse heat transfer method. As a longer time history is employed, it is as if the ends of the trough are being raised (see Fig. 7.7). Eventually there will be only one minimum, or one possible solution that will satisfy the given data set. This observation also shows that the data points at the end of the data set should have more of an influence on the optimization procedure than the points at the beginning of the data set.

In order to provide a way for the objective function to place more importance on certain data points, a new scaling factor was introduced in the least squares term of the objective function. The new objective function can be seen in Eq. 7.5. This new objective function can be skewed to give more weight to the beginning or end of the data set depending on the sign of the scaling factor, p. If p is negative, the objective function
Figure 7.6  Ideal Response of Fiber-Optic Temperature Sensor – 10 Times Longer Duration than Shock Tube Tests

Figure 7.7  Effects on the Objective Function of Lengthening the Testing Time
will be altered so the first data points affect the outcome of the optimizer more than the later data points. If $p = 0$, all data points will have an equal effect, and if $p > 0$, the latter data points will have a greater weight on the outcome of the optimizer.

$$S(h, T_i) = \sum_{i=1}^{M} \left( (1 + \frac{i}{M})^p \cdot (1 - \frac{Y_i}{T_i(h, T_i)})^2 \right) + c_1 (1 - \frac{\hat{h}}{h})^2 + c_2 (1 - \frac{T_i^*}{T_i})^2 \quad (7.5)$$

Sensitivity coefficients were defined to represent the change in temperature with respect to each unknown. The sensitivity coefficients are represented by

$$X_{i,h}(h) = \frac{\partial T_i}{\partial h} \quad \text{and} \quad X_{i,T}(T_i) = \frac{\partial T_i}{\partial T_i}. \quad (7.6)$$

It is preferred to have large, uncorrelated sensitivity coefficients. If the sensitivity is small, the temperature will not be affected very much by changes in the unknown parameter. The inverse process will be very sensitive to small errors in the measured values and the solution process will become increasingly difficult when the sensitivity is small. Figure 7.8 is a plot of the sensitivity coefficients for the measured data. The sensitivity with respect to $h$ was multiplied by $\frac{\hat{h}}{\Delta T_i}$ to ensure that the sensitivity coefficients were dimensionless. Figure 7.8 shows that the two lines are initially identical and after about 0.0005 seconds they start to separate. This indicates that the values of the sensitivity coefficients are strongly correlated initially, but become uncorrelated as time progresses. This again explains the existence of multiple possible solutions for short time periods after the passing of the shock wave. The sensitivity values also start out small and increase as time progresses.

One question that remained to be answered was how long the experiment needed to last in order to enable the use of the inverse method. An estimate of the necessary testing time can be determined by examining the sensitivity plots and determining how much time is needed to ensure the sensitivities are not correlated. A more in depth approach
was taken. To create a lower limit to the necessary testing time, an ideal data set was simulated using the theoretical model. This data set was created without any random noise. The points for the data set were generated with the initial temperature set to 294 K, the total temperature set to 960 K, and the film coefficient set equal to 8341.8 W/m²K.

The optimization routine was run iteratively with the initial guess for temperature assigned a value of 1000 K, a guess that is very close (within 40 K) to the value used to generate the data set. The optimizer was first run only using the first point from the data set. Then, the first two data points were used in the objective function. This continued until finally all of the data points were used. In this manner, it was determined how many data points were required to get an accurate response from the optimization routine. It can be seen from Fig. 7.9 that even if the initial estimate of the total temperature is within 40 K of the actual temperature, it would still take about 0.0002 seconds of testing time with this fiber-optic sensor to gather enough time history of the change in temperature for
the inverse heat transfer method to reach a valid conclusion. The plot reveals that there are three abnormalities within the solutions for this data set. These abnormalities repeated themselves throughout multiple runs of the optimizer, and were also noticed when different data sets were examined as well. These spikes in the results reiterate the instability of ill-posed problems. It is, therefore, recommended that, in order to ensure avoiding one of these abnormalities, multiple subsets are tried from each data set when this method is used with the experimental data.

Next, a comparison was made to understand what would happen if the initial estimate of the total temperature was not a very good estimate. Figure 7.10 shows the difference between an initial estimate within 40 K and an initial estimate that was 1540 K from the simulated total temperature. The results show that even with a very poor estimate of the total temperature, if the data set is perfect the inverse solution method would only require
about 0.0004 seconds of data (100 kHz sampling frequency) to accurately determine the total temperature.

![Graph showing the comparison of initial estimates of total temperature affecting required testing time.](image)

**Figure 7.10** Comparison of how the Initial Estimate of Total Temperature Affects Required Testing Time

Because it was possible to collect data with the Hyperscan data acquisition system at 1 MHz sampling frequency, a final comparison was made to see how the sampling frequency affected the results of this ideal simulation. It can be seen in Fig. 7.11 that increasing the sampling frequency by an order of magnitude only decreased the required testing time by about a third. For this data set, the 100 kHz sampling frequency would require about 0.00023 seconds of data while the 1 MHz sampling frequency would require about 0.00015 seconds of data. One benefit of having more data points can be found in the stability of the solution. With more data points, the stability of the solution process increased. There is only one spike set of solutions when the 1 MHz data set was used. Even this spike occurs before the solutions have reached a steady value.
A program was developed in Mathematica following the procedure outlined above and can be found in the appendix. This program was used to try to predict the total temperature in the shock tube using the measured data from run 1 with the five inch probe. When the algorithm was unconstrained a solution set of \( T_t = 626 \) K and \( \dot{h} = 32,950 \) W/m\(^2\)K was obtained. The total temperature value predicted by the normal shock relations was instead 960 K. Again the solution is not unique. As discussed earlier, the inverse heat transfer method is affected by errors in the measured values. An uncertainty of \( \pm 9 \) K is very substantial. Also, the theoretical model of the system must be a good match to the physical reality. In this preliminary study the theoretical model did not account for any heat lost from the wafer to the glass fiber. These errors contribute to finding an incorrect solution with the inverse heat transfer method.

Figure 7.11 Effect of Sampling Frequency on Required Testing Time
Table 7.1 lists some possible solutions from the trough in the contour plot of the objective function. When the total temperature system is constrained to be the expected value of 960 K, a film coefficient of 14,562 W/m²K is returned by the optimizer. Interestingly, this value for the film coefficient lies between the values for the two faces of the wafer predicted by the Fluent model.

<table>
<thead>
<tr>
<th>( T_t, \text{ K} )</th>
<th>( \dot{h}, \text{ W/m}^2\text{K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>36,603</td>
</tr>
<tr>
<td>700</td>
<td>25,738</td>
</tr>
<tr>
<td>800</td>
<td>19,866</td>
</tr>
<tr>
<td>900</td>
<td>16,181</td>
</tr>
<tr>
<td><strong>960</strong></td>
<td><strong>14,562</strong></td>
</tr>
<tr>
<td>1000</td>
<td>13,652</td>
</tr>
<tr>
<td>1100</td>
<td>11,808</td>
</tr>
<tr>
<td>1200</td>
<td>10,403</td>
</tr>
</tbody>
</table>

Figure 7.12 is a plot of the measured data points alongside the solution of the theoretical model with \( T_t = 960 \text{ K} \) and \( \dot{h} = 14,562 \text{ W/m}^2\text{K} \). This plot indicates that the film coefficient found by the optimizer is too low, because the heat flux at the beginning of the data set is too low. This is of secondary concern because if the theoretical model is adjusted, the value of the film coefficient returned by the optimizer will change. As time progresses, the theoretical model crosses the measured data and gives too high of an estimate for the wafer temperature. If the heat lost from the wafer to the fiber were accounted for, these two plots would align better.

Figure 7.13 displays the error associated with the measurements along with the theoretical model. As can be seen, the uncertainties in the measurements are very large. If the testing time of the experiment were greatly increased, it may be possible to get an answer that is close to the total temperature of the flow. Another approach would be to try to reduce the uncertainty in the measurements. A combination of reducing the
uncertainty and increasing the testing time would work best. However, for the data currently available, the inverse solution method cannot reliably predict the total temperature of the flow.

![Figure 7.12 Theoretical Model for the Inverse Method Compared to Measure Data Points](image1)

![Figure 7.13 Theoretical Model for the Inverse Method Compared to Measure Data Points with Measurement Uncertainties](image2)
Chapter 8. Computational Results and Comparison with Experiments

The flow around the probe at the end of the shock tube was modeled using Fluent. From the Fluent model the convection film coefficient associated with the heat flux form the air flow to the fiber-optic sensor was determined. Across the front of the wafer the film coefficient was determined to be 27,149 W/m²K and across the side of the wafer the film coefficient was 13,485 W/m²K. These values along with a bulk air temperature of 960 K were applied as the thermal loading conditions for the model of the fiber-optic sensor in ANSYS. The final results from this Fluent/ANSYS modeling can be seen in Fig. 8.1. This figure is a temperature contour plot of the fiber-optic sensor model generated with ANSYS and details the temperature distribution in the fiber-optic sensor 0.0005 seconds after the incident shock wave passed the tip of the sensor. The time history of the results of this calculation is found in the graph in Fig. 8.2.

Figure 8.1  Temperature Contour Plot of the Fluent/ANSYS Model
Figure 8.3 compares the fiber-optic measurements with the Fluent/ANSYS model prediction for the short probe configuration. This figure reveals that the Fluent/ANSYS model over-predicts the change in temperature for the fiber-optic sensors. This is due to the approximations made in order to create axisymmetric models. The area of the front face of the wafer was modeled as a circle with a diameter of 125 microns. In the physical sensor, the area of this front face is actually an 85 by 85 micron square. The ratio of these areas was found to be \( \frac{A_{\text{mod.el}}}{A_{\text{sensor}}}_{\text{front}} = 1.70 \). The thickness of the wafer in the model was changed in order to keep the volume of the wafer in the model equal to the volume of the wafer on the sensor. The thickness of the wafer in the model was 20.6 microns, while the thickness on the sensor was in reality 35 microns. The ratio for the area on the side of the wafer exposed to the flow was \( \frac{A_{\text{mod.el}}}{A_{\text{sensor}}}_{\text{side}} = 0.68 \). The Fluent model predicted that the heat flux to the front face of the wafer is greater than the heat flux to the side face. It

![Fluent/ANSYS Time History Results](image_url)
therefore, is logical that the Fluent/ANSYS model would over-predict the temperature response of the fiber-optic sensor because the area of the wafer is skewed to be greater in an area with an increased heat flux.

The uncertainty of the Hyperscan demodulation system when used with this type of sensor was reported as ±9 K. Since the Hyperscan system is a differential system, relying on the comparison of two laser signals, this uncertainty was presumed to be constant regardless of the measured temperature change. This assumes that both laser signals change in an identical manner no matter what temperature change is detected. Figure 8.4 shows that even though the Fluent/ANSYS model over-predicts the measured response of the sensor on the short probe, this prediction still lies within the uncertainty of the demodulation system.

In Fig. 8.5 the Fluent/ANSYS model is again seen to at first over-predict the temperature response of the sensor on the longer probe. As time progresses, the slope of the model prediction is reduced, while the slope of the measured data remains almost
constant. One possible explanation for this is found in how the bond between the wafer and the glass fiber was modeled. The ANSYS model assumed that the wafer material and the glass fiber were in perfect thermal contact. In reality there is likely some thermal resistance between the two materials. As the temperature of the wafer increases, the temperature gradient across this boundary increases as well. At first there will be little to no heat lost to the fiber from the wafer. As time progresses the amount of heat lost to the glass fiber will increase. Any thermal resistance between the wafer and the fiber will decrease the amount of heat lost to the fiber. The Fluent/ANSYS model could be allowing for too much heat to flow from the wafer into the fiber. This error would increase as the temperature of the wafer increases and cause the deviation seen in Fig. 8.5. When the measurement uncertainty is shown with the data from the longer probe, it can be seen that the Fluent/ANSYS is still a good fit during the duration for which the shock tube experiments were performed (see Fig. 8.6).

Figure 8.4  Comparison of Fiber-Optic Data and Fluent/ANSYS Model Results – Short Probe with Uncertainty of the Hyperscan Shown
Figure 8.5  Comparison of Fiber-Optic Data and Fluent/ANSYS Model Results – Longer Probe

Figure 8.6  Comparison of Fiber-Optic Data and Fluent/ANSYS Model Results – Longer Probe Uncertainty of the Measured Data Shown
Chapter 9. Discussion

This work detailed some efforts in developing a fiber-optic temperature sensor. Fiber-optic sensors are generally very small and highly sensitive. Their small size tends to give fiber-optic sensors a very rapid response, and in volatile environments, fiber-optic sensors are safer than electronic sensors since there is no risk of creating a spark. For these reasons, it is desirable to use fiber-optic sensors in experiments where volatile materials are being tested.

The temperature sensor discussed here consists of a glass fiber with an optical wafer attached to the end. The glass fiber is 125 microns in diameter, and this dimension determined the overall size of the sensor. The wafer on the physical sensor is a rectangular prism with a square area of 85 by 85 microns and a height of 35 microns. As the temperature of the wafer changes, so does the index of refraction and thickness of the wafer. A light signal is sent along the fiber. A portion of the light reflects from the fiber/wafer junction and a portion of the light reflects from the end of the wafer. These two reflections form an interferometric pattern which is sensitive to changes in the thickness and index of refraction of the wafer. The change in temperature of the wafer can be determined by monitoring how this interferometric pattern changes.

An unsteady computational thermal model of a fiber-optic sensor was developed using the software package ANSYS. This model was used to simulate the thermal response of the sensor to various inputs and was a useful design tool. Tests were performed in order to verify that the grid spacing was such as to minimize numerical errors in the model. The geometry of the sensor in the model was simplified from the geometry of the physical sensor in order to create an axisymmetric model. The wafer in the ANSYS model was approximated to cover the entire front surface of the glass fiber (a circular area with a 125 micron diameter), making the wafer in the model a short cylinder. The height of the cylinder was set at 20.6 microns in order to match the volume of the wafer in the model with the volume of the wafer on the physical sensor. This effectively increased the frontal area of the wafer in the model to be 1.7 times larger than
the frontal area of the wafer on the physical sensor. The side area of the wafer on the model was reduced to 0.68 times the side area of the wafer on the physical sensor.

Fiber-optic sensors were successfully tested in a shock tube environment to create a step change in temperature for testing the response of the fiber-optic temperature sensors. During the first experiments, the tip of the sensor was mounted flush with the end of the shock tube. A high-speed coaxial Medtherm thermocouple was also mounted similarly for comparison. These tests demonstrated that the fiber-optic sensor contained less noise than the Medtherm data. The response of the two sensors was approximately the same due to the testing configuration. Only a small temperature change was measured by these sensors in the short test time due to the low heat transfer from nominally stagnant air after the reflected shock. The stagnant air condition was applied to the ANSYS model and a prediction was obtained for the response of the sensor. The ANSYS prediction results were compared to the measurements and good agreement was obtained for the short time the gas remained stagnant.

While these initial tests showed the ability of the fiber-optic sensor to measure rapid temperature changes, they also revealed an anomaly not predicted by idealized shock tube relations. In each test, there appeared to be a temperature step change followed by a convective effect felt by the sensor. Shock tube theory predicts that the air is stagnant at the end of the shock tube after the incident shock wave is reflected. The convective effect was measured by the sensor about 0.1 milliseconds after the incident shock was reflected. Every run for this experimental configuration was consistent with this timing. To determine the cause of this anomaly, the shock tube was altered to allow for schlieren photographs to be taken as the shock reflected off the end. These photographs revealed that the flow at the end of the shock tube initially remained stagnant, as predicted by normal shock relations. However, as the reflected shock traveled back into the shock tube flow, turbulence developed behind the shock. After a short time, this turbulence reached the end of the shock. The schlieren pictures explain what was measured by the fiber-optic sensor. When the incident shock wave reflected off the end of the shock tube, the air at the end of the shock tube remained stagnant but experienced a step change in
temperature. A short time later (about 0.1 milliseconds) the turbulence in the air reached the end of the shock tube, and the sensor was subjected to convection heat transfer.

Further tests were designed to increase the heat transfer to the fiber-optic sensor. The heat transfer was increased by extending the temperature sensors on probes into the oncoming flow. The experiments were then intended to measure the temperature change as the incident shock wave passed the probe tip, and the end of the test was defined by the passing of the reflected shock wave over the probe tip. Defining the experiment in this manner excluded the turbulent portion of the flow behind the reflected shock in the experiment. The shock created in the shock tube simulated the pressure loading experienced at a distance of 3 m from the detonation of 22.7 kg of TNT. Two probes were designed and tested. One extended the sensor 2.54 cm (1.0 in) into the flow and the other protruded 12.7 cm (5.0 in) from the end of the shock tube. Again, probes were made for both the fiber-optic sensors and for the thermocouples so data from each type of sensor could be compared. Schlieren pictures were taken of the shock waves passing both a flat-tip probe and a pointed-tipped probe. These photographs give visual evidence of the benefit of the pointed-tip probe.

In the tests with the short probe, the fiber-optic sensor registered a temperature change of 36 K compared to a temperature change of about 11 K for the thermocouple before the reflected wave contacted the sensors, and the fiber-optic temperature sensors had a much faster response when compared to the thermocouple. The fiber-optic sensor experienced a change in temperature 3-4 times greater than the thermocouple. This test gives credence to the use of fiber-optic sensors. Also noted was the lack of noise present in the fiber-optic measurement when compared to the thermocouple data. The fiber-optic data from the short probe experiment were compared to the fiber-optic data from mounting the sensor flush with the end of the shock tube. When the sensors were mounted flush, the air temperature was predicted to change from 297 K to 1130 K. During the first 0.1 milliseconds for this test the sensor only changed 8 K. For the probe experiments, the air temperature was expected to change from 295 K to 960 K and the sensor recorded a temperature change of 36 K during the first 0.1 milliseconds. Even
though the air temperature changed more for the flush mounted configuration, extending the sensor into the flow increased the convection film coefficient and resulted in an increase in the change of temperature of the sensor that was over four times greater for the probe configuration.

The longer probe experiments increased the testing time for the experiments and yielded a temperature change of 145 K for the fiber-optic sensor, and only 38 K for the thermocouple before the reflected wave reached the probe tips. Again, the fiber-optic sensor recorded a temperature change 3-4 times greater than the thermocouple. The fiber-optic sensors were shown to have a faster response than the fastest commercially available thermocouple when the thermocouples were in probes of similar design. The rate of temperature rise for the fiber-optic sensors was $2.9 \times 10^5$ K/s compared to $0.76 \times 10^5$ K/s for the thermocouple. When using the sensor measurement to determine the flow temperature, the process becomes easier as the recorded temperature change increases. Therefore, the fiber-optic sensors are better suited to this process because of the faster response when compared to the thermocouple. The fiber-optic sensor recorded 22% of the overall 664 K change in total temperature of the flow, and the thermocouple only measured about 6% of this temperature change during the 0.5 millisecond tests.

An unsteady CFD model of the flow in the shock tube was created with Fluent, a popular commercial CFD program. The model of the flow around the probe tip was again an axisymmetric model. The shock in the model was generated by using the upstream boundary of the model as a piston. This surface was given the properties of a moving wall with the velocity of the flow as predicted by normal shock relations (696 m/s). The resulting modeled flow matched the conditions expected to be encountered by the sensors in the shock tube with a flow Mach number of 1.29 and a final total temperature of 960 K. The Fluent model was useful to calculate the convection film coefficient from the heat transfer from the flow to the wafer at the tip of the probe. An average film coefficient was determined for both the front and side faces of the wafer. The convection film coefficient was predicted to be 27,149 W/m$^2$K along the front face of the wafer and 13,485 W/m$^2$K along the side face of the wafer. These film coefficients
were then used as inputs to the ANSYS model to obtain a prediction for the response of the sensor in the shock tube. Although the Fluent/ANSYS model geometry was simplified, the prediction did a good job of matching the measured values within the uncertainty of the measurements. The slope of the prediction was slightly greater than the slope of the measured data, indicating that the heat flux to the wafer was over-predicted. Also, the computations over-predicted the heat lost from the wafer. Even with the inaccuracies of the model, the prediction still lies within the uncertainty of the measurement.

Preliminary work was completed for developing an algorithm to predict the total temperature of the air flow from the measured temperature values. This problem was attempted using the principles of inverse heat transfer. A least-squares approach with regularization was used comparing the data measured by the fiber-optic sensors with an idealized model of the temperature response of the wafer. For the short duration of the experiments, this approach resulted in a set of possible solutions that lie at the bottom of a trough in a contour plot of the objective function. When the optimizer was unconstrained, a solution was obtained where $T_t = 626$ K and $h = 32,950$ W/m$^2$K. These values defined the overall minimum for the objective function. Another point that lies at the bottom of the trough near the overall minimum is defined by the values $T_t = 960$ K and $h = 14,562$ W/m$^2$K, which are in good agreement with the expected values for both total temperature and film coefficient. It was determined that the film coefficient and total temperature were coupled for these short tests. If the duration of the tests were extended, these two quantities decoupled. The uncertainty of the measurement system and the adequacy of the simple thermal model employed contribute to the errors associated with the inverse heat transfer method. It was determined that the uncertainty in the temperature measurements was too large to allow for an accurate solution in the very short time of these shock tube tests. The inverse heat transfer process could be improved if the analytical expression describing the change in temperature of the sensor wafer provided a better simulation of the response of the sensor.
When exploring the use of the inverse heat transfer methods for interpreting the measured data, it was assumed that the film coefficient, $h$, was unknown. The problem is greatly simplified if the film coefficient is known beforehand. Even if the film coefficient can only be approximated, some bounds can be set limiting the optimization procedure. Correlations exist for some body shapes that relate the Mach number of the flow with the stagnation point heat transfer. As discussed in Ch. 5, White [40] gives one such correlation. This relationship is shown in Eq. 9.1 and states that the stagnation point heat flux is dependent on the Prandtl number, the density, viscosity and enthalpy at both the wall and edge of the boundary layer, and the stagnation point velocity gradient, $K$. This velocity gradient can be obtained by interpreting the graph also provided by White in Fig. 7-6 on page 597 of his book.

$$q_w = 0.763 \Pr^{-0.6} \left( \frac{\rho_e \mu_e K}{\rho_w \mu_w} \right)^{1/2} \left( \frac{\rho_w \mu_w}{\rho_e \mu_e} \right)^{0.1} (h_e - h_w)$$

(9.1)

Since the goal of the current measurement is to determine the total temperature of the flow and not the film coefficient, Eq. 9.1 and Eq. 9.2 can be used to provide an estimate for the film coefficient. This estimate can be used to provide limits on the value of $h$ for the optimizer. In utilizing these equations, a range of values for the free stream Mach number must be assumed unless additional measurements such as Pitot pressure are available.

$$h = \frac{q_w}{(T_e - T_w)} = \frac{q_w}{(h_e - h_w)} C^p$$

(9.2)

For the current case, the shape of the probe is a non-typical shape. The tip of the probe is conical with a small flat face, but the tip of the cylinders for which White provides data are either rounded or flat. Also, the Fluent model suggests that the flow around the wafer is affected by the larger diameter of the probe, not just the diameter of the wafer. Therefore, for this approximation, the tip was assumed to have a flat tip and
the diameter was assumed to be 0.00159 m (1/16 inch). Using White’s curve for a flat-tipped cylinder, and assuming that the product of density and viscosity are constant throughout the boundary layer thickness, values for the film coefficient were determined from Eq. 9.3.

\[
\bar{h} = 0.763 C_p \, \text{Pr}^{-0.6} \sqrt{\rho_w \mu_w K}
\]

(9.3)

A wide range was chosen for the Mach number behind the shock, 1 to 1.5. If the pressure ratio across the shock, \( P_{12} \), is allowed to increase infinitely, the maximum Mach number for the flow behind the shock is only 1.89. This range for the Mach number, \( 1 \leq M \leq 1.5 \), resulted in a range for the film coefficient of 7,500 W/m\(^2\)K to 23,300 W/m\(^2\)K. The corresponding total temperature range provided by the optimizer for these film coefficient values is 1,533 K to 737 K respectively. Although this is still a large temperature range, it represents a large reduction in the feasible computation area for the optimizer. Establishing limits for the optimizer by using available information about the film coefficient will help increase the accuracy of the inverse solution process. A better correlation for non-typical probe shapes could be obtained through further experiments or Fluent calculations.

As stated before, the inverse process was used to try to find both the total temperature and the convection film coefficient simultaneously. This same process has been shown to be applicable for finding the heat flux with good success [8, 14, 25, 26, 39]. The fiber-optic sensor will be able to measure the heat flux to the wafer very effectively. The timing issues associated with decoupling the dependence on total temperature and film coefficient can be avoided in this case, because there is only one unknown instead of two. This application has yet to be explored, but shows great promise.

### 9.1. Future Work

The experimental work completed supports the use of fiber-optic temperature sensors in place of thermocouples when extremely fast response times are required. Work is
currently underway to expand the shock tube experiments done at Virginia Tech to other facilities.

The **Fluent/ANSYS** model could be improved by accounting for thermal contact resistance between the wafer and the optical fiber. Also, generating a three dimensional model of the shock tube flow and the sensor including the complex geometry of the wafer on the fiber would result in a more accurate model. These changes to the model would be useful if more accurate predictions of the response of the sensor are needed.

However, the end goal in developing these sensors is to manufacture and market them as an alternative method for measuring temperature. A product requiring calculations performed with **Fluent** and/or **ANSYS** will not be useful to most end users of this product. Therefore, the development of a method or algorithm to determine the temperature of the medium being measured is of primary concern. The inverse heat transfer methods show promise if the uncertainty in the demodulation system can be reduced and/or if the testing times of the experiments are increased. A more realistic analytic model of the temperature sensor is also needed to increase the accuracy of this data analysis method. In particular, some treatment of the heat loss from the wafer to the fiber should be included. Also, information known about the film coefficient can be used to establish limits for the optimization process and more powerful methods of optimization can be explored if it is determined that the least-squares method is too simplistic.

The use of these sensors as instruments to measure heat flux should also be explored. The wafer acts essentially as a calorimeter. The inverse solution method can also be employed here as well. It will be easier to determine the heat flux from the temperature measurements than it is to find the total temperature. In this application, there is no need to worry about decoupling the temperature and film coefficient parameters. Instead, these are combined and only the heat flux needs to be determined.
References


Appendix

A. Shock Tube Uncertainties (Total Temperature, Mach number, Testing Time)

B. Mathematic Program for using the Inverse Solution Process
Appendix A-Shock Tube Uncertainties

The following tables were used to calculate the uncertainty in the predicted total temperature and the predicted Mach number for the flow in the shock tube experiments. The gas constants and specific heat ratios were known constants. The primary measurements were the atmospheric pressure in the room, the room temperature and the loading pressure of the driver section of the shock tube. The pressure and temperature of the driver section of the shock tube were monitored prior to bursting the diaphragms. The temperature of the driver section was allowed to reach thermal equilibrium with the room. It was assumed that the pressure and temperature in the driven end of the shock tube remained the same as the ambient conditions in the room. The pressure and temperature of both the driver and driven sections of the shock tube were all that were needed to predict the total temperature and Mach number of the flow in the shock tube. The shock tube relations described in Ch. 4 were used to determine the flow conditions.

The first column containing calculations contains the optimum values obtained assuming that the measurements are exact. A primary uncertainty was determined for each of the primary measurements. The pressure transducer used to measure the driver pressure was calibrated prior to the experiments within 0.5 psi. However, the maximum pressure for the equipment used during the calibration was only 110 psi. Since the loading pressure was about 450 psi during the experiments, the primary uncertainty for this transducer was increased from 0.5 to 5 psi. The primary uncertainties were used to perturb the values of the primary measurements one at a time for the next three columns containing the calculations. This shows the effect that the uncertainty of each measured value has on the final value of total temperature or Mach number. The differences between the optimum values and the values calculated with the uncertainties are then calculated. The overall uncertainty is determined by taking the square root of the sum of the squares of these differences.

The total temperature of the shock tube flow was predicted to be about 960 K for each of the tests. This value is accurate within ±5.4 K. When sensors were extended into the
flow, the Mach number of the flow was predicted to be 1.29 – 1.30. The uncertainty in
the Mach number calculation is determined to be only ±0.002.

### Worksheet for Uncertainty in Total Temperature

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Primary</th>
<th>Perturbation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncertainty</td>
<td>a+da,b,c</td>
</tr>
<tr>
<td>R1 (air)</td>
<td>287</td>
<td>287</td>
</tr>
<tr>
<td>γ1</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>R4 (helium)</td>
<td>2077</td>
<td>2077</td>
</tr>
<tr>
<td>γ4</td>
<td>1.667</td>
<td>1.667</td>
</tr>
</tbody>
</table>

### Primary measurements

- **Atmospheric pressure, mbar**:
  - 949.4 ± 0.4
  - 949.8 ± 0.4
  - 949.4 ± 0.4
  - 949.4 ± 0.4

- **Room temperature, K**:
  - 296.15 ± 1
  - 296.15 ± 1
  - 297.15 ± 1
  - 296.15 ± 1

- **Loading pressure, psi**:
  - 448 ± 5
  - 448 ± 5
  - 448 ± 5
  - 453 ± 5

### Intermediate results

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Result</th>
<th>Result</th>
<th>Result</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1 (Pa)</td>
<td>94940</td>
<td>94980</td>
<td>94940</td>
<td>94940</td>
</tr>
<tr>
<td>p4 (Pa)</td>
<td>308851.136</td>
<td>308851.136</td>
<td>308851.136</td>
<td>308851.136</td>
</tr>
<tr>
<td>ro1</td>
<td>1.117006226</td>
<td>1.117476814</td>
<td>1.11324716</td>
<td>1.117006226</td>
</tr>
<tr>
<td>beta1</td>
<td>0.142857143</td>
<td>0.142857143</td>
<td>0.142857143</td>
<td>0.142857143</td>
</tr>
<tr>
<td>alpha1</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>ro4</td>
<td>5.021676653</td>
<td>5.021676653</td>
<td>5.004777136</td>
<td>5.077722151</td>
</tr>
<tr>
<td>beta4</td>
<td>0.200059988</td>
<td>0.200059988</td>
<td>0.200059988</td>
<td>0.200059988</td>
</tr>
<tr>
<td>alpha4</td>
<td>3.99850075</td>
<td>3.99850075</td>
<td>3.99850075</td>
<td>3.99850075</td>
</tr>
<tr>
<td>a1</td>
<td>344.9537215</td>
<td>344.9537215</td>
<td>345.5356277</td>
<td>344.9537215</td>
</tr>
<tr>
<td>a4</td>
<td>1012.609312</td>
<td>1012.609312</td>
<td>1014.317493</td>
<td>1012.609312</td>
</tr>
</tbody>
</table>

Use solver and set "function" equal to p4/p1 by changing p2

<table>
<thead>
<tr>
<th>function</th>
<th>p4/p1</th>
<th>p2/p1</th>
<th>p2</th>
<th>T2</th>
<th>a2</th>
<th>u2</th>
<th>Flow Mach Number</th>
<th>Static Temperature, K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>32.53477076</td>
<td>32.52106903</td>
<td>32.53477076</td>
<td>32.89788204</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.854637725</td>
<td>8.852721425</td>
<td>8.854637725</td>
<td>8.905236805</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>32.53477076</td>
<td>32.52106903</td>
<td>32.53477076</td>
<td>32.89788204</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>84059.3056</td>
<td>840831.4809</td>
<td>84059.3056</td>
<td>845463.1823</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>719.6544444</td>
<td>719.5587289</td>
<td>722.084478</td>
<td>722.1815996</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>537.7333501</td>
<td>537.697589</td>
<td>538.6404583</td>
<td>538.6766811</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>695.9814821</td>
<td>695.8855969</td>
<td>697.1555408</td>
<td>698.508777</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.294287368</td>
<td>1.294195122</td>
<td>1.294287368</td>
<td>1.296712484</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>719.6544444</td>
<td>719.5587289</td>
<td>722.084478</td>
<td>722.1815996</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Final Result

| Total Temperature, K | 960.7645607 | 960.6024143 | 964.0087429 | 965.0459657 |
| change              | -0.162146404 | 3.244182207 | 4.281405034 |

### Final Uncertainty

5.374145394
### Worksheet for Uncertainty in Mach Number

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Primary Uncertainty</th>
<th>Perturbation</th>
<th>a+da,b,c</th>
<th>a,b+db,c</th>
<th>a,b,c+dc</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1 (air)</td>
<td>287</td>
<td>287</td>
<td>287</td>
<td>287</td>
<td>287</td>
</tr>
<tr>
<td>γ1</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>R4 (helium)</td>
<td>2077</td>
<td>2077</td>
<td>2077</td>
<td>2077</td>
<td>2077</td>
</tr>
<tr>
<td>γ4</td>
<td>1.667</td>
<td>1.667</td>
<td>1.667</td>
<td>1.667</td>
<td>1.667</td>
</tr>
</tbody>
</table>

### Primary measurements

- **Atmospheric pressure, mbar**
  - 949.4 ± 0.4
- **Room temperature, K**
  - 296.15 ± 1
- **Loading pressure, psi**
  - 448 ± 5

### Intermediate results

<table>
<thead>
<tr>
<th>p1 (Pa)</th>
<th>p4 (Pa)</th>
<th>ro1</th>
<th>beta1</th>
<th>alpha1</th>
<th>ro4</th>
<th>beta4</th>
<th>alpha4</th>
<th>a1</th>
<th>a4</th>
</tr>
</thead>
<tbody>
<tr>
<td>94940</td>
<td>3088851.136</td>
<td>1.117006226</td>
<td>0.142857143</td>
<td>6</td>
<td>5.021676653</td>
<td>0.200059988</td>
<td>3.99850075</td>
<td>344.9537215</td>
<td>1012.609312</td>
</tr>
<tr>
<td>94980</td>
<td>3088851.136</td>
<td>1.117476841</td>
<td>0.142857143</td>
<td>6</td>
<td>5.021676653</td>
<td>0.200059988</td>
<td>3.99850075</td>
<td>344.9537215</td>
<td>1012.609312</td>
</tr>
<tr>
<td>94940</td>
<td>3088851.136</td>
<td>1.11324716</td>
<td>0.142857143</td>
<td>6</td>
<td>5.004777186</td>
<td>0.200059988</td>
<td>3.99850075</td>
<td>344.9537215</td>
<td>1012.609312</td>
</tr>
<tr>
<td>94940</td>
<td>3088851.136</td>
<td>1.117006226</td>
<td>0.142857143</td>
<td>6</td>
<td>5.077722151</td>
<td>0.200059988</td>
<td>3.99850075</td>
<td>344.9537215</td>
<td>1012.609312</td>
</tr>
</tbody>
</table>

- **Intermediate results**

- **p4/p1**
  - 32.53477076 ± 32.52106903 ± 32.53477076 ± 32.89788204
- **p2/p1**
  - 8.854637725 ± 8.852721425 ± 8.854637725 ± 8.905236805
- **function**
  - 32.53477076 ± 32.52106903 ± 32.53477076 ± 32.89788204
- **p2**
  - 840659.3056 ± 840831.4809 ± 840659.3056 ± 845463.1823
- **T2**
  - 719.654444 ± 719.5587289 ± 722.084478 ± 722.1815996
- **a2**
  - 537.733501 ± 537.697589 ± 538.6404583 ± 538.6766811
- **u2**
  - 695.9814821 ± 695.8855969 ± 697.1555408 ± 698.508777

### Final Result

- **Flow Mach Number**
  - 1.294287368 ± 1.294195122 ± 1.294287368 ± 1.296712484
  - **change** ± 9.22456E-05 ± 2.79998E-13 ± -0.00242512

- **Final Uncertainty**
  - 0.00242687
Appendix B- Mathematica Program for using the Inverse Solution Process

Input:

change directory

Directory[]
SetDirectory["c:\Documents and Settings\Mike\My Documents\school\dissertation"]; SetDirectory["fiber-optic data from shock tube tests\good data"]; import data points and set up for optimization

pts=Import["s5rldata.csv","Table"]; << Graphics`Legend` V = 0.000035* (0.000085^2); A = 0.000085^2 + 4 * 0.000085 * 0.000035; ρ = 3160; Cp = 690; To = 273.15 + 22.64; n = Length[pts]; S = 0; α = 0.0001; points = Table[0, {i, 1, n}, {j, 1, 2}];
For \([i = 1, i \leq n,]\)
\[
\text{time}[i] = (i - 1) \times 0.00001;
\]
\[
\theta[i] = e^{-\frac{Ah^2 \text{time}[i]}{V \rho C_p}} (T_0 - T_\infty^2) + T_\infty^2;
\]
\[
Y[i] = \text{pts}[[i, 1]] + T_0;
\]
\[
\text{points}[[i, 1]] = \text{time}[i];
\]
\[
\text{points}[[i, 2]] = Y[i];
\]
\[
R[i] = Y[i] / \theta[i];
\]
\[
i++;
\]
\]

**optimization**

\[
c1=0.0055 \times 10^{-3};
\]
\[
h\text{guess}=10000;
\]
\[
T\text{guess}=440;
\]
\[
zh=1;
\]
\[
c2=0.0055 \times 10^{-3};
\]
\[
h\text{lim}=150000;
\]
\[
i=1;
\]
\[
S = \left( \sum_{j=1}^{n} \left( \frac{(1 - R[j])}{(n)} \right)^zh \times (1 - R[j])^2 \right)
\]
\[
+ c1 \times (1-h\text{guess}/h^2)^2 + c2 \times (1-T\text{guess}/T_\infty^2)^2;
\]

Print[" h\text{guess}=" , h\text{guess}," T\text{guess}=" , T\text{guess}];

Print[N\text{Minimize}[\{S,\{h2\geq 0, T_\infty^2 \leq 1500, h2 \leq \text{hlim}, T_\infty^2 \geq 200\}\}, \{h2, T_\infty^2\}];

**Output:**

\[
\text{hguess} = 10000 \quad \text{Tguess} = 44
\]
\[
\{0.00116708, \{h2\rightarrow 36037.3, T_\infty^2\rightarrow 603.662\}\}