I. INTRODUCTION

India and Pakistan are two of the world’s poorest countries, yet each devotes a substantial portion of its resources to defend itself against the other. Since their independence from Britain in 1947 they have fought three wars and each accuses the other of waging proxy wars by aiding and assisting terrorists and insurgents.

Over the past thirty years, real defense spending in both countries has continued to rise (figures 1 and 2). Pakistan now spends about $3 billion per year on defense—roughly 6 percent of GDP—which accounts for about 25 percent of total government expenditure (see figures 3 and 4). Indian defense spending is now about $10 billion—approximately 2.5 percent of GDP—and accounts for about 15 percent of government spending.

Figure 1
Figure 2
These are sizeable commitments to defense for countries with staggering social problems and for governments that face serious financial difficulties—Pakistan has been in negotiations with the International Monetary Fund for emergency funding to avoid defaulting on its international obligations, while the World Bank, among others, has been warning New Delhi of an impending debt trap as interest payments consume an ever larger share of government income. What drives these expenditures? Are they internally or externally driven? If externally, how do the countries interact with each other? Do they attempt to overwhelm each other or just keep pace? Does the enormous difference in the size of the economies—India’s economy is five to six times larger than Pakistan’s—affect how each country allocates resources toward defense?

To try to answer these questions, we first introduce defense economics through a graphical approach. We then apply to Indian and Pakistani defense spending data from 1966 to 1996 five mathematical models widely used in defense spending studies. Each
The model assumes a different underlying cause or combination of causes drive defense spending decisions. Three models explore the level of defense expenditure:

- The bureaucratic model asserts that defense spending is predominately internally driven by the defense establishment and the long-term nature of defense procurement.

- In the strategic model, defense spending is primarily a response to the external threat and limited by a resource constraint.

- The combined bureaucratic and strategic model attempts to measure the relative strengths of internal and external drivers.

Two models posit how countries adjust their defense spending in response to the spending of their rivals:

- In the classic competitive arms race model—also known as a Richardson model—rival states answer each other’s increases with increases of their own in an effort to stay ahead of one another.

- In the submissive arms race model, the larger of the rival countries tries to engage the smaller power in an economically ruinous arms race, but the smaller power opts out.
In fact, the influence of the larger power on the smaller wanes as the spending gap widens.

Our methodology will be to apply each model to each country. If the model performs well, we will assume that the underlying driver of defense expenditure or change in defense expenditure is present. If the model does not perform well, we will assume the driver is absent. Our goal is not to find the single “best” model, but to see if a consistent pattern of behavior emerges for each country through the combination of the models. For example, we might expect the bureaucratic and submissive models to perform well for a country whose security spending is driven primarily by internal factors, while the strategic and Richardson models should perform well for a country that is more responsive to the actions of its rival. The submissive model should also detect if a country is being overly aggressive.

We conclude that existing models do shed light on the defense spending behaviors of the two countries, although they are by no means the final word and have only limited value for forecasting. The patterns that emerge from empirical testing of the models indicate that

- India is far more sensitive to Pakistan’s spending than Pakistan is to India’s. India is concerned with maintaining a certain level of superiority over its rival, but shows little inclination to spend Pakistan into the ground.
• Pakistan has run up against its resource constraint and Pakistani leaders have opted to spend what they feel they prudently can on defense rather than try to engage India in an arms race that they would assuredly lose.

• On the other hand, Pakistan’ defense spending bureaucracy is stronger than India’s, so that Pakistan finds it more difficult to cut defense spending than does India.

II. A GRAPHICAL INTRODUCTION TO DEFENSE ECONOMICS

To introduce the reader to defense economics, we will use a simplified version of the graphical approach first used by Murray Wolfson (1985) and popularized by Charles H. Anderton (1990). Starting in the upper left graph of figure 5, we depict the government’s income constraint and its options for allocating its income between defense goods and services and non-defense goods and services, the classic choice between “guns-or-butter.”
Whatever the government decides to allocate toward defense goods—we leave aside for a moment why the government allocates as it does—contributes to the military stockpile in the lower left hand graph. The graph of the military stockpile as a function of defense spending does not begin at the origin because there are non-military goods with which a citizenry can defend itself—rocks, bottles, hunting rifles, and the like—and rises at less than a 45-degree angle because of depreciation.

The military stockpile translates into military power, or security, in the lower right hand graph. This function is curved to reflect the diminishing marginal returns of increasing stockpile given no change in technology. A country’s first tank, for example, represents a huge jump in capability, while the acquisition of a country’s 10,001st tank has a
negligible effect. Improvements in weapons technology flatten the curve—i.e., better weapons increase power more per unit than do inferior weapons.

The upper right hand graph traces a frontier of the government’s choices between non-defense goods and national security. Wolfson and Anderton demonstrate that it is quite simple to extend the analysis by adding utility isoquants to the graph to determine the government’s optimum mix of non-defense spending and national security. Because we are using the graphical approach only as an introduction to the field, we simply note the possibilities in that approach and proceed to the more relevant topic of how an adversary affects the analysis.

In most countries, there is downward pressure on the government’s budget constraint line (taxes are tolerated, but rarely welcomed), as well as pressures to shift the “guns-or-butter” mix toward butter. Are there any counter-balancing forces that prevent the “guns” choice from going to zero? Defense economists have identified two major sources of countervailing pressures, the strength of the defense establishment to protect or expand its influence and resources (Lucier, 1979)—a topic to which we will return later—and the behavior of the country’s adversary.

Figure 6 demonstrates how the existence of a rival influences defense-spending decisions. In the figure, we present a highly stylized depiction of the Cold War, wherein the United
States government has about 2.5 times the resources of the Soviet government and both countries are on the same stockpile and technology curves.

**Cold War Arms Race**

![Cold War Arms Race Diagram](image)

Figure 6

The problem confronting decision-makers in both countries is how to manage the difference in power or security (a security gap for the smaller power, a security margin for the larger) given the consumption-security frontiers of the two countries. The larger country might choose to maintain a certain sized security margin, so it will adjust its spending in response to its rival, or it may decide to engage in economic warfare by spending at levels far beyond the capability of its adversary. The smaller power may try to keep the security gap narrow which will require it to adjust its spending in response to the larger power, or it may opt out of an arms race and spend only what it believes it can prudently afford.
Figure 7 depicts the Indo-Pakistani situation prior to the nuclear tests conducted by both countries in the spring and summer of 1998. India’s huge size advantage—a GDP and central government expenditures about 6 times that of Pakistan—is readily apparent and places Pakistani decision-makers in a serious predicament. Islamabad has no chance of matching India rupee for rupee, yet remains in danger of being overwhelmed if New Delhi ever chose to engage Pakistan in an arms spending race.

![India-Pakistan, 1995-6](image)

This predicament could explain, at least in part, Pakistan’s no longer covert nuclear weapons program. Figure 8 shows how the adoption of nuclear weapons would shift
Pakistan’s technology curve outward, giving it the ability to match or even surpass its rival with the same level of spending.

**Pakistan Obtains Nuclear Weapons**

As is well known, however, it was India that broke the subcontinent’s nuclear ambiguity with nuclear weapons tests on May 11, 1998. To finish the graphical story, we present figures 9 and 10 showing the security situation in South Asia after the Indian tests. Figure 9 demonstrates how untenable the situation had become for Pakistan after the Indian test,
India Tests First . . .

leaving it little choice but to conduct its own tests to restore a new security balance depicted in figure 10.

. . . Leaving Pakistan Little Option
We now turn to other arms race and defense spending models. Several authors have categorized arms race and defense spending models. Intrilligator (1982) presented the most thorough classification scheme, identifying eight families of analysis used in the field, including:

- Differential equations
- Decision theory/control theory
- Game theory
- Bargaining theory
- Uncertainty
- Stability theory
- Action-reaction models
- Organization theory

Intrilligator noted that the model or technique used often depended on the field of defense economics the researcher is investigating. Organization theory, for example, is most often used when exploring how various competing vested interests hammer out a defense budget. Game theory is often employed for war-fighting models, while differential equations, control theory (optimizing behavior), and action-reaction models are the most commonly used techniques for modeling arms races.
At the other end of the classification schemes, Haken Wiberg (1989) grouped arms race and defense spending models into two groups—strategic models and deterministic models. Strategic models are based on the assumption that some agent (usually the government) engages in optimizing behavior (government’s utility, national welfare, etc.) given some constraint (government revenues, GDP). Deterministic models are usually based on or derived from the arms race model of Lewis F. Richardson (1960) and are the most extensively used family of models in defense economics.

III. THE LEVEL OF DEFENSE SPENDING: BUREAUCRATIC MODELS

The simplest model of military expenditure is the bureaucratic model of Lucier (1979). The model assumes that defense spending is characterized by a high degree of inertia because entrenched bureaucracies protect their positions, bureaucracies tend to base future budget decisions on small adjustments of past spending levels, and because of the nature of defense procurement, where very large and expensive programs are necessarily stretched out over several years, even decades. Lucier’s bureaucratic model of defense spending is simply:

\[
M_t = \beta_0 + \beta_1 M_{t-1} + e
\]  

(1)

where \( M \) is the amount of military expenditure in year \( t \).
We would expect this model to perform well where militaries are in control, exercise substantial clout, or the defense establishment is particularly large. We would expect the model to perform well for both India and Pakistan, reflecting the sizeable militaries in both countries, but better for Pakistan because the military is more influential in government and society than is the Indian military, which has a strong tradition of being apolitical. Our test results indicate a large degree of inertia in both countries, with the stronger t and F statistics for Pakistan suggestive of slightly more rigidity on the Pakistani side.

For India:  
\[ M_{i(t)} = 0.163 + 1.012 M_{i(t-1)} + e \]  
\[ t\text{-statistics} \quad .55 \quad 25.63 \]
\[ p\text{-value} \quad .59 \quad \text{near 0} \]
\[ AR^2 = .96 \quad F = 656.68 \quad \text{SigF = near 0} \]

For Pakistan:  
\[ M_{p(t)} = 0.064 + 1.003M_{p(t-1)} + e \]  
\[ t\text{-statistics} \quad .86 \quad 32.28 \]
\[ p\text{-value} \quad .40 \quad \text{near 0} \]
\[ AR^2 = .97 \quad F = 1042 \quad \text{SigF = near 0} \]

Here and throughout the paper, the subscript i refers to India and p to Pakistan. Ordinary Least Squares (OLS) was used with this model and throughout the paper. P-values are the significance level of the t-statistics (the closer to 0 the better the p-score), AR$^2$ is
adjusted $R^2$ (the closer to 1 the better) and SigF is the significance of the F score (the closer to 0 the better). The data is from official Indian and Pakistani sources for the fiscal years 1966/67 to 1996/97. India’s fiscal year begins on 1 April and Pakistan’s on 1 July. Current defense spending was deflated with the respective Wholesale Price Index (WPI) for each country following Lakshmi (1988) who argues that governments are more wholesale than retail purchasers of defense goods, and converted to dollars in the base year of 1981/82, to make the numbers more familiar for the reader.

IV. THE LEVEL OF DEFENSE SPENDING: STRATEGIC MODELS

Strategic models are descended from the work of Mancur Olsen and Richard Zeckhauser (1966), who were interested in burden sharing among countries in an alliance. In particular, they wanted to determine if some countries—particularly the small countries within NATO—were prone to free riding. They treated defense as an international public good and derived empirically testable propositions that defense expenditures depended on the relative prices of defense and nondefense goods, national income, the level of defense spending by allies, and the perceived threat. Following Sandler and Hartley (1995), we derive a simple strategic model.

The strategic model begins with the objective utility function

$$U = U(y, q, Q, T),$$

(4)
where \( y \) is nondefense goods, \( q \) is defense goods, \( Q \) is the contribution of allies to the country’s security, and \( T \) is its perceived threat. Neither India nor Pakistan is a member of any formal alliance, but Sandler and Hartley (1995) have shown that the Olsen-Zeckhauser model can be applied to countries without allies by reducing to zero the allied contribution to the subject country’s security. In the case of India and Pakistan, we reduce \( Q \) to zero and eliminate it from further consideration. The objective function is subject to the budget constraint

\[
I = p_y y + p_q q
\]  

(5)

where \( I \) is income, usually measured in GDP or government revenue terms, and \( p_y \) and \( p_q \) are the prices of nondefense and of defense goods respectively. Maximizing the utility function subject to the constraint yields the following first order conditions:

\[
\frac{\delta U}{\delta y} - \lambda p_y = 0 \tag{6}
\]

\[
\frac{\delta U}{\delta q} - \lambda p_q = 0 \tag{7}
\]

\[
I - p_y - p_q = 0 \tag{8}
\]

The first order conditions yield the following Marshallian demand function for defense goods:
which is the basis for econometric models in the form:

\[ M_{a(t)} = \beta_0 + \beta_1 Y(t) + \beta_2 \text{PRICE} + \beta_3 \text{THREAT}_{(t-1)} + e \]  

(10)

where \( M \) is military expenditure (also sometimes noted as DE for defense expenditure), \( Y \) is GDP, \( \text{PRICE} \) is the price differential between nondefense and defense goods, and \( \text{THREAT} \) is the military expenditure of the rival. If a country has allies, then the effect of the allied spending—the “Q” in equation (4)—on the spending decision of the country being studied would be included as a variable usually called SPILL for the “spill-in effect.”

In practice, \( \text{PRICE} \) is difficult to obtain and can be omitted if the researcher is confident that the price of defense and nondefense goods are inflating at about the same rate (Sandler and Hartley 1995). A vector \( J \) of additional key variables can also be added. \( J \) can contain dummy variables for such occurrences as war, military rule, or other decision variables that might affect a government’s decision to allocate resources toward or away from defense. The most common specification is linear, but double log has also been used (Murdoch and Sandler, 1990).
Sandler and Hartley (1995) present an extensive list of studies using the strategic model and its variants. We reproduce two of the results they cite to demonstrate the variety of techniques and variables used on the basic strategic theme. Murdoch and Sandler (1984) used seemingly unrelated regressions (SUR) for the US from 1961 to 1979 and obtained:

\[
M(t) = 67.322 + 0.035Y(t) - 0.236SPILL_{(t-1)} - 0.472DSPILL_{(t-1)}
\]

\[
t-scores\quad (3.78) \quad (2.14) \quad (-0.38) \quad (-0.376)
\]

where \(SPILL\) represents the amount of spending by US allies. \(D\) is a dummy equal to 1 for 1974-1979 and 0 for 1961-1973.

Fritz-Asmus and Zimmerman (1990) used ordinary least squares (OLS) for West Germany from 1961 to 1987:

\[
M(t) = 10.8504 - 0.0064Y(t) + 0.5778FSPILL_{(t-1)} - 0.5117D_{SPILL} - 0.0958NATOSPILL_{(t-1)} + 0.0495DNATOSPILL_{(t-1)} + 0.211THREAT_{(t-1)} + 0.9056POL
\]

\[
t-scores\quad (6.80) \quad (-1.31) \quad (2.65) \quad (-3.01)
\]

\[
(-7.44) \quad (2.65) \quad (6.78) \quad (1.21)
\]

where \(FSPILL\) is French military spending, \(D = 1\) for 1974 to 1987 and 0 otherwise, \(NATOSPILL\) is non-German military spending within NATO, and \(POL\) is a dummy
variable for the party in power and equals 1 for 1960 to 1970 and from 1984 to 1987, otherwise 0.

For India and Pakistan, we specified a linear function with national income (Y) and the lagged spending of the rival which is equivalent to the THREAT variable in the above equations. The results using OLS on data from 1966 to 1996 for each country were:

For India: \[ Mi(t) = .900 + .007Yi(t) + 2.155Mp(t-1) \] \hspace{1cm} (11)

- t-statistics: 2.93 1.69 5.37
- p-value: .01 .102 near 0
- AR^2 = .95  F = 264.83  SigF = near 0

For Pakistan: \[ Mp(t) = -.157 + .053Yp(t) + .104Mi(t-1) \] \hspace{1cm} (12)

- t-statistics: -1.54 5.74 2.33
- p-value: .13 near 0 .03
- AR^2 = .96  F = 342.31  SigF = near 0

The model performs well for both countries and some differences in behavior begin to emerge. The coefficient values and t-statistics of the rival’s lagged spending variable in both equations (11) and (12) suggests India is more concerned with Pakistani spending than Pakistan is with India’s. By contrast, Pakistan appears to be more influenced by its income level than is India. A possible explanation is that Pakistan, as the smaller power,
allocates as much as it feels it prudently can to defense regardless of what India does since it cannot possibly match Indian spending rupee for rupee. India, meanwhile, is very concerned with maintaining its position as the dominant regional partner and responds very strongly to Pakistani spending.

We can combine the bureaucratic and strategic models by adding the country’s own lagged spending to equation (10) as discussed in Sandler and Hartley (1995) and as done by Looney and Mehay (1990) for the US. The model then takes the form:

\[ M_a(t) = \beta_0 + \beta_1 Y + \beta_2 \text{PRICE} + \beta_3 \text{THREAT}_{(t-1)} + M_a(t-1) + e \quad a = (i,p) \quad (13) \]

For India and Pakistan we drop the PRICE variable as in (11) and (12) and present the he results for India and Pakistan from 1966 to 1996:

**Bureaucratic and Strategic Model: India**

\[ M_i(t) = .406 + 1.024 M_p(t-1) + .598 M_i(t-1) + .002 Y_i(t) \quad (14) \]

<table>
<thead>
<tr>
<th>t</th>
<th>1.41</th>
<th>2.27</th>
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<th>.47</th>
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<tr>
<td>p</td>
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<td>.032</td>
<td>.001</td>
<td>.642</td>
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</tbody>
</table>

\[ \text{AR}^2 = .96 \quad F = 261 \quad \text{SigF} = \rightarrow 0 \]
Bureaucratic and Strategic Model: Pakistan

\[ Mp(t) = .013\ -\ .008Mi(t-1) + .761Mp(t-1) + .020Yp(t) \quad (15) \]

<table>
<thead>
<tr>
<th>t</th>
<th>p</th>
<th>AR^2</th>
<th>F</th>
<th>SigF</th>
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<td>.14</td>
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<td>.97</td>
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<td>1.80</td>
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If we look at the two resulting equations together and highlight the parameters with reasonably good t-scores (p values of .2 or less, see Deger and Senn [1990] who argue that adopting a more liberal statistical standard is advisable when dealing with third world countries) we can continue to refine the story that began to emerge in the bureaucratic and the strategic models.

\[ M_i = .406 + .598M_i(t-1) + 1.024M_p + .002Y_{i(t)} \quad (14) \]

\[ M_p = .013 + .761M_p(t-1) - .008M_i + .020Y_{p(t)} \quad (15) \]

Both countries have significant defense bureaucracies reflected in the lagged spending variable, but Pakistan’s is much stronger. GDP also plays a significant role in Pakistan’s
defense spending decisions, suggesting Pakistan may be bumping up against a resource constraint. India remains more strongly influenced by the spending of its rival as in the strategic model. The large, significant coefficient value for its rival’s lagged spending suggests a determination on the part of New Delhi to maintain a certain size advantage over Pakistan, regardless of how much Islamabad spends.

A major critique of the bureaucratic and strategic models is that they lack any interaction between the rivals. For this we turn to arms race models.

V. CHANGES IN DEFENSE SPENDING: THE RICHARDSON COMPETITIVE ARMS RACE MODEL

The most widely used arms race model in defense economics is the simultaneous differential equation arms race model developed in the 1930s by British mathematician Louis F. Richardson (1960):

\[
M_A' = kM_B - aM_A + g
\]

\[
M_B' = lM_A - bM_B + h
\]

where \(M_j'\) is the instantaneous change in defense spending of country \(j\) \((j = A, B)\) with respect to time \((dM_j/dt)\) as a function of its reaction coefficient \((k\) and \(l)\) times the spending of its rival, minus the fatigue coefficient \((a\) and \(b)\) times its own spending, plus
a grievance factor \((g\text{ and }h)\) that measures the general level of hostility that one country has for another.

For empirical work the model is generally transformed into difference equations:

\[
\Delta M_A = kM_A t - aM_A t + g + e \tag{18}
\]

\[
\Delta M_B = lM_B t - bM_B t + h + e \tag{19}
\]

where \(\Delta M\) is the decision made this year to change defense spending for next year, or \(\Delta M_t + M_t = M_{t+1}\). The grievance factors \((g\text{ and }h)\) are the intercept terms. Results for India and Pakistan with the Richardson model using OLS were:

**Richardson Model: India**

\[
\Delta M_i(t) = -0.395M_i(t) + 1.145M_p(t) + 0.512 \\
\text{AR}^2 = 0.17 \quad F = 4.03 \quad \text{SigF} = 0.03
\]
Richardson Model: Pakistan

\[ \Delta M_p(t) = -0.061 M_i(t) + 0.032 M_p(t) + 0.123 \]

<table>
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AR^2 = -0.06  F = 0.13  Sig F = 0.88

If we compare the results of the equations as before--

\[ \Delta M_i(t) = 1.145M_i(t) - 0.395M_i(t) + 0.512 + e \]  \hspace{1cm} (20)

\[ \Delta M_p(t) = -0.061M_i(t) + 0.032M_i(t) + 0.123 + e \]  \hspace{1cm} (21)

--we find Richardson provides a reasonable fit for India with signs as predicted, two significant t-statistics and a significant F statistic. The AR^2 is very low, but we are dealing with very small numbers for the dependent variable relative to the independent variables. Small budget decisions each year—such as a decision to upgrade aircraft or pay a bonus to military officers—can have a major influence on the size of the change and we can see that the model is not capturing every one of these decisions.

Nevertheless, in a visual comparison of the models in-sample predictions with the observed results shows the model moves with the observation although it generally
underestimates the size of the change. The Richardson model for India supports the results of the strategic and bureaucratic-strategic model that India reacts to Pakistani spending. The Richardson model adds that there is a downward pressure on Indian defense spending from internal sources. This inference agrees with the results of Deger and Senn (1990) who found Indian defense spending limited by internal political pressures.

The Richardson model is a very poor fit for Pakistan, however. In fact, the negative AR², the lack of significant t-statistics, and the huge SigF value indicate the Richardson model is not at all appropriate for Pakistan. In the visual comparison of the predictions with the results, the flat line predictions suggests that the model is not picking up, or is not sensitive to, what causes Pakistani spending to change. If we treat the Richardson model as a test for competitive arms race behavior, then the poor performance of the Richardson model indicates that Pakistan in not engaged in competitive arms race behavior, a conclusion supported by the results of the bureaucratic and strategic models.

VI. CHANGES IN DEFENSE SPENDING: EMULATION, RIVALRY, AND SUBMISSIVE MODELS

If we would like to explore further patterns of behavior other than the competitive “tit-for-tat” type of arms race asserted in the Richardson model, we need to turn to the Richardson variants. Richardson and other arms race theoreticians have developed three
main variants of the basic Richardson model: the emulative model, the rivalry model, and the submissiveness model. The models are described below:

The Emulation Model: In this model, the countries react to the difference between their levels of spending in an effort to retain parity.

\[ \Delta M_A = k(M_B - M_A) - aM_A + g \]  
\[ \Delta M_B = l(M_A - M_B) - bM_B + h \]

If we designate the larger power as country A, then the gap term \((M_B - M_A)\) is negative, and A tends to throttle back its defense spending the larger the gap. For the smaller power, country B, the gap term \((M_A - M_B)\) is positive, so that B increases spending the larger the gap.

Rivalry: In this model, both countries react to the more aggressive power, country B. Sandler and Hartley (1995) suggest the gap coefficients \((m \text{ and } n)\) should be viewed as modifying the reaction coefficients \((k \text{ and } l)\), but Wolfson, who used this model extensively in his studies of the Cold War (Wolfson 1968, 1990) made no such claim.

\[ M_A(t) = m(M_B(t-1) - M_A(t-1)) + aM_A(t-1) + kM_B(t-1) + g \]  
\[ M_B(t) = n(M_B(t-1) - M_A(t-1)) + bM_B(t-1) + lM_A(t-1) + h \]
Submissive: In this model used by Zinnes and Gillespie (1973) in their study of the Cold War, the smaller power reacts less to the larger power the greater the gap in spending, while the larger power is more aggressive than in the standard Richardson model.

\[
\Delta M_a = [1-\phi[M_b - M_a]^p]kM_a - aM_a + g \quad (26)
\]

\[
\Delta M_b = [1-\gamma[M_a - M_b]^p]M_a - bM_b + h \quad (27)
\]

Although important theoretically, the empirical forms of the emulation and rivalry models, unfortunately, are observationally equivalent to the basic Richardson model.

For the emulation model

\[
\Delta M_a = k(M_b - M_a) - aM_a + g \quad (28)
\]

\[
\Delta M_b = kM_b - kM_a - aM_a + g \quad (29)
\]

\[
\Delta M_a = kM_b - (k + a)M_a + g \quad (30)
\]

\[
\Delta M_a = kM_b - mM_a + g \quad (31)
\]

which is born out by the empirical results. For India—

Emulation: \[\Delta M_t = 1.145(M_r - M_i)t + 0.749M_i + 0.512 \quad (32)\]

Richardson: \[\Delta M_t = 1.145 M_i - 0.395M_i + 0.512 \quad (14)\]
and all statistical inference tests are identical with the Richardson results. The prediction that in the emulation model the coefficient for fatigue term \(m, .749\) is the sum of the fatigue and reaction coefficients of the Richardson model \(a + k, 1.145 - .395\) is borne out empirically.

Similarly for Pakistan:

**Emulation:**
\[
\Delta M_P t = -.016(M_I - M_P) t + .016M_P t + .123
\]  
\[
(33)
\]

**Richardson:**
\[
\Delta M_P t = -.061 M_I t + .032M_P t + .123
\]  
\[
(15)
\]

For the rivalry model

\[
M_a t = k(M_a t - M_a t) + aM_a t + k'M_a t + g
\]  
\[
(34)
\]

\[
M_s t = kM_s t - kM_s t + aM_s t + k'M_s t + g
\]  
\[
(35)
\]

\[
M_s t = kM_s t + k'M_s t - kM_s t + aM_s t + g
\]  
\[
(36)
\]

\[
M_a t = (k + k')M_a t - (k + a)M_a t + g
\]  
\[
(37)
\]

\[
M_a t = cM_a t - dM_a t + g
\]  
\[
(38)
\]

As a result, we did not test the rivalry model, and Sandler and Hartley (1995) note that the rivalry model has never been empirically tested.
The empirical form of the submissive model, however, does not reduce to a basic Richardson equation.

\[ \Delta M_A = [1 - \phi(M_B - M_A)]^g k M_B - a M_A + g \]  
(40)

\[ \Delta M_A = k M_B - \phi k M_B (M_B - M_A) - a M_A + g \]  
(41)

\[ \Delta M_A = k M_B - \phi k (M_B^2 - M_A M_B) - a M_A + g \]  
(42)

\[ \Delta M_A = k M_B - m (M_B^2 - M_A M_B) - a M_A + g \]  
(43)

with a mirror equation for \( \Delta M_B \).

Differentiating \( \Delta M_A \) with respect to \( M_B \) yields:

\[ \frac{\delta \Delta M_A}{\delta M_B} = k - m(2M_B - M_A), \text{ or} \]  
(44)

\[ \frac{\delta \Delta M_A}{\delta M_B} = k + m(M_A - 2M_B) \]  
(45)

If A is the larger power, the submissive model says it will increase spending the greater its advantage. The results for India are:
If we compare these results with those of the Richardson model (equation 14) we note slightly improved AR2 and SigF results, but only one significant t-statistic—again, the spending of the rival—compared to all three in the Richardson model. Moreover, the sign of the gap term is opposite what the model predicts (equation 43) if the larger country is more aggressive than in a Richardson competitive arms race. The results of the model suggest that India is not overly aggressive in its arms spending.

If A is the smaller power, it will react less to B the greater B’s advantage.

The empirical results for India and Pakistan are:
The submissive model fits Pakistan better than the Richardson model. The AR2 is still abysmal .03, but SigF has improved from .88 to .3, the t-statistics are all acceptable, the signs are as predicted, and visual inspection shows the predictions of the model move generally in the direction of the observed results. The results also conform with what we would be expected from the results of the bureaucratic and strategic model (equation 15) that suggested that Pakistan spends what it can, given its resources, but does not try to match India.

VII. CRITIQUES OF ARMS RACE MODELS

Given our statistical results, it should come as no surprise, that the criticisms of the Richardson model and its variants are legion. Several authors note that empirical results using Richardson are almost always disappointing. Anderton (1985) provides the longest and harshest criticism. He believes the Richardson models are too deterministic, are poorly grounded in economic theory, and are far too imprecise for any realistic policy analysis because they use the fatigue term as a catchall for all the political and economic variables surrounding defense expenditure decisions. Moreover, Richardson lends itself to extended efforts at curve fitting and data-mining (for example, see Smith, 1989).

Richardson and strategic models both have severe econometric shortcomings. Sandler and Hartley (1995) state that autocorrelation and multicolinearity plague all of these
models. Stoll (1982) performed a Monte Carlo experiment in which he created two fictitious dyads. In one, the spending was driven purely by internal factors, while in the second, the two countries reacted solely to one another. He concluded that because arms expenditures move together in either case, multicolinearity will be present not as a “data problem,” but as a fact of life. In his empirical results, he found the Richardson model had difficulty apportioning variance between the internal and external variables in both cases, although it did better the more externally driven the dyad was. Our own results seem to support this conclusion. Richardson performed reasonably well for India, which we judge the more reactive, but poorly for Pakistan which we consider more internally driven.

Aware of these criticisms and recognizing that none of the models were producing results could withstand extensive clean-up procedures, we decided to accept the regressions presented in this paper as a first pass to identify the broad outlines of arms spending policies in the region. In an early phase of our research, however, we did calculate a Durbin-Watson statistic and used two stage least squares for one of the data runs which we produce below.

The regression was run on a large strategic model using nominal Indian rupees and which included dummy variables for the Afghan war (AFWAR), US participation in the Afghan war (AFWARUS), periods of military rule in Pakistan (MLAW), and a time variable. The Durbin-Watson statistic was calculated only for India.
Indian Defense Expenditure, nominal Indian rupees, 1967-1995

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>t-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-7.2071</td>
<td>-0.9085</td>
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<tr>
<td>GDPi</td>
<td>-0.0095</td>
<td>-1.2507</td>
</tr>
<tr>
<td>Mi(t-1)</td>
<td>0.94528</td>
<td>5.74191</td>
</tr>
<tr>
<td>Mp</td>
<td>1.11174</td>
<td>2.81572</td>
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<tr>
<td>MLAW</td>
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<td>-1.8543</td>
</tr>
<tr>
<td>TIME</td>
<td>0.43896</td>
<td>1.30968</td>
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AR2 = .99
F = 1273.58 near 0

AR2 = .99
F = 938.75 near 0

Durban-Watson statistic = 2.959, exceeds Du
We applied two stage least squares to a similar model for Pakistan

Pakistan Defense Spending, Nominal Indian Rupees, 1968-1995

**OLS**

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
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<th>P-value</th>
</tr>
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<tbody>
<tr>
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<td>Mi(t-1)</td>
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<td>Mp(t-1)</td>
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<td>AFWARU</td>
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<td>AFWAR</td>
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<tr>
<td>MLAW</td>
<td>-0.4714</td>
<td>-0.4146</td>
<td>0.68309</td>
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<td>TIME</td>
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<tr>
<td>GDPp</td>
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<td>2.99615</td>
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**2SLS**

<table>
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<th>Coefficient</th>
<th>t Stat</th>
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<td>AFWARU</td>
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<td>MLAW</td>
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<td>TIME</td>
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<td>GDPp</td>
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</table>

**AR2 = .99**

F = 668.27 SigF = near 0

**AR2 = .99**

F = 742.0 SigF = near 0

In both cases there was the expected degradation the statistical inference scores, but there was no material change in either story. Since we are more concerned with the broad thrust of behavior rather than the precise values of the coefficients, we judged corrective measures to OLS procedures to be not worth the effort for our purposes.
Despite its problems, Richardson models remain the standard arms race modeling tools, and have their defenders. Intrilligator and Brito (1989) demonstrate that Richardson equations can be derived from models with impeccable theoretical credentials given a few reasonable assumptions, including the stock alignment models of Boulding (1962), Intrilligator (1964), and McGuire (1977), the optimizing normative models of Brito (1972) and Simeon and Gruz (1975), and the differential game model of Gillespie (1976,1977).

Majeski and Jones (1981) were puzzled by the poor results of Richardson models because they believe the underlying logic to be so persuasive. Their study concluded that researchers might be forcing the Richardson model into bilateral relationships that are not, in fact, competitive arms races. This inference seems to be borne out in this study, where we found that Pakistan is not sensitive to Indian defense spending.

VIII. CONCLUSIONS

After applying eight models to several countries, Hollist (1977) found that no single model for defense spending was appropriate for all situations, but that different models incorporating different mixes of variables and forms were appropriate in different empirical contexts. Rather than searching for the one universal model--the Holy Grail of the model builders of the 1970s and 1980s--he concluded that smaller models designed to
detect specific behaviors probably would be more useful, especially in a multi-polar world.

Going beyond Hollist, we believe that applying several models to a single country or dyad has value in uncovering the underlying dynamics.

- In the case of India and Pakistan, we first applied the bureaucratic and strategic models and found India sensitive to Pakistani spending, both countries to have substantial bureaucratic inertia in their spending, and Pakistan to be resource limited.

- The Richardson model gave strong evidence that India behaved as if it were in a competitive arms race with Pakistan, but gave no indication that Pakistan was not influenced by Indian spending.

- The submissive model gave indicated that Pakistan was not in a “rupee-for-rupee” arms race with India, while suggesting that India was not out to overwhelm its rival, only to maintain a cushion.
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