Appendix V

Cracked Section Properties of the Pier Cap Beam with the Moment-Curvature Method and the ACI Equation for the Prestressed Concrete Girder Bridge

Figure V-1. The actual pier cap beam cross section [Maday, 2002]. All dimensions are in mm, and the 1:20 scale is no longer correct.
\[ T = C_C + C_S \]
\[ A_S f_y = b \int_{0}^{kd} f_c dy + A_s' f_s' \]
\[ A_s f_y - A_s' f_s' = b \int_{0}^{kd} f_c dy \]
Assume the compression steel will not yield:

\[
\frac{\varepsilon_c}{kd} = \frac{\varepsilon_y}{d-kd} \iff \varepsilon_c = \varepsilon_y \frac{kd}{d-kd}
\]

\[
\frac{\varepsilon_s'}{kd-d'} = \frac{\varepsilon_y}{d-kd} \iff \varepsilon_s' = \varepsilon_y \frac{kd-d'}{d-kd}
\]

\[
\varepsilon_y = \frac{f_y}{E_s} = \frac{420\text{MPa}}{200,000\text{MPa}} = 0.00210
\]

\[
\varepsilon_s' = 0.00210 \left( \frac{kd-100\text{mm}}{1576\text{mm-kd}} \right)
\]

\[
\varepsilon_0 = \frac{2f_c'}{E_c} = \frac{2(25\text{MPa})}{23500\text{MPa}} = 0.00213
\]

\[
A_s f_y - A_s' E_s \varepsilon_s' = b f_c' \left[ \frac{\varepsilon_y}{\varepsilon_0} - \frac{1}{3} \frac{\varepsilon_y^2}{\varepsilon_0^2} \frac{(kd)^3}{(d-kd)^2} \right]
\]
By plugging in these following values to the equation above,

\[ A_s = 6555 \text{mm}^2 (8-\#10\text{bars}) \]
\[ f_y = 420 \text{MPa} \]
\[ A_s' = 3277 \text{mm}^2 (4-\#10\text{bars}) \]
\[ E_s = 200,000 \text{MPa} \]
\[ \varepsilon_y' = 0.00210 \left( \frac{kd-100 \text{mm}}{1576 \text{mm}-kd} \right) \]
\[ b = 1270 \text{mm} \]
\[ f_{c'} = 25 \text{N/mm}^2 \]
\[ \varepsilon_y = 0.00210 \]
\[ \varepsilon_0 = 0.00213 \]
\[ d = 1576 \text{mm} \]

then a third-degree equation in terms of kd is obtained. The solutions of the third-degree equation are

\[ kd = 1189 \text{ mm or } kd = 330 \text{ mm or } kd = -432 \text{ mm} \]

kd = -432 mm is obviously ruled out, and kd = 1189 mm is ruled out because the reinforcing area at the bottom is less than that at the top, therefore kd is supposed to be less than half of 1781 mm, which is equal to 890.5 mm. Thus kd = 330 mm is the only possible solution.

Now the initial assumption that the compression steel will not yield must be checked:

\[ \varepsilon_y' = \varepsilon_y \left( \frac{kd-d'}{d-kd} \right) = 0.00210 \left( \frac{330 \text{mm}-100 \text{mm}}{1576 \text{mm}-330 \text{mm}} \right) = 0.000388 < \varepsilon_y = 0.00210 \]

Thus the initial assumption is correct.

Now \( \varepsilon_{c \max} \) must be checked to see if it is less than \( \varepsilon_0 \). If it is, then the stress block will have a parabolic shape.
\[
\varepsilon_{c,max} = \varepsilon_y \frac{kd}{d-kd} = 0.00210 \left( \frac{330\,mm}{1576\,mm - 330\,mm} \right) = 0.000556 < \varepsilon_0 = 0.00213
\]

Thus the stress block will be parabolic. The curvature at the yield point is

\[
\phi_y = \frac{\varepsilon_{c,max}}{kd} = \frac{0.000556}{330\,mm} = 1.6848 \times 10^{-6} / mm
\]

\[
\begin{align*}
\bar{y} &= -\frac{2}{3} \left( \frac{\varepsilon_{c,max}}{\varepsilon_0} \right) (kd) - \frac{1}{4} \left( \frac{\varepsilon_{c,max}}{\varepsilon_0} \right)^2 (kd) \\
&= -\frac{2}{3} \left( \frac{0.000556}{0.00213} \right) (330\,mm) - \frac{1}{4} \left( \frac{0.000556}{0.00213} \right)^2 (330\,mm) \\
&= 217\,mm
\end{align*}
\]
\[ C_c = b \int_0^{kd} f_c \, dy \]
\[ = bf_c \left[ \left( \frac{\varepsilon_{c, \text{max}}}{\varepsilon_0} \right) (kd) - \frac{1}{3} \left( \frac{\varepsilon_{c, \text{max}}}{\varepsilon_0} \right)^2 (kd) \right] \]
\[ = (1270 \text{mm}) (25N/mm^2) \left[ \left( \frac{0.000556}{0.00213} \right) (330 \text{mm}) - \frac{1}{3} \left( \frac{0.000556}{0.00213} \right)^2 (330 \text{mm}) \right] \]
\[ = 2,500,000N \]

\[ M_y = (d - kd + \bar{y}) C_c + C_y (d - d') \]
\[ = (1576mm - 330mm + 217mm)(2,500,000N) + (254,000N)(1576mm - 100mm) \]
\[ = 4.03 \times 10^9 Nmm \]

\[ EI_e = \frac{M_y}{\phi_y} = 4.03 \times 10^9 Nmm \times \frac{1.68 \times 10^{-6} / mm}{\phi_y} = 2.39 \times 10^{15} Nmm^2 \]

\[ I_c = 1.02 \times 10^{11} mm^4 \]

\[ \frac{I_c}{I_g} = \frac{1.02 \times 10^{11} mm^4}{\frac{1}{12} (1270mm)(1781mm)^3} = 0.170 \]

**ACI Equation Method**

\[ f_c' = 25N/mm^2 = 3626 psi \]
\[ f_c = 7.5\sqrt{f_c'} = 7.5\sqrt{3626 psi} = 452 psi = 3.12 N/mm^2 \]

\[ M_{cr} = \frac{f_c I}{y} \]
\[ M_{cr} = \frac{\left(3.12 N/mm^2\right) \left(\frac{1}{12}\right) (1270mm)(1781mm)^3}{890.5mm} \]
\[ = 2.09 \times 10^9 Nmm \]

This cracking moment must be compared with the maximum positive and negative moments in the pier cap beam, which are given in Table V-1.
Table V-1. The calculation for the maximum positive and negative moments in the pier cap beam.

<table>
<thead>
<tr>
<th>Load Type</th>
<th>Maximum Positive</th>
<th>Maximum Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Truck</td>
<td>4.03E+07</td>
<td>-1.13E+07</td>
</tr>
<tr>
<td>Design Tandem</td>
<td>2.75E+07</td>
<td>-7.68E+06</td>
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<tr>
<td>Two Design Trucks</td>
<td>7.02E+07</td>
<td>-1.96E+07</td>
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<td>Lane</td>
<td>5.51E+07</td>
<td>-1.54E+07</td>
</tr>
<tr>
<td>Controlling Load</td>
<td>7.02E+07</td>
<td>-1.96E+07</td>
</tr>
<tr>
<td>Live Load Effects</td>
<td>3.41E+08</td>
<td>-9.51E+07</td>
</tr>
<tr>
<td>Dead Load Effects</td>
<td>1.45E+09</td>
<td>-4.35E+08</td>
</tr>
<tr>
<td>LL+DL Effects</td>
<td>1.62E+09</td>
<td>-4.82E+08</td>
</tr>
</tbody>
</table>

Notes: Moments in Nmm.

\[ M_{cr} = 2.09 \times 10^9 \, Nmm > M_a = 1.62 \times 10^9 \, Nmm \] for the maximum positive moment.

\[ M_{cr} = 2.09 \times 10^9 \, Nmm > M_a = -4.82 \times 10^9 \, Nmm \] for the maximum negative moment.

Therefore according to the ACI Equation method,

\[ I_c = I_g \]

\[ = \frac{1}{12} (1270 \, mm)(1781 \, mm)^3 \]

\[ = 5.98 \times 10^{11} \, mm^4 \]