Chapter 3

Nonlinear Aspects of “Hard” Flutter of an HSCT Flexible Semispan Model

In this chapter, nonlinear aspects of a “hard” flutter incident that lead to the loss of a High Speed Civil Transport Flexible Semispan Model is analyzed using wavelet-based tools. An overview of the model is presented. Nonlinearities encountered in sweep forced vibration testings are identified. Dissimilar frequencies in the pressure profiles near the leading and trailing edges of the wing are identified. Intermittent quadratic coupling between the shock motions and the structural vibrations are identified using the wavelet-based bicoherence. Particularly, frequencies of the wing tip acceleration and shock motion are shown to be coupled in a $2 : 1$ ratio, which eventually lead to the loss of the model.

3.1 Analysis of High Speed Civil Transport Flexible Semispan Model

The Flexible Semispan Model (FSM) cost several hundred thousand dollars and several hundred person hours in design and assembly time. In addition, extensive use of the NASA Transonic Dynamics Tunnel (TDT), using a heavy gas, under the guidance of NASA engineers lead to unique, diverse, and comprehensive sets of data. These included acceleration, strain, and pressure signals, under subsonic, transonic, and supersonic conditions, as well as forced and unforced runs. The
extensive instrumentation provided a detailed description of the pressure and the structural motions of the model during all these runs. Over 800 data records, representing many hours of tunnel time, were examined in this research effort.

### 3.2 Model Description

The TDT is specially configured for flutter testing, with excellent model visibility from the control room and a rapid tunnel shutdown capability for model safety (bypass valves). A detailed diagram of the FSM is presented in Figure 3.1. The FSM was not intended to be a flutter clearance model but rather a model that would exhibit an HSCT-like flutter mechanism within the range of operation of the TDT. In order to induce flutter at around 200 psf, a 2.2 lb. mass was added to the aft wing tip section. This mass was fabricated out of tungsten and bonded into the outboard removable trailing-edge section of the FSM. In order to handle the added stress of the additional mass, additional local strengthening and reinforcements of the attachment surface between the main wing box and the outboard removable trailing-edge section were implemented [33].

The instrumentation layout consisted of 131 in situ unsteady pressure transducers located at the 10, 30, 60, and 95% span stations. In order to accommodate pressure instrumentation at the wing tip of the model, the original reference H airfoil thickness was increased to a constant four-percent thickness over the entire wing span. The leading and trailing edges were removable in order to access pressure instrumentation in those regions. Channels were carved into the foam core to accommodate the wiring for the instrumentation. Specially designed pressure transducer holders were used to eliminate any leakage around the transducer and to provide easy access to the transducers. The instrumentation also included 14 accelerometers installed throughout the wing. The FSM was instrumented with three bending strain gages and one torsion strain gage. All gages were located at mid-span, a region of high stress concentrations. One bending strain gage was placed near the leading edge and will be referred to as the forward strain gage or FWD. Another was placed near the trailing edge and will be referred to as the aft strain gage or AFT. The third was placed at mid-chord and will be referred to as the middle strain gage or MID. The torsion strain gage was placed at mid-chord.

A summary of the response characteristics under varying Mach number and dynamic pressure...
Figure 3.1: Flexible semi-span model (FSM) with instrumentation [33]

is presented in Figure 3.2 [33]. Clearly, the tests were performed for subsonic, transonic, and supersonic flow conditions. Numerical analysis, using MSC/NASTRAN as a linear aeroelastic solver, predicted a flutter boundary along the diagonal line marked “Analysis” [55]. The model was mounted on a turntable which was capable of harmonic forcing. Forced vibration tests were conducted at the conditions indicated by each of the “□”’s in the figure. The response of various other test conditions are indicated on the figure in two places. Increased response was noted when the test conditions traversed the shaded region between $M = 0.85$ and $M = 0.95$ and extending up to a dynamic pressure of 210 psf. Finally, a small region of significantly increased response is marked as the “Chimney” region around $M = 0.98$ with a dynamic pressure between $q = 160$ psf and $q = 245$ psf. At the top of the “Chimney” region, $M = 0.979$ and $q = 245.80$ psf, the model was lost due to the sudden onset of “hard” flutter.

### 3.3 Prognosis of Nonlinearity

Forced response tests were performed to investigate the harmonic response of the model under different operating conditions. Two different forcing schemes were used to excite the model. In one scheme, referred to as “sweep” tests, the forcing signal was a swept sinusoid with constant amplitude between .5° and 1.25° depending on the test. The instantaneous frequency of the signal was swept between either $5Hz - 25Hz$ or $5Hz - 30Hz$. In the second scheme the forcing signal consisted of a sinusoid with a constant frequency around $12Hz$. This scheme is referred to as a “dwell” test. This was used to force the model near one of its natural frequencies.

Subsonic, transonic, and supersonic tunnel conditions were analyzed in this work. The tunnel
conditions and excitation signals for the analyzed runs are presented in Table 3.1. The analysis of these signals consisted of determining the magnitude of the wavelet coefficients for each of the three wing tip accelerometers. The results are presented in Figures 3.3 - 3.8. For “sweep” experiments, three white lines are superimposed on the plots of the magnitude of the wavelet coefficients. The middle line represents the instantaneous forcing frequency, the top line represents twice the forcing frequency, and the bottom line represents half the forcing frequency. These lines are based on descriptions accompanying the data since the actual forcing functions were not made available. Nonlinearities are indicated when a local maximum in the model’s response coincides with either the top line (double the forcing frequency), or the bottom line (half the forcing frequency).

The magnitudes of the wavelet coefficients of the wing tip accelerations in Run 911 ($M = 0.7$ and $q = 162.34$ psf) from the FWD, MID, and AFT locations are shown in Figure 3.3. A swept sinusoid, from 5 to 25 Hz, with an amplitude of 0.5° was used to excite the model, and is superimposed on the wavelet coefficients. A strong quadratic nonlinearity is evident between .6 seconds and 2.5 seconds. As the excitation frequency varied between $5.4Hz$ to $6.6Hz$ a significant increase in response is observed over double that frequency, between $10.8Hz$ and $13.3Hz$. This can be seen in the plot.
Table 3.1: Mach number, dynamic pressure, and forcing signal for several forced excitation runs of the HSCT-FSM

<table>
<thead>
<tr>
<th>Run</th>
<th>Mach</th>
<th>Dynamic Pressure (psf)</th>
<th>Signal</th>
<th>Amp. (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>911</td>
<td>0.7</td>
<td>162.34</td>
<td>Sweep 5Hz − 25Hz</td>
<td>0.5</td>
</tr>
<tr>
<td>912</td>
<td>0.7</td>
<td>162.73</td>
<td>Sweep 5Hz − 25Hz</td>
<td>0.75</td>
</tr>
<tr>
<td>914</td>
<td>0.7</td>
<td>162.60</td>
<td>Dwell 11.75Hz</td>
<td>0.75</td>
</tr>
<tr>
<td>980</td>
<td>0.948</td>
<td>159.99</td>
<td>Sweep 5Hz − 30Hz</td>
<td>0.5</td>
</tr>
<tr>
<td>981</td>
<td>0.948</td>
<td>159.79</td>
<td>Sweep 5Hz − 30Hz</td>
<td>1.0</td>
</tr>
<tr>
<td>966</td>
<td>1.201</td>
<td>183.91</td>
<td>Sweep 5Hz − 30Hz</td>
<td>1.25</td>
</tr>
</tbody>
</table>

by noting the coincident of the largest level of wavelet energy with the line representing double the forcing frequency. The nonlinearity is equally strong in all three accelerometers. The model also shows a smaller nonlinear response at half the forcing frequency near $t = 24.5$ seconds where the model is forced at $21.3Hz$ and the model response has a frequency of $10.6Hz$.

Figure 3.3: Wavelet transform magnitude of FWD, MID, and AFT wing tip accelerations. The instantaneous forcing frequency is plotted in white, along with half its frequency and double its frequency; Run 911
Run 912 was conducted at nearly identical tunnel conditions, \( M = 0.7 \) and \( q = 162.73 \) psf, as Run 911. The difference from Run 911 is in the sweep excitation amplitude which was increased to 0.75°. Magnitudes of the wavelet coefficients of the three measured accelerations are presented in Figure 3.4. Again, nonlinear response at nearly twice the excitation frequency is noted at different instants between \([0.6 \text{ to } 6.5] \) seconds. This represents a forcing frequency range of 5.4 to 9.3 Hz and model response of approximately 10.8 to 18 Hz. The response is greater in the MID and AFT accelerometers as indicated by the red color near 18 Hz at 7 seconds. The model shows an intermittent linear response when forced near one of its natural frequencies (first bending mode), around 12 Hz. A slightly increased response is noted at half the forcing frequency, 14 Hz, when the model is forced at double is natural frequency, 28 Hz near \( t = 27 \) seconds. In addition, this run exhibits frequent nonlinear response around \( 12 \) Hz over the entire range of forcing frequencies. Since the model has several natural frequencies, it is likely that the forcing frequency excites the 12Hz mode by combination resonances.

![Figure 3.4: Wavelet transform magnitude of FWD, MID, and AFT wing tip accelerations. The instantaneous forcing frequency is plotted in white, along with half its frequency and double its frequency; Run 912](image)

Based on Runs 911 and 912, Run 914 was conducted with almost identical tunnel conditions, \( M = 0.7 \) and \( q = 162.60 \) psf, to the previous two runs. The model was forced with a dwell signal
at 11.75Hz and an amplitude 0.75°. The analysis results, in terms of the magnitude of the wavelet coefficients, are presented in Figure 3.5. All three accelerometers show uniform and persistent energy at the forcing frequency. While this run condition gives insight into the dynamics, better insight into the nonlinearities would have been observed by forcing the model with a dwell signal roughly half of the natural frequency. Results from such an excitation could have been sued to identify the source of nonlinearities. Additionally, the more subtle combination resonances could have been studied by forcing the model between 15 to 22 Hz.

![Figure 3.5: Wavelet transform magnitude of FWD, MID, and AFT wing tip accelerations; Run 914](image)

The model response under a similar dynamic pressure to the previous runs near transonic tunnel conditions ($M = 0.949$ and $q = 159.99$ psf) are examined in Run 980. The model was forced with a sweep from 5 to 30 Hz at an amplitude of 0.5°. The results are presented in Figure 3.6. The results show a strong linear response in the FWD, MID, and AFT accelerations between 6 and 9 seconds, over a frequency range between 10 and 12.5 Hz. A strong nonlinear response, at half the forcing frequency, is evident near 19 seconds at 10.5 Hz, and again near 25 to 27 seconds at 12 Hz, at all three locations. Nonlinear response is also intermittently evident between 12 and 16 seconds, in a frequency range between 10 and 12 Hz at all three locations. The distinct superharmonic response, as in runs runs 911 and 912, is not detected in this Run.

A second case of transonic flow is analyzed in Run 981, $M = .948$ and $q = 159.79$ psf. The
Figure 3.6: Wavelet transform magnitude of FWD, MID, and AFT wing tip accelerations. The instantaneous forcing frequency is plotted in white, along with half its frequency and double its frequency; Run 980

conditions and sweep signal are similar to those of Run 980, except the forcing amplitude is increased to 1.0°. The analysis results are presented in Figure 3.7. The accelerations at all three locations shows similar behavior. A strong linear response occurs between 7 and 11 seconds and again between 14 and 16 seconds at forcing frequencies between 10.8 to 14 Hz and 16.6 to 18.3 Hz. The model responds at half the forcing frequency between 23 and 24 seconds at a frequency of 12 Hz. Several other nonlinear resonances are observed over the interval of 10 to 12 Hz although they are not as strong as the linear response.

Model response to forced sweep excitations under supersonic flow conditions \((M = 1.201 \text{ and } q = 183.91 \text{psf})\) were also examined in Run 966, as presented in Figure 3.8. The excitation consisted of a swept sinusoid with a frequency between 5 to 30 Hz and an amplitude of 1.25°. All three accelerations exhibit a strong linear response when forced near the natural frequency between 11 Hz to 13 Hz. A linear response is also exhibited when the forcing frequency is equal to 16 Hz – 18 Hz, exciting a higher vibrational mode. A nonlinear response occurs in all three stations at the natural frequency of 11 Hz when the forcing frequency is near 22 Hz. The response is stronger in the MID and AFT stations than in the FWD station. In addition, nonlinear responses are also detected
between 10.5 and $12 \text{Hz}$ when the forcing frequency is above $14 \text{Hz}$.

Figure 3.7: Wavelet transform magnitude of FWD, MID, and AFT wing tip accelerations. The instantaneous forcing frequency is plotted in white, along with half its frequency and double its frequency; Run 981

Figure 3.8: Wavelet transform magnitude of FWD, MID, and AFT wing tip accelerations. The frequency of the forcing signal is plotted with white, along with half its frequency and double its frequency; Run 966
3.4 “Hard Flutter” Test Procedure

As discussed above, the FSM exhibited nonlinear behavior that varied significantly with tunnel conditions. This is particularly important since the model experienced a “hard” flutter event at \( M = 0.979 \) and \( q = 245.8 \) psf which resulted in the loss of the model [33].

Referring back to Figure 3.2, the model showed increased response over the small Mach interval designated as the “Chimney” region. The tunnel conditions that lead to the loss of the model are at the top of this region. Several tests leading up to the “hard” flutter conditions were recorded before the model was lost. Valuable information about the flutter mechanism and evolution of the fluid structure interactions can be gained by systematically studying the unsteady pressure records and the structural response in the runs leading up to “hard” flutter. Data from the final conditions, where “hard” flutter took place, were not made available for analysis. The test conditions of the specific runs examined in this section are shown in Table 3.2. The runs are ordered by increasing Mach number and dynamic pressure. Although Run 1066 is described as having slightly higher dynamic pressure than Run 1067, the runs are ordered to be consistent with the existing literature. During the actual testing, the tunnel conditions experience slight variations, in addition, the model experiences heating due to skin friction which yields a small amount of uncertainty.

<table>
<thead>
<tr>
<th>Run</th>
<th>Mach</th>
<th>Dynamic Pressure (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1062</td>
<td>0.922</td>
<td>226.58</td>
</tr>
<tr>
<td>1065</td>
<td>0.950</td>
<td>236.00</td>
</tr>
<tr>
<td>1066</td>
<td>0.962</td>
<td>240.11</td>
</tr>
<tr>
<td>1067</td>
<td>0.960</td>
<td>239.66</td>
</tr>
<tr>
<td>1068</td>
<td>0.979</td>
<td>245.80</td>
</tr>
</tbody>
</table>

In order to understand the structural interactions at the wing tip, the vibrational modes of the model were examined. The first four vibrational modes of the FSM, based on ground vibration tests, are shown in Figure 3.9. Three of the first four natural modes have a node line near the wing tip. Due to frequency shifts under tunnel conditions, it is likely that these modes exchanged energy and possibly resonated. Silva et al. [33] state that, during Run 1068, modes three and four merged, leading to a large dynamic response and finally loss of the model.

Three video captures from Run 1068 are presented in Figure 3.10. The first plot, 3.10a, shows...
the intact model before failure. The second and third captures show the model at the onset of “hard” flutter where the wing tip fractured and the model was lost. The second video capture, shown in Figure 3.10b, is the only frame that shows the failure region exclusively. The wing tip has fractured in a region consistent with a vibrational mode of two through four. The natural frequencies and mode shapes of the model are expected to vary from those determined by the ground vibrations tests, given the model interaction with the transonic flow. The final capture, presented in Figure 3.10c, shows the final state of the model. The full extent of the wing tip damage as well as the separated control surface is evident.

Figure 3.9: Experimentally determined vibrational modes of the FSM [33].
Figure 3.10: HSCT-FSM intact and failed during “hard” flutter in Run 1068. The damage initiated near the outer portion of the wing.
3.5 Time/Frequency Analysis

Plots of the wavelet transform magnitude of the fluctuations measured by the three bending strain gages, and one torsion gage during Runs 1062, 1065, 1066, 1067 are presented in Figure 3.11a, b, c, and d, respectively. All strain gages were located in a line at mid span and the torsion gage was located on that line near the mid chord. In Run 1062, Figure 3.11a, the FWD and MID strain gages show high-level energy peaks that are accompanied by streaks that extend over a range of frequencies between 12.5 and 17 Hz. These peaks take place at the same time in both gages. At the torsion and AFT strain gages, the energy is more concentrated near a frequency of 12.5 Hz.

In contrast with the one high-energy frequency range that characterizes the strain gage fluctua-
tions in Run 1062, the distribution of energy in Runs 1065 and 1066, presented in Figure 3.11b and c, shows two distinct ranges of frequency components with high energy peaks. In Run 1065, and in all bending strain gages, most of the energy is contained at the low frequency components between 5.7 and 6.8 Hz. A second component that contains energy is near 12.5 Hz. It extends at different times over a range between 11.5 and 13.5 Hz. The torsion gage shows energy in this range exclusively. While the level of energy in this high frequency range is at the same level as that at the lower frequencies in the AFT strain gage, it is much smaller in the MID and FWD strain gages.

In Run 1066 the FWD and MID strain gages show peaks in the frequency range between 12.5 and 14.3 Hz and near 5.4 Hz. At the AFT strain gage, the energy is distributed in two ranges between 11.3 and 13 Hz and between 5.4 and 6.6 Hz. At the torsion strain gage the energy is concentrated in a range between 11Hz and 13Hz.

The presence of two separate ranges with high-energy peaks is also noted in Run 1067. At the FWD and MID strain gages, the highest peaks extend over the range between 12.5 and 14.3 Hz, with smaller peaks at the lowest measured frequency of 0.5Hz. At the AFT strain gage, the highest peak is near the 12.5 Hz. Again, a smaller peak is noted to appear intermittently, at the lowest measured frequency of 0.5 Hz. Through comparisons of the magnitude of the wavelet coefficients for all measurements presented above, it is noted that the energy peaks in the AFT strain gage are always concentrated over a smaller range of frequencies than the peaks in the mid and forward strain gages.

A closer look at the two separated frequency ranges in both Runs 1065 and 1066 in the AFT
strain gage shows that, within the resolution of the wavelet analysis performed here, the two ranges of peaks have a 2 to 1 ratio. This ratio implies the possibility of nonlinear coupling of the frequency components in the structural response.

Wavelet analysis was also performed on the pressure fluctuations measured at the 95% span location for Runs 1062, 1065, 1066, 1067. Magnitudes of the wavelet coefficients of the pressure fluctuations measured on the upper surface in Run 1062 are shown in Figure 3.12. The leading edge has most of its energy concentrated in a frequency range between 12 and 14 Hz with very few events in the higher frequency range. Starting at $x/c = 10\%$ energy is distributed over a larger band from 5 to 15 Hz. At $x/c = 40\%$ the band between 5 and 15 Hz contains significant energy with many intermittent fluctuations at 20 Hz and above. The energy distribution follows the same characteristics through $x/c = 70\%$. At $x/c = 80\%$ and $x/c = 90\%$ fewer events above 20 Hz are present, and most of the energy is concentrated in the 5 to 15 Hz band.

The magnitude of the wavelet coefficients for the pressure fluctuations on the lower surface of the wing at $\eta = 95\%$ are shown in Figure 3.13. A trend similar to the one noted on the upper surface is observed. In all pressure taps, most of the energy is concentrated in a band between 5 and 15 Hz, and at $x/c = 10\%$, some events range from 20 Hz and above. At $x/c = 40\%$ more events ranging from 20 Hz and above are present. These characteristics remain, until a marked decrease in higher frequency events is noted at $x/c = 70\%$. At $x/c = 80\%$ and $x/c = 90\%$ the high frequency band contains some events at but fewer than at $x/c = 40\%$ to $x/c = 60\%$.

The pressure fluctuations for Run 1065 show different characteristics than those of Run 1062. The magnitude of the wavelet coefficients for Run 1065, upper surface, are shown in 3.14. The leading edge shows energy in a tight band from 11 to 15 Hz, with few events outside this range. The $x/c = 10\%$ record shows two distinct bands, one at 11 to 15 Hz and another near the lower frequency band around 5 Hz. The energy characteristics remain the same for $x/c = 20\%$, with less energy in the higher frequency band. At $x/c = 30\%$ and $x/c = 40\%$, a similar distribution of energy is observed, with several intermittent events that range in frequency from 15 Hz to 30 Hz and possibly higher. At $x/c = 50\%$, the low frequency band, from 5 to 7 Hz contains most of the energy although some energy is still present from 11 to 14 Hz. At pressure taps locations $x/c = 60\%$ to $x/c = 90\%$, the energy in the band from 11 Hz to 14 Hz decreases with increasing chord, while the energy in the lower frequency band, from 6 to 8 Hz remains constant.
The same analysis is repeated on the lower surface for Run 1065 and is shown in Figure 3.15. From $x/c = 10\%$ to $x/c = 30\%$, most of the energy is in the band from 11 to 14 Hz, with little energy at 6 to 8 Hz. At $x/c = 40\%$, the energy is distributed equally in the two frequency bands. There are also several intermittent events that range in frequency from 5 Hz to 30 Hz. At $x/c = 50\%$, the energy of the 12 to 14 Hz band has decreased and is insignificant over the surface between $x/c = 60\%$ to $x/c = 90\%$. The energy at all pressure taps from $x/c = 50\%$ to $x/c = 90\%$ is concentrated in the 6 to 8 Hz band.

Next, the analysis is repeated for the upper surface of the wing for Run 1066; the results are shown in Figure 3.16. Pressure taps from the leading edge to $x/c = 40\%$ show significant energy in the 11 to 14 Hz band. The energy in this band decreases at $x/c = 50\%$ through $x/c = 70\%$ and becomes insignificant at $x/c = 80\%$ and $x/c = 90\%$. The energy of the lower band from 5 to 8 Hz increases from $x/c = 20\%$ to $x/c = 40\%$, where energy is equally distributed between the two bands. The lower frequency band remains significant from $x/c = 50\%$ through $x/c = 90\%$. Several intermittent high frequency events, ranging from 5 to 30 Hz are apparent at $x/c = 50\%$ and $x/c = 60\%$.

The magnitude of the wavelet transforms of the pressures on the lower surface of the wing during Run 1066 are shown in Figure 3.17. From $x/c = 10\%$ through $x/c = 50\%$ the band between 12 and 14 Hz contains most of the energy. At $x/c = 40\%$ and $x/c = 50\%$, the lower frequency band from 5 to 8 Hz starts to show energy. Several intermittent events appearing in both frequency bands, and ranging up to 30 Hz are present at $x/c = 60\%$. Further back on the wing, from $x/c = 70\%$ to $x/c = 90\%$, the energy is contained in the low frequency band from 5 to 8 Hz.

The analysis is repeated for the upper surface of the wing, Run 1067, and is shown in Figure 3.18. From the leading edge to $x/c = 70\%$, a band from 12 to 14 Hz contains significant energy. The pressure taps at $x/c = 80\%$ and $x/c = 90\%$ show almost no energy in this band. From leading edge to $x/c = 50\%$, energy is concentrated in a single high frequency band. At $x/c = 60\%$ and $x/c = 70\%$, a lower frequency band is present from 5 to 8 Hz. At $x/c = 80\%$ and $x/c = 90\%$, the low frequency band extends from 0.5 to 6 Hz and no energy is shown in the high frequency band.

The analysis concludes with the magnitude of the wavelet transform for the pressure on the lower surface of the wing, during Run 1067, shown in Figure 3.19. From $x/c = 10\%$ to $x/c = 60\%$ energy is concentrated exclusively in a band from 11 to 15 Hz. At $x/c = 70\%$ two bands, 11 to
15Hz and 4 to 6 Hz have equal energy. At $x/c = 80\%$ and $x/c = 90\%$, energy is contained in the low frequency band which now extends from .5 to 6 Hz. In summary, the pressure data and the strain gage data indicate that the loading on the wing and the resulting motions of the wing contain different frequencies from fore to aft. The mechanism that causes the differences in wing loading and motions is inherently linked to the flutter of the model. This relationship is examined in more detail in the next section.
Figure 3.11: Wavelet transform magnitude of the FWD, MID, AFT bending strain gages, and mid torsion strain gage respectively, for Runs 1062 and 1065.
Figure 3.11: Wavelet transform magnitude of the FWD, MID, AFT bending strain gages, and mid torsion strain gage respectively, for Runs 1066 and 1067.
Figure 3.12: Wavelet transform magnitude of the pressure at $\eta = 95\%$, upper surface; leading edge to $x/c = 90\%$; Run 1062
Figure 3.13: Wavelet transform magnitude of the pressure at \( \eta = 95\% \), lower surface; \( x/c = 10\% \) to \( x/c = 90\% \); Run 1062
Figure 3.14: Wavelet transform magnitude of the pressure at $\eta = 95\%$, lower surface; leading edge to $x/c = 90\%$; Run 1065
Figure 3.15: Wavelet transform magnitude of the pressure at $\eta = 95\%$, lower surface; $x/c = 10\%$ to $x/c = 90\%$; Run 1065
Figure 3.16: Wavelet transform magnitude of the pressure at $\eta = 95\%$, upper surface; leading edge to $x/c = 90\%$; Run 1066
Figure 3.17: Wavelet transform magnitude of the pressure at $\eta = 95\%$, lower surface; $x/c = 10\%$ to $x/c = 90\%$; Run 1066
Figure 3.18: Wavelet transform magnitude of the pressure at $\eta = 95\%$, upper surface; leading edge to $x/c = 90\%$; Run 1067
Figure 3.19: Wavelet transform magnitude of the pressure at $\eta = 95\%$, lower surface; $x/c = 10\%$ to $x/c = 90\%$; Run 1067
3.6 Intermittent Fluid/Structure Coupling

A detailed analysis of the pressure records during Run 1065 gives insight into the cause of the uneven loading discussed in Section 3.5. A portion of the pressure record at $\eta = 95\%$ and $x/c = 20\%$ is presented in Figure 3.20. The time series shows small, high frequency fluctuations, but the pressure is roughly constant with a mean value near $-0.1$. The pressure at $x/c = 40\%$, presented in Figure 3.21, has a higher mean value (in an absolute sense), with brief events. The pressure further aft along the chord at $x/c = 60\%$, presented in Figure 3.22, shows a significantly different characteristic than the previous two stations. The record shows low frequency, high amplitude fluctuations. This is evidence of an unsteady shock passing over the pressure tap. Similar results are indicated at $x/c = 80\%$, as presented in Figure 3.23. Again, low frequency, high amplitude fluctuations are present along with other fluctuations.

The large fluctuations described in the pressure records can be explained by the presence of an unsteady shock near the trailing edge. The shock moves forward with a frequency of roughly 6 to 8 Hz. It travels forward over the chord, rarely traveling past $x/c = 40\%$, and never reaching $x/c = 20\%$.

A partial record of the leading edge wing tip acceleration for Run 1065 is shown in Figure 3.24. The signal shows high frequency fluctuations with varying amplitude and significant noise. A partial record of the mid chord wing tip acceleration, shown in Figure 3.25, shows less noise and portions of harmonic fluctuations with a varying amplitude as well.

The trailing edge wing tip acceleration, shown in Figure 3.26 is the least noisy, and shows two distinct types of fluctuations. Small amplitude, high frequency, mostly aperiodic fluctuations occur intermittently throughout the record. The interval between 7 and 7.5 seconds shows a particularly good example of these degenerate fluctuations. The second type of acceleration fluctuations are characterized by a larger amplitude and lower frequency. The lower frequency fluctuations occur in groups. An example of such fluctuations are those that appear during the interval between 15.5 and 16.5 seconds.
Figure 3.20: Partial pressure record, $\eta = 95\%$, $x/c=20\%$; Run 1065

Figure 3.21: Partial pressure record, $\eta = 95\%$, $x/c=40\%$; Run 1065
Figure 3.22: Partial pressure record, $\eta = 95\%$, $x/c=60\%$; Run 1065

Figure 3.23: Partial pressure record, $\eta = 95\%$, $x/c=80\%$; Run 1065
Figure 3.24: Acceleration of the leading edge wing tip; Run 1065

Figure 3.25: Acceleration of the wing tip at mid chord; Run 1065
Figure 3.26: Acceleration of the trailing edge wing tip; Run 1065
Figure 3.27: Power spectra of the pressure fluctuations at \( x/c = 20\%, 40\%, 60\%, \) and \( 80\%; \eta = 95\%; \) Run 1062

The power spectra of the pressure fluctuations at \( \eta = 95\% \) and \( x/c = 20\%, 40\%, 60\%, \) and \( 80\%, \) as measured in Run 1062, \( (M = 0.922 \) and \( q = 226.58 \text{ psf}) \) are shown in Figure 3.27. The energy level at \( x/c = 20\% \) is clearly an order of magnitude lower than that measured at the other chord locations. At all locations, the power is distributed between 7 Hz and 16 Hz with peaks at 6 Hz and 12 Hz.

The power spectra of the same wing locations shown above, for Run 1065 \( (M = 0.950 \) and \( q = 236.00 \text{ psf}) \), are shown in Figure 3.28. The power spectrum of pressure at \( x/c = 20\% \) shows small peaks between 6 and 8 Hz and a peak that spans 12.5 Hz. The power spectrum of the pressure at \( x/c = 40\% \) shows an increasing trend in the power at 6 to 8 Hz and a higher peak near 12.5 Hz. The power spectrum at \( x/c = 60\% \) and \( x/c = 80\% \) shows the same characteristics with higher amplitude. The unsteady shock motion is indicated by the peak near 6 Hz, while structural motion
Figure 3.28: Power spectra of the pressure fluctuations at $x/c = 20\%$, 40\%, 60\%, and 80\%; $\eta = 95\%$; Run 1065

is indicated by the higher frequencies around 12 Hz.

The power spectra of the pressure fluctuations at the same locations as above, measured in Run 1066 ($M = 0.962$ and 240.11 psf) are presented in Figure 3.29. The forward and second forward most stations ($x/c = 20\%$, and 40\%) show little evidence of shock motion as indicated by the low energy near 6 Hz. At $x/c = 60\%$, a slight increase in the energy associated with shock formation is present. At $x/c = 80\%$, the energy associated with shock motion is more than than order of magnitude higher than the energy measured at other stations. The shock is fully developed and intermittent at this station.

The power spectra from Run 1067 ($M = 0.960$ and 239.66 psf) measured at the same locations are presented in Figure 3.30. At all stations, the energy associated with shock motion is an order of magnitude lower than the energy in Run 1065 at $x/c = 80\%$, where a fully developed shock
Figure 3.29: Power spectra of the pressure fluctuations at $x/c = 20\%, 40\%, 60\%, \text{and } 80\%; \eta = 95\%; \text{Run 1066}$
Figure 3.30: Power spectra of the pressure fluctuations at $x/c = 20\%, 40\%, 60\%, \text{and } 80\%; \eta = 95\%; \text{Run 1067}$

Power spectra of the wing tip accelerations for Run 1062, ($M = 0.922 \text{ and } q = 226.58 \text{ psf}$) are presented in Figure 3.31. Energy is highly concentrated in a frequency band associated with structural motion, between 11 to 14 Hz at all locations. No significant power is present at any other frequencies. The power increases from the leading edge and doubles at the trailing edge.

The power spectra of the accelerations as measured in Run 1065 ($M = 0.950 \text{ and } q = 236.00 \text{ psf}$), at the leading edge, mid chord, and trailing edge wing tip locations are shown in Figure 3.32. Two significant peaks are present at 12 Hz and 14 Hz in all three records. The mid chord and trailing edge accelerometers show higher harmonic energy than that at the leading edge. The frequencies present in the acceleration records are roughly double the frequencies present in the pressure records which indicates a quadratic coupling between the acceleration of the wing and the
Figure 3.31: Power spectra of the leading edge, mid chord, and trailing edge wing tip accelerations; Run 1062
pressure at $x/c = 60\%$ and $x/c = 80\%$.

The power spectra of the accelerations for Run 1066 ($M = 0.962$ and 240.11 psf), at the leading edge, mid chord, and trailing edge wing tip locations are shown in Figure 3.32. The two distinct peaks that were evident in the previous run have separated further and now appear at 13 and 15 Hz. The magnitude of the power increases from leading edge to trailing edge and is slightly less than the power in the previous run.

The power spectra of wing tip accelerations for Run 1067 ($M = 0.960$ and 239.66 psf) are shown in Figure 3.34. At each of the wing tip locations, three peaks are present. The strongest is at 12.6 Hz followed by 13.5 Hz and the weakest is at 14.5 Hz. The magnitude increases from leading edge to trailing edge like in the previous run.
Figure 3.33: Power spectra of the leading edge, mid chord, and trailing edge wing tip accelerations; Run 1066
Figure 3.34: Power spectra of the leading edge, mid chord, and trailing edge wing tip acceleration; Run 1067
The Fourier cross-bicoherence for six combinations, between pressure fluctuations as measured at $x/c = 60\%$ and $x/c = 80\%$ and the leading edge, mid chord, and trailing edge accelerations in Run 1065 are shown in Figure 3.35. No bicoherence was detected between the pressure and acceleration as indicated by the absence of contour lines (0.5 : 0.1 : 0.9) in all plots. Based on this evidence, it may appear that coupling does not exist. On the contrary, intermittent coupling may be present but undetected because of the averages required to compute the Fourier-based bicoherence.

Figure 3.35: Fourier-based cross bicoherence between the pressure at $x/c = 60\%$ and the leading edge, mid chord and trailing edge wing tip accelerometers (top) and $x/c = 80\%$ and the leading edge, mid chord, and trailing edge wing tip accelerometers (bottom); Run 1065
A small portion of the pressure at $\eta = 95\%$, $x/c = 80\%$ and the trailing edge wing tip acceleration records are shown together in Figure 3.36. Notice how the pressure shows coherent fluctuations around 16 seconds, then again around 17.5 seconds. Coincident with the coherent motion of the pressure, the acceleration also experiences coherent motion, at a higher frequency, during the same time intervals. Both the pressure and the acceleration signals experience coherent motion simultaneously. In addition, when the motion is degenerate in one signal, it is degenerate in both signals.

The pressure at $\eta = 95\%$, $x/c = 80\%$ and its wavelet transform are shown in Figure 3.37. Energy is intermittently present in the 7 Hz band. There is no significant energy in the higher frequencies above 10 Hz. Harmonic motion is identified at a frequency of 7 Hz from 16.0 – 16.4 seconds and again from 17.5 – 18.25 seconds. The smaller, high frequency fluctuations that occur between 16.5 and 17.5 seconds do not have any significant harmonic content as shown by the absence of wavelet energy in that interval.
Figure 3.37: Pressure signal at $\eta = 95\%$, x/c=80\% and its wavelet transform magnitude. There is intermittent harmonic motion near 7 Hz around $t = 17.5$ seconds; Run 1065
The trailing edge wing tip acceleration and its wavelet transform magnitude are presented in Figure 3.38. The results show that the energy is contained in a band between 10 Hz and 20 Hz. There is significant harmonic content near 14 Hz between 16.0 to 16.5 seconds and again near 17.5 seconds. The acceleration has harmonic content that is intermittent in the same intervals at twice the frequency of the pressure signal.

The wavelet-based cross-bicoherence between the pressure and the acceleration during the interval from 16.1 seconds to 16.4 seconds is shown in Figure 3.39a. Bicoherence values above the statistical noise level are shown with contour lines of (.6:.1:.9). Coupling is detected at (7Hz, 7Hz, 14Hz) between the pressure and the acceleration. No other significant coupling is detected during the interval. The wavelet-based cross bicoherence is calculated again in the interval of 17.4 – 17.7 seconds. Again, the strongest coupling occurs at (7Hz, 7Hz, 14Hz) and no other significant coupling appears. The time series and wavelet transforms of the pressure and acceleration records indicate the absence of coupling in the interval between 16.5 – 17.5 seconds, as shown in Figure 3.40. This confirms that pressure and acceleration near the wing tip were intermittently and quadratically
coupled.

Figure 3.39: Wavelet cross-bicoherence between the pressure fluctuations at \( \eta = 95\% \), \( x/c=80\% \) and trailing edge wing tip acceleration during \( t = [16.1 - 16.4] \) seconds and \( t = [17.4 - 17.7] \) seconds. The cross-bicoherence shows intermittent coupling at \((7Hz, 7Hz, 14Hz)\) between the pressure and the acceleration; Run 1065
Figure 3.40: Wavelet cross-bicoherence between the pressure at $\eta = 95\%$, $x/c=80\%$ and trailing edge wing tip acceleration during $t = 16.5 - 17.5\text{s}$. The cross-bicoherence shows no coupling over this interval; Run 1065
3.7 Flutter Mechanism

The wavelet-based cross-bicoherence has shown intermittent quadratic coupling between the pressure and acceleration signals near the trailing edge. Using the intermittent coupling as starting point, a more detailed picture of the intermittent nonlinear interactions can be developed. Specifically, the pressure and acceleration signals are shown during a coupling event in Figure 3.41.

![Figure 3.41](image-url)

Figure 3.41: The pressure at \( \eta = 95\% \), \( x/c=80\% \) (top) and the trailing edge wing tip acceleration (bottom) during coupled motion. The accelerometer goes through four cycles while the pressure fluctuations go through two cycles.

Over the period between 17.4 and 17.7 seconds, the pressure undergoes two cycles. The acceleration undergoes four cycles over the same period. The pressure fluctuations indicate the formation of a shock near the aft of the wing and periodically moving forward and backward.
Figure 3.42: Pressure distribution over the wing at $\eta = 95\%$ at $t = 17.485$ seconds. The acceleration of the wing tip is marked with a “•”

The pressure distribution over the top surface of the wing at $\eta = 95\%$ at the specific time of $t = 17.486$ seconds is shown in the top portion of Figure 3.42. The sharp increase in pressure near the trailing edge, at $x/c = 80\%$, indicates a shock at the back of the wing. Coincident with this instantaneous pressure distribution, the acceleration of the trailing edge wing tip is at a minimum, as indicated by the “•” in the bottom portion of Figure 3.42.
As time progresses, the shock moves forward to give the instantaneous pressure distribution shown in the top portion of Figure 3.43. In this state, at $t = 17.556$ seconds, the pressure at the leading edge is unchanged, however, there is a sharp increase in pressure at the 40% chord. The shock is at its forward most position. The acceleration, shown in the bottom portion of the figure, has experienced one complete cycle and is again at a minimum as indicated by the “□”.

Figure 3.43: Pressure distribution over the wing at $\eta = 95\%$ at $t = 17.556$ seconds. The acceleration of the wing tip is marked with a “□”
The final configuration of the instantaneous pressure profile and the acceleration, at time $t = 17.620$ seconds is shown in Figure 3.44. The increase in pressure near the trailing edge indicates that the shock has returned to the back of the wing. The acceleration has completed another cycle and is again at a minimum as indicated by the “x”.
The unsteady shock and quadratically coupled trailing edge acceleration are shown at all three times simultaneously in Figure 3.45. Three instantaneous pressure profiles, each marked with a different symbol indicating the time, are shown in the top portion of the figure. The acceleration record is also marked with the same symbols denoting three specific times. The “•” indicates $t = 17.486$ seconds; the “□” indicates $t = 17.556$ seconds, and the “×” indicates $t = 17.620$ seconds. Since the flow is locally supersonic, the conditions upstream from $x/c = 40\%$ do not vary as the shock moves. The shock develops at the back, moves forward, and again appears at the back completing one single cycle while the acceleration completes two cycles. Based on the location of the node lines, shown previously in Figure 3.9, it is likely that the shock traversed a node line causing the nonlinear interaction demonstrated above.

An additional sequence of coupled shock motion and wing tip acceleration is shown in Figure 3.46. The wing is indicated by a thick black outline. Pressure taps at $\eta = 60\%$ and $\eta = 90\%$ are indicated by black dots. The pressure distribution on the top and bottom surfaces are indicated
by the colored strip. The height of the strip indicates the local pressure value as measured by
the pressure tap. A cubic spline interpolation was used to determine a continuous distribution of
pressure values. The color of the strip is redundant with the height of the strip, indicating the local
pressure value. The color scheme is as follows: purple indicates a low saturation of the color scale,
blue gradating to green gradating to red indicates increasing pressure, and finally yellow indicates
a high saturation of the color scale. The pressure tap at $\eta = 60\%$ and $x/c = 75\%$ recorded invalid
data as indicated by a constant and locally high value for the pressure at all times.

A similar color scheme shows the acceleration of the wing tip. A cubic interpolation between the
leading edge, mid chord, and trailing edge wing tip accelerations gives an estimate of the continuous
distribution of wing tip acceleration. The color and height redundantly represent the value of the
vertical acceleration. The surface contour of the physical model causes slight variations in the
pressure distribution along the wing. This is particularly evident at $\eta = 60\%$ and $x/c = 40\%$.

Slightly different $\Delta t$'s between the plots were required in order to illustrate the specific times
shown in the previous analysis. The unsteady shock motion at $\eta = 95\%$ and the coupled trailing
dge wing tip acceleration can be seen sequentially in the plots $a - p$. In plot $a$, the shock at
$\eta = 95\%$ is at its rearmost and the acceleration of the trailing edge is at its lowest. As time
progresses, the shock moves forward, and the acceleration is positive. The acceleration reaches
its highest point in plot $f$. The shock reaches its foremost position and the acceleration is at a
minimum in plot $h$. The shock decreases in strength at its foremost and simultaneously begins to
develop at the trailing edge while the acceleration becomes less negative, shown in plot $j$. This is
possible because of the contour of the wing’s surface. The shock is fully developed near the trailing
dge and the acceleration is at a maximum in plot $l$. The shock remains at the trailing edge while
the acceleration decreases in plot $n$. Finally, the shock begins to move forward slightly, while the
acceleration is at a minimum in plot $p$. 

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Figure 3.46: Pressure and acceleration records. The trailing edge wing tip acceleration goes through two cycles while the shock on the upper surface at $\eta = 95\%$ goes through one cycle, continued on the next page; Run 1065
Figure 3.46: Pressure and acceleration records. The trailing edge wing tip acceleration goes through two cycles while the shock on the upper surface at $\eta = 95\%$ goes through one cycle, continued from the previous page; Run 1065.

Run No. 1065; Time = 17.564sec.
Run No. 1065; Time = 17.572sec.
Run No. 1065; Time = 17.580sec.
Run No. 1065; Time = 17.588sec.
Run No. 1065; Time = 17.596sec.
Run No. 1065; Time = 17.604sec.
Run No. 1065; Time = 17.612sec.
Run No. 1065; Time = 17.620sec.