CHAPTER 4. PROPOSED MODEL FOR THE RESEARCH

The models developed by Athanassopoulos (1995) and Thanassoulis and Dyson (1992) are targeted towards service oriented organizations. However a similar approach can be used to address performance improvement issues in a manufacturing environment. The general nature of the problem addressed by all models are the same i.e. to resolve multiple and conflicting objectives within the organization operating at multiple levels of decision making.

4.1 MULTIPLE OBJECTIVE FRAMEWORK

There are a number of interdependencies among the components of the system (in this case, a manufacturing facility). These policies assume that there is no effect of other decision-making units. However, simultaneous analyses of all relevant decision making entities and policy objectives within a manufacturing facility are necessary to analyze the system in a broader perspective. Most of the traditional policy models (DEA, for example) are based on the assumption of individual decision making units aimed at achieving a one-dimensional objective.

A manufacturing facility with a series of processes has multiple objectives. The main objectives are to reduce costs, achieve maximum output, reduce waste and rework, enhance environmental performance etc. All the above objectives are not distinct from each other. To increase profits, production costs have to be reduced and output has to be maximum. To achieve the above objectives in a production plant with a number of processes, a simultaneous consideration of all the pertinent factors such as the achievement of plant goals, the achievement of process goals and the interdependencies among processes have to be taken into account.

A plant can have objectives both at the individual process level and at the plant level. The individual process objectives might be to reduce input costs and to meet production requirements.
Mostly these objectives are expressed as a target. A target is a desired level of the achievement of the objective. The process generally articulates these targets based on production orders and plans. The plant level objectives are of a similar nature, but larger in scope and dimension. Most manufacturing systems are large and complex\textsuperscript{6}. The control of management is divided into a hierarchy consisting of different levels. Each level is characterized by the length of the planning horizon and the kind of data required for the decision making process. Higher levels of hierarchy have long horizons and use highly aggregated data. Lower levels have shorter horizons and use more detailed information. Typically, managers of a manufacturing firm make production plans for finished products by considering forecasts of demand, sales, orders, raw material availability, inventory levels, and plant capacity. From the resulting high level plan, the requirements for the components that go onto the final products can be determined. The various departments that are responsible for the manufacture of the components schedule their activities so as to meet the requirements generated by the master production and materials requirement plans.

The achievement of the targets at the respective levels are complementary. As the achievement of the overall plant targets will depend on the performance of the individual processes, improving the effectiveness of the individual processes should impact the overall plant performance. This research analyzes the effectiveness of the individual plant processes in achieving the overall plant target. It also addresses the achievement of the global plant targets, meaning the accomplishment of plant objectives. The firm under consideration is a manufacturing firm with serial production processes. Therefore, it is only logical to expect that the outputs at the end of each of the processes be related. The outputs at the end of a process are sent to the next process for further processing. This research also addresses how balanced the production processes are. This will very clearly indicate if there are any bottlenecks in the production processes. Figure 4.1 shows the multiple objective frame work for a manufacturing facility with serial production processes.

To analyze and implement process improvement decisions, a simultaneous analysis of all the above objectives are needed. Traditional models (DEA) provide the capability to analyze decision-making units in isolation. However, in this case a multiple objective programming framework has to be considered.

As explained in chapter 2, the multiple objective model selected for the research is goal programming. Using goal programming the achievement of both individual and plant targets can be analyzed. The results obtained will be helpful in making decisions regarding performance of the plant in general. Based on the results, some suggestions can then be offered to improve the overall performance of the plant. This might mean improving the effectiveness of one or more of the processes which will be identified as

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**Fig 4.1 Contribution of the Individual Processes to the Global Targets**

OVERALL PLANT PERFORMANCE

GLOBAL PRODUCTION PLANNING RESOURCE TARGETS

GLOBAL PRODUCTION PLANNING OUTPUT TARGETS

MAXIMIZE GLOBAL EFFECTIVENESS

INPUTS

PROCESS 1

OUTPUTS

INPUTS

PROCESS K

OUTPUTS

MAXIMIZE PROCESS EFFECTIVENESS & LINE BALANCING
the bottlenecks in the sequence of operations. The goal programming model is explained in detail in the following section.

4.2 THE SERIAL-MANUFACTURING GOAL PROGRAMMING MODEL (SMGP)

The formulation of the model is given below. There are three objective functions, which indicate three goals, which a decision-maker can have. The first objective function focuses on the plant level targets (effectiveness). The second objective function addresses the process level targets (process effectiveness). The third objective function indicates the dependency of a process to its predecessor (line balance). Each of the goals have deviations associated with them. The sum of the deviations are normalized by their respective targets and are minimized in the objective functions. The formulation for process $k$ is given below:

$$
\text{Min} \sum_{i=1}^{m} W_i \frac{P_i^k}{GTX_{i,k}} + \sum_{r=1}^{s} W_r \frac{N_r^k}{GTY_{r,k}}
$$  \hspace{1cm} (4.1)

$$
\text{Min} \sum_{j=1}^{T} \left( \sum_{i=1}^{m} \left( w^x_i n_{ij}^{jk} + w^y_i P_i^{jk} \right) + \sum_{r=1}^{s} \left( w^x_r n_{ij}^{rk} + w^y_r P_r^{jk} \right) \right)
$$  \hspace{1cm} (4.2)

$$
\text{Min} \sum_{j=1}^{T} \left( \sum_{r=1}^{s} \left( w^x_r \xi_{ij}^{jk} \right) + \sum_{r=1}^{s} \left( w^y_r \eta_{ij}^{jk} \right) \right)
$$  \hspace{1cm} (4.3)

Subject to:

Plant-Level Planning Horizon Target Constraint (translated into a process constraint for the entire planning horizon):
\[
\sum_{j=1}^{T} \gamma_j^{ik} y_r^{jk} + \sum_{j=1}^{T} \gamma_j^{2k} y_r^{jk} + \ldots + \sum_{j=1}^{T} \gamma_j^{Tk} y_r^{jk} + (N_r^k - P_r^k) = GTY_r^k \quad r = 1, \ldots, s \tag{4.4}
\]

\[
\sum_{j=1}^{T} \gamma_j^{i,k} x_i^{jk} + \sum_{j=1}^{T} \gamma_j^{2k} x_i^{jk} + \ldots + \sum_{j=1}^{T} \gamma_j^{Tk} x_i^{jk} + (N_i^k - P_i^k) = GTX_i^k \quad i = 1, \ldots, m \tag{4.5}
\]

**Process Target Constraints:**

\[
\sum_{j=1}^{T} \gamma_j^{c,k} y_r^{jk} + \sum_{j=1}^{T} \gamma_j^{c,k} = \hat{y}_r^{c,k} \quad c = 1, \ldots, T, \quad r = 1, \ldots, s \tag{4.6}
\]

\[
\sum_{j=1}^{T} \gamma_j^{c,k} x_i^{jk} + \sum_{j=1}^{T} \gamma_j^{c,k} = \hat{x}_i^{c,k} \quad c = 1, \ldots, T, \quad i = 1, \ldots, m \tag{4.7}
\]

**Production-Line Balance Constraint:**

\[
\sum_{j=1}^{T} \gamma_j^{c,k} y_r^{jk} + \sum_{j=1}^{T} \gamma_j^{c,k} = \gamma_j^{c,k-1} \quad c = 1, \ldots, T, \quad k = 2, \ldots, K, \quad r = 1, \ldots, s \tag{4.8}
\]

\[
\sum_{j=1}^{T} \gamma_j^{c,k} = 1 \quad c = 1, \ldots, T, \quad j = 1, \ldots, T \tag{4.9 a}
\]

Or

\[
\gamma_j^{c,k} \geq 0 \quad c = 1, \ldots, T, \quad j = 1, \ldots, T \tag{4.9 b}
\]

Or

\[
0 \leq \gamma_j^{c,k} \leq 1 \quad c = 1, \ldots, T, \quad j = 1, \ldots, T \tag{4.9 c}
\]

\[
n_i^{jk}, p_i^{jk}, n_r^{jk}, p_r^{jk}, z_j^{jk}, \mu_r^{jk}, N_i^k, P_i^k, N_r^k, P_r^k \geq 0, \forall i, j \text{ and } r \tag{4.10}
\]

\[
w_i^n, w_i^p, w_r^n, w_r^p, w_r^s, W_i, W_r > 0, \quad \forall i, r, n \text{ and } p \tag{4.11}
\]

**Where**

- \( T \) is the total number of periods in the planning horizon.
- \( s \) is the total number of outputs produced by each process.
- \( m \) is the total number of inputs used by each process.
\( y_{r,k}^{j} \) is the amount of output \( r \) produced by process \( k \) during the \( j^{th} \) period.

\( x_{i,k}^{j} \) is the amount of input \( i \) used by process \( k \) during the \( j^{th} \) period.

\( \hat{x}_{i,k}^{j} \) is the \( j^{th} \) period process-level production-planning target for input \( i \) for process \( k \).

\( \hat{y}_{r,k}^{j} \) is the \( j^{th} \) period process-level production-planning target for output \( r \) for process \( k \).

\( \gamma_{j}^{c,k} \) is the activity parameter associated with \( j^{th} \) production period when process \( k \) is being evaluated during period \( c \) or another way of stating this when process \( k \) at time period \( c \) is the observation or decision making unit being evaluated. This activity parameter represents the weight ascribed to each observation or DMU in the planning cycle when production process \( k \) during period \( c \) is being evaluated. This is consistent with the notion in DEA where each observation or DMU is compared to a weighted combination of existing observations or DMUs.

\( P_{i,k}^{c} \) and \( N_{i,k}^{c} \) are the positive and negative deviations respectively associated with the \( i^{th} \) input plant-level target for the \( k^{th} \) process for the entire planning horizon.

\( P_{r,k}^{c} \) and \( N_{r,k}^{c} \) are the positive and negative deviations respectively associated with the \( r^{th} \) output plant-level target for the \( k^{th} \) process for the entire planning horizon.

\( GTX_{i}^{k} \) is the plant level target for the \( i^{th} \) input for the entire planning horizon and \( GTY_{r}^{k} \) is the global plant level target for the \( r^{th} \) output for the \( k^{th} \) process for the entire planning horizon.

\( n_{i,k}^{c} \) and \( p_{i,k}^{c} \) are the negative and positive deviations from the \( i^{th} \) input process targets when process \( k \) is evaluated during period \( c \).

\( n_{r,k}^{c} \) and \( p_{r,k}^{c} \) are the negative and positive deviations from the \( r^{th} \) output process targets when process \( k \) is evaluated during period \( c \).

\( \xi_{r}^{c,k} \) and \( \mu_{r}^{c,k} \) are the negative and positive deviations of the output of process \( k \) from the output of the immediately preceding process \( k-1 \) when process \( k \) is evaluated during period \( c \).

\( w_{i}^{n} \) and \( w_{i}^{p} \) are the user defined weights associated with the deviations \( n_{i,k}^{c} \) and \( p_{i,k}^{c} \) respectively.

\( w_{r}^{n} \) and \( w_{r}^{p} \) are the user defined weights associated with the deviations \( n_{r,k}^{c} \) and \( p_{r,k}^{c} \) respectively.
is the user defined weight associated with the deviation $\xi_{r,k}$.

As explained previously, the objective function has three parts. Equation (4.1) minimizes the sum of the positive deviations of the inputs and the negative deviations of the outputs from the targets at the plant level. Equation (4.2) minimizes the sum of the positive and negative deviations from the input and output targets at the process level. Equation (4.3) minimizes the sum of the negative deviations from the actual output of the preceding process. This is not applicable for the first process, as it has no predecessor. The first objective function represents the plant level goals, the second considers the process level goals, and the third establishes an output line balance among the processes as the firm under consideration is a serial-manufacturing firm.

The above goal programming approach gives the decision-maker the flexibility of attaching weights to the deviation variables, which are deemed as important. There is also flexibility in altering the priorities of the three goals. Each combination of priorities represents a different strategy in analyzing the firm under consideration. Three such combinations of priorities are considered. In the first strategy, the achievement of plant level goals (equation (4.1)) has the first priority. The process achievement goal (equation (4.2)) has the second priority and the production line-balance goal (equation (4.3)) has the last priority. In the first strategy, the master production requirements must be met first. This means that the effectiveness performance of the processes may be compromised. The production balance goal receives the lowest priority and indicates the extent to which bottlenecks are avoided given that the above two goals are already met.

In the second strategy, the avoidance of bottlenecks is considered more important that the effectiveness performance of the processes with plant level goals still having the first priority.

In the third strategy, the impact on the achievement of master production plans is assessed when the production efficiency is given the highest priority and avoiding bottlenecks the second priority. The plant level goal has the last priority.

In the second objective function (equation (4.2)), there is a possibility of increase in inputs and decrease in outputs to meet the plant level objectives. This is a fundamental deviation from the DEA approach, which allows only input decreases and output increases. This approach may be useful to a decision-maker whose ultimate responsibility
may be the achievement of the plant level master production requirements. A more specific case may be obtained by setting the weights on the input positive deviation variable and output negative deviation variable to zero. This would relate to the DEA assumption described in the Banker et al. (1984) model described in chapter 2.

Each of the deviation variables in the objective functions are normalized with respect to their targets. Since the firm under consideration did not track targets or goals on the input side, the actual input consumption was used as targets.

There are three sets of constraints corresponding to the three goals. The first set of constraints refers to the effectiveness of each process in contributing to the plant level goal. Equation (4.4) represents the weighted combination of period outputs for all the processes compared with the plant level output target. Equation (4.5) represents the weighted combination on the input side. The input plant level target is the sum of actual consumption for the process across the planning horizon. The second set of constraints addresses the operational effectiveness of each production process at a point in time with respect to its output and input targets represented by equations (4.6) and (4.7) respectively. Again, the input targets are the actual inputs used by each process during a period. The third set of constraint (equation (4.8)) represents the dependency of each process on its previous process. The targets for each of the constraints are the actual output of the previous process in the same time period. The activity parameters $\gamma_j$ for each time period are normalized (their sum equal to one). The convexity assumption makes the comparison with the Banker et al. (1984) model more relevant (equation 4.9 a)). However, two more variations of the above constraint are considered. The first variation leaves them unrestricted (equation (4.9 b)). The second variation restricts the activity parameters between zero and one (equation (4.9 c)). The evaluation of the processes was also conducted with respect to the actual outputs of the processes. In this case, the process targets are the actual outputs. In this context, it represents the achievement of operational efficiency. Numerical results were obtained by solving the above goal programming model as a sequential linear programming problem considering various strategies and variations of the activity parameters described above. The above goal programming models were developed using PC/SAS. The results of these goal-programming models are described in Chapter 6.