AN INTEGRATED APPROACH TO THE OPTIMAL RUNWAY EXIT LOCATIONS

by

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(ABSTRACT)

The airport capacity problem has recently received a great deal of attention due to airport congestion and delays. Capacity improvements of airfield and airspace component of an airport are currently being addressed by several researchers and federal and state agencies. The optimal location of runway turnoffs is the focus of this research. Although the current airport capacity limitations are dictated by airspace separation rules, it is expected that runway occupancy time (ROT) will become an important factor in the near future as the interarrival separations between landing aircraft are reduced.

The intent of this research is to show that the use of high speed exits on runway contributes to the reduction of ROT, and therefore provides enhancement in runway capacity. However, locating the high speed exits is a complex and dynamic problem stemming from the aircraft landing behavior. The landing behavior of an aircraft is affected by many factors such as approach speed, deceleration rate, design exit speed, airport elevation, wind, temperature, etc. Some of these factors are probabilistic in nature.

A simulation model and an optimization algorithm that take into consideration all above factors developed to address the problem. The simulation model consists of a series of dynamic equations of motion that models the aircraft landing behavior under various airport conditions, and determines the best exit location for that aircraft. The optimization model takes the simulation results as input for various aircraft mix, and finds a given number of exit locations which minimize the average ROT for the total aircraft fleet.
The integrated approach is implemented on a software package named REDIM (Runway Exit Design Interactive Model). REDIM runs on an IBM personal computer or compatible with VGA graphics card. In addition to the analysis functions, REDIM has many convenient features. For example, the flow of the program is controlled by menus such as Main Menu and Edit Menu. A data base file storing fifty one aircraft geometric characteristics is included in the package. Data base file and working data file are edited interactively. Output file containing analysis results is displayed on the screen with a format easy to understand.
Acknowledgements

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I also wish to express heartfelt thanks to my wife, Mi Kim, for her love and patience, and to my parents and parents-in-law who constantly pray for the successful completion of my studies. Without the love and support of my family members, my Masters degree would not have been achieved.
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1.0 INTRODUCTION

The subject of airport congestion and delays has received a great deal of attention in recent years due to the rapid growth of air transportation services coupled with a relatively stagnant airport infrastructure. Current statistics indicate that approximately three billion dollars are paid by air travelers due to system imposed delays in the United States alone [FAA, 1988]. These delays are likely to increase as air travel demand builds up from 416 billion passenger revenue miles flown in 1988 to an estimated 750 billion passenger revenue miles by the end of the century or equivalent to an average annual growth of roughly 6 percent [Aviation Week & Space Technology, 1988]. The problem is further aggravated when one considers that the current air transportation system has been operating in stagnant mode with almost the same infrastructure in terms of airport facilities since the early seventies. The Federal Aviation Administration (FAA) has estimated that eleven major airports now experience severe chronic operational delays -more than 20,000 hours of system imposed delays per year - as a result of traffic congestion. According to the FAA this number will increase to thirty two by the year 1996 and possibly fifty by the end of the century [FAA, 1988]. One fifth of these airport facilities will experience more than 50,000 hours of system imposed delays according to the same study. Delays at these key airports are not simply local problems; the effects ripple outward
to other airports with flights connecting to these hubs and ultimately to the entire air transport network [Transportation Research Board, 1988].

1.1 Problem Descriptions

The capacity of an airport facility is dictated by the critical capacity of the following four subsystems: 1) the airspace, 2) the runway, 3) the taxiway, and 4) the apron-gate subsystem, assuming a well designed ground access subsystem. Figure 1.1 shows the airport system breakdown.

Although the interrelations between these four components could be significant for certain airport configurations, it has been customary to study in detail each component independently and then select the most restrictive one as that defining the capacity of the facility. The airport capacity is currently limited by the terminal airspace subsystem capacity. The capacity of terminal airspace is determined by the landing aircraft separation rules which are applied to the final approach stage of flight. In order to increase the airspace capacity, several subjects such as aircraft wake vortex generation and final approach procedures are under investigation [FAA, 1988]. Table 1.1 shows the present and expected future separation criteria. The first row of each cell is the separation distance in nautical miles and the second row is the separation time in seconds assuming the approach speeds as depicted under the table.

The airport capacity, however, would not increase by much if the runway subsystem becomes the limiting constraint in the airport system once the airspace capacity problem is resolved. The runway capacity is determined primarily by runway occupancy time (ROT) and its variation from aircraft to aircraft. ROT is the time that an aircraft spends on the runway until a new landing or departure operation can processed.
Figure 1.1 Airport System Breakdown.
Barrer and Dielh [1988] quantified the potential increases to runway capacity resulting from improvements to the Air Traffic Control System (ATC) performance parameters (i.e., reducing in-trail landing separations, better planned runway exits, improved ground-based radar surveillance capability, etc.) and concluded that gains of up to 20% in the capacity of a single runway are possible if these control actions were to be implemented. Other studies support similar gains if advanced systems are used [Lebron, 1987; Simpson et al, 1988]. Table 1.2 illustrates the present and future ROT values of several aircraft classes estimated Barrer and Dielh [1988].

1.2 Scope of Research

While the ultimate goal is to relieve the airport congestion and delays, this thesis focuses on the runway landing capacity (or just runway capacity) problem. The runway capacity problem is expected to be crucial for the airport capacity in the near future with the improvement of the Air Traffic Control (ATC) systems and aircraft technology.

Runway capacity may be distinguished according to the operation scenarios: 1) landing operation only, 2) taking off operation only, and 3) mixed operation. While the third scenario is the most realistic one, assuming the first scenario is meaningful as a conservative approach since an aircraft usually spends more time on the runway during landing than during takeoff operations.
Table 1.1  Arrival In-Trail Separation Criteria.
Source: Barrer and Dielh, 1988

<table>
<thead>
<tr>
<th>Lead Aircraft</th>
<th>Current Values (nautical miles) (seconds)</th>
<th>Future Goal (1996) (nautical miles) (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small</td>
<td>Large</td>
</tr>
<tr>
<td>Small</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>82</td>
<td>64</td>
</tr>
<tr>
<td>Large</td>
<td>4.0</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>131</td>
<td>64</td>
</tr>
<tr>
<td>Heavy</td>
<td>6.0</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>196</td>
<td>129</td>
</tr>
</tbody>
</table>

Assumed speeds for three aircraft classes:
Small - 110 knots
Large - 140 knots
Heavy - 150 knots

Table 1.2  Arrival Runway Occupancy Times.
Source: Barrer & Dielh, 1989

<table>
<thead>
<tr>
<th>TERP Category</th>
<th>ROT Mean Value (seconds) Present</th>
<th>Future (1996)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>43</td>
<td>35</td>
</tr>
<tr>
<td>B</td>
<td>45</td>
<td>37</td>
</tr>
<tr>
<td>C</td>
<td>46</td>
<td>40</td>
</tr>
<tr>
<td>D</td>
<td>50</td>
<td>45</td>
</tr>
</tbody>
</table>
1.3 Research Objectives

The goal of the research is to develop a method which is able to find optimal exit locations on a runway which minimize the average ROT of a mixed aircraft fleet. The runway throughput (or potential) capacity is defined as the inverse of the average ROT of an aircraft fleet, thus can be maximized by minimizing the average ROT.

1.4 Proposed Approach

In order to find optimal exit location, an integration of a simulation model and an optimization model is suggested. It is well known that ROT's can be reduced by using high speed exit. A few mathematical models which attempt to find exit locations to maximize the runway capacity have been developed in the past. These models assumed that the probabilistic landing behavior of aircraft were well known. This assumption, however, restricts usability of the models, since finding out the landing behavior requires tremendous empirical data analysis efforts. Instead of finding out the landing behavior empirically, the use of a dynamic simulation model which mimics aircraft landing behavior is suggested. The simulation model is sensitive to the influencing factors such as design exit speed, landing weight, runway surface condition, etc. The optimization model which is developed in the thesis receives the simulation results as input, and finds the optimal runway exit locations for an aircraft population, taking into account the proportions of each aircraft type.

An important difference between the previous models and the proposed model is that the latter model considers only an upper bound location associated with reliability for an aircraft
type instead of the whole range of the location distribution. To assign an aircraft to an exit with certain probability of exiting is beneficial because the variation of ROT of the aircraft decreases. And thus the runway landing capacity increases if the separation time between two landing aircraft is determined based on only ROT of the former aircraft. Another difference is that our model deals with an aircraft type separately based on the runway surface conditions. Since the landing movement of an aircraft type on dry surface is quite different from that on wet surface, it is desirable to consider each case separately.

In order to formulate the optimization model, the following assumptions are made:

1. Taxiway capacity is large enough that the landing aircraft is not affected.
2. The proportions of each aircraft type is known and does not change during the period being studied.
3. The airspace is saturated, hence the aircraft makes a landing as soon as the former aircraft turns off runway completely.
4. An aircraft is considered to miss an exit if its ground speed exceeds the design exit speed of the exit.
2.0 LITERATURE REVIEW

This research focuses on the enhancement of runway landing capacity by reducing runway occupancy time (ROT) of an aircraft fleet. ROT of an aircraft primarily depends on exit location and secondarily on exit geometry. The literature review conducted in this thesis concentrated on four important aspects:

1. Previous studies of the high speed exit geometry
2. Mathematical models for finding optimal exit locations
3. Theoretical aspects of Dynamic Programming (DP)
4. Network location problem

The third subject is included to provide a better understanding of the DP model which is developed in this research to find optimal exit locations for a general mixed aircraft population. The fourth subject is included because runway exit location problem can be interpreted as a special type of network location problem.
2.1 Previous Studies of the High Speed Exit Geometry

For a high speed exit, the turnoff geometry is very important as it should not cause aircraft mishaps nor impair passenger comfort during the turning maneuver. As the turning radius of turnoff geometry increases, the safety and the level of comfort are improved, however the time for aircraft to exit the runway completely also increases, which is a negative effect. Thus, it is desirable to make the turning radius as small as possible provided that safety and passenger comfort limitations are not violated.

Horonjeff et. al. (1952), performed extensive experiments to find the acceptable turning radius at a given exit speed. The results concluded that:

1. Safe and comfortable turnoffs can be designed for aircraft exiting at 60 to 85 mph
2. The exit speed, not the passenger comfort, is the most significant factor determining the turning radius
3. The exit speed should not be much faster than 60 mph

This study also suggested two centered curves for turnoff geometry with specifications shown in Table 2.1 and Figure 2.1.

In 1970, FAA (1970, AC 150/5335-1A) made standards of high speed exits, angled exits with 30° and 45°. The FAA employed single centered curves with 1800 ft radius for 30° angled exit designed for transport type aircraft, and with 800 ft radius for 45° angled exit used for small aircraft. Figure 2.2 and 2.3 show the geometry of FAA standard high speed exits. Suggested exit speeds for 30° and 45° angled exits are 60 and 40 mph, respectively. Later FAA modified the standards slightly (1983, FAA AC 150/5300-12), however the shape of exit geometry remained almost same.

LITERATURE REVIEW
Table 2.1 Turnoff Geometry Specification Table.

<table>
<thead>
<tr>
<th>Exit Speed (MPH)</th>
<th>Radius of Entrance Curve (ft)</th>
<th>Length of Entrance Curve (ft)</th>
<th>Radius of Central Curve (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>1724</td>
<td>189</td>
<td>821</td>
</tr>
<tr>
<td>50</td>
<td>2936</td>
<td>236</td>
<td>1282</td>
</tr>
<tr>
<td>60</td>
<td>3138</td>
<td>283</td>
<td>1846</td>
</tr>
</tbody>
</table>

Figure 2.1 Turnoff Geometry Specification Graph.
Source: Horonjeff et. al., 'Exit Taxiway Locations and Design', 1958
Figure 2.2 30° High Speed Turnoff Geometry.
Source: FAA AC 150/5335-1A, 1970

Figure 2.3 45° High Speed Turnoff Geometry.
Source: FAA AC 150/5335-1A, 1970
Schoen et al. (1965) investigated the turnoff trajectory of high speed taxiing aircraft in an isolated basis. The resulting shape of the turnoff was a variable curvature geometry. Since the turning radius of this geometry decreases continuously, a little gain on ROT is expected. The end result was a computer program to calculate the (x, y) coordinates of the geometry, considering exit speed and aircraft turning ability.

The most recent research of turnoff geometry was conducted by Aviation Department staffs of Dade County, Florida (Carr et al., 1980, Witteveen, 1987, and Haury, 1987). They tested various types of geometry, lighting, and marking in an L1011 flight simulator. Figure 2.4 is the resultant turnoff geometry which has much difference like a wider throat than standard high speed exit geometry. Figure 2.5 shows the differences of the standard geometry and suggested geometry. The shaded area is the standard geometry. The new geometry was implemented in Miami International Airport, Baltimore-Washington International Airport, and Orlando International Airport, and is expected to be constructed in Cincinnati International Airport and the new Denver International Airport (Witteveen, 1987 and Haury, 1987).

### 2.2 Mathematical Models for Optimal Turnoff Locations

The earliest effort to make a model for the optimal runway exit locating problem is found in Horonjeff et al. (1959, 1960) works. The objective of their model is to find exit locations which maximize the landing acceptance rate of a runway in the saturated situation, assuming the aircraft arrival pattern is fixed as fixed time separation or fixed distance separation. The saturated situation means that aircraft try to land continuously with a separation rule. The acceptance rate is determined by:

\[
E(A_o) = \frac{1}{E(\delta)} \cdot \frac{1}{(1 + q)}, \tag{2.1}
\]
Figure 2.4 A Modified High Speed Turnoff Geometry.

Figure 2.5 Comparison of the Standard and Modified High Speed Turnoff Geometry.
where \( E(A_c) \) = the expected acceptance rate (aircraft/hr),
\( E(\delta) \) = the expected interarrival time (hr/aircraft),
\( q \) = weighted average of 'wave off' probabilities.

A wave off occurs if the previous aircraft remains on the runway when the next aircraft reaches the runway threshold. The expected acceptance rate can be maximized by minimizing the weighted average of wave off probabilities which is expressed as:

\[
q = \sum_{i=1}^{n} p_i \cdot q_i, \tag{2.2}
\]

where \( p_i \) = the proportion of aircraft type \( i \),
\( q_i \) = the wave off probability of aircraft type \( i \),
\( n \) = the number of aircraft types.

\( p_i \) should provided to the model, and \( q_i \) is calculated by:

\[
q_i = Pr \{ T_i > \delta \}, \tag{2.3}
\]

where \( T_i \) = runway occupancy time of aircraft type \( i \).

Since \( T_i \) is a function of exit locations \( (D_1, D_2, \cdots, D_m) \), \( q \) can be expressed as a function of exit locations, and thus, by calculus, the optimal exit locations, which minimize \( q \), are found. The equation for \( T_i \) involves bivariate random variables, \( (d_i, t_i) \). \( d_i \) and \( t_i \) are the distance and the time for aircraft type \( i \) to decelerate to the predetermined exit speed, respectively. The differential equations for optimal exit locations are not simple, and cannot be solved analytically. Hence, finding the optimal locations requires a numerical computation algorithm which spends a lot of computational time.

The joint distributions of \( (d_i, t_i) \) of every aircraft type are another input data for the model. The values of \( d_i \) and \( t_i \) vary according to the operational factors such as the design exit speed.
and the landing weight, and environmental factors such as runway surface conditions, altitude of airport, and temperature, even though we consider only one aircraft type. Hence the parameters of the joint distribution of $d_i$ and $t_i$ should be estimated again if an influencing factor is changed.

In 1974, Daellenbach (1974) developed a dynamic programming model which is equivalent to the Horonjeff's approach with some extensions. Horonjeff's model imposes a strict assumption on the aircraft arrival pattern. Daellenbach released the assumption, and permitted a generalized arrival pattern. He showed his model is more efficient in computational aspect and more flexible for modelling than Horonjeff's model. Daellenbach's model, however, also requires the joint distributions of $(d_i, t_i)$ as input. The data for estimating the parameters of the joint distributions are difficult to collect and almost impossible when the influencing factors vary.

In the same year, Joline (1974) developed another dynamic programming model to find the optimal number of exits and their locations with respect to the combined objective function of ROT and exit construction cost. He incorporated the ROT gain and the exit construction cost into an objective function by equating one second gain in ROT with $100,000 in construction cost. While Horonjeff's model and Daellenbach's model require the joint distributions of $(d_i, t_i)$ for each aircraft type, Joline's model needs only an univariate distribution of 'ideal exit location' for a mixed aircraft population. Joline classified aircraft into three categories based on the aircraft size, and found the distributions of ideal exit locations for these three aircraft classes based on the observations of aircraft landing operations at Chicago O'Hare Airport. The ideal exit location distribution for the entire aircraft population is found by combining the three distributions according to the proportions of the three aircraft classes. As mentioned earlier, there are several factors influencing the aircraft landing distance such as the design exit speed, landing weight, etc.. Joline's model, like the previous ones, makes the effects of these influencing factors hard to be incorporated.
ROT consists of the time from the runway threshold to the exit location and the time from the beginning of the turn to the clearance of the runway. The second term obviously varies according to turnoff trajectory. None of three models above, however, takes into account the ROT variation due to the change of the turnoff trajectory. The turnoff trajectory also varies according to design exit speed, aircraft turning ability, runway surface conditions, etc. Therefore an attempt is made in this study to bridge the gap.

The three models above implicitly assume that an aircraft type can use more than one exit for turnoff with different ROT and exiting probability. If we want to decide separation time between the landing aircraft based on the ROT of the aircraft, it is desirable to assign an aircraft to an exit with high exiting probability, say 95%. This situation is expected to occur in the near future with improvements of aircraft traffic control system and aircraft technology.

In 1985, Tosic et. al. proposed a model regarding the runway exit locations. The model, however, focuses on the minimum taxiing distance from the runway to the terminal area, which is irrelevant subject with the effort to increase the runway capacity by reducing the runway occupancy time.

2.3 Dynamic Programming

Dynamic programming (DP) is a mathematical technique to analyze sequential decision process, or systems that can be thus modelled. That is, the DP approach can be applied to time-sequenced decision problem such as periodic replenishment of inventory and annual investment plan, or to a ‘simultaneous optimization over n decision variables’ problem like an ordinary linear programming (LP) problem by breaking it into ‘n sequential optimization’ problem, if possible. For the decomposition case, a great concern should be given to the
conversion lest the optima of the ‘pieces’ do not yield the desired optimum of the ‘whole’ [Elmaghraby, 1989].

Dynamic Programming is a way of problem solving without a standard mathematical formulation in contrast to LP. Hence, the particular formation should developed for each problem [Hiller and Lieberman, 1988]. Any DP formulation includes, in general, the following elements:

1. Stage(s)
2. State(s)
3. Decision(s)
4. State transformation function
5. Immediate (stage) return function
6. Global return function

Dynamic Programming problems can be divided into finite or infinite number of stages at which the decisions are made. Each stage has a state or a set of states associated with it. The state(s) should contain all information necessary to make current decision(s). If a decision is made, the state of the current stage is transformed to the state of the next stage. The mapping governing this transformation is referred as state transformation function. The decisions made at each stage determine the return (or cost) of the stage. The incremental return occurred at each stage is referred as immediate (or stage) return. The return of a stage is defined as a function of state and decision of the stage, or a function of state, decision, and stage itself. The total return resulting from individual immediate returns is referred as global return. The global return function which is referred as objective function in other optimization techniques has a recursive property. That is, the global return function of stage \( n \) can be expressed as a function of global return function of stage \( n-1 \) and immediate return function of stage \( n \).
An important property of DP is the principle of optimality which ensures that DP approach finds the optimal solution correctly. The principle of optimality is formally expressed as: "Given the current state, an optimal policy for remaining stages is independent of the policy adopted in previous stages" [Hiller and Lieberman, 1986]. For this principle to hold, the global return function should be: 1) separable by the immediate returns and 2) monotone non-decreasing (or non-increasing) with respect to the stages. More rigorous development of the principle of optimality and illustrative examples are found in Denardo, 1982 and Elmaghraby, 1989.

2.4 Network Location Problem

When new facilities are to be constructed on a network, a question arises naturally: where should they be located? Network location problem concerns the question. Tansel et al. [1983 a, b] surveyed the studies regarding this problem and categorized them from several viewpoints. The problem is divided into point location and path location problem according to the characteristics of the facilities to be constructed; into single-objective and multi-objective problem according to the number of objective functions; into minisum and minimax problem according to the type of objective function, and so on.

We discuss more about point location, single-objective, minisum problems, because the runway exit location problem can be included in this category. Hakimi [1965] named this category as the p-median problem. P-median problem concerns with finding p locations for new facilities on a network to serve existing facilities with minimum total cost (distance). Locating warehouses on a road network to serve market areas with minimum travel distance is an example of the formulation.
Although the optimal runway exit location problem is a continuous optimization problem by its nature, it can be converted to a discrete optimization problem as described in Chap. 4. The solution space consists of primary candidates which are the best locations for each aircraft of fleet, and secondary candidates which are generated based on the primary candidates and minimum separation distance of two adjacent exits. By matching the exits to be constructed as new facilities, the runway as a network whose nodes are the exit candidates, and the aircraft fleet as the customers which should be served by new facilities, and which is located at runway threshold (or origin of the network), the optimal exit location problem becomes a special type of network location problem.

Erkut et. al. (1989) demonstrated that a specific type of network location problem could be formulated as a equivalent mathematical programming problem. That is, if a network has a tree structure, and the cost function is monotonic, then the network location problem can be formulated as linear programming problem, and thus the knowledge accumulated in mathematical programming area can be transferred to the network location area. However, the runway exit locating problem is not an above case, because in the runway exit locating problem, cost function is not monotonic, and minimum separation constraint between two adjacent exits is imposed.
3.0 THE SIMULATION MODEL OF INDIVIDUAL AIRCRAFT LANDING

The purpose of the simulation is to find the location and the time at which an aircraft decelerates to the predetermined design exit speed. The simulation model keeps track of the position and the time data from the runway threshold. Figure 3.1 shows the model of landing sequence which the simulation employs. The model divides the landing sequence into five phases:

1. Glide phase
2. The first free roll coasting phase
3. Braking phase
4. The second free roll coasting phase
5. Turnoff phase

After passing the runway threshold, an aircraft flies over the runway to touchdown with inherent air drag deceleration due to a flare maneuver (phase 1). The aircraft coasts on the runway for a few seconds with a constant speed (phase 2). The pilot begins to decelerate until
Figure 3.1 The Model of Aircraft Landing Sequence and Corresponding Velocity/Time Graph
the speed reaches a design exit speed (phase 3). The aircraft coasts on the runway for a few seconds at the desired exit speed before taking a turnoff (phase 4). And then, the aircraft begins to turn (phase 5). The ROT of the aircraft is the time passed from the runway threshold to the clearance of the runway. The simulation model calculates the mean and the standard deviation of the location where the aircraft finishes the phase 4, and calculates the mean and the standard deviation of the time when the aircraft completes the phase 5.

3.1 Glide Phase

The air distance is estimated indirectly from the basic aircraft geometric and performance characteristics. These characteristics are used to estimate the approach ($V_s$) and touchdown speeds ($V_{TD}$) for each aircraft selected by the user. Once the approach speed (or reference speed) is known, an estimation of the air distance can be made assuming a circular arc flare maneuver flown at constant load factor to transition from a constant rate of descent flown at constant descent flight angle $\gamma$ on final approach to a flat flight path tangent to the the runway. An analytical expression for the air distance can be found by equating the changes in kinetic and potential energy of the aircraft near the ground with the product of a retarding force $F_s$ and the air distance $S_{air}$ as shown in Eq. 3.1 [Nicolai, 1975; Torenbeek, 1981; Roskam, 1985].

$$S_{air} = \frac{H_{thres}}{\gamma} + \frac{V_{flare}^2}{2g(\gamma_{flare} - 1)} \tag{3.1}$$

where, $V_{flare}$ is the flare speed (taken as 95% of the approach speed), $\gamma$ is the effective descent flight path, $H_{thres}$ the threshold crossing altitude. For preliminary analyses the flare load factor has been set conservatively to 1.15 g's and $\gamma$ to 3 degrees to simulate a regular ILS approach flight path. Currently the dispersions in the air distance are set internally to fixed values that
depend upon the aircraft category being analyzed. The aircraft categories used in this research are consistent with those implemented in the FAA Terminal Operating Procedures (TERPS) and defined in Table 3.1. The underlying assumption in this respect is that slower aircraft will usually experience smaller touchdown dispersions than those of faster aircraft in absolute distance terms (this is not in contradiction to the fact that transport-type pilots might be more accurate in terms of touchdown point standard deviations). Actual measurements of lateral and longitudinal landing dispersions for transport-type aircraft made by Hosang [Hosang, 1975] suggest that for manual control landings the average touchdown dispersion (i.e., standard deviation) is about 171 meters (550 feet). Although little data is available in actual instrument meteorological conditions (IMC) it has been found that reduced touchdown dispersions prevail under this circumstances. Typical standard deviations under IMC conditions range between 107-76 meters (350-250 feet).

The advantage in estimating air distances relying on information pertaining each aircraft is two-fold: 1) frees the analyst from relying on field data for a particular aircraft that in most cases is not available or which could be implemented at a later stage for calibration of the model, 2) introduces more realistic variabilities in the touchdown locations for the entire landing aircraft population instead of assigning a fixed touchdown location to an entire aircraft category population. The method is also sensitive to specific airfield scenarios since more parameters have been accounted for. For example, short takeoff and landing aircraft can be assigned independently different values for the flare load factor and descent flight angle as they occur in practice thus affecting accordingly the air distance values estimated internally. The time consumed in the air phase ($T_{air}$) is a function of the touchdown location ($S_{air}$), the approach speed ($V_{app}$), and the touchdown speed ($V_{td}$). Assuming a normal distribution for the aircraft touchdown location, $T_{air}$ and its corresponding variance, $\sigma_{air}^2$, are given as follows:
### Table 3.1 Aircraft Approach Category Classification

Source: FAA, 1988

<table>
<thead>
<tr>
<th>Category</th>
<th>Landing Speed ($1.3 \cdot V_{\text{Stall}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>less than 91 Knots</td>
</tr>
<tr>
<td>B</td>
<td>From 91 to 120 Knots inclusive</td>
</tr>
<tr>
<td>C</td>
<td>From 121 to 140 Knots inclusive</td>
</tr>
<tr>
<td>D</td>
<td>From 141 to 165 Knots inclusive</td>
</tr>
<tr>
<td>E</td>
<td>166 Knots and higher</td>
</tr>
</tbody>
</table>

\[
T_{\text{air}} = \frac{2 \cdot S_{\text{air}}}{V_{\text{ap}} + V_{\text{td}}} \tag{3.2}
\]

\[
\sigma_{T_{\text{air}}}^2 = \left( \frac{2}{V_{\text{ap}} + V_{\text{td}}} \right)^2 \sigma_{S_{\text{air}}}^2 \tag{3.3}
\]

where, \( V_{\text{ap}} \) and \( V_{\text{td}} \) are the approach and touchdown speeds, respectively.
3.2 Free Roll Phases

Free roll distances arise in the aircraft landing operation at two different times: 1) between the air distance and the braking stage, \(T_{FR1}\), and 2) between the braking segment and the turnoff maneuver, \(T_{FR2}\). The first free roll distance tries to simulate an inherent human delay to arm and apply braking mechanisms such as thrust reversers, spoilers, or normal wheel braking. A conservative average value of three seconds has been allocated for this transition stage with a typical standard deviation of one second. The corresponding free roll distance \(S_{FR1}\) and its variance \(\sigma^2_{S_{FR1}}\) are as follows:

\[
S_{FR1} = V_{id} T_{FR1} \tag{3.4}
\]

\[
\sigma^2_{S_{FR1}} = V_{id}^2 \sigma^2_{T_{FR1}} \tag{3.5}
\]

Note that any reductions in aircraft speed during wheel spin-up have been neglected for the sake of simplicity.

The second transition segment tries to mimic a delay time arising from the proper suppression of braking action and a recognition time of the turnoff geometry prior to exiting the runway. Under all visibility conditions there is a delay time associated with the recognition of a high-speed turnoff and the decision of whether or not the current aircraft state (i.e., speed, braking status, etc.) are appropriate to negotiate the turn. The user has complete freedom to specify this delay time \(T_{RC}\). A nominal value of 2 seconds has been allocated for this parameter as a representative value under clear air and unlimited visibility conditions (CAVU). However, the analyst could increase this value accordingly to simulate low visibility scenarios. The end result being a correspondingly higher value for the total runway occupancy time (ROT), with a downrange displacement of the turnoff location.
A complementing assumption in this analysis is that free roll time, $T_{FR2}$ and its variance $\sigma^2_{T_{FR2}}$ are known. Then, the second free roll distance $S_{FR2}$ and its variance $\sigma^2_{S_{FR2}}$ are:

$$S_{FR2} = V_{brake_n} T_{FR2} = V_{exit} T_{FR2}$$  \hfill (3.6)$$

$$\sigma^2_{S_{FR2}} = \{V_{brake_n}\}^2 \sigma^2_{T_{FR2}}$$  \hfill (3.7)$$

where, $V_{brake_n}$ is the last braking speed integrated in the braking stage and $V_{exit}$ is the aircraft exit speed selected by the user.

### 3.3 Braking Phase

Under normal landing conditions, the braking segment constitutes the largest component of the Runway Occupancy Time (ROT). As such, it becomes necessary to estimate with some accuracy the braking distance if one is to have some confidence in the total distances covered by the aircraft on the ground. The previous requirement also stems from the incorporation of airport specific variables such as local runway slope and its effect on aircraft deceleration characteristics. The problem seems to be complicated by the fact that many aircraft parameters necessary to determine the forces and moments acting on the aircraft as it brakes are not only time dependent (e.g., thrust reverse forces, braking forces, parasitic drag contributions, etc.) but also aircraft specific in most instances (e.g., small reciprocating aircraft generally do not have thrust reverse capability whereas turbofan and large turbopropeller-driven aircraft do). The dilemma is then to use a model that will provide an accurate answer without going into the sophistication and computing expense of a higher-order model (i.e., 6-DOF model).
The braking algorithm used in the model integrates the local deceleration rate of the aircraft as it travels along the runway. The local deceleration is estimated from the runway initial conditions specified by the user in the Input portion of the program. At the same time a deceleration multiplier is computed throughout the integration process in order to correct the nominal aircraft deceleration due to local variations of runway slope. This simplistic model then treats the aircraft as a second order point mass model whose resultant deceleration is integrated forward in time to obtain the velocity/distance aircraft state at all points in time. The assumption of a constant uncorrected deceleration rate is justifiable if one realizes that in modern aircraft the deceleration rate is controlled by an antiskid system. The question is how we can estimate the deceleration rate for either each aircraft or for the entire aircraft population from the known runway conditions. As the reader recalls there are two different scenarios defined in the runway friction characteristics. The correlation of actual aircraft data is done backwards to estimate the necessary friction coefficient and its corresponding deceleration rate necessary to match the data published for some known conditions such as those corresponding to the aircraft maximum allowable landing mass (MALW) under dry pavement conditions [Janes's, 1988; Aviation Week & and Space Technology, 1988; Business and Commercial Aviation, 1988]. The wet condition braking analysis is performed with the introduction of a deceleration degradation multiplier, $c_f$, into the baseline deceleration equation (Eq. 3.9). The multiplier has been derived using NASA and ICAO empirical data [Yager and White, 1981; ICAO, 1986].

A second correction multiplier is also introduced in this analysis to correct the instantaneous deceleration due to variations in the local runway slope. The evaluation of this multiplier has been done outside the current REDIM Model using complete drag/thrust data for a Boeing 727-200 and for simplification purposes it is assumed to be constant for all the aircraft population.
\[ S_{brake} = \sum_{i=1}^{n} dt V_{brake_i} \]  

(3.8)

Furthermore, decomposing \( V_{brake_i} \) as a function of the instantaneous aircraft deceleration \( (a_{act}) \) and the deceleration correction factors for runway friction \( (cf_r) \) and runway slope \( (cf_s) \) we obtain,

\[ S_{brake} = (dt) (n) (V_{act}) + (dt)^2 \sum_{i=1}^{n} (n + 1 - i) \ cf_r f_c \ cf_s \ a_{act} \]  

(3.9)

where, \( n \) is the number of iterations computed in the simulation of the braking process and is determined when the \( V_{brake_i} \) is less than the mean exit speed \( V_{exit} \) specified by the user. The computation of the variance of \( S_{brake} \) denoted by \( \sigma_{S_{brake}}^2 \) is estimated as follows,

\[ \sigma_{S_{brake}}^2 = \left( (dt)^2 \sum_{i=1}^{n} (n + 1 - i) \ cf_r f_c \ cf_s \right)^2 \sigma_{a_{act}}^2 \]  

(3.10)

where, \( \sigma_{a_{act}}^2 \) is the variance of the deceleration rate which assumed as a constant fraction of deceleration and \( dt \) is the simulation step size. Note that the time consumed in the braking process and its variance are estimated according to Eq. 3.11.

\[ T_{brake} = \frac{S_{brake} (n)}{\sum_{i=1}^{n} V_{brake_i}} \quad \sigma_{T_{brake}}^2 = \frac{\sigma_{S_{brake}}^2}{(V_{brake})^2} \]  

(3.11)

THE SIMULATION MODEL OF INDIVIDUAL AIRCRAFT LANDING
3.4 Turnoff Phase

The turnoff algorithm integrates the aircraft path throughout the exit maneuver. The exit maneuver is initiated when the aircraft reaches the user-defined exit speed and finalizes with the complete clearance of the runway by the landing aircraft as shown in Fig. 3.2. In order to simplify the number of inputs to the model it is assumed that the aircraft wingtip point controls the time to clear the runway. This is generally true for all aircraft exiting at high speed. Exceptions to this rule are small aircraft and Short Takeoff and Landing Aircraft STOL (i.e., requiring abnormally large tailplane wingspans) exiting at low speed (e.g., less than fifteen meters per second). However, since the objective of this research is the investigation of high-speed turnoffs these exceptions would seldom occur and therefore the prediction of the clearing point can be done adequately with a single aircraft control point.

The characteristic motion of an aircraft turning at speeds where insignificant aerodynamic control can be exerted by conventional primary aerodynamic surfaces is simplified to the forces acting on the nose landing gear. An algorithm developed by Schoen et al. [Schoen et al., 1985] and used in a previous NASA research effort on this topic considers three side force contributions acting on the aircraft nose landing: 1) the centripetal force, 2) the aircraft inertia, and 3) the tire scrubbing resistance to the turn. Mathematically, the nondimensional contributions to the nose gear are,

\[ \mu_{\text{skid}} = \mu_{\text{le}} + \mu_c + \mu_{\text{sc}} \]  \hfill (3.12)

where, \( \mu_{\text{skid}} \) is the nose gear tire skid friction coefficient, \( \mu_{\text{le}} \) is the aircraft inertia contribution term to the nose gear side load, \( \mu_c \) is the centripetal acceleration contribution, and \( \mu_{\text{sc}} \) is the tire scrubbing resistance. Each of these contributions is calculated as follows:
\[ \mu_{az} = \frac{l_{zz} \alpha}{m \ g \ wb \ lm/100 \ (1 - lm/100)} \]  \hspace{1cm} (3.13)

It is noted from this equation that the term \(m \ g \ (1 - lm/100)\) represents the aircraft weight supported by the nose gear whereas \(wb \ (lm/100)\) is the moment arm from the aircraft center of mass to the nose gear.

\[ \mu_c = \frac{V^2}{g \ R} \]  \hspace{1cm} (3.14)

\[ \mu_{sc} = f(R, m) \]  \hspace{1cm} (3.15)

where, \(l_{zz}\) is aircraft moment of inertia about the vertical axis, in Kg-m-m, \(\alpha\) is the angular acceleration (rad/sec.) of the aircraft fuselage as it executes the turning maneuver, \(wb\) is the aircraft wheelbase (meters), \(lm\) is the aircraft mass supported by the main gear (in percent), \(g\) is the gravitational constant (m/sec-sec), \(m\) is the total aircraft mass (Kg.), \(V\) is the instantaneous speed (m./sec.) of the nose gear, and \(R\) is the instantaneous radius of the curve (m.).

Further breakdown of the angular acceleration yields for Eq. 3.13 the following,

\[ \mu_{az} = \frac{l_{zz} \left( \dot{V} R - V \dot{R} \right)}{m \ g \ wb \ lm/100 \ (1 - lm/100)} \]  \hspace{1cm} (3.16)

where, \(R\) represents the rate of change of the turning radius of curvature, \(\dot{V}\) if the instantaneous velocity rate of change of the nose gear, and \(R\) and \(V\) are the state variables of our system. A further simplification can be introduced if the term \(\dot{V} R\) is neglected on the grounds of very small values for the deceleration rate through the turnoff maneuver. Fact that has been found true in the empirical studies of Horonjeff and Hosang.

\[ \mu_{az} = \frac{l_{zz} \left[ -V \dot{R} \right]}{R^2 \ m \ g \ wb \ lm/100 \ (1 - lm/100)} \]  \hspace{1cm} (3.17)
solving for the rate variable, $\dot{R}$ and integrating over time it is possible to estimate the state variables of the motion,

$$\dot{R} = \frac{\mu_{l2} R^2}{\mu_{l2} V} \frac{m g \text{wb}}{100} \left(1 - \frac{1}{100}\right)$$ (3.18)

$$R_t = \int_0^t \dot{R} \, dt$$ (3.19)

$$X_t = \int_0^t V \cos(\psi) \, dt$$ (3.20)

$$Y_t = \int_0^t V \sin(\psi) \, dt$$ (3.21)

where, $X$ and $Y$ are the position coordinates of the vehicle as it progresses into the turn and $\psi$ is the heading angle that the nose gear makes with a global axis system centered about initial position of the turnoff path (Fig. 3.2).

It should be noticed that this simplification applies only to speeds up to about two thirds of the landing speed ($V_{\text{sa}}$) as this is known to be the threshold for significant aerodynamic control for conventional aircraft [Miller and Thomas, 1963]. Even with this restriction, the evaluation of runway turnoffs can be accomplished for a large range of aircraft speed values ranging from 10 to 45 m/sec. (22.3-100.4 MPH) for transport-type category aircraft. Turnoff designs above 45 m/sec. (100.4 MPH) are probably unlikely due to lateral space limitations following the turn. The inclusion of the lifting forces acting on the aircraft at high speed can be added to Eqn. 3.18 replacing the mass term by an equivalent force that accounts for the potentially large lifting forces experienced at high speeds.
Figure 3.2 Runway Clearance Point Nomenclature.
\[ \dot{R} = \frac{\mu_{lz}}{l_{zz}} R^2 \left( m - 0.5 \rho V^2 S C_l \right) g \text{ wb } lm/100 \left( 1 - lm/100 \right) \]  \tag{3.22} \\

where, \( \rho \) is the air density, \( C_l \) is the aircraft lift coefficient in ground effect and the landing flap configuration, \( S \) is the wing area and \( V \) is the aircraft speed.

The aforementioned algorithm has been modified in order to account for the large variations in skid friction coefficients observed for a large aircraft population. It is well documented in the literature that the skid friction coefficient is a function of aircraft tire pressure and aircraft speed, among other variables [Harrin, 1958; Wong, 1978]. A summary of this functional relationship is depicted graphically in Fig. 3.3 where four aircraft tire pressures are represented in this figure and they correlate well with the four different aircraft categories modeled in this research. The upper curve corresponds to a tire pressure of 50 PSI (pounds per square inch) and is representative of the characteristics of TERP A category aircraft. Similarly, the lower curve represents a 200 PSI tire pressure typical of current heavy aircraft (i.e., Boeing 747, DC-10, L-1011, etc.). In order to complement this algorithm a small forward deceleration rate can be introduced in order to account for the small speed losses expected while turning. The rolling friction opposing the motion of the aircraft on the ground introduces a deceleration rate proportional to the product of \( g \) and \( f_{roll} \) where this last term is the coefficient of rolling friction. For the sake of simplicity \( f_{roll} \) is taken constant with speed although it known to vary with tire speed as well. A typical value of .03 is used for \( f_{roll} \) for the base model.

\[ a_{roll} = g \ f_{roll} \]  \tag{3.23} \\

An Euler first-order integrating scheme is used to solve numerically the aircraft equations of motion through the turnoff maneuver. The time spent on the turn, \( T_{turn} \) is considered to be deterministic in nature. A baseline step size of one hundredth of second was found to offer accurate results within the desired computational time limitations for the program. The accuracy of the method is evident from Fig. 3.4 where the first-order solution is compared with
an equivalent Runge-Kutta fourth-order scheme. These solutions were obtained using the geometric and performance parameters of a Boeing 727-200 and as can be seen from the turnoff paths generated the results are within one half of a percent of each other (i.e., less than half a meter difference between both solutions at the end of a high-speed turnoff for a medium size transport aircraft). Another justification for the Euler algorithm was the desired accuracy in stopping the simulation as closely as possible to the runway clearance point (Fig. 3.2). With the current step size it is possible to ascertain the turnoff time (TOT) and the lateral range distance within very small windows, .01 seconds or .15 meters, for an aircraft traveling at 30 m/sec. (67.2 MPH) and reaching the runway clearance point with up to 30 degrees of total heading change.

The aircraft position coordinates in the turn ($X_{path}$ and $Y_{path}$), the aircraft speed ($V_{path}$), and the aircraft instantaneous heading ($\psi$) constitute state variables through the turnoff maneuver (Eqns. 3.20-22). These states are integrated forward in time to assess the instantaneous turning radius ($R_{path}$) and ultimately estimating the position changes experienced by an aircraft as a high-speed turnoff is negotiated. The aircraft is considered to have cleared the runway when its right wingtip has traveled the lateral distance equivalent to the sum of one half the runway width and the aircraft midspan. Once the turnoff path and times are estimated it is possible to ascertain the time from threshold crossing to the end of the turnoff maneuver. Since some of the distances and times involved in the process are random variables the net effect is that runway occupancy time (ROT) and the total distance to initiate the turnoff are both probabilistic in nature.

\[
S_{\text{turn}} = S_{\text{air}} + S_{\text{FR1}} + S_{\text{brake}} + S_{\text{FR2}} \tag{3.24}
\]

\[
\sigma_{S_{\text{turn}}}^2 = \sigma_{S_{\text{air}}}^2 + \sigma_{S_{\text{FR1}}}^2 + \sigma_{S_{\text{brake}}}^2 + \sigma_{S_{\text{FR2}}}^2 \tag{3.25}
\]

\[
T_{\text{turn}} = T_{\text{air}} + T_{\text{FR1}} + T_{\text{brake}} + T_{\text{FR2}} \tag{3.26}
\]
Figure 3.3 Skid Friction Variation with Tire Pressure and Speed.

Figure 3.4 Comparison of Track Simulation Results Using two Integrating Methods.
\[
\sigma^2_{\text{ROT}} = \sigma^2_{\text{ROT}} + \sigma^2_{\text{path}} + \sigma^2_{\text{turn}}
\]

\[
\text{ROT} = T_{\text{turn}} + T_{\text{path}}
\]

where, \( S_{\text{turn}} \) is the distance from the threshold to the initiation of the turnoff (i.e., exit location distance), \( \sigma^2_{\text{turn}} \) is the variance of this previous parameter, \( T_{\text{turn}} \) and \( \sigma^2_{\text{turn}} \) are the time consumed from threshold to the initiation of the turnoff and its corresponding variance, and \( \text{ROT} \) is the total runway occupancy time for a single aircraft with variance \( \sigma^2_{\text{ROT}} \).

### 3.5 Airport Environmental Variables

The airport environmental variables were defined in a single screen bearing a similar name. The environmental characteristics of interest are: 1) wind speed (NSPEED), 2) wind direction (WDIR), 3) airport elevation (AIRELV), 4) airport temperature (AIRTEMP), 5) runway orientation (RUNOR), 6) runway visual range (RVR), 7) runway width (RUNWID), and 8) distance to nearest taxiway (DISTT).

The wind vector is used in conjunction with the runway orientation to estimate the longitudinal and lateral wind components affecting aircraft operations. The longitudinal wind component affects the landing speeds of the aircraft population and as such has a direct impact in the runway occupancy time and turnoff locations. Regarding the use of a single wind vector as input to the model, the user is urged to execute the baseline program under the average prevailing wind conditions at the airport facility just as he/she would do under the average prevailing temperature.
Temperature and airfield elevation have a direct impact in the performance of the aircraft in the air and on the ground. Changes to the aircraft equivalent airspeed (EAS) due to temperature and field elevation can have large impact in the ROT and the turnoff location parameters as will be seen in Chapter 6 of this report. The model converts equivalent speeds (EAS) to true air speeds (TAS) to estimate the stalling ($V_{stal}$) and approach speeds and ultimately predict the aircraft landing roll performance. The runway width and runway distance to nearest taxiway are included in this set of parameters in order to estimate the time spent on the turnoff maneuver by each aircraft.

### 3.6 Aircraft Characteristics

The aircraft characteristics used in the model are shown in Table 3.2. These are necessary to estimate the aircraft performance on the ground as well as in the flare maneuver. The aircraft mass, wing area, and the maximum landing lift coefficient dictate the approach speed and hence affect the ROT and exit location. It is also used to estimate the second moment of inertia of the aircraft around the vertical axis ($I_{zz}$) ultimately influencing the turning aircraft capabilities through an exit. Roskam [Roskam, 1985] suggests a logarithmic relationship between these two parameters which seems to correlate very well for all aircraft TERP categories. The regression equation in metric units is shown in Eq. 3.2 where the aircraft mass is given in kilograms and the moment of inertia in kg-m-m.

The aircraft wheeltrack (ACFWT) is used to estimate the maximum track-in distance present during the turnoff maneuver. The track-in distance is defined as the perpendicular distance measured from the geometric center of the aircraft main gears to the imaginary path followed by the nose gear. Track-in distances are used to assure a sound geometric design of the
high-speed turnoff. It should be pointed out that in general track-in distances tend to be relatively small for very high speeds (i.e., > 30 m/sec.) However, for large aircraft and medium speeds they should be considered in the geometric design. Fig. 3.5 shows graphically the nomenclature used to model the aircraft kinetic behavior including the estimation of the track-in distance. As the dynamic simulation executes a sample record of the main gear position \((X_m, Y_m)\) is kept and the track-in distance estimated. A simple sorting routine searches for the largest value of track-in and this one is later transferred to the output module to calculate the corresponding turnoff geometry that satisfies the kinetic constraints of the turnoff track. As usual, a safety distance is selectively used to estimate the distance from the centerline of the the turnoff track to the edge of the pavement. No judgmental oversteering is implied in the program as this is certainly not recommended for an aircraft traveling at high-speed on the ground.

Another geometric parameter included here is the distance from the aircraft nose gear to the imaginary plane passing through the airplane wingtips. This distance is used as the controlling point to ascertain whether or not the aircraft has cleared the runway. A graphical description of some of these parameters is seen in Fig. 3.6.

\[
l_{zz} = \text{Antilog}_{10} \left(1.7215 \log_{10} (m) - 1.6730\right)
\]  

(3.29)
<table>
<thead>
<tr>
<th>Name</th>
<th>Variable</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft Mass</td>
<td>ACFMASS</td>
<td>Max. Landing Mass (Kg.)</td>
</tr>
<tr>
<td>Aircraft Wheelbase</td>
<td>ACFWB</td>
<td>in meters</td>
</tr>
<tr>
<td>Aircraft Wheeltrack</td>
<td>ACFWT</td>
<td>in meters</td>
</tr>
<tr>
<td>Aircraft Load on Main Landing Gear</td>
<td>ACFLM</td>
<td>At aft C.G. (%)</td>
</tr>
<tr>
<td>Aircraft Wing Area</td>
<td>ACFWA</td>
<td>Gross wing area (sq. m.)</td>
</tr>
<tr>
<td>Aircraft Maximum Lift Coefficient</td>
<td>ACFCL</td>
<td>At max. flap setting (dim.)</td>
</tr>
<tr>
<td>Distance from Nose Gear to Wingtip</td>
<td>NWTIP</td>
<td>in meters</td>
</tr>
</tbody>
</table>
Nomenclature

(Xcg, Ycg) Aircraft Center of Gravity Coordinates
(Xm, Ym) Coordinates of Main Landing Gear Geometric Center
(Xml, Yml) Left Main Landing Gear Position Coordinates
(Xmr, Ymr) Right Main Landing Gear Position Coordinates
Castor Castor Angle
V Aircraft Velocity Vector (at Center of Gravity)
Track-In Track-In Distance (from Nose Gear to Main Gear Geometric Center)

Figure 3.5 Aircraft Kinetic Behavior Nomenclature.

THE SIMULATION MODEL OF INDIVIDUAL AIRCRAFT LANDING 40
Figure 3.6 Aircraft Geometric Characteristics.

ACFWS  Aircraft Wing Span
ACFWB  Aircraft Wheel Base
NWTPLANE  Distance from Nose Gear to Wing Tip Plane
ACFWT  Aircraft Wheel Track
4.0 OPTIMIZATION MODEL FOR MIXED AIRCRAFT FLEET

The dynamic simulations of aircraft landing movements calculate the best turnoff locations for each aircraft in both dry and wet runway surface conditions. The best turnoff location is defined as the nearest location from the runway threshold where the aircraft decelerates to the pre-specified desirable exit speed with the pre-specified reliability. If the aircraft reduces its speed to the pre-specified exit speed before reaching the assigned turnoff location, the aircraft will be considered to exit the runway successfully. Reliability is defined as the probability that the aircraft exits the runway successfully. For example, if the reliability is specified as 90%, 90 aircraft out of 100 landing attempts will exit the runway successfully.

If an exit is constructed at the best turnoff location for an aircraft, the runway occupancy time (ROT) of the aircraft will be minimized without sacrificing the reliability. Though some exits constructed ahead of the best turnoff location can produce less ROT, it is not permissible to assign the aircraft to these exits, since reliability must be sacrificed.
Suppose there are five aircraft in consideration, the simulations of the aircraft landing movements will provide ten different turnoff locations for each aircraft and two runway surface conditions. The goal of an optimization algorithm is to find a few locations (e.g. 2 or 3) at which all the aircraft in consideration can exit the runway with the minimum weighted sum of ROT. Since each aircraft and each surface condition can have different relative frequency, the weighted sum of ROT should be minimized instead of total ROT. Figure 4.1 illustrates the best turnoff locations and their relative frequencies. Let \( l_i \) and \( w_j \) represent the best turnoff location and relative frequency for aircraft \( i \) and surface condition \( j \).

The optimization procedure in REDIM conducted with the following steps:

1. Generate the complete set of candidate locations.
2. Calculate the ROT of every aircraft for each candidate locations.
3. Find the optimal location(s) out of the candidates.
4. Assign aircraft to the optimal locations.

Step 1 and 2 are the data preparation for step 3 which is the mathematical optimization. The optimization in step 3 employs a dynamic programming technique. Step 4 is the interpretation of the optimization results into a practical solution. A flow chart of the optimization is depicted in Figure 4.2. The notations used in the flow chart are explained in the following sections.

### 4.1 Generation of a Complete Set of Candidates

Finding optimal turnoff locations is a continuous optimization problem. That is, an optimal turnoff location can be at any place on the runway. Fortunately, the theorem 1 of Appendix B
Figure 4.2 Optimization Procedure Flow Chart.
shows that the optimal solutions can be found by searching through a finite set of potential
turnoff locations. This set of potential solutions consists of two types of candidates: 1) primary
and 2) secondary candidates. Primary candidates are the best exit locations for each aircraft,
which are found during the simulations of individual aircraft landings. Secondary candidates
are exit locations for each aircraft i and surface condition j located at discrete distance, \( D_{\text{min}} \),
away from a primary candidate. Secondary candidates are generated as follows:

\[
l_{ij} = l_{ij} + k \times D_{\text{min}},
\]

where \( l_{ij} \) is the primary candidate for aircraft i, surface condition j.

\[
k = 1, 2, \ldots
\]

Under current FAA runway turnoff standards two adjacent turnoffs do not affect the runway
exit index unless they are separated by 229 m (750 ft.) from each other. In REDIM we have
added more flexibility by allowing the user to define the desired minimum distance \( D_{\text{min}} \) be-
tween adjacent exits. The primary and secondary candidates comprise the complete candi-
date set. Suppose a runway scenario with three aircraft in consideration, and the best
locations for each aircraft and two pavement conditions are \((1000, 1100, 1300, 1400, 1600, 1700)\).
In this example, the complete candidate set would be \((1000, 1100, 1229, 1300, 1329, 1400, 1458,
1529, 1558, 1600, \ldots, 1700, \ldots)\). Theorem 1 of Appendix A guarantees that the optimal lo-
cations should be some of the candidate set. Thus we need to examine only the candidate set
to find optimal locations instead of examining infinite points on the runway.

4.2 Estimation of Runway Occupancy Times

The simulations of landing movements provided the best exit locations for each aircraft con-
sidering the design exit speed, and the corresponding times required to reach those exit lo-
cations. The complete candidates were enumerated. The next step is to find out the time required to clear runway for every aircraft for every candidate. The time is denoted as \( T_{ijk} \).

That is, \( T_{ijk} \) is defined as the runway occupancy time when aircraft \( i \) takes turnoff candidate \( k \) on surface condition \( j \). Every \( T_{ijk} \) has three components, which are: 1) time to reach the best exit location ( \( T_b \) ), 2) time to travel from the best location to the candidate ( \( T_n \); subscript 'fr' stands for 'free roll'), and 3) time to clear the runway after the beginning of the turnoff ( \( T_{off} \)). Figure 4.3 illustrates the components of the \( T_{ijk} \).

If the best location of aircraft \( i \) is farther downrange than turnoff candidate \( k \) from the runway threshold, \( T_{ijk} \) would be set as 0, which means aircraft \( i \) is not able to take turnoff candidate \( k \). Otherwise, \( T_{ijk} \) would be calculated as the sum of \( T_b \), \( T_n \), and \( T_{off} \). \( T_b \) is calculated during the simulations. \( T_n \) is calculated assuming that the aircraft coasts on the runway 'without braking' until the speed of the aircraft is reduced to the taxiing speed which is specified by program user. 'Without braking' implies the rolling friction coefficient, \( f \), is equal to 0.03 (i.e. deceleration rate is \( 9.81 \, \text{m/s}^2 \) \( \times 0.03 \) \( = \) \( 0.2943 \, \text{m/s}^2 \)). Once taxiing speed is reached, the aircraft is assumed to travel on the runway with constant speed which is same as taxiing speed.

That is, \( T_n \) is calculated by:

\[
T_n = \begin{cases} 
\frac{2D_{fr}}{V_{ex} + V_{term}}, & \text{if } D_{fr} \leq D_{tax} \\
\frac{2D_{tax}}{V_{ex} + V_{tax}} + \frac{D_{fr} - D_{tax}}{V_{tax}}, & \text{if } D_{fr} > D_{tax}
\end{cases}
\]

(4.1)

where

- \( D_{fr} \) is an additional free roll distance
- \( D_{tax} \) is a distance to reach taxiing speed
- \( V_{ex} \) is the design exiting speed

*OPTIMIZATION MODEL FOR MIXED AIRCRAFT FLEET*
Estimation of Runway Occupancy Time for Secondary Candidates

Figure 4.3 The Components of $T_{ij}$ Time.

$T_b$ = Time to reach the desired exit speed
$T_{fr}$ = Free roll time to reach the nearest exit
$T_{off}$ = Time in the turnoff
$H_{thres}$ = Threshold crossing altitude
$V_{tax}$ is the taxiing speed

$V_{term}$ is a terminal speed after free roll when $D_{fr} \leq D_{tax}$

$D_{tax}$ is given by $(V_{tax}^2 - V_{sax}^2) / 2g$, and $V_{term}$ is calculated by $\sqrt{V_{tax}^2 - 2gD_{fr}}$ in case $D_{fr} \leq D_{tax}$

Suppose, for instance, the best turnoff location of aircraft $i$ and $T_i$ are calculated as 1000 m and 35 seconds, respectively, with the following input parameters:

- Desired exit speed = 30 m/s
- Taxiing speed = 7 m/s.

The distance to reach taxiing speed would be:

$$D = (30^2 - 7^2) / (2 \times g \times f), \text{ where } g = 9.81, f = 0.03$$

$$= 1445 \text{ m.}$$

If another candidate $k$ is located 1300 m downrange from the runway threshold, then $T_k$, the travel time to reach the new candidate would be:

$$T_k = \frac{(2 \times D_k)}{(V_i + V_k)}$$

where $D_k$ is free roll distance (300 m),

$V_i$ is initial speed (30 m/s),

$V_k$ is terminal speed ($\sqrt{V_{term}^2 - 2gD_{fr}} = 26.9 \text{ m/s}$)

$$= 10.5 \text{ seconds.}$$

The calculation of $T_{eff}$ is closely related to the turnoff geometry. That is, $T_{eff}$ is the travel time along the turnoff geometry from the beginning of the turn to the clearance of the runway. For the exact calculation, a numerical integration requiring large computational times is needed. Moreover, this integration should be executed for every $T_{ijk}$, unless $T_{ijk}$ is set as 0. $T_{eff}$ is therefore approximated by the method described in Appendix B, to reduce the computation time. $T_{eff}$ usually ranges from 6 seconds to 13 seconds according to the size and exit speed of the aircraft. The approximation scheme is as follows: First, the actual trajectory of turnoff phase is delineated by a two-radius compound curve for each aircraft. Second, the critical aircraft is found for each exit candidate. The critical aircraft requires the largest radius of cur-
ature among the aircraft which is permitted to take that exit candidates. Finally, $T_{en}$'s of all the permissible aircraft of each exit candidate are calculated using the trajectory of the critical aircraft as described in Appendix B.

4.3 Finding Optimal Locations

In this section, a technique to find optimal turnoff locations is described. The final goal is to find a given number of turnoff locations which minimize the total weighted sum of ROT from the set of candidates. The number of turnoff is provided by the user. The optimization task can be modeled as a specific linear programming model. A dynamic programming algorithm is applied to find the solution, since the dynamic programming algorithm is more efficient than the ordinary linear programming algorithm for our case.

4.3.1 Mathematical Model

Suppose $M$ different types of aircraft use a runway, then $2M$ different turnoff locations would be calculated for every aircraft and two runway surface conditions (dry and wet) during the simulation of landing movements. A complete set of exit candidates, which is indexed $k = 1$ to $K$, is generated based on the $2M$ initial locations. It is not always permissible to assign aircraft $i$ on surface condition $j$ to candidate $k$. Let us define $A(i,j)$ as a set of feasible candidates for aircraft $i$ on the surface condition $j$, for $i = 1$ to $M$, $j = 1$ to 2. If candidate $k$ is nearer from the threshold than the primary candidate for aircraft $i$ and surface condition $j$, the candidate $k$ does not belong to $A(i,j)$.
If exit candidate $k$ is selected to be built, the candidates which are within $D_{\text{min}}$ (213 m or 600 ft) from the candidate $k$ can not be constructed. Let us define $S(k)$ as the mutually exclusive set of candidates in which at most one candidate can be selected to be built, for $k = 1$ to $K$.

In order to the minimize the weighted sum of ROT, information about weights should be provided by the user. Let $a_{i}$ be the proportion of aircraft $i$ for $i = 1$ to $M$, and let $p_{j}$ be the probability of occurrence of the surface condition for $j = 1$ to $2$ (if $j = 1$, surface condition is dry, otherwise, surface condition is wet).

Suppose the number of exits to be built is set as $N$. The binary decision variables are defined as follows:

$$x_{k} = \begin{cases} 
1, & \text{if exit candidate } k \text{ is selected} \\
0, & \text{otherwise,} 
\end{cases} \quad \text{for } k = 1 \text{ to } K$$

$$y_{ijk} = \begin{cases} 
1, & \text{if aircraft } i \text{ is assigned to the exit candidates } k \text{ on surface condition } j \\
0, & \text{otherwise,} 
\end{cases} \quad \text{for } i = 1 \text{ to } M, j = 1 \text{ to } 2, k \in A(i,j)$$

Then, the model which attempts to design a feasible runway with the least total weighted runway occupancy time may be formulated as follows:

$$\text{Minimize } \sum_{i=1}^{M} \sum_{j=1}^{2} \sum_{k \in A(i,j)} a_{i} p_{j} T_{ijk} y_{ijk} \quad \{4.2\}$$

$$\text{subject to } \sum_{k \in A(i,j)} y_{ijk} = 1 \quad \text{for } i = 1,2,\ldots,M; j = 1,2 \quad \{4.3\}$$
\[
\sum_{k \in S(k)} x_k \leq 1 \quad \text{for } k = 1, 2, \ldots, K \tag{4.4}
\]

\[
\sum_{k=1}^{K} x_k \leq N \tag{4.5}
\]

\[
y_{ijk} \leq x_k \quad \text{for } i = 1, 2, \ldots, M; j = 1, 2; k \in A(i,j) \tag{4.6}
\]

\[
\bar{x}, \bar{y} \quad \text{binary} \tag{4.7}
\]

The objective function (Eq. 4.2) represents the aggregate expected runway occupancy time. Constraint (Eq. 4.3) requires that each aircraft type should be assigned to one (available) exit under each surface condition. Constraint (Eq. 4.4) ensures a feasible mix of exits, while constraint (Eq. 4.5) enforces a maximum limit to the total number of exits constructed. The fourth constraint (Eq. 4.6) asserts that only the constructed exits must be used, and lastly, Eq. 4.7 enforces the logical restrictions on the variables.

The same formulation given above may be used to model the problem of re-designing or modifying existing runways, by simply fixing the appropriate variables \(x_k\) to be one. This option can also be adopted for a priority enforcing choice of certain exits.

### 4.3.2 Dynamic Programming Formulation

Suppose the number of exits to be built is \(N\), the number of candidates is \(K\), and the candidates are sorted based on the distance from the threshold. For the dynamic programming (DP) formulation, one imaginary candidate need to be introduced. This imaginary candidate
is indexed 0, and is located 213 m ahead of the first candidate. The corresponding $T_{ijb}$ is set
as 0, for all (i,j). This means no aircraft can take exit 0. With the imaginary candidate, we can
observe the following characteristics of $T_{ijb}$:

1) There exists at least one $T_{ijb} = 0$ for all (i,j).
2) If $T_{ijb} > 0$, then $T_{ijk} > 0$, for $k \geq k_0$, for all (i,j).
3) $T_{ijb} \leq T_{ijk-1}$, for $k \in A(i,j)$, for all (i,j).

$D(k)$ is defined as the distance from candidate $k$ to candidate 1. Thus $D(0) = -213$ m, and $D(1) = 0$ m. Let us define another variable, which is denoted as $K_0$, a candidate index beyond
which at least one exit should be constructed. $K_0$ is determined by:

$$K_0 = \text{Max} \{ k ; T_{ijk-1} = 0 \text{ for some } (i,j) \} \leq K$$

$K_0$ ensures that each aircraft will be assigned to an exit, even if it is the largest aircraft. With
the variables defined in the previous section and above, the DP formulation is as follows:

**Stages:** Stage $q$ corresponds a situation in which up to $q$ exits can be located to the right
of the last exit already located. $q$ ranges from 1 to $N$. For $1 \leq q \leq N$, $(N-q)$
exits are assumed to have been constructed.

**States:** The state $s_q$ at stage $q$ is a candidate index, and corresponds to the right most
exit currently located. For $1 \leq q \leq N - 1$, the possible values of $s_q$ are $l_1, \ldots , K$, where $l_q$ is the smallest exit candidate index such that it is possible to con-
struct $(N-q)$ exits in candidate $1, \ldots , l_q$ subject to the $D_{min}$ separation restriction.
That is, $l_q$ is determined by:

$$l_q = \text{Min} \{ k ; D(k) \geq (N-q) \times D_{min} \}, \text{and}$$

If $q = N$, $s_q = 0$.

**Decisions:** Decision $d_q$ is another candidate index. Given stage $q$ and state $s_q$, the deci-
sion, $d_q$, corresponds to the next exit to be constructed to the right of $s_q$. Let '$d_q = 0'$ mean that no more exits are constructed. Then the possible values of

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\(d_q\) are 0, and \(L_q, \ldots, K\), for \(1 \leq q \leq N - 1\), where \(L_q\) is the smallest exit index such that \(D(L_q) - D(q) \geq D_{min}\), if it exists, for \(q = N, L_N = 1\).

Given any stage \(q\) and state \(s_q\), all aircraft-surface condition combinations \((i,j)\) for which \(T_{ijq} > 0\) would have been assigned to some existing exit, due to the characteristics of \(T_{ij}\). Hence, the problem decomposes into locating up to \(q\) more exits to the right of \(s_q\) with the minimum separation constraint, considering only \((i,j)\) combinations whose \(T_{ijq} = 0\), which implies that \((i,j)\) is not yet assigned. Since the optimum of this decomposed problem is independent of the previous decision, and depends only on \(q\) and \(s_q\), Bellman's principle of optimality holds, and thus, the DP application is valid.

With the stage, the state, and the decision defined above, some functions need to be defined for the complete DP formulation. These are:

**Immediate return function**

The return function \(c_q(s_q, d_q)\) is the 'immediate' stage cost incurred by making decision, \(d_q\), at stage \(q\) in state \(s_q\). This cost corresponds to the additional \((i,j)\) assignments which can be made with a given \(d_q\). Hence,

\[
c_q(s_q, d_q) = \begin{cases} 
\infty, & \text{if } D(d_q) - D(s_q) < D_{min}, \text{ and } d_q \neq 0 \\
\sum_{((i,j): T_{ijq} = 0, \text{ but } T_{ijd_q} > 0)} a_i p_j T_{ijd_q}, & \text{if } D(d_q) - D(s_q) \geq D_{min}, \text{ and } d_q \neq 0 \\
0, & \text{if } d_q = 0
\end{cases} \quad (4.8)
\]

**Stage transition function**

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\[ \tau_q(d_q) = \begin{cases} 
q - 1, & \text{if } d_q \neq 0 \\
0, & \text{if } d_q = 0 
\end{cases} \] \quad \text{[4.9]}

State transition function

\[ \tau_q(s_q, d_q) = \begin{cases} 
d_q, & \text{if } d_q \neq 0 \\
s_q, & \text{if } d_q = 0 
\end{cases} \] \quad \text{[4.10]}

Recursive formula

Defining \( f_q(s_q) \) to be the optimal accumulated return function with given input state \( s_q \) at stage \( q \), the recursive formula would be:

\[ f_q^*(s_q) = \min_{d_q} \{ c_q(s_q, d_q) + f_{q+1}(\tau_q(s_q, d_q)) \} \] \quad \text{[4.11]}

where the final condition is

\[ f_0^*(s_0) = \begin{cases} 
\infty, & \text{if } s_0 < K_0 \\
0, & \text{otherwise} 
\end{cases} \] \quad \text{[4.12]}

By iterating the recursive formula (4.11) with \( q \) from 1 to (N-1), we can find the optimal accumulated return (minimum weighted sum of ROT) for all possible states for each stage.

At the final iteration, or the last stage (\( q = N \)), the overall weighted sum of ROT is minimized, and then a sequence of optimal decisions, \( d_q^* (q = 1, \cdots, N) \), which minimizes the overall weighted sum of ROT is revealed. These \( d_q^* \)'s are the optimal exit candidate indices which we are looking for. Since the iteration is repeated \( N \) times and each iteration involves \( O(\text{RK}^3) \) computations, the algorithm is of polynomial complexity \( O(\text{NRK}^2) \).

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4.4 Aircraft Exit Assignment

By the DP technique, the optimal exit locations are found. The final step in the optimization sequence is to assign every aircraft-surface condition combination, (i,j), to an appropriate exit. This step is performed by making (i,j) to take the exit which is permissible, and requires minimum ROT.

4.5 A Simple Example

In this section, a simple example is discussed to illustrate the optimization procedure developed previously. Suppose three aircraft use a single runway, where two exits will be constructed. The aircraft types and the relative frequencies of operation are: 1) Learjet-31 (30%), 2) Airbus A300-600 (30%), and 3) Boeing B767-300 (40%). The desired exit speed for all the three aircraft is 30 m/s (67 MPH). The exit reliability factor is 90%. The chances of dry and wet conditions occurring are same. With these data, the simulation of landing movement calculates six primary exit candidates for three aircraft and two runway surface conditions. That is, the best exit location for the Learjet-31 on dry surface is 993 m, and on wet surface is 1081 m. The best location for the A300-600 on dry surface is 1508 m, and 1860 m is the best location under wet condition. The best location for B767-300 is 1596 m on dry surface, and 1763 m on wet surface. The corresponding ROT's are 27.4, 28.9, 39.0, 42.6, 40.7, 44.5 seconds, respectively.

Based on the primary candidates, thirteen more secondary candidates are found to comprise the complete candidate set assuming $D_{min}$ is 213 m (see the second row of table 4.1). These
fourteen exit candidates are: 993m, 1061m, 1206m, 1274m, \ldots 1976m (\textbf{STEP 1}). A $T_{ip}$ matrix is calculated as shown in Table 4.1 (\textbf{STEP 2}). The optimization is performed with the $T_{ip}$ data, and then 1061m and 1763m are selected as optimal exit locations (\textbf{STEP 3}). Finally, the aircraft are assigned to the selected exit locations as shown in Table 4.2, the weighted average ROT is calculated as 41.5 seconds.

\section*{4.6 Modified Algorithm for 'Improvement' Analysis}

The optimization algorithm described in section 4.1 to 4.4 was developed for design analysis which assumed no exits were available on the runway. With some modifications, this algorithm can be applied to an improvement analysis scenario in which some exits already exist on the runway and a few more exits will be added to reduce the ROT metric.

In this new procedure, the existing exit locations as well as the best locations are considered as primary candidates. The complete candidates are generated with the same principles used in design analysis, and then the candidates which are located within the $\pm D_{ml}$ range from the existing exits are eliminated. Stages, states, and decisions of DP formulation are same as those of the design analysis. The immediate return function should be changed to consider the effect of the existing exits. Suppose the ROT of aircraft $i$ should be accumulated as an immediate return of a decision, $d_{e}$, associated with a state, $s_{e}$. If there are some existing exits in the region of $(s_{e}, d_{e})$, and a existing exit requires less ROT than $d_{e}$ does, then the less ROT required by the existing exit is considered as an immediate return.
### Table 4.1 T\(_{\text{fl}}\) Data for Three Aircraft.

<table>
<thead>
<tr>
<th>Exit #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location (m)</td>
<td>1061</td>
<td>1208</td>
<td>1275</td>
<td>1449</td>
<td>1602</td>
<td>1756</td>
<td>1819</td>
<td>1973</td>
<td>1913</td>
<td>1976</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Learjet 35</td>
<td>dry</td>
<td>27.30</td>
<td>29.05</td>
<td>23.39</td>
<td>27.17</td>
<td>43.54</td>
<td>46.94</td>
<td>47.80</td>
<td>51.15</td>
<td>51.79</td>
<td>54.42</td>
<td>57.47</td>
<td>58.10</td>
<td>60.48</td>
<td>63.25</td>
<td>65.24</td>
<td>67.13</td>
<td>69.64</td>
<td>70.88</td>
</tr>
<tr>
<td>wet</td>
<td>0.00</td>
<td>15.00</td>
<td>15.19</td>
<td>15.79</td>
<td>15.48</td>
<td>15.46</td>
<td>15.30</td>
<td>15.26</td>
<td>15.94</td>
<td>15.58</td>
<td>15.53</td>
<td>15.14</td>
<td>15.68</td>
<td>15.29</td>
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<td>14.63</td>
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<td>13.79</td>
<td></td>
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<tr>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
</tr>
<tr>
<td>Boeing 767</td>
<td>dry</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
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<tr>
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<td>0.00</td>
</tr>
</tbody>
</table>

### Table 4.2 Aircraft Assignment Results

<table>
<thead>
<tr>
<th>Exit # Location (m)</th>
<th>Exit Type</th>
<th>ROT / RELIABILITY TABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(This is for Designing a New Runway)</td>
</tr>
<tr>
<td>A-300-600</td>
<td>dry</td>
<td>49.13</td>
</tr>
<tr>
<td>B-767-300</td>
<td>dry</td>
<td>47.33</td>
</tr>
<tr>
<td>Learjet-55C</td>
<td>dry</td>
<td>29.85</td>
</tr>
</tbody>
</table>

**ROT** = Runway Occupancy Time in Secs

Reliability in % = 98

Average ROT = 44.51

OPTIMIZATION MODEL FOR MIXED AIRCRAFT FLEET
4.7 Examples

Numerical examples are discussed to illustrate the optimization procedure developed previously. Suppose five aircraft types use a single runway, and the aircraft types and the proportions are: 1) Airbus A300-600 (20%), 2) British Aerospace BAe-146 (30%), 3) Boeing B727-200 (30%), 4) Boeing B747-200 (10%), and 5) McDonnell Douglas DC 10-30 (10%). The desired exit speed for all the five aircraft is 30 m/s (67 MPH). The exit reliability factor used is 90%. The chances of dry and wet conditions occurring are same (i.e. 50% each). With these data, the simulation of landing movement calculates ten primary exit candidates for five aircraft and two runway surface conditions.

4.7.1 Design problem

Suppose there is no exit currently. If two exits are decided to be constructed, the model will suggest 1745 m and 2348 m from the runway threshold as the optimal locations with the weighted average ROT of 52.5 seconds. If the number of exits increases to three, the optimal locations will be 1253 m, 1745 m, and 2348 m from the runway threshold, and the average ROT will decrease to 46.2 seconds. For four exits scenario, the optimal exits will be 1253 m, 1578 m, 1829 m, and 2348 m with the average ROT of 44.3 seconds. Figure 4.4 shows the optimal locations of two, three, and four exit cases and the corresponding average ROT. Table 4.3 shows the aircraft assignments to the exits for each case and the anticipated individual ROT for each aircraft assignment. For example, in four exits case, B 727 will use the second exit (1578 m) on dry runway surface, and use the third exit (1829 m) on wet runway surface. 41.1 seconds and 48.1 seconds are the corresponding ROT's, respectively. The exiting probability
of all the assignments are greater than or equal to the given reliability. The computation time for four exits case is about five seconds on the IBM PS/2 Model 50.

4.7.2 Improvement problem

Hypothesize an existing runway with three exits. Two of them are 30° angled exits located at 2000 m and 2500 m. The third is 90° angled exit located at 3000 m. The design exit speeds for 30° angled exit and 90° angled exit are assumed as 26.7 m/s (60 mph) and 8 m/s (18 mph), respectively. 26.7 m/s is the Federal Aviation Administration standard, and 8 m/s is selected because it is slow enough for an aircraft to make a right angle turn. The effectiveness of the existing runway can be evaluated with respect to ROT using the simulation model. Table 3 a) shows the evaluation results. For example, Boeing 787 can turn off the runway using both the first exit (2000 m) and the third exit (2500 m) when the runway is dry. 50.9 seconds and 70.5 seconds are the corresponding ROT’s assessed by the simulation model. For the average ROT calculation, only third exit ROT (70.5 seconds) is considered, because the cumulative exiting probability for the second exits is less than the desired reliability factor. The average ROT of existing runway is evaluated as 64.7 seconds.

Suppose one exit is to be added to the existing runway, then the additional exit should be located at 1578 m from the runway threshold, which reduces the average ROT to 51.3 seconds, using the design speed of 30 m/s. If two exits were added, the optimal locations of these would be 1253 m and 1745 m with the average ROT of 45.9 seconds. If one more exit is added, the third will located at 2213 m, and the average ROT will reduce to 45.3 seconds. Figure 4.5 shows the location of existing exits and additional exits. Table 4.4 shows the aircraft assignments and the individual ROT anticipated.
Figure 4.4 The Optimal Locations for Design Problems.
### Table 4.3 Aircraft Assignments / ROT Tables for Design Problems

#### a) 2 exits case

<table>
<thead>
<tr>
<th>EXIT # LOCATION (M)</th>
<th>1</th>
<th>2</th>
<th>Avg ROT = 52.5 seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1745</td>
<td>2348</td>
</tr>
<tr>
<td>A 300 dry</td>
<td>48.50</td>
<td>49.40</td>
<td></td>
</tr>
<tr>
<td>A 300 wet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bae 146 dry</td>
<td>58.70</td>
<td>56.10</td>
<td></td>
</tr>
<tr>
<td>Bae 146 wet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>727 dry</td>
<td>47.80</td>
<td>45.00</td>
<td></td>
</tr>
<tr>
<td>727 wet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B 747 dry</td>
<td>63.80</td>
<td>58.70</td>
<td></td>
</tr>
<tr>
<td>B 747 wet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DC 10 dry</td>
<td>46.80</td>
<td>70.40</td>
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</tr>
<tr>
<td>DC 10 wet</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### b) 3 exits case

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<tr>
<th>EXIT # LOCATION (M)</th>
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<th>3</th>
<th>Avg ROT = 46.2 seconds</th>
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<tbody>
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<td>1745</td>
<td>2348</td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>Bae 146 dry</td>
<td>37.20</td>
<td>35.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bae 146 wet</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>727 dry</td>
<td>47.80</td>
<td>49.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>727 wet</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>B 747 dry</td>
<td>63.80</td>
<td>58.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B 747 wet</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DC 10 dry</td>
<td>46.80</td>
<td>70.40</td>
<td></td>
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<tr>
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<td></td>
<td></td>
<td></td>
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</tr>
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</table>

#### c) 4 exits case

<table>
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<tr>
<th>EXIT # LOCATION (M)</th>
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<th>3</th>
<th>4</th>
<th>Avg ROT = 44.3 seconds</th>
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<tr>
<td></td>
<td>1253</td>
<td>1578</td>
<td>1828</td>
<td>2348</td>
<td></td>
</tr>
<tr>
<td>A 300 dry</td>
<td>41.60</td>
<td>48.40</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>A 300 wet</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bae 146 dry</td>
<td>37.20</td>
<td>35.80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bae 146 wet</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>727 dry</td>
<td>41.10</td>
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</tr>
<tr>
<td>727 wet</td>
<td></td>
<td></td>
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<tr>
<td>B 747 dry</td>
<td>63.80</td>
<td>58.70</td>
<td></td>
<td></td>
<td></td>
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<td>B 747 wet</td>
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</tr>
<tr>
<td>DC 10 dry</td>
<td>50.10</td>
<td>44.90</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>DC 10 wet</td>
<td></td>
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<td></td>
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</tr>
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</table>
Figure 4.5 The Existing/Optimal Locations for Improvement Problems.
Table 4.4 Aircraft Assignments / ROT Tables for Improvement Problems.

a) existing exits

<table>
<thead>
<tr>
<th>EXIT #</th>
<th>LOCATION (M)</th>
<th>TYPE</th>
<th>Avg ROT = 64.7 seconds</th>
<th>rot (exit prob, %)</th>
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<td></td>
<td></td>
<td></td>
<td>1 2 3</td>
<td>1 2 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2000 30-deg 2500 30-deg 3000 90-deg</td>
<td></td>
</tr>
<tr>
<td>A 300</td>
<td>dry</td>
<td>wet</td>
<td>61.0 (100)</td>
<td></td>
</tr>
<tr>
<td>BAE 146</td>
<td>dry</td>
<td>wet</td>
<td>85.8 (100)</td>
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</tr>
<tr>
<td>B 727</td>
<td>dry</td>
<td>wet</td>
<td>76.7 (100)</td>
<td></td>
</tr>
<tr>
<td>B 747</td>
<td>dry</td>
<td>wet</td>
<td>50.9 (66) 70.5 (38)</td>
<td></td>
</tr>
<tr>
<td>DC 10</td>
<td>dry</td>
<td>wet</td>
<td>58.4 (100)</td>
<td>10.7 (1)</td>
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</table>

b) 1 additional exit

<table>
<thead>
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<th>EXIT #</th>
<th>LOCATION (M)</th>
<th>TYPE</th>
<th>Avg ROT = 51.3 seconds</th>
<th>rot (exit prob, %)</th>
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<td></td>
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<td>1 2 3</td>
<td>1 2 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1978 2000 30-deg 2500 30-deg 3000 60-deg</td>
<td></td>
</tr>
<tr>
<td>A 300</td>
<td>dry</td>
<td>wet</td>
<td>41.06</td>
<td></td>
</tr>
<tr>
<td>BAE 146</td>
<td>dry</td>
<td>wet</td>
<td>50.90</td>
<td></td>
</tr>
<tr>
<td>B 727</td>
<td>dry</td>
<td>wet</td>
<td>41.10</td>
<td></td>
</tr>
<tr>
<td>B 747</td>
<td>dry</td>
<td>wet</td>
<td>58.30</td>
<td></td>
</tr>
<tr>
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<td>dry</td>
<td>wet</td>
<td>58.40</td>
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<table>
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<tr>
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<th>TYPE</th>
<th>Avg ROT = 45.9 seconds</th>
<th>rot (exit prob, %)</th>
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<td>1 2 3</td>
<td>1 2 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1253 1745 30-deg 2000 20-deg 3000 90-deg</td>
<td></td>
</tr>
<tr>
<td>A 300</td>
<td>dry</td>
<td>wet</td>
<td>48.30</td>
<td></td>
</tr>
<tr>
<td>BAE 146</td>
<td>dry</td>
<td>wet</td>
<td>70.20</td>
<td></td>
</tr>
<tr>
<td>B 727</td>
<td>dry</td>
<td>wet</td>
<td>47.80</td>
<td></td>
</tr>
<tr>
<td>B 747</td>
<td>dry</td>
<td>wet</td>
<td>70.50</td>
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</tr>
<tr>
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<td>dry</td>
<td>wet</td>
<td>46.80</td>
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<table>
<thead>
<tr>
<th>EXIT #</th>
<th>LOCATION (M)</th>
<th>TYPE</th>
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<th>rot (exit prob, %)</th>
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<tr>
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<td>1 2 3</td>
<td>1 2 3</td>
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<td></td>
<td></td>
<td></td>
<td>1251 1745 30-deg 2000 20-deg 2213 90-deg</td>
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<tr>
<td>A 300</td>
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<td>wet</td>
<td>48.30</td>
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<tr>
<td>B 727</td>
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<tr>
<td>B 747</td>
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<td>58.20</td>
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</tr>
<tr>
<td>DC 10</td>
<td>dry</td>
<td>wet</td>
<td>46.80</td>
<td></td>
</tr>
</tbody>
</table>
4.8 The Minimum Number of Exits

During the optimization the number of exits is assumed as user's input. This number might be determined according to airport regulatory guidelines and budgetary consideration among other factors. However, there is still interest of investigating what is the smallest value of \( N \) for which the problem admits a feasible solution. In this section two variations of this problem are addressed. In the first case, \( D_{mn} \) is assumed as 0, while in the second case, \( D_{mn} \) is assumed as some positive number.

Suppose there exist feasible ranges \( r (r=1 \text{ to } R) \) for each aircraft-surface condition combination. \( N \) exits are to be located on the runway at any points such that there is at least one exit for each feasible range. These exits should be separated by at least a distance of \( D_{mn} \). Let \( L_r \) and \( R_r \) be the left hand and the right hand interval end points for the range \( r \), respectively. \( L_r \) and \( R_r \) are actually distances measured from the start of the active runway threshold.

**Problem 1 \( (D_{mn} = 0) \)**

Given a set of aircraft feasible ranges \([ L_r, R_r ], r = 1, \ldots, R \), indexed in order of nondecreasing values of \( R_r \), \( r = 1, \ldots, R \) with minimum \( (L_r) = 0 \), and given a discrete set of \( K \) potential exit locations on \([0, R_{\max})\), determine the minimum number of exits such that there is at least one exit in each range \( r = 1, \ldots, R \).

**Solution Algorithm A1**

**Initialization** Let \( L = \{1, \ldots, R\}, \rho = 1, \) and \( N = 0 \). Go to Step 1.

OPTIMIZATION MODEL FOR MIXED AIRCRAFT FLEET
Step 1
Place an exit at the rightmost potential within the range $p$. Increment $N'$ by 1, and remove from the list $L$ all the ranges which contain the new exit location.

Step 2
If $L = \phi$, stop; $N'$ solves problem 1. Otherwise, put $p$ equal to the first remaining range index in $L$, and return to Step 1.

The algorithm is clearly finitely convergent. Moreover, its complexity is $O(RK)$, since the rightmost potential exit in each range and the number of ranges covered by each potential location can be found in $O(RK)$ time, and given this information the algorithm can be executed in $O(R)$ time.

Now, to validate the algorithm, examine the range $p = 1$ which has the leftmost right-hand endpoint. An exit should be located in this range at optimality by definition. Suppose this optimal location does not coincide with the right-hand endpoint, $k_r$. If the optimal location is moved rightwards to $k_r$, the ranges which were covered by previous optimal location will continue to be covered since $R_y \geq R_1$. Hence, the new exit will cover at least as many as the old one, and so must yield an alternative optimal solution. By the same argument, it follows that the range $p = 1$ cannot contain more than one exit, since otherwise, a feasible solution with at least one fewer exit can be obtained. Since an optimal solution should contain an exit located at $k_r$, it is possible to remove all ranges covered by this optimal exit location from the problem. Since the resultant problem has same form as the original problem with the reduced list of ranges which were not eliminated at Step 1, the same argument can be repeated for this reduced problem.

**Problem 2** ($D_{min} > 0$)
Problem 2 is the same as Problem 1 with additional restriction that the selected exit locations must be separated by a distance of at least $D_{min} > 0$ from each other.

Solution Algorithm A2
Initialization  Construct a dummy potential location \( k = 0 \) at the point \( -D_{\text{mix}} \) on the real line.

Denote \( l(k) \) as the position on the real line segment \([-D_{\text{min}}, R_{\text{r}}]\) at which potential exit \( k \) is located, \( k = 0, \ldots, K \), and define

\[
F(k) = \{ k' : l(k') \geq l(k) + D_{\text{min}}, \text{ and if } L_{r} > l(k), \text{ then } R_{r} \geq l(k') \text{ for each } r = 1, \ldots, R \},
\]

for \( k = 0, \ldots, K \).

Note that \( F(k) \) gives the feasible set of locations \( k' \) at which the next exit to the right of \( k \) can be located, assuming that an exit located at \( k \). Put \( N^* = 1 \) and let

\[
T = \{ k \in \{1, \ldots, K\}, \ : \ L_{r} \leq l(k) \forall r = 1, \ldots, R \}, \text{ and } \ E = \{ 0 \}.
\]

Step 1  Find \( L = \bigcup_{k \in \hat{E}} F(k) \)

Step 2  If \( L = \emptyset \), stop; Problem 2 is infeasible.

If \( L \cap T \neq \emptyset \), stop; \( N^* \) solves Problem 2.

Otherwise, put \( E = L \), increment \( N^* \) by 1, and return to Step 1.

Construct a graph \( G \) with \((K + 2)\) nodes \( 0, \ldots, K, K + 1 \), where the first \( K + 1 \) nodes correspond respectively to the potential exit locations \( 0, 1, \ldots, K \), and where node \( K + 1 \) is a dummy terminal node. Construct directed arcs \((k, K + 1)\) for all \( k \in T \) with a cost of zero, and arcs for all pairs of \((k_1, k_2)\) such that \( k_2 \in F(k_1) \) for \( k_1 = 1, \ldots, K \) with a cost of unity. Note that the set \( T \) is comprised of potential exit locations which must contain exactly one selected exit at optimality. This follows from the fact at least one exit must be located in the set \( T \) since \( L_{r} > k \) for some \( r = 1, \ldots, R \) for each \( k \notin T \). Moreover, if an exit is located at some \( k_1 \in T \) at optimality, then another exit cannot be located at a location \( k_2 > k_1 \in T \) since \( k \) covers all the ranges that \( k_2 \) does. Hence, any exit located in \( T \) is the rightmost one, and conversely the rightmost exit belongs to \( T \).
Similarly, if \( k_1 \) and \( k_2 \) are two adjacent exit location in \( \{0, \ldots, K\} \), \( k_1 < k_2 \), then \( k_1 \) should be contained in \( F(k_2) \). Consequently, Problem 2 by above construction becomes a shortest path problem on the above graph from node 0 to node \( K+1 \). Since Algorithm A2 is essentially Dijkstra’s algorithm applied to this shortest path problem with \( E \) being the set of scan eligible nodes and \( L \) being the set of newly labelled nodes at each stage (refer to Bazaraa et. al., 1990), Algorithm A2 is justifiable by the properties of Dijkstra’s algorithm. The determination of \( F(k) \) for all \( k = 0, \ldots, K \) takes \( O(K^2 + RK) \) effort, while the shortest path algorithm is of complexity \( O(K^2) \). Hence, Algorithm A2 is of complexity \( O(K \max(R, K)) \).

For the improvement of existing runway, the solutions of Problem 1 and 2 can be modified as follows: First, eliminate the aircraft ranges which are covered by existing exits. Second, eliminate the potential exit locations which are closer than \( D_{\text{min}} \) from any existing exits. The Problem 1 and 2 reduces to find the minimum number of exits which cover the remaining aircraft ranges from the surviving potential exit locations, and this can be solved as above.
5.0 REDIM - RUNWAY EXIT DESIGN INTERACTIVE MODEL

5.1 Software Functions and Features

REDIM is a fully interactive software package to determine the runway exit locations and their geometries which minimize the average ROT of a mixed aircraft fleet. Users can control the program flow, edit data files, begin an analysis, and view output via the menus, editing screens, and output screens provided by REDIM. The software is designed to be used by airport planners, managers, and designers, and has three major type of analysis functions:

1. Evaluation of an existing runway
2. Improvement of an existing runway
3. Design of a new runway

For all kinds of analyses above, an important input parameter is the reliability of aircraft exiting. Reliability is defined here as the probability that an aircraft takes the assigned exit suc-
cessfully at a exiting speed prespecified by the user. A basic policy of our approach is that an aircraft should be assigned to only one exit with a high reliability. By the policy, the variation of ROT of an aircraft is expected to decrease significantly, even though the average ROT may increase slightly. The reduction of the variation of ROT is very crucial specially when ROT is the most determinant factor of separation of two consecutive landing aircraft. This is expected to happen in a near future with improvement of air traffic control systems and aircraft technology. Reliability is remained as user’s decision, and is recommended to be high probability, say greater than 90 %.

The ‘evaluation’ option examines the effectiveness of a runway with respect to a weighted average ROT of an aircraft fleet using the runway. This option requires information about the existing runway configuration such as the number of exits, the locations of exit, and the types of exits as input data. The model produces the aircraft assignment to the existing exits with the corresponding ROT and probabilities of exiting, and the weighted average ROT as results of the analysis. When the average ROT is calculated for all the aircraft population, only one ROT is considered for each aircraft. For example, if aircraft i is able to take exit k and k+1 whose corresponding ROT’s and cumulative probabilities are \( t_k, t_{k+1}, \) and \( p_k, p_{k+1} \), respectively. If \( p_i \) is less than the reliability specified by the user, and \( p_{k+1} \) is greater than the desired reliability, then only \( t_{k+1} \) is considered as the representative ROT for aircraft i.

The ‘improvement’ option assumes that a few exits would be added to an existing runway. This analysis requires the number of existing exits, the locations of existing exits, the types of existing exits, and the number of new exits which will be constructed. The results are optimal exit locations, aircraft assignment to the existing and new exits, the weighted average ROT which is minimized by the optimal exit locations, and turn-off geometries of the exits.

The ‘design’ option assumes no exit exists on the runway. The number of new exits is the main input parameter for this analysis. The results of this type of analysis are similar to that
of 'improvement' option except for the fact that only the new exits are accounted for aircraft assignment in this analysis. A flow chart depicting input/output relationships for each analysis type are shown in Figure 5.1.

In the development of REDIM the following features are incorporated:

1. The software is menu driven. Users do not need to master how to operate microcomputers completely in order to use the software. They can control the program flows by selecting their choice from the menu screens provided by the software at each step.

2. The software includes a data base file name MAST.DAT for the sake of user's convenience. For every analysis, the user has to select aircraft types comprising an aircraft fleet. The data base file stores fifty one current aircraft names and data of nine items for every aircraft. If an aircraft is selected, the corresponding data will transferred to a working data file automatically. The working data file is a data file which contains a complete set of data necessary for analyses, and can be created and edited inside of the software. Similar provision has been made to modify the data base file as the addition of new aircraft will be necessary.

3. The software employs a simulation model and an optimization model for analyses. The simulation model mimics an individual aircraft landing behavior, and calculates the landing distance required and time passed for the aircraft to decelerate to the given exiting speed from the runway threshold. The optimization model finds a given number of exit locations which minimize the average ROT of a mixed aircraft fleet based on the simulation results. The simulations are performed for each aircraft comprising the aircraft fleet. The simulation model consists of a series of point mass aircraft equations of motion, and the optimization model utilizes a dynamic programming technique.

4. The software offers both graphical displays on the screen and hardcopy on the printer of the analysis results.
Figure 5.1 Input/Output Relationships for Three Major Analyses
5. The software has a limited input data validation facility. If some input data are out of feasible range, REDIM asks the user to correct them instead of producing unreasonable outputs.

6. The software has a brief on-line help facility which describes the choices of every menu.

7. The software can handle up to twenty five aircraft, and find up to ten optimal exit locations at a single analysis.

5.2 Software Structure and Procedures

REDIM can be broadly divided into six components and three accompanying data files: Main Menu, Input Module, Dynamic Module, Optimization Module, Output Module, and working data file, master data file, output file, as shown Figure 5.2.

Working data file stores all input data which is necessary to perform an analysis. Input data are classified into six categories for the user’s convenience. That is, when the user wants to change some of the input data, he has to review only a category instead of scanning the whole data file. The following paragraphs define the categories in more detail.

Analysis Type and Related Data

The program provides the user with three choices for the type of analysis to be performed. For each type of analysis, there are some accompanying data which vary depending on the user’s choice. That is, information about existing exits is necessary for the ‘evaluation’ or the ‘improvement’ analysis, but is not necessary for the ‘design’ analysis.

Aircraft Mix and Characteristics Data
Figure 5.2 The Structure of REDIM
In this category, the percentages of the aircraft comprising the aircraft mix population and their geometric characteristics are included.

**Airport Operational Data**

In this category, the free roll time between the touchdown and the beginning of braking, the free roll time between the end of braking and the beginning of turn off, taxiing speed, and their standard deviations are included. A safety factor for the pavement impending skid condition is also included in this category.

**Airport Environmental Data**

The following parameters are included in this category: wind speed, wind direction, airport elevation, airport temperature, runway orientation, minimum distance between two adjacent exits, runway width, and distance to the nearest taxiway.

**Runway Gradient**

In this category, runway length, and the effective gradient for every one tenth of runway are included.

**Weather and Exit Speed**

The relative frequency of dry and wet runway surface conditions are included in this category. The desired exit speeds of each aircraft category under each runway pavement condition are also included here.

The master data file is a data base file which stores all aircraft names and their geometric characteristics. This file can be edited with a procedure named 'edit master file' which is provided as a part of input module. The user can add new aircraft data, or change the contents of the file. If an aircraft is selected as a member of an aircraft population, its name and characteristics data will be transferred to the working data file automatically. The output file stores the results of the analysis. The contents of the file can be displayed graphically on the screen, or be printed on the print by procedures which is provided by output module. The
Main Menu is placed at the top of the structure. Main menu provides the user with the highest level of control over the program. It offers seven choices including four procedures (1 - 4):

1. Start a new problem
2. Edit data
3. Begin analysis
4. Edit master (data) file
5. Go to output module
6. Help
7. Quit

Choices 1, 2, and 4 lead the program flow to the Input Module, choice 3 leads to the Dynamic Module and Optimization Module which are followed by Output Module, and finally choice 5 connects the user to the Output Module directly. Figure 5.3 shows the actual screen of the Main Menu.

5.2.1 Input Module

The Input Module is a collection of subroutines which is necessary to perform three procedures: 1) start a new problem, 2) edit data, and 3) edit master file. All the procedures are conducted interactively with users. The user has control over the procedures via menu screens, and edit data files on the edit screens provided by the Input Module.

Start a New Problem

In this procedure, all the data necessary for the analysis should be provided by the user. Once the user enters this procedure, a complete set of values is expected before completing the entire input process. The first set of data which the user specifies is "Analysis Type and Re-
lated Data." The type of analysis is selected through a menu input method. That is, the user has to select one out of multiple choices displayed on the screen. Figure 5.4 shows the menu screen for analysis type selection, which is followed by the related data screen which might vary depending upon the analysis type selected. The second set of the data is "Aircraft Mix and Characteristics." The aircraft mix screen shows the names of the aircraft whose characteristics are included in master data file in table form as shown in Figure 5.5. The user inputs the percentages of the aircraft which comprise the aircraft population expected to be operated at the runway facility. Following the aircraft mix screen, several aircraft characteristics screens for each aircraft selected in the mix screen are displayed as shown in Figure 5.6. These screens are provided as a check for the user's review. All the values shown in this screen are transferred from the master data file. If the user does not want to change the values, he/she needs to press "Esc" key to proceed to the next step which might be another aircraft characteristics screen or airport environmental data screen.

The screens for "Airport Environmental Data" and the "Airport Operational Data" follow those for "Aircraft Mix and Characteristics Data". The screens for both of the two sets of the data have the similar format with the screen for the aircraft characteristics data. The next screen deals with "Runway Gradients." At this screen, the user specifies the runway length at the line where cursor is blinking, and inputs the gradients for every one tenth section of the runway at the proper position on the screen depicted in Figure 5.7. The final screen in "Start a New Problem" mode is designed for "Weather and Exit Speeds." On this screen, the relative frequencies of dry and wet runway pavement conditions, and the desired exit speeds of every aircraft category under each weather condition are specified, as shown in Figure 5.8.

**Edit Data**

This portion of the program allows the user to modify existing data files. If the user selects "2) Edit Data" mode at the main menu, the edit menu showing six groups of data, is displayed. In the "Edit Data" mode the user can select the group of data which he/she wants to change,
while in the “Start a New Problem” procedure the user should input all the data sequentially. The details for editing data are the same as in the “Start a New Problem” procedure Figure 5.3 shows the edit menu.

**Edit Master File**

While the function of “Edit Data” procedure is editing the working data file, the function of “Edit Master File” is editing the master data file which keeps the aircraft names and their geometric characteristics. If “Edit Master File” procedure is selected, a different edit menu for master data file appears. In this menu, there are two choices: 1) “Add a New Aircraft” and 2) “Change some Specific Data.” If the user chooses the first, he/she has to select one out of five aircraft categories (TERPS A-E) and input the new aircraft name. Then a screen for inputting aircraft characteristics appears. If the user opts for the second choice, he/she has to select one aircraft category and one aircraft name included in the category selected. Then a screen for editing aircraft characteristics appears.

### 5.2.2 Dynamic Module and Optimization Module

The Dynamic Module consists of several computational subroutines used to simulate aircraft movement during a landing operation. The purpose of the simulation is to find the location and the time at which an aircraft decelerates to the predetermined design exit speed. The simulation model keeps track of the position and the time data of aircraft from the runway threshold. The simulations of aircraft landing movements calculate the best turnoff locations for each aircraft in both dry and wet runway surface conditions. Suppose there are five aircraft in consideration, the simulations of the aircraft landing movements will provide ten different turnoff locations for each aircraft and two runway surface conditions. The goal of the optimization is to find a few locations (e.g. 2 or 3) at which all the aircraft in consideration can exit.
Figure 5.3 The Main Menu Screen

Figure 5.4 Screen for Analysis Type Selection
Figure 5.5 Screen for Aircraft Mix Data

Figure 5.6 Screen for Aircraft Characteristics Data
**Figure 5.7 Screen for Runway Gradients**

<table>
<thead>
<tr>
<th>RAW LENGTH (m)</th>
<th>3900</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRADIENT (°)</td>
<td></td>
</tr>
<tr>
<td>0 TO 300</td>
<td>9</td>
</tr>
<tr>
<td>600 TO 900</td>
<td>9</td>
</tr>
<tr>
<td>1200 TO 1500</td>
<td>0</td>
</tr>
<tr>
<td>1800 TO 2100</td>
<td>0</td>
</tr>
<tr>
<td>2400 TO 2700</td>
<td>0</td>
</tr>
</tbody>
</table>

**Figure 5.8 Screen for Weather and Exit Speeds**

<table>
<thead>
<tr>
<th>DAY</th>
<th>%</th>
<th>UFL</th>
</tr>
</thead>
<tbody>
<tr>
<td>TERFS A exit speed (m/s)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>TERFS B exit speed (m/s)</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>TERFS C exit speed (m/s)</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>TERFS D exit speed (m/s)</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>TERFS E exit speed (m/s)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

REDIM - RUNWAY EXIT DESIGN INTERACTIVE MODEL.
the runway with the minimum weighted sum of ROT. Since each aircraft and each surface condition can have different relative frequency, the weighted sum of ROT should be minimized instead of total ROT. The Optimization Module is a collection of computational subroutines to find the given number of optimal exit locations. More details of the Dynamic Module and the Optimization Module are developed in Chap. 3 and 4.

5.2.3 Output Module

The Output Module is structured so that at each stage the user is prompted with specific questions and guidelines that are to be followed in order to view the appropriate results. A significant feature in this module is that in each type of analysis, the screens clearly display information regarding the aircraft and the airport data. This information provided at the top
of each screen helps the user to identify the general information pertaining to a specific type of analysis. Users may get into the output module by selecting ‘go to output module’ in the main menu or by selecting ‘begin analysis’. The first screen of the output module is output menu which offers four options:

1. View the output
2. Print the report
3. Help
4. Go to main menu

Figure 5.10 depicts the Output Menu screen. In the following paragraphs, the procedures that are provided to the user in the Output Menu will be discussed in detail.

View the output

This procedure shows user the analysis results in various formats. Even though there exists a little difference depending on the analysis type, the output module produces five types of output screen:

1. ROT table
2. Exit location display
3. Exit geometry and its centerline coordinate table
4. Comparison of centerline geometry
5. ROT statistics

Figure 5.11 shows a example of ROT table containing aircraft assignments to exits and their corresponding ROT. For instance, a Boeing B737 is recommended to use the exit located 1554 m from the runway threshold both on dry condition and wet condition, and the corresponding ROT’s are 42.8 seconds and 40.6 seconds, respectively.
The screen for exit location display is shown in Figure 5.12. In this screen, the user may proceed to see the turnoff centerline comparison shown in Figure 5.13 or the exit geometry shown in Figure 5.14. At the exit geometry screen, the user may view the centerline coordinates as shown in Figure 5.15. The list of assigned aircraft type to the exit is displayed at upper left corner of this screen. The screen for ROT statistics shows user the ROT's of all aircraft comprising the aircraft population in a bar chart format. In this screen, the ROT's are compared each other and to the average ROT graphically.

Print the report

Instead of viewing a piece of input data and output results on the screen, the user can get a comprehensive report from the attached printer. The report includes all the input data and analysis results. Figure 5.17 is an example of the printout.
Figure 5.10 Output Menu Screen

Figure 5.11 ROT Table Screen
Figure 5.12 Exit Location Display Screen

Figure 5.13 Centerline Comparison Screen
Figure 5.14 Exit Geometry Screen

Figure 5.15 Centerline Coordinates Table Screen
Figure 5.16 ROT Statistics Screen

Figure 5.17 An Example of the Printed Output
1.2. Aircraft Mix and Characteristics

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>UAE</th>
<th>WT</th>
<th>TMC</th>
<th>MAW</th>
<th>L/D</th>
<th>FL</th>
<th>CL</th>
<th>MAX</th>
<th>HA</th>
<th>VA</th>
<th>WT</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA-35-112</td>
<td>10.0</td>
<td>1.5</td>
<td>3.0</td>
<td>77.4</td>
<td>577.0</td>
<td>486.0</td>
<td>1.57%</td>
<td>15.6</td>
<td>15.0</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>PA-28-161</td>
<td>75.0</td>
<td>2.8</td>
<td>3.0</td>
<td>82.3</td>
<td>1105.0</td>
<td>415.0</td>
<td>1.60%</td>
<td>15.0</td>
<td>16.7</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>BE-58</td>
<td>10.0</td>
<td>2.7</td>
<td>2.9</td>
<td>94.6</td>
<td>2590.0</td>
<td>79.15</td>
<td>1.486</td>
<td>18.5</td>
<td>13.5</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td>BE-300</td>
<td>10.0</td>
<td>4.6</td>
<td>5.2</td>
<td>89.0</td>
<td>656.0</td>
<td>857.2</td>
<td>2.076</td>
<td>28.2</td>
<td>16.0</td>
<td>6.3</td>
<td></td>
</tr>
<tr>
<td>TE-102C</td>
<td>10.0</td>
<td>5.2</td>
<td>5.5</td>
<td>86.5</td>
<td>5107.0</td>
<td>655.8</td>
<td>2.150</td>
<td>21.0</td>
<td>15.4</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>SAB-340</td>
<td>10.0</td>
<td>7.5</td>
<td>7.7</td>
<td>90.5</td>
<td>5220.0</td>
<td>1145.4</td>
<td>2.376</td>
<td>61.8</td>
<td>23.4</td>
<td>7.7</td>
<td></td>
</tr>
<tr>
<td>EMP-120</td>
<td>12.0</td>
<td>7.6</td>
<td>6.8</td>
<td>90.5</td>
<td>1290.0</td>
<td>1269.5</td>
<td>2.272</td>
<td>39.4</td>
<td>19.8</td>
<td>7.6</td>
<td></td>
</tr>
<tr>
<td>FOKKER-100</td>
<td>15.0</td>
<td>14.0</td>
<td>15.0</td>
<td>89.3</td>
<td>39915.0</td>
<td>1360.0</td>
<td>2.533</td>
<td>91.3</td>
<td>28.1</td>
<td>16.8</td>
<td></td>
</tr>
<tr>
<td>BAe-146</td>
<td>15.0</td>
<td>11.2</td>
<td>4.7</td>
<td>92.3</td>
<td>36740.0</td>
<td>1130.0</td>
<td>3.385</td>
<td>77.2</td>
<td>26.3</td>
<td>12.6</td>
<td></td>
</tr>
</tbody>
</table>

- WB = Wheelbase
- WT = Wheeltrack
- LM = Load on Main Gear
- LD = Landing Mass
- LM = Landing Run Distance
- CL = Max C.L.
- VA = Wing Area
- WS = Wing Span
- WT = Dist. Nose Gear to Wingtip

1.3. Operational Data

- 1st free roll time = 2.0 (sec) std. dev. = 0.5
- 2nd free roll time = 1.0 (sec) std. dev. = 0.2
- taking speed = 8.5 (m/s) std. dev. = 1.0
- safety fac. for skid = 50.0 (%) std. dev. = 0.0

1.4. Environmental Data

- wind speed = 0.0 (m/s) wind direction = 0.0
- airport elevation = 0.0 (m) temperature = 20.0 (°C)
- min. exit interval = 23.0 (sec) runway orientation = 0.0
- runway width = 45.0 (m) distance to taxiway = 200 (m)

1.5. Runway Gradients

- runway length = 2000 (m)
- gradients (%) = -0.3 , -0.1 , -0.5 , 0.5 , -0.5 , 0.5 , -0.5 , 0.5 , -0.5 , -0.5 , -0.5 , -0.5 , -0.5 , -0.5 , -0.5 , -0.5

1.6. Weather and Exit Speeds

- weather & exit speed (m/s)

<table>
<thead>
<tr>
<th>EXIT</th>
<th>&quot;A&quot;</th>
<th>&quot;B&quot;</th>
<th>&quot;C&quot;</th>
<th>&quot;D&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>probability (%)</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>TERPS A</td>
<td>20.0</td>
<td>20.0</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>TERPS B</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
<td>25.0</td>
</tr>
<tr>
<td>TERPS C</td>
<td>30.0</td>
<td>30.0</td>
<td>30.0</td>
<td>30.0</td>
</tr>
<tr>
<td>TERPS D</td>
<td>30.0</td>
<td>30.0</td>
<td>30.0</td>
<td>30.0</td>
</tr>
<tr>
<td>TERPS E</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Figure 5.17 An Example of the Printed Output (Continuation)
II. ANALYSIS RESULTS

II-1. Average ROT

average ROT = 45.91 (sec)

II-2. Exit Locations, Types, and Turn-off Assignment.

<table>
<thead>
<tr>
<th>Exit #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location (m)</td>
<td>0</td>
<td>727</td>
<td>1000</td>
<td>1495</td>
<td>2000</td>
</tr>
<tr>
<td>Type</td>
<td>90-d</td>
<td>new</td>
<td>90-d</td>
<td>new</td>
<td>90-d</td>
</tr>
<tr>
<td>PA-36-112</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dry</td>
<td>35.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wet</td>
<td>34.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>PA-28-162</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dry</td>
<td>36.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wet</td>
<td>35.7</td>
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<td></td>
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<td>BE-58</td>
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<td></td>
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<tr>
<td>dry</td>
<td>71.4</td>
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<td></td>
<td></td>
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<td>BE-300</td>
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<tr>
<td>dry</td>
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<td>CE-402C</td>
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</tr>
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</tr>
<tr>
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<td></td>
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</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>dry</td>
<td>41.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wet</td>
<td>39.0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>BAe-146</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>dry</td>
<td>46.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wet</td>
<td>45.1</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Figure 5.17 An Example of the Printed Output (Continuation)
### 11.3. Turn-off Centerline Geometries

<table>
<thead>
<tr>
<th>Exit 1</th>
<th>Exit 2</th>
<th>Exit 3</th>
<th>Exit 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Y</td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>5.1</td>
<td>0.2</td>
<td>19.8</td>
<td>0.3</td>
</tr>
<tr>
<td>10.1</td>
<td>0.7</td>
<td>39.3</td>
<td>2.6</td>
</tr>
<tr>
<td>15.2</td>
<td>1.5</td>
<td>58.1</td>
<td>6.7</td>
</tr>
<tr>
<td>20.3</td>
<td>2.8</td>
<td>76.1</td>
<td>12.7</td>
</tr>
<tr>
<td>25.4</td>
<td>4.3</td>
<td>93.1</td>
<td>20.5</td>
</tr>
<tr>
<td>30.4</td>
<td>6.3</td>
<td>107.0</td>
<td>29.6</td>
</tr>
<tr>
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<td>38.6</td>
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<td>11.7</td>
<td>140.1</td>
<td>47.5</td>
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<td>56.3</td>
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<td>66.9</td>
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<td>24.3</td>
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<td>75.4</td>
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<td>81.7</td>
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<td>38.1</td>
<td>213.3</td>
<td>89.8</td>
</tr>
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<td>71.0</td>
<td>48.7</td>
<td>227.2</td>
<td>97.9</td>
</tr>
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<td>76.1</td>
<td>56.1</td>
<td>240.8</td>
<td>105.7</td>
</tr>
<tr>
<td>82.1</td>
<td>64.1</td>
<td>254.2</td>
<td>113.5</td>
</tr>
<tr>
<td>90.5</td>
<td>72.1</td>
<td>267.3</td>
<td>121.0</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Exit 5</th>
</tr>
</thead>
<tbody>
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<td>X</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>5.1</td>
</tr>
<tr>
<td>10.1</td>
</tr>
<tr>
<td>15.2</td>
</tr>
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<td>20.3</td>
</tr>
<tr>
<td>25.4</td>
</tr>
<tr>
<td>30.4</td>
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<tr>
<td>90.5</td>
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<td>96.0</td>
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*Figure 5.17 An Example of the Printed Output (Continuation)*
6.0 SUMMARY and RECOMMENDATIONS

The end result of this research is to recommend a high-speed turnoff locations and geometries that will minimize the runway occupancy time under realistic airport scenarios. As was seen from Chapter 3 the model is able to predict turnoff locations and geometries that optimize the weighted average ROT parameter for a given set of airport circumstances. For the sake of producing practical and useful results, a simulation model and an optimization model were developed. The simulation model which consists of a series of dynamic equations of motion mimics the aircraft landing behavior considering the influencing factors above, and then finds the best exit location for individual aircraft. The optimization model takes the simulation results as input, and finds a given number of exit locations which minimize the average ROT of the aircraft fleet.

These models are implemented on a software package named REDIM (Runway Exit Design Interactive Model). REDIM runs on the IBM personal computer or compatible with VGA graphics card. In addition to the analysis functions, REDIM has many convenience features. For example, the flow of the program is controlled by menus, a data base file storing fifty one aircraft geometric characteristics is included in the package, data are edited interactively, and output results are displayed on the screen with well designed format.
6.1 Suggested High-speed Standard Geometry

In dealing with a new standard geometry our approach to the standardization problem is very similar to that confronted by the Horonjeff team three decades ago. If a standard is to be accepted by the aviation community it not only needs to be proven in simulators and in fields demonstrations, but also needs to address the needs of the builder in terms of a simple definition of the geometry. This is probably the most difficult task to address since a fully variable geometry is obtained as the result of the turning equations of motion of aircraft negotiating a high-speed turnoff. The specification of such a geometry (i.e., fully variable geometry) is difficult to justify in practice since every position coordinate in a two-dimensional plane needs to be known. From an operational point of view it is possible to approximate slow-varying turnoff geometries [i.e., spirals and cissoids] with large radius of curvature entrance curve followed by a reduced radius of curvature circular segment. This approach was suggested by Robert Horonjeff in the late 50’s (Horonjeff, et al., 1959) but interestingly enough his results have been not universally accepted by all the aviation authorities in terms of adopting a large entrance curve as geometric design standard. The current FAA practice uses a single radius of curvature to define the geometry of a high-speed turnoff (i.e., 1800 ft for 30-Deg. angled exits). In our findings with REDIM we have to acknowledge that Horonjeff’s suggestions were justifiable and that possibly the simplest approach to define a new standard is to consider two circular arcs with a common tangency point as a viable solution to approximate a fully variable turnoff geometry (see Fig. 6.1). This approach is revisited in this section to show the selection process behind the variable geometry standard.

From Fig. 6.1 it is seen that two radii of curvature defined $R_1$, $R_2$, and a turnoff exit angle, $\psi$ form the basis for the suggested approximation. The first radius of curvature approximates the jerk-limited curve corresponding to a specified entry speed ($V_e$) whereas the second one, $R_2$, models the aircraft “steady-state rotational” inertia characteristics as it negotiates the turnoff.
Through many simulations using REDIM it became evident that extracting two specific values of \( R \) an excellent approximation to this fully variable turnoff geometry could be obtained. The values of \( R_1 \) and \( R_2 \) then were obtained as a function of turnoff time and aircraft category.

The rationale behind the time factor in this recommendation is to account for the aircraft inertia resistance motion which can be categorized as a "pseudo-first order model" (see Eqns. 3.18-19 for \( \tilde{R} \)) where the radius of curvature changes slowly as a function of time. Looking at Fig. 6.2 it is observed that an equivalent "time constant" characterizing the aircraft rotational motion about the z axis as it negotiates a high-speed turnoff is proportional to the aircraft mass and moment of inertia about this axis among other factors. Knowing this fact a straight correlation between the values of \( R_2 \) and an extraction time were established. Table 6.1 summarizes the nominal extraction times used in REDIM to approximate the variable turnoff trajectory.

In REDIM nomenclature these times are labeled as easement curvature time, \( TR_1 \), and steady-state curvature time, \( TR_2 \). Note that for heavy transport-type aircraft (i.e., > 300,000 lbs) larger time lags to achieve a "steady-state" radius of curvature are a direct result of larger time constants in the model.

Fig. 6.1 also illustrates the two corresponding encompassing the approximate turnoff track. Arcs with lengths \( L_1 \) and \( L_2 \) are defined as follows,

\[
L_1 = R_1 \theta_1 \tag{6.1}
\]

\[
L_2 = R_2 \theta_2 \tag{6.2}
\]

where, \( L_1 \) and \( L_2 \) represent the turnoff characteristic lengths. \( R_1 \) and \( R_2 \) are the radii of curvature defining the turnoff, and \( \theta_1 \) and \( \theta_2 \) are the arcs defined by \( R_1 \) and \( R_2 \), respectively measured in radians. The turnoff arcs are characteristic for each aircraft since the transition and

**SUMMARY and RECOMMENDATIONS**
Figure 6.1 Definition of the New Turnoff Geometry.
Table 6.1 REDIM Model Extraction Times for Estimating $R_1$ and $R_2$.

<table>
<thead>
<tr>
<th>Category</th>
<th>$R_1$</th>
<th>$R_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 sec.</td>
<td>4 sec.</td>
</tr>
<tr>
<td>B</td>
<td>1 sec.</td>
<td>5 sec.</td>
</tr>
<tr>
<td>C</td>
<td>1 sec.</td>
<td>6 sec.</td>
</tr>
<tr>
<td>D</td>
<td>1 sec.</td>
<td>6 sec.</td>
</tr>
<tr>
<td>E</td>
<td>1 sec.</td>
<td>6 sec.</td>
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</table>

Radius of Curvature Time Histories

35 m./sec. Exit Speed
Wet Runway Surface Condition

- Piper Tomahawk
- Embraer 120
- BAe-146-200
- Boeing 747-400

Figure 6.2 Time Variations of Radius of Curvature for Representative Aircraft.

SUMMARY and RECOMMENDATIONS
runway clearance point are aircraft speed and geometry dependent. It should be kept in mind that \( L_1 \) is a linear function of aircraft speed if the jerk-limited equation is used and if the values of \( a_n \) and \( J_n \) are substituted in Eqn. 6.3.

\[
L_1 = \frac{a_n V_0}{J_n} = \frac{a_{\text{max}} V_0}{J_{\text{max}}}
\]  

(6.3)

The analyst, however, does not need to be concerned in REDIM since the actual turnoff track values are presented in tabular form. The approximation is primarily used to depict the geometry on the computer screen.

### 6.2 Recommendations for Further Research

An aspect that deserves our attention in this section is that dealing with some of the safety margins and assumptions made in the present modeling effort. In the overall conceptualization of REDIM safety margins were implemented in some of the dynamic module subroutines to account for the usual uncertainties associated with manual control tasks, such as the landing of an aircraft, the activation of braking devices, etc. However, the reduction of these uncertainties could significantly reduce the runway occupancy time (ROT) by reducing the margins of safety needed to cope with the original assumptions. This phenomena is similar to the anticipated reductions in the aircraft interarrival time (IAT) to the runway threshold through an improvement of the aircraft delivery accuracy (e.g., by reducing the final approach IAT separation buffers). The underlying assumptions made in this model have tried to establish a good balance between operational safety and the efficiency of the runway subsystem. This compromise was necessary simply because the model is expected to be applied in a variety of scenarios where the manual control uncertainties could be quite high. That is, the

**SUMMARY and RECOMMENDATIONS**
model could be either applied to small community airports where the proficiency and accuracy of the pilots might dictate slightly larger safety margins or to large transport-type airports where an increased number of automated landing rollout operations could take place in the future. It is expected that REDIM will be calibrated with the help of simulation and experimental results in order to gain more confidence in the output results of the model. This calibration is, in fact, one of the most important steps to follow the development of REDIM. It should be clearly understood by the analyst that scenario-specific factors such as obstructions, runway length, lighting conditions, etc. could affect significantly some of the results obtained for certain airports. For example, it is well known that the runway exit location and length have a large influence in ROT as pilots adjust their piloting behavior under scenario specific circumstances such as displaced thresholds and short runways. Therefore a series of empirical observations are recommended in order to modify REDIM to account for some of these human operational factors.
Bibliography


51. Witteveen, N. D., Modified Rapid Runway Exit Taxiways To Reduce Runway Occupancy Time, Presented at 21st International Air Transport Association Conference on System Demand and System Capacity, Montreal, Canada, September 1987.


# Appendix A. Glossary of Aircraft Characteristics

<table>
<thead>
<tr>
<th>CODE NAME</th>
<th>NAME</th>
<th>MAX ANGLE</th>
<th>WHEELSPAN</th>
<th>WHEELBASE</th>
<th>% LOAD</th>
<th>WING AREA</th>
<th>CM INERTIA</th>
<th>CL MAXIMUM</th>
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<tr>
<td>3 04.7 1 1</td>
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<td>24.80</td>
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<td>24.80</td>
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<td>24.80</td>
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<td>91.50</td>
<td>140.000</td>
<td>2.6400</td>
<td>140 029</td>
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</table>

**SUMMARY**

- **BOOM**
  - Code Name: B 3.0
  - Max Angle: 30.60
  - Wheelspan: 24.80
  - Wheelbase: 18.80
  - % Load: 91.50
  - Wing Area: 140.000
  - CM Inertia: 2.6400
  - CL Maximum: 120 029

**NOTES**

- The above table provides a summary of the aircraft characteristics for a specific code name (B 3.0).
- Each row represents a different parameter related to the aircraft's performance.
- The values for Max Angle, Wheelspan, Wheelbase, % Load, Wing Area, CM Inertia, and CL Maximum are provided.

---

**Appendix A. Glossary of Aircraft Characteristics**

103
<table>
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<tr>
<th>CODE NAME</th>
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<th>SPEED RANGE</th>
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<th>V. TOUCHDOWN</th>
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Appendix A. Glossary of Aircraft Characteristics
Appendix A: Glossary of Aircraft Characteristics

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<th>Aircraft Type</th>
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<th>Max. Speed</th>
<th>Cruise Speed</th>
<th>Turbine Type</th>
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</table>
Appendix B. Optimality Through a Discrete Search

Suppose there exist feasible ranges \( r (r = 1 \text{ to } R) \) for each aircraft-surface condition combination, where ROT's within each range are increasing from left to right. Assuming that \( N \) exits are to be located on the runway at any points such that there is at least one exit for each feasible range, and that the exits are separated by at least a distance of \( D_{\text{min}} \), the optimal exit locations, which minimize the weighted sum of ROT, can be found from a finite collection of points. Let \( L_r \) and \( R_r \) be the left hand and the right hand interval end points for the range \( r \), respectively. \( L_r \) and \( R_r \) are actually distances measured from the start of the active runway threshold. Define a set of breakpoints as points on the runway which are of the type \( L_r + q \cdot D_{\text{min}} \) for \( q \geq 0 \) and integer valued, for \( r = 1 \) to \( R \). Then the optimal locations are found from the set of breakpoints by the following theorem.

**Theorem 1** Assume that \( N \) is large enough so that the above problem has a feasible solution. Then at optimality, each location will coincide with some breakpoint.

**Proof** We will prove this by induction on the exit index. Consider the leftmost exit location. This exit must coincide with \( L_r \) for some \( r \in \{1, \ldots, R\} \) because if not, by sliding its location leftwards until it coincides with such a location, we will
maintain feasibility (since all aircraft which could take this exit can continue to do so), and the objective value will strictly improve. Inductively, suppose that the result is true for the location of exit \( t \in \{1, \ldots, N-1\} \). If exit \( t+1 \) coincides with some \( L_r \) for \( R \in \{1, \ldots, R\} \), then the result is true. If exit \( t+1 \) is at a distance \( D_{min} \) from exit \( t \) to its left, then by the induction hypothesis, and the construction of breakpoints, the result is again true. If neither of these cases holds, then we can slide the location of exit \( t+1 \) leftwards until one of these conditions holds, thereby maintaining feasibility and improving the objective value. Hence, the result must be true for the location of exit \( t+1 \), and this completes the proof.

**COROLLARY 1** For any pair of exits \( t \) and \( t+1 \) separated by a distance greater than \( D_{min} \), the location of exit \( t+1 \) must lie in \( \{L_1, \ldots, L_R\} \).

**PROOF** Evident from the proof of Theorem 1.

**COROLLARY 2** Given that the ROT's are nondecreasing, rather than strictly increasing within each feasible range, there exists an optimal solution in which the exit locations coincide with the defined breakpoints.

**PROOF** Evident from the proof of Theorem 1.

**COROLLARY 3** (Improvement problem) Given existing exit locations at points \( D_1, \ldots, D_n \), define additional breakpoints as the points \( D_i + q D_{min} \) for \( q \geq 1 \) and integer, for \( i = 1, \ldots, n \). Furthermore, delete from the set of breakpoints thus defined, those which lie at a distance less than \( D_{min} \) from an existing exit location (on either side of it). Then again, any optimal solution will have the new exit locations coinciding with these defined breakpoints.
PROOF

Can be constructed similar to that of Theorem 1.

REMARK

By Corollary 1, for $N$ and $C_{mn}$ small enough, optimal locations of exits will coincide with the points $L_r, r = 1, \ldots, R$. For larger values of these parameters, the other breakpoints will begin to play a role. This is of consequence since the points $L_r$ represent the critical locations given by the simulations of aircraft landing movement. Also, given our emphasis, Corollary 3 is of most importance.
Appendix C. Approximation of Turnoff Times

The estimation of the turnoff time plays a very important role in the mathematical optimization module of REDIM as the dynamic programming technique used tries to minimize a time related performance index. It was said in Chapter 4 of this report that in order to save valuable computational time it was necessary to approximate the time spent by aircraft in the turnoff maneuver under two scenario conditions (dry and wet). Furthermore, every secondary candidate solution (i.e., those generated from the actual aircraft landing simulations to comply with Bellman's principle of optimality as explained in Appendix B) has an associated turnoff time (TOT) for every aircraft and scenario condition and thus the estimation of these times would consume large amounts of time if performed through the complete simulation scheme used to estimate primary candidates and described in Section 3.3 of this report.

Since the geometry for every primary candidate is completely known from the simulation results it is possible to extract two representative values of the radius of curvature, R1 and R2, to approximate the turnoff geometry until the aircraft has cleared the runway as depicted in Fig. C.1. It should be emphasized that although this is an approximation the results are usually accurate if R1 and R2 are selected appropriately. In the late fifties Horonjeff (Horonjeff, 1958) used this scheme to approximate high-speed turnoff tracks with satisfactory results.
Through hundredths of simulations of the REDIM model its was observed that the values of R1 and R2 could be extracted from the turnoff simulation as a function of time and aircraft category. This segmentation per category was somewhat expected from equations 3.13-3.18 in Section 3.4 if one realizes that the aircraft turning capability is related to the inertia, centripetal and scrubbing forces resisting the aircraft turning motion. Results depicting the time rate of change variations of the radius of curvature for representative aircraft using REDIM are shown in Fig. C.2. It was then decided through examination of all the data to estimate R1 as the instantaneous radius of the curvature occurring one second after the turning maneuver started whereas R2 was varied selectively between four and six seconds depending upon the aircraft category. The four-second R2 is used with category A aircraft which display very fast behavior in the turnoff dynamics whereas the six-second R2 is used to predict heavy transport aircraft turnoff dynamics having larger time lags to achieve a “steady-state” radius of curvature. In REDIM nomenclature these times are labeled as easement curvature time, TR1, and steady-state curvature time, TR2.

Once the exact turnoff path is known the next step is to estimate the time required to clear the runway. This is done under the assumption that a turning aircraft decelerates due to rolling friction alone. Actual aircraft speed measurements performed by by Horonjeff [Horonjeff et al, 1959, 1960] and Hosang [Hosang, 1978] in high-speed taxiways show nearly constant deceleration rates similar to those associated with a moderate value of rolling friction alone. This can be attributed to the small aircraft castor angles present while negotiating a high-speed turnoff. A conservative value of $F_{\text{rel}}$ of 0.03 has been used throughout the program to model the rolling friction deceleration rates experienced by every aircraft.

Fig. C.3 illustrates how the turnoff time is estimated using two simple radii of curvature to approximate the actual turnoff track. Two turnoff arcs with lengths $L_1$ and $L_2$ are defined as follows.
Figure C.1 Turnoff Time Approximation Nomenclature.

Figure C.2 Time Variations of Radius of Curvature for Representative Aircraft.
\[ L_1 = R_1 \theta_1 \quad \text{(C.1)} \]

\[ L_2 = R_2 \theta_2 \quad \text{(C.2)} \]

where, \( L_1 \) and \( L_2 \) represent the turnoff characteristic lengths, \( R_1 \) and \( R_2 \) are the radii of curvature defining the turnoff, and \( \theta_1 \) and \( \theta_2 \) are the arcs defined by \( R_1 \) and \( R_2 \), respectively measured in radians. The turnoff arcs are characteristic for each aircraft since the transition and runway clearance point are aircraft speed and geometry dependent. A further simplification regarding the easement length, \( L_1 \), can be introduced using results derived from highway geometric design principles where the length of a spiral transition curve \( L_1 \), is made a function of exit speed. Horonjeff later on showed that a short transition spiral could well be approximated with a large radius of curvature segment and this approximation is easily implemented in the model [Horonjeff, 1959].

\[ L_1 = \frac{a_{\text{max}} V_{\text{exit}}}{J_{\text{max}}} = \frac{1.2 \text{ m/s}^2}{.55 \text{ m/s}^2} \times V_{\text{exit}} \quad \text{(C.3)} \]

The aircraft speed at the transition point between the two radii of curvature is obtained from Eqn. C.4 whereas the speed at the runway clearance point is shown in Eqn. C.5.

\[ V_{\text{tran}} = (V_{\text{exit}}^2 - 2 f_{\text{roll}} L_1)^{.5} \quad \text{(C.4)} \]

\[ V_{\text{final}} = (V_{\text{tran}}^2 - 2 f_{\text{roll}} L_2)^{.5} \quad \text{(C.5)} \]

where, \( V_{\text{exit}} \) is the desired aircraft exit speed (m./sec.), \( V_{\text{tran}} \) is the transition speed (m./sec.), \( V_{\text{final}} \) is the final speed at the runway clearance point, \( (X_c, Y_c) \), \( g \) is the gravity constant (m./sec.-sec.) and \( f_{\text{roll}} \) is the rolling friction coefficient (dimensionless). The travel time across each of the turnoff segments is estimated as shown in Eqns. C.6 and C.7.

\[ T_1 = \frac{V_{\text{exit}} + V_{\text{tran}}}{2} \quad \text{(C.6)} \]

Appendix C. Approximation of Turnoff Times
\[ T_2 = \frac{V_{\text{tran}} + V_{\text{final}}}{2} \tag{C.7} \]

where, \( T_1 \) and \( T_2 \) are the travel times from the start of the turnoff to the transition point \((X_t, Y_t)\) and from transition point to runway clearing point \((X_c, Y_c)\), respectively. The total turnoff time is the summation of these two previous contributions.

\[ T_{\text{tot}} = T_1 + T_2 \tag{C.8} \]

where, \( T_{\text{tot}} \) is the turnoff time until clearing the runway. This procedure to estimate the turnoff time is implemented for the secondary candidates whose locations are \( q(D_{\text{min}}) \) meters away from primary candidate solutions (for \( q = 1, 2, ..., N \)) as explained in Chapter 4 of this report.

It should be noticed that the secondary candidate solutions obtained for small aircraft far downrange from an active threshold will usually be unfeasible for large aircraft since these will not be able to negotiate the turnoff with the desired margin of safety. This process reduces even more the candidate set to be used in the optimization module.
Vita

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