REASONING FOR PUBLIC TRANSPORTATION SYSTEMS PLANNING:
USE OF DEMPSTER-SHAFER THEORY OF EVIDENCE

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ABSTRACT

Policy-makers of today’s public transportation investment projects engage in debates in which the reasonableness and clarity of their judgment are tested many times. How to recommend the transportation system that achieves project’s goals and different stakeholders’ needs in a most logical and justifiable manner is the main question of this dissertation.

This study develops a new decision-making approach, Belief Reasoning method, for evaluating public transportation systems in the planning process. The proposed approach applies a reasoning map to model how experts perceive and reason transportation alternatives to lead to the project’s goals. It applies the belief measures in the Dempster-Shafer theory of evidence as the mathematical mechanism to represent knowledge under uncertainty and ambiguity and to analyze the degree of achievement of stated goals.

Three phases are involved in implementing the Belief Reasoning method. First, a set of goals, a set of characteristics of the alternatives, a set of performances and impacts are identified and the reasoning map, which connects the alternatives to the goals through a series of causal relations, is constructed. Second, a knowledge base is developed through interviewing the experts their degree of belief associated with individual premises and relations, and then aggregating the expert opinions. Third, the model is executed and the results are evaluated in three ways: (i) the transportation alternatives are evaluated based on the degree of belief for achieving individual goals; (ii) the integrity of the reasoning process is evaluated based on the measures of uncertainty associated with information used; and (iii) the critical reasoning chains that significantly influence the outcome are determined based on the sensitivity analysis.
The Belief Reasoning method is compared with the Bayesian reasoning, which uses the probability measures as the measure of uncertainty. Also it is compared with the Analytical Hierarchy Process method, which uses a hierarchical tree structure and a weighting scheme. The numerical examples in transit planning are developed for comparison. The proposed Belief Reasoning method has advantages over these traditional evaluation and reasoning methods in several ways.

- Use of a reasoning map structure together with an inference process, instead of a tree structure together with a weighting scheme, allows modeling interdependency, redundancy and interactions among variables, usually found in transportation systems.
- Use of belief measures in Dempster-Shafer theory can preserve non-deterministic nature of inputs and performances as well as handle incomplete or partial knowledge of experts or citizens, i.e. “I don’t know” type opinion. The “degrees of belief” measures allow experts to express their strength of opinions in the conservative and optimistic terms. Such operation is not possible by the probability-based approach.
- Dempster-Shafer theory can avoid the scalability issue encountered in Bayesian reasoning. It can also measure uncertainty in the reasoning chains, and identify information needed for improving the reasoning process.
- Use of Dempster’s rule of combination, instead of the average operator in probability theory, to merge expert opinions about inputs or relations is a better way for combining conflicting and incomplete opinions.

In the dissertation, the Belief Reasoning method is applied in real-world Alternatives Analysis of a transit investment project. The results show its potential to analyze and evaluate the alternatives and to provide reasons for recommending a preferred alternative and to measure the uncertainty in the reasoning process.

In spite of some shortcomings, discussed in the dissertation, the Belief Reasoning method is an effective method for transportation planning compared with the existing methods. It provides means for the planners and citizens to present their own reasons and allows review and analysis of reasoning and judgments of all participating stakeholders. The proposed method can promote focused discourse among different groups of stakeholders, and enriches the quality of the planning process.
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CHAPTER 1

INTRODUCTION

1.1 Overview

Unlike the traditional motorized mode of individual traveler as the automobile, public transit—the transportation mode of aggregated travelers, is a major transportation mode in most urban cities and metropolitan areas in the U.S., and has been proposed as a transportation solution for the near- and long-term future. Around the nation in each year approximately 30 major transit investment projects are implemented (FTA, 2011a) and almost 200 transit alternatives studies are planned. (FTA, 2007)

Statistics of travel demand and transportation supply in the U.S. show that the demand of travel has been continued to outpace the supply of urban transportation infrastructures for many decades. Since 1970s the total vehicle miles of travel has increased approximately 170% (from 1.1 to 3.0 trillion vehicle miles), while transport supply has increased only 0.1% (from 3.7 to 4.0 million road mileage). These figures show the needs of new and efficient public transit infrastructure systems including vehicles, stations, way, management, and control in both short- and long-term metropolitan plans to manage growing demand, changes in travel pattern, and transportation constraints. (FHWA, 2008)

Furthermore, the comparison of demand and supply between transit and highway modes shows the trend for public transit systems. Since 1990s, transit demand has grown rapidly, far outpacing the growth in highway demand as well as population. Figure 1-1(a) shows that from 1995 through 2008 the transit passenger miles of travel increases by 36%, three times of the growth rate of U.S. population (14%) and 1.5 times the growth rate of vehicle miles of travel (24%). During the same time period, transit supply has substantially increased compared to the growth rate of highway supply. Figure 1-1(b) shows a significant increase in transit vehicle miles operated (45%), but a meager
increase in highway lane mileage (5%) from 1995 through 2008. As congestion builds up and gasoline prices increases increased transfer of federal funds from highway to transit projects may ensue, including a revision of government’s planning processes for more comprehensive consideration of public transportation modes.

![Figure 1-1: Changes in the U.S. Travel Demand and Supplies from 1995 to 2008](Source: APTA, 2010; FHWA, 2008; BTS, 2010; and U.S. Census Bureau, 2008)
Unlike the traditional planning and evaluation of highway infrastructure projects, the planning and evaluation of public transit infrastructure projects is a multi-criteria planning problem. The goals and performance measures of highway projects often reflect only the aspects related to the conditions that highway users experience, such as safety and economic efficiency buttressed by *Highway Capacity Manual.* (TRB, 2010). On the other hand, the goals and performance measures in the planning of transit projects consider the impacts that transit has on all types of road users, transit agencies, and the community served, such as land use, enhancement in walkability, reduction in congestion and environmental impacts, creation in job opportunities, and increase in property values. (TRB, 2003a; 2003b)

Policy-makers of today’s major public transportation investment projects are engaged in debates and face arguments about whether to build a new or extend an existing transit system, which transit technologies should be considered, which transit alternative is locally preferred, and which transit systems should be implemented. How to evaluate, present and recommend in a logical manner the most desirable transit system that meets the purposes and needs from diverse standpoints, and at the same time, satisfies multiple goals and objectives under uncertain information. These are increasingly challenging issues, and the main questions addressed in this dissertation.

### 1.2 Public Transportation Systems Planning and Evaluation Process

Researchers view a public transportation system a large-scale system. It is characterized by many elements that interact with each other. Planning a large-scale system is complicated because it must satisfy different groups of people with a wide range of views about benefits and needs, and about paying for its costs. Decision-making about a public transportation system is not straightforward and requires negotiations. Often times, the planning cannot be advanced because there is no consensus with regard to the goals and expected outcomes of a project.

Generally, planning of major public transport investment projects follows the steps shown in Figure 1-2. The process consists of two major sequential phases. The first
phase is a transit systems planning and forecasting process. It involves defining transportation-related problems, identifying the goals and objectives of a project to address the problem, developing a transportation model and collecting data of existing conditions, and forecasting the future travel demands and other conditions. The second phase is the evaluation and decision-making process. It involves defining a set of criteria and performance measures, formulating possible transit alternatives, screening and validating the alternatives to propose a set of potential alternatives for implementation. The proposed public transport alternatives are then analyzed in detail and compared to the No-Build (Baseline) alternative to evolve the locally preferred alternative, which is submitted for financial supports, and finally recommended for construction and operation.

Evaluation of transit systems and reaching the recommended alternative is embedded in three stages: alternatives screening process; alternatives analysis process; and project evaluation process for funding recommendations. These processes are labyrinthine, because they deal with both demand and supply characteristics of transit systems and their interactions. On the supply side, a set of transit alternatives is proposed, while on the demand side, a set of goals, criteria, or performance measures are defined by diverse groups of stakeholders—the users, transit agency, local government, and the general public—who benefit, pay or are otherwise affected by a project.

Traditional approaches to decision-making on transit systems are based on various unrealistic assumptions (Ribbons and Timothy, 2007). For example, the decision problem is assumed to be well structured; the evaluation objectives are assumed to be independent; the evaluation criteria are assumed to be quantifiable; the decision makers are assumed to be from a consistent group of individuals; all possible alternatives are assumed to be clearly defined; the decision-makers have complete knowledge of information needed when analyzing transit alternatives; and the alternative which gives the maximum utility is assumed to be the optimal solution. However in reality most transit decision makers have neither the complete information nor the rigid decision rules to make the “correct” decision.

In addition, traditional approaches seem to oversimplify the complex transit system by (i) aggregating performance measures and evaluating a system as a whole and (ii) omitting the analysts’ ambiguity. The project goals associated with each alternative
are rated directly; for example, the land use impacts of a light-rail transit is “Very High,” or the mobility of a bus alternative is “Low.” However in reality transit analysts and planners face with uncertainties in making a decision. Issues of current transit decision-making approaches are discussed in details in Chapter 2.

Figure 1-2: Transit Planning and Project Development Process
(Adapted from Vuchic, 2005; Sinha and Labi, 2007; FTA 2011)
1.3 Research Scope and Objectives

The scope of the dissertation is to propose a new public transit decision-making methodology that can evaluate transit alternatives in a logical manner, and at the same time, in the planning stage, accounts for analysts’ uncertainty and ambiguity about the system characteristics and performances of transit alternatives. The subject of this research has two main objectives.

1. To develop a reasoning methodology to improve the decision-making process in transport systems planning. The study proposes the use of reasoning map structure to explain the causalities between the proposed transit alternatives and the transit project’s goals. The study also develops mathematical mechanisms to evaluate alternatives and measure the integrity of the reasoning process and analyst’s uncertainty and ambiguity in transport project decision-making process in general.

2. To apply the proposed reasoning methodology to the real-world planning and decision-making of public transit systems under uncertainty and ambiguity.

The research contributes to three difficult evaluation issues: (i) development of the reasoning framework for appraising the merits of alternatives by building reasoning chains and mathematical mechanisms for evaluating their integrity; (ii) measurement of uncertainty and ambiguity of information used in decision-making process; and, for practice; and (iii) application of the proposed reasoning framework to solve an urban transit planning and decision-making problem under uncertainty and ambiguity.

1.4 Research Significance and Merits

The core of planning and decision-making related to transit systems is constructive discourse among the analysts, citizens, and decision makers. Today’s decision-making process is a participatory planning by all concerned citizens. To make
citizen participation most productive, public discourse which is often distracted and confused need become more focused and possible consequences and uncertainty must be clearly presented. The proposed mechanism helps the participants to focus on specific causal relations. The integrity of the decision is related to how uncertainty is treated and how the participants understand uncertainties and ambiguities involved.

The intellectual merits of this study are three-fold: (i) to understand the causal relationships of the affecting factors involved in and affected by proposed transit alternatives; (ii) to develop mathematical mechanisms that best capture the analysts’ knowledge and represent decision-makers’ reasoning in transit planning process; and (iii) to help derive the best course of action based on the available knowledge and discourse.

The proposed framework will bring significant impacts to the planning and decision-making of transit systems. Firstly, this study is of concern for today’s policy-makers, since they face numerous pressures and arguments in decision-making related to transit project development.

Secondly, this study benefits all stakeholders. Operators will be able to provide efficient transit facilities and services that meet regional and local plans. Users will be able to achieve maximum utility as a result of good and efficient transit systems. The public will be able to review and understand the integrity of planning process, and the ambiguities involved in the reasoning quantitatively and objectively.

Finally, this study is of interest for both the developed and the developing regions. For the developing regions, policy-makers are interested in the justification of a need for transit service, which is the first step of transit systems planning process. For the developed regions, their interest is shifted to the modification and improvement of the performance of the existing transportation system, which is critical for sustainable transportation and livable communities.

The proposed reasoning methodology can finally be applied to many other areas of decision making under uncertainty and ambiguity of information involving all types of inference, such as prediction, diagnosis, and control processes. Use of these mechanisms can promote construction of logical planning and decision-making processes, while enhancing the strategic use of information.
1.5 Research Organization

The research is organized into eight chapters. The relationship among chapters is presented in Figure 1-3. This first chapter introduces the research background, the research motivation, the research scope, and the research objectives and its contributions.

The second chapter presents the problem and justification of the research area, and presents the overall framework of the research. It accentuates the problems of transport planning, justifies the needs of this research, proposes the ideas to address the problems in transport planning together with the approaches and analytical algorithms, discusses the verification and validation process, and presents the expected outcomes of the research.

The third chapter reviews the relevant literatures. Three main aspects of the research are reviewed: (i) current methods and practices in planning and evaluation of transit systems, (ii) attendant reasoning and inference process; and (iii) mathematical theories of uncertainty. In the chapter’s first section, the characteristics and limitations of different planning and evaluation methods are identified. In the second section, the concept and applications of reasoning and inference process to evaluation and decision-making problems are reviewed. In the last section, different mathematical treatments of uncertainty including probability theory and evidence theory are presented along with their limitations.

The fourth chapter presents the first contribution of this dissertation: the proposed reasoning methodology under uncertainty and ambiguity. It is called ‘belief reasoning’ in this study. This chapter describes the main elements and characteristics of ‘belief reasoning’: the reasoning process, the mathematical mechanism to represent knowledge under uncertainty and ambiguity, the evaluation and interpretation of the results from the proposed methodology. This chapter also presents the step-by-step process to apply the proposed reasoning methodology to evaluate transit alternatives.
Following the development of the proposed reasoning methodology, the fifth, sixth, and seventh chapters demonstrate its verification and application. The fifth chapter presents the comparison of the belief reasoning method to traditional decision-making approaches including weighted linear method and the Bayesian reasoning method. The sixth chapter examines the application of the belief reasoning to the real-world transit alternatives analysis and project evaluation. This chapter evaluates transit alternatives, and measures the integrity and validity of information used in transit plans.

The seventh chapter, the last chapter, will summarize the findings of the dissertation and discuss the benefits of reasoning framework and the possibility of the applications of the proposed reasoning to other decision problems. This chapter will also present possible recommendation for further study.
CHAPTER 2

PROBLEM AND JUSTIFICATION OF THE RESEARCH

2.1 Research Framework

The framework of this research as presented in Figure 2-1 consists of two stages. One is the problem identification, and the other is the model development. In the first stage, the study starts from examining the reality and practices of planning and evaluation process of urban transportation systems and identifying related issues of transit system planning process. Two main issues of the transit systems planning are considered: (i) the lack of clear reasoning process; and (ii) the presence of various types of uncertainty in the planning process. As a result, two ideas are proposed in this study to address the problems. The first is a deliberative reasoning process, and the second is the recognition of uncertainty and ambiguity.

In the second stage, the research develops a methodology that integrates these two concepts. The proposed methodology is referred as a reasoning under uncertainty and ambiguity. It is called belief reasoning in this research. Belief reasoning uses (i) the reasoning map structure to model the planning and decision problem, and (ii) a belief measure in the Dempster-Shafer theory of evidence (DST) as a mathematical mechanism to represent knowledge about the decision outcomes under ambiguity. This belief reasoning methodology is then verified by comparing with two traditional decision-making methods: weighted linear method and Bayesian reasoning method, and validated by applying to the planning and evaluation of real-world transit alternatives. In the end, the decision support system is developed and the proposed reasoning method is incorporated into this decision-making tool. The final outcome of the research is a new decision-making method that, with improving reasoning, helps focus debates in planning and decision-making under the environment of uncertainty and ambiguity. The details of each element of the research framework are described in the Sections below.
2.2 Reality and Practices of Transit Planning and Evaluation

Public transportation interacts with many aspects of society. Planning and evaluation of public transportation systems is a multidisciplinary process governed by laws in many countries, which require the consideration of a comprehensive set of factors. In the U.S. the most recent endorsement in the Safe, Accountable, Flexible, Efficient Transportation Act: A Legacy for Users (SAFETEA-LU) requires transit planners and analysts to do more than analyze transportation models and estimate benefits and costs of the projects, but to consider a wide array of impacts to the society.
Public transportation is a public service supported by public resources. Sound planning and evaluation process is influenced by public input and reflect public purposes and needs. Their inclusion makes transit planning and evaluation processes complex.

This research recognizes several characteristics and consequences of the current practices of evaluation and planning of urban transportation systems. They are shown in Figure 2-2, and each of the elements is described below.

### Reality and Practices of Evaluation and Planning of Urban Transportation Systems

#### Natures
- Many stakeholders
- Many goals and objectives (land-use, finance, politics, equity, engineering)
- Many constraints
- Many unknowns (knowledge, data)

#### Consequences
- Many and conflicting opinions
- “Trust me solution”
- Lack of logical process
- Skepticism by the public
- Long-time to reach a decision
- Decision is always made under uncertainty

![Figure 2-2: Nature and Consequences of Transit Planning and Evaluation Process](image)

**Many stakeholders.** The decision problem involves a diverse group of stakeholders who are motivated by different purposes of public transportation systems. They are usually classified into three groups: users, operators, and the community.

**Many goals and objectives.** Different stakeholders have different perspectives and, as a result, they advocate conflicting goals and objectives. Table 2-1 shows the goals and objectives commonly used in transit planning and evaluation process.

**Many constraints.** Several constraints both internal and external, influence the planning and evaluation of transit projects; these include time constraints, financial constraints, political constraints, data and information constraints, and technological constraints.

**Many unknowns.** Planning and evaluation process relies largely on a predictive process where many key inputs are ambiguous and incomplete but the outputs are expected to be certain.
Table 2-1: Common Goals and Objectives in Transit Planning and Evaluation

<table>
<thead>
<tr>
<th>Transit Users</th>
<th>Transit Operators</th>
<th>Community</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum of travel time</td>
<td>Increase in passenger attraction</td>
<td>Quality of life</td>
</tr>
<tr>
<td>Minimum of user costs</td>
<td>Cost effectiveness</td>
<td>Employment Opportunities</td>
</tr>
<tr>
<td>Reduction of perceived travel time</td>
<td>Minimum of operating costs</td>
<td>Economic development</td>
</tr>
<tr>
<td>High mobility</td>
<td>Reduction of operating subsidy</td>
<td>Environmental impacts</td>
</tr>
<tr>
<td>High accessibility</td>
<td>Reliability of service</td>
<td>Community impacts</td>
</tr>
<tr>
<td>Availability of service</td>
<td>Safety and security</td>
<td>Fuel consumption and Imported Oil</td>
</tr>
<tr>
<td>Reliability and punctuality of service</td>
<td>Side effects</td>
<td>Livability and Sustainability</td>
</tr>
<tr>
<td>Convenience</td>
<td></td>
<td>Traffic Congestion</td>
</tr>
<tr>
<td>Comfort</td>
<td></td>
<td>Intermodal connectivity</td>
</tr>
<tr>
<td>Safety and security</td>
<td></td>
<td>Land use impacts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Equity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Accountability</td>
</tr>
</tbody>
</table>


The characteristics of the current evaluation and planning process of urban transportation system described above cause various undesirable consequences.

Many and conflicting opinions. A wide group of stakeholders have many and conflicting opinions. As a result, it is difficult to compromise and resolve conflicts. Most existing evaluation methods avoid this situation by using a small and consistent group of decision-makers in the evaluation process.

“Trust me solution”. Since only a small group of planners is involved in the evaluation process, the solution of the transport problem therefore reflects their preferences. However that solution may not be valid to other stakeholders or the public. Furthermore, the decision is traditionally made with the top-down approach; in other words, the solution is already in the decision-makers’ mind before a critical appraisal.

Lack of logical process. The question that transit planners are very often asked is how to justify the proposed transit project and to what extent the proposed solution achieves the goals of the project. Many decision approaches simplify the structure of the problem; and as a result, it is difficult to justify the decision or even know on what grounds it was made.
Skepticism by the public. When the transit planners are not able to justify their decision, the solution becomes questionable in the public’s mind and the merits of transit projects result in debates and with a large number of conflicting arguments.

Long time to reach a decision. The conflicts among different opinions in planning process usually delay the decision-making process, and sometimes cause the abandonment of the plans.

Decision is always made under uncertainty. Although conflicts exist and the arguments remain, most transportation planners are forced to recommend a decision under an uncertain environment. As a result, most transportation projects are often not as successful as expected. Pickrell (1990) showed evidence on the inaccuracy of travel demand and cost forecasting (over-estimated ridership and cost overruns) in the appraisal of transit projects. TCRP (2011) discussed the failure of several downtown transit circulators in the U.S.

The approach to analyze the proposed alternatives and evaluate their performance should be done as logically and transparently as possible, and should reflect the perceptions of the stakeholders and policy-makers.

2.3 The Problem

Review of the practices of planning and evaluation processes leads to two statements of the problem: the lack of a clear reasoning process, and the presence of many uncertainties in evaluation and planning process. These two problems are inherent in the political processes during planning and evaluation. Figure 2-3 shows the main elements affecting the issues of the current planning and evaluation process. The details of each element are described below.
2.3.1 Lack of Reasoning Process

Traditional planning and evaluation approaches do not have sufficient and reasoned justification for recommending a specific transit solution in the face of complicated chains of reasoning. Kanafani et al. (1994) describe the transportation planning problem as a deliberative process of negotiation and consensus building that is supported by rational analysis. It is essential to consider the public transit planning as a reason-building process to advance a certain course of actions. A series of actions which achieve goals is explained by chained relations in a reasoning structure.

There are three main reasons why the traditional practices do not provide sufficient room for the kind of reasoning needed in environment of limited understanding.

**Lack of knowledge.** Transit planners and analysts do not have sufficient knowledge either to identify a set of relevant variables or ‘drivers’ to build the chained reasons for outcomes. Without complete knowledge and information, it is simply impossible or at least unreliable for traditional approaches to justify the outcomes.

**Lack of reasoning structure.** None of the traditional transport planning and evaluation approaches structures the problem in such a way that the reasoning process can be modeled. The current procedures simplify the problem, and are sometimes considered as a “black box” where the inputs and the relations between inputs and outputs are not fully described or are results of imaginations.
Lack of quantitative measures of strengths. In reality, the goals and performance of alternatives are assessed and presented in either quantitative terms (e.g. monetary values, persons) or qualitative terms (e.g. “High,” “Medium,” or “Low”) in a decision table format. However, the question often remains to what extent the values are reliable and true or how much they can be trusted. These values are called the measures of strength of the goals and performance.

In actuality, transport analysts and planners develop implicit reasoning chains when evaluating transport alternatives; however, that reasoning is unstructured or undocumented. This causes difficulties in understanding the grounds for decision-making and lack a measure for inconsistencies, conflicts, and omissions without references to the complexity of the problem. Therefore, the traditional evaluation and planning approaches are inadequate for communicating decision reasoning in practice because they do not support the (supposed) determining decision variables and criteria in a fact-based manner.

This research views today’s transportation planning and evaluation process as a complex reasoning process which consists of various elements of both transportation and non-transportation aspects and involves diverse groups of stakeholders.

2.3.2 Presence of Various Types of Uncertainty

Planning and evaluation of (public) transportation systems involve uncertainty because the process deals with subjective judgments and perceptions, poorly-defined goals, criteria and performance, incomplete and unreliable data, insufficient knowledge and conflicting opinions, and different patterns and units of evidence. This study classifies the uncertainty into four sources.

Analyst uncertainty. Transit analysts may experience difficulties in proposing decisions due to the presence of multiple outcomes due to the lack of knowledge or conflicting opinions. As a result, they may not be able to specify a preference.
**Data uncertainty.** Evidence and model inputs of transportation systems which are in form of data, statistics, information, knowledge, perceptions, opinions, debates, critiques, are, as a rule, incomplete, inaccurate, approximate, imprecise, conflicted, and scattered. However, they are the data available and the analysts and engineers have to deal with them.

**Uncertainty in Prediction and Forecasting.** The predicted performance of transportation systems (e.g. predicted riderships, estimated costs, and future land-use impacts and economic activities) is not completely certain. These future values may be estimated inaccurately due to the limited capability of the transportation models. (North, 2005)

**Ambiguity in goals and constraints.** The goals and constraints of public transportation systems are not well defined. They tend to change over time and are biased by psychological, political, cultural, social and economical pressures. It is relatively difficult to explain whether the conclusions reached and decisions made satisfying and conforming to the goals.

**Ambiguity in outside factors.** Experts may reach their conclusions differently. Factors affecting or affected by the public transportation systems may vary between different standpoints. External factors may be excluded and overlooked.

In general, there are three types of uncertainty present in knowledge representations: randomness, vagueness, and ambiguity. Figure 2-4 shows the classification of information uncertainty. This research however aims at dealing with two types of uncertainty that commonly found in a decision making process: (i) ambiguity due to conflicting opinions (discord); and (ii) ambiguity due to the lack of knowledge (non-specificity.)

The first type, ambiguity due to conflicting opinions, arises because of the inability to draw a conclusion of the status of the variable (e.g. “High” or “Low”). There is no agreement among experts; in one opinion “The environment impact of the project is high,” and in the other “The environmental impact is low.”
For the second type, ambiguity due to the lack of knowledge arises because of inability to specify the state of variables; an expert opinion simply is “I don’t know” (whether the environmental impact is Low, Medium, or High).

![Types of Uncertainty for Knowledge Representations](Adapted from Klir and Wierman, 1999)

Uncertainty and ambiguity always play a significant role in the decision-making, particularly in the public transportation decision making process. This research encourages the need of mathematical and other mechanisms that can handle uncertainty and ambiguity and represent different patterns of evidence associated with uncertainties.

### 2.4 Approaches to the Problem

This research proposes a new decision-making framework to improve the planning and evaluation process of urban transportation systems. The proposed methodology applies (i) a reasoning map structure in information logic, and (ii) a belief measure in evidence theory.
2.4.1 Reasoning Map Structure

This research constructs the decision structure using a reasoning map instead of the hierarchical structure used in traditional transport decision-making approaches. The reasoning map presents the chain of reasoning of a collection of systems variables, and seeks to explain the causalities and interrelationships among the characteristics of the alternatives, evaluation criteria, and goals of the project. A reasoning map structure can deliberatively link the characteristics of alternatives to a set of goals, and justify the recommendations for transit alternatives in a logical manner. It also helps transport planners to develop a form of knowledge, to understand the cause-and-effect relationships of the variables in the system, and to derive conclusions from available information.

The reasoning map in transportation planning connects four basic elements in transportation systems as shown in Figure 2-5. They are predicted characteristics related to the transportation system \(D\), the characteristics of the transit alternative \(S\), the performance of the transit alternatives \(P\), and the goals of the project \(G\). The concepts and characteristics of this reasoning map structure are presented in Chapters 3 and 4, respectively.

![Figure 2-5: Structure of Evaluation Process Using Reasoning Maps](image-url)
2.4.2 Mathematical Framework for Knowledge Representation

Different types of uncertainty require different mathematical representations of uncertainty treatment. Probability theory deals with uncertainty due to randomness (that is risk); fuzzy set theory deals with vagueness; and possibility theory and evidence theory deals with ambiguity.

Traditionally, probability has been the approach used to deal with risk (and even uncertainty) in decision-making process. Probability represents the degree of belief in terms of the frequency of occurrences based on the evidence presented. Nonetheless, in reality when analysts evaluate alternatives, they experience evidence in the form of data, information, opinions, and critiques, which are usually vague, incomplete, conflicting, and scattered. The traditional probability theory may not be sufficient and appropriate to model and work with such weaker state (that is uncertainty) of information and knowledge.

The evidence theory, Dempster-Shafer theory of evidence (DST), is a generalized mathematical theory to deal with the all types of uncertainty, particularly uncertainty involving ambiguity (or ignorance). This type of uncertainty is always present in decision-making about large-scale systems like public transit system. DST is suited for eliciting opinions from different stakeholders and measuring uncertainties involving non-specificity and conflict in which the traditional probability theory is not. DST uses belief measures instead of probability measures to represent the degree of belief (or degree of support) of occurrences. The basic axioms and characteristics of DST are presented in Chapter 3.

2.4.3 Reasoning under Uncertainty and Ambiguity: Use of Evidence Theory

This research develops a reasoning methodology by applying a reasoning map structure and Dempster-Shafer theory of evidence to reason decision-making under uncertainty and ambiguity. The models developed can be useful in understanding the effect of information on the decision-making process, and also in evaluating the performance of alternatives.
The important steps of the proposed reasoning methodology are (i) identifying the variables and goals of the decision problem, (ii) developing causalities and reasoning chains, (iii) collecting knowledge about the proposed alternatives, (iv) eliciting experts’ knowledge about the reasoning chains and assigning the degree of belief to each relation; (v) measuring uncertainty of the inference and validity of knowledge and reasoning chains; (vi) measuring the degrees of goal achievement, (vii) deriving the overall performance of an alternative and comparing alternatives; and (viii) determining the critical chains and interpreting the results. The details of the proposed methodology are presented in Chapter 4.

2.5 Verification and Validation Process

In order to examine the reasonableness of the proposed belief reasoning method, the study compares its capability and benefits to those of the two decision-making approaches.

The first is the Analytical Hierarchy Process (AHP), which uses a hierarchical structure to model the decision problem and evaluates the alternatives based on the weighted linear function. The weighted linear function is the most widely-used evaluation method in transit decision-making problems. The study develops a numerical example to illustrate the benefits and limitations between the hierarchy structure and the proposed reasoning map structure.

The second decision-making approach, which is relevant to this study, is the Bayesian reasoning (the traditional way to reason knowledge under uncertainty). This probability-based reasoning has been applied to many decision-making applications (Wiboonsak and Peng, 2004; Bayraktar and Hastak, 2009), but it has not been applied in a transit decision-making context. The study also develops a numerical example to demonstrate the calculation procedures of the Bayesian reasoning and the proposed belief reasoning methods.

In order to test the practicability of the belief reasoning method, the study applies it to the real-world transit project. The study selects the transit alternatives analysis of the
Columbia Pike transit corridor project in Northern Virginia as a case study. The alternatives analysis of this project is tested. Local transit planners and experts are interviewed to collect knowledge about the alternatives and the reasoning process and to evaluate the alternatives.

### 2.6 Product and Testing

To facilitate practical use of the proposed methodology, the study designs and develops a computerized decision support system (DSS) that integrates the proposed belief reasoning methodology for evaluating transit proposals. This computerized tool applies the belief reasoning method to replicate the Federal Transit Administration New Starts evaluation and rating process. The DSS is designed under *Microsoft Visual Basic* object-oriented programming language. This tool is developed to support transit decision-makers in evaluating and prioritizing transit proposals as well as to help transit analysts understand the data needed and the input-output process of the proposed method. Nevertheless, this decision-making tool will be presented and published elsewhere.

### 2.7 Outcomes

The outcome of the research is the development of the new decision-making methodology that can present reasoning patterns and take into consideration the uncertainty involving ambiguity in planning of (public) transportation systems. The proposed methodology should be useful for improving reasoning in the evaluation process and focused debate among different stakeholders under ambiguous environments.
CHAPTER 3

TRANSIT PLANNING: A LITERATURE REVIEW

This chapter reviews the literature on four aspects of transportation planning. In order to keep the review in focus, it concentrates on transit planning, but it is broadly applicable to transportation planning in general. The first section examines the transit systems planning and evaluation process. The second section discusses different methods to evaluate transportation systems. The third section reviews the mathematical theories about uncertainty. The last section studies the reasoning and inference process for decision-making problems in transportation.

3.1 Transit Systems Planning and Evaluation Process

In the U.S. a new major transit investment project—either the development of a new fixed-guideway transit system or the extension of an existing transit system—must be developed through a major transit investment planning and project development process. This planning and project development process begins with the systems planning and is followed by the alternatives evaluation process.

Transit systems planning is the comprehensive and coordinated transportation planning process executed by different transportation planning agencies, including metropolitan planning organizations, state department of transportation, local government, and local transit agencies. The systems planning process results in (i) the identification of existing transportation problems, future needs, and transportation policies; and (ii) the identification of a wide range of conceptual transportation alternatives and investment plans.

Transit project development is a long process normally under the leadership of the local transit agencies in collaboration with local governments and the general public. The
process is often conducted in a vibrant political and institutional setting. The goal of the evaluation process is the provision of sound and objective information for decision-making. (FTA, 2007a)

3.1.1 Historical Background of Planning and Evaluation Process

Since 1970s, the Urban Mass Transportation Administration (UMTA), now known as the Federal Transit Administration (FTA) has been engaged in consistent efforts to initiate and improve major transit investment planning and project development processes and to allocate a share of the nation’s capital investment to urban public transportation systems. This has resulted what is known as the New Starts program (Duff et al., 2010). The processes have then been incorporated in Federal legislation.

The original policy for system planning and evaluation process by UMTA required projects to be cost-effective only. However, due to limited resources, requirements for a multi-year local commitment were established later. In 1980s, many projects requested, and the policy introduced an approach for comparing competing transportation projects using a cost-effectiveness index. The policy under the Surface Transportation and Uniform Relocation Assistance Act of 1987 (STURAA) established a set of criteria, which new starts projects had to meet: cost-effectiveness and local financial commitment.

The FTA New Starts program was first established by the Intermodal Surface Transportation Efficiency Act of 1991 (ISTEA). Under this law, substantial changes were made on the criteria in the evaluation process. The new policy required projects to be justified based on an evaluation of multiple measures: mobility improvements, environmental benefits, cost-effectiveness, and operating efficiencies.

The FTA New Starts program was further refined under the Transportation Equity Act for the 21st Century (TEA-21). The evaluation and rating process remained the same as in ISTE A except for the change in warrants of projects to be evaluated and the progression in stage-wise approval of projects (Emerson, 2002).

Recently, the FTA New Starts program has been continued by the Safe, Accountable, Flexible, Efficient Transportation Equity Act: A Legacy for Users
(SAFETEA-LU). A refinement in a planning and project development process was made. Figure 3-1 presents the planning and development process of New Starts projects. (FTA, 2007b)

Figure 3-1: FTA New Starts Planning and Project Development Process

(Adapted from FTA, 2007b)
The FTA *New Starts* planning and project development process is a continuing process with multi-stage evaluation and decision-making actions. Three main stages of evaluation and decision actions are carried out for the development of major transit investment projects. They are the alternatives screening process, the alternatives analysis process, and the FTA New Starts evaluation for funding recommendation process.

(1) *Alternatives Screening Process*

The intention is to assess the potential for transit alternatives to meet the purpose and need of the project. The outcome of this initial screening is a recommended set of transit alternatives that potentially meet the project benchmarks. FTA (2007a) suggests that this set of alternatives must address the purpose and need, goals and objectives, and must include a baseline alternative, all possible alternatives with a range in costs. The screening process evaluates three aspects: (i) mode-specific improvements (e.g. implementing travel demand management strategies, improvement of existing transit systems); (ii) technologies: road-based (e.g. regular bus, bus rapid transit) and rail-based (e.g. streetcar, light rail, high-speed rail, automated guided transit); and (iii) alignments—based on predicted station location and existing right-of-way. (Parsons Brinckerhoff, 2006)

(2) *Alternatives Analysis Process*

The purpose of the Alternatives Analysis (AA) process is to evolve and recommend the *locally preferred alternative* (LPA) that best meets the goals of the transit project development and addresses locally identified transportation problems in a specific corridor. It involves the development and selection of the most promising technology and alignment for the system design from the recommended list of alternatives. FTA (2007a) suggests that the set of alternatives should include a ‘No-Build’ alternative, at least one non-guideway transportation system management alternative (e.g. HOV or TSM alternatives), and one or more ‘Build’ alternatives.

It is noted that in the AA process, the transit alternatives must be reasonably comparable, and the evaluation must be conducted considering impartial evaluation criteria and using an explicable decision method. (SEPTA, 2002; DMJM, 2005)
(3) Funding Recommendation Process

The purpose of the funding recommendation process is to justify (i) whether a major transit investment project is financially feasible for construction, and (ii) whether it is timely to recommend for implementation compared to other competing projects. FTA evaluates several major transit investment projects around the nation every fiscal year. The LPA of each transit project requested for federal funding is evaluated through the New Starts planning and project development process.

After the analysis of alternatives, the LPA of a transit project—the alternative that best meet local needs and project purposes—FTA may advance the project for Preliminary Engineering (PE) in the New Starts program. In the PE phase, the project scope, cost estimates, financial plans with local funding commitments, and environmental impact assessment of the transit proposal are presented in order for FTA to advance the proposal into the Final Design (FD) phase. In the FD phase, the readiness for construction is evaluated. Finally, FTA examines the possibility of a multi-year commitment to fund a proposal through the Full Funding Grant Agreement. (FTA, 2007b)

Although the purpose, evaluation criteria, and alternatives considered in the three stages (alternatives screening, alternatives analysis, and project evaluation) are different, the nature and characteristics of the evaluation process are similar. This is described in the next section.

3.1.2 The Nature of Evaluation Process

Evaluation of public transportation systems (either comparison of transit alternatives in the project development process or comparison of transit projects in the funding recommendation process) is complicated because the problem deals with interaction of two aspects of transit systems: demand-side and supply-side.

On the demand side, data and information about the anticipated travel demand and systems requirements are analyzed. They include (i) goals and objectives obtained from local government’s policy and the public; and (ii) existing local conditions including travel demand and its predictions, transportation routes and networks, and socioeconomic and environmental conditions in the targeted service areas. The outcome
of the demand side includes the list of project goals, evaluation criteria, and performance measures, including demand, which are defined for diverse groups of stakeholders who benefit from or are affected by the project: the users, the transit agency, the local governments, and the general public. (Meyer and Miller, 2000; Vuchic, 2005)

On the supply side, the basic elements of the proposed transit options, including the physical and operational characteristics of alternatives, which affect operational performances, life-cycle costs, service quality, and impacts of transit systems on the environment, are considered in the analysis of alternatives. The outcome of the supply side is the list of candidate transit options. Figure 3-2 shows the interaction between the demand and supply sides in a public transportation decision-making process.

Given the list of goals, requirements, and evaluation criteria from the demand-side and the list of candidate transportation options, the decision is made based on the selected evaluation method.

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**Figure 3-2: Interaction of Supply and Demand Sides in Evaluation Process**

(Source: Adapted from Vuchic, 2005)
3.2 Transportation Evaluation Methods

There are several methods available for the evaluation of major transportation investment projects. They range from a free-form debate of the alternatives to a very structured and complicated analysis of project performance. They can be categorized into four groups based on the levels of complexity of analysis and evaluation measures.

3.2.1 Checklist and Warrant-Based Methods

The checklist and warrant-based approaches are the simplest methods of evaluation. They require decision-makers to assess whether an alternative meets the list of requirements. For checklist method the list of requirements is defined, while for the warrant-based method, minimum criteria for each of the requirements are defined. The U.S. Federal Transit Administration currently specifies minimum performance criteria and standards that transit proposals must be met in order for a funding recommendation of the Very Small Starts program (FTA, 2009b) They include the total capital cost, number of transit riders, and the peak and off-peak frequencies of the proposed transit system.

However, the checklist and warrant-based methods may not result in the best transit alternatives being ranked highest; in fact the methods do not rank the alternatives at all. They only provide feasible alternatives which would advance to detailed analysis. In these approaches, definition of requirements is very important and must be clearly made.

3.2.2 Economic Evaluation Methods

Economic evaluation methods, such as the Benefit Cost Analysis (BCA) and Cost Effectiveness Analysis (CEA) methods, have been widely used in evaluation of transportation projects, particularly when the budget is strict and the major constraint in decision making process. (AASHTO, 1977; UMTA, 1984) These methods estimate benefits and costs and compare the cost-effectiveness of each of the alternatives.
BCA converts all economic and non-economic attributes into monetary values and presents the economic justification in terms of economic measures, such as benefit-cost ratio and net present value (TRB, 2002). However, the fairness of conversion in BCA method is questionable and the economic measures are often uncertain. (Johnston and Deluchi, 1989)

CEA estimates benefits and costs of the transportation projects without estimating a scalar index and presents the economic justification in terms of several cost-effectiveness measures. Fielding et al. (1978) proposed three efficiency measures (revenue vehicle hours per vehicle, revenue vehicle hours per employee, operating expense per revenue vehicle hour), four effectiveness measures (revenue passengers per service area population, percent of population served, total passengers per vehicle, and revenue passengers per revenue hour), and two overall measures (operating expenses per total passengers, and operating expenses per revenue passenger.) In spite of its multi-dimensionality, CEA is not sufficient for the evaluation of public transportation investment projects where many benefits are socialized or related to the environment, and perhaps most importantly, it does not account for uncertainty in the chosen measures.

These economic evaluation methods only consider a single objective (financial aspects), but cannot accommodate diverse social objectives which are not easily measurable in monetary values. In addition, these methods do not examine long-term sustainability of a public transportation plan whose benefits are far less than its costs.

3.2.3 Multi-Criteria Decision-Making Methods

A large number of multi-criteria decision-making (MCDM) methods have been proposed to incorporate the needs of different stakeholders involved in decision-making process. MCDM methods use a numerical or analytical model to find the alternative that would best meet a wide variety of criteria. They transform both qualitative and quantitative measures into a single objective value. Among various MCDM methods, the following are applied in transportation planning.
(1) **Indexing and Scoring Method**

The indexing (or scoring) method is the simplest and least analytical MCDM method, which requires analysts to rate the potential performances of each option according to a list of criteria. (Triantaphyllou, 2000; Figueira et al., 2005) The performance of alternatives is rated by either linguistic terms (e.g. high, low, good, very good) or numerical scores (e.g. 1-5 or 1-100). The scores are then combined to analyze the overall performance by using aggregate operators, e.g. an average operator or a weighted linear function. This model is mathematically expressed as:

\[
\text{Rating} = \sum_{i=1}^{n} (w_i \cdot x_i) 
\]

where \( x_i \) = the numerical score of criterion \( i \), \( w_i \) = the additive weight of criterion \( i \) and \( \sum w_i = 1 \).

The U.S. FTA has developed the evaluation and rating process according to the stipulation of SAFETEA-LU to evaluate and rate the FTA New Starts proposals. (FTA, 2009a; 2009b; 2009c; 2010; 2011b; 2011c) This process is a multi-measure approach which utilizes an indexing and weighting scheme and considers two broad categories of evaluation criteria: the financial commitment and project justification. For the financial commitment, three attributes are considered: Non-New Starts funding share, capital funding plan, and operating funding plan. For the project justification, six attributes are involved: mobility improvements, environmental benefits, cost effectiveness, operating efficiency, economic development, and land use impacts. The attributes considered in the FTA process include both qualitative and quantitative assessments. For each attribute, an ordinal rating scale from 1 to 5: 1 for Low, 2 for Medium-Low, 3 for Medium, 4 for Medium-High, and 5 for High. The scores are then aggregated using the predefined relative weights and decision rules to receive the overall rating of each project. The proposals will be recommended to advance to the next phase, if they receive at least the Medium rating.

This method however presents several challenging issues. Kikuchi and Kronprasert (2011) examine the issues of the mathematical operations in the FTA process. That study finds that the weighted linear method in FTA process is sensitive to many factors: (i) the use of integer scores and rounding operations increase the chances that the
overall score is higher; (ii) the rigid breakpoints of the performance ranges affect the 
overall score; and (iii) the values of weights can easily be biased and mislead the decision 
makers.

The issue of assigning the values of weights in the weighted linear method is 
always questionable to decision makers in any multi-objective, multi-criteria decision-
making applications. In decision theory, the weights represent the relative importance or 
preference of one objective relative to another objective. For example, “the objective i is 
two times as important as the objective j (w_i = 2w_j)” However, it is difficult to measure 
the degree of importance or preference when two objectives have different units or are 
redundant; such as the operating costs per passenger mile and operating costs per 
passenger hour.

In statistics, there is a trend in which weights represent the relative difference in 
the measurement error. The more precise the performance measures, the higher its 
weight; or the smaller the variance (uncertainty), the higher the weight. Ghilani and Wolf 
(2006) define that the weights are inversely proportional to variances (or uncertainty,) 
w=1/\sigma^2. However in reality when dealing with subjective and non-commensurate 
objectives where data come from expert opinions, the uncertainties cannot be presented 
by variance.

Furthermore, the indexing and scoring methods always require an informed group 
of individuals for evaluating and rating the projects. Some experts argue that the linear 
scoring methods are prone to biases and leave room for speculative “political inputs” in 
deciding the weights assigned to each attribute. (FTA, 2009c)

(2) Analytical Hierarchy Process Method

An Analytical Hierarchy Process (AHP) method is a multi-criteria decision 
analytical tool to determine the weights for criteria (or objectives) in a comprehensive 
manner. (Saaty, 1980) The AHP method decomposes a complex decision problem into a 
hierarchical tree as shown in Figure 3-3. The analysis of AHP has four steps: (i) 
identification of goals, criteria (and sub-criteria), and alternatives and construction of the 
hierarchical structure of the problem; (ii) assignment of the relative weights (preference 
or importance) to criteria using pair-wise comparison; (iii) calculation of the absolute
weight of an individual criterion; and (iv) determination of the final ranking among alternatives.

However, AHP has several limitations. (Saaty, 1995; 2005; Parajuli and Wirasinghe, 2001; Banai, 2006; Zak, 2005, 2007; Arslan, 2009) Firstly, the hierarchical structure shows the classification of criteria and sub-criteria; however, it does not explain the dependent relationships between attributes. Secondly, the evaluation criteria in the same group are assumed to be independent; that is, there is no redundancy (interrelationship) among the criteria.

Thirdly, computational efforts for AHP are proportional to the scale of the decision problem. AHP is burdensome for experts when many evaluation criteria are involved or when many alternatives are compared because the number of pair-wise comparison increases polynomially with the number of criteria and alternatives. For example, if the number of criteria and alternatives on each level in Figure 3-3 increases from 3 to 4, the total number of pair-wise comparison increases from 39 to 124.

Finally, AHP assumes that experts are able to indicate the relative importance (weights) between attributes and that the relative weights are reciprocal (i.e. if $A$ is $m$ times preferred to $B$, then $B$ is $1/m$ times preferred to $A$.

![Figure 3-3: Structure of Analytical Hierarchy Process](image-url)
(3) Elimination and Choice Expressing Reality Method

An Elimination and Choice Expressing Reality (ELECTRE) method is a MCDM method based on the outranking relation. This method has two steps: (i) construction of outranking relations by comparing the performance of each pair of alternatives; and (ii) ranking or selecting alternatives. ELECTRE uses the concept of preference or indifference thresholds to define the preference relationships between two alternatives with respect to a particular criterion. The method uses weights of criteria to represent their importance. (Roy, 1991)

The limitation of the ELECTRE method is that it does not provide the numerical results, which helps decision-makers distinguish the difference between alternatives. Further, ELECTRE is usually limited to small decision problems and many experts have difficulties for defining the thresholds. (Zak, 2005)

(4) Utility Theory Additive Method

A Utility Theory Additive (UTA) method utilizes additive utility functions to aggregate multiple criteria into a composite measure. It uses a linear form of a weighted sum of marginal utility.

\[ U(g) = \sum_{i=1}^{n} w_i u_i(g_i) \]  

(3-2)

where \( w_i \) is the weight of criteria \( i \), and the \( u_i(g_i) \) is a normalized function of marginal utility function of criteria \( i \), \( u_i(g_i) \in [0, 1] \). For each criterion, the marginal utility function is derived as a non-decreasing function from the information given by a subjective ranking of decision maker’s preferences.

The UTA method has five steps: (i) definition of alternatives and the estimation the ranges of the performance values of alternatives associated with each criterion; (ii) assigning the weights of criteria; (iii) construction of the utility function; and (iv) estimation of the attributes which optimize each utility function using a linear programming technique; and (v) ranking and analysis of sensitivity. (Jacquet-Lagreze and Siskos, 1981)
Zak (2005) argued that the UTA method is limited if the number of alternatives is small because the alternatives may receive the same ranking. Many experts have difficulties for defining subjective preference (or indifference) for each criterion.

(5) Multi-Objective Mathematical Programming Method

The multi-objective mathematical programming (MOMP) method is the analytical MCDM method which constructs a decision problem as an optimization problem and then solves for the optimum by using mathematical programming techniques. The algorithms most often used are integer programming and goal programming methods. (Friesz, 1980)

MOMP is solved for the set of preferable transportation investment alternatives among feasible and limited alternatives according to multiple objectives to be achieved, and under several resource constraints. Consider the \( n \) decision problems \( x_1, x_2, \ldots, x_n \) of different alternatives according to \( m \) objectives \( Z_1, \ldots, Z_m \) under \( p \) resource constraints \( B_1, \ldots, B_p \). Mathematically, the model can be formulated as (Teng and Tzeng, 1993; 1996)

\[
\text{maximize } Z_i(x) = (Z_1(x), \ldots, Z_m(x)) = \sum_{j=1}^{n} G_{ij}x_j 
\]

subject to

\[ A_{k,i}x_j \leq B_k; \ x_j = 0 \text{ or } 1 \]  \hspace{1cm} (3-4)

where \( G_{ij} \) is a linear coefficient of decision variable \( x_j \) for objective \( i, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, p. \)

However, the MOMP method can be used for selecting alternatives due to the use of 0-1 scheme. The analysis method is based on the assumption that all objectives can be mathematically formulated without uncertainty and it requires a strict computational effort to solve for optimal solutions.

3.2.4 Limitations of Current Transit Evaluation Methods

The review of the classical evaluation methods reveals that there is no standard method. Each of these evaluation methods has been used for different purposes and
functions. Table 3-1 summarizes the main characteristics of the transit evaluation methods.

**Table 3-1: Summary of Current Transportation Evaluation Methods**

<table>
<thead>
<tr>
<th>Evaluation Methods</th>
<th>Descriptions</th>
<th>Limitation</th>
</tr>
</thead>
</table>
| 1. Checklist and Warrant-based method                    | • Simple and does not require detailed analysis  
• Assesses whether each option meets certain minimum criteria or requirements | • Does not rank nor evaluate alternatives  
• Cannot distinguish the alternatives that slightly differ or are in the same category |
| 2. Economic Evaluation methods                           | • Provide information about the economic justification  
• Require that benefits and costs are in monetary values  
• Compares the economic measure or cost-effectiveness measures of each of the transportation alternatives | • Does not represent the benefits and impacts to the society  
• Many benefits and costs are hard to be quantified and many subjective assumptions are made through the analysis |
| 3. Multi-criteria decision-making (MCDM) methods          | • Aggregate different objectives and criteria into one global objective utility function  
• Rates the potential effectiveness of each option according to a list of criteria in numerical scale or in linguistic terms  
• Analyzes the overall performance by combining the performance associated with different objectives | • The weight values are hard to determine and justify  
• The overall rating is sensitive to the demarcation setting |
| 3.2 Analytical Hierarchy Process (AHP) method             | • Decomposes a complex decision problem into a hierarchical structure  
• Models the preference using pair-wise comparison to determine the relative weights at each level of hierarchy  
• Ranks the alternatives based on the weighted utility | • AHP is complicated when many alternatives and criteria are involved.  
• Use relative weights for the objectives and selection of their values and interpretation can be a problem.  
• Difficult in conceptualizing the meaning of the overall function |
| 3.3 Elimination and Choice Expressing Reality (ELECTRE) method | • Compares the preference or indifference between a pair of alternatives  
• Ranks the alternatives, but does not present the numerical results | • Ranks the alternatives, but does not present the numerical results for each alternative.  
• Difficult to determine utility function and justify the weight values in a utility function |
| 3.4 Utility Theory Additive (UTA) method                  | • Measures the preferences of a policy maker as an additive utility  
• Used for ranking in choice problems with a large number of alternatives |   |
| 3.5 Multi-Objective Mathematical programming (MOMP) method | • Formulated as an optimization problem  
• Recommends the alternative that maximizes the achievement of objectives and satisfies the resource constraints | • Difficult to formulate an optimization function and justify the weight values in an optimization function |
The premise of most evaluation methods is to replicate mathematically human decision-making process. The mathematical mechanisms in these traditional transportation evaluation methods are restrictive. Firstly, they assume that decision makers’ knowledge about the alternatives is complete, and the expert judgments are consistent and accurate. Secondly, they ignore the redundancy (“double counting”) among the objectives and criteria; as a result, normally, a small number of objectives and criteria are considered for comparing alternatives. Thirdly, they disregard the interrelationships among system parameters and system performance. The objectives and criteria are assumed to be independent of the properties of the alternatives. Finally, some of the evaluation methods consider only system performance factors that can be expressed quantitatively, and exclude those that are qualitative or difficult to quantify.

Decision-making in transportation is more complex than the classical decision methods can model. The traditional evaluation methods need to be modified so that they can account for the psychological framework of individual decision makers and also enable sound and reasoned decision-making with the maximum use of available, normally partial information under uncertainty.

3.3 Mathematical Theories about Uncertainty

When a proposition “x is A.” is considered, two groups of information-based uncertainty are related: (i) uncertainty associated with information that defines the classification of the outcome set A, and (ii) uncertainty associated with information that defines the characteristics of the variable x. There are two groups of mathematical theories for representing information-based uncertainty. (Kikuchi and Pursula, 1998)

The first group is the set theory in which the information is defined in terms of a set of possible outcomes. The classical set and fuzzy set theories are the two mathematical theories to utilize this information for the definition of outcomes. The set theory deals with the characteristics and classification of the outcomes; for example, the outcome of travel time is classified into “less than 10 minutes,” “10-30 minutes,” and “greater than 30 minutes,” or it is classified into “Low,” “Medium,” and “High.”
The measure theory evaluates how much an outcome is supported by evidence. Probability theory, Possibility theory, and Evidence theory are the three mathematical methods for measuring uncertainty. Each deals with different types of evidence; probability theory deals with mutually exclusive and comprehensive evidence; possibility theory deals with consonant (or nested) evidence; and evidence theory deals with both consistent and conflicting evidence.

Probability theory is the most established measure theory that deals with the uncertainty associated with randomness. It presents the degree of support of outcomes through a probability distribution, $p$ where $p: X \to [0, 1]$ and $\sum p(x) = 1$ where $x \in X$. It measures the degree of belief of proposition by probability measure $p$. Evidence in probability theory must be singleton (i.e., points to only one outcome.) The probability of the outcome can be defined by three approaches: (i) the uncertainty maximization approach; (ii) the relative frequency-based approach in a large number of trials; and (iii) subjective. Based on these three approaches, the probability of 0.5 can imply (i) equal (50/50) chance of occurrence (or total ignorance), (ii) observed 500 (out of 1,000) times of occurrence; or (iii) the subjective probability (or belief) of 0.5.

Possibility theory is another type of measure theory that deals with the uncertainty associated with non-specificity. It presents the degree of support of outcomes through the possibility distribution, $r$ where $p: X \to [0, 1]$ and $\max \{p(x)\} = 1$ where $x \in X$. It measures the degree of belief of proposition by a range between a possibility measure ($\text{Pos}$) and necessity measure ($\text{Nec}$). $\text{Pos}$ denotes the degree of belief in an optimistic view and $\text{Nec}$ denotes the degree of belief in a pessimistic view. Evidence in possibility theory must be nested. More details of possibility theory can be found in Zadeh (1978), Dubois and Prade (1988), and Klir (2006).

Evidence theory, which is the most recent mathematical treatment of uncertainty that has been developed to deal with the uncertainty associated with ambiguity. It presents the degree of support of outcomes through belief distribution, $m$. Evidence theory was pioneered by Dempster in the 1960s and Shafer in the 1970s. It is also known as Dempster-Shafer theory of evidence (DST). (Dempster 1967; 1968; Shafer, 1976; Yager, 1994; 2002)
DST, which is the core mathematical framework in this study, is a generalization of traditional probability theory. DST can deal with ambiguity whereas the probability theory cannot because DST has a capability to deal with all patterns of evidence. DST allows degrees of support (beliefs) to be assigned to one or more sets of outcomes in propositions, instead of one mutually exclusive outcome of propositions as in probability theory. The Dempster-Shafer theory can handle evidence with different levels of precision and ambiguity and is discussed in more detail next.

3.3.1 Patterns of Evidence

Evidence, which is the basic representation of knowledge, enables the analysts and decision makers to determine the degree of belief of a proposition, to draw a conclusion and make a judgment about a complex system. Evidence is presented in several forms, such as data, information, and knowledge. In this study, the terms: evidence, information, knowledge are used interchangeably.

Uncertainty is closely related to quality and quantity of knowledge or evidence. Types of uncertainty are based on patterns of evidence leading to a set of outcomes. In knowledge representation, evidence which is the information about \( x \) in a proposition “\( x \) is \( X_i \)” is classified into four patterns as shown in Figure 3-4. (Sentz and Ferson, 2002) In this figure, \( X_1, X_2, X_3, X_4 \) are the possible outcomes from a set and the circles present different pieces of evidence or information \( E_1, E_2, E_3, E_4 \) in the form of proposition “\( x \) is \( X_i \).”

**Exclusive evidence.** Each piece of evidence points to one outcome and none of them supports the other. Figure 3-4(a) shows the mutually exclusive sets: “\( E1: x \) is \( X_1 \),” “\( E2: x \) is \( X_2 \),” “\( E3: x \) is \( X_3 \),” and “\( E4: x \) is \( X_4 \).”

**Nested or consonant evidence.** The smallest piece of evidence completely supports its next larger piece. Figure 3-4(b) shows the consonant sets: “\( E1: x \) is \( X_1 \) or \( X_2 \) or \( X_3 \) or \( X_4 \),” “\( E2: x \) is \( X_2 \) or \( X_3 \) or \( X_4 \),” “\( E3: x \) is \( X_4 \),” And “\( E4: x \) is \( X_3 \) or \( X_4 \).” The piece of evidence \( E3 \) is a nested to evidence \( E4, E2, \) and \( E1 \), respectively.
**Consistent evidence.** At least one piece of evidence completely supports all other pieces. Figure 3-4(c) shows the example of consistent sets: “$E1: x$ is $X_1$ or $X_2$”, “$E2: x$ is $X_2$”, “$E3: x$ is $X_2$ or $X_3$”, and “$E4: x$ is $X_2$ or $X_3$ or $X_4$”. It shows that the piece of evidence $E2$ is supported by evidence $E1$, $E3$, and $E4$.

**Arbitrary evidence.** None of pieces of evidence completely supports all other pieces of evidence. Figure 3-4(d) shows the example of arbitrary sets: “$E1: x$ is $X_1$ or $X_3$”, “$E2: x$ is $X_2$ or $X_3$”, “$E3: x$ is $X_3$ or $X_4$”, and “$E4: x$ is $X_4$”. None of any pieces of evidence completely supports the other.

![Figure 3-4: Types of Evidence](image)

**3.3.2 Representation of Evidence in Evidence Theory**

Three basic elements are involved in Dempster-Shafer theory of evidence: frame of discernment ($\Theta$), basic probability assignment ($m$), Belief ($Bel$) and Plausibility ($Pl$) functions.
(1) **Frame of Discernment**

The frame of discernment (Θ) is a set containing all mutually exclusive outcomes of set X. Each outcome in Θ, called a focal element, represents a proposition that can be either true or false. The power set of X, $2^X$, contains all possible subsets of Θ. This power set is called a body of evidence. If the set X is \{X₁, X₂, X₃\}, then eight sets: \{X₁\}, \{X₂\}, \{X₃\}, \{X₁, X₂\}, \{X₁, X₃\}, \{X₂, X₃\}, X, and $\emptyset$ are included in a body of evidence.

(2) **Basic Belief Assignment**

A basic belief assignment (or mobius representation, m) is a characteristic function of any set. In DST, a basic probability mass, $m(X)$ replaces a probability mass, $p(X)$ in the probability theory. A basic belief assignment $m$ is defined on a body of evidence $2^X$, whereas a probability $p$ is defined on $X$. The difference between the basic belief assignment ($m$) and probability ($p$) is that $m$ can be given to any proposition rather than given to one proposition in probability theory. A basic belief assignment ($m$) is characterized by four following axioms (Klir and Wierman, 1999; Ayyub and Klir, 2006).

\[
m: 2^X \rightarrow [0, 1] \tag{3-5}
\]

\[
m(\emptyset) = 0 \tag{3-6}
\]

\[
\sum_{X \in \Theta} m(X) = 1 \tag{3-7}
\]

\[
m(X) + m(\overline{X}) \leq 1 \tag{3-8}
\]

Equation (3-4) defines that the range of the belief mass is between 0 and 1. Equation (3-5) implies there is no confusion at all and Equation (3-6) is the exhaustion of all beliefs. Equation (3-7) shows that a basic belief mass ($m$) is a non-additive measure. In other words, there is no relationship of the belief mass between a set and its complement, $m(X)$ and $m(\overline{X})$. This differs from a probability mass ($p$), which defines $p(X) + p(\overline{X}) = 1$. Figure 3-5 compares a graphical representation between probability and belief distribution of DST where $X = \{X₁, X₂, X₃\}$. In Figure 3-5(a), $p(X₁) + p(X₂) + p(X₃) = 1$. In Figure 3-5(b), $m(X₁) = m₁$, $m(X₁ \cup X₂) = m₂$, and $m(X₂ \cup X₃) = m₃$; in other words, there is no piece of evidence that specifies to $X₂$ only and $X₃$ only.
(3) Belief and Plausibility Functions

The belief function (Bel) represents the believability or the weight of evidence that supports an outcome. The degree of belief, Bel(A), represents the total amount of justified support given to (the set of) outcome, A. It represents the level of confidence. The plausibility measure (Pl) represents the plausibility or the weight of evidence that does not oppose a particular set of outcome. The degree of plausibility, Pl(A), represents the total amount of potential support given to the set of outcome, A. Belief and plausibility functions are both defined on the same body of evidence $2^X$.

$$Bel : 2^X \rightarrow [0, 1] \text{ and } Pl : 2^X \rightarrow [0, 1]$$ (3-9)

The belief and plausibility functions are derived from the basic probability assignment ($m$). The degree of belief of a set of outcome $A$, Bel($A$), is measured by summing all the basic probability assignments given to the proposition $B$ of the set of outcome $A$ where $B \subseteq A$ and $B \neq \emptyset$. The degree of plausibility of the set of outcome $A$, Pl($A$), is measured by summing all the basic probability assignment of the focal elements $B$ that intersect the focal element $A$.

$$Bel(A) = \sum_{B \subseteq A} m(B)$$ (3-10)

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B)$$ (3-11)
3.3.3 Combination of Evidence in Evidence Theory

One of the important mathematical operators in evidence theory is the rule of combination of evidence—the aggregate operator to combine different pieces of evidence. In evidence theory, Dempster’s rule of combination (DRC) is the most commonly used for combining multiple pieces of evidence (or information) given by independent sources. Let \( m_1 \) and \( m_2 \) are basic belief assignment from two sources, Sources 1 and 2, and then the combined basic belief assignment is:

\[
m(A) = m_1(U) \oplus m_2(V) = \frac{\sum_{U \cap V \subseteq A} m_1(U) \cdot m_2(V)}{1 - \sum_{X \cap Y \subseteq \emptyset} m_1(X) \cdot m_2(Y)}
\]

(3-12)

where \( A \neq \emptyset \).

The numerator of Equation (3-12) is the sum of the product of the belief values associated with evidence from two sources that supports set \( X \). The denominator is a normalizing factor which is the sum of the product of the belief values associated with all the possible combinations of evidence that are not in conflict.

The following shows the example of the DRC. Consider the set \( X = \{X_1, X_2\} \). Let \( m_1 \) and \( m_2 \) are basic belief assignments of two pieces of evidence (or two experts):

\[
m_1(\{X_1\}) = 0.2, \quad m_1(\{X_2\}) = 0.2, \quad m_1(\{X_1, X_2\}) = 0.6, \quad m_2(\{X_1\}) = 0.5, \quad m_2(\{X_2\}) = 0.3, \quad \text{and} \quad m_2(\{X_1, X_2\}) = 0.2.
\]

Figure 3-6 shows the basic belief assignment of each focal element associated with individual piece of evidence (or expert) \( m_1 \) and \( m_2 \), and the combined evidence \( m \) (between \( m_1 \) and \( m_2 \))
The basic belief assignment of each focal element of the combined evidence is calculated as:

\[
m(X_1) = \frac{m_1(X_1) \cdot m_2(X_1) + m_1(X_1) \cdot m_2(X_{1 \cup X_2}) + m_1(X_{1 \cup X_2}) \cdot m_2(X_1)}{1 - [m_1(X_1) \cdot m_2(X_2) + m_1(X_2) \cdot m_2(X_1)]}
\]

\[
m(X_1) = \frac{0.10 + 0.04 + 0.30}{1 - (0.06 + 0.10)} = 0.524
\]

\[
m(X_2) = \frac{m_1(X_2) \cdot m_2(X_2) + m_1(X_2) \cdot m_2(X_{1 \cup X_2}) + m_1(X_{1 \cup X_2}) \cdot m_2(X_2)}{1 - [m_1(X_1) \cdot m_2(X_2) + m_1(X_2) \cdot m_2(X_1)]}
\]

\[
m(X_2) = \frac{0.06 + 0.04 + 0.18}{1 - (0.06 + 0.10)} = 0.333
\]

\[
m(X_{1 \cup X_2}) = \frac{m_1(X_{1 \cup X_2}) \cdot m_2(X_{1 \cup X_2})}{1 - [m_1(X_1) \cdot m_2(X_2) + m_1(X_2) \cdot m_2(X_1)]}
\]

\[
m(X_{1 \cup X_2}) = \frac{0.12}{1 - (0.06 + 0.10)} = 0.143
\]
Belief and plausibility measures of each focal element can be obtained as follows:

\[
\begin{align*}
Bel(X_1) &= \sum_{x | x \subseteq X_1} m(X) = 0.524, \quad Pl(X_1) = \sum_{x | x \cap X_1 \neq \emptyset} m(X) = 0.524 + 0.143 = 0.667 \\
Bel(X_2) &= \sum_{x | x \subseteq X_2} m(X) = 0.333, \quad Pl(X_2) = \sum_{x | x \cap X_2 \neq \emptyset} m(X) = 0.333 + 0.143 = 0.476 \\
Bel(\Theta) &= \sum_{x | x \subseteq \Theta} m(X) = 0.143, \quad Pl(\Theta) = \sum_{x | x \cap \Theta \neq \emptyset} m(X) = 0.524 + 0.333 + 0.143 = 1.0
\end{align*}
\]

### 3.3.4 Measures of Uncertainty in Evidence Theory

Uncertainty and evidence (or knowledge) is related. In Evidence Theory, uncertainty is the lack of knowledge. The degree of uncertainty is measured by two measures: (i) non-specificity measure—how much the opinions are non-specific; and (ii) Discord measure—how much they are conflict with one another. The derivation of these two measures are found in Klir and Wierman (1999) and Ayyub and Klir (2006).

#### (1) Non-specificity Measure

The measure of non-specificity, \(N(m(A))\), refers to ambiguity due to imprecise knowledge. The measure of non-specificity increases when the degree of belief of “I don’t know” state increases and the degrees of belief of all specific states decrease. It is given by

\[
N(m(A)) = \sum_{A \subseteq X} m(A) \cdot \log_2 |A|
\]  

(3-13)

where \(|A|\) is the cardinality (or size) of subset \(A\) and \(\log_2 |A|\) is the bits of information needed to find out the solution from the 0-1 problem. If the evidence is completely ignorance, \(m(A) = 1\), then the amount of uncertainty due to non-specificity is equal to the total bits of information needed.
(2) Discord Measure

The measure of discord, \( D(m) \) or conflict refers to ambiguity due to conflicting information. The measure of discord increases when the degrees of belief of two or more states are even. These two uncertainty measures are calculated as follows.

\[
D(m(A)) = - \sum_{A \in X} m(A) \log_2 \left( \sum_{B \in X} m(B) \frac{|A \cap B|}{|B|} \right) 
\]

(3-14)

where \(|A|\) and \(|A \cap B|\) are the cardinality (or size) of subset \(A\) and intersection between subsets \(A\) and \(B\), respectively.

(3) Total Uncertainty

In addition to the two measures of uncertainty presented, the integrity associated with information can also be measured by the total amount of uncertainty. It is the sum of those two. It is expressed by:

\[
TU(m) = N(m) + D(m) 
\]

(3-15)

Table 3-2 shows the increase of non-specificity measure, \(N(m)\) with respect to different patterns of belief distribution. \(N(m)\) gets the lowest value when all the belief mass points to single outcome (see Column 2), while \(N(m)\) gets the highest value in the case of complete ignorance or “I don’t know.” In this case, all the belief masses are assigned to every outcome, \(m(X_1 \cup X_2 \cup X_3) = 1\). (see the last column)

Table 3-3 shows the increase of discord measure, \(D(m)\) with respect to different patterns of belief distribution. \(D(m)\) receives the lowest value when all the belief masses point to only one outcome (see Column 2), while \(D(m)\) receives the highest value when the belief mass is uniformly distributed over the body of evidence or equal probability in Bayesian structure, \(m(X_1) = m(X_2) = m(X_3) = 1/|X|\). (see the last column)
Table 3-2: Effect of Belief Distribution Patterns on Measure of Non-specificity

<table>
<thead>
<tr>
<th>Pattern</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m(X_1))</td>
<td>1</td>
<td>0.8</td>
<td>0.6</td>
<td>0.333</td>
<td>0</td>
</tr>
<tr>
<td>(m(X_2))</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(m(X_3))</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(m(X_1 \cup X_2))</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.333</td>
<td>0</td>
</tr>
<tr>
<td>(m(X_1 \cup X_3))</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(m(X_2 \cup X_3))</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(m(X_1 \cup X_2 \cup X_3))</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0.333</td>
<td>1</td>
</tr>
</tbody>
</table>

- Non-specificity, \(N(m)\): 0.000, 0.200, 0.317, 0.862, 1.585
- Discord, \(D(m)\): 0.000, 0.122, 0.732, 0.293, 0.000
- Total Uncertainty, \(TU(m)\): 0.000, 0.322, 1.049, 1.155, 1.585

Table 3-3: Effect of Belief Distribution Patterns on Measure of Discord

<table>
<thead>
<tr>
<th>Pattern</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m(X_1))</td>
<td>1</td>
<td>0.80</td>
<td>0.60</td>
<td>0.5</td>
<td>0.333</td>
</tr>
<tr>
<td>(m(X_2))</td>
<td>0</td>
<td>0.20</td>
<td>0.40</td>
<td>0.25</td>
<td>0.333</td>
</tr>
<tr>
<td>(m(X_3))</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.25</td>
<td>0.333</td>
</tr>
<tr>
<td>(m(X_1 \cup X_2))</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(m(X_1 \cup X_3))</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(m(X_2 \cup X_3))</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(m(X_1 \cup X_2 \cup X_3))</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- Non-specificity, \(N(m)\): 0.000, 0.000, 0.000, 0.000, 0.000
- Discord, \(D(m)\): 0.000, 0.722, 0.971, 1.500, 1.585
- Total Uncertainty, \(TU(m)\): 0.000, 0.722, 0.971, 1.500, 1.585

3.3.5 Comparison of Evidence Theory with Other Mathematical Theories

A comparison of different mathematical frameworks: probability theory, possibility theory, and evidence theory is presented here. Each mathematical theory has unique characteristics. Table 3-4 compares the main characteristics of three mathematical frameworks. As discussed, there are three mathematical frameworks to cope with
uncertainty. It is therefore very important to select the most suitable framework to represent knowledge and to measure the uncertainty in question.

### Table 3-4: Comparison of Probability, Possibility, and Evidence Theories

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Probability Theory</th>
<th>Possibility Theory</th>
<th>Dempster-Shafer Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element</td>
<td>$x \in X$</td>
<td>$x \in X$</td>
<td>$x \subseteq X$</td>
</tr>
<tr>
<td>Function</td>
<td>$p(x)$</td>
<td>$r(x)$</td>
<td>$m(x)$</td>
</tr>
<tr>
<td>Evidence pattern</td>
<td>Conflicting evidence</td>
<td>Nested evidence</td>
<td>Mixed evidence</td>
</tr>
</tbody>
</table>

#### Evidence pattern

- **Evidence pattern**
  - **Sets of outcome**
    - **X1**
    - **X2**
    - **X3**
    - **X4**

#### Distributions

- **Probability distribution**
  - $p(x)$
- **Possibility distribution**
  - $r(x)$
- **Belief distribution**
  - $m(x)$

#### Degree of support

- **Probability, $Prob$**
  - $\sum_{x \in X} p(x) = 1$
- **Possibility, $Pos$**
  - $\max_{x \in X} \{r(x)\} = 1$
- **Plausibility, $Pl$**
  - $\sum_{x \in X} m(x) = 1$

#### Uncertainty measure

- **Non-specificity = 0, Conflict $\geq 0$**
- **Non-specificity $\geq 0$, Conflict = 0**
- **Non-specificity $\geq 0$, Conflict $\geq 0$**

#### Total ignorance

- **Probability, $Prob$**
  - $p(x) = \frac{1}{|X|}$ or uniform distribution
- **Possibility, $Pos$**
  - $r(x) = 1$
- **Belief, $Bel$**
  - $m(x) = 1$ if $x = X$; $m(x) = 0$; otherwise

For many reasons DST is the most suitable mathematical theory for decision-making in transportation planning. Firstly, most knowledge and information in transportation applications are partial and subjective. DST has ability to handle all mixed
patterns of evidence, which the probability or possibility theories cannot. Secondly, when an expert is in favor of one outcome in reality, it does not mean he is completely against its complement. Often times, in decision-making process, there is conflict among experts’ knowledge or historical information, the DST allows measuring uncertainty due to non-specificity, conflict and compromising views among different experts’ opinions, and helping resolve or at least clarify their conflicts.

Thirdly, DST can define different level of “I don’t know.” It distinguishes between information with uncertainty (“I don’t know because I’m not sure”) and lack of information (“I don’t know because it doesn’t say”). Consider a set of two outcomes \( X = \{ X_1, X_2 \} \). If an expert is slightly not sure (e.g. the degree of “I don’t know” is 0.2), then \( m(X_1)=0.4, m(X_2)=0.4, \) and \( m(X_1 \cup X_2)=0.2 \). If an expert totally lacks of information, then \( m(X_1)=0, m(X_2)=0, \) and \( m(X_1 \cup X_2)=1 \). However, in traditional probability theory, the probability values for these two cases are assumed to be 0.50, \( p(X_1)=0 \) and \( p(X_2)=0.5 \). Finally, DST contains the principle of uncertainty. It maximizes the use of information available without making any additional assumptions.

### 3.3.6 Applications of Evidence Theory in the Transportation Applications

The application of DST is in its infancy, particularly in dealing with real-world applications. Most studies related to DST focused only on the rule of combination to fuse or aggregate information from multiple sources. Such studies are limited to two applications. The first is the applications related to data analysis in computer sciences, and the second is the applications related to decision analysis.

In data analysis applications, several past studies apply the DST as a data fusion technique to aggregate incomplete data from multiple databases, such as, Faouzi et al. (2009), Yi et al. (2002), and Klein et al. (2002). DST has been increasingly applied to geographical information system data. Many studies use the aggregate operator in DST to fuse geospatial information in GIS for classification and data mining as reviewed by Malpica et al. (2007).

In decision making, DST has been recently introduced to solve multi-criteria decision-making (MCDM) problems in many applications, such as landfill site selection.
50

(Tayyebi et al, 2010), and plant location (Deng et al, 2010). In MCDM, the aggregate
operator in DST is applied in determining the criteria weights in the evaluation process.
DST helps reduce uncertainty between experts’ viewpoints and reach compromises on
individual weights for each criterion. For example, DST has been applied in an analytical
hierarchy process (AHP) technique to prioritize decision alternatives in multi-criteria
decision making environments. This DS/AHP method allows decision makers to identify
significance levels for a group of decision alternatives rather than a single alternative in
conventional AHP. (Beynon et al., 2000; 2001; Hua et al., 2008)

Although DST has been applied in recent past in several decision situations, its
use has been limited to the rule of combination to aggregate databases from different
sources during data preprocessing. There has been no comprehensive study to apply the
framework of the entire evidence theory to real-world problems, particularly in the
transport sector. Such an application is sketched out in the next section and explained in
detail and compared with the probabilistic approaches in the subsequent chapters.

3.4 Reasoning and Inference Process

Reasoning and inference are central to human decision-making that enable
humans (i) to develop a form of knowledge about a phenomenon; (ii) to understand the
relationships in it; and (iii) to infer conclusions from best available information. Logical
reasoning is applicable in many areas, such as logic, philosophy, science, and psychology.
(Aliev, 2001; Halpern, 2003) Two basic features involved in the reasoning process are
the reasoning map structure, and the mathematical mechanism to represent reasons.

3.4.1 Reasoning Map Structure

A reasoning map presents a chain of reasoning of a collection of propositions (or
states of variables) and describes the cause-and-effect relationships among them. A
reasoning map is defined as a directed acyclic graph network consisting of a set of boxes
and a set of links as presented in Figure 3-7. This diagram is useful in decision analysis
because it is easy to understand and is applicable in brainstorming and compromising. (Merkhofer et al., 1990; Schwartz et al., 2006; Bayraktar and Hastak, 2009)

![Typical Reasoning Map](image)

**Figure 3-7: Typical Reasoning Map**

Each box denotes the system variable $X$ and its states in form of a proposition “$X$ is $X_i$”, while each link denotes the cause-effect relation, input-output relation, time-related phenomena, or inference between two variables $X$ and $Y$ and their states in form of the rule “if $X$ is $X_i$, then $Y$ is $Y_j$.”

The reasoning in decision-makers’ mind is commonly expressed in the IF-THEN logical form, “IF …, THEN …” The proposition which follows the IF-clause is called the premise, while that which follows the THEN-clause is called the conclusion. The relationship between these two statements is called the rule. For example, knowing that the implication “IF $X$ is $X_i$, THEN $Y$ is $Y_j$” is true and a proposition “$X$ is $X_i$” is true, the belief of the proposition “$Y$ is $Y_j$” is inferred as shown in Figure 3-8.

![Elements of Reasoning](image)

**Figure 3-8: Elements of Reasoning**
3.4.2 Mathematical Mechanisms for Reasoning

How to measure the belief of the premises, relations, and conclusions in the reasoning process is the main issue, after the decision makers develop the reasoning map. Evidence plays the most important role to measure the belief of propositions involved in a reasoning system. Collecting and combining pieces of evidence and applying them in a logical manner will help decision makers understand the property of the system and causalities among system variables.

In reality a piece of evidence conveys different types of uncertainty; therefore, an evidential reasoning process requires different mathematical frameworks to handle different types of uncertainty. (Shafer, 1987; Yager, 1987) Several mathematical mechanisms in reasoning under uncertainty are available in the literature.

(1) Bayesian Reasoning

Reasoning based on Bayesian probability theory is one of the most popular approaches for reasoning under uncertainty. It uses the probability theory and statistical evidence (observation) to infer the degree of belief (truth) of a proposition. Bayesian reasoning uses probability as a measure of belief, and it uses conditional probabilities (either subjective or frequency-based) and Bayes’ theorem to propagate the degree of belief of propositions of variables along the reasoning chains. In Bayesian reasoning, these propositions are assumed to be mutually exclusive and exhaustive. (Darwiche, 2009) The details of the limitations of the traditional Bayesian reasoning will be discussed in Chapter 5.

(2) Rule-Based Reasoning

The rule-based reasoning is the most basic inference engine which uses “if-then-else” rule statement in reasoning process. This rule-based reasoning is also based on probability theory; however, it uses subjective probabilities rather than frequency-based probabilities. The rule-based reasoning quantifies uncertainty as subjective probabilities
expressed in a linguistic manner of reasoning. For example, “always” = 0.99, “usually” = 0.85, “sometimes” = 0.20, and “Never” = 0. (Simpson, 1944)

One of the most popular rule-based reasoning models is the MYCIN model, which is an expert system for the diagnosis and treatment of blood meningitis and infections developed by medical research teams. (Shortliffe, 1975) The MYCIN model uses a certainty factor (ranging from -1 to +1) derived from subjective probabilities to infer the degree of truth of the propositions.

The major drawbacks of this rule-based reasoning are (i) the model is ad hoc in which the certainty factor is not based on a strong mathematical foundation; and (ii) the MYCIN model is limited to short and simple reasoning chains; as a result, the errors could obtain when dealing with practical problems (Lee, 1987)

(3) Fuzzy Reasoning

Fuzzy reasoning is the reasoning with vague information. It uses the fuzzy sets theory to represent approximate data about the propositions, and fuzzy rule-based logic to represent approximate relations in order to infer a conclusion. The concept is similar to fuzzy inference system (FIS), but it is applied to a complex decision-making problem. Fuzzy reasoning allows imprecise linguistic terms to be represented. Fuzzy reasoning uses the membership function to measures the degree of truth (belief) of the propositions. (Klir and Folger, 1988; Terano et al., 1992; Klir and Yuan, 1995; Tanaka, 1997; Zimmermann, 2001; Grabisch et al., 2010)

(4) Belief Reasoning

Belief reasoning, which is proposed in this study, is a generalized reasoning under uncertainty and ambiguity. It uses the Dempster-Shafer theory of evidence and imprecise and incomplete expert opinions to infer the degree of belief of proposition. This will be elaborated in detail in the next chapter.
This chapter presents the proposed belief reasoning methodology. The first section explains the basic elements of belief reasoning methodology in transportation planning. The second section discusses the calculation procedures used in the evaluation of transportation alternatives. The last section presents the step-by-step approach to reasoning for public transportation system planning.

4.1 Structure and Elements of Belief Reasoning in Transit Planning

The belief reasoning in transit planning applies two concepts: a reasoning map in information logic and a belief measure in information theory. This proposed reasoning process contains two main features. The first is the elements of reasoning map and the second is the representations of knowledge in a reasoning map.

4.1.1 Elements of Reasoning Map

A reasoning map (or an argument map structure), which is one of the Artificial Intelligence applications in conflict resolutions, is used to model the causalities of variables in decision-making systems. This method allows us to reflect perceptions of decision-makers about the variables and to model how decision-makers are thinking and making decisions. The reasoning map structure, which is a directed acyclic graph, consists of a set of boxes (or nodes) and links as described in Chapter 3 and as shown in Figure 4-1.

For transportation planning and decision-making applications, the study uses a reasoning map structure to connect from a set of transit system characteristics to a set of
transit project’s goals. Boxes (or nodes) in a reasoning map for transit planning are classified into four categories based on their purposes and functions.

**Decision nodes** ($D$) represent the parametric characteristics of transportation alternatives that affect transportation system requirements. These decision nodes are considered as starting nodes in the reasoning map. Different transit alternatives or projects are defined by their physical and operational characteristics; for example, right-of-way, vehicle capacity, propulsion system, headway, and so on.

**Exogenous factor nodes** ($E$) represent the variables that are not related to the characteristics of the transportation alternatives, but directly affect the system requirements and system performance, such as predicted aggregate demand, land-use characteristics, and car ownership. These nodes are also the starting nodes in the reasoning map.

**Consequence nodes** ($C$) represent the expected planned performances and impacts, for example, accessibility, level of service, cost of the transit alternatives, air pollution emissions. The consequence nodes are considered as intermediate nodes which connect to both ‘starting nodes’ $D$ and $E$ and ‘end nodes’ $G$.

**Goal nodes** ($G$) also called ‘end nodes’ represent the goals of the transit project, and are considered the end nodes of the reasoning chains.

A transportation decision problem can be considered as comprising two components: a set of objectively defined transit alternatives and a set of subjectively defined goals and objectives of the project. The relationship between the decision variables of alternatives (decision nodes) and the goals of the project (goal nodes) can be described by attributes and performance. Such attributes (intermediate nodes) form the bridge between two components. Figure 4-1 shows the structure of reasoning chains for evaluating an alternative.

A map (or network) structure has high flexibility for formulating a problem in a large-scale system and defining the relations among variables associated with the alternatives and goals of the system. This structure resolves the issue of redundancy where one variable affects more than one goal and when one performance measures may
be dependent on more than one system variables. This issue is found in most other multi-criteria decision-making approaches as discussed in Chapter 3.

**Figure 4-1: Reasoning Map Structure**

4.1.2 Representation of Knowledge in Belief Reasoning

Prior to inferring the states of variables $C$ and $G$ along the reasoning chains using belief reasoning, two knowledge representations are needed. The first is the knowledge about every premise (i.e. starting nodes $D$ and $E$ in the reasoning map) and the second is the knowledge about every relation (from $D$ to $C$, $E$ to $C$, $C$ to $C$, and $C$ to $G$).

Knowledge about premises and relations is presented by the strength of evidence (or the degrees of belief or the truth values) as expressed by the experts, the public, or predicted by the analysts. In evidence theory, they are expressed by a basic belief assignment, $m$-value, between 0 and 1.

*(1) Knowledge about Premises*

For the premises “$x$ is $X_i$” of nodes $D$ and $E$, the knowledge is specified by attaching the prior beliefs (truth values), $m(X)$, associated with the states of the system.
variables ($D$ and $E$). For example, a belief that the service headway of Light Rail is short with $m=0.80$; or that the operating speed of Bus Rapid Transit is medium with $m=0.65$ are expressions of prior beliefs (truth values).

The knowledge about each premise is presented by the belief distribution associated with respect to the possible outcomes of a variable $X$. It represents the degree of belief associated with each state of variable ($D$ or $E$) supported by evidence, e.g. “$x$ is $X_1$ with the belief of $m(X_1)$” or “$x$ is $X_1$ or $X_2$ with the belief of $m(X_1 \cup X_2)$.” Figure 4-2 shows a typical belief distribution for a premise $X$, consisting of three possible outcomes $X_1$, $X_2$, and $X_3$.

![Figure 4-2: Representation of Premise (Belief Distribution)](image)

(2) Knowledge about Relations

For the relations “IF $x$ is $X_1$, then $y$ is $Y_1$”, $Y$ being a consequence variable $C$, the knowledge is specified by attaching the degrees of belief, $m(Y|X)$, associated with the causal relations or inference. For example, if the number of transit station/stop is medium or high, then the reliability of transit travel time is low, with the degree of belief 0.9. If the population density along the transit corridor is high, then transit ridership is medium or high, with the degree of belief 0.60.

The knowledge about each relation is presented by the conditional basic belief assignment associated with the causality between the outcomes of parent node $X$ and those of child node $Y$. Figure 4-3 shows the examples of the relationships between two nodes $X$ and $Y$; relationship $R1$: ($X_1$ or $X_2$) $\rightarrow$ ($Y_1$) and $R2$: ($X_2$ or $X_3$) $\rightarrow$ ($Y_2$ or $Y_3$). The
degree of belief associated with the relation is assigned by an expert. Figure 4-4 shows a conditional basic belief assignment table, \( m(Y|X) \) considering three outcomes. This matrix is the representation of knowledge about a relation \( X \rightarrow Y \). It is the counterpart of conditional probability table in traditional Bayesian reasoning. This relation matrix called conditional belief table will allow analysts to present different levels of ambiguity (or “I don’t know”) about the relations. The light-shaded area is the one considered in the traditional Bayesian reasoning and the entire matrix describes the relations and beliefs that can be possible in the evidence theory.

![Figure 4-3: Representation of Relation](image)

<table>
<thead>
<tr>
<th>( X \rightarrow Y )</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
<th>( Y_3 )</th>
<th>( Y_1 \cup Y_2 )</th>
<th>( Y_1 \cup Y_3 )</th>
<th>( Y_2 \cup Y_3 )</th>
<th>( Y_1 \cup Y_2 \cup Y_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( X_2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( X_3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( X_1 \cup X_2 )</td>
<td>Various Levels of Ignorance (Non-specific Knowledge)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( X_1 \cup X_3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( X_2 \cup X_3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( X_1 \cup X_2 \cup X_3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total Ignorance</td>
</tr>
</tbody>
</table>

![Figure 4-4: Representation of Relation (Conditional Belief Table)](image)

### 4.2 Calculation Procedures in Belief Reasoning

Once a reasoning map structure is developed and knowledge about premises and relations are solicited and obtained, the following calculation procedures are used for evaluating transportation alternatives for assisting and clarifying decision-making.
4.2.1 Aggregation of Expert Knowledge

Given multiple experts (or information sources) involved in the planning process, knowledge from different experts is combined by using Dempster’s rule of combination (DRC). DRC is an aggregate operator in evidence theory used for combining two belief distributions from independent sources. Let $m_1$ and $m_2$ be belief distributions of a premise “$x$ is $X_p$” from two experts, Experts 1 and 2, and $X_p$ is one element of the ‘power set’ (the power set consisting of all outcomes, the left column of matrix in Figure 4-4).

$$m(X_p) = m_1(X_U) \oplus m_2(X_V) = \frac{\sum_{x_U, x_V | x_U \cap x_V = X_p} m_1(X_U) \cdot m_2(X_V)}{1 - \sum_{x_U, x_V | x_U \cap x_V = \emptyset} m_1(X_U) \cdot m_2(X_V)}$$  \hspace{1cm} (4-1)

In the above equation $X_U$ and $X_V$ represent the possible outcomes of $X$ from Experts 1 and 2 (the ‘power set’), respectively. This aggregate operator of Equation (4-1) takes into account the degrees of belief of the outcomes that are supported by both experts; in other words, it excludes the conflicting outcomes in the calculation. For instance, if two experts are strictly conflict; e.g. Expert 1 says “$x$ is $X_1$ with $m_1(X_1) = 1$” and Expert 2 says “$x$ is $X_2$ with $m_2(X_2) = 1$”, then there is no compromising solution, and $m(X_p)$ would be undefined. This formulation is the generalization of the ‘AND’ aggregate operator in probability theory when combining two probability distributions.

4.2.2 Belief Propagation

Given knowledge of a premise $A$ and a relation $A \rightarrow B$, the belief of conclusion $B$ is calculated by:

$$m(B_p) = \sum_{A_p \subseteq A} m(B_p | A_p) \cdot m(A_p)$$  \hspace{1cm} (4-2)
where \( A_p \) and \( B_p \) are the power set of states of parent node \( A \) and child node \( B \), respectively. \( m(A_p) \) is the degree of belief of premise \( A_p \) and \( m(B_p|A_p) \) is the conditional basic belief assignment of \( B_p \) with respect to \( A_p \).

Using this inference concept, the degree of belief can be propagated to other variables (consequences) along reasoning chains. Three patterns of inference are found in the reasoning chains as shown in Figure 4-5.

Pattern 1 represents the serial reasoning chain connecting between one node to another node, (“\( X \) is \( X_i \)” and (“If \( X \) is \( X_i \), then \( Y \) is \( Y_i \)”)) Pattern 2 represents the parallel reasoning chains connecting from multiple nodes to one node, (“\( W \) is \( W_i \)”, and “\( X \) is \( X_i \)” and (“If \( W \) is \( W_i \), then \( Y \) is \( Y_i \),” and “If \( X \) is \( X_i \) then \( Y \) is \( Y_i \)”)) Pattern 3 represents the branch reasoning chains connecting from one node to multiple nodes, (“\( X \) is \( X_i \)” and (“If \( X \) is \( X_i \), then \( Y \) is \( Y_i \)” and “if \( X \) is \( X_i \), then \( Z \) is \( Z_i \)”)) The calculation procedures of each inference are described as follows.

**Figure 4-5: Patterns of inference**

(1) **Pattern I: Serial Reasoning Chain**

Given a serial reasoning chain connecting one parent node to one child node as shown in Figure 4-5, the degree of belief of the lower node is calculated using Equation (4-3).

\[
m(Y_p) = \sum_{X_p \subseteq X} m(Y_p | X_p) \cdot m(X_p)
\]  

(4-3)
where $X_p$ and $Y_p$ are a power set of states of node $X$ and node $Y$, $X_p \subseteq X$ and $Y_p \subseteq Y$, respectively. $m(X_p)$ and $m(Y_p | X_p)$ are the basic belief assignment of premise $X_p$ and the conditional basic belief assignment of $Y_p$ with respect to $X_p$, respectively.

(2) Pattern II: Parallel Reasoning Chains

Given parallel reasoning chains connecting from many parent nodes to one child node as shown in Figure 4-5, the degree of belief of the child node is calculated by first using Equation (4-3) to calculate the degree of belief of the child node from each parent node, and then combining them by the DRC in Equation (4-1) to obtain the combined basic belief assignment of the child node. The equations are as follows.

$$m_{W \rightarrow Y}(Y) = \sum_{W_p \subseteq W} m(Y_p | W_p) \cdot m(W_p)$$

$$m_{X \rightarrow Y}(Y) = \sum_{X_p \subseteq X} m(Y_p | X_p) \cdot m(X_p)$$

$$m(Y_p) = m_{W \rightarrow Y}(Y) \oplus m_{X \rightarrow Y}(Y_p) = \frac{\sum_{U \cup V \subseteq Y \cup Y_p} m_{W \rightarrow Y}(U) \cdot m_{X \rightarrow Y}(V)}{1 - \sum_{U \cup V \subseteq Y \cup Y_p} m_{W \rightarrow Y}(U) \cdot m_{X \rightarrow Y}(V)}$$

where $W_p$ and $X_p$ are a power set of states of parent nodes $W$ and $X$ and $Y_p$ is a power set of states of child node $Y$, $W_p \subseteq W$, $X_p \subseteq X$ and $Y_p \subseteq Y$. $m_{W \rightarrow Y}(Y_p)$ and $m_{X \rightarrow Y}(Y_p)$ are the degree of belief of conclusion $Y_p$ influenced by the premises $W$ and $X$, respectively.

(3) Pattern III: Branch Reasoning Chains

Given a branch reasoning chain connecting from one parent node to many child nodes as shown in Figure 4-5, the degree of belief of each child node is calculated by using Equation (4-3). The branch reasoning chain is considered as many serial reasoning chains.
\[ m(Y_p) = \sum_{X_p \subseteq X} m(Y_p \mid X_p) \cdot m(X_p) \] (4-7)

\[ m(Z_p) = \sum_{X_p \subseteq X} m(Z_p \mid X_p) \cdot m(X_p) \] (4-8)

where \( X_p \) is a power set of states of parent node, and \( Y_p \) and \( Z_p \) are a power set of states of child nodes \( Y \) and \( Z \), respectively. \( X_p \subseteq X \), \( Y_p \subseteq Y \), and \( Z_p \subseteq Z \).

4.2.3 Measure of Goal Achievement

Using the calculation procedure in the previous section, the belief values are propagated from the starting nodes to the end “Goal” nodes through a series of relations, and the basic belief assignment of each goal \( k \) is determined, \( m(G_k) \).

The degree of achievement of individual goal is calculated by two measures, Measure of Belief (\( Bel \)) and Measure of Plausibility (\( Pl \)). The former, \( Bel(G_k) \), indicates the conservative measure of achieving the goal, while the latter, \( Pl(G_k) \), indicates the optimistic measure of goal achievement. Both measures can be derived from \( m(G_k) \).

\[ Bel(G_k) = \sum_{G_k \subseteq G_k} m(G_{k,p}) \] (4-9)

\[ Pl(G_k) = \sum_{G_k \subseteq G_k \cap G_k \neq \emptyset} m(G_{k,p}) \] (4-10)

where \( G_{k,p} \) is a power set of states of goal \( k \), \( G_{k,p} \subseteq G_k \).

4.2.4 Identification of Critical Reasoning Chains and Information Needs

Given the chains of reasoning and the truth values (or the belief distribution) attached to each attribute in a reasoning map, one can determine the strength and weakness of the reasoning chains; in other words, one can indicate whether a given reasoning chain is more influential (or less uncertain) than the other chains.
The study proposes an importance measure ($I_k$) to determine how much each attribute $k$ in a reasoning map affects the degree of goal achievement. This importance measure is calculated by the difference of the Measure of Belief ($Bel$) of goal achievement when all the variables in $D$, $E$, and $C$ exist and when one of them, variable $k$, does not exist.

$$I_k = Bel_0(\text{Goal}) - Bel_k(\text{Goal})$$ (4-11)

The higher the importance measure of the attribute, the higher that attribute affects on the goal achievement. The determination of critical reasoning chains is conducted backward (from the goal node to the decision nodes) by comparing the importance measures among its preceding nodes and selecting the preceding node that has the highest importance measure.

### 4.3 Approaches to Reasoning for Transit Planning Process

The analysis process of reasoning for transit planning consists of three phases. The first phase is the construction of the reasoning map structure (ideally a composite base map drawn together by the experts). The second phase is the elicitation of expert opinions about the system characteristics and causalities (the m-values, which normally are different for each expert or informant). The last phase is the model execution for evaluating transit alternatives. Figure 4-6 shows the steps of analysis of this approach.
4.3.1 Construction of the Reasoning Map

Two steps are needed to construct the reasoning map. First, a set of transit project’s goals and the collection of variables that describe the characteristics of the systems and the forecast travel conditions are identified in the planning process (factors D and e and G). The set of goals (end nodes in a reasoning map) are defined based on the
purposes and needs of the proposed transit projects. The definition of alternatives and the project description define the variables (starting nodes $D$ and $E$) in the reasoning map.

Second, the reasoning chains (links in a reasoning map) that determine the causalities are constructed by bridging the relationships between variables (nodes) and the goals. The chains of reasoning are developed using the descriptions of performance measures and arguments about transit alternatives documented in the transportation plan and reports. The content in the transit plans and reports is categorized into a series of the reasoning processes by the experts and informed by public input. The chains of reasoning can be input-output relations, cause-and-effect relations, or inferences for particular actions. The degree of complexity of reasoning chains depends on the content of plan. The output of this step is the reasoning map that accounts for the performance of transit alternatives. The reasoning maps are justified by experts, informed by public input, before further analysis.

Different experts may have different opinions in constructing the map. The maps from individual experts, which can include citizen groups, are aggregated by keeping all the links and assuming ignorance of links which an expert does not specify. Figure 4-7 shows an example of combined map from two mapping sources. It is important to note that the map after combining must be acyclic graph; in other words, links $A\rightarrow B$ and $B\rightarrow A$ are not allowed.

![Combined Map Diagram](image)

**Figure 4-7: Example of Combination of Reasoning Map**
4.3.2 Elicitation of Expert Opinions

Once the reasoning chains are developed, the knowledge (or evidence) about every premise (parent node) and every relation (link) in the reasoning chains are elicited by aggregating opinions from experts and planners and citizen groups through face-to-face interview. These “informants” assign probabilities to every state in premise (D and E) and every state in relation shown in Figure 4-1.

For example, consider that the transit headway, X, has three outcomes, Low ($X_1$), Medium ($X_2$), and High ($X_3$), and passenger waiting time, Y, has also three outcomes, Short ($Y_1$), Medium ($Y_2$), and Long ($Y_3$). Table 3-3 shows knowledge about the premise “Headway,” and Table 4-2 shows knowledge about the relation “If Headway, then Waiting time.” Knowledge as “I don’t know” (i.e., $X_1$ or $X_2$ or $X_3$) is useful in transportation planning and conserved in the proposed approach. Different degrees of “I don’t know” (subsets of the power set X or Y) are possible due to incomplete measurement or imperfect evidence.

Knowledge about a premise and a relation from different experts is aggregated through the Dempster’s rule of combination (DRC). For example, the truth values of C in Figure 4-7 can be calculated by aggregating knowledge of C from three sources: C given A of Expert 1, C given A of Expert 2, and C given B of Expert 2.

### Table 4-1: Example of Knowledge about Premise

<table>
<thead>
<tr>
<th>Type</th>
<th>States of Outcome of “Headway”</th>
<th>Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Outcome</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>$m(X_1)$</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td>$m(X_2)$</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>$m(X_3)$</td>
<td></td>
</tr>
<tr>
<td><strong>Non-specific Outcome</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Different levels of “I don’t know”)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low or Medium</td>
<td>$m(X_1 \cup X_2)$</td>
<td></td>
</tr>
<tr>
<td>Medium or High</td>
<td>$m(X_2 \cup X_3)$</td>
<td></td>
</tr>
<tr>
<td>Low or Medium or High (Total ignorance)</td>
<td>$m(X_1 \cup X_2 \cup X_3)$</td>
<td></td>
</tr>
</tbody>
</table>
## Table 4-2: Example of Knowledge about Relation

<table>
<thead>
<tr>
<th>Type</th>
<th>States of Outcome of “Headway → Waiting time”</th>
<th>Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Outcome</td>
<td>IF “Headway is Low, THEN Waiting time is …”</td>
<td></td>
</tr>
<tr>
<td>Short</td>
<td>$m(Y_1</td>
<td>X_1)$</td>
</tr>
<tr>
<td>Medium</td>
<td>$m(Y_2</td>
<td>X_1)$</td>
</tr>
<tr>
<td>Long</td>
<td>$m(Y_3</td>
<td>X_1)$</td>
</tr>
<tr>
<td>Non-specific Outcome</td>
<td>(Different levels of “I don’t know”)</td>
<td></td>
</tr>
<tr>
<td>Short or Medium</td>
<td>$m(Y_1 \cup Y_2</td>
<td>X_1)$</td>
</tr>
<tr>
<td>Medium or Long</td>
<td>$m(Y_2 \cup Y_3</td>
<td>X_1)$</td>
</tr>
<tr>
<td>Short or Medium or Long (Total ignorance)</td>
<td>$m(Y_1 \cup Y_2 \cup Y_3</td>
<td>X_1)$</td>
</tr>
</tbody>
</table>

### 4.3.3 Execution of the Model and Evaluation of Alternatives

Once the knowledge about every premise of individual transportation alternatives and every relation in a given reasoning map is elicited, the following calculation process is performed to evaluate transportation alternatives. It includes (i) determining the degrees of achievement of individual goals through the belief propagation; (ii) measuring the uncertainty of information in the reasoning process; (iii) calculating the composite degree of achievement of the project; and (iv) identification of critical reasoning chains.

1. **Determination of the Degree of Achievement of Individual Goals**

   Using the proposed operation to propagate the truth values from the premises to the conclusions, the degrees of achievement of individual goals are eventually obtained. Generally when the chains indicate consistent knowledge the truth value of the outcome increases, but if the chains indicate conflicting knowledge the truth value of the outcome decreases.

2. **Measuring Uncertainty of Information in Reasoning Process**

   Measuring uncertainty of information helps identify information needs in the reasoning chains and promotes focused public discourse in the decision-making process.
In this step, the amount of information-based uncertainty in a reasoning chain is quantified. It is measured by the quality and quantity of knowledge and information given to the transport analyst.

(3) Calculating the Composite Degree of Achievement of the Alternatives

To evaluate and compare transit alternatives, the composite degree of goal achievement is calculated for each alternative using the DRC aggregate operator. Generally the higher the overall degree of achievement by the alternative, the higher priority it has.

(4) Identification of the Critical Reasoning Chains

Identifying the strong and weak reasoning chains in a reasoning process is necessary to justify the validity of reasoning. This process helps decision-makers determine which characteristics of alternatives affect the decision, and provide information how to improve the alternatives in order to reach a higher degree of achievement.

4.4 An Illustrative Example

This section presents an illustrative example of reasoning process. Consider a mapping structure with five attributes: two parent (Decision or Exogenous) nodes (A and B), two intermediate nodes (C and D), and one child (goal) node (E), and five relations $A \rightarrow C$, $B \rightarrow C$, $B \rightarrow D$, $C \rightarrow E$, and $D \rightarrow E$ as shown in Figure 4-8.

Given the belief value ($m$) of each parent node and link, the truth values of the subsequent nodes are calculated based on the formulas of inference. Figure 4-8 shows the example of calculation process, where the input and inference are chained. The truth value of $E$ is calculated by following: $A \rightarrow C$, $B \rightarrow C$, $B \rightarrow D$, $C \rightarrow E$, and $D \rightarrow E$. 
The belief value of node C is propagated from nodes A and B and can be calculated by Equations (4-13) and (4-14), respectively. Equation (4-15) is the aggregated value of belief of node C.

\[
m_{A\rightarrow C}(C_i) = \sum_{A_i \in A} m(C_i \mid A_i) \cdot m(A_i) \quad (4-12)
\]

\[
m_{B\rightarrow C}(C_i) = \sum_{B_i \in B} m(C_i \mid B_i) \cdot m(B_i) \quad (4-13)
\]

\[
m(C_i) = m_{A\rightarrow C}(C_i) \oplus m_{B\rightarrow C}(C_i) = \frac{\sum_{X,Y \mid X \cap Y = C} m_{A\rightarrow C}(X) \cdot m_{B\rightarrow C}(Y)}{1 - \sum_{X,Y \mid X \cap Y = \emptyset} m_{A\rightarrow C}(X) \cdot m_{B\rightarrow C}(Y)} \quad (4-14)
\]

The belief value of node D is calculated from the belief value of node B as expressed in Equation (4-16).

\[
m_{B\rightarrow D}(D_i) = \sum_{B_i \in B} m(D_i \mid B_i) \cdot m(B_i) \quad (4-15)
\]

The belief value of the end node E is propagated from nodes C and D and can be calculated by Equations (4-17) and (4-18), respectively. The belief values of E from two chains are combined using DRC as shown in Equation (4-19).

\[
m_{C\rightarrow E}(E_i) = \sum_{C_i \in C} m(E_i \mid C_i) \cdot m(C_i) \quad (4-16)
\]

\[
m_{D\rightarrow E}(E_i) = \sum_{D_i \in E} m(E_i \mid D_i) \cdot m(D_i) \quad (4-17)
\]

\[
m(E_i) = m_{C\rightarrow E}(E_i) \oplus m_{D\rightarrow E}(E_i) = \frac{\sum_{X,Y \mid X \cap Y = E} m_{C\rightarrow E}(X) \cdot m_{D\rightarrow E}(Y)}{1 - \sum_{X,Y \mid X \cap Y = \emptyset} m_{C\rightarrow E}(X) \cdot m_{D\rightarrow E}(Y)} \quad (4-18)
\]
The truth value (or belief value) of individual variable is presented by two measures: Measure of Belief (Bel) and Measure of Plausibility (Pl). They can be derived from the truth value, \( m \).

\[
Bel(A) = \sum_{A_i | A_i \subseteq A} m(A_i), \ldots, Bel(E) = \sum_{E_i | E_i \subseteq E} m(E_i)
\] (4-19)

\[
Pl(A) = \sum_{A_i | A_i \cap A \neq \emptyset} m(A_i), \ldots, Pl(E) = \sum_{E_i | E_i \cap E \neq \emptyset} m(E_i)
\] (4-20)

The amount of uncertainty associated with each variable is determined by the non-specificity measure, \( N(m) \), and the discord measure, \( D(m) \). The former measures the “I don’t know” portion and the latter measures the conflict in information. \( N(m) \) and \( D(m) \) vary from 0 to the logarithm base 2 of the cardinality, which is the maximum bit of information needed to specify the outcome: \( 0 \leq N(m) \leq \log_2 |A| \) and \( 0 \leq D(m) \leq \log_2 |A| \)

![Figure 4-9: Graphical Representation of Belief Reasoning](image)

In this example, there are three chains of reasoning: \( A \rightarrow C \rightarrow E \), \( B \rightarrow C \rightarrow E \), and \( B \rightarrow D \rightarrow E \) as shown in Figure 4-10.
To determine the critical reasoning chains, the importance measure of nodes \( C \) and \( D \), \( I_C \) and \( I_D \), are compared. The link between nodes \( C \) to \( E \) and nodes \( D \) to \( E \) is eliminated one at a time.

\[
I_C = Bel_0(E_p) - Bel_C(E_p) \\
I_D = Bel_0(E_p) - Bel_D(E_p)
\]  

(4-21)  

(4-22)

Compare between \( I_C \) and \( I_D \). If \( I_C > I_D \), then link \( C \rightarrow E \) is selected and link \( D \rightarrow E \) is dropped, and vice versa. Assume the former holds, and then links \( A \rightarrow C \) and \( B \rightarrow C \) are compared in the next stage. Figure 4-11 demonstrates the procedure to determine the critical reasoning chains.
The proposed belief reasoning brings about a new dimension to transportation planners and experts when evaluating transportation alternatives and projects. It provides useful information to the planning process. It has several advantages and addresses limitations of existing evaluation approaches. In the next chapter, the proposed belief reasoning approach will be compared to the traditional evaluation approaches. It will then be applied to the real-world transportation planning problem in the following chapter.
CHAPTER 5

VERIFICATION OF BELIEF REASONING FOR TRANSIT PLANNING

This chapter presents the verification process of the proposed Belief Reasoning. The study justifies two aspects of the proposed methodology: the structure of the decision-making problem and its mathematical mechanism. The study discusses the comparison between the reasoning map structure and the traditional hierarchical tree structure, and presents the similarities and discrepancies between the proposed Dempster-Shafer Belief Reasoning and the Bayesian Reasoning. Examples in transportation planning are presented to demonstrate these comparisons.

5.1 Decision-Making Structures in Transportation Planning

When evaluating a large-scale complex system like an urban transportation system, modeling a decision structure is one of the most important phases in decision-making process. It defines how decision-makers and analysts can address the decision problem and which information and data would be needed in the evaluation. Different decision structures may lead to different views about the decision alternatives. Therefore, it is desirable to model the system with the robust decision structure.

In general there are two ways to model such a large-scale complex system. The first manner, which is usually found in the traditional MCDM approaches in transportation planning, decomposes the system into independent subsystems and models the attributes of a system in a hierarchical tree structure. The other manner, which does not segregate the system, defines the relationships among variables in the system and models them in a mapping (or network) structure. The reasoning map structure has recently been applied in a few decision-making problems. This study is among the few to
use a reasoning map structure to model a decision-making process, and is the first to apply to transportation planning processes.

5.1.1 Hierarchical Tree Structure

A hierarchical tree structure is the simplified decision structure. It decomposes the attributes (variables) of the system into different levels (e.g. goals, criteria, sub-criteria) and classifies them into categories. The most well-known decision-making methods that use this structure are the scoring method and Analytical Hierarchy Process (AHP) method. These methods use the weighting schemes to evaluate the composite performances of individual alternatives. The additive weights ($\sum w = 1$) associated with the attributes in the same category are assigned to aggregate the performance values to the upper level. In these methods, the attributes are assumed to be independent and exclusive.

The hierarchical tree structure is favored by most planners and analysts because of its visualization of the decision problem, and the simple mathematical operator to analyze. However, this structure has several unrealistic assumptions. Firstly, the structure presents the classification of attributes, but it neither explains the dependent relationships nor the positive and negative effects between them. Secondly, a hierarchical structure cannot handle the issue of redundancy attributes in which one attribute may belong to two categories, which is always found in real-world problems.

Finally, the assignment of the values of weight is problematic. The weight can represent either the absolute value, which the attribute contributes to its class or the relative importance of one attribute relative to another attribute in the same class. However with a tree structure it is difficult to measure or compare the degree of contribution or the degree of importance when two attributes are subjective, have different units, or are redundant. This method leaves room for speculative “political inputs” in deciding the assignment of weights to each attribute.
5.1.2 Reasoning map structure

This study proposes a reasoning map structure because causality between attributes affecting transportation systems is not always linear or proportional. The reasoning map structure enhances the limitations over the traditional tree structure. It presents cause-and-effect relationships between outcomes of attributes and allows recognition of interrelationships among attributes. It allows one attribute to be affected by multiple causes. Further, the reasoning map structure does not require the additive weighing scheme, which is highly sensitive to the inputs during evaluation.

Figure 5-1 and Figure 5-2 respectively show the graphical representation of the hierarchical tree structure and reasoning map structure used in decision-making system. These figures show that the relation in a tree structure has to be one-to-one. In other words, an element can belong to only one category. On the other hand, a reasoning map structure allows one-to-one, one-to-many or many-to-one relations.

In planning of public transportation systems, there is always a direct relationship between the goals of a transit project and the physical and operational characteristics of the proposed transit alternatives. The limitations of the hierarchical structure and the presence of cause-and-effect relationships in transit systems motivate this study to examine the application of a reasoning map structure to transit planning process.
Figure 5-1: A Typical Hierarchical Tree Structure

Figure 5-2: A Typical Reasoning map structure
5.2 Decision Mechanisms in Transportation Planning

The main input to decision-making in transportation planning is to evaluate the overall performance of individual alternatives and prioritize them from technical viewpoint. Different decision structures as discussed in the previous section use different decision mechanisms to evaluate the overall performance. Figure 5-3 shows the steps in the evaluation process and the mechanisms applied under different decision structures. The steps are the following.

Step 1. Define the physical and operational characteristics of each of the proposed alternatives
Step 2. Develop a decision structure (either a hierarchical tree structure or a reasoning map structure) to model the system variables and connect them to the goals of the project.
Step 3. Provide inputs to the evaluation process.
  - For a hierarchical structure, the inputs are the performance values predicted by experts and transportation models. The scores are then assigned to each performance.
  - For a mapping structure, the inputs are the ‘truth’ values associated with the characteristics of each alternative and the ‘truth’ values associated with the causal relations along the reasoning chains.
Step 4. Execute the model.
  - For a hierarchical structure, the weights are assigned to each element of the system and the performance scores are aggregated for each alternative.
  - For a reasoning map structure, the ‘truth’ values are propagated through the inference process from the characteristics of the alternative to the goal of the project.
Step 5. Evaluate and compare the overall performance of all the proposed alternatives.
5.2.1 Weighting Methods

A weighting method is a simplified method to present a linear relation between the individual performances and overall performance: \( Y = w_1P_1 + w_2P_2 + \ldots \) where \( Y \) is the overall performance score; \( w_i \) is the weight for attribute \( i \); and \( P_i \) is the performance of attribute \( i \). It is based on certain assumptions.

Under a hierarchical tree structure, the model uses the weighting scheme to combine the performance values of attributes in the same class. The weight value presents the degrees of contribution of attributes in that class. The model assumes a linear correlation structure, and as a result, may not be able to capture the non-linearity inherent in complex real-world problems.

The underlining assumption is that the performance of an attribute can be used only once in the evaluative function, in other words, no double counting is permitted. This is rarely possible in a complex system in which performance of an attribute can affect more than one goal. Secondly, the information and knowledge used for evaluating performance is assumed to be complete and consistently interpreted between the alternatives and the experts; in other words, the analysts are certain about the predicted
performance and the values of the weights for all the alternatives. In reality, most important measures are difficult to quantify, and as a result, varying degrees of ambiguity exist in the minds of the analysts as to the performance and the values of the weights.

It is important to note that the weighting method can be applied to different types of performance values, although it has been applied to only the deterministic performance values. Table 5-1 shows various weighting methods based on types of performance values. Among them, the weighted average and AHP are the two most applicable in transportation planning because these two methods deal with deterministic values; no uncertainties are involved.

Table 5-1: Comparison of Different Types of Weighting Methods

<table>
<thead>
<tr>
<th>Methods</th>
<th>Types of Performance Values</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighted average</td>
<td>Deterministic, absolute values</td>
<td>Scores 1-5, 1-100</td>
</tr>
<tr>
<td>Analytical Hierarchy Process</td>
<td>Deterministic, relative values</td>
<td>Priority value (0-1)</td>
</tr>
<tr>
<td>*Fuzzy weighted average</td>
<td>Language-based measures with membership function</td>
<td>Linguistic terms, e.g. high(0.7), medium(0.5)</td>
</tr>
<tr>
<td>*Weighted probability distribution</td>
<td>Non-deterministic values with probability distribution</td>
<td>Scores with probability values attached</td>
</tr>
<tr>
<td>*Weighted belief distribution</td>
<td>Non-deterministic values with belief distribution</td>
<td>Linguistic terms, e.g. high(0.7), medium(0.5)</td>
</tr>
</tbody>
</table>

Note: The first two methods have been widely used in transportation applications, but not the last three methods (*).

5.2.2 Inference Methods

Rather than applying a linear weight function to refer one (preceding) variable to other (subsequent) variable, or one performance value to the overall performance, the inference method considers the causal relation between the two: \( Y = f(X) \) where \( Y \) is the subsequent variable; \( X \) is the preceding variable, and \( f(.) \) is the relational function between inputs and outputs.

There are many ways to present this causality or input-output relationship, \( Y = f(X) \). When the relation is exact and certain, then it can be presented as a mathematical function, such as multi-linear regression \( Y = f(X_1, X_2, \ldots) \), logit regression functions \( Y = f(\ln(X_1), \ln(X_2), \ldots) \). It is important to note that although this type of inference exists, but
it is difficult to model an exact relation for a large-scale transportation system. To address this issue, many Artificial Intelligence (AI) methods, such as Neural Network, are applied to model this process. However, the AI methods generally lack a logical justification.

When the relation is uncertain or very difficult to present as a mathematical function, then the uncertainty-based inference methods are recommended. The inference methods are modeled as a reasoning process. Several methods are available in the literature, e.g. rule-based inference, fuzzy inference, and Bayesian inference as discussed in Chapter 3. These inference methods model the relation through a set of “IF-THEN” rules instead of an exact mathematical expression. The inference methods have been applied to many applications, but none of them has been applied to transportation planning applications.

This study proposes another type of inference method when the relations and the inputs are incomplete and not completely known. The proposed method uses only available information without making any additional assumptions or constrains experts for opinions. It allows experts to admit their ignorance.

Table 5-2 compares the characteristics of inputs and relations needed for different inference methods, and Table 5-3 categorizes the traditional evaluation methods. Figure 5-4 presents how a weighting method can be replaced by an inference method.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Characteristics of Relations</th>
<th>Characteristics of Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule-Based Inference</td>
<td>Deterministic relations with the degree of certainty</td>
<td>Deterministic relations with the degree of certainty</td>
</tr>
<tr>
<td>Fuzzy Inference</td>
<td>Approximate relations expressed by language-based rules</td>
<td>Vague or approximate inputs, expressed by linguistic terms</td>
</tr>
<tr>
<td>Bayesian Reasoning</td>
<td>Non-deterministic relations with a complete information on the conditional probabilities</td>
<td>Non-deterministic inputs attached by probability measures</td>
</tr>
<tr>
<td>D-S Belief Reasoning (Proposed method)</td>
<td>Non-deterministic but incomplete relations</td>
<td>Non-deterministic but incomplete inputs attached by belief measures</td>
</tr>
</tbody>
</table>
Table 5-3: Classification of Evaluation Methods

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Relations</th>
<th>Weighting Methods</th>
<th>Inference Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>Weighted linear function</td>
<td>Regression functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Analytical Hierarchy Process</td>
<td>Rule-based reasoning</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fuzzy weighted function</td>
<td>Fuzzy reasoning</td>
<td></td>
</tr>
<tr>
<td>Non-deterministic</td>
<td>Weighted probability distribution</td>
<td>Bayesian Reasoning</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Weighted belief distribution</td>
<td>&quot;D-S Belief Reasoning&quot;</td>
<td></td>
</tr>
</tbody>
</table>

Note: * Proposed method

(a) Weighting Methods
(b) Inference Methods

Figure 5-4: Example of under Different Decision Structures

In the following sections, the study verifies the proposed method by comparing its capability with the other traditional transit decision-making methods. First, in Section 5.3 the proposed method is compared with the conventional weighting method through a transit mode selection problem. Second, in Section 5.4 the proposed method is compared with the well-known Bayesian reasoning through a viability of Personal Rapid Transit discussion.
5.3 Comparison of Belief Reasoning with Weighting Methods

The traditional methods for transit mode selection evaluate the overall goal of a transit mode applying weights to different attributes of the performance. The schemes used to apply the weights vary, for example, the weighted linear function, and the Analytical Hierarchy Process (AHP).

5.3.1 Numerical Example: Selection of Transit Mode

The reasoning method and weighting method are compared by applying to evaluate the transit modes proposed as a circulator transit system for a large commercial development. The system is to support transit riders from and to four metro stations. The goal is to provide a fast and convenient transit system connecting between the metro stations and developments in the commercial centers.

In this example, three transit mode alternatives are compared: personal rapid transit (PRT), circulator bus (BUS), and streetcar (SCR) modes. Three groups of stakeholders are considered: transit planners, users, and the community. The planner group regards five criteria (in red): fare revenues, capital costs, operating costs, transit riders, and system reliability. The user group regards six criteria (in blue): accessibility, mobility, availability, capacity, comfort, and safety. The community group regards four criteria (in green): economic impact, aesthetic quality, environmental condition, and quality of life. These criteria used are taken from TCRP Report 100: Transit Capacity and Quality of Service Manual. The proposed transit modes are defined based on ten characteristics: right-of-way, type of supports, vehicle size, vehicle speed, type of power, vehicle operation, stopping operation, type of service, headway, and station location.

5.3.2 Evaluation by Weighting Methods

For weighting methods, the attributes of the system are classified into different levels (commonly classified as goals, stakeholders, criteria, and sub-criteria) and they are related to the overall goal through a hierarchical tree structure. Figure 5-5 shows the
hierarchical tree structure for evaluating the most desirable transit mode in this example. The hierarchical structure consists of four levels (overall goal, stakeholders, criteria, and alternatives.) It is important to note that the characteristics of transit modes are hidden, not presented in this structure.

Figure 5-5: An Example for Hierarchical Structure for Transit Planning

The steps of evaluation and results based on AHP are as follows. First, the pair-wise comparisons among stakeholders are conducted, and the weight associated with each stakeholder group is calculated. Second, the pair-wise comparisons among criteria with respect to each stakeholder are made, and the weight associated with each criterion is calculated. Third, the pair-wise comparisons among transit modes are made, and the priority value associated with each transit mode with respect to each criterion is computed. Finally, the weights associated with criteria are normalized and the priority value of each transit mode is obtained using the weighted linear function.

\[
Z_k = \sum_{s,c} p(S)p(C | S)p(k | C, S) \quad (5-1)
\]

where \( p(S) \) = the priority (or weight) values (from 0 to 1) of stakeholder group \( S \), \( \Sigma p(S) = 1 \). \( p(C|S) \) = the priority (or weight) values (from 0 to 1) of criteria \( C \) with respect to
stakeholder group \( S \), \( \sum_C p(C | S) = 1 \). \( p(k | C, S) \) = the priority (or weight) values (from 0 to 1) of transit mode \( k \) with respect to criterion \( C \), \( \sum_k p(k | C, S) = 1 \).

The steps for determining the priority (or weight) values through pair-wise comparisons from different experts and the calculation results from the AHP method are described in Appendix A.

Figure 5-6 and Figure 5-7 show the weight values of different stakeholder groups and criteria associated with each stakeholder, respectively. Figure 5-8 shows the final priority values of three transit modes. They are 0.270 for PRT, 0.389 for Bus, 0.340 for Streetcar alternatives. The result shows only the ranking (relative preference) among three transit modes. The bus alternative is more preferable than Streetcar and PRT systems.

![Figure 5-6: AHP Priority Weights on Stakeholders](image)
Priority Weights on Criteria associated with ‘Benefits to Users’ (Level 3)

(a) Users

Priority Weights on Criteria associated with ‘Benefits to Planners’ (Level 3)

(b) Planners

Priority Weights on Criteria associated with ‘Benefits to the Community’ (Level 3)

(c) Community

Figure 5-7: AHP Priority Weights on Criteria with respect to Different Stakeholders
5.3.3 Issues of Weighting Methods

Using a hierarchical structure, each attribute is assumed to be independent of each other, and belongs to only one group. This assumption makes it difficult for analysts to classify the attributes especially those in transportation systems when they are related to several groups. Figure 5-9 shows the issues found when applying a hierarchical structure to transportation planning process.

Figure 5-9(a) shows the example of interdependency issues (the red dotted line). The improvement in mobility not only benefits to users but also gains the benefits to the community through enhancing the quality of life and improving environmental quality. Under this consideration, the ‘mobility’ should be classified under both ‘Benefits to users’ and ‘Benefits to the community.’

In Figure 5-9(b) the red dotted lines show an example of interactions among attributes. Accordingly, accessibility and mobility belong to users’ benefits; service area and walking time belong to accessibility; and travel time and transit riders belong to mobility. However, service area and transit riders are highly correlated, and so are walking time and travel time.
Moreover, the weighting methods require exact values of performance measures from the analysts and transportation models, and consistent weights among experts. However in reality, a wide range of uncertainty of performance measures and conflict among expert opinions exist during evaluation.

Using a weighting scheme, the weight values attached to each attribute are additive. This assumption requires a normalization process which may bias the real meaning of the weight values. Since the weight attached to one attribute is relative to that to other attributes in the same category, the ranking is very sensitive to the weight values.
Figure 5-10 show the effect of weight values of stakeholders on the priority values of three transit modes and the most desirable transit mode. When the weight value for the planner group increases (or is the highest among three groups), then the bus alternative is the most desirable. When the weight value for the user group increases, the PRT alternative is desirable. When the weight value for community group increases (or the weight values for both user and planner groups decrease), the streetcar alternative is recommended.

It is clear from the above and Figure 5-10 that the ranking of alternatives can be easily manipulated by the assignment of the weights. These weight values are always debatable and require policy makers to select their preferred weight values; the preferred transit mode can be selected by selecting the “right” weight values.

Note: weight for the community = 1 - (weight for users + weight for planners)

Figure 5-10: Effect of Weight Values on the Most Desirable Transit Mode

5.3.4 Evaluation by the Proposed Belief Reasoning Method

To overcome the issues in hierarchical tree structure and a weighting scheme previously discussed, the reasoning map structure and inference method are proposed. The reasoning map method connects the attributes of the system into a series of chained reasons starting from the decision variables of transit modes to the overall goal of the project. Figure 5-11 shows the reasoning map structure for evaluating the most transit modes in this example. In the proposed reasoning map structure, eight attributes are added to express the causalities: fleet size, travel time, auto users, traffic congestion, land use pattern, transit-oriented development (TOD), emission, and gasoline consumption.
Using a reasoning map structure, the classification of variables is not needed because it allows presenting the interrelationships among attributes in the system. The reasoning map structure addresses the redundancy issue in evaluation.

Figure 5-11 for example shows that improved mobility not only increases the benefit to the users due to travel time saving, but also attracts more transit riders which benefits to the planners as well as reduces automobile users. This in turn benefits the community by reducing environmental impacts and improving quality of life. In addition to evaluating the overall goal for each transit mode, the proposed method allows analysis and judgments about the validity of reasoning and the uncertainty and usefulness of information be evaluated for each goal.

Based on this reasoning map, the degrees of belief of every premise and relation are assigned by one professional and, while consistent and as objective as possible, they are hypothetical. The degrees of goal achievement for each transit mode are shown in Table 5-4. The results show that the Bus alternative has the highest degree of achievement of overall goal (0.52), followed by PRT (0.38) and streetcar (0.32). Although the ranking among alternatives is similar to that of the weighting method in the previous section, the degrees of belief in achievement of three of them are low. It can be implied that none of single alternative achieves the goal of the project. However, the Bus alternative’s superiority over PRT and Streetcar is now much clearer and stronger, and uncertainty regarding its goal achievement is the lowest and much lower than that of PRT.

Table 5-5 compares the final results obtained from the weighting AHP method and the proposed Belief Reasoning method. It shows that the Belief Reasoning evaluates transportation alternatives similar to the rating methods, but Belief Reasoning provides the degree of belief for achieving goals. AHP method however provides the prioritization of alternatives, but does not rate or evaluate each alternative.
Figure 5-11: Reasoning map structure for Transit Planning

Table 5-4: Degree of Overall Goal Achievement Using the Reasoning Map Structure

<table>
<thead>
<tr>
<th>States of Outcome</th>
<th>Alternatives</th>
<th>PRT</th>
<th>Bus</th>
<th>Streetcar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree of belief for “High” level achievement of the overall goal</td>
<td>0.38</td>
<td>0.52</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>Degree of belief for “Medium” level achievement of the overall goal</td>
<td>0.31</td>
<td>0.31</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>Degree of belief for “Low” level achievement of the overall goal</td>
<td>0.20</td>
<td>0.10</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>“I don’t know”</td>
<td>0.12</td>
<td>0.07</td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>

Table 5-5: Comparison of the Results between AHP and Belief Reasoning

<table>
<thead>
<tr>
<th>Transit Modes</th>
<th>AHP Priority</th>
<th>Rating Method</th>
<th>Belief Reasoning Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal Rapid Transit</td>
<td>0.271</td>
<td>Medium</td>
<td>Medium (0.31-0.43)</td>
</tr>
<tr>
<td>Circulator Bus</td>
<td>0.389</td>
<td>High</td>
<td>High (0.52-0.59)</td>
</tr>
<tr>
<td>Streetcar</td>
<td>0.340</td>
<td>Medium</td>
<td>Medium (0.46-0.54)</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Using the proposed reasoning map from Figure 5-11, the decision-makers can explain the reasons for supporting transit alternatives. Figure 5-12 shows the example of the critical reasons (in red) in the entire map for supporting PRT alternative.

Figure 5-13 summarizes the reasons for supporting individual transit alternatives. Each map is extracted from Figure 5-11 using the method explained in Chapter 4. Figure 5-13(a) corresponds to the red critical reasons in Figure 5-12. The extraction shows the most important chains of reasoning supporting the alternatives.

PRT alternative does not achieve the ‘High’ level of satisfaction for overall goal achievement because it provides ‘Low’ benefits to the planners. PRT does not have enough cost recovery. This is because (i) the predicted PRT ridership is not high enough to yield satisfactory fare revenue, and (ii) the capital cost is high due to substantial investment on the elevated guideway system. The planners may cut down the fleet size to reduce the investment cost; however, with a smaller fleet size the capacity will be less, PRT will be less comfortable, and, as a result, the transit ridership will decrease. Consequently, the system has low cost recovery.

Bus alternative is recommended if one alternative has to be implemented because, in the planners’ opinion, it provides the most benefits, although it only moderately satisfies the users and is expected to provide rather ‘low benefits’ to community. Bus has low capital costs because it operates on the existing shared lane. Street improvements and land use acquisition are not required. It is noted that Bus does not meet the community’s goal because it is not expected to improve quality of life and traffic congestion; however, that target is not the only purpose of this system and the community does not, in fact, have strong opinion against the bus.

Streetcar alternative does not achieve the ‘High’ level of satisfaction for overall goal achievement, although the capacity cost is less than PRT and it supports the community’s goals more than Bus. Based on the expert opinions, the benefit to the community is ‘Medium’ because it will bring higher aesthetic quality to the development, but the quality of life is not improved, in part because the area is automobile-oriented. The investment cost of streetcar still receives the ‘Low’ rating from the planners’ standpoint.
Figure 5-12: Reasons for Supporting PRT Alternative

(a) ‘PRT’ Alternative

(b) ‘Bus’ Alternative

(c) ‘Streetcar’ Alternative

Figure 5-13: Reasons for Supporting Individual Transit Alternatives
5.4 Comparison of Belief Reasoning with Bayesian Reasoning Methods

To compare between the Bayesian Reasoning and the proposed Belief Reasoning, a numerical example in transit planning is tested. A simple case study is selected in comparison because the Bayesian method requires a complete set of conditional probabilities for all links in the reasoning chains (shown in Figure 5-11). This is a formidable (in fact impossible) requirement and has all the uncertainties it seeks to avoid. Therefore, to illustrate the method, the study is restricted to the examination of the viability of Personal Rapid Transit (PRT) system in a commercial development.

5.4.1 Numerical Example: Viability of Personal Rapid Transit System

The reasoning map to justify the viability of the PRT systems in a commercial development is constructed based on the Viability of Personal Rapid Transit in New Jersey and Virginia study (Carnegie et al., 2007; Virginia DRPT, 2009). PRT uses small automated electric vehicles and provides on-demand service and non-stop travel between origin and destination. The vehicles are operated on a narrow exclusive guideway. (Cottrell and Mikosza, 2008) The proponents of PRT state that it is an efficient transit mode for a wide range of urban transportation applications. Figure 5-14 shows the reasoning map for the limited case being studied. The map consists of 24 attributes and 22 relations connecting from the decision variables to the five goals.

The degree of achievement of the five goals (compatible with dense urban form, low system cost, improved mobility, improved privacy, and better environment) are achieved by nine characteristics of PRT (small stations, small tracks, automated operation, small vehicles, lightweight vehicles, electric vehicles, direct trip, non-stop operation, and short headway.) Two approaches, Bayesian reasoning and Dempster-Shafer Belief reasoning, are compared.
5.4.2 Evaluation by Bayesian Reasoning Method

For Bayesian reasoning, each variable in a reasoning map is assumed to have three states, e.g. “high,” “medium,” and “low” or “large,” “medium,” and “small.” After the truth values (probabilities) are assigned by experts to the PRT characteristics and to the causal relations, the strength of opinions of achieving individual goals is calculated by Bayesian reasoning.

The truth values of the premises and causalities along the reasoning chains are measured by the *Probability* measures (*p*). Figure 5-15 presents the reasoning map for PRT with probabilities for the traditional Bayesian reasoning method. The results show, for PRT, the degree of goal achievement for Bayesian reasoning as follows: 0.89 for highly compatible with dense urban form; 0.43 for low system costs; 0.79 for high mobility; 0.65 for high privacy; and 0.99 for high environment quality.

Figure 5-14: Reasoning Map for PRT Viability
5.4.3 Issues of Bayesian Reasoning

Three main issues of Bayesian reasoning are discussed when applying it to real-world transportation planning process.

(1) **Handling Uncertainty “I don’t know”**

When an expert lacks of knowledge or says “I don’t know” about the characteristics of transit modes and/or causalities among attributes during evaluation, the analysts handle this type of uncertainty differently. In traditional probability theory, such as the Bayesian reasoning, this information is eliminated in the analysis by regarding it as “bad” data, or not allowing the experts to admit uncertainties in their opinions. However, this lack of knowledge may be true in real-world applications.
If “I don’t know” is modeled within Bayesian theory, the degree of belief (or probability) is assumed to be uniformly distributed as shown in Figure 5-16. The weight of evidence of one-third is assigned to each of the three states. However, this assumption makes it difficult to distinguish between the lack of knowledge and full knowledge with equal probabilities.

![Figure 5-16: Representation for “I don’t know” in Bayesian Reasoning](image)

On the other hand, the Dempster-Shafer theory of evidence allows presenting “I don’t know” with different degrees and different levels of uncertainty. Figure 5-17 shows that when “I don’t know” implies “Low or Medium or High,” different degrees of “I don’t know” can be presented, e.g. 1.0 or 0.1. In Figure 5-17(a), the evidence is ‘total ignorance’; it does not support any exclusive outcome at all. In Figure 5-17(b), the evidence is ‘partial ignorance’; it supports the specific outcomes with certain degrees of belief, but it also has a certain degree of not supporting any of the outcomes.
Figure 5-17: Different Degrees of “I don’t know” in the Proposed Method

Figure 5-18 shows that “I don’t know” opinion of the experts can be presented by different levels of “I don’t know,” e.g. “Low or Medium,” “Medium or High,” or “Low or Medium or High.” In Figure 5-18(a), the “I don’t know” opinion implies non-specific outcomes among three outcomes “Low or Medium or High.” The degree of belief of 0.25 is ‘totally’ unspecified to any outcomes. However, In Figure 5-18(b) the “I don’t know” opinion may be ‘partially’ specified to some of the outcomes. For example, of the 0.25 “I don’t know” degree of belief, 0.05 is attributed to “Low or Medium,” 0.05 attributed to “Medium or High,” and 0.15 is ‘totally’ unspecified. This interpretation is useful in modeling reasoning processes.
Figure 5-18: Different Levels of “I don’t know” in the Proposed Method

(2) Inferring the Conclusion

The traditional Bayesian inference method is limited by the issue of scalability when constructing the reasoning map. (Betsi, 2011) Bayesian reasoning is not feasible for constructing a large-scale decision-making system with complicated causal relations because it requires a complete set of conditional probability tables relating all combinations of outcomes of the premises to the outcomes of the conclusions. The simple example of conditional probability table is as follows. If headway is short, then vehicle occupancy is high (0.1), medium (0.2), and low (0.7); if headway is moderate, then
vehicle occupancy is high (0.2), medium (0.6), and low (0.2); and if headway is low, then vehicle occupancy is high (0.5), medium (0.4), and low (0.1). It requires three (3) probability distributions for one relation.

The situation is more cumbersome when many attributes are used to infer the conclusion. For instance, if four attributes are related to one conclusion, and each attribute and outcome have three states, then the total of $81 = 3^4$ combinations of the states of inputs, or 81 probability distributions, are needed and the conditional probability table is a 81x3 matrix. These numerous assessments become tedious work for experts to enter the degree of belief for all combinations. Moreover, the assessments would increase unreliability when dealing with a sizable system or combining multiple experts (Bensi, 2011).

On the other hand, the proposed reasoning method in which experts can provide partial knowledge and “I don’t know” allows them to view each causal relation separately. The conclusions from individual relations are derived separately and the final conclusion is obtained by aggregating the conclusions from different relations. In addition, one can assign the belief value to “I don’t know” when no information about any relation is available.

(3) Aggregation of Expert Opinions

The conventional evaluation methods in transportation planning assume that the experts are consistent in their attitude and opinions or have small biases when evaluating transportation alternatives. Neither the methods nor the experts recognize clearly or explicitly handle conflicting opinions. In Bayesian probability theory, the expert opinions are aggregated using the average operator. This averaging method treats the judgments from different experts equally regardless the strength of expert opinions.

The proposed reasoning method can deal with the case when experts have different strengths of opinions. The proposed method uses the Dempster’s rule of combination (DRC) to aggregate the opinions. DRC naturally builds up the belief value (or eliminate the degree of ignorance) of experts when supporting pieces of evidence are combined.
For example, if one expert has strong belief about an outcome and the other expert has very strong belief about it, then DRC will provide the very strong (stronger than both of two experts separately) about the outcome. However, if one expert has strong belief (is quite certain) about an outcome and the other expert has weak belief (or is uncertain) about it, then DRC will indicate higher support for the former. This is consistent with natural feelings when making a compromise or negotiating between two expert opinions. There is an explicit assumption that the expert with a “strong belief” can substantiate his view. Therefore, the transport analysts must be on guard against unsubstantiated “strong beliefs”. This guarding against superficial “strong beliefs” is by no means specific to DRC, but is present in all evaluation methods. In fact, DRC reasoning is the method that allows the transport analysts question weights and strength of beliefs. Detailed examples for aggregating opinions from multiple experts are presented in Chapter 6 when applying the method to the real-world case study.

5.4.4 Evaluation by the Proposed Belief Reasoning Method

The same application example (i.e. the Viability of Personal Rapid Transit system) is analyzed by the proposed method. In addition to the three states (“High,” “Medium,” and “Low”) applied in Bayesian reasoning, the state of “I don’t know” is added. This “I don’t know” state presents the non-specific state of these three states. It is important to note that different levels of “I don’t know,” e.g. “High or Medium,” “Medium or Low,” or “High or Medium or Low” can be incorporated. However, this example considers as the combination of the three.

The degrees of belief of the premises and causalities along the reasoning chains are measured by the basic probability assignments \( m \), which can be presented in conservative and optimistic terms: the Belief measure \( Bel \) and Plausibility measure \( Pl \). These two measures are calculated from Equations (3-10) and (3-11). Figure 5-19 presents the reasoning map based on the proposed method. The results show the degrees of goal achievement as follows: 0.73-0.87 for the “high” compatibility with dense urban form; 0.38-0.43 for “low” system cost; 0.60-0.81 for “high” mobility; 0.48-0.79 for “high” privacy; and 0.95-0.98 for “high environment quality. The lower value and the
upper value indicate Belief (conservative) and Plausibility (optimistic) values, respectively.

Further for the proposed Belief Reasoning, the information-based uncertainty associated with individual variable along the reasoning chains can be measured. These include the Non-specificity measure, the Discord measure, and the Total Uncertainty measure, the sum of the two. These measures can be calculated from Equations (3-13) through (3-15). Table 5-6 summarizes the degrees of achievement and the values of uncertainty measures associated with individual goals.

Figure 5-19: Illustrative Example of Belief Reasoning
The summary results show that the weight of evidence, given by the experts, does not support the goal of “low system cost” (0.38-0.43) and uncertainty about this information is very high, nearly the maximum. In other words, the experts believe that, compared to the other goals, the “low system cost” goal is the most difficult to achieve. On the other hand, the goal of “high environmental quality” is supported by the experts (0.95-0.98). The experts are confident that PRT can contribute to high environmental quality.

Among five goals of the transportation system, the “System cost” has also the highest discord measure. This is because the experts are conflicted in their opinions on the capital costs of the PRT system; although the operating costs of the PRT system are expected to be small, the total investment costs seem to be arguable and, many believe, high.

Among five goals, the “Privacy” and “Mobility” have relatively high values of non-specificity measure. The opinions on privacy are highly non-specific because the experts are not sure how many users the PRT vehicles will serve, and the vehicle occupancy is therefore questionable. Experts also have different opinions on how much the PRT system contributes to the travel time reduction.

According to the total uncertainty, the “System cost,” “Mobility,” and “Privacy” are the three goals whose achievement is highly questionable by the PRT system. In particular, the “System cost” has a very high total uncertainty (1.565); it is almost close to the theoretical maximum value (1.595).
There are two approaches to improve the validity of the reasoning and the accuracy of the assessment of the presented problem. First, in the above case, the reasoning map should be expanded to incorporate all aspects of the PRT system, such as the cost estimates, technological development, and operational designs. Second more expert knowledge should be collected and collated.

It is important to indicate that the nature of the results from the proposed Belief Reasoning method is different from the Bayesian reasoning method. The degrees of achievement of individual goals present the weight of evidence of which the experts support individual goals. For example, \( Bel(\text{High mobility}) = 0.60 \) suggests the conservative degree of support by the experts for high mobility with PRT. \( Pl(\text{High mobility}) = 0.81 \) suggests the optimistic degree of support by the experts for high mobility with PRT.

Table 5-7 compares the degree of achieving the goals of this example between Bayesian and the proposed Dempster-Shafer Belief Reasoning methods. When using traditional deterministic evaluation approaches, the goals are rated only by their state of outcome, such as “High” or “Low.” Nevertheless, when using reasoning under uncertainty, the degrees of goal achievement are attached. For Bayesian reasoning, the degrees of achievement are single-valued. For the Belief Reasoning, these values range from conservative to optimistic. Belief Reasoning also gives values for non-specificity and discord, which are not available from Bayesian reasoning. In this simplified case the results show that the degrees of goal achievement from the two methods are similar. However, the Bayesian reasoning required much more information than the Belief Reasoning method. In this example, the Bayesian results are on the “optimistic” side.

<table>
<thead>
<tr>
<th>Goals</th>
<th>Goal</th>
<th>Bayesian Reasoning</th>
<th>Proposed Belief Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal 1: Compatibility to Urban Form</td>
<td>High</td>
<td>High (0.89)</td>
<td>High (0.73-0.87)</td>
</tr>
<tr>
<td>Goal 2: System Costs</td>
<td>Low</td>
<td>Low (0.43)</td>
<td>Low (0.38-0.43)</td>
</tr>
<tr>
<td>Goal 3: Mobility</td>
<td>High</td>
<td>High (0.79)</td>
<td>High (0.60-0.81)</td>
</tr>
<tr>
<td>Goal 4: Privacy</td>
<td>High</td>
<td>High (0.65)</td>
<td>High (0.48-0.79)</td>
</tr>
<tr>
<td>Goal 5: Environmental Quality</td>
<td>High</td>
<td>High (0.99)</td>
<td>High (0.95-0.98)</td>
</tr>
</tbody>
</table>

Table 5-7: Comparison of Degree of Goal Achievement with Reasoning Methods
5.4.5 Effect of Incorporating the “I don’t know” Notion

The unique characteristic of the proposed method is to incorporate the notion of “I don’t know” in the premises and causal relations in the reasoning process. The study found that 70-80% of transportation experts have only partial knowledge about these matters and assign that belief to the non-specific state of outcome, i.e. “I don’t know.” A sensitivity analysis was carried out to examine the effect of the notion of “I don’t know” in the premises and causal relations in the reasoning chains. It is expected that the certainty about the goal outcomes decreases as the degrees of belief assigned to “I don’t know” for the premises and causal relations increase.

To prove this, the study assumes the truth value of “I don’t know” of every premise and relation in this example is the same and increases from 0 to 1 at the increment of 0.1. Figure 5-20 shows how the values of Belief (Bel) measure and Plausibility (Pl) measure for achievement of different goals with respect to the degree of belief of “I don’t know.”

When the degree of belief of “I don’t know” is close to 0, or the knowledge is totally certain, then the Belief (Bel) measure and the Plausibility (Pl) measure are equal and they are the same as the Probability (Prob) measure in Bayesian reasoning. This means that the proposed Belief Reasoning inference method is equivalent to Bayesian inference method without “I don’t know”.

When the degree of belief of “I don’t know” increases, the value of Bel (the lower line) decreases and the value of Pl (the upper line) increases, and hence the difference between the Pl and Bel measures is wider. The greater this difference the greater the degree of uncertainty.

When the degree of belief of “I don’t know” is close to 1 or the knowledge approaches total ignorance, then Bel = 0 and Pl = 1. This means complete uncertainty. In other words, there is no concrete evidence that supports a goal (Bel = 0), but because, at the same time, there is no evidence against it, the achievement of that goal is still plausible (Pl = 1).
(a) Goal 1: ‘Compacted Land Use’

(b) Goal 2: ‘Low System Costs’

(c) Goal 3: ‘High Mobility’

(d) Goal 4: ‘High Privacy’

(e) Goal 5: High Environmental Quality

Figure 5-20: Effects of “I don’t know” on the Degree of Achievement of Goals
The following sections present the effects of incorporating “I don’t know” notion on different uncertainty measures including Non-specificity, Discord, and Total Uncertainty. The non-specificity measure shows the extent of incompleteness of expert opinions. The greater the degree of belief of “I don’t know,” the higher the value of non-specificity measure is. The discord measure shows degree of conflict in expert opinions.

(1) Effect of Incorporating “I don’t know” Notion on Non-specificity Measure

Figure 5-21 shows the effect of change in the degree of belief of “I don’t know” on non-specificity. When it equals 0, knowledge is certain, and the value of Non-specificity ($N(m)$) for a goal is 0. This means that the experts are absolutely confident about the outcome, which is always the case for deterministic approach. On the other hand, when the degree of belief of “I don’t know” approaches 1, the knowledge is unspecified and the value of Non-specificity measure approaches the maximum uncertainty value ($\log_2 3 = 1.595$ for a case of three states of outcome: “High,” “Medium,” and “Low”).

![Figure 5-21: Effects of “I don’t know” on the Non-specificity Measure](image-url)
(2) Effect of Incorporating “I don’t know” Notion on Discord Measure

The value of Discord measure \((D(m))\) of individual goals represents the range of the experts’ opinions for the goal outcomes. This value is equivalent to entropy in probability theory, which is the measure of disorder (or randomness) in outcomes. Figure 5-22 shows the effect of the change in the degree of belief of “I don’t know” on the discord measure.

Among five goals the value of discord measure for the “environmental quality” goal is the lowest and for the “system cost” goal the highest. This means that with certainty in expert opinions (or the degree of belief of “I don’t know” = 0), the outcome of “environmental quality” goal is deterministic; there is no variation in the outcome of “environmental quality” goal. On the other hand, with certainty in expert opinions, the outcome of “system cost” goal shows highly conflicting views among experts.

When the uncertainty in the experts’ mind increases (or the degree of belief of “I don’t know” increases), the value of discord for the “system cost” goal gradually decreases. This is because the “I don’t know” opinion weakens the strength of expert opinion and implies willingness to “negotiate” with those having other opinions. Therefore, when the degree of belief in “I don’t know” increases the discord among opinions decreases.

On the other hand, the value of discord for “environmental quality” increases when the degree of belief in “I don’t know” is small—that is there is little uncertainty—and gradually decreases when it is large. This is because the initial value of discord is close to 0; all experts believed that the PRT will definitely provide high environmental quality. When some experts say “I don’t know” with a degree of confidence they actually increase the degree of conflict among experts since they are not sure whether the environment quality can be “Medium,” or “Low.”

Further, as the degree of belief of “I don’t know” notion increases, the values of discord measure for the goals converge to zero. This is because all experts have the same opinion of “I don’t know”, and there is no conflict among them.
(3) Effect of Incorporating “I don’t know” Total Uncertainty Measure

According to Figure 5-23, when the degree of belief in “I don’t know” increases, the total uncertainty increases to the maximum amount of information needed. In this example, each goal has three (3) states, “High,” “Medium,” and “Low.” The maximum amount of information needed is $\log_2(3) = 1.585$. 

Figure 5-22: Effects of “I don’t know” on the Discord Measure

Figure 5-23: Effects of “I don’t know” on the Total Uncertainty Measure
This analysis is particular useful for reasoning processes in transportation planning with uncertainty and ambiguity in the outcomes and/or two or more opinions. The impacts of data collection or knowledge enhancement on the integrity of reasoning can be analyzed through the proposed Belief Reasoning method but not with other reasoning or evaluation methods as the examples and comparisons with AHP and Bayesian reasoning show.

5.5 Summary

The proposed reasoning method is verified by comparing it with traditional evaluation methods in decision-making problems. The traditional methods and the proposed method are different in modeling the decision structure. The following discusses the advantages of the proposed method.

The proposed Belief Reasoning method has advantages over the weighting methods, such as indexing and scoring method and AHP method in three ways. First, the proposed method allows decision-makers to evaluate the performances of transportation alternatives as well as to measure the validity of the reasoning chain and the degree of belief of achieving goals. Second, the proposed reasoning method can handle non-deterministic characteristic of values of performance measures (e.g. operating costs can be low or medium or high, depending on the design), as well as the nature of uncertainty of experts’ judgment (e.g. “I don’t know” notion). Third, the reasoning map realistically models the decision problem in human thinking process. It allows modelling the interdependency and redundancy among system variables. This is not the case in the traditional multi-criteria decision-making methods. Thus, Belief Reasoning has a higher flexibility to model the decision issues in a large-scale system.

The proposed Belief Reasoning method has advantages over the Bayesian reasoning in four ways. First, the proposed method can handle the “I don’t know” type opinion provided by the experts (or citizens). The mathematical mechanism for dealing with this type of uncertainty is the Dempster-Shafer theory. This “I don’t know” opinion is usually present in expert judgments. Experts usually have only partial knowledge about
the input values or conditions, and the only honest opinion is “I don’t know.” Second, the proposed method can evaluate the alternatives although knowledge is incomplete. Unlike the Bayesian reasoning where a complete set of information about causal relations is required, the proposed method can use all available information. Third, the proposed method uses the Dempster’s rule of combination (DRC) to merge different expert opinions about inputs or causal relations in a reasoning map. This combination rule is naturally better for combining conflicting opinions. Finally, the proposed method can measure uncertainty along the reasoning chains and identify additional information for improving the reasoning process.

Comparison of the four reasoning methods (rule-based reasoning, fuzzy reasoning, Bayesian reasoning, and the proposed Belief Reasoning) shows that, the proposed Belief Reasoning method has relatively strong theoretical background because it is based on generalized probability theory. The mathematical mechanism may look complex, but in fact it is far less complicated than Bayesian reasoning in large-scale decision-making systems. The downside of the proposed method is complexity of the mathematical mechanism when different levels of “I don’t know” opinions are present. Therefore, it is recommended that the levels “I don’t know” opinion are minimized by the data gathering process. Table 5-8 has a summary comparison of the applicability of the different reasoning methods.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Theories</th>
<th>Rule-Based Reasoning</th>
<th>Fuzzy Reasoning</th>
<th>Bayesian Reasoning</th>
<th>(Proposed) Belief Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical Background</td>
<td>Weak</td>
<td>Moderate</td>
<td>Strong</td>
<td>Strong</td>
<td></td>
</tr>
<tr>
<td>Computational Complexity</td>
<td>Low</td>
<td>Moderate</td>
<td>Moderate</td>
<td>Moderate</td>
<td></td>
</tr>
<tr>
<td>Model Setup</td>
<td>Low</td>
<td>Moderate</td>
<td>Moderate</td>
<td>Moderate</td>
<td></td>
</tr>
<tr>
<td>Model Input Requirements</td>
<td>High</td>
<td>Partial</td>
<td>High</td>
<td>Partial</td>
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<tr>
<td>Model Execution</td>
<td>Low</td>
<td>Moderate</td>
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<tr>
<td>Ease of Application</td>
<td>Easy</td>
<td>Moderate</td>
<td>Substantial</td>
<td>Moderate</td>
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</tr>
<tr>
<td>Amount of Training in Theory for Analysts</td>
<td>Little</td>
<td>Moderate</td>
<td>Substantial</td>
<td>Substantial</td>
<td></td>
</tr>
<tr>
<td>Amount of Training in Application for Analysts</td>
<td>Little</td>
<td>Moderate</td>
<td>Substantial</td>
<td>Substantial</td>
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</tbody>
</table>

Table 5-8: Comparison of Applicability of Different Reasoning Methods
CHAPTER 6

VALIDATION OF BELIEF REASONING IN TRANSIT PLANNING

This chapter presents the validation process of the proposed belief reasoning method. The study applies the proposed methodology to the alternatives analysis process in a real-world evolving urban public transportation project in the U.S. In this chapter, the evaluation of transit technology alternatives of the case study is presented, the reasoning for public transportation decision is discussed, and the validity of reasoning based on expert opinions is measured.

6.1 Introduction

In major urban transit investment projects, there is a supposed direct relationship between the final project goals (such as cost, mobility, land use impact, environmental benefits) and the decisions made on the planning stages regarding the characteristics of transit alternatives including vehicles, stations, guideway, management, and control. The main objective of this chapter is to present the real-world application of the proposed belief reasoning method for assisting transit agencies and citizens in understanding the chains of reasoning in decision-making and developing viable transit alternatives corresponding to specific project goals. The locally preferred alternative from the transit alternatives analysis should promote optimal performance trade-offs in a transit investment project by considering the causal relations among the characteristics of the alternative that can best meet the system performance goals while observing the constraints specific to the project.

This study develops the reasoning framework to analyze the decision (already made) in the Alternatives Analysis process of a planned transit project. Face-to-face interviews were conducted to compile opinions of experts and citizens from different
organizations, including local transit planners and metropolitan regional planners. The reasoning maps are developed and the integrity of the reasoning process is then measured. The proposed transit alternatives are evaluated and finally the analysis of the expert knowledge is performed.

The subsequent sections discuss this application in detail. First, the project profiles including the project background, the goals of the project, and the definition of proposed alternatives, are discussed. Second, the data collection and expert panels are described. Third, the reasoning model and the model results are presented. Finally, the discussion of the results is presented.

6.2 Project Profile

The Columbia Pike transit corridor in Northern Virginia is selected as a case study for transit alternatives analysis. (PTI, 2005; 2011) The Columbia Pike Transit Initiative project and plan have been initiated by the Washington Metropolitan Area Transit Authority, along with Arlington and Fairfax Counties since last decade to improve a transit system in this corridor. This corridor is situated in Washington Metropolitan Area connecting between the Pentagon/Pentagon City area and Bailey’s Crossroads area.

6.2.1 Project Information

The corridor of Columbia Pike or Virginia State Highway 244 contains mixed land uses for residences, employment, and commerce. The transit corridor passes through several major activity centers and the regional transit stations including the Skyline complex, Bailey’s Crossroads shopping center near Leesburg Pike, Pentagon and Pentagon City metro stations and shopping centers. Figure 6-1 shows the map of the study area and Figure 6-2 shows the map of the proposed transit alignment.
Figure 6-1: Map of Study Area

Figure 6-2: Map of Proposed Transit Corridor Alignment

(Source, Pike Transit Initiative, 2010)
6.2.2 Definition of Transit Alternatives

During the Alternatives Analysis process of this transit project in 2005, two ‘Build’ Alternatives, ‘Bus Rapid Transit (BRT)’ and ‘Streetcar’, along with the Baseline Alternative (of improving the existing bus system) were proposed and evaluated. The definitions of alternatives considered in this study are based on the Columbia Pike Initiative transit plan in 2005.

The proposed BRT and Streetcar technologies are compatible with urban form and are consistent with the County plan. Both transit modes meet the criteria in the screening process. They are surface transit systems operated in the mixed traffic shared lane. The horizontal and vertical alignments are accommodated within the existing roadway and compatible with pedestrian street frontage.

Many features of these two proposed ‘Build’ alternatives are similar. They would be operated on the existing travel lane and major street improvements would not be required. Curb-to-curb roadway and pedestrian safety are planned for both alternatives. Transit signal priority and real-time information would be implemented. Station locations and conceptual designs for both ‘Build’ alternatives would be the same.

The unique characteristics of these two alternatives are as follows. Bus Rapid Transit technology consists of rubber-tired vehicles operated in a mixed traffic on a paved way. The proposed electric-diesel hybrid BRT vehicles are 40 to 60 feet long and can carry approximately 60 to 120 passengers.

Streetcar technology consists of light rail steel-wheeled vehicles running on at-grade concrete track slab with other traffic. The proposed streetcar vehicles are 30 to 70 feet long with low-floor and wide doors and they can carry 45 to 190 passengers per vehicle. They receive power from an overhead electric wire.

6.2.3 Goals of the Project

Five goals are considered to be achieved by the proposed transit alternatives. (PTI, 2010) They are (i) Improvement of mobility within the corridor; (ii) Enhancement of community and economic development; (iii) Livability and long-term sustainable
community; (iv) Development of an integrated multi-modal transport system; and (v) Provision of safe environment for the citizens and all travel modes.

To achieve these goals the following objectives are taken into account. For the ‘Mobility’ goal, the proposed ‘Build’ transit alternative should provide adequate transportation capacity and choice to meet future travel demand as well as high quality of transit service along the corridor. For the ‘Economic Development’ goal, the proposed transit alternative should promote local economic revitalization and support the growth of employment within the corridor. For the ‘Livability and Sustainability’ goal, the proposed alternative should promote long-range private investment and sustainable development-oriented transit as well as support economically vibrant and environmentally sustainable focused communities along the corridor. For the ‘Multi-modal Transport System’ goal, the proposed alternative should be well-integrated into a region-wide multi-modal transportation system and improve regional transportation connections. For the ‘Safety’ goal, the proposed alternative should enhance community safety from both personal security and transportation operational safety.

6.3 Belief Reasoning Model

The proposed Dempster-Shafer belief reasoning method is applied to compare the two ‘Build’ Alternatives and evaluate their relative degree of support of the goals of the project. The method consists of two stages. First, the reasoning map was constructed, and second, the validity of reasoning was measured. In the first stage, the variables that have a potential effect on the five goals of the project and their causal relations were identified from a series of interviews with experts, and reviews of transit plans and reports, and relevant literature. An expert panel was brainstormed to customize and validate the reasoning map. In the second stage, experts were asked to assign the degrees of belief to the characteristics of alternatives and to the causal relations among variables in the proposed reasoning map.

An expert panel in this study was designated from three groups: local and regional transit planners, local experts, and public transit scholars. A group of 10 experts is
involved in the survey. Face-to-face interviews with individual experts were carried out. Each of the selected experts has expertise in transit project evaluation and planning and possesses graduate degrees in engineering or urban planning.

The proposed reasoning map is composed of 91 variables. Table 6-1 provides the list of variables used in the reasoning map for comparing the ‘Streetcar’ and ‘Bus Rapid Transit’ Alternatives. The state of outcomes, preceding nodes and subsequent nodes of individual variables are also listed in this table. These variables are classified into four categories. There are:

- 22 decision variables (D1-D22) describing the operational, propulsion, and physical characteristics of transit alternatives
- 2 exogenous variables (X1-X2) describing the variables that do not explain the characteristics of transit modes, but affect the reasoning chains
- 62 consequences—20 consequences for Goal 1 (M1-M20), 11 for Goal 2 (E1-E11), 21 for Goal 3 (L1-L21), 5 for Goal 4 (I1-I5), and 5 for Goal 5 (S1-S5)
- 5 goals of the project (G1-G5)

For each consequence and goal, five possible states of outcomes are set, i.e. “Much Higher,” “Higher,” “Same,” “Lower,” “Much Lower” in order to compare the measure of uncertainty among these variables. The “I don’t know” state is added to represent to imply non specific opinion.
Table 6-1: List of Variables in the Reasoning Map

<table>
<thead>
<tr>
<th>No.</th>
<th>ID</th>
<th>Description</th>
<th>States of Outcomes</th>
<th>Parent Nodes (Causes)</th>
<th>Child Nodes (Effects)</th>
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<tbody>
<tr>
<td></td>
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<td><strong>Decision Nodes</strong></td>
<td></td>
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<tr>
<td>1</td>
<td>D1</td>
<td>Right-of-way</td>
<td>Exclusive/Semi-exclusive/Shared lane/I don’t know</td>
<td>-</td>
<td>M2, E1</td>
</tr>
<tr>
<td>2</td>
<td>D2</td>
<td>Guideway type</td>
<td>Fixed guideway/Flexible guideway/I don’t know</td>
<td>-</td>
<td>M3, E3, L4</td>
</tr>
<tr>
<td>3</td>
<td>D3</td>
<td>Support type</td>
<td>Rail-based support/Dual/Pavement-based support/I don’t know</td>
<td>-</td>
<td>M4</td>
</tr>
<tr>
<td>4</td>
<td>D4</td>
<td>Vehicle capacity</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>-</td>
<td>M1</td>
</tr>
<tr>
<td>5</td>
<td>D5</td>
<td>System appearance</td>
<td>Very good/Good/Fair/Poor/Very poor/I don’t know</td>
<td>-</td>
<td>M6</td>
</tr>
<tr>
<td>6</td>
<td>D6</td>
<td>System security</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>-</td>
<td>M6</td>
</tr>
<tr>
<td>7</td>
<td>D7</td>
<td>Transit line length</td>
<td>Very long/Long/Medium/Short/Very short/I don’t know</td>
<td>-</td>
<td>M7, E6</td>
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<tr>
<td>8</td>
<td>D8</td>
<td>Service area</td>
<td>Very large/Large/Medium/Small/Very small/I don’t know</td>
<td>-</td>
<td>M6, M18, E4, E5</td>
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<tr>
<td>9</td>
<td>D9</td>
<td>Service headway</td>
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<tr>
<td>10</td>
<td>D10</td>
<td>Station location</td>
<td>Very close/Close/Medium/Sparse/Very sparse/I don’t know</td>
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<tr>
<td>11</td>
<td>D11</td>
<td>Station user friendliness</td>
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</tr>
<tr>
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<td>D12</td>
<td>Multi-door boarding</td>
<td>Yes/No/I don’t know</td>
<td>-</td>
<td>M9</td>
</tr>
<tr>
<td>13</td>
<td>D13</td>
<td>Low-floor vehicles</td>
<td>Yes/No/I don’t know</td>
<td>-</td>
<td>M9</td>
</tr>
<tr>
<td>14</td>
<td>D14</td>
<td>Fare collection system</td>
<td>Off-board/On-board/I don’t know</td>
<td>-</td>
<td>M9</td>
</tr>
<tr>
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<td>D15</td>
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<td>L6</td>
</tr>
<tr>
<td>16</td>
<td>D16</td>
<td>Technological advance</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
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<td>L7, L16</td>
</tr>
<tr>
<td>17</td>
<td>D17</td>
<td>Mean of propulsion</td>
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<td>Closeness to Transit Centers</td>
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</tr>
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<td>D20</td>
<td>Station design</td>
<td>Very good/Good/Fair/Poor/Very poor/I don’t know</td>
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<td>S1</td>
</tr>
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<td>21</td>
<td>D21</td>
<td>Vehicle acceleration/deceleration</td>
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<td>-</td>
<td>S4</td>
</tr>
<tr>
<td>22</td>
<td>D22</td>
<td>Control/Braking system</td>
<td>Very good/Good/Fair/Poor/Very poor/I don’t know</td>
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<td>S3</td>
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<td><strong>Exogenous Factor Nodes</strong></td>
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<tr>
<td>23</td>
<td>X1</td>
<td>Support by existing local buses</td>
<td>Very good/Good/Fair/Poor/Very poor/I don’t know</td>
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<td>X2</td>
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<td>25</td>
<td>M1</td>
<td>Transit line capacity</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D4</td>
<td>M15</td>
</tr>
<tr>
<td>26</td>
<td>M2</td>
<td>Vehicle operating speed</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D1</td>
<td>M5</td>
</tr>
<tr>
<td>27</td>
<td>M3</td>
<td>Possibility of vehicle passing</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D2</td>
<td>M5</td>
</tr>
<tr>
<td>28</td>
<td>M4</td>
<td>Ride quality</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D3</td>
<td>M6</td>
</tr>
<tr>
<td>29</td>
<td>M5</td>
<td>Corridor travel time</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M2, M3</td>
<td>M12, M13, L13, I4</td>
</tr>
<tr>
<td>30</td>
<td>M6</td>
<td>Attraction to transit users</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D5, D6, D8, M4</td>
<td>M12, M13</td>
</tr>
<tr>
<td>31</td>
<td>M7</td>
<td>Activity centers served</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D7</td>
<td>M13, M16, E4, S2</td>
</tr>
<tr>
<td>32</td>
<td>M8</td>
<td>Transit service frequency</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D9, DD</td>
<td>M13, M19</td>
</tr>
<tr>
<td>33</td>
<td>M9</td>
<td>Ease of passenger boarding</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D12, D13, D14</td>
<td>M11</td>
</tr>
<tr>
<td>34</td>
<td>M10</td>
<td>Station accessibility</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D10, D11</td>
<td>M20, E2</td>
</tr>
<tr>
<td>35</td>
<td>M11</td>
<td>Vehicle accessibility</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M9</td>
<td>M20</td>
</tr>
<tr>
<td>36</td>
<td>M12</td>
<td>Potential for auto users shifted to transit users</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M5, M6</td>
<td>M14, L12</td>
</tr>
<tr>
<td>37</td>
<td>M13</td>
<td>Transit ridership</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M5, M6, M7, M8</td>
<td>M14, E2, L13</td>
</tr>
<tr>
<td>38</td>
<td>M14</td>
<td>Total person throughput</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M12, M13</td>
<td>M15</td>
</tr>
<tr>
<td>39</td>
<td>M15</td>
<td>Transportation capacity</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M1, M14</td>
<td>G1</td>
</tr>
<tr>
<td>40</td>
<td>M16</td>
<td>Transportation choices for some O-D pairs</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M7</td>
<td>M17</td>
</tr>
<tr>
<td>41</td>
<td>M17</td>
<td>Transportation choices</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M16</td>
<td>G1</td>
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<td>42</td>
<td>M18</td>
<td>Service area coverage</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D7, D8</td>
<td>M19</td>
</tr>
<tr>
<td>43</td>
<td>M19</td>
<td>Quality of service for local trips</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M8, M18</td>
<td>G1</td>
</tr>
<tr>
<td>44</td>
<td>M20</td>
<td>Service for transit dependent</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M10, M11</td>
<td>G1</td>
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<td><strong>Consequence Nodes with respect to 'Economic Development'</strong></td>
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<td></td>
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<tr>
<td>45</td>
<td>E1</td>
<td>Adaptability to higher ridership</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D4</td>
<td>E10</td>
</tr>
<tr>
<td>46</td>
<td>E2</td>
<td>Opportunity for transit-oriented development</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M10</td>
<td>E8, E9, L1, L3</td>
</tr>
<tr>
<td>47</td>
<td>E3</td>
<td>Permanence of transit service</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D2</td>
<td>E9</td>
</tr>
<tr>
<td>48</td>
<td>E4</td>
<td>Opportunity for land redevelopment</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D8, M7</td>
<td>E4</td>
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<tr>
<td>49</td>
<td>E5</td>
<td>Increment of economic growth</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M8</td>
<td>E7</td>
</tr>
<tr>
<td>50</td>
<td>E6</td>
<td>Connectivity to jobs&amp;educational opportunities</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M7</td>
<td>G2</td>
</tr>
<tr>
<td>51</td>
<td>E7</td>
<td>Local economic impacts</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>E5</td>
<td>G2</td>
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<tr>
<td>52</td>
<td>E8</td>
<td>Focused development</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>E2, E11</td>
<td>G2</td>
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<tr>
<td>53</td>
<td>E9</td>
<td>Possibility of private investment</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>E2, E3</td>
<td>G2</td>
</tr>
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<td>54</td>
<td>E10</td>
<td>Support population and employment growth</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>E1</td>
<td>G2</td>
</tr>
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<td>55</td>
<td>E11</td>
<td>Consistency with County economic plan</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>E4, E8</td>
<td>G2</td>
</tr>
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<td>States of Outcomes</td>
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<td>Child Nodes (Effects)</td>
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<tr>
<td>56</td>
<td>L1</td>
<td>Enhanced walkability</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>E2</td>
<td>L2</td>
</tr>
<tr>
<td>57</td>
<td>L2</td>
<td>Promoting Ped/Bike focused communities</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>L1</td>
<td>G3</td>
</tr>
<tr>
<td>58</td>
<td>L3</td>
<td>Consistency with adopted corridor initiatives</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>E2</td>
<td>E19</td>
</tr>
<tr>
<td>59</td>
<td>L4</td>
<td>Fleet size required</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D4, D9</td>
<td>L6, L16</td>
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<td>L5</td>
<td>Guideway costs</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D2</td>
<td>L15</td>
</tr>
<tr>
<td>61</td>
<td>L6</td>
<td>Vehicle cost</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D15, L5</td>
<td>L15</td>
</tr>
<tr>
<td>62</td>
<td>L7</td>
<td>Technical reliability</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D16</td>
<td>L17, S5</td>
</tr>
<tr>
<td>63</td>
<td>L8</td>
<td>Gasoline consumption</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D17</td>
<td>L14</td>
</tr>
<tr>
<td>64</td>
<td>L9</td>
<td>Noise pollution</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D17</td>
<td>L14</td>
</tr>
<tr>
<td>65</td>
<td>L10</td>
<td>Air pollution</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D17</td>
<td>L14</td>
</tr>
<tr>
<td>66</td>
<td>L11</td>
<td>Visual impacts of the community</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D18</td>
<td>L14</td>
</tr>
<tr>
<td>67</td>
<td>L12</td>
<td>Income equity of transit riders</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M12</td>
<td>G3</td>
</tr>
<tr>
<td>68</td>
<td>L13</td>
<td>User benefits</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M5, M13</td>
<td>L18</td>
</tr>
<tr>
<td>69</td>
<td>L14</td>
<td>Environmental benefits</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>L8, L9, L10, L11</td>
<td>L20, L21</td>
</tr>
<tr>
<td>70</td>
<td>L15</td>
<td>Capital costs</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>L4, L6</td>
<td>L18</td>
</tr>
<tr>
<td>71</td>
<td>L16</td>
<td>Operating costs</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D16, L5</td>
<td>L18</td>
</tr>
<tr>
<td>72</td>
<td>L17</td>
<td>Maintenance costs</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>L7</td>
<td>L18</td>
</tr>
<tr>
<td>73</td>
<td>L18</td>
<td>Long-term benefits of investment</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>L13, L14, L15, L16, L17</td>
<td>L19</td>
</tr>
<tr>
<td>74</td>
<td>L19</td>
<td>Long-term public and private investment</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>L3, L18</td>
<td>G3</td>
</tr>
<tr>
<td>75</td>
<td>L20</td>
<td>Environmentally sustainable communities</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>L14</td>
<td>G3</td>
</tr>
<tr>
<td>76</td>
<td>L21</td>
<td>Adverse environmental impacts</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>L14</td>
<td>G3</td>
</tr>
<tr>
<td>77</td>
<td>I1</td>
<td>Service to regional activity centers</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D19</td>
<td>G4</td>
</tr>
<tr>
<td>78</td>
<td>I2</td>
<td>Accessibility to regional activity centers</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M7</td>
<td>I3</td>
</tr>
<tr>
<td>79</td>
<td>I3</td>
<td>Connections to transit centers</td>
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<td>I2</td>
<td>G4</td>
</tr>
<tr>
<td>80</td>
<td>I4</td>
<td>Transit travel to regional destination</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M5</td>
<td>I5</td>
</tr>
<tr>
<td>81</td>
<td>I5</td>
<td>Transit ridership to regional activity centers</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>I4</td>
<td>G4</td>
</tr>
<tr>
<td>No.</td>
<td>ID</td>
<td>Description</td>
<td>States of Outcomes</td>
<td>Parent Nodes (Causes)</td>
<td>Child Nodes (Effects)</td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
<td>-----------------------------------------------</td>
<td>----------------------------------------------------------</td>
<td>-----------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>82</td>
<td>S1</td>
<td>Station visibility</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D20</td>
<td>S2</td>
</tr>
<tr>
<td>83</td>
<td>S2</td>
<td>Personal security</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>S1</td>
<td>G5</td>
</tr>
<tr>
<td>84</td>
<td>S3</td>
<td>Accident rates with other travel mode</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D22</td>
<td>S4</td>
</tr>
<tr>
<td>85</td>
<td>S4</td>
<td>Safe operations for travelers</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>D21, S3</td>
<td>G5</td>
</tr>
<tr>
<td>86</td>
<td>S5</td>
<td>Safe environment for staffs and employees</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>S5</td>
<td>G5</td>
</tr>
</tbody>
</table>

**Goal Nodes**

<table>
<thead>
<tr>
<th>No.</th>
<th>ID</th>
<th>Description</th>
<th>States of Outcomes</th>
<th>Parent Nodes (Causes)</th>
<th>Child Nodes (Effects)</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>G1</td>
<td>Improved mobility</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>M15, M17, M19, M20</td>
<td>-</td>
</tr>
<tr>
<td>88</td>
<td>G2</td>
<td>Economic development</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>E6, E7, E9, E10, E11</td>
<td>-</td>
</tr>
<tr>
<td>89</td>
<td>G3</td>
<td>Livability and long-term sustainability</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>L2, L12, L19, L20, L21</td>
<td>-</td>
</tr>
<tr>
<td>90</td>
<td>G4</td>
<td>Integrated multi-modal transport system</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>I1, I3, I7</td>
<td>-</td>
</tr>
<tr>
<td>91</td>
<td>G5</td>
<td>Safe environment</td>
<td>Much Higher/Higher/Same/Lower/Much Lower/I don’t know</td>
<td>S2, S4, S5</td>
<td>-</td>
</tr>
</tbody>
</table>
6.4 Model Results

This section presents the results of this case study based on the proposed mathematical mechanism. First, the reasoning map configuration is presented. Second, the belief values of reasoning and goal achievement and their uncertainty measures are discussed. Finally, the critical reasons supporting the goals are determined.

6.4.1 Reasoning Maps

Figure 6-3 through 6-7 present the reasoning maps for individual goals. The variables in these figures correspond to those in Table 6-1 above. In each reasoning map, the boxes on the left column present the decision variables of the transit technology, and the boxes on the right column show the goals of the project. The intermediate boxes which connect between decision variables and goals are a series of interrelated consequences and performances.

For visualization purpose, the map of each goal is illustrated separately, although, in the analyses that follow, some variables in one map may be connected to the other maps. For example, ‘Increase in transit riders’ is related to ‘Mobility’ (Goal 1) and ‘Economic Development’ (Goal 2), or ‘Improvement in corridor travel time’ is related to Goals 1 and 3.
Figure 6-3: Reasoning Map for Goal 1 “Mobility”

Figure 6-4: Reasoning Map for Goal 2 “Economic Development”
Figure 6-5: Reasoning Map for Goal 3 “Livability and Sustainable Community”

Figure 6-6: Reasoning Map for Goal 4 “Integrated Multimodal Transport System”

Figure 6-7: Reasoning Map for Goal 5 “Safe Environment”
6.4.2 Degrees of Belief and Uncertainty Measures

In this example, two Build alternatives are compared. The degrees of belief that the Streetcar alternative achieves the goals of the project relative to the Bus Rapid Transit alternative are evaluated. The following section summarizes the degrees of belief and uncertainty measures associated with goal nodes and consequence nodes, and discusses the results.

Table 6-2 presents the degrees of belief of achieving individual goals and their associated uncertainty measures. Compared to the BRT alternative, the Streetcar alternative supports all the goals of the corridor project at different degrees of achievement: 0.95 for ‘Higher’ mobility, 0.98 for ‘Higher’ economic development, 1.00 for ‘Higher’ livability and sustainability, (but only) 0.19 for ‘Higher’ multi-modal transport system, and 1.00 for ‘Higher’ safe environment.

The values of the uncertainty measure reflect the expert opinions. The values of the non-specificity measure are very low (0.00-0.02). They imply that the expert panel (the planner group) as a group decision is rather confident with their opinions. They do have conflicting opinions about the integrated multimodal transport system (Goal 4); some argue for and some against that streetcar and bus alternatives support similar connectivity to regional transport system. The value of discord measure of this goal is relatively high (0.694). The lack of non-specificity for multimodal transport goal simply means that each expert was quite certain on his opinion about the multimodality of the transport system, but these opinions conflicted with each other as shown by the discord measure.

Table 6-2: Degrees of Belief and Uncertainty Measures of Goals of the Project

<table>
<thead>
<tr>
<th>Goals</th>
<th>Degrees of Belief of Outcomes</th>
<th>Uncertainty Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Much Higher</td>
<td>Higher</td>
</tr>
<tr>
<td>Goal 1: Mobility</td>
<td>0.000</td>
<td>0.952</td>
</tr>
<tr>
<td>Goal 2: Economic Development</td>
<td>0.005</td>
<td>0.982</td>
</tr>
<tr>
<td>Goal 3: Livability and Sustainable Community</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Goal 4: Multi-modal Transport</td>
<td>0.000</td>
<td>0.187</td>
</tr>
<tr>
<td>Goal 5: Safe Environment</td>
<td>0.000</td>
<td>0.997</td>
</tr>
</tbody>
</table>
Table 6-3 presents the degree of belief about states of outcomes and uncertainty measures for consequence variables corresponding to Nodes 25-86 in Table 6-1. Since there are five states of outcomes for each variable, the maximum uncertainty measure is 2.322 (\(=\log_2 5\)) according to information theory. Thus, the non-specificity measure, the discord measure, and their sum, the total uncertainty measure, range between 0.000 and 2.322.

The values of non-specificity measure of these variables are relatively low (0.00-0.25), and at the same time, the degrees of belief (strength of opinion) of “I don’t know” notion are small (0.00-0.16). Together they imply that the experts have ‘strong’ opinions; they are certain about their reasoning and judgment.

Nevertheless, there are some controversial issues in reasoning. For example, among consequences with respect to Mobility goal (Goal 1) the most controversial variables among experts are ‘Corridor travel time (M5),’ and ‘Service for transit dependent (M20).’ Some experts believe that the Streetcar alternative would improve corridor travel time and service for transit dependent more than the BRT alternative, but some of them believe that they would provide the same service.

For the Economic development (Goal 2), there are no significant conflicts among the experts. They believe that economic development would follow the ‘Build’ alternative, but they are less specific about the consequence outcomes from the Economic development goal than those of the Mobility goal. For example, the experts believe that the streetcar would ‘Support the population and employment growth (E10),’ but they are not sure how much more that support (“Much Higher” or “Higher”) would be over the BRT alternative. The non-specificity measure of this variable is 0.256.

Among the consequences with respect to the Livability and Sustainability goal (Goal 3), the experts have least knowledge about the “Maintenance costs (L17).” The values of non-specificity and discord for it are the highest, 0.112 and 1.378, respectively.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Degrees of Belief of Outcomes</th>
<th>Uncertainty Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Much Higher</td>
<td>Higher</td>
</tr>
<tr>
<td>M1: Transit line capacity</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>M2: Vehicle operating speed</td>
<td>0</td>
<td>0.11</td>
</tr>
<tr>
<td>M3: Possibility of vehicle passing</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M4: Ride quality</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>M5: Corridor travel time</td>
<td>0</td>
<td>0.38</td>
</tr>
<tr>
<td>M6: Attraction to transit users</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M7: Activity centers served</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M8: Transit service frequency</td>
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<td>0.79</td>
</tr>
<tr>
<td>M9: Ease of passenger boarding</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>M10: Station accessibility</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M11: Vehicle accessibility</td>
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</tr>
<tr>
<td>M12: Potential for auto users shifted</td>
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<td>0</td>
</tr>
<tr>
<td>M13: Transit ridership</td>
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<td>0.95</td>
</tr>
<tr>
<td>M14: Total person throughput</td>
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</tr>
<tr>
<td>M15: Transportation capacity</td>
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<td>M16: Transit choices for local trips</td>
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</tr>
<tr>
<td>M17: Transportation choices</td>
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</tr>
<tr>
<td>M18: Service area coverage</td>
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</tr>
<tr>
<td>M19: Quality of service for local trips</td>
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</tr>
<tr>
<td>M20: Service for transit dependent</td>
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</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Degrees of Belief of Outcomes</th>
<th>Uncertainty Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Much Higher</td>
<td>Higher</td>
</tr>
<tr>
<td>E1: Adaptability to higher ridership</td>
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</tr>
<tr>
<td>E2: Opportunity for TOD</td>
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<td>0.85</td>
</tr>
<tr>
<td>E3: Permanence of transit service</td>
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<tr>
<td>E4: Opportunity for land redevelopment</td>
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</tr>
<tr>
<td>E5: Increment of economic growth</td>
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</tr>
<tr>
<td>E6: Connectivity to jobs and educational opportunities</td>
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<td>0.19</td>
</tr>
<tr>
<td>E7: Local economic impacts</td>
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<td>0.90</td>
</tr>
<tr>
<td>E8: Focused development</td>
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</tr>
<tr>
<td>E9: Possibility of private investment</td>
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<tr>
<td>E10: Support pop. and emp. growth</td>
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<tr>
<td>E11: Consistency with County plan</td>
<td>0</td>
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### Variables
- Degrees of Belief of Outcomes
- Uncertainty Measures

#### Degrees of Belief of Outcomes
<table>
<thead>
<tr>
<th>Variables</th>
<th>Much Higher</th>
<th>Higher</th>
<th>Same</th>
<th>Lower</th>
<th>Much Lower</th>
<th>I don’t know</th>
<th>Non-specificity</th>
<th>Discord</th>
<th>Total</th>
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<tbody>
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<td>L1: Enhancing walkability</td>
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<td>0.82</td>
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<tr>
<td>L2: Promoting Ped/Bike focused communities</td>
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<td>0.81</td>
<td>0.11</td>
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<tr>
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<tr>
<td>L5: Fleet size required</td>
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<td>0.00</td>
<td>0.04</td>
<td>0.95</td>
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<td>L6: Total vehicle costs</td>
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<td>0.79</td>
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<td>0.02</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.000</td>
<td>0.139</td>
<td>0.139</td>
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<tr>
<td>L9: Noise pollution</td>
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<td>0.97</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>L10: Air pollution</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0.000</td>
<td>0.111</td>
<td>0.111</td>
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<tr>
<td>L11: Visual impacts of community</td>
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<td>0.13</td>
<td>0</td>
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<td>0</td>
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<td>0.559</td>
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<tr>
<td>L12: Range of household income of transit riders</td>
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<td>0</td>
<td>0.99</td>
<td>0</td>
<td>0</td>
<td>0.002</td>
<td>0.046</td>
<td>0.047</td>
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<tr>
<td>L13: User benefit</td>
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<td>0.91</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.000</td>
<td>0.513</td>
<td>0.513</td>
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<tr>
<td>L14: Environmental benefits</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>L15: Capital costs</td>
<td>0.73</td>
<td>0.27</td>
<td>0.00</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.004</td>
<td>0.845</td>
<td>0.849</td>
</tr>
<tr>
<td>L16: Operating costs</td>
<td>0</td>
<td>0.02</td>
<td>0.18</td>
<td>0.80</td>
<td>0</td>
<td>0</td>
<td>0.003</td>
<td>0.830</td>
<td>0.833</td>
</tr>
<tr>
<td>L17: Maintenance costs</td>
<td>0</td>
<td>0.02</td>
<td>0.19</td>
<td>0.61</td>
<td>0.12</td>
<td>0.06</td>
<td>0.112</td>
<td>1.378</td>
<td>1.490</td>
</tr>
<tr>
<td>L18: Long-term benefits of investment</td>
<td>0.49</td>
<td>0.48</td>
<td>0.03</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.000</td>
<td>1.161</td>
<td>1.161</td>
</tr>
<tr>
<td>L19: Long-term public and private investment</td>
<td>0.14</td>
<td>0.84</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.032</td>
<td>0.648</td>
<td>0.680</td>
</tr>
<tr>
<td>L20: Environmentally sustainable communities</td>
<td>0</td>
<td>0.99</td>
<td>0.00</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.010</td>
<td>0.006</td>
<td>0.017</td>
</tr>
<tr>
<td>L21: Adverse environmental impacts</td>
<td>0</td>
<td>0.45</td>
<td>0.51</td>
<td>0.04</td>
<td>0</td>
<td>0</td>
<td>0.000</td>
<td>1.202</td>
<td>1.202</td>
</tr>
</tbody>
</table>

#### Uncertainty Measures
- Non-specificity: 0.000
- Discord: 0.000
- Total: 0.000

#### Consequences with respect to 'Livability and Sustainable Community' (Goal 3)

#### Consequences with respect to 'Integrated Multimodal Transport System' (Goal 4)

#### Consequences with respect to 'Safe Environment' (Goal 5)
6.4.3 Critical Reasons

A sensitivity analysis is performed to identify the critical reasoning chains that most influence the recommendation of ‘Streetcar’ alternative. The study follows the method proposed in Chapter 4 to calculate the importance measure of individual variable $k$ in the reasoning map. This importance measure ($I_k$) is calculated by the difference of the degree of belief of goal achievement between the case when variable $k$ is eliminated from the map and when it is included along with the other variables. The critical chains contain the variables that highly support the degree of belief of achieving the goal.

Figure 6-8 through Figure 6-12 illustrate the critical reasoning chains that influenced the most the achievement of individual goals. With respect to the ‘Mobility’ goal, the Streetcar alternative receives ‘Higher’ achievement than Bus alternative because streetcars ride on smooth steel rails with higher riding quality than buses which may be sensitive to irregularity in the pavement. It gives streetcars higher attraction to residents, commuters, and visitors. The number of transit users would increase; the streetcar would increase the capacity of the corridor and carry more transit passengers.

With respect to the ‘Economic Development’ goal, the Streetcar alternative would achieve ‘Higher’. It is believed that a fixed rail infrastructure is a permanent and long-term transit investment. It would become a corridor landmark and community resource. It would bring more private investment and would encourage more economic activities along the corridor.

With respect to the ‘Livability and Sustainability’ goal, the Streetcar alternative is believed to be a more sustainable solution, economically and environmentally, for this corridor than the BRT alternative. It is believed ‘Highly’ that the streetcar would maximize the long-term benefits of investment due to its lower costs of operation and continued growth in demand.

With respect to ‘Multi-modal transport system’ goal, the Streetcar alternative is preferable—though with conflicting opinions—because it would provide better accessibility and connectivity from local origins to regional transportation centers. With respect to ‘Safe environment’ goal, the Streetcar alternative is believed to possess a more advanced security technology for. People would feel more secured when using streetcars.
Figure 6-8: Critical Path for Achieving “Mobility” Goal

Figure 6-9: Critical Path for Achieving “Economic Development” Goal
Figure 6-10: Critical Path for Achieving “Livability and Sustainability” Goal

Figure 6-11: Critical Path for Achieving “Multimodal Transport System” Goal

Figure 6-12: Critical Path for Achieving “Safe Environment” Goal
6.5 Discussion

This section describes the uniqueness of the proposed belief reasoning method. It answers two main questions for applying the reasoning map method in decision-making process: (i) how individual expert opinions would affect the aggregated expert opinion or the group decision; and (ii) how the configuration of the reasoning map would affect the degree of belief of the overall goal.

6.5.1 Effect of Aggregation of Expert Opinions

In transportation planning, different experts have different opinions. However, traditional methods assume that the evaluation and decisions made by analysts and experts have consistent attitudes. This assumption avoids dealing with conflict, uncertainty and transparent negotiation.

(1) Aggregation of Belief from “Consistent” Opinions

The traditional way to combine the belief (or probability) value from different experts is to apply averaging. Let two experts have high degree of belief. For example, if one says the degree of belief is 0.8 for a “Medium” rating, the other says the degree of belief is stronger, say 0.9, then the aggregated degree of belief is 0.85 for a “Medium” rating. The average operator always takes the compromising solution between the two.

The Dempster’s rule of combination (DRC) in the proposed belief reasoning method however is more intuitive. If two pieces of evidence (expert opinions) support each other, then the experts build up their belief and confidence, and the updated degree of belief value is higher than the original two. Figure 5-20 compares and demonstrates the effect of aggregating ‘consistent’ expert opinions between DRC and average operators. Consider the decision problem with two states: YES or NO (with the possibility for ‘I don’t know’ (IDK)). Each expert has the same belief values, e.g. the belief values of each expert is: \( m(\text{YES}) = 0.4, \) \( m(\text{NO}) = 0.1, \) and \( m(\text{IDK}) = 0.5. \) After combining more pieces of evidence (expert opinions), \( m(\text{YES}) \) increases, \( m(\text{NO}) \)}
decreases, and $m$(IDK) decreases. Although $m$(IDK) is higher than $m$(YES), $m$(IDK) decreases as the number of experts increases because “I don’t know” in this case means either YES or NO. For the same example, if the traditional average operator is used, the update belief values remain the same as the original values.

The DRC allows experts learning from the others and building up or updating their degree of belief when new piece of evidence (or information) arrives.

![Graphs showing aggregation of consistent and conflicting opinions](image)

(a) Dempster’s Rule of Combination  
(b) Average Operator

Figure 6-13: Aggregation of “Consistent” Opinions by Different Operators

(2) Aggregation of Belief from “Conflicting” Opinions

Consider two experts have conflicting opinions; for example, one states high and the other states low degree of belief for “Medium” rating. Both traditional average and proposed DRC methods result in the compromising degree of belief between the two experts. The aggregate opinion seems to be realistic for both methods. However, the DRC would reduce the degree of belief of “I don’t know” while the average operator would keep “I don’t know” in the aggregated opinion. DRC seems to be more intuitive because when new piece of evidence arrives, the experts usually recognize whether it supports or opposes their opinions instead of keeping the ambiguity.

Figure 6-14 compares and demonstrates the effect of aggregating ‘conflicting’ expert opinions between DRC and average operators. Consider the same decision problem in previous section. Assume two groups of experts have different belief values, Group 1: $m_1$(YES) = 0.4, $m_1$(NO) = 0.1, and $m_1$(IDK) = 0.5; and Group 2: $m_2$(YES) = 0.1,
$m_2(\text{NO}) = 0.4$, and $m_2(\text{IDK}) = 0.5$. This figure presents the aggregated degree of belief obtained from DRC and average operators when adding one expert opinion from Group 1 and Group 2 alternatively.

The results show that for DRC both $m(\text{YES})$ and $m(\text{NO})$ converges 0.5, and $m(\text{IDK})$ decreases and converges 0. For average operator, $m(\text{YES})$ and $m(\text{NO})$ converges 0.25, the average between 0.4 and 0.1, and $m(\text{IDK})$ remains the same since the original values from both experts are 0.5. DRC provides a realistic result after negotiations. When combining two totally conflicting opinions, the group decision believes in both outcomes the same and they are 0.5 for both; as a result, even a nuanced recommendation for decision cannot be made on this evidence.

![Graph](image)

(a) Dempster’s Rule of Combination  
(b) Average Operator

Figure 6-14: Aggregation of “Conflicting” Opinions by Different Operators

The following sections present two examples of aggregation of expert opinions obtained from the case study. One is the example for aggregating consistent opinions, and the other is the example for aggregating conflicting opinions.

(3) Application Example for Combining Consistent Opinions

Table 6-4 presents the case when all ten experts have consistent opinions. They support that the vehicle capacity of streetcar is “Higher” than that of BRT but at different degree of belief; some assign a high degree of belief for “Higher vehicle capacity,” but some state non-specific opinion “Much Higher or Higher.”
After aggregating opinions by the Dempster’s rule of combination (DRC), the degree of belief associated with “I don’t know” notion is reduced (or eliminated), and the degree of belief associated with “Higher vehicle capacity” is increased (or strengthened), \( m(Higher) = 1.0 \).

This characteristic of the proposed method is unique. It allows individuals to admit their uncertainty, but at the end the group decision becomes specific without “I don’t know”. This situation replicates the nature of negotiation. Since each expert at least believes in “Higher vehicle capacity,” as a result the group decision believes in it. On the other hand, if the average operator is used, each expert would treat equally and the “I don’t know” remains; in other words, the ambiguity would still exist for recommending a decision.

<table>
<thead>
<tr>
<th>State of “Vehicle Capacity”</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Average</th>
<th>DRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Much Higher (A)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0.05</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Higher (B)</td>
<td>0.8</td>
<td>0.9</td>
<td>0.8</td>
<td>0.8</td>
<td>1</td>
<td>0.9</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0.57</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>About the Same (C)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Lower (D)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Much Lower (E)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>I don’t know (A or B)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.9</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.19</td>
<td>0</td>
</tr>
<tr>
<td>I don’t know (A or B or C)</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>I don’t know (A or B or C)</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>1</td>
<td>0.15</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Non-Specificity</th>
<th>Discord</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.32</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>0.32</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>0.46</td>
<td>0.27</td>
<td>0.15</td>
</tr>
<tr>
<td>0.23</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>Total Uncertainty</td>
<td>0.48</td>
<td>0.35</td>
</tr>
<tr>
<td>0.48</td>
<td>0.35</td>
<td>0.73</td>
</tr>
<tr>
<td>0.35</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>0.23</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>0.15</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2.23</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.23</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.23</td>
<td>-</td>
<td>2.23</td>
</tr>
</tbody>
</table>

It is also important to note that the representation of expert’s opinion indicates his uncertainty. Consider the degrees of belief from Experts 8 and 9. Both of them hold uncertain opinions. Expert 8 assigns the equal degree of belief (50/50) to “Much Higher” and “Higher.” Expert 9 is also not sure about “Much Higher” (A) or “Higher” (B), but he assigns the degree of belief of 1.0 to “I don’t know” (A or B) notion instead. The total amount of uncertainty of these two experts is the same, but we define the type of uncertainty differently. Expert 8 provides conflicting opinion, while Expert 9 provides non-specific evidence. This discrepancy would not be explained by the traditional probability theory.
(4) Application Example for Combining Conflicting Opinions

Table 6-5 presents the case when experts have conflicting opinions. Five experts (Experts 1 to 5) believe that the fare collection system of the streetcar is “About the same” as that of the BRT but with different degrees. Four experts (Experts 6 to 9) believe in “Better” fare collection system. Expert 10 states “I don’t know.”

After aggregating opinions, DRC vanishes the degree of belief associated with “I don’t know” notion, $m(\text{I don’t know}) = 0$ as shown in the last column. The five experts classified as Group 1 would build their belief on “About the same” notion; the four experts classified as Group 2 would build their belief on “Better” notion; and the last expert would be dictated by these two groups. The group decision would still have conflicting opinions between “About the Same” and “Better” notions, but stronger belief in “About the Same” due to a higher degree of belief from experts in the first group.

Table 6-5: Example of Aggregating Conflicting Expert Opinions

<table>
<thead>
<tr>
<th>State of “Vehicle Capacity”</th>
<th>Expert No.</th>
<th>Aggregated Opinions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>Average</td>
</tr>
<tr>
<td>Much Better (A)</td>
<td>0 0 0 0 0 0 0 0 0 0 0</td>
<td>0 0</td>
</tr>
<tr>
<td>Better (B)</td>
<td>0 0 0 0 0.7 0.9 1 0.8 0 0 0.34</td>
<td>0.17</td>
</tr>
<tr>
<td>About the Same (C)</td>
<td>0.8 0.9 0.8 1 0.7 0 0 0 0 0 0.42</td>
<td>0.83</td>
</tr>
<tr>
<td>Lower (D)</td>
<td>0 0 0 0 0 0 0 0 0 0 0</td>
<td>0 0</td>
</tr>
<tr>
<td>Much Lower (E)</td>
<td>0 0 0 0 0 0 0 0 0 0 0</td>
<td>0 0</td>
</tr>
<tr>
<td>I don’t know (B or C)</td>
<td>0 0 0 0 0 0 0 0 0 0 1</td>
<td>0.10 0</td>
</tr>
<tr>
<td>I don’t know (A or B or C or D or E)</td>
<td>0.2 0.1 0.2 0 0.3 0.3 0.1 0 0.2 0 0.14 0</td>
<td></td>
</tr>
<tr>
<td>Non-Specificity</td>
<td>0.46 0.23 0.46 0 0.70 0.70 0.23 0 0.46 1.00 - 0</td>
<td></td>
</tr>
<tr>
<td>Discord</td>
<td>0.27 0.12 0.27 0 0.43 0.43 0.12 0 0.27 0 - 0.65</td>
<td></td>
</tr>
<tr>
<td>Total Uncertainty</td>
<td>0.73 0.36 0.73 0 1.13 1.13 0.35 0 0.73 1.00 - 0.65</td>
<td></td>
</tr>
</tbody>
</table>

An important issue is worth pointing out here. The average for belief “About the same” (C) for Group 1, Experts 1-5, is 0.84, while the average for belief “Better” (B) for Group 2, Experts 6-9, is 0.85. And, one Expert’s belief for C or B is “I don’t know”. However, when aggregating all the information the consensus favors belief “About the same”.

Thus, in summary, when more experts or stakeholders are involved, or more pieces of evidence are available, ambiguity about the decision situation decreases and the
group’s decision becomes more specific. This can also be seen from the decrease in “I don’t know” value. It implies that “I don’t know” of an individual expert or stakeholder becomes “We know” as a group decision.

6.5.2 Effect of Reasoning Map Configuration

How complex the reasoning map ought to be is the challenging question. Too small reasoning map may be easier for calculation purpose, but may contain high uncertainty when eliciting expert opinions. Too large reasoning map may require substantial computational effort, but may replicate better experts’ thinking process. Generally the reasoning map is based on experts or decision-makers because the maps would describe the variability in dispositional goal preferences. However, to maximize the benefits of the proposed method, the following criteria should be taken into consideration when constructing a reasoning map.

- All important variables should be included in a reasoning map particularly the decision variables that distinguish the characteristics of different transit alternatives;
- The reasoning map should be direct and easy to visualize and explain;
- The uncertainty associated with each link should be as small as possible for individual experts. In other words, an expert should be as certain as possible about each relation, otherwise more variables should be added;
- Citizens or citizen groups should also be involved in drawing up the reasoning map and expressing their views and beliefs about the alternatives;
- The higher the number of participating experts and citizens, the higher the specificity of the goal achievement. In other words, participation decreases ambiguity and the degree of “I don’t know” and, at the same time, increases the strength of collective opinion about the alternatives.

Careful attention must be made when conducting the reasoning chains. Different configurations of reasoning chains influence the degree of achievement of goal. The
following sections describe the differences between the sequential and parallel reasoning chains.

(1) Sequential Reasoning Chains: Short vs. Long Series

A sequential reasoning chain has a unique characteristic. When the variables are connected as a sequence as shown in Figure 6-15, the degree of belief of the conclusion has high a tendency to decrease due to the fact that the degree of belief of the subsequent node would be less than (or equal to) that of the preceding node. The degree of belief of the variables diminishes along the sequential chains. The degrees of belief are propagated by the product operator.

\[
m(Z) = m(Z | Y) \cdot m(Y | X) \cdot m(X | W) \cdot m(W)
\]

(6-1)

When comparing between short (Map 1) and long (Map 2) sequential reasoning chains as shown in Figure 6-15, the longer chains of reasons (or the more intermediate nodes) are involved, the lower degree of belief of the conclusion is.

To maximize the confidence of each expert if the degree of belief of link \( W \rightarrow Z \), \( m_{W \rightarrow Z} \), assigned by an expert is already certain, then it is not necessary for an expert to add any new intermediate node between variables \( W \) and \( Z \). On the other hand, if the degree of belief of link \( W \rightarrow Z \) has low certainty, then an expert may find the other reasons (links) to support the end (goal) node. In this case, he or she may connect links \( W \rightarrow X \), \( X \rightarrow Y \), and \( Y \rightarrow Z \).

\[
\begin{align*}
\text{Map 1} & \\
W & \rightarrow & Z \\
\text{Map 2} & \\
W & \rightarrow & X & \rightarrow & Y & \rightarrow & Z
\end{align*}
\]

**Figure 6-15: Comparison of Short and Long Series of Reasoning Chains**
(2) Parallel Reasoning Chains: One-to-One vs. Many-to-One

A parallel reasoning chain has different characteristic from the sequential in the proposed method. When the variables are connected as a parallel as shown in Map 2 of Figure 6-16, the degree of belief of the conclusion would be either increase or decrease depending on how the added parallel causes support or oppose the degree of belief of the subsequent node. The degrees of belief are aggregated by the DRC operator.

\[ m(Z) = m_{W\rightarrow Z}(Z) \oplus m_{X\rightarrow Z}(Z) \oplus m_{Y\rightarrow Z}(Z) \]  

(6-2)

When comparing between one-to-many (Map 1) and many-to-one (Map 2) reasoning chains as shown in Figure 6-16, the many-to-one chains of reasons are usually increase the degree of belief of the subsequent node unless there is a highly conflicting evidence (cause) connecting to the subsequent node.

To maximize the confidence of each expert if the degree of belief of link \( W \rightarrow Z \), \( m_{W\rightarrow Z} \), assigned by an expert is already certain, then an expert may not seek for additional evidence connecting to \( Z \). On the other hand, if the degree of belief of link \( W \rightarrow Z \) has low degree of certainty, then he or she may seek for other reasons (pieces of evidence) to support the subsequent node. In this case, he or she may connect links \( X \rightarrow Z \), and \( Y \rightarrow Z \).

![Figure 6-16: Aggregation of “Conflicting” Opinions by Different Operators](image)

6.5.3 Limitations of the Belief Reasoning Method

The belief reasoning method is sensitive to the reasoning map structure. Similar to traditional evaluation approaches, the proposed approach is flexible for experts to
deliberately manipulate the goal achievement. However, the reasoning map method is more transparent way for experts (either a planner group or citizen group) to present their own reasons. The proposed method would allow the experts to build their own reasoning chains, but at the same time, it is easy for the opponents to debate by adding conflicting opinions or opposing chains.

The proposed Belief Reasoning method is susceptible to ‘group think’. When many experts that have the same attitude are involved, then the group decision becomes very strong. Therefore, it is highly desirable to have several groups of stakeholders in planning work to construct reasoning maps and to assign beliefs on those maps.

The benefit of the proposed method is its approach to deal with uncertainty and ambiguity of the experts and citizens alike. Like in every work it is necessary for experts, citizens and decision-makers to express their knowledge openly and acknowledge their uncertainties as honestly and genuinely as possible. The final point to note is that the proposed Belief Reasoning method provides the same results as the traditional methods if no uncertainty is involved.

6.6 Summary

This chapter presented the application of the proposed Belief Reasoning method for assisting transportation planners to evaluate transit alternatives, to reason about the recommended transit alternative, and to measure the validity of reasoning in evaluation. The Alternatives Analysis of the real-world transit investment project in the U.S. was selected as a case study to validate the proposed Belief Reasoning method.

The decision model is created under the reasoning map structure and the evaluation is developed under the Dempster-Shafer theory of evidence. The proposed approach is designed to collect inputs from multiple experts and citizens and allows incorporating “I don’t know” opinion from different parties. The reasoning map is drawn and modified by a group of experts first. The degrees of belief, which represent the confidence of the opinions associated with each causality, are assigned next. The mechanism for propagating the degrees of belief along the chains is applied, and the
degree of belief for achieving the goals is obtained as a result. Finally, a measure of uncertainty associated with each variable and goal is calculated to assess the quality of information and the reasons for selecting the preferred alternative.

In this case study, the reasoning map is quite large. It was developed based on detailed analysis. The following two observations are central and important in the context of this application. Detailed information about characteristics of alternatives was available and considered, and professionally worked through by the experts before their interviews and in drawing up the reasoning map. Significant consequences, which contribute to the goals of the project, were discussed and anticipated. They are reflected in the reasoning map and show the underlying thinking of the experts. It is likely that in a real-world application a smaller reasoning map would evolve over time and would not only be justified for planning purposes, but also be clarified to the decision situation.

The credibility of the proposed Belief reasoning method does, however, depend on one critical hypothesis. The experts—and also participating citizens—should be genuine and honest about their judgments and openly admit their level of uncertainty, their degree of “I don’t know”. During the planning process the reasoning map should be reviewed by experts and stakeholders for reasonableness, comprehensiveness, clarity and economy (parsimony) in carrying out the final evaluation. This evolutionary aspect of the reasoning map and its application in evaluation and comparing alternatives was not possible during this dissertation work.

The proposed Belief Reasoning method is a very useful and illuminating method for transportation planning purposes in which stakeholders can review the experts’ and decision-makers’ reasoning, judgments and evaluation as a process. The proposed method can promote focused discourse among the transportation planners and citizens because it reveals the degree of belief and uncertainty, and how much trust can be placed for a proposed transit alternative to achieve the goals set for it. The method also identifies the critical logical links in the reasoning process.
CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

This chapter presents the conclusions of the proposed Belief Reasoning development and its verification and validation for planning of public transportation systems. This chapter also discusses the future study and the potentials of the proposed methodology for other applications in the context of planning and decision-making.

7.1 Conclusions

A major effort in this dissertation was expended in understanding the process of evaluating and supporting decision-making in transportation planning. An extensive review of the literature of the existing transportation evaluation and decision-making process was carried out. This led to the recognition of three fundamental steps for planning of transportation systems: (i) understanding the cause-and-effect relationships among system variables; (ii) modeling the decision-making structure; and (iii) evaluating the goal achievement of the alternatives.

Depending on the pattern of knowledge, the nature of uncertainty associated with this knowledge, and the structure of the decision model, there are several different approaches to conduct the evaluation of transportation investment projects. It is common to the existing approaches for planning of urban public transportation systems that they lack a clear reasoning framework in the evaluation process and they disregard the uncertainty and ambiguity in expert judgments.

In transportation planning, expert knowledge is haphazard and incomplete. The primary hypothesis in this study is that if the amount of uncertainty associated with information can be measured and the critical reasons for supporting or opposing a
transportation alternative can be determined, then the decision-making process could be improved and become more robust.

The main focus of the dissertation is to develop a new methodology that recognizes the reasoning process about preferences among transportation alternatives and the uncertainties in experts’ minds during reasoning, and then using this incomplete information for evaluating transport alternatives in a systematic and logical manner. The contents and key findings of this dissertation can be divided into four extended areas including the development, verification, validation, and advantages and disadvantages of the proposed methodology.

7.1.1 Development of the Belief Reasoning Methodology

A new decision-making approach in the planning process for evaluating public transportation systems is developed. It is called the “Belief Reasoning” or the “Dempster-Shafer Belief Reasoning.” The proposed methodology applies two theoretical concepts. First, the reasoning map is developed to describe decision structure to model the transportation decision-making systems. Second, the Dempster-Shafer theory of evidence is applied as a mathematical mechanism to analyze the degree of goal achievement of transportation alternatives under the reasoning map structure.

The reasoning map, which is an Artificial Intelligence application in conflict resolutions, is used to structure a goal-oriented transportation decision-making system by connecting the decision variables or characteristics of transportation alternatives (the premises) to the goals of the transit investment project (conclusions) through a series of chains of performance, criteria, and objectives. This reasoning map structure can describe a large-scale transportation system and define the causal relations among variables of the system. It can incorporate both independent and interdependent relations through different rules and operators.

The Dempster-Shafer theory of evidence, which is a generalized mathematical framework for representing knowledge and handling uncertainties, is used to represent expert opinions about the premises and the relations among relevant performances and impacts using a reasoning map. This theory is an extension of Bayesian probability
theory. The knowledge elicited from experts is defined as the “degree of belief” (or truth value), \( m \)-values, between 0 and 1, attached to the premises and relations instead of a (conditional) probability measure. This theoretical mechanism can incorporate incomplete and ambiguous knowledge and the notion of “I don’t know (IDK)” in reasoning. The Dempster-Shafer theory can also measure the quality of available knowledge and determine uncertainty in variables with two measures of uncertainty, non-specificity and discord.

The proposed approach consists of three important phases: (i) construction of reasoning map from the premises to the goals; (ii) elicitation and assignment of expert knowledge on the reasoning map; and (iii) evaluation of the alternatives achievement of the goals. Construction of the reasoning map involves defining the goals of the project and the decision variables of transportation alternatives and relating the decision variables to the goals of the planning problem. The reasoning map may be designed by experts, from planned documents or by the affected citizens. The reasoning map structure assists in presenting how the stakeholders, including the planning experts, perceive, interpret, and reason with their preferences to achieve the goals through the decision-making process involving many actors.

Elicitation of expert knowledge consists of collecting opinions from multiple experts (and stakeholders) about their beliefs in the premises and relations in the reasoning map. The method requires that the experts express their opinions in terms of ‘degree of belief’ in the premises and relations. Different levels of knowledge can be incorporated as evidence, e.g. complete, fragmented, non-specific (or “I don’t know”). The belief values assigned by the experts (and stakeholders) are aggregated using the Dempster’s rule of combination (DRC) operator. Unlike the conventional average operator, DRC has a unique characteristic to combine conflicting and “I don’t know” type opinions from multiple sources. It naturally builds up the degrees of belief if the opinions support each other, and, as a result this mutual support, causes the disappearance of the “I don’t know”, the lack of knowledge.

Evaluation of the transportation alternatives consists of propagating the degrees of beliefs from the premises to the goals via a series of causal relations. At the end the proposed method provides (i) the degrees of belief that the transportation alternatives
achieve the (individual) goals; (ii) the reasons for supporting the preferred alternative; and (iii) the measures of validity of reasoning and uncertainty associated with information used in evaluation.

7.1.2 Verification of the Belief Reasoning Methodology

The capability and mechanisms of the proposed methodology is compared in the study with other decision-making approaches: (i) the Analytical Hierarchy Process (AHP) method, one of the weighting methods commonly applied in transportation decision-making process, and (ii) the Bayesian reasoning method, the most well-known reasoning process.

A close examination of the weighting methods is conducted. It shows that the traditional weighting methods, such as the rating methods and the AHP, use a tree structure to decompose the system to several hierarchical subsystems and use additive weights ($\sum w=1$) to aggregate the performance values or preference values of the subsystems. These mechanisms assume that criteria or performances representing the subsystems are independent and individually share the contribution to the overall system.

The weighting methods are compared with the proposed reasoning method through a numerical example for selecting the most desirable transit mode. The comparison shows that the traditional weighting methods: (i) overlook the issue of redundancy and interdependency among criteria or performance, and ignore the interactions among system variables; (ii) recognize only deterministic performance (or priority) values without incorporating the expert’s attitude about uncertainty; and (iii) usually require the analysts to perform sensitivity analysis with the change of weight values and examine the variation of the overall performance of transit alternatives to see whether the results are stable; and (iv) provide insufficient reasoning and evidence for recommending a transit alternative.

The reasoning map methods to describe the decision-making structure give more flexibility to the analysts to model the decision-making situation and to define the interrelationships among system variables. The reasoning map allows the planners and others to give a realistic picture of their thinking and decision-making processes. It
addresses the issues of “double-counting” and interdependence among performance characteristics. The method encourages the planners to present non-deterministic characteristic of performance values and their uncertainty and handle them in a systematic manner.

From among the several reasoning methods available in the literature, the study examines the characteristics of Bayesian reasoning and compares it with the proposed Belief Reasoning method. Bayesian reasoning represents knowledge with probability measures. It assumes that the data and knowledge about premises and causal relations in the reasoning map are complete. Bayesian reasoning has been widely applied in estimation and prediction problems where data are available, but not in decision-making situations.

The Bayesian reasoning method is compared with the proposed Belief Reasoning method using a numerical example for evaluating the viability of a PRT system. The comparison shows that the Bayesian reasoning: (i) requires complete knowledge about premises and relations; (ii) overlooks incomplete knowledge or lack of information; (iii) cannot easily model reasoning in a large-scale decision-making problem; and (iv) provides no measure for the validity of the reasoning process.

The proposed Belief Reasoning method, which uses the Dempster-Shafer theory of evidence, has a capability to address incomplete knowledge. Incomplete knowledge, ignorance, is generally present in expert judgments in transportation planning. Both experts and citizens have limited knowledge. The Belief Reasoning method effectively handles “I don’t know” type opinion. Ignorance can be represented with different levels and degrees of ignorance. Due to its flexibility to deal with partial knowledge, the proposed method has no scalability issue to model a large-scale system. The Belief Reasoning method gives the same result as the Bayesian reasoning method if complete knowledge exists.

In addition, the Belief reasoning method, which uses the Dempster’s rule of combination (DRC) to aggregate experts’ opinions about inputs and causal relations in a reasoning map, can also effectively handle conflicting opinions. The proposed Belief Reasoning can measure uncertainty associated with information in the reasoning chains and identify information needed for improving and clarifying the reasoning process.
7.1.3 Validation of the Belief Reasoning Methodology

The application of the proposed Belief Reasoning method is tested in an Alternatives Analysis for a real-world transit investment project. In this case study, the Belief Reasoning method is used for evaluating transit alternatives, examining the reasoning of the planners for recommending a transit alternative, and assessing the validity of this reasoning.

The reasoning maps were developed based on detailed analyses and reviewed by a group of planners. These planners representing different stakeholders were interviewed to elicit their knowledge and opinions. It was found that 70-80% of these experts’ “knowledge” is actually ignorance; individual experts (planners) are not sure about the ‘truth value’ of the premises and the causal relationships toward the goals. A substantial degree of “I don’t know” belief exists. The results of the application show that although experts individually acknowledge uncertainty in the premises and their relationships, the degrees of belief and consistency in these beliefs, for achieving most of the project’s goals are very high, except for one goal (Integrated multimodal transport system) that is surrounded by significant conflicts. It appears that all the participating experts have about the same attitudes and preferences in their minds. Further, the critical reasoning chains (the reasons most supportive individual goals) are consistent with experts’ minds according to the method.

Two interesting points are identified from this application. First the study shows how a group’s degree of belief is updated with a new piece of evidence arrives or arrival of a new expert. The DRC mechanism replicates updating of group belief and decision more naturally than the average operator. When the new opinion or evidence supports the earlier one, the combined degree of belief is higher than its earlier value. When the new opinion is conflicting with the earlier, then the combined degree of belief is a compromise between the two and likely to support the stronger and more specific belief (or less unspecific belief) than the weaker or less specific belief or opinion.

Second, the study shows the effect of the reasoning map configuration. The proposed Belief Reasoning method treats each link connecting to the conclusion as a piece of evidence. When a single reason affects a conclusion (one-to-one reasoning
chain), the belief in that conclusion is lower. However, when more pieces (links) of evidence reasons exist, i.e. several reasons affect a conclusion, the belief in the conclusion could be stronger or weaker based on the strength of the additional reasons. This is not the case in Bayesian reasoning where the belief in conclusion is always lower. The Bayesian Reasoning will provide indecisive results.

### 7.1.4 Advantages and Disadvantages of the Belief Reasoning Methodology

The main advantages of the Belief Reasoning method are (i) the potential to model the planners’ (and the stakeholders’) reasoning process and evaluate (transportation) alternatives in the planning of large-scale (transportation) systems; (ii) flexibility to handle different patterns of knowledge and opinions including incomplete, approximate, ignorant, and conflicting, elicited from multiple actors; and (iii) capability to measure different types of uncertainty associated with knowledge in order for the experts and citizens to focus debates and improve analyses. The proposed method is useful and illuminating at a planning level of transportation systems and particularly in a decision-making processes where (i) multiple experts or actors are involved; (ii) knowledge of individual experts is fragmented; and (iii) opinions among them are conflicting.

The possible drawbacks of the Belief Reasoning method are two-fold. First, the evaluation results can be deliberately manipulated by the analysts/planners, although that manipulation would be more transparent than in the traditional evaluation approaches. This is because the way the analysts reasoning is illustrated and the knowledge they express is presented in a reasoning map. Thus, the results depend on how the analysts customize their reasoning maps, and how truthful they are about their knowledge.

Second, the mechanism to calculate the degree of belief for achieving goals is susceptible to ‘group think’. The underlying concept of the proposed mechanism leads to believing the stronger opinions and suspecting the weaker opinions. The stronger opinions dominate the decision. However, this seems to be true in any decision-making process. Therefore, for the method to be constructive, it is important that “strong beliefs” are frank and close to the “truth.”
Therefore, it is desirable in applying the proposed method that several groups of stakeholders, possibly representing different views and values, are involved in planning work. The success of the proposed Belief Reasoning method depends on the agreement on the reasoning maps and the integrity of knowledge used on assigning belief values those maps. During the planning process, the reasoning map should be reviewed by several groups of stakeholders for reasonableness, comprehensiveness, and clarity. The experts and participating citizens should speak out honestly and genuinely about their judgments and openly admit their level of uncertainty in their opinions.

7.2 Implications of the Research

This research provides a wide range of opportunities for researchers and practitioners in transportation and urban planning.

Implication of Theory. This research elaborates the concepts and characteristics of Dempster-Shafer theory of evidence. It helps researchers to understand and apply how to represent, analyze, and combine data, information, knowledge, and opinions which are incomplete, uncertain, partial or conflicting from different sources. The research is one of the first studies that fully applies this new theory to deal with expert knowledge under uncertainty and ambiguity in transportation applications. The research highlights the significance of “I don’t know” type of opinions. Such knowledge, although possess uncertainty, is always present in human’s mind and used in analysis.

Implication of Transportation Planning and Decision-Making. This research proposes a new technique in transportation and urban planning process. It puts forward the logical reasoning structure for planning and decision-making of urban transportation systems, and underline the treatment of “I don’t know” opinions from experts and citizens. The values added of the proposed method to the transportation planning are the presentation of reasoning in planning process and the recognition and preservation of analysts’ and data uncertainty.
7.3 Recommendations for Further Studies

In traditional transportation planning, success is usually measured by tangible results, but results are achieved after implementing good decisions. Thus, success in transportation planning depends upon quality of the decision-making process. Under uncertain environment it is not easy to measure the level of success ex-ante based on analytical or statistical methods. The logical justifications should be brought into play. This study proposes a logical reasoning method for transportation systems planning and decision-making.

7.3.1 Recommendations

This method is new in this area and the dissertation suggests three issues that warrant further research.

Reasonableness of Decision Structure and Criteria. A structured decision problem attempts to define how decision-makers’ think and reach their preference for an alternative over another. A “Bottom-Up” approach and public participation process are highly recommended to construct the decision structure or reasoning maps in this research. Several groups of stakeholders should be involved in the planning process and review the decision structure and criteria.

Integrity of Knowledge and Data. Knowledge of experts and planners, and the data used in transportation models are crucial elements in evaluating transportation alternatives. It is very important that knowledge and data should be used to its maximum extent and as honestly as is humanly possible. Otherwise the intent of analyses and resulting recommendations may be biased or distorted. Methods to elicit and present expert opinions should be transparent and existence of ‘black boxes’ in evaluation should be minimized. Revealed preference surveys and open discussions among the planning participants are recommended—but subjected to analyses of the kind developed in this dissertation.
Practical Use of the Proposed Methodology. While the proposed method is promising in many ways, it requires more research and practical applications. Compared to a weighting scheme or probability theory, the Dempster-Shafer theory of evidence is a relatively new concept. A substantial amount of training in theory, applications and experiences are necessary.

7.3.2 Challenges and Future Studies

In order to achieve the maximum strengths of this proposed methodology in transportation planning, many further studies is challenging to be explored.

Development of the Decision Support System. One of the keys to success of the proposed methodology is the development of a decision support system (DSS) for transport analysts and experts to understand the evaluation process of this methodology. The design and development of the decision support system utilizing this approach will be further presented elsewhere.

Development of the Expert System. The next challenge of the implementation of the proposed method is the development of an expert system (ES) for transport policy-makers to make the decision and recommend transport alternatives; for example an expert system for recommending the FTA New Starts transit projects. The key components of an expert system may include the development of a unified reasoning map and generalized knowledge bases.

Development of the Dynamic Belief Reasoning for Long-Term Planning. In this dissertation, the proposed method is applied to a one-time decision-making process. However, transportation decision-making process is a multi-year planning process. The development of the Dynamic Belief Reasoning to evaluate alternatives over time is challenging.
REFERENCES


The steps of analyzing the priority value in Analytical Hierarchy Process (AHP) method and the calculation results according to the example shown in Chapter 5 are presented in this Appendix.

A.1 AHP Calculation

The principal idea of AHP is to determine the priority among attributes through an eigenvector from the matrix of pair-wise numbers which represent the expert judgment. This matrix, denoted as $A$, consists of $a_{ij}$ the number indicating the strength of the attribute $C_i$ when compared with the attribute $C_j$.

$$A = (a_{ij}) \text{ for } i, j = 1, 2, \ldots, n$$

This matrix is a reciprocal and diagonal n-by-n matrix where $a_{ij} = 1/a_{ij}$ ($a_{ij} > 0$). The matrix $A$ is formed as:

$$A = (a_{ij}) = \begin{bmatrix}
1 & a_{12} & \ldots & a_{1n} \\
1/a_{12} & 1 & \ldots & a_{2n} \\
: & : & \vdots & : \\
1/a_{tn} & 1/a_{2n} & \ldots & 1
\end{bmatrix}$$

Consider five experts are involved in assigning the weights of four criteria ($C_1, C_2, C_3, C_4$). Each expert conducts the pair-wise comparison between two criteria, and the pair-wise comparison matrices ($A_1, A_2, A_3, A_4, A_5$) are given below.
### Step 1: Determine the comparison matrix $A$

The number of each cell $(a_{ij})$ indicating the relative preference (or importance) of criterion $C_i$ with respect to $C_j$ from individual pair-wise comparison questionnaire, is averaged to determine the aggregated matrix $A$. The geometric mean is introduced because of the reciprocal scale used in AHP. The individual pair-wise comparison matrix is shown below.

\[
a_{i,j} = \sqrt[n]{(a_{i,j})_1 \times (a_{i,j})_2 \times \ldots \times (a_{i,j})_n} \quad \text{where} \ n \ \text{is the number of experts.}
\]

\[
a_{j,i} = \frac{1}{a_{i,j}}
\]
Then, we obtain the aggregated pair-wise comparison matrix \((A)\).

\[
\begin{array}{cccccc}
  & C_1 & C_2 & C_3 & C_4 & \text{Total} \\
 C_1 & 1 & 3.728 & 1.888 & 3.898 & 10.514 \\
 C_2 & 0.268 & 1 & 0.341 & 1.431 & 3.040 \\
 C_3 & 0.530 & 2.930 & 1 & 2.702 & 7.162 \\
 C_4 & 0.256 & 0.699 & 0.370 & 1 & 2.325 \\
 \text{Total} & 2.054 & 0.357 & 3.599 & 9.031 & 23.041 \\
\end{array}
\]

**Step 2: Determine and normalize the weights \(w_j\)**

The weight of criteria \((w_j)\) is calculated by raising the value of matrix \(A\) to powers \(k\), and then normalizing it. The process is iterative until the weights of criteria are stable.

\[
(w_j)_k = \lim_{k \to \infty} \frac{A^k e}{e^T A^k e}
\]

where \(e = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{n \times 1}\)

\[
(w_j)_k = \lim_{k \to \infty} \frac{1}{\sum_j (a_{ij})} \sum_j (a_{ij})
\]

\[
\begin{array}{cccccc}
  & C_1 & C_2 & C_3 & C_4 & \text{Total} & (w_j)_1 \\
 C_1 & 1 & 3.728 & 1.888 & 3.898 & 10.51 & 0.4563 \\
 C_2 & 0.268 & 1 & 0.341 & 1.431 & 3.04 & 0.1320 \\
 C_3 & 0.530 & 2.930 & 1 & 2.702 & 7.16 & 0.3108 \\
 C_4 & 0.256 & 0.699 & 0.370 & 1 & 2.33 & 0.1009 \\
 \text{Total} & 23.04 & 1.0000
\end{array}
\]

\[
\begin{array}{cccccc}
  & C_1 & C_2 & C_3 & C_4 & \text{Total} & (w_j)_2 \\
 C_1 & 1 & 3.728 & 1.888 & 3.898 & 10.51 & 0.4563 \\
 C_2 & 0.268 & 1 & 0.341 & 1.431 & 3.04 & 0.1320 \\
 C_3 & 0.530 & 2.930 & 1 & 2.702 & 7.16 & 0.3108 \\
 C_4 & 0.256 & 0.699 & 0.370 & 1 & 2.33 & 0.1009 \\
 \text{Total} & 23.04 & 1.0000
\end{array}
\]
The weights of criteria \( w_j \) are finally converged. They are 

The priorities of 
\[ C_1 = 0.4733 \]
\[ C_2 = 0.1248 \]
\[ C_3 = 0.2968 \]
\[ C_4 = 0.1050 \]

The following steps, Steps 3 and 4 are used to verify whether the expert judgment is consistent.

*Step 3: Determine the eigenvalue of comparison matrix, \( \lambda_{\text{max}} \)*

Gass (1985) proposed an approximate way to estimate the maximum eigenvalue of matrix (\( \lambda_{\text{max}} \)). Let \( w_i \) be the corresponding AHP priorities. The process starts from
summing each column of the matrix and then multiplying each sum by the corresponding \( w_i \). Finally, \( \lambda_{\text{max}} \) is calculated by summing these \( n \) products as shown below.

\[
\lambda_{\text{max}} = w_1 \sum_{i=1}^{n} a_{1i} + w_2 \sum_{i=1}^{n} a_{i2} + \ldots + w_n \sum_{i=1}^{n} a_{in}
\]

\[
= (0.4733)(2.054) + (0.1248)(8.357) + (0.2968)(3.599) + (0.1050)(9.031)
\]

\[
= 4.0326
\]

If the expert judgment is consistent, then \( \lambda_{\text{max}} = n \).

**Step 4: Determine the consistency of comparison matrix**

The consistency of the expert judgment in \( n \)-by-\( n \) matrix is measured by the deviation of \( \lambda_{\text{max}} \) from \( n \). This consistency measure (\( CI \)) evaluates the closeness of the derived scale from a ratio scale. It is calculated by.

\[
CI = \frac{\lambda_{\text{max}} - n}{n - 1}
\]

Golden et al. (1989) outlined the average of consistency indices called random inconsistency index (\( RI \)) for each size of matrix \( n \) where the \( a_{ij} \) were drawn randomly from the values \( 1/9, 1/8, \ldots, 1, 2, \ldots, 9 \). Table A.1 shows the random inconsistency index for each matrix size.

**Table A.1: Random Inconsistency Index (RI)**

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( RI )</td>
<td>0.00</td>
<td>0.00</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
<td>1.51</td>
<td>1.54</td>
<td>1.56</td>
<td>1.57</td>
<td>1.59</td>
</tr>
</tbody>
</table>

*Source: Golden et al. (1989)*

For this example, the consistency index is:

\[
CI = \frac{\lambda_{\text{max}} - n}{n - 1}
\]
\[
= \frac{4.0326 - 4}{4 - 1} = 0.0107
\]

The consistency ratio (\(CR\)) is defined as the ratio of the \(CI\) to \(RI\). Gass (1985) suggested that the consistency ratio less than or equal to 0.10 (or 10\%) is acceptable, otherwise, the judgment must be revised.

For this example, the value of random inconsistency (\(RI\)) at \(n\) equals to 4 is 0.90.

\[
CR = \frac{CI}{RI}
\]

\[
= \frac{0.0107}{0.90} = 0.0119
\]

This consistency ratio \(CR < 0.10\), thus, the expert judgment is consistent and acceptable.

\section*{A.2 AHP Results for Transit Mode Selection}

The aggregated pair-wise comparison matrix for each of attributes in a hierarchical tree structure.

\textit{1) Comparison among Stakeholders}

Priority Values (or Weights) among Stakeholders

<table>
<thead>
<tr>
<th>Stakeholders</th>
<th>Users</th>
<th>Operators</th>
<th>Community</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Users</td>
<td>1</td>
<td>0.484</td>
<td>1.719</td>
<td>0.282</td>
</tr>
<tr>
<td>Operators</td>
<td>2.064</td>
<td>1</td>
<td>2.825</td>
<td>0.540</td>
</tr>
<tr>
<td>Community</td>
<td>0.582</td>
<td>0.354</td>
<td>1</td>
<td>0.177</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>1.000</td>
</tr>
</tbody>
</table>
2) Comparison among Criteria

Priority Values (or Weights) among Criteria to Users

<table>
<thead>
<tr>
<th>Benefits to Users</th>
<th>Accessibility</th>
<th>Mobility</th>
<th>Availability</th>
<th>Capacity</th>
<th>Comfort</th>
<th>Safety</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accessibility</td>
<td>1</td>
<td>1.00</td>
<td>0.72</td>
<td>1.38</td>
<td>1.48</td>
<td>1.38</td>
<td>0.184</td>
</tr>
<tr>
<td>Mobility</td>
<td>1.00</td>
<td>1</td>
<td>0.72</td>
<td>1.25</td>
<td>1.38</td>
<td>1.25</td>
<td>0.176</td>
</tr>
<tr>
<td>Availability</td>
<td>1.38</td>
<td>1.38</td>
<td>1</td>
<td>1.48</td>
<td>1.48</td>
<td>1.38</td>
<td>0.219</td>
</tr>
<tr>
<td>Capacity</td>
<td>0.72</td>
<td>0.80</td>
<td>0.68</td>
<td>1</td>
<td>1.38</td>
<td>1.25</td>
<td>0.154</td>
</tr>
<tr>
<td>Comfort</td>
<td>0.68</td>
<td>0.72</td>
<td>0.68</td>
<td>0.72</td>
<td>1</td>
<td>0.80</td>
<td>0.125</td>
</tr>
<tr>
<td>Safety</td>
<td>0.72</td>
<td>0.80</td>
<td>0.72</td>
<td>0.80</td>
<td>1.25</td>
<td>1</td>
<td>0.142</td>
</tr>
</tbody>
</table>

Total: 1.000

Priority Values (or Weights) among Criteria to Operators

<table>
<thead>
<tr>
<th>Benefits to Operators</th>
<th>Fare Revenues</th>
<th>Capital Costs</th>
<th>Operating Costs</th>
<th>Riders</th>
<th>System Reliability</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fare Revenues</td>
<td>1</td>
<td>0.29</td>
<td>0.68</td>
<td>1.43</td>
<td>3.88</td>
<td>0.160</td>
</tr>
<tr>
<td>Capital Costs</td>
<td>3.47</td>
<td>1</td>
<td>4.36</td>
<td>3.68</td>
<td>5.24</td>
<td>0.487</td>
</tr>
<tr>
<td>Operating Costs</td>
<td>1.46</td>
<td>0.23</td>
<td>1</td>
<td>1.15</td>
<td>2.95</td>
<td>0.161</td>
</tr>
<tr>
<td>Riders</td>
<td>0.70</td>
<td>0.27</td>
<td>0.87</td>
<td>1</td>
<td>3.16</td>
<td>0.138</td>
</tr>
<tr>
<td>System Reliability</td>
<td>0.26</td>
<td>0.19</td>
<td>0.34</td>
<td>0.32</td>
<td>1</td>
<td>0.055</td>
</tr>
</tbody>
</table>

Total: 1.000

Priority Values (or Weights) among Criteria to the Community

<table>
<thead>
<tr>
<th>Benefits to Operators</th>
<th>Economic</th>
<th>Aesthetic Quality</th>
<th>Environment</th>
<th>Quality of Life</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic</td>
<td>1</td>
<td>1.38</td>
<td>1.00</td>
<td>0.80</td>
<td>0.254</td>
</tr>
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### 3) Comparison among Transit Modes

#### Priority Values among transit modes with respect to “Accessibility”

<table>
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<tr>
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<th>Priority</th>
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#### Priority Values among transit modes with respect to “Mobility”

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#### Priority Values among transit modes with respect to “Availability”

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#### Priority Values among transit modes with respect to “Capacity”

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#### Priority Values among transit modes with respect to “Comfort”

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<td>PRT</td>
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Priority Values among transit modes with respect to “Safety”

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Priority Values among transit modes with respect to “Fare Revenue”

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Priority Values among transit modes with respect to “Capital Cost”

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Priority Values among transit modes with respect to “Operating Cost”

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<td>1.246</td>
<td>0.725</td>
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Priority Values among transit modes with respect to “Transit Riders”

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Priority Values among transit modes with respect to “System Reliability”

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Priority Values among transit modes with respect to “Economic”

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Priority Values among transit modes with respect to “Aesthetic Quality”

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Priority Values among transit modes with respect to “Environment”

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Priority Values among transit modes with respect to “Quality of Life”

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<td>0.330</td>
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</tr>
<tr>
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APPENDIX B

DETERMINATION OF CRITICAL REASONING CHAINS

The step-by-step analysis for determining the critical reasoning chains is demonstrated in this Appendix. This demonstration is referred to the reasoning process for transit alternatives analysis of the case study presented in Chapter 6. The reasoning map of the ‘Mobility’ goal (Goal 1) as shown in Figure 6-3 is used as a reference. The process to determine the critical reasoning chains is backward from the goal node to the starting nodes. Figure B-1 shows the reasoning map of the ‘Mobility’ goal.

Figure B-1: Demonstration of Critical Reason Determination
Based on the proposed reasoning map, the degree of belief in “High Mobility” is 0.992. There are four causal nodes (or boxes) connecting to the “Mobility” goal node: “Transportation capacity (M15),” “Transportation choices (M17),” “Quality of service (M19),” and “Service for transit dependent (M20).”

Among these four nodes, the “Transportation capacity (M15)” has the highest influence to the mobility because this node has the highest $I_k$ value. These values can be derived as follows. If each of these four nodes is eliminated from the map, the degree of belief in the “High Mobility” changed. They are 0.637 for M15, 0.999 for M17, 0.919 for M19, and 0.990 for M20 as shown in Figure B-2. Thus, the $I_k$ for these four causal nodes are +0.355 (0.992-0.637) for M15, –0.007 for M17, +0.073 for M19, and +0.002 for M20. The + sign means that the causal node positive effect the degree of belief, while the – sign implies the negative effect on the degree of belief of the conclusion.

![Diagram](image-url)

Figure B-2: Demonstration of Critical Reason Determination
The “Transportation capacity (M15)” node is selected as a critical (highly supporting) factor for “High Mobility.” Next, two causal nodes relating to “Transportation capacity” are considered. Using the same calculation procedure, the critical factors are determined one-by-one back to the starting nodes. Figures B-3 through B-7 illustrate the critical reasoning chains (in red).

Figure B-3: Demonstration of Critical Reason Determination
Figure B-4: Demonstration of Critical Reason Determination
Figure B-5: Demonstration of Critical Reason Determination
Figure B-6: Demonstration of Critical Reason Determination
Figure B-7: Demonstration of Critical Reason Determination