Three-Dimensional Nonlinear Dynamics of a Moored Cylinder to be Used as a Breakwater

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(ABSTRACT)

A three-dimensional, nonlinear dynamic analysis is conducted on a fully submerged, rigid, solid cylinder to be used as a breakwater. The breakwater could potentially be used as a single cylinder to protect small structures. Alternatively, multiple cylinders could be positioned in series to protect shorelines, harbors, or moored vessels from destructive incident water waves. The cylinder is positioned with its axis horizontal and is moored to the seafloor with four symmetrically placed massless mooring lines connected at the ends of the cylinder. The mooring lines are modeled as both linearly elastic (“regular”) springs and compressionless springs. All six degrees of freedom of the structure are considered. The breakwater is modeled in air with a net buoyant force acting through the cylinder’s center of gravity. The six “dry” natural frequencies of the structure are computed. Both linear and nonlinear free vibrations of the structure are considered. Linear damping is used to model the fluid and mooring damping effects. Normal and oblique harmonic wave forces at various frequencies and amplitudes are applied to the cylinder. The effects of the forcing amplitude and frequency, and the coefficient of damping, on the motion of the breakwater are studied. The results show that more erratic behavior occurs for the breakwater with compressionless springs, mainly due to the development of snap loads in the mooring lines.
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