Chapter 6
Airspace Planning and Collaborative Decision
Making Model

6.1. Model Formulation
6.1.1. First Formulation (APCDM-1)

We summarize below the APCDM having the $C_3$ conflict resolution constraints and linear equity formulation.

Minimize

$$\sum_{f=1}^{F} \sum_{p \in P_f} c_{fp} x_{fp} + \sum_{s \in S} \sum_{n=0}^{\pi_s} \mu_{sn} y_{sn} + \mu' x' + \mu'' \sum_{a=1}^{\tau} \omega_a \left[1 - F_a(x) \right] + \sum_{s \in S} \gamma_s w_s + \sum_{(P,Q) \in A} \varphi_{PQ} z_{PQ}$$ (6.1a)

subject to:

$$\sum_{p \in P_f} x_{fp} = 1, \quad \forall f = 1, \ldots, F$$ (6.1b)

$$\sum_{(f,p) \in C_u} x_{fp} \leq n_s, \quad \forall i = 1, \ldots, I_s, s = 1, \ldots, S$$ (6.1c)

$$w_s = \frac{1}{|H|} \sum_{(f,p) \in \Omega_s} t_{fp} x_{fp}, \quad \forall s = 1, \ldots, S$$ (6.1d)

$$n_s - w_s = \sum_{n=0}^{\pi_s} n y_{sn}, \quad \forall s = 1, \ldots, S$$ (6.1e)
\[ \sum_{n=0}^{\pi} y_{sn} = 1, \quad \forall s = 1, \ldots, S \]  
(6.1f)

\[ x_p + x_Q \leq 1, \quad \forall (P, Q) \in FC \]  
(6.1g)

\[ \sum_{(P, Q) \in M_{sk}} z_{PQ} \leq 1, \quad \forall (s, k) \]  
(6.1h)

\[ x_p + x_Q - z_{PQ} \leq 1, \quad \forall (P, Q) \in A \]  
(6.1i)

\[ x_p + x_Q + x_R \leq 2, \quad \forall (P, Q, R) \in T_{NC} \]  
(6.1j)

\[ E_{\alpha}(x) = \frac{D_{\max} \sum_{f \in A_p} c_{f}^* - \sum_{f \in A_p, p \in P_f} c_{fp} x_{fp}}{(D_{\max} - 1) \sum_{f \in A_p} c_{f}^*}, \quad \forall \alpha = 1, \ldots, \alpha \]  
(6.1k)

\[ E_{\alpha}^e(x) = E_{\alpha}(x) - \left( \sum_{a=1}^{\pi} \omega_a E_{\alpha}(x) \right), \quad \forall \alpha = 1, \ldots, \alpha \]  
(6.1l)

\[ v_{\alpha} \geq -E_{\alpha}^e(x), \quad v_{\alpha} \geq E_{\alpha}^e(x), \quad \forall \alpha = 1, \ldots, \alpha \]  
(6.1m)

\[ x^e = \sum_{a=1}^{\pi} \omega_a v_{\alpha} \]  
(6.1n)

\[ z_{PQ} \geq 0, \quad x_{fp} \text{ binary}, \quad \forall p \in P_f, \forall f = 1, \ldots, F, \]  
\[ y_{sn} \geq 0, \quad \forall n = 1, \ldots, \pi, s = 1, \ldots, S, \quad E_{\alpha}(x) \geq 0, \quad \forall \alpha = 1, \ldots, \alpha, \quad n_s \leq \pi_s, \forall s = 1, \ldots, S, \]  
\[ x^e \leq v^e, \quad -E_{\max}^e \leq E_{\alpha}^e(x) \leq E_{\max}^e, \quad \forall \alpha = 1, \ldots, \alpha. \]  
(6.1o)

Table 6-1 below summarizes the parameters and constants required for this formulation.
### Objective Function Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{fp}$</td>
<td>Cost to execute flight plan $fp$</td>
<td>(5.4)</td>
</tr>
<tr>
<td>$c_{f0}$</td>
<td>Cost to cancel flight $f$</td>
<td>(5.5)</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>Airspace monitoring (penalty) cost factor for sector $s$</td>
<td>(4.9)</td>
</tr>
<tr>
<td>$\mu_{sn}$</td>
<td>(Penalty) cost when peak monitoring workload in sector $s$ exceeds the average by $n$</td>
<td>(4.6e)</td>
</tr>
<tr>
<td>$\varphi_{PQ}$</td>
<td>(Penalty) cost to resolve an enroute conflict between aircraft $P$ and $Q$</td>
<td>(4.16)</td>
</tr>
<tr>
<td>$\mu^f$</td>
<td>(Penalty) cost factor associated with the level of collaboration equity achieved</td>
<td>(5.18a)</td>
</tr>
<tr>
<td>$\mu^D$</td>
<td>(Penalty) cost factor associated with the total (weighted) collaboration efficiency achieved</td>
<td>(5.18a)</td>
</tr>
</tbody>
</table>

### Constants in Constraints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>H</td>
<td>$</td>
</tr>
<tr>
<td>$t_{fp}^*$</td>
<td>Length of time flight plan $fp$’s trajectory occupies sector $s$</td>
<td>(4.8)</td>
</tr>
<tr>
<td>$\overline{n}_s$</td>
<td>Maximum allowable peak monitoring workload (simultaneous flight occupancies) in sector $s$</td>
<td>(4.10)</td>
</tr>
<tr>
<td>$D_{max}$</td>
<td>Maximum allowable ratio for any airline of CDM-selected surrogates to individually optimized surrogates</td>
<td>(5.9)</td>
</tr>
<tr>
<td>$c^*_f$</td>
<td>Optimal (lowest cost) surrogate from those offered for flight $f$</td>
<td>(5.7)</td>
</tr>
<tr>
<td>$E_{max}^c$</td>
<td>Maximal limit for each airline’s absolute collaboration equity</td>
<td>(5.18e)</td>
</tr>
</tbody>
</table>

**Table 6-1: APCDM-1 Parameter Summary**

Observe that the objective function (6.1a) contains \([|F| + (\overline{n}_s + 3)|S| + 3\overline{\alpha} + |A| + 1]\) variables (assuming $\overline{n}_{s_1} = \overline{n}_{s_2}$ $\forall s = 1, \ldots, S$). Furthermore, there are \([|F| + 4|S| + 6\overline{\alpha} + |A| + C_2\text{ constraints} + C_3\text{ constraints} + |FC| + |C_u| + 2]\) constraints (not including the lower and upper bounds specified in (6.1t)) for this model formulation.

### 6.1.2. Second Formulation (APCDM-2)

The second formulation incorporates the generalized $C_4$ conflict resolution constraints, by replacing (6.1i) and (6.1k) with:
\[
\sum_{(P,Q) \in \mathcal{M}_d} z_{PQ} \leq r_s, \quad \forall (s,k) \tag{6.2a}
\]
\[
\sum_{Q \in \mathcal{J}_d(P)} z_{(PQ)} \leq r_x P, \quad \forall P \in N_{sk} : |J_{sk}(P)| \geq r_s + 1, \quad \forall (s,k). \tag{6.2b}
\]

There are \([|F| + 4|S| + 6\bar{c} + |A| + |C_2| \text{ constraints} + |C_4| \text{ constraints} + |FC| + |C_{1f}| + 2]\) constraints (not including the lower and upper bounds specified in (6.1t)) for this model formulation.

### 6.2. Objective Function Parameters

#### 6.2.1. Airspace Monitoring Penalty Function

We consider costs associated with average workload in terms of the related ATC sector crew expertise necessary to safely perform operations as workload increases from zero to the sector capacity, \(\pi_s\). The ATC infrastructure cost is taken to be a “sunk cost” (which, along with airspace geometry, determines \(\pi_s\)) that is insensitive to variances in workload.

Each sector within an ARTCC generally employs a three-person ATC crew of Certified Professional Controllers (CPC). The crew consists of a Flight Data Controller, a Radar Controller, and a Radar Associate/Non-Radar Controller [36]. When ATC workload is minimal, relatively inexperienced crews (having minimal salaries) might be used, whereas more experienced crews might be necessary for increased workloads to safely perform more complex operations. When operating at sector capacity, the most qualified crews (having maximal salaries) might likely be used to handle such highly stressed operations.

The FAA publishes vacancy announcements for CPC positions. For example, the New York ARTCC advertisement dated September 2002 seeks to fill multiple CPC positions having annual salaries ranging from $90,268 to $126,375 [19], with the median salary being $108,322.
Suppose that each controller can monitor an average of five aircraft simultaneously, throughout the horizon of $|H|$ minutes. Then sector $s$ would require $\frac{w_s}{5}$ controllers working $|H|$ minutes. Assuming that each controller works 40 hours per week and 50 weeks per year, and a per-controller annual cost of $216,644$, including a 100% employee overhead cost (hiring, training, medical benefits, retirement plans, etc.), the per-minute per-controller cost would be $\left(\frac{216,644}{50 \times 40 \times 60}\right) = 1.805$. Hence, the cost per unit of average workload, $w_s$, for sector $s$ for some horizon $H$ can be taken as

$$\gamma_s = \left(\frac{1.805}{5}\right) |H| = 0.361 |H|, \quad \forall s = 1, \ldots, S. \quad (6.3)$$

### 6.2.2. Peak Airspace Monitoring Penalty Function

We use a similar rationale as in the foregoing subsection to generate a suitable penalty function to be associated with peak sector occupancy workloads. We assume that the convex increasing rate penalty function $\mu_{sa}$ is a quadratic function of $n$, say, $f(n) \equiv \frac{1}{2} a_n n^2$. Suppose further that we let the penalty ascribed to a peak workload exceeding the average by one unit be $\frac{\gamma_s}{5}$, based on the five aircraft monitoring axiom.

Hence, $\mu_{s1} \equiv f(1) = \frac{a_s}{2} = \frac{\gamma_s}{5}$. Furthermore, since $f(n) - 2f(n-1) + f(n-2) \equiv f''(n) = a_s$, we take

$$\mu_{s0} = 0; \quad \mu_{s1} = \frac{1}{5} \gamma_s;$$

$$\mu_{sj} = 2 \mu_{s(j-1)} - \mu_{s(j-2)} + \frac{2}{5} \gamma_s = \left(\frac{\gamma_s}{5}\right) (j)^2, \quad \forall j = 2, \ldots, \bar{n}, \quad \forall s = 1, \ldots, S, \quad (6.4)$$

where $\gamma_s$ is given by (6.3).
6.2.3. Conflict Resolution Penalty Function

Conflict resolution activities increase the workload beyond that required for basic airspace monitoring activities. Once again we shall utilize the foregoing methodologies to formulate a suitable penalty function associated with the necessity to resolve these conflicts.

Avoiding mid-air aircraft collisions is perhaps the most critical aspect of an Air Traffic Controller’s duties. Hundreds of lives and millions of dollars in property and assets are potentially at risk if procedures are not followed precisely. Conflict resolution can therefore be a high stressful ATC controller task. Ideally, only the most highly qualified and experienced controllers would perform these actions, hence we shall ascribe a penalty cost that is commensurate with (theoretically) having an additional highly-qualified controller on duty during the length of time that conflict resolution activities are taking place. Using the maximal salary from [19], similar to the derivation in Section 6.3.1., this yields a per-minute penalty cost of \( \frac{$126,375}{(50)(40)(60)} = $1.053 \) for conflict resolution actions.

Suppose that an ATC controller, on average, initiates conflict resolution actions 10 minutes prior to the projected aircraft separation violation. The controller issues new vectors to the affected aircraft, monitors compliance, and issues another set of vectors to the aircraft to return them to their respective nominal trajectories. Assuming symmetric conflict avoidance trajectories, from and back to the nominal flight plan trajectories, conflict resolution activities persist for approximately 20 minutes for each conflicting pair of aircraft. Accordingly, let the conflict resolution penalty function be given as

\[
\varphi_{P,Q} = I_{P,Q}($1.053)(20) = ($21.06)I_{P,Q}, \quad \forall (P,Q) \in A ,
\]

(6.5)

where \( I_{P,Q} \) is the intensity of the conflict \((P,Q)\). In this study, we take \( I_{P,Q}=1 \) for Level-1 conflicts as defined in Chapter 3, and \( I_{P,Q}=4 \) for Level-2 conflicts, to reflect a quadratic intensity increase corresponding to the half-sized separation box (in the horizontal plane) that is violated in this instance.
Remark: We can modify (6.5) to account for additional conflict geometry as determined by the PAEM, as discussed previously in Section 4.9. Conflict resolution actions may differ based on relative headings. For example, the advance warning of a potential separation violation is significantly shortened when aircraft are approaching head-on versus approaching on near parallel trajectories. An intruder moving along the cross-track axis of a focal aircraft’s trajectory might avoid the conflict by either increasing or decreasing its air speed, rather than altering its trajectory. We shall explore such conflict geometry implications in subsequent research.

6.2.4. Collaboration Efficiency and Equity Penalty Factors

Recall from the discussion in Section 5.2. that \( \sum_{\alpha} \omega_\alpha E_\alpha (x) \leq 1 \). Observe that \( 0 \leq x^e \leq 0.5 \). Furthermore, we have \( x^e = 0.5 \) when, for example, given an even number of airline participants (and equal weighting factors, \( \omega_\alpha \) for all airlines) the model solution yields 100% collaboration efficiency for \( \bar{\alpha}/2 \) airlines and 0% collaboration efficiency for the remaining \( \bar{\alpha}/2 \) airlines.

Let us assign values to the factors \( \mu^e \) and \( \mu^D \) in (6.1a) in such a way that we obtain a penalty of, say 10%, with respect to the total flight plan costs obtained through each airline’s respective individually optimizing strategy (i.e. \( \sum_{f=1}^{F} c^*_f \)), for any feasible solution that yields worst-case values for either the \( \omega \)-mean collaboration efficiency or \( \omega \)-mean absolute collaboration equity. That is, let

\[
\mu^e = \mu^D = \mu_0 \sum_{f=1}^{F} c^*_f ,
\]

where, say, \( \mu_0=0.1 \). We shall study the sensitivity of flight plan selection at optimality with respect to varying values (using the 10% rationale as the baseline) of these two parameters.
6.3. Passenger Load Estimates

We assign a passenger load estimate, $l_f$, based on the passenger capacity of the type of aircraft used for each flight $f$, and an estimated 75% load factor, and use this information for (5.3) and (5.6). Table 6-2 presents the capacities and the corresponding passenger load estimates for each aircraft type considered in the APCDM.

<table>
<thead>
<tr>
<th>AIRCRAFT TYPE</th>
<th>LABEL</th>
<th>CAPACITY</th>
<th>$l_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boeing 707</td>
<td>B707</td>
<td>180</td>
<td>135</td>
</tr>
<tr>
<td>Boeing 727 (all series)</td>
<td>B727</td>
<td>189</td>
<td>142</td>
</tr>
<tr>
<td>Boeing 737</td>
<td>B73F/J/S</td>
<td>130</td>
<td>98</td>
</tr>
<tr>
<td>Boeing 747 - 100/200/300</td>
<td>B747</td>
<td>380</td>
<td>285</td>
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<tr>
<td>Boeing 757</td>
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<tr>
<td>Boeing 767</td>
<td>B767</td>
<td>250</td>
<td>188</td>
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<tr>
<td>Boeing 777</td>
<td>B777</td>
<td>300</td>
<td>225</td>
</tr>
<tr>
<td>McDonnell-Douglas DC-8</td>
<td>DC8x</td>
<td>180</td>
<td>135</td>
</tr>
<tr>
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<td>McDonnell-Douglas MD-80</td>
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<td>Lockheed L-1011</td>
<td>L101</td>
<td>250</td>
<td>188</td>
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</table>

Table 6-2: Aircraft Passenger Load Estimates