Chapter 4

Discrete-continuum Analysis of Shear Banding in the Direct Shear Test

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The direct shear test is a widely used method of determining peak and critical state strength parameters for soil and for soil-manufactured material interfaces. However, stress conditions inside the box during shear are not well known. This paper presents a new micromechanics-based approach that modifies existing relationships between plane strain and direct shear friction angles taking into account the effects of non-coaxiality between the principal stress and the principal strain increment direction and the deviation of zero extension direction from horizontal. The results show that anisotropies of fabric and contact force rotate significantly after shearing starts, and reach their maximum at peak state. Strain localization is found to initiate from the side boundaries and extend slowly towards the middle of the box, and eventually develop into a distinct shear band along the middle plane around peak state. Simulation data are in good agreement with the proposed extended flow rule, with better agreement obtained when the effect of non-coaxiality is considered. Deviation of zero linear extension direction at peak from the horizontal is found to be less than 3 degrees and thus verifies the practice of measuring volume changes at the boundary in the direct shear device.

KEY WORDS: Anisotropy, Constitutive relations, Deformation, Failure, Numerical modeling and analysis, Shear strength
4.1. Introduction

The direct shear apparatus (DSA) is one of the most widely used geotechnical testing devices. It has played an important role in the history of geotechnical engineering and in our understanding of soil mechanics. Its use dates back about 150 years when Alexandre Collin conducted direct shear tests for slope stability analyses (Skempton 1949). Major criticisms of the direct shear box test include non-uniform stress and strain applied to the sample (Terzaghi and Peck 1948; Hvorslev 1960; Saada and Townsend 1981) and ambiguity in interpreting shear strength parameters at failure (Morgenstren and Tchalenko 1967). For these reasons, the direct shear test fell out of favor in the geotechnical community.

In recent years the direct shear test has seen a resurgence in use, primarily because tests are simple and are typically of lower cost than a triaxial test. Furthermore, standard tests (ASTM D 5321-02) to determine the strength of geosynthetic interfaces are based on the direct shear test. Potts et al. 1987 used finite element analyses to show that stresses and strains within the final failure zone are fairly uniform and progressive failure effects were found to be minor, despite the non-uniform stresses and strains within the box before failure. It was concluded that peak shear strength from DSA is very close to that obtained in ideal simple shear.

4.1.1. Previous research work on constitutive behavior of granular soils

Radiography observations by Jewell and Wroth 1987 show that a uniform band of deforming sand across the sample center exists at the peak state, and the direction of zero linear extension is nearly horizontal. Jewell and Wroth also presented relationships between plane strain friction angle $\phi_p$, direct shear friction angle $\phi_d$, critical state plane strain friction angle $\phi_{crit}$ and dilation angle $\psi$ by introducing Davis’s equation and Rowe’s stress-dilatancy relationship. This Rowe-Davis framework is based on three major assumptions, which are generally valid for dense sand at peak stress ratio: (1) the deforming sand inside the shear band is sufficiently uniform to be described by a single state of stress and strain increment; (2) the incremental strain is plastic and direction of zero linear extension is horizontal; and, (3) the orientation of principal stress is coaxial with that of principal strain increment. An improved version of this framework is given by Lings and Dietz 2004.
Extensive experimental and numerical data exists in the literature showing the importance of non-coaxiality in prediction of strain localization, stress-dilatancy and energy dissipation behavior of granular soils (Arthur et al. 1977; Miura et al. 1986; Gutierrez et al. 1993; Gutierrez and Ishihara 2000). Gutierrez and Ishihara 2000 pointed out that constitutive relations cannot be sufficiently formulated in the principal stress space unless non-coaxiality behavior is taken into account. Furthermore, energy dissipation calculated from the principal stress and plastic strain increment tensors is overestimated in case of non-coaxial flow.

4.1.2. Primary work of the current paper

This study presents additional evidence that justifies the validity of DSA data at peak state and why results are comparable to simple shear within the shear band. An extended Rowe-Davis framework is introduced which accounts for the effects of non-coaxiality behavior and deviation of zero extension direction from horizontal. The paragraphs below describe development of relations for stress, strain, flow rule and plane strain angle of friction accounting for non-coaxiality. Microscopic quantities are related to continuum-based quantities through theories of shear-induced anisotropy and homogenization of granular material. A series of numerical experiments based on discrete element simulations of a direct shear box test are described.

4.2. Theoretical Framework

4.2.1. Stress tensor and shear-induced anisotropy

The average stress tensor (Rothenburg and Selvadurai 1981; Christoffersen et al. 1981) acting on a granular assembly can be computed as follows:

\[
\sigma_{ij} = \frac{1}{V} \sum_{k=1}^{N_c} f_i^k l_j^k
\]  

(4-1)

where \( f_i^k \) is the contact force acting at the \( k \)-th contact point between the two particles; \( l_j^k \) is the branch vector connecting the centroids of two particles forming the \( k \)-th contact point (Figure
4.1); and $N_v$ is the total number of contacts in the volume $V$. A contact is created only when it transmits contact forces, $f$.

Rothenburg 1980 introduced the two-dimensional, second order density distribution tensors of fabric ($F_{ij}$), average contact normal force ($N_{ij}$) and average contact shear force ($S_{ij}$) to describe the anisotropy of fabric and contact forces in a granular system,

$$F_{ij} = \int_0^{2\pi} E(\theta)n_i n_j d\theta = \frac{1}{N_c} \sum_{k=1}^{N_c} n_i^k n_j^k ,$$

(4-2)

$$N_{ij} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\bar{f}_n(\theta)}{f_0} n_i n_j d\theta = \frac{1}{N_c} \sum_{k=1}^{N_c} \frac{f_n^k}{f_0} n_i^k n_j^k ,$$

(4-3)

$$S_{ij} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\bar{f}_s(\theta)}{f_0} t_i n_j d\theta = \frac{1}{N_c} \sum_{k=1}^{N_c} \frac{f_s^k}{f_0} t_i^k n_j^k ,$$

(4-4)

where, $E(\theta)$, $\bar{f}_n(\theta)$ and $\bar{f}_s(\theta)$ are density distribution functions of contact normals, contact normal force and contact shear force, respectively; $f_n$ and $f_s$ are contact normal force and shear force, respectively; $n = (\cos \theta, \sin \theta)$ is unit contact normal vector, and $t = (-\sin \theta, \cos \theta)$ is the vector perpendicular to $n$; and $\bar{f}_0$ is the average contact normal force:

$$\bar{f}_0 = \frac{1}{2\pi} \int_0^{2\pi} \bar{f}_n(\theta) d\theta = \frac{1}{N_c} \sum_{k=1}^{N_c} f_n^k .$$

(4-5)

The following Fourier series expressions of $E(\theta)$, $\bar{f}_n(\theta)$ and $\bar{f}_s(\theta)$ were proposed by Bathurst & Rothenburg 1990:

$$E(\theta) = \frac{1}{2\pi} [1 + a \cos 2(\theta - \theta_n)] ,$$

(4-6)

$$\bar{f}_n(\theta) = \bar{f}_0 [1 + a_n \cos 2(\theta - \theta_n)] ,$$

(4-7)

$$\bar{f}_s(\theta) = \bar{f}_0 [a_s - a \sin 2(\theta - \theta_s)] ,$$

(4-8)

where $a$, $a_n$, $a_s$ and $a_w$ are the coefficients of contact normal, contact normal force and contact shear force anisotropies, respectively; $\theta_n$, $\theta_s$ and $\theta$ are the principal directions of contact normal, contact normal force and contact shear force, respectively. These expressions account for the second order tensor only.
With $E(\theta)$, $\bar{f}_n(\theta)$, $\bar{f}_s(\theta)$ and contact vector length distribution $\bar{l}_j(\theta)$, Equation (4-1) can be written as follows:

$$\sigma_{ij} = m_v \int_0^{2\pi} \left[ \bar{f}_n(\theta)n_i + \bar{f}_s(\theta)t_i \right] \bar{l}_j(\theta)E(\theta)d\theta,$$  \hspace{1cm} (4-9)

where $m_v = N_c / V$ is the contact density. For a granular assembly of spherical particles with a narrow size distribution, Equation (4-9) can be expressed as:

$$\sigma_{ij} = m_v \bar{l}_0 \int_0^{2\pi} \left[ \bar{f}_n(\theta)n_i + \bar{f}_s(\theta)t_i \right] E(\theta)d\theta,$$  \hspace{1cm} (4-10)

where $\bar{l}_j(\theta) = \bar{l}_0 n_j = \bar{l}_0 \sin \theta$; $\bar{l}_0$ is the average contact vector length over the assemblage.

Substituting Equation (4-6) – (4-8) into Equation (4-10) and integrating, one obtains:

$$\sigma_{11} = p \left[ 1 + \frac{a \cdot a}{2} \cos 2(\theta_u - \theta_n) + \frac{1}{2} (a \cos 2(\theta_u) + a_n \cos 2(\theta_u) + a_s \cos (2\theta_t)) - \frac{a \cdot a}{2} \sin 2(\theta_u) \right],$$  \hspace{1cm} (4-11)

$$\sigma_{22} = p \left[ 1 + \frac{a \cdot a}{2} \cos 2(\theta_u - \theta_n) - \frac{1}{2} (a \cos 2(\theta_u) + a_n \cos 2(\theta_u) + a_s \cos (2\theta_t)) + \frac{a \cdot a}{2} \sin 2(\theta_u) \right],$$  \hspace{1cm} (4-12)

$$\sigma_{12} = p \left[ \frac{1}{2} (a \sin 2(\theta_u) + a_n \sin 2(\theta_u) + a_s \sin (2\theta_t)) + \frac{a \cdot a}{2} \cos 2(\theta_u) + \frac{a \cdot a}{2} \sin 2(\theta_u - \theta_n) - a_u \right],$$  \hspace{1cm} (4-13)

$$\sigma_{21} = p \left[ \frac{1}{2} (a \sin 2(\theta_u) + a_n \sin 2(\theta_u) + a_s \sin (2\theta_t)) + \frac{a \cdot a}{2} \cos 2(\theta_u) - \frac{a \cdot a}{2} \sin 2(\theta_u - \theta_n) + a_u \right],$$  \hspace{1cm} (4-14)

where

$$p = \frac{m_v \bar{l}_0 \bar{l}_n}{2}.$$  \hspace{1cm} (4-15)

The moment equilibrium condition in the granular assemblage requires:

$$\int_0^{2\pi} \bar{f}_s(\theta)E(\theta)d\theta = 0.$$  \hspace{1cm} (4-16)

With Equation (4-16), the following equation can be derived:

$$\frac{a \cdot a}{2} \sin 2(\theta_u - \theta_n) - a_u = 0.$$  \hspace{1cm} (4-17)

Substituting Equation (4-17) into Equation (4-13) and (4-14) one obtains:
\[ \sigma_{zz} = \sigma_{zz} = p \left[ \frac{1}{2} (a \sin(2\theta_a) + a_n \sin(2\theta_n) + a_s \sin(2\theta_s) + a_n^2 \cos(2\theta_n)) \right]. \quad (4-18) \]

Equations (4-11), (4-12) and (4-18) imply that the average stress tensor within a granular system is a function of the anisotropy of contact normals and contact forces. The shear strength of the granular assemblage depends on its capacity to develop the anisotropy of fabric, contact normal forces and contact shear forces.

4.2.2. Strain tensors

The spatial discretization approach is the most often used method to compute strains (Thomas 1997; Dedecker et al. 2000; Cambou et al. 2000; Bagi and Bojtar 2001). For the current problem involving large strain localization inside the granular media, the Green-St. Venant strain tensor, \( E_{ij} \), can be expressed as:

\[ E_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i} + u_{k,i} u_{k,j}) \], \quad (4-19) \]

where \( u_{i,j} \) is the displacement gradient tensor. It is based on the deformation measure related to the reference configuration. The second order term in Equation (4-19) can be neglected with little error and the strain tensor is computed by:

\[ e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \]. \quad (4-20) \]

The mesh free method used in this study employs a grid type discretization over the reference configuration. A schematic diagram of the approach is illustrated in Figure 4.2. This method differs from the mesh free method of O’Sullivan et al. 2002 in that no interpolation functions are used, making it simpler. A rectangular grid is generated that serves as the continuum reference space superimposed over the volume of particles. Then each grid point is assigned to an individual particle \( j \) which has the following property:

\[ \frac{d_{ij}}{r_j} \leq \frac{d_{ij}}{r_i} \quad (i = 1, 2 \cdots N_p; i \neq j), \quad (4-21) \]
where \( r_i \) is the radius of particle \( i \); \( d_i \) is the distance between the grid point and the centroid of particle \( i \); \( N_p \) is the total number of particles within the volume [Figure 4.2(a)]. If the ratio of the distance between particle centroid and its associated grid point to the particle radius is the least among all the particles, then the grid point is considered a point on the rigid body of the particle. Displacement of the grid point is calculated by:

\[
\begin{align*}
    u^x_i &= u^x_p + d\left(\cos(\theta_0 + \omega) - \cos(\theta_0)\right) \\
    u^y_i &= u^y_p + d\left(\sin(\theta_0 + \omega) - \sin(\theta_0)\right)
\end{align*}
\]  

(4-22)

where \( u^x_p, u^y_p \) and \( u^x_i, u^y_i \) are the x and y component of displacement of grid point and particle centroid, respectively; \( d \) is the distance between the grid point and the particle centroid; \( \theta_0 \) is the initial phase angle of the position of grid point relative to the particle centroid; and \( \omega \) is the accumulated rotation of the particle [Figure 4.2(b)].

4.2.3. Stress-dilatancy relationship

Rowe (1962) presented the stress-dilatancy relationship:

\[
\frac{\sigma_1}{\sigma_3(1 + d\nu/d\varepsilon_1)} = \tan^2\left(\frac{\pi}{4} + \frac{\phi_\mu}{2}\right)
\]  

(4-23)

where \( d\nu \) and \( d\varepsilon_1 \) are the volumetric strain increment and major principal strain increment respectively; \( \phi_\mu \) is the interparticle friction angle. The direction of particle movement takes the fixed angle of \( \frac{\pi - \phi_\mu}{2} \) from the major principal stress direction. Niiseki 2001 formulated Rowe’s equation on the plane of maximum strength mobilization using optimality theory. He showed that \( \phi_\mu \) depends not only on mineral surface of particles but also on deformation mechanisms. By relating the internal friction angle to the dilation angle, Niiseki also pointed out that particle movement directions actually change during strain hardening deformation.

Non-coaxiality behavior is observed in granular systems involving significant stress rotations. An extended stress-dilatancy relationship, accounting for the non-coaxiality effects, is used herein that is based on Niiseki 2001. The detailed derivation is presented in Appendix A.
This extended stress-dilatancy equation (flow rule) is written as:

\[
\tan \left( \frac{\pi}{4} + \frac{\phi_{ps}}{2} \right) = \tan \left( \frac{\pi}{4} + \frac{\phi_{crit}}{2} \right) \tan \left( \frac{\pi}{4} + \frac{A}{2} \right),
\]

It can also be written as:

\[
\sin \phi_{ps} = \frac{\sin \phi_{crit} + \sin A}{1 + \sin \phi_{crit} \sin A},
\]

or

\[
\frac{1 + \sin \phi_{ps}}{1 - \sin \phi_{ps}} = \frac{1 + \sin \phi_{crit} + \sin \psi \cos 2\Delta}{1 - \sin \phi_{crit} - \sin \psi \cos 2\Delta}
\]

Where \( \Delta \) is the angle between the principal stress and the principal strain increment direction, and \( A \) is the nominal dilation angle. Equations (4-24) to (4-26) are the extended Rowe’s stress-dilatancy relationships taking into account non-coaxiality. The nominal dilation angle \( A \) takes the place of the actual dilation angle, \( \psi \). It is usually smaller than the actual dilation angle due to non-coaxiality effects.

### 4.2.4. Plane strain and direct shear angles of friction

In Appendix B, the relationship between plane strain and direct shear angles of friction for the most general case are derived. The following equations are for the case where the principal stress direction does not coincide with the principal strain increment direction, and the zero linear extension direction deviates from horizontal:

\[
\sin \phi_{ps} = \frac{\tan \phi_{ds}}{\cos(\alpha - 2\Delta) + \sin(\alpha - 2\Delta) \tan \phi_{ds}},
\]

and

\[
\sin \phi_{ps} \approx \frac{\tan \phi_{ds}}{\cos(\delta - \psi - 2\Delta) + \sin(\delta - \psi - 2\Delta) \tan \phi_{ds}},
\]

where \( \alpha \) and \( \delta \) are defined in Figure 4.3 and Appendix B.
From Figure 4.3, we can obtain two orientations of shear band at peak stress ratio. The Mohr-Coulomb solution assuming the shear band lies in the plane of maximum mobilized strength:

\[ \theta_c = \frac{1}{2}(\phi_m - \beta); \]  

(4-29)

and the Roscoe solution assuming shear band lies in the direction of zero linear extension:

\[ \theta_r = \frac{1}{2}(\psi - \alpha). \]  

(4-30)

where \( \theta_c \) and \( \theta_r \) denotes the angle between the shear band and the horizontal direction.

4.3. Numerical Analysis of Direct Shear Box

4.3.1. DEM model of direct shear box

Discrete element simulations of a direct shear test were performed using PFC2D (Itasca Inc. 2002). Particle-level, and thus continuum-based, information such as stress and strain can be determined from DEM analysis. The DEM model of a direct shear box shown in Figure 4.4 is 88 mm long and 56 mm high. The top and bottom boundaries are made up of continuous triangular “sawtooth” asperities that prevent circulation of particles.

Specimens are consolidated to equilibrium under a specified normal stress during the initial confinement stage. Interparticle friction coefficients during the consolidation phase are selected to achieve a desired target initial density. The lower the initial density is, the higher the interparticle friction is required, and as a result, the higher the initial built-in shear stress is. All but the top boundaries are fixed. The top boundary rotates and moves in the vertical direction to maintain constant normal stress. Shearing occurs by displacing the lower half of the box to the right at a constant velocity of 1 mm per minute. Horizontal stress, horizontal displacement and vertical displacement are measured at the boundaries.

The Hertz-Mindlin contact model was implemented in all the simulations. An additional particle rolling resistance model (Wang et al. 2004) was also applied at both particle-particle contacts and particle-boundary contacts. Physical constants used in the simulations include: Particle density 2650 kg/m\(^3\); shear modulus and Poisson’s ratio of the particles, 29 GPa and 0.3
respectively; critical normal and shear viscous damping coefficient, both 1.0; time step $5.0 \times 10^{-5}$ sec; and interparticle and particle-boundary friction coefficient during shear, 0.5 and 0.9 respectively.

4.3.2. Numerical experiments

A series of numerical experiments were performed in which particle size distribution ($D_{\text{max}}/D_{\text{min}}$), initial relative density ($D_r$), interparticle friction coefficient ($\mu_p$) during consolidation and normal stress ($\sigma_n$) were varied to examine their effects on the shear banding behavior. The values of each variable used in the experiments are listed in Table 4.1. It should be mentioned that particles are generated with diameters randomly selected between $D_{\text{max}}$ and $D_{\text{min}}$, resulting in a nearly linear particle size distribution. However, the median particle diameter $D_{50}$ in all cases remained 0.7 mm. Sixteen evenly spaced particle columns extending from bottom to top of the box are marked before shear. Displacements and velocities of these column particles were recorded during each test. A typical profile of a deformed column, which is representative of the deformation of the whole assemblage, is shown in Figure 4.5. A nearly uniform shear band at the middle plane of the box is apparent.

<table>
<thead>
<tr>
<th>Variable</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{\text{max}}/D_{\text{min}}$</td>
<td>3.0</td>
<td>1.1</td>
<td>3.0</td>
<td>3.0</td>
<td>1.1</td>
<td>1.1</td>
<td>3.0</td>
<td>1.1</td>
<td>3.0</td>
<td>1.1</td>
<td>3.0</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>0</td>
<td>0</td>
<td>0.3</td>
<td>0</td>
<td>0.3</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_n$ (KPa)</td>
<td>300</td>
<td>300</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>25</td>
<td>25</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>$D_r$ (%)</td>
<td>112</td>
<td>118</td>
<td>82</td>
<td>89</td>
<td>80</td>
<td>96</td>
<td>65</td>
<td>87</td>
<td>53</td>
<td>46</td>
<td>46</td>
</tr>
</tbody>
</table>

The stress and strain conditions inside the shear band are recorded using a 28 mm high sampling window symmetrical about the middle plane of the box, as shown in Figure 4.5. The length of the window is initially equal to the length of the box but decreases during the shear process by an amount equal to the shear displacement. Inside the sampling window, the average stress can be calculated by Equation (4-1); the anisotropies of fabric and contact normal and shear forces can be calculated by Equation (4-2) – (4-4) and approximated by their Fourier series
expressions in Equation (4-6) – (4-8); and, the strain field can be calculated by Equation (4-19) or Equation (4-20). For a granular assembly with narrow size distribution (Experiments B, E, F, H and J), the average stress can also be calculated by Equation (4-11), (4-12) and (4-18) using anisotropy parameters. Thereafter, the calculated average stress and strain values can be used to verify the theoretical relationships presented.

4.3.3. Stress-displacement relation

Typical stress – displacement relations from simulations of a dense sample (Experiment D) and a medium dense sample (Experiment I) are shown in Figure 4.6. Besides the conventional curve of direct shear stress ratio measured at the boundary, two additional curves representing the plane strain stress ratio and direct shear stress ratio inside the window are also included. It is clear that the peak stress ratio is higher and occurs at a lower displacement in the dense sample than in the medium dense sample. More apparent post peak strain-softening behavior is also observed in the dense sample.

The direct shear and plane strain stress ratios calculated by Equation (4-11), (4-12) and (4-18) are based on the average stress conditions inside the sampling window. These are lower than the boundary-measured values which actually represent the stress conditions at the middle plane of the box. The sampling window extends to the mid height of the upper and lower boxes. This incorporates a number of particles that lie outside of the intense shear zone. Use of a smaller window closer to the shear zone will give results closer to the boundary measurements. For example, the difference between the stress ratios measured at the boundaries and inside the window at peak is only 2% when the sampling window height is reduced to 2 mm. The shapes of the direct shear and plane strain stress ratios curves are quite similar to that of the conventional curve. As expected, there is little difference between the two new curves throughout the whole process. However, it is not implied that the direct shear and plane strain angle of friction are nearly equal in general. This result is due to the two-dimensional plane strain simulation condition that does not explicitly include the effect of the intermediate stress.

Figure 4.6 also shows the volume change measured by the vertical displacement of the top boundary versus shear displacement. For the dense sample, continuous dilation is observed during shear until the end of the test when dilation rate approaches zero, while for the medium
dense sample with lower confining stress, critical state with zero volume change was reached earlier with less total dilation. The greatest dilation rate occurs at the peak state for both cases.

4.3.4. **Strength and dilatancy behavior**

The peak and critical state strength ratios and dilation angles for the eleven numerical simulations measured at the boundaries and inside the windows are summarized in Table 4.2 and Table 4.3, respectively. It should be noted that the peak dilation angles measured at the boundaries are calculated using the vertical incremental displacement of the top wall, implicitly assuming that the zero linear extension direction is horizontal at peak; while the peak dilation angles inside the sampling window are strictly based on Equation (B-5). It can be seen that the peak dilation angles inside the window are larger than those at the boundaries, suggesting that greater dilation occurs inside the plastic shear zone than can be measured at the boundary due to averaging effects. Also included in Table 4.3 are the non-coaxiality angle $\Delta$ and the corresponding nominal dilation angle $A$ at peak.

The variations of peak strength ratios and dilation angles with particle size distribution, initial relative density and normal stress used are illustrated in Figure 4.7. Three major conclusions can be drawn that are in agreement with fundamental soil mechanics principles: (1) the more uniform the granular material, the lower the peak strength ratio and dilation angle; (2) the higher the initial relative density, the higher the peak strength ratio and dilation angle; (3) the higher the normal stress, the lower the peak strength ratio and dilation angle. A combination of these three effects results in the final peak strength and dilation angles shown in Figure 4.7.

4.3.5. **Shear induced anisotropy**

Bathurst and Rothenburg 1990 show the shear strength of a granular assemblage depends on its capacity to develop the anisotropy of fabric and contact forces. Figure 4.8 illustrates the evolution of anisotropies of fabric, contact normal force and contact shear force inside the shear zone. It can be clearly seen that all three types of anisotropies grow rapidly after shearing starts, and reach their maximum at peak state. It indicates that the shear strength of granular material increases until peak. After peak state, the anisotropies of contact normal force and contact shear force decrease rapidly. Besides the magnitude, the orientations of the anisotropies also rotate.
significantly during shear, especially before peak. For all stages, the orientation of contact normal force anisotropy was found to be essentially coincident with the major principal stress direction. Contact force chains inside the shear box at peak and critical stages are shown in Figure 4.9 for visualization.

Table 4.2. Peak and critical state strength ratios and dilation angles from simulation boundary measurements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak $\tau/\sigma$</td>
<td>0.618</td>
<td>0.584</td>
<td>0.63</td>
<td>0.713</td>
<td>0.54</td>
<td>0.585</td>
<td>0.638</td>
<td>0.656</td>
<td>0.623</td>
<td>0.508</td>
<td>0.626</td>
</tr>
<tr>
<td>Peak $\phi_{ds}$ (deg)</td>
<td>31.7</td>
<td>30.3</td>
<td>32.2</td>
<td>35.5</td>
<td>28.4</td>
<td>30.3</td>
<td>32.5</td>
<td>33.3</td>
<td>31.9</td>
<td>26.9</td>
<td>32.0</td>
</tr>
<tr>
<td>Critical state $\tau/\sigma$</td>
<td>0.46</td>
<td>0.43</td>
<td>0.44</td>
<td>0.42</td>
<td>0.4</td>
<td>0.4</td>
<td>0.43</td>
<td>0.42</td>
<td>0.45</td>
<td>0.4</td>
<td>0.43</td>
</tr>
<tr>
<td>Critical state $\phi_{ds}$ (deg)</td>
<td>24.7</td>
<td>23.3</td>
<td>23.7</td>
<td>22.8</td>
<td>21.8</td>
<td>21.8</td>
<td>23.3</td>
<td>22.8</td>
<td>24.2</td>
<td>21.8</td>
<td>23.3</td>
</tr>
<tr>
<td>Peak $\psi$ (deg)</td>
<td>5.7</td>
<td>5</td>
<td>8.8</td>
<td>10.5</td>
<td>4.4</td>
<td>5</td>
<td>9.2</td>
<td>5</td>
<td>9.2</td>
<td>3.7</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Table 4.3. Peak and critical state strength ratios and dilation angles measured inside the sampling window

<table>
<thead>
<tr>
<th>Parameter</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak ($\tau/\sigma$)$_{ps}$</td>
<td>0.452</td>
<td>0.402</td>
<td>0.408</td>
<td>0.47</td>
<td>0.387</td>
<td>0.427</td>
<td>0.418</td>
<td>0.456</td>
<td>0.42</td>
<td>0.346</td>
<td>0.407</td>
</tr>
<tr>
<td>Peak ($\tau/\sigma$)$_{ds}$</td>
<td>0.447</td>
<td>0.4</td>
<td>0.405</td>
<td>0.467</td>
<td>0.381</td>
<td>0.415</td>
<td>0.413</td>
<td>0.455</td>
<td>0.415</td>
<td>0.341</td>
<td>0.403</td>
</tr>
<tr>
<td>Peak $\phi_{ps}$ (deg)</td>
<td>24.3</td>
<td>21.9</td>
<td>22.2</td>
<td>25.2</td>
<td>21.2</td>
<td>23.1</td>
<td>22.7</td>
<td>24.5</td>
<td>22.8</td>
<td>19.1</td>
<td>22.1</td>
</tr>
<tr>
<td>Peak $\phi_{ds}$ (deg)</td>
<td>24.1</td>
<td>21.8</td>
<td>22.0</td>
<td>25.0</td>
<td>20.9</td>
<td>22.5</td>
<td>22.4</td>
<td>24.5</td>
<td>22.5</td>
<td>18.8</td>
<td>21.9</td>
</tr>
<tr>
<td>Critical state ($\tau/\sigma$)$_{ps}$</td>
<td>0.315</td>
<td>0.244</td>
<td>0.24</td>
<td>0.24</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.24</td>
<td>0.26</td>
<td>0.256</td>
<td>0.24</td>
</tr>
<tr>
<td>Critical state ($\tau/\sigma$)$_{ds}$</td>
<td>0.313</td>
<td>0.24</td>
<td>0.24</td>
<td>0.234</td>
<td>0.24</td>
<td>0.24</td>
<td>0.25</td>
<td>0.24</td>
<td>0.26</td>
<td>0.244</td>
<td>0.236</td>
</tr>
<tr>
<td>Critical state $\phi_{ps}$ (deg)</td>
<td>17.5</td>
<td>13.7</td>
<td>13.5</td>
<td>13.5</td>
<td>14.0</td>
<td>14.0</td>
<td>13.5</td>
<td>14.6</td>
<td>14.4</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>Critical state $\phi_{ds}$ (deg)</td>
<td>17.4</td>
<td>13.5</td>
<td>13.5</td>
<td>13.2</td>
<td>13.5</td>
<td>13.5</td>
<td>14.0</td>
<td>13.5</td>
<td>14.6</td>
<td>13.7</td>
<td>13.3</td>
</tr>
<tr>
<td>Peak $\psi$ (deg)</td>
<td>5.8</td>
<td>7.5</td>
<td>9.1</td>
<td>13</td>
<td>6.1</td>
<td>7</td>
<td>10</td>
<td>6.3</td>
<td>10</td>
<td>4.2</td>
<td>13</td>
</tr>
<tr>
<td>Peak $\Delta$ (deg)</td>
<td>-2.4</td>
<td>-3.58</td>
<td>-5.6</td>
<td>0.9</td>
<td>0.8</td>
<td>1.3</td>
<td>-5.36</td>
<td>-5.6</td>
<td>-4.23</td>
<td>1.8</td>
<td>-2</td>
</tr>
<tr>
<td>Peak A (deg)</td>
<td>5.8</td>
<td>7.4</td>
<td>8.9</td>
<td>13.0</td>
<td>6.1</td>
<td>7.0</td>
<td>9.8</td>
<td>6.2</td>
<td>9.9</td>
<td>4.2</td>
<td>13.0</td>
</tr>
</tbody>
</table>
Contact shear force does not exist in Experiment D before shear due to zero interparticle friction used in the consolidation stage to achieve high relative density. For cases with lower initial relative density, interparticle friction is introduced, resulting in less isotropic stress condition and finite built-in shear stress at the end of consolidation. This results in a finite value of contact shear force anisotropy before shear. However, this has little influence on the subsequent shearing process.

Figure 4.10 presents two sets of data from a dense sample (Experiment D) and a medium dense sample (Experiment I), showing the evolution of anisotropy parameters during shear. It can be seen that, for both cases, magnitude of contact normal anisotropy varies slightly and thus has much less effect on the macroscopic shear strength than those of contact normal force and shear force anisotropies. The contact normal and contact normal force anisotropy orientations agree well with each other since the contact distribution is defined only by contacts that transmit forces. However, some deviation is observed between the orientations of contact shear force anisotropy and contact normal anisotropy. This corresponds to the situation where there is a finite value of $a_w$ and the distribution of contact shear force is non-symmetric. However, this effect is not great and value of $a_w$ is nearly zero, as evident in Figure 4.10.

4.3.6. Strain localization inside the shear band

Strain localization inside the box during shear can be visualized by individual particle displacements and shear strain distributions recorded at different shear displacements. The horizontal displacement of four selected particle columns and shear strain contours from Experiment D at initiation of shear, pre-peak, peak and post-peak state, are shown in Figure 4.11. Due to the boundary constraint, strain localization begins at the two side boundaries. However, for the material located in the middle of the box, uniform deformation dominates during initial shearing, as can be seen from Figure 4.11(a). A linear profile of particle displacement is observed on Columns 8 and 9. This is in contrast with the curved profiles of Columns 2 and 15 due to boundary effects.

Figure 4.11 provides supporting evidence that shearing condition inside the middle of the direct shear box is ideal simple shear during the initial loading. As shearing proceeds, the plastic shear zone extends slowly toward the middle of the box from the boundaries. Meanwhile,
nonlinear stress-displacement behavior starts to occur. At the pre-peak state, some shear distortion has taken place even in the two most middle columns [(Figure 4.11(b)], indicating the plastic shear band extends over the length of the middle plane. The shear band cannot be seen in Figure 4.11(b) due to the resolution of the figure. However, at peak state [Figure 4.11(c)], a distinct shear band is observed, which becomes continuous and thicker with post-peak strain softening [(Figure 4.11(d)]. The shear strain distributions inside and outside the shear band are both uniform, indicating progressive failure is of minor importance in a dense sample. It is interesting to find that the overall shape of the shear band is slightly curved and deviates from the horizontal. Besides the major shear band, a secondary inclined shear zone forms near the left side boundary. This minor shear zone develops from the early stage of shearing and is caused by the boundary constraint.

The shear strain distributions of two medium dense samples (Experiments J and K) at peak and postpeak state are shown in Figure 4.12. In contrast with dense samples, the plastic shear zones in medium dense samples are more dispersed. This is mainly attributed to the less uniform stress and strain conditions inside the sample with lower relative density.

4.3.7. Verification of extended Rowe/Davis framework

A comparison between peak and critical state data from Table 4.2 and Table 4.3 from numerical simulations and the extended flow rule are presented in Figure 4.13. Two sets of data are plotted: data calculated inside the sampling window that account for the effect of non-coaxiality and data measured at the boundaries, neglecting the effect of non-coaxiality. Three series of laboratory direct shear test data for Ottawa 20/30 sand, C-33 concrete sand and 0.7 mm glass beads (Dove and Jarrett 2002) are also plotted for comparison. The laboratory test data are given in Table 4.4. It can be seen that both the simulation and laboratory test data agree well with the flow rule. Numerical experiment data from the sampling window fit the flow rule better than those measured at the boundary. The small difference between the window measurements and boundary measurements means that the effect of non-coaxiality is minor for granular material at peak state. However, for shear loading before peak state when rotation of principal stress is significant, the effect of non-coaxiality can not be neglected.
The shear band orientations at peak predicted by Equation (4-29) and (4-30) for all the numerical experiments are listed in Table 4.5. The Mohr- Coulomb solution and Roscoe solution yield the upper and lower bound values of orientations measured from the horizontal, respectively. It should be noted that these values represent the average orientations of shear band inside the window and do not apply to the local minor shear zones existing inside the samples, especially those due to boundary effects. Since the Roscoe solution actually represents the direction of zero linear extension, one can see from Table 4.4 that the deviations of zero linear extension from the horizontal at peak are all less than 3 degrees, with most of them less than 1 degree. This justifies the assumption of horizontal zero linear extension direction made at peak for dense and medium dense granular soils in the literature, and verifies the soundness of boundary measurement in the direct shear test.

Table 4.4. Laboratory direct shear test data on Ottawa 20/30 sand, C-33 Concrete Sand and 0.7 mm glass beads (Dove & Jarrett, 2002)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ottawa 20/30 sand</td>
</tr>
<tr>
<td>σn (KPa)</td>
<td>95.4</td>
</tr>
<tr>
<td>Peak ϕds (deg)</td>
<td>40.0</td>
</tr>
<tr>
<td>Critical state ϕds (deg)</td>
<td>27.9</td>
</tr>
<tr>
<td>Peak ψ (deg)</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 4.5. Orientation of shear band predicted by Mohr-Coulomb and Roscoe’s solution

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>θC (deg)</td>
<td>3.9</td>
</tr>
<tr>
<td>θR (deg)</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Note: Positive angles indicate counterclockwise rotations from horizontal; Negative angles indicate clockwise rotations from horizontal.
4.4. Conclusions

This paper presents a discrete-continuum study of granular soil in direct shear test. An extended Rowe-Davis framework is presented that describes the internal stress, dilatancy and shear banding characteristics of granular soils in the general case where non-coaxiality behavior and non-horizontal zero linear extension exist. Availability of internal stress and strain from DEM modeling makes it possible to verify the extended framework.

Anisotropies of fabric, contact normal force and contact shear force increase and rotate significantly after shearing starts, and reach their maximum at peak state. After peak, the shear strength decreases rapidly as anisotropies of contact normal force and contact shear force decrease. The plane strain and direct shear stress ratios based on average stress tensors are found to be similar, and have the same variation with shear displacement as the boundary measured stress ratio. Therefore it is concluded that Equation (4-11) (4-12) and (4-18) can be used to interpret the mechanical response of a granular material.

Strain localization is found to initiate from the side boundaries due to geometric constraints and extend towards the middle of the box after displacement begins. In the central portion of the sample during initial displacement, conditions similar to ideal simple shear exist. A distinct shear band develops along the middle of the plane at peak state. Shear bands in dense samples are more continuous, concentrated and uniform than those in medium dense samples.

Peak and critical state data from simulations are found to be in good agreement with the extended flow rule. Better agreement is obtained when the effect of non-coaxiality is taken into account. This effect is important during pre-peak shear displacement as significant stress rotation takes place. The shear band orientation predicted from the Roscoe equation suggests deviation of zero linear extension direction from the horizontal at peak state is less than 3 degrees. Therefore, the volume change measurements made at the boundary of the DSA should reflect the volume changes occurring in the sample.

Acknowledgement

This work was supported by National Science Foundation award number CMS-200949. This support is gratefully acknowledged.
Notation

The following symbols are used in this paper:

- $A =$ nominal dilation angle considering non-coaxiality effects
- $a =$ second order coefficient of contact normal anisotropy
- $a_n =$ second order coefficient of average contact normal force anisotropy
- $a_s , a_w =$ second order coefficient of average contact shear force anisotropy
- $c = \cos 2\Delta , \text{ Gutierrez-Ishihara non-coaxiality parameter}$
- $D_{\text{max}}, D_{\text{min}} =$ maximum and minimum particle diameter
- $D_r =$ relative density
- $D_{50} =$ median particle diameter
- $d =$ distance between the grid point and the centroid of particle
- $dV =$ volumetric strain increment
- $d\varepsilon_1 , d\varepsilon_2 =$ major and minor principal strain increment
- $E_{ij} =$ Green-St. Venant strain tensor
- $E(\theta) =$ density distribution function of contact normal
- $e_{ij} =$ Green-St. Venant strain tensor neglecting the second order term
- $F_{ij} =$ second order density distribution tensor of fabric
- $f =$ contact force vector
- $f_n , f_s =$ contact normal force and shear force
- $\tilde{f}_n (\theta) =$ density distribution function of average contact normal force
- $\tilde{f}_s (\theta) =$ density distribution function of average contact shear force
- $\tilde{f}_0 =$ average contact normal force
- $K =$ energy ratio
- $l =$ branch vector
- $l(\theta) =$ distribution of contact vector length
- $\tilde{l}_0 =$ assembly average contact vector length
- $m_i = N_c / V , \text{ contact density}$
- $N_c =$ total number of contacts
\( N_j \) = second order density distribution tensor of average contact normal force

\( N_p \) = total number of particles within the volume

\( n \) = unit contact normal vector

\( r \) = radius of particle

\( S_{ij} \) = second order density distribution tensor of average contact shear force

\( s \) = mean stress, \((\sigma_1 + \sigma_2)/2\)

\( t \) = unit contact tangent vector

\( t \) = deviator stress, \((\sigma_1 - \sigma_2)/2\)

\( u_{i,j} \) = displacement gradient tensor

\( u_x^p, u_y^p \) = x and y component of displacement of particle centroid

\( u_x^p, u_y^p \) = x and y component of displacement of grid point

\( V \) = assembly area (or volume)

\( \alpha \) = geometrical angle, \( \sin \alpha = (dV/2 - d\varepsilon_{yy})/(d\gamma/2) \)

\( \beta \) = geometrical angle, \( \sin \beta = (\sigma_{xx} - \sigma_{yy})/2t \)

\( \Delta \) = angle between principal stress and principal strain increment direction

\( \delta \) = geometrical angle, \( \sin \delta = -d\varepsilon_{yy}/(d\gamma/2) \)

\( \phi_{crit} \) = critical state plane strain friction angle

\( \phi_{ds} \) = direct shear friction angle

\( \phi_{ps} \) = plane strain friction angle

\( \phi_{\mu} \) = interparticle friction angle

\( \mu_p \) = interparticle friction coefficient

\( \theta_a \) = second order principal direction of contact normal anisotropy

\( \theta_c \) = Mohr-Coulomb angle of shear band from the horizontal direction

\( \theta_n \) = second order principal direction of average contact normal force anisotropy

\( \theta_r \) = Roscoe angle of shear band from the horizontal direction

\( \theta_s \) = second order principal direction of average contact shear force anisotropy
$\theta_0$ = initial phase angle of the position of grid point relative to the particle centroid

$\sigma_{ij}$ = stress tensor

$\sigma_n$ = normal stress

$\sigma_1$, $\sigma_3$ = major and minor principal stress

$\omega$ = accumulated particle rotation

$\psi$ = dilation angle

References


ASTM D5321-02, “Standard Test Method for Determining the Coefficient of Soil and Geosynthetic or Geosynthetic and Geosynthetic Friction by the Direct Shear Method,” ASTM, West Conshohocken, PA, USA.


Figure 4.1. $k$-th contact point between two particles.
Figure 4.2. Schematic diagram of the meshfree method: (a) Association of a grid point with certain particle; (b) displacement of grid point and its associated particle.
Figure 4.3. Mohr’s circles: (a) stress; (b) incremental strain.
Figure 4.4. DEM model of a direct shear box.
Figure 4.5. Deformed columns inside the shear box, Experiment D, 5mm shear displacement.

(Bold solid line: shear box; Bold dashed line: window for stress and strain analysis)
Figure 4.6. Typical stress-displacement relationships for dense and medium dense well-graded sample.
Figure 4.7. Variation of peak strength ratios and dilation angles against initial relative density and normal pressure. Solid and open symbols represent $D_{\text{max}}/D_{\text{min}}$ of 3.0 and 1.1, respectively. Bubble area represents normal stress, with the largest bubble area equal to 300 KPa.
Figure 4.8. Evolution of anisotropies of contact normal, contact normal force and contact shear force inside the shear zone during the shear process (Experiment D): (a) Before shear, 0 mm displacement; (b) At peak state, 2.3mm displacement; (c) At steady state, 10mm displacement.
Figure 4.9. Contact force chains inside the shear box (Experiment D): (a) At peak state, 2.3 mm displacement; (b) At critical state, 10 mm displacement.
Figure 4.10. Evolution of magnitudes and orientations of anisotropies during shear.
Figure 4.11. Evolution of strain localization during shear (Experiment D).
Figure 4.12. Shear band formation inside the medium dense sample: (a) Experiment J, at peak (2.4 mm displacement); (b) Experiment J, post peak (5 mm displacement); (c) Experiment K, at peak (2 mm displacement); (d) Experiment K, post peak (5 mm displacement).
Figure 4.13. Peak and steady state data from numerical simulations and laboratory tests.