Micromechanics of Granular Media: A Fundamental Study of Interphase Systems

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Abstract

The interphase is a localized region adjacent to a manufactured inclusion that is surrounded by granular soil. These regions are ubiquitous in civil infrastructure and often are components of large-scale composite systems. The interphase region influences load-deformation behavior of the entire composite system. However, mechanisms that control the mechanical behavior of the interphase region and, in turn, control the composite structure behavior, are not clearly understood. Few relationships exist for predicting interphase behavior from properties of granular materials and the inclusion surface that can be measured in the laboratory.

A two dimensional discrete element model of a general interphase system was developed and validated against laboratory data. Numerical experiments are conducted with varying soil to inclusion relative geometry. A new micromechanics-based approach, which utilizes microscopic quantities to explain the mechanics of granular media from a continuum point view, is adopted to investigate the mechanisms that underlie the interphase behavior.

It is shown that the grain to inclusion surface relative geometry controls the degree of granular media strength mobilization by controlling development of fabric and contact force anisotropy inside the interphase region. A unique bilinear relationship exists between the mobilized granular media strength and the principal direction of average contact force anisotropy at the interface between the particles touching the surface and the inclusion. These findings suggest the problem is one of contact and cannot be solved using purely geometric correlations, as past research presumed. A fundamental mechanism of behavior, long sought in geomechanics problems, is presented. Publications resulting from this research are significant and original contributions to the geoengineering, material science, geophysics and granular physics literature.
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