A Nondimensional Scaling Parameter for Predicting Pressure Wave Reflection in Stented Arteries

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Coronary stents have become a very popular treatment for cardiovascular disease, historically the leading cause of death in the United States. Stents, while successful in the short term, are subject to high failure rates (up to 24% in the first six months) due to wall regrowth and clotting, probably due to a combination of abnormal mechanical stresses and disruption of the arterial blood flow.

The goal of this research was to develop recommendations concerning ways in which stent design might be improved, focusing on the problem of pressure wave reflections. A one-dimensional finite-difference model was developed to predict these reflections, and effects of variations in stent and vessel properties were examined, including stent stiffness, length, and compliance transition region, as well as vessel radius and wall thickness. The model was solved using a combination of Weighted Essentially Non-Oscillatory (WENO) and Runge-Kutta methods. Over 100 cases were tested. Results showed that reasonable variations in these parameters could induce changes in reflection magnitude of up to ±50%. It was also discovered that the relationship between each of these properties and the resulting wave reflection could be described simply, and the effect of all of them together could in fact be encompassed by a single non-dimensional parameter. This parameter was titled “Stent Authority,” and several variations were proposed. It is believed this parameter is a novel way of relating the energy imposed upon the arterial wall by the stent, to the fraction of the incident pressure energy which is reflected from the stented region.
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A = lumen area of vessel
A₀ = some reference area, typically diastolic area
b = an empirically determined fitting constant used in the stent authority relation
D = bending stiffness coefficient
E = modulus of elasticity
E₀ = modulus of elasticity of the vessel wall
E₁ = maximum modulus of elasticity of the stent
f = pulse frequency, or heart rate
F = list of dependent variables, functions of dependent variables q
h = vessel wall thickness
L = total length of the stent, including transition regions
P = fluid pressure
P₀ = some reference pressure, typically minimum diastolic pressure
ΔP = pulse pressure, the difference in pressure between diastole and systole
q = list of independent variables
r = inner radius of vessel
r₀ = some reference radius, typically maximum inner radius at systole
t = time
u = flow velocity in the x-direction
w = inward deflection of vessel wall from rest
x = distance along the x-direction

β = coefficient relating pressure and area
δ = length of the compliance transition region
υ = Poisson’s ratio
ρ = fluid density

SA = stent authority
TSA = total stent authority
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A Nondimensional Scaling Parameter for Predicting Pressure Wave Reflection in Stented Arteries

Introduction

Cardiovascular disease has historically been the leading cause of death in the United States [1], and accounts for approximately one third of all deaths worldwide [2]. In the US, coronary artery disease in particular accounts for over fifty percent of these deaths, and in 2004 was the single largest killer [1]. As a result, investigation into the causes of and treatments for this disease has become a high priority among researchers. Arterial stenoses, one common form of coronary artery disease, are of great importance since they can result in an interruption in blood flow to the heart; common causes are the build up of plaque within arteries or the thickening of the arterial wall. Intravascular stents are a type of implanted prosthetic device that has received increasing acceptance among the medical community for the treatment of these problems. Inherently, stent implantation alters the mechanical environment of the vessel introducing a complicated strongly coupled fluid solid interaction. Despite the wide use of stents, our understanding of the associated governing mechanics is still lacking. This paper addresses a new method for predicting the performance of an intravascular stent based on the interaction between the fluid and mechanical properties of the system.

Stents, while successful in the short term, are subject to high failure rates due to restenosis and thrombosis. In one study, Knight et al. [5] found that in patients receiving a balloon angioplasty and subsequent stenting, restenosis resulted in a 24% failure rate for the procedure over a six month span, better than angioplasty alone (53%), but still leaving much room for improvement. It is believed that the high failure rates are partly due to abnormal wall stresses [6, 7, 8], and disruption of the flow around the stent [9, 10, 11, 12] which can be caused by both irregular geometry and compliance mismatch between the stent and the artery. Although progress in suppressing acute stent failure has been made using drug-eluting stents [13, 14, 15], this technique does not address the underlying mechanical and fluidic disruption. Despite their problems, stents are currently the most effective treatment for many types of coronary artery disease, and research shows that stent design remains an important factor in the long-term success of the implant [15, 16, 17, 18, 19, 20]. Therefore, pursuit of improvements in stent design is of great importance. As of yet, analytical models remain simplistic, and innovations have resulted mainly from experimental observations or trial-and-error clinical studies.

One major weakness of many computational approaches was that they examined only the mechanics of the walls or stent. Chua et al. [21] used LS-DYNA and ANSYS to study the initial expansion of a realistic stent model under varying inflation pressures, focusing on differences in strut stresses and final stent geometry. However, their analysis omitted any consideration of the effect of the vessel walls, and did not examine the coupling of the mechanical system to the fluid pressure after implantation.

Petrini et al. [22] developed finite element models of two commercial stents and examined their behavior when expanded into curved vessels. They were able to successfully predict the rotational moments required to generate a given angle of curvature, as well as predict the mechanical stresses within the stent, but their analysis neglected the contribution of the walls and the delivery catheter.
Other researchers have focused on the behavior of the blood flow within the system, considering only the geometry of the stent and wall, and not their coupled motion. To keep the simulations manageable, approximations such as restricted domains, steady flow, and simplified geometries are also common in many studies.

Berry, Santamarina, et al. [9] looked at the formation of vortices and stagnation regions between stent struts using a 2-D model of flow over a flat rigid plate. They found that large stagnation and separation regions could form between the struts for part or all of each cardiac cycle, and this behavior was dependent on stent size and spacing. Their observations agreed well with dye flow measurements they performed on a similar experimental setup.

The effect of coronary stent implantation on wall shear stresses (WSS) was examined by LaDisa et al. [23] in a three-dimensional CFD model of a slotted-tube stent in a rigid artery. Blood flow was imposed as steady parabolic profiles, neglecting the pulsatile variation. They also observed stagnation zones next to stent struts, and noted a decrease in minimum shear stress of up to 77% as compared to unstented arteries. The lowest values were localized near strut junctions and downstream of the stent outlet.

Fewer studies have looked at combining the effects of both the fluid mechanics and the elastic behavior of the vessel walls. Without the addition of the compliance of the vessel walls, the propagation of pressure waves through the arterial system is completely absent, and the effect of any pressure reflections from regions of increased stiffness, such as a stent or plaque, is missed. While there is a wealth of sources detailing computational and experimental efforts concerning pressure wave reflections in bifurcating vessels and arterial trees, there has not been a similar degree of investigation into reflections from areas of increased stiffness.

Tortoriello and Pedrizzetti [24] examined the effects of stent implantation on WSS using an axisymmetric 2-D numerical fluids-solids model, relying on a 1st order perturbative method to handle the walls, rather solving the full solids problem. Their analysis of pulsatile flows revealed that the compliance mismatch and overexpansion caused by the stent both enhanced the flow disruption in the stented region, reducing the minimum WSS and causing rapid variations in flow near the stent ends.

Alderson and Zamir [25] used analytical techniques to derive expressions for the magnitude of a sinusoidal pressure waveform in a compliant vessel with discrete segments of variable stiffness. They noted that reflections raised the pressure amplitude upstream of the rigid section, and that increased stiffness and segment length amplified this effect. However, this model considered only reflection, and not fluid velocities or wall deflections, and did not allow for arbitrary axial distributions of elastic modulus, radius, or thickness. Results were also limited to sinusoidal driving pressures, though more complex waveforms might be constructed using a Fourier-type wave summation.

Formaggia et al. [26] also investigated a one dimensional numerical model of a flexible artery with variable stiffness. Although mostly concerned with the development of the numerical techniques themselves, they did present some qualitative examinations of the effect of stent stiffness and length on the pressure within the vessel, and found that stiffer stents created larger upstream reflections, as did lengthening the prosthesis. However, their tests did not use physiologically correct values for the dimensions and physical properties of the system. In addition, their model was less able to deal with sudden discontinuities in the wall stiffness, and they did not examine the effect of variations of wall geometry. Nevertheless, the concepts outlined in their work provided a valuable guide in the effort presented here.
It was the goal of this research to develop an analytical tool to address some of these limitations by considering the interaction between the fluid mechanics of the blood and the solid mechanics of the vessel walls. The model developed here includes the coupling between the elastic motion of the walls and the flow in the stented artery, and allows for the specification of arbitrary vessel properties (wall thickness, radius, stiffness) along its length as well as the introduction of arbitrary pressure waveforms at the inlet. This model was then used to evaluate possible improvements in stent design. As the main measure of comparison was to be pressure wave reflection, consideration of the full three-dimensional flow was unnecessary and great gains in computational efficiency were achieved by focusing the analysis to a one-dimensional solution.

Methods

Governing Equations

The problem of wave propagation in a compliant artery can be modeled by an equation for continuity, and one for momentum. Since pressure wave transmission and reflection are inherently inviscid phenomena, the effects of the fluid viscosity can safely be omitted from the momentum equation. As a result these two equations form a system of the form

$$\frac{\partial q}{\partial t} + \frac{\partial F}{\partial x} = 0$$  \hspace{1cm} (1)

with

$$q = \begin{bmatrix} A \\ u \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} uA \\ \frac{1}{2}u^2 + \frac{P}{\rho} \end{bmatrix},$$  \hspace{1cm} (2)

which has three independent variables in x and t: area (A), pressure (P), and velocity (u). Here t and x are time and position, while fluid density, \(\rho\), is taken to be constant equal to the normal density of human blood, (1057 kg/m³).

To complete the solution, a third relationship is required. For this problem, an equation relating the applied internal pressure in the vessel to the wall deflection was derived based on Timoshenko and Woinowski-Krieger’s work on cylindrical shells [27].

$$\frac{d^2}{dx^2} \left( D \frac{d^2 w}{dx^2} \right) + \frac{Eh}{r^2} w = P,$$  \hspace{1cm} (3)

where D is defined as

$$D = \frac{Eh^3}{12(1-\nu^2)}$$  \hspace{1cm} (4)

Here, \(r\) is the inner radius of the vessel (which can be rewritten in terms of A), the wall thickness is given by \(h\), and \(w\) represents the deflection of the wall from neutral. The mechanical properties of the vessel wall are given by \(E\), the elastic modulus, and \(\nu\), the Poisson’s ratio. Both are functions of position, x, but are constant with time.

Using Equation (3), an examination of the contribution of the fourth order bending term to the overall deflection curves was conducted. For this comparison, the full solution to the equation was compared to results obtained by considering only the linear response of the vessel walls. Figure 1 shows the results of one such test for the Cordis CS30-035 stent design that will be used as a reference case below. Qualitatively it appears that the results are reasonably close,
and cause only a localized disturbance in the shape of the transition region between the stent border and the unstented vessel. While the magnitude of the relative errors, as referenced to the maximum deflection for the vessel (see Figure 2), appear large locally, it was decided this was an acceptable compromise, and that the first-order hoop term was the dominant effect in the conditions being examined. Equation (3) can then be written as a linear relationship between the applied internal pressure, and the square root of the vessel area:

\[ P(x) = \beta(x)\left(\sqrt{A(x)} - \sqrt{A_0(x)}\right) + P_0, \tag{5} \]

where \( \beta = \frac{4Eh}{3r_0^2 \sqrt{\pi}} \) if the vessel wall is assumed to be incompressible. The suitability of this approximation is further reinforced by a consideration of the derivatives in Equations (1) and (2), since the pressure and area terms are present only as first-order derivatives in \( x \).

Solution Method

In order to solve Equation (1) numerically, a weighted essentially non-oscillatory (WENO) scheme [28, 29] of order 3 was chosen for the spatial discretization. The details of the method used are described in [28] and [29]. These schemes have several advantages when solving problems with a mix of smooth regions and discontinuities. With the sudden changes in wall stiffness introduced by the stent, surrounded by large regions of little or no changes in properties, this problem is exemplary of these conditions. In this case, the third-order method used can maintain up to fifth-order accuracy throughout the region of interest without introducing spurious oscillations in the solution.

Time discretization was performed using a fourth-order, five-stage Runge-Kutta method, as described in [30] and [31]. This form was chosen to help minimize the memory requirements of the algorithm.
Boundary Conditions

Because the 3rd order WENO method requires the existence of 2 nodes left and right of each calculated node, it was necessary to generate extra points beyond the region of interest at each end of the domain. This was handled by reusing the current value of the first or last node in the domain, as appropriate, until there were sufficient nodes to perform the calculations near the boundaries.

In order to drive the flow through the vessel, it was assumed that the behavior of the system was governed by the entry of a pressure wave at the proximal entrance, to which the vessel area and velocity would respond. To simulate this, the pressure at either end was calculated as a function of time for each trial. This forcing function was then translated into an equivalent induced fluid velocity and the associated wall deflection was calculated by Equation (5). During the calculation of the solution, boundary conditions based on the method of characteristics were imposed upon the two end points.

Mesh Generation

For this study, the region of greatest interest is localized to the area surrounding the location of the stents (and the associated sharp gradient in wall compliance), while most of the remaining problem domain is smooth, containing no sudden changes in vessel properties. Initial trials indicated that generation of pressure wave reflection and other flow structures was limited to the area near regions of changing compliance, as was most of the interaction of intersecting pressure waves. To take advantage of this fact, the solver was modified to incorporate a variably spaced grid. Figure 3 shows the grid chosen for this study.
The size of the time step used was calculated dynamically for each simulation based on the material properties of the vessel and grid spacing used in that trial. For stability, a maximum Courant number (CFL) of 1.0 was chosen. For each node in the problem domain the value for $\Delta t$ required to yield the desired CFL was calculated, and from this set a $\Delta t$ that would maintain a CFL less than or equal to 1.0 throughout the domain was chosen. Smaller values for $\Delta x$ and stiffer vessels walls both required smaller $\Delta t$'s. Because the regions of maximum stiffness and minimum grid spacing typically intersected, this led to very small values for $\Delta t$ being required; typically $\Delta t$ fell between $1\times10^{-5}$ and $1\times10^{-6}$.

Repeated testing of the solution at varying grid densities showed that for sufficiently fine grids, the solution was largely independent of grid selection. It was important, however, to maintain finer resolution around the stent ends, or the compliance transition would introduce time-independent local oscillations in the solution at these points.

Cases of Interest

The conditions chosen for the setup of the numerical model were based upon average values for left coronary arteries in humans aged 40 to 59 years [32]. Table 1 lists these base values. The pulse pressure, $\Delta P$ was based upon a systolic pressure of 120 mmHg and a diastolic pressure of 80 mmHg. Because the numerical model responds to changes in pressure, but the absolute pressure is unimportant, the lower bound for pressure was set at 0 mmHg for computational simplicity. The shape of the input pressure wave was chosen to be a series of half sine wave pulses with a constant resting pressure between each pulse. Figure 4 shows a plot of the waveform that was used as an input to the system at the vessel inlet.

The presence of the stent was handled by assuming the stent and the vessel walls together acted as a single, composite material. The effective stiffness of the stent-vessel system was calculated by numerical simulation of its mechanical properties in a commercial FEM package. The stent modeled was a standard Palmaz-type stent, similar to Cordis’s CROWN stent. The results showed the effective modulus was approximately 50 MPa.

For the numerical model the stiffness along the length of the vessel was assumed to be a smoothly varying function, $E(x)$. Figure 5 shows a schematic of $E(x)$ for one of the cases tested. The baseline value corresponding to the vessel stiffness, $E_0=2.19$ MPa, was the same for all tests. The maximum stiffness of the stent-vessel system, $E_1$, depended on the exact case tested. In order to help understand how the stent scaling parameters interact, the stiffness ratio, $E_1/E_0$, for each trial will be compared instead of the actual value of $E_1$. Because of the way the stent edges interacted with the vessel walls, the compliance was assumed to transition gradually from $E_0$ to $E_1$, and the transition length, $\delta$, was varied depending on the test case. This mimicked the way real stents can have struts of differing number and design at the two ends. To help compare stents with the same total stent length, but different transition lengths, a quantity called the effective stent length was defined to be the distance between the midpoints of the two transition regions.
Table 1:  Base vessel and circulatory properties used for the simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse pressure, ΔP</td>
<td>40 mmHg</td>
</tr>
<tr>
<td>Pulse frequency, f</td>
<td>1.33 Hz (80 bpm)</td>
</tr>
<tr>
<td>Fluid density, ρ</td>
<td>1057 kg/m³</td>
</tr>
<tr>
<td>Base expanded radius, r₀</td>
<td>1.75 mm</td>
</tr>
<tr>
<td>Vessel wall thickness, h</td>
<td>0.69 mm</td>
</tr>
<tr>
<td>Vessel modulus of elasticity, E₀</td>
<td>2.19x10⁶ Pa</td>
</tr>
</tbody>
</table>

The Cordis CS30-035 CROWN stent was chosen to serve as reference point for the geometric properties of the system [33]. This stent is 30 mm long when fully expanded, and has an inner radius of 1.75 mm. The vessel wall thickness, as mentioned above, remained 0.69 mm. A value of 3.0 mm was found to be a good estimate for the compliance transition zone length. The bold values in Table 2 correspond to these reference conditions. Based upon these values, several other values were chosen for each of these five parameters in order to create a reasonable range of conditions for testing.

Table 2:  Vessel properties used for testing. The values for the Cordis CS30-035 stent, which were used as a reference for each characteristic, are shown in bold.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Tested Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness ratio, E₁/E₀</td>
<td>2x, 5x, <strong>22.9x</strong>, 100x, 1000x</td>
</tr>
<tr>
<td>Total stent length, L (mm)</td>
<td>15, 22, <strong>30</strong>, 37, 45</td>
</tr>
<tr>
<td>Compliance transition zone length, δ (mm)</td>
<td>1, 2, <strong>3</strong>, 4, 5, 6</td>
</tr>
<tr>
<td>Vessel radius, r₀ (mm)</td>
<td>1.0, <strong>1.75</strong>, 3.0, 4.0, 5.0</td>
</tr>
<tr>
<td>Wall thickness, h (mm)</td>
<td>0.25, 0.5, <strong>0.69</strong>, 1.0</td>
</tr>
</tbody>
</table>
Figure 4: Pressure wave that was used as the input to the system.

Figure 5: Diagram shows how the various test parameters, such as total stent length and transition length, influence the variation in stiffness along the length of the stent.
Results

Figure 6 provides a summary of the relative magnitudes of the pressure reflections for some of the cases examined using the methods described above. In the trials shown here several groups were constructed by selecting a single parameter from Table 2 to change while the rest were held constant, allowing the relative effect of each change to be evaluated. It was found that varying several of the parameters examined could cause the magnitude of the pressure wave reflected from the stented region to vary more than 50% from the reference calculated for a Cordis CS30-35 CROWN stent, while changing the transition length had a relatively small effect on the reflections. For all cases tested, the reflected pressure wave demonstrated a sudden increase in pressure to the maximum magnitude, followed by a region during which the pressure slowly decreased towards an equal minimum value, and then a sudden correction back to zero. Figure 7 shows a plot of the pressure reflection recorded over time as the wave passes by a fixed point upstream of the stent. A comparison of this waveform shows it to be the same length and shape as the incident pressure waveform, but shifted 90 degrees out of phase.
Figure 7: Plot of the shape of the reflected pressure wave for the reference case, which was typical of all tests. For each case the shape was similar, and only the magnitude of the pressure variation changed.

In a quantitative sense, the magnitude of the reflected pressure wave was found to vary linearly with length of the stent being tested. Figure 8 shows this relationship for several different values of the maximum vessel stiffness. As the stiffness of the vessel rose, the system became more sensitive to the length of the stent, and consequently the reflection increased more quickly with increasing length.

Figure 8: The magnitude of the reflected pressure wave increased linearly with total stent length if maximum stiffness was held constant.
Similarly, when the total length of the stent was held constant, but the length of the transition region was increased, the pressure reflection decreased in magnitude (Figure 9). These two parameters, stent length and transition length, can be lumped into a measurement of effective stent length. If this done, it can be seen that a linear relationship gives a good approximation of the reflection, but does not capture the entire behavior of the system. Figure 10 illustrates these results for a stent of maximum stiffness of 50 MPa ($E_1/E_0 = 22.8$).

![Figure 9: Pressure wave reflection decreased linearly with increasing transition length for a stent of constant total length.](image)

![Figure 10: There is a correspondence between the reflection and the effective length of the stent, but the relationship is only approximately linear.](image)
Next, the effect of the geometry of the vessel walls was examined by varying the inner radius and the wall thickness. As can be seen in Figure 11, for a constant stent and wall stiffness the magnitude of the reflected pressure wave increased approximately proportional to the square root of the vessel radius. Similarly, Figure 12 shows how the reflection decreased with the inverse of the square root of the wall thickness. Taken together, these two relationships suggested using a single non-dimensional parameter, \((R/h)^{1/2}\), to describe the effect of the vessel geometry on the magnitude of the pressure waves reflected from an implanted stent. Figure 13 shows how the calculation of this non-dimensional geometric parameter causes the results plotted in Figure 11 and Figure 12 to collapse onto a single line.

Figure 11: The pressure reflection increased with increasing vessel radius. This relationship appeared to be linear in the square root of the radius.

Figure 12: The magnitude of the reflected pressure wave decreased geometrically with increasing wall thickness. Once again, the geometry appears to be related to the reflection by the exponent of 0.5.
Figure 13: Combining the inner vessel radius with the wall thickness into a single non-dimensional parameter yielded a linear relationship between the reflection and the geometry of the vessel.

The effect of the stiffness of the stent relative to the vessel was also examined for several stent lengths between 15 to 45 mm. Figure 14 shows the pressure reflections calculated for each of these tests. From inspection of this plot, the relationship appears to be similar to a hyperbolic tangent, with the reflection assumed to be zero for a vessel in which any “stent” had the same compliance as the surrounding tissue, and asymptotically reaching a maximum value as the stiffness increased towards infinite.

Figure 14: The relationship between the pressure reflection versus the stiffness ratio appears to be asymptotic. When plotted on a logarithmic scale, the curve closely resembles the plot of the hyperbolic tangent.
This suggested the development of a new non-dimensional parameter, here referred to as “stent authority,” which might obey a relationship of the form

\[ SA(x) = \tanh \left[ \log \left( b \left( \frac{E_1}{E_0} - 1 \right) + 1 \right) \right]. \tag{6} \]

The stent authority can be calculated at every point in the vessel, and has a value between 0 (no stent or other arterial hardening present) and 1 (infinite stiffness). For the cases examined here the constant \( b \) was calculated to be approximately 2.198.

As can be seen in Figure 15, applying this equation resulted in a series of linear relationships between the peak stent authority (\( E_1 \) is defined as the maximum stiffness for each stent) and the magnitude of the reflected pressure wave. It is apparent that the length of the stented region also plays a role in determining the wave reflection and so, to account for variations in stiffness along the length of the vessel, the “Total Stent Authority” was also calculated by a simple integral,

\[ \text{total } SA = \int_L SA(x)dx, \tag{7} \]

which has the effect of collapsing all the points in Figure 15 onto a single line. This concept of total stent authority also corrects the difficulties encountered in Figure 10 above. Figure 16 depicts the linear relationship between the pressure wave reflection and the total stent authority for each case. The plot includes tests examining variations in stent length, stiffness, and transition zone length.

![Figure 15: For a given stent length, the magnitude of the pressure reflection was observed to increase linearly with the stent authority as measured in the center of the stent. The reflection was more dependent on the stent stiffness when stent length was greater.](image-url)
Figure 16: Total stent authority for each case corresponds very closely to the magnitude of the reflected pressure wave for that case.

The above formula neglects the effects of vessel geometry shown in Figure 11 and Figure 12, however. To account for variability in vessel radius and thickness of the vessel a term was added to the definition of stent authority, yielding an equation for “Geometric Stent Authority,”

\[
GSA = \sqrt{\frac{r_0}{h}} \int_0^L \tanh \left[ \log \left( b \left( \frac{E_i}{E_0} - 1 \right) + 1 \right) \right] dx. 
\]  

(8)

It can be seen from Figure 17 that computing this parameter for each of the cases examined in this paper resulted in a simple linear relationship between the geometric stent authority and the resulting wave reflection.

Figure 17: When the effect of the vessel geometry is included, every case tested collapses onto a single linear relationship between the reflection and the geometric stent authority.
Discussion

The results above demonstrate that variation in intravascular stent design can have a significant effect on the magnitude of the pressure waves reflected from the region of higher compliance created by that stent. Figure 6 showed that for the cases tested here this effect could be at least plus or minus 50% from the reference design. Thus, if a reduction in the pressure wave reflection caused by a stent is desired several factors should be considered in its design.

The first such consideration should be the total length of the stent chosen for implantation. It is clear from Figure 8 that the reflection is proportional to the stent length. As a result, the smallest stent that will provide adequate coverage of the vessel lesion should be chosen when planning an operation. This reinforces the results of several clinical studies that found a clear link between the length of the implanted stent and the likelihood that the patients would suffer from an adverse clinical outcome in the first few months [17,20].

Related to this is the stiffness of the stent itself. As would be expected, the model examined here predicts that the reflection drops off as the compliance of the stent approaches that of the vessel wall. This suggests that it would be wise to design stents to be as compliant as possible without compromising the intended function of the prosthesis. This is supported by clinical research [19], and many modern stents are less rigid than older designs.

Additionally, it has been shown that a gradual transition region can act to reduce the mechanical wall stresses imposed upon the artery, possibly improving the acute and chronic response of the vessel to the procedure [8]. One of the goals of this research was to determine if a similar reduction might be obtained for the wave reflection. However, these results show no benefit to a gradual compliance transition region at the ends of the stent, except as it reduces the effective length and total stent authority of the device. The magnitude of the reflection is unaffected by the transition zone length, and a design that remains rigid from end to end will not cause a negative effect.

The results of this model also suggest that the reflections will be greatest in vessels that have a larger radius and smaller wall thickness. It is not clear if there is a limit below which the reflected waves, already small, no longer have a physiological impact on the arterial system. Experimental research needs to be done to examine this issue.

Taken together, examination of the above effects led to the creation of a single a non-dimensional parameter referred to here as the “stent authority.” This number appears to be a measure of how much of the incident pressure energy from the blood flow will be captured and reflected back upstream, relative to the energy the stent imposes upon the vessel. Assuming that experimental testing verifies this relationship, this is an extremely useful result, since it suggests that it is possible to quickly predict the magnitude of the pressure wave that will be reflected from a discontinuity in a vessel with compliant walls. For the conditions tested here, the resulting proportionality constant was approximately 3.46 mmHg per unit of Authority (GSA), but in general the precise relationship will depend on the magnitude of the incident pressure wave as well as the exact shape of the waveform in the time domain.

Another consideration is the relatively small magnitude of the reflected pressure waves compared to the magnitude of the fluctuations normally transmitted downstream in a healthy artery. Typical blood pressures can vary up to 40 mmHg per beat at a frequency of 1 to 2 Hz. The reflections predicted by this model for conditions matching these ranged from approximately 0.035 to 0.42 mmHg, or in other words at most only 1% of the total incident pressure variance is reflected proximally. Also, though not presented here, use of this model predicts similar reflection magnitudes for vessels containing reductions in lumen area. This suggests that
although the presence of irregularities, such as stenoses or coronary stents, along the length of a vessel does in fact lead to reflection of pressure waves back upstream, these reflections may not have much influence in-vivo for determining the subsequent health of an artery. If the presence of abnormal pressure reflections does lead to negative clinical outcomes, the role is probably not purely from the magnitude of the transmitted waves. Instead, it may have more to do with the introduction of sharp gradients in pressure within the vessel, as seen in Figure 7. An examination of the Navier-Stokes equations suggests that these pressure gradients may be related to the generation of vorticity and wall shear stress within the vessel, and can have an effect out of proportion to the associated magnitude of the pressure waves themselves. If this relationship proves valid, the geometric stent authority might then become a useful predictor of not only the magnitude of pressure reflections, but also the vorticity and wall shear stresses induced by the presence of a stent in an artery.

Conclusions

A one-dimensional numerical model for the reflection of pressure waves within a compliant, stented artery was created. The method chosen for this solution was a combination of an optimally 5th order WENO method for spatial derivatives and a 4th order Runge-Kutta method for time derivatives. This model was used to develop several relationships governing the magnitude of the pressure pulse reflected upstream off an implanted stent. It was found that an increase in relative stent stiffness, overall stent length, or lumen radius, or a decrease in arterial wall thickness, could lead to an increase in the magnitude of the wave reflection. These results suggest that to reduce wave reflections physicians should consider using only the smallest stent length that will successfully treat their patient. Additionally, because it was shown that more compliant stents create less wave reflection, designers of new prosthesis should favor designs that offer less rigidity. It was also shown that effect of the stent was proportionally greater in larger vessels. However, in contrast to analysis of mechanical wall stresses, it was not found that allowing a more gradual transition region between the vessel wall and the full stiffness of the stent had any impact on the magnitude of the wave reflection once the effect of the reduction in effective stent length was included in the analysis.

The total effect of all these properties was summarized in a single parameter referred to here as the “geometric stent authority.” This number took the form shown in Equation (8), and appears to relate the energy imposed upon the vessel by the stent to the energy of the reflected pressure wave. If confirmed experimentally, this parameter could offer a simple way to evaluate the magnitude of the pressure wave that will be reflected from a new stent design without needing to perform detailed CFD or other similar numerical simulations.

Work is now planned to experimentally verify the conclusions of this numerical model in-vitro. Also, an extension of this work to axially symmetric or fully 3-D flow, elimination of the inviscid flow assumption, and consideration of the non-linear constitutive behavior of both blood and the arterial walls could provide important insights into additional physical behavior as well as help confirm the conclusions of this study.
References


Appendix A: Simulation Code

The following code was originally written and compiled for FORTRAN. Additional details are available upon request.

```fortran
USE DFLIB

IMPLICIT REAL*8(A-H,O-Z)
parameter(nt=100001,nx=404,ntshort=10001)
common /block1/ x(nx),t(nt)
common /block2u/ u(nx,2)
common /block2p/ p(nx,2)
common /block2a/ A(nx,2)
common /block1/ area(-1:nx),uvel(-1:nx)
common /block3/ pin(nt),uin(nt),ain(nt)
common /block3a/ prr(nt),urr(nt),arr(nt)
common /block4/ fa(nx),fu(nx)
common /block5/ faj(nx,2),fuj(nx,2)
common /block6/ ao(-1:nx),beta(-1:nx),YM(-1:nx)
common /block6a/ r0(nx),c2(nx)
common /block7/ cin(nt),crr(nt)
common /block8t/ tsave(ntshort)
common /block8p/ psave(nx,ntshort)
common /block8u/ usave(nx,ntshort)
common /block8a/ asave(nx,ntshort)
common /block8g/ x_xi(nx),dxi
common /block9/ /dxi
common /blocka/ /ro, rho
common /blockb/ /ro, rho
common /blockc/ /ro, rho
common /blockd/ /ro, rho
character*128 path
integer(2) status
integer(4) num
logical(4) success

namelist /param1/ x1,x2,xst1,xst2,delta,ro,ho,dr

namelist /param2/ cn,n,m

namelist /param3/ pmin,pmax,tp,npulse

namelist /param4/ Eo,E1,rho

ccc SET WORKING DIRECTORY
ccc examine commandline for valid paths
ccc try to change to that directory, quit if fail
ccc default directory is ./

if (NARGS( ).eq.2) then
   CALL getarg(1, path, status)
else
   path   = '.
   status = 1
end if

success = ChangeDirQQ(path)

write(*,* success)
write(*,* path(1:status)
if (success) then
   write(*,'(a)') 'path='//path(1:status)
   write(*,'(a)') 'READING:'
   write(*,'(a)') 'path='//path(1:status)//options.inp'
   write(*,'(a)') 'grid.dat'
   write(*,'(a)') 'WRITING:'
   write(*,'(a)') 'path='//path(1:status)//stiffness.inp'
else
   num = GetLastErrorQQ()
   if (num.eq.ERR$NOMEM) then
      stop 'stopping: ERR$NOMEM'
   elseif (num.eq.ERR$NOENT) then
```

21
write(*,*) path(1:status)//' is not a directory'
stop 'stopping: ERR$NOENT'
else
stop 'stopping: unknown error'
end if
end if

ccc OPEN files from working directory
open(16, file='grid.dat')
read(16,*) n
read(16,*) ximin, ximax
read(16,*) (x(j), j=1,n)
dx1=(ximax-ximin)/dfloat(n-1)
Hg=9.81d0*13.6d0
open(5, file='options.inp')
open(6, file='options.out')
open(7, file='properties.out')
open(8, file='scale.out')
open(9, file='stiffness.inp')
read(5, param1)
read(5, param2)
read(5, param3)
read(5, param4)
if(n.gt.nx) then
write(*,*)' n>nx - insufficient allocation for space variables'
stop
end if
pi=4.d0*datan(1.d0)
pmin=pmin*Hg
pmax=pmax*Hg
read(9,*) (YM(j), j=1,n)
read(10,*) (r0(j), j=1,n)
do 210 j=1,n
YM(j)=YM(j)*Eo
210 continue
ccc testing int division
write(*,*) 1,1
do 1234 j=2,100
write(*,*) j,(j-2)/10+2
1234 continue
ccc Young's modulus as a function of x
Ay=10.d0
By=-15.d0
Cy=6.d0
do 212 j=1,n
YM(j)=Eo
if (x(j).gt.xst1.and.x(j).lt.xst2) then
YM(j)=E1
if (x(j).gt.xst1.and.x(j).lt.(xst1+delta)) then
eta=(x(j)-xst1)/delta
YM(j)=Eo+(E1-Eo)*(AY+By*eta+CY*eta**2)*eta**3
end if
if (x(j).lt.xst2.and.x(j).gt.(xst2-delta)) then
eta=(xst2-x(j))/delta
YM(j)=Eo+(E1-Eo)*(AY+By*eta+CY*eta**2)*eta**3
end if
end if
212 continue
write(6,777) (x(j),YM(j)/Eo,j=1,n)
777 format(2E15.6)
cccc Initial area distribution as a function of x
Assumptions: (1) If the pressure (= pmax) is uniform (not a function of x)
cccc the area, with radius = rmax (to be determined), is assumed to be calculated
cccc by inflation of radius=r0(x) with no stent (YM(j)=YM(1))
cccc (2) If the pressure (= pmin) is uniform (not a function of x)
cccc the area is variable [ = ao(x)] to be determined such that its radius at x=0
is specified by \( r_0 \). If \( YM = \text{constant} \), \( a_0 \) may not equal \( \pi r_0^2 \) everywhen.

First, compute \( r_{max} \):

\[
\begin{align*}
  j &= 1 \\
  dP &= p_{max} - p_{min} \\
  c_1 &= 4.0 \cdot h_0 \cdot dsqrt(\pi)/3.0 \\
  a_0(j) &= \pi r_0(j)^2 \\
  \beta(j) &= c_1 \cdot YM(j)/a_0(j) \\
  r_{max} &= r_0(j) + dP/(dsqrt(\pi) \cdot \beta(j)) \\
  c_2 &= dsqrt(\pi) \cdot r_{max}
\end{align*}
\]

\begin{verbatim}
  ao(j) = pi*r0(j)**2  \\
  beta(j) = c1*YM(j)/ao(j)  \\
  rmax = r0(j) + dP/(dsqrt(pi)*beta(j))  \\
  c2 = dsqrt(pi)*rmax
end do
\end{verbatim}

First, compute \( r_{max} \) (assume \( YM[j] = YM[1] \)):

\[
\begin{align*}
  ao(j) &= \pi r_0(j)^2 \\
  \beta(j) &= c_1 \cdot YM(j)/a_0(j) \\
  r_{max} &= r_0(j) + dP/(dsqrt(\pi) \cdot \beta(j)) \\
  c2(j) &= dsqrt(\pi) \cdot r_{max}
\end{align*}
\]

Second, compute \( a_0(x) \) and then \( \beta(x) \) exactly using quadratic formula:

\[
\begin{align*}
  \text{bracket} &= (c_1 \cdot YM(j)/dP)^2 + 4.0 \cdot c_1 \cdot c_2(j) \cdot YM(j)/dP \\
  \theta &= 1.0/2.0 \cdot (-c_1 \cdot YM(j)/dP + dsqrt(\text{bracket})) \\
  a_0(j) &= \theta \cdot \theta \\
  \beta(j) &= c_1 \cdot YM(j)/(a_0(j) \cdot ao(j)) \\
  r_{x} &= dsqrt(ao(j)/pi) \\
  r_{x1} &= dmax1(r_{x},r_{x1})
\end{align*}
\]

end do

write(" *, *") r_{x1}/ro

OLD METHOD (OBSOLETE):

Second, compute \( \beta(x) \) and \( a_0(x) \), initially assume uniform \( a_0(x) \), and then iterate to improve both \( \beta \) and \( a_0 \):

\begin{verbatim}
  ao(j) = pi*r0**2
end do
sqpi = dsqrt(pi)
c1 = 4.0*ho*dsqrt(pi)/3.0
iterate
  do ki = 1, 10
    do 214 j = 1, n
      beta(j) = c1*YM(j)/ao(j)
    continue 214
    rx1 = 0.0
    do j = 1, n
      rx = r_{max} - (p_{max} - p_{min})/(sqpi*beta(j))
      ao(j) = pi*r**2
      rx1 = dmax1(rx,rx1)
    end do
    write(" *, *") ki,rx1/ro
  end do
\end{verbatim}

open (7, file = 'properties.out')
write (7, 603)
write (7, 604)
603 format('x(j),ao(j),YM(j),beta(j),j=1,n)
& YM(j) (Pa) beta[j] (Pa/m)'
604 format(4e15.6)
change this to a check
dt = 1e9
open (8, file = 'scale.out')
write (8, 201)
write (8, 202)
calculate the timestep needed to fulfill stability
write (*, *) ' requested dt = ', dt
do 216 j = 1, n
bbeta=beta(j)
aao=ao(j)
aaa=ao(j)
call pofa(ppp,aaa,dpda,aao,bbeta,1)
cro=dsqrt(aao*dpda/rho)
dx=x_xi(j)*dxi
dt1=cn*dx/cro
write(8,203) x(j),cro,dt1
dt=dmin1(dt1,dt)
216 continue
write(*,*),'required dt=',dt
ccc find the nskip needed to reduce dtsave to dt
nskip = 1
217 do 217 while (dtsave/nskip.gt.dt)
nskip = nskip + 1
217 enddo
dt = dtsave/nskip
write(*,*),'dt=',dt
write(*,*),'dtsave=',dtsave
write(*,*),'nskip=',nskip
ccc setup time scale and BC's
open(13,file='bc.out')
write(13,607)
write(13,608)
c602 format(i5,4e15.6)
206 format(1e14.6e3)
606 format(i15,4e15.6)
607 format('   i (count)    t (sec)    uin (m/s)',
&      '       pin (Pa)    ain (m^2)',
&      ' ---------------  ---------------  ---------------'
&      ' ---------------  ---------------  ---------------')
c itime   = 1
moffset = 0
if ( (nt).gt.(m) ) then
call calcBCs(1,m,dt)
crites = 1.0d0
write(13,606)(i,t(i),uin(i),pin(i)/hg,ain(i),
& write(13,602)(i,t(i),urr(i),prr(i)/hg,arr(i),
& i=1,m)

else
 call calcBCs(1,nt,dt)
 write(13,606)(i,t(i),uin(i),pin(i)/hg,ain(i),
 & i=1,nt)
 write(13,602)(i,t(i),urr(i),prr(i),arr(i),
 & i=1,nt)
end if

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call pofa(ppp,aaa,dpda,aao,bbeta,-1)
cr1=dsqrt(aaa*dpda/rho)
ppp=p(2,i-1)
aaa=a(2,i-1)
aao=ao(2)
bbeta=beta(2)
call pofa(ppp,aaa,dpda,aao,bbeta,-1)
cr2=dsqrt(aaa*dpda/rho)
glrw1=4.d0*cr1-u(1,i-1)
glrw2=4.d0*cr2-u(2,i-1)

if ( (itime-moffset*nt).gt.(nt) ) then
  moffset = moffset+1
  if ( (itime+nt-1).gt.(m) ) then
    call calcBCs(itime,m,dt)
c    write(20,206) ( t(i),
    c     &      i=itime-nt*moffset,m-nt*moffset,nskip)
c  end if
  write(20,206) ( t(m-nt*moffset),
  c     &      i=itime-nt*moffset,(itime+nt-1)-nt*moffset)
celse
  call calcBCs(itime,(itime+nt-1),dt)
c  write(13,606)(i+nt*moffset,t(i),uin(i),pin(i)/hg,ain(i),
  c     &      i=itime-nt*moffset,(itime+nt-1)-nt*moffset)
cend if
end if
if ( ((itime-2)/nskip+2).gt.((noffset+1)*ntshort) ) then
  dump memory to disk to make more room
write(20,206) ( tsave(i), i=1,ntshort )
write(22,206) ((psave(j,i), j=1,n), i=1,ntshort )
write(23,206) ((asave(j,i), j=1,n), i=1,ntshort )
write(24,206) ((usave(j,i), j=1,n), i=1,ntshort )
noffset = noffset+1
end if

c assuming nskip=10, save routine
c overwrites all but the 11th(2), 21st(3), ....
c similarly for nskip=100, etc
c itime=1 should already be saved
do j=1,n
tsave(((itime-2)/nskip+2-noffset*ntshort) )=t(itime-nt*moffset)
psave(j, ((itime-2)/nskip+2-noffset*ntshort) )=p(j,1)/hg
asave(j, ((itime-2)/nskip+2-noffset*ntshort) )=a(j,1)/ao(1)
usave(j, ((itime-2)/nskip+2-noffset*ntshort) )=u(j,1)
end do

c write(*,*) itime,((itime-2)/nskip+2),t(itime)
1000 continue

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& write(23,206) ((asave(j,i), j=1,n),
& i=1, (m-2)/nskip+2-noffset*ntshort )
& write(24,206) ((usave(j,i), j=1,n),
& i=1, (m-2)/nskip+2-noffset*ntshort )

C206   format(1e14.6e3)
207   format(i5,1e14.6e3)

END

subroutine pofa(p,a,dpda,ao,beta,ind)
    IMPLICIT REAL*8(A-H,O-Z)
    common /pppp/ pmin
    if(ind.eq.1) then
        ccc   given a find p and partial dp/da
        p=beta*(dsqrt(a)-dsqrt(ao))+pmin
        dpda=0.5d0*beta/dsqrt(a)
    else
        ccc   given p find a and partial dp/da
        a=((p-pmin)/beta)+dsqrt(ao)**2
        dpda=0.5d0*beta/dsqrt(a)
    end if
    return
end

subroutine pofak9(p,a,dpda,ao,ind)
    IMPLICIT REAL*8(A-H,O-Z)
    itmax=100
    eps=0.1e-6
    pi=4.d0*datan(1.d0)
    sigma=1172.63d0
    beta=8.36483d0
    rmh=9.5d0
    if(ind.eq.1) then
        ccc   given area, find pressure
        r=dsqrt(a/pi)
        ro=dsqrt(ao/pi)
        rr=r/ro
        expr=dexp(beta*(rr-1.d0))
        top=sigma*(expr-1.d0)
        bot=rmh*rr**2-0.5d0
        p=top/bot
        dadr=2.d0*pi*r
        dtop=sigma*beta*expr/ro
        dbot=2.d0*rmh*r/ro**2
        dpdr=(bot*dtop-top*dbot)/bot**2
        dpda=dpdr/dadr
    else
        ccc   given pressure, find area
        ccc   initial guess for r
        ro=dsqrt(ao/pi)
        rr=1.001
        do 10 it=1,itmax
            bot=rmh*rr**2-0.5d0
            rro=rr
            rr=1.0d0+dlog(1.0d0+(p/sigma)*bot)/beta
            if(dabs(rr-rro).lt.eps) go to 11
            10   continue
        write(6,*) it
        11   continue
        r=rr*ro
        a=pi*r**2
        expr=dexp(beta*(rr-1.d0))
        top=sigma*(expr-1.d0)
        bot=rmh*rr**2-0.5d0
        dadr=2.d0*pi*r
        dtop=sigma*beta*expr/ro
        dbot=2.d0*rmh*r/ro**2
        dpdr=(bot*dtop-top*dbot)/bot**2
        dpda=dpdr/dadr
        end if
        return
    end

subroutine weno (imax1,dt1)
    implicit real*8(a-h,o-z)
    PARAMETER(nx=404,NSTAGE=5)
COMMON /QQQQ/ Q1(-1:NX),Q2(-1:NX),Q3(-1:NX)
COMMON /QPQP/ QP1(NX),QP2(NX),QP3(NX)
common /gggg/ _x_(nx),dx1
COMMON /NIJK/ IMAX
COMMON /STEP/ DX,DT,CFL
common /blk1/ area(-1:nx),uvel(-1:nx)
common /blk2/ rhs(nx,2)
common /blk3/ ark(0:nstage),brk(nstage)
imax=imax1
dt=dt1
ark(0)=0.d0
ark(1)=0.141789047d0
ark(2)=-1.92115169d0
ark(3)=-1.69784469d0
ark(4)=-1.51418344d0
brk(1)=0.14965902d0
brk(2)=0.37921031d0
brk(3)=0.8295502d0
brk(4)=0.69945045d0
brk(5)=0.15305724d0
call weno_points
DO 34 I=-1,IMAX+2
  Q1(I)  = area(i)
  Q2(I)  = uvel(i)
34   CONTINUE
CALL RK45
DO 50 I=2,IMAX-1
  area(i)= Q1(I)
  uvel(i)= Q2(I)
50   CONTINUE
return
end

subroutine RK45
implicit real*8(a-h,o-z)
PARAMETER(nx=404,NSTAGE=5)
COMMON /QQQQ/ Q1(-1:NX),Q2(-1:NX),Q3(-1:NX)
COMMON /QPQP/ QP1(NX),QP2(NX),QP3(NX)
common /gggg/ _x_(nx),dx1
COMMON /STEP/ DX,DT,CFL
COMMON /NIJK/ IMAX
common /blk2/ rhs(nx,2)
common /blk3/ ark(0:nstage),brk(nstage)
DO 150 KK=1,NSTAGE
  call res
  DO 140 I=2,IMAX-1
    qp1(i)=rhs(i,1)+ark(kk-1)*qp1(I)
    qp2(i)=rhs(i,2)+ark(kk-1)*qp2(I)
 140 CONTINUE
  DO 143 I=2,IMAX-1
    Q1(i)=q1(i)+dt*brk(kk)*qp1(I)
    Q2(i)=q2(i)+dt*brk(kk)*qp2(I)
 143 CONTINUE
150 CONTINUE
return
END

subroutine weno_points
implicit real*8(a-h,o-z)
PARAMETER(nx=404,mn=2)
COMMON /NIJK/ IMAX
common /block6/ ao(-1:nx),beta(-1:nx),YM(-1:nx)
common /blk1/ area(-1:nx),uvel(-1:nx)
do j=1,2
  ao(j-2)=ao(1)
  beta(j-2)=beta(1)
  area(j-2)=area(1)
  uvel(j-2)=uvel(1)
  ao(imax+j)=ao(imax)
  beta(imax+j)=beta(imax)
  area(imax+j)=area(imax)
  uvel(imax+j)=uvel(imax)
end do
return
end

cccccccccccccccccccccccccccccccccccccc
cc Compact scheme
subroutine res

implicit real*8(a-h,o-z)
PARAMETER(nx=404,mm=2)
COMMON /QQQQ/ Q1(-1:NX),Q2(-1:NX),Q3(-1:NX)
COMMON /STEP/ DX,DT,CFL
COMMON /NIJK/ IMAX
common /block6/ ao(-1:nx),beta(-1:nx),YM(-1:nx)
common /qqgg/ x_xi(nx),dx1
common /dddd/ cro,rho
common /blk2/ rhs(nx,2)
common /work1/ split,press,Fnp,Fnm
common /work2/ fp1,fp2,fp3,sp1,sp2,sp3
common /work3/ fml,fm2,fm3,sm1,sm2,sm3
common /work4/ FL1,FL2,FL3
common /work5/ FR1,FR2,FR3
common /work6/ wfl,wr
common /work7/ fn
dimension split(-1:nx),press(-1:nx)
dimension Fnp(-2:2,-1:nx,mm),Fnm(-2:2,-1:nx,mm)
dimension spl(-1:nx,mm),sp2(-1:nx,mm),sp3(-1:nx,mm)
dimension fml(-1:nx,mm),fm2(-1:nx,mm),fm3(-1:nx,mm)
dimension sml(-1:nx,mm),sm2(-1:nx,mm),sm3(-1:nx,mm)
dimension fl1(-1:nx,mm),fl2(-1:nx,mm),fl3(-1:nx,mm)
dimension frl(-1:nx,mm),fr2(-1:nx,mm),fr3(-1:nx,mm)
dimension wfl(-1:nx,mm),wr(-1:nx,mm)
dimension fn(-1:nx,mm)
eps=1.d-6

do 112 i=-1,imax+1
A = 0.5d0*(q1(i)+q1(i+1))
bbeta = 0.5d0*(beta(i)+beta(i+1))
aao = 0.5d0*(ao(i)+ao(i+1))
dpda = 0.5d0*bbeta/ddsqrt(A)
C = ddsqrt(A*dpda/rho)

following line caused problems, orginally u=0
U = 0.5d0*(q2(i)+q2(i+1))
U = 0
split(i) = 1.0d0*(abs(u)+c)

112 continue

do 114 i=-1,imax+2
A = q1(i)
bbeta = beta(i)
aao = ao(i)
p=bbeta*(ddsqrt(A)-ddsqrt(aao))+pmin
press(i)= p

114 continue

do 120 i=-1,imax-0

do 120 j=-2,2
A = q1(i+j)
U = q2(i+j)
bbeta = beta(i+j)
aao = ao(i+j)
p = press(i+j)

Fnp(j,i,1)  = U*A              +split(i)*A
Fnp(j,i,2)  = 0.5d0*U*U+P/rho  +split(i)*U
Fnm(j,i,1)  = U*A              -split(i-1)*A
Fnm(j,i,2)  = 0.5d0*U*U+P/rho  -split(i-1)*U

120 continue

do 100 m=1,mm

do 100 i=1,imax-0

fp1(i,m)= 0.5d0*(fnp(-2,i,m)-4.d0*fnp(-1,i,m)+3.d0*fnp(0,i,m))
fp2(i,m)= 0.5d0*(fnp( 1,i,m)+fnp(1,i,m))
fp3(i,m)= 0.5d0*(fnp( 2,i,m)-4.d0*fnp( 1,i,m)+3.d0*fnp(0,i,m))

sp1(i,m)= fnp(-2,i,m)-2.d0*fnp(-1,i,m)+fnp( 0,i,m)
sp2(i,m)= fnp( 1,i,m)-2.d0*fnp( 0,i,m)+fnp(-1,i,m)
sp3(i,m)= fnp( 2,i,m)-2.d0*fnp( 1,i,m)+fnp(-1,i,m)

fm1(i,m)= 0.5d0*(fnm(-2,i,m)-4.d0*fnm(-1,i,m)+3.d0*fnm(0,i,m))
fm2(i,m)= 0.5d0*(fnm( 1,i,m)+fnm(1,i,m))
fm3(i,m)= 0.5d0*(fnm( 2,i,m)-4.d0*fnm( 1,i,m)+3.d0*fnm(0,i,m))

sm1(i,m)= fnm(-2,i,m)-2.d0*fnm(-1,i,m)+fnm( 0,i,m)
sm2(i,m)= fnm( 1,i,m)-2.d0*fnm( 0,i,m)+fnm(-1,i,m)
sm3(i,m)= fnm( 2,i,m)-2.d0*fnm( 1,i,m)+fnm( 0,i,m)

100 continue

hh=0.5d0
hh2=1.d0/8.d0

c hh=0.0d0
ccc
hh2=0.0d0/8.0d0
ccc
do 200 m=1,mm
ccc
do 200 i=1,imax-0
ccc
fL1(i,m)=fnp(0,i,m)+hh*fp1(i,m)+hh2*sp1(i,m)
ccc
fL2(i,m)=fnp(0,i,m)+hh*fp2(i,m)+hh2*sp2(i,m)
ccc
fL3(i,m)=fnp(0,i,m)+hh*fp3(i,m)+hh2*sp3(i,m)
ccc
fR1(i,m)=fnm(0,i,m)-hh*fm1(i,m)+hh2*sm1(i,m)
ccc
fR2(i,m)=fnm(0,i,m)-hh*fm2(i,m)+hh2*sm2(i,m)
ccc
fR3(i,m)=fnm(0,i,m)-hh*fm3(i,m)+hh2*sm3(i,m)

200 continue
ccc
CL1=1.d0/16.d0
ccc
CL2=10.d0/16.d0
ccc
CL3=5.d0/16.d0
ccc
cr1=5.d0/16.d0
ccc
cr2=10.d0/16.d0
ccc
cr3=1.d0/16.d0
ccc
do 300 m=1,mm
ccc
do 300 i=1,imax-0
ccc
rL1=(eps+fp1(i,m)**2+sp1(i,m)**2)**2
ccc
rL2=(eps+fp2(i,m)**2+sp2(i,m)**2)**2
ccc
rL3=(eps+fp3(i,m)**2+sp3(i,m)**2)**2
ccc
rR1=(eps+fm1(i,m)**2+sm1(i,m)**2)**2
ccc
rR2=(eps+fm2(i,m)**2+sm2(i,m)**2)**2
ccc
rR3=(eps+fm3(i,m)**2+sm3(i,m)**2)**2
ccc
betaL1=CL1/rL1
ccc
betaL2=CL2/rL2
ccc
betaL3=CL3/rL3
ccc
betaR1=CR1/rR1
ccc
betaR2=CR2/rR2
ccc
betaR3=CR3/rR3
ccc
sumL=betaL1+betaL2+betaL3
ccc
omegal1=betaL1/sumL
ccc
omegal2=betaL2/sumL
ccc
omegal3=betaL3/sumL
ccc
sumR=betaR1+betaR2+betaR3
ccc
omegar1=betaR1/sumR
ccc
omegar2=betaR2/sumR
ccc
omegar3=betaR3/sumR
ccc
wfL(i,m)=omegal1*fL1(i,m)+omegal2*fL2(i,m)
ccc
&+omegal3*fL3(i,m)
ccc
wfR(i,m)=omegar1*fR1(i,m)+omegar2*fR2(i,m)
ccc
&+omegar3*fR3(i,m)
ccc
300 continue
ccc
do 500 m=1,mm
ccc
do 500 i=1,imax-1
ccc
fn(i,m)=0.5d0*(wfL(i,m)+wfR(i+1,m))
ccc
500 continue
ccc
visc=0.0d-0/rho
ccc

do 544 m=1,mm
ccc
do 544 i=2,imax-1
ccc
dx=x_xi(i)*dxi
ccc
rhs(i,1)=-(fn(i,1)-fn(i-1,1))/dx
ccc
rhs(i,2)=-(fn(i,2)-fn(i-1,2))/dx
ccc
$ -visc*q2(i)/q1(i)
ccc
544 continue
ccc
return
ccc
end
ccc
ccccccccccccccccccccccccccccccccccccccccccc
ccc Original Explicit? Method
ccc
ccccccccccccccccccccccccccccccccccccccccccc
subroutine res2
ccc
implicit real*8(a-h,o-z)
ccc
PARAMETER(nx=404,mm=2)
ccc
COMMON /QQQQ/ Q1(-1:NX),Q2(-1:NX),Q3(-1:NX)
ccc
COMMON /STEP/ DX,DT,CFL
ccc
COMMON /NIJK/ IMAX
ccc
common /pppp/ pmin
ccc
common /block6/ ao(-1:nx),beta(-1:nx),YM(-1:nx)
ccc
common /gggg/ x_xi(nx),dxi
ccc
common /dddd/ cro,rho
ccc
common /blk2/ rhs(nx,2)
ccc
common /work1/ split,press,Fnp,Fnm
ccc
common /work2/ fp1,fp2,fp3,sp1,sp2,sp3
ccc
common /work3/ fm1,fm2,fm3,sm1,sm2,sm3
ccc
common /work4/ fL1,fL2,fL3
ccc
common /work5/ fR1,fR2,fR3
ccc
common /work6/ wfl,wfr
ccc
common /work7/ fn
ccc
dimension split(-1:nx),press(-1:nx)
dimension Fnp(-2:2,-1:nx,mm),Fnm(-2:2,-1:nx,mm)
dimension fp1(-1:nx,mm),fp2(-1:nx,mm),fp3(-1:nx,mm)
dimension spl(-1:nx,mm),sp2(-1:nx,mm),sp3(-1:nx,mm)
dimension fm1(-1:nx,mm),fm2(-1:nx,mm),fm3(-1:nx,mm)
dimension sm1(-1:nx,mm),sm2(-1:nx,mm),sm3(-1:nx,mm)
dimension fL1(-1:nx,mm),fL2(-1:nx,mm),fL3(-1:nx,mm)
dimension fR1(-1:nx,mm),fR2(-1:nx,mm),fR3(-1:nx,mm)
dimension wfl(-1:nx,mm),wFr(-1:nx,mm)
dimension fn(-1:nx,mm)
eps=1.d-6
do 112 i=-1,imax+1
  A = 0.5d0*(q1(i)+q1(i+1))
  bbeta = 0.5d0*(beta(i)+beta(i+1))
  aao = 0.5d0*(ao(i)+ao(i+1))
  dpda = 0.5d0*bbeta/dsqrt(A)
  C = dsqrt(A*dpda/rho)
  U = 0
  split(i) = 1.0d0*(abs(u)+c)
112  continue
ccc  flux splitting
do 114 i=-1,imax+2
  A = q1(i)
  bbeta = beta(i)
  aao = ao(i)
  p=bbeta*(dsqrt(A)-dsqrt(aao)) + pmin
  press(i) = p
114  continue
do 120 i=1,imax-0
  j=-2,2
  A = q1(i+j)
  U = q2(i+j)
  bbeta = beta(i+j)
  aao = ao(i+j)
  p = press(i+j)
  Fnp(j,i,1) = U*A + split(i)*A
  Fnp(j,i,2) = 0.5d0*U*U+P/rho + split(i)*U
  Fnm(j,i,1) = U*A - split(i-1)*A
  Fnm(j,i,2) = 0.5d0*U*U+P/rho - split(i-1)*U
120  continue
ccc
do 100 m=1,mm
  do 100 i=1,imax-0
    fp1(i,m)= 0.5d0*(fnp(-2,i,m)-4.d0*fnp(-1,i,m)+3.d0*fnp(0,i,m))
    fp2(i,m)= 0.5d0*(fnp( 1,i,m)-     fnp(-1,i,m)                )
    fp3(i,m)=-0.5d0*(fnp( 2,i,m)-4.d0*fnp( 1,i,m)+3.d0*fnp(0,i,m))
  100  continue
hh=0.5d0
hh2=1.d0/8.d0
c
hh=0.0d0
c
hh2=0.d0/8.d0

do 200 m=1,mm
  do 200 i=1,imax-0
    fl1(i,m)=fnp(0,i,m)+hh*fp1(i,m)+hh2*sp1(i,m)
    fl2(i,m)=fnp(0,i,m)+hh*fp2(i,m)+hh2*sp2(i,m)
    fl3(i,m)=fnp(0,i,m)+hh*fp3(i,m)+hh2*sp3(i,m)
    fm1(i,m)=fnp(0,i,m)+hh*fm1(i,m)+hh2*sm1(i,m)
    fr1(i,m)=fnp(0,i,m)+hh*fm2(i,m)+hh2*sm2(i,m)
    fr2(i,m)=fnp(0,i,m)+hh*fm3(i,m)+hh2*sm3(i,m)
  200  continue
ccc
CL1=1.d0/16.d0
CL2=10.d0/16.d0
CL3=5.d0/16.d0
CR1=5.d0/16.d0
CR2=10.d0/16.d0
CR3=1.d0/16.d0
do 300 m=1,mm
  do 300 i=1,imax-0
    rl1=(eps*fp1(i,m)**2+sp1(i,m)**2)**2
    rl2=(eps*fp2(i,m)**2+sp2(i,m)**2)**2
  300  continue
\[
\begin{align*}
rl3 &= (\varepsilon + fp3(i,m)^2 + sp3(i,m)^2)^2 \\
rr1 &= (\varepsilon + fm1(i,m)^2 + sm1(i,m)^2)^2 \\
rr2 &= (\varepsilon + fm2(i,m)^2 + sm2(i,m)^2)^2 \\
rr3 &= (\varepsilon + fm3(i,m)^2 + sm3(i,m)^2)^2 \\
\beta l1 &= CL1/rl1 \\
\beta l2 &= CL2/rl2 \\
\beta l3 &= CL3/rl3 \\
\beta r1 &= CR1/rr1 \\
\beta r2 &= CR2/rr2 \\
\beta r3 &= CR3/rr3 \\
\sum l &= \beta l1 + \beta l2 + \beta l3 \\
\omega l1 &= \beta l1/\sum l \\
\omega l2 &= \beta l2/\sum l \\
\omega l3 &= \beta l3/\sum l \\
\sum r &= \beta r1 + \beta r2 + \beta r3 \\
\omega r1 &= \beta r1/\sum r \\
\omega r2 &= \beta r2/\sum r \\
\omega r3 &= \beta r3/\sum r \\
wfl(i,m) &= \omega l1*fL1(i,m) + \omega l2*fL2(i,m) + \omega l3*fL3(i,m) \\
wfr(i,m) &= \omega r1*fR1(i,m) + \omega r2*fR2(i,m) + \omega r3*fR3(i,m)
\end{align*}
\]
common /nnnn/ npulse, n

write(*,*) 'calcBCs, istart=',istart,', iend=',iend
pi=4.d0*datan(1.d0)
do 20 i=istart,iend
  t(i-istart+1)=dt*(i-1)
20 continue

cccc  boundary condition at j=1, right-running pulse with (pmin-pmax)
cccc  ap is wrong if alpha is not 1
alpha=1.d0
  ap=(pmax-pmin)*dsqrt(2.d0)*dexp(alpha*pi/4.d0)
  tpt=npulse*tp
do 14 i=1,iend-istart+1
  ipulse=t(i)/tp+1
  if(ipulse.le.npulse) then
    taw=pi*(t(i)-float(ipulse-1)*tp)/tp1
    if(taw.gt.pi) then
      pin(i)=pmin
    else
      taw1=0.25d0*taw/0.165d0
      if(taw1.gt.pi) taw1=pi
      pin(i)=pmin+ap*dexp(-alpha*taw)*dsin(taw1)
    end if
  end if
14 continue

ccc  compute area corresponding to inflow pressure pulse
aa0=ao(1)
bbeta=beta(1)
aaa=ao(1)
call pofa(ppp,aaa,dpda,aa0,bbeta,-1)
cro=dsqrt(aa0*dpda/rho)
do 114 i=1,iend-istart+1
  ppp=pin(i)
call pofa(ppp,aaa,dpda,aa0,bbeta,-1)
cr=dsqrt(aaa*dpda/rho)
  ain(i)=aaa
  uin(i)=(pin(i)-pmin)/(rho*cr)
  cin(i)=cr
  uin(i)=4.d0*(cin(i)-cro)
114 continue

cccc  boundary condition at j=n, left-running pulse with zero amplitude!!
ap=0.d0
aa0=ao(n)
bbeta=beta(n)
aaa=ao(n)
call pofa(ppp,aaa,dpda,aa0,bbeta,-1)
cro=dsqrt(aa0*dpda/rho)
do 16 i=1,iend-istart+1
  ppp=prr(i)
call pofa(ppp,aaa,dpda,aa0,bbeta,-1)
cr=dsqrt(aaa*dpda/rho)
  arr(i)=aaa
  crrr(i)=cr
  urrr(i)=(prr(i)-pmin)/(rho*cr)
16 continue
return
END
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
Vita

John J. Charonko, III was born on October 8, 1979 in Martinsburg, West Virginia. At the age of five he moved to Charleston, West Virginia, where he remained until his graduation from George Washington High School in 1998. After graduation, John attended college at Virginia Tech in Blacksburg, Virginia, where he was awarded the Dan H. Pletta Award for Most Outstanding Senior Project. He graduated Summa Cum Laude in the Spring of 2002 and earned the degree of Bachelor of Science from the Department of Engineering Science and Mechanics, with minors in Computer Science and Mathematics. John remained at Virginia Tech for his Master’s research, and is currently pursuing a Ph.D. in Biomedical Engineering. In addition to his primary research in computational modeling and cardiovascular fluid mechanics, his research has also included experimental work in flow control and the application of Digital Particle Image Velocimetry (DPIV) to a wide variety of experimental conditions. His hobbies and interests include downhill skiing, swing dancing, music, and computers.