Wall shear stress measurements in arterial flows

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ABSTRACT

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Cardiovascular disease is responsible for the majority of morbidity and mortality in the United States. Physiologically healthy flow rarely displays turbulent behavior, thereby maintaining normal shear levels. The presence of vortical flow structures, however, alters the hemodynamical characteristics within the system, which has significant effect upon shear stress (SS) and wall shear stress (WSS) levels, as well as particle residence times. The relationship between these hemodynamic parameters and vascular injury response is of great relevance to understanding the cardiovascular disease process.

In this work, new methods and algorithms are developed and presented for resolving, both globally and locally, the spatial and temporal variations of shear stress (SS) and WSS for in vitro models of the human cardiovascular system. Advancements in global measurements are based on improving the accuracy of SS and WSS estimation from time-resolved Digital Particle Image Velocimetry (DPIV) velocity measurements. A new velocity derivative method, the fourth-order noise-optimized compact-Richardson implicit scheme, has been developed, overcoming the obstacle of minimizing both the bias and random error in temporal/spatial derivative estimations. The resulting error is on the same order as the velocity measurement error for global measurements which results in an order of magnitude accuracy improvement. The method has been extended to WSS measurements, and combined with a new method of mirroring/reflecting a flow field over its boundary in order to achieve higher-order estimation. For moving boundaries an edge detection cross-correlation algorithm has been developed and characterized, yielding sub-pixel accuracy in measuring dynamic wall position prior to estimating WSS. An original microelectromechanical system (MEMS) WSS sensor capable of delivering high sensitivity, frequency response and accurate WSS measurements has been developed and characterized in this work.
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Graduate school for me has been an uphill climb, filled with roadblocks and pitfalls. Without the support of my family and friends, I would not have survived. So, I would like to thank my family for their support, especially my brother, who has been suffering through graduate school himself. Thank you to my parents for raising me to be a successful person. Thank you Jennifer, for your support over the years. Thank you to all of my friends (Donkey, Yazacus, Olga, Jose, The Brade, Dubaya, Steven, Chardonkalonk, Karri, Sam, Clay, Kevin, Potter, etc. – there are too many of you to mention). Thank you to my committee: Dr. Pavlos Vlachos, you have supported me through all these years; Dr. Demetri Telionis, without your kind words surely I would not have made it through to this date, Dr. Joel Berry, your commitment to my education is much appreciated, Dr. Donald Leo, your easy going nature has always made you a good mentor, and Dr. YongWoo Lee, your dedication to your research and your students is inspiring.

And now I can finally move on to my regrets. Graduate school, like any institution for higher education, is intended to be an environment for learning and enrichment. Based upon my observations and experiences, however, graduate school is more of a job type environment that tests one's limits and pushes them as far as they can bend before they break. A PhD comes at an opportunity cost, so I will now express what that cost was to me in the form of my regrets over the course of my PhD. My first regret is all of the times I have had to neglect the aforementioned people that I acknowledged. Most of them have been very good to me and I regret not being able to spend more time with them. Also, I regret not having enough time to continue my efforts as the President of BookAid, a foundation that I founded at Virginia Tech to send used textbooks overseas to universities in third world countries. Over the first year of BookAid’s existence I managed to send over 5,000 textbooks overseas. Since then, I have learned quickly that unless it is going in my dissertation, then it is not a priority in graduate school. There are surely hundreds of students who I could have better served in this respect had it not been a physical impossibility for me to continue my contributions, and thus, I apologize to those that could have had better resources. Second, I regret that my health has suffered due to agonizing stress levels and month after month without proper sleep, which for the successful graduate student is supposed to be 4 to
6 hours. Furthermore, I regret that my lung function has become so poor over the past one-and-a-half years from the chronic bronchitis I have developed due to this stress and exposure to particle filled environments that I have had to go on strong medications. This brings me to my next regret, which is that for the most part I have had to give up playing soccer competitively because of my compromised lung function, and I may not ever be able to play competitively at any level again. Finally, I should point out that I do regret not truly becoming a biomedical engineer. Throughout my education I had aspirations of saving lives through biomedical research. However, my work at the Complex Thermo-Fluid Sciences Laboratory has prepared me for more of a fundamental fluid mechanics research position, and as such I plan to work for the Navy.
Chapter 1

1 BACKGROUND AND INTRODUCTION

Cardiovascular disease is responsible for the majority of morbidity and mortality in the United States. While physiologically healthy flow shows few detrimental affects, if any, it is well known that certain abnormal flow patterns can initiate and accelerate atherosclerotic disease progression. Physiologically healthy flow rarely displays turbulent behavior, thereby maintaining normal shear levels. The presence of vortical flow structures, however, alters the mechanical environment within the vasculature, particularly fluid-fluid shear stress (SS) and wall shear stress (WSS). Abnormal SS levels can be detrimental to blood particulates, and if high enough can damage red blood cells (RBCs) and activate platelets, leading to thrombus and/or plaque formation, while abnormal WSS levels can have a detrimental effect on endothelial cells (ECs). Abnormally low or complex spatio-temporally varying shear levels are found in bifurcations, curved vessels, and stenoses, and are most susceptible to atherosclerosis (Kleinstreuer et al, 2001; Liepsch, 2002; Shalman et al, 2002; Resnick et al, 2003; Sho et al, 2004; Passerini et al, 2003; Silber et al, 2001; Severyn et al, 2004; Takeda et al, 2001; Scholz and Wolfgang, 2004). The complexities of the systems of interest and limited technologies have made it difficult to quantify SS and WSS levels experimentally for these flows. Novel methodologies with high spatial and temporal resolution will more readily allow SS and WSS quantification for healthy and pathological cardiovascular flows using experimental fluid mechanics techniques. These results will be essential to the validation of analytic and numerical models.

Herein, we use Digital Particle Image Velocimetry (DPIV), a state-of-the-art global flow measurement system that allows provides planar velocity measurements with high spatial and temporal resolution, making it extremely useful for cardiovascular flows. Furthermore, advances in sensor technologies have resulted in the development of new classes of microelectromechanical systems (MEMS) for WSS measurement. However, the available sensors do not provide robust, dynamic WSS measurements, making them almost inapplicable to biological flows. This work presents the development and characterization of novel methodologies and technologies for
quantifying SS and WSS for steady and pulsatile flows in *in-vitro* experiments. First, velocity derivative estimation is investigated to determine the accuracy associated with fluid-fluid SS. A fourth-order noise-optimized compact-Richardson implicit scheme has been developed for velocity derivative estimation, overcoming the obstacle of minimizing both the bias and random error in temporal/spatial derivative estimations. The resulting error is on the same order as the velocity measurement error for global measurements which results in an order of magnitude accuracy improvement. After quantifying the accuracy associated with fluid-fluid SS, the presence of a boundary is considered, in order to extend the developed methods to estimation of WSS. WSS estimation accuracy is broken down into three components: (1) boundary point detection; (2) near-wall velocity measurement; and (3) wall velocity derivative estimation. For moving boundaries, an edge detection cross-correlation algorithm has been developed and characterized, yielding sub-pixel accuracy in measuring dynamic wall position prior to estimating WSS. Near-wall velocity measurement accuracy is quantified for laminar pipe flow, showing large bias error due to the high shear. A method of mirroring/reflecting a flow field over its boundary is proposed for achieving higher-order estimation of WSS, resulting in accuracy on the order of the near-wall velocity measurement uncertainty. Finally, an original microelectromechanical system (MEMS) WSS sensor based on ionic polymer transducers (IPTs) was developed, allowing direct, dynamic WSS measurements within complex biological flows.

The application of the tools developed herein is validated for physiologically relevant flows, including stenotic flow conditions and flow over endothelial cell (EC) monolayers. This effort provides a set of tools to guide future research with potential application to *in-vivo* measurements, as well as to validate the results of previous works in the literature. This chapter serves as a brief introduction to the analysis of complex wall-bounded flows in the cardiovascular system, and provides an outline of this dissertation as follows. Chapters 2 through 4 present the development and characterization of the tools necessary for investigating shear stresses in biological flows, and Chapters 5 and 6 show the major applications of the tools in this effort. The following sections provide summaries of each chapter, including the major contributions.
1.1 Chapter 2: Improvements on the accuracy of derivative estimation from DPIV velocity measurements

Velocity derivative estimation from time resolved DPIV flow fields plays an integral role in the analysis of the shear levels acting upon blood constituents, including platelets and red blood cells. The magnitudes of these quantities determine their physiological responses, which can lead to the formation of aggregates and potentially atherosclerotic lesions. The method utilized has a significant effect upon the estimation accuracy. Previous work by Fouras and Soria (1998), Luff et al. (1999), and Foucaut and Stanislas (2002) evaluated the performance of several conventional velocity derivative evaluation methods, including finite-difference, least square fit direct velocity differentiation, Richardson extrapolation, and higher-order compact schemes. All three studies concluded that any improvement in bias error using higher-order methods is outweighed by the effect of random error noise propagation. Thus, it is important to develop and utilize novel methodologies that achieve higher accuracy with lower-order noise amplification. The methods developed in this task will be utilized throughout this effort in order to optimize the accuracy of the in-vitro measurements.

Accuracy of SS estimation from in-plane experimental velocity measurements is investigated with particular application to DPIV. Simulations of known flow fields are used to quantify and predict, a priori, errors associated with velocity measurement noise amplification and method bias error due to spatial sampling resolution. A novel, adaptable, hybrid estimation scheme combining compact finite difference and Richardson extrapolation schemes is proposed for improved derivative estimation. The scheme delivers higher-order truncation error with minimal (better than second-order) noise amplification. This effort has been published as “Improvements on the accuracy of vorticity estimation from DPIV velocity measurements” in Experiments in Fluids.

1.2 Chapter 3: Improvements on the accuracy of WSS estimation from DPIV

WSS estimation is critical to the analysis of cardiovascular flows, due to its relevance to the initiation and progression of cardiovascular disease. Abnormal WSS levels can lead to injury to ECs lining the vessel walls, making it susceptible to plaque formation. WSS estimation is analyzed in terms of the following three components: (a) determination of the exact boundary points; (b) DPIV velocity measurement uncertainty in the near wall region; and (c) velocity derivative error.
The knowledge obtained and methods developed will be utilized throughout this effort in order to optimize the accuracy of the in-vitro measurements. The first aim of this effort is to quantify the accuracy associated with determination of boundary position. Edge-detection routines, such as Canny’s algorithm, Sobel method, or Laplacian of Gaussian, have traditionally been implemented to determine boundary point location. These methods give discrete edge information, offering pixel accuracy only. A gradient cross-correlation method that uses image gradient information in a statistical cross-correlation has been developed, yielding sub-pixel accuracy. The second aim is to quantify the accuracy of wall velocity estimation from DPIV. To this end, quantification of the bias and random errors of DPIV data sets has been conducted for Poiseuille (steady pipe) flow. The third aim of this effort is to quantify and reduce the uncertainty of velocity derivatives for wall shear estimations. Although previous efforts and Chapter 2 address the issue of vorticity estimation uncertainty, the accuracy of wall shear stresses for bounded flows has yet to be quantified. The utilization of higher-order schemes has been explored. In particular, implicit compact finite difference schemes are employed as alternatives to conventional schemes, for achieving higher accuracy in determining velocity derivatives from discrete flow field data. The use of higher-order methods is shown to be beneficial in the reduction of the bias error associated with these measurements. However, the accuracy is compromised by errors in near-wall velocity measurement. Part of this chapter has been published as a conference technical paper in the ASME 2004 Fluids Engineering Division conference.

1.3 Chapter 4: A dynamic wall shear stress sensor based on ionic polymer transducers

Current WSS sensor technologies are unable to provide accurate, direct, dynamic measurements in biological flows. Furthermore, they are not robust enough to be used in particulate rich environments such as blood flow through arteries. In this effort, an active ionic polymer MEMS-WSS sensor has been designed, calibrated, and implemented for dynamic WSS measurements in in-vitro studies of healthy and pathological vascular flows. These dynamic measurements are compared with dynamic DPIV measurements described in the previous sections. The two measurement modalities are used in parallel to quantify vortical and turbulent flow structures within the vasculature. In particular, measurement accuracy in dynamic shear on the order of 4.92% with respect to a full scale measurement of +/-3Pa and frequency response up
to 140 Hz is demonstrated. Measurements in a stenosed flow and through a mechanical heart valve implant reveal disturbed flow conditions, including elevated shear levels and high-frequency WSS fluctuations. This chapter has been submitted to *Experiments in Fluids* and is under review. The device has been submitted for intellectual property disclosure and is in the patent process. Further development is necessary to apply these sensors to *ex-vivo* or *in-vivo* investigations.

1.4 Chapter 5: In-vitro investigation of flow through stenotic vessels – coronary flow

Coronary flow requires careful consideration as the flow and pressure waveforms are distinctly out of phase due to the systolic contraction of the myocardium and aortic valve leaflet position. Quantification and identification of vortical and turbulent flow structures in coronary arteries has been performed for comparison with and validation of analytical/numerical models and in-vivo experiments. DPIV is employed to deliver global, time-resolved flow field measurements with kilohertz sampling. This is the first experiment to deliver sufficiently high spatial and temporal resolution to justify comparison with and validation of numerical and analytical studies of similar flows.

For these experiments, the two relevant parameters governing the flow waveforms are the Reynolds number and Womersley parameter. Based upon the analysis of Berry et al. (2000), the flow waveform and pulse rate are dynamically scaled for Re ranging between 150 and 450 and $\alpha$ ranging between 2.7 and 3.7. These ranges represent resting and exercise conditions, and are investigated for both healthy and stenosed vessels. Four mm transparent elastic vessels have been fabricated with physiological compliance for the experiment. A glycerin-water mixture was used to match the viscosity of blood, while maintaining optical clarity. Flow alterations due to 50% and 75% (by diameter) glass stenoses were investigated. Analysis of the flow induced shear stress levels within the vessel and on the vessel walls is presented to provide qualitative and quantitative justification for the relationship between vortical flow structures, shear stresses, and atherosclerotic disease progression. For the 75% stenosis, transition to turbulence is noted due to the interaction between the shear layer formed between the jet issued from the neck of the stenosis and the separation regions downstream. Kelvin-Helmholtz instabilities that arise within the shear layer cause vortices to grow and shed from the shear layer. Lowered shear levels are noted in the
1.5 Chapter 6: Characterization of near-wall flow over endothelial cell monolayers

The interaction between blood, ECs, and the vessel wall presents significant challenges in determining the role of fluid induced WSS on EC mechanotransduction. WSS has been directly related to EC injury, platelet aggregation, and ultimately atherosclerotic lesions. The presence of an EC monolayer, due to its morphology and surface chemistry, vastly alters the near-wall fluid dynamics, thereby changing the WSS magnitude to which the layer is subjected under both healthy and disturbed conditions. Global flow quantification of near-wall fluid mechanics over an EC monolayer lined surface versus a control surface has been performed. The control surface has no coating, while the test surface is coated with an EC monolayer. The two surfaces form a rectangular microchannel. A schematic of the experimental setup is shown in Figure 1.1. The experiment was carried out for healthy cells and cells treated for 24 hours with tumor necrosis factor-α (TNF-α) to simulate an inflamed or diseased condition. DPIV measurements were conducted within the microchannel with a resolution of 5 microns/pixel to yield measurements as close as 20 microns from the wall. WSS reduction on the order of 45% was observed on the EC monolayer coated surface.

![Figure 1.1. Effect of EC monolayer on near-wall fluid mechanics](image)

1.6 References


2 IMPROVEMENTS ON THE ACCURACY OF DERIVATIVE ESTIMATION FROM DPIV VELOCITY MEASUREMENTS

This chapter was published in *Experiments in Fluids (Etebari and Vlachos, 2005)*. It is included herein with kind permission of Springer Science and Business Media.

Abstract

Accuracy of out-of-plane vorticity estimation from in-plane experimental velocity measurements is investigated with particular application to Digital Particle Image Velocimetry (DPIV). Simulations of known flow fields are used to quantify errors associated with amplification of the velocity measurement noise and method bias error due to spatial sampling resolution. A novel, adaptable, hybrid estimation scheme combining implicit compact finite difference and Richardson extrapolation schemes is proposed for improved vorticity estimation. The scheme delivers higher-order truncation error with less noise amplification than an explicit second order finite difference scheme. Finally, a complete framework for predicting, *a priori*, the random, bias and total error of the vorticity estimation on the basis of the error of the resolved velocities and the choice of differentiation scheme is developed and presented.

*Keywords: PIV, velocity measurements, vorticity, experimental fluid mechanics*
### 2.1 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A, A_k )</td>
<td>constants for Richardson extrapolation</td>
</tr>
<tr>
<td>( \alpha, \beta, a, b, \varepsilon )</td>
<td>constants for the compact schemes</td>
</tr>
<tr>
<td>( i, j, k )</td>
<td>indices</td>
</tr>
<tr>
<td>( \theta, r, z )</td>
<td>coordinate axes (subscripts)</td>
</tr>
<tr>
<td>( r )</td>
<td>radius</td>
</tr>
<tr>
<td>( \Delta, \Delta x )</td>
<td>spatial sampling resolution</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>circulation (set to 1000 mm(^2)/sec for Oseen vortex solution)</td>
</tr>
<tr>
<td>( V_{\text{ref}} )</td>
<td>characteristic velocity scale</td>
</tr>
<tr>
<td>( L )</td>
<td>characteristic length scale (set to 130 mm for Oseen vortex solution)</td>
</tr>
<tr>
<td>( U )</td>
<td>velocity in ( i ) direction</td>
</tr>
<tr>
<td>( V )</td>
<td>velocity in ( j ) direction</td>
</tr>
<tr>
<td>( \omega )</td>
<td>out-of-plane vorticity</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>standard deviation</td>
</tr>
<tr>
<td>( \delta )</td>
<td>uncertainty</td>
</tr>
<tr>
<td>( \lambda_0 )</td>
<td>noise transmission ratio</td>
</tr>
<tr>
<td>( K )</td>
<td>coefficient (numerator) of the noise transmission ratio</td>
</tr>
<tr>
<td>( \gamma, \phi )</td>
<td>constants for power law</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>noise amplification coefficient (probabilistic quadratic approach)</td>
</tr>
<tr>
<td>( \chi )</td>
<td>constants from velocity derivative schemes evaluated in probabilistic quadratic approach</td>
</tr>
<tr>
<td>( \text{FD2} )</td>
<td>2(^{\text{nd}})-order central finite difference scheme</td>
</tr>
<tr>
<td>( \text{Chapra-4} )</td>
<td>4(^{\text{th}})-order Chapra scheme</td>
</tr>
<tr>
<td>( \text{comp-4} )</td>
<td>4(^{\text{th}})-order compact scheme</td>
</tr>
<tr>
<td>( \text{comp-6} )</td>
<td>6(^{\text{th}})-order compact scheme</td>
</tr>
<tr>
<td>( * )</td>
<td>denotes noise optimization</td>
</tr>
<tr>
<td>( \text{Rich-4}^* )</td>
<td>noise optimized 4(^{\text{th}})-order Richardson extrapolation scheme</td>
</tr>
<tr>
<td>( \text{Rich-6} )</td>
<td>6(^{\text{th}})-order Richardson extrapolation scheme</td>
</tr>
<tr>
<td>( \text{CR4}^* )</td>
<td>noise optimized 4(^{\text{th}})-order hybrid compact-Richardson extrapolation scheme</td>
</tr>
</tbody>
</table>

### 2.2 Introduction

Digital Particle Image Velocimetry (DPIV) is currently one of the most widely used methods for noninvasive optical flow diagnostics. Recent advancements in cross-correlation DPIV methodologies deliver higher-accuracy sub-pixel velocity measurements (Lourenco and Krothapalli, 1995; Westerweel, 1997; Huang et al. 1997; Nogueira et al. 1999; Hart 2000; Werely and Meinhart 2001; McKenna and McGillis 2002). In addition, the advancement of available instrumentation allows resolution of a broader range of scales in the flow. The experimentally determined flow fields can be used to evaluate additional flow parameters, including vorticity (Luff
et al., 1999; Etebari et al., 2003; Fouras and Soria, 1998; Foucaut and Stanislas, 2002), shear stress (Etebari et al., 2004a; Etebari et al., 2004b), and dissipation rate (Saarenrinne and Piirto, 2000; Sheng et al., 2000). The quantification of these hydrodynamic parameters and their spatial variations are essential to the understanding of unsteady and/or turbulent flows. However, the method chosen for estimating the velocity derivatives has significant effect upon the accuracy of estimation of these quantities, which greatly depends on the length scales present in the flow and the velocity random error. It is necessary to quantify and further reduce the error associated with the indirect measurement of the velocity derivatives. This paper aims to develop novel methodologies that achieve higher accuracy over a large range of flow scales and sampling resolutions without noise amplification.

Previous work by Fouras and Soria (1998) and Luff et al. (1999) evaluated the performance of finite-difference and least-square fit direct velocity differentiation algorithms on the estimation of vorticity. Both groups studied the effects of velocity measurement random noise on vorticity estimation. Both studies concluded that the use of algorithms involving some type of local smoothing on either the velocity or vorticity fields improved the estimation accuracy to 3-8% total error when there was 10% absolute random error present in the velocity field. Both groups came to the conclusion that the simple second-order central finite difference (FD2) scheme gave the best tradeoff in performance between bias and random error. Although these two quantities are dependent upon the order of the method truncation error, they act in opposition. Higher-order methods generally offer lower bias error, while lower-order methods reduce random error propagation. To aid in the design of new schemes, Fouras and Soria (1998) proposed a method for estimating \textit{a priori} the propagation of random error associated with each method. The method adequately predicts the noise amplification in the form of an analytically determined non-dimensional random error transmission ratio.

Foucaut and Stanislas (2002) evaluated the effect of spatial sampling resolution on the vorticity measurement accuracy to determine the bias error due to each method. All schemes inherently apply a low-pass filter based upon their truncation error. Under the presence of measurement noise, as is the case with experimental data, the schemes also impose a high-pass filter, causing them to act as band-pass filters. The transfer functions for each were compared with respect to the wavenumber within the flow in application to turbulent flows. Overall, the group concluded that the range of scales in the flow is critical to the choice of derivative estimators.
Based upon the spatial resolution of the system and the noise present, they provided criteria to guide the appropriate choice. However, for turbulent flow fields they found little improvement in the vorticity estimation accuracy of higher-order schemes compared to the FD2 approach, based upon the filter characteristics of the various schemes. In particular, they noted that the FD2 scheme exhibits the same high cutoff frequency as PIV under optimized conditions. Information at higher frequencies is treated as noise that should be filtered by the derivative scheme to avoid noise amplification. Thus, a benefit from higher-order schemes is not realizable due to their high cutoff frequencies.

Given the limitations mentioned above, in the present effort we quantify and improve simultaneously the bias and random error of indirect derivative measurements resulting from DPIV data. In particular, the use of implicit higher-order compact finite difference schemes is investigated, as they offer the best performance over ranges of frequencies. Such schemes have been used extensively in direct numerical simulation (DNS) and large eddy simulations (LES) (Nagarajan et al., 2003). Here, we develop a new method for derivative estimation that combines a fourth-order compact scheme with a Richardson extrapolation to form a hybrid compact-Richardson scheme. This work follows the methodology set forth by Fouras and Soria (1998) to investigate the effect of random error transmission and spatial sampling resolution, and provides additional consideration for vortical flows. The transfer function of the method is investigated following the analysis of Foucaut and Stanislas (2002) to determine the applicability of the hybrid scheme to turbulent flow fields. Finally, we propose an \textit{a priori} method for providing an initial estimate of the total error for use in altering and adapting the scheme to suit the needs of a particular system.

### 2.3 Methodology

\textit{Vorticity estimation schemes}

The uncertainties of three conventional methods were evaluated in this study for comparison with the higher-order compact schemes. The first method utilized is second-order, central finite differencing (FD2), a commonly used scheme that delivers second-order truncation error. The scheme takes the form:
Here, $U$ and $V$ are the in-plane velocity components, $\Delta$ is the vector grid spacing (this analysis considers only homogeneous/uniform grid spacing in both directions), and $\omega_z$ is the out-of-plane vorticity component. The two main methods investigated by Luff et al. (1999) were included as well for comparison with the compact schemes. The first of these is an 8-point vorticity estimation using circulation, calculated as the line integral of the dot product of tangential velocity with the outward normal for a location, $(i,j)$:

$$\omega_z(i,j) = \frac{1}{\Delta} \left[ \frac{U_{i,j+1} - U_{i,j}}{2} + \frac{V_{i+1,j} - V_{i,j}}{2} \right] \quad (2.1)$$

The circulation method is presented herein for comparison only, as it offers only vorticity determination as opposed to velocity derivative estimations in general. The second method is a fourth-order central difference (fourth order accurate) approximation (denoted here as Chapra-4) developed by Chapra (1998), as given by Eq. (2.2):

$$\omega_z(i,j) = \frac{1}{4\Delta} \left[ (U_{i,j+1} - U_{i,j}) + (V_{i+1,j} - V_{i,j}) + 0.5(-U_{i,j+1} + V_{i+1,j} - U_{i,j} - V_{i,j+1}) + U_{i-1,j-1} - V_{i,j-1} + U_{i+1,j+1} + V_{i,j+1}) \right] \quad (2.2)$$

Fourth- and sixth-order accurate, implicit, compact finite-difference schemes (denoted here as comp-4 and comp-6) developed by Lele (1990) were utilized as higher-order methods for evaluating the velocity derivatives. The schemes used in this work are generalizations of the Pade scheme (1990), as given by Eq. (2.4):

$$\omega_z(i,j) = \frac{1}{\Delta} \left[ \frac{U_{i,j+2} - 8U_{i,j+1} + 8U_{i,j+1} - U_{i,j+1}}{12} + \frac{V_{i+2,j} - 8V_{i+1,j} + 8V_{i+1,j} - V_{i+2,j}}{12} \right] \quad (2.3)$$
\[
\beta U'_{i-2} + \alpha U'_{i-1} + U'_i + \alpha U'_{i+1} + \beta U'_{i+2} = \frac{c}{6\Delta} U_{i+3} - U_{i-3} + \frac{b}{4\Delta} U_{i+2} - U_{i-2} + \frac{a}{2\Delta} U_{i+1} - U_{i-1}
\] (2.4)

where \(\alpha, \beta, a, b,\) and \(c\) are constants that are solved for by substitution of Taylor series coefficients, with \(U\) and \(U'\) representing the values of the velocity and implicitly-determined local spatial derivative values. The values of the parameters \(\alpha, \beta, a, b,\) and \(c\) are shown below in Table 2.1.

**Table 2.1. Constants for compact schemes**

<table>
<thead>
<tr>
<th>Method</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compact-4</td>
<td>0.25</td>
<td>0</td>
<td>1.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Compact-6</td>
<td>1/3</td>
<td>0</td>
<td>14/9</td>
<td>1/9</td>
<td>0</td>
</tr>
</tbody>
</table>

Results for only the comp-6 scheme are presented here as the two methods yield nearly identical results for the simulations studied herein. It should be noted that the coefficient values of \(b\) and \(c\) for the fourth-order accurate compact scheme are zero, which thereby allows twice the order accuracy as the FD2 estimator with the same computational stencil size. Sustaining a small stencil will allow more accurate estimation for the small scales in the flow.

In addition to the schemes presented above, two of the Richardson extrapolation algorithms presented by Foucaut and Stanislas (2002) are evaluated in this study, namely the sixth-order formulation and a fourth-order scheme with coefficients optimized for minimization of noise error. The Richardson schemes take the form:
\[
\frac{\partial U}{\partial x_i} = \frac{1}{A} \sum_{k=1,2,4,8} A_k \frac{U_{i+k} - U_{i-k}}{2k\Delta x_i}
\]  

(2.5)

The coefficients for the Richardson schemes used in this study are given below in Table 2.2. The asterisk denotes a noise-optimized set of coefficients.

Table 2.2. Constants for Richardson extrapolation schemes

<table>
<thead>
<tr>
<th>Order</th>
<th>( A )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_4 )</th>
<th>( A_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>45</td>
<td>64</td>
<td>-20</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4*</td>
<td>1239</td>
<td>272</td>
<td>1036</td>
<td>0</td>
<td>-69</td>
</tr>
</tbody>
</table>

The fourth-order noise-minimizing Richardson extrapolation scheme (Rich-4*) reduces the random error transmission by combining a range of second-order central difference schemes with varying spatial samplings using an optimized set of coefficients. The coefficients can be optimized to minimize either the bias error or the random error, but not both. By incorporating a larger stencil into the scheme and varying the weighting of each component, the Rich-4* scheme reduces the effect of velocity random noise on the vorticity estimation, at the expense of increase in bias error. The Rich-6 method, conversely, utilizes coefficients that are optimized for minimum bias error. A more complete presentation of these methods can be found in Foucaut and Stanislas (2002). An alternative method for estimating the vorticity is a least-squares-fit, direct velocity differentiation, which is not presented here. These results can be found in detail in Fouras and Soria (1998), which presents a comparison with the central differences method presented in this study.

Compact-Richardson formulation

All of the conventional schemes presented above suffer from the inherently opposing effects of random and bias error, both of which are highly dependent upon the order of the
method and scales present within the flow. As the order of truncation error of the method is increased, accuracy at high wavenumbers is improved at the cost of increased noise amplification. Accordingly, previous efforts have not been able to increase method order beyond the simple FD2 scheme in practice. An ideal estimator would minimize the total error over a large range of flow scales without a strong dependence upon spatial sampling resolution. However, this would imply that we could reduce the bias error without a subsequent increase in the random error and vice versa.

The purpose of this work is to deliver a higher-order scheme with less noise amplification and less bias error than the FD2 scheme over the range of flow scales relevant to DPIV. To this end, we hypothesized that combining the most favorable characteristics of the higher order implicit schemes and the noise-minimizing schemes could reduce the error dependencies and the overall error simultaneously, with the goal of outperforming the FD2 scheme. In particular, we propose to utilize the fourth-order compact scheme (low bias error) in a noise-optimized Richardson extrapolation (low noise amplification), using a spatial sampling summation of the derivatives. This is a hybrid compact-Richardson formulation. The scheme can be represented as:

\[ \frac{\partial U}{\partial x_i} = \frac{1}{A_k} \sum_{k=1,2,4,8} A_k U_{i,k-grid} \]

where \( U_{i,k-grid} \) is the implicitly computed derivative from the compact scheme for \( k \)-grid spacing.

The method is implemented as follows. For a flow field described by \( N \times N \) vectors, for \( k = 1 \) corresponding to an array of vectors of length \( N \), the fourth-order compact scheme is solved using the smallest grid spacing and all \( N \) vectors. For \( k = 2 \) the compact scheme is solved twice using the arrays comprised of every other vector 1, 3, 5, ..., \( N \) etc. and 2, 4, 6, ..., \( N \). The same process of skipping samples is used to solve the approximation for \( k = 4 \). The results from each spatial sampling are then averaged according to their respective weights and location \((i,j)\) within the flow field. Here, we present a noise-optimized, fourth-order, hybrid compact-Richardson scheme (which we denote as CR4*), and compare its performance versus the conventional schemes. The coefficients for this method vary slightly from those for the fourth-order, noise-optimized, Richardson method presented above, and are given below in Table 2.3. The \( A_4 \) and \( A_8 \) values are switched from the conventional Richardson scheme to reduce the stencil size and allow
computation on relatively small vector field grids (less than 100 x 100 vectors). This change prevents major influence of the boundary methods affecting the entire estimations within the flow due to the implicit nature of the scheme. Overall, it was observed that the alteration makes little difference, with the cutoff wavelengths (presented in Section 2.6) affected by less than 0.2%.

| Table 2.3. Constants for hybrid compact-Richardson scheme |
|-------------|---------|---------|---------|---------|---------|
| Order      | $A$    | $A_1$  | $A_2$  | $A_4$  | $A_8$  |
| 4*         | 1239   | 272    | 1036   | -69    | 0      |

2.4 Monte-Carlo simulations: Oseen vortex

Error analysis is performed using simulations of known flow fields. Following the work of Fouras and Soria (1998) and Luff et al. (1999), an Oseen vortex is used as a representative flow field with known analytical velocity and vorticity distribution. This provides a baseline for the evaluation of the different schemes and allows comparison with previously published results. The analytical in-plane velocity field for the Oseen vortex in polar coordinates is:

$$U_\theta = \frac{\Gamma}{2\pi r} [1 - e^{-r^2/2L^2}]$$

$$U_r = 0$$

(2.7)

The analytical out-of-plane vorticity distribution is:

$$\omega_z = \frac{1}{r} \frac{\partial}{\partial r} (rU_\theta) - \frac{1}{r} \frac{\partial}{\partial \theta} (U_r) = \frac{\Gamma}{2\pi L^2} e^{-r^2/2L^2}$$

(2.8)

A series of velocity fields for the Oseen vortex flow were generated for a range of spatial samplings. The range of samplings is achieved by varying the parameter $\Delta/L$, where $\Delta$ corresponds to the spatial resolution and $L$ is the characteristic length scale of the vortex core. The resolution varied from detailed to coarse, where the coarsest sampling equaled the Nyquist frequency ($\Delta/L = 0.5$). The corresponding exact velocity and vorticity magnitudes for a spatial resolution of $\Delta/L = 0.05$ (fine grid) are shown in Fig. 2.1 (a) and (b), respectively. The values of $\Delta/L$ used in this study range from 0.05 to 0.49 for the eight flow fields.
2.5 Random error analysis

Fouras and Soria (1998) presented a methodology for predicting \textit{a priori} the ratio of the normalized random vorticity error to the normalized random velocity error. This ratio, $\lambda_\omega$, is given by Eq. (2.9), and is referred to as the non-dimensional random error transmission ratio. This ratio represents the amplification of the velocity random error in the vorticity estimation.

$$
\lambda_\omega = \frac{\sigma(\omega_z)}{V_{ref}/L} \frac{\sigma(U)}{V_{ref}} = L \frac{\sigma(\omega_z)}{\sigma(U)} = L \frac{\delta(\omega_z)}{\delta(U)} \tag{2.9}
$$

Here, $V_{ref}$ and $L$ are the characteristic velocity and length scales, respectively, $\delta(U)$ is the velocity measurement random error, $\delta(\omega_z)$ is the corresponding vorticity uncertainty, and is estimated using a standard error propagation, as follows. The uncertainty in the resulting calculation of the quantity $\delta(\omega_z)$, when $\delta(U)$ is a function of several independent variables ($U_i$), is given in Eq. (2.10). The root-sum-square (RSS) uncertainty in each variable, $\delta X_i$, is defined as twice the standard deviation, $\sigma(X_i)$, of the measurement variable.
\[ \delta(\omega) = \left[ \sum_{i=1}^{N} \left( \frac{\delta(\omega_i)}{\delta(U_i)} \delta(U_i) \right)^2 \right]^{1/2} \]  \hspace{1cm} (2.10)

In general, the transmission ratio takes the form:

\[ \lambda_o = K \frac{1}{\Delta/L} \]  \hspace{1cm} (2.11)

where \( K \) is a coefficient unique to each scheme. For the comp-4 scheme of a regular rectangular grid, a similar analysis yields the random error transmission ratio:

\[ \lambda_o = \frac{a}{\Delta/L} \]  \hspace{1cm} (2.12)

where \( a \) is the coefficient of the \( i+1 \) term in the compact scheme. Analysis for the comp-6 scheme yields the same transmission ratio given in Eq. (2.12). The value of the coefficient \( a \) for the comp-4 and comp-6 schemes is 1.5 and 1.556, respectively. Table 2.4 shows the values of \( K \) for all of the methods investigated herein.

<table>
<thead>
<tr>
<th>Method</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Richardson-4*</td>
<td>0.4723</td>
</tr>
<tr>
<td>Comp-Rich-4*</td>
<td>0.7085</td>
</tr>
<tr>
<td>2nd-FD</td>
<td>1</td>
</tr>
<tr>
<td>Chapra</td>
<td>1.3437</td>
</tr>
<tr>
<td>Richardson-6</td>
<td>1.4395</td>
</tr>
<tr>
<td>Compact-4</td>
<td>1.5</td>
</tr>
<tr>
<td>Compact-6</td>
<td>1.556</td>
</tr>
</tbody>
</table>

The value of \( K \) is inversely proportional to the degree of smoothing. Based upon this analysis, the CR-4* method exhibits better smoothing than the FD2 method.

Fig. 2.2 displays the transmission ratios versus spatial sampling resolution for the schemes studied herein in addition to the results presented by Fouras and Soria (1998). The solid lines
correspond to the analytically derived transmission ratios, while the overlaying data points are from simulations using the Oseen vortex flow field, superimposed with white noise with a magnitude of 10% with respect to the local velocity. It is clear that there is good correlation between the analytic expressions and simulated data. Of the methods presented here, the Rich-4* has the lowest random error transmission ratio, which coincides with the 13-point least-square fit scheme. The most important observation is that the CR4* scheme has approximately one half the transmission error of all of the higher-order methods, including the comp-4 and comp-6, the Rich-6, and the Chapra-4. Moreover, this transmission error is significantly less than that for the FD2 scheme. This is despite the fact that the differentiation scheme employs high order truncation error finite differences. This observation implies that proper optimization of the CR4* coefficients compensates for the noise amplification properties of the 4th order compact schemes.

Figure 2.2. Random error transmission ratio as a function of $\Delta/L$.

Foucaut and Stanislas (2002) presented a similar *a priori* estimation of the random error for each method using a probabilistic quadratic approach. Interestingly, the random error transmission
estimated using their method predicted a decrease in random error transmission as the order of the compact scheme increased (from sixth- to tenth-order). This violates the general premise that bias and random error have opposing effect. This prediction led Foucaut and Stanislas (2002) to speculate that the tenth-order compact scheme might offer smoothing effects comparable to those of the FD2 scheme with almost no bias error. However, even in their study using real data, the effect was not realized. Using the current analysis, it can be shown that the coefficient of the random error transmission ratio for the eighth- and tenth-order compact schemes are 1.6727 and 1.6832, respectively, predicting an increase in the noise amplification with an increase in the order of the compact scheme. Therefore, from the discrepancy realized in their study, combined with the results shown here, we conclude that the random error transmission ratio is the appropriate method for prediction of random error transmission in comparison with the probabilistic quadratic approach.

2.6 Bias error analysis

Analysis for the seven numerical schemes in computing the vorticity field from Monte-Carlo simulations was performed to determine the global and local effects of spatial sampling and radial position on vorticity bias error within the Oseen vortex. As the flow fields have no added noise, only the bias error of each method is quantified. All results are normalized to the local value of the vorticity. The figures displaying normalized averaged error have been averaged in space. Fig. 2.3 shows the average normalized bias error for each method for various spatial resolutions. The results indicate that the comp-6 and Rich-6 schemes deliver less bias error compared to the other methods investigated herein. In particular, for the lowest spatial sampling equal to the Nyquist frequency (Δ/L = 0.5) the sixth-order schemes yield less than one-half of the error of the Chapra-4 formulation, one of the best performing conventional schemes. The compact-6 scheme yields a bias error of 0.295% versus 0.650% for the Chapra-4 formulation at this resolution. The Rich-4* scheme yields higher-order accuracy at high spatial sampling (less than 1% error up to Δ/L = 0.21), but quickly diverges at poor spatial sampling resolutions, crossing the FD2 and 8-point circulation formulations at sampling resolutions of Δ/L = 0.21 and 0.25, respectively. The fourth-order noise optimized CR4* scheme, however, yields results that are nearly identical to the Chapra-4 scheme,
with a maximum error of 1.02%. This indicates that the CR4* scheme developed, although having inherently noise-optimized properties, outperforms conventional schemes in bias error. This is attributed to the utilization of the compact scheme in place of the central finite difference in the Richardson extrapolation.

Figure 2.3. Vorticity bias error (%) as a function of spatial resolution

Fig. 2.4 shows the relationship between bias error in the vorticity measurement and position within the core of the vortex for the highest sampling resolution ($\Delta/L = 0.05$). It can be seen that the sixth-order schemes again outperform the other methods, displaying an error of less than 0.01% regardless of the measurement location. The 8-point circulation scheme and the conventional central difference schemes rapidly increase their error as the measurement location approaches the center of the vortex, thus underestimating the local vorticity value. This trend continues for the other spatial resolutions of up to 0.49 (not shown here). These results are in agreement with Fouras and Soria (1998). The conventional schemes overestimate the local vorticity value outside of the core of the vortex ($r/L > 1.5$). Although the magnitudes of the errors outside of the core appear to be on the same order as those within, the higher levels are due to normalization with the
local vorticity value, which approaches zero outside of the vortex core. Although previous studies have normalized the data with the maximum or mean vorticity value to avoid this feature, it is more representative of the performance of the methods to express the accuracy with respect to the local values. Very importantly, the CR4* scheme, although exhibiting random error transmission comparable to the 8-point circulation, Rich-4*, and FD2 schemes, is relatively unaffected by the sampling resolution and local velocity gradient.

![Figure 2.4. Vorticity bias error (%) vs. position (Δ/L = 0.05)](image)

Fouras and Soria (1998) used a power law model to approximate the dependence of the bias error on the sampling resolution, Δ/L at the center of the vortex. This can be expressed as:

\[
\frac{-\omega_{\text{bias}}(0)}{\omega_{\text{exact}}(0)} = \gamma \left( \frac{\Delta}{L} \right)^\delta
\]  

(2.13)

Following the same analysis we present in Fig. 2.5 the corresponding log-log plots. A linear relationship between the bias error and the spatial sampling resolution is illustrated. The coefficients and exponents for these fitted relationships are summarized below in Table 2.5.
Table 2.5. Constants for power laws

<table>
<thead>
<tr>
<th>Method</th>
<th>Coefficient</th>
<th>Power</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd-FD</td>
<td>0.2416</td>
<td>1.99</td>
<td>2</td>
</tr>
<tr>
<td>8-point circ.</td>
<td>0.3828</td>
<td>2.0084</td>
<td>2</td>
</tr>
<tr>
<td>Chapra</td>
<td>0.1478</td>
<td>3.9532</td>
<td>4</td>
</tr>
<tr>
<td>Richardson-4*</td>
<td>2.2592</td>
<td>3.4579</td>
<td>4</td>
</tr>
<tr>
<td>Richardson-6</td>
<td>0.1994</td>
<td>5.7816</td>
<td>6</td>
</tr>
<tr>
<td>Comp-Rich-4*</td>
<td>0.3951</td>
<td>4.6519</td>
<td>4</td>
</tr>
<tr>
<td>Compact-6</td>
<td>0.0709</td>
<td>5.8088</td>
<td>6</td>
</tr>
</tbody>
</table>

The results for the second-order finite difference scheme are equivalent to those of the 9-point least squares fit. Fouras and Soria (1998) noted that the coefficient of the power law increased with the number of points used in the least-squares interpolation, corresponding to increased smoothing and consequently increased bias error. However, because their study was limited to second-order schemes their resulting model was a quadratic power law for all schemes. Thus they were not able to identify the significance of the exponent of the power law model. Using an analytical approach, Foucaut and Stanislas (1998) noted the same relationship for the vorticity.
error as a function of distance from the center of an Oseen vortex. Motivated by both of these previous efforts and by investigating more differentiation schemes with varying truncation order and form, results indicates that the power law exponent and the order of truncation error of the method are approximately equal. This observation cannot be generalized for every differentiation scheme that can be possibly used without detailed analytical or experimental validation. However, for the schemes presented herein it does provide a simple and direct way to approximate the power law exponent from the order of the truncation error of the method of choice. Based on this, it appears that the CR4* scheme behaves as a fifth-order method, due to the summation of various space derivatives. This supports the contention that the combination of the Richardson extrapolation with the Compact schemes is responsible for simultaneously reducing the noise amplification and decreasing the bias error.

This contribution is further extended by determining a relationship between the power law coefficient, $\gamma$, and the transmission ratio coefficient, $K$. Inherently, the bias error and random error of each method must be inversely proportional, as indeed is the case. Fig. 2.6 shows a plot of $\gamma$ versus the coefficients of the transmission ratios, $K$ on a log-log scale.

![Figure 2.6. Relationship between the coefficient of the power law and the transmission ratio](image-url)
The relationship takes the form of a power law. The coefficient, \( \gamma \), can be related to the random error transmission ratio by the following power law expression:

\[
\gamma = \frac{0.2746}{K^{2.34}}
\]  

(2.14)

Although this relationship cannot be generalized beyond the schemes investigated herein and does not eliminate the need for rigorous error analysis for each differentiation scheme, it can be used as a simple and easy to apply preliminary *a priori* estimator of the associated error. The coefficient of the power law for a particular method can be estimated with previous knowledge of only the transmission ratio, a quantity that can be evaluated *a priori* using the analysis of Fouras and Soria (1998).

### 2.7 Application to complex flow fields

To this point, only error for a simple flow field consisting of a single, large-scale vortex has been considered. Inherently, an organized flow of this type does not require a higher-order scheme to estimate the vorticity due to the lack of small-scale flow structures. The focus of this paper is to improve upon conventional methods by incorporating higher-order schemes to obtain higher accuracy velocity derivative estimations in turbulent flow fields. To investigate the applicability of the methods studied herein to a range of wavenumbers, we follow the methodology set forth by Foucaut and Stanislas (2002), who analytically decompose the contributions of bias and random error in terms of truncation error and a noise amplification term. They utilize a probabilistic quadratic approach (Neuilly, 1998) to estimate the noise amplification coefficient, \( \varepsilon \), given by:

\[
\varepsilon^2 = \frac{2}{\chi^2} \sum_{i=1}^{n/2} \chi_i^2
\]  

(2.15)

where \( n \) is the order of the method, and \( \chi \) and \( \chi_i \) are related to the coefficients in the schemes. The noise amplification term is then \( \varepsilon (\sigma_u / \Delta x) \), where \( \sigma_u \) is the measurement noise level. Based upon this noise amplification term and the truncation error of the method, the transfer function can be determined.
Because DPIV employs cross-correlation interrogation windows to obtain a measurement, the averaging effect associated with the method provides a low-pass filtering effect. Based upon this inherent limitation, Foucaut and Stanislas (2002) compared the transfer functions of the various schemes to determine which scheme displays the closest high cutoff wavenumber to that of their DPIV system. It is important to note that the results presented by the group for the transfer function of the system are applicable only to that particular DPIV system in the configuration in which it was utilized. This cutoff wavenumber will change depending upon the system parameters.

Following this analysis, we present the transfer function for the CR4* scheme for comparison with the FD2 scheme, the traditional fourth-order compact scheme and the fourth-order noise optimized Richardson extrapolation. These results are illustrated in Fig. 2.7, which displays the transfer function versus the wavenumber, \( k \Delta x \). The FD2 scheme was chosen as the optimal derivative estimator for the study conducted by Foucaut and Stanislas (2002) because of its high cutoff wavenumber, 1.392, which was closest to that of their DPIV system, 1.4. The -3 dB line indicates the crossing points which determine the low and high cutoff wavenumbers. As can be seen, the FD2 scheme does have a high cutoff wavenumber that is optimal according to the corresponding cutoff wavenumber of 1.4 for DPIV. However, the shape of the filter prevents the method from performing optimally. The slow roll-off in the transfer function begins around a value of \( k \Delta x = 0.5 \) and continues to decrease nearly linearly through \( k \Delta x = \pi \). An ideal filter would maintain a value of one until the cutoff wavenumber, and drop immediately to zero. The resulting error can be viewed in terms of two contributions. The discrepancy of the transfer function value below a value of one from a wavenumber of zero through the cutoff wavenumber is manifested in the bias error. The discrepancy between the transfer function values and zero above the cutoff wavenumber serves to amplify noise, as the measurements from DPIV in this range represent scales that cannot be resolved. The Rich-4* scheme represents the best method for providing smoothing above the cutoff wavenumber, but has significant roll-off (bias) below this wavenumber. The comp-4 scheme displays nearly ideal characteristics below the cutoff wavenumber, but has a cutoff wavenumber nearly twice that for the optimized DPIV system, resulting in noise amplification. These results are in agreement with previous works. The CR4* scheme is essentially a combination of the best features of these two conventional methods.
The advantages of the CR4* scheme become apparent upon further inspection. The transfer function of the scheme maintains a value very near one, and does not begin a significant roll-off until after a value of \( k \Delta x = 1 \). The roll-off is more pronounced than the conventional schemes, as the transfer function reaches and remains below 0.2 for \( k \Delta x = 1.5 \) and above. The high cutoff wavenumber for the CR4* scheme is 1.27, which is near that for the optimized DPIV system. To summarize, there are two observable improvements of the CR4* scheme over the FD2 scheme. First, the bias of the transfer function is improved between \( k \Delta x = 0.5 \) and the cutoff wavenumber for DPIV, which is consistent with a decrease in error at higher wavenumbers. Second, the noise amplification is drastically reduced for wavenumbers above the DPIV cutoff, where the velocity measurements are unreliable and noisy (Foucaut and Stanislas, 2002).
2.8 Application: Monte-Carlo simulations of vortical flow fields

Although the transfer function gives a prediction of the behavior of the methods in turbulent flows, it is more convenient to illustrate their performance in an artificially generated vortical flow field with known noise level. The following analysis will serve to illustrate the combined effects of bias and random error for a complex flow field, and support the arguments made thus far in this paper.

In this section, we treat random error propagation as a secondary effect specific to the system used in obtaining the measurements. The sources of these errors in the velocity field have been studied in detail, and error-reducing algorithms have been presented (Huang et al. 1997; Nogueira et al. 1999; Hart 2000; Marxen et al. 2000; Werely and Meinhart 2001; McKenna and McGillis 2002). With emphasis on the bias error, the range of wavelengths that the system is capable of resolving is limited by the field of view, dictating the largest resolvable wavelength, and the spatial resolution, dictating the smallest resolvable wavelength. Maximization of the dynamic ranges of DPIV systems, while maintaining high-accuracy and low-noise measurements has been investigated thoroughly by Adrian (1997), among others.

As noted earlier, most conventionally utilized algorithms exhibit the least accuracy as the measurement location approaches the center of a vortex, a region characterized by high velocity gradient. To investigate the effect of superimposed turbulent flow structures of varying scales, a series of flow fields with a specified range of wavenumbers were generated as given by the following analytical expressions:

\[
\begin{align*}
    u &= \sum_{k_i}^{k_{\text{max}}} -13 \sin(k\pi x) \cos(k\pi y) \\
    v &= \sum_{k_i}^{k_{\text{max}}} 13 \cos(k\pi x) \sin(k\pi y)
\end{align*}
\]

(2.16)

where \( k \) corresponds to the wavenumber index. Then, \( k_{\text{max}} = 1 \) gives a single vortex within our flow field with a characteristic length scale equal to the span of the domain, while \( k_{\text{max}} = 2 \) adds the superposition of four vortices one-quarter the size to the \( k_{\text{max}} = 1 \) flow field, and so on. The
ratios of the largest wavelength to the smallest wavelength are restricted such that they fall well within the measurable range of scales as described above.

The effect of random noise on the vorticity estimation for the vortical flow fields was investigated by the addition of white noise with magnitude 10% onto the velocity vectors. The total average error for a summation of wavenumbers from 1 through 7 is presented below in Figure 2.8 as a function of the sampling resolution. As anticipated, in the case of over sampling the data ($\Delta/L = 0.05$), the total error is dominated by noise amplification, causing the higher-order schemes to have errors on the order of 100% or more. The minimum error for these schemes occurs at the poorest spatial sampling resolution ($\Delta/L \sim 0.5$), where the poor sampling acts as a low-pass filter against the high noise levels. Conversely, the schemes that provide significant smoothing are less affected by the noise at the highest spatial sampling resolution, but display significant error as the sampling resolution decreases and the complexity of the flow increases. Over a small range of sampling resolutions the error is minimized, outside of which the error quickly rises to undesirable levels.

The CR4* scheme developed in this work displays the least dependence upon the spatial sampling resolution, as the total error remains nearly constant and on the order of magnitude of the level of the random noise. There is some dependence upon the sampling resolution due to the transmission of random error in the range of spatial samplings very near the over-sampled condition ($\Delta/L = 0.05$), and due to the bias error towards the lower spatial sampling resolutions ($\Delta/L \sim 0.5$). Another interesting observation is that the next best overall method for the range of resolutions investigated is the FD2 scheme, as it too maintains a relatively flat total error profile. This result is in agreement with previous works that came to the conclusion that the FD2 scheme is usually the best overall choice when comparing conventional methods for vortical flow fields. The magnitude of the error, however, is nearly twice that for the CR4* scheme up to a sampling resolution of $\Delta/L = 0.35$. 

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2.9 \textit{A priori} total error prediction

A topic of great importance in the design of a derivative estimator is the prediction of total error of the method, assuming there is some previous knowledge of the system parameters. In this work we have expanded upon the work of Fouras and Soria (1998) in estimating the random and bias components of error. Both of these are functions of the spatial sampling resolution, $\Delta/L$, and the form of the method employed. In Section 2.5 we developed Eqs. 2.13 and 2.14 for estimating the bias error. Combining these results with the random error transmission ratio we can, in principle, develop an estimate of the total error under a particular set of system parameters at given spatial sampling resolutions. The total error is expressed as the square root of the sum of the squares of the random and bias error components:

$$\varepsilon_{\text{total}} = \left(\varepsilon_{\text{random}}^2 + \varepsilon_{\text{bias}}^2\right)^{1/2}$$  \hfill (2.17)
Substituting the \emph{a priori} relations into this expression we get:

\[
\mathcal{E}_{\text{total}} = \sqrt{\left(\frac{\sigma(u) \lambda_o}{L \Delta / L}\right)^2 + \left(\frac{u_{\text{max}}}{L} \gamma \left(\frac{\Delta}{L}\right)^4\right)^2}
\]  \hspace{1cm} (2.18)

Here, \(u_{\text{max}}/L\) is the characteristic vorticity scale used to dimensionalize the bias error term for a given flow field, and the values of \(\gamma\) (determined from Eq. 2.14) have been normalized according to the corresponding characteristic vorticity scale for the Oseen vortex from which they were derived. Figure 2.9 shows the total error prediction curve versus the Monte-Carlo simulations for the vortical flow field investigated in the previous section. The data points refer to the simulation results. The curves show good agreement with the simulation results, particularly for the first flow field, which is similar to the Oseen vortex. This is expected as the bias error (second term in Eq. (2.18)) is derived from the Oseen vortex simulations. For the more complex flow fields where a range of length scales are present in the flow, the equation underestimates the bias error. This is because the characteristic vorticity scale is a function of the maximum velocity only and thus does not incorporate all of the information that is present in the flow field. However, the general agreement of the data does support the argument that the random and bias errors can be estimated for complex flow fields using minimal information.

These results validate the applicability of the error analysis methodology proposed herein. Although not intended to replace a detailed error analysis, which should be performed for each differentiation method, Eqs. 2.14 and 2.18 do establish a framework to estimate \emph{a priori} the error of the vorticity calculation in a simple and easy to implement manner.
2.10 Conclusions

Most previous works (Luff et al. 1999; Fouras and Soria 1998; Foucaut and Stanislas 2002) have emphasized the need for estimators with lower-order smoothing capabilities, with the general conclusion that the FD2 scheme is adequate for most applications. The results from this study are in agreement with this conclusion. However, it is the conclusion of this work, contrary to those of previous studies, that it is in fact possible to obtain higher-order accuracy and better than second-order smoothing characteristics over the range of scales resolved by DPIV.

The estimation of vorticity from DPIV measurements using conventional schemes, from second-order to sixth, and a novel hybrid fourth-order compact-Richardson scheme developed herein is explored. The noise optimized fourth-order compact-Richardson scheme presents a new alternative to conventional methods. Whereas previous works have concluded that accuracy cannot be improved beyond a second-order finite difference method due to noise amplification, the hybrid compact-Richardson scheme offers nearly 30% less noise amplification while simultaneously reducing the bias error. The hybrid CR4* scheme successfully reduces the dependence of the estimation upon both the spatial sampling resolution at a particular scale of
flow and the measurement random noise. The superior performance of the CR4* scheme is further manifested by comparing its transfer function with those for the FD2, comp-4, and Rich-4* methods. The CR4* scheme provides three critical improvements, namely: (1) the bias error below the cutoff wavenumber is minimized; (2) the slope of the filter at the cutoff wavenumber is closer to that for an ideal filter; and (3) the transfer function is nearly zero beyond the cutoff wavenumber, resulting in significantly reduced noise amplification.

Furthermore, the hybrid CR4* scheme is versatile in its application, as the cutoff wavenumber can be adjusted to meet the characteristics of a given DPIV system with simple modifications of the coefficients. It is important to note that the scheme used herein has not been optimized; the actual coefficients used are determined from a conventional fourth-order Richardson extrapolation scheme that has been optimized for minimal noise amplification. Further investigation is necessary to determine if the coefficients of the hybrid scheme can be altered to provide improved performance, a topic for future work. In addition, it will be necessary to investigate parametrically the use of various order implicit compact schemes in the various noise-optimized Richardson extrapolations to determine if it is possible to further decrease method bias error and minimize noise amplification.

The random error transmission ratio presented by Fouras and Soria (1998) was found to be a more appropriate and reliable tool for predicting \textit{a priori} the propagation of random uncertainty into the vorticity estimation compared to the probabilistic quadratic approach proposed by Foucaut and Stanislas (2002). The transmission ratio allows for simple and effective comparison between schemes. Expanding upon this effort, we present a method for estimating the method bias error and total error. Although this estimation is not as robust an indicator as the transfer function, our preliminary results do indicate that \textit{a priori} bias and random error estimation is possible. It should be noted that these tools provide rough estimations when designing or adjusting a derivative estimation scheme to save time in the initial design stage. Furthermore, our conclusions do not extend beyond the schemes studied here. Care should be taken to ensure that a proper and thorough error analysis accompanies the design of each new scheme. Overall, this analysis provides a complete framework for predicting,\textit{a priori}, the random, bias and total error of the vorticity estimation on the basis of the error of the resolved velocities and the choice of differentiation scheme.
2.11 Acknowledgments

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2.12 References


Chapter 3

3 ON THE ACCURACY OF WALL SHEAR STRESS USING DPIV

Wall shear stress measurements are important for a variety of fluid mechanics phenomena and engineering applications ranging from estimation of viscous drag to the regulation of endothelial cell function in arterial flows (Naughton and Sheplak, 2002; Chew et al, 1998; MacLean and Schetz, 2003; Nowak, 2002; Wooton, 1999). Although DPIV has emerged over the past years as the method of choice for global non-invasive optical flow diagnostics (Abiven and Vlachos, 2002; Willert and Gharib, 1991; Werely and Meinhart, 2001; Raffel et al, 1998), the issues associated with the indirect estimation of wall shear stresses from DPIV measured velocities have not been sufficiently addressed (Meunier and Leweke, 2003; Noguiera et al, 1999; Huang et al, 1997). The challenge is even more significant in the presence of deformable and dynamically moving boundaries. In particular, such measurements require accurate determination of wall position, near-wall velocity measurements using DPIV algorithms, and the indirect estimation of the velocity derivatives in order to evaluate the shear stress. Dynamically moving boundaries, whether rigid or compliant, require special consideration, as the boundary position must be determined accurately as a function of space and time. It is necessary to quantify the accuracy of each measurement that contributes to the error of the wall shear stress estimation.

In this work, we decompose the problem into the three objectives: (a) determination of the exact boundary points (b) DPIV velocity measurement uncertainty in the near wall region. (c) velocity derivative error. Methodologies and improvements addressing each aspect individually are proposed and a systematic parametric study of the related error is performed. To our knowledge, this is the first detailed parametric effort to quantify the errors associated with wall shear stress estimation from DPIV velocities.
3.1 Introduction

Digital Particle Image Velocimetry (DPIV) has emerged as a standard in global flow measurements. In this technique, a fluid is seeded with neutrally buoyant, reflective microparticles and interrogated with a sheet of light expanded from a high-power laser. Reflected light from the particles is recorded using a digital CMOS camera with high frame rate (generally 30 Hz). Time-resolved DPIV systems such as the one developed and employed by our group use CMOS cameras with kHz sampling rates (Abiven and Vlachos, 2002). These images are then used to resolve the flow field using sophisticated statistical cross-correlation techniques.

The first aim of this effort is to quantify the accuracy associated with determination of boundary position. Conventional DPIV systems utilize cameras with CCD sensors, which suffer from a leakage effect where the intensity from saturated pixels will spill over into neighboring pixels. This situation can be avoided by employing a CMOS sensor, which truncates saturated pixels without leakage into the neighboring pixels (Abiven and Vlachos, 2002). This technology allows more accurate determination of the boundaries, and is a fundamental element in developing a system that is capable of resolving near-wall flows. Once accurate image information has been obtained, edge-detection routines, such as Canny's algorithm, Sobel method, or Laplacian of Gaussian, have traditionally been implemented to determine boundary point location (Canny, 1986). These methods give discrete edge information, offering pixel accuracy only. Herein we present a stochastic algorithm that expands upon these methods to improve their accuracy and allow determination of moving boundaries.

The second aim is to create an experimental DPIV data set that is shear rate dependant and quantify the accuracy of the velocity estimation. It is necessary to quantify the velocity error prior to evaluating the error associated with wall shear stress estimation. Velocity measurement error for DPIV has been quantified in the past; a brief review is presented here. Even in the presence of strong velocity gradients the accuracy of the velocity measurements is better than 0.1 pixels (px) for state-of-the-art systems. However, in the case of wall-bounded flows averaging effects can significantly affect the measurement due to very strong localized shear rates. The particle pattern displacements are determined with sub-pixel resolution from the cross-correlation peak for each interrogation window, using a Gaussian fit that is applied to the correlation coefficient matrix.
Detailed analyses of these methods can be found in Abiven and Vlachos (2002), Willert and Gharib (1991), Wereley and Meinhart (2001), and Raffel et al., (1998). For near wall measurements the cross-correlation signal to noise ratio is often compromised by reduced seeding density compared to the outer flow.

Evaluation of near-wall velocities from experimentally determined DPIV flow fields is most affected by the effect of spatial averaging due to window interrogation size. This is particularly true as the shear rate increases, which effectively skews the correlation matrix, the consequence of which is the inability of the conventionally utilized Gaussian three-point estimator to accurately determine the true displacement at the center of the window (Marxen et al., 2000). The signal is dominated by the flow away from the boundary due to higher velocities and better seeding density causing a biasing of the displacement estimation.

The third aim of this effort focuses on the uncertainty of velocity derivatives for wall shear estimations. Although previous efforts have explored the related aspect of vorticity estimation uncertainty, they have not address the accuracy of wall shear stresses for bounded flows (Etebari and Vlachos, 2003; Fouras and Soria, 1998; Luff et al., 1999; Foucaut and Stanislas, 2002). These previous studies have utilized finite-difference and least-square fit direct velocity differentiation algorithms, as well as higher-order compact and Richardson extrapolation methods, for the estimation of these quantities. However, these studies were performed for unbounded flows. The only parameters in these studies are velocity measurement uncertainty and spatial sampling resolution, which govern the ever-opposing effects of random error propagation and bias error. The wall position, although not accounted for in previous efforts, is very pertinent to the present study. Variations in this parameter produce dynamic changes in the spatial proximity of the measured velocity vectors with respect to the wall. The use of algorithms involving local smoothing on either the velocity or vorticity fields has traditionally been recommended to reduce random error propagation, which was found to be the dominant source of error in the estimation. In particular, a simple second-order central difference solution has been shown to offer the best trade-off between random and bias error. Recent efforts (Etebari and Vlachos, 2003; Fouras and Soria, 1998; Foucaut and Stanislas, 2002) have identified several advantages associated with the utilization of higher-order schemes. We utilize fourth- and sixth-order compact finite difference
schemes as alternatives to conventional schemes, for achieving higher accuracy in determining velocity derivatives from discrete flow field data (Lele, 1990). Higher order schemes are superior in reducing the bias error, which is the most significant source of error for near wall measurements.

In this study we address the first and third issues pertinent to wall shear determination, and attempt to quantify the errors associated with each. First, challenges in determining the exact location of points along a compliant, moving wall are noted, and a novel cross-correlation based method is presented to provide improved edge detection. Finally, methods for processing the information near the boundary in order to provide higher accuracy for wall shear estimation are proposed, and an estimation of the overall measurement accuracy is presented.

### 3.2 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>correlation matrix</td>
</tr>
<tr>
<td>$g$</td>
<td>interrogation window image</td>
</tr>
<tr>
<td>$a$</td>
<td>local intensity value</td>
</tr>
<tr>
<td>$L$</td>
<td>characteristic length scale</td>
</tr>
<tr>
<td>$s$</td>
<td>displacement</td>
</tr>
<tr>
<td>$U$</td>
<td>velocity in horizontal direction</td>
</tr>
<tr>
<td>$V$</td>
<td>velocity in vertical direction</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>spatial sampling resolution</td>
</tr>
<tr>
<td>$\tau$</td>
<td>shear stress</td>
</tr>
<tr>
<td>$\omega_z$</td>
<td>out-of-plane vorticity</td>
</tr>
<tr>
<td>$i,j,o$</td>
<td>subscripts describing measurement point location</td>
</tr>
<tr>
<td>$FD2$</td>
<td>2nd-order central finite difference scheme</td>
</tr>
<tr>
<td>$Chapra-4$</td>
<td>4th-order Chapra scheme</td>
</tr>
<tr>
<td>$comp-4$</td>
<td>4th-order compact scheme</td>
</tr>
<tr>
<td>$comp-6$</td>
<td>6th-order compact scheme</td>
</tr>
<tr>
<td>$Rich-4^*$</td>
<td>noise optimized 4th-order Richardson extrapolation scheme</td>
</tr>
<tr>
<td>$Rich-6$</td>
<td>6th-order Richardson extrapolation scheme</td>
</tr>
<tr>
<td>$CR4^*$</td>
<td>noise optimized 4th-order hybrid compact-Richardson extrapolation scheme</td>
</tr>
</tbody>
</table>
3.3 Methodology

Cross-correlation edge detection

Determination of the exact location of points along the boundary is critical in wall shear estimation, particularly in the case of moving boundaries. Edge detection routines yield information with pixel accuracy only, thus resulting in an uncertainty of +/- 0.5 pixels. This accuracy is further compromised in the presence of noise, which is present in all DPIV recorded images. Edge detection routines use explicit searching algorithms to find points with high intensity gradient or near zero second derivative (Canny, 1986). As a result, the estimation is subject to the error in the position used to determine the velocity gradient.

We expand upon conventional edge detection schemes by integrating image gradient information in a stochastic algorithm inspired by conventional DPIV analysis. We denote the method a Cross-Correlation Edge Detection (CCED) as it utilizes a traditional statistical cross-correlation to resolve the edge image displacements in a manner similar to established PIV algorithms (Willert and Gharib, 1991; Westerweel, 1997). The edge displacement represents a transfer function operating between two consecutive images. However, edges correspond to low wavenumber deterministic coherent shapes. Thus such an input will not allow appropriate system identification. We account for this detriment as follows: two original images are converted into gradient images using a central difference approximation, effectively applying a high pass filter to the image information. This process is illustrated in Figure 3.1.

![Figure 3.1. (a) original image (left); (b) gradient image (right)](image)
In effect by performing a gradient operation over the image we achieve an initial estimation of the edge location and remove the low wavenumber information, amplifying the high frequency image noise. Subsequently, the displacement for each edge point is determined by performing a cross-correlation between two interrogation windows centered at the measurement location, illustrated in Figure 3.1. This process is the same one used in conventional DPIV systems. Following the analysis of Westerweel (1997), the deformation of the object (boundary) in the image can be regarded as a black-box system acting on the input signal of the original boundary pattern, \( G(X',t') \), to deliver the output signal corresponding to the displaced boundary pattern, \( G(X,t'') \), according to the displacement given by \( s(X,t') \). This process is described schematically in Figure 3.2.

\[
G(X',t') \rightarrow [s(X',t')] \rightarrow G(X,t'')
\]

Figure 3.2. Black box system schematic of deformation

The system represents a linear transformation; thus the output signal can be expressed as the convolution of the input signal with the impulse response, \( H \), of the system (Westerweel, 1997):

\[
G''(X) = \int H(X,X')G'(X')dX'
\]

(3.1)

where the impulse response is shift invariant, given by (Westerweel, 1997):

\[
H(X',X'') = \delta[X''-X']
\]

(3.2)

From linear system theory, the impulse response of the black-box system is then the cross-covariance \( R_{GG'} \) of a random input signal with the corresponding output signal (Westerweel, 1997):

\[
R_{GG'}(s) = H*R_{G'}(s)
\]

(3.3)
where $R_G$ is the auto-covariance of the input signal and the \* denotes a convolution integral. The deformation can be determined by utilizing a cross-correlation of the two images. Spatial averaging is obtained by utilizing interrogation windows. The discrete form of the cross-correlation function is given as:

$$R(m, n) = \frac{\sum_{j=0}^{M-1} \sum_{i=0}^{N-1} g_1(i, j) g_2(i + m) (j + n)}{\sum_{j=0}^{M-1} \sum_{i=0}^{N-1} g_1(i, j) \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} g_2(i, j)}$$  (3.4)

where $g_1$ and $g_2$ are the signals within the interrogation windows.

Using a Gaussian three-point estimation, sub-pixel accuracy can be obtained after determination of the peak. The corresponding equation for estimating the true location, $x_c$, from the location of the peak of the correlation matrix, $x_o$, is given by Eq. (3.2). $R_o$ is the correlation value at that point (Marxen et al, 2000).

$$x_c = x_o + \frac{\ln R_{o-1} - \ln R_{o+1}}{2(\ln R_{o-1} + \ln R_{o+1} - 2 \ln R_o)}$$  (3.5)

A series of images simulating moving boundaries, as shown in Figure 3.1 (a), were generated to evaluate the sub-pixel accuracy of the method. The resulting RMS, mean, and total error, for window sizes of 32, and 64 pixels square are displayed in Figures 3.3-3.5 and plotted against pixel displacement. It is important to note that the total error of the method is below 0.1 pixels in resolving sub-pixel displacements, delivering better than the 0.5 pixel uncertainty of conventional edge detection. Furthermore, the RMS error is almost an order of magnitude less than the mean error. These results are in agreement with the typical performance of conventional DPIV algorithms for velocity evaluation.
Figure 3.3. RMS error vs. sub-pixel displacement

Figure 3.4. Mean error vs. sub-pixel displacement
Only sub-pixel displacements are investigated herein. It is assumed in this study that the temporally varying wall displacement is on the same order of magnitude as the flow velocities, so that both displacements can be measured. This is a necessary requirement for the near-wall flow investigation.

It should be noted that the method provides erroneous results if the original images are used in the cross-correlation in lieu of the edge detection images, due to the low signal-to-noise-ratio (SNR). Converting the original image into an image comprised of its gradient information effectively converts the low-frequency image information into a higher-frequency. The useful information for the correlation is contained in the variations in pixel intensities, thus the gradient information is most useful for this application. The total error in displacement estimation of the correlation of the original images is shown with the corresponding plot for the gradient images in Figure 3.6. The estimation from the original images is clearly on the order of the actual displacement. Figure 3.7 displays a few correlation coefficients for both algorithms through the peak location. The original image correlation biases the estimation towards zero displacement due
to the low frequency object information, which increases the width of the correlation peak. This result can be expected following the analysis of Westerweel (1997). The gradient correlation displays a peak that clearly lies between zero and one pixel displacement, as expected, reducing the biasing effect.

Figure 3.6. Mean error vs. displacement
Figure 3.7. Correlation planes of original images and gradient images

Accuracy of near-wall velocity estimation

Near-wall velocity measurements using DPIV are prone to error due to velocity gradients within the interrogation window, velocity gradients that are strong in one direction only (i.e. perpendicular to the wall), and poor near-wall seeding density. In generating an experimental data set that covers a range of wall shear rates, it is necessary to first quantify the error associated with the velocity measurement. The parabolic velocity profile of a Poiseuille flow through a channel or pipe makes it an ideal candidate for this type of analysis. Clearly, the shear rate is then linear as a function of the channel/pipe height. For a pipe flow with radius, R, the axial velocity is given by:

\[ u(r) = \left[ 1 - \left( \frac{r}{R} \right)^2 \right] u_0 \]  \hspace{1cm} (3.6)

where \( u_0 \) is the maximum velocity, which is located at the centerline of the pipe \((r = 0)\). The nominal shear rate is then given by
Time-resolved DPIV was employed to investigate flow through a 4 mm diameter pipe. Images were acquired at 1 kHz with 100 µsec laser pulse separation, and a spatial resolution of 18 microns per pixel. Due to the anisotropic shear of the flow, rectangular windows 64x8 pixels in size were used to perform cross-correlation to evaluate the velocity field. The use of irregular size windows reduces the spatial averaging effect of the cross-correlation in the direction characterized by a high velocity gradient. Mean validation using rectangular windows of 5x3 vectors has been applied to the data to remove spurious vectors outside of a deviation threshold. Shear rates ranging up to 0.23 were evaluated, where the shear rate is defined as the non-dimensional wall velocity gradient. The data were taken at 60 diameters downstream of the inlet to the tube to ensure that the flow was fully developed. Flow profiles superimposed on normalized axial velocity contours for a shear rate of 0.12 are shown below in Figure 3.8, where R is the radius of the tube. The velocity profiles are parabolic in form and show signs of fully developed flow (no variation in axial direction). In addition, near-wall biasing of the velocity vectors is noted.

The DPIV velocity flow profiles were investigated for the each of the shear rates investigated by computing the mean and RMS axial velocity at each point. The difference between the mean velocity and analytic velocity is then the bias error of the method (accuracy), while the RMS fluctuation is the random error of the method (precision). Figure 3.9 shows the analytic and
averaged DPIV flow profile (average of 2000 instantaneous flow profiles) as a function of the radius from the center of the pipe. The curves show excellent agreement, with biasing in the near-wall region. Within the high shear region, $0.6 < r < 0.95$, the velocity measured is lower than the analytical velocity. This biasing is due to the higher signal in the cross-correlation associated with slower moving particles that stay within the interrogation windows longer than faster moving particles. On the wall, the no-slip condition is imposed and so it is not a measurement from DPIV, but rather an assumption that is used in estimating the wall shear stress. For static boundaries, as is the case here, the wall position uncertainty is $\pm 0.5$ pixels, which can affect the estimation accuracy. The effect of random error is noted in the error bars on each point. The fluctuations increase as the measurement location approaches the wall.

![Figure 3.9. Analytical and DPIV time-averaged flow profile for a shear rate of 0.12](image)

The mean bias and random error associated with the velocity profile measurements were computed for each of the shear rates, presented in Figure 3.10. The analytical curve is determined based upon the mean centerline velocity. The error values are determined with respect to the local velocity value prior to averaging. The bias error is responsible for most of the total error.
associated with the measurement, with the mean bias error less than 10% over the entire range of shear rates. Most of the error is from the near wall region, where the actual measurement bias is on the order of 10 to 20% of the local analytic value. The lower shear rates have larger associated errors, but the mean absolute error levels off at roughly 4% for shear rates greater than 0.03. The DPIV data sets provide a set of data with known uncertainty so that the velocity derivative on the wall can be estimated.

![Figure 3.10. Mean bias and RMS error as a function of shear rate for Poiseuille flow](image)

**Accuracy of velocity derivative estimation**

Many conventional systems utilize lower order schemes, such as second-order central differences, to estimate velocity derivatives. The widespread use of these schemes results from their ability to filter the random noise levels inherent from the velocity estimation. However, our error analysis for the near wall velocity estimation demonstrated that the bias error is the dominant contributor. Thus we propose the use of higher-order compact schemes to address the converse problem: the high levels of bias error associated with the lower-order methods. In order to deliver highly accurate estimations of wall shear from experimental DPIV data, the level of bias error
associated with the method must be reduced, particularly in the case where the spatial sampling resolution is insufficient.

Following the efforts completed in Chapter 2, the methods employed herein are the fourth- and sixth-order accurate implicit compact finite difference schemes, which exhibit spectral-like resolution, entailing that they have higher accuracy over a larger range of scales than explicit finite difference algorithms. The methods are implemented implicitly in order to obtain a higher-order truncation error (Lele, 1990). The compact schemes used in this work are generalizations of the Pade scheme, and take the form:

\[
\beta f_{i-2} + \alpha f_{i-1} + f_i + \alpha f_{i+1} \beta f_{i+2} = \\
\frac{c f_{i+3} - f_{i-3}}{6h} + b \frac{f_{i+2} - f_{i-2}}{4h} + a \frac{f_{i+1} - f_{i-1}}{2h}
\]

(3.6)

where \( \alpha \), \( \beta \), \( a \), \( b \), and \( c \) are constants that are solved for by substitution of Taylor series coefficients, with \( f \) and \( f' \) representing the values of the velocity and implicitly-determined local derivative values. The values of the parameters \( \alpha \), \( \beta \), \( a \), \( b \), and \( c \) are shown below in Table 3.1.

**Table 3.1. Constants for compact schemes**

<table>
<thead>
<tr>
<th>Method</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( a )</th>
<th>( B )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compact-4</td>
<td>0.25</td>
<td>0</td>
<td>1.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Compact-6</td>
<td>1/3</td>
<td>0</td>
<td>14/9</td>
<td>1/9</td>
<td>0</td>
</tr>
</tbody>
</table>

Results for only the comp-6 scheme are presented here as the two methods yield nearly identical results for the simulations studied herein. It should be noted that the coefficient values of \( b \) and \( c \) for the fourth-order accurate compact scheme are zero, which thereby allows twice the order accuracy as the second-order finite (FD2) estimator with the same computational stencil size. Sustaining a small stencil will allow more accurate estimation for the small scales in the flow.

The error of these methods, for unbounded flows, has been quantified in detail for vorticity and velocity derivative estimation in previous works (Fouras and Soria, 1998; Foucaut and Stanislas, 2002; Etebari and Vlachos, 2005). The evaluation of wall-bounded velocity derivatives
requires additional consideration in order to determine the values of the derivative components. It is normally acceptable to use a finite difference scheme with a truncation error order, one order of magnitude lower than that of the method. Thus, the second-order methods utilize a first-order upwind scheme on the boundaries, given by:

\[
\frac{\partial u}{\partial y} = \frac{u_{i+1,j} - u_{i,j}}{\Delta y}
\]  

(3.7)

\(u\) is the axial velocity and \(\Delta y\) is the vector grid spacing. The fourth-order compact scheme utilizes a third-order accurate upwind scheme of the form:

\[
f'_{1} + \alpha f'_{2} = \frac{1}{h} (a_{1}f_{1} + b_{1}f_{2} + c_{1}f_{3} + d_{1}f_{4})
\]  

(3.8)

where \(a_{1}, b_{1},\) and \(c_{1}\) are given by the following relationships, which are solved for by substituting Taylor-series expansions into the upwind scheme, \(h\) is the vector spacing, and \(f'\) is one of the derivative components comprising the vorticity.

\[
a_{1} = -\frac{3 + \alpha_{i} + 2d_{1}}{2}
\]

\[
b_{1} = 2 + 3d_{1}
\]

\[
c_{1} = \frac{1 - \alpha_{i} + 6d_{1}}{2}
\]

\[
\alpha_{i} = 2
\]

\[
d_{1} = 0
\]  

(3.9)

For the sixth-order compact scheme, boundary conditions must be set for two points on each boundary, since it requires two adjacent points to either side of the point of interest in order to solve for the derivative. Accordingly, the fourth-order compact scheme is used for the inner boundary points, and the third-order upwind scheme is utilized on the outer boundary points. Again, the derivatives must be solved for, implicitly.
Conventional schemes have also been evaluated, including the second-order accurate, central finite difference formulation (FD2):

\[ \frac{\partial u}{\partial y} = \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta y} \quad (3.10) \]

A fourth-order central difference (fourth order accurate) approximation (denoted here as Chapra-4) developed by Chapra (1998), was investigated:

\[ \frac{\partial u}{\partial y} = \frac{U_{i,j+2} - 8U_{i,j+1} + 8U_{i,j-1} - U_{i,j-2}}{12\Delta y} \quad (3.12) \]

Two Richardson extrapolation algorithms, presented by Foucaut and Stanislas (2002), are evaluated in this study, namely the sixth-order formulation and a fourth-order noise-optimized scheme. These take the form:

\[ \frac{\partial u}{\partial y} = \frac{1}{A_k} \sum_{k=1,2,4,8} A_k \frac{u_{i+k} - u_{i-k}}{2k\Delta y} \quad (2.19) \]

The coefficients for the Richardson schemes used in this study are given below in Table 3.2. The asterisk denotes a noise-optimized set of coefficients.

**Table 3.2.** Constants for Richardson extrapolation schemes

<table>
<thead>
<tr>
<th>Order</th>
<th>$A$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_4$</th>
<th>$A_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>45</td>
<td>64</td>
<td>-20</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4*</td>
<td>1239</td>
<td>272</td>
<td>1036</td>
<td>0</td>
<td>-69</td>
</tr>
</tbody>
</table>

The fourth-order noise-minimizing Richardson extrapolation scheme (Rich-4*) combines a range of second-order central difference schemes with varying spatial samplings using an optimized set of coefficients, which minimize either the bias error or the random error. A more complete presentation of these methods can be found in Foucaut and Stanislas (2002) and Etebari.
and Vlachos (2005). A least-squares-fit, direct velocity differentiation approach (Fouras and Soria, 1998) delivers lower-order estimation than the second-order central difference scheme, and is therefore not of interest to this effort and not considered herein.

In order to evaluate the accuracy of the differentiation schemes, we perform Monte-Carlo simulations by generating a synthetic vortical flow field with a physical solid boundary, similar to that presented by Noguiera et al (12) but modified in order to satisfy the no-slip and no-penetration conditions on the wall. The shear stress along the wall is then proportional to the slope of the velocity profile along the boundary. A set of 8 representative flow fields were generated and used to evaluate the performance of each method. The spatial sampling resolution, $\Delta/L$, was varied for each flow field, such that the resolution varied from very fine ($\Delta/L = 0.05$) to very coarse ($\Delta/L = 0.5$), representing the Nyquist frequency. The flow field is given by:

$$S_x = \sin\left(\frac{2\pi x}{41}\right)$$

(3.10)

The nominal shear rate for the flow is then 0.15, although the velocity derivative algorithms are not functions of the shear rate, but rather the spatial sampling resolution within the flow field.

The average errors on the boundary of the velocity derivative are presented in Figure 3.16 as a function of the spatial sampling resolution, $\Delta/L$. $\Delta$ is the spacing between vectors and $L$ is the characteristic length scale of the flow, corresponding to the span of the smallest vortical flow structure, in this case the boundary layer thickness. The first-order upwind scheme quickly diverges from the known solution along the boundary as the spatial sampling decreases. The sixth-order compact scheme maintains an error of less than 1% for high spatial samplings, reaching a maximum bias error of 5.2% as $\Delta/L$ approaches 0.5. The compact-Richardson formulation exhibits low bias error for high spatial sampling ($\Delta/L < 0.3$), but quickly diverges for low spatial sampling ($\Delta/L > 0.3$). It is important to note that the velocity profiles used to determine these uncertainties have no associated bias or random error, and are merely sampled at different points, unlike the DPIV data set explored in the previous section, which has both bias and random error.
To determine the effect of velocity uncertainty on the wall shear estimation, 10% Gaussian random noise was added to the velocity vectors and averaged for 1000 flow profiles. The resulting error distribution is shown in Figure 3.12. The curves have the same form as for the case with no random noise, with higher associated errors. The compact scheme methods show large deviations, due to the nature of the implicit schemes on the boundaries. The sixth-order compact scheme begins to display oscillations as well, which are also consequence of using the implicit solution on the discontinuity in the flow field, i.e. the boundary.
An alternative method was developed herein based upon the reflection, or mirroring, of the velocity vectors across the boundary, effectively extending the flow field. To illustrate the proposed methodology, Figure 3.13 shows the reflection process in a more generalized manner for a single point along the boundary, with the near-wall velocities parallel to the wall. The direction of the velocity vectors is reversed in order to avoid discontinuity of the velocity and its derivative along the boundary. All of the derivative estimation schemes can then be used to determine the wall shear, since their derivative estimations within the flow (away from the boundaries) all differ.
Figure 3.14 shows the average normalized errors for the velocity derivatives on the boundary for the flow field with the reflected vectors. The method results in a significant reduction of bias error, on the order of less than 0.5% for most of the range of resolutions rendering the method independent of the spatial sampling. The compact-Richardson scheme reaches a maximum of under 0.2% at a sampling resolution equal to the Nyquist frequency compared to almost 50% for the case with no vector reflection. The accuracy is increased greatly for the poor spatial samplings, thus reducing the dependence of the estimation accuracy on the distance between the boundary and the nearest resolvable vector. This result can be attributed to the ability of the system to utilize the higher-order derivative estimation schemes to determine the derivative at the wall as if it were a point in an unbounded region of the flow. The methods retain their higher-order truncation error because the respective boundary methods are not used to evaluate the derivatives. The fourth-order noise-optimized Richardson scheme exhibits the highest error, which yields less than 1.6% error at the lowest spatial sampling resolution.
As in the previous case, 10% Gaussian random noise was added to the velocity vectors and averaged over 1000 flow profiles. The mean error distributions are shown in Figure 3.15. Clearly the effect of the noise is visible, particularly for the higher spatial sampling ($\Delta/L < 0.2$) in the higher-order schemes which do not offer any smoothing. The fourth-order, noise-optimized Richardson scheme offers good smoothing at the high spatial sampling resolutions ($\Delta/L < 0.2$), but has high error at poor sampling resolutions ($\Delta/L > 0.2$). The fourth-order, noise-optimized compact-Richardson scheme offers the best performance over the entire range of sampling resolutions.
WSS estimation from DPIV data

WSS analysis was performed for the Poiseuille flow profiles experimentally obtained from DPIV, using both the no-slip condition and the reflected vector method. For the no-slip condition, only the second-order finite difference scheme yields reasonable error (less than 30%) over the range of shear rates investigated (Figure 3.16). The implicit compact schemes are far more erratic, however, yielding as much as 100% error depending upon the flow field. The poor performance of these methods can be attributed to the previously mentioned oscillations due to the implicit solution near the boundary. The errors in the reflected vector method are shown in Figure 3.17. The vector reflection causes all of the methods to collapse near one another, reducing the overall error in the compact schemes drastically. For most of the shear rates the errors remain well below 20%, which is quite reasonable considering the bias error on the velocities themselves was on the order of 5-10%. Considering how little the data is spread, one is led to conclude that the bias error from the velocities and the wall position uncertainty is responsible for the majority of the total error.
It is important to note that the error does not show a strong trend with respect to the shear rate. As noted above, the accuracy does depend upon both the spatial sampling and the random noise. In this case the spatial sampling is held constant, and each flow profile has the same parabolic form with the only difference being a scaling factor that is a function of the centerline velocity. The average error over the entire set of data for each method was determined to be between 14.9% and 15.9%.
3.4 Conclusions

The accuracy of boundary point determination was investigated, and a correlation-based method for obtaining sub-pixel displacement accuracy in its determination is developed herein. The method utilizes statistical cross-correlation to give a stochastic estimation of the sub-pixel location of the edge points. The method is found to be accurate to within 0.1 pixels as compared to an uncertainty of 0.5 pixels for conventional edge detection. The effect of averaging in velocity vector estimation due to interrogation window size is investigated, and the effect of increasing shear rate is evident in the biasing of the velocities.

A DPIV data set for Poiseuille flow through a 4 mm tube has been obtained, and its error quantified. The high shear region near the wall is subject to large bias errors due to biasing of the cross-correlation towards lower velocities. Finally, the bias error associated with wall derivative estimation was quantified. The use of fourth- and sixth-order compact schemes in estimating
velocity derivatives from DPIV measurements is shown to deliver improved accuracy compared with conventionally utilized explicit schemes. This is true for the entire range of spatial sampling resolutions investigated when used with the reflected velocity vector method developed herein. Previous studies suggest that methods utilizing a form of smoothing should be used to reduce this propagated error. However, it was shown herein that the bias error in the near-wall velocity measurement is roughly twice the corresponding random error. Thus, it is suffice to state that the bias error should be considered the most important parameter in the analysis, precipitating the need to employ higher-order methods. The compact schemes are less susceptible to errors introduced by poor sampling resolution or strong velocity gradients. In particular, the error of the measurements is less than 2% for the range of spatial samplings investigated.

3.5 References


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4 A DYNAMIC WALL SHEAR STRESS SENSOR BASED ON IONIC POLYMER TRANSDUCERS

This paper is in under review at Experiments in Fluids.

Abstract

This chapter presents the first implementation of a novel class of dynamic, time-resolved, direct skin friction measurement sensor using active ionic polymer transducers. These sensors contain no moving parts, directly measure shear, and can be surface mounted with minimal flow intrusion. A complete characterization of the sensor dynamic response and accuracy is presented. We demonstrate measurement accuracy in fluctuating shear on the order of 4.92% over a range of stresses of +/- 3 Pa and signal-to-noise-ratio on the order of 60 dB. The frequency response of the sensor is over 10 kHz; however, due to experimental limitations we were not able to calibrate for frequencies higher than 140 Hz. These sensors were shown to be insensitive to vibration or pressure. An automatic change of impedance compensation approach is proposed that allows in-situ recalibration of the sensors. The results demonstrate the potential for ionic polymer sensors to provide accurate, high frequency measurements of shear in turbulent boundary layers.

Keywords:

wall shear stress measurement, turbulent boundary layers
There is an increasing need for novel technologies that deliver easy to implement, accurate, robust, time-resolved measurements of skin friction in harsh environments. Turbulent skin friction is the major contributor of viscous drag on any body operating in a fluid. Recent developments in micromachining processes have led to the design of a class of microelectromechanical (MEMS) sensors that attempt to measure wall shear based on the requirements set forth by the turbulent flow fields of interest. Namely, the sensors should deliver spatial resolution on the order of 100 µm and frequency bandwidth on the order of 10 kHz (Sheplak et al, 2002b). Conventional technologies have been unable to provide sufficiently accurate measurements over this range of fluid velocity fluctuation scales due to problems with sensitivity, frequency response, cross-axis sensitivity, and electromagnetic interference (EMI). A robust technique for measuring wall shear stress is needed to characterize the structure of the turbulent boundary layers in order to assist the development and assess the effectiveness of viscous drag reduction flow control methods.

The objective of this work is to develop and demonstrate the feasibility of a novel class of direct wall shear sensing technology based on ionic polymer membrane transducers. To verify the effectiveness of this new technology, we experimentally investigate its effectiveness in resolving dynamic (time-resolved) measurements of skin friction with application to high-Reynolds number flows. Herein we present a first implementation of the sensor development with dynamic
calibration and characterization of the sensor design. This paper will give an introduction to the mechanism of transduction of active ionic polymers, followed by methodology and results for a dynamic oscillating Stokes-layer calibration, and an analysis of the advantages and disadvantages of this sensor versus conventional skin friction sensing technologies.

4.2 Background

Several technologies currently exist for measuring skin friction. However, none have been able to deliver dynamic, time-resolved skin friction measurements with sufficiently high frequency response, accuracy, durability, and cost-efficiency. Current technologies include hot-wire/film sensors, oil-film interferometry, liquid crystal coatings, and floating element, optical, and thermal microelectromechanical systems (MEMS). A thorough review is beyond the scope of the current effort; however the most promising of these technologies will be discussed. Readers are directed to Naughton and Sheplak (2002) for a thorough review of shear stress sensor technologies.

MEMS sensors have presented the most promise in novel shear stress sensor design. Three types of MEMS sensors exist, each with distinctly different characteristics in their sensing modes, accuracies, and frequency responses. Namely, these are floating element, thermal-based, and optical MEMS. Floating element MEMS sensors measure the integrated force on a flexible membrane supported by thin flexures, typically on the order of 100 to 500 µm in size. The advantage of this technique is that it is a direct measurement of skin friction; however, the sensors suffer from error associated with misalignment between the floating element and gap, pressure gradients, cross-axis sensitivity to acceleration and vibration inputs, and debris that may become trapped in the gap between the floating element and the sensor mount. Moreover, the floating element must be sufficiently flexible to resolve forces on the order of 10 pN, compromising its durability (Naughton and Sheplak, 2002). MacLean and Schetz (2003) present a numerical study on the operation of such a sensor and find that the theoretical overall error in the presence of pressure gradients is 17%. Sheplak et al (2001) present a dynamic calibration of a floating element MEMS sensor using a plane wave tube setup to generate a Stokes-layer oscillation for dynamic calibration with a fluctuating shear of 0.1 Pa at 100 Hz up to 1 Pa at 10 kHz; however, frequencies above 4 kHz were not obtained due to the structural resonance of the tube at this frequency. The response of the sensor was shown to be linear over most of the range of shear stresses and
frequencies. The dynamic calibration results were presented in the form of a bode plot of the magnitude of the frequency response function. However, an error analysis of the actual signal in the time-domain was not presented.

Thermal-based sensors operate based upon the transduction of temperature fluctuations to a voltage signal in the same manner as hot-wire anemometers. The major drawback of this technology is that the frequency response of thermal-based sensors is difficult to characterize due to the frequency-dependent heat conduction to the substrate, further complicating the calibration process. Sheplak et al (2002) characterized a thermal-based MEMS sensor under both static and dynamic shear environments. Static calibration up to 1.7 Pa was achieved via a laminar flow channel. Dynamic calibration was performed using the plane-wave tube with Stokes-layer oscillation described above, with a theoretical fluctuating wall shear stress envelope of 0.9 mPa at 0.2 Hz to 6.1 mPa at 7 kHz. A noise floor of 100 nV/Hz$^{1/2}$ was observed, with a resolution of 9 µPa/Hz.

The third type of MEMS sensor utilizes ultrasound or optical Doppler effect to estimate the shear stress by measuring a frequency shift caused by the motion of particles near the wall (Gharib et al., 2002; Fourgette et al., 2001&2003; Arik et al., 2004; Modarress et al., 2002; Modarress et al., 2004; Nowak, 2002). Ultrasound Doppler velocimetry (UDV) sensors measure the Doppler frequency shift between emitted and reflected signals. Their optical counterpart, based on optical MEMS (MOEMS), calculates wall shear from the scattered Doppler frequency of particles passing through a diverging fringe pattern with the predefined fringe divergence rate. Both sensor types provide direct measurements of the local wall shear, are non-invasive, are sufficiently accurate under optimal conditions, and can respond to high-frequency fluctuations. However, the measurements are significantly limited by the particle seeding, which limits the data rate (Nowak, 2002, Naughton and Sheplak, 2002). Moreover, the methods are currently limited by the difficulty to fabricate a sensor with a probe volume that is contained entirely within the viscous sublayer (Naughton and Sheplak, 2002). In flows where natural seeding is employed, the accuracy of the measurement may be affected by the size and properties of the particles. Nowak (2002) estimated the total error of UDV to be +/- 8.4 %. Fourgette et al. (2001) present static calibration results for a MOEMS shear stress sensor with a probe volume located 66 µm above the wall surface. The group theoretically estimated accuracies on the order of 99% for low Reynolds
number (laminar) boundary layer flows. However, the accuracy of the method decreases rapidly as the Reynolds number increases due to the large probe volume height with respect to the height of the viscous sublayer. They estimate accuracy as low as 70% depending upon the ratio of these two quantities.

### 4.3 Active Ionic Polymers

Active ionic polymer sensors can provide a novel alternative to conventional devices as a result of their robustness (no moving parts), direct sensing ability, high frequency response, and high sensitivity to mechanical deformation. Ionic polymer transducers consist of an ionomer membrane plated with conductive metal layers. These materials exhibit electromechanical coupling under the application of electric fields and imposed mechanical deformation (Oguru et al., 1992, Shahinpoor et al., 1998, Sadeghipour et al., 1992). Newbury and Leo (2002) and Bennett and Leo (2003) have shown that these materials demonstrate sensitivities two orders of magnitude higher in charge-sensing mode than piezoelectric polymers such as PVDF. At the same time, they have the high compliance and durability of a polymer. Moreover, they can be constructed in any configuration to suit specific applications.

![Active ionic polymer schematic](image)

**Figure 4.1. Active ionic polymer schematic**

Ionic polymer membranes contain charged side groups that are covalently bound to the polymer chain. A phase separation between hydrophilic “ion clusters” and the hydrophobic backbone results in a charge aggregation that leads to ionic selectivity. As a result, only certain charged groups, either cations or anions, can be transported through the polymer. Charge aggregation due to mechanical deformation gives rise to the transducer sensing capabilities. Figure
4.1 illustrates the process by which mechanical deformation of the transducer causes charge redistribution via cluster deformation.

Due to the large capacitance of ionic polymer transducers (on the order of 2 to 5 mF/cm²) the voltage generated across the thickness is very small. On the other hand, using charge sensing or current sensing circuits has proven to be effective in increasing the signal-to-noise ratio of the sensor (Newbury and Leo, 2003a, Bennett and Leo, 2003). Thus, a charge amplifier was utilized in this work to condition the signal generated by the ionic polymer transducer.

4.4 Methods and Facilities

Sensor fabrication

The ionic polymer material consists of a Nafton™-117 membrane that is plated on both surfaces by the procedure described in Akle et al (2003). From a large piece of material, small strips are cut to be used as wall shear sensors. In order for ion conduction and electromechanical coupling within the material to occur, the polymer must be swollen with an appropriate solvent. Traditionally, the ionic polymer sensors have been used in a water-saturated state. However, due to its volatility, water limits the processing of the sensors and the environments in which they can be used. Bennett and Leo (2004) have recently shown that ionic liquids can be used to replace the water. Ionic liquids have the advantage that they have an immeasurable low vapor pressure and are highly stable, even at high temperatures.

After the sensor strips are swollen with ionic liquid, they are packaged into a useable configuration (Figure 4.2). A copper-clad polyimide substrate is used to mount the sensor. Two electrical traces are etched onto this substrate in order to make contact with the surface electrodes on the sensor. The sensor is attached to this substrate using a conductive epoxy. In order to connect the top surface to the positive electrode, a 25 micron-thick gold wire is attached using a drop of conductive epoxy that is on the order of 50 microns in thickness. For a turbulent flow this surface may affect measurements in turbulent flow fields, where the scales of interest may be as low as 10 microns. This issue has been addressed by our group, and future prototypes are being designed with direct connections between sensor electrode and substrate connections. Finally, the sensor is laminated with a thin (2.5 µm) Mylar film. This serves to protect the sensor from the surrounding environment and improves its robustness and reliability. Sensors have been
fabricated with square sensing elements between 1 and 3 mm in width. The sensing element depicted in Figure 2 is approximately 2 mm x 2 mm in size. Fabricating larger sensors does increase the sensitivity, making it easier to perform a dynamic calibration. Future work will involve employing self-assembly methods to fabricate smaller sensors in order to resolve smaller scale flow structures.

Figure 4.2. A prototype ionic polymer wall shear sensor: picture (left) and schematic (right)

4.5 Oscillating Stokes layer test apparatus

An oscillating Stokes layer apparatus was designed for performing dynamic calibration of the active ionic polymer sensor. The Stokes oscillating plate provides a well-known analytical solution for a plate harmonically oscillating in its own plane with velocity:

\[ u_w = u_o \sin \Omega t \]  

where \( \Omega \) represents the frequency of oscillation. The non-dimensional wall shear stress (\( y = 0 \)) is:

\[ \tau_w = -\frac{1}{\sqrt{2}} [\sin(T) + \cos(T)] \]  

Figure 4.3 shows a diagram of the experimental setup for the calibration. The oscillation is generated by a cam driven by a motor / gearbox. The gearbox is used as a speed increaser and has a ratio of 50:1. The motor is driven by a 12 V battery and a speed controller. The eccentricity of the cam is 125 \( \mu \)m, which generates 250 \( \mu \)m peak-to-peak oscillatory plate motion. A laser
vibrometer is used to measure the motion of the plate, with a measurement resolution better than 25 µm/sec. Although any fluid can be used in the bath, the results shown herein are for water at room temperature only. The calibration setup is separated into two parts: a drive section and a test section. In practice, the two sections are placed on separate tables and are connected only by a single shaft. This arrangement minimizes the effect of transferred vibration on the measured signal. The presence of a Stokes layer has been verified using DPIV.

The vibrometer readings are numerically differentiated in order to obtain the shear stress from the Stokes equations and phase shifted according to the Stokes equations as well. The data is acquired with sufficiently high sampling rate (2 kHz) to ensure that aliasing does not occur (greater than 15 times the highest calibration frequency).

![Figure 4.3. Stokes layer calibration setup](image)

Although necessary, it is very difficult to characterize a Stokes layer calibration fixture of this type due to the small penetration length and a lack of adequate instrumentation to allow a full characterization. In order to verify the presence of a Stokes layer, DPIV was performed with 1 kHz sampling rate. 2 micron silvered glass spheres were used as flow tracers, with a magnification of 2.5 microns per pixel. Figure 4.4 shows the wall velocity profiles through a single cycle of oscillation driven at 30 Hz as measured by DPIV. The vector spacing was 20 microns. Clearly, the existence of a Stokes layer is evident.
For the calibration, the output voltage of the charge amplifier, $V(t)$, is compared with the analytically estimated shear stress, $\tau(t)$, using a least-square fit that minimizes the relation:

$$\sum_{n=1}^{N} \left[ C \times V(t) - \tau(t) \right]^2,$$

where $C$ is the calibration coefficient. The measurement error for a given calibration coefficient can then be calculated as:

$$\frac{\tau(t) - C \times V(t)}{\tau(t)}.$$  

(4.4)

### 4.6 Results

**Dynamic calibration**

The IPT wall shear stress sensor was dynamically calibrated over a range of frequencies from 20 to 140 Hz with 1 kHz sampling rate over a 5 second period. By converting Eq. (4.2) into dimensional form, it can be shown that the wall shear stress magnitude is proportional to the product of the velocity magnitude, $u_0$, and the square root of the frequency of oscillation. This is analogous to the product of the pressure magnitude and the square root of the frequency of oscillation in the acoustically driven plane-wave excitation setup of Sheplak et al. (2001).
Preliminary calibration was performed at each frequency by comparing the sensor output with the analytically predicted shear stress using least-squares fit. A charge amplifier circuit was used to condition the sensor output. The analytic wall shear was determined directly from the laser vibrometer displacement measurement using the relations developed in the previous section. An interesting feature of the initial calibration was a distinct roll off of the sensitivity with increasing frequency. This characteristic is unfavorable, as it is desirable to have a flat response with respect to frequency. We attribute this behavior to the interaction between the highly capacitive transducer and the charge amplifier in the signal conditioning circuit. Particularly, the impedance mismatch between the two may compromise the high frequency response, resulting in a 20 dB/decade roll off in frequency response. Sheplak et al. (2002) noted similar trends for a thermal MEMS sensor, observing a 40 dB/decade roll off in frequency response. Such results are indicative of a highly damped second-order system. Frequency dependence was also observed by Sheplak et al. (2001) for a floating element MEMS sensor, which displayed a 5 dB decrease in amplitude between 200 Hz and 4 kHz. Most of the frequency roll off was observed in the lower frequency range (< 500 Hz), and it was unclear as to whether this trend continues for frequencies below 200 Hz.

In order to address the frequency dependent roll off of the sensitivity, a differentiator stage was introduced into the signal conditioning to counteract the 20 dB/decade roll off. Figure 4.4 shows calibration results for the same transducer using the modified signal conditioning (new sensor) versus the original signal conditioning (old sensor). Both calibrations were performed in water and have been normalized by their maximum value to yield a nominal sensitivity for comparison of the two response curves. Clearly, the frequency dependent roll off is significantly reduced with the modified signal conditioning arrangement, although a small decay is still observed.
The frequency dependence of the sensor output can be described as follows. The sensor may be modeled as a circuit consisting of resistor and capacitor elements. In doing so one finds that the interaction of the sensor with the charge amplifier circuit will introduce frequency dependence into the input / output transfer function. This frequency dependence arises due to the fact that the sensor is not a pure capacitor. One can consider the analogy to a piezoelectric sensor coupled to a charge amplifier. Because piezoelectric sensors are essentially pure capacitors and possess a small capacitance (typically nanofarads to microfarads), no frequency dependence will be observed in the input / output transfer function. However, based on classical models of piezoelectric sensors, one will find that for a sensor whose impedance matches that of the ionomeric wall shear sensor, frequency dependence will be observed. A typical impedance curve for the ionic polymer transducer is shown below in Figure 4.6.
The ability to obtain a flat frequency response allows analysis in the time domain of the sensor output versus the analytical shear stress as determined from the laser vibrometer signal. To obtain a single calibration coefficient for the prototype sensor, all of the calibration data is stored point by point for the time-varying sensor output versus the time-varying analytical wall shear stress values. A least squares fit routine (Eq. (4.4)) is then used to correlate the input with the output. Figure 4.7 illustrates this process, showing the sensor output versus the wall shear stress for over 100,000 points. For this sensor, the calibration coefficient was determined to be 0.51 Pa/V. The $R^2$ value for the fit was 0.97, and the mean overall error with respect to the full-scale range was determined to be 4.92%. Figure 4.8 shows the error distribution (with respect to the full-scale range) over the range of shear stresses investigated. The data have been binned to 0.1 Pa increments in order to show the mean absolute error for each shear value. The error remains below 8% for the entire range of shear stress values, with most of the values below 5%. It should be noted that to the best of our knowledge this is the lowest wall shear stress error reported under dynamic calibration conditions yet to date.
4.7 Sensitivity coupling

In order for a wall shear sensor to be effective, it must not only exhibit a high sensitivity to shear, but must also exhibit a low sensitivity to any unwanted inputs. Sensor response to vibration, pressure, and temperature has presented limitations in the shear sensing capabilities of MEMS...
sensors (Sheplak et al., 2001&2002; Schober et al., 2004). In order to demonstrate the ability of the ionic polymer sensors to reject unwanted signals, several key experiments have been performed.

_Vibration_

In order to evaluate the effect of vibration, a polymer sensor and an accelerometer were both mounted to a flat acrylic plate. The plate was struck with a modal hammer fitted with a load cell. A Fourier analyzer was used to measure the input signal from the modal hammer and the output from the polymer and accelerometer. The frequency response functions between the impulse force input and both the polymer and accelerometer were then computed as shown in Figure 4.9. As can be seen, the accelerometer response correlates well with the vibration of the plate. Three vibrational modes can be clearly seen in the response and the coherence is near 1.0 for most of the frequency range. However, the polymer sensor output does not show correlation with the plate vibration as indicated by its flat response and very low coherence. Although no calibration sensitivities have been applied to the data, the sensitivity of the polymer sensor to vibration appears to be negligible.
Pressure

In order to explore the effect of dynamic pressure on sensor output, a polymer sensor was subjected to a pressure input from a speaker placed at a distance of 1 meter from the sensor. A microphone was used to measure the pressure generated by the speaker. A schematic of the test setup is shown in Figure 4.10. Based on the response of the microphone, the incident pressure on the polymer sensor was determined to be on the order of 1 Pa over frequencies ranging from 100 Hz to 5 kHz. The response of the polymer sensor to this pressure was found to be less than 0.02 V/Pa. The shear sensitivity of the ionic polymer sensors is typically about 2 V/Pa. Therefore, the sensors are about 100 times more sensitive to wall shear than to pressure. Accordingly, the effect of dynamic pressure on the shear response of the sensor is negligible; however, its effect in water has yet to be quantified, though it is anticipated to be the same in both media.
Because the electromechanical coupling in ionic polymer sensors arises from motion of the mobile ions within the polymer, their sensitivity is expected to change with temperature. This change in the sensitivity is caused by a change in the mobility of the ions, which is also manifested as a change in the electrical impedance of the sensors. These effects are illustrated in Figs. 4.11 and 4.12 for two sensors. The results were obtained by performing multiple calibrations on two sensors in the Stokes layer fixture at different temperatures. The water temperature and sensor impedance were measured immediately prior to and immediately after calibration to ensure measurement accuracy. As can be seen in Figure 4.11, the high frequency impedance of the sensors decreases as the temperature is increased, leading to an increase in sensitivity.
Figure 4.11. Electrical impedance (@ 300 Hz) vs. temperature for two ionic polymer sensors.

Figure 4.12. Sensitivity vs. temperature for two ionic polymer sensors.
The sensitivity is linked to the high frequency impedance through a logarithmic relationship:

\[ \text{sensitivity} = -\ln(\text{impedance}) + \alpha \]  

(4.5)

The first term in Eq. (4.5) is a universal relationship true for all ionic polymer sensors. The constant term \( \alpha \) is a parameter that is unique to each sensor determined by the manufacturing and packaging processes and can be obtained by a single calibration. This parameter is related to the ability of the sensor to physically interact with its environment and will be higher for sensors with higher sensitivity. For the two sensors tested here, the \( \alpha \) parameter is found to be 7.6 for sensor A and 7.1 for sensor B. Accordingly, sensor A exhibits slightly higher sensitivity to wall shear than sensor B. However, the similarity of the \( \alpha \) parameters for the two sensors demonstrates repeatability of our manufacturing process. The measured sensitivity versus impedance is shown in Figure 4.13, along with the logarithmic fit (Eq. (4.5)), where the impedance was altered by controlling the temperature, resulting in the change in sensitivity.

By using Eq. (4.5) and the \( \alpha \) parameters that were measured for the two sensors, the sensitivity of the sensors was predicted from the measured impedance over a range of temperatures, as shown in Figure 4.14. The data points are the measured sensitivity at each temperature, and the curves are the sensitivities predicted from the impedance using Eq. (4.5). It is
important to clarify that the curves are not regression lines. Rather, the relationship is used here as an automatic calibration approach, as it accurately predicts the change in the sensors sensitivity over a range of temperatures from the measured impedance. This allows on-site (i.e. while deployed/mounted) calibration of the sensors.

As shown from the previous results, the sensors can be used effectively over a range of temperatures. It was shown that the impedance change was caused by a temperature change. However, the logarithmic relationship between sensitivity and impedance holds true regardless of the origin of the impedance change. For this reason, the approach described here can potentially be used to compensate for any change in sensitivity that is caused by a change in impedance. Automatic calibration can likely be used to correct for drift due to unforeseen changes, such as aging of the sensors. By using the impedance to compensate for changes to the sensitivity, the need to calibrate the sensor each time it is used is eliminated. The elegance of this approach stems from the ease with which the impedance of the sensors may be measured. The accuracy of this method has not been estimated. Further experiments are necessary to verify repeatability of the method and to obtain sufficient data points in order to perform adequate quantification of the error associated with the method.

Transducer degradation would have a detrimental effect on the auto-calibration procedure. The major issue associated with degradation is due not to the Nafton™ polymer membrane itself,
but rather to changes in the amount of solvent with which the membrane is swollen. Traditionally, water has been used as a solvent for ionic polymer transducers. However, water has a large vapor pressure, making it difficult to control the sensitivity of the transducer. In order to overcome this limitation ionic liquids were used. Ionic liquids are salts that exist in their liquid state. They have immeasurably small vapor pressure and are inherent ion conductors. The use of ionic liquids in the sensor design has shown no effects of degradation, and measurements are currently being conducted to prove this in the long term.

4.8 Application 1: Turbulent boundary layers

An experiment was conducted in the Virginia Tech Fluid Mechanics Laboratory water tunnel to replicate classic theoretical and experimental skin friction coefficient results for a sharp edge flat plate boundary layer. A bare (unpackaged) ionic polymer of 90- micron thickness, with dimensions 4 x 4 mm, and a piezoelectric film sensor (PVDF), with dimensions 6 x 12 mm, serving as the current state-of-the-art in direct displacement sensing materials were tested for shear response. The sensors were flush mounted 1.5 meters downstream from the leading edge and time-resolved signals were recorded at 20 kHz sampling rate for 15 seconds. Reynolds numbers ranging from the laminar flow regime (Re = 300,000) to fully turbulent flow (Re = 1.1 x 10^6) were investigated. No response was realized for the PVDF sensor, while the ionic polymer sensor showed significant response to wall shear. Several time records of the fluctuating wall shear measurements are shown below in Figure 4.15 for transitional and fully developed turbulent boundary layer flow. The development of turbulence is evident from the time series, and becomes clearer in the spectral domain, shown in Figure 4.16 for a few of the Re numbers investigated. The energy of the power spectra clearly increases with Re, as expected.
Distinct spikes are noticeable, particularly for the lower Re number flows. The presence of these spikes can be attributed to EMI caused by the tunnel motor operating frequency, which determines the free-stream velocity. For the higher Re number flows, the spikes are almost negligible, due to the high shear signals present. A limitation of the study was that Kapton tape was used to insulate the polymer to prevent short-circuiting between the surfaces of the ionic polymer. However, this encapsulating layer is stiffer than the polymer itself, reducing the amplitude of its response to shear. The motor frequency noise thus was realized to be on the order of magnitude of the shear, particularly for the lower Re flows. In addition, a level of 60 Hz noise was present in the signals due to the surrounding circuitry.
4.9 Application 2: Stenotic vessels

Locations associated with low or complex spatio-temporally varying WSS, on the other hand, are the most susceptible to atherosclerosis (Kleinstreuer et al, 2001; Liepsch, 2002). Frequently, these abnormal shear levels are found in bifurcations, curved vessels, and stenoses. The presence of stenosis, in particular, alters the hemodynamic characteristics of the system, which alters wall shear stress (WSS) levels and effects disease progression. It is therefore essential to accurately quantify the spatio-temporal variations in WSS within the vasculature in order to better diagnose disease progression and also improve implantable devices.

Ionic polymer transducer sensors have the potential to be used for \textit{in vivo} experiments. Our present efforts serve to demonstrate our success in implementing these sensors for \textit{in vitro} biomedical applications. Herein, we utilize our novel micro-sensor in performing direct measurement of WSS in a pulsatile flow, as well as upstream and downstream of a stenotic vessel. To demonstrate the feasibility of the WSS sensor for a pulsatile flow, a 25 mm diameter pipe flow was designed such that DPIV measurements were taken simultaneously with direct WSS measurements using two previously calibrated sensors on the inner wall of the tube. WSS was estimated from the DPIV data using a 2\textsuperscript{nd}-order central finite difference scheme. The comparison
between the DPIV WSS estimation and the two WSS sensors is shown in Figure 4.17. The data agree to within approximately 20%.

![Figure 4.17. Time records of sensor WSS measurement and DPIV WSS estimation for a pulsatile flow](image)

WSS levels were quantified in compliant vessels 30 mm in length using an *in vitro* flow loop that generated peripheral and coronary flow waveforms. The vessels used in these experiments were fabricated in-house using an elastomeric silicone (Sylgard 184, Dow Corning Corporation) with 4 mm inner diameter and 0.5 mm wall thickness. A symmetric glass stenosis with a length of 20 mm (5 diameters) was inserted into the vessel to provide a 90% blockage by area. A computer controlled gear pump generated a physiologic waveform through the vessel.

The Reynolds numbers investigated in this study range from 150 to 350, with corresponding Womersley parameters, $\alpha$, ranging from 2.7 to 3.7. The flow rate was measured upstream of the vessel using an ultrasonic flow meter (Transonic Systems Inc. Model T110). Additionally, the pressure upstream of the vessel was measured using a pressure transducer (Omega PX302). LabView was used to control the gear pump and acquire data from the flow meter, pressure transducer, and ionic polymer WSS sensors. The sensors were flush mounted 2.5 diameters upstream, 2.5 diameters downstream and 5 diameters downstream of the neck of the stenosis. The sensors were fabricated such that they were 200 microns in height, with surface
dimensions of less than 1 mm by 3 mm. A small Kapton film backing was attached to each sensor to prevent it from responding to deformation due to vessel expansion.

Figure 4.18(a) displays time records of normalized WSS measurements from the three IPT sensors for pulsatile flow with a Reynolds number of 350. The upstream sensor displays smooth pulsatile flow, with a few high-frequency oscillations noted during the deceleration phase of the flow. The sensor located five diameters downstream, on the other hand, shows the presence of high frequency fluctuations in the acceleration and forward phases of the flow. These are associated with the generation of turbulent flow structures. These are made particularly apparent in the power spectral density plots shown in Figure 4.18(b). Furthermore, the sensor response appears to have a peak located at approximately 35 Hz, which may be related to the shear layer instability between the high speed flow in the neck of the stenosis and the slow moving flow in the recirculation regions.
In summary, this work represents the first demonstration of direct, dynamic WSS measurements in stenotic vessels. This achievement, although a major leap forward, is only the first step in understanding the complex phenomena associated with the atherosclerotic disease process. Future, ongoing comparisons with DPIV will provide benchmark validation of the sensor response, further validating their applicability in biological flows.

4.10 Application 3: Mechanical heart valves (MHVs)

Abnormal shear levels are often found in mechanical heart valves, bifurcations, curved vessels, and stenotic vessels. The mechanics of these flows have been quantified in detail through numerical and experimental investigations over the past two decades, and WSS sensors have been used in limited in vitro experiments (Bluestein et al, 2000). However, direct measurements of WSS have yet to be performed in cardiovascular applications. In this effort, we demonstrate application of our novel micro-sensor in providing direct measurement of WSS upstream and downstream of a prosthetic bi-leaflet valve in an aortic configuration. More importantly, we were successful in attaching the sensors on the valve leaflets, allowing for the first time direct quantification of WSS values on the surface of the leaflets. This achievement represents a leap forward in WSS measurement technologies.

The flow past artificial heart valves drastically changes the flow conditions and induces disturbed flow, resulting in vortical or turbulent flow. Bluestein et al. revealed vortex shedding as a
mechanism for free emboli formation past mechanical heart valves. Turbulence as well as high shear rates and deformation were significant factors implicating thrombus formation. Two-dimensional LDV and PIV systems have been used for in vitro testing of turbulence characteristics of mechanical heart valve (MHV) prostheses. Liu et al. (1996) studied the turbulent flow characteristics of three bileaflet aortic valves using LDV. They found that the Reynolds shear stresses of all three prosthetic valves induced minor damage to red blood cells, but directly damaged platelets, thus increasing the possibility of thrombosis. Additionally, the smallest turbulence length scale, offering a more reliable estimate of the effects of turbulence on blood cell damage, was found to be three times the size of red blood cells and five times the size of platelets. This suggests that there is more direct interaction with the blood cells, thus causing more damage. Although such studies have quantified shear stresses in the flow, none of them have being able to present direct dynamic measurements of WSS on MHV leaflets.

In this effort, we conducted both steady and pulsatile flow experiments in a circular vessel (both rigid and flexible), which housed a MHV. A St. Jude Medical (SJM) bileaflet mechanical valves (diameters of 25-31mm), the most commonly implanted heart valve prosthesis, were used in this study. The Reynolds numbers investigated ranged from 1000 to 5000, with volumetric flow rates ranging from 1.5 to 6 L/min. A gear pump was used to drive the flow, while an ultrasonic flow meter was used to measure the flow rate through the vessel (Transonic Systems Inc., Model T110). Two pressure transducers (Omega PX302) were positioned for measurement both upstream and downstream of the valve. Ionic polymer sensors (2 mm width by 5 mm height) were flush-mounted on the valve leaflets surfaces which are rigid, but also on the vessel wall upstream and downstream of the valve. Figure 4.19 serves to illustrate the experimental setup as well as a picture of an active ionic polymer positioned on one of the valve leaflets. One sensor was placed 20 mm upstream of the valve. Two sensors were placed 20 mm downstream such that each corresponded to the same circumferential position as one of the sensors on the valve leaflets. Sensor output was acquired using LabView software with 2 kHz sampling rate for 50 seconds for each case.
Figure 4.19 displays normalized time records of WSS measurements from two ionic polymers positioned on the back face (downstream side) of the two leaflets of the SJM MHV for pulsatile flow at 60 bpm with a flow rate of 3.5 L/min. Measurements taken by a sensor upstream and two sensors downstream of the valve are also shown in the figure. The upstream time record (green curve) shows a purely pulsatile waveform, as expected. The time records on the valve leaflets show a drastic increase in the WSS on the leaflets with respect to the upstream sensor, with the right leaflet experiencing approximately 100% increase in peak WSS and the left almost 300%. Furthermore, there are two distinct peaks in the signals that are in phase with each other, corresponding to the opening and closing of the valve leaflets. High frequency fluctuations are also noted by inspection of the time records. Power spectral densities computed for the signals are shown in Figure 4.21. The high frequency fluctuations are apparent from the spectra, with higher magnitude noted for the right leaflet at high frequencies, probably corresponding to turbulent fluctuations. These are diminished by the time the flow has reached the downstream sensors. However, some of the energy magnitude at the driving frequency is lost through the valve, as can be observed by comparing with the power for the upstream sensor.
Figure 4.20. Time series of normalized WSS measurements

Figure 4.21. Power spectral density of normalized WSS measurements
4.11 CONCLUSIONS

Dynamic wall shear measurements are essential for the characterization of near wall flow behavior. The nature of turbulent flow imposes strict requirements in sensing capabilities for the sensor. In particular, the sensor must be small in size, possess a high frequency response, have a high sensitivity to shear, be non-invasive to the flow, robust, reliable and easy to install with the minimum alteration on the vehicle. In this work, we introduce and characterize a new type of wall shear sensor based on active ionic polymers. The new sensor presents favorable design characteristics, including high frequency response, sensitivity, durability, and can potentially be miniaturized or implemented in MEMS devices.

Dynamic calibration of shear stress sensors is a topic that has received increasingly more attention in the past several years. The frequency response of a sensor must be characterized in order to determine its performance and applicability to turbulent flows. Dynamic calibration is presented for the sensor developed using the analytically defined wall shear stress for a Stokes layer generated on the surface of an oscillating flat plate. The sensor response shows good agreement with the analytically derived shear stress from the displacement measurements. The time-resolved measurements show agreement with a mean accuracy of 4.92% over a shear range of +/- 3 Pa for frequencies up to 140 Hz.

Cross sensitivity of the sensor has been characterized with respect to vibration, pressure, and temperature. The effects of vibration and dynamic pressure are found to be negligible. It should be noted that the sensor does not respond to static pressure. A novel automatic calibration scheme is demonstrated whereby the sensitivity of the sensor can be easily predicted from the measured electrical impedance of the sensor. This process can be conducted real-time, allowing auto-calibration prior to each use of the sensor. This approach has been shown to be effective at compensating for the effects of temperature and may also be able to compensate for aging or degradation of the sensor itself. However, any errors due to this procedure will affect the overall sensor measurement accuracy.

This work presents the first implementation of ionic polymer transducers for the measurement of wall shear stress. Active ionic polymers represent a promising new technology in resolving wall shear stress fluctuations in a turbulent boundary layer. The sensor characterized in this work demonstrates accuracy comparable to current state-of-the-art technologies.
scales of interest in turbulent boundary layers are on the order of microns, precipitating the need for sensors with physical dimensions on the order of microns. Future work involves using state-of-the-art self-assembly technologies to fabricate micron-sized sensors, thereby increasing the spatial range of the sensor. Further application in biological flows will be conducted in the future.

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4.13 References


Chapter 5

5 EXPERIMENTAL TIME-RESOLVED DPIV ANALYSIS OF PULSATILE FLOW IN STENOTIC ARTERIAL AND CORONARY ARTERIES

5.1 Background and Introduction

The presence of stenosis in the vasculature is responsible for significant alterations in blood flow patterns, including flow regions characterized by high shear rates, flow separation, and turbulence. A thorough understanding of these phenomena has clinical merit because of the established relationship between hemodynamics and vascular pathology (Fung, 1993; McDonald, 1974). Flow through moderate stenosis is characterized by accelerated flow through the throat of the stenosis issued in the form of a jet. Flow separation downstream of the constricted lumen results in the formation of recirculation regions, characterized by low wall shear stress (WSS) and large residence times (Bluestein et al, 1999; Banarjee, 2003). Particle residence time within the separated flow region is of particular significance due to the inherent link between platelet aggregation and thrombus formation (Kleinstreuer et al, 2002). A stagnation point is located at the distal end of the recirculation zone and has been linked to thrombus formation (Schoephoerster, 1993).

A shear layer is formed between the jet and the recirculation region, which may be responsible for platelet activation (Bluestein et al, 1999; Banarjee, 2003). Wall shear stress values as high as 1500 to 3000 dynes/cm² have been predicted at the throat of the stenosis in coronary flow models (Banerjee et al., 2003). The interaction between the jet and the separation bubble is similar to flow over a backward facing step or a diverging tube in that a shear layer develops between the jet and separation region. This type of interaction is governed by Kelvin-Helmholtz instabilities and is unstable in nature (Becker et al, 2005, Lesieur, 1997). The resultant flow field is further complicated by the pulsatile nature of the flow and three-dimensional (3D), compliant boundaries that are the arterial walls.

Accurate characterization of unsteady flow within and downstream of stenotic lesions is necessary in understanding the mechanics of atherosclerotic disease formation and progression. It
is of critical importance, therefore, to identify the strengths and weaknesses of available flow quantification methods in complex flow measurements. Although ideally one would like to make such measurements \textit{in-vivo}, the lack of non-invasive measurement technologies with sufficient spatial and temporal resolution has hampered such efforts. Invasive techniques such as Doppler ultrasound yield as high as 90\% accuracy; however the spatial resolution is compromised along with problems of aliasing, sample volume, transit time and scattering (Garbini et al., 1982). MRI studies have indicated that signal is lost in the neighborhood of stenosis, possibly due to turbulence (Oshinski et al., 1995).

Numerical methods offer the advantage of being versatile in nature, yielding high spatial and temporal resolution and the ability to simulate flow through complicated physiologic geometries obtained through medical imaging techniques (Boutsianis et al, 2004; Cebral et al., 2002). However, due to their inherent unsteadiness, the flows of interest are generally three-dimensional and transitional in nature. As a result, commercially available software packages are insufficient for resolving such flow fields, precipitating the need for in-house developed solutions to the full Navier-Stokes equations (Sherwin, and Blackburn, 2005). Furthermore, vessel compliance and dynamic motions require the use of unstructured grids to accurately model the complex fluid-structure interaction. Numerical studies do, however, offer much valuable information, and upon experimental validation could potentially reach clinical acceptance for use in disease prediction and patient-specific treatment.

Analytical studies of stenotic flows (Back et al., 1992; Smith et al., 1979; Alan et al., 1982; Resse and Thompson, 1998) have been used to estimate various flow parameters, including WSS, and have served as the foundation for further investigations. With recent advances in computing and numerical methods, numerous computational models have been applied to the problem. Vortex formation has been noted during the deceleration phase of flow (Rosenfeld and Einav, 1995) with periodic generation of turbulence observed (Siegal et al., 1994; Mittal et al., 2001&2003). In particular, frequency spectra from large eddy simulations performed by Mittal et al (2001) indicated that vortex shedding downstream of a constriction occurs at distinct high frequencies. Post stenotic flow has been shown to be more sensitive to changes in percent occlusion in the case of symmetric stenoses than asymmetric stenoses (Long et al., 2001). Mittal et al. (2003) noted the formation of Kelvin-Helmholtz instabilities at high Re in the post-stenotic shear layer between the jet and recirculation regions. Although these studies have provided a wealth of information
understanding the phenomenon of post-stenotic flow, most studies have been oversimplified in nature and furthermore, most of these works have been conducted for aortic flow conditions. Banerjee (2003) recently numerically investigated coronary flow numerically, finding shear rates as high as 3500 dynes/cm², complex interactions between the separated shear layer wave and the wall shear layer flow, and a diminishing of the shear layer vortices in the distal vessel at low Re (100-230) through viscous damping.

Experimental efforts, on the other hand, have difficulties in implementation, or in resolving the flow field spatially and temporally. The effect of a stenosis in the flow has been studied in limited experimental works over the past two decades (Young and Lloyd, 1983; Ahmed and Giddens, 1984; Asakura and Karino, 1990; Ojha et al., 1989, Bluestein, 1999), and transition to turbulence has been observed, depending upon the flow conditions and severity of the stenosis. However, most of these experiments were conducted in large or scaled-up model stenoses under steady flow conditions due to the difficulty associated with generating physiological waveforms in-vitro. Souffi et al. (1998) noted a sensitivity of the flow characteristics to the flow and pressure wave forms. In light of this observation, special attention must be given to blood flow in the coronaries in comparison with arterial flow because of the phase angle between flow and pressure waveforms.

The objective of this work is to perform in-vitro experiments to investigate flow through model coronary vessels with moderate and severe stenosis. This is the first experiment of this type, incorporating physiologic flow and pressure waveforms in 4 mm vessels with physiological compliance. Recirculation and transition to turbulence are investigated and discussed in detail.

Transition to turbulence for pulsatile flows

Transition in unsteady pipe flow is poorly understood in the field of fluid mechanics. Herein, we address this topic briefly to introduce the significant results from previous works. Separation of unsteady flows is a complicated event that is dependent upon several flow parameters, including geometry, waveform, frequency, and Reynolds number (Re), and involves the formation of vortical flow structures. Transition to turbulence in unobstructed tubes results from the inherent instability of the velocity profiles. The transition appears to depend on the time-averaged boundary layer thickness, on the formation of helical vortices and on the interaction between opposing vortical structures on the walls of the tube. In particular, helical vortical structures with a
wavelength equal to three times the time-averaged boundary layer thickness have been noted (Das and Arakeri, 1998), with transition occurring for high Re, and particularly during the deceleration phase (Ghidaoui and Kolyshkin, 2002; Greenblatt and Moss, 2003, Akhavan et al., 1991). Transition in occluded tubes/vessels, on the other hand, is highly dependent upon the stability of the shear layer downstream of the stenosis, with turbulent bursting events noted during the forward phase of flow (Siegal et al., 1994; Rosenfied and Einav, 1995; Mittal et al, 2001; Mittal et al, 2003, Banerjee et al, 2003, Beratlis et al, 2005). Overall, the physics of these phenomena are poorly understood, and require sophisticated experimental and numerical models to elucidate the mechanism by which momentum is transferred to smaller flow scales.

5.2 Methodology

Time-Resolved Digital Particle Image Velocimetry

Digital Particle Image Velocimetry (DPIV) is the state-of-the-art in global flow measurement techniques, delivering planar flow velocity measurements with high spatial resolution. A detailed review of the range of applications and current advancements in DPIV is beyond the scope of this paper. Review articles by Adrian (1991), Grant (1997), and Willert and Gharib (1991) provide comprehensive summaries. DPIV involves illuminating a flow field with a high-power laser by expanding the beam in a planar sheet and imaging neutrally buoyant particle flow tracers in the flow field. Image pairs corresponding to the laser pulse separation between acquisitions are acquired using a high-speed digital camera and are then cross-correlated to determine local flow velocities within the area of interrogation. Spatial cross-correlation yields a correlation plane between sub-samples from the original images, the peak of which corresponds to the displacement of the flow tracers between frames. It is important to note that the pulse separation can be set such that the laser and the camera have the same repetition and frame rates, respectively, or can be set independently to occur at closer pulse pairs, a technique known as “frame-saddling”. In the former all image acquisitions are acquired with equal pulse separation from one another, and thus each frame can be correlated with the next; however, the magnification and the sampling rate of the camera then dictate the largest resolvable velocity. In the case of the latter, the first pulse occurs at the end of the exposure in the first image of the pair and the second immediately afterwards in the beginning of the second exposure. As a result, higher flow velocities can be
acquired, but the actual sampling rate of the measured velocities is half that of the maximum frame rate of the camera.

This distinction is of great significance to the experiment due to the need to measure high velocities with high magnification, while maintaining sufficiently high sampling rate such that frequencies up to 200 Hz or more can be detectable (below the Nyquist frequency). High flow velocities and small flow structures complicate the study of turbulent flow in stenotic arteries, particularly for smaller vessels such as the coronary arteries. For pulsatile flow through a 4 mm diameter vessel at a mean Re of 350, one can expect peak flow velocities on the order of 0.75 m/s, with a mean velocity on the order of 0.35 m/s. Within a constriction of 50% or 75% by diameter stenosis, the peak and mean velocities increase to roughly 4 and 10 times, respectively, as can be readily estimated from the law of conservation of mass. In order to resolve these peak velocities while maintaining sufficiently high spatial resolution to identify flow structures one fortieth the size of the unobstructed diameter, the pulse separation needs to be on the order of 100 µsec. In this experiments the time step between laser pulses was set to 100 µsec, while the camera images were acquired at 500 image pairs per second. This allows identification of flow structures developing over a time step of 2 msec. The magnification utilized in this experiment was 18 µm/pixel, and flow tracers with mean diameter of 10 microns were used. Our system utilizes an X-stream 5.0 camera (IDT) with a 1280x1024 sensor and a 60W Yag laser (Lee Lasers).

Experimental setup

The vessels used in these experiments are fabricated in-house using an elastomeric silicone (Sylgard 184, Dow Corning Corporation) with 4 mm inner diameter and 0.5 mm wall thickness. The manufacturing process involves the following steps: (a) proportioned amounts of elastomeric silicone and hardener are mixed with a ratio of 15:1 (compliance tests of mixtures varying between 8:1 and 20:1 revealed that 15:1 reproduced physiologic compliance); (b) a vacuum pump is then used to remove any air bubbles, and the mixture is subsequently injected into a mold consisting of a 4 mm diameter stainless steel inner rod supported within a 5 mm diameter (inner diameter) glass outer cylinder; (c) the mold is sealed and the vessel is cured for 24 hours at room temperature. Glass stenoses with outer diameter of 4 mm and length of 20 mm (5 diameters) were inserted into
the vessel, representing a constriction with 100% compliance mismatch. The two stenosis models were investigated provided 50% and 75% blockage by diameter, corresponding to approximately 70% and 90% blockage by area, respectively.

The experimental setup was comprised of a computer controlled gear pump connected to a 30 cm long compliant vessel. The tube was contained in a Plexiglas enclosure filled with water. The working fluid was a 60-40 water-glycerin mixture \( \rho = 1060 \, \text{kg/m}^3 \) and \( \nu = 4 \times 10^{-6} \, \text{m}^2/\text{s} \). This mixture served two important purposes. First, the index of refraction is close to that of the vessel and stenosis, removing the presence reflections from the walls of the model (Budwig, 1994). Second, the density and viscosity match closely with those for blood, eliminating the need to scale the Reynolds number, \( \text{Re} \), or the Womerseley parameter, \( \alpha \). To model the physiological environment within the vessel, the experimental setup as shown in Figure 5.1 was used. A gear pump drives the flow through the vessel, working against a downstream pressure head. A fluid capacitance was constructed using a balloon, which served as the impedance presented by vasculature (Yazdani et al, 2004). A solenoid valve is actuated to simulate myocardial contractions, resulting in flow and pressure waveforms that are 180 degrees out of phase, representing coronary flow and pressure waveforms, as opposed to peripheral arterial. The flow conditions used for the study corresponded to physiologic rest, mild exercise, and exercise states. The Reynolds numbers used for these conditions were 250, 350, and 450 respectively, with corresponding \( \alpha \) values of 2.7, 3.2, and 3.7, corresponding to 60, 90, and 120 beats/min. The flow rate was measured upstream of the vessel using an ultrasonic flow meter (Transonic Systems Inc. Model T110). Additionally, the pressure upstream of the vessel was measured using a pressure transducer (Omega Engineering Corp). LabView was used to control the gear pump and solenoid valve, as well as to acquire data from the flow meter and pressure transducer. Time resolved DPIV measurements were taken at 500 Hz for 5 diameter long segments of the flow. The data were processed using rectangular windows (due to the high ratio of axial to radial velocities) of 64 x 8 pixels in size. The vector spacing was set to 4 pixels (72 microns) in both the axial and radial directions. Although this represents an over-sampling in the axial direction, it allows for calculation of eigenvalues using equal spacing in both dimensions.
**Figure 5.1.** Test section consists of a compliant vessel with rigid stenosis insert. A gear pump delivers the flow, while an ultrasonic flow meter and a pressure sensor monitor flow rate and pressure. A downstream pressure head and capacitive section create a physiologically significant flow, and a solenoidal valve provides coronary type waveform (flow and pressure 180 degrees out of phase) when actuated.

**Limitations of the experiment**

There are several limitations of the experiment that must be accounted for in the analysis. First, the entrance length is significantly larger than typical for cardiovascular flows, which are rarely fully developed. Second, although the density and viscosity of the fluid used in this study match those for blood, the fluid is Newtonian. Third, the vessel was fixed in a straight configuration, while coronary arteries display dynamic curvature. Fourth, the transition in compliance between the vessel and stenosis is a step function rather than a smooth transition. Finally, velocity measurements are taken at only at the center of the vessel and the velocity components measured are confined to the plane. These limitations will be addressed in future efforts by fabricating more realistic models, applying dynamic curvature, using particle-rich mixtures, and using volumetric velocity measurement methods.
**Reversal coefficient**

To determine the mean, minimum, and maximum recirculation lengths, a flow reversal coefficient was computed based on the axial velocity at each point in space. For each point in all instantaneous flow fields, a value of 1 is assigned if the flow is forward and 0 if the flow is reversed. This is averaged over the entire ensemble of DPIV frames. Thus, a flow that is always forward will have a reversal coefficient that is 1, and a flow that is always fully reversed will have a value of 0. Thus, the reversal coefficient indicates the fraction of time for which the flow is reversed.

**Reynolds stress estimation**

The Reynolds stress tensor, defined as time-average of the product of the fluctuating velocities, \( \overline{u'v'} \), is a flow parameter of significant importance to the study of turbulent fluid-fluid and fluid-wall shear stresses. To this end, we analyze flows with fluctuating velocity components by decomposing the velocity component, \( u \), into a mean (time averaged) component \( \overline{u} \), and its fluctuating component \( u' \). This is the well known in fluid mechanics community as Reynolds decomposition. For a planar flow field, the velocity components in the axial and radial (vertical) directions can then be represented as \( u = \overline{u} + u' \) and \( v = \overline{v} + v' \). An important requirement of the time averaging is that the mean velocity components must be taken over a significantly large amount of time such that the time averages of the fluctuating components are zero, corresponding to steady turbulent flow (where the mean flow is stationary), i.e. \( \overline{u'} = 0 \) and \( \overline{v'} = 0 \). For pulsatile flows this is clearly not the case, as the axial velocity, \( u' \), is inherently non-stationary, with a variance that changes as a function of time. The \( v' \) component does not exhibit the pulsatility of \( u' \), and could be considered to be almost stationary; however, the product \( u'v' \) is non-stationary. Thus, estimation of the Reynolds stress will be biased by a strong pulsatility if taken from the raw velocity signals.

To remove the pulsatility from the fluctuating velocity components, we took the approach of applying a high-pass filter to the \( u' \) and \( v' \) time-series. Because the driving frequencies for the experiment are low (less than 4 Hz), the high-pass filter can be set with a low cut-off frequency in order to preserve as much spectral information as possible. For the flow cases investigated herein, a Butterworth filter with a cut-off frequency of 4 Hz was utilized such that only frequency...
information 4 Hz and above was retained. It should be noted, however, that the filtered time-records are still not completely stationary, as the turbulent fluctuations only occur during the forward phase of the flow as periodic bursts.

Estimation of skin friction

It is notoriously difficult to accurately measure wall shear stress from DPIV flow fields (Etebari et al., 2003). In this work, two methods were used to estimate the skin friction on the vessel walls. These include direct estimation from DPIV velocity fields using a fourth-order compact scheme (Lele, 1990), and estimation from the filtered RSS profiles. The former yields the total skin friction, while the latter only gives the turbulent contribution. For the $du/dy-C_f$ formulation, $U_{neck}$ is taken to be the mean velocity within the neck of the stenosis, and the formula is given by:

$$C_f = \frac{\tau_w}{\sqrt{\frac{1}{2} \rho U_{neck}^2}}$$

(5.1)

In a recent effort, Fukagata et al. (2002) developed a method to estimate wall shear directly from Reynolds stresses. This approach uses the Reynolds averaged Navier-Stokes equation to analytically derive the skin friction coefficient. The formulation is given as:

$$C_f = \frac{4(1-\delta_d)}{Re_\delta} + 2\int_0^1 2(1-y)(-\overline{u'v'})dy - 2\int_0^1 (1-y)^2 (\overline{T_x + \frac{\partial \overline{\tau}}{\partial t}})dy$$

(5.2)

where $Re_\delta$ is the Reynolds number based upon the boundary layer thickness, defined for steady flows only. The summation of the first term in the formulation, $4(1-\delta_d)/Re_\delta$, and the third term gives the laminar contribution, which, for steady flows, can be approximated as:

$$C_{f, lam} \approx 3.3/Re_\delta$$

(5.3)

$Re_\delta$ for this experiment is not quantifiable due to the pulsatile nature of the flow. The second term in the formulation is the turbulent contribution, which requires only $a priori$ knowledge of the Reynolds stresses within the boundary layer. This term is readily computed from the DPIV flow fields, given proper estimation of the Reynolds stress as described in the previous section. The use of this formulation yields the turbulent contribution to the wall shear stress without the need to
indirectly estimate the wall shear stress using the velocity derivative on the wall. The group validated the scheme for circular pipe and rectangular duct flows, with and without active control in the form of oscillatory blowing and sucking on the wall. This method was employed on the data obtained during this experiment.

Identification of coherent structures

It is well known that coherent vortical structures are prevalent within a transitional/turbulent boundary layer, and that these structures are responsible for the majority of skin friction, due to ‘bursting’ events, and turbulence production. Jeong and Hussain (1995) developed a vortex identification scheme that uses as a criterion for the detection of coherent structures the second eigenvalue, $\lambda_2$, of the tensor given by $S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj}$. Essentially, the method entails finding the negative extrema of $\lambda_2$. Here, $S_{ij}$ and $\Omega_{ij}$ correspond to the symmetric and asymmetric components, respectively, of the velocity gradient tensor, $\nabla = \partial / \partial x_j$. By definition, $S_{ij} = (u_{i,j} + u_{j,i})/2$ and $\Omega_{ij} = (u_{i,j} - u_{j,i})/2$. The necessity for this analysis arises from the difficulty associated with resolving streamwise vortical structures embedded within the boundary layer. Whereas schemes utilizing velocity vectors, such as vorticity estimation, have a preferential orientation in the detection of coherent structures, the eigenvalue method does not. As the near wall environment is characterized by a large degree of shear, vorticity fails to differentiate between the velocity gradient on the wall and the presence of an actual vortex. Using $\lambda_2$ as the criterion avoids this difficulty. The group concluded that the RMS of the eigenvalue, $\lambda_2^\prime$, is more indicative of the statistical probability of the presence of a coherent structure than its mean.

A second coherent structure identification method, known as the swirling strength ($\lambda_{ci}$), involves estimation of the imaginary part of the complex conjugate eigenvalue of the velocity gradient tensor, $\nabla u^*$ (Zhou et al, 1999). The method is frame independent, and the eigenvalue is complex only where the streamlines of the flow are locally circular or spiraling. Thus, $\lambda_{ci}$ values that deviate from zero correspond to regions within a coherent structure. $\lambda_{ci}^2$ is analogous to enstrophy, and can be used to identify vortical structures as well. An advantage of the method is that the theoretical threshold for identifying coherent structures is zero and therefore not arbitrary.
5.3 Results and Discussion

Time-resolved DPIV flow fields have been processed for the three test conditions corresponding to rest, mild exercise, and exercise conditions for both peripheral and coronary arterial waveforms. For each test case, data was acquired throughout the stenosis and for as many diameters downstream as was necessary to capture the dynamics of the shear layers until they were sufficiently dissipated and the flow had relaminarized. The field of view of the system allowed for time-resolved measurements over a distance of approximately five diameters. Thus, data was acquired within the stenosis (-2.5<x/D<2.5), and the camera was subsequently moved downstream a distance of 5D in order to acquire data downstream of the occlusion. Time-averaged data have been superimposed next to one another to provide a representation of the statistical flow quantities over the entire area of interrogation.

Figure 5.2 displays flow velocity profiles at various locations (x/D) with shear stress in dynes/cm² for the 50% stenosis with (a) peripheral and (b) coronary arterial flow waveforms. For both flow waveforms, an accelerated flow is observed within the neck of the stenosis, creating a jet that exists to at least 5D and for the Re 450 cases exceeds 8D. Significantly elevated shear levels are noted within the neck of the stenosis, which generates a shear-layer ring that persists downstream within the flow, until the jet expands and reattaches with the walls at the end of the recirculation regions. Shearing stresses as high as 300 dynes/cm² are noted on the neck of the stenosis. The flow profiles in both cases are remarkably similar in spite of the phase difference in the pressure waveform. Large separation regions are noted in all cases, that vary with Re. The shear layer tube remains parallel with the vessel axis up to Re 350, but begins to show a deflection toward a preferred wall (top wall) of the vessel for Re 450. This deflection can be attributed to the highly unstable nature of the flow (Sherwin and Blackburn, 2005). In particular, the axial perturbation mode of the flow generates a cross-flow (radial) pressure gradient and flow, creating a mild Coanda-type attachment to the separation region, also known as the “wall-attachment effect”. The Coanda effect is a well known fluid dynamic phenomenon that describes the tendency of a moving fluid to attach itself to a wall due to the viscous forces originating at the wall. The resulting relaminarized flow profile is skewed. This behavior was also observed by Beratlis et al (2005).
Figure 5.2. Time-averaged flow profiles and shear stress contours, 50% stenosis

(a) peripheral flow

(b) coronary flow
Flow profiles for the 75% stenosis cases (Figure 5.3) reveal the same trends, with peak mean shear stresses as high as 1,000 dynes/cm² noted within the stenosis. A much stronger jet region is observed, with separation zones that are more easily noted. An interesting difference between the peripheral and coronary arterial waveforms is that in the peripheral flow waveform case (Figure 5.3a), the length of the jet region decreases with increasing Re, while the opposite is true of the coronary waveform case (Figure 5.3b). The jet stability is therefore a function of Re and the phase offset between the flow and pressure waveforms.

(a) peripheral flow
The lengths of the recirculation regions vary in a similar manner as the jet lengths, as their persistence is dependent upon the existence of the jet. Figure 5.4 and Figure 5.5 show contours of the reversal coefficient for 50% and 75% stenosis, respectively, for (a) peripheral and (b) coronary arterial flow waveforms. The 50% peripheral case (Figure 5.4a) shows little dependence upon Re, with strongly reversed flow (dark blue contours) within the separation zone that extends to a distance of \( x/D = 5 \). For the 50% coronary case (Figure 5.4b) the length of the separation zones increases with Re, with mildly reversed flow (light blue contours) noted within these regions, and the length of the separation zones extending as far as \( x/D = 12 \) for the Re 450 case. For the Re 450 case, a strong Coanda-type redirection of the jet towards the bottom wall is also noted. For the 75% peripheral case (Figure 5.5a), the length of the separation zones clearly decreases with increases with increasing Re. Thus, the increase in momentum reduces the magnitude of the adverse pressure gradient on the walls. For the Re 250 case a strong Coanda-type redirection of the jet towards the top wall is also noted, so much so that all of the recirculation is confined to the bottom wall. This is also noted in the Re 350 case, but the separation zones become once again symmetrical for the Re 450 case. The recirculation region heights are much larger than in the 50% stenosis case, as expected. For the 75% coronary case (Figure 5.5b), the recirculation length is
noted to increase with Re, with stronger Coanda-type redirection occurring towards the top wall for the Re 350 and particularly Re 450 cases. The phase offset between the flow and pressure waveforms is responsible for the drastic changes in the separation zone lengths noted between peripheral and coronary arterial waveforms. This phenomenon can be explained as follows. The high systolic (no flow phase) pressure of the coronary-type pressure waveform affects the slow moving fluid near the walls greatest due to its low momentum, preventing it from having a strong recirculation velocity. For the peripheral flows this is not the case, and the separated regions gain energy from the jet, allowing for strong recirculation. The most stable recirculation zones appear to be those for the lower peripheral Re (250 and 350). Furthermore, an interesting observation is that the recirculation regions persist throughout the entire circle. No vortex shedding was observed, due to the low range of $\alpha$. Thus, for the peripheral flows the flow reversal within the recirculation regions is enhanced during the diastolic (no flow) phase by the drop in arterial pressure. Conversely, the pressure drop occurs during the diastolic (forward flow) phase for the coronary flow, retarding the flow near the walls. As a result, the separated region of flow for the coronary case is essentially a dead region with almost no flow.
Figure 5.4. Reversal coefficient, 50% stenosis

(a) peripheral flow

(b) coronary flow

Figure 5.4. Reversal coefficient, 50% stenosis
For the 75% stenosis, the presence of transitional/turbulent flow structures was observed from the DPIV data. To investigate the unsteady dynamics of the flow at various locations along the vessel, spectra of the axial velocity were computed for cross-sections of the flow located at x/D=-
2.5, 2.5, 7.5, and 12.5 for the 50\% stenosis, peripheral and coronary flow (Figure 5.6 and Figure 5.7) and the 75\% stenosis, peripheral and coronary flow (Figure 5.8 and Figure 5.9). The spectra are very consistent across the radius for each x/D locations between the peripheral and coronary cases. The 50\% stenosis cases exhibit no transition to turbulence, but show some fluctuations in the two shear layers at x/D=2.5D with some fluctuations in the coronary case at x/D=7.5D. The 75\% cases show rapid transition to turbulence, as noted by the broadband frequency of the axial velocities. The turbulent flow structures can be attributed to the growth of Kelvin-Helmholtz instabilities in the shear layer formed between the jet and the separation zone. For the 50\% case the instabilities are sufficiently damped such that they do not grow. The 75\% case results in rapid growth of the instabilities, with the intensity of fluctuation of the turbulent region increasing with Re and the length of its influence decreasing (smaller x/D). Viscous forces, interaction between the two shear layers and interaction between the turbulent region and the elastic walls serve to dampen the flow structures and allow the flow to relaminarize. Mittal et al. (2001) noted similar turbulent transition downstream of a stenosis, and was able to identify possible characteristic frequencies associated with vortex shedding or shear layer instability, which could possibly be used for clinical diagnosis given an appropriate sensor. However, the spectra are broadband, making it difficult to identify a dominant frequency.
Figure 5.6. Normalized PSD for 50% stenosis, peripheral flow, velocities normalized by mean neck velocity at centerline.
Figure 5.7. Normalized PSD for 50% stenosis, coronary flow, velocities normalized by mean neck velocity at centerline
Figure 5.8. Normalized PSD for 75% stenosis, peripheral flow, velocities normalized by mean neck velocity at centerline
Reynolds stress analysis

Unfiltered and filtered Reynolds shear stress (RSS) contours, i.e. the $(\overline{i_2})=(1,2)$ term in the Reynolds stress tensor, corresponding to $\overline{uv'}$, were computed for the cases investigated. Figure 5.10 and Figure 5.12 display the high-pass filtered RSS for (a) peripheral and (b) coronary flow waveforms. For comparison, Figure 5.11 displays the unfiltered RSS for peripheral flow for the 75% stenosis. The contours have been normalized with the mean centerline neck velocity. The unfiltered contours are biased by the pulsatile fluctuations of the flow, with momentum transferred between the axial and radial directions via the contraction (red above blue contours) and expansion...
(blue above red contours) of the jet. These signals are not stationary, and thus are a bias on the estimation. Only high frequency information should be used in the estimation, so the high-pass filtered data should contain only information from turbulent fluctuations. The filtered contours for the 50% stenosis show effectively zero RSS (with some noise between the data sets due to edge effects of DPIV), as expected. There is essentially no mixing or transfer of momentum between layers of fluid, and the flow thus remains laminar for all practical purposes at the values of Re and $\alpha$ investigated. The 75% stenosis filtered RSS shows regions associated with high transfer of momentum between the axial and radial directions, and the location of this region correlates well with Re. A higher Re is associated with a rapid growth rate of Kelvin-Helmholtz instabilities, which then cause interaction between the shear layers and subsequent transition followed by relaminarization. The Re 350 case for peripheral flow (75% stenosis) exhibits the largest distance for this occurrence ($x/D \sim 10$), while the rest of the cases occur at $x/D < 8$. The contours are consistent with the velocity spectra, and it appears that the filtering has removed the extraneous pulsatile information. The Re 350 case (75% stenosis) appears to have the highest associated RSS in comparison with the other two cases, for both peripheral and coronary flow waveforms. This result may be a function of Re, percent stenosis, and vessel diameter.
Figure 5.10. Filtered Reynolds stresses, 50% stenosis, velocities normalized by mean neck velocity at centerline.
Figure 5.11. Raw, unfiltered Reynolds stresses, 75% stenosis, peripheral flow, velocities normalized by mean neck velocity at centerline.
Figure 5.12. Filtered Reynolds stresses, 75% stenosis, velocities normalized by mean neck velocity at centerline.

(a) peripheral flow

(b) coronary flow
**Skin friction coefficient estimation**

The two methods given by Eq. (5.1) and the second term in Eq. (5.2) were used to estimate total and turbulent skin friction on the vessel walls. The direct estimation from DPIV velocity fields is denoted as \( \frac{du}{dy}C_f \) and is estimated using the fourth-order noise-optimized compact-Richardson scheme with reflected vectors, as described in the previous chapters. The estimation from the filtered RSS profiles is denoted as RSS-Cf. The former yields the total skin friction, while the latter only gives the turbulent contribution. For the 50% case, where the walls were subjected to only laminar flow, only the DPIV estimation was used, shown in Figure 5.13 for \( x/D = 2.5 \) to \( x/D = 13.5 \), with mean levels for the healthy cases indicated. Clearly, the peripheral flow experiences a much higher magnitude of the skin friction coefficient within the separated region due to the high amount of flow reversal, and in fact is higher within the separation region than further downstream as exhibited by the Re 250 case. The coronary flow shows an order of magnitude less \( C_f \) than the peripheral flow levels, as the flow is nearly stagnant on the walls. For Re 250, coronary flow, \( C_f \) rapidly increases after the reattachment point (\( x/D = 5 \)). Clearly, the low WSS levels for the coronary waveform persist for a very long distance downstream (\( x/D > 13 \)), which would be extremely detrimental to endothelial cell health. It is expected that \( C_f \) increases up to the level at which the other profiles appear to converge (\( C_f \sim 0.01 \)).

![Figure 5.13](image)

*Figure 5.13. Skin friction, \( C_f \), for 50% stenosis*
The turbulent and total $C_f$ profiles for the 75% stenosis are shown in Figure 5.14 for (a) peripheral and (b) coronary flow waveforms. The turbulent contribution rises sharply for the Re 350 and 450 cases for both flows in the regions characterized by high levels of near-wall RSS, providing 100% of the wall experienced skin friction. The total skin friction is maintained outside of these regions by the near-wall laminar flow, with significantly lower shears occurring immediately downstream of the stenosis. The Re 250 case for both flows has the smallest overall turbulent skin friction contribution. For both flow cases the total skin friction is elevated as compared with the 50% stenosis, and a large difference between the $C_f$ values experienced by coronary and peripheral flows for the Re 250 case, with the coronary flow showing $C_f$ very near that for the Re 350 and, in particular, the Re 450 cases. The elevated skin friction persists downstream to further than $x/D=13$, with about three times that estimated for the 50% stenosis. There are slight discrepancies between the two methods, as the turbulent component does exceed the total $C_f$ estimation at some points; however, this small discrepancy can be attributed to noise in the experimental data and different smoothing kernels associated with each method. In the cases investigated, wall deflection was not an issue, as the relatively low compliance of the model coronary arteries results in less than 4 or 5% deflection under application of a physiologic pressure wave. However, it should be noted that the method of evaluating the skin friction directly from the velocity gradient, $du/dy$, is prone to errors associated with the difficulty in determining the true wall location and due to difficulty in measuring the near-wall velocity. This results in a measurement uncertainty that is on the order of 10%, as demonstrated in the previous chapter.
Figure 5.14. Skin friction, $C_f$, for 75% stenosis computed from velocity gradient and turbulent contribution, $C_{f_T}$, computed from Reynolds stresses.
Coherent structure identification

The two coherent structure methods, $\lambda_2$ and $\lambda_{ci}$, were evaluated for the cases investigated. An instantaneous contour is shown in Figure 5.15 for 75% stenosis, Re 350 coronary flow within the stenosis. The growth of vortical structures within the shear layers are evident, and are most likely in the form of ring-like structures that begin as symmetric rings that become asymmetric as they tilt within the shear layer ring. The positioning of the vortical structures appears to follow this trend, as they begin one above the other at the neck of the stenosis, and then follow an alternating pattern further downstream. The downstream pattern looks quite similar to the wake of a circular cylinder. Time-averaged contours for the 50% stenosis are presented in Figure 5.16 showing RMS contours $\lambda_{ci}$ for (a) peripheral and (b) coronary flow. The presence of flow structures within the shear layers are clearly made evident as the RMS is highest in these layers. Mixing of the two layers is not observed even though the structures are convected downstream. This finding supports the results of the velocity spectra and Reynolds stress analysis. Time-averaged contours for the 75% stenosis are presented in Figure 5.17, showing RMS contours $\lambda_{ci}$ for (a) peripheral and (b) coronary flow. Again, the shear layers are visible from the RMS contours. However, for these cases the shear layers do mix, resulting in regions composed of almost entirely coherent structures. These regions agree very well with the velocity spectra and, in particular, the Reynolds stress analysis, with the turbulence observed to occur in the same regions for both analyses. $\lambda_2$ contours show almost identical information. To demonstrate, time-averaged contours for the 75% stenosis are presented in Figure 5.18, showing RMS contours $\lambda_2$ for coronary flow.

Figure 5.15. $\lambda_{ci}$ contour for 75% stenosis, Re 350, arterial flow
(a) peripheral flow

(b) coronary flow

Figure 5.16, $\lambda_{ci}$ RMS for 50% stenosis
Figure 5.17. $\lambda_{ci}$ RMS for 75% stenosis

(a) peripheral flow

(b) coronary flow

Figure 5.17. $\lambda_{ci}$ RMS for 75% stenosis
Spectra of $\lambda_i$

Of particular interest in the current effort is determination of the dominant frequencies for the various cases, which could potentially be used in a clinical environment for diagnosis of stenosis severity. Due to the transitional/turbulent nature of the flow, it is difficult to differentiate between these dominant frequencies and broadband noise due to turbulent breakdown of these structures. The spectra of $\lambda_2$ and $\lambda_{ci}$ were investigated to determine if the frequencies associated with the vortical structures could be determined. These parameters can be thought of as filtered vorticity fields, as they remove the mean shear due to the presence of the shear layer, as well as random fluctuations that are not in the form of a coherent structure or eddy. Thus, the dominant frequencies of their spectra should correspond to the frequencies associated with the shear layer.
instabilities. The spectra of $\lambda_2$ are rather uninteresting, due to the nature of the coherent structure identification method. $\lambda_2$ only has meaning at the core of a vortical structure, thus causing its local fluctuation to be represented as an impulse function when a structure passes over the point of interest, returning back to zero when it has passed by. Thus, its spectrum is broadband in nature, and will only yield meaningful information if the interrogation point is large enough to capture each of the structures as they pass. The spectra of $\lambda_{ci}$, on the other hand, are useful, as its time record captures the rise and decay of the swirling strength of the structures, and has a nonzero value at all points within the structure. Spectra for peripheral flow are shown for 50% stenosis in Figure 5.19 for (a) peripheral and (b) coronary flow, and for 75% stenosis in Figure 5.20 for (a) peripheral and (b) coronary flow. Distinct spikes in the spectra are noticeable near 20-30, 50-60, 100-120, and 140-160 Hz, particularly for the 50% stenosis.
Figure 5.19. Normalized $\lambda_{ci}$ PSDs for 50% stenosis

peripheral flow: (a) Re 250 (left); (b) Re 350 (middle); (c) Re 450 (right)

coronary flow: (a) Re 250 (left); (b) Re 350 (middle); (c) Re 450 (right)
5.4 Conclusions

This work represents the first comprehensive, experimental analysis of flow through stenosed coronary and peripheral arterial flows. Time-resolved DPIV was employed to obtain spatio-temporal quantification of flow structures within a 1:1 scale model with physiologic compliance for resting, mild exercise, and exercise conditions. Such an experiment has not been conducted.
previously to date, and is necessary for validation of numerical and analytical tools due to the complex pulsatile and transitional nature of the flow.

The results confirm the strong dependence of the flow on the stenosis geometry. A shear layer is formed between the jet issued from the neck and the separation zones downstream. The nature of these recirculation regions, i.e. length, symmetry, etc., varies between the coronary versus peripheral conditions and the mild (50% by diameter) versus severe (75% by diameter) stenosis geometries. In particular, the effect of a coronary condition (flow and pressure waves 180° out of phase), flow within the recirculation regions is retarded to a near zero velocity, resulting in very low skin friction/WSS. Thus, based on the low WSS hypothesis for endothelial cell damage and atherosclerosis development, coronary conditions are more harmful than peripheral conditions. Furthermore, it was noted that although stenosis geometry dictated whether transition to turbulence occurred, coronary conditions caused recirculation length to increase with Re, while the converse was true for arterial flow. Recirculation lengths were noted to be as short as five diameters and as long as twelve to thirteen diameters downstream of the neck of the stenosis.

Although instabilities are noted within the shear layers from the velocity spectra and coherent structure analyses, they are damped quickly by the viscosity of the fluid. Only for the severe stenosis was the shear layer strong enough to allow the structures to grow in the form of inviscid Kelvin-Helmholtz instabilities. The coherent structure methods employed herein proved successful in identifying the vortical structures within the high shear layers, allowing quantification of their dominant frequencies. These were noted to occur at different bands depending upon the flow conditions, showing particularly high energy for the severe stenosis at frequencies in the ranges near 25 Hz, 55 Hz, 85 Hz, and 135 Hz. This information may provide some information as to the dynamics of the flow.

Finally, the total skin friction and its turbulent component have been quantified from the DPIV data. The former, although an indirect estimation, is evaluated directly from the velocity fields, while the later requires estimation of the Reynolds stresses. A high-pass filter approach was used to remove the pulsatile frequency from each of the velocity components prior to computation of the Reynolds stress distribution. The filtered information allowed estimation of
the turbulent component, which represents the first time it has been estimated for a flow with a dominant/driving frequency. The results show elevated skin friction due to the turbulent

5.5 References


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6 CHARACTERIZATION OF NEAR-WALL FLOW OVER ENDOTHELIAL CELL MONOLAYERS

6.1 Background and Significance

Flow mediated changes in cell shape and arrangement are among the earliest noticeable evidence of endothelial cell (EC) mechanotransduction. Traditionally, emphasis has been placed on the dependence of cell arrangement on applied shear magnitude and direction, with end morphology believed to minimize stress transduction to the cells. Recently, evidence of an endothelial significant layer (ESL) (Smith et al., 2003; Damiano et al., 2002&2004) has cast some doubt on pre-existing theories, in that results have shown that a near-wall layer on the EC surface causes the transmission of wall shear stress (WSS) to diminish to zero. Moreover, the effective roughness of the EC layer surface profile is responsible for surface and near-wall shear stress distribution modification (Barbee et al., 1995) and changes in mass transport. Although the existing evidence has partly explained "how" EC mechanotransduction occurs, correlations provided for various hemodynamic indices leave several questions unanswered.

Relationship between biomechanical stimuli and vascular disease

The relationship between alterations to the stress environment to which ECs are exposed and atherosclerotic development has been investigated over the past few decades in hopes of determining the mechanisms by which the disease is initiated and progresses. Atherosclerosis is most commonly found in locations characterized by abnormal blood flow patterns, including regions of separated flow, stagnation flow (such as at reattachment points), vortical, and turbulent flow. Through this observation the integral link between hemodynamic parameters describing the flow field and the onset of atherosclerotic disease formation is established. Altered flow patterns are found in regions with complex geometries, such as curved, diverging (expanding), and bifurcating vessels. As a result of blood-vessel wall interactions and the pulsatile nature of the
flow, the hemodynamic indices in these regions, including wall shear stress (WSS), wall shear stress
gradient (WSSG), particle residence times (PRT), and normal pressure gradients (NPG), tend to
reach extreme values.

Throughout the years, researchers have striven to relate these indices, as well as other
factors, to atherosclerosis in order to provide reliable predictors of disease progression. EC
damage has been related to regions characterized by abnormal shear levels, including either low or
extreme values of WSSG (Kleinstreuer et al, 1996; Lei et al, 1996; Ojha, 1993; Ojha, 1994), and
extreme values of the oscillatory shear index (OSI) (Ku et al, 1985; He and Ku, 1996). However,
other indices in these flow regions can be considered to be “abnormal” and have been linked to
the onset of EC damage and vascular disease. These include normal and circumferential pressures
in the form of mechanical stresses (Thubrikar and Robicsek, 1995; Salzar et al, 1995) and increased
PRTs (Karino et al, 1990; Pritchard et al, 1995; Buchanan and Kleinstreuer, 1998). Because all of
these parameters are so integrally linked, it has been difficult for researchers to isolate the most
important indices.

A second important observation is that endothelial cell morphology is directly influenced
by hemodynamic parameters, such as WSS, WSSG, and PRT. This is particularly apparent in
complex, unsteady flow fields that may be developing spatially and temporally. Numerous studies
have noted that endothelial cell morphology undergoes significant changes under conditions of
mechanical stress. Most of these have been conducted under conditions of steady WSS (Zeigler et
that although the altered flow environment is sensed by the nucleus very quickly with activation of
transcription factors (Nagel et al, 1999), changes in endothelial cell morphology can take as long as
72 hours to be completed. In particular, ECs tend to elongate and align themselves with the flow
direction.

Although these changes are attributed to changes in the WSS exerted upon the cells, it
should be noted that as the WSS increases, the near-wall PRT of fluid elements and blood
particulates decreases. Therefore, it is indeed possible that the elongation occurs in order to
increase residence times of nutrients such as oxygen, which requires direct contact between the
blood and ECs to achieve diffusion into the endothelium. In fact, it has been noted that the
elongation of the cells may increase the permeability of blood substrates into the endothelium
Further studies incorporating a variety of unsteady and pulsatile flow conditions have revealed additional information (Zeigler et al, 1995; Kataoka et al, 1998). Even under no-flow conditions, ECs healthily form a confluent monolayer, which can be shown readily via in vitro cell culturing, and the morphology of the cells is still in an elongated form but with no directional alignment. It would be anticipated, therefore, that under low WSS conditions, EC morphology would resemble that of an in vitro condition. However, previous works have noted that ECs take on a rounded shape under separated flow conditions (Nerem et al, 1986; DePaolo et al, 1992), particularly near the separation and reattachment points, otherwise known as stagnation points. At these points the nearby fluid velocity is zero, thus the level of blood mixing is also zero and nutrient exchange by diffusion between fluid elements is therefore not enhanced by the flow patterns. There is thus an inconsistency between EC morphology noted at points with no flow (stagnation points) and under in vitro culture conditions with no fluid flow, where gravity is the only force exerted upon the cells. It is interesting to note, however, that the WSS in both of these cases is inherently zero. Therefore, other forces/mechanisms may be responsible for the changes in cell shape. Moreover, none of these studies considered the effect of the EC monolayer surface on the near-wall hemodynamic indices.

Recent works have shown that there exists an endothelial significant layer, also known as the glycocalyx, that reduces the WSS magnitude on the EC surface to zero (Smith et al., 2003; Damiano et al., 2002&2004). The glycocalyx is composed of a layer of membrane-bound macromolecules (Pries et al., 1997), and makes up the interface between the blood and endothelium. According to Smith et al (2003) the layer is on the order of 0.3-0.5 µm in height, appears to be very nearly impermeable, impenetrable by leukocyte microvilli, and is regulated by the underlying ECs. Moreover, the glycocalyx is believed to play an integral role in reducing inflammation by preventing leukocyte adhesion in low WSS environments. Inflammatory agents, such as TNF-α have been shown to degrade the glycocalyx, increasing cell permeability to macromolecules (Henry and Duling, 2000). Thus, the glycocalyx acts as a protective barrier, preventing the adhesion of blood constituents. Furthermore, it may play an important role in mechanotransduction, an extensive review of which is presented by Tarbell and Pahakis (2006). The authors describe the transduction mechanism of the glycocalyx as a ‘wind in the trees’ analogy – wind, i.e. fluid force, is sensed by the branches of the trees, i.e. glycosaminoglycan (GAG) side
chains that extend from proteoglycans (PGs), and the signal is transmitted to the ground, i.e. cell membrane, through the trunk (core protein). Similar to a person walking through a forest on a windy day, the ground/cell is shielded from the force of the moving fluid, as the force is completely dissipated before reaching the surface. Based upon these results, it appears that WSS is not transmitted to the EC walls due to the presence of the glycocalyx. To date, only a few fluid mechanic studies have considered either cell morphology or EC surface chemistry effects in determining the WSS levels to which the ECs are subjected (Barbee et al, 1995; Smith et al, 2001; Damiano et al, 2002&2003). Therefore, the assumption of smooth-wall surfaces in EC monolayer studies brings into question the validity of EC mechanotransduction theories which are based upon these shear levels. It is of critical importance to accurately characterize the mechanical environment to which ECs are exposed, with particular regard to fluid shear.

6.2 Methodology

In this work, we employed our time-resolved Digital Particle Image Velocimetry (DPIV) system, a state-of-the-art flow quantification method that delivers global flow measurements with high time and spatial resolution. DPIV provides planar measurements of the instantaneous velocities along a plane that cuts the flow field. In a typical experimental arrangement, a high power laser beam (60W Yag laser, Lee Lasers) is guided through a series of mirrors and lenses and is opened up as a thin sheet intersecting the area of interest. The flow is seeded with neutrally buoyant 2-micron diameter silver coated spheres, which serve as flow tracers. The particle diameter is such that the particles accurately follow the flow. A high frame rate camera is employed to capture the instantaneous images. The sampling rate and particle motion are then used to determine the velocity field within the area of interest (Adrian, 1991; Grant, 1998). The velocity fields can then be post-processed to determine various quantities throughout the flow, including shear stress, kinetic energy, and turbulent dissipation rate.

A frame-saddling approach (Grant, 1998) was employed, as described in detail in Chapter 5. The laser pulse separation was set to 100 microseconds, while the image pair acquisition rate was 500 Hz using a camera with a 1280x1024 pixel CMOS sensor (IDT-PIV, model X-Vision 4). A spatial resolution of 5 microns/pixel was investigated. A laminar flow chamber was fabricated such that an EC monolayer is seeded onto the bottom wall of the chamber, as shown in Figure
6.1. DPIV was conducted on a vertical plane within the channel to resolve the flow velocities for steady, laminar flow. The height of the channel was 1.5 mm to ensure laminar flow. An ultrasonic flow meter measured the flow rate through the channel (Transonic Systems Inc., Model T110).

![Image of DPIV setup]

**Experimental Setup**

A laminar flow channel has been designed and fabricated using transparent Lexan to maintain optical access. The flow channel is constructed such that it has a gap width of 1.5 mm and can be easily disassembled to insert glass slides with EC monolayers cultured on the surface. A rendered view of the flow chamber is shown in Figure 6 (top), along with an image of the flow chamber in a DPIV configuration (bottom). A gear pump is used to drive the flow through the chamber, and is controlled by a computer using LabView software. The flow rate will be measured with an ultrasonic flow meter (Transonic Systems Inc. Model T110), and the pressure through the flow loop will be measured using a pressure transducer (Omega Engineering Corp.). Water was used for the fluid within the flow chamber.

**Cell cultures**

EC monolayers were cultured onto glass slides inserted within the flow chamber for this study. Human brain microvascular ECs were isolated and cultured as described by Lee et al (2001). RPMI 1640-based medium with 10% fetal bovine serum (HyClone Laboratories, Inc.), 10% NuSerum (Becton Dickinson), 30 µg/ml of EC growth supplement (ECGS, Beckton Dickinson), 15U/ml heparin, 2 mM L-glutamine, 2 mM sodium pyruvate, nonessential amino acids, vitamins, 100U/ml penicillin, and 100 µg/ml streptomycin (all reagents, Gibco BRL) were
used as the cell culturing medium (Lee et al, 2001). The cultures were incubated at 37°C with high humidity and 5% CO₂. Half of the samples were treated with 10 ng/ml TNF-α, a proinflammatory cytokine, for 24 hours to attempt to induce injury to the glycocalyx (Henry and Duling, 2000). Figure 6.2 shows images of the EC monolayers prior to and after treatment with TNF-α. No significant change in morphology is noticeable.

Figure 6.2. EC monolayer (a) prior to treatment (left); and (b) after 24 hours treatment with TNF-α (right)

6.3 Results and Discussion

A series of experiments investigating steady flow through the channel were quantified using our time-resolved DPIV system. Flow through the laminar flow channel has been characterized with no EC monolayer present. Results for this arrangement are shown in Figure 6.3, showing time-averaged u-velocity profiles, and Figure 6.4, showing time-averaged WSS contours, using the implicit scheme set forth in Chapter 3. The results demonstrate that the same flow profiles and shear stresses are experienced on both walls of the channel. The profile is smooth and parabolic as expected.
Figure 6.3. Time-averaged u-velocity flow profiles and contours through laminar flow channel with no EC monolayer

Figure 6.4. Time-averaged WSS contours through laminar flow channel with no EC monolayer

Figure 6.5 shows time-averaged flow profiles for the healthy EC monolayer case, with the monolayer located on the $Y/h=-1$ surface, with $Y/h=1$ as control. The profile is blunter than in the previous case, with low velocities measured near the EC covered wall. This results in a profile that has an inflection point near the wall. The control wall ($Y/h=1$) shows higher near-wall velocities. The effect of the flow alteration is made evident by computing the SS, shown in Figure 6.6. Although the SS levels are nearly equivalent between top and bottom walls, the bottom wall appears to experience much lower WSS due to the fact that the shear reaches its maximum further into the flow rather than on the wall itself. In fact, a drag reduction of $44\% \pm 4.5\%$ is estimated.
Figure 6.5. Time-averaged u-velocity flow profiles and contours through laminar flow channel with healthy EC monolayer (Y/h) = -1

Figure 6.6. Shear stress contours: Y/L = -1 has healthy EC monolayer, control wall is Y/L = 1 (no ECs)

The flow over the TNF-α treated EC monolayer does not show this drastic decrease, as is evident from the flow profiles and u-velocity contours shown in Figure 6.7, with corresponding SS shown in Figure 6.8.
6.4 Conclusions

Evidence of an ESL has put to question theories of EC mechanotransduction, as previous results have shown that a near-wall layer on the EC surface causes the transmission of wall shear stress (WSS) to diminish to zero (Smith et al., 2003; Damiano et al., 2002&2004). Herein, the effect of the EC monolayer surface profile and surface layer make-up is investigated from an aggregate standpoint. Overall, a significant effect of reduced viscous stress is noted for flow over a healthy EC monolayer, which can be attributed to these two surface characteristics. Drag reduction of the order of 45% was measured quantitatively. Similar results were not noted for an EC monolayer treated with TNF-α, most likely due to degradation of the glycocalyx. Thus, the present results support the argument that the glycocalyx is responsible for a reduction in the WSS transmitted to the cells. However, the results are still premature and require further investigation in order to assure the repeatability of the measurements.

6.5 References


7 CONCLUSIONS

WSS measurements are extremely difficult under even the simplest flow conditions and geometrical configurations. As a result, most experimental fluid mechanics studies have avoided such measurements, particularly in application to DPIV flow fields and dynamic measurements using wall-mounted sensors. In this work, methodologies have been established to allow for SS and WSS measurements with both spatial and temporal resolution. The established tools have been implemented in a set of biological flows in order to quantify hemodynamic indices and to demonstrate the possible applications for in-vitro laboratory experiments. In the future, it is possible that the methods could be extended to in-vivo applications. The use of the velocity derivative schemes developed in this work could potentially be applied to clinical flow data from imaging-based flow quantification methods such as Magnetic Resonance Velocimetry. In-vivo measurements generally have low spatial sampling resolution due to coarse image spatial resolution, precipitating the need for higher-order methods for evaluating velocity gradients. The WSS sensors developed in Chapter 4 were shown to have the potential to be used in realistic clinical settings, and might eventually find use as a probe on a catheter or as part of an instrumented implant.

The specific contributions of this work are listed below.

1) Velocity derivative error analysis (Chapter 2)
The accuracy of in-plane velocity derivative estimation was quantified for unbounded planar Monte-Carlo flow fields, with application to DPIV. Simulations of vortical flow fields were used to quantify and predict, a priori, errors associated with velocity measurement noise amplification and method bias error due to spatial sampling resolution.

2) Novel velocity derivative estimation scheme (Chapter 2)
A novel, adaptable, hybrid estimation scheme combining compact finite difference and Richardson extrapolation schemes (fourth-order noise-optimized compact-Richardson scheme) was developed for improved derivative estimation. The scheme delivers higher-order truncation error with minimal (better than second-order) noise amplification.
3) A novel Cross-Correlation Edge-Detection (CCED) method (Chapter 3)

A gradient cross-correlation method was developed that uses image gradient information in a statistical cross-correlation to track the motion of boundary points. The use of a three-point Gaussian fit allows sub-pixel accuracy to be obtained for boundary displacement measurement.

4) Wall shear rate estimation error analysis (Chapter 3)

The uncertainty of velocity derivatives for wall shear estimations has been quantified for the Poiseuille flow fields, as well as Monte-Carlo simulated flow fields. The methods were implemented directly on the flow fields with the no-slip condition imposed, as well as with a mirrored flow field containing reflected vectors near the boundary. In the Monte-Carlo simulations, the fourth-order noise-optimized compact-Richardson scheme with reflected vectors provides the best WSS estimation of all methods, achieving higher accuracy with low noise-amplification. For real DPIV data the estimation error is significantly compromised by the near-wall velocity bias error.

5) A novel WSS sensor based on Ionic Polymer Transducers (Chapter 4)

A novel class of transducers has been developed for non-invasive, dynamic, time resolved, direct WSS measurements. This technology is based on active ionic polymer membranes, a type of smart material. The accuracy of the sensors was quantified for dynamic measurements. Measurement accuracy in dynamic shear on the order of 5% with respect to a full scale measurement of +/-3Pa and frequency response up to 140 Hz was demonstrated. These sensors have been applied to biological flows, including stenotic vessels and mechanical heart valves, as well as a boundary layer experiment.

6) Quantification of flow in stenotic arteries

DPIV tools developed in this work were used in measuring fluid-fluid and fluid-wall shear stress for flow through stenotic coronary and peripheral arterial flows. WSS levels, turbulent WSS contribution, and recirculation lengths were quantified for these flows, with low WSS levels noted in the separation regions and elevated WSS levels noted further downstream due to transition to turbulence of the flow. Kelvin-Helmholtz instabilities were elucidated within shear layers emanating from the neck of the stenosis. A coherent structure analysis was applied to analyze the growth and subsequent shedding of the vortical structures, and frequency information was presented, showing distinct frequency peaks related to this occurrence.
7) **Quantification of WSS over healthy and altered EC monolayers**

The presence of an EC monolayer, due to its morphology and surface chemistry, vastly alters the near-wall fluid dynamics, thereby altering the WSS magnitude to which the layer is subjected under both healthy and pathological conditions. DPIV was used to investigate flow over healthy and TNF-α treated EC monolayers, with altered flow noted on the EC coated surfaces, indicating a reduction in WSS over the EC surface on the order of 45%.

**Future work and directions**

In this work, the tools necessary for quantifying SS and WSS in complex biological flows were developed. However, a few opportunities exist for continuing and extending these efforts. In particular, further developments are necessary to improve upon the design of the IPT based WSS sensor. Modifications to the sensor packaging could be made to remove the gold epoxy junction on the sensor surface, reducing the irregularities of the surface profile. Moreover, an improved calibration fixture with a piezoelectric transducer to drive the oscillating Stokes layer could improve the quality of the calibration and the accuracy associated with the dynamic calibration. Potentially, the sensor could be applied to in-vivo applications if mounted to a biomedical implant, such as a mechanical heart valve, or to a catheter type probe.

The DPIV results in the stenotic flows offer valuable information to its diagnosis. In particular, the spectral information for both the velocities and the coherent structures is directly related to the hydrodynamic and acoustic signatures associated with these flows. The high frequency response of the IPT sensor makes it a promising candidate for this type of diagnostic application. It is possible to redesign the WSS sensor as an acoustic sensor, which could then be used to detect stenosis. This type of technology would have a huge impact on the biomedical field, and could potentially save millions of lives.

Finally, the results presented for flow over endothelial cells shows an interesting phenomenon which is worthy of further investigation. It would be interesting to investigate additional cell lines and additional damaged cell conditions to compare with the conditions investigated herein. Furthermore, the difference between flows over EC monolayers using water compared with a more protein-rich fluid, such as cell culture media, has yet to be quantified. The
protein content of the fluid could affect the boundary later flow profile over the EC monolayers due to protein adhesion to the glycocalyx. Finally, it would be useful to quantify these flows using a microDPIV system to provide added spatial resolution.