TECHNIQUES FOR USING 3D TERRAIN SURFACE MEASUREMENTS FOR VEHICULAR SIMULATIONS

Zachary Ray Detweiler

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Dr. John B. Ferris, Chair
Dr. Steve C. Southward
Dr. Saied Taheri
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Abstract

Throughout a ground vehicle development program, it is necessary to possess the loads the vehicle will experience. Unfortunately, actual loads are only available at the conclusion of the program, when the vehicle has been built and design changes are costly. The design engineer is challenged with using predicted loads early in the design process, when changes are relatively easy and inexpensive to make. It is advantageous, therefore, to accurately predict these loads early in the program, thus improving the vehicle design and, ultimately, saving time and money. The prediction of these loads depends on the fidelity of the vehicle models and their excitation. The focus of this thesis is the development of techniques for using 3D terrain surface measurements for vehicular simulations. Contributions are made to vehicle model parameter identification, terrain filtering, and application-dependent interpolation methods for 3D terrain surfaces.

Modeling and simulation are used to improve and shorten a vehicle’s development cycle, thus, saving time and money. An important aspect in developing a vehicle model is to identify the parameters. Some parameters are easily measured with readily available tools; however, other parameters require dismantling the vehicle or using expensive test equipment. Initial estimates of these difficult or costly to obtain parameters are made based on similar vehicle models or standard practices. In this work, a parameter identification method is presented to obtain a better estimate of these inaccessible parameters using measured terrain excitations. By knowing the excitations to the physical vehicle, the simulated response can be compared to measured response, and then the vehicle model’s parameters can be optimized such that the error between the responses is minimized. Through this process, better estimates of the vehicle’s parameter are obtained, which demonstrates that measured terrain can improve vehicle development by increasing the accuracy of parameter estimates.
The principal excitation to any ground vehicle is the terrain, and by obtaining more accurate representations of the terrain, vehicular simulation techniques are advanced. Many simple vehicle models use a point contact tire model, which performs poorly when short wavelength irregularities are present because the model neglects the tire’s mechanical filtering properties. Therefore, a filter is used to emulate a tire’s mechanical filtering mechanism and create an effective terrain profile. In this work, terrain filters are evaluated to quantify their effect on the sprung mass response of the dynamic simulation of a seven degree of freedom vehicle model.

In any vehicular simulation, there is a balance between analytical expense and simulation realism. This balance often limits simulations to 2D terrain profile excitations, but as computing power increases the computational expense decreases. Thus, 3D terrain excitations for vehicular simulation are a tool for advancing simulation realism that is becoming less computationally expensive. Three dimensional terrain surfaces are measured with a non-uniform spacing in the horizontal plane; therefore, application-dependent gridding methods are developed in this work to interpolate 3D terrain surface to uniform grid spacing. The uniform grid spacing allows 3D terrain surfaces to be used more efficiently in any vehicular simulation when compared to non-uniform spacing.
Dedication

To my wife Nicole, father Terry, and mother Marti...
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Contents

Abstract ................................................................................................................................. ii

Dedication ............................................................................................................................. iv

Acknowledgements ............................................................................................................... v

List of Figures ....................................................................................................................... x

List of Tables ......................................................................................................................... xii

1 Introduction ....................................................................................................................... 1

2 Background ....................................................................................................................... 5

2.1 Terrain Measurement ................................................................................................. 5

2.2 Vehicle Modeling ......................................................................................................... 7

3 Parameter ID .................................................................................................................... 9

3.1 Introduction .................................................................................................................. 9

3.2 Parameter Optimization .............................................................................................. 9

3.2.1 Model Excitations .................................................................................................. 10

3.2.2 Initial Parameter Estimates ...................................................................................... 13

3.2.3 Optimization .......................................................................................................... 18

3.3 Results ......................................................................................................................... 18

3.4 Discussion ................................................................................................................... 20
3.5 Conclusions ................................................................. 21

4 Terrain Filters ............................................................. 22

4.1 Introduction .............................................................. 22

4.2 Background ............................................................. 23

4.2.1 Tire Envelopment .................................................... 23

4.2.2 Model Response ..................................................... 24

4.2.3 Error Calculations ................................................... 25

4.3 Terrain Types ............................................................ 26

4.3.1 Smooth Asphalt ...................................................... 27

4.3.2 Belgian Block ......................................................... 28

4.3.3 Speed Bump .......................................................... 29

4.4 Terrain Filters ........................................................... 30

4.4.1 Moving Average ...................................................... 31

4.4.2 Low-Pass Butterworth ............................................. 32

4.4.3 Morphological ....................................................... 33

4.5 Results ................................................................. 34

4.5.1 Smooth Asphalt ...................................................... 35

4.5.2 Belgian Block ........................................................ 37

4.5.3 Speed Bumps ........................................................ 38

4.6 Discussion ............................................................. 38
5 Interpolation Methods ................................................................. 43

5.1 Introduction ................................................................................. 43

5.2 Background ................................................................................. 44

5.2.1 Terrain Measurement ............................................................. 44

5.2.2 Geostatistics ............................................................................. 45

5.3 Uniform Grid Spacing ............................................................... 47

5.3.1 Attributes Being Evaluated .................................................... 47

5.3.1.1 Computational efficiency ................................................... 48

5.3.1.2 Outlier sensitivity ............................................................... 48

5.3.1.3 Location sensitivity ............................................................ 48

5.3.1.4 Trend sensitivity ................................................................. 49

5.3.2 Interpolation Methods ............................................................ 49

5.3.2.1 Mean .................................................................................. 50

5.3.2.2 Median ................................................................................. 50

5.3.2.3 Inverse Distance to a Power ............................................... 52

5.3.2.4 Kriging ............................................................................... 53

5.4 Desired Attributes and Applications .......................................... 53

5.4.1 Desired Attributes ................................................................. 54

5.4.2 Desired Applications ............................................................. 55
5.4.3 Preferred Methods..............................................................................56

5.4.3.1. Mean...............................................................................................56

5.4.3.2. Median.............................................................................................57

5.4.3.3. Inverse Distance to a Power ..........................................................57

5.4.3.4. Kriging ............................................................................................58

5.5 Discussion .............................................................................................59

5.6 Conclusions ...........................................................................................60

6 Conclusions ...............................................................................................61

References ....................................................................................................63

Appendix A Nomenclature.............................................................................67

Appendix B Equations of Motion 7-DOF Model........................................67

Appendix C Parameters 7-DOF Model .......................................................70
List of Figures

Figure 1.1: Photograph of Belgian Block with Measured Rendering Overlaid (photo by author March 23, 2009) ................................................................. 2

Figure 2.1: Host Vehicle and VTMS (photo by author September 12, 2007) .... 6

Figure 2.2: 7-DOF Vehicle Model (Reprinted with permission from SAE paper 2009-01-1197 © 2009 SAE International) ......................................................... 8

Figure 3.1: Speed Bump used for Vehicle Excitation (photo by author March 11, 2009) ........................................................................................................ 11

Figure 3.2: Measurement Delay ..................................................................... 12

Figure 3.3: Speed Bump Model Excitations ................................................. 13

Figure 3.4: Initial Model Results .................................................................. 15

Figure 3.5: Results after Detrending .............................................................. 17

Figure 3.6: Comparison of Error Plots .......................................................... 20

Figure 4.1: Point Contact Tire Model ............................................................. 24

Figure 4.2: FRF of 7-DOF Model ................................................................. 25

Figure 4.3: PSD of Terrain Profiles ............................................................... 27

Figure 4.4: Smooth Asphalt Excitation ......................................................... 28

Figure 4.5: Belgian Block Excitation ............................................................. 29

Figure 4.6: Speed Bump Excitation ............................................................... 30

Figure 4.7: Frequency Response of Moving Average Filter ....................... 32
Figure 4.8: Frequency Response of Low-Pass Butterworth Filter ......................... 33

Figure 4.9: Morphological Filtering ...................................................................... 34

Figure 4.10: Smooth Asphalt Response ............................................................... 36

Figure 4.11: PSD of Belgian Block ...................................................................... 39

Figure 4.12: Unsprung Mass Acceleration for Different Terrain Filters ............... 41

Figure 5.1: Non-uniform Spacing (photo by author April 13, 2009) ................. 44

Figure 5.2: 10 mm Uniform Grid Nodes ............................................................. 47

Figure 5.3: Contour Plots: (a) Location Sensitive (b) Trend Sensitive .............. 49

Figure 5.4: 3D Terrain Surface of Rough Road .................................................. 56

Figure 5.5: 3D Surface of Smooth Highway ....................................................... 57

Figure 5.6: Digital Photo and 3D Surface of Gravel Terrain (photo by author April 13, 2009) ............................................................................................................. 58

Figure 5.7: 3D Surface of Belgian Block inside Digital Photo (photo by author March 23, 2009)......................................................................................................... 59
# List of Tables

Table 1: Result of Optimized Parameters ........................................................... 19

Table 2: Results of Terrain Filters on Smooth Asphalt ................................. 35

Table 3: Results of Terrain Filters on Belgian Block .................................... 37

Table 4: Results of Terrain Filters on Speed Bumps ....................................... 38

Table 5: Desired Attributes ........................................................................ 54

Table 6: Desired Applications ..................................................................... 55

Table 7: Preferred Methods .................................................................... 56

Table 8: Vehicle Model Parameters .......................................................... 70
1 Introduction

This thesis is the culmination of two years of research that is focused on developing techniques to use 3D terrain surface measurements in vehicular simulations. The Vehicle Terrain Performance Laboratory has developed the Vehicle Terrain Measurement System (VTMS), capable of measuring a 3D terrain surface over a width of four meters with horizontal spacing of five millimeters by five millimeters and a vertical resolution of one millimeter [1, 2]. This system acquires high-fidelity terrain surface measurements that are an enabling technology to pursue advance vehicular simulations so that the vehicular development cycle is made more efficient. Figure 1.1 is an example of a high-fidelity terrain surface measurement from the VTMS. In this figure, the large rectangle is a digital photograph of Belgian block, and the superimposed, tilted, small square is a rendering of the corresponding measured data. The rendering is generated by loading a measured terrain surface into a commercial visualization software package. In the figure, the measurement detail is seen in the cracks between the blocks to the roughness of each block.
These terrain measurements are used to advance the development of ground vehicles. For example, throughout a chassis development program, it is necessary to possess load data representing severe customer usage to ensure that the chassis will perform as required. Unfortunately, actual loads are only available at the conclusion of the program. The design engineer is challenged with using predicted chassis loads early in the design process, when changes are relatively easy and inexpensive to make, and measured chassis loads late in the program when changes to the design are extremely costly. It is advantageous, therefore, to accurately predict these target chassis loads early in the program and to maintain a consistent process for predicting chassis loads as the design changes throughout the program [3]. Through modeling and simulation, these loads are accurately predicted earlier in the design process to save both time and money [4].
The main excitation to the vehicle chassis is the terrain, although cornering and aerodynamic loads also play an important role [5]. Non-deformable terrain imposes a unilateral geometric boundary constraint on rolling tires to which the chassis responds by generating loads, moments, motions, deformations, etc. Clearly then, accurate simulations require accurate terrain excitations. The terrain is a consistent vehicle excitation throughout the design process; therefore, the excitations used for preliminary simulations must match the excitations used for physical simulation later in the program. This consistent excitation is realized through the data acquired by the VTMS. This work develops techniques to use terrain measurements for vehicular simulations. Specifically, the contributions of this work are summarized below.

1. This work demonstrates measured terrain excitations are better for parameter identification compared to simple excitations.
2. This work demonstrates that sprung mass response (vertical displacement, roll, and pitch) is insensitive to typical terrain filters.
3. This work develops a suggested practice for application-dependent interpolation methods for 3D terrain surface measurements that ensures 3D terrain surfaces are uniformly spaced with the appropriate resolution for a given application.

The chapters of this work are organized as follows.

Chapter 2 examines the background for two topics that are used throughout this thesis. First, the background on terrain measurement is presented. Next, vehicle modeling is presented with respect to basic ride dynamics. Chapter 3 presents a parameter identification process that is developed and implemented. In this chapter, a vehicle model’s parameters are optimized by minimizing the error between the measured vehicle response and a model’s response that is excited by the same terrain as the real vehicle. This chapter concludes by demonstrating how the optimized parameters better estimate the model’s response. Chapter 4 examines terrain filters that are implemented to emulate a real tire’s mechanical filter properties. This work is examined over various
terrain types, including smooth asphalt, Belgian block, and speed bumps. This chapter concludes by discussing the effectiveness of each terrain filter.

Chapter 5 examines application-dependent interpolation methods for creating uniformly spaced 3D terrain surfaces. The lessons learned from a literature review of geostatistics are discussed. Then, a framework for judging interpolation methods based on computational efficiency and the sensitivity to outliers, location, and trends is presented. The interpolation methods that are considered are developed. Next, the interpolation methods are discussed in terms of their attributes and preferred applications. The future possibilities for this research are discussed and finally recommendations are made for application-dependent interpolation methods. Finally, Chapter 6 summarizes the conclusions and major contributions of this thesis.
2 Background

2.1 Terrain Measurement

Most modern terrain measurement systems are inertial systems that establish an inertial reference point via some combination of accelerometers, a distance measurement instrument (DMI), an inertial measurement unit (IMU), a Differential Global Positioning System (DGPS), and an inertial navigation system (INS). Relative height is measured from the inertial reference point to the terrain surface with a non-contact sensor (e.g., a laser), and horizontal position is determined from a DGPS or a DMI. Since the 1960s, profiling systems have recorded 2D terrain profiles [1, 6]. These systems use single-point lasers to measure a sequence of longitudinally spaced points along the terrain. The result is a single ‘line’ of data [7-9]. Systems using single-point lasers typically create a uniformly spaced 2D terrain profile (distance traveled vs. height). The 2D profiles are implemented as excitations to simple vehicle models to provide extremely useful estimates of ride quality for quality control of road construction. In fact, the Federal Highway Administration uses 2D profiles for the Long-Term Pavement Performance study to monitor the health and ride quality of roads throughout the United States [10].

Recent computational advances and advances in laser technology have produced 3D laser measurement systems, which acquire relative distance measurements in one of three ways. The first method uses time-of-flight distance calculations [11-13]. Systems using this method have a long range (up to 100 meters) and an accuracy of 5 millimeters. The second method uses triangulation techniques to measure a 3D surface [12-15]. Systems using this method have a short range (less than 0.5 meters) and sub-millimeter accuracy. The last method is through a phase modulation technique [1, 2, 16-18]. Systems using this method have a range of 2 meters and millimeter accuracy.

In 2007, the Vehicle Terrain Performance Lab of Virginia Tech developed the VTMS, which uses a scanning laser to provide high-fidelity 3D terrain measurements [1,
This system uses a 3D scanning laser that employs a phase modulation technique to measure relative distance. The scanning laser offers many advantages over other systems, such as a larger standoff height (2 meters) between the terrain and laser, allowing both on-road and off-road terrain measurement [16-18]. This system scans about one thousand points across a transverse path that is over four meters wide, thus capturing an entire lane width of a typical road. The phase modulation scheme originally was limited by an ambiguity interval of 100 millimeters. The ambiguity interval limits the ability to measure height that are 50 millimeters above or below the nominal scanning surface. However, recent advances employ two wavebands of phase modulation thus increasing the ambiguity interval to 3 meters; this far exceeds the mobility of the host vehicle. These 3D terrain surface measurement systems establish a new level of precision and resolution that results in new applications for terrain measurements [1, 2, 12, 13, 15, 19, 20]. The host vehicle and VTMS used in this work are seen in Figure 2.1.
2.2 Vehicle Modeling

A model must be created that accurately emulates the behavior of a real vehicle before load predictions can be generated. Complex vehicle dynamics models, created using commercial multi-body dynamics software, have produced convincing simulation results [21]; however, to date, they are computationally expensive [22] and impractical for obtaining information concerning basic ride dynamics. Early vehicle models used quarter car and half car models to simulate ride dynamics [23]. These models are useful for preliminary suspension design, but they lack the DOF required to accurately represent the coupled rigid-body motions of the sprung mass.

Kim and Ro show that a simple 7-DOF vehicle model, developed using model reduction and deduction techniques, is able to accurately portray vehicle ride dynamics [24-26]. The 7-DOF model includes the vertical motion of the four, lumped-parameter, unsprung masses. Additionally, the bounce, pitch, and roll degrees of freedom of the sprung mass are included. As seen in Figure 2.2, the 7-DOF model is able to fully describe a vehicle’s primary ride dynamics, but retains a parsimonious number of DOF for computational efficiency. The equations of motion for the 7-DOF model are included in Appendix B and the definition of the model parameters are presented in Appendix C.
The 7-DOF model uses a point contact tire model, where a linear spring is used to represent the tire stiffness. Simple 2D terrain profiles are used to excite the model; the aerodynamic load contributions are not included. This assumption is reasonable as long as the host vehicle’s velocity is low, as is the case for typical durability testing and most ride evaluations. Higher host vehicle speeds, such as highway applications, are outside the scope of this work; however, the equations of motion could be augmented by the aerodynamic loads in future work. A wide range of excitations were used to validate the particular 7-DOF model used in this work, including harmonic and step inputs to ensure the model behaves as expected [27].
3 Parameter ID

3.1 Introduction

Some vehicle parameters, such as mass, track, and wheelbase, are easily measured, but other parameters, such as pitch and roll inertias, are more difficult to acquire. Measuring these values may require dismantling the vehicle or using a large inertial properties test rig. A parameter optimization routine could be used with a vehicle on a shaker rig that allows complete control over the input to the system; however, if such equipment is not available then the next best option is using measured terrain. This chapter develops a parameter identification method to attain better estimates of the vehicle parameters that are difficult to measure. This work seeks to demonstrate that measured terrain is a suitable excitation to use for parameter identification. This method is developed as follows. First, vehicle parameters are initially estimated based on standard practices or on known values from similar vehicles as described in Section 3.2.2. Next, a physical vehicle is excited using a known (measured) surface and the vehicle’s sprung mass responses are measured over that surface. The terrain that excited that physical vehicle is also used to excite the vehicle model. Responses from the vehicle model are directly compared to the responses measured on the physical vehicle. By optimizing the vehicle parameters such that the error between the model’s response and the real vehicle’s response is minimized, a better estimate of the vehicle parameters is obtained. The method is demonstrated on the VTMS in Figure 2.1. Finally, the chapter discusses potential model improvements followed by concluding remarks.

3.2 Parameter Optimization

The parameter optimization is this work is not a new concept [24, 27-29], but it builds on the previous works to use measured terrain for the parameter. This process is necessary to obtain better estimates of vehicle parameters for the VTMS and to obtain a basic vehicle model for the study presented in Chapter 0. The methodology presented to
optimize a vehicle model’s parameters is developed in this section through an example using the 7-DOF vehicle model presented in Section 2.2, and is used in this chapter to model the host vehicle seen in Figure 2.1.

For the example in this work, the vehicle that measures the terrain is also the vehicle that is modeled; however, this is not required. All that is required is for the location of the test vehicle during the test run to be synchronized with the location of the terrain. This can be easily done if the test vehicle has a GPS or INS.

### 3.2.1 Model Excitations

Traditionally, vehicle simulations are excited by combinations of ramps, harmonic waves, steps, and a variety of other deterministic excitations [30]; however, these excitations fail to adequately describe real terrain. Therefore, in this work, real terrain measurements are used to excite the 7-DOF model. The terrain measurements over rough terrain are advantageous because they have wider bandwidth than simple excitations, such as a speed bump. Therefore, the result from optimizing about simple excitations will be compared with optimizing about various measure excitations. The VTMS is mounted on the rear of the host vehicle, as seen in Figure 2.1, and measures the 3D terrain surface as well as the sprung mass response (via an IMU/INS mounted at the vehicle’s center of mass). Typically, the highways traveled by a passenger vehicle are very smooth and do not contain large localized events that would significantly excite a vehicle’s suspension. Terrain profiles are selected such that the sprung-mass of the vehicle is excited by larger amplitude vibrations. The larger amplitude excitations make it easier to identify the vehicle’s response from signal noise. One example of an excitation used to obtain high amplitude vibration is speed bumps placed on smooth asphalt. A sample speed bump is seen in Figure 3.1, where the top half of the picture is a rendering of the measured terrain and the bottom half is a digital photograph of the speed bump. Runs are taken with in various excitation orientations and host vehicle velocities to excite the different modes of the sprung mass, such as pitch and roll. For an explanation of the runs used in this work, see Section 3.3.
While the 3D terrain surface is needed for high-fidelity tire model applications, it is computationally expensive; consequently, 2D terrain profiles are extracted from the 3D terrain surface measurements for excitation of the 7-DOF model. In this chapter, the 2D terrain profiles are extracted from the measured 3D terrain surface simply by using the laser point corresponding to the center of each tire contact patch. Others have considered techniques that use the 3D terrain surface below the tire’s contact patch to create a 2D terrain profile and have shown convincing results for estimating unsprung mass accelerations [31, 32]. This work is only considering the basic ride dynamics of the sprung mass; therefore, the assumption is made that the center point of the contact patch is sufficient.

The longitudinal 2D terrain profile is acquired at 1000 Hz, but the vehicle’s response is sampled at 100 Hz. Because the 7-DOF model is used to evaluate basic ride dynamics and the typical secondary ride frequency is 15 Hz, the 2D terrain profile can be reduced to 100 Hz without adversely influencing the dynamics. To avoid aliasing, the 2D terrain profile is low-pass filtered using a 10\textsuperscript{th} order Butterworth filter with a 100 Hz break frequency and then down sampled to 100 Hz. Therefore, the sample rate of the excitations and measured response are the same.

The excitations to the front and rear wheels on the same side of the vehicle are identical except for a time delay. This requires the path of the vehicle to be straight.
during a test run. Because the laser is measuring the terrain after the wheels have passed over it, a time delay is used to synchronize the terrain measurement with the wheel excitation. The time delay is simply the distance from the wheel center to the laser measurement divided by vehicle’s longitudinal velocity, $v$, which is obtained from the INS. An example of this is shown in Figure 3.2, where the distance offset, $d$, is between the wheel center and the laser measurement, and $v$ is the velocity of the host vehicle. The wheel hits the excitation at some time, and then the laser measures the excitation at some time later, $\Delta t$, which is equal to the distance, $d$, divided by the velocity, $v$. This process neglects the effect of the pitch dynamics of the vehicle on the longitudinal position of the excitation. The maximum pitch experienced is 0.01 radians, and the typical height is 2 meters thus the maximum error in the longitudinal position of the excitation due to pitch is 20 millimeters. This error is acceptable because the contact length of the tire is approximately 200 millimeter, and a 20 millimeter error still places the point within the contact length of the tire.

![Figure 3.2: Measurement Delay](image)

Consider the measured terrain profiles used to excite the vehicle models depicted in Figure 3.3. The speed bump is located on a nearly flat asphalt parking lot, both of the vehicle’s front wheels are excited by the bump at approximately the same moment, and
the rear wheels are excited approximately half of a second later because the wheel base is 2.888 meters and the velocity of the vehicle is approximately 5 meters/second.

Figure 3.3: Speed Bump Model Excitations

3.2.2 Initial Parameter Estimates

When building a vehicle model, it is easy to measure certain vehicle parameters, such as the tracks, wheelbase, mass, and center of mass in the horizontal plane. More difficult parameters to obtain, such as inertia values, damping ratios, and spring stiffnesses, are estimated using standard practices or comparable vehicles. In the example presented in this work, the tracks, wheel base, total vehicle mass, and the center of mass (CM) in the horizontal plane are measured. The inertia values, damping coefficients, and tire and spring stiffnesses are estimated based on standard practices or similar vehicles [24, 29]. These initial parameter estimates are displayed in Appendix C.
As an initial estimate, the dampers are modeled as linear. This approximation is highly idealized, as damper behavior is not strictly viscous nor do they behave linearly. The equations of motion for a simple model can be implemented in any numerical solver (e.g., Simulink®). The model is coarsely verified using basic deterministic excitations (e.g., step and sinusoid). This preliminary check exposes any fundamental design or programming issues.

Next, the initial parameters are tested on a measured terrain excitation. All proceeding plots in this chapter use the model excitations displayed in Figure 3.3. Initial experimental results are shown in the top three plots of Figure 3.4; where, from top to bottom, vertical displacement, pitch, and roll are displayed. The actual vehicle’s sprung-mass response is measured using a six-axis sensor (e.g., an IMU/INS) mounted at the CM of the host vehicle. The lower portion of Figure 3.4 shows the root-mean-square (RMS) error between the experimental response and the model response, and the normalized RMS error is displayed in the legends. The initial results demonstrate that signals drift apart because the initial conditions for the model are set to zero, but the real vehicle are not zero. This drift could be eliminated by examining the derivatives (vertical velocity, pitch rate, and roll rate) of these signals; however, this is not done because there is interest is capturing long wavelength content that is only available in the displacement signals.
Figure 3.4: Initial Model Results
All modeled and measured responses are detrended, which makes their mean value zero thus removing any offset in the signals caused by initial conditions. This can be easily seen by comparing the second plot (Roll) from Figure 3.4 (before detrending) to Figure 3.5 (after detrending). In the first figure, the measured and modeled signals exhibit offsets that are not the same; however, after detrending, the signals both have a mean value of zero. Detrending also removes any linear trends in the signal and, thus removing the linear component of drift.

Also a settling time of two seconds is implemented to allow the model’s response to converge to the measured response before error calculations begin. An approximation for the settling time is found by assuming the first natural frequency of the system is 1.5 Hz and the damping ratio is 0.33; hence, the time constant is approximately two seconds. The time constant represents the time it takes for the system to settle from an impulse and, in this case, settle from its initial conditions.

Figure 3.5 shows the results after the signals have been detrended, and a settling time has been implemented. The responses show improved agreement, which is evident by the RMS error values. The RMS error of the vertical displacement, pitch, and roll is reduced by an order of magnitude or more.
Figure 3.5: Results after Detrending
3.2.3 Optimization

The root-mean-square (RMS) of the error is a suitable method for comparing the motions of the actual vehicle to the motions of the simulated vehicle. Therefore, the cost function for the optimization is the sum of normalized RMS error between the model’s response and the measured response for vertical displacement, pitch, and roll. From this point forward, the sum of normalized RMS error between the model’s response and the measured response for vertical displacement, pitch, and roll is referred to as overall RMS error. Using the initial values for pitch and roll inertia, tire and suspension stiffnesses, and the damping coefficients, an optimization method is used to ascertain a more accurate estimation of these parameters. The optimization algorithm used is a bounded nonlinear constrained minimization, which is implemented with \texttt{fmincon} function in MATLAB® by applying initial parameter estimates as well as upper and lower bounds for each parameter. This function determines the local minimum; therefore, the initial estimates and bounds must be as accurate as possible. Also the optimization is run multiple times using new initial parameters to increase the likelihood that a global minimum is found. The optimized parameters are displayed in Appendix C.

3.3 Results

For the optimization process, two sets of runs are identified: a set for identification and a set for validation. The set of runs for identification are used for the optimization. Then, the set of runs for validation are used to verify that the parameters obtained from the optimization improve the model’s performance on a separate set of runs. The set of validation runs is never included in the optimization.

In this experiment, there are two sets of identification runs. The first is the simple set that consists of the vehicle driving over one or two speed bumps with velocities varying from 5 meters/second (12 MPH) to 10 meters/second (24 MPH) at varying approach angles. This set of runs is used to tune the parameters. The second set of identification runs consists of various measured terrains, such as smooth asphalt, Belgian
block, gravel parking lots, and speed bumps with velocities varying from 5 meters/second (12 MPH) to 10 meters/second (24 MPH).

Table 1 displays a summary of the results of simulations using the optimized parameters. From this table, it can be seen that using both measured and simple excitation for the identification set of runs produce similar improvement (7.50% and 5.25%). However, using measure excitations for the identification perform much better over the validation set of run compared to when simple excitations were used for identification. Figure 3.6 displays a comparison of the error for a specific run when using the initial parameters, the parameters optimized using a simple excitation, and the parameter optimized using various measured excitations. It is clear that when measured terrain is used for the optimization the error is less than when simple excitations are used. This demonstrates that the parameters are better estimate of the vehicle model’s parameters.

Table 1: Result of Optimized Parameters

<table>
<thead>
<tr>
<th>Set of Runs</th>
<th>Average improvement in Overall RMS error</th>
<th>Average improvement in Overall RMS error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Using various measured excitations)</td>
<td>(Using simple excitations)</td>
</tr>
<tr>
<td>Identification</td>
<td>7.50%</td>
<td>5.25%</td>
</tr>
<tr>
<td>Validation</td>
<td>4.18%</td>
<td>1.09%</td>
</tr>
</tbody>
</table>
This work provides the basis for developing a model to identify vehicle system parameters. Assuming the vehicle’s springs are linear throughout the suspension travel is reasonable as long as the suspension is not excited to the extent that bump stops are encountered. The vehicle’s dampers will exhibit strongly hysteretic, asymmetric, and otherwise nonlinear behavior that cannot accurately be modeled as linearly viscous. The dampers should be tested on a shock dynamometer and the results could be implemented into the model to obtain better agreement. To a lesser degree, nonlinear tire elasticity properties would also improve the simulation.
The VTMS installed on the back of the host vehicle records data approximately every five millimeters longitudinally at a typical host vehicle velocity of five meters/second. This sample spacing detects small stones and other foreign objects, which will have a significant impact on the response of a vehicle model, but will be enveloped by the host vehicle’s tires. The measurement system uses a laser to optically sample the terrain profile; whereas, the physical tire encounters the terrain profile physically and mechanically samples the terrain. Methods such as frequency or morphological filtering of the terrain excitations are discussed in greater detail in Chapter 0. These filters could make the 7-DOF model more robust when traveling over terrain with small localized disturbances.

3.5 Conclusions

In addition to validating measured terrain data, this work demonstrates that a vehicle model can be integrated with an optimization algorithm to provide accurate estimates of vehicle parameters, which may be difficult and expensive to physically measure. This work also demonstrates that measured terrain is a better excitation to use for parameter identification than simple excitations. This developed method is especially useful for varying spring stiffnesses, damping ratios, unsprung mass, inertias, track widths, and wheelbases throughout the design process. The model is computationally efficient and can be easily implemented. This process can be used as a development tool to find model parameters such that the model’s response matches a target response over a measured terrain.

The work has further emphasized the usefulness of the terrain measurement system for simulation purposes when estimating vehicle response. The simulations can be used to estimate chassis loading target early in the design process. The cost of the vehicle development process can be substantially reduced due to the availability of more accurate estimates of loads early in the design process.
4 Terrain Filters

4.1 Introduction

Simple vehicle simulations are often limited because of the complexity of tire dynamics. Advanced tire models show convincing results [22, 33-36]; however, these models are difficult and costly to parameterize. Simple tire models are easy to implement, but do not produce consistent simulation results. Therefore, when selecting a tire model there is a tradeoff between analytical complexity and simulation realism [37].

Simple point contact models commonly are used and model the stiffness of the tire as a single linear spring, which requires a point deflection for excitation. This modeling assumption neglects the physical mechanism by which tires interact with the terrain. The tire acts as a mechanical filter that attenuates small localized disturbances (such as a small stone) that do not cause significant excitation to the vehicle. The point contact tire model lacks this mechanical filtering mechanism. To account for this phenomenon, a filter is implemented on the terrain profile before it is used to excite the point contact tire model. In this way, the profile filter emulates how a tire mechanically filters the terrain. This filter enables simple tire models to better estimate a tire’s response and, therefore, improves the validity of simple vehicle models.

However, when the sprung mass response is being evaluated, the vehicle acts as a low-pass filter. Small localized disturbances have minimal effect on sprung mass response; therefore, terrain filters have little effect on sprung mass response and unfiltered terrain profiles are sufficient excitations for these simulations.

The primary objective of this work is to quantify the effect of filtering 2D terrain profiles on sprung mass response. The remainder of this work is developed as follows. First, a brief background on tire envelopment is presented, and then the terrain types examined in the work are presented. Next, each method for filtering 2D terrain profiles is
developed and examined over the previously discussed terrain types. The responses generated by the model are compared to the measured response and conclusions are drawn based on these results. This chapter concludes with recommendations based on the results of this study.

4.2 Background

4.2.1 Tire Envelopment

The point contact tire model has been used extensively [37, 38] and consists of modeling the tire as a spring in contact with the terrain at a single point below the unsprung mass (labeled $m_u$), as seen in Figure 4.1. The point contact tire model is a good approximation for smooth long wavelength terrain profiles, but results in an over prediction of vertical response for sharp, short wavelength irregularities. Therefore, the geometry of a terrain surface with short wavelength irregularities is not a sufficient excitation for a point contact tire model [35]. To address this problem, Davis uses an effective ground plane [39]. The effective ground plane redefines the terrain beneath the tire to account for tire envelopment and creates an input that is sufficient for a point contact tire model. The effective height of the effective ground plane is defined as the distance over which the wheel center moves vertically in order to keep the vertical load constant when rolling over an obstacle [34]. It has been shown that the point contact tire model is a valid approximation for longer wavelengths (greater than 3 meters) and gradual slope (less than 5%) profile irregularities [40].
4.2.2 Model Response

It is useful to evaluate the frequency response of the system as displayed in Figure 4.2. The sprung mass responses (vertical displacement, pitch, and roll) of a vehicle exhibit a low frequency response due to the vehicle system (suspension, tire, etc.) filtering much of the high frequency content. This is examined for the 7-DOF vehicle model. Using the same white noise excitation at each wheel input, the vehicle model is excited, and the sprung mass vertical displacement is calculated. The frequency response function (FRF) is found for this system, and it is displayed in Figure 4.2. It is apparent that the output signal rolls off quickly after the first natural frequency (approximately 1.2 Hz). This FRF will be reexamined in the Discussion section of this chapter.
4.2.3 Error Calculations

In this work, measured responses are compared modeled responses of the vehicle being excited by the terrain. The error between these signals is evaluated using the root-mean-square (RMS) of the error, which highlights the difference in the magnitude of the signals. The goodness of fit of the model’s response to the measured response is judged by the correlation coefficient, which represents a normalized measure of the strength of the linear relationship between the vectors [41]. Compared to the RMS, the correlation coefficient is more sensitive to the phase of the signals. The value of the correlation coefficient ranges from minus one to one; where a value of zero occurs when the signals are uncorrelated, a value of one occurs when the signals have a perfect linear relationship,
and a value of minus one occurs when the signals have a perfect linear relationship, but with a negative slope.

A settling time is required before the error between the signals is analyzed. This settling time is used to allow the model’s response to converge from its initial conditions to the measured vehicle response. In this work, a settling time of two seconds is used. The explanation for selecting two seconds can be found in Section 3.2.2. The vertical displacement, pitch, and roll of the sprung mass are measured and modeled in this work. These signals represent the basic ride dynamics that the 7-DOF model is estimating.

4.3 Terrain Types

The remainder of the work examines the 7-DOF model excited by a wide variety of terrain types. This chapter verifies the robustness of the terrain filters in a realistic environment. The terrain types considered in this work include smooth asphalt, rough gravel, smooth gravel, dirt trail, Belgian block, and speed bumps on asphalt. Three of these types are developed in detail: smooth asphalt, Belgian block, and speed bumps. These three excitations represent typical excitations to any simulation: smooth, sinusoidal, and impulse. These excitations are meant to test the vehicle model’s and terrain filter’s ability to respond to varying wavelengths of terrain excitations.

It is helpful to examine the frequency content of each terrain type. Figure 4.3 displays the power spectral density (PSD) estimate of the three terrain types discussed. All signals are detrended before the PSD is created to remove any linear trends in the profiles, such as a constant grade. It must be mentioned that all three terrain profiles were collected at a host velocity of 5 meters/second (12 MPH), and the runs on Belgian block and the speed bump are approximately 10 seconds in duration, whereas the smooth asphalt is 30 seconds. It is apparent the Belgian block (green signal) has the most energy of the three signals, and is clearly above the others throughout the frequencies above 0.5 Hz. The smooth asphalt (blue signal) clearly has the least energy. The speed bump is in between the two other signals. The speed bump is a large transient event whose energy is distributed throughout the frequency band.
The following subsections present plots and descriptions of each of the three terrain types. It is important to note that the three preceding figures contain a view of the terrain that is scaled the same so that the three plot can be compared objectively.

### 4.3.1 Smooth Asphalt

When using smooth asphalt as an excitation, long (greater than 3 meter) wavelengths are present. Figure 4.4 is a plot of smooth asphalt terrain. The lower plot is a zoomed in on a 10 meter section of the terrain; notice there is little roughness in the profile.
4.3.2 Belgian Block

When using Belgian block as an excitation, short (less than 3 meter) wavelengths are present. Figure 4.5 is a plot Belgian block terrain. The lower plot is a zoomed in on a 10 meter section of the terrain; notice the roughness in the profile compared to the smooth asphalt excitation.
4.3.3 Speed Bump

When using a speed bump as an excitation, the input is a half-sinusoid. The speed bump has a width of 0.3 meters and a height of 0.06 meters. At higher speeds this is effectively an impulse into the system. For example, at 5 meters/second the point contact tire model engages the speed bump for 0.06 seconds. Figure 4.6 is a plot of smooth asphalt terrain with a speed bump placed the 26 meter mark. The lower plot is a zoomed in on a 10 meter section of the terrain; notice the sharp disturbance at the 26 meter mark. This is the same speed bump that is in Figure 3.1.
4.4 Terrain Filters

In this work, three terrain filters are examined to judge their ability to emulate the way in which a real tire mechanically filters the terrain and their effect on sprung mass response. The filters are spatial moving average, low-pass frequency, and morphological. These filters create an effective terrain profile that is used as the excitation for a point contact tire model. The filters are first developed, and then the results are examined in the next section. It is important to point out that this chapter considers only longitudinal 2D terrain profiles, and does not use any 3D terrain surfaces. Future works may use 3D terrain surfaces when considering tire envelopment, but this work does not.
The parameters for each filter are selected through an optimization process that is used to minimize the overall RMS error. This ensures that these filters perform to the best of their ability.

4.4.1 Moving Average

A spatial moving average filter is used in the calculation of the International Roughness Index (IRI), which is a measure of the roughness of a road based on a quarter-car simulation [42]. The IRI is the standard by which the Federal Highway Administration accesses the quality of highway surfaces. For IRI calculations, the length of the moving average filter is 250 millimeters, which is meant to be similar to a typical passenger tire’s contact length. In this work, Equation 1 is used to implement a moving average filter on the terrain profile. For an explanation of all nomenclature see Appendix A. In this equation, \( k \) is the index of the first point in contact length, \( m \) is the index of the last point, and \( n \) is the total number of points in the contact length.

\[
\hat{z}(i) = \frac{1}{n} \sum_{j=k}^{m} z(j) 
\]

Equation 1

In the frequency domain, the moving average filter has the frequency response shown in Figure 4.7. This assumes a constant velocity of 5 meters/second (18 kph), a sample rate of 100 Hz, and a filter width of 250 millimeters. Because the sample rate is set at 100 Hz, the moving average filter is indirectly influenced by velocity because the number of points that are averaged is dependent on the velocity of the vehicle.
4.4.2 Low-Pass Butterworth

A low-pass Butterworth filter is a temporal filter; whereas, the moving average is a spatial filter. This directly incorporates the speed of the vehicle into the filter because the original profile consists of only distance traveled and height. In this work, a 6 Hz 10\textsuperscript{th} order low-pass Butterworth filter is used. The low-pass Butterworth filter has a maximally flat magnitude response [43, 44]. The flatness assures that data below the cutoff frequency are not attenuated. Figure 4.8 shows a frequency response plot of the Butterworth filter used in this work. The frequency response below 6 Hz is flat and then it rolls off.
4.4.3 Morphological

A morphological filter is used to create an effective terrain profile by filtering the terrain with a structuring element [37]. Researchers in the field of metrology (the science of measurement) use discrete-space morphological filters on surface measurements regularly [45, 46]. This knowledge is applied to 2D terrain profiles by applying dilation with a circular structuring element to emulate the tire; however, the circle is rigid unlike a real tire. Dilation is a procedure where the center of the structuring element is lowered onto the signal directly above a point until the structuring element comes in contact with any point in the signal. The path of the center of the structuring element is output of the dilation. Figure 4.9 demonstrates the dilation of two signals with a circular structuring element over short wavelength terrain (a) and a step input (b). The circle contacts the
profile at its highest point and the path of the circle’s center is the effective profile. This process smoothes short wavelength terrain and also elongates the sharp edges of step inputs as a real tire does. In this work, the morphological filter is implemented in MATLAB®, and the radius of the circle used for filtering is 300 millimeter.

![Figure 4.9: Morphological Filtering](image)

**4.5 Results**

The results are presented with respect to the three terrain types described previously. These results quantify the effects of filtering 2D terrain profiles on sprung mass response. A discussion and conclusions based on these results is in the preceding sections.
4.5.1 Smooth Asphalt

Table 2 displays the results of the terrain filters’ effect on the 7-DOF simulation excited by smooth asphalt terrain. The terrain filters show no significant improvement in the RMS error for runs on smooth asphalt terrain. This is expected because smooth asphalt, as seen in Figure 4.4, does not contain the short wavelength disturbances that the filters would attenuate.

Table 2: Results of Terrain Filters on Smooth Asphalt

<table>
<thead>
<tr>
<th>Terrain Filter</th>
<th>Average RMS Error Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving Average</td>
<td>0.11%</td>
</tr>
<tr>
<td>Low-pass Butterworth</td>
<td>0.02%</td>
</tr>
<tr>
<td>Morphological</td>
<td>0.03%</td>
</tr>
</tbody>
</table>

Figure 4.10 shows the measured and modeled responses of the vehicle traveling over smooth asphalt as well as the error for those signals. It is evident that the signals show excellent agreement by the high correlation coefficient values. The RMS error between the signals is also plotted, and, in the legends, the normalized RMS error is displayed. For all three responses, the error is minimal.
Figure 4.10: Smooth Asphalt Response
4.5.2 Belgian Block

Table 3 displays the results of the terrain filters’ effect on the 7-DOF simulation excited by Belgian block terrain. The terrain filters show marginal results on the Belgian block terrain. The moving average filter improves the RMS error by an average of 0.25%, which is not large enough to make a significant effect. The low-pass Butterworth filter shows no improvement in the RMS error. The morphological filter, on average, makes the RMS error worse by 1.45%.

Table 3: Results of Terrain Filters on Belgian Block

<table>
<thead>
<tr>
<th>Terrain Filter</th>
<th>Average RMS Error Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving Average</td>
<td>0.24%</td>
</tr>
<tr>
<td>Low-pass Butterworth</td>
<td>0.05%</td>
</tr>
<tr>
<td>Morphological</td>
<td>-1.45%</td>
</tr>
</tbody>
</table>
4.5.3 Speed Bumps

Table 4 displays the results of the terrain filters’ effect on the 7-DOF simulation excited by speed bumps. Again the terrain filters show marginal results. The moving average and low-pass Butterworth filter show only minimal improvement, but the morphological filter improves the RMS error by 4.3% on average. This is by far the most significant improvement seen in this study.

Table 4: Results of Terrain Filters on Speed Bumps

<table>
<thead>
<tr>
<th>Terrain Filter</th>
<th>Average RMS Error Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving Average</td>
<td>0.06%</td>
</tr>
<tr>
<td>Low-pass Butterworth</td>
<td>0.21%</td>
</tr>
<tr>
<td>Morphological</td>
<td>4.30%</td>
</tr>
</tbody>
</table>

4.6 Discussion

This study was started with the expectation the terrain filters would make a significant impact on sprung mass response; however, after conducting the study, the initial expectations were found to be false. By stepping back and examining the system closer, the reason for this was found. The main purpose of terrain filters is to attenuate high frequency content; however, the FRF of the system (see Figure 4.2) shows that the sprung mass response of the system is not greatly affected by the high frequency content in terrain excitations. Also because the terrain profiles have much less power content at high frequencies (see Figure 4.3), filtering the terrain profiles has minimal effect on sprung mass response. Figure 4.11 shows a PSD of Belgian block terrain. The blue line is terrain that is unfiltered, and the green line is terrain that has been low-pass Butterworth filtered at 6 Hz. This figure demonstrates that high frequency content is attenuated by the filter and by examining the FRF is seen that this high frequency content
has a minimal effect on sprung mass response when compared to the low frequency content.

Figure 4.11: PSD of Belgian Block

Further investigation is needed to examine the effect of terrain filters, and this study showed the sprung mass response is not sufficient for the investigation; therefore, unsprung mass response should be examined. Figure 4.12 demonstrates the effect of terrain filters on unsprung mass acceleration over Belgian block terrain. In all the figures, the blue line is the unsprung mass acceleration when the terrain profile is not filtered, and the red line is when the terrain is filtered. From top to bottom, the filters used are a 250 millimeter moving average, a 6 Hz low-pass Butterworth, and a morphological circle with a radius of 300 millimeters. It must be mentioned that the parameters of each filter have been arbitrarily selected and will have a large effect on the
result plotted below. These plots are meant to demonstrate a concept that the terrain filters will effect unsprung mass, and an investigation is required to find which filter best emulates the measured response. From all three plots it can be easily shown that when unsprung mass response is being examined, terrain filters have a large effect. Therefore, the study presented in this chapter should be reexamined in future work using unsprung mass response instead of the response of the sprung mass. This would allow for a more detailed examination of the terrain filters.
Figure 4.12: Unsprung Mass Acceleration for Different Terrain Filters
This study is a preliminary look at tire envelopment. Development of a constraint mode tire model is ongoing [47]. This tire model uses a linear radial spring tire model to captures the enveloped shape of the tire then allows for nonlinear springs and dampers to be implemented to find the force generated by the tire.

Ultimately as computational resources advance, the use of 3D tire models will increase. Therefore, 3D terrain measurement will replace 2D as the excitation for most vehicular simulations. The next chapter develops application-dependent interpolations methods for 3D terrain surfaces that advance the possibilities of vehicular simulation.

4.7 Conclusions

The results of this chapter show that over all three terrain types the average improvement is much less than 1%, which demonstrates that sprung mass response (vertical displacement, roll, and pitch) is insensitive to typical terrain filters. This study shows the minimal effect that terrain filters have on sprung mass response. Three terrain filters were discussed including a moving average, low-pass Butterworth, and morphological. It is discussed that while these filters have a minor effect on sprung mass response, they have a significant effect on unsprung mass response. The important contribution of this chapter is that accurately capturing high frequency content in terrain profiles is not necessary because the sprung mass response of the 7-DOF model is not sensitive to this frequency content.
5 Interpolation Methods

Parts of following chapter were originally published in the Proceedings of the 16th International Conference of the International Society for Terrain Vehicle Systems, 2008, and parts are in review for publishing in the Journal of Terramechanics. It is reprinted here with permission from the International Society for Terrain Vehicle Systems (ISTVS).

5.1 Introduction

Three dimensional terrain surface measurements are of interest to many groups, from civil engineers evaluating highway roughness to automotive engineers performing multi-body dynamic simulations to evaluate chassis loads. The quality of the terrain surface measurements has a large effect on all analyses; therefore, the terrain measurement must be as accurate as possible. Currently, high-fidelity terrain measurement systems produce terrain surfaces being used in many advanced applications, such as 3D tire models [22, 27]. These terrain measurement systems are an enabling technology to pursue more advanced modeling and simulation applications. However, 3D terrain surface measurements are not collected with uniform spacing in the horizontal plane, and uniformly spaced data is required for efficient data storage. This storing efficiency translates to an increase in computational speed and more practical simulation times.

This work examines application-dependent interpolation methods to convert non-uniformly spaced data to uniformly spaced data. The remainder of this work is presented as follows. First, background information on terrain surface measurement and interpolation methods is presented. Next, uniform grid spacing is examined in the context of 3D scanning laser systems. Then, interpolation methods to convert non-uniformly spaced data to uniform spacing are examined for desired attributes and
applications. The work concludes with a discussion of future work, and conclusions and recommendations for application-dependant interpolation methods.

5.2 Background

5.2.1 Terrain Measurement

Terrain measurement systems include a 3D scanning laser that collects data transversely through the application of a rotating prism [1, 16]. Figure 5.1 illustrates how the scanning laser acquires a single transverse scan. The data are collected at equal angles of rotation, shown as $\theta$ in Figure 5.1. When a flat surface is being scanned, the transverse spacing between measurements points will be closer at the center of the scan than at the edges of the scan. This issue is exacerbated when there are local disturbances in the surface being measured. Figure 5.1 illustrates the transverse spacing locations for such a surface, shown as $x_1$, $x_2$, and $x_3$. Due to the undulations in the surface, transverse spacing between points $x_1$ and $x_2$ is significantly less than the spacing between points $x_2$ and $x_3$. In the longitudinal direction, the spacing is controlled by speed of the host vehicle and, to a lesser degree, the yaw and pitch rates.

![Figure 5.1: Non-uniform Spacing (photo by author April 13, 2009)](image-url)
These new 3D terrain surface measurement systems present two challenges. First, these systems collect nearly one million data points per second, corresponding to approximately one gigabyte of processed data per minute of acquisition time. Second, variations in the speed of the vehicle and undulations in the terrain cause the data to be non-uniformly spaced. When the host vehicle travels at five meter per second (18 kph), these non-uniformly spaced data have an average horizontal resolution of five millimeters by five millimeters [1, 2]. This work addresses both these challenges by developing techniques to use application-dependent interpolation methods to convert non-uniformly spaced data to uniformly spaced data, thus greatly reducing the quantity of data without undue reduction in the precision.

5.2.2 Geostatistics

Researchers in mathematical geostatistics have studied interpolation methods to convert non-uniform measurements to a uniform grid when creating topographical maps for applications such as water runoff management. The modeling of 3D terrain surfaces is often called digital terrain modeling (DTM). Non-uniformly spaced 3D data is converted to uniformly spaced data by interpolating the unknown height at some uniform grid node in the horizontal plane. These typically have a horizontal resolution of one meter or greater. DTM have become widespread and commonly used in surveying and civil engineering [48]. The lessons learned from DTM are applied to uniform grid spacing of 3D terrain surfaces for vehicular applications.

Several researchers have addressed the issues that affect the accuracy of the terrain height estimation. In 2001, Heesom showed that the decreased resolution of the uniform grid spacing adversely affected the accuracy [49]. These results are reasonable; a coarser resolution for a data set may not capture localized terrain events. Yanalak showed that an insufficient number of measured points decreases the accuracy of a DTM [48]. Again, greater sampling density allows an increase in the resolution and precision in the estimation of the interpolated data points. Of course, there is an effective upper
limit, depending on computational resources, on the number of data points that can be collected to accurately estimate the height of each uniform grid node.

Several statistical methods have been used to estimate the terrain heights. In 2000, Katzil showed that a simple linear estimation (mean) can be used to find uniform grid nodes without adversely affecting the accuracy of results if dense measured point spacing exists [50]. Katzil’s results demonstrate that mean statistic can be used effectively for grid interpolation estimates when a computationally efficient method is needed. The conclusion of several works is that Kriging is the best overall method for creating uniform spacing [51-53]. However, these conclusions depend on factors such as roughness or measured spacing and, depending on these factors, different methods may work better for different applications [50-52]. It must also be noted that several works have highlighted the inaccuracies of Kriging, but even these critics of Kriging admit that when sufficient measured points are in the region being interpolated, the inaccuracies are eliminated [51, 52].

It is important to select the proper interpolation method, but it is even more important to choose a proper search area. The search area must be chosen such that they overlap to ensure the continuous nature of the surface [50, 52]. This is demonstrated in Figure 5.2 where each blue ‘x’ represents a measured point, the red dots ‘.’ represent uniform grid nodes, and the red circle encloses each search area used for interpolation of the height at that uniform grid node. The overlapping search areas are shown in this example. The scope of this work does not include the effects of search area; it exclusively examines the effects of the interpolation methods. Therefore, the search area is chosen based on current literature such that neighboring search areas share a minimum of one point [50-52].
5.3 Uniform Grid Spacing

Typical terrain surface measurement systems have on average five millimeter spacing in the horizontal plane, but the data are non-uniformly spaced. The horizontal spacing for the interpolated grid data is determined based on the particular application for which they will be used. Two categories for resolution are considered in this study: fine and coarse. In this work, fine resolution is defined as less than 25 millimeters and coarse resolution is 25 millimeters and greater. Fine resolution is required for high-fidelity applications such as vehicle dynamics simulations using advanced tire models such as CDTire or FTire. Ten millimeter resolution is sufficient for the bandwidth of these models [22, 33, 36]. An example of a coarse resolution application is road roughness evaluations. In this application, the ASTM standard E 950 requires a 25 mm sample interval for Class 1 status among inertial profilers [54]. Coarse resolution allows for large reductions in the amount of data used for the intended application.

5.3.1 Attributes Being Evaluated

In the context of DTM, absolute accuracy can never be achieved because many factors must be considered. Accuracy is typically evaluated by how close the interpolated height at a uniform grid node is to the absolute height at that grid node; therefore, many attributes affect this evaluation of accuracy [49]. Each of the four
interpolation methods examined in this work is evaluated according to the prevalence of each of four attributes: computational efficiency, outlier sensitivity, location sensitivity, and trend sensitivity.

5.3.1.1. Computational efficiency

Computational efficiency is the relative computational load required to implement a particular interpolation method. That is, the required computation time to implement an interpolation method on a given 3D terrain surface. This computation time can vary by more than an order of magnitude depending on the choice of method.

5.3.1.2. Outlier sensitivity

Outlier sensitivity is the relative influence of outliers on the statistics being used for each method. An outlier is defined as an observation that is 1.5 times the inner quartile range from the closest quartile; if an observation is 3 times the inner quartile range from the closest quartile, then it is classified as an extreme outlier [55]. The desirability of this attribute depends on the application. If the application requires roughness to be captured, then this attribute would be desirable since roughness measures are sensitive to outliers. For example, sensitivity to outliers can be beneficial for tire envelopment where the effect of roughness is studied. On the other hand, if global trends in the topology are being studied, then the interpolation method should be insensitive to outliers.

5.3.1.3. Location sensitivity

Location sensitivity is the relative influence of topographic location on the statistics being used for each method. When an interpolation method is sensitive to location, the measured points are weighted based distance to the grid node. This results in greater localized variation results in the interpolated points, which is manifest in contour plots as a “bull’s eye” effect [56]. Figure 5.3 demonstrates the effects of location sensitivity and trend sensitivity. In this figure, both contour plots were created from the
same set of non-uniformly spaced data. Figure 5.3a was created using an interpolation method that is sensitive to location. Notice the “bull’s eye” type circles lying within the area circled in blue. These localized variations or roughness are due to the location sensitivity of the interpolator.

5.3.1.4. Trend sensitivity

Trend sensitivity is the relative influence of topographic trends on the statistics being used for each method. Trend sensitivity is also dependent on location, but puts additional dependency on topographic trends in the data. Figure 5.3b was created using an interpolation method that is sensitive to trends. Notice the smooth transition in the area circled in blue. These smooth transitions are a result of the interpolator being sensitive to trends versus just location.

![Figure 5.3: Contour Plots: (a) Location Sensitive (b) Trend Sensitive](image)

5.3.2 Interpolation Methods

Each of the four interpolation methods (mean, median, Inverse Distance to a Power, and Kriging) are examined for use on 3D terrain surfaces. There are many other interpolation methods, but this work focuses on these four methods and their application to ground vehicle simulation applications. For each of the interpolation methods, \( \hat{z}(x, y) \) is the interpolated height at grid node \((x, y)\), \( n \) is the total number of measured points.
within the search area, and \(z_i(x_i, y_i)\) is the measured height at horizontal location \((x_i, y_i)\) of the \(i^{th}\) point in the search area. Additional nomenclature is defined in Appendix A. Specific equations are presented for each method in terms of these definitions.

### 5.3.2.1. Mean

The simplest method to interpolate the height of a grid node is to use the mean statistic (arithmetic average); the formula used to calculate the mean is shown in Equation 2.

\[
\hat{z}(x, y) = \frac{1}{n} \sum_{i=1}^{n} z_i(x_i, y_i)
\]

Equation 2

The mean interpolation method is computationally efficient. One key attribute of the mean statistic is that it is sensitive to outliers; therefore, it is appropriate for applications in which roughness is considered. The mean interpolation method is only influenced by heights within the search area; it is insensitive to horizontal location and topographic trends.

### 5.3.2.2. Median

Another simple method to estimate the height for grid nodes is to use the median statistic. The formula for the median is presented in Equation 3, where \(n/2\) is the index of the middle measured point when ordered by height, and \(z_n/2\) is the height of this middle measured point.

\[
\hat{z}(x, y) = z_{n/2}
\]

Equation 3
The median interpolation method is computationally efficient, requiring only a sorting process to be completed. The median statistic differs from the mean in that it is completely unaffected by outliers (at most, the middle two ordered values are considered), making it unsuitable for roughness applications. The median is similar to the mean statistic in that it is insensitive to horizontal location and topographic trends.
5.3.2.3. Inverse Distance to a Power

The Inverse Distance to a Power method is implemented by calculating the mean height value after weighting the values based on their distance to the uniform grid. The algorithm for this method is presented in Equation 4, where $P$ is the power to which the distance is raised, and $d_i(x_i, y_i)$ is the horizontal distance from the $i^{th}$ measured point in the search area to the corresponding uniform grid node. This method allows the power, $P$, to be adjusted depending on the application. Typically, the value of $P$ is chosen to be one or two; however, higher values are often used in applications that emphasize roughness. If the power is chosen to be zero, then the method reduces to the mean statistic and is insensitive to horizontal location; in contrast, as the value for the power increases, the location sensitivity increases since more weight is placed on the points that are closer to the grid node.

\[
\hat{z}(x, y) = \frac{\sum_{i=1}^{n} \frac{z_i(x_i, y_i)}{d_i(x_i, y_i)^P}}{\sum_{i=1}^{n} \frac{1}{d_i(x_i, y_i)^P}}
\]

Equation 4

The Inverse Distance to a Power method is computationally inefficient because it requires calculating the distance to each measured point, and it is sensitive to outliers because it weights each height based on the horizontal location, but not the value of the height itself (in contrast to the median statistic, which weights each height based on its relative order among all heights in the search area). The Inverse Distance to a Power method does not weight topographic trends in the data, which creates a “bull’s eye” effect around measured points in contour plots; such as the effect shown in Figure 5.3b.
5.3.2.4. Kriging

The Kriging method is the most popular method in geostatistics for interpolating heights for digital terrain models [51, 52]. The Kriging method builds on some of the same principles (location sensitivity) as inverse distance to a power, but enhances the method by adding weights ($\lambda$) to the points so that the variance of the elevation is minimized. The fundamental attribute of this approach is that it seeks to find trends in the data. Equation 5 expresses the formula used in the Kriging method, where $\lambda_i$ is the minimized weighting for point $i$.

$$\hat{z}(x, y) = \sum_{i=1}^{n} \lambda_i z_i(x_i, y_i)$$

Equation 5

The Kriging method is computationally inefficient because it includes a minimization algorithm to minimize the variance. The Kriging method is insensitive to outliers, which tend to increase the variance in the estimate. The Kriging method is sensitive to location; however, it is also sensitive to topographic trends. Because the Kriging method is sensitive to topographic trends in the data, there is a reduction of the “bull’s eye” effect on contour plots as demonstrated by the example shown in Figure 5.3b. Many works have concluded that Kriging is the most accurate interpolation method [51-53].

5.4 Desired Attributes and Applications

Selecting the correct interpolation method depends on the application of the 3D terrain surface so that computational resources are used effectively and efficiently. In this work, resolution has been previously divided into two categories: fine and coarse. The characteristics of the terrain surface are divided into two categories according to their application: roughness and global trends. In this work, roughness is used to describe terrain used in applications where sharp discontinuities are important (such as gravel terrain for tire envelopment studies). Global trends are used to describe applications of terrain where smoothness is important and can be characterized by longer wavelengths.
Applications where global trends are important include highways and rough roads where only the long wavelengths are being studied (e.g., a Belgian block road where each block is rough, but the long wavelength trends are being studied). The tables throughout this section evaluate desired attributes and applications in the four categories described previously.

### 5.4.1 Desired Attributes

When using coarse resolution, typical search areas comprise copious measured points (typically over 80). These data are being reduced to one uniform grid point; therefore, a computationally efficient interpolation method is desirable. When using fine resolution, the typical search area encloses fewer measured points (often as few as 6). It is important that the interpolator is sensitive to location and the topographic trends because there are few points from which to interpolate. When the application focuses on a terrain surface that is characterized by roughness, such as gravel, outlier sensitivity is important because outliers that are the actual terrain features being studied must not be attenuated. When using a terrain surface that is characterized by global trends, the interpolation method should be insensitive to outliers that typically have a short wavelength. Table 5 summarizes which attributes are desired for each category of application.

<table>
<thead>
<tr>
<th>Desired Attributes</th>
<th>Roughness</th>
<th>Global Trends</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coarse Resolution</strong></td>
<td>Computational Efficiency;</td>
<td>Computational Efficiency</td>
</tr>
<tr>
<td></td>
<td>Outlier Sensitivity</td>
<td></td>
</tr>
<tr>
<td><strong>Fine Resolution</strong></td>
<td>Location Sensitivity;</td>
<td>Location Sensitivity;</td>
</tr>
<tr>
<td></td>
<td>Outlier Sensitivity</td>
<td>Trend Sensitivity</td>
</tr>
</tbody>
</table>
5.4.2 Desired Applications

Vehicle simulations using simple tire models allow the use of coarse resolution because of the limitations on the tire model being used. However, roughness is important in durability studies because it can damage a vehicle and, therefore, must be accounted for. Typically, the International Roughness Index (IRI) is used for road smoothness evaluation. The IRI is a single number that represents the amount of suspension travel that a quarter car vehicle model, with a specified set of parameters, will experience when traveling over a section of road. When calculating this value, the standard spacing is 25 millimeters or greater [54]. This allows for coarse resolution to be used. IRI calculations are also sensitive to longer wavelengths, from 0.3 – 130 meters [57], so that global trends are important to this application. Fine resolution is required for more advanced tire modeling. Tire models must study the interaction between the tire and the surface, which is typically called the tire contact patch. Tire modeling can examine different properties of the tire. For example, if tire envelopment is to be studied, then roughness is important because the model is examining local deformation of the tire. However, if the tire model is used to study vehicle dynamics, then the global trends of the road are more important than the localized roughness. Table 6 provides examples of applications corresponding to each category.

Table 6: Desired Applications

<table>
<thead>
<tr>
<th>Desired Applications</th>
<th>Roughness</th>
<th>Global Trends</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse Resolution</td>
<td>Durability</td>
<td>IRI</td>
</tr>
<tr>
<td>Fine Resolution</td>
<td>Tire Modeling</td>
<td>Tire Modeling</td>
</tr>
</tbody>
</table>
5.4.3 Preferred Methods

The suggested interpolation method for each category is developed in this section, as summarized in Table 7. Each interpolation method is discussed briefly then a figure is shown to demonstrate a terrain on which that the method is applied.

Table 7: Preferred Methods

<table>
<thead>
<tr>
<th>Preferred Methods</th>
<th>Localized Roughness</th>
<th>Global Trends</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse Resolution</td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>Fine Resolution</td>
<td>Inverse Distance to a Power</td>
<td>Kriging</td>
</tr>
</tbody>
</table>

5.4.3.1 Mean

The mean interpolation method is best used for coarse resolution when localized roughness is present. This is a computationally efficient method and is sensitive to outliers, making it appropriate in durability studies. Figure 5.4 is an example of a rough road with bumps and dips where the mean was used to interpolate. This road could be used as an input into durability simulation.

Figure 5.4: 3D Terrain Surface of Rough Road
5.4.3.2. **Median**

The median interpolation method is best used for coarse resolution when global trends are of interest. This method is computationally efficient and is insensitive to outliers. Therefore, using the median interpolator creates a smoother road than the mean interpolator would. This method is appropriate when characterizing roughness via the IRI. Figure 5.5 is an example of a smooth highway where the median method has been used to interpolate.

![Figure 5.5: 3D Surface of Smooth Highway](image)

5.4.3.3. **Inverse Distance to a Power**

Inverse Distance to a Power is the best interpolation method for fine resolutions when localized roughness is present. This method is sensitive to both location and outliers in the data. This method is good for tire modeling and particularly tire envelopment. An example of a gravel terrain where Inverse Distance to a Power was used to interpolate is seen in Figure 5.6.
5.4.3.4. Kriging

Kriging is the best interpolation method for fine resolutions where global trends are important. This method is sensitive to location and topographic trends in the data. This method works well for tire modeling, particularly for vehicle dynamics studies because the Kriging interpolator captures the small details such as each Belgian block and characterizes global trends, such as long wavelength sinusoidal undulations. Figure 5.7 shows a section of a Belgian block road, which Kriging was used to interpolate. Note that the large rectangle is a digital photograph; and the superimposed, tilted, small square is a rendering of the corresponding measured data.
Figure 5.7: 3D Surface of Belgian Block inside Digital Photo (photo by author March 23, 2009)

5.5 Discussion

One possible expansion of this work is an investigation into the effect of search area definition in conjunction with interpolation method to emulate a tire’s mechanical filtering properties. Presently, the extraction of 2D terrain profiles from 3D terrain surfaces is being investigated for road smoothness evaluation. The application of this method is expected to result in a 2D terrain profile that emulates the mechanical filtering properties of a tire. In this way, the method retains the topology information in the 3D surface that is required for the implementation of a tire model and effectively captures this information in the computational efficient 2D profile. Additional research into the effects of terrain surfaces on ground vehicle dynamics is also ongoing.
5.6 Conclusions

This work examines interpolation methods for generating uniformly spaced terrain surfaces from non-uniformly spaced 3D terrain surfaces. Each method has its advantages and works effectively with different applications. The conclusions of this work are as follows.

1. The mean interpolation method is best for durability applications.
2. The median interpolation method is best for road smoothness evaluation.
3. The Inverse Distance to a Power method is best for tire modeling when tire envelopment is being studied.
4. The Kriging interpolation method is best for tire modeling when vehicle dynamics are being studied.

These conclusions enable interpolation methods to be used to create high-fidelity 3D surfaces from non-uniformly spaced measurements. These methods enable high-fidelity 3D terrain measurements to be used in a spectrum of applications from simple road smoothness evaluation to advanced tire modeling. By advancing the ability to use 3D terrain surfaces for ground vehicle applications, it is anticipated that a virtual proving ground can be developed that will shorten the design cycle for vehicles by creating more accurate simulations.
6 Conclusions

This research presented in this thesis is the culmination of two years of research to develop techniques to ultimately enable the use of 3D terrain surface measurements in vehicular simulations. The major contributions of this work are as follows:

1. Demonstrate measured terrain excitations are better for parameter identification compared to simple excitations.
2. Demonstrate that sprung mass response (vertical displacement, roll, and pitch) is insensitive to typical terrain filters.
3. Develop a suggested practice for application-dependent interpolation methods for 3D terrain surface measurements that ensures 3D terrain surfaces are uniformly spaced with the appropriate resolution for a given application.

This work proves that a vehicle model excited by measured terrain can be integrated with an optimization algorithm to provide accurate estimates of vehicle parameters, which may be difficult and expensive to physically measure. This developed method is especially useful for varying spring stiffnesses, damping ratios, unsprung mass, inertias, track widths, and wheelbases throughout the design process. This process can be used as a development tool to find model parameters such that the model’s response matches a target response.

The results of an empirical study show that overall three terrain types the average improvement is much less than 1%, which quantifies the inconsequential effect terrain filters have on the sprung mass response. Three terrain filters were discussed including a moving average, low-pass frequency, and morphological. It is discussed that while these filters have an inconsequential effect on sprung mass response, they have a significant effect on unsprung mass response, and an additional investigation is needed to examine this. The important contribution of this study is that accurately capturing high
frequency content in terrain profiles is not necessary because the sprung mass response of the 7-DOF model is not sensitive to this frequency content.

This research presents a suggested practice for application-dependent interpolation methods of vehicular applications. These methods create uniformly spaced 3D terrain surface that are of resolution that is efficient for the desired application. These methods enable high-fidelity 3D terrain measurements to be used in a spectrum of applications from simple road smoothness evaluation to advanced tire modeling. By advancing the ability to use 3D terrain surfaces for ground vehicle applications, it is anticipated that a virtual proving ground can be developed that will shorten the design cycle for vehicles by creating more accurate simulations.
References


28. Ziegenmeyer, J.D., *Estimation of Disturbance Inputs to a Tire Coupled Quarter-car Suspension Test Rig* 2007, Virginia Polytechnic Institute and State University Blacksburg, VA.
Appendix A   Nomenclature

\( d \)  
Distance

\((x, y)\)  
Location of a data point in the horizontal plane

\( z \)  
Vertical height of terrain profile

\( i, j, m \)  
Indices

\( n \)  
Number of data points

\( P \)  
Power for weighting

\( X \)  
Easting

\( Y \)  
Northing

\( \hat{z} \)  
Height of uniform grid node or filtered terrain profile

Appendix B   Equations of Motion 7-DOF Model

Equation [3] sums the vertical forces on the sprung mass.

\[
\begin{align*}
    m_s \ddot{x}_s &+ c_f (\dot{z}_{s,fr} - \dot{z}_{u,fr}) + c_f (\dot{z}_{s,fl} - \dot{z}_{u,fl}) + c_r (\dot{z}_{s,rr} - \dot{z}_{u,rr}) \\
    &+ c_r (\dot{z}_{s,rl} - \dot{z}_{u,rl}) + k_f (z_{s,fr} - z_{u,fr}) + k_f (z_{s,fl} - z_{u,fl}) \\
    &+ k_r (z_{s,rr} - z_{u,rr}) + k_r (z_{s,rl} - z_{u,rl}) = 0
\end{align*}
\]

[3]

\[ l_p \ddot{\theta}_s + c_f a(\dot{z}_{s,fr} - \dot{z}_{u,fr}) + c_f a(\dot{z}_{s,fl} - \dot{z}_{u,fl}) - c_r b(\dot{z}_{s,rr} - \dot{z}_{u,rr}) - c_r b(\dot{z}_{s,rl} - \dot{z}_{u,rl}) + k_f a(z_{s,fr} - z_{u,fr}) + k_f a(z_{s,fl} - z_{u,fl}) - k_r b(z_{s,rr} - z_{u,rr}) - k_r b(z_{s,rl} - z_{u,rl}) = 0 \]  

Equation [5] sums the moments about the roll axis.

\[ l_r \dot{\Phi}_s + c_f t_f (\dot{z}_{s,fr} - \dot{z}_{u,fr}) - c_f t_f (\dot{z}_{s,fl} - \dot{z}_{u,fl}) + c_r t_r (\dot{z}_{s,rr} - \dot{z}_{u,rr}) - c_r t_r (\dot{z}_{s,rl} - \dot{z}_{u,rl}) + k_f t_f (z_{s,fr} - z_{u,fr}) + k_f t_f (z_{s,fl} - z_{u,fl}) + k_r t_r (z_{s,rr} - z_{u,rr}) - k_r t_r (z_{s,rl} - z_{u,rl}) = 0 \]  


\[ m_{u,f} \ddot{z}_{u,fr} - c_f (\dot{z}_{s,fr} - \dot{z}_{u,fr}) - k_f (z_{s,fr} - z_{u,fr}) + k_{tf} z_{u,fr} = k_{tf} z_{0,fr} \]  

\[ m_{u,f} \ddot{z}_{u,fl} - c_f (\dot{z}_{s,fl} - \dot{z}_{u,fl}) - k_f (z_{s,fl} - z_{u,fl}) + k_{tf} z_{u,fl} = k_{tf} z_{0,fl} \]  

\[ m_{u,r} \ddot{z}_{u,rr} - c_r (\dot{z}_{s,rr} - \dot{z}_{u,rr}) - k_r (z_{s,rr} - z_{u,rr}) + k_{tr} z_{u,rr} = k_{tr} z_{0,rr} \]  

\[ m_{u,r} \ddot{z}_{u,rl} - c_r (\dot{z}_{s,rl} - \dot{z}_{u,rl}) - k_r (z_{s,rl} - z_{u,rl}) + k_{tr} z_{u,rl} = k_{tr} z_{0,rl} \]  


\[ \dot{z}_{s,fr} = \dot{z}_s + a \dot{\theta}_s + t_f \Phi_s \]  

\[ \dot{z}_{s,fl} = \dot{z}_s + a \dot{\theta}_s - t_f \Phi_s \]  

\[ \dot{z}_{s,rr} = \dot{z}_s - b \dot{\theta}_s + t_r \Phi_s \]  

68
\[ \dot{z}_{s, rl} = \dot{z}_s - b \dot{\theta}_s - t_r \dot{\phi}_s \] 

[13]


\[ z_{s, fr} = z_s + a \theta_s + t_f \Phi_s \] 

[14]

\[ z_{s, ft} = z_s + a \theta_s - t_f \Phi_s \] 

[15]

\[ z_{s, rr} = z_s - b \theta_s + t_r \Phi_s \] 

[16]

\[ z_{s, rl} = z_s - b \theta_s - t_r \Phi_s \] 

[17]
Appendix C  Parameters 7-DOF Model

Table 8: Vehicle Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Initial Value</th>
<th>Optimized Value</th>
<th>Percent Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>Total vehicle mass (kg)</td>
<td>2,710</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Sprung mass (kg)</td>
<td>2,430</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$m_{us}$</td>
<td>Unsprung mass of each wheel assembly (kg)</td>
<td>70</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$k_{s,f}$</td>
<td>Front suspension stiffness (N/m)</td>
<td>32,000</td>
<td>42,843</td>
<td>33.88</td>
</tr>
<tr>
<td>$k_{s,r}$</td>
<td>Rear suspension stiffness (N/m)</td>
<td>35,000</td>
<td>43,024</td>
<td>22.93</td>
</tr>
<tr>
<td>$k_t$</td>
<td>Tire stiffness (N/m)</td>
<td>240,000</td>
<td>248,660</td>
<td>3.61</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Damping coefficient front suspension N/(m/s)</td>
<td>2,900</td>
<td>3,477</td>
<td>19.90</td>
</tr>
<tr>
<td>$c_r$</td>
<td>Damping coefficient rear suspension N/(m/s)</td>
<td>3,200</td>
<td>4,218</td>
<td>31.81</td>
</tr>
<tr>
<td>$a$</td>
<td>Distance front axle to center of mass (m)</td>
<td>1.63</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$b$</td>
<td>Distance rear axle to center of mass (m)</td>
<td>1.25</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$t_f$</td>
<td>Track width front axle (m)</td>
<td>1.55</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$t_r$</td>
<td>Track width rear axle (m)</td>
<td>1.57</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$I_p$</td>
<td>Pitch inertia (kg-m$^2$)</td>
<td>1,134</td>
<td>1,239</td>
<td>39.24</td>
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<tr>
<td>$I_r$</td>
<td>Roll inertia (kg-m$^2$)</td>
<td>2,938</td>
<td>3,694</td>
<td>25.73</td>
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<td>$\theta_s$</td>
<td>Vehicle Pitch (rad)</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Vehicle Roll (rad)</td>
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<td>-</td>
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</tr>
<tr>
<td>$z_s$</td>
<td>Sprung Mass Displacement (m)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$z_{u,fr}$</td>
<td>Unsprung Mass Displacement (Front, Right) (m)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$z_{u,fl}$</td>
<td>Unsprung Mass Displacement (Front, Left) (m)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$z_{u,rr}$</td>
<td>Unsprung Mass Displacement (Rear, Right) (m)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$z_{u,rl}$</td>
<td>Unsprung Mass Displacement (Rear, Left) (m)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$z_{0,fr}$</td>
<td>Terrain Excitation (Front, Right) (m)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$z_{0,fl}$</td>
<td>Terrain Excitation (Front, Left) (m)</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$z_{0,rr}$</td>
<td>Terrain Excitation (Rear, Right) (m)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$z_{0,rl}$</td>
<td>Terrain Excitation (Rear, Left) (m)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>