A Photoelastic Investigation into the Effects of Cracks and Boundary Conditions on Stress Intensity Factors in Bonded Specimens

by

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An investigation into the influence of cracks in bonded specimens is conducted. Photoelastic specimens containing a bondline are subjected to a constant displacement boundary condition created by bonded end grips. Specimens containing various crack orientations are analyzed to determine stress intensity factors at the induced crack tips. Specimens containing interface and sub-interface cracks were investigated. Two global geometries were used in this investigation, square and rectangular. The constant displacement boundary condition was induced on the specimen through dead weights hung from bonded aluminum end grips. Stress intensity factors were determined using photoelastic techniques. The stress intensity factors were examined to determine trends in the results as a function of changes in geometry. The effects of the induced boundary condition, the specimen geometry, and the bondline were investigated. The results from this investigation were compared to known solutions with a similar specimen geometry. These tests exhibited influences from the bondline, the boundary conditions, and the specimen geometry. The bondline tended to decrease the stress intensity factor for specimens with small crack lengths and tended to increase the stress intensity factor for specimens containing long crack lengths. As the crack length increased so too did the stress intensity factor. A reduction in the bondline to crack distance with sub-interface crack specimens caused a reduction in the stress intensity factor. A reduction in the global height of the specimen caused a reduction in the stress intensity factor also. The results from this investigation will aid in the understanding of the influence of interface and sub-interface cracks in bonded specimens.
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1.0 Introduction

An investigation into the effects of both crack length and boundary conditions involving bonded photoelastic materials is performed. Specimens containing various crack lengths and boundary conditions are analyzed using common photoelastic techniques. The quantity under investigation for the various specimens is the stress intensity factor for the initiated crack tips. The intentions behind this study are to determine any trends present in stress intensity factors as a result of changing specimen geometries and boundary conditions and to compare these results to analysis, which is being performed using similar geometries. The specimens under investigation are configured such that their material properties are similar to that of solid rocket propellant. The specimens in this investigation all contain two adherends bonded together using an adhesive that has a similar elastic modulus to that of the adherends. The presence of the bondline within the specimen geometry is of particular interest to this study. The boundary conditions are prescribed to each specimen by attaching rigid aluminum end grips. The specimens are then analyzed under a dead load. Cracks of various sizes are initiated into the specimens and the resulting stress intensity factors are determined. The need for this study is to qualitatively understand the influence of cracks and crack sizing on the effectiveness of solid rocket propellant. The presence of cracks in solid rocket propellant can influence the direction and amount of burn within the rocket. Solid rocket propellant is designed to have a specific configuration and size. When cracks are introduced into the propellant, the size and shape of the original design for the propellant is lost. This investigation deals with stress intensity factors, which can be used to help explain the severity of the associated crack in a given material. This study will involve the effects of the bondline, the boundary conditions, and the specimen geometry. The study hopes to quantify the effects of cracks in and near bondlines present in solid rocket propellant. This will be accomplished by performing tests on double edge crack specimens and specimens with off bondline cracks. These two types of specimens allow for a wide range of applications to actual solid rocket propellant cracks.

This thesis will begin by discussing both the specimen components and the mechanics of fracture involved. A development of the equations and analysis techniques used for this study will then be presented. The development of the equations used in this study will start from basic
fracture mechanics, which will then be substituted into to photoelastic equations. The methods of data acquisition and analysis will be discussed. Lastly, the results and conclusions from this study will then be presented and explained.
2.0 Analytical and Experimental Considerations

2.1 Brief History

The basic principles behind photoelasticity have been around for nearly 200 yrs.\textsuperscript{1} Phototelasticity involves the retardation of polarized light within a transparent material. The retardation of the light creates fringes of light and dark shades. It has been determined that these fringes are proportional to the stresses within the material. Implementation of the stress optic law, discussed in more detail in section 2.4.1, allows for determination of these stresses. The method of photoelasticity is used in this study because it has been proven to be effective and very representative of the stress field within the material.

One of the main aspects involved in this study is the presence of a bondline within the specimen geometry. The presence of a bondline and a crack within that bondline, is better known as interface fracture mechanics. In the current study two adherends of the same material were bonded together. However, most of the previous research has been done using a bimaterial model involving two dissimilar adherends. Of the earliest works dealing with interface fracture mechanics was an analytical model created by Williams\textsuperscript{2} in 1959. The result of William’s research was the determination of an oscillating singularity near the crack tip. More research was performed during the 1960’s and 1970’s to either incorporate or refute the presence of this oscillating singularity. Of particular interest was the research performed by Comninou\textsuperscript{3,4}. Comninou incorporated the oscillating singularity into a more comprehensive model that better patterned the actual behavior of a crack within a bondline. More and more research has been done in the area of interface fracture mechanics because more advanced models can be created and determined due to the aid of computer technology. In 1988, Rice\textsuperscript{5} discussed the validity of implementing a stress intensity factor at the crack tip within an interface. Rice expanded upon the previous models that had been developed to better define the stress intensity factors associated with an interface crack. Two researchers that have had a large impact on the field of interface fracture mechanics have been Hutchinson and Suo. Individually, these two researchers have authored dozens of papers and together have co-authored several papers \textsuperscript{6}. In 1992, Hutchinson and Suo \textsuperscript{7} published a comprehensive analysis and literature review of subjects dealing with interface fracture mechanics. A great amount of detail was spent developing and
expanding equations to characterize the stresses at and near a crack within an interface. In recent years, much more research has been done to verify results that have been discussed in previous years. Such methods as numerical evaluation, boundary element method, finite element method, and contour integrals have all been implemented to help characterize interface fracture mechanics\(^8,9,10,11,12,13,14\).

The field of interface fracture mechanics has also been extended toward cracks that parallel interfaces. This class of interface fracture mechanics is also known as subinterface fracture mechanics. The amount of research dealing with subinterface fracture mechanics has increased steadily in the last couple of years. One of the first papers published in this field dealt with a crack paralleling an interface between a bimaterial specimen\(^15\). As is the case with interface fracture mechanics, Hutchinson plays a major role in the development of this field. As more precise techniques for solving interface and subinterface fracture mechanics problems developed, the complexity of the research also increased. Papers dealing with interface fracture and subinterface fracture involved in the same study became more popular\(^16\). One study which dealt with both interface and subinterface fracture was performed by Finlayson\(^17\). Finlayson’s study incorporated photoelastic techniques to quantify the effects of interface and subinterface cracks in solid rocket propellant. It was Finlayson’s investigation that prompted the need for the further study in the area of photoelasticity when dealing with interface and subinterface cracks in solid rocket propellant.

### 2.2 Solid Rocket Propellant

Solid rocket propellant is a particulate composite, composed of a soft rubber matrix and hard polyhedral particles. In its initial state the composite is in a liquid state and is poured into a form, which contains a rubber liner. As the liquid cools a joint is formed between the rubber liner and the hardening propellant. This joint may be subject to cracking along the created bondline due to the induced forces within the rocket casing. It is this joint that is being modeled in this research. The solid rocket propellant is considered to be a rubber like material in its cured state. The nature of the solid propellant creates an interesting crack growth phenomenon. When a crack is created in the composite under some load, the crack tip begins to blunt. After the blunting has reached a maximum, the crack then begins to grow. The process of blunt-growth-
blunt-growth is a highly non-linear phenomenon. In order to create a more simplified test situation, optical grade polyurethane (E = 4.14 MPa and ν = 0.50) is used to model the solid rocket propellant. By utilizing the polyurethane material the complexities associated with the crack growth phenomena are greatly reduced. The growth of the crack is not the focus of this research, instead this research focuses on the severity of the associated crack within the polyurethane. The polyurethane is a photoelastic material which is soft, transparent, and easily machinable. The polyurethane simulates a linearized propellant material within which a crack exists. Therefore, the polyurethane material makes an accurate and useful model for the solid propellant. The study of various cracks in solid rocket propellant is of great interest because small cracks in the propellant or in the bondline can cause great influences on both the rate and direction of burn in the rocket itself. However, as stated above, the actual solid propellant is not being used in this study. The results and conclusions obtained from this study will aid in the understanding of crack growth and the impact these cracks have on solid rocket propellant.

2.3 Bondline Adhesive

A photoelastic adhesive is used to create the necessary geometries for the specimens, which will be used to simulate the rubber liner and propellant bondline. The manufacturer formulates the adhesive, such that its material properties approach the material properties of the polyurethane. Once the total curing time for the adhesive has been reached, the adhesive is transparent. As a result of these facts, the adhesive is considered and designed to be a photoelastic material. The adhesive layer contains small bubbles throughout, which is not unlike the real bondline present between the solid propellant and the rubber liner. Even though the adhesive is designed with similar material properties to the polyurethane, the adhesive has a slightly higher elastic modulus (E = 6.90 MPa). The adhesive is formed into a thin layer and when it is compared in size to the polyurethane adherends, it can be considered to be a thin bondline and not a third adherend.
2.4 Analytical Considerations

The governing equations used for determination of stress intensity factors from the photoelastic fringe patterns are presented and developed in this section. The formulation of stress intensity factors from fracture mechanics will be discussed first. The Smith extrapolation will be implemented, which allows for determination of the apparent stress intensity at the crack tip using a two-parameter method. The reading of data and calculation of the resulting final stress intensity factor will then be discussed. Lastly, the assumption of whether to consider plane stress or plane strain conditions, in this study for the given specimen geometries, will be discussed.

2.4.1 Stress Intensity Factor

Equations used in photoelastic calculations to determine stress intensity factors must be related to fracture mechanics. To do this, first consider a two-dimensional case with stresses near a crack tip as shown in Fig. 2.1.

![Figure 2.1: Generalized 2-D Crack Tip Stress Field](image)

The linear-elastic fracture mechanics equations that follow describe the state of stress at any point within the body:

\[
\begin{align*}
\sigma_x &= \frac{K_I}{\sqrt{2\pi r}} \sin \theta \\
\sigma_y &= \frac{K_I}{\sqrt{2\pi r}} \cos \theta \\
\sigma_{xy} &= 0 \\
\end{align*}
\]
\[
\sigma_x = \frac{K_I}{\sqrt{2\pi r}} G_1 - \sigma_x^o \\
\sigma_y = \frac{K_I}{\sqrt{2\pi r}} G_2 - \sigma_y^o \\
\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} G_3 - \sigma_{xy}^o
\] (2.1)

where \(\sigma_i^o\) are the non-singular stress components, \(K_I\) is the mode I stress intensity factor, and \(G_1, G_2, G_3\) are functions of \(\theta\) and are defined as follows:

\[
G_1 = \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\
G_2 = \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\
G_3 = \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} 
\] (2.2)

In this study only Mode I crack displacements are considered and therefore the equations shown above are in a simplified state. These equations are based on linear elastic theory with an applied far field stress. Higher order terms are not considered because their magnitudes are considered small near the crack tip region. For a development of these equations with mode II crack displacements involved refer to Smith and Olaosebikan\(^{18}\). The equations above can be simplified further because of the direction of applied stresses. For all cases under investigation in this research a tension stress was introduced, which allows for simplifications of \(\sigma_x^o = -B_o\) and \(\sigma_y^o\) and \(\sigma_{xy}^o\) both set equal to zero. This substituted and reduced form is shown as Equation 2.3.

\[
\sigma_x = \frac{K_I}{\sqrt{2\pi r}} G_1 + B_o \\
\sigma_y = \frac{K_I}{\sqrt{2\pi r}} G_2 \\
\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} G_3 
\] (2.3)
The fracture mechanics equations of a given stress state will now be related to the stress optic law for photoelasticity. This is done by combining the stress optic law, for a two dimensional case, as seen in equation 2.4:

\[
\tau_{\text{max}} = \frac{\sigma_1 - \sigma_2}{2} = \frac{Nf}{2t} \quad (2.4)
\]

where \( \sigma_1 \) and \( \sigma_2 \) are the principal normal stresses, \( \tau_{\text{max}} \) is the maximum principal shear stress, \( t \) is the thickness of the sheet, \( N \) is the fringe order, and \( f \) is the material fringe value. The last two quantities are photoelastic parameters and can easily be determined. The fringe order, \( N \), is visually measured from the fringe pattern produced from the applied stresses. The material fringe value, \( f \), is a unique quantity which is specific per photoelastic material. The determination of the fringe value for a given material is discussed in appendix A. Equations for the relationships between principal stresses and applied stresses in a body are shown as equation 2.5:

\[
\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \sigma_{xy}^2} \quad (2.5)
\]

and are substituted into equation 2.4 which yields the form below:

\[
\tau_{\text{max}} = \frac{Nf}{2t} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + 4\sigma_{xy}^2} \quad (2.6)
\]

which can be written in another form as:

\[
\left(2\tau_{\text{max}}\right)^2 = \left(\frac{Nf}{t}\right)^2 = \left(\sigma_x - \sigma_y\right)^2 + 4\sigma_{xy}^2 \quad (2.7)
\]

This is the final format of the equation used to relate the stress optic law to the applied stresses within the material. This final form contains only two unknown parameters encompassed within the applied stress terms, which are \( K_1 \) and \( B_0 \). These two parameters are determined using the Smith extrapolation method.
2.4.2 Smith Extrapolation Method

The Smith extrapolation method is used to determine the two unknown parameters in the final equation from the previous section, $K_I$ and $B_o$\textsuperscript{19}. For this technique, data is taken from the near crack tip fringe pattern along the $\theta = \frac{\pi}{2}$ line. Figure 2.2 shows the coordinate system that is used for the reading of data.

![Crack Tip Coordinate System](image)

Figure 2.2: Crack Tip Coordinate System

After substitution of equations 2.3 and the implementation of the data read line angle, equation 2.7 converts to:

$$
(2\tau_{\text{max}}) = \left( \frac{Nf}{t} \right) = \left( \frac{K_I^2}{2\pi r} + \frac{K_I B_o}{\sqrt{\pi r}} + B_o^2 \right)^{\frac{1}{2}} \tag{2.8}
$$

To apply the Smith extrapolation method, the higher order terms must be eliminated. The elimination of the higher order terms will be accomplished by implementing a polynomial expansion, which has a general form of:

$$(p + q)^n = p^n + np^{(n-1)}q + \ldots + \text{higher order terms} \tag{2.9}$$

with:

$$n = \frac{1}{2} \quad ; \quad p = \frac{K_I^2}{2\pi r} \quad ; \quad q = \frac{K_I B_o}{\sqrt{\pi r}} + B_o^2$$
After substitution and simplification of the polynomial expansion the stress intensity factor relation becomes:

$$\tau_{\text{max}} = \frac{K_I}{\sqrt{8\pi r}} + \frac{B_0}{\sqrt{8}}$$

(2.10)

The final form for the equation from the Smith extrapolation method is obtained by multiplying by \(\sqrt{8\pi r} / \sigma\sqrt{\pi a}\), which yields:

$$\frac{K_{ap}}{\sigma\sqrt{\pi a}} = \frac{K_I}{\sigma\sqrt{\pi a}} + \frac{B_0}{\sigma} \sqrt{\frac{r}{a}}$$

(2.11)

where \(K_{ap} = 2\tau_{\text{max}}\sqrt{2\pi r}\) and is the apparent stress intensity factor at the crack tip, \(\sigma\) is the far field stress, and \(a\) is the crack length. This final format is in the form of \(y = mx + b\) and is the equation of a line whose intercept is \(K_I\) and whose slope is \(\frac{B_0}{\sigma}\). The plotting and application of this final equation to the photoelastic data is discussed in the subsequent section.

### 2.4.3 Data Line

Once the data have been read from the produced fringe pattern, the data are plotted on a chart with \(K_I\) and \(\sqrt{\frac{r}{a}}\) axes. The data points \((N, r)\) are converted using the equations mentioned from the previous section. The data are investigated to determine the most linear portion. A least squares fit is passed through the linear portion of the data and its y-axis intercept is determined. The y-axis intercept is the value of the stress intensity factor at the crack tip. The following plot demonstrates the application of the least squares fit to the linear portion of the data.
The plot shows all the data that was read from the fringe pattern, the least squares fit to the linear portion of the data, and the intercept value. The method described was used as the primary means for data plotting and stress intensity factor determination. However, a few cases arose where the straight-line curve fit was not the best means for stress intensity factor determination. These cases arose as a result of pre-load induced stresses within the specimen. Test cases involving both a pre-load induced stress specimen and a stress free specimen were investigated. The stress intensity factors for both cases were determined and found to be within a predictable scatter range of one another. The pre-load stresses within the specimens were a result of aged adhesive used for the bondline construction. In these cases, the data were plotted and a curve was fit to the entire set of data. The slope of the least squares curve fit at the last data point, closest to the crack, was calculated. A line was then plotted which continued at the calculated slope until the y-axis was reached. As in the previous method, the intercept is the stress intensity factor at the crack tip. The following two plots demonstrate this second means of stress intensity factor determination.
Figure 2.4: Non-linear Fringe Data

Figure 2.5: Extended Slope Curve Fit
Close examinations of the previous two plots show no clear linear zones. The data used for the curve fit was determined by comparing the range in the above and below crack data zones. In all cases, the stress intensity factor was calculated using the linear zone method for the data zone below the crack. This data range was then used for above crack data zone, which utilized the nonlinear curve fit. By using the same data range for the above data zone as the below data zone some consistence for stress intensity factor determination was achieved. However, there is still some uncertainty associated with the nonlinear curve fit method. The previous plot illustrates the nonlinear data zone, the use of the least squares fit to the fringe data, the application of the extended slope curve, and the apparent stress intensity factor.

### 2.4.4 Fringe Data

The fringe data used in the plotting and calculations is read from a photograph of the induced stress field for both the global and the crack tip regions. Below is a photograph of a typical specimen’s crack tip region with the data zones used for the calculations labeled.

![Figure 2.6: Typical Fringe Pattern](image)

The fringe order for each fringe present in the induced stress field is determined by counting each black fringe present in order. The fringe counting is begun at a free square corner because at this place, in the stress field, there is no shear stress present. The free square corner fringe is given a value of zero for its fringe order. Each black fringe appearing in sequence thereafter is
given the next subsequent whole number (1, 2, etc.). The white fringes present in the fringe pattern are also used for fringe data. However, white fringes are given values of 0.5, 1.5 etc. The fringe order determination described in this section is used when the background of the photograph is black. The photograph can also be taken such that the background is white, in which case, the white fringes are given the whole number and the black fringes are given the half-number designations. The fringe data was taken within a specific data region of $\sqrt{\frac{r}{a}}$ from 0.3 to 0.6. This data region was chosen because it contains the most linear fringe data. This region was also chosen so the measurements from one specimen to another would be consistent.

### 2.4.5 Fringe Folding

An interesting global fringe pattern develops as a result of the enforced loading conditions. The polyurethane specimens are epoxied to rigid aluminum end grips. A dead load is hung from the end grips which creates a constant displacement boundary condition. It is assumed that the end grips are much more rigid than the polyurethane and therefore this constant displacement of the end grips creates a constant tension force within the polyurethane. The application of this tension force on the polyurethane creates a fringe folding effect. This fringe folding effect is a result of the non-singular stress components parallel to the crack within the polyurethane. However, as the crack tip is approached the singular stress region dominates and the fringe folding effect disappears. The figure below exhibits the fringe folding phenomenon. The top fringe pattern in the figure would be a result of an applied hydrostatic stress, while the bottom fringe pattern is a result of an applied tension stress.
The figure illustrates how the fringe folding effect disappears as the crack tip is approached. This is commonly seen when fringe folding has occurred and only mode I displacements are involved. Even with the presence of this fringe folding, the data reading line orientation is not affected. Data is still read from fringes along a line normal to the crack plane and passing through the crack tip.

### 2.4.6 Thickness Effect

For result comparison purposes, the thickness of the specimens had to be taken into account. The thickness of the test specimens affected crack tip blunting, crack edge curling, and the state of stress present. The crack tip blunting is the rounding of the induced crack tip while the crack edge curling is a rounding of the crack edges at the plate surface due to the applied stresses. All of the specimen thicknesses were measured to be 12.7 mm. For this depth of thickness neither a plane stress nor a plane strain assumption could be made directly. The plane stress assumption can be utilized with a very thin sheet while the plane strain assumption can be utilized with a sheet of infinite thickness. The thicknesses of the specimens investigated in this study fell in between a plane stress and a plane strain assumption. The thickness of the specimens created a plane stress situation at the edges while a plane strain situation existed.
toward the middle. A correction factor was then developed to allow for the results of these experiments to be compared to known plane stress solutions. A single edge crack specimen was machined such that its dimensions were identical to those of a theoretical specimen that was previously studied. A single edge crack specimen was used for the correction factor because it is considered the most ideal and realistic comparison case. A single crack was then introduced into the side of the specimen using a razor blade. The stress intensity factor of the crack was determined for a small range of crack sizes. The single edge specimen was machined with the geometry shown below:

![Diagram of Single Edge Crack Specimen](image)

- $w = 38.1$ mm
- $t = 12.7$ mm
- $h = 127$ mm
- $b = 101.6$ mm
- $P = 26.9$ N

This simple test produced results that were 7.8% higher than an accurate two-dimensional solution of a plane stress case obtained by Gross, Srawley, and Brown (1964) through boundary collocation. The 7.8% was then used as a correction factor in all tests that were performed, which then allows for a more direct comparison of these test results to known plane stress results.
2.5 Procedure

2.5.1 Model

All of the specimens that were examined were created from a commercially available photoelastic material. Specifically, PSM-4 sheets were used for the construction of the specimens. PSM-4 sheets are manufactured such that their elastic modulus is roughly 4.14 MPa and their material fringe value is roughly 1.0 MPa/fringe/mm. These materials properties were determined per PSM-4 sheet used in this study. The material property determination is discussed in Appendix A. All of the specimens involved in this study were cut from PSM-4 sheets, which had thicknesses of 12.7 mm. The adhesive used for the creation of the bondlines is also commercially available, PC-12. PC-12 is manufactured to have an elastic modulus of 6.90 MPa. The effect of this mismatch in elastic modulus will be discussed in a later section. Lastly, two aluminum end grips were manufactured and used to induce the boundary conditions on the specimens. The end grips were attached with a commercially available epoxy. The end grips were manufactured such that the PSM-4 blocks would be inserted at a depth of 12.7 mm. The figure below illustrates the 12.7 mm deep slot used.

![Figure 2.9: Aluminum End Grip](image)

The epoxy was applied to both the PSM-4 blocks and to the entire surface area within the slots in the end grips. The epoxy was allowed to cure at room temperature overnight.

The specimens involving double edge cracks were created by cutting two rectangular blocks of equal size from a PSM-4 sheet. The edges of the blocks were then milled to create the exact required dimensions and straight uniform edges. The block sizes ranged from a 63.5 mm by 101.6 mm blocks to 38.1 mm by 101.6 mm blocks. The blocks were then cleaned to assure there was not any excess dust or machine lubrication on them. The adhesive was then mixed as per the manufacturers specified amounts for both the hardener and the resin. The resin and hardener were heated separately and then mixed to create the PC-12 adhesive. The mixture was
centrifuged to eliminate as many bubbles within the mixture as possible. The two PSM-4 blocks, to be used for the specimen creation, were lined up to one another and spaced apart by the thickness of a razor blade, 0.229 mm. A piece of cellophane tape was used to temporarily connect the two PSM-4 blocks. The cellophane tape also acted as a mold for the adhesive to be poured into. Another razor blade was then inserted into the gap between the blocks to create the desired crack within the bondline. Prior to insertion between the block, the razor blades were sprayed with a commercially available Teflon spray. The spray was necessary for the removal of the razor blades at a later time. The two razor blades, within the bondline area, were then adjusted such that the desired crack dimensions were met. The razor blades acted as mold boundaries for the adhesive and were used for crack sizing within the bondline. The figure below is the double edge crack specimen geometry used throughout this research.

![Diagram of Double Edge Crack Specimen Geometry]

T = 12.7 mm  
t = 0.229 mm  
w = 101.6 mm  
h = 50.8 mm or 25.4 mm  
a = crack length

The mixed adhesive and the specimen with the tape were placed in an oven at a temperature of 170 degrees Fahrenheit. The adhesive is then poured at the center of the gap between the two blocks. By doing this, the adhesive was allowed to slowly work its way down and out toward the
edges of the created mold. This process eliminated the creation of any new bubbles within the bondline. Once the adhesive level had reached the PSM-4 block height, the temperature was reduced to 155 degrees Fahrenheit and cured overnight. Once the curing was complete the specimen was removed from the oven and allowed to cool for roughly 4 hours. The cellophane tape and razor blades were then removed slowly so as not to disturb the created bondline.

In the cases dealing with single edge crack specimens, two PSM-4 blocks were cut and cleaned as previously mentioned and positioned at a razor blade distance apart. The cellophane tape was then used to attach the two blocks temporarily to one another. The razor blades within the bondline area were then removed as to create a solid complete bondline. The cellophane tape mold was filled at the center until the entire mold was full of adhesive. The entire assembly was then heated in the same manner as the double edge crack specimens. Once the specimen and bondline had cooled, the crack geometry was then created. Straight aluminum bars were attached to the specimen by using rubber bands. These bars acted as guides so a razor blade could be used to cut into the specimen to create the desired crack geometry. Both the aluminum bars and the razor blade were then removed from the specimen. The figure below is the specimen geometry used for the single edge crack specimens.

![Single Edge Crack Specimen Geometry](image)

T = 12.7 mm  
t = 0.229 mm  
w = 101.6 mm  
h = 50.8 mm or 25.4 mm  
a = crack length  
d = offset distance  

Figure 2.11: Single Edge Crack Specimen Geometry
The last set of specimens that were studied had double edge cracks and contained no bondlines. These specimens were cut and cleaned as previously mentions. However, the PSM-4 blocks used for these specimens were epoxied directly into the aluminum end grips. The rubber bands and straight aluminum bars were then utilized along with a razor blade to induce the cracks into the specimen. The figure below illustrates the geometry for the no bondline specimens.

![Figure 2.12: No Bondline Specimen Geometry](image)

- $T = 12.7 \text{ mm}$
- $t = 0.229 \text{ mm}$
- $w = 101.6 \text{ mm}$
- $h = 25.4 \text{ mm}$
- $a = \text{crack length}$

**2.5.2 Loading Method**

All of the specimens involved in this study were loaded with dead weights. However, there were two slightly different loading procedures for the two different types of specimens, off-bondline crack and double edge crack specimens. The double edge specimens were loaded with a single centrally located dead load. One central located dead load is used because it is assumed the aluminum end grips are much more rigid than the photoelastic specimens and therefore the applied dead load would create a constant displacement across the end grip/PSM-4 boundary.
The same dead weight load of 75.0 N was used for all of the double edge crack specimens with an overall 101.6 mm by 101.6 mm specimen geometry. For the double edge crack specimens whose geometry was 50.8 mm by 101.6 mm, dead weight loads ranging from 71.2 N to 48.9 N were used. The difference in magnitudes of the applied dead loads was not an issue because the resulting stress intensity factors were nondimensionalized for load. Figure 2.13 illustrates the double edge crack specimen loading setup.

A slightly different loading setup was used for the single edge crack specimens. Two different dead loads were applied to the end grips at equal distances from the center line. A commercially available bubble level was then attached to the specimen by a rubber band. The two dead loads were adjusted such that the bubble level read a flat level surface. The two different dead loads were used because of the unsymmetrical geometry of the specimen. If a centrally located single dead load would have been used a slight moment would have been created on the specimen. This slight moment would have caused different boundary conditions than the constant displacement boundary condition that was sought. Figure 2.14 illustrates the single edge crack specimen loading setup.
Figure 2.13: Double Edge Crack Specimen Loading Setup
Fine adjustment to the two different weights was done by adding small amounts of lead shot. In either the double edge crack case or the single edge crack case, the total load on the specimen was calculated by summing the dead loads, the hanging assembly, and one half of the total specimen weight. The total load result was used in a nominal stress intensity factor calculation.
The result of this calculation was then used to nondimensionalize the experimental stress intensity factor at the crack tip. The stress intensity factors were nondimensionalized in order to compare to known theoretical solutions and discount any effects from slight changes in the overall applied weight. Both types of loading setups involved a single pin located in the top end grip for the hanging of the specimens. It was determined that the single pin, which is able to swivel, would be suitable for the loading setup through a clamped end loading setup. One double edge crack specimen and one single edge crack specimen were clamped at the top so that the specimens were rigidly attached to the polariscope. The stress intensity factors were determined and it was found that the clamped end condition had identical results to the single pin end condition. Therefore, the single pin end condition was considered applicable and was used for all loading setups. However, for specimens involving long single edge cracks the clamped end would be desired.

### 2.5.3 Data Acquisition

For acquisition of data, the entire loading setup is placed within a circular polariscope. Figure 2.15 is a schematic of a typical circular polariscope.  

![Figure 2.15: Circular Polariscope Arrangement](image)
The circular polariscope can be oriented such that either the isochromatics or the isoclinics can be seen. Isoclinics are fringes that show the direction of constant principal stress within the specimen while isochromatics are fringes of constant differences between principal stresses. Throughout this study the isochromatics were the main focus. A circular polariscope involves four separate light polarizers. In this study, the quarter wave planes that are shown in the schematic are not used for fringe production. The isochromatics can be seen as either white fringes or dark fringes depending on the orientation of the polarizer to the analyzer. When the isochromatics are dark, the specimen is said to be in a dark field. A light field occurs when the isochromatics are seen as white. This study involved both the dark and light field orientations of the specimen. The dark fringes within the stressed specimen are not completely black and therefore a monochromator was utilized. The monochromator is a filter used to convert all dark colored fringes into dark black fringes. The monochromator is attached to a camera and when used with black and white film creates a noticeable contrast between the light and dark fringes. Once the loaded specimen was placed within the polariscope and the desired field was created, photographs were taken of both the global and a near crack tip fringe patterns. Photographs of both the light and dark fields of the loaded specimen were taken. The photographs were then developed and scanned into a computer for the actual acquisition of data. The distances of fringes from the crack tip were measured from the photograph using a commercially available computer program known as NIH Image 1.61b8. The fringe number and distance measurements were then imported into Microsoft Excel. The data was then transformed, using the equations discussed previously, and plotted to determine the stress intensity factor at the crack tip under investigation. This procedure was performed for all specimens involved in this study.

3.0 Results and Conclusions

This section contains the results and conclusions from this research. Specifically, this section contains the induced fringe patterns, the resultant stress intensity factors, and a comparison of these results to known analytical solutions. There were two main types of specimens studied in this research, the bonded double edge crack specimens and the bonded single edge crack specimens. These two types of specimens will be presented and discussed
individually. Lastly, a discussion of the results will be presented in order to explain the effects of the bondline, boundary conditions, and specimen dimensions.

3.1 Double Edge Crack Specimens

Double edge crack specimens were chosen for this research because of their symmetric geometry properties and the large amount of useable data that can be read from each specimen. The double edge specimen allows for four different data zones for the determination of the stress intensity factor. The double edge crack geometry was also chosen because there are many different known solutions for the stress intensity factor as a function of the geometry. There were two double edge specimen geometries studied in this research, square and rectangular specimens. The square specimens were studied first and then cut down to create the rectangular specimens. A third type of double edge specimen was studied to aid in the understanding and determination of the bondline effect. Three specimens, which contained no bondlines, were created and studied. These specimens had a rectangular geometry and were used for comparison versus the rectangular bondline specimens.

3.1.1 Specimen Configuration and Results

The square (101.6 mm by 101.6 mm) double edge specimens were all loaded with the same dead load of 75.0 N. A photograph of the global fringe pattern and the near crack tip fringe patterns were both taken of each specimen. The global fringe pattern was used to visually determine the effects of the boundary conditions and the specimen geometry. The near crack tip fringe patterns were used for the stress intensity factor determination. The figures that follow are photographs of the global and crack region fringe patterns. There were six different crack size specimens studied. The figures that follow are in order of increasing crack length with cracks ranging in size from 7.94 mm to 27.9 mm. The photographs are all shown with a white background as to allow for ease in viewing of the crack within the specimen. The figures show a close up of the left crack, the global view (entire specimen), and a close up of the right crack.
Figure 3.1: Square Double Edge Crack Specimen, $a = 7.94$ mm

Figure 3.2: Square Double Edge Crack Specimen, $a = 12.7$ mm
Figure 3.3: Square Double Edge Crack Specimen, $a = 17.46$ mm

Figure 3.4: Square Double Edge Crack Specimen, $a = 20.6$ mm
Figure 3.5: Square Double Edge Crack Specimen, $a = 25.4$ mm

Figure 3.6: Square Double Edge Crack Specimen, $a = 27.9$ mm
Figure 3.11: Rectangular Double Edge Crack Specimen, $a = 25.4$ mm

Figure 3.12: Rectangular Double Edge Crack Specimen, $a = 27.9$ mm
After the square double edge crack specimens were analyzed, they were then cut down to form rectangular double edge crack specimens (50.8 mm by 101.6 mm). The specimens were dead loaded with various different dead loads. This was done as to produce useful fringe patterns. Specimens with longer crack sizes were dead loaded with less weight as not to produce dense fringe patterns. The figures that follow are photographs of the global and crack region fringe patterns for the rectangular double edge crack specimens. Again, the global fringe pattern was used for a visual investigation while the crack region photographs were used for the stress intensity factor determination.
Figure 3.7: Rectangular Double Edge Crack Specimen, $a = 7.94$ mm

Figure 3.8: Rectangular Double Edge Crack Specimen, $a = 12.7$ mm
Figure 3.9: Rectangular Double Edge Crack Specimen, $a = 17.46$ mm

Figure 3.10: Rectangular Double Edge Crack Specimen, $a = 20.6$ mm
Figure 3.11: Rectangular Double Edge Crack Specimen, $a = 25.4$ mm

Figure 3.12: Rectangular Double Edge Crack Specimen, $a = 27.9$ mm
The determination of the stress intensity factors for the rectangular double edge specimens was done in the same manner as the square specimens. The table that follows contains the results from the rectangular specimens. Again, the stress intensity factor is shown for both the left and right cracks. The percent difference between the two cracks is also shown and again none of the percent differences is over the desired 5% scatter. The normalized stress intensity factors and the corrected normalized stress intensity factors are also shown. As stated before, different dead loads were used with different specimens so the dead load per specimen is also shown in the table. The normalization of the stress intensity factors takes in account the differences in the dead loads.

Table 3.2 – Rectangular Double Edge Specimen Results

<table>
<thead>
<tr>
<th>a (mm)</th>
<th>a/b</th>
<th>P (N)</th>
<th>left K (MPa√m)</th>
<th>right K (MPa√m)</th>
<th>average K (MPa√m)</th>
<th>% difference in K</th>
<th>corrected K/Ko</th>
<th>corrected K/Ko</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.94</td>
<td>0.156</td>
<td>75.0</td>
<td>6.89</td>
<td>7.49</td>
<td>7.19</td>
<td>4.17</td>
<td>0.93</td>
<td>0.86</td>
</tr>
<tr>
<td>12.70</td>
<td>0.250</td>
<td>75.0</td>
<td>10.57</td>
<td>10.16</td>
<td>10.37</td>
<td>1.98</td>
<td>0.94</td>
<td>0.87</td>
</tr>
<tr>
<td>17.46</td>
<td>0.344</td>
<td>52.7</td>
<td>8.65</td>
<td>8.58</td>
<td>8.62</td>
<td>0.41</td>
<td>0.98</td>
<td>0.90</td>
</tr>
<tr>
<td>20.6</td>
<td>0.406</td>
<td>50.7</td>
<td>10.24</td>
<td>9.25</td>
<td>9.75</td>
<td>5.08</td>
<td>1.00</td>
<td>0.93</td>
</tr>
<tr>
<td>25.4</td>
<td>0.500</td>
<td>50.7</td>
<td>12.47</td>
<td>12.78</td>
<td>12.63</td>
<td>1.23</td>
<td>1.18</td>
<td>1.09</td>
</tr>
<tr>
<td>27.9</td>
<td>0.549</td>
<td>51.0</td>
<td>12.91</td>
<td>13.45</td>
<td>13.18</td>
<td>2.05</td>
<td>1.22</td>
<td>1.12</td>
</tr>
</tbody>
</table>

\[ h = 25.4 \text{ mm} \]
\[ b = 50.8 \text{ mm} \]

The last type of double edge crack specimens that were studied in this research were no bondline rectangular double edge crack specimens. Three specimens with different size crack lengths were studied as to determine their stress intensity factors. The results from these specimens were then compared to the rectangular double edge crack specimens to better understand the effect of the bondline. These specimens were constructed by using a razor blade to cut the desired crack size into the photoelastic material. The figures that follow are the photographs of the no bondline fringe patterns. Again, the figures show the left crack region, the global view, and the right crack region.
Figure 3.13: No Bondline Double Edge Crack Specimen, $a = 10.72$ mm

Figure 3.14: No Bondline Double Edge Crack Specimen, $a = 19.86$ mm

Figure 3.15: No Bondline Double Edge Crack Specimen, $a = 28.6$ mm
A close investigation of the induced cracks in the no bondline specimens reveals non-uniform crack boundaries. The crack edges are not straight and seem to have some oscillation associated with them. The non-uniform crack edge has no effect on the resultant stress intensity factor as long as the crack tip is normal to the specimen edge. The non-uniform crack edges arose as a result difficulty in cutting the cracks into the photoelastic material with the razor blade. Several tests were performed to verify that the non-uniform crack edges have no effect on the resultant stress intensity factor. The non-uniform crack edges only effects the desired length of the induced crack. The length of the crack is measured from the free edge to the crack tip.

The results from the no bondline rectangular double edge crack specimens are shown in the Table 3.3. As in the two previous cases the stress intensity factors for both the left and right cracks and the percent differences are shown in the table. The normalized stress intensity factor and the corrected normalized stress intensity factor are also shown in the table.

<table>
<thead>
<tr>
<th>a (mm)</th>
<th>a/b</th>
<th>P (N)</th>
<th>left K (MPa√m)</th>
<th>right K (MPa√m)</th>
<th>average K (MPa√m)</th>
<th>% difference in K</th>
<th>K/Ko</th>
<th>corrected K/Ko</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.72</td>
<td>0.211</td>
<td>75.0</td>
<td>9.68</td>
<td>9.32</td>
<td>9.50</td>
<td>1.895</td>
<td>1.000</td>
<td>0.92</td>
</tr>
<tr>
<td>19.86</td>
<td>0.391</td>
<td>75.0</td>
<td>14.36</td>
<td>15.26</td>
<td>14.81</td>
<td>3.04</td>
<td>1.147</td>
<td>1.056</td>
</tr>
<tr>
<td>28.60</td>
<td>0.563</td>
<td>75.0</td>
<td>21.1</td>
<td>21.5</td>
<td>21.3</td>
<td>1.058</td>
<td>1.373</td>
<td>1.266</td>
</tr>
</tbody>
</table>

$h = 25.4 \text{ mm}$
$b = 50.8 \text{ mm}$

All the results that have been shown in this section will be compared to known analytical solution involving stress intensity factors. The no bondline specimens will also be compared to the rectangular double edge crack specimen to investigate the effect of the bondline. The comparison will be done using the corrected normalized stress intensity factors.
3.1.2 Experimental Comparisons

There are many solutions available for the comparison of double edge crack specimen results dealing with stress intensity factors. The determination of which solution to use for comparison purposes centers around the induced boundary conditions involved. In all the experiments involved in this study a dead load was put on the specimen in order to create a uniform displacement boundary condition. The uniform displacement boundary condition was thought to occur on the specimen as a result of the rigidity of the aluminum end grips when compared to the rigidity of the photoelastic material. As a result of this fact, the stress intensity factors from this experiment were compared to an analytical solution produced by Bowie (1964). Bowie’s solution involves a uniform displacement boundary condition with a material whose Poisson’s ratio is 0.33. This research was performed on a photoelastic material whose Poisson’s ratio was 0.5. Bowie also presented, in the same paper, data for normalized stress intensity factors as a function of crack length with a Poisson’s ratio of 0.25. The solutions for the two Bowie cases, with Poisson’s ratios of 0.25 and 0.33, became co-linear after an a/b = 0.35 and differed only slightly with short crack lengths. For this reason the mismatch in Poisson’s ratio for the experimental results and Bowie’s solution was considered negligible. The correction factor, which is used on all of the experimental results, takes into account any thickness effects from the specimens. Bowie’s solution involves a square double edge crack specimen in a plane stress state with similar geometry to that of the experimental specimen geometry and is shown below in the following table:

<table>
<thead>
<tr>
<th>a/b</th>
<th>K/Ko</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1563</td>
<td>1.060</td>
</tr>
<tr>
<td>0.250</td>
<td>1.042</td>
</tr>
<tr>
<td>0.344</td>
<td>1.044</td>
</tr>
<tr>
<td>0.406</td>
<td>1.062</td>
</tr>
<tr>
<td>0.500</td>
<td>1.100</td>
</tr>
<tr>
<td>0.550</td>
<td>1.120</td>
</tr>
</tbody>
</table>

Table 3.4 – Bowie’s solution (no bondline, plane stress theory)
These results are then compared to the square double edge crack specimen results from this experiment. Below is a plot of both the experimental results and Bowie’s solution.

Figure 3.16: Comparison of Square Double Edge Crack Specimen Results

The experimental results follow the same general pattern as Bowie’s solution does; however, the experimental results have more dramatic changes. The possible reasons for the differences in the two will be discussed in the next section. The rectangular double edge crack specimen’s results were then compared to both Bowie’s solution and to the square double edge crack specimen’s results. The purpose of this comparison was to determine the effect of changing the geometry. The comparison was done with the Bowie square specimen results as in the previous case. Below is a plot of the rectangular double edge crack specimen’s results, the square double edge crack specimen results, and Bowie’s solution.
The last set of double edge specimen results for comparison was the no bondline specimens. The no bondline specimens had the same geometry as the rectangular double edge crack specimens so these two sets of results were compared to one another. In this comparison case both the pattern and the actual stress intensity factor results could be compared. Below is a plot of the no bondline specimen’s results and the rectangular double edge crack results.
The possible reasons for the differences in the two result cases will be discussed in the subsequent section.

### 3.1.3 Discussion

In all three of the double edge crack specimen cases there are only a few factors that could have created differences from the known solution. These factors include the presence of the bondline, the induced boundary condition, the specimen geometry, and the photoelastic material properties. A discussion about the results from the three double edge crack cases will be presented in this section. The possible reasons for any differences will be discussed for each case.

The square double edge crack specimen results were presented in comparison to the known Bowie square specimen solution. The plot (Fig. 3.16), which was shown, compared the
two curves on a non-dimensional plot. The square double edge crack specimen’s results were compared to the Bowie solution because they both involve the same geometry and boundary condition of constant displacement. As mentioned previously, a correction factor was implemented to correct for any effects due to the thickness. The comparison plot for the square double edge crack specimen displayed two curves with a similar pattern of increasing stress intensity factor with increasing crack length. The experimental curve had more extreme changes in slope than did the known solution. This was a result of the bondline effect and the applied boundary conditions. A close investigation of the global fringe patterns for the double edge crack specimens reveals some end grip or boundary slippage. If no slippage were present the near boundary fringes would have been smooth and symmetric. If the boundaries were attached to the specimen perfectly, smooth fringes would have originated from the corners and grown toward the middle of the specimen. However, the global fringe patterns also seem to suggest that the slippage effect seems to fix itself as the fringes approach the center of the specimen. The half size specimens will help prove that this is true. The double edge crack specimens contained no shear mode and all of the results fell within the desired 5% scatter. The global and local fringe patterns only exhibited the fringe folding effect. The general pattern for the double edge square specimens was that with increasing crack length the stress intensity factor also increased. The global fringe patterns seemed to suggest that as the crack length was increased the slippage effect seemed to decrease. This was most likely a result of the stress within the specimen dominating over the slight slippage within the boundary.

After the square specimens had been analyzed, the specimens were cut to rectangles and analyzed once again. These non-dimensional results were then compared to both the square experimental results and to the Bowie solution, which involved square specimen geometry (Fig. 3.17). By reducing the height of the specimen the stress intensity factors were also reduced. These results can not be compared directly to Bowie’s solution however; the overall trend can be compared. The same general pattern for the stress intensity factors as a function of crack length was present, with an increase in crack length the stress intensity factor increases. As was the case with the square specimens, some boundary slippage occurred. The global fringes patterns for the rectangular specimens displayed the slippage effect because the near boundary fringes were not symmetric or smooth. The global fringe patterns also showed that the slippage effect was more predominant in the specimen with shorter crack lengths. As in the rectangular
specimen case, the slippage effect does seem to decrease with increasing crack length. This is most likely a result of the dominance of the stresses within the specimen. The rectangular specimen results can be compared to the square specimen results because the entire specimen is the same except for the overall height of the specimen. The rectangular specimen results were much lower than the square specimen results. This is a result of the geometry of the specimen and the slippage within the boundary. The shorter specimen seemed more susceptible to the slippage effect as compared to the square longer specimens. This occurred because the shorter specimens had less material within the specimen to work out the slippage effect. The global and near crack tip fringes patterns revealed no shear mode but did show the fringe folding effect. Again, the overall pattern is with increasing crack length the stress intensity factors also increase. All of the rectangular double edge crack specimen results fell within the desired 5% scatter.

The last set of double edge crack specimens involved no bondlines. These specimens were constructed with the same geometry as the rectangular specimens with a bondline. This was done such that a direct comparison between the results could be done. The no bondline specimens and the rectangular double edge crack specimens were constructed from the same material, had the same geometry, and the same applied boundary condition. The only difference between the two cases was the presence of the bondline and therefore any differences in the results should be a result of the bondline effect. The comparison plot (Fig. 3.18) of these two cases reveals similar overall curve patterns, with increasing crack length the stress intensity factor increases. However, the no bondline specimen’s stress intensity factors were higher than the results from the specimens with a bondline. This suggests that the bondline tends to absorb some of the applied stresses within the specimen due to the bondlines higher elastic modulus. The presence of a bondline decreases the stress intensity factor at a given crack length. As in the other two cases previous discussed, the no bondline specimen’s fringe patterns involved some slippage characteristics. The slippage was apparent because of the unsymmetrical and non-smooth fringes near the end grip boundaries. The global fringe patterns also displayed the decrease in slippage effect as the crack length was increased. The near crack tip fringe patterns illustrated some wandering of the crack edges. As mentioned previously, this is a result of the creation of the cracks and does have any effect on the stress intensity factor as long as the crack tip is normal to the free edge. As in the other two cases previously discussed, there was the fringe folding effect with no shear modes present.
In all three of the cases discussed thus far, the overall trend is that the stress intensity factor increases with increasing crack length. However, the rate of increase of the stress intensity factors is more than should be expected. The more rapid increase is a result of the boundary condition and the bondline effect. The end grips were designed to impart a uniform displacement boundary condition on the specimens. The presence of the slippage within the boundary created more of a uniform load boundary condition. The uniform load boundary is created as a result of some specimen boundary edges under going more stress than others. The result of the uniform load boundary condition was a reduction in the stress intensity factors for shorter crack sizes. The uniform load boundary condition seemed to help the shorter crack specimens and intensify the longer crack specimen stress intensity factors.

3.2 Off-Bondline Single Edge Crack Specimens

After the double edge specimens had been analyzed and the bondline effect had been seen, further study was needed to better understand the effect of the bondline on specimens involving different sized cracks at different location. The motivation behind the single edge crack specimens was to try to quantify the impact of cracks near and below the bondline. Results from the double edge crack specimens showed a lowering of the stress intensity factor for small sized cracks. The single edge crack specimens were then constructed and analyzed to gain a better understanding of this bondline effect. Several different specimen geometries were chosen for the single edge crack specimens. As implied from the name of the specimens, these specimens contained one crack, which was near and below the bondline.

3.2.1 Specimen Configuration

As was the case with the double edge crack specimens, there were two global specimen geometries studied, a 101.6 mm X 101.6 mm specimen and a 50.8 mm X 101.6 mm specimen. Both specimen geometries contained bondlines and were loaded with the dead weight loads. There were three cases of single edge crack specimens involved in this research. Two cases involved square specimens with cracks of varying length at two different distances from the
bondline, while the third case was the rectangular specimen. The results from these three single edge crack specimen cases were compared to two known solutions, one with a uniform displacement boundary condition and one with a uniform stress boundary condition. The overall trends in the stress intensity factors as a function of crack length and specimen geometry were also examined. The results from all three cases were plotted and compared versus one another and versus the two known solutions to determine the trends and effects of changing the specimen geometry and the bondline effect.

The first set of single edge crack specimens that were constructed and analyzed were square specimens with a crack to bondline distance of 2.58 mm. The crack was extended and analyzed three times with the crack being located just below the bondline. The photographs which follow are of the global and near crack tip fringe patterns.
Figure 3.19: Square Single Edge Specimen, $d = 2.58$ mm, $a = 2.78$ mm

Figure 3.20: Square Single Edge Specimen, $d = 2.58$ mm, $a = 8.33$ mm

Figure 3.21: Square Single Edge Specimen, $d = 2.58$ mm, $a = 12.70$ mm
The second case of single edge crack specimens studied involved square specimens whose crack to bondline distance was 1.19 mm. As in the previous case, the crack length was extended and analyzed three times. The specimens were constructed, loaded, and analyzed in the same manner as in the previous case with the one change of the bondline to crack distance. The photographs that follow are the global and near crack tip fringes patterns of these specimens.

Figure 3.22: Square Single Edge Specimen, d = 1.19 mm, a = 2.98 mm

Figure 3.23: Square Single Edge Specimen, d = 1.19 mm, a = 7.95 mm
The last case of single edge crack specimens which were constructed and analyzed were rectangular specimens with a bondline to crack distance of 2.18 mm. The results from this case will be used to compare to the results from the square specimens with a bondline to crack distance of 2.58 mm. As in the previous two cases, three different crack lengths were cut and analyzed. The photographs that follow are of the global and near crack tip fringe patterns for this last case of single edge crack specimens.
Figure 3.26: Square Single Edge Specimen, $d = 2.18$ mm, $a = 7.54$ mm

Figure 3.27: Square Single Edge Specimen, $d = 2.18$ mm, $a = 12.70$ mm
3.2.2 Results and Comparisons

As mentioned previously, the results from the three single edge crack cases were compared to two known solutions. The three cases were also compared versus one another to determine any trends due to the changes in geometry. The table which follows contains the results from all three of the single edge crack cases.

Table 3.5 – Single Edge Specimen Results

<table>
<thead>
<tr>
<th>name</th>
<th>a (mm)</th>
<th>d (mm)</th>
<th>h (mm)</th>
<th>K top (MPa√m)</th>
<th>K bottom (MPa√m)</th>
<th>K (MPa√m)</th>
<th>Ko (MPa√m)</th>
<th>K/Ko</th>
<th>corrected K/Ko</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP6</td>
<td>2.78</td>
<td>2.58</td>
<td>101.6</td>
<td>8.19</td>
<td>8.78</td>
<td>8.48</td>
<td>8.01</td>
<td>1.059</td>
<td>0.976</td>
</tr>
<tr>
<td>SP8</td>
<td>8.33</td>
<td>2.58</td>
<td>101.6</td>
<td>17.50</td>
<td>16.77</td>
<td>17.13</td>
<td>13.87</td>
<td>1.235</td>
<td>1.139</td>
</tr>
<tr>
<td>SP11</td>
<td>12.70</td>
<td>2.58</td>
<td>101.6</td>
<td>24.66</td>
<td>26.01</td>
<td>25.34</td>
<td>17.11</td>
<td>1.481</td>
<td>1.365</td>
</tr>
<tr>
<td>SP7</td>
<td>2.98</td>
<td>1.191</td>
<td>101.6</td>
<td>8.06</td>
<td>7.86</td>
<td>7.96</td>
<td>8.29</td>
<td>0.960</td>
<td>0.885</td>
</tr>
<tr>
<td>SP9</td>
<td>7.95</td>
<td>1.191</td>
<td>101.6</td>
<td>14.63</td>
<td>15.22</td>
<td>14.92</td>
<td>13.53</td>
<td>1.103</td>
<td>1.017</td>
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<tr>
<td>SP10</td>
<td>13.10</td>
<td>1.191</td>
<td>101.6</td>
<td>22.60</td>
<td>21.65</td>
<td>22.12</td>
<td>17.39</td>
<td>1.272</td>
<td>1.173</td>
</tr>
<tr>
<td>SP12</td>
<td>2.58</td>
<td>2.78</td>
<td>50.8</td>
<td>6.35</td>
<td>7.13</td>
<td>6.74</td>
<td>7.71</td>
<td>0.874</td>
<td>0.806</td>
</tr>
<tr>
<td>SP13</td>
<td>7.54</td>
<td>2.78</td>
<td>50.8</td>
<td>13.26</td>
<td>11.75</td>
<td>12.51</td>
<td>13.17</td>
<td>0.950</td>
<td>0.876</td>
</tr>
<tr>
<td>SP14</td>
<td>12.70</td>
<td>2.78</td>
<td>50.8</td>
<td>20.13</td>
<td>17.97</td>
<td>19.05</td>
<td>17.08</td>
<td>1.116</td>
<td>1.028</td>
</tr>
<tr>
<td>SP15</td>
<td>2.78</td>
<td>2.18</td>
<td>101.6</td>
<td>9.32</td>
<td>7.68</td>
<td>8.47</td>
<td>7.80</td>
<td>1.086</td>
<td>1.002</td>
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<tr>
<td>SP16</td>
<td>8.74</td>
<td>2.18</td>
<td>101.6</td>
<td>19.86</td>
<td>12.69</td>
<td>16.28</td>
<td>13.83</td>
<td>1.177</td>
<td>1.085</td>
</tr>
<tr>
<td>SP18</td>
<td>13.89</td>
<td>2.18</td>
<td>101.6</td>
<td>27.91</td>
<td>21.16</td>
<td>24.55</td>
<td>17.44</td>
<td>1.407</td>
<td>1.298</td>
</tr>
</tbody>
</table>

The table contains an extra set of specimen data, located in the last three rows. This set of data was used as a verification to confirm the results from previous tests. The verification specimen was constructed with an identical geometry to that of the square single edge specimen with a crack to bondline distance of 2.58 mm. The verification specimens were studied to demonstrate the repeatability of the results from this research. On average the percent difference from the 2.58 mm specimen’s results to the 2.18 mm specimen’s results was less than 4.8%. This slight percent difference fell within the desired scatter for experimental results of 5%. The
normalization of the stress intensity factors was done in a similar manner as in the previous cases. The two known solutions that were used for comparison are solutions obtained by Bowie and Neal (1965). Their solution involves the resultant stress intensity factors obtained through conformal mapping. The geometry used in their analysis contained no bondline and a crack that was located at the mid-span of one of the free edges. The table below is from Bowie and Neal and is the data that was used for the single edge crack specimen comparisons.

Table 3.6 – Bowie and Neal Comparison Data

<table>
<thead>
<tr>
<th>a/w</th>
<th>K/Ko uniform stress</th>
<th>K/Ko uniform displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>1.14</td>
<td>1.12</td>
</tr>
<tr>
<td>0.05</td>
<td>1.17</td>
<td>1.12</td>
</tr>
<tr>
<td>0.075</td>
<td>1.20</td>
<td>1.10</td>
</tr>
<tr>
<td>0.1</td>
<td>1.24</td>
<td>1.08</td>
</tr>
<tr>
<td>0.125</td>
<td>1.30</td>
<td>1.08</td>
</tr>
</tbody>
</table>

The known solutions and the experimental results were then plotted on the same non-dimensional graph in order to determine any trends. A discussion of the results is presented in the subsequent section. Below is a plot of the experimental results and the two known solutions used for comparison.
3.2.3 Discussion

The fringe patterns from the single edge cracked specimens revealed the boundary slippage that was apparent with the double edge crack specimens. Again, this slippage was a result of imperfections, mainly air bubbles, in the epoxy used to attach the photoelastic material to the aluminum end grips. The slippage was apparent in the global fringe patterns because of the unsymmetrical and non-smooth near boundary fringes. For all of the single edge specimens the near crack tip fringe patterns showed no shear modes, however, a fringe folding effect was seen. The near crack tip fringe patterns also showed a fringe compression and straightening due to the bondline. Visually it appears that the bondline tends to hinder the fringe folding effect. The stress intensity factor results show the normal 5% scatter in data so the compression and straightening due to the bondline causes no stress intensity alterations.

As mentioned previously, some of the single edge crack specimens involved some near bondline residual stresses. These residual stresses were apparent in the global fringe pattern and
were present within the specimen before the dead weight was loaded on the specimen. These residual stresses were a result of improper bondline creation. However, the residual stresses present did not influence the resultant stress intensity factor. Several specimens were studied to determine the influence of these residual stresses. Results from specimens containing the residual stresses and results from specimens that contained no residual stresses were compared and found to lie within the desired 5% data scatter. The residual stresses did cause a change in the procedure for stress intensity factor determination. The extended slope method was used for stress intensity factor determination, for the residual stress specimen cases, for the data zone located between the bondline and the crack. The normal least squares fit method was used to determine the stress intensity factor for the data zone located below the crack. The two stress intensities were compared and found to be on average 5% different in magnitude. Therefore the specimens involving the residual stresses were considered to be useful and able to produce meaningful data.

The comparison of the results to the two known solutions aided in the understanding of the bondline and boundary condition effects. In all three of the single edge crack cases, as the crack length increased so did the normalized stress intensity factor. This is no surprise because the same trend was seen with the double edge crack specimens. However, when compared to the uniform displacement solution, the trends were in opposite directions. The results tended to agree more in pattern and direction with the uniform stress solution. There are two reasons why the results from the single edge crack specimen agreed more with the uniform stress solution. The effect of the bondline and the boundary condition influenced the results. The bondline either weakened or strengthened the stress intensity factor depending on the length of the crack. Shorter crack lengths seemed to be helped by the presence of the bondline and thus lowering the stress intensity factor, while the presence of the bondline increased the stress intensity factor for longer length cracks. The length between the bondline and the crack also caused changes in the normalized stress intensity factors. As the distance between the crack tip and the bondline was increased the normalized stress intensity increased. It seems the bondline tends to shield the crack from the induced stress field. Lastly, the global geometry of the specimen influenced the magnitude of the normalized stress intensity factors. The square single edge crack specimen had higher values of the normalized stress intensity factors as compared to the shorter rectangular specimens. The normalized stress intensity factor decreased with a
reduction in the specimen height. This result is no surprise because Torvik\textsuperscript{24} saw a similar phenomenon in single edge crack specimen that contained no bondlines. Therefore, a reduction in normalized stress intensity factors with a reduction in specimen height seems logical. The single edge crack specimens were studied to better understand the effect of the bondline and the induced boundary conditions.
4.0 Summary

Even though there were two different specimen geometries involved in this research there are a few common themes to the results. As expected, as the crack length is increased so does the normalized stress intensity factor. Another trend in the results was with a reduction in overall specimen height the normalized stress intensity factor decreases. In both geometries the bondline influenced the magnitudes of the normalized stress intensity factors. The bondline tended to shield shorter cracks from the applied stresses, while the normalized stress intensity factor for longer crack specimens was increased. The crack to bondline distance also tended to influence the magnitude of the stress intensity factor. As the crack to bondline distance increased, the stress intensity factor increased. None of the specimens involved in this research appeared to have any shear modes associated with them. However, the fringe folding effect that is a result of the applied loads was present in all cases. Lastly, imperfections in the glued aluminum end grips were apparent from the global fringe patterns. These imperfections changed the desired induced boundary condition from constant displacement to more of a constant stress boundary condition. The results from the single edge crack specimens agreed with this conclusion because the trend in experimental results matched much closer to the uniform stress boundary condition case. The results and conclusions from this research will be used to qualitatively understand the impact of cracks on bonded specimens with an induced uniform displacement boundary condition. The results and conclusions will also be used to gain a better understanding of the influence of bondlines on soft rubber-like materials such as solid rocket propellant.
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Appendix A: Determination of Material Properties

Several sheets of PSM-4 were used to create the specimens involved in this research. For each sheet, both the material fringe value and the elastic modulus were determined. These properties tend to change slightly from sheet to sheet due to influences from the environment and PSM-4 formulation. The determination of these two material properties was done by analyzing a four point bend specimen. There are several different specimen geometries that are currently used to determine the material fringe value. The four point bend specimen was chosen because both material properties can be determined simultaneously and the four point bend specimen allows for a large data zone. In the four point bend specimen a large constant moment area is created. The constant moment area is the zone for which data is collected for the material fringe value calculation. All the four point bend specimens involved in this research had the same geometry.

![Four Point Bend Specimen Geometry](image)

- $a = 0.5 \text{ in}$
- $b = 1.5 \text{ in}$
- $L = 8 \text{ in}$
- $t = 0.5 \text{ in}$
- $h = 1 \text{ in}$

$P = \text{applied dead load}$

$W = \text{weight of bend specimen}$

Figure A.1: Four Point Bend Specimen Geometry

The four point bend specimen was placed within the circular polariscope and dead weight loaded. The dead load was connected to the bend specimen through the use of pins and beams.
attached at the two interior holes. The two exterior holes on the bend specimen were used to attach rigid supports. The constant moment region was created between the two interior holes. The determination of the material fringe value was accomplished by relating the stress state within the specimen to the stress optic law. The stress optic law relates to the state of stress as follows:

\[
\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_{yy}}{2} = \frac{nf}{2t}
\]  

(A.1)

The state of stress must now be related to the applied loads and the geometry.

\[
\sigma_{yy} = \frac{Mx}{I} = \frac{12Mx}{th^3}
\]  

(A.2)

with \( M = Pb + \frac{W}{L}(0.125L^2 - 0.5aL) \) measured from the mid-span of the beam. All the calculations are considered at the mid-span of the beam because this where the data will be read from. Simplification and rearrangement of the terms yields the final form of the equation for the material fringe value.

\[
f = \left[ \frac{12x}{h^3n} \right] \left[ Pb + \frac{W}{2} \left( \frac{L}{4} - a \right) \right]
\]  

(A.3)

All of the quantities are known in the previous equation except for \( x/n \). This quantity is read from the produced fringe pattern, with \( x \) being the distance to a particular fringe and \( n \) being the fringe number. The bend specimen allows for multiple points of data to be collected and averaged which creates a very accurate material fringe value calculation. Below is a picture of a typical bend specimen’s fringe pattern.
The thick black fringe at the center of the fringe pattern is the zero fringe or the neutral axis. Each subsequent fringe and distance is recorded and plotted. The slope of the line from a chart of x vs. n is determined and used in the material fringe value calculation. As stated before the elastic modulus calculation can be performed simultaneously with the material fringe value calculation. The elastic modulus calculation is performed using the induced deflection of the specimen from the dead load. The mid-span deflection is measured from a photograph. The formula that relates the mid-span deflection to the elastic modulus of the material is determined from linear elastic beam theory. The initial equation utilized from beam theory is as follows:

\[
\frac{d^2x}{dz^2} = \frac{M(z)}{EI}
\]  

(A.4)
The equation of the moment as a function of distance is substituted into the equation, integrated, and the elastic modulus is solved for. The final form of the elastic modulus equation is as follows:

\[
E = \frac{6 \left[ P \left( abL - a^2b - \frac{bL^3}{4} \right) + W \left( \frac{a^3}{3} + \frac{a^4}{12L} - \frac{aL^2}{24} - \frac{5L^4}{192} - \frac{a^3L}{2} \right) \right]}{th^\delta} \tag{A.5}
\]

All quantities are known except for the deflection, which is measured and substituted into the equation. Below is a picture that illustrates the deflection of the bend specimen as a result of the applied dead load.

![Figure A.3: Global Bend Specimen View](image)

The measured deflection is the distance the loaded specimen’s edge has moved from the unloaded state. The material fringe value is used in all the stress intensity factor calculations in this study. The elastic modulus is not used in any other calculation however, the elastic modulus is important for comparison purposes.
Vita

Born on January 15, 1976 in Newport News, Virginia, Kevin Gloss was raised and educated in the same region. The author is the son of Blair Gloss and Nancy Gloss, sibling to Martin and Mandy Gloss. The author began his collegiate studies during the fall of 1994 at Virginia Polytechnic Institute and State University and received his Bachelor’s of Science in Engineering Science and Mechanics. The author then began graduate studies in the Department of Engineering Mechanics at Virginia Polytechnic Institute and State University. The author intends to pursue a career in industrial research and development.