Application of visco-hyperelastic devices in structural response control

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Application of visco-hyperelastic devices in structural response control

(Anantha Narayan Chittur Krishna Murthy)

Abstract

Structural engineering has progressed from design for life safety limit states to performance based engineering, in which energy dissipation systems in structural frameworks assume prime importance. A visco-hyperelastic device is a completely new type of passive energy dissipation system that not only combines the energy dissipation properties of velocity and displacement dependent devices but also provides additional stability to the structure precluding overall collapse.

The device consists of a viscoelastic material placed between two steel rings. The energy dissipation in the device is due to a combination of viscoelastic dissipation from rubber and plastic dissipation due to inelastic behavior of the steel elements. The device performs well under various levels of excitation, providing an excellent means of energy dissipation. The device properties are fully controlled through modifiable parameters.

An initial study was conducted on motorcycle tires to evaluate the hyperelastic behavior and energy dissipation potential of circular rubber elements, which was preceded by preliminary finite element modeling. The rubber tires provided considerable energy dissipation while displaying a nonlinear stiffening behavior. The proposed device was then developed to provide additional stiffness that was found lacking in rubber tires.

Detailed finite element analyses were conducted on the proposed device using the finite element software package ABAQUS, including parametric studies to determine the effect of the various parameters of device performance. This was followed by a nonlinear dynamic response history analysis of a single-story steel frame with and without the device to study
the effects of the device in controlling structural response to ground excitations. Static analyses were also done to verify the stabilizing effects of the proposed device. Results from these analyses revealed considerable energy dissipation from the device due to both viscoelastic as well as plastic energy dissipation.

Detailed experimental analyses on the proposed device, finite element analyses of the device on multistory structures have been put forth as the areas of future research. It may also be worthwhile to conduct further research, as suggested, in order to evaluate the use of scrap tires which is potentially a very valuable structural engineering material.
Dedicated to all the lives lost in numerous earthquakes all over the world
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Good things in life come to an end soon, but leave behind memories that are cherished for years. Graduate studies and research at Virginia Tech were probably one the most memorable in my life and as I write this, I feel a sense of relief at the culmination, but a sudden feeling of void.

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I would also like to thank all my friends in Blacksburg who have been with me during both the high tides and the low tides. I might run out of words in thanking Hardik, Hari, Rakesh, Anand, Nitin and many others with whom I have spent countless hours discussing a whole plethora of subjects. I would also like to thank John Ryan for all the help that he provided me at the Structures Lab in the summer, lab technicians Brett Farmer and Dennis Huffman for their help and Yasser Ibrahim for being my mentor during my baptism with ABAQUS.

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Blacksburg, April 2005

Anantha Narayan

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1 Something original that I came up with during the long hours spent ‘supposedly’ doing research.
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List of symbols

The following is a list of the most important symbols that appear in the subsequent chapters. Symbols not included in this list are explained where they first occur. The number refers to the page where the symbol first appears.

\[ A \quad \text{Final area, 81} \]
\[ A_o \quad \text{Original area, 81} \]
\[ t_i \quad \text{Breadth of the viscoelastic material, 67} \]
\[ c, C \quad \text{Damping constant, 19} \]
\[ d \quad \text{Depth of the viscoelastic material, 67} \]
\[ E \quad \text{Modulus of elasticity, 72} \]
\[ E_d \quad \text{Energy dissipated due to supplemental devices, 8} \]
\[ E_o \quad \text{Energy dissipated per cycle, 12} \]
\[ E_h \quad \text{Irrecoverable energy due to dissipation from the inherent damping, 8} \]
\[ E_i \quad \text{Energy input into the system from the earthquake, 8} \]
\[ E_k \quad \text{Kinetic energy, 8} \]
\[ E_s \quad \text{Recoverable strain energy, 8} \]
\[ F \quad \text{Total force, 18} \]
\[ F_n \quad \text{Normal force, 10} \]
\[ F_o \quad \text{Total damper force, 38} \]
\[ F_f \quad \text{Total frictional force, 10} \]
\[ G \quad \text{Shear modulus, 72} \]
\[ G'' \quad \text{Shear loss modulus, 12} \]
\[ k \quad \text{Stiffness, 18} \]
\[ K \quad \text{Bulk modulus, 72} \]
\[ K_f \quad \text{Loss stiffness, 39} \]
$K_s$  Storage stiffness, 39

$l$  Final length, 79

$l_o$  Original length, 79

$t$  Time, 82

$t_i$  Thickness of inner steel ring, 67

$t_o$  Thickness of outer steel ring, 67

$V$  Volume of viscoelastic material, 12

$W$  Strain energy density, 18

$\mu$  Coefficient of friction, 10

$\gamma_o$  Shear strain in the viscoelastic material, 12

$\Delta$  Displacement, 18

$\delta$  Phase angle, 38

$\omega$  Frequency of loading, 38

$\nu$  Poisson’s ratio, 72

$\sigma$  Stress, 74

$\varepsilon$  Strain, 74
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Chapter 1

Introduction

1.1 Preliminary remarks

Intense ground motion during earthquakes causes structures to deform beyond the limits of linearly elastic behavior and the ensuing dynamic instability can cause excessive structural damage, and even overall collapse. Early structural design laid emphasis on life safety, with little or no concentration on damage control. In recent years, structural damage control has taken a central role in seismic design of civil structures. It is almost impossible and economically unfeasible to design a structure to withstand the full effect of earthquake-induced forces. Traditional design relies on the energy dissipation from the yielding of the structural members. However, this leads to severe localized damage in a few regions and causes serviceability issues.

There are methods to reduce the effect of ground motion such as viscous dampers, base isolators and other types of passive, active and semi-active control devices. The passive control devices were the earliest to be developed and have been used extensively in seismic protection systems because they require relatively less maintenance and need no external power to operate.

The most commonly used passive control devices in structural systems are base isolators, tuned mass dampers, and energy absorbing elements. In the base isolation approach, the building is decoupled from the horizontal component of the ground motion by introducing a layer of a low horizontal stiffness between the structure and the foundation. The first dynamic mode of the isolated structure produces deformation only in the isolation
system. The higher modes, however, cause deformation in the structure, but are usually not of great significance in short to medium height structures. Tuned mass dampers transfer the energy from the input ground motion to auxiliary oscillators that counteract the sway in the structure. On the contrary, the passive energy dissipation systems work by absorbing the dynamic energy through discrete elements called dampers, thereby reducing the energy dissipation demands on the primary structural members and minimizing possible structural damage. The most commonly used passive energy dissipation systems include friction dampers, metallic yielding devices, viscous fluid dampers and viscoelastic solid dampers. This research focuses on the use of modified viscoelastic dampers for structural response control.

1.2 Visco-hyperelastic device

Viscoelastic materials have been used in vibration control for a long time, and they have been deployed in high-rise buildings to resist wind loads. These materials are sturdy and reliable prompting research and analyses for their use in seismic applications. Solid viscoelastic materials dissipate energy through shear deformation and axial strains when loaded cyclically. This property has been utilized to come up with a new passive energy dissipation system

A visco-hyperelastic device is a completely new type of seismic resistant system comprising a solid viscoelastic material sandwiched between two steel rings that provides increased stiffness at higher levels of displacement, without significantly increasing the system forces at low levels of displacements. The increased stiffness provided by the device compensates for loss of stiffness in the primary structural members due to damage and negative geometric stiffness associated with the P-Δ effect. These devices not only provide increased stability, but also enable dissipation of dynamic energy. The energy dissipation is due to a combination of plastic dissipation due to yielding in the steel elements and creep dissipation in the viscoelastic material. These devices have a nonlinear stiffening stress-strain relationship and can be implemented in a structural system as part of toggle braces [Constantinou et al., 1997] or diagonal braces. The device is described in detail in Chapter 4.
1.3 Aims and scope

The aim of this work is to study the energy dissipation aspects of a viscoelastic material and its application for seismic structural response control. The long range goal of this research project is to develop a passive energy dissipation system for use in low and mid-rise structures for seismic protection and high rise buildings to mitigate wind-induced vibrations.

The first part of this research focuses on hyperelastic behavior of materials and geometric shapes. These include: preliminary finite element modeling in SAP2000\(^2\) and experimental studies on a tire to verify hyperelastic behavior. The second part is devoted to the development of a new visco-hyperelastic device, and its implementation in a structural system. Detailed finite element analyses and parametric studies including the viscoelastic and hyperelastic material properties of rubber, and inelastic behavior of steel elements were developed in ABAQUS\(^3\). This device was then tested in a single story single bay steel frame under real unscaled El Centro and Northridge ground motion records.

1.4 General structure of thesis

To get an overview of the structure of this thesis, the contents of the chapter are presented below.

In *Chapter 2*, an extensive literature study is presented on damping in structures and passive energy dissipation systems. The study includes the different types of passive energy systems used such as base isolation, friction dampers, metallic yielding dampers, viscous fluid dampers and viscoelastic dampers.

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\(^2\) *SAP2000* is a registered trademark of Computers and Structures Inc., 1995 University Avenue, Suite 540, Berkeley, CA 94704 U.S.A. Internet: http://www.csiberkeley.com

\(^3\) *ABAQUS* is a registered trademark of Hibbitt, Karlsson & Sorensen, Inc., 1080 Main Street, Pawtucket, RI 02860-4847, U.S.A. Internet: http://www.abaqus.com
In Chapter 3, the energy dissipation mechanics of rubber compounds is presented. Rubber manufacturing processes are also described in detail. The widely used rubber compounds, their constituents and applications are also discussed in detail.

In Chapter 4, a detailed review of the hyperelastic behavior of materials is presented along with the influence of geometry on hyperelastic behavior. Results from experiments on three different motorcycle tires are presented and discussed.

In Chapter 5, the proposed visco-hyperelastic device is described in detail along with sketches of its geometry and modifiable parameters. Details of the connection of the device to the bracing system are also discussed.

In Chapter 6, a detailed finite element analysis of the device is presented with study on the effect of the parameters mentioned in Chapter 5 on the performance of the device. The finite element analyses were conducted using the finite element software package ABAQUS incorporating the inelastic behavior of steel and the energy dissipation properties of the viscoelastic material.

In Chapter 7, the device was implemented in a single-story single-bay frame in a toggle brace configuration, and its effect on controlling the structural response under ground motions was studied. The results from this analysis are compared with results from the same frame subjected to ground motions without the presence of the device.

In Chapter 8, the research is summarized and conclusions from the study are stated and directions for further research are stated.

In Appendix A, the derivation of the equations for the toggle-brace-damper configuration is presented.
Chapter 2

Literature Review

In studying the dynamic behavior of passive energy dissipation systems, two issues need to be addressed for reliable and accurate prediction of structural response in the presence of dampers. The first being a detailed analysis of the proposed device including parametric studies and the second being the effect of these devices in controlling structural behavior. A brief background review and literature survey of related research works is presented in this chapter.

2.1 Background

Seismic design has progressed a long way from special detailing of plastic hinge locations for energy dissipation due to yielding, to passive energy dissipation systems such as viscoelastic dampers, friction dampers, metallic yielding devices etc. for additional energy dissipation in structures. Conventional seismic design has also progressed from merely satisfying the life safety limit state to performance based engineering, wherein multiple limit states are considered.

Damping refers to the ability of a material or a structure to dissipate energy and steadily diminish the amplitude of vibration. Damping is still a very abstract quantity, even though considerable research has been done to accurately represent and quantify it. There are a number of means by which a structure can dissipate energy, some of which include the opening and closing of microcracks in concrete, friction between steel connections, and friction between structural and nonstructural components of a building. Damping in a structure is mathematically expressed in terms of a damping ratio $\xi$, which encompasses all
the damping mechanisms in a structure. Unlike the mass and stiffness of a structure that can be calculated from the dimensions and sizes of structural elements, it is almost impossible to arrive at an accurate value of damping due to the above mentioned reasons. Damping is at best, only an idealization.

In general most engineering materials dissipate energy during cyclic deformation. Under cyclic deformation, the material stress-strain response is in the form of hysteresis loops, the area under which gives a rough estimate of the energy dissipation in the material. Most engineering materials, within their elastic limit have small hysteresis loops in comparison to viscoelastic materials which have larger hysteresis loops due to their inherent ability to dissipate energy under all levels of vibration [Nashif et al., 1985]. The typical behavior of viscoelastic and metallic materials, dissipating energy due to creep and relaxation and due to yielding, respectively, are shown in Figure 2.1.

![Hysteresis plots for viscoelastic behavior and metallic yielding](image)

**Figure 2.1: Hysteresis plots for viscoelastic behavior and metallic yielding**

The effect of introducing energy dissipation mechanisms in a structure is very significant. Most structures have damping ratios $\xi$ in the region of 1-5% critical. Small increases in the damping ratios have great implications on the response of the building, as shown in Figure 2.2, which is a pseudoacceleration response spectrum for the El Centro ground motion. The higher the damping ratio, the lower the acceleration, and the lower the
base shear. Similar reductions occur for pseudovelocity and displacement. It can be seen that the acceleration values drop down from 0.92 g to 0.53 g for a change in damping from 5% critical to 20% critical, with a decrease of 42%.

![Figure 2.2: Effect of damping in structures](image)

**Figure 2.2: Effect of damping in structures**

Newmark and Hall [1985] recommended damping ratios (Table 2.1) to be used in the analysis of various structures. These values encompass the various damping mechanisms in a structure, exclusive of supplemental damping added to the system like active and passive energy dissipation systems. It is of interest to note the increase in damping ratio from 2-3% to 5-7% in the case of welded steel as the stress level increases from a working stress level to a point where yielding occurs. Plastic energy dissipation is of considerable importance in
seismic design and the proposed device incorporates both plastic dissipation and viscoelastic dissipation to provide an ideal combination for a passive energy system.

<table>
<thead>
<tr>
<th>Type of structure</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working stress level (1/2 yield point)</td>
<td></td>
</tr>
<tr>
<td>Welded steel</td>
<td></td>
</tr>
<tr>
<td>Prestressed concrete</td>
<td>2-3%</td>
</tr>
<tr>
<td>Well reinforced concrete (slight cracking)</td>
<td></td>
</tr>
<tr>
<td>Reinforced concrete with considerable cracking</td>
<td>3-5%</td>
</tr>
<tr>
<td>Bolted or riveted steel</td>
<td></td>
</tr>
<tr>
<td>Timber</td>
<td>5-7%</td>
</tr>
<tr>
<td>At or just below yield point</td>
<td></td>
</tr>
<tr>
<td>Welded steel</td>
<td>5-7%</td>
</tr>
<tr>
<td>Prestressed concrete (without loss of prestress)</td>
<td></td>
</tr>
<tr>
<td>Reinforced concrete</td>
<td>7-10%</td>
</tr>
<tr>
<td>Bolted or riveted steel</td>
<td></td>
</tr>
<tr>
<td>Bolted timber</td>
<td>10-15%</td>
</tr>
<tr>
<td>Nailed timber</td>
<td>15-20%</td>
</tr>
</tbody>
</table>

### 2.2 Passive energy dissipation systems

During a seismic event, a considerable amount of energy is input into a structure. The law of conservation of energy imposes the restriction that the energy must either be absorbed and/or dissipated by the structure. Most structures have an inherent damping in them which results in some of this energy being dissipated, but a large amount of energy is absorbed by the structure, undergoing severe deformations and maybe even collapse. The structural performance can be improved by providing additional means of energy dissipation. The
classical energy conservation law was described by Uang and Bertero (1988) in the following form for structural engineering applications

\[ E_I = E_k + E_r + E_s + E_d \]  

(2.1)

where \( E_I \) is energy input into the system from the earthquake, \( E_k \) is the kinetic energy, \( E_s \) is the recoverable strain energy, \( E_h \) is the irrecoverable energy due to dissipation from the inherent damping and \( E_d \) is the energy that is dissipated due to supplemental devices.

Passive energy dissipation systems offer additional means of energy dissipation in the structure by introducing mechanical devices in the structural framing system. Passive energy dissipation systems, unlike active energy dissipation systems, do not require external power to generate system control forces and hence, are easier and cheaper to implement in a structure. Motion control forces are generated due to the relative motion or relative velocity between the points of attachment. The former are called ‘displacement dependent devices’ (or rate independent) and the latter ‘velocity dependent devices’ (or rate dependent).

### 2.2.1 Seismic isolation

Seismic isolation or base isolation refers to an earthquake protection system where the structure is isolated from the ground by materials of low horizontal stiffness which includes, high damping rubber pads and sliding systems. Base isolation systems work on the principle of increasing the ‘flexibility’ of the structure. The low horizontal stiffness of the isolator helps to decouple the structure from the horizontal component of the ground motion. At the onset of the ground motion, the isolator deforms in shear and the structure above moves laterally, however, with very little relative motion of the structural components.

A typical base isolator [Naeim and Kelly, 1999] configuration is shown in Figure 2.3. The steel plates are used to prevent the rubber from bulging so that the isolator can support more load with lesser deformations. The lead core is used as an energy dissipater, which converts the kinetic energy from the motion to heat. It has low yield shear strength but
sufficiently high shear stiffness. It deforms plastically under shear deformations, and its post yielding behavior is essentially elastic-plastic.

![Diagram of base isolator configuration]

Figure 2.3: Typical base isolator configuration [Courtesy: MCEER]

### 2.2.2 Hysteretic devices

Hysteretic devices dissipate energy due to a mechanism that is independent of the rate of loading, unlike viscoelastic devices. Metallic dampers, which dissipate energy due to yielding of metals and friction dampers, which dissipate energy due to coulomb friction constitute these types of devices.

Yielding of metals beyond their elastic range contributes to significant energy dissipation, a concept that was explored extensively by Kelly et al. [1972] and Skinner et al. [1975]. Two different configurations of metallic dampers used in structural seismic protection are the X-shaped metallic damper [Bergman and Goel, 1987; Xia et al., 1992; Whittaker et al., 1991] and the triangular plate damper [Tsai et al., 1993]. The energy dissipation in these devices is due to the yielding of large volumes of steel, placed between the beam and the bracing members. These devices are also referred to as ADAS (Added Damping And Stiffness). These devices reduce the energy demand on the other structural members, and also add considerable damping to the structure.
Friction dampers, on the other hand, dissipate energy due to friction between two sliding surfaces. The effect of frictional damping was first studied by Mayes and Mowbray [1975] and Keightley [1977]. Frictional dampers for structural engineering application were first developed by Pall [1980] and the response of frames with friction dampers were studied by Pall and Marsh [1982]. Further research has been conducted by Aiken and Kelly [1990] using copper alloy friction pads. The principle of these devices is that energy is dissipated due to friction between two surfaces. The total frictional force \( F_t \) is independent of the area of contact, and the frictional force and the normal force \( F_n \) are proportional to one another with the coefficient of friction \( \mu \) being the proportionality constant. Equation 2.2 represents this mathematically.

\[
F_t = \mu F_n \tag{2.2}
\]

Even though \( \mu \) is treated as a proportionality constant, it is seldom constant. It depends on the material that is used and also on the sliding interface. The theory of dry friction is not mathematically well developed and hence, use of these devices relies heavily on experimental verification.

### 2.2.3 Viscoelastic devices

Viscoelastic dampers are devices that consist of a combination of a viscoelastic material used in conjunction with steel plates. These devices dissipate energy mainly due to the shear deformations in the viscoelastic material. A typical viscoelastic damper consists of viscoelastic layers bonded with steel plates as shown in Figure 2.4. The energy is dissipated due to relative motion between the outer flanges and the center plate, which results in shear strains in the viscoelastic material.

The viscoelastic systems could be either viscoelastic solid dampers or fluid viscous dampers. The behavior of viscoelastic dampers are influenced by many parameters including temperature, frequency of loading, strain rate and number of cycles of deformation. Temperature has an inverse effect of energy dissipation, with energy dissipated per cycle
decreasing with an increase in temperature. The solid viscoelastic devices dissipate energy in the form of heat while undergoing shear deformations.

Figure 2.4: Typical viscoelastic solid damper configuration [Constantinou et al., 1998]

The energy dissipated by a viscoelastic damper depends on the amount of axial or shear strains to which it is subjected. The energy dissipated per cycle for a viscoelastic material subjected to sinusoidal loading is given by:

\[
E_D = \pi \gamma_o^2 G'' V
\]

where \( E_D \) is the energy dissipated, \( \gamma_o \) is the shear strain in the material, \( G'' \) is the shear loss modulus given by \( G'' = G' \tan \delta \), where \( G' \) is the shear storage modulus and \( \delta \) is the phase angle between shear stress and shear strain, and \( V \) is the volume of the material. From
equation 2.3, the dependence on the volume of the viscoelastic material and the shear strains to which it is subjected can be noted. Hence, for the optimum performance of these devices, it must be placed at locations where structural deformations are expected to be high. Singh and Moreschi [2002] have presented genetic algorithms on the optimal size and location of viscous and viscoelastic dampers. Solid viscoelastic dampers are relatively inexpensive, and easy to implement in a structure. They also reduce displacement demand on the structure, minimizing nonstructural damage.

Viscous fluid dampers, on the other hand, dissipate energy by the constrained flow of a viscous liquid (often compressible liquid silicon) through cylindrical orifices machined into a piston head [Schwahn and Delinic, 1988]. The resistance force arises from a pressure difference on either side of the piston as it moves within the casing. The force is proportional to $|\dot{u}|^\alpha$ where $\dot{u}$ is the velocity of the piston head and $\alpha$ is a coefficient ranging from 0.2 to 2.0. Considerable experimental research on shake tables with steel frame and concrete building models has been conducted by Constantinou and Symans [1992, 1993], Reinhorn et al. [1995] and Tsopelas et al. [1995]. The fluid viscous damper used in the above research consisted of a stainless steel piston with a bronze orifice head and an accumulator, filled with silicon oil. The piston head utilizes specially made passages to alter fluid flow in order to control the damper force.

Extensive experimental work has been conducted on viscoelastic and fluid viscous dampers in Japan and the United States by Lin et al [1991], Aiken et al. [1993], Chang et al. [1995], Fujita et al. [1992], Kirekawa et al. [1992] on full scale and scaled models of reinforced concrete and steel frames.

### 2.2.4 Previous work

The idea of amplifying device displacements was studied by Ibrahim [2005] in the visco-plastic device. The device was a modification of the scissor-jack configuration, in which a high-damping viscoelastic material was sandwiched between two steel channels bent in a configuration in order to amplify the deformations in the device. This was done in order to
obtain large tensile and compressive strains in the viscoelastic material. The device provided energy dissipation due to creep in rubber and plastic behavior in steel.

The device presented in this research is a modification of the visco-plastic device. This device, like the visco-plastic device, provides energy dissipation from the viscoelastic behavior of rubber and yielding in steel. However, one important aspect of this device is its nonlinear hardening behavior. The device provides additional stiffness as the structure is on the verge of collapse due to the loss of stiffness in the primary structural members, a feature that is shown in Chapter 7. Stability is one of the key concerns in structural engineering and this device aims at providing stability to the structure at high levels of deformation. The device also has low stiffness at low levels of deformation with increasing stiffness at higher levels of deformation. One of the disadvantages in conventional dampers is that they often act as ‘seismic attractors’ imposing higher base shear demands at service level loads.

2.2.5 Innovative damper configurations

The dampers can be installed in different locations on a structure, and in various configurations. The dampers, commonly implemented in a structure in bracings, provide supplemental damping, and in some cases provide additional stiffness also.

The dampers can be installed in a variety of ways which include using them in diagonal braces, chevron braces, toggle-brace-dampers and scissor jack configurations. Each one of them has their own advantages and limitations. The various configurations are shown in Figure 2.5. Implementing the damper in a diagonal brace is one of the simplest means, and has been in use for a long time for seismic protection of buildings. The biggest disadvantage in them is the very high amount of damper strength and force required due to its orientation. The displacement in the devices in diagonal configuration is always less than the interstory drift. Hence, it is not an optimum way to use viscoelastic dampers compared to other configurations where there is device displacement amplification, causing more axial and shear strains in the viscoelastic material. The chevron brace, on the other hand, causes device displacements equal to the interstory drift.
Innovative damper configurations have been developed by Constantinou et al. [1997] including the toggle-brace and the scissor-jack configurations. These configurations aim at magnifying device displacements in order to exploit the behavior of viscoelastic devices. The magnifications can be in the range of 2.0-5.0 times the interstory drifts. The toggle-brace consists of two steel linkage elements which are not collinear and the damper is linked diagonally to these elements. The steel elements are linked to each other by a pin, which provides no rotational restraint. Accordingly, small interstory drifts are magnified in the
direction of the damper, increasing the effect of the dampers. The scissor-jack configuration also employs the same principle of amplifying displacements, though in a different manner.

Based on the available configurations, the toggle-brace configuration would be most ideal for implementing the proposed visco-hyperelastic device in the structural system because of the amplification in device displacements, and the corresponding increase in the axial strains in the viscoelastic material. The toggle-brace-damper configuration has been explained in detail in Chapter 7 and its derivation is presented in Appendix A.

2.3 Summary

Passive energy dissipation systems are supplemental energy dissipation mechanisms added to a structure in order to improve its seismic performance. There are many types of passive energy systems and a brief overview of these systems available for seismic protection was discussed along with the benefits and drawbacks. The proposed visco-hyperelastic device utilizes dissipation due to both the viscoelastic properties and the metallic yielding. The device can be manufactured readily in structural fabrication shops, and have a considerable effect on controlling the structural response as is seen in Chapters 6 and 7.
Chapter 3

Engineering properties of rubber

3.1 Preliminary remarks

The single most important property of elastomers like rubber is their ability to undergo large elastic deformations, that is, their ability to stretch and return to their original shape in a reversible way. Elastomers such as natural rubber are amorphous, isotropic polymers to which various ingredients are added, and following subsequent heating and reactions, these materials possess various desirable properties such as high tensile strength, damping etc. Rubber has been used for vibration isolation for a long time, because of its inherent ability to dissipate large amounts of energy due to shear deformations and axial strains.

There are two basic properties of an engineering material: modulus (stiffness) and damping (ability to dissipate energy). The law of conservation of energy states that energy is neither created nor destroyed; it is always transferred from one form to another. Energy is dissipated to the surroundings as heat due to internal friction. The magnitude of energy dissipated depends on the constituents of rubber. Elastomers, in their natural state, are not very useful engineering materials. Hence, it becomes necessary to ‘synthesize’ to introduce useful properties in them.

3.2 Hyperelasticity and viscoelasticity of rubber

Rubber consists of relatively long network of polymeric chains which have a very high degree of flexibility and mobility. The high deformability of rubber is due to its flexibility and mobility (ability of chains to slide past one another). When a stress is imposed on a rubbery
material, the chains alter their configurations instantaneously. The network structure of these chains forces them to act monolithically. As a result, rubber can very often be stretched up to ten times its original length, and upon removal of the force it returns to its original length with very little permanent deformation. The molecular theories that form the basis of rubber elasticity are beyond the scope of this thesis.

Rubber is a highly nonlinear material, and strictly speaking no portion of the stress-strain curve obeys the Hooke’s law. However, the stress-strain relation can be assumed to be linear over small values of strains, but there is considerable argument over the limits of ‘small’ strains. Rubber vulcanizates (rubber compounds subjected to vulcanization) contain large amounts of reinforcing fillers and hence, have considerable initial stiffness, which then softens before stiffening again, giving rise to a S-shaped stress-strain curve which is typical of filled rubber as shown in Figure 6.1. Material behavior can be divided into two classes, [1] Time independent behavior (nonlinear elastic behavior) and [2] Time dependent behavior (creep and viscoelasticity) which are discussed herein.

3.2.1 General theory of large elastic deformations

A general theory of stress-strain relations for rubber like elastomers was developed by Rivlin [1956], assuming that the material behavior is isotropic in elastic behavior in the unstrained state and incompressible in bulk. The measures of strain, are given by three strain invariants, given as follows

\[
J_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3 \\
J_2 = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 - 3 \\
J_3 = \lambda_1^2 \lambda_2^2 \lambda_3^2 - 1
\]  

(3.1)

where \( \lambda_1, \lambda_2, \lambda_3 \) denote the principal stretch ratios, defined as the ratio of the stretched length to the unstretched length of the edges of a cubical element. For incompressible

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4 Further reference can be found in Engineering with rubber, How to design rubber components, 2nd edition, Alan Gent, chapter 3; pp 50-63.
materials, the volume remains constant and hence $\lambda_1\lambda_2\lambda_3 = 1$. Hence, the strain energy density ($W$) is a function of $J_1$ and $J_2$ only. This results in the following equation:

$$W = C_1 J_1 + C_2 J_2$$  \hfill (3.2)

where $C_1$ and $C_2$ are constants. This particular form of strain energy function was proposed by Mooney [1940] and is referred to as the Mooney-Rivlin equation. It is one of the most commonly used strain energy relations for the finite element modeling of rubber.

### 3.2.2 Viscoelastic behavior

Rubber exhibits time dependent behavior and can be modeled as a viscoelastic material with its properties depending on both temperature and time. Under conditions of constant stress, rubber creeps (increase in deformation with time), and under conditions of constant strain there is relaxation (decay in stress with time). These are due to a combination of physical and chemical relaxation processes in rubber. The physical process is due to the viscoelasticity of rubber and occurs approximately as a linear function of log time. The chemical process is due to modification of the crosslinks and alteration in the network of chains and occurs as a linear function of time.

An ideal linear elastic solid obeys Hooke’s law; stress is proportional to strain. An ideal viscous liquid obeys Newton’s law: stress is proportional to rate of change of strain with time. Viscoelasticity is a combination of an elastic and viscous behavior. A Hookean solid can be expressed as a linear spring with the following relation:

$$F = k \Delta$$  \hfill (3.3)

where, $F$ is force, $k$ is the spring rate and $\Delta$ is the deformation.
Newton’s law of viscosity can be written in the following form:

\[ F = c \left( \frac{d\Delta}{dt} \right) \]  

(3.4)

where \( c \) is the damping constant. Viscoelastic behavior has been expressed in the form of two mechanical models, namely the Maxwell model and the Voigt (or Kelvin) model. The Maxwell model consists of a spring and a dashpot in series, while the Voigt model consists of a spring and a dashpot in parallel as shown in Figure 3.1.

![Figure 3.1: Maxwell and Voigt models for viscoelasticity](image)

In the Maxwell model, application of a load causes a sudden deflection in the elastic spring which is followed by creep in the dashpot. In the case of deformation, the reaction is first offered by the spring, followed by stress relaxation in the dashpot according to the exponential law. In the case of the Voigt model, the spring and the dashpot are in parallel and hence sudden application of load will not cause immediate deflection in the spring due to the viscous behavior of the dashpot. Deformation builds up gradually, with the spring taking a greater share of the load. The dashpot displacement relaxes exponentially.
3.3 Energy dissipation in rubber

Under cyclic loading rubber dissipates energy due to hysteresis. Filled rubbers undergo stress-induced softening, due to progressive collapse of crosslinks and due to separation between rubber molecules and the reinforcing fillers. The most important factors causing the hysteresis effects in rubber are discussed herein.

3.3.1 Friction

Rubber is composed of a network of chains, and when it is loaded the molecules rearrange themselves due to the imposed load. This results in the sliding of the chains relative to one another. This phenomenon is called internal friction (or) internal viscosity and is a temperature dependent phenomenon. An increase in temperature leads to an increased mobility, resulting in a reduced viscosity and hence, reduced hysteresis.

3.3.2 Stress softening

Stress softening refers to a reduced stiffness of rubber and change in damping characteristics due to repeated loading. This is often referred to as the Mullins’ Effect [Mullins, 1969]. If an elastomer is subjected to a uniaxial strain, the stiffness remains unchanged at strains higher than the previously applied strains; however, they have lesser stiffness at strains lower than the previously applied strains. Stress softening could be due to the rearrangement of the molecular network, microstructural damage under stress and due to void formation.

3.3.3 Crystallization

Large extensions and retractions lead to the formation of crystallized regions in the elastomer. Crystallization often results in increased strength. Natural rubber is one such example of a material that has a low modulus at small strain with tensile strength of thousands of pounds per square inch after crystallization at high strain.

3.3.4 Structural breakdown

In filled rubbers, the carbon black particles tend to break down due to mutual interactions and this breakdown of the matrix/infill bond due to loading leads to considerable hysteresis in rubber.
3.4 Rubber compounding

Compounding of rubber is a complex multidisciplinary science involving materials physics, organic and polymer chemistry, inorganic chemistry and chemical reaction kinetics. Compounded rubber has many unique characteristics not found in other materials, such as dampening properties and high elasticity. An elastomer is a high molecular weight liquid with low elasticity and strength. Vulcanization or curing is a process of chemically linking the network of chains to form a tough elastic solid. This results in an increase in stiffness and strength, while the hysteresis decrease. The compounding of rubber starts with the choice of an elastomer, filler, crosslinking chemicals and various additives, which when added results in a ‘compound’ having the desired characteristics.

3.4.1 Sulfur curing

Sulfur is the most commonly used vulcanizing agent. It is carried out by heating rubber mixed with sulfur under pressure. The rubber compounds subjected to this treatment are called vulcanizates. For sulfur to effectively crosslink a rubber, an elastomer must contain double bonds with allylic hydrogens. Commonly used elastomers such as Butyl rubber, Nitrile rubber, Styrene Butadiene rubber satisfy this requirement. Vulcanization can be done either using a soluble or an insoluble form of sulfur. Crosslinking with sulfur is usually ineffective and takes a long time to cure. To increase the rate and efficiency of curing, accelerators such as thiozoles, xanthates and thiurams are added. However, the accelerators should be selected in such a way that they delay the onset of vulcanization so that the shaping process is complete.

Mechanical aspects of rubber depend on the crosslink density. Modulus and hardness increase proportional to the crosslink density. Crosslinking however, reduces the hysteresis, because it reduces the sliding between the networks of chains. At high crosslink levels, chain motions become restricted, and the network is incapable of dissipating energy, resulting in brittle fracture at low elongation. The crosslink density should be high enough such that it prevents failure by viscous flow and low enough to prevent brittle fracture, providing the required dissipation at the same time.
3.4.2 Filler systems

Fillers or reinforcement aids, such as carbon black, clays and silicas are added to rubber to improve material properties. Particle surface area is a very important parameter for fillers. Particles with large surface area are useful since they have more interaction with rubber and close particle-to-particle spacing. Carbon black and silica are two of the most commonly used fillers.

Carbon black is chemically linked with rubber by shear mixing. The interactions between rubber and carbon black vary in magnitude, with some chains chemically bonded to the rubber while others have physical bonds of varying strength. To provide the greatest strength, the carbon black must be broken down into fine aggregates and dispersed thoroughly in the rubber, requiring mixing at high shear stresses. Carbon black reduces the melt elasticity, increasing the processability, in addition to enhancing the strength.

The addition of silica to a rubber compound improves tear strength, improves adhesion of the compound to other components and reduction in heat buildup. Silica, in comparison to carbon black, does not provide the same level of reinforcement for the same particle size. However, the addition of silica improves hysteresis and tear strength in the rubber compound.

3.5 Mechanical aspects of high damping rubber

High damping rubber (HDR) is manufactured from the vulcanization of Natural Rubber (NR) with the addition of carbon black, plasticizers, oils, resins and consequently introduces specific characteristics such as hardening properties, energy absorbing properties and maximum strain dependency of stress evolution [Yoshida et. al, 2004].

Research on natural rubber for isolating buildings began in 1976 as a joint venture by Earthquake Engineering Research Center (EERC), now called Pacific Earthquake Engineering Research center (PEER) and the Malaysian Rubber Producers Research Association (MRPRA). Furthermore, high damping rubber has been developed for specific
applications in base isolation from earthquakes [Kelly, 1997]. The tensile and shear strains are highly nonlinear for HDR. They show a high initial stiffness due to the presence of high amounts of reinforcing filler, and the stiffness remains a constant before increasing towards the end. This could be attributed to the finite extensibility of the chains and also due to strain crystallization [Fuller et al., 1996].

The high damping rubber, manufactured by Yokohama Rubber Co., used in this research is based on experimental investigation by Yoshida et al [2004] and Amin et al. [2002]. The device undergoes cyclic deformations, and hence the results from tension and compression tests are necessary. The experimental results used in modeling the rubber are shown in Figure 3.2. The configuration of the device is such that there are tensile and compressive strains induced simultaneously during any stage of the loading cycle.

![Graph showing stress vs strain for HDR](attachment://Figure_3.2.png)

**Figure 3.2: Uniaxial tension-compression tests on HDR [Amin et al., 2002]**

HDR, like other elastomeric materials, exhibits a time-dependent behavior, referred to as viscoelasticity. HDR creeps under the effect of constant stress and relaxes under the effect
of constant strain. Multi-step relaxation tests were conducted to determine the time-
dependent behavior, the results of which are shown in Figure 3.3.

![Figure 3.3: Relaxation tests on HDR [Yoshida et al., 2004]](image)

### 3.6 Summary

Natural rubber (NR) is a very versatile material and has been used in many engineering
applications such as dock fenders, bridge bearings etc. The properties of NR can be
modified by adding fillers, reinforcing materials, oils and resins referred to as compounding.

Rubber dissipates energy due to various mechanisms such as friction, stress
softening, structural breakdown and crystallization. High damping rubbers have been
developed specifically for engineering applications and they involve modification of
properties of Natural Rubber based on the requirements.
Chapter 4

Experimental investigation of a tire and its hyperelastic behavior

4.1 Preliminary remarks

Numerous structural devices have been studied for controlling the dynamic response of structures. One of the primary disadvantage of some of these devices is that they act as ‘seismic attractors’, increasing the amount of system forces at service level loads. Rigid bracings are one such example, which help reduce the drift in the structure but add stiffness to the structure, increasing the base shear demands. New devices are being investigated for optimizing structural efficiency by controlling the dynamic behavior and minimizing behavioral disadvantages. Hyperelastic devices are intended to overcome these deficiencies in damping systems. These devices have a non-linear force-deformation relation as shown in Figure 4.1, providing increased stiffness at higher levels of deformation in the structure.

Hyperelasticity is useful in increasing the structural stability while avoiding increased base shear that occurs with linear stiffening devices [Saunders, 2004]. Devices that add stiffness to the structure increase the base shear demands in the structure, thereby increasing the strength of the structural members. Hyperelastic material behavior provides the best alternative in terms of having low strength requirements at service loads, and providing the additional stiffness at higher displacement levels. These devices are akin to nonlinear hardening devices [Oesterle, 2003]. Hyperelastic devices could be implemented in a structural system in the form of diagonal braces. A realistic brace design could consist of a set of cross-braces linked at the center by a round elastic element as shown in Figure 4.2.
Figure 4.1: Force-deformation relation for a hyperelastic element

Figure 4.2: Implementation of the tire in a diagonal brace
The equations of circular displacement dictate that the lengthening of a circular element under tension would be greater than the corresponding shortening. The circular element would continue to gain resistance as the displacement increases, thereby providing the required additional stiffness.

### 4.2 Tire as a hyperelastic device

In human history, the wheel is considered one of the greatest inventions. Wheeled vehicles were recorded in Sumeria in 3500 BC, Assyria in 3000 BC, and central Europe in 1000 BC. The discovery of vulcanization by Charles Goodyear in 1839 and the industrial revolution in Europe and North America enabled the tire to evolve from a rubberized canvas covering a rubber tube to a complex fabric, steel, and elastomeric composite.

#### 4.2.1 Basic tire design

A tire is essentially a cord/rubber composite. Tires have plies of reinforcing cords extending transversely from bead to bead, above which is a layer of textile/steel wire belt which lies under the tread. A typical tire cross-section is shown in Figure 4.3.

![Figure 4.3: Typical tire cross-section. (Courtesy: Muscle Car News)](image)
4.2.2 Tire construction

A tire is an assembly of a series of components, each of which serve a specific purpose. Figure 4.3 illustrates the key components of an automobile tire [Kovac, 1978], which are described herein.

Tread

It is the wear resistance component of the tire in contact with the road. It must provide good traction, skid resistance and good cornering characteristic with minimum noise generation and low heat buildup. Tread components consist of blends of natural tuber, PBD, SBR, compounded with carbon black, oils and vulcanizing chemicals.

Sidewall

It serves to protect the casing from side-scuffing, controls the vehicle/ride characteristics, and assists in tread support. Sidewall compounds consist of natural rubber, SBR, and PBD along with carbon black and a series of oils and organic chemicals.

Beads

There are non-extensible steel wire loops which anchor the plies and also lock the tire onto the wheel assembly so that it will not slip or rock on the rim.

Plies and liners

Plies are textile or steel cords extending from bead to bead and thereby serving as a primary reinforcing material in the tire casing. Liners are butyl rubber or derivatives of such polymers that are used to retain the compressed air inside the tire.

---

5 Abbreviation for polybutadiene
6 Abbreviation for styrene-butadiene rubber
Belts

These are layers of textile or steel wire lying under the tread and serve to stiffen the casing, thereby allowing improved wear performance and handling response, better damage resistance and protection of the ply cords.

4.3 Preliminary model

Prior to the experimental analysis of the tire, a rudimentary finite element model of a circular rubber element was created and analyzed in SAP2000. The rubber element was modeled using thin shell elements and the properties of rubber were incorporated using a Young’s Modulus ($E$) of 1 ksi and a Poisson’s ratio ($\nu$) of 0.49.

The rubber element was subjected to pseudo-static uniaxial tension, with the load applied at a selected few nodes on diagonally opposite ends. The tire was supported on X-Y rollers which prevented any displacement in the Z plane. This, however, is not realistic since the tire sidewalls not only move in the X and Y directions, but also move in the Z directions. Figure 4.4 shows the three dimensional model of the rubber element and one of its displaced configurations respectively.

![Figure 4.4: Preliminary finite element model of a circular rubber element](image)

Figure 4.4: Preliminary finite element model of a circular rubber element
The analysis was run as a nonlinear, large displacement analysis to account for the expected hyperelastic behavior and the large nodal displacement due to the deformation in the rubber. The convergence tolerance had to be increased in order to account for the non-convergence of nodal displacements, which could be due to the inability to model accurately the large displacements in a rubber-like materials and elastomers.

The models were subjected to pseudo-static tensile loads, in order to generate the force-deformation plots for the circular element as shown in Figure 4.5.

![Figure 4.5: Force-deformation relation for the SAP2000 circular element model](image)

### 4.4 Testing Program

The objectives of the experimental analyses were to determine the hyperelastic properties of a automobile tire and evaluate its performance under a series of static and dynamic loading steps. Three sets of these tests were conducted involving one pseudo-static test and ten dynamic tests. The tires used in these tests were discarded rear motorcycle tires of various
sizes manufactured by Dunlop™, chosen so as to represent a wide range of tire tread wear and tear.

The tests are designated TR1-K555, TR2-D404 and TR3-K591 and the specimens numbered 1 – 3 respectively. K555, D404, K591 represent the tire’s model number as specified by the manufacturer.

Table 4.1: Summary of test specimen parameters

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Size</th>
<th>Load/speed index</th>
<th>Overall Diameter (in.)</th>
<th>Overall width (in.)</th>
<th>Full tread depth (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR1-K555</td>
<td>170/80-15 MC</td>
<td>77H</td>
<td>25.98</td>
<td>6.5</td>
<td>9/32</td>
</tr>
<tr>
<td>TR2-D404</td>
<td>170/80-15 MC</td>
<td>77H</td>
<td>25.43</td>
<td>6.71</td>
<td>9/32</td>
</tr>
<tr>
<td>TR3-K591</td>
<td>130/90-16 MC</td>
<td>64V</td>
<td>24.86</td>
<td>5.35</td>
<td>7/32</td>
</tr>
</tbody>
</table>

4.4.1 Test Frame

The test was conducted in the Virginia Tech Structures and Materials Testing Laboratory. The test frame used to test the tires was composed of two 16ft-0in. tall W21x68 steel columns fixed at the base to reaction-floor beams at 8ft-0in. on center. Two 8ft-0in. long W24x94 steel beams spanned between the columns at approximately 13ft-0in. above the reaction floor, and were bolted to each flange of the columns. A 55,000 pound capacity MTS dynamic actuator was hung from the W24’s at the midpoint, such that the load from

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7 K555,D404, K591 are copyright of The Goodyear Tire & Rubber Company, 1144 East Market Street, Akron, OH 44316-0001. Internet: http://www.goodyear.com
the actuator was equally distributed between the two beams. The actuator was positioned directly above a beam in the reaction floor. Two small three-plate steel fixtures were fabricated. One was bolted to the actuator and the other was bolted to the reaction beam directly below the actuator. These fixtures were used to attach the tire to the test setup, using eye bolts and threaded rods. (Figure 4.6)

4.4.2 Instrumentation

To achieve the desired information, displacement and load was measured continuously. The MTS\textsuperscript{8} actuator used is equipped with an internal LVDT\textsuperscript{9} and a full scale capacity load cell. In addition, two draw-wire transducers were attached to the test frame to measure the lateral displacement in the tire on both sides. Measurement of the vertical and lateral deformation

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\textsuperscript{8} MTS is a trademark of MTS Systems Corporation, 14000 Technology Drive, Eden Prairie, MN 55344-2290, U.S.A. Internet: http://www.mts.com

\textsuperscript{9} Abbreviation for Linear Variable Differential Transformer
of the tire was necessary to observe the behavior of the tire under quasi-static and dynamic loadings.

The signals transmitted from all four devices were converted in real-time and recorded directly by a *Micrometasurements System 6000* data acquisition device. The System 6000 is capable of recording a signal up to 10000 times per second. The rate of recording required was determined to be 100 records per second, or one reading every 0.01 seconds for the dynamic tests and 500 readings per second for static tests.

The MTS LVDT was calibrated using a dial type stand extensometer. The range of the LVDT is +/- 3.5 inches, and was shown to be accurate within 0.001 in. across the range. The load cell was also calibrated to ensure that accurate data would be recorded. The draw-wire transducers were calibrated using a dial type stand extensometer to an accuracy of within 0.001 in.

### 4.4.3 Loading protocol

The test specimens were first subjected to a peak displacement of 2.5 in. by means of a quasi-static ramp loading, in which deformation was applied at a rate of 0.1 in. /sec. The tire was then pulled to a touch load varying from 50-120 lbs. to prevent any slack in the tire and subjected to a series of ten displacement-controlled loading functions having the following form for ten complete cycles each:

\[
y(t) = y \sin \omega t
\]  

(4.1)

In this function, \( y(t) \) is the displacement with respect to time in inches, \( y \) is the amplitude of the sinusoidal loading, \( \omega \) is the frequency of oscillation in radians per second, and \( t \) is time in seconds. Figure 4.7 represents graphically a sample loading of 0.75 in. amplitude at a frequency of 0.25 Hz.

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10 System 6000 is a trademark of Vishay Measurements Group GmbH, Geheimrat-Rosenthal-Str. 100, Selb, D-95100. Germany. Internet: http://www.vishay.com
4.4.4 Test setup and procedure

Tests were conducted on different sizes of used motorcycle tires with varying tread and side wall thicknesses. The system tested was aimed towards application in developing countries and hence, the setup and fabrication of different appurtenances were done keeping in mind the ease of use and economy of materials used. The tires will be implemented in the diagonal braces in a structural system, using cables whose tautness can be adjusted using turnbuckles.

Holes of ¾ in. diameter were drilled on diagonally opposite points on the tire; through which threaded rods were attached. Two 3 in. x 6 in. plates were used to hold the threaded rods with the tire in order to prevent the punching of the nuts when the tire was loaded. Rubber mats were placed on the inside of the tire to allow for even seating of the plates. The threaded rods were used to allow for adjustments depending on the size of the tire, instead of having to adjust the actuator. The threaded rods were coupled to eye bolts using ¾ in. couplers. The eyebolts were used to suspend the tire from the head of the
actuator and were also used secure the other end to the reaction floor. Details are shown in Figures 4.8-4.10.

Figure 4.8: Top Connection Details

Figure 4.9: Bottom Connection Details

Figure 4.10: Connection details of seating plate and tire
4.4.5 Static test procedure

As mentioned previously, all tires were tested under static loading prior to loading them dynamically. The specimens were loaded to a peak displacement of 1.75 in., corresponding to the maximum amplitude of the dynamic sinusoidal excitation, by a ramp load. All the tests were displacement-controlled, in other words, the peak displacement was the input and the load was measured corresponding to the deflection.

The tire shortening, in the $x$ direction, orthogonal to the loading, was measured using a LVDT connected to the System 6000. Linear displacement transducers measure the deflections using an extendible wire that spins on a spool. The data was scanned into the data acquisition system at a rate of 500 scans per second.

Once the tire was secured to the plates at the top and bottom, the load cell in the MTS was set to zero. The wire pots were attached to the sides of the tire, and set to zero displacement. After all the measuring devices were “zeroed” out, the static tests were initiated. A loading and unloading rate of 0.1 in./sec was used for the pseudo-static load.

4.4.6 Dynamic test procedure

Tires were attached to the test frame for the dynamic tests in the same manner as they were for the static tests. Once the tire was secured, a touch load of 80 lb – 100 lb was applied to keep the tire taut. The set point (point about which the sinusoidal excitation oscillates) had to be changed for each displacement amplitude set, so as to prevent the tire from becoming slack and rebounding against the base plate during the dynamic excitation. This limited the maximum amplitude of the sinusoidal excitation that could be applied to the tire. The test setup for the dynamic test was similar to that of the static tests with the only difference being in the data scan rate, which was reduced to 100 scans per second.

Multiple dynamic tests were run on each specimen under varying displacement amplitudes and frequencies. Each test was run for ten cycles in order to ascertain any degradation in behavior over repeated cycles. The focus of the dynamic tests was to simulate
a variety of loading conditions on the tires to determine the response for each displacement amplitude and frequency. Table 4.2 shows a summary of the test parameters for each test set.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Frequency (Hz)</th>
<th>Amplitude (in.)</th>
<th>No. of Cycles</th>
<th>Data Scan Rate (scans/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static 1</td>
<td>-</td>
<td>1.75</td>
<td>-</td>
<td>500</td>
</tr>
<tr>
<td>Dynamic - 1</td>
<td>0.1</td>
<td>0.75</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Dynamic – 2</td>
<td>0.25</td>
<td>0.75</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Dynamic – 3</td>
<td>0.5</td>
<td>0.75</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Dynamic – 4</td>
<td>0.75</td>
<td>0.75</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Dynamic – 5</td>
<td>1.0</td>
<td>0.75</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Dynamic – 6</td>
<td>0.1</td>
<td>1.00</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Dynamic – 7</td>
<td>0.25</td>
<td>1.00</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Dynamic – 8</td>
<td>0.5</td>
<td>1.00</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Dynamic – 9</td>
<td>0.1</td>
<td>1.75</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Dynamic – 10</td>
<td>0.25</td>
<td>1.75</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>

4.4.7 Force-deformation

Force deformation plots are beneficial in determining the efficiency of the tire system. From the shape and consistency of the hysteresis loops, it can be determined whether the system is effective in dissipating energy and if the system would break down during the course of cyclic loading. The area within a hysteresis loop is equivalent to the amount of energy that the device is dissipating. Also, the plots could be used to determine the approximate stiffness of the system.

To produce these force-deformation plots, sinusoidal load of amplitudes varying from 0.75 in. – 1.75 in. and frequencies varying from 0.1 Hz to 1 Hz were applied on the tire system for 10 cycles each and the data was recorded, which produced the hysteresis loops. Higher amplitudes and frequencies could not be tested due to the limitations in the testing
system, which limited the tests to a maximum displacement of 1.75 in. and a maximum frequency of 1.0 Hz.

4.4.8 Computation of the damping constant

The energy dissipation potential and the damping properties of a passive energy dissipation system are mathematically quantified as a single term, referred to as the coefficient of damping ‘c’ or damping constant.

Solid viscoelastic devices are velocity-dependent devices, wherein the response of the device depends on the frequency and amplitude of the excitation. Their force-displacement response is a function of the relative velocity at the ends of the device. The properties of viscoelastic devices can be obtained by tests of the dampers to a displacement controlled harmonic loading.

\[ u(t) = u_0 \sin(\omega t) \]  

(4.2)

where \( u_0 \) is the experimentally controlled peak displacement amplitude and \( \omega \) is the circular frequency of loading. The resisting force in the damper lags behind the displacement as follows:

\[ F(t) = F_0 \sin(\omega t + \delta) \]  

(4.3)

where \( F_0 \) is the maximum total damper force recorded during the test, and \( \delta \) is the phase angle expressed in radians. Damping results in a phase shift \( (\delta) \) between the displacement and the resulting force, and under cyclic loading results in a force-deformation curve. Equation 4.3 can be mathematically rewritten as:

\[ F(t) = F_0 \sin(\omega t) \cos(\delta) + F_0 \cos(\omega t) \sin(\delta) \]  

(4.4)
The force response in the damper can be mathematically expressed as:

\[ F(t) = K_s u_o \sin(\omega t) + K_v u_o \cos(\omega t) \]  \hspace{1cm} (4.5)

where,

**Storage Stiffness** \( K_s = \frac{F_o}{u_o} \cos(\delta) \) \hspace{1cm} (4.6)

**Loss Stiffness** \( K_l = \frac{F_o}{u_o} \sin(\delta) \) \hspace{1cm} (4.7)

The resisting force in the damper consists of two parts, an effective spring force in phase with the displacement, and a damping force 90 degrees out of phase with the displacement. The phase angle (\( \delta \)) experimentally computed from the force-deformation plot as:

\[ \delta = \sin^{-1}\left(\frac{F_{x=\text{zero}}}{F_o}\right) \] \hspace{1cm} (4.8)

The velocity across the damper can be obtained by differentiating the displacement in the damper, \( \dot{u}(t) = u_o(t)\omega \cos(\omega t) \) and Equation 4.5 can be rewritten as

\[ F(t) = K_s u(t) + \frac{K_l}{\omega} \dot{u}(t) \] \hspace{1cm} (4.9)

\[ F(t) = K_s u(t) + C \ddot{u}(t) \] \hspace{1cm} (4.10)

where the effective damping constant \( C = \frac{K_l}{\omega} \)

The damping constant is frequency dependent; however, it does not always vary inversely with the frequency as suggested by equation 4.10.
Illustration

The damping constant for the tire specimen TR1-K555 for which the loading frequency is 0.5 Hz (3.142 radians/sec) and peak displacement is 0.75 in. has been calculated [refer Figure 4.11]. The measured peak resisting force $F_0$ is 304.45 lb and the resisting force at zero displacement $F_{u=zero}$ is 47.308 lb.

Using the relations aforementioned, the phase shift ($\delta$) between the damping force and the displacement was calculated from the recorded force-displacement curve for the tire specimen.

$$\delta = \sin^{-1} \left( \frac{47.308}{304.45} \right) = 8.939^\circ$$

The loss stiffness ($K_l$) and the storage stiffness ($K_s$) for the peak displacement ($u_0$) of 0.75 inches were calculated. These were later used to plot the spring force and damping force components of the force relation for the damper. The effective damping constant $C$ for the tire for the given excitation frequency was also computed.

$$K_l = \frac{304.45}{0.75} \sin(8.939^\circ) = 63.077 \text{ lb/in.}$$

$$K_s = \frac{304.45}{0.75} \cos(8.939^\circ) = 401.003 \text{ lb/in.}$$

$$C = \frac{63.077}{3.142} = 20.078 \text{ lb-sec/in.}$$

The force response of the tire has been split into its two components, the spring force and the damping force and plotted as shown in Figure 4.12.
Figure 4.11: Hysteretic behavior of a nonlinear viscoelastic damper

Figure 4.12: Time history of damping response for the tire specimen TR1-K555
4.5 Results and discussion

4.5.1 Static tests

The static tests were performed on each specimen as mentioned earlier. The recorded force
and displacement from each test were plotted, which were used to ascertain the hyperelastic
behavior of the tires.

4.5.2 Dynamic tests

The dynamic tests were conducted to ascertain the response of the tire to dynamic excitation
and to quantify the energy dissipation potential in the tires. The force-displacement curves
from each test were used to compute the phase angle ($\delta$) and the damping constant as
shown in the previous section. The area under the hysteresis loop provided a rough estimate
of the energy dissipation in the tire.

4.5.3 Discussion

From the force-deformation plots, it can be seen that the tires not only behave as a
hyperelastic device, but also have very good energy dissipation potential. The tests also
confirmed the mathematical equations of circular displacement, according to which the
elongation in the tire would be greater than the corresponding shortening. In each of the
force-displacement curves for dynamic excitation, the curves sloping down from the left to
the right represent the total ‘contraction’ the tire, and the curves sloping up from left to right
represent the total ‘elongation’ in the tire as illustrated in Figure 4.13.

The results from the testing are shown in Figures 4.14 - 4.46, from which it can be noted
that the nonlinearity in the plots increases at higher displacements, indicating an increase in
the stiffness at higher displacement amplitudes. The anomaly in the 0.75 in. force-
deformation plots at 1.0 Hz was due difficulty in preventing the tire from going slack, which
caused the eye-bolt at the bottom to rebound against the base plate, causing the effect as
seen in Figures 4.19 and 4.41.
Total elongation \( \Rightarrow \Delta_{\text{elongation}} = \delta_{\text{elongation}} \)

Total contraction \( \Rightarrow \Delta_{\text{contraction}} = \delta_{\text{contraction, left}} + \delta_{\text{contraction, right}} \)

Figure 4.13: Displaced configuration of the tire and definition of terms
4.5.4 TR1-K555 results

Figure 4.14: Pseudo-static force-deformation plot, TR1-K555

Figure 4.15: Force-deformation plot at y = 0.75 in. and $\omega = 0.1$ Hz, TR1-K555
Figure 4.16: Force-deformation plot at $y = 0.75$ in. and $\omega = 0.25$ Hz, TR1-K555

Figure 4.17: Force-deformation plot at $y = 0.75$ in. and $\omega = 0.5$ Hz, TR1-K555
Figure 4.18: Force-deformation plot at $y = 0.75$ in. and $\omega = 0.75$ Hz, TR1-K555

Figure 4.19: Force-deformation plot at $y = 0.75$ in. and $\omega = 1.0$ Hz, TR1-K555
Figure 4.20: Force-deformation plot at $y = 1.0$ in. and $\omega = 0.1$ Hz, TR1-K555

Figure 4.21: Force-deformation plot at $y = 1.0$ in. and $\omega = 0.25$ Hz, TR1-K555
Figure 4.22: Force-deformation plot at y = 1.0 in. and $\omega = 0.5$ Hz, TR1-K555

Figure 4.23: Force-deformation plot at y = 1.75 in. and $\omega = 0.1$ Hz, TR1-K555
Figure 4.24: Force-deformation plot at $y = 1.75$ in. and $\omega = 0.25$ Hz, TR1-K555

4.5.5 TR2-D404 Results

Figure 4.25: Pseudo-static force-deformation plot, TR2-D404
Figure 4.26: Force-deformation plot at $y = 0.75$ in. and $\omega = 0.1$ Hz, TR2-D404

Figure 4.27: Force-deformation plot at $y = 0.75$ in. and $\omega = 0.25$ Hz, TR2-D404
Figure 4.28: Force-deformation plot at $y = 0.75$ in. and $\omega = 0.5$ Hz, TR2-D404

Figure 4.29: Force-deformation plot at $y = 0.75$ in. and $\omega = 0.75$ Hz, TR2-D404
Figure 4.30: Force-deformation plot at y = 0.75 in. and $\omega = 1.0$ Hz, TR2-D404

Figure 4.31: Force-deformation plot at y = 1.0 in. and $\omega = 0.1$ Hz, TR2-D404
Figure 4.32: Force-deformation plot at \( y = 1.0 \) in. and \( \omega = 0.25 \) Hz, TR2-D404

Figure 4.33: Force-deformation plot at \( y = 1.0 \) in. and \( \omega = 0.5 \) Hz, TR2-D404
Figure 4.34: Force-deformation plot at \( y = 1.75 \) in. and \( \omega = 0.1 \) Hz, TR2-D404

Figure 4.35: Force-deformation plot at \( y = 1.75 \) in. and \( \omega = 0.25 \) Hz, TR2-D404
4.5.6 TR3-K591 Results

Figure 4.36: Pseudo-static force-deformation plot, TR3-K591

Figure 4.37: Force-deformation plot at y = 0.75 in. and $\omega = 0.1$ Hz, TR3-K591
Figure 4.38: Force-deformation plot at $y = 0.75$ in. and $\omega = 0.25$ Hz, TR3-K591

Figure 4.39: Force-deformation plot at $y = 0.75$ in. and $\omega = 0.5$ Hz, TR3-K591
Figure 4.40: Force-deformation plot at \( y = 0.75 \) in. and \( \omega = 0.75 \) Hz, TR3-K591

Figure 4.41: Force-deformation plot at \( y = 0.75 \) in. and \( \omega = 1.0 \) Hz, TR3-K591
Figure 4.42: Force-deformation plot at y = 1.0 in. and $\omega = 0.1$ Hz, TR3-K591

Figure 4.43: Force-deformation plot at y = 1.0 in. and $\omega = 0.25$ Hz, TR3-K591
Figure 4.44: Force-deformation plot at $y = 1.0$ in. and $\omega = 0.5$ Hz, TR3-K591

Figure 4.45: Force-deformation plot at $y = 1.75$ in. and $\omega = 0.1$ Hz, TR3-K591
4.5.7 Damping constant calculations

As mentioned in Section 4.4.8, the damping constant ‘C’ for the tire can be calculated from the force-displacement plots under harmonic loading. The results of these calculations are presented in Table 4.3. It can be seen that the damping constant does not change considerably with increase in peak displacement amplitude. The change in damping constant with frequency for the tire is significant, with the damping decreasing as the frequency of loading increases. Specimen TR2-D404 was much larger than the other specimens and therefore had a larger damping constant.
Table 4.3: Effect of loading frequency and displacement amplitude on the damping constant $^{11}$

<table>
<thead>
<tr>
<th>TR1-K555</th>
<th>$\delta$</th>
<th>TR2-D404</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_0$</td>
<td>$F_{u=0}$</td>
<td>$F_o$</td>
<td>$K_f$</td>
</tr>
<tr>
<td>in.</td>
<td>rad/sec</td>
<td>lb</td>
<td>lb</td>
</tr>
<tr>
<td>0.75</td>
<td>0.63</td>
<td>35.86</td>
<td>281.56</td>
</tr>
<tr>
<td>0.75</td>
<td>3.14</td>
<td>47.31</td>
<td>304.45</td>
</tr>
<tr>
<td>0.75</td>
<td>4.71</td>
<td>35.86</td>
<td>301.40</td>
</tr>
<tr>
<td>0.75</td>
<td>6.28</td>
<td>38.92</td>
<td>293.77</td>
</tr>
<tr>
<td>1.00</td>
<td>0.63</td>
<td>45.78</td>
<td>377.70</td>
</tr>
<tr>
<td>1.00</td>
<td>1.57</td>
<td>51.87</td>
<td>390.67</td>
</tr>
<tr>
<td>1.00</td>
<td>3.14</td>
<td>41.20</td>
<td>399.83</td>
</tr>
<tr>
<td>1.75</td>
<td>0.63</td>
<td>70.96</td>
<td>685.20</td>
</tr>
<tr>
<td>1.75</td>
<td>1.57</td>
<td>56.46</td>
<td>769.14</td>
</tr>
<tr>
<td>1.75</td>
<td>6.28</td>
<td>22.13</td>
<td>300.00</td>
</tr>
</tbody>
</table>

$^{11}$ The nomenclature used in the table is consistent with the definitions mentioned a priori
The energy dissipation systems are usually tested at frequencies that correspond to the fundamental frequency of the structure in which they would be installed. For example, if the device is intended for use in a ten story structure that has a fundamental frequency \( \omega \) of approximately 6.28 radians/sec, then the device would have to be tested under that same excitation frequency \( \omega \). It can be seen from Table 4.3 that the damping constant \( C \) decreases at higher frequencies, implying that the device would not be very effective for structures having low fundamental periods (low to mid rise structures). Hence, the damping in the device would have to be increased by the methods suggested in Section 4.6. The device in its current configuration is only suited for structures having higher fundamental periods of vibration.

### 4.6 Summary

An experimental investigation of motorcycle tires was conducted to validate the hyperelastic behavior and involved analysis on experimental data obtained from static and dynamic analysis on the tires. When a tire is subjected to an increasing tensile force, the resistance offered to deformation increases with deformation. In other words, there is increased stiffness at higher displacement.
The force deformation plots establish the hyperelastic behavior and the energy dissipation capability of the tire over a range of displacements. Because the hysteresis loop did not break down over a period of 10 cycles, the reliability of the tire system was not compromised.

Tires possess an inherent ability to dissipate energy. Studies have been conducted to determine the hysteresis behavior of rubber. Some possible means of energy dissipation in the tires could be internal friction due to rearrangement of the molecular structure under applied load and subsequent sliding of chains; due to opening and closing of the cracks present in the worn out tire; or due to stress softening at higher levels of deformation due to multi-chain rearrangement, microstructural damage and microvoid formation.

However, there were some issues that cropped up during this study. The first and foremost was the load carried by the tire at peak displacement, which was not as high as would be required for a system to withstand seismic forces. Studies have to be conducted to improve the load carrying capacity of the tire. Common sense suggests that a bigger tire would be capable of carrying higher loads, but only further tests would ascertain it. Local bucking of the tire was observed at high displacements and the effect of this bucking on the energy dissipation capacities should be studied to determine whether the local bucking needs to be prevented. Research needs to be done in order to determine whether the inherent energy dissipation capacity of the tire can be further enhanced by infilling the tire with materials like crushed aluminum cans, stone aggregates etc.

4.6.1 Recommendations

The strength of the tire in tension should be high, in order to be able to carry the seismic loads, so that they can be used in a lateral dissipative bracing. The strength of the tire can be increased by using one of the following proposed methods.

4.6.1.1 Tires in series

The strength of the bracing could be increased by using tires in series as shown in Figure 4.47, whereby the strength would be expected to be double the strength of one tire. Intuitively, one might expect an increase in the energy dissipation of the bracing due to frictional damping, if the tires are in contact. This method should be relatively easy to
implement considering the varieties of tire sizes available with varying diameter and varying thicknesses.

4.6.1.2 Use of helical coils

The strength of the tire and/or its damping characteristics may be improved by using helical coils (also called wire rope) of steel inside the tire as shown in Figure 4.47. These coils would be placed within the tire such that they provide an additional increase in the strength and/or energy dissipation of the tire in tension.

![Figure 4.47: Innovative configurations to increase the load carrying capacity in a tire](image)

(a) Tires in series  
(b) Use of helical coils
Chapter 5

Visco-Hyperelastic Device

The visco-hyperelastic device used in this research provides energy dissipation by combining the effects of creep dissipation from the viscoelastic material and plastic dissipation due to the inelastic behavior of the steel elements. It incorporates the qualities of both displacement-dependent and velocity-dependent devices. During low levels of excitation, the device dissipates energy almost entirely due to the axial strains in the viscoelastic material, and at higher levels of excitation, energy is dissipated by a combination of creep dissipation and dissipation due to yielding in the inelastic steel elements. The device not only dissipates energy, but also provides additional stiffness to the structure at higher levels of deformation due to its hyperelastic behavior. The increased stiffness provided by these devices compensates for the loss of stiffness in the structural elements due to severe yielding and negative geometric stiffness associated with the P-Δ effects.

Energy dissipation in the viscoelastic materials is pronounced at increased levels of axial strains in the element. The concept of amplifying displacement in order to magnify damping effects is utilized by Constantinou et al. [1997] in a toggle brace system. Charney and McNamara [2002] exploited the same idea in order to control the structural response of a 39-story office building, in which viscous fluid dampers were used in a toggle brace configuration. The toggle brace configuration consists of a damper placed diagonally and linked to two non-collinear steel elements, which causes any small interstory drift to be amplified and improves the device performance. The toggle brace configuration is discussed in detail in Chapter 7.
5.1 Ellipse

When a circular element is loaded under uniaxial tension or compression, the displaced shape is usually an ellipse. The displacement of the device is governed by the equation for an ellipse, which determines the deformation in the direction of loading $\delta_{DL}$ and the corresponding displacement in the direction orthogonal to the loading $\delta_{ODL}$ in the device under tensile loading.

An ellipse, as shown in Figure 5.1, is a curve that is the locus of all points in the plane, the sum of whose distances $r_1$ and $r_2$ from two fixed points $F_1$ and $F_2$ (the foci) separated by a distance of $2c$ is a given positive constant $2a$ [Hilbert and Cohn-Vossen, 1999]. This results in a two-center bipolar coordinate equation

$$r_1 + r_2 = 2a$$  \hspace{1cm} (5.1)

The standard form of the equation of an ellipse with center at $(h, k)$, semimajor distance $a$ and semiminor distance $b$ (assuming $b < a$) is

$$\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1$$  \hspace{1cm} (5.2)

The semimajor distance $a$ is always greater than the semiminor distance $b$ for an ellipse. For a circular ring subjected to uniaxial compressive load (Figure 5.2), the change in the horizontal and vertical diameters is given by:

$$D_h = \frac{WR^3}{EI} \left(\frac{2}{\pi} - \frac{k_1}{2}\right)$$

$$D_v = -\frac{WR^3}{EI} \left(\frac{\pi k_1}{4} - \frac{2}{\pi}\right)$$  \hspace{1cm} (5.3)
where, $D_H$ and $D_V$ are the changes in horizontal and vertical diameters respectively, $W$ is the applied load, $R$ is the radius to the centroid of the cross section, $E$ is the modulus of elasticity, $I$ is the moment of inertia of the cross section and $k_1, k_3$ are constants which can be taken as unity if there is no correction in hoop stress or shear deformation required. For any given load and sectional properties, the change in vertical diameter is greater than the change in the horizontal diameter. This implies that the displacement in the direction of loading is greater than the displacement in the orthogonal direction.

This governs the displacement of the device. The displacement in the direction of loading is always greater than the displacement in the corresponding orthogonal direction. The elongation and contraction occur simultaneously under tensile and compressive loading. The elongation produces tensile stresses in the viscoelastic material and the contraction causes compressive stresses. When the device expands or contracts, the entire volume of viscoelastic material is subjected to tensile and compressive strains simultaneously.

![Figure 5.1: Ellipse parameters](image-url)
5.2 Device Constituents

The visco-hyperelastic device comprises of a viscoelastic material sandwiched between two circular steel rings of same or varying thickness as shown in Figure 5.3. The viscoelastic material is bonded to the steel rings and hence, it behaves monolithically under tensile and compressive loading. The device behaves identically under tension and compression, thus enabling the use of only one device under cyclic loading.

Under low levels of seismic excitation, the device dissipates energy primarily through the axial strains in the viscoelastic material. The steel rings remain elastic, and do not dissipate energy due to inelastic behavior. The axial stiffness of the device is primarily due to the outer steel ring, and the axial strain in the viscoelastic material can be controlled by varying the thickness of the steel rings. The force-deformation relation for the device is nonlinear with increased stiffness at higher levels of deformation and lower stiffness at low levels of deformation. This type of behavior is desired since the device would not increase the base
shear demands at service level loads as much as a conventional device would, and also provide the additional stiffness needed to compensate for the loss of stiffness of the structural elements at higher deformation demands.

Under higher levels of seismic excitation, there is plastic dissipation due to inelastic behavior of the steel elements, in addition to the creep dissipation in the viscoelastic material. It is desirable to vary the thickness of the steel elements, such that the creep dissipation and the plastic dissipation are almost equal, so as to extract the best behavior from the device.

5.3 Device parameters

The device behavior can be controlled by varying one or more of the following parameters.

- Breadth ($b$) and depth ($d$) of viscoelastic material
- Thickness of the inner ($t_i$) and outer ($t_o$) steel rings
- Type of steel (mild or high strength steel)
- Diameter ($d_o$) of the device
- Properties of the filler

The effect of these parameters on the device response is presented in detail in the next chapter with detailed finite element analyses under various types of loading.

5.4 Structural implementation of the device

The device would be implemented in a structural framework as a part of the bracing which could either be in diagonal, chevron or toggle configurations. The device could be attached to these bracing members either in the form of bolted or welded connections as shown in Figure 5.4. The designs of the connections were not a part of this research and the connections discussed herein are only proposed details.
In the bolted connection, the connecting rod would pass through the steel rings and the viscoelastic material and would be bolted on both the inside and outside of the steel rings using seating plates using high strength bolts. In the case of a welded connection, the connecting rod would be welded to the outer steel ring using high strength welds. The use of these connections for the device would require considerable experimental investigation. It can also be seen that implementing a welded connection would be easier compared to the bolted connections.

Figure 5.3: The visco-hyperelastic device
Figure 5.4: Device connection details
Chapter 6

Finite element analysis of the device

6.1 Introduction

In the previous chapter, the details of the proposed visco-hyperelastic device were discussed. A detailed finite element model was generated and analyzed to completely understand the behavior of the device. This was followed by parametric studies to determine the effect of the rubber depth, the ring thickness and the steel type on the performance of the device. This chapter discusses the modeling of the device and the parametric studies using the finite element software package ABAQUS. The various material models available in ABAQUS that were used to incorporate the viscoelastic behavior of rubber and inelastic behavior of steel are also discussed.

The rubber core, including its hyperelastic and viscoelastic properties, was modeled using three dimensional solid elements. The inelastic behavior of the steel elements was modeled using the quadrilateral shell elements utilizing the Von Mises yield surface. The steel elements were modeled as quadrilateral shell elements and incorporated as skin reinforcement around the rubber core. These are explained in detail in sections 6.3.1 and 6.3.2. The nonlinear effects under large deformations of the viscoelastic material were also considered.

In order to model the effects of various parameters on the device behavior accurately, the device was then studied under the effects of different types of quasi-static tensile and compressive loads and harmonic loads.
6.2 ABAQUS/CAE environment

ABAQUS/CAE is the visual modeling environment of ABAQUS. It is a forms-based, mouse-driven menu system for the operation of all tasks associated with the finite element model, such as modeling the geometry, automatic meshing, assigning load steps and boundary conditions, element selections and assigning materials.

ABAQUS/CAE is divided into functional units called modules. Each module consists of only those tools that are relevant to a specific portion of the modeling task. The modules are arranged in a logical sequence that may be followed to create the model.

Part

The part module allows the user to create individual parts by sketching their geometry directly in the CAE environment. It also allows the user to import geometries created from other geometric modeling programs.

Property

A section definition contains information about the properties of a part. The property module is used to create various sections such as solids, shells and truss/beam elements having their own cross-sectional geometry. Material properties are also defined and assigned to the various sections in this module.

Assembly

An assembly is a collection of various parts. The parts are created in their own local axes. In the assembly module, instances of these parts are created and positioned relative to each other in a global coordinate system. An ABAQUS model can contain any number of parts but only one assembly.

Step

The analysis steps and the output requests are configured in the step module. The step module is used to create various loading types such as static, dynamic, linear perturbation
and frequency analysis. The numerical time steps in the analysis are also defined in this module.

Interaction

The interaction module is used to specify mechanical and thermal interactions between parts of a model. One such example could be contact between two surfaces. Constraints such as tie, equation and rigid body motion are defined in this module. ABAQUS/CAE does not recognize mechanical contact unless it is explicitly defined.

Load

The load module allows the user to specify loads, boundary conditions and load cases. Loads and boundary conditions are step-dependent, which means that it is necessary to specify the step in which these conditions are active. Boundary conditions that are going to remain unchanged throughout an analysis, such as a fixed base restraint in a frame, can be specified in the initial step.

Mesh

The mesh module is used to generate the finite element mesh for the model. The mesh module is also used to select the type of element to be used in the analysis. ABAQUS also has automatic mesh control and adaptive meshing for advanced modeling.

Job

The job module is used to analyze the model created using the above steps. The job module allows an interactive job submission and monitoring. Multiple models and runs may be submitted and monitored simultaneously depending on the way the software is configured.

Visualization

Once the analysis has been completed, the visualization module is used to graphically display the results of the finite element models. The deformed configuration, stress plots and other output variables requested in the step module can be plotted and written to output data files using this module.
6.3 Modeling the visco-hyperelastic device

The visco-hyperelastic device was modeled as a solid rubber core sandwiched between two steel rings. The rubber was modeled incorporating the viscoelastic and the hyperelastic properties. The plasticity of the steel rings was modeled using the true stress$^{12}$ and true strain$^{13}$ values for mild steel and high strength steel.

6.3.1 Rubber modeling

Force equals stiffness times the deflection is probably the first equation that an engineer encounters. This assumption is however, only valid for linearly elastic materials. Rubber and other elastomers undergo large elastic deformations but they are highly nonlinear in nature. Hence, elastic modulus is almost never used in the modeling of rubber. The Poisson’s ratio for elastomers is between 0.499 and 0.5 which means that when a rubber block is compressed, its volume remains virtually unchanged unless very high pressures are applied, instead the block expands laterally and the volume remains unchanged. From the equations of linear elasticity,

\[
K = \frac{E}{3(1-2\nu)} \quad (6.1)
\]

\[
G = \frac{E}{2(1+\nu)} \quad (6.2)
\]

where \(K\) is the bulk modulus, \(E\) is Young’s Modulus (or) modulus of elasticity, \(G\) is the shear modulus and \(\nu\) is the Poisson’s ratio.

\[12\text{ True stress is defined as the ratio of the applied load (P) to the instantaneous cross-sectional area (A) i.e., } \sigma_T = \frac{P}{A}\]

\[13\text{ True strain is defined as the sum of all the instantaneous engineering strains i.e., } \varepsilon_t = \int_{t_0}^{t_f} \frac{dl}{l}\]
In equation 6.1, if the Poisson’s ratio is assumed to be 0.5, corresponding to an incompressible material, then the bulk modulus becomes infinity. This assumption also dictates that $E = 3G$. This is however not true for most elastomers, making closed form solutions impossible. Therefore, the equations of linear elasticity are no longer valid for an elastomer.

6.3.1.1 Hyperelasticity

Hyperelasticity refers to materials which can experience large elastic strain that is recoverable. Rubber and many other polymer materials fall in this category. The constitutive behavior of hyperelastic materials are usually derived from the strain energy potentials which are discussed in detail in the following section. ABAQUS has many built-in energy functions that can be used to model rubber hyperelasticity accurately. Hyperelastic materials behave elastically along a nonlinear stress-strain curve and are defined by a cubic polynomial relationship for the force and deformation of the related element as shown in Figure 6.1. The deformation of hyperelastic materials remains elastic up to large strain values (often well over 100%).

![Figure 6.1: Typical force-deformation curve for a hyperelastic material](image)
ABAQUS makes the following assumptions when modeling a hyperelastic material:

- The material behavior is elastic.
- The material behavior is isotropic.
- The material is incompressible unless otherwise specified.
- The simulation will include nonlinear geometric effects.

6.3.1.1.1 Hybrid elements

ABAQUS has a special family of elements to model hyperelastic behavior called ‘hybrid’ elements, which must be used to model fully the incompressible behavior in hyperelastic materials. An incompressible material response cannot be modeled with regular elements (except in the case of plane stress elements) because the pressure stress in the element is indeterminate. If the material is incompressible, then it cannot undergo any volume change. As a result, the stress cannot be calculated based on the displacement in the nodes. Hybrid elements include an additional degree of freedom to calculate the stress in the element directly. These ‘hybrid’ elements are identified by the letter ‘H’ in their name, for example, the hybrid form of a 20-node brick element, C3D20, is called C3D20H.

6.3.1.2 Hyperelastic material models

Stresses and strains are related to one another in ABAQUS through strain energy density function \( W \)\(^{14} \) instead of Young’s modulus and Poisson’s ratio. The stress in the material can then be obtained from the following relation:

\[
\sigma = \frac{\partial W}{\partial \varepsilon} \tag{6.3}
\]

where \( \sigma \) and \( \varepsilon \) are the stress and the strain in the element respectively.

\(^{14}\) Strain energy density is the amount of energy stored elastically in a unit volume of a material under a state of strain.
Several material models are available in ABAQUS for modeling hyperelasticity, which are listed below:

- The polynomial form and its various cases – the reduced polynomial form, the neo-Hookean form, the Mooney-Rivlin form and the Yeoh form.
- The Ogden form
- The Arruda-Boyce form
- The Van der Waals form

### 6.3.1.2.1 Polynomial form and particular cases

The polynomial form of the strain energy potential is the most commonly used strain energy density function, which can be mathematically represented as:

\[
W = \sum_{i+j=1}^{N} C_{ij} (\overline{T}_1 - 3)^i (\overline{T}_2 - 3)^j + \sum_{i=1}^{N} \frac{1}{D_i} (J_{el} - 1)^{2i}
\]  

(6.4)

where \(W\) is the strain energy density function, \(\overline{T}_1, \overline{T}_2\) are the measure of distortion in the material, \(C_{ij}\) describes the shear behavior of the material, \(D_i\) introduces the material incompressibility and \(J_{el}\) is the elastic volume strain. The parameter \(N\) can take up to six values; however, values of \(N\) greater than 2 are rarely used.

Regardless of the value of \(N\), the initial shear modulus \(\mu_o\) and the bulk modulus \(K_o\) depend only on the polynomial coefficients of order \(N=1\).

\[
\mu_o = 2(C_{01} + C_{10})
\]  

(6.5)

and

\[
K_o = \frac{2}{D_1}
\]  

(6.6)
If $N=1$, so that only the linear terms in the deviatoric strain energy are retained, the Mooney-Rivlin form is recovered:

$$W = C_{10} (T_1 - 3) + C_{01} (T_2 - 3)$$  \hspace{1cm} (6.7)

If $C_{01}$ is also zero, the material is referred to as neo-Hookean. The neo-Hookean model often serves as the prototype model for hyperelastic behavior in the absence of accurate experimental data. Yeoh [1993] studied the effect of the second order invariance on the general polynomial series expansion, and stated that the sensitivity of the strain energy function to changes in the second invariant is generally much smaller than the sensitivity to changes in the first invariant. In addition, the $T_2$ dependence was difficult to measure, and hence it was preferable to neglect it completely instead of using erroneous values. The Yeoh form can be viewed as special case of a reduced polynomial with a value of $N=3$.

$$W = \sum_{i=1}^{3} C_{i0} (T_i - 3)^i + \sum_{i=1}^{3} \frac{1}{D_i} (J_{el} - 1)^{2i}$$  \hspace{1cm} (6.8)

### 6.3.1.2.2 Ogden form

The Ogden strain energy potential is expressed in terms of the principal stretches. In ABAQUS the following formulation is used:

$$W = \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i} \left( \bar{\lambda}_i^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3 \right) + \sum_{i=1}^{N} \frac{1}{D_i} (J_{el} - 1)^{2i}$$  \hspace{1cm} (6.9)

where $\bar{\lambda}_i = J^{-\frac{1}{3}} \lambda_i$, $\lambda_i$ is a principal stretch ratio, and $N, \mu_i, \alpha_i$ and $D_i$ are material parameters which may be functions of temperature. $J_{el}, J$ and $D_i$ have the same definitions as in the polynomial form.
6.3.1.2.3 Arruda-Boyce form

The hyperelastic Arruda-Boyce potential has the following form:

\[
W = \mu \sum_{i=1}^{4} \frac{C_i}{\lambda_{m}^{2i-2}}\left(T_i' - 3\right) + \frac{1}{D}\left(\frac{J_{el}^2 - 1}{2} - \ln J_{el}\right)
\]  
(6.10)

where

\[
C_1 = \frac{1}{2}, \quad C_2 = \frac{1}{20}, \quad C_3 = \frac{11}{1050}, \quad C_4 = \frac{19}{7000}, \quad C_5 = \frac{519}{673750}
\]

The deviatoric part of the strain energy density comes from Arruda and Boyce [1993]. This model is also known as the eight-chain model, since it was developed starting out from a representative volume element where eight springs emanate from the center of a cube to its corners.

6.3.1.2.4 Van der Waals form

The hyperelastic Van der Waals potential, also known as the Kilian model, has the following form:

\[
W = \mu \left\{-(\lambda_{m}^2 - 3)\left[\ln(1 - \eta) + \eta\right] - \frac{2}{3} a \left(\frac{\tilde{I} - 3}{2}\right)^{\frac{3}{2}} + \frac{1}{D}\left(\frac{J_{el}^2 - 1}{2} - \ln J_{el}\right)\right\}
\]  
(6.11)

where \(\tilde{I} = (1 - \beta)\tilde{T}_1 + \beta\tilde{T}_2\) and \(\eta = \sqrt{\frac{\tilde{I} - 3}{\lambda_{m}^2 - 3}}\). The parameter \(\beta\) represents a linear mixture parameter combining \(\tilde{T}_1\) and \(\tilde{T}_2\) into \(\tilde{I}\). The global interaction parameter ‘\(a\)’ models the interaction between the chains.

\[
a = \frac{2C_{01}}{3\mu} + \frac{\lambda_{m}^2}{\lambda_{m}^3 - 1}
\]  
(6.12)
6.3.1.3  Modeling viscoelastic behavior

Viscoelasticity, as the name implies, is a generalization of elasticity and viscosity. Rubber exhibits a rate-dependent behavior and can be modeled as a viscoelastic material, with its properties depending on temperature and creep (creep, stress relaxation). Creep is an increase in deformation under constant stress. Stress relaxation, on the other hand, is the decrease in stress with time under a constant strain.

The viscoelastic model offered in ABAQUS can be described by a one dimensional model subjected a time varying strain $\gamma (t)$. The stress response $\tau (t)$, assuming viscoelastic behavior is given by:

$$\tau = G \left( \gamma + \int_0^t g(t-s)\dot{\gamma}(s)ds \right)$$  \hspace{1cm} (6.13)

where $G$ is the long term elastic shear modulus and $g(t)$ is the ‘shear relaxation function’. The time dependent $\tau$-$\gamma$ relationship is characterized by a time dependent relaxation modulus $G_R(t)$:

$$\tau(t) = G_R(t)\gamma$$  \hspace{1cm} (6.14)

For time history analysis, ABAQUS assumes that the material is defined by the Prony series expansion of the relaxation modulus:

$$G_R(t) = G_0 \left( 1 - \sum_{i=1}^N \bar{G}_i^P \left( 1 - e^{-t/\tau_i^G} \right) \right)$$  \hspace{1cm} (6.15)

where $G$ is the long term shear modulus, $N, G_i^P$ and $\tau_i^G$ are material constants and

$$G_0 = G + \sum_{i=1}^N G_i^P$$
The viscoelastic properties can be defined in *ABAQUS* by specifying any one or more of the following parameters:

- Parameters of the Prony series representation of the relaxation moduli.
- Shear and volumetric creep tests.
- Shear and volumetric relaxation tests.

### 6.3.2 Modeling steel elements

The steel elements in the device were modeled as an elastic-plastic material. Steel has an approximately linear elastic behavior at low strain magnitudes, however, at higher stress magnitudes; it displays a nonlinear, inelastic behavior which is referred to as plastic behavior. The plastic behavior of a material is defined by its yield point and post-yield hardening.

The deformation of a metal prior to reaching its yield stress is elastic in nature and can be recovered upon unloading. Beyond the yield stress, the metal undergoes a permanent deformation. Under ground motions, inelastic behavior of steel if often expected leading to considerable energy dissipation due to yielding. It is therefore, important to model the inelastic behavior of the steel elements. In order to define the plasticity in *ABAQUS*, true stress and true strain values are used. Quite often material data are supplied using value of nominal stress and nominal strain. The relationship between the nominal strain and true strain is expressed as:

\[
\varepsilon_{\text{nominal}} = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1
\]  

(6.16)

The above expression can be simplified as follows:

\[
\varepsilon_{\text{true}} = \ln(1 + \varepsilon_{\text{nominal}})
\]  

(6.17)

Assuming material incompressibility, there is no volume change and hence \( l_0A_0 = lA \). The instantaneous area is related to the original area as follows
\[ A = A_0 \frac{l_0}{l} \]  

(6.18)

Substituting the above equation into the definition of true stress gives

\[ \sigma = \frac{F}{A} = \frac{F}{A_0} \frac{l}{l_0} = \sigma_{\text{norm}} \left( \frac{l}{l_0} \right) = \sigma_{\text{norm}} \left( 1 + \varepsilon_{\text{norm}} \right) \]  

(6.19)

The plastic strain in the material can be obtained from the following relation:

\[ \varepsilon^{\text{pl}} = \varepsilon' - \frac{\sigma}{E} \]  

(6.20)

where \( \varepsilon^{\text{pl}} \) is the true plastic strain, \( \varepsilon' \) is the true total strain, \( \sigma \) is the true stress, and \( E \) is the Young’s modulus.

### 6.3.3 Modeling the device

The device was modeled using steel and viscoelastic materials. The rubber block was modeled using experimental test data from Yoshida et al. [2004] and Amin et al. [2002], which was used by ABAQUS to compute the strain energy constants using a least squares method. Unlike plasticity data, the test data for the hyperelastic material is specified as nominal stress and nominal strain values. The experimental tests which ABAQUS can use to fit data are:

- Uniaxial tension and compression.
- Equibiaxial tension and compression.
- Planar tension and compression (pure shear)
- Volumetric tension and compression

The hyperelastic properties of the rubber core was modeled using the results from the material tests on high damping rubber by Amin et al. [2002] and Yoshida et al. [2004], which includes uniaxial tension, biaxial tension and uniaxial compression tests on the high damping
rubber. The viscoelastic material behavior was modeled from relaxation tests conducted on high damping rubber by Yoshida et al. [2004]. The properties of rubber used in modeling the device are presented in Table 6.1. The inelastic behavior of the steel elements were modeled using the true stress-strain values for mild steel and high strength steel as given in Table 6.2.

### Table 6.1: Rubber properties used in material modeling

<table>
<thead>
<tr>
<th>Nominal stress (ksi)</th>
<th>Nominal Strain</th>
<th>Stress (ksi)</th>
<th>Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.2175</td>
<td>-0.5</td>
<td>0.450</td>
<td>0.1</td>
</tr>
<tr>
<td>-0.1595</td>
<td>-0.4</td>
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</tr>
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<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

The rubber block was modeled using 20-node quadratic brick, hybrid, linear pressure, reduced integration elements (*C3D20RH*). The steel ring was modeled as ‘skin reinforcement’ (Figure 6.2) using 8-node doubly curved shell, reduced integration elements (*S8R*). Skin reinforcement defines a skin that is bonded to the surface of an existing part and specifies its engineering properties.
### Table 6.2: Nominal stress and plastic strain for steel

<table>
<thead>
<tr>
<th>True stress (ksi)</th>
<th>Plastic strain</th>
<th>True stress (ksi)</th>
<th>Plastic strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.05</td>
<td>0</td>
<td>29.029</td>
<td>0</td>
</tr>
<tr>
<td>51.07</td>
<td>0.0193</td>
<td>35.67</td>
<td>0.0235</td>
</tr>
<tr>
<td>61.39</td>
<td>0.0467</td>
<td>42.63</td>
<td>0.0474</td>
</tr>
<tr>
<td>72.77</td>
<td>0.0928</td>
<td>54.23</td>
<td>0.0935</td>
</tr>
<tr>
<td>81.39</td>
<td>0.1366</td>
<td>63.67</td>
<td>0.1377</td>
</tr>
<tr>
<td>83.10</td>
<td>0.1795</td>
<td>69.6</td>
<td>0.18</td>
</tr>
<tr>
<td>84.61</td>
<td>0.2202</td>
<td></td>
<td></td>
</tr>
<tr>
<td>87.00</td>
<td>0.2594</td>
<td></td>
<td></td>
</tr>
<tr>
<td>88.26</td>
<td>0.2971</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 6.2: Skin reinforcements**

In Figure 6.2, the beam has a solid core and a skin of a different material on the top and bottom faces. In the mesh module, solid elements can be added to the solid core and shell elements to the skin reinforcements. The solid and shell elements share the same nodes.
6.4 Parametric studies

There are many parameters that govern the performance of the device. It is therefore necessary to study the effects of these parameters under various loading types (Table 6.3) and determine the optimum values for each of the following parameters:

- Type of steel (mild or high strength steel)
- Rubber depth ($d$)
- Thickness of the inner ($t_i$) and outer ($t_o$) steel rings

<table>
<thead>
<tr>
<th>Loading type</th>
<th>Force (kips)</th>
<th>Duration (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-static tension</td>
<td>$F = \frac{f}{60} \cdot 20^k$</td>
<td>60</td>
</tr>
<tr>
<td>Pseudo-static compression</td>
<td>$F = \frac{f}{60} \cdot 20^k$</td>
<td>60</td>
</tr>
<tr>
<td>Harmonic sinusoidal</td>
<td>$F = 20^k \sin(\pi t)$</td>
<td>6</td>
</tr>
</tbody>
</table>

6.4.1 Effect of steel type

The plastic energy dissipation in the device depends on the type of steel that is used. Mild steel (A36) with a yield strength of 36 ksi, density of 0.283 lb/cu. in. and high strength steel (A592) with a yield strength of 50 ksi, density of 0.287 lb /cu. in. were used for the analysis. The device displacements and energy dissipation were studied to ascertain the effect of the steel type of the behavior of the device.

The effective device diameter was kept constant at 45 in. The rubber breadth and depth are 9 in. and 12 in. respectively. The inner and outer steel ring thickness is kept as 1 in., and steel type is varied from mild steel in the first analysis to high strength steel in the second. The device is then tested under quasi-static tensile and compressive loading and harmonic loading. Figure 6.3 shows the device behavior under tension and compression.
(a) Device under tension

(b) Device under compression

Figure 6.3: Displaced configuration of the device under tension and compression
At the end of the tensile loading, the displacement in the direction of loading ($\delta_{DL}$) in the device was 0.74 in. and 0.70 in. for mild steel and high strength steel respectively. The device displacement in the direction orthogonal to the loading ($\delta_{ODL}$) was 0.64 in. and 0.62 in. for mild steel and high strength steel respectively. The plastic dissipation was 0.60 kip-in. and 0.0 kip-in. and the viscoelastic dissipation was 1.46 kip-in. and 1.35 kip-in. for the two cases respectively. The results of the analyses are shown in Figure 6.4 and 6.5.

The device was then subjected to a quasi-static compressive loading under the parameters mentioned above and the results are shown in Figure 6.6 and 6.7. The device displacements $\delta_{DL}$ and $\delta_{ODL}$ were 0.77 in., 0.73 in. and 0.64 in. and 0.62 in. for mild steel and high strength steel respectively. The creep dissipation in the viscoelastic material was 1.57 kip-in. and 1.44 kip-in. respectively and the dissipation due to inelastic behavior in steel was 0.65 kip-in. and 0.0 kip-in. for the two test cases respectively.

Under sinusoidal harmonic loading of $\pi$ radians/second, the peak displacement $\delta_{DL}$ under tension and compression were 0.71 in. and 0.74 in. respectively for mild steel and 0.68 in. and 0.71 in. for high strength steel. The energy dissipation due to yielding in steel was 5.80 kip-in. and 0.0 kip-in and creep dissipation in the viscoelastic material was 7.08 kip-in. and 6.43 kip-in. for the two test cases respectively. The results are shown in Figure 6.8 and 6.9.

Based on the parametric study, it is seen that the type of steel did not have any influence on the device displacements under the applied loading. The energy dissipation in this device is designed to be a combination of viscoelastic dissipation in the rubber and plastic dissipation in the steel elements. High strength steel, due to its higher yield strength, does not offer considerable dissipation due to yielding at low and medium levels of displacement. However, using mild steel potentially adds an additional source of energy dissipation to complement the energy dissipation in the rubber. High strength steel would dissipate energy due to yielding, but the device displacements would have to be considerably higher. Energy dissipation would be only due to the viscoelastic material, but the use of mild
steel would not only provide energy dissipation at low levels of deformation, but would also
be considerably cheaper. Accordingly, the device is designed with mild steel.

The energy plots for each of the above analyses are presented, with the input energy
\( \text{ALLWK}^{15} \), the creep dissipation \( \text{ALLCD}^{16} \) and the plastic dissipation \( \text{ALLPD}^{17} \) plotted as
functions of time. \textit{DL displacement} refers to the displacement in the device in the direction
of loading and \textit{ODL displacement} refers to the corresponding displacement in the
orthogonal direction.

\textsuperscript{15} ALLWK represents the input energy on the system from the displacement controlled loading
\textsuperscript{16} ALLCD represents the energy dissipated by the device due to creep in the viscoelastic material
\textsuperscript{17} ALLPD represents the energy dissipated due to the inelastic behavior of the steel in the device
Figure 6.4: Effect of steel type of device displacement under tensile loading
Figure 6.5: Effect of steel type on energy dissipation under tensile loading
Figure 6.6: Effect of steel type of device displacement under compressive loading
Figure 6.7: Effect of steel type on energy dissipation under compressive loading

(a) Mild Steel

(b) High strength steel
Figure 6.8: Effect of steel type on device displacement under harmonic loading

(a) Mild steel

(b) High strength steel
Figure 6.9: Effect of steel type on energy dissipation under harmonic loading
6.4.2 Effect of rubber depth

To study the effect of the rubber depth on the performance of the device, the rubber depth was varied from 6.0-15.0 in. keeping the other parameters constant. The device diameter was taken as 45.0 in. and the steel rings had a thickness of 1.0 in. and were made of mild steel. The breadth of rubber was maintained constant at 9.0 in. The device was first subjected to a quasi-static tensile loading followed by a quasi-static compressive load and a sinusoidal harmonic load. The dissipation from the viscoelastic material is a function of the volume of the material; however, the device aspect ratio\(^{18}\) has a more controlling influence on the device stiffness and performance.

The effect of the rubber depth on the device displacement and energy dissipation were studied. First, the device with varying rubber depth was tested under a tensile loading. The results of this analysis are shown in Figure 6.10. The displacement in the direction of loading \((\delta_{DL})\) was 2.52 in., 1.21 in., 0.74 in. and 0.52 in. for rubber depths of 6.0 in., 9.0 in., 12.0 in. and 15.0 in. respectively. The corresponding displacement in the orthogonal direction \((\delta_{DOL})\) was 2.04 in., 1.0 in., 0.64 in. and 0.48 in. respectively. The energy dissipation due to the creep dissipation in the viscoelastic material was 6.45 kip-in., 2.56 kip-in., 1.46 kip-in. and 1.11 kip-in. for the four cases respectively. The energy dissipation due to plastic dissipation in the steel elements was 12.23 kip-in., 3.83 kip-in., 0.60 kip-in. and 0.19 kip-in. respectively. The energy dissipation plots are shown in Figure 6.12.

The device was then subjected to a pseudo-static compressive loading. The results of this analysis are shown in Figure 6.11. The device displacement \(\delta_{DL}\) was 2.30 in., 1.04 in., 0.64 in. and 0.48 in. for rubber depths of 6.0 in., 9.0 in., 12.0 in. and 15.0 in. respectively. The corresponding device displacement \(\delta_{DOL}\) was 3.16 in., 1.34 in., 0.77 in. and 0.57 in. respectively. The energy dissipation due to the creep dissipation in the viscoelastic material was 9.32 kip-in., 3.01 kip-in., 1.57 kip-in. and 1.17 kip-in. for the four cases respectively. The energy dissipation due to plastic dissipation in the steel elements was 16.75 kip-in., 4.71 kip-in., 1.66 kip-in. and 0.71 kip-in. respectively.

\(^{18}\) Aspect ratio is the ratio of the breadth of the viscoelastic material to its depth.
in., 0.65 kip-in. and 0.22 kip-in. respectively. The energy dissipation plots are shown in Figure 6.13.

Devices with rubber depths of 6.0 in., 9.0 in., 12.0 in. and 15.0 in. were subjected to sinusoidal loading of frequency $\pi$ radians/second. The device displacements under tension were 2.35 in., 1.11 in., 0.71 in. and 0.53 in. for the various cases respectively. The energy dissipation at the end of three cycles due to the creep in the viscoelastic material was 29.27 kip-in., 11.41 kip-in., 7.08 kip-in. and 5.18 kip-in. for the four cases respectively. The energy dissipation due to inelastic behavior of the steel elements was 115.40 kip-in., 30.78 kip-in., 5.80 kip-in. and 1.35 kip-in. respectively. The analysis plots are shown in Figure 6.14 and 6.15.

Based on the parametric study, the rubber depth has a considerable effect of the device stiffness and energy dissipation. The device displacement is directly dependent on the rubber depth, and smaller the depth of viscoelastic material, smaller is its volume, and consequently, smaller the viscoelastic dissipation. The smaller the rubber depth, the greater is the plastic dissipation (Figure 6.12 a, b) even at low levels of displacement. The device is designed to dissipate energy primarily due to viscoelastic behavior of the rubber at low levels of displacement and due to a combination of plastic energy dissipation and creep at higher displacements. Hence, the rubber depth of 12 in. is the best suited for the visco-hyperelastic device for a rubber breadth of 9 in. since it provides an combined optimum amount of creep dissipation from the viscoelastic material and plastic energy dissipation from steel.

The energy plots for each of the analyses are presented, with the input energy ($\text{ALLWK}^{19}$), the creep dissipation ($\text{ALLCD}^{20}$) and the plastic dissipation ($\text{ALLPD}^{21}$) plotted as functions of time.

---

19 ALLWK represents the input energy on the system from the displacement controlled loading
20 ALLCD represents the energy dissipated by the device due to creep in the viscoelastic material
21 ALLPD represents the energy dissipated due to the inelastic behavior of the steel in the device
Figure 6.10: Effect of rubber depth on displacement of the device under tensile loading

Figure 6.11: Effect of rubber depth on displacement of the device under compressive loading
Figure 6.12: Effect of rubber depth on energy dissipation of the device under tensile loading
Figure 6.12: Contd…

(c) Rubber depth = 12.0 inches

(d) Rubber depth = 15.0 inches

Figure 6.12: Contd…
Figure 6.13: Effect of rubber depth on energy dissipation of the device under compressive loading
(c) Rubber depth = 12.0 inches

(d) Rubber depth = 15.0 inches

Figure 6.13: Contd…
Figure 6.14: Effect of rubber depth on displacement of the device under harmonic loading
(c) Rubber depth = 12.0 inches

(d) Rubber depth = 15.0 inches

Figure 6.14: Contd…
Figure 6.15: Effect of rubber depth on energy dissipation under harmonic loading
Figure 6.15: Contd…

(c) Rubber depth = 12.0 inches

(d) Rubber depth = 15.0 inches

Figure 6.15: Contd…
6.4.3 Effect of steel ring thickness

The viscoelastic material is sandwiched between two circular steel rings. The thickness of the steel rings can be varied to control the device stiffness, strength and energy dissipation. The cross section of the steel elements also controls the weight of the device. The effect of ring thickness on the device performance has been studied by varying the inner and outer thicknesses, keeping one of them constant while varying the other. The steel rings were made of mild steel.

The breadth and depth of rubber used was 9 in. and 12 in. respectively. The device diameter was maintained constant at 45 in. The steel ring thickness was varied from 0.75-1.25 in. During the first set of analyses, the inner ring thickness was kept constant at 0.75 in. and the outer ring thickness was varied from 0.75-1.25 in. During the second set of analyses, the outer ring thickness was kept constant at 0.75 in. and the inner ring thickness was varied from 0.75-1.25 in. The effect of these parametric changes on device displacement and energy dissipation was studied under tensile, compressive and harmonic loads.

The device was first analyzed under pseudo-static tensile loads keeping the inner ring thickness constant at 0.75 in. and outer ring thickness varied from 0.75-1.25 in. The device displacements \((\delta_{DL}, \delta_{ODL})\) were (1.46, 1.14) in., (0.77, 0.68) in. and (0.45, 0.40) in. for outer ring thicknesses of 0.75, 1.0, 1.25 in. respectively. The creep dissipation was 3.86 kip-in, 1.26 kip-in and 0.46 kip-in. and the plastic dissipation were 3.96 kip-in., 0.67 kip-in., and 0.12 kip-in. for the three cases respectively. For the next set, the outer ring was kept constant at 0.75 in. and the inner ring thickness was varied from 1.0-1.25 in., with the ring thicknesses of 0.75 in. being common to both test cases. The device displacement \((\delta_{DL}, \delta_{ODL})\) were (1.34, 1.02) in., and (1.29, 0.98) in. for inner ring thicknesses of 1.0 and 1.25 in. respectively. The creep dissipation was 4.09 kip-in and 4.18 kip-in. and the plastic dissipation were 3.22 kip-in. and 2.95 kip-in. for the two cases respectively. The results are shown in Figure 6.16 and 6.17.

Under the compressive load, the device displacement \(\delta_{DL}\) was 1.62 in., 0.81 in. and 0.47 in. for inner ring thickness of 0.75 in. and outer ring thickness of 0.75, 1.0 and 1.25 in.
respectively. The corresponding displacement $\delta_{ODL}$ was 1.06 in., 0.68 in. and 0.42 in. respectively as shown in Figure 6.18. The energy dissipation, as shown in Figure 6.19, due to the viscoelastic material were 4.49 kip-in., 1.36 kip-in. and 0.48 kip-in. and the plastic dissipation in steel were 4.69 kip-in., 0.73 kip-in. and 0.15 kip-in. for the three cases respectively. For a constant outer ring thickness of 0.75 in. and inner ring thickness of 1.0 in. and 1.25 in. the displacement $\delta_{DL}$ was 1.46 and 1.40 in. respectively. The corresponding displacements $\delta_{ODL}$ were 1.06 in. and 1.0 in. The creep dissipation in the viscoelastic material was 4.68 kip-in. and 4.76 kip-in. and the plastic dissipation was 3.78 kip-in. and 3.45 kip-in. respectively for the two cases.

The device was then subjected to sinusoidal harmonic loading of $\pi$ radians/second. The displacements $(\delta_{DL}, \delta_{ODL})$ were (1.38, 1.06) in., (0.75, 0.66) in. and (0.45, 0.40) in. for inner ring thickness of 0.75 in. and outer ring thickness of 0.75 in., 1.0 in. and 1.25 in. respectively. Viscoelastic dissipation and plastic dissipation were 17.49, 6.11, 0.45 kip-in. and 34.02, 6.54 and 0.20 kip-in. respectively for the three cases. For a constant outer ring thickness of 0.75 in. and inner ring thicknesses of 1.0 in. and 1.25 in., the device displacements $\delta_{DL}$ and $\delta_{ODL}$ were 1.26, 1.19 in. and 0.96, 0.92 in. respectively. Creep dissipation in the viscoelastic material was 18.58 kip-in. and 19.08 kip-in. and dissipation due to yielding was 26.36 kip-in. and 23.52 kip-in. respectively. The displacement and the energy dissipation plots are shown in Figure 6.20 and 6.21 respectively.

The device displacement was found to be proportional to the outer ring thickness, with the inner ring having negligible effect. The smaller the outer ring thickness, the greater was the displacement of the device and correspondingly greater the viscoelastic and creep dissipation. It was also noticed that higher the inner ring thickness in comparison to the outer ring thickness, greater was the tensile and compressive strains in the viscoelastic material. The inner ring remained circular (due to its higher thickness), while the outer ring became elliptical, which caused a corresponding increase in the strains in the viscoelastic material. The stiffness can be decreased and the energy dissipation increased by decreasing the thickness of the outer ring and vice versa. For the proposed device, it was decided to use an inner ring thicker than the outer ring.
Figure 6.16 [a-e]: Effect of ring thickness on device displacement under tensile loading

(a) Inner ring=0.75 in., outer ring=0.75 in.

(b) Inner ring=0.75 in., outer ring=1.0 in.
(c) Inner ring = 0.75 in., outer ring = 1.25 in.

(d) Outer ring = 0.75 in., inner ring = 1.0 in.

Figure 6.16: Contd...
(e) Outer ring = 0.75 in., Inner ring = 1.25 in.

Figure 6.16: Contd…

(a) Inner ring = 0.75 in., outer ring = 0.75 in.

Figure 6.17 [a-e]: Effect of ring thickness on energy dissipation under tensile loading
(b) Inner ring = 0.75 in., outer ring = 1.0 in.

(c) Inner ring = 0.75 in., outer ring = 1.25 in.

Figure 6.17: Contd…
Figure 6.17: Contd…

(d) Outer ring = 0.75 in., inner ring = 1.0 in.

(e) Outer ring = 0.75 in., Inner ring = 1.25 in.
Figure 6.18 [a-e]: Effect of ring thickness on device displacement under compressive loading

(a) Inner ring=0.75 in., outer ring=0.75 in.

(b) Inner ring=0.75 in., outer ring=1.0 in.
(c) Inner ring=0.75 in., outer ring=1.25 in.

(d) Outer ring=0.75 in., inner ring=1.0 in.

Figure 6.18: Contd…
(e) Outer ring = 0.75 in., Inner ring = 1.25 in.

Figure 6.18: Contd…

(a) Inner ring = 0.75 in., outer ring = 0.75 in.

Figure 6.19 [a-e]: Effect of ring thickness on energy dissipation under compressive loading
(b) Inner ring = 0.75 in., outer ring = 1.0 in.

(c) Inner ring = 0.75 in., outer ring = 1.25 in.

Figure 6.19: Contd…
Figure 6.19: Contd…

(d) Outer ring = 0.75 in., inner ring = 1.0 in.

(e) Outer ring = 0.75 in., Inner ring = 1.25 in.
Figure 6.20 [a-e]: Effect of ring thickness on device displacement under harmonic loading.

(a) Inner ring=0.75 in., outer ring=0.75 in.

(b) Inner ring=0.75 in., outer ring=1.0 in.
(c) Inner ring=0.75 in., outer ring=1.25 in.

(d) Outer ring=0.75 in., inner ring=1.0 in.

Figure 6.20: Contd…
(e) Outer ring = 0.75 in., Inner ring = 1.25 in.

Figure 6.20: Contd…

(a) Inner ring = 0.75 in., outer ring = 0.75 in.

Figure 6.21 [a-e]: Effect of ring thickness on energy dissipation under harmonic loading
Figure 6.21: Contd…

(b) Inner ring = 0.75 in., outer ring = 1.0 in.

(c) Inner ring = 0.75 in., outer ring = 1.25 in.
(d) Outer ring = 0.75 in., inner ring = 1.0 in.

(e) Outer ring = 0.75 in., Inner ring = 1.25 in.

Figure 6.21: Contd…
6.5 Summary

A detailed three-dimensional analysis on the device was conducted using the finite element software package ABAQUS, incorporating the viscoelastic and hyperelastic properties of rubber and the inelastic behavior of the steel elements. The effects of various device parameters on the device performance were studied under the effect of quasi-static tensile and compressive and harmonic loads. The rubber breadth was not studied because the device displacements depend not only on the ring thickness but also on the device diameter. Hence, only the rubber depth was studied keeping the rubber breadth a constant. However, studies need be conducted on varying the inner diameter of the device, and hence the rubber breadth, and its effect on the device performance. The parameters analyzed were

- Steel type (mild steel or high strength steel)
- Rubber depth
- Thickness of steel rings

Based on the results of the parametric study, it was decided to use mild steel because of the additional energy dissipation provided due to inelastic behavior of the steel elements. Also, it was noted that there was no drastic difference in device displacement due to the use of high strength steel in comparison to mild steel.

The energy dissipated by a viscoelastic material depends on the volume of the material and an optimum aspect ratio. Various rubber depths ranging from 6.0-15.0 in. with constant width of 9.0 in. were studied, and based on the performance of the device, it was decided to use a rubber depth of 12.0 in. since it provided a right balance of creep dissipation and plastic dissipation.

The effect of the steel ring thickness on the device performance was also studied. It was noted that the outer steel ring thickness determined the stiffness and strength of the device and the thickness of the inner ring determined the energy dissipation in the device. It was also decided to use a thicker inner ring in comparison to the outer ring to provide more energy dissipation in the device due to the viscoelastic material.
Chapter 7

Analysis of a single story frame with a visco-hyperelastic device

7.1 Introduction

In the previous chapter, a detailed finite element analysis was conducted on the proposed visco-hyperelastic device, including a study on the various parameters controlling the device performance. The device provided considerable energy dissipation with stable viscous and hysteretic damping. In this chapter, the effectiveness of the device in controlling the response of the structure to dynamic excitations is studied. The structure analyzed is a single story single bay steel frame, with the device implemented in a toggle-brace configuration [Constantinou et al., 1997]. The device performance is studied by comparing the results from an analysis of a bare steel frame with the results from an analysis of the same frame incorporating the visco-hyperelastic device. The modeling of the frame and its subsequent analysis was conducted using the finite element package ABAQUS.

7.2 Toggle-Brace-Damper system

Energy dissipation in a structure can be achieved in two possible ways, the first being in the form of specially detailed plastic hinge regions of beams and columns which dissipate energy due to inelastic behavior. These, however, cause permanent damage to the lateral force resisting system and prevent its subsequent reuse, which is undesirable. The other viable mechanism involves the introduction of additional energy dissipation systems in the
structure which are not part of the gravity load carrying frame. The concept of adding energy dissipation device (called ‘dampers’) to improve seismic response has been adopted by engineers as an integral part of a structure’s seismic protection system.

Solid viscoelastic materials dissipate energy due to shear deformation and axial strains in the element. The dissipation increases with the level of axial strains and hence, their effectiveness increases at higher axial strains. It is therefore, desirable to increase the displacements in the device. The proposed device uses a solid viscoelastic material sandwiched between two steel rings. The amplification of device displacement not only improves the damping characteristics, but allows for use of devices with lesser capacity.

The proposed visco-hyperelastic device can be implemented in a structural system in a diagonal brace, a chevron brace (Figure 7.1) or as a toggle brace. The toggle brace configuration would result in the best performance because of the displacement magnification that it provides. The device displacement would be equal to the interstory drift in the case of a chevron brace, and would be less than the interstory drift in the case of a diagonal brace.

![Diagram](image)

(a) Diagonal brace configuration  
(b) Chevron brace configuration

Figure 7.1: Brace configurations for the visco-hyperelastic device
7.2.1 Toggle-brace theory

Several configurations have been proposed for amplifying displacements, and can be used in energy dissipation systems. One of them is a lever mechanism [Hibino et al., 1989] and the other is based on a slider-crank mechanism [Constantinou et al., 1997].

Figure 7.2 illustrates the toggle-brace-damper system. The system consists of toggles ABC which are configured as a shallow truss. The damper is placed along one of the diagonals as shown.

![Tested toggle-brace-damper configuration](image)

Figure 7.2: Tested toggle-brace-damper configuration
When the frame sways to the right (in the $u^+$ direction), the point B displaces to the top, causing a rotation in the toggles. The resulting rotation causes displacements in the device. The displacements in the device are related to the interstory drift through simple equations. The connection at point B is to be designed as a true pin, and hence there is no bending in the system. Figure 7.3 shows the undeformed and deformed configurations of the toggle.

![Deformed toggle configuration](image1)

(a) Deformed toggle configuration

![Undeformed toggle configuration](image2)

(b) Undeformed toggle configuration

Figure 7.3: Deformed toggle configuration

Under the displaced configuration of the toggle, point B moves upward and assuming inextensible members would result in the following equation, which has been derived in detail in Appendix A.

\[
l_2^2 = h^2 + l_1^2 + (l + u)^2 - 2hl_1 \sin(\alpha \pm \phi) - 2(l + u)l_1 \cos(\alpha \pm \phi)
\]  

(6.21)
The displacement in the device, which would be equal to the movement of the point B to the deformed configuration, is given by the following equation.

\[ u_D = \pm l_1 \left[ \sqrt{1 + \left( \frac{\sin^2 (\alpha + \theta)}{\sin^2 \theta} \right)} - \frac{2 \cdot \sin (\alpha + \theta) \cos(\alpha \pm \phi)}{\sin \theta} \right] - \frac{\sin \alpha}{\sin \theta} \]  

(6.22)

Equations 7.1 and 7.2 reveal a complex nonlinear relationship between the story drift \( u \) and the device displacement \( \delta \). However, they can be simplified further as the rotation angle \( \phi \) and the displacement \( u \) are small compared to the structural dimensions. Based on this simplification, the device displacement and the interstory drift are related to one another by the following relation

\[ \delta = fu \]  

(6.23)

The quantity \( f \) is the displacement amplification factor, and depends only on the inclination of the toggles and is independent of its dimensions. Extensive research on the toggle-brace-damper system has been conducted by Constantinou et al. [1997, 2001] including the study on the position of these dampers and the effect of the inclination of the toggles on the amplification factor.

The toggle brace configuration can be used for frames where the interstory drift is less than the displacement in the toggles at which they ‘straighten out’, or in other words \( \alpha + \beta = 90^\circ \) in the deformed configuration. The limit on the frame displacement is given by the following relation:

\[ u = \left[ \sqrt{(l_1 + l_2)^2 - h^2} \right] - l \]  

(6.24)
7.3 Analysis of a single story plane frame

In order to determine the effect of the visco-hyperelastic device on the structural response under dynamic excitation, the device was attached to a single story single bay steel frame in a toggle configuration (Figure 7.2) and subjected to a series of displacement controlled harmonic loads of varying frequencies at varying amplitudes of roof displacement. The steel frame was also studied under unscaled real acceleration records of the Northridge (1994) and the El Centro (1940) earthquakes.

The steel frame used for the analysis had a bay width of 144 in. and a height of 144 in. The toggles were inclined at an angle of 31°, and had a length of 105 in. The toggle brace connections at the beam-column joint were combined welded-bolted that allowed for some rotation in order to attain toggle geometry and had a pin connecting them at the middle. The damper was inclined to the base at an angle of 45°. The device used in this analysis was 45 in. in diameter with a 0.5 in. outer ring and a 0.75 in. inner ring. The rubber breadth and depth were 9.0 in. and 12.0 in. respectively. Figure 7.2 shows the details of the tested model.

7.3.1 Harmonic loading

The frame was subjected to sinusoidal harmonic loads of peak displacement varying from 1.0-4.0 in. at frequencies varying from $\pi$-3$\pi$ radians/second. The material damping in the structural steel was assumed to be 2% critical. Frame reaction forces, damper displacement and damper force were recorded from each analysis. The input energy, dissipation due to creep in rubber and plastic dissipation were also obtained from each of the analyses.

From the results of the analyses, it was noted that at low levels of drift, the energy dissipation was predominantly due to the creep in the viscoelastic material. However, at higher levels of displacement, the energy dissipation was due to a combination of creep dissipation and viscoelastic dissipation. The force-deformation plots for the dampers reveal higher energy dissipation at higher levels of roof displacement. The average displacement amplification was found to be 1.65.
7.3.1.1 Loading frequency of $\sin \pi t$

The steel frame was subjected to roof displacements varying from 1.0-4.0 in. at a frequency of $\pi$ radians/second. For a displacement of 1.0 in. at the roof, the energy dissipated due to creep in the viscoelastic material and plastic dissipation were 2.36 kip-in. and 0.59 kip-in. respectively. The force in the damper, along its own axis was 7.58 kips with a displacement of 1.56 in. The amplification factor is the ratio of the device displacement to the story displacement, which is equal to 1.56.

For a displacement of 2.0 in. at the roof, the creep dissipation and plastic dissipation were found to be 7.61 kip-in. and 6.44 kip-in. respectively. The damper force was 16.27 kips, and the device displacement was 3.26 in. with an amplification factor of 1.63.

At a roof displacement of 3.0 in., the energy dissipated due to creep and plastic dissipation were 18.83 kip-in. and 25.12 kip-in. respectively. The damper force was 22.81 kips, and the displacement in the device was 5.08 in. with an amplification factor of 1.69.

For a roof displacement of 4.0 in., the energy dissipated due to creep and plastic dissipation were 19.96 kip-in. and 59.30 kip-in. respectively. The damper force was 36.82 kips with a device displacement of 7.08 in. and an amplification factor of 1.77.

7.3.1.2 Loading frequency of $\sin 2\pi t$

The loading frequency was changed to $2\pi$ radians/second and the frame was again subjected to displacement amplitudes of 1.0-4.0 in. For a displacement of 1.0 in. at the roof, the energy dissipated due to creep in the viscoelastic material and plastic dissipation were 4.51 kip-in. and 1.13 kip-in. respectively. The force in the damper, along its own axis was 7.97 kips with a displacement of 1.58 in. The amplification factor is the ratio of the device displacement to the story displacement, which is equal to 1.58.

For a displacement of 2.0 in. at the roof, the creep dissipation and plastic dissipation were found to be 18.15 kip-in. and 9.59 kip-in. respectively. The damper force was 16.54 kips, and the device displacement was 3.28 in. with an amplification factor of 1.64.
At a roof displacement of 3.0 in., the energy dissipated due to creep and plastic dissipation were 30.94 kip-in. and 35.33 kip-in. respectively. The damper force was 23.99 kips, and the displacement in the device was 5.12 in. with an amplification factor of 1.69.

For a roof displacement of 4.0 in., the energy dissipated due to creep and plastic dissipation were 38.01 kip-in. and 87.75 kip-in. respectively. The damper force was 39.21 kips with a device displacement of 7.12 in. and an amplification factor of 1.78.

### 7.3.1.3 Loading frequency of $\sin 3\pi t$

The steel frame was subjected to a sinusoidal displacement controlled harmonic loading of amplitudes varying from 1.0-4.0 in. at a frequency of $3\pi$ radians/second.

At a displacement of 1.0 in. at the roof, the energy dissipated due to creep in the viscoelastic material and plastic dissipation were 5.91 kip-in. and 1.47 kip-in. respectively. The force in the damper, along its own axis was 7.74 kips with a displacement of 1.53 in. The amplification factor is the ratio of the device displacement to the story displacement, which is equal to 1.53.

For a displacement of 2.0 in. at the roof, the creep dissipation and plastic dissipation were found to be 23.62 kip-in. and 10.27 kip-in. respectively. The damper force was 17.08 kips, and the device displacement was 3.18 in. with an amplification factor of 1.59.

At a roof displacement of 3.0 in., the energy dissipated due to creep and plastic dissipation were 40.00 kip-in. and 49.71 kip-in. respectively. The damper force was 5.65 kips, and the displacement in the device was 24.27 in. with an amplification factor of 1.64.

For a roof displacement of 4.0 in., the energy dissipated due to creep and plastic dissipation were 48.88 kip-in. and 118.44 kip-in. respectively. The damper force was 38.58 kips with a device displacement of 6.90 in. and an amplification factor of 1.72.
The energy plots for each of the above analyses are presented, with the input energy \((\text{ALLWK}^{22})\), the creep dissipation \((\text{ALLCD}^{23})\) and the plastic dissipation \((\text{ALLPD}^{24})\) plotted as functions of time. The force-displacement relation for the visco-hyperelastic device under the displacement controlled loading at the roof is also plotted.

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22 ALLWK represents the input energy on the system from the displacement controlled loading
23 ALLCD represents the energy dissipated by the device due to creep in the viscoelastic material
24 ALLPD represents the energy dissipated due to the inelastic behavior of the steel in the device
Figure 7.4: Energy plot and force-displacement relation for the damper at a loading frequency of $\pi$ radians/second and a roof displacement of 1.0 in.
Figure 7.5: Energy plot and force-displacement relation for the damper at a loading frequency of $\pi$ radians/second and a roof displacement of 2.0 in.
Figure 7.6: Energy plot and force-displacement relation for the damper at a loading frequency of $\pi$ radians/second and a roof displacement of 3.0 in.
Figure 7.7: Energy plot and force-displacement relation for the damper at a loading frequency of $\pi$ radians/second and a roof displacement of 4.0 in.
Figure 7.8: Energy plot and force-displacement relation for the damper at a loading frequency of $2\pi$ radians/second and a roof displacement of 1.0 in.
Figure 7.9: Energy plot and force-displacement relation for the damper at a loading frequency of $2\pi$ radians/second and a roof displacement of 2.0 in.
Figure 7.10: Energy plot and force-displacement relation for the damper at a loading frequency of $2\pi$ radians/second and a roof displacement of 3.0 in.
Figure 7.11: Energy plot and force-displacement relation for the damper at a loading frequency of $2\pi$ radians/second and a roof displacement of 4.0 in.
Figure 7.12: Energy plot and force-displacement relation for the damper at a loading frequency of $3\pi$ radians/second and a roof displacement of 1.0 in.
Figure 7.13: Energy plot and force-displacement relation for the damper at a loading frequency of $3\pi$ radians/second and a roof displacement of 2.0 in.
Figure 7.14: Energy plot and force-displacement relation for the damper at a loading frequency of $3\pi$ radians/second and a roof displacement of 3.0 in.
Figure 7.15: Energy plot and force-displacement relation for the damper at a loading frequency of $3\pi$ radians/second and a roof displacement of 4.0 in.
7.3.2 Ground Excitations

The performance of the steel frame with the visco-hyperelastic device was analyzed and compared with the results from the same steel frame without the device under different ground excitations that have different characteristics such as frequency content and peak ground accelerations, to study the effect of the visco-hyperelastic device on controlling the structural response. The analyses were performed using the finite element software package *ABAQUS*.

The real unscaled acceleration records of the El Centro (1940) and the Northridge (1994) earthquakes were used for the nonlinear dynamic response history analysis of the steel frame. The input ground motion time history that was used is given in Figure 7.13. The frame was analyzed with rigid base conditions, and the material damping in the structural steel was taken as 2% of critical. The inelastic behavior accounted for the plastic dissipation in the steel rings of the device, while the viscoelastic material model accounted for the creep dissipation in the rubber.

The first 15 seconds of response of the structure with the device under the ground excitation was studied and compared with the frame without the device. The time period of the structure without the device was 0.499 seconds and the time period of the structure with the device decreased to 0.417 seconds as a result of the additional stiffness provided by the toggle-brace-damper configuration. The acceleration was measured at the roof of the structure and interstory displacement was computed as the difference between the displacement at the roof and the displacement at the base.

First, the single story single bay plane frame was analyzed under the El Centro ground motion. The peak interstory drift was reduced from 2.01 in. to 0.61 in. with a reduction percentage of 69.6%. The peak acceleration at the roof was reduced from 1.17g to 0.49g with a reduction percentage of 58.1%. Figure 7.17 shows the comparison between the response of the steel frame with and without the device.
The steel frame was then analyzed under the Northridge ground motion. The peak interstory drift was reduced from 3.28 in. to 0.90 in. with a reduction percentage of 72.5%. The peak acceleration at the roof was reduced from 1.62g to 0.84g with a reduction percentage of 48.1%. Figure 7.18 shows the comparison between the response of the steel frame with and without the device.

7.3.3 Stabilizing effect of the device

The visco-hyperelastic device has an increasing stiffness at higher deformation levels. This is very useful in preventing structural collapse and adding more stability to the structure. The earthquake event causes severe inelastic behavior in the structural elements, which lose their strength at higher deformation levels, often leading to collapse.

The effect of these devices was studied under a static roof displacement of 8 in. The force-deformation relation for the structure at the roof was plotted and it the stiffening effect of these devices is noted as in Figure 7.19.
Figure 7.16: Input ground motion time histories

(a) Northridge ground motion record

(b) El Centro ground motion record
Figure 7.17: Effect of device on the response of a single story frame under the El Centro ground motion
Figure 7.18: Effect of device on the response of a single story frame under the Northridge ground motion
7.4 Summary

In order to investigate the effect of the visco-hyperelastic device on structural response, a single story single bay steel frame was subjected to displacement controlled harmonic loading and ground excitations. The device was implemented in the structure in a toggle-brace configuration, which resulted in device displacement amplifications of about 1.6 times the interstory drift.

Nonlinear dynamic response history analyses under the El Centro and Northridge ground motions were conducted on the steel frame with and without the device to evaluate its performance. The device resulted in considerable decrease in the interstory drift and on the peak acceleration at the roof of the frame.
Chapter 8

Conclusions and recommendations

8.1 Summary

Research was conducted on a visco-hyperelastic device for structural seismic response control incorporating the properties of both velocity dependent and displacement dependent devices. The device consists of a combination of a viscoelastic material (rubber) sandwiched between two mild steel rings.

The proposed device provides considerable energy dissipation while behaving as a hyperelastic device. First, the hyperelastic and energy dissipation properties of circular rubber elements were studied by experimental testing of three motorcycle tires under varied dynamic loads. The force-deformation relations from these tests showed considerable energy dissipation under various levels of excitation with a pronounced hyperelastic behavior at higher levels of deformation. Following this, a comprehensive three dimensional finite element model of the device was developed using the finite element software package ABAQUS incorporating the hyperelastic and viscoelastic properties of rubber, and the inelastic behavior of steel. Experimental results of uniaxial tension-compression and relaxation tests on high damping rubber compounds from literature were used in modeling the device. The inelastic behavior of steel was considered using the Von Mises yield criterion. The analyses were conducted as nonlinear large displacement analyses.

A parametric study was conducted to determine the effect of the steel type, rubber depth and steel ring thickness on the device performance. These parameters were evaluated
under pseudo-static tensile and compressive, and harmonic loads. Based on the parametric studies, the following observations were made:

- Mild steel dissipated more energy due to yielding compared to high strength steel. The device displacements were independent of the type of steel used. The high strength steel required very high device displacements to yield, thereby rendering them ineffective in energy dissipation at moderate levels of deformation, which is not desired.
- The energy dissipation due to creep in the viscoelastic material depends on the volume of viscoelastic material used. At low levels of deformation, the smaller the depth of the viscoelastic material, the larger was the plastic dissipation due to yielding in mild steel. The rubber depth also had an effect on the device displacements, with displacement reducing as rubber depth increased. Accordingly, a reasonable rubber depth was chosen in order to have a combined optimum dissipation from both steel and rubber.
- The device displacement depends on the thickness of the outer steel ring, with the inner ring having negligible effect. However, the amount of compressive and tensile strains developed in the viscoelastic material depends on the thickness of the inner steel ring. This is due to the fact that greater inner ring thickness in comparison to the outer ring thickness, lesser is its tendency to become elliptical in comparison to the outer ring, and hence, greater the strains in the viscoelastic material.

8.2 Conclusions

Based on the experimental analyses of three motorcycle tires under harmonic loading, the following conclusions can be drawn.

- Tires have considerable energy dissipation capacity and a pronounced hyperelastic behavior at higher levels of deformation.
- The hysteresis loops for the tire did not break down over a number of cycles of loading and hence, the reliability of the tire system is not compromised.
The energy dissipation in the tires could be due to the opening and closing of the cracks in the treads, rearrangement of the molecular structure under applied load, sliding of the chains or due to stress softening at higher levels of deformation.

The strength of a single tire system was not found to be as high as would be required in seismic applications. Some recommendations have been proposed to increase the strength of the tire such as nested tire configuration and using helical coils inside the tire.

Based on the detailed analyses on the device and study on the response of a single-story frame, the following conclusions can be drawn:

- The visco-hyperelastic device incorporates energy dissipation due to viscoelastic material behavior and yielding in the steel elements which can be controlled by various device parameters such as the rubber depth, steel type and the thickness of the steel rings.
- The device performance can be enhanced by amplifying the axial strains in the viscoelastic material. This can be achieved by using them in a toggle-brace-damper configuration, which not only reduces damper force requirements, but also increases the energy dissipation in the device.
- The device exhibits a stabilizing effect on structures that may get severely damaged during an earthquake. The device provides increasing stiffness at higher levels of deformation, and precludes an impending collapse.
- Structural response of a single-story steel frame was reduced considerably under the presence of these dampers. There was significant reduction in the peak accelerations and interstory drifts in the structure. The device performed admirably dissipating a lot of energy due to creep and plastic behavior.

### 8.3 Future work

No analytical work is complete without substantiating experimental work. Hence, it is necessary to extensively test this device before it can be implemented in structural
frameworks. Prior to designing the proposed visco-hyperelastic device, studies on various available rubber compounds must be conducted in order to compare mechanical properties such as tensile strength, modulus of elasticity, rupture strength, recoverable strains etc. Experimental tests on rubber should also be conducted in order to facilitate the development of detailed finite element models.

The connection details proposed in this research require considerable testing before they can be implemented. This study would also include tests on adhesives used to bond the rubber with the steel and its behavior under repeated loading. Once the viscoelastic material and the device connection details have been selected, the device should be experimentally tested under various loading frequencies in order to compare its results with those from the finite element model.

The concept of using tires as hyperelastic devices need to be explored since it can provide an alternate means of energy dissipation in structural systems at a fraction of a cost of conventional dampers. Fillers and reinforcing materials to increase the strength of a tire also needs exploration, since it does provide a viable means of seismic and wind mitigation.
References


Appendix A

The derivation for the expressions for a toggle-brace configuration (Figure 7.2) is presented here. Figure A shows the deformed configuration of the toggles.

\[ A \]

In the following derivation, it is assumed that the members are inextensible and the preservation of length of members is used.

Consider \( \Delta B'DC' \). Using the Pythagoras theorem, we get:

\[ B'C'^2 = B'D^2 + DC'^2 \] (A.1)

Using the deformed configuration of the toggle-brace, and from \( \Delta AB'E \), we get:

\[ EG = (l + u) - l_1 \cos(\alpha + \phi) \]
\[ C'D = h - l_1 \sin(\alpha + \phi) \] (A.2)
Using Equation A.2 in Equation A.1, we get:

\[ l_2^2 = [(l + u) - l_1 \cos(\alpha + \phi)]^2 + [h - l_1 \sin(\alpha + \phi)]^2 \]

\[ l_2^2 = (l + u)^2 - 2l_1(l + u)\cos(\alpha + \phi) + [l_1 \cos(\alpha + \phi)]^2 + h^2 - 2hl_1 \sin(\alpha + \phi) + [l_1 \sin(\alpha + \phi)]^2 \]

Simplifying the above, we get

\[ l_2^2 = (l + u)^2 + h^2 + l_1^2 - 2l_1(l + u)\cos(\alpha + \phi) - 2hl_1 \sin(\alpha + \phi) \quad (A.3) \]

The displacement in the damper is obtained by calculating the distance $BB'$, represented as $u_D$. Based on the deformed configuration of the toggle and using the law of cosines we get:

\[ (FB + u_D)^2 = AF^2 + AB'^2 - 2 \cdot AC \cdot AB \cdot \cos(\alpha + \phi) \quad (A.4) \]

From the law of sines, we get:

\[ \frac{FB}{\sin \alpha} = \frac{l_1}{\sin \theta} \quad (A.5) \]

Substituting Equation A.5 and other terms in Equation A.4, we get:

\[ \left( u_D + \frac{l_1 \sin \alpha}{\sin \theta} \right)^2 = l_1^2 \left( \frac{\sin(\alpha + \theta)}{\sin \theta} \right)^2 + l_1^2 - 2 \cdot l_1^2 \frac{\sin(\alpha + \theta) \cos(\alpha + \phi)}{\sin \theta} \quad (A.6) \]

Simplifying further we get:

\[ u_D = \pm l_1 \sqrt{1 + \frac{\sin^2(\alpha + \theta)}{\sin^2 \theta} - 2 \cdot \frac{\sin(\alpha + \theta) \cos(\alpha + \phi)}{\sin \theta}} - \frac{\sin \alpha}{\sin \theta} \quad (A.7) \]