Compound Aircraft Transport Study: Wingtip-Docking Compared to Formation Flight

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(ABSTRACT)

Compound Aircraft Transport (CAT) flight involves two or more aircraft using the resources of each other; a symbiotic relationship exists consisting of a host, the mothership aircraft and a parasite, the hitchhiker aircraft. Wingtip-docked flight is just as its name implies; the two aircraft are connected wingtip-to-wingtip. Formation flight describes multiple aircraft or flying objects that maintain a pattern or shape in the air. There are large aerodynamic advantages in CAT flight. The aforementioned wingtip-docked flight increases total span of the aircraft system, and formation flight utilizes the upwash from the trailing wingtip vortex of the lead aircraft (mothership) to reduce the energy necessary to achieve and/or maintain a specific flight goal for the hitchhiker and the system.

The Stability Wind Tunnel (6 X 6 X 24 foot test section) at Virginia Tech, computational aerodynamic analysis with the vortex lattice method (VLM), and a desktop aircraft model were used to answer questions of the best location for a hitchhiker aircraft and analyze stability of the CAT system. Wind tunnel tests implemented a 1/32 scale F-84E model (hitchhiker) and an outboard wing portion representing a B-36 (mothership). These models were chosen to simulate flight tests of an actual wingtip-docked project, Tom Tom, in the 1950s. That project was terminated after a devastating accident that demonstrated a possible “flapping” motion instability. The wind tunnel test included a broad range of hitchhiker locations: varying spanwise gap distance, longitudinal or streamwise distance, and vertical location (above or below wing) with respect to a B-36-like wing. The data showed very little change in the aerodynamic forces of the mothership, and possibilities of large benefits in lift and drag for the hitchhiker when located slightly aft and inboard with respect to the mothership. Three CAT flight configurations were highlighted: wingtip-docked, close formation, and towed formation. The wingtip-docked configuration had a 20–40% performance benefit for the hitchhiker compared to solo flight. The close formation configuration had performance benefits for the hitchhiker approximately 10 times that of solo flight, and the towed formation was approximately 8 times better than solo flight.

The VLM analysis completed and reenforced the experimental wind tunnel data. A modified VLM program (VLM CAT) incorporated multiple aircraft in various locations as well as additional calculations for induced drag. VLM CAT results clearly followed the trends seen in the wind tunnel data, but since VLM did not model the fuselage, has assumptions like a flat wake, and is an inviscid computation it did not predict the large benefits or excursions as seen in the wind tunnel data. Increases in performance for the hitchhiker in VLM CAT were on the order of 3 to 4 times that of the hitchhiker in solo flight, while the wind tunnel
study saw up to 10 times that of solo flight. VLM CAT is a valuable tool in supplying quick analysis of position and planform effects in CAT flight.

Modifications to a desktop F-16 dynamic simulation have been developed to investigate the stability of wingtip-docked flight. These modifications analyze the stability issues linked with sideslip angle, $\beta$, as seen by the Tom Tom Project test pilot, when he entered docking maneuvers with 5 degrees yaw to simulate a “tired pilot”. The wingtip-docked system was determined to have an unstable aperiodic mode for $\beta < 0.0$ degrees and an unstable oscillatory mode for $\beta \geq 2.0$ degrees. There is a small range of $\beta$ that is a stable oscillatory mode, $0.0 < \beta \leq 2.0$ degrees. The variables, altitude and speed, yield little effect on the stability of the system. The sensitivity analysis was indeterminate in distinguishing a state driving the instability, but the analysis was conclusive in verifying the lateral-longitudinal (roll-pitch) coupled motion observed by test pilots in wingtip-docked flight experiments. The parameter with the largest influence on the instability was the change in pitch angular acceleration with respect to roll angle.

The aerodynamic results presented in this study have determined some important parameters in the location of a hitchhiker with respect to a mothership. The largest aerodynamic benefits are seen when the hitchhiker wingtip is slightly aft, inboard and below the wingtip of the mothership. In addition, the stability analysis has identified an instability in the CAT system in terms of sideslip angle, and that the wingtip-docked hitchhiker is coupled in lateral and longitudinal motion, which does concur with the divergent “flapping” motion about the hinged rotational axis experienced by the Tom Tom Project test pilot.
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Chapter 1

Introduction

Compound Aircraft Transport (CAT) flight involves two or more aircraft using the resources of each other; a symbiotic relationship exists consisting of a host, the mothership aircraft and a parasite, the hitchhiker aircraft. CAT flight is useful in transporting a hitchhiker(s), usually of a smaller size, with a unique mission that the mothership is unable to perform. CAT flight proposes to achieve the mission requirements with minimal energy losses to the system. The types of CAT flight studied in this work utilize aerodynamic properties to maximize and even increase the potential energy of the system. Some types of CAT flight to be discussed are: wingtip-docked, carried and/or towed, and formation flight.

Wingtip-docked flight is just as its name implies; the two aircraft are connected wingtip-to-wingtip, as demonstrated in Figure 1.1.

![Figure 1.1: Example of Wingtip-Docked Flight Associated with the Project TipTow [1].](image)

Both carried and towed arrangements imply that the hitchhiker is completely null of any propulsion, all engines are off. But the true distinction between carried and towed flight
is that the former does not support its own weight through aerodynamic lift, and the latter does. For example, a small aircraft might be carried on a trapeze mechanism slung beneath the belly of a bomber, and a glider would be tethered to a powered vehicle. Now the glider does not hang from the powered vehicle rather it trims the control surfaces to maintain a level position and utilizes lift to support its weight. Wingtip-docked flight can be a type of towed flight if the engines are off; all the historical experiments in wingtip docking were towed. Figures 1.2 and 1.3 are examples of carried and towed flight, respectively.

Figure 1.2: Example of Carried Flight Associated with the Fighter Conveyor (FICON) Project [2].

Figure 1.3: Example of Towed Flight Associated with the Eclipse Project [3].
Formation flight describes multiple aircraft or flying objects that maintain a pattern or shape in the air. This pattern or shape can signify a message, like the US Air Force missing man formation or be used to aerodynamically benefit the overall system. The latter is the type to be studied and discussed in this work. Migratory geese use the aerodynamic benefits of formation flight by flying in a vee pattern as seen in Figure 1.4.

Figure 1.4: Example of Formation Flight in Nature with Migrating Geese.

1.1 CAT Flight Advantages

Wingtip-Docked Flight

Performance benefits achieved in wingtip-docked flight are keyed to the increase in span. Lift increases with span or aspect ratio, $AR = \frac{b^2}{S}$. From a perspective of the mothership as a reference span, the increased span or surface area, $S$, for wingtip-docked flight allows for the pressure differential produced by the airfoil shape to act on a larger area ($F = PA$). A theoretical approach for this increase in lift due to an increase in span is based on two-dimensional airfoil theory. By definition a two-dimensional airfoil is one of infinite span, thus the lift acting on it is spanwise constant. A three-dimensional wing of finite span must obey the laws of nature and so the lift must vanish at the tips. Techniques utilizing two-dimensional airfoil theory are often employed for calculations, because the span of a wing is typically much greater than the chord (a large aspect ratio). The drop-off in lift is manifested as instantaneous, creating a rectangular lift distribution. In real-life this drop-off in lift cannot manifest itself instantaneously, but degrades smoothly across the span of the wing in a curved path; Figure 1.5 sketches the two types of lift distribution. The total lift is the area under the lift distribution curve, and in comparing the two-dimensional lift distribution to the three-dimensional lift distribution, one can clearly see the two-dimensional rectangular distribution has the greatest possible area and lift ($lb > \frac{\pi}{4}lb$). So, the larger the span or aspect ratio, the closer the lift is to two-dimensional theory and thus the higher the total lift.
Increase in span also plays a role in reducing the induced drag, $C_{D_i}$, and by assuming an elliptic wing loading (Oswald’s efficiency factor, $e = 1.0$) with identical planforms ($b_1 = b_2$, $c_1 = c_2$, $W_1 = W_2$, ...), that reduction could be as much as 50% (Figure 1.6).

Of course, in real life several additional factors would inhibit actually meeting this ideal value. Total drag is the sum of the parasite or form drag, $C_{D_o}$, plus the induced drag, $C_{D_i}$, plus the wave drag due to compressible effects (incompressibility is assumed for this work, especially in the docking procedure). Parasite drag is directly proportional to the wetted area of the aircraft, $S_w$, so having a system of more than one aircraft would increase the total wetted area and thus the parasite drag. Whether or not there is a decrease in total drag would depend on the span and wetted area ratio ($\sim b/S_w$) of the aircraft to be docked.

Another advantage of decreasing induced drag is increasing the ratio of the lift to drag or $L/D$, which is the aerodynamic parameter relied most upon to determine flight performance. Taking a drag polar of the form used previously in Figure 1.6:

$$C_D = C_{D_o} + \frac{C_L^2}{\pi A Re}$$

$$C_{D_i} = \frac{C_L^2}{\pi A Re}$$

The maximum $L/D$ occurs when the parasite drag equals the induced drag, $C_{D_o} = C_{D_i}$. So,

$$C_D = 2C_{D_o},$$

and

$$C_L = (C_{D_o}\pi A Re)^{1/2}.$$  

The maximum $L/D$ is then

$$\left(\frac{L}{D}\right)_{\text{max}} = \left(\frac{C_L}{C_D}\right)_{\text{max}} = \left(\frac{\pi A Re}{4C_{D_o}}\right)^{1/2}.$$
Parasite drag is a function of skin friction ($C_F$), wetted ($w$) area, and wing or reference area, $S$, 

$$C_D = C_F S_w / S.$$ 

Incorporating this, the best $L/D$ is 

$$\left( \frac{L}{D} \right)_{\text{max}} = \left( \frac{C_L}{C_D} \right)_{\text{max}} \sim \left( \frac{\pi \epsilon}{4C_F} \right)^{1/2} \left( \frac{b^2}{S_w} \right)^{1/2}.$$

$b^2 / S_w$ is defined as the wetted aspect ratio, $AR_w$, so maximum $L/D$ is proportional to the square root of the wetted aspect ratio. Applying this to the example in Figure 1.6, $S = S_w$, this value is doubled with the span, so the wetted aspect ratio is also doubled leading to $L/D \propto \sqrt{2}$. Note that for any aircraft planform, the wetted aspect ratio would be doubled.
if the two aircraft were identical and docked wingtip-to-wingtip. So, the maximum $L/D$ for two identical aircraft docked wingtip-to-wingtip is increased from the individual solo value by up to $\sim 40\%$. Three aircraft would indicate up to $\sim 75\%$ [4].

**Towed and/or Carried Flight**

The benefits of carried or towed flight are evident, but solely so for the hitchhiker. The hitchhiker expends zero energy for transit to the mission platform. There is an obvious trade-off in this situation between the typically larger mothership (M) and the smaller hitchhiker (HH). For carried flight the energy, $E$, changes could be defined as,

\[
E_{\text{Total}} = E_{M_{\text{solo}}} + E_{HH_{\text{solo}}}
\]

\[
E_{M_{\text{solo}}} = E_{M_{\text{mission}}} + E_{M_{\text{transit}}}
\]

\[
E_{HH_{\text{solo}}} = E_{HH_{\text{mission}}} + E_{HH_{\text{transit}}}
\]

\[
E_{M_{\text{solo}}} > E_{HH_{\text{solo}}}
\]

\[
\Delta E_{HH} = E_{HH} - E_{HH_{\text{transit}}}
\]

\[
E_{HH_{\text{CAT}}} = E_{HH_{\text{mission}}} = \Delta E_{HH}
\]

\[
\Delta E_{M} = E_{M} + E_{HH_{\text{CAT}}}
\]

\[
E_{HH_{\text{CAT}}} = E_{HH_{\text{CAT}}}(\text{Weight}_{HH}, \text{Drag}_{HH})
\]

\[
E_{Total_{\text{CAT}}} = E_{M} + \Delta E_{M}
\]

\[
E_{Total_{\text{CAT}}} \leq E_{Total}.
\]

So is the total change in the system energy, $E$, for carried CAT flight less than or equal to that of the original solo system, and if not are those losses small or acceptable? Acceptable losses would be if the hitchhiker has a unique capability, say to dock with a damaged satellite, then the mothership would have to accept the performance losses that she encounters attaining a reasonable altitude to launch a single-stage orbiter hitchhiker.

Towed flight removes the weight factor of the hitchhiker, because the hitchhiker carries its own weight through aerodynamic lift. Gliders commonly use this technique to reach soaring altitudes, and similar to the previous general energy-exchange example, an orbiter spacecraft could save energy by being towed to some launch altitude before using its own power. A towed-flight system would be more prone to energy losses due to parasite drag, in that the entire surface or wetted area, $S_w$, of the hitchhiker is exposed to the fluid air, unlike carried where partial or none of the hitchhiker wetted area is exposed to the fluid air. Also, the tether device could propose drag penalties unacceptable for towed-flight institution.
Formation Flight

The advantages of formation flight are keyed to the energy savings of the hitchhiker by flying in the trailing vortices of the lead aircraft or mothership, much like surfing a wave. The hitchhiker would fly off-center of the mothership, like migrating birds do in a vee, attempting to align the hitchhiker wingtip with the region of beneficial upwash. The addition of an upward velocity would increase lift and reduce the induced drag by decreasing the induced flow angle. All of this translates into a total reduction of the energy necessary to stay aloft and cover a specified range. The hitchhiker would have better fuel efficiency and increased range. Figure 1.7 shows significant drag reductions for the trailing hitchhiker based on calculations by Hoerner. He assumed a maximum $L/D$ flight condition ($C_D = 2C_{D_0}$), using a vortex lattice method, and allowing for development of the wingtip trailing vortices [5]. The best location for a swept formation is not staggered wingtip-to-wingtip, but at a location with slight lateral overlapping. Birds can be seen to fly with this overlap and have been recorded by ornithologists as manifesting the fuel savings through a decreased heart rate up to 14.5% [6].

![Figure 1.7: Formation Flight Induced Drag Reductions with Three Identical Aircraft](image)

The disadvantage of this for humans is the energy required to maintain a position within the upwash. The same power and energy that is so beneficial saved in the upwash is dangerous in the downwash, just a short distance away. Many aircraft have been destroyed by inadvertently coming into contact with a powerful trailing vortex of another aircraft. Since the mothership is continually in motion under her influence and that of the perturbing-viscous fluid air, then her trailing vortices are also in motion—varying location and diffusing over time. The long and short is the pilot would get extremely fatigued “jockeying the throttle”, and any savings would be unimportant if the mission could not be performed. The possible
aerodynamic benefits in formation flight are far greater than the aforementioned wingtip-docked flight, but for humans the losses and dangers (thus far) outweigh any advantages. So historically, more focus has been placed on wingtip-docked flight. Recently with the advent and ever increasing capability of computers coupled with Global Positioning System (GPS) satellites, new light is being shed on the true feasibility of formation flight for humans as a energy saving tool.

1.2 Historical Review of CAT Flight

The attractiveness of CAT flight has evolved with military needs. During and shortly after WWII, there was an apparent necessity to have long range fighter escorts for bombers. The range of some bombers built in the late 1940s was 10,000 miles, and the range of the fighters was a tenth of that or less. With the advent of in-air-refueling, the long range fighter escorts were supplied without the complicated mechanisms and hairy flight conditions of wingtip-docked flight. Long range missions to China from US Air Bases would propose advantages for the US in terms of Korea, but it was the Cold War that brought the most interest to the concept. The Cold War was focused on reconnaissance (recon), knowing what the other had and being prepared to retaliate against it. The B-36 Peacemaker or Big Stick was able to fly higher than the Soviet planes, therefore it was equipped with cameras (RB-36) to photograph “targets deep within the Soviet Union”. But, the Soviet began to develop anti-aircraft defenses, so the parasitic fighter concept was revisited. This time to act as a recon vehicle fitted with cameras to be deployed from the RB-36-to dash into protected anti-aircraft areas in the Soviet Union, take strategic pictures, and then to rendezvous with the RB-36 to fly home \[2\]. Two conflicting ideas to perform this task emerged: the first had the bomber carry the fighter in the bomb bay and the second coupled two or more fighters to the wingtips of the bombers. Both had merit and were flight tested. Subsequent sections review the experiments and interesting anecdotes that preceded the current work.

1.2.1 Wingtip-Docked Flight

From the available literature, one could conclude that the concept and first experiments of wingtip-docked or coupled flight were conceptualized by the Germans. Known experiments (though not well-documented) were conducted by the German Air Ministry at the end of WWII (1944-1945). These experiments flew two light-equal-size planes that were coupled with a rope connection. Also, experiments with two light aircraft and a small transport were performed in Germany under the direction of Dr. Richard Vogt. Since neither experiment is well documented, the information lends itself to the conclusion that these two experiments conducted in Germany around the end of WWII, could be one in the same.

Dr. Richard Vogt, a German aircraft designer, was sent to the US after WWII under
the “auspices of the US-sponsored Project Paperclip”[7]. Vogt first proposed his idea of “something for nothing” to a fellow German immigrant Ben Hohmann, a German WWII engineer and test pilot who now worked at Wright Field (Wright-Patterson AFB). Vogt’s idea was for wingtip coupled fuel tanks to extend the range of conventional aircraft up to 30%. The increased aspect ratio by adding the fuel tanks would decrease the induced drag and offset any increase in drag due to additional wetted area. Eventually, Vogt envisioned the usefulness of coupled aircraft. Fighters attached to bombers could be permanent escorts with little or no penalty to the range of the bomber. He officially presented his idea to personnel at Wright Field in 1947, and funding was awarded [7].

The ultimate goal was to experiment with a B-29/F-84 combination, but to get some quick answers on the feasibility of the project an existing Douglas C-47A (42-23918∗) cargo plane and a Q-14B (44-68334, a pre-war version of the PQ-14 Culver Cadet) target plane were modified. The modifications were very simple. A single-joint, cantilever arm with a receiver ring at the end was implanted with local structural reinforcement. A similar attachment to the Q-14, with a lance at the end was implanted. Figure 1.8 is a schematic of the engagement mechanism. The idea was that the Q-14 would back the lance into the receiver ring on the C-47. This took the six degrees of freedom, the Q-14 had in solo flight and reduced it to three. No locking device was employed; the system relied on the drag of the Q-14 to hold it in place. To disengage, the Q-14 would simply increase power to advance forward.

On August 19, 1949 the C-47 and Q-14 completed a short coupling, though it was unsuccessful in demonstrating controlled flight. The Q-14 was flown by Major Clarence E. “Bud” Anderson, a veteran P-14 WWII fighter pilot and contemporary of General Chuck Yeager. Anderson being inexperienced in wingtip coupling (considering this was the first flight of this particular type and configuration), spent 30 minutes trying to engage smoothly and when that did not work he flew the Q-14 very close and then reduced power more quickly than normally desired. This method did work and the two aircraft were coupled, but the

\[\text{Figure 1.8: Schematic of C-47/Q-14 Wingtip Engagement Mechanism[1]}\]

*Numbers of this fashion occurring after a aircraft designation represent the serial number of the aircraft used
immediate 90 degree nose-down position that the Q-14 took in relation to the C-47, just did not seem quite right. Therefore, Anderson advanced power to immediately disengage. The whole episode only lasted a few seconds, but it was more than enough to end tests for that day, since the wingtip of the C-47 was slightly bent[1]. Afterward, some modifications were made to the hook-up mechanism, consisting of moving the receiver ring outward from three inches to nineteen inches. So, by lengthening the receiver ring outward, the Q-14 was moved out of the strong wingtip vortex of the C-47, and on October 7, 1949 a more successful coupling was made. Four flights were actually made that day; the longest was five minutes. Figure 1.9 is a photograph of flight tests with the C-47/Q-14 combination.

One important difference from flying wingtip docked or solo flight was in the control system. When docked, the elevator became the primary control. The ailerons, which normally control roll, were ineffective, and the elevator controlled pitch and roll. At one time, the inboard aileron was disconnected, but roll control via ailerons was only slightly improved. The transition from aileron to elevator for roll control, was odd at first, but a skilled pilot quickly adapted (though “hand’s off” flying was not truly possible)[1]. Attempts were made to incorporate some autopilot control of the elevator to keep the flap angle small, but this was unsuccessful and beyond the original scope of the program. The jargon of flap angle was adopted to described the roll angle, \( \eta \), about the longitudinal wingtip connection.

Figure 1.9: Aerial Photograph of C-47/Q-14 Wingtip Docking Flight Tests[1]
axis. When the docking aircraft connected and rotated about this axis, the motion was like “flapping”.

The program with the C-47/Q-14 combination ended in October 1950 without any performance data collected, but the accomplishments were significant [1]:

- 231 wingtip couplings
- 28:35 total coupled flight hours
- 17 pilots familiarized with technique
- 56 night couplings (2:09) flight hours
- Proved feasibility of concept
- Showed smooth air was required to couple
- Demonstrated that pilots could be trained
- Elevator shown to be primary control (ailerons ineffective)

In the summer of 1950, the next set of wingtip coupling experiments were ready to be flight tested, known as Project MX 1018 or *Tip Tow*. The Republic Aviation Corporation of Long Island, NY had modified a B-29A bomber and two straight-wing F-84D fighter jets. The outer wing panels of the B-29 were replaced with a towing or retrieving mechanism, similar to the C-47/Q-14 mechanism in that it had a lance device and an extended capture piece associated with it. It consisted of a hydraulically actuated cylinder or retractable boom that would extend out from the wing of the B-29, the F-84 then flew forward with a lance to hook up to the receiver at the end of the boom. The connection would lock (though motion in pitch, roll, and yaw was viable) and then pull the F-84 in to be locked at an aft position, that had been rubber sealed. At that point, the F-84 was only capable of roll about the longitudinal axis for the wingtip connection. The capability to roll about this longitudinal axis is the aforementioned flapping motion. It was hoped that a simple method relating the lance rotation or flap angle to elevator movement could provide automated flight control. Figure 1.10 is a schematic of the engagement mechanism employed for *Tip Tow*.

The first coupled flight was July 21, 1950 in Long Island, NY again by Anderson, with the B-29 bomber and one F-84 fighter, only the right-hand F-84 had the necessary modifications at that time. Four successful engagements were made, and the previous Figure 1.1 is a photograph of the flight test.

An important aspect of this arrangement was the fragility in comparison to the C-47/Q-14 combination. The B-29 wing was more flexible than the C-47, and if the connection was performed too roughly then the B-29 wing would oscillate structurally in a lateral-longitudinal direction. This twisting of the B-29 wing was controlled by physically pitching the F-84. In these flights, performance data was collected at altitudes of 10,000, 15,000, and 20,000 ft with banking right and left at ten degrees, airspeed from 156-195 knots for
a docked time of 1 hour and 40 minutes. Overall Project *Tip Tow* completed 43 couplings with 2 pilots and a total 15 hours of towed flight.

With the basic B-29 mission profile as a baseline, when carrying two F-84s the loss in range was only 7.5%. But, if the profile was optimized aerodynamically the loss was only 2.9%. Theoretically, the range of the B-29 should be increased slightly if the rubber seals are truly airtight. Figure 1.11 is the graph of the data collected from the *Tip Tow* Project.

In the early part of 1953, Republic Aviation Corporation reactivated the flight testing of the B-29/F-84 combination with system improvements. The fighters had mechanical doors to close the air inlets of the engines during towing to reduce “windmilling” drag [1]. The major change was the installation of an automatic-electric flight control system to control the flapping angle. The required damping frequencies for the flight control system were determined though actual docked flight tests where the fighter pilot would induce a pulse and then let go. Needless to say this was dangerous. As Anderson said, “I could not envision ever making a pitch pulse of any magnitude while coupled in towed flight, nor could I imagine letting go of the control stick if there were any flapping action at all” [1]. Six flights were made between March and April of 1953 in Farmingdale, Long Island. These flight tests only employed the left-hand F-84, because electrical power could not be received on the right-
hand side. On April 24, 1953, Major Davis flew the left-hand F-84 into fully-locked and coupled flight with the B-29 (the right-hand F-84 was uncoupled). When the system was stabilized and trimmed, the automatic flight-control system was activated momentarily from the fighter. This resulted in violent pitching of the F-84 and then flapping upward onto the main wing spar of the B-29, and before the two aircraft could separate, the nose of the F-84 was sheared off forward of the cockpit. The B-29 spiraled into the Peconic Bay, and the F-84 followed shortly after. There were no survivors.

During this same time period between 1952 and 1953, General Dynamics Corporation, the Convair Division (Consolidated Vultee) at Forth Worth, TX was contracted for a wingtip docking project, code-named *Tom Tom*, involving a KRB-36F (49-2707) and two RF-84F swept-wing fighters (51-1848 and 51-1849, Thunderflash, a derivative of the F-84E, Thunderstreak). Originally conceived as a method of long range bomber escorts, it evolved into a recon vehicle system. The fighters could be carried, released, and recovered. The advantages of this configuration were to increase the number of targets, to carry more weaponry, and to provide more effective penetration into enemy territory. The modifications were limited to the right wingtip of the B-36 and the left wingtip of one F-84. The wingtip of the B-36 was replaced with podied articulated hook-up arms (in pitch and roll, but not yaw). The appropriate wingtip of the F-84 was replaced with “jaws” that clamped shut on the B-36 towing arm and pulled the F-84 into the B-36 wingtip to lock into place. Only the aforementioned flapping movement was possible for the F-84. Figure 1.12 is three photographs of the modifications made to the F-84 and the B-36 to show how the docking device was utilized.

In mid-1952, initial flights were made to test approach patterns for the F-84 to the B-36, and in early 1953 the first hook-up took place with the F-84 (51-849). Figure 1.13 is
Figure 1.12: (Top left) The Jaw Mechanism on the F-84, (Top right) the Pod Appendage on the B-36 Wing, (Bottom) the Engagement Mechanism for Tom Tom Clamping onto the Articulated Arm Extending from the Pod Appendage [2].

an artist’s rendition of the two aircraft in Tom Tom engaging in a docking maneuver. The hook-ups were short and the flying environment was difficult and dangerous. “Problems encountered were primarily aerodynamic and not mechanical” [7]. Due to strong wingtip vortices of the B-36, the small F-84 had to fly in these extremely hazardous conditions.
Also, the swept wings required the pilot to look backward over his shoulder to complete a hook-up even though he was flying forward. This took some practice and resulted in a lot of stiff necks. There were about ten hook-ups, and no maneuvers were made in flight. The average mission was about 2 hours, including take-off, hook-up, and landing. There were three planes involved: the B-36, the F-84, and a Convair 240 for observation with all 240 seats occupied by engineers. Since the idea was to increase range of the fighters with little or no loss in range of the bomber, drag was measured in several flight conditions: below, at, and above cruise conditions and at altitudes of 10,000 and 20,000 feet. The fighter was towed with engines shut down, flapping down at lower speeds and up at higher speeds. This is an inherently stable phenomena, since as the right wing of the fighter dipped down the angle with respect to the free stream flow (angle of attack) was increased creating lift on the right wing and restoring it to level flight. The opposite occurred when the unattached wing flapped upward. The results were favorable; there was little or no loss in fuel or speed of the B-36 and essentially no loss in range.

On a day in late 1953, test pilot Beryl A. Erickson (who was considered one of the most famous and versatile Convair test pilots) “flew the wingtip of the B-36 home with [him] that day” [8]. The way the mission was setup, the fighter pilot would have to sit in the tiny cockpit of the F-84 for as long as 18 hours until it was time for him to even begin his part of the mission. So, in an effort to simulate a “tired” pilot (or injured), Erickson attempted a hook-up with a positive yaw angle of approximately 5 degrees (F-84 toed into B-36) and within 2.5 flapping cycles, less than 3 seconds, six feet of the B-36 wingtip had been torn off by the F-84 [9]. Fortunately, Erickson and the B-36 pilot all returned safely to Carswell for an uneventful landing. Not so fortunate, the program was officially terminated less than a month later, before much (if any) study could be conducted on the reasons for such violent apparent instability.

Towards the end of the Tom Tom Project, the tensions between the Chinese and our influence in Korea were escalating. The Pentagon wanted the capability to reach China from US bases by extending the range of the B-36. The idea was to add floating wingtip extensions to the B-36, increasing its range by 30% and span to 400 feet. It was probably because of the increased span, and 400-foot-wide taxiways necessary to accommodate this aircraft, that the modified B-36 was never built.

Another component of the wingtip-docked flight was the Long Tom Project, conducted between 1955-56. Beechcraft won a contract to modify an existing military Beech L-23 with small fuel-carrying wingtip “floating panel” extensions. It was successful in demonstrating significant improvements in range [7].

1.2.2 Towed and/or Carried Flight

In the 1920s the United States Army Air Force (USAAF) experimented with airships, dirigibles, aerostats, or blimps (the terms are many, but Zeppelin is only correct if the vehicle
was made by Zeppelin) to carry other aircraft. Dirigibles have advantages similar that of a helicopter; they could take-off and land with or without a runway and in remote or rough terrain. They stay aloft for days, not just hours, and carry large items for little cost in performance, but their top speeds are very slow in compared to airplanes. In the early 1930s, Curtiss F9C-2 Sparrowhawk fighters (built especially for these flight tests) were successful in flying from a trapeze-like apparatus slung beneath US Navy dirigibles, the Akron and Macon. The disaster with the Third Reich’s Hindenburg on May 6, 1937 appeared to cause the end for any wide military or commercial usage of dirigibles. Figure 1.14 is a cartoon of the minuscule Sparrowhawk and its accompanying dirigible. Also around this time, the Soviet Union was conducting experiments with wing and fuselage mounted fighters.

In the summer of 1944, the USAAF proposed the idea of a fighter being carried by a bomber. In early 1945, the USAAF’s Air Technical Services Command (ATSC), sent out the idea of building an ultra-light-weight parasite fighter that could be carried in the bomb bay of B-36 bombers, which were to come online in the late 1940’s. McDonnell Aircraft, a young and eager company, submitted a proposal in March of 1945, and in October two prototypes were ordered by the USAAF under the designation XP-85. In July 1948, the US Air Force took delivery of the first XP-85 (now designated the XF-85, Fighter in lieu of Pursuit) at Muroc Field (Edwards AFB). The aircraft was nicknamed the Goblin because of James Smith McDonnell’s belief in the spirit world, one thought was that a “swarm of Goblins” [10] released from the B-36 could chase the enemy away. Figure 1.15 is the XF-85 Goblin (46-0524) with Ed Schoch, the only test pilot of the Goblin, seen to the far left.

The XF-85 aircraft was [10]:

- 16 feet 3 inches long
- unfolded wing-span of 21 feet and 1.5 inches and folded span of 5 feet
- wing area of 90 square feet
- maximum unhook weight of 4,550 pounds
• powerplant 3,000 pounds J-34-We-22 turbojet
• could be launched from an altitude of 48,200 feet

The B-36 was not ready, so for test flights the EB-29B (44-84111) called Monstro (after the whale that swallowed Pinochio) was fitted with the trapeze system of the B-36. A pit was built into the tarmac to load the Goblin onto the trapeze device and into Monstro. This was complicated and consisted of a horse collar to secure the nose of the Goblin. Figure 1.16 shows the Goblin being lifted and fully retracted into the belly of the B-29, Monstro.

Seven flights were conducted that began on August 23, 1948, and only three connected due to the turbulence surrounding the bombbay. On the first flight, test pilot Ed Schoch missed hooking the trapeze and hit the canopy of the Goblin against the trapeze. This broke the canopy and “knocked off his helmet” [10]. The flights that did not connect to the trapeze had to perform a belly landing; the Goblin had not been equipped with landing gear. By 1949, it was apparent that the Goblin was not going to be the answer for the fighter escort issue, so on October 24, that program was terminated. On a queer note, the XF-85 Goblin is featured as one of the World’s Worst Aircraft [10].

The Fighter Conveyor Project (FICON) achieved much greater success than the Goblin. The goal of the FICON project was very similar to the wingtip coupled projects, in that the range of fighters would be increased. But, a plus (to some) for FICON was that the
pilots would be able to rest comfortably inside the bomber during the 18 hour flight; even capabilities of resupply were possible. The F-84E was modified with a nose-mounted hook, so the aircraft could be captured and stored within the belly of the B-36. Figure 1.17 gives a close-up of the hook used in FICON to attach the F-84 to the trapeze device in the B-36. The F-84 did not completely fit into the belly, but just enough for the canopy section so that the pilot could enter and exit the fighter aircraft. One problem or source of damage was that the horizontal tail surface repeatedly came into contact with the rubber bumpers installed in the bomb bay. Actual flight testing took place at Edwards AFB in California between November 29, 1955 and April 27, 1956 with Major James Rudolph as project pilot and Charles Neyhart as project engineer. Figure 1.18 shows FICON in operation again with Bud Anderson at the helm of the F-84. The RF-84K limited the ground clearance of the B-36, but range was only slightly downgraded. The flight tests were considered successful, yet the project was terminated. Erickson, who worked on the Tom Tom Project and was also a test pilot for FICON called the system a “tinkertoy easy to perform the engagement”
and believed the test pilots “deliberately tore up the system because they didn’t like the idea of [just] riding in a B-36”. Phase I of FICON took place at Eglin AFB in Florida and used a GRB-36F (49-2707) and a F-84E (49-2115, the aforementioned Thunderstreak with swept wings) all from the 31st Fighter Escort Group. Phase II of this project incorporated the prototype YF-84F (49-2430), and Phase IV incorporated the RF-84K (52-7258). This F-84 swept wing model had anhedral in the horizontal tail and spoilers to interface with the ailerons. The RF-84K had great improvements in the control and powerplant devices, “that had long plagued the F-84F”. Improvements like lateral control (spoilers) and pitch-up in accelerated maneuvers (anhedral in tail) were incorporated.

The towed flight experiment depicted in Figure 1.3, the Eclipse project, which ended in 1998, utilized the advantages of towed flight for flight tests on a lifting body. The techniques necessary in designing a vehicle to perform space operations are aerodynamically very different than traditional terrestrial flight, therefore a lifting body is employed much like the shuttle. Recently, focus has been on designing a Reusable Launch Vehicle (RLV) to replace the shuttle and achieve single-stage orbit, but there are difficulties and losses in attaining orbit in a single stage. Replacing the first stage of the shuttle by towing a RLV to a specified attitude could advance the feasibility of the concept.

1.2.3 Formation Flight

Formation flight is the oldest type of CAT flight being widely practiced by our feathered friends long before the Wright Brothers or any human civilization. Birds use the benefits of upwash from trailing vortices of a point bird to cover long distances in their seasonal mi-
Analysis has shown for 25 birds flying in formation the induced drag reduction could be as large as 65% with a corresponding range increase of 71% [12].

Though man has performed formation flight often, he does not usually use all the benefits, as birds do. Mostly formation flight has been used to show-off in airshows, like the Air Force’s Thunderbirds or Navy’s Blue Angles, or to signify a mission, like bombings or missing man. Today with the rapid development of computer technology, tools like Global Positioning System (GPS) and Inertial Navigational System (INS), the energy the pilots used “jockeying the throttle” to maintain position could be minimized, if not eliminated. Therefore, NASA Dryden, Boeing’s Phantom Works, and the University of California at Los Angeles (UCLA) have put much effort into the Autonomous Formation Flight (AFF) project. Figure 1.19 is the cover of the March 2002 issue of Aerospace America which depicts the AFF project in action with a pair of F/A-18s. A team of students and faculty at UCLA developed a prototype navigational system combining GPS and inertial measurement units (IMUs), allowing for position and velocity measurements as well as attitude sensing abilities. This system was integrated to form the Formation Flight Instrumentation System and then fitted to the F/A-18s with radio data modems, so that the aircraft could trade position information. The system had the capability of position accuracy to within 10 cm [6].

On December 5, 2001 a test pilot flew a modified F/A-18 in the trailing vortex upwash of a lead aircraft for 96 minutes; reducing fuel consumption by 12% at an altitude of 40,000 ft. This was the last and longest AFF flight. Previous flights showed even greater benefits at lower altitudes; fuel consumption was reduced by 19.9 – 17.7% at 25,000 ft. The wingtip of the trailing F/A-18 is in a region of best location below and slightly inboard of the wingtip of the lead aircraft. The results from AFF could be advantageous not only to military operations, but commercial operations too. Fighter aircraft in formation would
receive optimal fuel savings flying at a distance 300 ft apart, but commercial airliners would increase that distance up to a mile apart due to their much stronger trailing vortices. Typical commercial airliners flying in formation could achieve a drag savings of 10% on a New York to Los Angeles route translating to a half a million dollar savings per aircraft per year. Also, the air polluting emissions that add to the degenerating greenhouse effect could be reduced: carbon dioxide by 10% and nitrous oxide by 15% [6].

The funding for AFF ended just short of the planned autonomous flight in the summer of 2002 (previous flights only demonstrated the ability to find and stay in the beneficial upwash). Autonomy is the key to making formation flight feasible. The trailing F/A-18 test pilot repeatedly commented on the tiresome work necessary to keep the guidance needles (cross-hairs) on the heads-up display (HUD) display in the beneficial position. Any hope of a commercial application would require autonomous flight; if the airliners are separated by a mile, it would likely be impossible for the pilot to even see the other airliner [6].

The overall idea of formation flight has been known for quite some time, but as mentioned, until recently it has not been feasible. Using advanced computer technology with GPS and INS the autopilot of the aircraft would maintain the sweet-spot position, thereby reducing drag, energy loss, and fuel usage. A gain in range of up to 40% is theoretically possible. The AFF project had a reasonable goal of 10% fuel savings and achieved 12% [6].

1.3 Motivation for Current CAT Flight Study

In recent years and events the US military has had reason to focus towards distant regions like the Middle East, Africa, China and so forth, either for military conflict or support or humanitarian aid. In these regions, the US may not have the luxury of an air base, aircraft carrier, or air bases and carriers of allies. There would be a great advantage in time, cost, and defensive stance to supply “beans, bullets, and bandaids” to these distant regions.

Sectary of Defense, Donald Rumsfield began his appointment under President George W. Bush by announcing changes in the current military strategy through the Department of Defense (DOD). What is pertinent to this work is the military’s need for faster deployment capability. Rumsfield is quoted:

Given these developments, we believe there is reason to explore enhancing the capabilities of our forward deployed forces in different regions to defeat an adversary’s military efforts with only minimal reinforcement. We believe this would pose a stronger deterrent in peacetime, allow us [the US] to tailor forces for each region, and provide capability to engage and defeat adversaries’ military objectives wherever and whenever they might challenge the interests of the U.S. and its
allies and friends. [13]

The armed forces particularly require a system that can be rapidly deployed to meet the increasing need for high mobility—strategically, operationally, and tactically. High mobility is one of the core functions for the Army’s Brigade Combat Team (BCT) that has become essential in command for trouble spots overseas and even more so with the developing War on Terrorism and Operation Enduring Freedom. A task to air-deliver 20 tons safely, over 4000 miles, non-stop, has been presented. This delivery could not only include tanks, and ammunition, but also people/soldiers and supplies/humanitarian aid—"beans, bullets, and band aids". To achieve this mission several types of Compound Aircraft Transport flight system have been proposed, consisting of a larger mothership and a smaller hitchhiker(s). The types in this study are wingtip-docked and formation flight.

This project has been sponsored by the Lockheed Martin Aeronautics Company (LMCO) through the Defense Advanced Research Program Agency (DARPA), and LMCO has proposed the use of vertical take-off and landing (VTOL) aircraft as the hitchhikers, and a transport like the C-5 Galaxy. An additional proposal is the possibility of the hitchhiker(s) as Unmanned Aerial Vehicles (UAVs). UAVs are becoming important tools in intelligence, surveillance, and reconnaissance (ISR) for the armed forces, and their capabilities could foreseeably extend into the civilian world as well. The actual aircraft to ultimately be employed in the project are not yet defined. Also, still on the drawing board is whether or not to modify existing aircraft or build new ones.

The work described here deals specifically with determining the best position, longitudinally, laterally, and vertically for the hitchhiker with respect to the mothership and with the analysis of the stability of the wingtip-docked system. To accomplish this, wind tunnel testing, inviscid aerodynamic computation with the vortex lattice method (VLM), and flight dynamic modelling techniques are employed. These matters are discussed in the following Chapters.
Chapter 2

Wind Tunnel Experiments

Wind tunnel tests were conducted using models representing the aircraft and conditions in the Tom Tom Project of the 1950s discussed in Section 1.2.1. The goal was to measure forces and the moments on two models representing the mothership and the hitchhiker, and to record flow visualization of the trailing vortex interaction. The goal also included studying any factors through measurements or visualizations that could be perceived as leading to system instability, like the “flapping” motion seen in Tom Tom. The models were a retail-bought 1/32 scale F-84E, Thunderstreak, for the hitchhiker and an in-house manufactured composite wing to represent the outboard wing-section of the B-36, Peacemaker, the transport wing, for the mothership.

2.1 Experimental Setup

The tests were conducted in the Stability Wind Tunnel of Virginia Tech, VT, with a test section six feet tall by six feet wide by twenty-four feet long (6’ X 6’ X 24’). Figure 2.1 is a schematic of the wind tunnel. The swept-wing F-84 and transport wing configuration was setup vertically in the test section using the tunnel floor as a plane of symmetry. Figure 2.6 is an example of the general test setup. By representing only the outer portion of the B-36 wing (transport wing) minimized construction time and allowed for a higher Reynolds number. Also, boundary layer trip strips were adhered to each lifting surface tripping the boundary layer through transition to turbulence, thus ensuring a simulation of a reasonable flight Reynolds number.

The F-84 model and sting balance combination were mounted on a traverse mechanism. The traverse mechanism is a staple tool of this wind tunnel facility; it is mobilized by a stepper motor and has vertical and horizontal motion in a cross-sectional plane perpendicular to the streamwise flow. A step of 0.25 inches was standard for these tests. For this experiment, the operator’s attention needed to be focused so that the models would not
collide in returning to the origin. Anywhere from $1 - 0.5\%$ resolution was lost due to gear backlash; these losses are acceptable.
2.1.1 F-84 Model Setup

It was necessary to bore out a section of the fuselage of the 1/32 scale F-84E model to fit a sting balance inside securely. A sheath piece was previously designed and milled, but the development of a mechanism to insert and remove the sting balance was necessary. The simple design utilized Allen-type screws of different diameter to true-up (smaller diameter screw) and push out (larger diameter screw) the balance from a hole bored out of the model nose. Photographs of the finished modifications on the F-84 Model are shown in Figure 2.7. Table 2.1 gives a comparison of the full-scale and sub-scale wind tunnel model. The swept-wing variant 1/32 scale F-84E has a wing span of 12.5 inches and is only differentiated from the F-84F by the propulsive changes not geometric changes.

![Figure 2.3: Modifications for F-84 Model to Internally Incorporate Six-Component Sting Balance](image)

2.1.2 Transport Wing Setup

The transport wing was built specifically for these tests. It had comparable B-36-to-F-84 ratios of tipchord, and a similar leading edge sweep and wingtip airfoil shape of the B-36. Table 2.1 compares the actual dimensions for the B-36 to the transport wing (TW) wind tunnel model.
Table 2.1: Comparison of Wind Tunnel Model Scale to Actual Scale

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Transport Wing Model</th>
<th>B-36</th>
<th>F-84E Model</th>
<th>F-84F</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_{tip})</td>
<td>5.0 in</td>
<td>21.0 ft</td>
<td>2.4 in</td>
<td>6.25 ft</td>
</tr>
<tr>
<td>AR</td>
<td>4.0</td>
<td>11.1</td>
<td>4.26</td>
<td>4.72</td>
</tr>
<tr>
<td>(\Lambda_{LE})</td>
<td>15.(^\circ)</td>
<td>15.(^\circ)6.5(^\prime)</td>
<td>45.(^\circ)</td>
<td>45.(^\circ)</td>
</tr>
<tr>
<td>Airfoil</td>
<td>63–420(^1)</td>
<td>63(420)–517(^1)</td>
<td>HSL(^2)</td>
<td>HSL(^2)</td>
</tr>
</tbody>
</table>

The smaller aspect ratio for the wind tunnel version of the B-36 model is because again only the outboard section of the B-36 wing was to be modelled, and the variation in airfoil section and leading edge sweep, \(\Lambda_{LE}\), are attributed to construction limitations.

The transport wing model used in the wind tunnel experiments was constructed to simulate the lift distribution in the wingtip region of the B-36, where the interaction of the wingtip vortices of the mothership and hitchhiker would occur. The NACA 63–420 airfoil was chosen based on information of the B-36 wingtip and the availability of the airfoil coordinate points. To ensure that the wingtip region on the wind tunnel model was comparable to the actual B-36, a crude vortex lattice method (VLM) comparison was made. An NACA paper supplied VLM data on the B-36 \[14\], and a readily available VLM code at VT supplied the data on the transport wing model. VLM produces a lift curve slope \((C_L/\alpha)\), thus an appropriate angle of attack was chosen based on data in the same NACA paper. The paper listed a washout angle on the B-36 wing of 2 degrees, and note was made that data on the B-36 was calculated at an angle of attack with respect to the zero lift angle of attack, \(\alpha_o = -1.0\) degrees. So an \(\alpha = 1.0\) degrees was used for the calculations on the wind tunnel transport wing model \[14\]. The resulting lift distribution over the semi-span of the B-36 and the entire transport wing model is plotted in Figure 2.4. The wingtip region highlighted in the lower right-hand corner shows good agreement between that of the transport wing and B-36 lift distribution; thus, the use of this transport wing as a model of the B-36 wingtip has been validated.

The transport wing was constructed of foam and composite graphite. The angle of attack of the transport wing was held constant to simulate the relative rigidity of a large bomber in steady, straight, and level flight to a small fighter nearby. This angle of attack was chosen based on a cruise lift coefficient like the Tom Tom Project. The cruise flight conditions were approximately at altitude of 20,000 ft, 330 mph, and the B-36 weight with maximum fuel (Tom Tom flights were short) and no payload or stores, 324,000 lbs, thus

\[
C_L = \frac{W}{\frac{1}{2} \rho_\infty U_\infty^2 S} \approx 0.55,
\]

\(S\) is the B-36 wing planform area of 4772 ft\(^2\). From a vortex lattice code, the lift curve slope \((dC_L/\alpha)\) was determined and an angle of attack, \(\alpha\), was 6.8 degrees. The airfoil employed,

\(^1\)NACA Airfoil

\(^2\)High Speed Laminar Airfoil
NACA 63-420, was cambered with a zero lift angle of attack, $\alpha_o = -2.5^\circ$, so the angle of attack of the transport wing positioned in the wind tunnel should be 4.3 degrees. But, due to the difficulty in actually measuring the angle of attack in the wind tunnel, the focus was on simply reproducing the cruise $C_L$ of approximately 0.55.

2.1.3 Coordinate System

The orthogonal coordinate system ($x$, $y$, and $z$) used in testing was nondimensionalized by the average chord of the F-84E model ($c_{ave} = \frac{c_{root} + c_{tip}}{2} = 2.94$ inches). The nondimensional coordinate system was defined with Greek notation, ($\xi, \eta, \zeta$).

The angle of attack, $\alpha$, roll, $\phi$, and sideslip angle, $\beta$, followed the standard aircraft definition for these angles throughout this study. $\alpha$ is measured from the projection of the aircraft velocity vector into the xz plane to the longitudinal axis of the aircraft and is positive when the vertical component, $w$, of the vector is positive (nose up). $\beta$ is measured from aircraft longitudinal axis to the projection of the aircraft velocity vector into the xy plane and is positive when the lateral component, $v$, of the velocity vector is positive (nose left). $\phi$ is measured as the rotation of the lateral aircraft axis about the aircraft longitudinal axis; wings level or parallel to tunnel floor as the zero location. Figure 2.5 shows the coordinate system and angle definitions, respectively.
2.2 Instrumentation

The F-84 model was mounted onto a six-component sting balance (ID # 8106(4.00-y-36-081)), and the transport wing was mounted onto a four-component strut balance. Figures 2.6 and 2.7 are photographs of the six-component sting balance and the four-component balance, respectively.

Measurements from the balances and traverse mechanism were collected through a data acquisition system consisting of a Measurements Group 2310 Signal Conditioning Amplifier unit for each pair of strain gages on the balances and a National Instruments AT-MIO-16-XE-10 Data Acquisition Card installed in a Pentium III computer. Software for data acquisition is written using LabView 4.0 under the Microsoft Windows 2000 environment.

The centerpiece for the VT Stability Wind Tunnel operation system is a SCXI-1001 Mainframe from National Instruments that allows for installation of up to 12 SCXI modules.
performing signal conditioning and sampling of the input signals: tunnel pressure, temperature, and speed. Each of these modules isolate and amplify 8 differential analog voltages having an input range of -10 to +10 volts. The 8 inputs are read by a National Instruments AT-MIO-16-XE-10 Data Acquisition Card installed in a Pentium III computer; the same as that used for the balances and traverse. This results in a total of 32 isolated differential analog input channels with an analog to digital conversion resolution of 16 Bits. The software for data acquisition is written using LabView 4.0 under the Microsoft Windows 2000 environment, again the same as that used for the balances and the traverse. Collected data was stored directly to the departmental mainframe for data reduction. Detailed information on the wind tunnel facilities is located on the World Wide Web at < www.aoe.vt.edu >; Figure 2.8 are photographs of the DAQ system on the tunnel control platform [15].

Most of the wind tunnel instrumentation is standard and is considered to have minimal measurement errors. Of course there is uncertainty in the accuracy of the wind tunnel measurements, as well as the inevitable losses in instrumentation, but the nature of these experiments is relative, to compare large values in determining a best location for the hitchhiker, and observe trends in the forces and moments on the models while in a region of wingtip vortex interaction. So a full uncertainty analysis was not necessary to validate this work.

### 2.2.1 Model Balances as Measuring Devices

Two types of balances were used: one internal and one external. The internal sting balance measured the forces and moments on the F-84 model, and the external strut balance
measured those on the transport wing.

The six-component sting balance measures six quantities: normal, axial, and side force, as well as rolling, pitching, and yawing moments. The simplified schematic in Figure 2.9 explains the idea behind using scales to measure forces and moments. It is important that the wires or scales be positioned equally apart and be perpendicular both to each other and to the lateral plane (xy plane) of the model. This is to avoid errors in offset due to misalignment. The normal force, \( N \), is equal to the sum of the vertical forces, that carry equal weight, \( N = C + D + E \), the side force, \( Y = F \), and the axial force, \( A' = B + A \). The rolling moment, \( RM = (C - D) \times b/2 \), pitching moment, \( PM = E \times c \), and yawing moment, \( YM = (A - B) \times b/2 \) [16].
The strut external balance measures forces and moments with the same idea, but for these tests, it is only a four component balance. A four component balance does not measure pitching moment and side force; therefore, referring to Figure 2.9 it would only have scales for A, B, C, and D. With reference to the transport wing, the lift and drag forces were measured as well as the rolling and yawing moments.

These balances work on the same principles described previously, but each scale measures a moment directly, not a force. Most wind tunnel balances operate in this fashion. The advantage is that the reference point where the forces and moments act is not necessary; the aerodynamics center of the models does not need to be known. This method is convenient because the balance is often located away from the model like the external balance, and it would normally be very difficult to measure. Figure 2.10 is a cantilever beam with two pairs of strain gages, that represent the scales. The normal force, \( N = (M_r - M_f)/d \), where \( d \) is the distance between the gages, and the subscripts \( r \) and \( f \) refer to rear and front, respectively. Side force is calculated with the same arrangement. This holds because the strain gages are setup as a differential circuit, and for the moments, the strain gages are set up in a summing circuit (see following section and Figure 2.11). So that \( M_{ref} = M_f + \frac{x_{ref}}{d}(M_r - M_f) \), where \( x_{ref} \) is measured from the front gage station and is usually set in the calibration. Rolling moment uses a torque rod or flat plate at 45 degrees with strain gages mounted on the side faces. An axial force is directly proportional to longitudinal strain gage output. The use of strain gages for scales is common, and they are employed for these tests, therefore some discussion is necessary.

**Strain Gages**

A strain gage is an element that senses the change in strain and converts it to an electric output, where strain is the ratio of the change in length of a specimen to the original
The strain gage, itself, stretches and compresses with movement, and this changes the resistance of the material in the strain gage, yielding a change in electric output. Strain gages are configured in a Wheatstone Bridge circuit because it can measure changes in resistance; a Wheatstone Bridge is shown in Figure 2.11[17].

Strain is related to resistance change by Equation 2.1,

\[ \varepsilon = \frac{\Delta R}{R_o(GF)}; \tag{2.1} \]

where, \( \varepsilon \) is the strain, \( R \) is the resistance, and \( GF \) is the gage factor. Gage factor is the measurement of sensitivity of a strain gage. For a typical cantilever beam arrangement with two active gages with equal and opposite strains, the output voltage for a bridge circuit is

\[ E_{out} = \frac{E_{excit}GF\varepsilon}{2}. \tag{2.2} \]

\( E_{out} \) and \( E_{excit} \) are the output voltage and the excitation voltage, respectively. The strain in a cantilever beam with a load at the end is

\[ \varepsilon = \frac{FLd}{2EI}; \tag{2.3} \]

where \( E \) is the Young’s Modulus Elasticity, and \( I \) is the moment of inertia. Figure 2.12 shows the basic configuration.

Substituting Equation 2.3 into Equation 2.2 gives Equation 2.4,

\[ E_{out} = E_{excit} \frac{GF FLd}{2 2EI}. \tag{2.4} \]
Temperature effects on strain measurement can skew gage output. An increase or decrease in temperature can expand or compress the material that the strain gage is attached to, and this produces a temperature-induced resistance change. An advantage of a Wheatstone Bridge using a pair of identical strain gages cemented 180 degrees apart (Figure 2.12) is that those temperature-induced resistance changes are cancelled (assuming the material has a uniform thermal expansion coefficient). Two pairs of identical strain gages (4 total strain gages) and cross-sectional area properties, $I/d$, (Figure 2.10) would add to the overall accuracy of temperature compensation. Take Equations 2.1 and 2.2 and expand based on Figure 2.10, then Equation 2.5 results.

$$\frac{E_{out}}{E_{excit}} = \frac{R_1R_3 - R_2R_4}{(R_1 + R_2)(R_4 + R_3)}$$ (2.5)

The pairs are located on opposite legs, so if the thermal resistivity of the material was the same throughout then the temperature-induced resistance change would be the same for each leg. Equation 2.5 shows that the numerator, $R_1R_3 - R_2R_4$ would be zero.

Generally, the signal from the strain gage must be gain amplified, excited, filtered, and balanced. In this work the multi-purpose Measurements Group 2310 signal conditioning amplifier was used for each pair of strain gages with an excitation of 5V, filtering at 100Hz, and a gain of 100[17].

### 2.2.2 Flow Visualization Test Methods

The flow visualization was performed using tufts attached to the wingtips of the F-84 model and the transport wing model. The tufts were approximately two to three inch pieces of yellow woolen yarn, which are clearly visible in Figure 2.13 as well as other figures throughout the wind tunnel results. The tufts were attached to the wingtips with electrical tape, thus minimizing damage to the surface of the models. Due to the delicacy of this
arrangement, the wind tunnel speed or dynamic pressure was reduced to 1.5 inches of water. When there was flow through the tunnel test section the yellow tufts can be easily seen rotating in the direction of the wingtip vortices. As the F-84 model was traversed near and far from the transport wingtip, the tufts would interact just as the wingtip vortices did. A qualitative analysis of this nature recorded the data with a VHS video camera, which was edited and digitized at a later date. This qualitative analysis also helped to set the distinct regions for the force and moment measurements; these were the regions where there was flow interaction between the transport wing and the F-84 model.

2.3 CAT Wind Tunnel Experimental Procedures

The testing consisted of force and moment measurements focusing on the changes seen by the F-84 model, and flow visualization of the trailing vortex interaction between the F-84 and transport wing. All measurements were taken at a tunnel dynamic pressure, \( q = 4.5 \text{ inches of water} \) (\( \sim 100 \text{ mph} \)), and all flow visualization was performed at a lower tunnel speed, \( q = 1.5 \text{ inches of water} \) (so as not to lose the tufts).

2.3.1 Testing Configurations

These configurations were analyzed at various angle of attack and sideslip angle combinations. The angles chosen were constrained by the mechanical setup seen in the left of Figure 2.5, that were approximately -6, -4, 0, +4, +6 degrees for angle of attack and +9, +2.5, 0, -2.5, 9 degrees for sideslip angle. The angle of attack was measured with respect to the F-84 fuselage centerline or longitudinal plane of symmetry referenced to the tunnel walls, and sideslip angle was measured with respect to the F-84 or lateral plane of symmetry referenced to the tunnel floor. Wind tunnel experimentalists will sympathize, “approximately” must be be emphasized because of the difficulty in measuring angles by eyeballing levels, tape measurements, and assuming that the tunnel floor and walls are straight; therefore, \( C_L \) is employed to more accurately define the reference condition for all configurations.

The testing consisted of a broad scope of parameters and configurations that relied on the flow visualization to define the areas of importance. The parameter study was organic throughout the testing procedure and was not completely specified until data reduction was finalized.

Configuration One: Wingtip-Docked CAT Flight

The first test configuration started with the origin or zero location of the F-84-wingtip-quarter-chord-to-transport-wingtip-quarter-chord. The F-84 was moved in the negative \( \zeta \)-direction which is below the transport wing and in the negative \( \eta \)-direction which is toward
the transport wingtip as if coming up and in for docking; the $\eta \times \zeta$ grid was $2 \times 1$ inches square. The angle of attack and sideslip variations were approximately -4, 0, +4 degrees and 0, -2.5, -9 degrees, respectively. $\alpha$ variation corresponds to a $C_L = -0.03, 0.48, 0.98$, respectively. Figure 2.5 shows the test setup.

**Configuration Two: Close Formation CAT Flight**

The second test configuration was set up with one transport wing chordlength, 5.0 inches, separation between the trailing edge and the leading edge of the F-84 model ($\xi = 3.0$). This was termed close formation flight, because the hitchhiker takes advantage of the mothership trailing vortex upwash while still maintaining the possibility of being rigidly connected to the mothership. This arrangement can also utilize the energy savings of docked or carried flight by flying at conditions of minimal power: engines off or ideal. The tests employed the same pitch and yaw combinations as the first test configuration, and the zero location was in the F-84 model wingtip-to-transport wingtip plane as seen in Figure 2.13, this is the same zero-plane as that in configuration one. The F-84 was now capable of moving inboard of the transport wingtip one inch, and the grid was also expanded to include positive $\zeta$; the $\eta \times \zeta$ grid was $2 \times 4$ inches square. Figure 2.13 shows the setup.

**Configuration Three: Closer Formation CAT Flight**

The third test configuration was the same as the second, but now the distance between the trailing edge of the transport and the leading edge of the F-84 was reduced by half, 2.5 inches or $\xi = 2.2$. This was chosen more on the basis to simulate docking procedures, but it can be used to further analyze a close and rigidly-connected formation flight.

**Configuration Four: Wingtip-Docked plus Roll-Hinge Angle**

The fourth test configuration added roll to the F-84 at a zero location quarter-chord-to-quarter-chord with the two wings in the same plane like the first and second tests. This test hoped to validate some work on stability done by Professor W.C. Durham at Virginia Tech in modelling the flapping or hinge angle ($\eta'$) and pitch angle. Roll angle ($\phi$) of the F-84 and the hinge angle ($\eta'$) are of opposite signs, and combinations of $\phi$ at -4 degrees and $\eta'$ at 45 degrees, and $\phi$ at +4 degrees and $\eta$ at -45 degrees were tested with the same grid pattern as the first test configuration. Figure 2.14 shows the setup.

**Configuration Five: Towed Formation CAT Flight**

Finally, the fifth test configuration attempted to simulate towed formation flight by moving the nose of the F-84 back 15 inches or $\xi = 10.0$ (three chordlengths of the transport
wing) from the trailing edge of the transport wing. In this configuration, the hitchhiker could be connected to the mothership by a tether or unconnected maintaining flight in the mothership trailing vortex manually or through automation. The traverse made a slice though the flow field in the plane perpendicular to the free-stream or $\xi - \zeta$ plane, the F-84 nose in-plane with the transport wingtip, recording force and moment measurements, as well as flow visualization. Figure 2.15 shows the setup.

### 2.4 Wind Tunnel Results

The methods for data reduction are detailed in Appendix A.1. The resulting data for the F-84 model at an approximately zero degree sideslip and angle of attack is presented in this section, and complementary data for other sideslip angles is located in Appendix A.2. Again to alleviate some of the uncertainty of orientating the angles, a more accurate reference to the lift coefficient will be used throughout this text to designated the F-84 model orientation.

The plots show data in terms of standard nondimensional aerodynamic coefficients.
Figure 2.14: Testing Configuration Four: Wingtip-Docked Flight plus Roll

Figure 2.15: Testing Configuration Five: Towed Formation CAT Flight (No flow: tufts hanging down)
based on the free-stream conditions and planform geometry of the F-84 model for lift ($C_L$),
drag ($C_D$), rolling moment ($C_l$), and lift-to-drag ($\frac{(L/D)}{(L/D)_{solo}}$) ratio versus spanwise
location, $\eta$, of the F-84 model for several vertical locations above and below the transport
wing, $\zeta$. $(L/D)_{solo}$ denotes a value for the F-84 model (or transport wing) when the two models
are far removed, relative to the wind tunnel test section, from each. Each vertical or $\zeta$
position has a symbol that is constant for all the plotted wind tunnel data. For example
when the F-84 model and transport wing are in the spanwise-plane, this is the zero $\zeta$ location
represented with the red *’s. If the F-84 model is above the transport wing then $\zeta$ is a positive
and below is negative. For a positive $\zeta$ the symbols are open, and for negative $\zeta$ the symbols
are closed, but the symbol for the absolute value of $\zeta$ is the same. Take another example,
in Figure 2.23 the $\zeta$ positions in the legend of 0.17 and -0.17 are represented by an open
and closed green circle, respectively. There are three sets of data presented in detail for the
F-84 model depicting three streamwise positions for the F-84 model, wingtip-docked to the
transport wing or Configuration One, close to the transport wing or Configuration Two and
towed from the transport wing or Configuration Five, $\xi = 0.0$, $3.0$, and $10.0$.

The data is best analyzed through the lift-to-drag ratio. Lift and drag are coupled in
determining maximum aerodynamic performance. Viewing only one as a benefit determiner,
while the other may be subject to adverse effects, leads to an inaccurate study of the system.
For this particular configuration involving CAT flight, a comparison between the aircraft in
solo flight to the aircraft in CAT flight is important in determining the overall benefits of CAT
flight, therefore a lift-to-drag ratio between solo and CAT flight is defined, $\frac{(L/D)}{(L/D)_{solo}}$.
$L/D$ without a subscript represents the ratio for the configuration tested. The solo values
were determined through asymptotic estimation as the F-84 model is moved away from the
transport wing, and are $L/D \approx 18$, and $C_L \approx 0.49$ for the F-84 model.

Rolling moment coefficient is also presented because of control surface deflection
limitations in maintaining level or trimmed flight in a strong vortex field.

Flow Visualization

First, consider some of the flow visualization results. Figures 2.16 through 2.4 are
progressive still shots of the flow visualization video for the configurations to be discussed:
wingtip-docked (Configuration One), close formation (Configuration Two), towed formation
(Configuration Five). The transport wingtip vortex and the right or inboard wingtip vortex
of the F-84 model are counter rotating and it often appears as though the F-84 vortex is
stationary in many of the videos, even stopping and then rotating in the opposite direction
to match the transport wing vortex rotation.

In Figure 2.16, time progresses from left to right in Frames 1, 2, and 3. Frame 1
shows the wingtip vortices of the F-84 model and the transport wing being unaffected by
one another. As the F-84 model is moved inboard toward the transport wing in Frame 2,
the right or inboard wingtip vortex of the F-84 model is being drawn into the wingtip vortex
of the transport wing. In Frame 3 the right wingtip vortex of the F-84 model is completely sucked in to the wingtip vortex of the transport wing. The transport wing appears to have the strength to cause the F-84 model vortex to become stationary, depicted by the tuft being drawn into the core of the transport wing vortex. The point being the vortex of the transport wing is much stronger in comparison to the F-84, and dangers do exist in the region close to the transport wing trailing vortex. Another thing to note is the tightness of the transport wing vortex. When the F-84 is aligned with the transport wing as in the second test configuration, the F-84 moves barely a quarter of the transport wing chord and it is out of the vortex influence.

In Figure 2.17, displays a similar time progression in Frames 1, 2, and 3 as the F-84 model traverses toward the transport wing. Frame 1 shows the F-84 model not far from the transport wingtip, and it is apparently unaffected by the transport wingtip vortex. As the F-84 begins to move toward the transport wing, Frame 2 shows some distortion and flutter in the tuft of the F-84 model right wingtip vortex. Frame 3 depicts the tuft fluttering about a stationary position; again the strength of the transport wingtip vortex has overcome that of the smaller F-84 model.

Then, in the last flow visualization video at $\xi = 10.0$ in Figure 2.4 the F-84 must move 3 or 4 times the transport chord to be out of the vortex influence. Compare the relatively large distance between the F-84 model and the transport wing in Frame 1 of Figures 2.17 and 2.4. In Frame 1 of Figure 2.4, the tuft on the F-84 right wingtip is almost straight or stationary and thus still under the influence of the transport wing trailing vortex. As the F-84 model moves toward the transport wing in Frame 2, the tuft attached to the right wingtip of the F-84 model remains stationary, and continues to do so as the F-84 model moves further inboard of the transport wingtip and into her downwash region. It is here that visual recognition was made of the F-84 model right wingtip vortex periodically rotating with the transport wingtip vortex, which was opposite to its usual rotational direction.

The videos for all tests are in .mpg (Moving Pictures Experts Group) file format and are located at <www.aoe.vt.edu/groups/cat>. The interesting things to note in the flow visualization videos is how the smaller trailing vortex from the F-84 is sucked into the more powerful vortex of the transport wing, which appears to be unaffected by the smaller aircraft. The left wingtip vortex of the F-84 model generally did not appear to be affected by the transport wingtip vortex in any configuration.
Configuration One: Wingtip-Docked CAT Flight

The arrangement is shown in Figure 2.5 and again the results are presented in terms of forces and moments on the F-84 model versus spanwise distance, $\eta$, for several vertical locations above and below the transport wing, $\zeta$. Figures 2.19 through 2.22 give the data for lift, drag, and rolling moment coefficient as well as the $(L/D)/(L/D)_{solo}$ ratio defined previously.

In Figure 2.19, the F-84 closes the spanwise distance between it and the transport wingtip from an $\eta = 0.68$ to an $\eta = 0.0$ ($\eta = 0.0$ being a wingtip-docked position). So, referring to Figure 2.19 in that manner, the lift first increases gradually and then more rapidly beginning at an $\eta \approx 0.3$ and $C_L \approx 0.55$. At $\eta = 0.0$ the span between the F-84 and
Figure 2.18: Configuration Five: *Towed Formation*, $\xi = 10.0$: Frame 1, $\eta \approx 1.0$, Frame 2, $\eta \approx 0.0$, and Frame 3, $\eta \approx 2.0$

Figure 2.19: $C_L$ vs. $\eta$, Spanwise Distance for Configuration One
transport wing would be continuous, and that, as theory predicts, is the point of maximum lift, $C_{L_{\text{max}}} \approx 0.59$. Figure 2.19 also shows data for several vertical locations, $\zeta$, of the F-84 model with respect to the transport wing. A hitchhiker aircraft docking to a mothership would approach from below, so to simulate this docking procedure only the vertical positions of the F-84 model below the transport wing, $-\zeta$, were tested. The red *’s represent the zero vertical location where the F-84 model and transport wing are in the same spanwise-plane, $\zeta = 0.0$. The data for all vertical locations follows the same general trend established by the $\zeta = 0.0$ location. The green crosses in Figure 2.19 represent the data of the F-84 at a vertical location of $\zeta = -0.26$, only 26% of the F-84 model average chord (2.94 inches). This relatively small negative $\zeta$ value yields an overall increase in lift compared to the other F-84 $\zeta$ positions.

The drag data in Figure 2.20 is quite invariant as the F-84 model moves from $\eta = 0.68$ to $\eta = 0.0$, closing in on the transport wing for a docked position. The drag of the F-84 model is banded averaging approximately $C_D \approx 0.27$. There is no vertical, $\zeta$, or spanwise $\eta$ location of distinguishable advantage in consistently minimizing drag.
Figure 2.21: $C_l$ vs. $\eta$, Spanwise Distance for Configuration One

The magnitude of rolling moment in Figure 2.21 increases as the F-84 moves spanwise in to a docked position. This should be intuitive to the reader due to the clockwise rotation for the left wingtip vortex of the transport wing. The air flow must circulate from the high pressure region (lower surface) to the low pressure region (upper surface), thus an upwash is hitting the inboard wing (or right wing) of the F-84 model. The changes in rolling moment occur quicker than the changes in lift in Figure 2.19. But like the variation in lift, the magnitude of the rolling moment begins to increase very rapidly at approximately an $\eta = 0.3$ and $C_l \approx -0.013$. Also like the lift data in Figure 2.19, the green +’s that correspond to a vertical position, $\zeta = -0.26$, of the F-84 model below the transport wing have the greatest increase in roll. The data for the two vertical positions furthest below the transport wing, the green crosses, $\zeta = -0.26$, and the red triangles, $\zeta = -0.34$, deviate from the general path at an $\eta \approx -0.1$. The rolling moment data decreases in magnitude for those two points. Perhaps the right wing of the F-84 model is coming into contact with the downwash of the transport wingtip vortex or is moving out of the influence region. The maximum rolling moment seen is at an $\eta \approx 0.0$, wingtips aligned, $C_l = -0.025$. The negative sense of rolling moment variation is based on the standard aircraft coordinate system; the upwash of the
transport wingtip vortex is impacting the inboard or right wing of the F-84 model, thus the F-84 is rolling counterclockwise (left wingtip down) and that is defined as negative rolling moment. The trends in magnitude between lift and rolling moment coincide, but though increases in lift are beneficial, the increases in rolling moment are not. It is the zero vertical position for the F-84 model, the red *’s, $\zeta = 0.0$, that shows the least variation in rolling moment compared to the non-zero vertical positions. Ideally rolling moment should equal zero far away, where a trimmed flight condition exists. If the aircraft could not be trimmed (i.e. not deflect the control surfaces enough), the pilot might lose control or become fatigued trying to stay straight and level. So, the large benefits in lift for a docked position as seen in Figure 2.19 may not be reasonable to achieve, because of the inability to maintain trimmed flight.

![Figure 2.22: $L/D$ vs. $\eta$, Spanwise Distance for Configuration One](image)

The last set of data shown for the wingtip-docked configuration (Configuration One) is in Figure 2.22. It plots the $L/D$ data for the F-84 model as a ratio to the solo value of $L/D$ for the F-84 model. Again this is as the F-84 model moves spanwise, left to right ($\eta = 0.68 \rightarrow 0.0$), toward the transport wing to a docked position for various vertical positions below the transport wing. Since the drag is fairly constant in Figure 2.20, and
the lift increases as the F-84 moves toward the transport wingtip in Figure 2.19, the increase of $L/D$ in Figure 2.22 is logical. The $L/D$ data in Figure 2.22 shows no distinguishing advantage in a vertical position, $\zeta$, for the F-84 model. The zero vertical position, the red *’s, $\zeta = 0.0$, at $\eta = 0.0$ or wingtips-docked has the highest $(L/D)/(L/D)_{solo}$ value at $\approx 1.45$; the other data points were unable to reach $\eta = 0.0$ because the gear backlash in the traverse discussed earlier. As mentioned in Chapter 1, the aerodynamic parameter $L/D$ is viewed as the determining flight performance parameter, thus Figure 2.22 is the most important in determining the benefits of this flight configuration compared to solo flight. This benefit for wingtip-docked flight is shown as an approximate 20%–40% increase in performance.

**Configuration Two: Close Formation CAT Flight**

For this configuration the F-84 model is moved downstream slightly to an $\xi = 3.0$ as in Figure 2.13. This corresponds to a longitudinal gap between the leading edge of F-84 model wingtip and the trailing edge of the transport wingtip equal to one transport wing chordlength (5 inches or 40% of the F-84 model span). Figures 2.23 through 2.26 give the data for this configuration, and it is presented like the data for Configuration One, previously; lift, drag, roll and $L/D$, plotted versus the spanwise gap, $\eta$, between the F-84 model and the transport wing for various locations of the F-84 model above and below the transport wing. Remember the transport wing is fixed, only the F-84 is capable of moving, so, when shifted downstream, the F-84 model is able to move inboard of the transport wingtip, $-\eta$ locations (slightly, both models have wing sweep). For clarity, the location of the wingtip-to-wingtip plane is highlighted by a bold line along the vertical axis at $\eta = 0.0$. The vertical positions above and below the transport wing, $\zeta$, cover a broader range than that for Configuration One, now including positive values of $\zeta$ with the F-84 model moving above the transport wing. Recall in the introduction of this section, positive values of $\zeta$ are represented as open symbols, while the mirrored negative $\zeta$ value is represented as a closed symbol.

The lift data in Figure 2.23 increases as the F-84 is moved in and continues to increase as the F-84 wingtip moves inboard of the transport wingtip, $-\eta$. The lowest $\eta$ value is $-0.34$, 34% of the average chord for the F-84 model, only 8% of the F-84 span and 5% of the transport wing span. This is a relatively small distance. A position for the F-84 model wingtip slightly inboard of the transport wingtip means that more of the F-84 is enveloped in the upwash of the transport wing; it is well known that the core of a wingtip vortex rolls-up slightly inboard of the wingtip. If the F-84 moves further inboard of the transport wingtip, the downwash would begin to contribute negatively to the lift of the F-84. This switch to a downwash region is already evident in Figure 2.23 with the introduction of a levelling-off or peak in the lift at $\eta \approx -0.2$.

The variation in lift for different vertical locations, $\zeta$, fans out as the F-84 model nears the transport wingtip and continues to do so inboard of the wingtip. The fanning or spreading out of the lift data for $\eta < 0.25$ can be attributed to the F-84 contact with
the transport wingtip vortex, which has begun to diffuse radially due to the viscous nature of real-life fluid flow. A radial diffusion would create a larger region of influence for the transport wingtip vortex to act on the F-84, thus the fanned out lift data compared to the wingtip-docked lift data. This fanning pattern is carried throughout the close formation data.

The lift data for $\eta = \zeta = 0.0$ in Figure 2.23 is more or less identical corresponding to the wingtip-docked configuration ($\xi = 0.0$) in Figure 2.19. Compare the red *’s in Figure 2.23 at the wingtip-to-wingtip point, $\eta = 0.0$, (the bold vertical line) with the red *’s in Figure 2.19 at the far left also $\eta = 0.0$, the lift values are, for all practical purposes, equal, $C_L \approx 0.60$ on Figure 2.19 , and $C_L \approx 0.59$ on Figure 2.23. The lift appears invariant with respect to longitudinal or stagger position, and the only theorem known to the author that relates constant inviscid forces with variation in longitudinal position is the Munk Stagger Theorem. The Munk Stagger Theorem states that – *The total induced drag of a system of lifting surfaces is not changed when the elements are moved in the streamwise direction*, and for this theorem to hold the free stream conditions as well as vertical and lateral position...
of the staggered aircraft must remain the same. But, the phenomenon noted in the wind tunnel relates invariance in lift not induced drag, so a conjecture is posited. Induced drag is directly proportional to lift by definition, \( C_{D_i} = C_L^2/\pi AR e \), and for this conjecture to have some weight two strict assumptions must be true: one that the variation in lift on the transport wing must be small, and two, that the lift distribution or Oswald’s efficiency factor, must be invariant for differing stagger positions. The Oswald’s efficiency factor, \( e \), quantifies the percentage deviation that the lift distribution is from an elliptic distribution or a minimal induced drag, \( e = 1.0 \). The first assumption has already been shown to be plausible, but the second is less clear. From the Munk Stagger Theorem it should be evident that the induced drag distribution over the system remains constant for variation in stagger position of the lifting surfaces, but the individual surfaces could have variation in induced drag distribution. This is nothing more than a conjecture that requires further study, but it is curious phenomenon noted in the wind tunnel testing.

In Figure 2.24, the drag data, for the F-84 in Configuration Two or close formation is presented. Compared to Figure 2.20 for the wingtip-docked configuration, \( \xi = 0.0 \), the overall drag has decreased. All data in Figure 2.24 is less than \( C_D \approx 0.2 \), and in Figure 2.20 the drag data is banded between \( C_D = 0.3 \) and \( C_D = 0.2 \). Though the lift data described
above spoke of the Munk Stagger Theorem for lift and induced drag, the drag data here is for total drag which includes not only the induced drag but also the parasite drag. So the reduction in total drag for a change in longitudinal position is reasonable. The maximum for drag in Figure 2.24 is for a spanwise location for the F-84 at about \( \eta = 0.5 \) and \( C_D = 0.02 \), while the minimum is at \( \eta = -0.34 \) and \( C_D \approx 0.005 \). This value is very near zero. Upwash can cause a negative drag or positive thrust, thus at some point \( C_D = 0 \) leading to an infinite \( L/D \) at that same point.

Whether, in general, above or below the transport wing is more beneficial in drag for the F-84 is not as clear as that for the lift data for Configuration Two. The vertical position data is clustered until the F-84 wingtip passes the transport wingtip, \( \eta = 0.0 \) (the bold vertical line), and then clearly the red asterisks and green open circles depart and decrease rapidly. Referring to the legend in Figure 2.24, the red asterisks are at a zero vertical position for the F-84 model, \( \zeta = 0.0 \), and the green open circles are at a vertical position for the F-84 model slightly above the transport wing, \( \zeta = 0.17 \). Perhaps also worthy of note, the next best vertical position for the F-84 is the mirror of the open circles, the closed green circles. A conclusion could be drawn that a slightly off-center location for the F-84 is best in terms of drag for close formation flight.

In Figure 2.25, the trend in rolling moment for the F-84 wingtip outboard of the transport wingtip, \( \eta > 0.0 \), appears to be similar to that for the wingtip-docked configuration in Figure 2.21. For both configurations, the magnitude of the rolling moment at the furthest right spanwise position, \( \eta = 0.68 \), is \( C_l \approx 0.005 \) increasing as the F-84 model moves toward the transport wing. There is a short continuation of the increasing rolling moment to a maximum magnitude of \( C_l \approx 0.027 \) and then a decrease or roll reversal tending towards zero. As the F-84 wingtip moves inboard of the transport wingtip, it begins to become subject to the downwash. This would cause the F-84 to roll in the opposite direction, thus the roll data follows the levelling-off or peak in the lift data, Figure 2.23.

The best vertical position, \( \zeta \), for the rolling moment data is furthest away from the transport wing and the disturbances of her wingtip vortex. The best rolling moment value would be zero for trimmed flight, thus expending no extra energy in control surface deflections. The maximum rolling moment is at the vertical position of zero, \( \zeta = 0.0 \), and the magnitude of rolling moment decreases consistently with increasing vertical separation; following \( \eta = 0.0 \), the red asterisks, for \( \zeta = \pm 0.17 \) are the green open circles for \( \zeta = \pm 0.34 \), and then the red triangles and so forth.

The lift and roll data from both Configuration One and Two show a trade-off in flight performance benefits for the F-84 model. To have benefits in lift, the F-84 needs to be close to the transport wingtip vortex, but the least penalty in the rolling moment is to be far away from that vortex.

In Figure 2.26, the \( L/D \) data is presented as a ratio, as previously, to the solo \( L/D \) value for the F-84. This is plotted versus spanwise separation, \( \eta \) between the F-84 and the fixed transport wing. Compared to the wingtip-docked data in Figure 2.22, this data for
close formation is dramatically more beneficial. As the F-84 moves toward the transport wing, $\eta > 0.0$, the data varies little compared to the variation of the data inboard of the transport wingtip, $\eta < 0.0$. Focusing first on the $L/D$ data for the F-84 model outboard of the transport wingtip, $\eta > 0.0$ or to the right of the bold vertical line, it shows two to three times increase in flight performance compared to that of the wingtip-docked in Figure 2.22. At the wingtip-docked plane, $\eta = 0.0$, the zero vertical position represented by the red *’s, $\zeta = 0.0$, $(L/D)/(L/D)_{solo} \approx 4.0$ this corresponds to a 300% increase in flight performance from solo flight of the F-84. As the F-84 wingtip passes inboard of the transport wingtip, $\eta = 0.0$, the majority of the data asymptotically approaches the $(L/D)/(L/D)_{solo}$ value seen at the wingtip-to-wingtip point, $\eta = 0.0$ and $\zeta = 0.0$, which is 4.0, but two of the vertical locations take-off inboard of the transport wingtip. Those two vertical positions happen to correspond with the two decreasing rapidly in the drag data (Figure 2.24), thus it is evident that the reduction in drag is driving the benefits in close formation flight. The beneficial vertical positions are the wingtip-to-wingtip plane, $\zeta = 0.0$ and the vertical plane slightly above the transport wing at $\zeta = 0.17$. The corresponding $C_D$ values in Figure 2.24 travel very close to zero, and at zero the $L/D$ data would be infinite. This is caused by the upwash of the trailing vortex from the transport wing, and it would be tight and powerful close to
the transport wingtip, because dissipative forces have had little time to affect the trailing wingtip vortices. This should explain the large increase in \((L/D)/(L/D)_{solo}\). At the wingtip-to-wingtip plane, \(\eta = 0.0\), an \(\approx 700\%\) increase from solo flight for the F-84 is seen. And, up to a 1100\% increase is seen at the vertical position slightly above the transport wing, \(\zeta = 0.17\) and \(\eta = -0.34\). Though close formation shows a large benefit aerodynamically to the system, the proximity of the F-84 to the transport wing may be too close for the F-84 to control his position. Air traffic controllers keep airliners up to a mile apart because the trailing wingtip vortices of a large airliner can be powerful enough to destroy a airliner of equal or lesser size.

**Configuration Five:** *Towed Formation CAT Flight*

Figures 2.27 through 2.30 show the same type of wind tunnel data previously discussed, but now for the towed formation or Configuration Five, which is shown in Figure 2.15. The F-84 has been moved downstream three chordlengths of the transport wing or five of the F-84 (\(\xi = 10.0\)). It was set up for a slightly different purpose (towed or tethered

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Figure 2.26: \(L/D\) vs. \(\eta\), Spanwise Distance for Configuration Two
flight), thus the focal point was with the F-84 nose aligned with transport wingtip. This spanwise position, \( \eta = -2.13 \), is highlighted with a bold vertical line located about mid-way on all the plots for this configuration. Also highlighted is the same wingtip-docked plane, \( \eta = 0.0 \), to the far right of the data. With reference to the plotted data, each Configuration, One, Two, and Five, presented is sequentially shifted to the left in the plane perpendicular to the free stream; for Configuration One \( \eta = 0.0 \) is at the far left, for Configuration Two \( \eta = 0.0 \) is in the middle, and for Configuration Five \( \eta = 0.0 \) is to the far right. Another note on this towed configuration data is that only data for one vertical position was acquired, that is in the wingtip-docked plane, \( \zeta = 0.0 \). The sweep through this plane was larger than that for the other data, \( \pm \) a F-84 semi-span from the F-84 nose to transport wingtip, \( \eta = -2.2 \).

![Figure 2.27: \( C_L \) vs. \( \eta \), Spanwise Distance for Configuration Five](image)

In Figure 2.27, the lift data for the F-84 in towed formation is presented versus spanwise location, \( \eta \). The data point to the far right of the plot, \( \eta = -0.43 \) and \( C_L \approx 0.77 \), is closest for comparison to \( \eta = -0.34 \), the far left point in Figure 2.23, with a \( C_L \) value of approximately 0.64. This lift value seems to comply with the conjecture proposed with reference to the lift data for Configurations One and Two based loosely on the Munk Stagger Theorem concerning induced drag. From that far right point in Figure 2.27 as the F-84 moves
further inboard of the transport wingtip, the lift data decreases smoothly and appears to be coming to a plateau, where the F-84 would be fully enveloped by the downwash of the transport wing. The lift is actually negative at the minimum, $C_L \approx -0.25$.

![Figure 2.28: $C_D$ vs. $\eta$, Spanwise Distance for Configuration Five](image)

In Figure 2.28, the drag data for the F-84 in a towed configuration is presented for various spanwise locations. This data is more interesting than the lift data. The far right data point in Figure 2.28 coincides with the far left data point in Figure 2.24, with $C_D$ values of approximately 0.005. As the F-84 continues to move inboard of the transport wingtip, the drag rises and falls twice with a sharp dip just after the nose of the F-84 passes the transport wingtip, $\eta \approx -2.25$ and $C_D \approx 0.004$. It then appears to plateau at about $C_D \approx 0.01$ for awhile, and the data rises to a maximum of $C_D \approx 0.015$ at $\eta = -3.8$. The drag data recorded for Configuration One in Figure 2.20 is still the largest at almost double the maximum recorded here.

The rolling moment data for the F-84 in towed formation is presented in Figure 2.29. The rolling moment behavior is similar to the lift data in Figure 2.27; it is highest to the far right at $C_l \approx 0.045$ and lowest passing through the wingtip-to-wingtip plane at $C_l = 0.0$. If the F-84 passes spanwise from the beneficial upwash region to the non-beneficial
downwash region of the transport wing trailing vortex, then it should be quite logical that
the rolling moment would change direction. The velocity component shifts from upwash
pushing on the inboard wing of the F-84 model to the outboard wing. Ultimately, in the
more uniform downwash region the F-84 model should be trimmed and rolling moment
should approximately equal zero. The rolling moment data appears to define two plateaus,
one at \( C_l \approx 0.04 \) and two at \( C_l \approx -0.02 \). Also, the curious dip in the drag data in Figure 2.28
is manifested here at the same spanwise location, \( \eta = -2.25 \), as a peak valued at \( C_l \approx 0.008 \).
Having a discontinuity in the drag and roll data at the same point very close to the fuselage
leads to the hypothesis that this is simply vortex-fuselage interaction with the circulatory
motion being disrupted or transformed by the fuselage cylindrical shape.

In Figure 2.30, the \( L/D \) data for the F-84 to the solo F-84 \( L/D \) is presented versus
the various spanwise locations, \( \eta \). The data seems to be driven by the interesting drag
data in Figure 2.28, with the drag dips now manifested as \( L/D \) peaks at spanwise locations
of \( \eta \approx -1.25 \) and \( \eta \approx -2.25 \). These discontinuities seem amplified in comparison to the
drag data. The maximum flight performance benefit of \( (L/D)/(L/D)_{solo} \approx 8 \) is seen at the
first peak, \( \eta \approx -1.25 \), and the second peak, \( \eta = -2.25 \), shows \( (L/D)/(L/D)_{solo} \approx 5 \). The
Figure 2.30: $L/D$ vs. $\eta$, Spanwise Distance for Configuration Five

$(L/D)/(L/D)_{solo}$ is a minimum at the far left data point, $\eta = -3.8$ at approximately -1.0. Overall, the $L/D$ follows the general trend seen in lift (Figure 2.27), but the excursions in drag data (Figure 2.28) are present.

2.4.1 Summary of Wind Tunnel Results

The wind tunnels experiments showed that the close formation, Configuration Two, yielded the greatest ($\sim 1100\%$) aerodynamic benefit through large reductions in drag. The wingtip-docked configuration showed aerodynamic benefits (20-40%) when the F-84 and transport wing were tip-to-tip based on the lift force. The towed formation showed aerodynamic benefits ($\sim 800\%$) driven by drag almost as large as the close formation. Unfortunately, this is not the whole story; other factors must be considered to conclude what is the best location for a hitchhiker with respect to the mothership. As mentioned previously, the issue of hitchhiker control is a factor in feasibility; that is why the roll data is presented as well. The large magnitude of the velocities close to a wingtip trailing vortex could render a smaller hitchhiker uncontrollable, particularly in roll due to the circulatory nature of a vortex.
flow field.

Several combinations of data for angle of attack, $\alpha$, and sideslip, $\beta$, were also conducted like positive $\alpha$ and $\beta$, negative $\alpha$ and positive $\beta$, negative $\alpha$ and $\beta$ and so forth. The positive and negative values of $\alpha$ corresponded to a $C_L$ of 0.98 and -0.03, again this is for the F-84 model in solo flight. It should be clear that data for $-\alpha$ would produce a decrease in flight performance with respect to solo flight (with $\alpha \geq 0.0$ degrees), but this data is included for a complete study. The trends in this data followed that of the presented data. All of this data is located in Appendix A.2.

Similar data was plotted for the transport wing for lift, drag, lift-to-drag ratio ($(L/D)_{solo}$ for the transport wing is 4.2), but due to the large size of the transport wing compared to the F-84 model, there was relatively little change in the forces and moments on the transport wing as the F-84 model was moved. This is emphasized by the flow visualization, which saw an unaffected trailing wingtip vortex from the transport wing when the F-84 model was moved in close to her wingtip. The transport wing did see benefits and not losses in aerodynamic performance, 20 – 80 %, but this data compared to the corresponding performance increases in the F-84 (300 – 1100 %) are very small. That is why the data presented here is for the F-84 only. All of the transport wing data is in Appendix A.2.
The computational aerodynamic analysis for compound aircraft transport flight presented here utilizes the vortex lattice method (VLM) for an incompressible and inviscid flow field about a finite wing. The goal of this effort is to develop a complement to the experimental data that will aid in understanding and interpreting the data, and also a simple tool for detailing a hitchiker location of maximum aerodynamic benefit. Undoubtedly, all of the idiosyncracies seen in the experiments discussed in Chapter 2 will not be accurately simulated because the VLM does not account for the real-life viscous effects present in the wind tunnel.

The vortex lattice method is similar to and sometimes categorized as a panel method, because it represents the body surfaces as a set of quadrilateral panels. The panels are distributed with a finite number of singularities whose strengths are unknown. The system yields a linear set of algebraic equations that can be solved exactly for the singularity strengths through the flow tangency boundary condition. The flow tangency boundary condition results because the surface shape must follow a streamline, which by definition requires that the velocity is tangent everywhere along the streamline,

\[ d\vec{S} \times d\vec{V} = 0, \]

where \( d\vec{S} \) is a segment along the streamline and \( d\vec{V} \) is the velocity vector associated with that segment. These strengths can then be integrated over the surface to determine the total forces and moments associated with pressure changes on the body due to the flow field [18].

Vortex lattice method was chosen because CAT flight deals with geometries of low aspect ratio and highly swept wing fighter aircraft as hitchhikers. This eliminated the possible use of VLM’s much simpler cousin lifting line theory (LLT) postulated by Prandtl in the early 1900s. Some fundamental assumptions of LLT are: the wing is unswept, the aspect ratio is large, the wing is thin, and the vortex induced spanwise velocity is much less than the vortex induced downward velocity or downwash; \( \bar{c} << b, \frac{t}{\bar{c}} << 1, \) and \( q << w. \) These assumptions allow for a bound distribution of vortices, a vortex sheet, along the wing to be represented as
a single vortex filament along the quarter chord or aerodynamic center and for the discrete chordwise wing segments to be treated as two-dimensional airfoil sections. Low aspect ratio negates the first assumption and highly swept negates the last assumption; a thin wing is still assumed \[19\].

The bound vortex sheet on the upper and lower surfaces can be represented as a single vortex sheet bound to the mean camberline of the finite wing. This simplification is validated through the thin wing assumption which includes not only a \( \frac{t}{c} \ll 1 \), but also that the airfoil radius of curvature is elongated; elongation allows a line segment following the airfoil shape to be considered normal to the chordline. The angle of attack is considered to be small so that the velocity perturbations in the flow field are much less compared to the free-stream velocity, \( u, v << U_\infty \). Finally, if the simplified boundary condition can be expanded into a Taylor series then, with the application of the stated assumptions, the only term of importance is the velocity perturbation at \( v(x, 0) \), i.e. the mean camberline \[19\]. The flow tangency boundary condition is thus transferred to the mean camberline, and as a result, VLM is considered to be invariant with wing thickness or viscous effects (the viscosity effects tend to offset the thickness). These techniques have yielded good agreement with empirical data. An additional advantage of VLM is that it is the simplest of the panel methods; the enclosed surface of the finite wing is not modelled with a source and a sink, (a doublet), which has twice as many equations required to be solved \[18\].

Full simulation of the aerodynamic problems of interest here would require treatment of the viscous effects which surely become important as the wings of the mothership and hitchhiker come close together. This would necessitate a three-dimensional, Navier-Stokes code and elaborate gridding techniques. Such an effort was deemed outside the scope of this work.

### 3.1 Vortex Lattice Method Theory

The Vortex Lattice Method employs horseshoe vortices compiled into a lattice structure of trapezoidal panels. Horseshoe vortices are composed of three vortex filaments. This is based on the theorems of Helmholtz and Kelvin plus knowledge that circulation produces lift. The force produced by a vortex of circulation, \( \Gamma \), subject to a velocity, \( \vec{V} \) is

\[ \vec{F} = \rho \vec{V} \times \vec{\Gamma}. \]

A typical horseshoe vortex is shown in Figure 3.1 with general orthogonal coordinates \[19\].

One filament is finite and is bound at the quarter chord or aerodynamic center of each panel. This location is chosen because the total pressure forces from thin airfoil theory act at the quarter chord. In lifting line theory, the bound vortex filaments are coincident with the span of the finite wing and in panel methods, VLM in this case, the individual vortex filaments span each trapezoidal panel. The two remaining vortex filaments are semi-infinite,
starting at the quarter chord and extending in the free-stream direction to infinity. The system of vortices representing the wing surface are the bound vortex system; bound to the wing surface. Figure 3.2 is an example of the vortex system arrangement for LLT and VLM [18].

Helmholtz’s third vortex theorem states that in a fluid flow free of dissipative effects, a vortex filament cannot end in a fluid; it either extends to infinity, forms a closed loop, or ends on the boundaries of the fluid. So, that is why the semi-infinite vortex filaments must extend to infinity. More on the reason for this horseshoe arrangement is the fact that lift, and thus circulation, is not uniform across the wing span and ultimately goes to zero at the wingtips. This creates a curved spanwise lift/circulation distribution. Since this circulation cannot just disappear; the change in circulation, $\Delta \Gamma$, is continuously shed from the bound vortex creating a vortex sheet which extends to infinity. This sheet rolls-up spanwise into trailing vortices at the wingtips due to the pressure differential between the upper (lower pressure) and lower (higher pressure) surfaces. The vortex sheet can be assumed to be an infinitesimally thin sheet of discontinuity, and this sheet can be estimated by a series of discrete vortex lines of strength, $d\Gamma$, extending to infinity and parallel to the free stream. The trailing vortex system is termed the free vortex or more commonly the wake [18]. The free vortex is assumed parallel or flat with respect to the free stream, because the free stream velocity is usually much greater than the downwash velocity, $U_\infty >> w$. This means all the lift is produced by the vortex bound to the mean camberline of the lifting surface. In cases of complicated induced flow fields, it might be necessary to incorporated the wake deformations to yield an accurate solution for lifting forces [20].

Another part of the vortex system associated with the finite wing is the starting vortex. The starting vortex completes the circulation circuit, so Kelvin’s circulation conservation theorem is not broken. Kelvin states, similar to Helmholtz’s vortex theorem, that in
the absence of dissipative forces if circulation is zero it remains zero at all times, and if it is finite it retains that value for all times; the material or substantial derivative is zero, $\frac{D\Gamma}{Dt} = 0$. Recall that the material derivative is the local time rate of change plus convective rate of change for a material, in this case $\Gamma$. An airfoil, initially without circulation when introduced to a velocity field, has lift and thus circulation, but the total system of circulation–consisting of a clockwise circulation of the bound vortex enclosing the airfoil and the counterclockwise circulation of the starting vortex–remains zero \[19\]. Figure 3.3 diagrammatically shows the starting vortex. The starting vortex is diffused with the flow over time and is neglected in these methods, because all cases are assumed to be at a constant angle of attack and at a
The final goal is to determine the pressure forces and moments on the finite wing. This is calculated from the circulation, $\Gamma$, or the strength of the vortex by the Kutta-Joukowski theorem in Equation 3.1 for the lift on a two-dimensional wing section (an airfoil) which is

$$l(y) = \rho_\infty U_\infty \Gamma(y).$$  \hspace{1cm} (3.1)

The Kutta-Joukowski theorem is derived from a two-dimensional body of arbitrary cross-section in a uniform flow, and the force, lift, acts normal to the free stream velocity, $U_\infty$. Integrating the sectional lift, $l(y)$ from Equation 3.1, over the span of the wing yields the
total lift.

\[ L = \int_{-\frac{b}{2}}^{\frac{b}{2}} \rho \infty U \infty \Gamma(y) dy. \]  

(3.2)

The vortex drag or induced drag can be derived with the aid of a small angle approximation and the Kutta-Joukowski theorem,

\[ d_i(y) = l(y) \tan(\alpha_i(y)) \approx l(y) \alpha_i(y), \]  

(3.3)

where \( \alpha_i(y) \) is the angle induced by the downwash velocity,

\[ \alpha_i(y) = \arcsin\left(\frac{-w(y)}{U \infty}\right) \approx \frac{w(y)}{U \infty}. \]  

(3.4)

Substituting these geometric definitions into Equation 3.1 yields

\[ d_i(y) = -\rho \infty w(y) \Gamma(y), \]  

(3.5)

and integrating over the span of a finite wing determines the total induced drag, \( D_i \),

\[ D_i = \int_{-\frac{b}{2}}^{\frac{b}{2}} -\rho \infty w(y) \Gamma(y) dy. \]  

(3.6)

The next question is how to solve for the circulation, \( \Gamma(y) \); \( \rho \infty \) and \( U \infty \) are constant properties of the flow field. The key to the circulation solution is the flow tangency boundary condition. The flow must be tangent to the wing surface at the control point since the panelling represents a solid body and thus a streamline. To implement this boundary condition, the flow velocity normal to the surface (i.e. the control point) must be zero. For a wing cross-section at some angle of attack, \( \alpha \), the normal velocity has two components: the downward velocity or downwash, \( w \), induced by the vortex and the free-stream, \( U \infty \). Mathematically, \( w + U \infty \sin \alpha = 0 \approx w + U \infty \alpha \) for small \( \alpha \); this is valid because this is an inviscid analysis and viscous effects only become relevant at large \( \alpha \) near stall.

In two-dimensions, the velocity induced at a perpendicular location \( r \) by a vortex filament is defined as

\[ U = \frac{\Gamma}{2\pi r}, \]  

(3.7)

where \( r \) is the distance between the control point and vortex filament core. The incident angle or angle of attack between the wing surface and the free stream, \( \alpha \), is

\[ \alpha \approx \sin(\alpha) = \frac{U}{U \infty} = \frac{\Gamma}{2\pi r U \infty}, \]  

(3.8)

again taking \( \alpha \) as small. Defining the sectional lift, \( l(y) \), with the sectional lift coefficient, \( C_l \), and utilizing the two-dimensional thin airfoil lift curve slope, \( C_{l\alpha} = 2\pi \), the lift is

\[ l = \frac{1}{2} \rho \infty (U \infty)^2 c \]  

\[ 2\pi \alpha = \rho \infty U \infty \Gamma. \]  

(3.9)
Inserting Equation 3.8 into Equation 3.9 and solving for \( r \) yields,

\[
    r = \frac{c}{2}.
\]  

(3.10)

\( c \) is the panel chordlength; therefore, the control point location is \( \frac{3}{4} c \). Recall that the vortex filament is located at the quarter chord. Now with the flow tangency boundary condition, the individual vortex strengths, circulation, induced by all the other horseshoe vortices for each panel can be solved through a set of linear equations determined by the Biot-Savart Law [18].

The Biot-Savart Law calculates the velocity induced by a vortex of strength \( \Gamma \) at a location \( \vec{r} \) which is

\[
    d\vec{V} = \frac{\Gamma_n (d\vec{l} \times \vec{r})}{4\pi r^3}.
\]  

(3.11)

Employing the definition for a cross product, \( A \times B = |AB| \sin \theta \) Equation 3.11 leads to

\[
    d\vec{V} = \frac{\Gamma_n \sin(\theta) dl}{4\pi r^3},
\]  

(3.12)

where \( \theta \) is the angle between \( d\vec{l} \) and \( \vec{r} \).

The vortex filaments in question are three straight segments of the horseshoe vortex, where the normal distance between each segment, \( dl \), and a point \( C \) is \( r_p \) as in Figure 3.4.

\[
    r = \frac{r_p}{\sin(\theta)} dl = r_p \left( \csc^2(\theta) d\theta \right).
\]  

(3.13)

This with Equation 3.16 and integrating yields,

\[
    V = \frac{\Gamma_n}{4\pi r_p} \int_{\theta_1}^{\theta_2} \sin \theta d\theta = \frac{\Gamma_n}{4\pi r_p} \left( \cos \theta_1 - \cos \theta_2 \right).
\]  

(3.14)

Now using the nomenclature defined in Figure 3.4, the velocity induced by a vortex filament can be determined for a control point, \( C \).

To apply this knowledge to the VLM horseshoe vortex system, consisting of velocity vectors, \( \vec{V}_{AB}, \vec{V}_{A\infty}, \) and \( \vec{V}_{B\infty} \), one follows the position vectors defined in Figures 3.1 and 3.5 and then sums the three velocity vectors of the horseshoe vortex to calculate the total induced velocity vector,

\[
    \vec{V}_{m,n} = \vec{C}_{m,n} \Gamma_n.
\]  

(3.15)

The influence coefficient matrix, \( \vec{C}_{m,n} \) is based on the geometry of the nth horseshoe vortex and its distance from the control point of the mth panel which is defined through
the Biot-Savart Law in Equations 3.11 through 3.18. The velocity at the $mth$ control point induced by all other horseshoe vortices is the sum of Equation 3.12 [18],

$$\vec{V}_m = \sum_{n=1}^{2N} \vec{C}_{m,n} \Gamma_n.$$  \hspace{1cm} (3.16)

The flow tangency boundary condition states $w_m = -U_\infty \alpha$, and applying this to Equation 3.16, the $2N$ equations can be solved for $\Gamma_n$. Summing the set of $\Gamma_n$ for each chordwise strip of panels and applying the Kutta-Joukowski theorem yields the sectional lift. Integrating the sectional lift coefficient over the span of the wing yields total lift per radian ($C_{L\alpha}$ per radian).

The pitching moment is the cross product of the position vector from the leading edge or nose of the wing to the quarter chord and the lift force. The moment arm for the pitching moment is the axial or chordwise distance from the wing root leading edge to the location of the bound vortex of each panel, the quarter chord of each panel, therefore the pitching moment is easily determined knowing the lift and the panelling of the aircraft. In VLM, it is given as a slope like the lift coefficient, $C_{M\alpha}$ per radian [18].

The circulation on each panel is known, $\Gamma_n$, and the induced drag can then be determined based on the velocity induced at the quarter chord of the $mth$ panel by $n - 1$ panels. A vortex cannot induce a velocity on itself. The only change in Equation 3.16 is the influence
coefficient matrix which replaces the control point location, \( r = \frac{3c}{4} \) with \( r = \frac{c}{4} \).

\[
\tilde{W}_m = \sum_{n=1}^{2N} D_{m,n} \Gamma_n \tag{3.17}
\]

This induced velocity is the downwash for each panel, \( w_m \), and with Equation 3.5 the sectional

Figure 3.5: Example of Vector Elements for Horseshoe Vortex [18]
induced drag, $d_i$, can be calculated. Integrating, thus summing in the same manner as lift the sectional induced drag and total induced drag per $\alpha^2$ (per radian squared) can be determined [21].

VLM yields consistently good results compared to empirical data for lift, but drag as calculated above is notoriously somewhat lower than the minimum induced drag, for the case of an elliptic lift distribution, $\frac{C_{D_i}}{C_L^2} < \frac{1}{\pi \alpha R}$. The error for this method presented in Kalman [21] is small, between 2-3%. The error would increase with wing sweep, $\Lambda$. Tulinius [22] mathematically proved that for the induced drag integrated from the bound vortex to be equal to the induced drag computed in the Trefftz plane; the panelling must be parallel and perpendicular to the free stream (i.e. unswept). The Trefftz plane is a location in the wake far downstream, where the downwash is equal to twice the downwash induced at the bound vortex as seen in Figure 3.6. Thus, the induced flow is invariant in the streamwise direction, and can be considered two-dimensional in the y-z plane normal to the free stream. The key in the induced drag error is the discontinuity in vorticity or circulation that occurs across a "kinked" panelling [22]. The distribution of vorticity on the wing is continuous in real life and any attempt to discretize this continuous vortex sheet into panels leads to a logarithmic singularity in the downwash. This creates an error in the induced drag, but not the lift. Many have adopted an induced drag correcting scale factor based on the energy in the Trefftz plane. Kalman et al [21] define the ratio, $\frac{C_{D_{i,w}}}{C_{D_{i,V}}}$, where $C_{D_{i,w}}$ is the wake drag integral given in Equation 3.18 which sums the chordwise components of the normal force vector, and $C_{D_{i,V}}$ is calculated by the downwash from VLM,

$$C_{D_i} = \frac{1}{2S} \int_{-s}^{s} c_L \alpha_i dy.$$  \hspace{1cm} (3.18)

$\alpha_i$ does not vary with lift on an unswept wing as it does on a swept wing, therefore Equation...
3.18 is most easily evaluated in the Trefftz plane where downwash, thus induced angle, is constant. This correcting scale factor method is commonly used for calculating the induced drag from the bound vortex, but it is not reliable near the tip, especially for variable leading edge angle planforms. Calculating the induced drag in the Trefftz plane is the only rigorous method. Unfortunately for this work, we do not have the luxury of calculating the induced drag in the wake because another aircraft is present, the hitchhiker, and the mathematical methods or tricks to create an equally-spaced, perpendicular, and parallel panelling are difficult, and no clear documentation has surfaced. Fortunately, the changes in induced drag on a hitchhiker in formation flight are much greater than any error documented by sweeping panels; therefore, the original method for calculating induced drag based on the downwash at the bound vortex is used here to determine trends in formation flight as observed in the wind tunnel.

3.2 A Compound Aircraft Transport Flight Vortex Lattice Method

The baseline VLM code extended to CAT mapped the techniques described in Bertin and Smith’s *Aerodynamics for Engineers* [18]; this particular code was originally written by Professor William H. Mason in 1989. The code required the axial location (x-direction) of the leading and trailing edge root and tip as well as the spanwise location (y-direction) of the root and tip for the wing planform. This coordinate system was referenced with respect to a plane of symmetry originating at the leading edge centerline as in Figure 3.7 and following Figure 3.7. Also input are the number of spanwise (NSPAN) and chordwise (NCHRD) divisions to create a lattice structure of $N$ (NSPAN by NCHRD) trapezoidal panels.

With this basic information, one-dimensional arrays were created for the x, y, and z locations of the panels, horseshoe vortices, and control points in the plane of symmetry. The arrays were then sent to a subroutine, VHORSE, twice to calculate the appropriate influence coefficient matrix, $\vec{C}_{m,n}$ from Equation 3.18, based on the Biot-Savart Law described in the previous section. VHORSE is called once with the given coordinates and then again with the mirrored coordinates ($Y = -Y$) to incorporate the total influence at each control point from the $2N$ panels. Then, applying the flow tangency boundary condition at the panel $\frac{3c}{4}$ or control point the circulation for each panel was solved through the Gaussian Elimination technique for a linear set of algebraic equations. The known circulations were summed and the Kutta-Joukowski theorem applied to determine the sectional and total lift and pitching moment coefficient slopes. The output consists of the wing planform and panel geometry, as well as the circulation of each panel with the spanwise lift distribution and the total coefficient slopes, $C_{L,\alpha}$ and $C_{M,\alpha}$ per radian.

The modifications here began with the geometric reconstruction of the VLM code to accept multiple aircraft, which were represented as planar, flat, surfaces without camber.
The system origin remained the same at the mothership leading edge centerline. Each additional aircraft or hitchhiker was input based on its individual origin at the hitchhiker leading edge centerline. Geometric symmetry is a requirement of each aircraft. The location of the hitchhiker is then referenced to the mothership by inputting the coordinates of the hitchhiker origin with respect to the mothership origin in terms of her coordinate system. The arrays had to be system symmetric since undoubtedly the hitchhiker lift distribution would not be symmetric, therefore it is significant to note the great care that was necessary in creating the left-to-right-reading geometric arrays to be sent to VHORSE. Some general book-keeping in summing circulation for lift coefficients required a division of two on the hitchhiker, since again the lift distribution cannot be expected to be symmetric. The layout for the original VLM and the VLM modified for CAT flight is in Figure 3.7.

![Coordinate System](image)

Figure 3.7: Coordinate System for (a) the Original VLM and (b) the VLM Modified for CAT Flight

The accuracy of the method and code was first proven by moving two identical aircraft far apart with the code yielding the same results. This technique for proof of accuracy was utilized with an unswept wing of aspect ratio 4.0 and the well-known Warren 12 planform in Figure 3.8 for a swept wing. Very slight discrepancies were found due to the coarse lattice or grid, but overall the results had acceptable errors. Data for proof of accuracy is tabulated
in Table 3.1 for a $5 \times 5$ lattice or grid ($NSPAN = NCHRD = 5$).

<table>
<thead>
<tr>
<th></th>
<th>CAT VLM</th>
<th>Unmodified VLM</th>
<th>Planform</th>
<th>% Error in $C_{L_{\alpha}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mothexhip</td>
<td>3.8549</td>
<td>-0.9042</td>
<td>3.8528</td>
<td>-0.9037 $AR = 4.0$</td>
</tr>
<tr>
<td>Hitchhiker</td>
<td>3.8563</td>
<td>-0.9046</td>
<td>3.8528</td>
<td>-0.9037 $AR = 4.0$</td>
</tr>
<tr>
<td>Mothexhip</td>
<td>2.8765</td>
<td>-3.3155</td>
<td>2.743</td>
<td>-3.100 Warren</td>
</tr>
<tr>
<td>Hitchhiker</td>
<td>2.8120</td>
<td>-3.2716</td>
<td>2.743</td>
<td>-3.100 Warren</td>
</tr>
</tbody>
</table>

The wind tunnel data specifically analyzed aerodynamic trends in CAT flight through lift-to-drag ratio, but the original VLM code only calculates lift and not drag. So, the technique described in the previous section for calculating induced drag was applied. The code determines the circulation for each panel in the CAT system, but induced drag requires the downwash, $w_{m,n}$, of each panel as well. By changing the influence coefficient matrix using the known circulations to solve for panel downwash, the induced drag can be calculated by the rearranged Kutta-Joukowski theorem in Equation 3.1. So the subroutine, VHORSE, was called twice more (4 times total) with the modification for $r$ in the Biot-Savart Law (Equation 3.11 from the previous $r = \frac{\alpha}{4}$ to $r = \frac{\alpha}{4}$). This changed only the axial location sent to the subroutine making it the center of the bound finite horseshoe vortex segment, the panel quarter chord. The downwash for each panel was determined based on the velocity induced by all other system horseshoe vortices. These values can be integrated over the wing to yield sectional and total induced drag coefficients. Note, the total induced drag slope would have units of $C_{D_{\alpha,2}}$ per radian squared; recall $C_{D_{\alpha}} \propto C_{L_{\alpha}}^2$. The proof of accuracy for induced drag has much less data available for comparison. The Kalman et. al. paper [21] documented a
\( \frac{C_{D_i}}{C_L} = 0.155375 \) value for an unswept wing with an aspect ratio of 2.0 and VLM CAT yielded \( \frac{C_{D_i}}{C_L} = 0.15537 \); this is very good agreement. To test for swept wings the comparison relied on that fact that two aircraft (mothership and hitchhiker) of the same geometry far apart should have the same induced drag values. LLT was also employed successfully for general in-the-ball-park comparison.

A manual, an example with input and output files, and a copy of the VLM CAT program are presented in Appendices B.1, B.2, and D, respectively.

### 3.3 VLM CAT Results Compared to Wind Tunnel Data

To make a reasonable comparison between the VLM CAT results and the wind tunnel data, a proper lift-to-drag ratio needed to be determined for the VLM CAT results. Currently the VLM supplies a \( C_L \) and a \( C_{D_i,2} \). First, an estimate for parasite drag, \( C_{D_o} \) was calculated following two different methods; total drag is \( C_D = C_{D_o} + C_{D_i} \) plus drag due to compressibility effects or wave drag which would be negligible at the speeds considered here. Method one followed the techniques presented in Shevell’s *Fundamentals of Flight* [4]. This requires the summation of \( C_{D_o} \) for each aircraft component based on wetted area, \( S_{wet} \), skin friction as a function of Reynolds Number, and pressure effects due to thickness. It can be reasonably assumed that the wing is the major lift-producing component on an aircraft, thus an \( L/D \) ratio from the VLM CAT should compare more favorably to the wind tunnel data by simply adding the parasite drag due to the majority of the F-84, the fuselage and wing. The steps in calculating this value are in Figure 3.9.

The second method assumed a point of maximum performance for the hitchhiker at \( C_L = 0.48 \) (the \( C_{L_{solo}} \) in the wind tunnel), which is theoretically when \( C_{D_o} = C_{D_i} \). Parasite drag is typically considered invariant with a constant-Reynolds number flow field, so \( C_{D_o} \) of the F-84 was chosen to equal \( C_{D_i} \) of the F-84 when far from the transport wing, \( C_{D_o} \approx 0.01 \).

Next, since the coefficients are output from VLM as slopes with respect to an angle of attack, \( \alpha \), from the wind tunnel tests needs to be specified. Recall in the previous chapter the difficulty in measuring accurately the angle of attack; a reference solo \( C_L \) was chosen in lieu of an \( \alpha \). So, the reference solo hitchhiker \( C_{L_{solo}} = 0.48 \) was applied to the VLM output \( C_{L_{solo}} \) to determine a corresponding flight \( \alpha \),

\[
C_{L_{solo}} = \frac{dC_L}{d\alpha} \implies \alpha = \frac{C_{L_{solo}}}{C_{L_{solo}}},
\]

One more value must be defined before the VLM CAT and wind tunnel data are ready to be compared. The wind tunnel data defined a lift-to-drag ratio based on the solo values of the hitchhiker, \( (L/D)/(L/D)_{solo} \). The value in the wind tunnel data was \( (L/D)_{solo} = 18 \), but this value is not the same for VLM CAT, since only the wing geometries were implemented.
\[ C_{D_o} = \sum_i K_i C_{f_i} S_{WET_i} / S_{REF} \]

- **\( K_i \): correction factor for pressure drag - based on thickness**
- **\( C_{f_i} \): skin friction coefficient**

\[ S_{WET_i} = S_{\text{expected}} \times 2 \times 1.02 \]

- **wing**
  - \( C_{f_{\text{wing}}} = 0.005 \text{ at } Re = 10^6 \)
  - \( K_{\text{wing}} = 1.15 \text{ for } 0.1 < t/c < 0.12 \)
  - \( S_{\text{WET}_{\text{wing}}} = c_{\text{ave}} \times \frac{b}{2 \sin \Delta_{LE}} \times 2 \times 1.02 = 53.0 \text{ in}^2 \)
  - \( S_{\text{REF}} = b c_{\text{ave}} = 36.72 \text{ in}^2 \)
  - \( C_{D_{\text{wing}}} = 0.0083 \)

- **fuselage + wing**
  - \( C_{f_{\text{fus}}} = 0.005 \text{ at } Re = 10^6 \)
  - \( K_{\text{fus}} = 1.20 \text{ for } L/D \approx 7.0 \)
  - \( S_{\text{WET}_{\text{fus}}} = \pi d_{\text{ave}} l = 64.0 \text{ in}^2 \)
  - \( S_{\text{REF}} = b c_{\text{ave}} = 36.72 \text{ in}^2 \)
  - \( C_{D_{\text{fus}}} = 0.010 \)

\[ C_{D_o} = C_{D_{\text{wing}}} + C_{D_{\text{fus}}} = 0.0083 + 0.010 \approx 0.018 \]

Figure 3.9: Steps in Calculating Parasite Drag, \( C_{D_o} \) Based on Wetted Area, \( S_{\text{wet}} \), of F-84 Wing and Fuselage [4]

The geometry input into the VLM CAT program is tabulated below in Table 3.2 and is based on the wing planform of the wind tunnel models described in the previous chapter.

The number of chordwise and spanwise panels for the mothership and the hitchhiker in VLM CAT were determined mainly through a simple grid refinement study comparing the lift curve slope, \( C_{L_{\alpha}} \), between the original VLM and VLM CAT with the aircraft far apart. The steps in the grid refinement study are in Table 3.3, and the resulting spanwise by chordwise (NSPAN X NCHRD) panelling selected was 20 X 5 for the mothership and 25 X 5 for the hitchhiker based on symmetry of the individual aircraft. These values were chosen from the grid refinement study and computational time with an error \( \leq 3.0\% \) being considered adequate. A small increase in accuracy that requires a much larger computational time may not be a viable trade-off for this work. A diagrammatic view of the panelling applied to the mothership and hitchhiker planforms used here is in Figure 3.10.
Table 3.2: Geometric Dimensions for Wind Tunnel Models to Input in VLM CAT

<table>
<thead>
<tr>
<th>Wind Tunnel Model Dimensions</th>
<th>$\Lambda_{LE}$</th>
<th>$\Lambda_{TE}$</th>
<th>$c_{root}$</th>
<th>$c_{tip}$</th>
<th>$\frac{b}{2}$</th>
<th>$AR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mothership</td>
<td>15.0</td>
<td>15.0</td>
<td>5.0 in</td>
<td>5.0 in</td>
<td>20.0 in</td>
<td>4.0</td>
</tr>
<tr>
<td>Hitchhiker</td>
<td>45.0</td>
<td>30.0</td>
<td>3.5 in</td>
<td>2.4 in</td>
<td>6.25 in</td>
<td>4.24</td>
</tr>
</tbody>
</table>

VLM Input in Inches

<table>
<thead>
<tr>
<th>VLM Input in Inches</th>
<th>YROOT</th>
<th>YTIP</th>
<th>XRTLE</th>
<th>XRTTE</th>
<th>XRTPLET</th>
<th>XTITPE</th>
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</thead>
<tbody>
<tr>
<td>Mothership</td>
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<td>20.0</td>
<td>0.0</td>
<td>5.0</td>
<td>5.36</td>
<td>10.36</td>
</tr>
<tr>
<td>Hitchhiker</td>
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<td>6.25</td>
<td>0.0</td>
<td>3.5</td>
<td>6.25</td>
<td>8.65</td>
</tr>
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</table>

Table 3.3: VLM CAT Grid Refinement Study

<table>
<thead>
<tr>
<th>NSPAN</th>
<th>NCHRD</th>
<th>VLM CAT</th>
<th>VLM $C_{L,n}$</th>
<th>$%$ error</th>
</tr>
</thead>
<tbody>
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<td>Mothership</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>4.6125</td>
<td>4.4150</td>
<td>4.5</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>4.6129</td>
<td>4.4150</td>
<td>4.5</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>4.7302</td>
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<tr>
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<td>4.5457</td>
<td>4.4150</td>
<td>3.0</td>
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<tr>
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<td>10</td>
<td>4.5466</td>
<td>4.4150</td>
<td>3.0</td>
</tr>
<tr>
<td>30</td>
<td>5</td>
<td>4.5221</td>
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<td>2.4</td>
</tr>
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<td>10</td>
<td>4.5233</td>
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<td>4.5100</td>
<td>4.4150</td>
<td>2.1</td>
</tr>
<tr>
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<td>10</td>
<td>4.5114</td>
<td>4.4150</td>
<td>2.2</td>
</tr>
<tr>
<td>Hitchhiker</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>3.2504</td>
<td>3.1574</td>
<td>3.0</td>
</tr>
<tr>
<td>20</td>
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<td>3.1574</td>
<td>3.0</td>
</tr>
<tr>
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<td>3.1574</td>
<td>3.2327</td>
<td>2.4</td>
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<tr>
<td>30</td>
<td>10</td>
<td>3.2356</td>
<td>3.1574</td>
<td>2.5</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
<td>3.2239</td>
<td>3.1574</td>
<td>2.1</td>
</tr>
</tbody>
</table>

The VLM CAT results clearly demonstrate similar trends in aerodynamic benefits as for CAT flight in the wind tunnel. VLM CAT was modelled for the three distinct formations: wingtip-docked flight, close-formation flight, and towed-formation flight, focused on in Chapter 2. All VLM CAT results are plotted directly onto those depicted in Chapter 2 for $C_L$, $C_D$, and $(L/D)/(L/D)_{solo}$ in Figures 2.22 through 2.30 for the experimental wind tunnel data. To clearly show the comparison between the wind tunnel data and VLM CAT results only one vertical location, $\zeta$, is presented for each configuration. Configuration One is at the wingtip-to-wingtip location and the subsequent locations progress the hitchhiker further downstream from the mothership. VLM CAT results are distinguished from experimental data by dashed or solid lines connecting the points.

1This data was all determined at a $NSPAN = 20$ and $NCHRD = 5$
Figure 3.10: VLM CAT Panelling Presented in Configuration Five: Towed Formation (Axisymmetric)

**Configuration One:** Wingtip-Docked CAT Flight

Figure 3.11: Configuration One: Wingtip-Docked Formation, $\xi = 0.0$, VLM CAT Compared to Experimental Data; $C_L$ vs. $\eta$, Spanwise Distance

The VLM CAT results are plotted in the same method as the experimental wind
tunnel data. The $C_L$, $C_D$, and $L/D$ data output from VLM CAT are plotted versus the spanwise or gap separation, $\eta$, between the wingtip of the F-84 and the wingtip of the transport wing. The symbols on the connecting lines correspond to the same various vertical locations for the F-84 wing above and below the transport wing as the wind tunnel data. Also like the wind tunnel data the transport wing, is fixed and the F-84 is moved relative to her. The $L/D$ ratio data in Figure 3.13 for differing parasite drag calculations are distinguished with a dashed line for $C_{D_o} = 0.01$ and solid line for $C_{D_o} = 0.018$.

The VLM CAT lift data compared to the wind tunnel lift data, in Figure 3.11, with the mothership wingtip and hitchhiker wingtip in-plane, $\zeta = 0.0$, shows very good agreement. In Figure 3.12, the VLM CAT induced drag is plotted with the wind tunnel total measured drag, and the offset is about 0.01. Referring to Figure 3.13, the offset in $(L/D)/(L/D)_{solo}$ data between induced and total drag must be closest to the value calculated with a parasite drag based on the planform of the F-84 wing and fuselage, i.e. $C_{D_o} = 0.018$. It is the $L/D$ ratio value calculated using that $C_{D_o}$ which compares most favorably with the wind tunnel data in Figure 3.13. Recall that the drag for this configuration in Figure 2.20 was quite invariant. One can thus conclude that the aerodynamic benefit is driven by the inviscid force, lift; VLM can predict forces in an inviscid flow field very well.
Figure 3.13: Configuration One: *Wingtip-Docked Formation*, $\xi = 0.0$, VLM CAT Compared to Experimental Data; $L/D$ vs. $\eta$, Spanwise Distance

The maximum aerodynamic benefits for this configuration are at the wingtip-to-wingtip location, $\eta = 0.0$. The performance increase is 60–80% for the VLM CAT results with the smaller parasite drag value and within the same range predicted by the wind tunnel data, 20 – 40%, with larger value of parasite drag. Though only one vertical location is presented for clarity, the VLM CAT results coincide for all points outboard of the wingtip-to-wingtip location, $\eta = 0.0$. So, an important differing point is that the wind tunnel data shows an overall increase in aerodynamic benefit for a vertical position slightly below the transport wing, but VLM CAT does not predict this.

**Configuration Two: Close Formation CAT Flight**

In Figures 3.14 through 3.16, the VLM CAT results and wind tunnel data for $C_L$, $C_D$, and $(L/D)/(L/D)_{solo}$ in Configuration Two, close formation, is plotted versus the spanwise or gap separation, $\eta$. The hitchhiker has moved downstream one chordlength of the transport wing or mothership between the trailing edge of the transport wing to the leading edge of F-84 model wing, $\xi = 3.0$. So, the hitchhiker has the capability to move inboard of the transport wingtip, thus the spanwise range is $0.68 \leq \eta \leq -0.34$. The wingtip-to-wingtip...
plane is highlighted by the bold vertical line at $\eta = 0.0$. In Figures 3.14, 3.15, and 3.16, the data is presented for a vertical location with the F-84 slightly above the transport wing at $\zeta = 0.17$. In Figure 3.17, the data is presented for a vertical location of the F-84 further above at $\zeta = 0.34$. This change in $\zeta$ is because Figure 3.17 is depicting a different range of $(L/D)/(L/D)_{solo}$ than Figure 3.16.

The VLM CAT results for the outboard locations of the hitchhiker wingtip, $\eta > 0.0$, in Figures 3.14 and 3.16 support the wind tunnel data trends well. Inboard of the transport wingtip, $\eta < 0.0$, the lift VLM CAT results in Figure 3.14 diverge from the wind tunnel data, but do, in general, support the overall increasing trend of the wind tunnel data. Figure 3.15 plots the wind tunnel data and the VLM CAT results in drag for a vertical position of $\zeta = 0.17$, the hitchhiker is slightly above the transport wing. Only the induced drag, which is directly from the VLM CAT calculations is presented for comparison, and it follows the trend of the wind tunnel data. For an F-84 position inboard of the transport wingtip, $\eta < 0.0$, the induced drag coincides closely with the wind tunnel data for a small period between $\eta \approx -0.1$ and $\eta \approx -0.25$. For $\eta$ smaller than -0.25 the VLM CAT induced drag over predicts the wind tunnel data and for $\eta$ larger than 0.0 the VLM CAT data under predicts the wind tunnel data. Of course, since the VLM CAT lift results in Figure 3.14 are not in concise agreement with the wind tunnel, then in follows that the induced drag data
from VLM CAT would not be in concise agreement either.

The lift behavior is revisited in Figure 3.16 when the \((L/D)/(L/D)_{solo}\) VLM CAT results inboard of the transport wingtip, \(\eta = 0.0\), clearly do not pick up the large benefits in performance as seen in the wind tunnel data. The wind tunnel data measured benefits up to 1100% while the VLM CAT results only yield benefits about 100–300%. Figure 2.24 of the wind tunnel drag data shows a large reduction in drag for hitchhiker wingtip locations inboard of the mothership wingtip, while the lift data in Figure 2.23 is very similar to that in Figure 2.19. This being the case, it is clear that reduction in drag must be the driver in increasing the aerodynamic benefits, and since VLM cannot accurately predict benefits caused by viscous effects, then the results in Figure 3.16 are expected. The VLM CAT results like the wind tunnel results show a significant increase in flight performance benefit from close formation flight compared to wingtip-docked. VLM CAT shows a rise in aerodynamic benefit from 20 – 40% to \(\sim 100–300\%\) benefit. In Figure 3.16, the VLM CAT results with the smaller parasite drag \((C_{D_o} \approx C_{D_{i,solo}})\) are more comparable to the wind tunnel data. Figures 3.16 and 3.17 highlight different ranges of wind tunnel \((L/D)/(L/D)_{solo}\) data. The VLM CAT results do not pick up the benefits in the larger range \((0.0 \leq (L/D)/(L/D)_{solo} \leq 14.0)\) in Figure 3.16, but do in the smaller range \((0.0 \leq (L/D)/(L/D)_{solo} \leq 6.0)\) in Figure 3.17.
Recall that the wind tunnel results in Figures 2.27 through 2.30 for the towed configuration had the most interesting data. The drag was characterized by dips in Figure 2.28 (or peaks in Figure 2.30 for \((L/D)/(L/D)_{solo}\)) distinctly at two spanwise locations, \(\eta = -1.25\) and \(\eta = -2.25\). As in Configuration Two for VLM CAT, these results for VLM CAT in Figures 3.18 through 3.20 also do not pick up such nuances in the wind tunnel data. However, the path defined by all the VLM CAT results for this configuration catch the main trends seen in the wind tunnel.

In VLM, trailing vortices cannot intersect a control point, and the fineness of the grid necessary in CAT made it difficult for the trailing horseshoe vortices of the mothership not to intersect the control points of hitchhiker, when the hitchhiker is inboard of the mothership wingtip. Thus, the simple solution was to separate the mothership and hitchhiker vertically, slightly. The VLM CAT results with the connecting line has green \(\circ\)'s as symbols, these symbols represent the same vertical location as the green \(\circ\)'s in the wind tunnel data, above and below the transport wing at \(\zeta = \pm 0.17\). To reiterate, there is no wind tunnel data at a vertical location of \(\zeta = \pm 0.17\), only at \(\zeta = 0.0\). But, VLM CAT results at \(\zeta = \pm 0.17\) are presented, due to current VLM CAT limitations in calculating circulation in the wingtip-to-
The VLM CAT results for lift are presented in Figure 3.18 and like the previous VLM CAT lift data in Configuration Two, the inboard locations of the F-84, $\eta < 0.0$, the VLM CAT data diverges from the wind tunnel data. But again the overall trend is duplicated, the lift decreases on the F-84 as it is moved into a downwash region inboard of the transport wingtip. Oddly, the VLM CAT induced drag data presented in Figure 3.19 with the wind tunnel total drag, is nearly identical. Throughout this study the VLM CAT results are smoother than the measured wind tunnel data, and perhaps this data depicts that difference the best. The $(L/D)/(L/D)_{solo}$ at the far right of Figure 3.20 is the highest. Using $C_{D_o} = 0.018$, $(L/D)/(L/D)_{solo} \approx 2.3$ and then falls smoothly as the hitchhiker wingtip moves inboard of the mothership wingtip to a minimum at $(L/D)/(L/D)_{solo} \approx 0.5$. The wind tunnel data represented in Figure 3.20 rises above and below the maximum and minimum values from VLM CAT. So one difference in the experimental wind tunnel data and the computational VLM CAT data is the magnitude, and another is the smoothness in the computational results compared to the experimental data. It is important to mention, VLM CAT cannot model the fuselage of the F-84, therefore if the spikes in wind tunnel results are due to fuselage-vortex interaction, then VLM CAT would not predict these effects.
The Munk Stagger Theorem conjecture proposed in the wind tunnel results section in explanation of the lift data similarity between the close and towed formation (Figures 2.23 and 2.27) was that there was a lift rise in the small spanwise gap, $C_L \approx 0.68$ at $\eta = -0.34$ to $C_L \approx 0.77$ at $\eta = -0.45$. Now view Figure 3.20 at the far right wingtip-to-wingtip location, where $\eta = 0.0$; the VLM CAT results clearly show a continued rise with decreasing $\eta$ to the maximum value $(L/D)/(L/D)_{solo} \approx 2.3$.

### 3.4 Summary of VLM CAT Results

The VLM CAT results model the general trends of the wind tunnel data well. The aerodynamic benefits in the wind tunnel experiments derived from viscous force or drag reduction are not predicted as well as the inviscid aerodynamic benefits. The VLM CAT results in Figures 3.11 and 3.13 are almost identical to the wind tunnel data, which showed only strong variation in lift and not in drag. It is clear that the correct choice or method in calculating parasite drag could be an important factor in determining the correct total drag for VLM CAT and wind tunnel comparison. Parasite drag is complicated to calculate and
is usually estimated by subtracting induced drag and wave drag (which can be reasonably calculated) from the measured total drag. Other issues associated with the complex flow field of wingtip-docked and formation flight could negate the assumption of invariant parasite drag. For instance, the flow field in the small gap when the two aircraft are close together might cause boundary layer separation or thickening both varying the values of skin friction from that based on free-stream conditions \[23\]. Another assumption in this VLM is that of a flat wake, all lift is produced on the quarter chord of each panel. Flow fields of this nature with aircraft flying close behind another would most likely have some substantial deformation in their wakes, and incorporating the wake deformation with respect to the freestream would undoubtedly yield results more comparable to the wind tunnel data.

At some locations in the drag data (Figures 3.15 and 3.19), the VLM CAT induced drag was greater than the total wind tunnel drag and at other locations, the VLM CAT induced drag was greater than the total wind tunnel drag. So VLM CAT is overpredicting and underpredicting induced drag. In overprediction, the VLM CAT is not completely describing the flow fields, which could be attributed to the assumption of flat wake. A deformable wake could add to the accuracy of the flow field in VLM CAT. In underpredicting, the VLM CAT is not representing the effects of viscosity, and the flat-wake assumption is most likely still a factor. Viscosity cannot be modelled in VLM by its definition, only a full
Figure 3.20: Configuration Five: Towed Formation Flight, $\xi = 10.0$, VLM CAT Compared to Experimental Data; $L/D$ vs. $\eta$, Spanwise Distance

Three-dimensional Navier-Stokes code could describe all the viscous effects. But, regardless of the uncertainty in calculating total drag and associated assumptions, the VLM CAT program does supply valuable knowledge and insight in quickly determining the aerodynamic benefits for CAT flight in terms of position of the hitchhiker with respect to the mothership and in terms of different geometries or planforms.
Chapter 4

Simulation of Dynamics for Wingtip-Docked Flight

According to the account of Mr. B.A. Erickson, the F-84 test pilot for Project Tom, approximately six feet of the left wingtip on the B-36 was “ripped off” when he attempted a “tired pilot” simulation with a sideslip toed-in to the mothership of approximately five degrees (positive with respect to hitchhiker being docked by its left-wingtip) in April of 1953 [8]. This catastrophic failure was apparently caused by a violent flapping motion [9]. So, to study the instability perceived in wingtip-docked flight, and address an obvious question of pilot induced oscillation (PIO), a fighter aircraft desktop model simulation was altered through modifications in the equations of motion (EOM). The desktop model is an F-16 model in FORTRAN from Aircraft Control and Simulation [24] which numerically linearizes the system through small perturbation theory to create an influence coefficient matrix. The eigenvalues of this matrix that represent the dynamic modes of the system, can then be determined and examined for any instability. The modifications were made to the six-degree-of-freedom (6DOF) equations of motion (EOM) to constrain the left wingtip to straight line motion, as if joined to a mothership in straight flight. The 6DOF aircraft is therefore a three-degree-of-freedom (3DOF) aircraft with respect to the mothership.

The sensitivity of the eigenvalues or modes to changes in various states as well as the states or state rates driving the eigenvalues can be analyzed regardless of stability. Of course, in determination of an unstable mode, the sensitivities and driving factors of that mode will be the most important. Based on the recollections of Mr. Erickson, the flight parameter, sideslip ($\beta$), will receive the most focus here. Other flight parameters such as speed and altitude are included in this stability analysis.
4.1 Techniques for System Stability Analysis

The modelling of an aircraft is a combination of experimental and mathematical techniques. The mathematical model supplies the equations of motion (EOM). These are ordinary differential equations that use a set of variables or states to describe the motion of the aircraft system, and therefore the changes of the states through time. The experimental data, from wind tunnel tests, is the database for these variables or states. Equation 4.1 represents a set of ordinary differential equations,

\[ \dot{x} = Ax + Bu. \]  \hspace{1cm} (4.1)

\( \dot{x} \) and \( x \) are vectors of the time rate of change for the states and the states themselves, respectively. \( u \) is the input control state vector, and \( A \) and \( B \) are influence coefficient matrices that describe the change of each derivative with respect to each state (i.e. \( \frac{\partial \dot{x}_i}{\partial x_j} \)) and control.

Twelve differential equations describe the motion of a rigid body aircraft, and since the motion is outlined in orthogonal coordinates, one can think of four sets of equations. The force equations, assuming the thrust vector is small, are the aircraft accelerations,

\[
\begin{align*}
\dot{u} &= \frac{1}{m} (X + T) - g \sin \theta + r v - q w \\
\dot{v} &= \frac{Y}{m} + g \sin \phi \cos \theta - r u + p w \\
\dot{w} &= \frac{Z}{m} + g \cos \phi \cos \theta + q u - p v.
\end{align*}
\]  \hspace{1cm} (4.2)

The moment equations for principal axes are the aircraft angular accelerations:

\[
\begin{align*}
\dot{p} &= \frac{L - (I_{zz} - I_{yy}) q r}{I_{xx}} \\
\dot{q} &= \frac{M + M_T - (I_{xx} - I_{zz}) p r}{I_{yy}} \\
\dot{r} &= \frac{N - (I_{yy} - I_{xx}) p q}{I_{zz}}.
\end{align*}
\]  \hspace{1cm} (4.3)

The kinematic equations are the angular rates of the Euler angles:

\[
\begin{align*}
\dot{\phi} &= p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \\
\dot{\theta} &= q \cos \phi - r \sin \phi \\
\dot{\psi} &= q \sin \phi \sec \theta + r \cos \phi \sec \theta.
\end{align*}
\]  \hspace{1cm} (4.4)
The navigational equations with respect to the motion of the Earth are:

\[
\begin{align*}
\dot{x}_E &= u \cos \theta \cos \psi + v (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) + w (\cos \phi \sin \theta \cos \psi + \cos \phi \cos \psi) \\
\dot{y}_E &= u \cos \theta \sin \psi + v (\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) + w (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\
\dot{z}_E &= -u \sin \theta + v \sin \phi \cos \theta + w \cos \phi \cos \theta 
\end{align*}
\]

(4.5)

The variables in these equations represent the velocity components, \(u, v,\) and \(w,\) Euler angles, \(\phi, \theta,\) and \(\psi,\) the aerodynamic forces and moments, \(X, Y, Z, L, M,\) and \(N,\) and the angular velocities, \(p, q,\) and \(r.\) The subscript \(T\) and \(E\) signify thrust and the Earth, respectively [25]. These twelve equations are nonlinear and are solved with numerical integration, typically using a Runge-Kutta method. Runge-Kutta methods are derived from \(nth\) degree Taylor polynomial in two variables. A fourth order is typical with a truncation error of \(O(h^4), h\) is the step size [26].

Equation 4.1 is linear, but the equations of motion described are not. To obtain a linear set of equations, the matrices \(A\) and \(B\) must be defined as constant coefficients without any products of the states or controls involved. These matrices are a set of partial differential equations that describe the change of each state derivative to every other state and control variable (as previously mentioned). Linearized equations of motion greatly simplify the stability analysis of a system, which is the focus of this work. Assuming the aircraft only makes small deviations from a steady-state condition, then the well-known theory of small disturbances or perturbations can be applied. The equations of motion may be linearized usually about a steady reference condition by expanding the equations into a power series and retaining only the linear terms. Now the powerful stability analysis tools for a square matrix, the eigenvalues and eigenvectors, can be readily determined and employed.

Eigenvectors, \(v,\) are defined as the non-zero solutions to Equation 4.6,

\[A v = \lambda v.\]

(4.6)

\(A\) is a square matrix, and \(\lambda,\) the eigenvalue, can be solved by rearranging Equation 4.6 into a characteristic polynomial through Equation 4.7,

\[det(A - \lambda I) = 0.\]

(4.7)

The eigenvalues and eigenvectors construct the solution to a first-order differential equation like Equation 4.1. For example, Equation 4.8

\[\dot{x} = Ax\]

(4.8)

has solutions in the form,

\[x(t) = x_0 e^{\lambda t}.\]

(4.9)

\(x_0\) is the state vector, which is also \(x(t)\) at \(t = 0,\) and \(\lambda\) is the eigenvalue. The stability of the system is simply obtained from the real parts of the eigenvalues. If all eigenvalues have
negative real parts, then the system is stable; if any eigenvalue has a positive real part, then
the system is unstable.

To accurately model and analyze the stability of a wingtip-docked aircraft system, the
proper equations of motion must be determined and linearized, and then the eigenvalues
must be calculated from the influence coefficient matrix, \( A \) in Equation 4.1. Each eigenvalue
represents a mode. There are three types of modes possible: \( \lambda = \pm n \) which is monotonic, and
\( \lambda = n \pm i\omega \) which is oscillatory; \( \lambda = \pm i\omega \) is an undamped oscillation. From the eigenvalues
such criteria for the mode as period, \( T = \frac{2\pi}{\omega} \), time to half or double, cycles to double or half,
natural frequency \( (\omega_n = (\omega^2 + n^2)^{\frac{1}{2}}) \), logarithmic decrement, and damping ratio \( (\zeta = \frac{\omega_n}{\omega}) \) can
be calculated. Noting the definition of the damping ratio, if the real part of the eigenvalue
is negative the mode is positively damped and thus stable, and if the real part is positive
then the mode is negatively damped and thus unstable. If the eigenvalue is complex then
the mode has a period and so is oscillatory, such as the phugoid and short period modes [27].

The information from linearization of the influence coefficient matrix \( A \) can answer
another question about a system besides that of stability. With the employment of the
eigenvectors of \( A \), defined in Equation 4.6, the unforced initial condition response to a
change in one state can be calculated. This would lead to the determination of the dominant
state in a particular mode. For this work the variable in question is sideslip, \( \beta \), which can be
defined as a state. The variation of \( \beta \) creates variation in the other states, which leads to a
variation in the eigenvalues or stability of the system. To determine the relative influence of
an eigenvalue to a change in a single variable or state, one should refer to Equation 4.10 – a
solution to the first-order differential equation in Equation 4.8 – \( x_0 = x(0) \) is the eigenvector
for that particular eigenvalue, \( \lambda \). It is common to diagonalize Equation 4.10 as

\[
\begin{align*}
\mathbf{x}(t) &= M\mathbf{q}(t) \\
M &= \begin{bmatrix}
x_{01} & x_{02} & \cdots & x_{0n} \\
0 & 0 & \cdots & 0 \\
0 & e^{\lambda_2 t} & \cdots & 0 \\
0 & 0 & \cdots & e^{\lambda_n t} \\
\end{bmatrix} \\
\mathbf{q}(0) &= M^{-1}\mathbf{x}(0).
\end{align*}
\]

(4.10)

Focusing on the latter part of Equation 4.10, a change in one state or one flight parameter
like \( \beta \) can be determine a \( \Delta x(0) \). This would yield a variation in each state with respect to
that one change.

By elaborating on the above analysis to calculate the variation in the eigenvalues,
particularly an unstable eigenvalue, one can determine the change in each element of the
\( A \) matrix. In Equation 4.8, the definitions of eigenvalues and vectors and methods to be
described in a subsequent section can all be manipulated to determine the driving states in an instability.

4.2 Flying Qualities for Wingtip-Docked Flight: Pilot Induced Oscillation (PIO)

Due to the complexities and differences of the wingtip-docked flight to typical flight, it is relevant to have some discussion of aircraft flying qualities and PIO.

PIO is a dangerous nonlinear phenomenon inadvertently instigated by the pilot. Two reasons are categorized as key elements for PIO. One is a pilot’s overcompensation to a sudden and unprepared event like a wind gust or slat deployment. The pilot is reacting in an effort to precisely control the aircraft, but in doing so he induces a mode that is uncontrollable and perhaps unrecoverable. Two, the pilot inputs a control that is 180° out of phase with the actual control operation due to delays in the control gain feedback either in terms of the pilot’s perception or in the mechanical control delivery of his commands [28].

An aircraft or aircraft system that is prone to PIO is considered to have poor flying qualities, and the only real indication that an aircraft or aircraft system is prone to PIO is through pilot-in-the-loop simulation. The flying qualities of an aircraft or aircraft system is essentially based on pilot opinion, and over time with numerous opinions a loose numerical scale, rating aircraft handling qualities from 1 to 10, has been derived, the Cooper-Harper Rating Scale [25]. Given the opinion of the author and the information gathered from historical flight tests of wingtip-docked flight, the flying quality of the wingtip-docked system is between 5 and 7 placing it on the cusp of Level I and II, which is categorized with significant pilot workload compensation to maintain and/or achieve a mission [25]. So perhaps the question of Mr. Erickson inadvertently triggering a PIO should be rephrased to, is the wingtip-docked system prone to PIO?

4.3 Techniques for Wingtip-Docked Stability Analysis

An F-16 desktop model was modified to act as if the left wingtip was docked to another aircraft—therefore stationary in translation—but could rotate freely about the connecting point, thus making the docked flight a three degree of freedom (3DOF) system. This connecting point, p, was assumed to be in the plane of the center of gravity and has a position vector, \( r_p = -\frac{b}{2}j \). Figure 4.1 depicts the position vector of \( r_p \) as well as various angles and forces associated with a 6DOF aircraft system.

First the mothership is taken to be flying steady, straight, and level with a northward velocity of \( V \). Since the mothership is assumed in steady, straight, and level flight, her
orientation is the same as with respect to the Earth coordinate system. And this being the case, the mothership velocity, $V$, can be transformed to the velocity components of the hitchhiker through a matrix of Euler angles relating the mothership body-axis system to the hitchhiker wind-axis system. Equation set 4.11 works through the mathematical logistics.

$$
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix} = T_{M,H} \begin{bmatrix}
V \\
0 \\
0
\end{bmatrix}
$$

$$
T_{M,H} = \begin{bmatrix}
\cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\
(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) & (\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) & \sin \phi \cos \theta \\
(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) & (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) & \cos \phi \cos \theta
\end{bmatrix}
$$

$$
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix} = \begin{bmatrix}
V \cos \theta \cos \psi \\
V (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) \\
V (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)
\end{bmatrix}.
$$

(4.11)

The subscripts $M$ and $H$ represent mothership and hitchhiker, respectively. The hitchhiker aerodynamic angles, angle of attack, $\alpha$, and sideslip, $\beta$, can be defined in Equation 4.12 with
the transformed velocities,

\[
\alpha = \arctan \left( \frac{w}{u} \right), \\
\beta = \arcsin \left( \frac{v}{V} \right).
\]  

(4.12)

\(\alpha\) is the angle between the projection of velocity vector onto the aircraft x-z plane and the x-axis of the aircraft; it is positive when the \(w\)-component of the velocity is positive (nose up). \(\beta\) is the angle between the projection of the velocity vector onto the aircraft x-y plane and the x-axis; it is positive with the velocity vector from the right (nose left).

With \(\alpha\) and \(\beta\) defined in terms of the hitchhiker, the values can be sent to the look-up tables of wind tunnel data to accurately determine the aerodynamic forces (\(X, Y,\) and \(Z\)) and moments (\(L, M,\) and \(N\)). The most evident alteration for the equations of motion is the moment. The moment is now referenced about the left wingtip not the center of gravity (c.g.), and the aerodynamic forces, \(X\) and \(Z\), apply an additional moment, \(-\Delta N\) and \(+\Delta L\), respectively to the system. Because of the chosen position vector of \(p\), there is no change in the pitching moment, \(M\). Both \(-\Delta N\) and \(+\Delta L\) have moment arms equal to half the wingspan, \(r_p = -\frac{b}{2}\). Principal axes were assumed and the parallel axis theorem was applied to shift the moments of inertia to \(p\). The parallel axis theorem in Equation 4.13 states,

\[
I_{x'x'} = I_{xx} + mr^2,
\]

(4.13)

where \(m\) is the mass of the body and \(r\) is the distance from the original axis to the new axis, and for this problem, \(r = r_p\),

\[
I_{x'x'} = I_{xx} + \frac{mb^2}{4}.
\]

(4.14)

The subscript notation in Equation 4.13 is for an arbitrary axis shift, where the prime represents the new axis. There was no change in the \(y\)-axis moment of inertia, again because of the chosen position vector.

The states, \(x\), involved are the Euler angles, and the angular velocities. Also the power is included, because the thrust vector can apply additional moments as well, so it is necessary for linearization, but has no significance in determining the stability of the aircraft system.

\[
x = [\phi \, \theta \, \psi \, p \, q \, r \, \text{power}].
\]

The program from Stevens & Lewis’ Aircraft Control and Simulation [24] first asks for flight conditions (\(V, h,\) etc.), trims the six degree of freedom aircraft, then it either linearizes the system and/or flies the aircraft for various changes in the states and control states. It solves the state rates for one condition and then sends them to a Runge-Kutta integrator to solve over time [24]. Because a traditional aircraft flies as a six degree of freedom system and a wingtip-docked aircraft flies as a three degree of freedom system, the aircraft was first
trimmed as if flying traditionally and then trimmed as if docked. The cost function had to be manipulated for the seven states in the 3DOF system.

From the situation Mr. Erickson described in the Tom Tom accident, the instability of the system was instigated with sideslip, \( \beta \); therefore, in the F-16 desktop model, it was necessary for \( \beta \) to be a user-defined variable, as part of the initial flight condition. \( \beta \) is not one of the seven states for this model, so if \( \beta \) is specified then, \( \psi \), the state variable and heading angle, must calculated from \( \beta \). This was performed as a defined constraint, when the model is trimmed. Part of Equation 4.11 defines the y-component of the aircraft velocity as follows,

\[
v = V(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi)
\]

Then solving for \( \psi \) in terms of \( \beta \) (recall Equation 4.11) yields Equation 4.15,

\[
\psi = \arccos\left(\frac{\sin \beta}{\sqrt{(\sin \phi \sin \theta)^2 + (-\cos \phi)^2}}\right) + \arctan\left(\frac{-\cos \phi}{\sin \phi \sin \theta}\right).
\]

The solution for \( \psi \) is complicated and involves the trigonometric function in terms of exponentials, and the polar form of these complex relations.

### 4.4 Examination of Wingtip-Docked Flight with a Desktop Model

To examine the stability of the system, a sweep of sideslip (\( \beta \)) was performed at various combinations of altitude (\( h \)), and velocity (\( V \)), the total velocity of the system (and northward velocity of the mothership). The altitudes were 10,000 ft, 15,000ft, and 20,000ft. These values are reasonable for this system; too high would limit the control of the fighter aircraft, and too low would leave little margin for error on the part of the pilot. Also this altitude range is consistent with the previously conducted experiments. The velocity ranges from 400 ft/s to 900 ft/s (\( \approx \) 240 knots to 533 knots) in increments of 100 ft/s. A fighter does not control well at low speeds, but the mothership (a subsonic bomber,cargo, or transport like a B-36) does not fly trans- or supersonically. The range of \( \beta \) varied from case to case, under the constraint of a maximum thirty degrees deflection on the control surfaces when trimmed, namely the ailerons and rudder, \(-5.0 \leq \beta \leq 10.0\) degrees. The maximum allowable \( \beta \) decreased with speed and altitude. \( \beta \) was varied incrementally by one degree, and in regions of unusual change, it was varied as little as a tenth of a degree. Each run or case was linearized with seven states, \( \phi, \theta, \psi, p, q, r \), and power to yield the influence coefficient matrix, \( A \), in Equation 4.1. The \( A \) matrix was then imported into MATLAB and with the use of several small programs common to MATLAB toolboxes, the eigenvalues, eigenvectors, natural frequency, damping ratio, and relative eigenvalue influence evaluated from Equation 4.10, \( q(0) \), were calculated and stored. The eigenvalues were plotted in the
complex plane at constant altitude and speed as a function of $\beta$. The variations in the initial unforced response or relative influence of the eigenvalue were plotted with respect to $\beta$ for each state.

**4.5 Results for Dynamic Modelling of Wingtip-Docked Flight with a Desktop Model**

As the previous section described, the eigenvalues are the key to understanding the stability of the system. Again the eigenvalues were plotted in the complex plane at a constant altitude and speed for various $\beta$. From the resulting graphs, altitude and speed are flight parameters that do not greatly effect the stability of the system. This is concluded because the overall shape of each eigenvalue path does not vary with altitude and speed. Significant variation does appear with $\beta$. Figure 4.2 is approximately the middle of the test envelope at an altitude of 15,000 ft and a speed of 600 ft/s, and is representative of all the data.

![Figure 4.2: $\lambda_i$ for Influence Coefficient Matrix, $A$, for Various $\beta$ at an Altitude of 15,000 ft and a Speed of 600 ft/s](image)

Three distinct oscillatory modes or complex conjugate pairs (recall the seventh state power is dropped in the stability analysis) are evident in Figure 4.2. Two of the pairs have negative real parts for all $\beta$, and are therefore stable, but the last pair has positive real parts for $\beta$ greater than approximately 1.5 degrees (this slightly varies with each case) and is
Table 4.1: The Ranges of Sideslip, $\beta$, for which Eigenvalues, $\lambda_5$ and $\lambda_6$, are Stable and Unstable

<table>
<thead>
<tr>
<th>$\beta$ deg.</th>
<th>$\lambda_5$</th>
<th>$\lambda_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.0 – 0.0</td>
<td>stable: aperiodic</td>
<td>unstable: aperiodic</td>
</tr>
<tr>
<td>0.0</td>
<td>split</td>
<td>split</td>
</tr>
<tr>
<td>0.0 – 2.0</td>
<td>stable: periodic</td>
<td>stable: periodic</td>
</tr>
<tr>
<td>2.0 &gt;</td>
<td>unstable: periodic</td>
<td>unstable: periodic</td>
</tr>
</tbody>
</table>

therefore unstable. Eigenvalues $\lambda_5$ and $\lambda_6$, for $\beta < 0.0$, are two aperiodic modes, one stable ($\lambda_5$) and one unstable ($\lambda_6$). At $\beta \approx 0.0$, $\lambda_5$ and $\lambda_6$ meet on the real axis to briefly create a stable oscillatory mode, and then for $\beta \sim > 2.0$ degrees, the oscillatory mode is pushed onto the right-half of the complex plane and is unstable. Since there is one unstable root, the entire system is unstable. Table 4.1 states the approximate ranges of $\beta$ that are stable and unstable for $\lambda_5$ and $\lambda_6$, as well as the type of mode depicted. For reference concerning Table 4.1 see Figure 4.2. There is a pocket of stability for the system when the complex conjugate mode for $\lambda_5, 6$ is between approximately $0.0 < \beta < 2.0$ degrees. But a range of two degrees is perhaps too small for the hitchhiker to maintain controllability in the tumultuous mothership wingtip vortex region. So, at this point, it appears Mr. Erickson is somewhat “off the hook”, though pilot induced oscillations (PIO) can still inadvertently induce a dynamic instability from an otherwise controllable static instability.

Figures 4.3 through 4.6 are a sampling of the data. As mentioned previously, the overall shape is the same and the figures are the outer corners of the envelope tested. Note, not all cases included the negative values of $\beta$, since Mr. Erickson only spoke of a positive or toed-in $\beta$ and due to the aforementioned limitations in control surface deflections, $\delta_a$ and $\delta_r$. All data is presented in Appendix C.2. Lower speeds seem to contract the eigenvalue paths on the real axis, and at lower altitudes the paths seem less smooth. An example of the input and output for this program is in Appendix C.3 and the actual program is in Appendix D.

The states dominating the instability seen in $\lambda_5, 6$ can be determined through the methods described in Equation set 4.10, which solve a linear ordinary differential equation with the eigenvectors from matrix $A$. This information can then be applied for each state and analyzed over a range of $\beta$, which is the parameter in question. The range of $\beta$ was -5.0 – 8.0 degrees in increments from 1.0 degrees to 0.1 degrees depending on the interest in the region. In Figures 4.7 and 4.8, the states that the unstable mode have relative influence on the changes are the angles and angular rates for roll and pitch, $\phi, \theta, p$, and $q$. Unfortunately little real information on the dominating states for the unstable mode can be extracted from these plots. The magnitude of the differences between the states is small in comparison to analyzes like this. For example, this analysis for the well-known short period reveals the dominating states, $\alpha$ and $q$, by an order of magnitude of a hundred [25]. But the states concerning roll and pitch about the hitchhiker left wingtip rotational axis are the dominating states in Figures 4.7 and 4.8 for the unstable mode, and the Tom Tom Project spoke of pitch...
Figure 4.3: $\lambda_i$ for Influence Coefficient Matrix, $A$, for Various $\beta$ at an Altitude of 10,000 ft and a Speed of 400 ft/s

Figure 4.4: $\lambda_i$ for Influence Coefficient Matrix, $A$, for Various $\beta$ at an Altitude of 20,000 ft and a Speed of 400 ft/s
and roll being coupled by the elevators controlling pitch and roll (ailers ineffective) [8]. Also Mr. Erickson distinctly described a “flapping” motion at the time of the accident about
the longitudinal hinge axis between the B-36 and F-84 [9].

Figure 4.7: Relevant Dominance of Mode, \( \lambda_5 \), to the States

Figure 4.8: Relevant Dominance of Mode, \( \lambda_6 \), to the States

4.6 The Driving State for Wingtip-Docked Flight Instability

The previous sections have shown the wingtip-docked system to be unstable. So additional knowledge is needed to show how the states and state rates vary with respect to each other (i.e. how the influence coefficient matrix changes—thus how the eigenvalues change). This analysis mentioned briefly in Section 4.1 can determine the factors driving the system instability. Knowledge of this kind would allow a control designer to build an automated system to counter the instability, and thus make an unstable system stable in the eyes of the pilot. So, the sensitivity of the unstable eigenvalue to the variation each state is a necessary analysis.

The eigenvalues \( \lambda_i, i = 1 \ldots n \) for the influence coefficient A matrix at several combinations of sideslip angle, altitude, and speed are known from the previous study. The analysis begins with the question:

\[
\frac{\partial \lambda_i}{\partial x} =?,
\]  

(4.16)

where \( x \) is a dummy variable representing the various states. By the definition of an eigenvalue, Equation 4.6:

\[
\begin{align*}
u_i[\lambda_i I - A] &= 0 \\
[\lambda_i I - A]v_i &= 0,
\end{align*}
\]

(4.17)
where \( u_i \), a row vector, and \( v_i \), a column vector, are the left and right eigenvectors, respectively. Clearly \( u_i[\lambda_i I - A]v_i = 0 \), so the derivative with respect to any variable is

\[
\frac{\partial u_i[\lambda_i I - A]v_i}{\partial x} = 0. 
\]

(4.18)

Expanding Equation 4.18,

\[
\frac{\partial u_i[\lambda_i I - A]v_i}{\partial x} = \frac{\partial u_i}{\partial x}[\lambda_i I - A]v_i + u_i \frac{\partial[\lambda_i I - A]}{\partial x}v_i + u_i[\lambda_i I - A] \frac{\partial v_i}{\partial x} = 0. 
\]

(4.19)

The first and third terms on the right-hand side are identically zero based on Equation 4.17, leaving only the second term

\[
u_i \frac{\partial[\lambda_i I - A]}{\partial x} v_i = 0.
\]

(4.20)

Expanding the partial derivative and solving for the eigenvalue partial derivative yields,

\[
\frac{\partial \lambda_i}{\partial x} = \frac{u_i \frac{\partial A}{\partial x} v_i}{u_i \cdot v_i},
\]

(4.21)

where

\[
\frac{\partial A}{\partial x} = \frac{\partial a_{jk}}{\partial x} j, k = 1 \ldots n.
\]

(4.22)

A change in an eigenvalue is

\[
\Delta \lambda_i = \frac{\partial \lambda_i}{\partial a_{jk}} \Delta a_{jk} = \frac{u_i \frac{\partial A}{\partial a_{jk}} v_i \Delta a_{jk}}{u_i \cdot v_i}.
\]

(4.23)

where \( \frac{\partial A}{\partial a_{jk}} \Delta a_{jk} \) is the change in the \( A \) matrix between two flight systems for variation in one variable, \( \beta \), and it is a matrix of the same size as \( A \) with all elements zero except the \( jk \) element.

A MATLAB code was written that would determine seven \( \Delta \lambda_i \)'s for each element of the \( \frac{\partial A}{\partial a_{jk}} \Delta a_{jk} \) matrix, yielding a 7 X 49 matrix. Fortunately this quantity of information is not necessary. Only the information on the unstable mode is required, \( \lambda_5,6 \), also the seventh row and column need not be considered because the solution is trivial (\( \lambda_7 = -1 \)). The system consists of a 2 X 49 \( \Delta \lambda_i \) matrix, and even most of these can be eliminated because they are identically zero.

Two systems of different \( \beta \) values were considered, the second system or larger \( \beta \) value was used as the reference, and thus its eigenvectors were employed for calculation. The largest \( \Delta \lambda_i \) value corresponded to \( j, k = 5,1 \) element of the influence coefficient matrix, \( A \), which represents \( \frac{\partial \dot{q}}{\partial \phi} \). This is the partial derivative of the pitching angular acceleration, \( \dot{q} \), with respect to the roll angle, \( \phi \). The change in the state rate, \( \dot{q} \), with respect to the change in state, \( \phi \), drive the instability associated with variations in \( \beta \). A detailed example of this process is located in Appendix C.1 for a unit change in \( \beta \).
To determine the variation importance of the driving factors in the instability, it is worthwhile to map their development through the wingtip-docked desktop model. Knowing that the state rate $\dot{\phi}$ by definition is a function the state $\phi$ from analysis following Equation 4.1, then it is only necessary to map the development of that state. The Euler angle, $\phi$, is determined by trimming the aircraft for a given altitude, velocity, and sideslip. First it is trimmed numerically for the 6DOF model by summing the squares of the kinematic and moment angular accelerations to a small tolerance ($TOL << 1$), and then it is trimmed for the 3DOF model, as if docked, by summing the squares of the moment angular accelerations also to a small tolerance ($TOL << 1$). Actually trimming for a 6DOF model first, appears to have little significance in the overall process, but it is good practice since in reality the hitchhiker would be ideally flying in a trimmed condition before docking procedures began.

The roll angle, $\phi$, is then used to determine the $y$ and $z$ components of velocity, $v$ and $w$, for the hitchhiker (Equation 4.11). In turn the flight angles $\alpha$ and $\beta$ are determined by Equation 4.13 ($\beta$ of course should be the same as originally specified) and are thus functions of $\phi$. The aerodynamics forces ($X$, $Y$, and $Z$) and moments ($L$, $M$, and $N$) are functions of $\alpha$ and $\beta$ which are values necessary for the look-up tables of wind tunnel data. The kinematic equations are directly functions of $\phi$, while the moment equations are indirectly functions of $\phi$ through the aerodynamic forces and moments that directly define $\dot{\phi}$. Figure 4.9 is a flow chart attempting to describe this development of $\phi$. It is relevant to point out key differences in typical flight and wingtip-docked flight. Typically the aerodynamic angles, $\alpha$ and $\beta$, determine the motion of the aircraft, and the Euler angles, $\phi$, $\theta$, and $\psi$, are only used...
in determining the gravity vector or direction down towards the Earth. Refer to the Equation sets 4.2 and 4.6, \( \phi \), \( \theta \), and \( \psi \) appear in the gravity term and the navigation equations with respect to Earth. For the wingtip-docked flight the Euler angle are very significant. They determine the aerodynamic angles, \( \alpha \) and \( \beta \) through the hitchhiker velocity components, \( u \), \( v \), and \( w \).

Figure 4.10 graphs the variation in the unstable eigenvalue with respect to the variation in \( \beta \) for several changes in state rates to states, \( \frac{\partial k}{\partial x} \). The coefficients for partial derivatives of rolling and pitching angular accelerations with respect to \( \phi \) and \( \psi \) are not small, but the aforementioned \( \frac{\partial q}{\partial \phi} \) is clearly driving the unstable eigenvalue into the right-half of the complex plane. Figure 4.10 emphasizes the dominance of the \( \frac{\partial q}{\partial \phi} \) coefficient.

![Figure 4.10: Changes in the Unstable Eigenvalue versus Changes in \( \beta \).](image)

4.7 Summary of Wingtip-Docked Flight Dynamic Simulation

The stability analysis for wingtip-docked flight has proven that there exists an unstable mode with respect to the variation in sideslip, \( \beta \), as perceived in Project Tom Tom. There is a very small range for \( \beta \), slightly toed-in to the mothership, where the system is stable (0.0 < \( \beta \) < 2.0 degrees), and clearly in such a tumultuous region as the wingtip vortex of the large mothership, maintaining this stable position, with or without automated flight control, would be difficult.
The analysis attempting to determine the states influencing and/or driving the unstable mode was not determinate. It did clearly establish the coupled nature of the system between roll and pitch motion, but it do not clearly establish a single state of unquestionable dominance. These results concur with the accounts of several test pilots in multiple wingtip-docked flight tests. For instance, at the point right before the hitchhiker is rigidly docked and locked into place, the hitchhiker and mothership are connected at the wingtips by a ball joint; the ailerons are ineffective as roll control surfaces and the elevators control both pitch and roll. Also the recollection of the *Tom Tom* crash by Mr. Erickson, described a severe roll or "flapping" motion onset by a yawing motion. All of this is reinforcement for the results of the wingtip-docked stability analysis, that wingtip-docked flight is complicated from typical flight by a coupling in the lateral and longitudinal aircraft motions.

The system was proven to be unstable, but this in no way means that the system is unfliable. Fighter pilots maintain control of their typically unstable aircraft routinely, but as mentioned previously, the wingtip-docked system operates completely different than typical aircraft. So again the question of PIO proneness for the wingtip-docked flight is readdressed. The key elements discussed in Section 4.2 for PIO described the pilot overcompensating and being $180^\circ$ out of phase with his control inputs and the actual control output. The pilot of a wingtip-docked system might very well revert to the control methods of typical flight, like using the ailerons to compensate for a sudden roll caused by a gust or yawing motion, when it is the elevators that actually control roll motion in wingtip-docked flight. Such instinctual reactions could drive the wingtip-docked system into uncontrollable or unrecoverable instability. From the collected data and analysis, it seems quite logical that the wingtip-docked flight system would be prone to PIO, but to validate this the next step, for future work, would be a pilot-in-the-loop simulation.
Chapter 5

Summary and Discussion

The armed forces require a system that can be rapidly deployed to meet the increasing need for high mobility—strategically, operationally, and tactically. High mobility is one of the core functions for the Army’s Brigade Combat Team (BCT) that has become essential in trouble spots overseas and even more so with the developing War on Terrorism and Operation Enduring Freedom. A task to air-deliver 20 tons, safely, over 4000 miles, non-stop, has been presented. This delivery could not only include tanks, and ammunition, but also people/soldiers and supplies/humanitarian aid—“beans, bullets, and bandaids”. To achieve this mission several types of Compound Aircraft Transport (CAT) flight systems have been proposed, consisting of a larger mothership and a smaller hitchhiker(s). The types considered in this study are wingtip-docked and formation flight. Questions posed particularly for this study are: where is the best location, aerodynamically, for the hitchhiker with respect to the mothership? And what are the instabilities, if any, in the system? And in conjunction with that, what are the flight states driving the instability?

Chapter 1 described the basic aerodynamic benefits seen in CAT flight: increasing lift and reducing the energy necessary to maintain and/or achieve a goal. This chapter also reviewed previous flight tests for CAT flight. Most of these were begun on the basis of permanent fighter escorts for bombers in WWII, and it evolved into a possible reconnaissance systems during the Korean and Cold War. Little flight data was recorded. Two of the tests for a wingtip-docked system, Projects Tip Tow and Tom Tom ended catastrophically. From the historical references it was clear that a wingtip-docked system was feasible and controllable, but the efforts involved for the pilot were fatiguing. The Tom Tom Project is the model for this study; it consisted of an F-84F hitchhiker and a B-36 mothership and was conducted in the early 1950s. The lead test pilot for the F-84 F was Beryl A. Erickson who described a violent “flapping” motion about the wingtip-docked hinge line that appeared to be caused by a positive or toed-in sideslip angle of the F-84 at the docking point. The team was attempting to simulate a “tired or injured” pilot [9], but unfortunately it resulted in structural failure of the B-36 wing—six feet was torn off that Mr. Erickson flew home.
with [1]. Chapter 1 also discussed some modern advancements with formation flight with GPS and computer technology, particularly the Autonomous Formation Flight Project with two F/A-18’s. The hitchhiker test pilot reiterated the fatigue in maintaining the favorable location in the mothership trailing vortex upwash.

Chapter 2 discussed the experimental tests performed in the Stability Wind Tunnel with a 6’ X 6’ X 24’ test section at Virginia Tech. These tests consisted of a 1/32 scale F-84E model with an internally mounted six-component sting balance and an in-house composite manufactured wing—representing the outboard section of the B-36—with an externally mounted 4-component strut balance (lift, drag, roll, and yaw). The entire system was mounted vertically with the tunnel floor as a plane of symmetry. The B-36 like-wing or transport wing was fixed at the center of the test section, and the F-84 model was movable in the plane perpendicular to the free stream by means of a traverse mechanism.

Post data reduction, three configurations were determined to be of the most interest: the wingtip-docked or Configuration One, the close formation or Configuration Two, and the towed/far formation or Configuration Five. The data was presented as forces and moments of the F-84 model plotted against the spanwise displacement or gap separation between the fixed transport wing and the movable F-84 model for various F-84 positions above and below the transport wing, zero being the location with both model wingtips in-plane. The wingtip-docked configuration showed improvements in flight performance driven by the inviscid force, lift. Creating a ratio of $L/D$ for the F-84 in CAT flight with respect to the $L/D$ for a solo flight yielded a 20-40% increase in flight performance at the wingtip-docked position, where the gap distance between the F-84 and transport wing is zero.

The tests for flight in close formation had the F-84 model moved downstream one chordlength of the transport wing with the capability of moving inboard of the transport wingtip. The F-84 showed essentially no increase in lift data, but a very large reduction in drag. The idea for close formation flight was that the hitchhiker could advantageously remain docked or connected to the mothership and utilize the upwash of her trailing wingtip vortex. This was without question shown in the drag reduction for the F-84 model wingtip slightly inboard of the transport wingtip, where drag reaches very close to zero. Upwash can actually produce thrust on a trailing aircraft, thus drag must at some point pass through zero and $L/D \rightarrow \infty$. The reduction in drag drives the increase in flight performance, and the same two vertical locations of the F-84 drag reduction data show improvements on the order of 200-300%. The two vertical locations were at zero, the wingtip-to-wingtip docked plane, and at a position slightly above the transport wingtip.

Finally, the towed configuration showed the most interesting trends. The tests were set up to simulate the towed flight of a hitchhiker, therefore the focal point is the nose of the F-84 aligned with the transport wingtip trailing edge, but data is presented in the same previous coordinate system. The F-84 was moved spanwise approximately a semi-span inboard and outboard of the transport wingtip. Outboard moving in, the lift increased and drag decreased on the F-84, and then the lift began to decrease and the drag began to
increase until the F-84 was significantly in the downwash of the transport wing and there
the lift and drag levelled-off. The most interesting part of the results was two spikes that
appeared in the drag and rolling moment data (thus manifesting in the $L/D$ data too) near
the F-84 nose to transport wingtip-plane, speculation placed the spikes in the category of
vortex-fuselage interaction.

Chapter 3 computationally analyzed CAT flight with a vortex lattice method (VLM)
to model the wings of the mothership and hitchhiker. A VLM program was modified to
incorporate multiple aircraft in various locations, and the computation of induced drag
was added. It is well-known that induced drag calculated in VLM from a swept panelling
has errors, but the magnitude of these errors are much much less in comparison to the
magnitude of the changes in CAT flight to be modelled. The wing geometry and locations in
the wind tunnel tests were implemented into VLM, and the results clearly depicted the same
trends. The VLM predictions and wind tunnel data for the wingtip-docked configuration,
which was driven by inviscid lift improvements were almost identical. The close formation
predictions showed an increase in flight performance like the wind tunnel data, but the
very large magnitude of improvement was not predicted. For the towed formation, VLM
results followed the trends in the experimental, but did not pick up the spikes as seen in
the wind tunnel. This only further supported the hypothesis that the spikes are a result of
fuselage-vortex interaction; the fuselage was not modelled in VLM.

For all the configurations, the total wind tunnel drag was compared to the VLM
CAT calculated induced drag, and the $L/D$ wind tunnel data was compared to two $L/D$’s
determined from VLM CAT. The two $L/D$’s from VLM CAT analysis were determined
through two methods of calculating parasite drag. For the close and towed formations, the
lower parasite drag, $C_{D_o}$, compared more favorably to the wind tunnel data, and this $C_{D_o}$
calculation utilized a maximum performance benefit estimation where $C_{D_o} = C_{D_i}$. The
VLM CAT $L/D$ results for the wingtip-docked configuration compared more favorably with
the larger $C_{D_o}$ calculation, which utilized estimations of wetted area and skin friction to
determine $C_{D_o}$. At some locations in the drag data, the VLM CAT induced drag was greater
than the total wind tunnel drag, and at other locations, the VLM CAT induced drag was
greater than the total wind tunnel drag. So VLM CAT is overpredicting and underpredicting
induced drag. In overprediction, the VLM CAT is not completely describing the flow fields,
which could be attributed to the assumption of flat wake. A deformable wake could add
to the accuracy of the flow field in VLM CAT. In underpredicting, the VLM CAT is not
representing the effects of viscosity, and the flat-wake assumption is most likely still a factor.
Viscosity cannot be modelled in VLM by its definition, only a full three-dimensional Navier-
Stokes code could describe all the viscous effects.

Chapter 4 analyzed the stability of wingtip-docked flight for the hitchhiker as a func-
tion of sideslip. An F-16 desktop model modified the equations of motion to act as if docked
with the left wingtip stationary in translation. For toed-in and -out (positive and negative)
values in sideslip as well as increments of velocity and altitude, the system was linearized
with respect to seven states, Euler angles, angular velocities, and thrust, and the resulting
changes in eigenvalues were plotted versus sideslip. An unstable aperiodic mode exists for 
\( \beta < 0.0 \) and an unstable oscillatory mode exits for \( \beta > 2.0 \) degrees, thus the wingtip-docked
system is almost always unstable in terms of sideslip. A sensitivity analysis led to the states
that drive this instability. The changes in eigenvalues are most sensitive to the change in
pitch angular acceleration \( (\dot{q}) \) with respect to roll angle \( (\phi) \). This all seems in accordance
with the accounts of Mr. Erickson. When docked, just before rigidly locked into place, the
ailerons became ineffective and the elevators controlled pitch and roll, and it was a very
short period with an undamped flapping motion that destroyed the system. The very short
or possible shortening period is concurrent with an acceleration and a flapping motion is
concurrent with a roll angle; the coupling of pitch and roll, longitudinal and lateral motion,
is an already-known component of wingtip-docked flight.

In conclusion the most aerodynamically beneficial location for a hitchhiker with re-
spect to a mothership is aft, off-center (below), and slightly inboard of the mothership based
on the wind tunnel testing. This is congruent with the flight test data of the Autonomous
Flight Formation Project which shows a region of maximum upwash, thus maximum bene-
fit, inboard and beneath the lead aircraft’s trailing wingtip vortex \([6]\). The VLM reenforced
these findings and supplies a vital tool in quick analysis for CAT flight in terms of posi-
tion and planform. The stability analysis shows an undeniable instability in wingtip-docked
flight for toed-in values of sideslip. This emphasizes the need for automated flight control in
wingtip-docked flight for coupling of the pitch and roll motions.

In the opinion of the author the quickest and most viable implementation for CAT
flight to relieve some of the issues in the tumultuous Middle East is that of a wingtip-docked
system. This is for two reasons, the modifications to existing aircraft would be minimal and
confined to the wingtips, and with modern computers, an automated flight control system
would be simple. The relation between flap angle and the flight angle of attack and sideslip
has already been established as a simple transformation of coordinate systems. Also in the
opinion of the author, the most aerodynamically beneficial position being close formation,
could easily be modified for the hitchhiker to be docked to the mothership. If the hitchhiker
was flying in formation very close or tight with respect to the mothership, perhaps a simple
sturdy connecting mechanism could be developed to hold the hitchhiker in the most beneficial
region of the mothership trailing vortex. This would eliminate the technical difficulties of
maintaining formation flight that have thus far constrained formation flight (close or far) to
small time periods and skilled pilots. The connection would have to be rigid, to maintain
the hitchhiker in the proper location, and thus would have to be short in length to validate
it structurally. The rolling moments caused by the mothership wingtip trailing vortex on
the hitchhiker would be an important factor in determining that length, but a connection
mechanism that was rigid and very long in length would be heavy and subject to greater
drag penalties. Thus, the arrangement to reap the greatest performance benefits for the
hitchhiker and the mothership would, in the opinion of the author, be close formation flight.

The aerodynamic benefits are undoubtedly greatest for formation flight. The upwash
is an energy saver like no other, but the computational techniques in automated flight control
necessary to maintain a hitchhiker aircraft in that upwash have not yet been developed. The benefits in formation flight can be more reasonably applied to industries outside the military. The reductions in fuel and environmentally hazardous emissions would save the commercial airline industry a significant amount of money, translating to cost savings for the consumer as well. Included in the commercial airline industry are the cargo suppliers. Futuristically, one could envision squadrons of unmanned aerial vehicles flying in a vee formation peeling-off out of formation to: deliver soldiers, goods or ammunition to soldiers, relief to victims, or intelligence, reconnaissance, and surveillance (IRS) information. The commercial airliners big or small could travel in formation and passengers could reach any destination (with an airport) after a pleasant non-stop flight. In the same respect the cargo or mail industry would be able to deliver mail directly to its destination. Not only could aircraft peel-off out of the formation, but also could join the formation at anytime. Compound Aircraft Transport flight has many applications and advantages, and with the continuing advancement of computers, CAT flight could become a reality.
Bibliography


Appendix A

Wind Tunnel Data
A.1 Wind Tunnel Data Reduction

A.1 Wind Tunnel Data Reduction:

The raw data consisted of measurements from the sting and strut balance as well as the conditions of the tunnel. The input format was as follows:

\[ x, y, z, \alpha, \beta, Q, T, P, CH_1, CH_2, CH_3, CH_4, CH_5, CH_6, CH_7, CH_8, CH_9, CH_{10} \]

Elements 1–3 are spatial quantities in inches, and 4–5 are angles in degrees. These elements \((x, y, z, \alpha, \beta)\) are inputed by the user and stored. Elements 6–8 \((Q, T, P)\) are the tunnel condition values dynamic pressure, temperature, and pressure in volts. Elements 9–18 are the channel readings from the sting and strut balance in volts. Elements 9–14 are the six components of the sting balance inside the F-84 model. Those components are the forward pitch (FP, in-lbs), aft pitch (AP, in-lbs), forward yaw (FY, in-lbs), aft yaw (AY, in-lbs), roll (in-lbs), and axial (lbs). Elements 15–18 are the four components of the strut balance that the transport wing is mounted. Those components are the forward pitch (FP, in-lbs), aft pitch (AP, in-lbs), forward yaw (FY, in-lbs), and aft yaw (AY, in-lbs). The tare values for the channels are taken for each run and subtracted, and in all cases these tare values are small.

The calibration curves used to reduce this data to forces and moments are as follows:

\[
\begin{align*}
\text{StingBalance} & \quad \text{StrutBalance} \\
FP &= 20.040(CH_1) - 0.0965 & FP &= 416.7(CH_1) + 0.3542 \\
AP &= 19.685(CH_2) - 0.0965 & AP &= -833.3(CH_2) - 0.4167 \\
FY &= 19.861(CH_3) - 0.0765 & FY &= 833.3(CH_3) - 0.0833 \\
AY &= 19.840(CH_4) - 0.0933 & AY &= 833.3(CH_4) + 0.0833 \\
\text{Roll} &= -5.105(CH_5) + 0.005105 \\
\text{Axial} &= 6.510(CH_6) + 0.0013
\end{align*}
\]

The normal \((FN, \text{lbs})\), axial \((FA, \text{lbs})\), and side forces \((FS, \text{lbs})\) and the rolling \((\text{in-lbs})\), pitching \((\text{in-lbs})\), and yawing \((\text{in-lbs})\) moments were determined as describe in section 3.1.1. The distance between the strain gages on the sting balance, \(d_{\text{sting}}\), is 4.2484 inches (supplied by the manufacturer), and for the strut, \(d_{\text{strut}} = 9.5\) inches. So the forces and moments for the balance would be as follows for the F-84:

\[
\begin{align*}
FN &= \frac{AP - FP}{d_{\text{sting}}} \\
FS &= \frac{AY - FY}{d_{\text{sting}}} \\
FA &= Axial \\
\text{Roll} &= \text{Roll} \\
Pitch &= FP + (0.783FN) \\
Yaw &= FY + (0.783FS)
\end{align*}
\]

And the transport wing:

\[
\begin{align*}
FN &= \frac{AP_{TW} - FP_{TW}}{d_{\text{strut}}} \\
FA &= \frac{AY - FY_{TW}}{d_{\text{strut}}}
\end{align*}
\]

All the values above are with respect to the balances and not the models. For instance the
side force (FS) on the sting balance is actually comparable to the lift on the F-84. So the force transformation is with respect to the F-84 model is:

\[
\begin{bmatrix}
\text{Drag} \\
\text{Side} \\
\text{Lift}
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha \cos \beta & \sin \beta & \sin \alpha \cos \beta \\
-\sin \beta \cos \alpha & \cos \beta & -\sin \alpha \sin \beta \\
-\sin \alpha & 0 & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
FA_{F-84} \\
FN_{F-84} \\
FS_{F-84}
\end{bmatrix}
\]

This is synonymous with a body to wind transformation. The transport wing transformation is much simpler. The lift is equal to the normal force, FN, and the drag is equal to the axial force, FA.

The moment values for the F-84 with respect to the model are as follows:

\[L = \text{Roll}\]
\[M = \cos \beta \text{Yaw}\]
\[N = \cos \beta \text{Pitch}\]

The dimensional values are to be non-dimensionalized as aerodynamic force and moments commonly are. Therefore the nondimensionalized quantities are:

\[C_L = \frac{\text{Lift}}{QS}\]
\[C_D = \frac{\text{Drag}}{QS}\]
\[C_S = \frac{\text{Side}}{QS}\]
\[C_l = \frac{L}{QS^2}\]
\[C_M = \frac{M}{QS^2}\]
\[C_N = \frac{N}{QS^2}\]

Q is the dynamic pressure, and the geometric values are:

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A.2 Additional Wind Tunnel Data

Configuration One: Wingtip-Docked Formation

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F-84 model $C_{L_{solo}}$ = 0.48

Note: Wind tunnel data on the F-84 model for lift, drag, roll, and lift-to-drag ratio to solo lift-to-drag $((L/D)/(L/D)_{solo})$ is located in Chapter 2.
Figure A.1: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.2: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.3: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Coordinates

\[\begin{array}{ccc}
\xi & \eta & \zeta \\
0.0 & 0.0-0.68 & 0.0 \\
\end{array} \]

Angles (deg.)

\[\begin{array}{ccc}
\alpha & \beta & \phi \\
4.0 & 0.0 & 0.0 \\
\end{array} \]

F-84 model \(C_{L_{\text{soln}}}\) 0.98
Figure A.4: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.5: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.6: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.7: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.8: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.9: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.10: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location (\(\eta\)) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
### Coordinates

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### F-84 model $C_{L_{solo}}$

-0.03
Figure A.11: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.12: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.13: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.14: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.15: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.16: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.17: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location \((\eta)\) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Configuration One: Wingtip-Docked Formation

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F-84 model \(C_{L_{solo}}\) | 0.48
Figure A.18: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.19: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.20: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.21: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.22: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.23: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.24: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location \((\eta)\) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
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### F-84 model $C_{L_{sato}}$

0.98
Figure A.25: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.26: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.27: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.28: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.29: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.30: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.31: $\frac{(L/D)}{(L/D)_solo}$ vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation.
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**F-84 model** $C_{L_{solo}}$ -0.03
Figure A.32: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.33: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.34: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.35: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.36: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.37: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.38: \(\frac{L/D}{(L/D)_{solo}}\) vs. Spanwise Location (\(\eta\)) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Configuration One: Wingtip-Docked Formation

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F-84 model $C_{L_{solo}}$ | 0.48
Figure A.39: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.40: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.41: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.42: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.43: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.44: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.45: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location (\(\eta\)) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
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Angles (deg.) | α | β | φ
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F-84 model $C_{L_{soln}}$ | 0.98
Figure A.46: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.47: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.48: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.49: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.50: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.51: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.52: $(L/D)/(L/D)_{solo}$ vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
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| F-84 model $C_{L_{solo}}$ | -0.03 |
Figure A.53: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.54: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.55: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.56: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.57: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation

Figure A.58: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
Figure A.59: \( \frac{(L/D)}{(L/D)_{solo}} \) vs. Spanwise Location (\( \eta \)) of F-84 Model at Various Vertical Positions for Wingtip-Docked Formation
## Configuration Two: Close Formation

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| F-84 model $C_{L_{solo}}$ | 0.48 |

Note: Wind tunnel data on the F-84 model for lift, drag, roll, and lift-to-drag ratio to solo lift-to-drag $(L/D)/(L/D)_{solo}$ is located in Chapter 2.
Figure A.60: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.61: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.62: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
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Figure A.63: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.64: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.65: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.66: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.67: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.68: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.69: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location \((\eta)\) of F-84 Model at Various Vertical Positions for Close Formation
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Figure A.70: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.71: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.72: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.73: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.74: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.75: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.76: \( \frac{(L/D)}{(L/D)_{solo}} \) vs. Spanwise Location (\( \eta \)) of F-84 Model at Various Vertical Positions for Close Formation
Configuration Three: Half Close Formation

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F-84 model $C_{L_{solo}} \sim 0.98$
Figure A.77: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.78: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.79: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.80: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.81: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.82: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.83: \( (L/D)/(L/D)_{solo} \) vs. Spanwise Location (\( \eta \)) of F-84 Model at Various Vertical Positions for Close Formation
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F-84 model $C_{\mu_{\text{solo}}}$ -0.03
Figure A.84: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.85: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.86: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.87: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.88: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation

Figure A.89: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Figure A.90: $(L/D)/(L/D)_{solo}$ vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Close Formation
Configuration Four: Wingtip-Docked Roll Formation

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F-84 model $C_{L_{solo}}$ | 0.98
Figure A.91: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation

Figure A.92: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation
Figure A.93: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation

Figure A.94: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation
Figure A.95: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation

Figure A.96: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation
Figure A.97: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location \((\eta)\) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation
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### F-84 model $C_{L_{satu}}$

-0.03
Figure A.98: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation

Figure A.99: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation
Figure A.100: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation

Figure A.101: Rolling Moment ($C_l$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation
Figure A.102: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation

Figure A.103: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation
Figure A.104: \( (L/D)/(L/D)_{\text{solo}} \) vs. Spanwise Location (\( \eta \)) of F-84 Model at Various Vertical Positions for Wingtip-Docked Roll Formation
Configuration Five: Towed Formation

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F-84 model $C_{L_{solo}}$ 0.48

Note: Wind tunnel data on the F-84 model for lift, drag, roll, and lift-to-drag ratio to solo lift-to-drag $(L/D)/(L/D)_{solo}$ is located in Chapter 2.
Figure A.105: Side Force ($C_Y$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Towed Formation

Figure A.106: Pitching Moment ($C_M$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Towed Formation
Figure A.107: Yawing Moment ($C_N$) vs. Spanwise Location ($\eta$) of F-84 Model at Various Vertical Positions for Towed Formation
Configuration One: Transport Wing Data for Wingtip-Docked Formation

Note: Wind tunnel data on the Transport Wing for lift, drag and lift-to-drag ratio to solo lift-to-drag \( ((L/D)/(L/D)_{solo}) \). \( (L/D)_{solo} \) for transport wing is 4.2.

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F-84 model \( C_{L_{solo}} \) 0.48
Figure A.108: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation

Figure A.109: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
Figure A.110: $(L/D)/(L/D)_{solo}$ vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
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## F-84 model $C_{L_{soln}}$ 0.98
Figure A.111: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation

Figure A.112: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
Figure A.113: \(\frac{(L/D)}{(L/D)_{solo}}\) vs. Spanwise Location (\(\eta\)) of Transport Wing for Wingtip-Docked Formation.
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Figure A.114: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation

Figure A.115: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
Figure A.116: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location \((\eta)\) of Transport Wing for Wingtip-Docked Formation
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0.48
Figure A.117: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation

Figure A.118: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
Figure A.119: \( \frac{(L/D)}{(L/D)_{solo}} \) vs. Spanwise Location (\( \eta \)) of Transport Wing for Wingtip-Docked Formation
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| F-84 model $C_{L_{s,olo}}$ | 0.98 |
Figure A.120: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation

Figure A.121: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
Figure A.122: \( \frac{(L/D)}{(L/D)_{solo}} \) vs. Spanwise Location \( \eta \) of Transport Wing for Wingtip-Docked Formation
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Figure A.123: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation

Figure A.124: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
Figure A.125: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location (\(\eta\)) of Transport Wing for Wingtip-Docked Formation
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### F-84 model $C_{L_{sola}}$

| F-84 model $C_{L_{sola}}$ | 0.48 |
Figure A.126: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation

Figure A.127: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
Figure A.128: \( (L/D)/(L/D)_{solo} \) vs. Spanwise Location \( (\eta) \) of Transport Wing for Wingtip-Docked Formation
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F-84 model $C_{L_{soln}}$ 0.98
Figure A.129: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation

Figure A.130: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
Figure A.131: \( (L/D)/(L/D_{solo}) \) vs. Spanwise Location \((\eta)\) of Transport Wing for Wingtip-Docked Formation
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**F-84 model** $C_{L_{solo}}$ = -0.03
Figure A.132: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation

Figure A.133: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
Figure A.134: $\frac{(L/D)}{(L/D)_{solo}}$ vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Formation
Configuration Two: Transport Wing Data for Close Formation

Note: Wind tunnel data on the Transport Wing for lift, drag and lift-to-drag ratio to solo lift-to-drag \((L/D)/(L/D)_{solo}\). \((L/D)_{solo}\) for transport wing is 4.2.

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F-84 model \(C_{L_{solo}}\) 0.48
Figure A.135: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Close Formation

Figure A.136: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Close Formation
Figure A.137: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location \((\eta)\) of Transport Wing for Close Formation
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Figure A.138: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Close Formation

Figure A.139: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Close Formation
Figure A.140: \( \frac{(L/D)}{(L/D)_{solo}} \) vs. Spanwise Location (\( \eta \)) of Transport Wing for Close Formation
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Figure A.141: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Close Formation

Figure A.142: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Close Formation
Figure A.143: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location \((\eta)\) of Transport Wing for Close Formation
Configuration Three: Transport Wing Data for Half Close Formation

Note: Wind tunnel data on the Transport Wing for lift, drag and lift-to-drag ratio to solo lift-to-drag \((L/D)/(L/D)_{solo}\). \((L/D)_{solo}\) for transport wing is 4.2.

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F-84 model \(C_{L_{solo}}\): 0.48
Figure A.144: Lift Force \( (C_L) \) vs. Spanwise Location \( (\eta) \) of Transport Wing for Close Formation

Figure A.145: Drag Force \( (C_D) \) vs. Spanwise Location \( (\eta) \) of Transport Wing for Close Formation
Figure A.146: \( \frac{(L/D)}{(L/D)_{solo}} \) vs. Spanwise Location (\( \eta \)) of Transport Wing for Close Formation
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### F-84 model $C_{L_{solo}}$

-0.03
Figure A.147: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Close Formation

Figure A.148: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Close Formation
Figure A.149: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location ($\eta$) of Transport Wing for Close Formation
Configuration Four: Transport Wing Data for Wingtip-Docked Roll Formation

Note: Wind tunnel data on the Transport Wing for lift, drag and lift-to-drag ratio to solo lift-to-drag \( (L/D)/(L/D)_{solo} \). \( (L/D)_{solo} \) for transport wing is 4.2.

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| F-84 model \( C_{L_{solo}} \) | 0.48 |
Figure A.150: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Roll Formation

Figure A.151: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Roll Formation
Figure A.152: $(L/D)/(L/D)_{solo}$ vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Roll Formation
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Figure A.153: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Roll Formation

Figure A.154: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Wingtip-Docked Roll Formation
Figure A.155: \( (L/D)/(L/D)_{solo} \) vs. Spanwise Location \( (\eta) \) of Transport Wing for Wingtip-Docked Roll Formation
Configuration Five: Transport Wing Data for Towed Formation

Note: Wind tunnel data on the Transport Wing for lift, drag and lift-to-drag ratio to solo lift-to-drag \((L/D)/(L/D)_{solo}\). \((L/D)_{solo}\) for transport wing is 4.2.

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F-84 model \(C_{L_{solo}}\) 0.48
Figure A.156: Lift Force ($C_L$) vs. Spanwise Location ($\eta$) of Transport Wing for Towed Formation

Figure A.157: Drag Force ($C_D$) vs. Spanwise Location ($\eta$) of Transport Wing for Towed Formation
Figure A.158: \((L/D)/(L/D)_{solo}\) vs. Spanwise Location \(\eta\) of Transport Wing for Towed Formation
Appendix B

VLM CAT
B.1 VLM CAT Manual

The VLM code for CAT (vlmcat.f) is written in the programming language Fortran 77; a copy of it is in Appendix D.

1. The code accepts input from the screen for the geometry of the mothership, the number of spanwise and chordwise divisions for the mothership planform, respectively; the number of hitchhiker(s) – it then asks if the hitchhiker(s) have identical geometry and/or spacing between each other–the hitchhiker(s) geometry, the number of spanwise and chordwise divisions for the hitchhiker planform, respectively, and finally for the hitchhiker origin relative to the mothership in her coordinate system.

2. With this information the program then extracts typical aircraft geometry like aspect ratio, leading edge sweep, reference area, etc.

3. Now the panel geometry is built, and arrays are built for horseshoe vortex and control point locations to be sent to the subroutine VHORSE. The geometry data to be sent to the VHORSE subroutine must read left to right from the centerline of the entire system–the right half of the mothership wing + one full hitchhiker. This requires some manipulation of the input data for the $X_{12}, Y_{12}, X_{22}, Y_{22}, \ldots$ must be mirrored, therefore, several counter and/or place holder arrays of no significant purpose appear throughout the program. The geometry is written to the file GEOM.DAT.

4. The subroutine VHORSE follows the VLM techniques described in Bertin & Smith [18]. Implementing the flow tangency boundary condition at the control point, the subroutine determines the influence coefficient matrix $\vec{C}_{m,n}$ from Equation 3.. Note, VHORSE is called twice to mirror the left-hand-side of the wing; all vortices influence the other.

5. The influence coefficient matrix is sent to the subroutine GAUSS that employs Gaussian Elimination to solve for the linear set of unknown circulations equations. The circulation values are stored in the last column of the $C$ matrix, which are written to the file GAMMA.DAT.
6. Now that the circulation values are known for all control points, the downwash velocity induced by each horseshoe vortex on each panel can be determined (a vortex filament does not induce a velocity on itself). The subroutine \texttt{VHORSE} is called once again, but now with the x-location of the spanwise mid-point for the finite segment of the horseshoe vortex (\texttt{XVCPT}). And also \texttt{VHORSE} is called twice here to incorporate the wing symmetry. The outputted influence coefficient matrix $\mathbf{\hat{D}}_{m,n}$ is multiplied by the known circulation array to yield the downwash, which is written to the file \texttt{DOWN.DAT}.

7. Summing the strips or chordwise panel circulation values the sectional lift coefficient and pitching moment are calculated, per radian, and written to the \texttt{COEF.DAT} file. Similarly the downwash times the circulation is summed for each strip to calculate the induced drag coefficient per radians squared; this is also written to \texttt{COEF.DAT}.

8. All the circulation values are summed to determine the total coefficients for lift and pitching moment per radian, written to \texttt{COEF.DAT}. Similarly for total induced drag coefficient with respect to downwash.

**INPUT VARIABLES:** listed in order inputted  
All coordinates follow that in Figure B.1  
\textbf{YROOT}: The y-coordinate of the wing root  
\textbf{XRTLE}: The x-coordinate of the wing root leading edge  
\textbf{XRTTE}: The x-coordinate of the wing root trailing edge  
\textbf{YTIP}: The y-coordinate of the wingtip  
\textbf{XTIPLE}: The x-coordinate of the wingtip leading edge  
\textbf{XTIPTE}: The x-coordinate of the wingtip trailing edge  
\textbf{NSPAN}: The number of spanwise divisions for the symmetric right-hand portion of the wing. Double for total number of spanwise divisions on wing  
\textbf{NCHRD}: The number of chordwise divisions for the wing  
\textbf{NAC}: The number of hitchhiker(s), this value does not include the mothership.  
\textbf{XSTR}: The x-location of the hitchhiker origin with respect to the mothership origin in terms of her coordinates  
\textbf{YSTR}: The y-location of the hitchhiker origin with respect to the mothership origin in terms of her coordinates  
\textbf{ZSTR}: The z-location of the hitchhiker origin with respect to the mothership origin in terms of her coordinates
GEOMSPC: Logical \( Y \) for yes and \( N \) for NO. The hitchhiker(s) are of equal spacing from each other including the mothership, and their geometry is identical. The origin of hitchhiker 1 with respect to the mothership origin in her coordinates are \((X_{STR}, Y_{STR}, Z_{STR})\), hitchhiker 2 is \((2\times X_{STR}, 2\times Y_{STR}, 2\times Z_{STR})\) and so forth. For hitchhiker 1, \( Y_{ROOT2}(1), X_{RTLE2}(2), X_{RTEE2}(2), \ldots\) and for hitchhiker 2, \( Y_{ROOT2}(2), X_{RTLE2}(2), X_{RTEE2}(2)\)

NGEOMSPC: Logical \( Y \) for yes and \( N \) for NO. The spacing between each hitchhiker(s) and the geometry is not equal.

GEOM: Logical \( Y \) for yes and \( N \) for NO. The geometry of each hitchhiker is identical.

SPC: Logical \( Y \) for yes and \( N \) for NO. The spacing between each hitchhiker is equal, again this includes the spacing between the mothership and the first hitchhiker.

Throughout the entire code, the mothership is denoted with a 1 and the hitchhiker(s) as an array with a 2, i.e. \( Y_{ROOT1} \) for the mothership and \( Y_{ROOT2}(N_{NAC}) \) where \( N_{AC} \) is the number of hitchhiker aircraft

\$ S \$ preceding a listed input value refers to the SCREEN, and \$ H \$ following a listed input value refers to a HOLD like a place holder.

**INTERNAL CODE VARIABLES** list alphabetically

**AR:** The aspect ratio of the wing defined as span squared divided by the wing reference area: \( AR = \frac{A_{B2}}{S_{REF}} \)

**ARRAY:** Dummy counter array

**B2:** Semi-span of wing: \( Y_{TIP} - Y_{ROOT} \)

**C1:** The chord of the left-hand-side of each chordwise strip

**C2:** The chord of the right-hand-side of each chordwise strip

**C:** The influence coefficient matrix determined from the flow tangency boundary condition in the \textsc{Vhorse} subroutine. It has \( NTOT \times NTOT \) elements and after solving for the circulation through Gaussian Elimination \( (NTOT \times NEQNS + 1 \) elements), the last column of the matrix is the array \text{GAMMA}.

**CAV:** The average chord of the wing, \( \frac{S_{REF}}{2} \)

**CCPT:** The chord at the control point of the panel

**CDI:** The total induced drag of the wing per radians squared

**CDILOC:** The local induced drag of the wing per radian squared

**CHRD:** The panel chord through the control point

**CL:** The total lift coefficient per radian

**CCLCA:** The local lift coefficient divided by the average chord of the wing

**CLLOC:** The local lift coefficient per radian

**CM:** The total pitching moment coefficient per radian reference from leading edge

**COEFH:** The constant value \( \frac{1.0}{107} \) applied in the \textsc{Vhorse} subroutine

**CREF:** The reference wing chord, equal to \( CAV \)

**CROOT:** The wing root chord, \( X_{RTEE} - X_{RTLE} \)

**CTIP:** The wingtip chord, \( X_{TIPT} - X_{TIPLLE} \)

**CV:** The same as the \( C(N_{CHRD}, N_{SPAN}) \) matrix, but to determine the downwash at the spanwise mid-point of the finite segment-horseshoe vortex
DELC1: The left-hand-side of each chordwise strip divided by the number of inputted spanwise divisions, $C_1/X_{NSPAN}$

DELC2: The right-hand-side of each chordwise strip divided by the number of inputted spanwise divisions, $C_2/X_{NSPAN}$

DELCPT: The change in control point x-location

DELTAY: The width of the panel

DOWN: The matrix for downwash

DXDYLE: The change in x-root location leading edge to x-tip leading edge divided by the change in y-root location to y-tip location, $(XTIPE – XRTLE)/(YROOT – YTIP)$ (this is zero if the wing has no leading edge sweep, $A_{LE} = 0 \implies XRTLE = XTIPE$

DXDYTE: Like $DXDYLE$ but with respect to the trailing edge, TE

ETA: The non-dimensionalized spanwise coordinate, $\eta = YCPTB2$

GAMMA: The array for the circulation $\Gamma$

GAUSS: A Gaussian Elimination subroutine capable of solving a set of linear equations. In this program for circulation, the $GAMMA$ array.

L: Dummy matrix for sorting.

NEQNS: $NTOT$ sent to the GAUSS subroutine

NTOT: The total number of equations to be solved which is the same as the total number of control points

PI: 3.1415926...

SREF: The reference area or wing area, $SREF = 4B^2 \ast AR$

SUM: The series sum of $GAMMA$, counter for total coefficients

SUMCDI: The series sum of downwash, $DOWN$, for each chordwise strip of panel, $NSPAN$, for section induced drag

SUMCL: The series sum of circulation, $GAMMA$ for each chordwise strip of panel, $NSPAN$, for sectional lift

SUMCM: The series sum of circulation, $GAMMA$ for each chordwise strip of panel, $NSPAN$, for sectional pitching moment

SUMD: The series sum of $DOWN$, counter for total induced drag coefficient

TAPER: The root chord divided by the tip chord

UR: x-velocity component at control point for the right-hand-side of the wing. Outputted from the VHORSE subroutine.

UL: x-velocity component at control point for the left-hand-side of the wing. Outputted from the VHORSE subroutine.

UVR: x-velocity component at the spanwise mid-point on the finite segment of the horseshoe vortex for the right-hand-side of the wing. Outputted from the VHORSE subroutine and used to determine the downwash.

VR: y-velocity component at control point for the right-hand-side of the wing. Outputted from the VHORSE subroutine.

VL: y-velocity component at control point for the left-hand-side of the wing. Outputted from the
**VHORSE** subroutine.

**VVR:** y-velocity component at the spanwise mid-point on the finite segment of the horseshoe vortex for the right-hand-side of the wing. Outputted from the **VHORSE** subroutine and used to determine the downwash.

**WVR:** z-velocity component at the spanwise mid-point on the finite segment of the horseshoe vortex for the left-hand-side of the wing. Outputted from the **VHORSE** subroutine and used to determine the downwash.

**WR:** z-velocity component at control point for the right-hand-side of the wing. Outputted from the **VHORSE** subroutine.

**WL:** z-velocity component at control point for the left-hand-side of the wing. Outputted from the **VHORSE** subroutine.

**WV:** DOW

**WVR:** y-velocity component at the spanwise mid-point on the finite segment of the horseshoe vortex for the right-hand-side of the wing. Outputted from the **VHORSE** subroutine and used to determine the downwash.

**WVR:** y-velocity component at the spanwise mid-point on the finite segment of the horseshoe vortex for the left-hand-side of the wing. Outputted from the **VHORSE** subroutine and used to determine the downwash.

**X1:** An array of the x-location of the left-hand-side corner of the horseshoe vortex as in Figure xx for point A

**X1:** An array of the x-location of the right-hand-side corner of the horseshoe vortex as in Figure xx for point A

**X1LEG:** The x-location on the leading edge for the left-hand side of each chordwise strip

**X1TEG:** The x-location on the trailing edge for the left-hand side of each chordwise strip

**X2LEG:** The x-location on the leading edge for the right-hand side of each chordwise strip

**X2TEG:** The x-location on the trailing edge for the right-hand side of each chordwise strip

**XCPT:** An array of the x-location of the control point in Figure xx for point A

**XCPTLE:** The x-location of the panel leading edge in the plane of the control point or at the panel spanwise mid-point.

**XCPTTE:** The x-location of the panel trailing edge in the plane of the control point or at the panel spanwise mid-point.

**XLESWP:** The leading edge sweep, $\Lambda_{LE}$

**XNCHRD:** $NCHRD$

**XNC:** Counter for $NCHRD$

**XNS:** Counter for $NSPAN$

**XNSPAN:** $NSPAN$

**XSTAR:** The x-location of the hitchhiker origin with respect to the mothership origin in terms of her coordinates as an array of NNAC elements

**XTESWP:** The trailing edge sweep, $\Lambda_{TE}$

**XVCPT:** An array of the x-location for the spanwise mid-point location for the finite segment of the horseshoe vortex

**Y1:** An array of the y-location of the left-hand-corner of the horseshoe vortex as an array of NTOT elements

**Y2:** An array of the y-location of the right-hand-corner of the horseshoe vortex as an array

**YCPT:** An array of the y-location of the control point
YCPTG: YCPT
YINBD: Y1 scalar,
YOUTBD: Y2 scalar
YSTAR: The y-location of the hitchhiker origin with respect to the mothership origin in terms of
her coordinates as an array of $NNAC$ elements
Z1: An array of the z-location of the left-hand-corner of the horseshoe vortex
Z2: An array of the z-location of the right-hand-corner of the horseshoe vortex
ZCPT: An array of the z-location of the control point of the horseshoe vortex
ZSTAR: The z-location of the hitchhiker origin with respect to the mothership origin in terms of
her coordinates as an array of $NNAC$ elements

Output Data listed as referenced
COEF.DAT: This output file contains all the data for sectional lift and drag as well as the dimen-
sional and non-dimensional spanwise coordinates. The total coefficients, $CL, CDI, CM$ are listed
at the end per radian for $CL$ and $CM$ and per radian squared for $CDI$.

GEOM.DAT: This output file contains all the geometry data for each aircraft (i.e. $SREF, TAPER$ . . .),
the panel horseshoe geometry (i.e. $X1, X2, Y1, \ldots XCPT, YCPT$).

GAMMA.DAT: This output file contains all the circulation values on each panel for entire
system and individual aircraft.

DOWN.DAT: This output file contains all the downwash values on each panel for the
entire system and individual aircraft.

TOTALCOEF_*.DAT: Optional output file for total coefficients only for plotting. Read
left to right starting with mothership $CL1, CDI1, CM1$, hitchhiker 1 $CL2(1), CDI2(1), CM2(1)$,
hitchhiker 2 $CL2(2), CDI2(2), CM2(2)$, and so forth.

To follow is a sample of the input, then output for the Warren-12 planform discussed in
Chapter 2 with $NSPAN = NCHRD = 10$, spacing equal to $XSTR = 2chord$ and $YSTR = 1span$, 
Z-axis out of page

Figure B.1: Definition of Some Variables in VLMCAT.F

and geometry of the mothership and hitchhiker the same.
Appendix B.2

INPUT

VORTEX LATTICE PROGRAM:

COMPOUND AIRCRAFT TRANSPORT

INPUT VROOT,XRTLE,XRTTE FOR MOTHERSHIP: AIRCRAFT 1
0.0,0.0,1.5
INPUT YTP,XTIPE,XTIPTE FOR MOTHERSHIP: AIRCRAFT 1
1.41421,1.91421,2.41421
INPUT NSPAN AND NCHRD FOR MOTHERSHIP: AIRCRAFT 1
10,10

NUMBER OF HITCHIKER AIRCRAFT WRT SYMMETRY, NAC
1

TO ANSWER YES, TYPE: Y and TO ANSWER NO, TYPE: N

DO ALL HITCHIKER(S) HAVE SAME GEOMETRY?
Y

ARE ALL HITCHIKER(S) SPACED EVENLY?
Y

HITCHIKER(S) ORIGIN IS ON THE LE OF THE CENTERLINE
FOR ALL HITCHIKERS

INPUT VROOT,XRTLE,XRTTE FOR HITCHIKER: 1
0.0,0.0,1.5
INPUT YTP,XTIPE,XTIPTE FOR HITCHIKER: 1
1.41421,1.91421,2.41421
INPUT NSPAN AND NCHRD FOR HITCHIKER: 1
10,10

INPUT LOCATION OF EACH HITCHIKER ORIGIN WRT
MOTHERSHIP ORIGIN, X, Y, Z, FOR HITCHIKER:

CAREFUL, MOTHERSHIP OR HITCHIKER(S)

TRAILING VORTICES CANNOT INTERSECT HITCHIKER(S) CP
1
3.91421,2.8242,3.0
NTOT 300
OUTPUT

COEF.DAT

SECTIONAL CHARACTERISTICS FOR MOTHERSHIP: AIRCRAFT 1

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SECTIONAL CHARACTERISTICS FOR HITCHHIKER(S):

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COEFFICIENTS FOR MOTHERSHIP: AIRCRAFT 1

CL ALPHA PER RAD. = 2.8767
CM ALPHA PER RAD. = -3.2925
CDI ALPHA PER RAD. = 0.8279

COEFFICIENTS FOR HITCHHIKER(S):

CL ALPHA PER RAD. = 3.2771
CM ALPHA PER RAD. = -3.9380
CDI ALPHA PER RAD. = 0.4271
### GEOM.DAT

**PLANFORM PROPERTIES FOR MOTHERSHIP: AIRCRAFT 1**

- \( SREF = 2.8242 \)
- \( AR = 2.824 \)
- \( LE \) SWEEP = 53.58405
- \( TE \) SWEEP = 32.91949
- \( TAPER \) RATIO = 0.3333

**PLANFORM PROPERTIES FOR HITCHHIKER(S): 1**

- \( SREF = 2.8242 \)
- \( AR = 2.824 \)
- \( LE \) SWEEP = 53.58405
- \( TE \) SWEEP = 32.91949
- \( TAPER \) RATIO = 0.3333

**PANEL DATA FOR MOTHERSHIP: AIRCRAFT 1**

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**PANEL DATA FOR HITCHHIKER(S): 1**

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**GAMMA.DAT**

GAMMA FOR MOTHERSHIP: AIRCRAFT 1

RESULTS: GAMMA

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GAMMA FOR HITCHHIKER(S): 1

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### DOWNWASH.DAT

**DOWNWASH FOR MOTHERSHIP: AIRCRAFT 1**

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**DOWNWASH FOR HITCHHIKER(S): 1**

RESULTS: DOWNWASH

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Appendix C

Dynamic Simulation of Wingtip-Docked Desktop Model
C.1 Example of Matrix $A$ Element Driving An Unstable Mode for a Wingtip-Docked Configuration

Which element of matrix $A$ is most likely responsible for driving the unstable mode in the wingtip-docked configuration. Take to flight condition with unstable roots at $\beta = 3.0$ degrees and $\beta = 4.0$ degrees, corresponding to $A_3$ and $A_4$, respectively.

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0.0070246 & 0.052239 \\ 0 & 0 & 0 & 0 & 0.99108 & -0.13327 \\ 0 & 0 & 0 & 0 & 0.13345 & 0.99246 \\ 1.1134 & -27.798 & -2.1824 & -0.19496 & -2.58 & 0.04046 \\ 0.26183 & -4.9145 & -0.70549 & 0 & -1.459 & 0.0 \\ 0.0637581 & 3.1881 & -2.6893 & 0.0064822 & -0.053926 & -0.13968 \end{bmatrix}$$

The unstable eigenvalue in $A_3$ is $5.93e - 02 \pm j5.20e - 01$.

$$A_4 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0.0092384 & 0.051523 \\ 0 & 0 & 0 & 0 & 0.9843 & -0.17649 \\ 0 & 0 & 0 & 0 & 0.17673 & 0.98565 \\ 1.5331 & -27.76 & -3.5433 & -0.19562 & -2.5666 & 0.039037 \\ 0.34792 & -4.8637 & -0.92933 & 0 & -1.459 & 0.0 \\ 0.026379 & 3.2559 & -2.5879 & 0.0080674 & -0.049824 & -0.13945 \end{bmatrix}$$

The unstable eigenvalue in $A_4$ is $0.13631 \pm j0.60342$. Using $A_4$ as the reference matrix for $\lambda_i = 0.13631 \pm j0.60342$,

$$u^{r_i} = \begin{cases} 0.078328 + j0.00036718 \\ -0.54792 + j0.52838 \\ -0.057794 + j0.097565 \\ 0.054787 - j0.10004 \\ -0.21344 + j0.58159 \\ 0.027426 - j0.063552 \end{cases}$$

$$v_i = \begin{cases} 0.83977 - j0.1094 \\ 0.044222 - j0.015027 \\ 0.069635 - j0.028064 \\ 0.17911 + j0.48983 \\ -0.019517 + j0.030981 \\ 0.023312 + j0.033195 \end{cases}$$

Recall from Chapter 4:
\[ \Delta \lambda_i = \frac{\partial \lambda_i}{\partial a_{jk}} \Delta a_{jk} = \frac{u_i \frac{\partial A}{\partial a_{jk}} v_i \Delta a_{jk}}{u_i \cdot v_i} \]

where \( \frac{\partial A}{\partial a_{jk}} \Delta a_{jk} \) is the \( DA \) j by k matrix consisting of only the element in the \( jk \) position and \( \Delta a_{jk} \) is simply \( DA = A_4 - A_3 \),

\[
DA = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0.0022138 & -0.000716 \\
0 & 0 & 0 & 0 & 0 & -0.006778 & -0.04322 \\
0 & 0 & 0 & 0 & 0 & 0.043277 & -0.006806 \\
0.41961 & 0.0373 & -1.5609 & -0.00655 & 0.01342 & -0.0014228 \\
0.086095 & 0.05085 & -0.22384 & 0 & 0.00311 & 0 \\
-0.037372 & 0.06771 & 0.10144 & 0.0015852 & 0.004102 & 0.000234
\end{bmatrix}
\]

The elements of significance were in rows 4–6 and column 1, corresponding to the derivatives of \( p, q, \) and \( r \) with respect to \( \phi \) for the wingtip-docked configuration. The \( \Delta \lambda_i \) for these entries are

\[
\Delta \lambda_{4i} = \frac{\partial \lambda_i}{\partial a_{41}} \Delta a_{41} = -0.041728 - j0.40966 \\
\Delta \lambda_{5i} = \frac{\partial \lambda_i}{\partial a_{51}} \Delta a_{51} = 0.11391 + j0.44456 \\
\Delta \lambda_{6i} = \frac{\partial \lambda_i}{\partial a_{61}} \Delta a_{61} = 0.004316 + j0.021834
\]

Based on this information elements \( a_{51} \) (corresponding to the \( \frac{\partial q}{\partial \phi} \)) is the greatest cause for the real part of the eigenvalue moving further into the right-half of the complex plane. The imaginary part of the eigenvalue changes in \( a_{51} \) is almost completely offset by that in \( a_{41} \).
C.2 Additional Data on Stability Analysis of Wingtip-Docked Desktop Model

These are additional plots not presented in Chapter 4, of the eigenvalues in the complex plane for the entire envelope analyzed for the stability of the wingtip-docked desktop model.

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<th>Speed (ft/s)</th>
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<td>20,000</td>
<td>400 $-$ 900 : 100</td>
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Figure C.1: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 10,000 ft, 500 ft/s for a $\pm$ Range of Sideslip $\beta$

Figure C.2: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 10,000 ft, 600 ft/s for a $\pm$ Range of Sideslip $\beta$
Figure C.3: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 10,000 ft, 700 ft/s for a ± Range of Sideslip $\beta$

Figure C.4: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 10,000 ft, 800 ft/s for a ± Range of Sideslip $\beta$
Figure C.5: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 15,000 ft, 400 ft/s for a ± Range of Sideslip $\beta$

Figure C.6: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 15,000 ft, 500 ft/s for a ± Range of Sideslip $\beta$
Figure C.7: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 15,000 ft, 700 ft/s for a $\pm$ Range of Sideslip $\beta$.

Figure C.8: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 15,000 ft, 800 ft/s for a $\pm$ Range of Sideslip $\beta$. 
Figure C.9: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 15,000 ft, 900 ft/s for a ± Range of Sideslip $\beta$
Figure C.10: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 20,000 ft, 500 ft/s for a ± Range of Sideslip $\beta$

Figure C.11: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 20,000 ft, 600 ft/s for a ± Range of Sideslip $\beta$
Figure C.12: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 20,000 ft, 700 ft/s for a $\pm$ Range of Sideslip $\beta$

Figure C.13: Eigenvalues, $\lambda$, for Wingtip-Docked Desktop Model at 20,000 ft, 800 ft/s for a $\pm$ Range of Sideslip $\beta$
C.3 Wingtip-Docking Model Example Input and Output
Input_Output:

Trim it up for yaw y
VT (G1/G3) = 400
BETA (G1/G3) = -2.0

****** TRIMMING 6-DOF MODEL ******

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<th>Rudder</th>
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Angle of attack 0.123039E+00 Sideslip angle 2.000000E+00
Power Controlled 1.5065493E+01
Initial cost function 3.00075E+00 Final cost function 5.06228E+13

More Iterations? (Y/N) : n

2.000000 3.4906558E-02 0.00000000E+00 400.0000

****** TRIMMING 3-DOF MODEL ******

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Angle of attack 0.835323E+00 Sideslip angle 2.000000E+00
Power Controlled 1.358493E+01
Initial cost function 9.37825E-02 Final cost function 1.35552E-15

More Iterations? (Y/N) : n

****** DONE TRIMMING ******

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Save Trim Data to file? n

Linearize it? y

Number of State Equations to linearize? 7

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P=7, Q=8, R=9, W=10, VE=11,
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WIP=12, PONER=13
Linearized_Dynamics.DAT

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0.142720E+00 & 0.000000E+00 & 0.000000E+00 & 0.000000E+00 & 0.999135E+00 \\
0.214912E+00 & 0.142720E+00 & 0.000000E+00 & 0.000000E+00 & 0.895050E+00 \\
0.966141E-01 & 0.000000E+00 & 0.000000E+00 & 0.000000E+00 & 0.919330E+00 \\
0.106878E+01 & 0.315867E+01 & -0.790099E+01 & 0.000000E+00 & 0.000000E+00 \\
\end{bmatrix} \]

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\[ C2 = [0 1 0 0 0 0 0]; \]
\[ C3 = [0 0 1 0 0 0 0]; \]
\[ C4 = [0 0 0 1 0 0 0]; \]
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\[ C6 = [0 0 0 0 0 1 0]; \]
\[ C7 = [0 0 0 0 0 0 1]; \]
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Appendix D

Appendix D: Program Codes
**Program for VLM CAT**

W.H. Mason, February 1989

Classic Vortex Lattice Method

Model Problem: Trapezoidal Wing

First Modified by Sam Magill 020602

Multiple Aircraft: Mothership + Hitchhiker(s)

System Symmetry About Centerline of Mothership

All Coordinates Inputted About Individual Aircraft Centerline

COMMON /COF/ C(5001,5002), NEQNS

COMMON /CONST/ PI, COEFH

DIMENSION Y1(800), Y2(800), Z1(800), Z2(800), X1(800), X2(800),
    XVCPT(800), YCPT(800), ZCPT(800), XCPT(800), GAMMA(800), CHRD(200)

DIMENSION Y11(800), Y21(800), Z11(800), Z21(800), X11(800), X21(800),
    XVCPT1(800), YCPT1( 800), ZCPT1(800), XCPT1(800), GAMMA1(800),
    CHRD1(200)

DIMENSION Y12(800,800), Y22(800,800), Z12(800,800), Z22(800,800),
    X12(800,800), X22(800,800),
    XVCPT2(800,800), YCPT2(800,800), ZCPT2(800,800),
    XCPT2(800,800), GAMMA2(800,800),
    CHRD2(200,200)

DIMENSION YROOT2(800), XRTLE2(800), XRTTE2(800), YTIP2(800),
    XTIPLE2(800), XTIPTE2(800), NSPAN2(800), NCHRD2(800)

DIMENSION XNSPAN2(800), XNCHRD2(800), DXDYLE2(800), DXDYTE2(800),
    YINBD2(800), CL2(800), CM2(800), ARRAY(800),
    YOUTBD2(800), X1LEG2(800), X2LEG2(800), X1TEG2(800),
    X2TEG2(800), YCPTG2(800), DELCPT2(800), NTOT2(800),
    L(800), XCPTLE2(800), XCPTTE2(800), C12(800), C22(800),
    CCPT2(800), DELC12(800), DELC22(800), SUMCL2(800), SUMCM2(800),
    ETA2(800,800), CLLOC2(800,800), CCLCA2(800,800), SUM2(800)

DIMENSION DOWN1(800), DOWN2(800,800), CD12(20),
    SUMC12(20), SUMD2(20), CILOC2(400,40), MW(1000),
    CV(1000,2000)

REAL XSTR, YSTR, ZSTR

LOGICAL GEOMSPC, NGEOMSPC, GEOM, SPC

PI = 3.1415926585

COEFH = 1.0/(4.0*PI)

IREAD = 5

IWRIT = 6

OPEN (UNIT = 1, FILE = 'GEOM.DAT', STATUS = 'NEW')

OPEN (UNIT = 2, FILE = 'GAMMA.DAT', STATUS = 'NEW')

OPEN (UNIT = 3, FILE = 'DOWN.DAT', STATUS = 'NEW')

OPEN (UNIT = 4, FILE = 'COEF.DAT', STATUS = 'NEW')

DEFINE PLANFORM FOR MOTHERSHIP: AIRCRAFT 1

WRITE (IWRIT, 1010)

READ (IREAD, *) YROOT1, XRTLE1, XRTTE1

WRITE (IWRIT, 1020)

READ (IREAD, *) YTIP1, XTIPLE1, XTIPTE1

WRITE (IWRIT, 1030)

READ (IREAD, *) NSPAN1, NCHRD1

DEFINE PLANFORM FOR HITCHHIKER: FOLLOW 'IF STATEMENT' IF ALL HITCHHIKERS
HAVE SAME GEOMETRY AND EQUAL SPACING; IF JUST HAVE SAME GEOMETRY; IF JUST HAVE
EQUAL SPACING; IF ARE NEITHER EQUALLY SPACED OR SAME GEOMETRY

WRITE (IWRIT, 1017)

READ (IREAD, *) NAC

WRITE (IWRIT, 1021)

READ (IREAD, 99) ANS

WRITE (IWRIT, 1019)

READ (IREAD, 99) SPACE

WRITE (IWRIT, 1016)

GEOMSPC = .FALSE.

NGEOMSPC = .FALSE.

GEOM = .FALSE.

SPC = .FALSE.

HITCHHIKERS HAVE SAME GEOMETRY AND EQUAL SPACING

IF ((ANS.EQ.'Y').AND.(SPACE.EQ.'Y')) THEN

GEOMSPC = .TRUE.

END IF

IF ((ANS.EQ.'Y').AND.(SPACE.EQ.'N')) THEN

GEOM = .TRUE.

END IF

IF ((ANS.EQ.'N').AND.(SPACE.EQ.'Y')) THEN

SPC = .TRUE.

END IF

IF ((ANS.EQ.'N').AND.(SPACE.EQ.'N')) THEN

NGEOMSPC = .TRUE.

END IF

GEOMETRIC PARAMETERS WITH 'S' PREFIX ARE SCREEN INPUT VALUES
C

IF (GEOMSPC) THEN
  NNAC = 0
  WRITE (IWRIT, 1) 'FOR ALL HITCHHIKERS'
  WRITE (IWRIT, 1015) NAC
  READ (IREAD, *) SYROOT2, SXRTLE2, SXRTTE2
  WRITE (IWRIT, 1025) NAC
  READ (IREAD, *) SYTIP2, SXTIPLE2, SXTIPTE2
  WRITE (IWRIT, 1035) NAC
  READ (IREAD, *) SNSPAN2, SNCHRD2
  WRITE (IWRIT, 1036) NAC
  READ (IREAD, *) XSTR, YSTR, ZSTR
C
C     XSTRH, YSTRH, ZSTRH ARE COUNTERS
C
XSTRH = 0.0
YSTRH = 0.0
ZSTRH = 0.0
C
DO 601 NNAC = 1, NAC
    YROOT2(NNAC) = SYROOT2
    XRTLE2(NNAC) = SXRTLE2
    XRTTE2(NNAC) = SXRTTE2
    YTIP2(NNAC) = SYTIP2
    XTIPLE2(NNAC) = SXTIPLE2
    XTIPTE2(NNAC) = SXTIPTE2
    NSPAN2(NNAC) = SNSPAN2
    NCHRD2(NNAC) = SNCHRD2
XSTAR(NNAC) = XSTRH + XSTR
YSTAR(NNAC) = YSTRH + YSTR
ZSTAR(NNAC) = ZSTRH + ZSTR
XNSPAN2(NNAC) = NSPAN2(NNAC)
XNCHRD2(NNAC) = NCHRD2(NNAC)
DXDYLE2(NNAC) = ((XTIPLE2(NNAC) - XRTLE2(NNAC))/
                  (YTIP2(NNAC) - YROOT2(NNAC)))
DXDYTE2(NNAC) = ((XTIPTE2(NNAC) - XRTTE2(NNAC))/
                  (YTIP2(NNAC) - YROOT2(NNAC)))
601 CONTINUE
END IF
C
C     HITCHHIKERS ARE UNEQUALLY SPACED AND DIFFERENT GEOMETRY
C
IF (GEOMSPC) THEN
  NNAC = 0
  WRITE (IWRIT, 1) 'FOR ALL HITCHHIKERS'
  WRITE (IWRIT, 1015) NAC
  READ (IREAD, *) SYROOT2, SXRTLE2, SXRTTE2
  WRITE (IWRIT, 1025) NAC
  READ (IREAD, *) SYTIP2, SXTIPLE2, SXTIPTE2
  WRITE (IWRIT, 1035) NAC
  READ (IREAD, *) SNSPAN2, SNCHRD2
  WRITE (IWRIT, 1036) NAC
  READ (IREAD, *) XSTR, YSTR, ZSTR
C
C     XSTRH, YSTRH, ZSTRH ARE COUNTERS
C
XSTRH = 0.0
YSTRH = 0.0
ZSTRH = 0.0
C
DO 602 NNAC = 1, NAC
    YROOT2(NNAC) = SYROOT2
    XRTLE2(NNAC) = SXRTLE2
    XRTTE2(NNAC) = SXRTTE2
    YTIP2(NNAC) = SYTIP2
    XTIPLE2(NNAC) = SXTIPLE2
    XTIPTE2(NNAC) = SXTIPTE2
    NSPAN2(NNAC) = SNSPAN2
    NCHRD2(NNAC) = SNCHRD2
XSTAR(NNAC) = XSTRH + XSTR
YSTAR(NNAC) = YSTRH + YSTR
ZSTAR(NNAC) = ZSTRH + ZSTR
XNSPAN2(NNAC) = NSPAN2(NNAC)
XNCHRD2(NNAC) = NCHRD2(NNAC)
DXDYLE2(NNAC) = ((XTIPLE2(NNAC) - XRTLE2(NNAC))/
                  (YTIP2(NNAC) - YROOT2(NNAC)))
DXDYTE2(NNAC) = ((XTIPTE2(NNAC) - XRTTE2(NNAC))/
                  (YTIP2(NNAC) - YROOT2(NNAC)))
602 CONTINUE
END IF
C
C     HITCHHIKERS ARE EQUALLY SPACED BUT DIFFERENT GEOMETRY
C
IF (SPC) THEN
  NNAC = 0
  WRITE (IWRIT, 1) 'FOR ALL HITCHHIKERS'
  WRITE (IWRIT, 1015) NAC
  READ (IREAD, *) SYROOT2, SXRTLE2, SXRTTE2
  WRITE (IWRIT, 1025) NAC
  READ (IREAD, *) SYTIP2, SXTIPLE2, SXTIPTE2
  WRITE (IWRIT, 1035) NAC
  READ (IREAD, *) SNSPAN2, SNCHRD2
  WRITE (IWRIT, 1036) NAC
  READ (IREAD, *) XSTR, YSTR, ZSTR
C
C     XSTRH, YSTRH, ZSTRH ARE COUNTERS
C
XSTRH = 0.0
YSTRH = 0.0
ZSTRH = 0.0
C
DO 600 NNAC = 1, NAC
    YROOT2(NNAC) = SYROOT2
    XRTLE2(NNAC) = SXRTLE2
    XRTTE2(NNAC) = SXRTTE2
    YTIP2(NNAC) = SYTIP2
    XTIPLE2(NNAC) = SXTIPLE2
    XTIPTE2(NNAC) = SXTIPTE2
    NSPAN2(NNAC) = SNSPAN2
    NCHRD2(NNAC) = SNCHRD2
XSTAR(NNAC) = XSTRH + XSTR
YSTAR(NNAC) = YSTRH + YSTR
ZSTAR(NNAC) = ZSTRH + ZSTR
XNSPAN2(NNAC) = NSPAN2(NNAC)
XNCHRD2(NNAC) = NCHRD2(NNAC)
DXDYLE2(NNAC) = ((XTIPLE2(NNAC) - XRTLE2(NNAC))/
                  (YTIP2(NNAC) - YROOT2(NNAC)))
DXDYTE2(NNAC) = ((XTIPTE2(NNAC) - XRTTE2(NNAC))/
                  (YTIP2(NNAC) - YROOT2(NNAC)))
600 CONTINUE
END IF
C
C     HITCHHIKERS ARE EQUALLY SPACED AND SAME GEOMETRY
C
IF (SFC) THEN
  NNAC = 0
  WRITE (IWRIT, 1) 'FOR ALL HITCHHIKERS'
  WRITE (IWRIT, 1015) NAC
  READ (IREAD, *) SYROOT2, SXRTLE2, SXRTTE2
  WRITE (IWRIT, 1025) NAC
  READ (IREAD, *) SYTIP2, SXTIPLE2, SXTIPTE2
  WRITE (IWRIT, 1035) NAC
  READ (IREAD, *) SNSPAN2, SNCHRD2
  WRITE (IWRIT, 1036) NAC
  READ (IREAD, *) XSTR, YSTR, ZSTR
C
C     XSTRH, YSTRH, ZSTRH ARE COUNTERS
C
XSTRH = 0.0
YSTRH = 0.0
ZSTRH = 0.0
C
DO 603 NNAC = 1, NAC
    YROOT2(NNAC) = SYROOT2
    XRTLE2(NNAC) = SXRTLE2
    XRTTE2(NNAC) = SXRTTE2
    YTIP2(NNAC) = SYTIP2
    XTIPLE2(NNAC) = SXTIPLE2
    XTIPTE2(NNAC) = SXTIPTE2
    NSPAN2(NNAC) = SNSPAN2
    NCHRD2(NNAC) = SNCHRD2
XSTAR(NNAC) = XSTRH + XSTR
YSTAR(NNAC) = YSTRH + YSTR
ZSTAR(NNAC) = ZSTRH + ZSTR
XNSPAN2(NNAC) = NSPAN2(NNAC)
XNCHRD2(NNAC) = NCHRD2(NNAC)
DXDYLE2(NNAC) = ((XTIPLE2(NNAC) - XRTLE2(NNAC))/
                  (YTIP2(NNAC) - YROOT2(NNAC)))
DXDYTE2(NNAC) = ((XTIPTE2(NNAC) - XRTTE2(NNAC))/
                  (YTIP2(NNAC) - YROOT2(NNAC)))
603 CONTINUE
END IF
C
READ (IREAD,*) YROOT2(NNAC), XRTLE2(NNAC), XRTTE2(NNAC)
WRITE (IWRIT,1025) NNAC
READ (IREAD,*) YTIP2(NNAC), XTIPLE2(NNAC), XTIPTE2(NNAC)
WRITE (IWRIT,1035) NNAC
READ (IREAD,*) NSPAN2(NNAC), NCHRD2(NNAC)
XSTAR(NNAC) = XSTRH + XSTR
XSTRH = XSTAR(NNAC)
YSTAR(NNAC) = YSTRH + YSTR
YSTRH = YSTAR(NNAC)
ZSTAR(NNAC) = ZSTRH + ZSTR
ZSTRH = ZSTAR(NNAC)
XNSPAN2(NNAC) = NSPAN2(NNAC)
XNCHRD2(NNAC) = NCHRD2(NNAC)
DXDYLE2(NNAC) = ((XTIPLE2(NNAC) - XRTLE2(NNAC)))/
&                  (YTIP2(NNAC) - YROOT2(NNAC))
DXDYTE2(NNAC) = ((XTIPTE2(NNAC) - XRTTE2(NNAC)))/
&                  (YTIP2(NNAC) - YROOT2(NNAC))

603 CONTINUE
END IF

C
C
C
C     PLANFORM PROPERTIES FOR MOTHERSHIP: AIRCRAFT 1
C
CROOT1 = XRTTE1 - XRTLE1
CTIP1 = XTIPTE1 - XTIPLE1
B21 = YTIP1 - YROOT1
SREF1 = (CROOT1 + CTIP1)*B21
AR1 = (2.*B21)**2/SREF1
XLESWP1 = 180./PI*ATAN(DXDYLE1)
XTESWP1 = 180./PI*ATAN(DXDYTE1)
TAPER1 = CTIP1/CROOT1
CAV1 = SREF1/(2.*B21)
XREF1 = XRTLE1
CREF1 = SREF1/(2.*B21)
WRITE(1,1045)
WRITE(1,1050) SREF1,AR1,XLESWP1,XTESWP1,TAPER1

C
C     PLANFORM PROPERTIES FOR HITCHHIKER: AIRCRAFT 2
C
NNAC = 0
DO 604 NNAC = 1,NAC
CROOT2(NNAC) = XTIP2(NNAC) - XRTLE2(NNAC)
CTIP2(NNAC) = XTIPTE2(NNAC) - XRTLE2(NNAC)
B22(NNAC) = YTIP2(NNAC) - YROOT2(NNAC)
SREF2(NNAC) = (CROOT2(NNAC) + CTIP2(NNAC))*B22(NNAC)
AR2(NNAC) = (2.*B22(NNAC))**2/SREF2(NNAC)
XLESWP2(NNAC) = 180./PI*ATAN(DXOYLE2(NNAC))
XTESWP2(NNAC) = 180./PI*ATAN(DXOYTE2(NNAC))
TAPER2(NNAC) = CTIP2(NNAC)/CROOT2(NNAC)
CAV2(NNAC) = SREF2(NNAC)/(2.*B22(NNAC))
WRITE(1,1046)NNAC
WRITE(1,1055) SREF2(NNAC),AR2(NNAC),XLESWP2(NNAC),
&                  XTESWP2(NNAC),TAPER2(NNAC)
604 CONTINUE

C
C     DEFINE GEOMETRY FOR MOTHERSHIP: AIRCRAFT 1
C
DELTAY1 = (YTIP1 - YROOT1)/XNSPAN1
N = 0
DO 50 NS = 1,NSPAN1
XNC1 = NC
Z11(N) = 0.0
Z21(N) = 0.0
XCP1(N) = XRTLE1 + (YCPT1 - YROOT1)*DXDYLE1
YCP1(N) = YTIP1 + DELTAY1/2.
WRITE(1,1040) N,Y11(N),Y21(N),X11(N),X21(N),XCPT1(N),YCPT1(N)
50 CONTINUE
NTOT1 = N

C
C     DEFINE GEOMETRY FOR HITCHHIKER: AIRCRAFT 2
C
DELTAY2(NNAC) = (YTIP2(NNAC) - YROOT2(NNAC))/
&                 (XNSPAN2(NNAC))
N = 0
NSPN2 = NSPAN2(NNAC)
NCHD2 = NCHRD2(NNAC)
DO 51 NS = 1,NSPN2
WRITE(1,1075)NNAC
DELTA22(NNAC) = (YTIP2(NNAC) - YROOT2(NNAC))/
&                 (XNSPAN2(NNAC))
N = 0
NPN2 = NCHRD2(NNAC)
DO 51 NS = 1,NPN2
WRITE(1,1076)NNAC
YOUTB2(NNAC) = YROOT2(NNAC) + (XNSPAN2(NNAC) - 1.0)*DELTA22
XOUTB2(NNAC) = XRTLE2 + DELTA22*DXDYLE2
XOUTT2(NNAC) = XRTTE2 + DELTA22*DXDYTE2
WRITE(1,1071)NNAC
DELTA22(NNAC) = DELTA22(NNAC) + DELTAY2(NNAC)}
YCPT2(NNAC) = YINBD2(NNAC) + DELTAY2(NNAC)/2.
XILEG2(NNAC) = XRTLE2(NNAC) + (YINBD2(NNAC) -
& YROOT2(NNAC))*DXDYLE2(NNAC)
XZLEG2(NNAC) = XRTLE2(NNAC) + (YINBD22(NNAC) -
& YROOT22(NNAC))*DXDYLE2(NNAC)
XITEG2(NNAC) = XRTTE2(NNAC) + (YINBD2(NNAC) -
& YROOT2(NNAC))*DXDYTE2(NNAC)
XZTEG2(NNAC) = XRTTE2(NNAC) + (YOUTBD2(NNAC) -
& YROOT2(NNAC))*DXDYTE2(NNAC)
XCPTLE2(NNAC) = XRTLE2(NNAC) + (YCPTG2(NNAC) -
& YROOT2(NNAC))*DXDYLE2(NNAC)
XCPTTE2(NNAC) = XRTTE2(NNAC) + (YCPTG2(NNAC) -
& YROOT2(NNAC))*DXDYTE2(NNAC)
C12(NNAC) = X1TEG2(NNAC) - X1LEG2(NNAC)
C22(NNAC) = X2TEG2(NNAC) - X2LEG2(NNAC)
CCPT2(NNAC) = XCPTTE2(NNAC) - XCPTLE2(NNAC)
CHRD2(NS,NNAC) = CCPT2(NNAC)
DELC12(NNAC) = C12(NNAC)/XNCHRD2(NNAC)
DELC22(NNAC) = C22(NNAC)/XNCHRD2(NNAC)
DELCPT2(NNAC) = CCPT2(NNAC)/XNCHRD2(NNAC)

DO 51 NC = 1,NCHD2
N = N + 1
XNC2(NNAC) = NC
Z12(N,NNAC) = 0.0
Z22(N,NNAC) = 0.0
ZCPT2(N,NNAC) = 0.0
Y12(N,NNAC) = YINBD2(NNAC)
Y22(N,NNAC) = YOUTBD2(NNAC)
YCPT2(N,NNAC) = YCPTG2(NNAC)
X12(N,NNAC) = X1LEG2(NNAC) + (XNC2(NNAC) - 1.0)
&                 *DELC12(NNAC) + 0.25*DELC12(NNAC)
X22(N,NNAC) = X2LEG2(NNAC) + (XNC2(NNAC) - 1.0)
&                 *DELC22(NNAC) + 0.25*DELC22(NNAC)
XCPT2(N,NNAC) = XCPTLE2(NNAC) + (XNC2(NNAC) - 1.0)
&                 *DELCPT2(NNAC) + 0.75*DELCPT2(NNAC)
XVCPT2(N,NNAC) = XCPT2(N,NNAC) - .5*DELCPT2(NNAC)
WRITE(1,1040) N,Y12(N,NNAC),Y22(N,NNAC),
&                  X12(N,NNAC),X22(N,NNAC),
&                  XCPT2(N,NNAC),YCPT2(N,NNAC)
51 CONTINUE

NTOT2 = N
NTT2 = NTOT2 + NCHD2
COUNT = NTOT1
I = 0
J = 0
K = 0
NTT2 = NTT2 + NCHD2
605 CONTINUE
5
C     ADD VORTEX GEOMETRY FROM MOTHERSHIP (AIRCRAFT 1) AND HITCHHIKER(S)
C     ADD MOTHERSHIP TIMES 2 B/C OF SYMMETRY FOR THE INDIVIDUAL HITCHHIKER
C     RECALL ORIGIN OF HITCHHIKER IS AT LE CENTERLINE
C     THESE LOOPS REORGANIZE X ARRAY OF HITCHHIKER TO READ LEFT TO RIGHT
DO 801 I = 1,NTT2
X1(I) = X11(I)
X2(I) = X21(I)
Y1(I) = Y11(I)
Y2(I) = Y21(I)
Z1(I) = Z11(I)
Z2(I) = Z21(I)
YCPT(I) = YCPT1(I)
XVCPT(I) = XVCPT1(I)
550 CONTINUE
801 CONTINUE

B = NCHP2*(J+1)
S = 1
C
800 CONTINUE
5
C     ADD VORTEX GEOMETRY FROM MOTHERSHIP (AIRCRAFT 1) AND HITCHHIKER(S)
C     ADD MOTHERSHIP TIMES 2 B/C OF SYMMETRY FOR THE INDIVIDUAL HITCHHIKER
C     RECALL ORIGIN OF HITCHHIKER IS AT LE CENTERLINE
C     THESE LOOPS REORGANIZE X ARRAY OF HITCHHIKER TO READ LEFT TO RIGHT
DO 605 I = 1,NTOT1
X1(I) = X11(I)
X2(I) = X21(I)
Y1(I) = Y11(I)
Y2(I) = Y21(I)
Z1(I) = Z11(I)
Z2(I) = Z21(I)
YCPT(I) = YCPT1(I)
XVCPT(I) = XVCPT1(I)
550 CONTINUE
605 CONTINUE

I = 0
&          + XSTAR(NNAC)
XVCPT(I+(COUNT+NTT2)) = XVCPT2(I,NNAC) + XSTAR(NNAC)
YCPT(I+(COUNT+NTT2)) = YCPT2(I,NNAC) + YSTAR(NNAC)
YCPT(I+(COUNT)) = YCPT2(I,NNAC) + YSTAR(NNAC)
ZCPT(I+(COUNT+NTT2)) = ZCPT2(I,NNAC) + ZSTAR(NNAC)
ZCPT(I+(COUNT)) = ZCPT2(I,NNAC) + ZSTAR(NNAC)

C

551 CONTINUE
COUNT = COUNT + (2* NTOT2(NNAC))
552 CONTINUE
C
WRITE(1,1076)
DO 609 N = 1, NTOT
WRITE(1,1040) N,Y1(N),Y2(N),X1(N),X2(N),XCPT(N),YCPT(N)
609 CONTINUE
C
C     CHECK FOR NO OVERLAP BETWEEN TRAILING VPRTICES AND CP
C
NNAC = 0
NTT2 = 0
COUNT = NTOT1
DO 606 NNAC = 1,NAC
NTT2 = NTOT2(NNAC)
DO 608 J = 1, NTT2
DO 607 I = 1, NTOT1
IF((Y1(I).EQ.YCPT(COUNT+J))
& .OR.(Y2(I).EQ.YCPT(COUNT+J))) THEN
WRITE(IWRIT,1056)
STOP
END IF
607 CONTINUE
IF((Y1(COUNT+J).EQ.YCPT(COUNT+(2*NTT2)+J)).OR.
& (Y2(COUNT+J).EQ.YCPT(COUNT+(2*NTT2)+J))) THEN
WRITE(IWRIT,1056)
STOP
END IF
608 CONTINUE
COUNT = COUNT + (2* NTT2)
606 CONTINUE
C
C     DEFINE THE DOWNWASH AT EACH CONTROL POINT - M
C     DUE TO THE VORTICES AT EACH N
C
DO 100 M = 1,NTOT
DO 100 N = 1,NTOT
C
CALL VHORSE(XCPT(M),YCPT(M),ZCPT(M),X1(N),Y1(N),Z1(N),
X2(N),Y2(N),Z2(N),UVR,VVR,WVR)
C
Y1N      =  -Y1(N)
Y2N      =  -Y2(N)
CALL VHORSE(XCPT(M),YCPT(M),ZCPT(M),X1(N),Y1N,Z1(N),
X2(N),Y2N,Z2(N),UVL,VVL,WVL)
C
CV(M,N)   =  -WVL + WVR
100 CONTINUE
C
DO 120 N = 1,NTOT
C(N,NTOT+1) = -1.0
120 CONTINUE
C
NEQNS = NTOT
NRHS = 1
CALL GAUSS(NRHS)
C
DO 101 M = 1,NTOT
DO 101 N = 1,NTOT
C
CALL VHORSE(XVCPT(M),YCPT(M),ZCPT(M),X1(N),Y1(N),Z1(N),
X2(N),Y2(N),Z2(N),UVR,VVR,WVR)
C
Y1N      =  -Y1(N)
Y2N      =  -Y2(N)
CALL VHORSE(XVCPT(M),YCPT(M),ZCPT(M),X1(N),Y1N,Z1(N),
X2(N),Y2N,Z2(N),UVL,VVL,WVL)
C
CV(M,N)   =  ( -WVL + WVR)
101 CONTINUE
C
DO 121 M = 1,NTOT
WV(M) = 0.0
DO 122 N = 1,NTOT
WV(M) =WV(M)+(CV(M,N)*C(N,NEQNS+1))
122 CONTINUE
121 CONTINUE
C
SUMCL1    = 0.0
SUMCM1    = 0.0
SUMCL2(1)    = 0.0
SUMCM2(1)    = 0.0
SUMCL    = 0.0
SUMCM    = 0.0
SUMCDI1 = 0.0
SUMCDI2(1) = 0.0
C
GAMMA AND DOWNWASH FOR MOTHERSHIP
C
WRITE(2,1081)
WRITE(2,1080)
WRITE(3,1083)
WRITE(3,1085)
DO 140 N = 1,NTOT1
GAMMA1(N) = C(N,NEQNS+1)
DOWN1(N) = WV(N)
SUMCM1 = SUMCM1 + (XREF1 - XVCPT1(N)) *GAMMA1(N)
SUMCL1 = SUMCL1 + GAMMA1(N)
SUMCD11 = SUMCD11 + (DOWN1(N) *GAMMA1(N))
WRITE(2,450) N,GAMMA1(N)
WRITE(3,450) N,DOWN1(N)
140 CONTINUE
C
C GAMMA AND DOWNWASH FOR HITCHHIKER(S)
C
NNAC = 0
COUNT = NT0T1
DO 553 NNAC = 1, NAC
WRITE(2,1082)NNAC
WRITE(2,1080)
WRITE(3,1084)NNAC
WRITE(3,1085)
NTT2 = NT0T2(NNAC)
N = 0
DO 141 N = 1, NTT2
GAMMA2(N,NNAC) = C(COUNT+N,NEQNS+1)
GAMMA2(N+NTT2,NNAC) = C(COUNT+N+NTT2 ,NEQNS+1)
DOWN2(N,NNAC) = WV(COUNT+N)
DOWN2(N+NTT2,NNAC) = WV(COUNT+N+NTT2)
SUMCM2(NNAC) = SUMCM2(NNAC) +
XREF2(NNAC) - XVCPT2(N,NNAC) +
(GAMMA2(N+NTT2,NNAC)+GAMMA2(N,NNAC))
SUMCL2(NNAC) = SUMCL2(NNAC)
+ (GAMMA2(N,NNAC)+DOWN2(N,NNAC))
SUMCDI2(NNAC) = SUMCDI2(NNAC) +
(GAMMA2(N+NTT2,NNAC)+GAMMA2(N,NNAC))
WRITE(2,450) N,GAMMA2(N,NNAC)
WRITE(2,450) N+NTT2,GAMMA2(N+NTT2,NNAC)
WRITE(3,450) N,DOWN2(N,NNAC)
WRITE(3,450) N+NTT2,DOWN2(N+NTT2,NNAC)
141 CONTINUE
COUNT = COUNT + (2 * NTOT2(NNAC))
553 CONTINUE
450 FORMAT(I6,E20.7)
C
C MOTHERSHIP SECTIONAL LIFT COEFFICIENT
C
WRITE(4,1121)
WRITE(4,1120)
DO 160 NS = 1,NSPAN1
SUM1 = 0.0
SUMD1 = 0.0
DO 150 NC = 1,NCHRD1
N = (NS - 1)*NCHRD1 + NC
SUMD1 = SUMD1 + (DOWN1(N)*GAMMA1(N))
150 SUM1 = SUM1 + (GAMMA1(N))
C IF (NS.LE.NSPN2) THEN
YCPT2(NS,NNAC) = YCPT2(N,NNAC)
ETA2(NS,NNAC) = (YCPT2(N,NNAC))/B22(NNAC) - 1.0
ELSE
YCPT2(NS,NNAC) = YCPT2(NS-NSPN2,NNAC)+B22(NNAC)
ETA2(NS,NNAC) = (YCPT2(NS-NSPN2,NNAC))/(B22(NNAC)
END IF
IF (NS.LE.NSPN2) THEN
CHRD2(NS,NNAC) = CHRD2(NS-NSPN2,NNAC)
ELSE
CHRD2(NS,NNAC) = CHRD2(NS-NSPN2,NNAC)
END IF
CLLOC2(NS,NNAC) = 2./CHRD2(NS,NNAC)*SUM1
CCLCA2(NS,NNAC) = CHRD2(NS,NNAC)*CLLOC2(NS,NNAC)
CDILOC2(NS,NNAC) = -CLLOC2(NS,NNAC)*SUMD1
WRITE(4,1141) NS,YCPT2(NS,NNAC),
ETA2(NS,NNAC),
CLLOC2(NS,NNAC),CCLCA2(NS,NNAC),
CDILOC2(NS,NNAC)
160 CONTINUE
C
C TOTAL COEFFICIENTS FOR MOTHERSHIP
C
CL1 = 4.0*DELTAY1/SREF1*SUM1
CM1 = 4.0*DELTAY1/SREF1/CREF1*SUM1
554 CONTINUE
C
C HITCHHIKER SECTIONAL LIFT COEFFICIENT
C
NNAC = 0
NSPN2 = 0
NCHD2 = 0
DO 555 NNAC = 1,NAC
WRITE(4,1122)
WRITE(4,1120)
NSPN2 = NSPAN2(NNAC)
NCHD2 = NCHRD2(NNAC)
J = 0
DO 161 NS = 1,NSPN2*2
SUM2(NNAC) = 0.0
SUMD2(NNAC) = 0.0
DO 151 NC = 1,NCHD2
N = (NS - 1)*NCHD2 + NC
SUMD2(NNAC) = SUMD2(NNAC) + (DOWN2(N,NNAC)
+ (GAMMA2(N,NNAC)+DOWN2(N,NNAC))
151 SUM2(NNAC) = SUM2(NNAC) + (GAMMA2(N,NNAC))
161 CONTINUE
555 CONTINUE
C
C TOTAL COEFFICIENTS FOR HITCHHIKER
C
CL2(NNAC) = 2.0*DELTAY2(NNAC)/SREF2(NNAC)*SUMCL2(NNAC)
CM2(NNAC) = 2.0*DELTAY2(NNAC)/SREF2(NNAC)/
CREF2(NNAC)*SUMCM2(NNAC)
CD2(NNAC) = -2.0*DELTAY2(NNAC)/SREF2(NNAC)*SUMCD2(NNAC)
CDI1 = -4.0 * DELTAY1 / SREF1 * SUMCDI1
WRITE(4,1061)
WRITE(4,1060) CL1,CM1,CDI1
C
TOTAL COEFFICIENTS FOR HITCHHIKER
C
NNAC = 0
DO 556 NNAC = 1, NAC
WRITE(4,1062) NNAC
WRITE(4,1060) CL2(NNAC),CM2(NNAC),CDI2(NNAC)
556 CONTINUE
C
99 FORMAT (A1)
1000 FORMAT (//5X, 'VORTEX LATTICE PROGRAM:'//5X,
&             'COMPOUND AIRCRAFT TRANSPORT'/)
1010 FORMAT(/'INPUT YROOT,XRTLE,XRTTE FOR MOTHERSHIP: AIRCRAFT 1'/)
1020 FORMAT(/'INPUT YTIP,XTIPLE,XTIPTE FOR MOTHERSHIP: AIRCRAFT 1'/)
1030 FORMAT(/'INPUT NSPAN AND NCHRD FOR MOTHERSHIP: AIRCRAFT 1'/)
1015 FORMAT(/'INPUT YROOT,XRTLE,XRTTE FOR HITCHHIKER:', I3/)  
1016 FORMAT(/'HITCHHIKER(S) ORIGIN IS ON THE LE OF THE CENTERLINE'/)  
1017 FORMAT(/'NUMBER OF HITCHHIKER AIRCRAFT WRT SYMMETRY, NAC'/)  
1018 FORMAT(/'DO ALL HITCHHIKER(S) HAVE SAME GEOMETRY?'/) 
1019 FORMAT(/'ARE ALL HITCHHIKER(S) SPACED EVENLY?'/) 
1021 FORMAT(/'TO ANSWER YES, TYPE: Y and TO ANSWER NO, TYPE: N'/) 
1025 FORMAT(/'INPUT YTIP,XTIPLE,XTIPTE FOR HITCHHIKER:', I3/)  
1035 FORMAT(/'INPUT NSPAN AND NCHRD FOR HITCHHIKER:',I3/)  
1036 FORMAT(/'INPUT LOCATION OF EACH HITCHHIKER ORIGIN WRT'//,
& 'MOTHERSHIP ORIGIN, X, Y, Z, FOR HITCHHIKER:'//,
& 'CAREFUL, MOTHERSHIP OR HITCHHIKER TRAILING VORTICES CAN INTERSECT HITCHHIKER(S) CP'//,
& 'MOTHERSHIP OR HITCHHIKER TRAILING VORTICES CAN INTERSECT HITCHHIKER(S) CP'//,
& 'NUMBER OF HITCHHIKER ORIGIN WRT',//,
& 'CAREFUL, MOTHERSHIP OR HITCHHIKER TRAILING VORTICES CANNOT INTERSECT HITCHHIKER(S) CP'//,
& 'PROGRAM WILL BE TERMINATED'/)
1060 FORMAT(//3X,'COEFFICIENTS FOR MOTHERSHIP: AIRCRAFT 1'//3X)
1061 FORMAT(//3X,'GAMMA FOR MOTHERSHIP: AIRCRAFT 1'//3X)
1062 FORMAT(//3X,'GAMMA FOR HITCHHIKER(S): ',I3//3X)
1063 FORMAT(//3X,'RESOLUTION: GAMMA'//5X,'N',9X,'GAMMA')
1064 FORMAT(//3X,'DOWNWASH FOR MOTHERSHIP: AIRCRAFT 1'//3X)
1065 FORMAT(//3X,'DOWNWASH FOR HITCHHIKER(S): ',I3//3X)
1066 FORMAT(//3X,'RESULTS: DOWNWASH'//5X,'N',9X,'DOWNWASH')
1069 FORMAT(4X,'NS',7X,'Y',7X,'ETA',8X,'CL -LOC',7X,'CCLCA',
& 7X,'CDI -LOC')
1070 FORMAT(3X,'SECTIONAL CHARACTERISTICS FOR MOTHERSHIP:
& AIRCRAFT 1'//3X)
1071 FORMAT(3X,'SECTIONAL CHARACTERISTICS FOR HITCHHIKER(S):',I3//3X)
1120 FORMAT(3X,'PANEL DATA FOR MOTHERSHIP: AIRCRAFT 1'//3X,' N
& Y1         Y2  ',4X,
& X1          X2         XCPT       YCPT')
1121 FORMAT(3X,'PANEL DATA FOR ENTIRE SYSTEM'//3X,' N
& Y1         Y2  ',4X,
& X1          X2         XCPT       YCPT')
1140 FORMAT(4X,I2,3X,F7.3,3X,F7.4,2X,F11.5,2X,F11.5,2X,F11.5)
IF (IMAX .NE. IM)                      GO TO 140
DO 130        J = IM,NTOT
TEMP          = A(IM,J)
A(IM,J)       = A(IMAX,J)
A(IMAX,J)     = TEMP
130 CONTINUE
C
C                   ELIMINATE (I-1)TH UNKNOWN FROM
C                   ITH THRU (NEQNS)TH EQUATIONS
C
140 DO 150        J = I,NEQNS
R             = A(J,IM)/A(IM,IM)
DO 150        K = I,NTOT
150 A(J,K)        = A(J,K) - R*A(IM,K)
C
C                   BACK SUBSTITUTION
C
DO 220        K = NP,NTOT
A(NEQNS,K)    = A(NEQNS,K)/A(NEQNS,NEQNS)
DO 210        L = 2,NEQNS
I             = NEQNS + 1 - L
IP            = I + 1
DO 200        J = IP,NEQNS
200 A(I,K)        = A(I,K) - A(I,J)*A(J,K)
210 A(I,K)        = A(I,K)/A(I,I)
220 CONTINUE
RETURN
END
C====================================================================
SUBROUTINE VHORSE(XPT,YP T,ZPT,X1N,Y1N,Z1N,X2N,Y2N,Z2N, &
                  UHORSE,AHORSE,WHORSE)
C
C     COMPUTE DOWNWASH AT A POINT XPT, YPT, ZPT DUE TO A
C     UNIT STRENGTH HORSESHOE VORTEX AT X1N,Y1N,Z1N - X2N,Y2N,Z2N
C
COMMON /CONST/PI,COEFH
C
C     THE BOUND VORTEX
C
D1 = SQRT((XPT-X1N)**2+(YPT-Y1N)**2+(ZPT-Z1N)**2)
D2 = SQRT((XPT-X2N)**2+(YPT-Y2N)**2+(ZPT-Z2N)**2)
T21= ((X2N-X1N)*(XPT-X1N)+(Y2N-Y1N)*(YPT-Y1N)+ &
       (Z2N-Z1N)*(ZPT-Z1N))/D1
T22= ((X2N-X1N)*(XPT-X1N)+(Y2N-Y1N)*(YPT-Y1N)+ &
       (Z2N-Z1N)*(ZPT-Z1N))/D2
T2  = T21 - T22
C
T1I = (YPT-Y1N)*(ZPT-Z2N)-(ZPT-Z1N)*(YPT-Y2N)
T1J = (XPT-X1N)*(ZPT-Z2N)-(ZPT-Z1N)*(XPT-X2N)
T1K = (XPT-X1N)*(YPT-Y2N)-(YPT-Y1N)*(XPT-X2N)
T2I = T1I**2 + T1J**2 + T1K**2
T2J = 0.0
T2K = 0.0
T2  = T2I - T22
IF (T2  .LE. 0.1E-5) WRITE (10,100) T2I,T1I,T1J,T1K,XPT,X1N,X2N, &
                           YPT,Y1N,Y2N
IF (T2I .LE. 0.1E-5) GO TO 10
C
10 CONTINUE
C
UBND = T2*I1X
VND  = T2*I1Y
WND  = T2*I1Z
C
C THE A TO INFINITY VORTEX
C
CA1 = (ZPT - Z1N)**2 + (YPT - Y1N)**2
CA2 = SQRT(CA1 + (XPT - X1N)**2)
COEFA = (1.0 + (XPT - X1N)/CA2)/CA1
UAI =  0.0
VAI = -(ZPT - Z1N)*COEFA
WAI = -(Y1N - YPT)*COEFA
C
C THE B TO INFINITY VORTEX
C
CB1 = (ZPT - Z2N)**2 + (YPT - Y2N)**2
CB2 = SQRT(CB1 + (XPT - X2N)**2)
COEFB = (1.0 + (XPT - X2N)/CB2)/CB1
C
UBI =  0.0
VBI = -(ZPT - Z2N)*COEFB
WBI = -(Y2N - YPT)*COEFB
C
UHORSE = COEFA*(UBND + UAI + UBI)
AHORSE = COEFH*(UBND + VAI + VBI)
WHORSE = COEFH*(WND + WAI + WBI)
C
RETURN
END
C====================================================================
SUBROUTINE VHORSE(XPT,YP T,X1N,Y1N,Z1N,X2N,Y2N,Z2N, &
                  UHORSE,AHORSE,WHORSE)
Program for Dynamic Simulation of F-16 Desktop Model for Wingtip-Docked Flight

Main program:

PARAMETER (NN=20, MM=20, NOP=20)
EXTERNAL F, F6, SF16, SF6, TRIMMER, RK4,
         JACOB, FDX, FDU, YDX, YDU, LAW
C
C F IS THE F16 AERO PART, RETURNS XD (DX/DT)
C TRIMMER IS THE TRIMMER
C RK4 IS THE INTEGRATOR
C JACOB IS THE LINEARIZER
C FDX, FDU, YDX, YDU SUPPORT JACOB
C Modified UNIT=13 to FILE='Trimdata.dat', Samantha Magill 10-24-01
C
REAL XCG
REAL THTL, EL, AIL, RDR
REAL AN,DUM2,DUM3,QBAR,AMACH,VT,ALPHA,THETAD,QD
C
REAL X(NN), XD(NN), U(MM), Y(NOP)
REAL YREF(NOP)
C   COMMON/REFS/ YREF
C
C X AND XD (DX/DT) ARE DECODED AS FOLLOWS:
C
C   1    2    3    4    5    6  7  8  9  10     11   12   13
C   VT ALPHA BETA PHI THETA PSI P  Q  R VNORTH VEAST VUP POWER
C
C LINEAR VELOCITIES IN FT/S, ANGLES IN RADIANS, ANGLE RATES IN RAD/S
C
COMMON/ STATE/ X, XD
C
C U DECODED AS FOLLOWS:
C
C       1       2       3       4
C   THROTTLE ELEVATOR AILERON RUDDER
C
C THROTTLE IS ZERO TO ONE, AERO CONTROLS IN DEGREES
C
COMMON/ CONTROLS/ U
C
C XCG IS THE LOCATION OF THE CENTER OF GRAVITY, % CHORD
C LAND IS NOT USED (ARTIFACT OF THE TRANSPORT MODEL)
C
COMMON/PARAM/ XCG
C
COMMON/OUTPUT/ Y
C
I, J, AND K ARE COUNTERS
C
INTEGER I, J, K
C
T IS TIME, DT IS THE STEP SIZE FOR INTEGRATION
C
REAL T,DT
C
ITER IS ITERATION COUNTER, NITER IS NUMBER OF ITERATIONS TO PERFORM
C SMITE IS SMITE QUEUES, SMITES IS NUMBER OF SMITED TO PERFORM
C
INTEGER ITER, NITER, SMITE, SMITES
C
TAB IS A TAB, USED TO FORMAT OUTPUT, ANSR AN ANSWER TO PROMPT
C TITLE IS A HEADER FOR OUTPUT FILES
C
CHARACTER*1 TAB, ANSR
CHARACTER*4 REPLY
CHARACTER*40 TITLE
C
COMMON/CONSTS/ PI, DTOR, RTOD, TAB, T
C
ANSS IS A GENERIC LOGICAL, DOTRIM IS A DO-TRIM FLAG,
C DOSIM A DO-SIM FLAG, DOLIN AN DO-LINEARIZATION FLAG
C
LOGICAL ANS, DOTRIM, DOSIM, DOLIN
C
LOGICAL DOREAD
C
COMMON/LGCLS/DOTRIM, DOLIN, DOSIM
C
COMMON/LINEAR/ UNDAGES
C
INTEGER UNDAGES, UNDAGS, UNDADS, UNDAPN, UNDAPM, UNDAPM
C
REAL ANEAD(400), ANEAD1(400)
C
COMMON/AERO/ UNDAGES
C
CSAM CALL OutWindowScroll(9999)
C BE TIDY
C
DO 850 I=1, NN
X(I)  = 0.0
850    XD(I) = 0.0
DO 851 I=1, MM
851    U(I) = 0.0
DO 852 I=1, NOP
852    Y(I) = 0.0

TAB=CHAR(9)
C
PI IS PI, RTOD IS RADIANS TO DEGREES, DTOR IS DEGREES TO RADIANS
C
PI = ATAN(1.0)*4.0
RTOD = 180.0/PI
DTOR = PI/180.0
C
301 CONTINUE
C
504  WRITE(6,103)
C   READ(5,*, ERR=504) XCG
536  WRITE(6,106)
DOTRIM = .FALSE.
READ(5,99, ERR=536) ANSR
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
&    (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) DOTRIM = .TRUE.
C
C INITIAL CONDITIONS
C
T = 0.0
C
IF (DOTRIM) THEN
C
506      WRITE(6,401)
C    READ(5,*, ERR=506) GAMMAD
GAMMAD = 0.0
RADGAM= GAMMAD/RTOD
SINGAM= SIN(RADGAM)
C ROLL RATE
C 507      WRITE(6,207)
C    READ(5,*, ERR=507) P
P = 0.0
RR = P/RTOD

C
C PITCH RATE
C 508 WRITE(6,208)
C ROLE(I,*) , ERR=508) Q
Q = 0.0
PR = Q/RTOD
X(7) = PR
C TURN RATE
C 509 WRITE(6,402)
C READ(5,*, ERR=509) TR
TR = 0.0
C NON-ZERO TURN RATE; COORDINATED TURN?
C IF (TR.NE.0.0) THEN
C 510 WRITE(6,403)
C COORD = .FALSE.
C READ(I,99, ERR=510) ANSR
C END IF
C NON-ZERO ROLL RATE; STABILITY AXIS ROLL?
C IF (RR.NE.0.0) THEN
C 511 WRITE(6,404)
C STAB = .FALSE.
C READ(I,99, ERR=511) ANSR
C END IF
C
C 512 WRITE(6,201)
C READ(I,*, ERR=531) VT
C VT = 400.0
X(1) = VT
C 513 WRITE(6,202)
C READ(I,*, ERR=513) ALPHA
C ALPHAF = ALPHA
X(2) = ALPHA/RTOD
X(14) = X(2)
C 514 WRITE(6,203)
C READ(I,*, ERR=514) BETA
X(3) = BETA/RTOD
C 515 WRITE(6,204)
C READ(I,*, ERR=515) PHI
X(4) = PHI/RTOD
C 516 WRITE(6,205)
C READ(I,*, ERR=516) THETA
X(5) = THETA/RTOD
C 517 WRITE(6,206)
C READ(I,*, ERR=517) PSI
X(6) = PSI/RTOD
C 518 WRITE(6,207)
C READ(I,*, ERR=518) P
X(7) = P/RTOD
C 519 WRITE(6,208)
C READ(I,*, ERR=519) Q
X(8) = Q/RTOD
C 520 WRITE(6,209)
C READ(I,*, ERR=520) R
X(9) = R/RTOD
C 521 WRITE(6,212)
C READ(I,*, ERR=521) H
X(12) = H
C 522 WRITE(6,213)
C READ(I,*, ERR=522) THTL
U(1) = THTL
C 523 WRITE(6,214)
C READ(I,*, ERR=523) EL
U(2) = EL
X(15) = U(2)
C 524 WRITE(6,215)
C READ(I,*, ERR=524) AIL
U(3) = AIL
C 525 END IF
C TRIMMING = .FALSE.
C IF (DOREAD) THEN
C 530 WRITE(6,330)
C READ(I,*, ERR=530) ROW
C 531 WRITE(6,331)
C READ(I,*, ERR=531) X(13)
C END IF
C
C IF (DOREAD .AND. (REPLY.NE.'DATA')) GOTO 1514
C IF (DOREAD) THEN
C DO 1532 I=1,NN
C 1532 READ(13,*) X(I)
C DO 1533 I=1,MM
C 1533 READ(13,*) U(I)
C CLOSE(13)
C IF (DOREAD .AND. (REPLY.NE.'DATA')) GOTO 1531
C IF (DOREAD) THEN
C 1532 READ(I,*, ERR=532) X(I)
C DO 1533 I=1,MM
C 1533 READ(I,*, ERR=533) U(I)
C END IF
C
DO READ (5,*, ERR=527) RDR
U(4) = RDR

1514     END IF

THROUGH WITH INITIALIZATION
CALL AIR DATA COMPUTER
CALL ADC(VT,H,AMACH,QBAR)
QS=QBAR*S
Y(4)= QBAR
Y(5)= AMACH

IF (DOTRIM) THEN
WRITE(6,*) ' ' 
WRITE(6,*) '****** TRIMMING 6-DOF MODEL ******'
WRITE(6,*) ' ' 
CALL TRIMMER(6,SF6) ! Trim 6 DOF model in SSSLF
X(13)= TGEAR(U(1))
WRITE(6,*) ' ' 
WRITE(*,*) BETA, X(3) , V, VT
WRITE(6,*) '****** TRIMMING 3-DOF MODEL ******'
WRITE(6,*) ' ' 
CALL TRIMMER(3,SF16) ! Trim 3 DOF model
WRITE(6,*) ' ' 
WRITE(6,*) '****** DONE TRIMMING ******'
WRITE(*,*) BETA, X(3), V, VT
TRIMMING = .FALSE.
END IF

DOREAD = .FALSE.
IF (DOTRIM) THEN
WRITE(6,3301)
READ(5,99, ERR=1631) ANSR
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
     &       (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) DOREAD = .TRUE.
ENDIF

IF (DOREAD) THEN
OPEN(UNIT=13,FILE='Trimdata.dat',STATUS='NEW')
WRITE(13,*) 'DATA'
DO 1632 I=1,NN
1632       WRITE(13,*) X(I)
DO 1633 I=1,MM
1633       WRITE(13,*) U(I)
CLOSE(13)
ENDIF

GET REFERENCE VALUES OF OUTPUT VARIABLES
CALL F(0.0, X, XD)
DO 343 I=1,NOP
343   YREF(I) = Y(I)

DOLIN = .FALSE.
READ(5,99, ERR=531) ANSR
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
     &      (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) DOLIN = .TRUE.
END IF

IF (DOLIN) THEN
DO合作 = .FALSE.
DO READ (5,*, ERR=527) RDR
U(20), V, W, Y = RDR

W(20) = U(20)
W(21) = V
W(22) = W
W(23) = Y

DO READ (5,*, ERR=527) RDR
U(20), V, W, Y = RDR

W(20) = U(20)
W(21) = V
W(22) = W
W(23) = Y

DO 164 I=1,NCU
164      READ(5,*) JOU(I)
WRITE(6,*) 'Beginning B Matrix Calculations ... '
CALL JACOB(FDU, F, X, XD, U, IOX, JOU, BMAT, NRX, NCU)
DO 704 I=1, NRX
704     WRITE(3,799) (BMAT(NRX*(J-1)+I), J = 1, NCU)
WRITE(3,*) 'C1 = [1 0 0 0 0 0 0];'
WRITE(3,*) 'C2 = [0 1 0 0 0 0 0];'
WRITE(3,*) 'C3 = [0 0 1 0 0 0 0];'
WRITE(3,*) 'C4 = [0 0 0 1 0 0 0];'
WRITE(3,*) 'C5 = [0 0 0 0 1 0 0];'
WRITE(3,*) 'C6 = [0 0 0 0 0 1 0];'
WRITE(3,*) 'C7 = [0 0 0 0 0 0 1];'
WRITE(3,*) 'D = [0];'
CLOSE(3)
886  WRITE(6,887)
READ(5,99, ERR=886) ANSR
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
     &       (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) DO 4750 1=1,HEX
4750   WRITE(6,*) 'Beginning A Matrix Calculations ...
CALL JACOB(FDX, F, X, XD, X, IOX, JOX, AMAT, NRX, NRX)
DO 703 I=1, NRX
703     WRITE(3,799) (AMAT(NRX*(J-1)+I) , J = 1, NRX )
WRITE(3,*) 'C1 = [1 0 0 0 0 0 0];'
WRITE(3,*) 'C2 = [0 1 0 0 0 0 0];'
WRITE(3,*) 'C3 = [0 0 1 0 0 0 0];'
WRITE(3,*) 'C4 = [0 0 0 1 0 0 0];'
WRITE(3,*) 'C5 = [0 0 0 0 1 0 0];'
WRITE(3,*) 'C6 = [0 0 0 0 0 1 0];'
WRITE(3,*) 'C7 = [0 0 0 0 0 0 1];'
WRITE(3,*) 'D = [0];'
CLOSE(3)
886  WRITE(6,887)
READ(5,99, ERR=886) ANSR
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
     &       (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) DO 4750 1=1,HEX
4750   WRITE(6,*) 'Beginning A Matrix Calculations ...
CALL JACOB(FDX, F, X, XD, X, IOX, JOX, AMAT, NRX, NRX)
DO 703 I=1, NRX
703     WRITE(3,799) (AMAT(NRX*(J-1)+I) , J = 1, NRX )
WRITE(3,*) 'C1 = [1 0 0 0 0 0 0];'
WRITE(3,*) 'C2 = [0 1 0 0 0 0 0];'
WRITE(3,*) 'C3 = [0 0 1 0 0 0 0];'
WRITE(3,*) 'C4 = [0 0 0 1 0 0 0];'
WRITE(3,*) 'C5 = [0 0 0 0 1 0 0];'
WRITE(3,*) 'C6 = [0 0 0 0 0 1 0];'
WRITE(3,*) 'C7 = [0 0 0 0 0 0 1];'
WRITE(3,*) 'D = [0];'
CLOSE(3)
886  WRITE(6,887)
READ(5,99, ERR=886) ANSR
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
     &       (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) DO 4750 1=1,HEX
4750   WRITE(6,*) 'Beginning A Matrix Calculations ...
CALL JACOB(FDX, F, X, XD, X, IOX, JOX, AMAT, NRX, NRX)
DO 703 I=1, NRX
703     WRITE(3,799) (AMAT(NRX*(J-1)+I) , J = 1, NRX )
WRITE(3,*) 'C1 = [1 0 0 0 0 0 0];'
WRITE(3,*) 'C2 = [0 1 0 0 0 0 0];'
WRITE(3,*) 'C3 = [0 0 1 0 0 0 0];'
WRITE(3,*) 'C4 = [0 0 0 1 0 0 0];'
WRITE(3,*) 'C5 = [0 0 0 0 1 0 0];'
WRITE(3,*) 'C6 = [0 0 0 0 0 1 0];'
WRITE(3,*) 'C7 = [0 0 0 0 0 0 1];'
WRITE(3,*) 'D = [0];'
CLOSE(3)
886  WRITE(6,887)
DOLIN = .FALSE.

CALL F_SetDefaultFileName('States & Controls')
OPEN(UNIT=2, FILE='States_&_Controls', STATUS='NEW', RECL=400)
WRITE(2,99) '*'
WRITE(2,100) TAB, TAB, TAB, TAB, TAB, TAB, TAB, TAB, TAB, TAB,
&   TAB, TAB, TAB, TAB, TAB, TAB, TAB

CALL F_SetDefaultFileName('State Rates')
OPEN(UNIT=12, FILE='State_Rates', STATUS='NEW', RECL=400)
WRITE(12,99) '*'
WRITE(12,193) TAB, TAB, TAB, TAB, TAB, TAB, TAB, TAB, TAB, TAB,
&   TAB, TAB, TAB

CALL F_SetDefaultFileName('State Rates')
OPEN(UNIT=22, FILE='OUTPUTS', STATUS='NEW', RECL=400)
WRITE(22,99) '*'
WRITE(22,*) 'T', TAB, 'Y(1)', TAB, 'Y(2)', TAB, 'Y(3)', TAB, 'Y(4)', TAB,
& 'Y(5)',  TAB, 'Y(6)', TAB, 'Y(7)', TAB, 'Y(8)', TAB, 'Y(9)',
& TAB, 'Y(10)', TAB, 'Y(11)', TAB, 'Y(12)', TAB, 'Y(13)', TAB,
& 'Y(14)', TAB, 'Y(15)', TAB, 'Y(16)', TAB, 'Y(17)', TAB, 'Y(18)',
&  TAB, 'Y(19)', TAB, 'Y(20)
GO TO 1221

DO I=1,NN
YREF(I) = X(I)
END DO
Q = YREF(4)
DO I = -20, 20
YREF(4) = Q + I*DTOR
CALL F(0.0, YREF, XD)
WRITE(12,191) YREF(4), TAB, Y(1), TAB, Y(2), TAB, Y(3), TAB,
&               XD(4), TAB, XD(5), TAB, XD(6), TAB,
&               XD(7), TAB, XD(8), TAB, XD(9), TAB,
&               Y(4), TAB, Y(5), TAB, Y(6), TAB,
&               Y(7)
END DO

1221 CONTINUE

WRITE(6,101)
READ(5,*, ERR=501) DT

WRITE(6,102)
READ(5,*,ERR=502) NWRITES

WRITE(6,104)
READ(5,*,ERR=503) NITER

C BEGIN SIMULATION

501  WRITE(6,3303)
READ(5,99, ERR=501) ANSR
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
&    (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) THEN
WRITE(6,112)
READ(5,*, ERR=501) IDELTA
WRITE(6,3304) X(IDELTA)
READ(5,*, ERR=501) DELTA
X(IDELTA) = X(IDELTA)+DELTA
WRITE(6,3306)
READ(5,99, ERR=501) ANSR
ENDIF
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
&    (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) GOTO 501

502  WRITE(6,3305)
READ(5,99, ERR=502) ANSR
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
&    (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) THEN
WRITE(6,1112)
READ(5,*, ERR=502) IDELTA
WRITE(6,3304) U(IDELTA)
READ(5,*, ERR=502) DELTA
U(IDELTA) = U(IDELTA)+DELTA
WRITE(6,3306)
READ(5,99, ERR=502) ANSR
ENDIF
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
&    (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) GOTO 502

1599  WRITE(6,3305)
READ(5,99, ERR=1599) ANSR
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
&    (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) THEN
WRITE(6,1112)
READ(5,*, ERR=1599) IDELTA
WRITE(6,3304) U(IDELTA)
READ(5,*, ERR=1599) DELTA
U(IDELTA) = U(IDELTA)+DELTA
WRITE(6,3306)
READ(5,99, ERR=1599) ANSR
ENDIF
IF ((ANSR.EQ.'T').OR.(ANSR.EQ.'t').OR.
&    (ANSR.EQ.'Y').OR.(ANSR.EQ.'y')) GOTO 1599

C IC based on real part of eigenvectors

VT= X(1)
ALPHA= X(2)*RTOD
BETA= X(3)*RTOD
PHI= X(4)*RTOD
THETA= X(5)*RTOD
PSI= X(6)*RTOD
P= X(7)*RTOD
Q= X(8)*RTOD
R= X(9)*RTOD
POW= X(13)

WRITE(2,192) T,      TAB, VT,   TAB, ALPHA, TAB, BETA, TAB,
&              PHI,    TAB, THETA, TAB, PSI,  TAB, P,      TAB, Q,    TAB, R,    TAB,
&              X(10),  TAB, X(11), TAB, X(12), TAB, POW,  TAB,
&              THTL,   TAB, EL,   TAB, AIL,   TAB, RDR

WRITE(12,191) T, TAB, XD(1), TAB, XD(2), TAB, XD(3), TAB,
&               XD(4), TAB, XD(5), TAB, XD(6), TAB,
&               XD(7), TAB, XD(8), TAB, XD(9), TAB,
&               XD(10), TAB, XD(11), TAB, XD(12), TAB,
&               XD(13)

WRITE(22,*) T, TAB, Y(1), TAB, Y(2), TAB, Y(3), TAB, Y(4), TAB,
& Y(5), TAB, Y(6), TAB, Y(7), TAB, Y(8), TAB, Y(9), TAB, Y(10),
& TAB, Y(11), TAB, Y(12), TAB, Y(13), TAB, Y(14), TAB, Y(15),
& TAB, Y(16), TAB, Y(17), TAB, Y(18), TAB, Y(19), TAB, Y(20)

DO 2 IWRITE=1,NWRITES

DO 1 ITER=0,NITER

CALL RK4(F,T,DT,X,XD,NN)
1     CONTINUE

VT= X(1)
ALPHA= X(2)*RTOD
BETA= X(3)*RTOD
PHI= X(4)*RTOD
THETA= X(5)*RTOD
PSI= X(6)*RTOD
P= X(7)*RTOD
Q= X(8)*RTOD
R= X(9)*RTOD
POW= X(13)

WRITE(12,191) T, TAB, VT, TAB, ALPHA, TAB, BETA, TAB,
&               PHI,    TAB, THETA, TAB, PSI,  TAB, P,      TAB, Q,    TAB, R,    TAB,
&               X(10),  TAB, X(11), TAB, X(12), TAB, FON,  TAB,
&               X(13),  TAB, X(14), TAB, X(15), TAB, X(16),
&               X(17),  TAB, X(18), TAB, X(19), TAB, X(20)

WRITE(12,191) T, TAB, VT, TAB, ALPHA, TAB, BETA, TAB,
&               PHI,    TAB, THETA, TAB, PSI,  TAB, P,      TAB, Q,    TAB, R,    TAB,
&               X(10),  TAB, X(11), TAB, X(12), TAB, FON,  TAB,
&               X(13),  TAB, X(14), TAB, X(15), TAB, X(16),
&               X(17),  TAB, X(18), TAB, X(19), TAB, X(20)

CALL RK4(F,T,DT,X,XD,NN)
1     CONTINUE

VT= X(1)
ALPHA= X(2)*RTOD
BETA= X(3)*RTOD
PHI= X(4)*RTOD
THETA= X(5)*RTOD
PSI= X(6)*RTOD
P= X(7)*RTOD
Q= X(8)*RTOD
R= X(9)*RTOD
POW= X(13)
DO WHILE (PHI.GT.180.)
  PHI = PHI - 360.
END DO
DO WHILE (PHI.LT.-180.)
  PHI = PHI + 360.
END DO
THETA= X(5)*RTOD
PSI=   X(6)*RTOD
DO WHILE (PSI.GT.180.)
  PSI = PSI - 360.
END DO
DO WHILE (PSI.LT.-180.)
  PSI = PSI + 360.
END DO
P=     X(7)*RTOD
Q=     X(8)*RTOD
R=     X(9)*RTOD
POW=   X(13)

WRITE(2,192) T,      TAB, VT,   TAB, ALPHA,TAB, BETA,TAB,
&              PHI,    TAB, THETA,TAB, PSI,  TAB,
&              P,      TAB, Q,    TAB, R,    TAB,
&              X(10),  TAB, X(11),TAB, X(12),TAB, POW,  TAB,
&              THTL,   TAB, EL,   TAB,AIL,   TAB, RDR

WRITE(12,191) T, TAB, XD(1), TAB, XD(2), TAB, XD(3), TAB,
&               XD(4), TAB, XD(5), TAB, XD(6), TAB,
&               XD(7), TAB, XD(8), TAB, XD(9), TAB,
&               XD(10), TAB, XD(11), TAB, XD(12), TAB,
&               XD(13)

WRITE(22,*) T, TAB, Y(1), TAB, Y(2), TAB, Y(3), TAB, Y(4), TAB,
& Y(5), TAB, Y(6), TAB, Y(7), TAB, Y(8), TAB, Y(9), TAB, Y(10),
& TAB, Y(11), TAB, Y(12), TAB, Y(13), TAB, Y(14), TAB, Y(15),
& TAB, Y(16), TAB, Y(17), TAB, Y(18), TAB, Y(19), TAB, Y(20)

2 CONTINUE
CLOSE(2)
CLOSE(12)
C
C END IF DOSIM THEN ...}

C
C **************************************************************************
Subroutines:

C MODIFIED BY Samantha Magill OCT 23, 2001 for Tipdocking Simulation
DOUBLE PRECISION T,DMIN,DMIN(5)
EXTERNAL F,ADC,SO,EC,ED,THRUST,ADC,SO,EC,ED,THROTTLE
COMMON/PARAM/XCG
COMMON/CONTROLS/THTL,EL,AIL,RDR
COMMON/OUTPUT/YOUT

REAL X(*), XD(*), D(9)
LOGICAL DBUG

REAL S, B, CBAR, RM, XCGR, JXX, JYY, JZZ,
& C1, C2, C3, C4, C5, C6, C7, C8, C9,
& RTOD, G,
& XCG, THTL, EL, AIL, RDR,
& AMACH, Q, ALPHA,
& S8, S7, S6, S5, S4, S3, S2, S1,
& T3, T2, T1,
& CXMOM, XMOM, XFOR,
& CZMOM, ZMOM, ZFOR,
& DUM, WDOT, VDOT,
& UDOT, AZ, AY, QSINPHI, GCOSTHETA, RMQS,
& QSB, QS, W, V, U,
& COSPSI, SINPSI, COSPHI, SINPHI, COSTHETA,
& SINTHETA, COSBETA, CQ, B2V,
& TVT, CNT, CMT, CPOW, CW, QBAR,
& ALT, P, PSI, R, Q, B, T, VT, THETA, PHI, BETA,
& CN, CM, CL, CZ, CY, CX, THRUST, TGEAR, VT, TIME,
& DAREA, DLRA, DLRA, DURLA,
& ALT, P, PSI, R, Q, B, T, VT, THETA, PHI, BETA,
& CN, CM, CL, CZ, CY, CX, THRUST, TGEAR, VT, TIME,
& DAREA, DLRA, DLRA, DURLA,

DATA S, B, CBAR, RM, XCGR, JXX, JYY, JZZ/
&  300,30, 11.32, 1.57e-3, 0.35, 152875.0, 55814.0, 206479.0/
DATA C1, C2, C3, C4, C5, C6, C7, C8, C9/
& -.9855, 0.0, 6.5475e-06,
&  0.0, 0.96039, 0.0, 1.792e-05, 0.46978, 4.8467e-06/
DATA RTOD,G / 57.29578, 32.17/

COMMON/PARAM/XCG
COMMON/CONTROLS/THTL,EL,AIL,RDR
COMMON/OUTPUT/YOUT

XCG = 0.25

C Assign state & control variables

VT    = X(1)
PHI   = X(4)
THETA = X(5)
P     = X(7)
Q     = X(8)
R     = X(9)

SINTHETA = SIN(THETA)
COSTHETA = COS(THETA)
SINPHI   = SIN(PHI)
COSPHI   = COS(PHI)
SINPSI   = SIN(PSI)
COSPSI   = COS(PSI)

U     = X(1)*COSTHETA*COSPSI
V     = X(1)*SINPHI*SINTHETA*COSPSI
W     = X(1)*COSPHI*SINTHETA*COSPSI

AZ    = 0.5*(Q*SINPHI + R*COSPHI)
AY    = 0.5*(Q*COSPHI - R*SINPHI)
AZ    = 0.5*(Q*SINPHI + R*COSPHI)/COSTHETA

ADD Damping derivatives : 

TVT= 0.5/VT
B2V= B*TVT
CQ = CBAR*Q*TVT

CALL DAMP(ALPHA,D)
CXT=  CXT + CQ  *   D(1)
CYT=  CYT + B2V * ( D(2)*R + D(3)*P )
CZT=  CZT + CQ  *   D(4)
CLT=  CLT + B2V * ( D(5)*R + D(6)*P )
CMT=  CMT + CQ  *   D(7)               + CZT * (XCGR-XCG)
CNT=  CNT + B2V * ( D(8)*R + D(9)*P )  - CYT * (XCGR-XCG) * CBAR/B

C Get ready for state equations

QS    = QBAR * S
QSB   = QS * B
RMQS  = RM * QS
GCOSTHETA  = G * COSTHETA
QSINPHI  = Q * SINPHI

AY    = RMQS * CYT
AZ    = RMQS * CZT

C Kinematics

XD(4) =  P + (SINTHETA/COSTHETA)*(Q*SINPHI + R*COSPHI)
XD(5) =  Q*COSPHI - R*SINPHI
XD(6) = (Q*SINPHI + R*COSPHI)/COSTHETA

C Forces

ZFOR  = CZT*QS + (G/RM)*COSTHETA*COSPHI
ZMOM  = ZFOR*B/2.0
CZMOM = ZMOM/QSB
XFOR  = CXT*QS - (G/RM)*SINTHETA + T
XMOM  =-XFOR*B/2.0
CXMOM = XMOM/QSB
CLT = CLT + CZMOM
CNT = CNT + CXMOM

XD(7) =  (QSB*CLT - (JZZ-JYY)*Q*R)/JXX
XD(8) =  (QS*CBAR*CMT - (JXX-JZZ)*P*R)/JYY
XD(9) =  (QSB*CNT - (JYY-JXX)*P*Q)/JZZ

C Navigation
C
T1 = SINPHI * COSPSI
T2 = COSPHI * SINTHETA
T3 = SINPHI * SINPSI
S1 = COSTHETA * COSPSI
S2 = COSTHETA * SINPSI
S3 = T1 * SINTHETA - COSPHI * SINPSI
S4 = T3 * SINTHETA + COSPHI * COSPSI
S5 = SINPHI * COSTHETA
S6 = T2 * COSPSI + T3
S7 = T2 * SINPSI - T1
S8 = COSPHI * COSTHETA
C
XD(10) = (B/2) * (P * (COSPHI * SINTHETA * COSPSI + SINPHI * SINPSI)
&                   - R * COSTHETA * COSPSI)  ! North Speed
XD(11) = (B/2) * (P * (COSPHI * SINTHETA * SINPSI - SINPHI * COSPSI)
&                   - R * COSTHETA * SINPSI)  ! East Speed
XD(12) = (B/2) * (P * COSPHI * COSTHETA + R * SINTHETA)  ! Vertical Speed
C
AN = -AZ/G
ALAT = AY/G
RETURN
END
C
**************************************************************************
C
SUBROUTINE ADC(VT, ALT, AMACH, QBAR)
REAL VT, ALT, AMACH, QBAR
REAL R0, TFAC, T, RHO, PS
DATA R0/2.377E-3/
TFAC = 1.0 - 0.703E-5 * ALT
T = 519.0 * TFAC
IF (ALT.GE.35000.0) T = 390.0
RHO = R0 * (TFAC**4.14)
AMACH = VT / SQRT(1.4 * 1716.3 * T)
QBAR = 0.5 * RHO * VT * VT
PS = 1715.0 * RHO * T
RETURN
END
C
**************************************************************************
C
FUNCTION TGEAR(THTL)
IF (THTL.LE.0.77) THEN
TGEAR = 64.94 * THTL
ELSE
TGEAR = 217.38 * THTL - 117.38
END IF
RETURN
END
C
**************************************************************************
C
FUNCTION PDOT(P3, P1)
IF (P1.GE.50.0) THEN
IF (P3.GE.50.0) THEN
T = 5.0
P2 = P1
ELSE
P2 = 60.0
T = RTAU(P2 - P3)
END IF
ELSE
IF (P3.GE.50.0) THEN
T = 5.0
P2 = 40.0
ELSE
P2 = P1
T = RTAU(P2 - P3)
END IF
END IF
PDOT = T * (P2 - P3)
RETURN
END
C
**************************************************************************
C
FUNCTION RTAU(DP)
IF (DP.LE.25.0) THEN
RTAU = 1.0
ELSE IF (DP.GE.50.0) THEN
RTAU = 0.1
ELSE
RTAU = 1.9 - 0.036 * DP
END IF
RETURN
END
C
**************************************************************************
C
FUNCTION THRUST(POW, ALT, RMA)  ! ENGINE THRUST MODEL
REAL A(0:5, 0:5), B(0:5, 0:5), C(0:5, 0:5)
DATA A/
&  1060.0,  670.0,  880.0, 1140.0, 1500.0, 1860.0,
&   635.0,   425.0,   690.0, 1010.0, 1330.0, 1700.0,
&    60.0,    25.0,   345.0,  755.0, 1130.0, 1525.0,
& -1020.0,  -710.0,  -300.0,  350.0,  910.0, 1360.0,
& -2700.0, -1900.0, -1300.0, -247.0,  600.0, 1100.0,
& -3600.0, -1400.0,  -595.0, -342.0, -200.0,  700.0/
C MIL DATA
DATA B/
& 12680.0,  9150.0,  6200.0,  3950.0,  2450.0,  1400.0,
& 12680.0,  9150.0,  6313.0,  4040.0,  2470.0,  1400.0,
& 12610.0,  9312.0,  6610.0,  4290.0,  2600.0,  1560.0,
& 12640.0,  9839.0,  7090.0,  4660.0,  2840.0,  1660.0,
& 12390.0, 10176.0,  7750.0,  5300.0,  3250.0,  1900.0,
& 11680.0,  9848.0,  8050.0,  6100.0,  3800.0,  2310.0/
C MAX DATA
DATA C/
& 20000.0, 15000.0, 10800.0,  7000.0,  4000.0,  2500.0,
& 21420.0, 15700.0, 11225.0,  7323.0,  4435.0,  2600.0,
& 22700.0, 16860.0, 12250.0,  8154.0,  5000.0,  2835.0,
& 24240.0, 18910.0, 13760.0,  9285.0,  5700.0,  3215.0,
& 26070.0, 21075.0, 15975.0, 11115.0,  6860.0,  3950.0,
& 28886.0, 23319.0, 18300.0, 13484.0,  8642.0,  5057.0/
H = 0.0001 * ALT
I = INT(H)
IF (I.GE.5) I = 4
DH = H - FLOAT(I)
RM = 5.0 * RMA
M = INT(RM)
IF (M.GE.5) M = 4
DM = RM - FLOAT(M)
CDH = 1.0 - DH
S = B(I, M) * CDH + B(I + 1, M) * DH
T = B(I, M + 1) * CDH + B(I + 1, M + 1) * DH
TMIL = S + (T - S) * DM
IF (POW.LT.50.0) THEN
S = A(I, M) * CDH + A(I + 1, M) * DH
T = A(I, M + 1) * CDH + A(I + 1, M + 1) * DH
TIDL = S + (T - S) * DM
THRUST = TIDL + (TMIL - TIDL) * POW * 0.02
ELSE
S = C(I, M) * CDH + C(I + 1, M) * DH
T = C(I, M + 1) * CDH + C(I + 1, M + 1) * DH
TMAX = S + (T - S) * DM
THRUST = TMIL + (TMAX - TMIL) * (POW - 50.0) * 0.02
END IF
SUBROUTINE DAMP(ALPHA, D)
REAL A(-2:9,9), D(9)
DATA A/
  -0.267, -0.110,  0.308,  1.34,   2.08,   2.91,   2.76,
  2.05,   1.50,   1.49,   1.83,   1.21,
  0.882,  0.852,  0.876,  0.958,  0.962,  0.974,  0.819,
  0.483,  0.590,  1.21,  -0.493, -1.04,
 -0.108, -0.108, -0.188,  0.110,  0.258,  0.226,  0.236,
  0.344,  0.362,  0.611,  0.529,  0.298, -2.27,
 -8.80, -25.8,  -28.9,  -31.4,  -31.2,  -30.7,  -27.7,
 -28.2,  -29.0,  -29.8,  -38.3,  -35.3,
 -0.126, -0.026,  0.063,  0.113,  0.208,  0.230,  0.319,
  0.437,  0.680,  0.100,  0.447, -0.330,
 -0.360, -0.359, -0.443, -0.420, -0.383, -0.375, -0.339,
  0.061,  0.052, -0.013, -0.024,  0.050,  0.150,  0.138/
S= 0.2*ALPHA
K= INT(S)
IF (K.LE.-2) K= -1
IF (K.GE.9)  K=  8
DA= S-FLOAT(K)
L= K+ INT( SIGN(1.1,DA) )
DO 1, I=1,9
  1    D(I)= A(K,I)+ABS(DA)*(A(L,I)-A(K,I))
RETURN
END

C **************************************************************************
C
FUNCTION CX(ALPHA,EL)
REAL A(-2:9,-2:2)
DATA A/
  -0.099, -0.081, -0.081, -0.063, -0.025, 0.044, 0.097,
  0.113,  0.145,  0.167,  0.174,  0.166,
 -0.048, -0.038, -0.040, -0.021,  0.016, 0.083, 0.127,
  0.137,  0.162,  0.177,  0.179,  0.167,
 -0.022, -0.020, -0.021, -0.004,  0.032, 0.094, 0.128,
  0.130,  0.154,  0.161,  0.155,  0.138,
 -0.026, -0.024, -0.020, -0.005,  0.006, 0.062, 0.087,
  0.084,  0.100,  0.104,  0.091,
 -0.003, -0.073, -0.021, -0.041,  0.052, 0.084,
  0.053,  0.043,  0.063,  0.047,  0.046/
S= 0.2*ALPHA
R= INT(S)
IF (R.LE.-2) R= -1
IF (R.GE. 9) R=  8
S= EL/12.0
M= INT(S)
IF (M.LE.-2) M= -1
IF (M.GE. 2) M=  1
DE= S-FLOAT(M)
N= M+ INT( SIGN(1.1,DE) )
T= A(K,M)
U= A(K,N)
V= T + ABS(DA) * (A(L,M) - T)
W= U + ABS(DA) * (A(L,N) - U)
CX= V + (W-V)  * ABS(DE)
RETURN
END
C **************************************************************************
C
FUNCTION CY(BETA,AIL,RDR)
CY= -0.02*BETA + 0.021*(AIL/20.0) + 0.086*(RDR/30.0)
RETURN
END
C **************************************************************************
C
FUNCTION CZ(ALPHA,BETA,EL)
REAL A(-2:9)
DATA A/
  0.770,  0.241, -0.100, -0.416, -0.731, -1.053,
 -1.366, -1.646, -1.917, -2.120, -2.248, -2.229/
IF (M .EQ. 0) M = 1
M = INT(S)
L = K + INT(SIGN(1.1, DA))
IF (K.GE. 9) K = 8
K = INT(S)
S = 0.2*ALPHA
DA = S - FLOAT(K)
IF (K.LE.-2) K = -1
K = INT(S)
S = 0.2*ALPHA
C
DUM = V + (W - V) * ABS(DB)
U = A(K, N)
T = A(K, M)
DB = S - FLOAT(M)
IF (M .GE. 6) M = 5
IF (M .EQ. 0) M = 1
M = INT(S)
S = 0.2*ABS(BETA)
DA = S - FLOAT(K)
IF (K.GE. 9) K = 8
IF (K.LE.-2) K = -1
K = INT(S)
S = 0.2*ALPHA
C
FUNCTION CN(ALPHA, BETA)
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FUNCTION CN(ALPHA, BETA)
C **************************************************************************
C 'SUBROUTINE TRIMMER (NV, COST)
PARAMETER (NN=20, MM=20)
EXTERNAL COST
CHARACTER*1 ANS
REAL S(6), DS(6)
COMMON/ STATE/  X(NN), XDOT(NN)
COMMON/ CONTROLS/ U(MM)
DATA RTOD /57.29577951/
C
S(1)= U(2)
S(2)= U(3)
S(3)= U(4)
S(4)= U(1)
S(5)= X(2)
S(6)= X(4)
DS(1) = 1.0
DS(2) = 1.0
DS(3) = 1.0
DS(4) = 0.2
DS(5) = 0.02
DS(6) = 0.02
NC= 1000
SIGMA = -1.0
10    F0 = COST(S)
CALL SMPLX(COST,NV,S,DS,SIGMA,NC,F0,FFIN)
FFIN = COST(S)
WRITE(*,'(/11X,A)')'Throttle         Elevator         Ailerons
 &      Rudder'
WRITE(*,'(9X,4(1PE14.6,3X),/)') U(1), U(2), U(3), U(4)
WRITE(*,'(/11X,A)')'Phi              Theta            Psi'
WRITE(*,'(9X,3(1PE14.6,3X),/)') RTOD*X(4), RTOD*X(5),
 &  RTOD*X(6)
WRITE(*,99)'Angle of attack', RTOD*X(2),'Sideslip angle',
 & RTOD*X(3)
WRITE(*,99) 'Power Commanded ', TGEAR(U(1))
WRITE(*,99)'Initial cost function ',F0,
 &  'Final cost function',FFIN
99   FORMAT(2(1X,A22,1PE14.6))
DO I = 1,6
DS(I) = DS(I)/2.0
END DO
40   WRITE(*,'(/X,A,$)') 'More Iterations ? (Y/N) : ' 
READ(*,'(A)',ERR= 40) ANS
C **************************************************************************
C 'SUBROUTINE TRIMMIC (NV, COST)
PARAMETER (HH=50, HH=50)
EXTERNAL COST
CHARACTER*1 ANS
REAL S(6), DS(6)
COMMON/ STATE/  X(NN), XDOT(NN)
COMMON/ CONTROLS/ U(MM)
DATA RTOD /57.29577951/
C
S(1)= U(2)
S(2)= U(3)
S(3)= U(4)
S(4)= U(1)
S(5)= X(2)
S(6)= X(4)
DS(1) = 1.0
DS(2) = 1.0
DS(3) = 1.0
DS(4) = 0.2
DS(5) = 0.02
DS(6) = 0.02
NC= 1000
SIGMA = -1.0
10    F0 = COST(S)
CALL SMPLX(COST,NV,S,DS,SIGMA,NC,F0,FFIN)
FFIN = COST(S)
WRITE(*,'(/11X,A)')'Throttle         Elevator         Ailerons
 &      Rudder'
WRITE(*,'(9X,4(1PE14.6,3X),/)') U(1), U(2), U(3), U(4)
WRITE(*,'(/11X,A)')'Phi              Theta            Psi'
WRITE(*,'(9X,3(1PE14.6,3X),/)') RTOD*X(4), RTOD*X(5),
 &  RTOD*X(6)
WRITE(*,99)'Angle of attack', RTOD*X(2),'Sideslip angle',
 & RTOD*X(3)
WRITE(*,99) 'Power Commanded ', TGEAR(U(1))
WRITE(*,99)'Initial cost function ',F0,
 &  'Final cost function',FFIN
99   FORMAT(2(1X,A22,1PE14.6))
DO I = 1,6
DS(I) = DS(I)/2.0
END DO
40   WRITE(*,'(/X,A,$)') 'More Iterations ? (Y/N) : ' 
READ(*,'(A)',ERR= 40) ANS
IF (ANS .EQ. 'Y' .OR. ANS .EQ. 'y' .OR. ANS .EQ. '/') GO TO 10
IF (ANS .EQ. 'N' .OR. ANS .EQ. 'n') RETURN
GO TO 40
END

C**************************************************************************
C
SUBROUTINE CONSTR(X) ! used by COST, to apply constraints
DIMENSION X(*)
DATA DTOR/1.745329251994330e-02/

X(7) = 0.0 ! P
X(8) = 0.0 ! Q
X(9) = 0.0 ! R

BETA = BETA*DTOR
SINPHI = SIN(X(4))
COSPHI = COS(X(4))
SINTHETA = SIN(X(5))

V = X(1)**2*(SINPHI*SINTHETA*COSPHI**2-COSPHI*SINPSI)

A = -COSPHI
B = SINPHI*SINTHETA
C = SIN(X(3))
EPS = ATAN2(A,B)
ANG = ACOS(C/SQRT(A*A+B*B))
X(6) = ANG + EPS ! Sideslip is BETA

RETURN
END

C**************************************************************************
C
FUNCTION SF16(S)
PARAMETER (NN=20)
REAL S(*)
COMMON/STATE/ X(NN), XDOT(NN)
COMMON/CONTROLS/THTL, EL, AIL, RDR

EL =  S(1)
AIL =  S(2)
RDR =  S(3)
CALL CONSTR(X)
CALL F(TIME,X,XDOT)
SF16 =  XDOT(7)**2 + XDOT(8)**2 + XDOT(9)**2

RETURN
END

C**************************************************************************
C
SUBROUTINE SMPLX(FX,N,X,DX,SD,M,Y0,YL)

REAL X(*), DX(*)
DIMENSION XX(32), XC(32), Y(33), V(32,32)

NV=N+1
DO 2 I=1,N
DO 1 J=1,NV
1 V(I,J)=X(I)
2 V(I,I+1)=X(I)+DX(I)
Y0=FX(X)
Y(1)=Y0
DO 3 J=2, NV
3 Y(J)=FX(V(1,J))
K=NV
4 YH=Y(1)
YL=Y(1)
NH=1
NL=1
DO 5 J=2,NV
IF (Y(J).GT.YH) THEN
  YH=Y(J)
  NH=J
ELSEIF (Y(J).LT.YL) THEN
  YL=Y(J)
  NL=J
ENDIF
5 CONTINUE
YB=Y(1)
DO 6 J=2,NV
6 YB=YB+Y(J)
YB=YB/NV
D=0.0
DO 7 J=1,NV
7 D=D+(Y(J)-YB)**2
SDA=SQRT(D/NV)
IF ((K.GE.M).OR.(SDA.LE.SD)) THEN
  SD=SDA
  M=K
  YL=Y(NL)
  DO 8 I=1,N
  8 V(I,NL)=X(I)
  RETURN
ENDIF
DO 10 I=1,N
10 XC(I)=0.0
DO 9 J=1,NV
9 IF (J.NE.NH) XC(I)=XC(I)+V(I,J)
10 XC(I)=XC(I)/N
DO 11 I=1,N
11 X(I)=XC(I)+XC(I)-V(I,NH)
K=K+1
YR=FX(X)
IF (YR.LT.YL) THEN
  DO 12 I=1,N
  12 XX(I)=X(I)+X(I)-XC(I)
  K=K+1
  YE=FX(XX)
  IF (YE.LT.YR) THEN
    Y(NH)=YE
    DO 13 I=1,N
    13 V(I,NH)=XX(I)
  ELSE
    Y(NH)=YR
    DO 14 I=1,N
    14 V(I,NH)=X(I)
  END IF
  GOTO 4
ENDIF
Y2=Y(NL)
DO 15 J=1,NV
15 IF ((J.NE.NL).AND.(J.NE.NH).AND.(Y(J).GT.Y2)) Y2=Y(J)
IF (YR.LT.YH) THEN
  Y(NH)=YR
  DO 16 I=1,N
  16 V(I,NH)=X(I)
  IF (YR.LT.Y2) GO TO 4
ENDIF
DO 17 I=1,N
17 XX(I)=0.5*(V(I,NH)+XC(I))
K=K+1
YC=FX(XX)
IF (YC.LT.YH) THEN
  Y(NH)=YC
  DO 18 I=1,N
  18 V(I,NH)=XX(I)
ENDIF
RETURN
END

C**************************************************************************
C
SUBROUTINE SMPLX(FX,N,X,DX,SD,M,Y0,YL)

REAL X(*), DX(*)
DIMENSION XX(32), XC(32), Y(33), V(32,32)

NV=N+1
DO 2 I=1,N
DO 1 J=1,NV
1 V(I,J)=X(I)
2 V(I,I+1)=X(I)+DX(I)
Y0=FX(X)
Y(1)=Y0
DO 3 J=2, NV
3 Y(J)=FX(V(1,J))
K=NV
4 YH=Y(1)
YL=Y(1)
NH=1
NL=1
DO 5 J=2,NV
IF (Y(J).GT.YH) THEN
  YH=Y(J)
  NH=J
ELSEIF (Y(J).LT.YL) THEN
  YL=Y(J)
  NL=J
ENDIF
5 CONTINUE
YB=Y(1)
DO 6 J=2,NV
6 YB=YB+Y(J)
YB=YB/NV
D=0.0
DO 7 J=1,NV
7 D=D+(Y(J)-YB)**2
SDA=SQRT(D/NV)
IF ((K.GE.M).OR.(SDA.LE.SD)) THEN
  SD=SDA
  M=K
  YL=Y(NL)
  DO 8 I=1,N
  8 V(I,NL)=X(I)
  RETURN
ENDIF
DO 10 I=1,N
10 XC(I)=0.0
DO 9 J=1,NV
9 IF (J.NE.NH) XC(I)=XC(I)+V(I,J)
10 XC(I)=XC(I)/N
DO 11 I=1,N
11 X(I)=XC(I)+XC(I)-V(I,NH)
K=K+1
YR=FX(X)
IF (YR.LT.YL) THEN
  DO 12 I=1,N
  12 XX(I)=X(I)+X(I)-XC(I)
  K=K+1
  YE=FX(XX)
  IF (YE.LT.YR) THEN
    Y(NH)=YE
    DO 13 I=1,N
    13 V(I,NH)=XX(I)
  ELSE
    Y(NH)=YR
    DO 14 I=1,N
    14 V(I,NH)=X(I)
  END IF
  GOTO 4
ENDIF
Y2=Y(NL)
DO 15 J=1,NV
15 IF ((J.NE.NL).AND.(J.NE.NH).AND.(Y(J).GT.Y2)) Y2=Y(J)
IF (YR.LT.YH) THEN
  Y(NH)=YR
  DO 16 I=1,N
  16 V(I,NH)=X(I)
  IF (YR.LT.Y2) GO TO 4
ENDIF
DO 17 I=1,N
17 XX(I)=0.5*(V(I,NH)+XC(I))
K=K+1
YC=FX(XX)
IF (YC.LT.YH) THEN
  Y(NH)=YC
  DO 18 I=1,N
  18 V(I,NH)=XX(I)
ENDIF
RETURN
END
C
C **************************************************************************
C
SUBROUTINE JACOB (FN,F,X,XD,V,IO,JO,ABC,NR,NC)
DIMENSION X(*),XD(*),V(*),IO(*),JO(*),ABC(*)
EXTERNAL FN, F
LOGICAL FLAG, DIAGS
CHARACTER*1 ANS
REAL*8 FN,TDV
DATA DEL,DMIN,TOLMIN,OKTOL /.01, .5, 3.3E-5, 8.1E-4/
C
DIAGS= .TRUE.
WRITE (*, '(1X,A,$)'), 'Diagnostics ? (Y/N, "/"= N) '
READ(*,'(A)') ANS
IF (ANS .EQ. '/' .OR. ANS .EQ. 'N' .OR. ANS .EQ. 'n')
& DIAGS=.FALSE.
IJ=1
DO 40 J=1,NC
DV=AMAX1( ABS( DEL*V(JO(J)) ), DMIN )
DO 40 I=1,NR
FLAG= .FALSE.
1
TOL= 0.1
OLTOL= TOL
TDV= DBLE( DV )
A2= 0.0
A1= 0.0
A0= 0.0
B1= 0.0
B0= 0.0
D1= 0.0
D0= 0.0
IF (DIAGS .OR. FLAG) WRITE(*,'(/1X,A8,I2,A1,I2,11X,A12,8X,A5)')
& 'Element ',I,',',j, 'Perturbation','Slope'
DO 20 K= 1,18
! iterations on TDV
A2= A1
A1= A0
B1= B0
D1= D0
A0= FN(F,XD,X,IO(I),JO(J),TDV)
B0= AMIN1( ABS(A0), ABS(A1) )
D0= ABS(A0-A1)
IF (DIAGS.OR.FLAG) WRITE(*,'(20X,1P2E17.6)') TDV, A0
IF (K.LE.2) GO TO 20
IF (A0.EQ.A1 .AND. A1.EQ.A2) THEN
ANSR= A1
GO TO 30
END IF
IF (A0.EQ.0.0) GO TO 25
10
IF (D0.LE.TOL*B0 .AND. D1.LE.TOL*B1) THEN
ANSR= A1
OLTOL= TOL
IF (DIAGS.OR.FLAG) WRITE(*,'(1X,A16,F8.7)') 'Met
&
tolerance = ', TOL
IF (TOL.LE.TOLMIN) THEN
GO TO 30
ELSE
TOL= 0.2*TOL
GO TO 10
END IF
END IF
20
TDV= 0.6D0*TDV
25
IF (OLTOL.LE.OKTOL) THEN

19
20

ELSE
DO 20 J=1, NV
DO 19 I=1,N
V(I,J)=0.5*(V(I,J)+V(I,NL))
IF (J.NE.NL) Y(J)=FX(V(1,J))
K=K+N
ENDIF
GO TO 4
END

C
C **************************************************************************
C
DOUBLE PRECISION FUNCTION FDX(F,XD,X,I,J,DDX)
REAL*4 XD(*), X(*), Y(20)
DOUBLE PRECISION T, DDX, XD1, XD2
EXTERNAL F
TIME= 0.0
T
= DBLE( X(J) )
X(J)= SNGL( T-DDX )
CALL F(TIME,X,XD)
XD1 = DBLE(XD(I))
X(J)= SNGL( T+DDX )
CALL F(TIME,X,XD)
XD2 = DBLE( XD(I) )
FDX = (XD2-XD1)/(DDX+DDX)
X(J)= SNGL( T )
RETURN
END
C
C **************************************************************************
C
DOUBLE PRECISION FUNCTION FDU(F,XD,X,I,J,DDU)
PARAMETER (NIN=10)
REAL*4 XD(*), X(*), Y(20)
COMMON/CONTROLS/U(NIN)
DOUBLE PRECISION T, DDU, XD1, XD2
EXTERNAL F
TIME= 0.0
T
= DBLE( U(J) )
U(J)= SNGL( T-DDU )
CALL F(TIME,X,XD)
XD1 = DBLE(XD(I))
U(J)= SNGL( T+DDU )
CALL F(TIME,X,XD)
XD2 = DBLE( XD(I) )
FDU = (XD2-XD1)/(DDU+DDU)
U(J)= SNGL( T )
RETURN
END
C
C **************************************************************************
C
DOUBLE PRECISION FUNCTION YDX(F,XD,X,I,J,DDX)
PARAMETER (NOP=20)
REAL*4 XD(*), X(*)
COMMON/OUTPUT/ Y(NOP)
DOUBLE PRECISION T, DDX, YD1, YD2
EXTERNAL F
TIME= 0.0
T
= DBLE( X(J) )
X(J)= SNGL( T-DDX )
CALL F(TIME,X,XD)
YD1 = DBLE(Y(I))
X(J)= SNGL( T+DDX )

40

30

21

GO TO 30
ELSE IF (.NOT.FLAG) THEN
WRITE(*,'(/1X,A)') 'Failed to Converge *****'
FLAG= .TRUE.
GO TO 1
ELSE
WRITE(*,'(1X,A,$)') 'Your best guess : '
READ(*,*,ERR=21) ANSR
FLAG= .FALSE.
GO TO 30
END IF
ABC(IJ)= ANSR
IF (DIAGS) WRITE (*,'(27X,A9,1PE13.6)'),'Answer = ', ANSR
IJ= IJ+1
RETURN
END


CALL F(TIME, X, XD)
YD2 = DBLE(Y(I))
YDX = (YD2 - YD1) / (DDX + DDX)
X(J) = SNGL(T)
RETURN
END

C**************************************************************************
C
DOUBLE PRECISION FUNCTION YDU(F, XD, X, I, J, DDU)
PARAMETER (NIN = 10, NOP = 20)
REAL*4 XD(*), X(*)
COMMON/CONTROLS/U(NIN)
COMMON/OUTPUT/Y(NOP)
DOUBLE PRECISION T, DDU, YD1, YD2
EXTERNAL F
TIME = 0.0
T = DBLE(U(J))
U(J) = SNGL(T - DDU)
CALL F(TIME, X, XD)
YD1 = DBLE(Y(I))
U(J) = SNGL(T + DDU)
CALL F(TIME, X, XD)
YD2 = DBLE(Y(I))
YDU = (YD2 - YD1) / (DDU + DDU)
U(J) = SNGL(T)
RETURN
END

C**************************************************************************
C
SUBROUTINE RK4(F, TT, DT, XX, XD, NX)
INTEGER NN
PARAMETER (NN = 30)
REAL XX(*), XD(*), X(NN), XA(NN), Y(20)
INTEGER M, NX
REAL TT, DT, Q, T
EXTERNAL F
CALL F(TT, XX, XD)
DO 1 M = 1, NX
XA(M) = XD(M) * DT
1 X(M) = XX(M) + 0.5 * XA(M)
T = TT + 0.5 * DT
CALL F(T, X, XD)
DO 2 M = 1, NX
Q = XD(M) * DT
X(M) = XX(M) + 0.5 * Q
XA(M) = XA(M) + Q + Q
CALL F(T, X, XD)
DO 3 M = 1, NX
Q = XD(M) * DT
X(M) = XX(M) + Q
XA(M) = XA(M) + Q + Q
TT = TT + DT
CALL F(TT, X, XD)
DO 4 M = 1, NX
4 XX(M) = XX(M) + (XA(M) + XD(M) * DT) / 6.0
RETURN
END

C**************************************************************************
C
SUBROUTINE ADC(VT, ALT, AMACH, QBAR)
DATA S, B, CBAR, RM, XCGR, HE/300, 30, 1.132, 160.0/
DATA C1, C2, C3, C4, C5, C6, C7, C8, C9/-0.767, 0.0, 1.053E-4, 0.0, 0.9604, 0.0, 1.792E-5, -0.7340, 1.585E-5/
DATA RTOD, G / 57.29578, 32.17/
XCG = 0.25
C Assign state & control variables
VT = X(1)
ALPHA = X(2) * RTOD
BETA = X(3) * RTOD
PHI = X(4)
THETA = X(5)
PSI = X(6)
P = X(7)
Q = X(8)
R = X(9)
ALT = X(12)
POM = X(13)
C Air Data Computer and engine model
CALL ADC(VT, ALT, AMACH, QBAR)
CPOW = TGEAR(THTL)
XD(13) = PDOT(POW, CPOW)
T = THRUST(POW, ALT, AMACH)
C Look-up tables and component buildup
CXY = CX(ALPHA, EL)
CXY = CY(BETA, AIL, RDR)
CXY = CZ(ALPHA, BETA, EL)
CXY = CLT(ALPHA, BETA) + DLDA(ALPHA, BETA) * DAIL
CXY = CMT(ALPHA, EL)
CXY = CNT(ALPHA, BETA) + DNDA(ALPHA, BETA) * DAIL
C Add damping derivatives:
C
TVT = 0.5 / VT
B2V = B * TVT
CXY = CBAR * Q * TVT
CALL DAMP(ALPHA, D)
CXY = CXY + CXY * D(1)
CXY = CXY + B2V * (D(2) * R + D(3) * P)
CXY = CXY + CXY * D(4)
CXY = CXY + B2V * (D(5) * R + D(6) * P)
CXY = CXY + CXY * D(7) + CZT * (XCGR - XCG)
CXY = CXY + B2V * (D(8) * R + D(9) * P) - CYT * (XCGR - XCG) * CBAR / B
C Get ready for state equations
C OSTA = ODS(K(1))
V = VT * COS(K(2)) * CSRA
V = VT * SIN(K(2)) * SBSA
S = SIN(THETA)
C = CTH
B = SPH
C = CPSB
Q = QBAR * S
R = COS(PSI) * B
Q = QBAR * C
CXR = C * CTR
CXR = Q * S
CXR = QBAR * C
CXR = QBAR * Q
CXR = ODS(K(1))
AY = RMQS * CYT
AZ = RMQS * CZT

C Force Equations
UDOT = R\*V - Q\*W - G\*STH + RM * (QS * CXT + T)
VDOT = P\*W - R\*U + GCTH * SPH + AY
WDOT = Q\*U - P\*V + GCTH * CPH + AZ

DUM = (U\*U + W\*W)

XD(1) = (U\*UDOT + V\*VDOT + W\*WDOT)/VT
XD(2) = (U\*WDOT - W\*UDOT) / DUM
XD(3) = (VT*VDOT - V*XD(1)) * CBTA / DUM

C Kinematics
XD(4) = P + (STH/CTH)*(QSPH + R*CPH)
XD(5) = Q*CPH - R*SPH
XD(6) = (QSPH + R*CPH)/CTH

C Moments
XD(7) = (C2*P + C1*R + C4*HE)*Q + QSB*(C3*CLT + C4*CNT)
XD(8) = (C5*P - C7*HE)*R + C6*(R*R-P*P) +QS*CBAR*C7*CMT
XD(9) = (C8*P - C2*R + C9*HE)*Q + QSB*(C4*CLT + C9*CNT)

C Navigation
T1= SPH * CPSI
T2= CPH * STH
T3= SPH * SPSI
S1= CTH * CPSI
S2= CTH * SPSI
S3= T1  * STH - CPH * SPSI
S4= T3  * STH + CPH * CPSI
S5= SPH * CTH
S6= T2*CPSI + T3
S7= T2  * SPSI - T1
S8= CPH * CTH

XD(10) = U * S1  + V * S3  + W * S6   ! North Speed
XD(11) = U * S2  + V * S4  + W * S7   ! East Speed
XD(12) = U * STH - V * S5  - W * S8   ! Vertical Speed

C OUTPUTS
AN = -AZ/G
ALAT = AY/G
RETURN
END

C **************************************************************************
FUNCTION SF6(S)
PARAMETER (NN=20)
REAL S(*)
COMMON/STATE/ X(NN), XDOT(NN)
COMMON/CONTROLS/THTL, EL, AIL, RDR
EL  =  S(1)
AIL =  S(2)
RDR =  S(3)
THTL=  S(4)
X(2)=  S(5)
X(4)=  S(6)
X(13)= TGEAR(THTL)
CALL   CONSTR6(X)
CALL   F6(TIME,X,XDOT)
SF6 =    XDOT(1)**2 + 100.0*( XDOT(2)**2 + XDOT(3)**2 )
&    + 10.0*( XDOT(7)**2 + XDOT(8)**2 + XDOT(9)**2 )
RETURN
END

C **************************************************************************
SUBROUTINE CONSTR6(X)   ! used by COST, to apply constraints
CONDNUM: XI(4) = XI(1) * XI(2) ! THETA = ALPHA
XI(6) = XI(2) ! PSI = BETA
XI(1) = 0.0 ! P
XI(2) = 0.0 ! Q
XI(3) = 0.0 ! R
RETURN
END
Vita

Samantha Anne Magill was born May 29, 1975 in Myrtle Beach, SC to Arthur and Marie Magill. In 1993 she graduated Myrtle Beach High School, and attended Auburn University in Auburn, AL for college, for which she graduated Cum Laude in June of 1997 with a Bachelor’s in Aerospace Engineering. From their she continued her education at Virginia Tech in Blacksburg, VA and received a Master’s of Science in Aerospace Engineering in July of 1999. After receiving her Ph.D., she plans to relocate to the Los Angeles, CA area to work with AeroVironment, Inc. beginning in August of 2002.

Her parents continue to reside in Myrtle Beach, SC, where her father is a retired airline pilot and her mother is an active community member. She has one sibling, a younger sister, who graduated with a Bachelor’s of Fine Arts from the University of Georgia in 1999 and now works as an artist with an interior design company. She still resides in the Athens, GA area.