High Frequency Modeling and Experimental Analysis for Implementation of Impedance-based Structural Health Monitoring

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A promising structural health monitoring (SHM) method for implementation on real world structures is impedance-based health monitoring. An in-service system is envisioned to include on board processing and perhaps wireless transfer of data. Ideally, a system could be produced as a slap-on or automatically installed addition to a structure. The research presented in this dissertation addresses issues that will help make such a system a reality. Although impedance-based SHM does not typically use an analytical model for basic damage identification, a model is necessary for more advanced features of SHM, such as damage prognosis, and to evaluate system parameters when installing on various structures. A model was developed based on circuit analysis of the previously proposed low-cost circuit for impedance-based SHM in combination with spectral elements. When a three-layer spectral element representing a piezoceramic bonded to a base beam is used, the model can predict the large peaks in the impedance response due to resonances of the bonded active sensor. Parallel and series connections of distributed sensor systems are investigated both experimentally and with the developed model. Additionally, the distribution of baseline damage metrics is determined to assess how the large quantities of data produced by a monitoring system can be handled statistically. A modification of the RMSD damage metric has also been proposed that is essentially the squared sum of the Z-statistic for each frequency point. Preferred excitation frequencies for macro-fiber composite (MFC) active sensors are statistically determined for a long composite boom under development for use in rigidizable inflatable space structures.
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Chapter 1

Introduction

1.1 Background and Motivation

The current information age encourages the desire for immediate gratification when a question is raised. People expect to have information at their fingertips. High value is placed on internet search engines that can locate desired answers to questions quickly. Personal data assistants and mobile smart phones allow users to query information wherever they may be. The U.S., along with much of the world, has become an information driven society. This thirst for information applies to engineering structures as well. Engineers responsible for the integrity of structures, and users of the structures would like to be able to determine the status of structures wherever the structure may be. Hence, there is currently a large interest in structural health monitoring (SHM) systems. One structural health monitoring technique that has shown promise in the past several years is impedance-based SHM, which is the focus of this dissertation.

Recent events have highlighted the need for SHM. The most compelling example is the NASA Columbia disaster. Shortly after launch on January 16, 2003, a large piece of insulating foam dislodged from the external fuel tank and struck a leading edge on a wing of the Columbia orbiter. The impact was not detected immediately. However, analysis of videos and pictures of the launch the next day discovered the piece of foam hitting the left wing. The images were not clear enough to determine exactly where the wing was hit. Further high resolution pictures of the wing were requested to assess the location and thus severity of damage. Certain areas of the orbiters thermal protection system, including the carbon-carbon leading edge, would be more affected by a hit than others due to factors such as the angle of impact and the thickness and material of the section.
Unfortunately, requests for these pictures were denied by management. Later analysis determined this was due to a variety of reasons including: lack of communication through the proper channels, bureaucratic issues such as reluctance to deviate from the planned timeline and the belief that the thermal protection tiles (different from the carbon-carbon leading edge) would withstand all impact scenarios from the size of foam and speed it was traveling calculated from the initial images. This required the engineers to prove that the orbiter was not safe to fly normally to justify the need for the pictures. This was impossible without additional information. Mathematical modeling was used to estimate the amount of possible damage to the orbiter, but was determined likely to be small and localized and would not cause structural problems during reentry. However, it was not possible to know the space craft would be safe for sure without knowing where the impact occurred. Subsequently, the Columbia orbiter attempted to return without any corrective action. On February 1, 2003, the orbiter broke apart and the crew perished when superheated gasses penetrated the wing at the foam impact site during reentry into the earth’s atmosphere. A detailed report was produced by NASA after the accident and is available at http://caib.nasa.gov/.

The Columbia accident highlights many of the motivations for SHM systems. Managers were reluctant to take any action that could have saved the Columbia because of cost and schedule considerations. They were relying on a time based inspection plan, which involved assessing structural condition only during launch with limited imaging resources and on the ground after a mission. Analysis after the accident showed that a structural health monitoring system could have changed the outcome of the mission. A SHM system could have provided the engineers with additional information that would have at least allowed them to indicate to the management team the need for additional images of the leading edge of the orbiter wing. An advanced system could have identified the location and extent of damage without additional images, though in the case of large damage, NASA would have probably visually inspected to confirm a SHM system’s findings. By automatically interrogating a structure, a system would have not delayed any
mission objectives until damage had been detected. NASA is now actively pursuing SHM systems for their remaining fleet and future vehicles.

Many other examples provide motivation for developing SHM technology. Farrar et al. (2005) presents a summary of safety and economic reasons why users of high-capital-expenditure structures should adopt a high level of SHM to be able to provide a damage prognosis solution. Such structures include commercial and military aircraft, helicopters and civil structures subjected to seismic loading. Balageas (2006) summarizes important SHM benefits such as reduced maintenance costs due to disassembly for inspections, minimization of human errors, and lower, as well as constant, maintenance costs. Monitoring aging bridges is suggested by Phares et al. (2006) as a prime use of SHM.

Since there are many potential applications for SHM and many possible methods of constructing a SHM system a significant amount of research has been conducted in the area. Currently, there are several recurring conferences on SHM including the Stanford Workshop on Structural Health Monitoring, European Workshop on Structural Health Monitoring, DAMAS, and SPIE’s International Symposium on NDE for Health Monitoring and Diagnostics to name a few. In addition, researchers in the area now produce a journal solely focused on SHM, the International Journal of Structural Health Monitoring.

As SHM becomes more developed, researchers continue to increase the capability of systems. The well accepted levels of SHM are:

1. Detect damage
2. Detect and locate damage
3. Detect, locate and quantify damage
4. Detect, locate, quantify and predict remaining life (prognosis) (Rytter, 1993)
5. Combine Level 4 with smart structures to form self-evaluating systems
6. Combine Level 4 with smart structures to form self-healing systems
7. Combine with active control to form simultaneous health monitoring and control. (Inman, 2001)
With increasing complexity more information is needed. Related to these levels, Worden et al. (2005) have suggested five axioms of SHM, briefly:

1. Damage detection requires a comparison between system states.
2. Detection and location (levels 1 and 2) can be accomplished in an unsupervised learning mode. Identifying type and quantity of damage can only be done in a supervised learning mode.
3. If feature extraction is not done carefully, the more sensitive a monitored feature is to damage, the more sensitive it will be to environmental conditions.
4. The more sensitive an algorithm, the less noise rejection capability it will have.
5. The size of damage that can be detected is inversely proportional to the frequency range excited.

The fifth has often been attributed to work presented by Nokes and Cloud (1993).

Much of the current research is concentrated on extending the level of SHM as well as moving the techniques from a laboratory environment to real world structures. This is the focus of this dissertation for impedance-based SHM.

1.2 Research Objectives

The aim of the research performed is to address issues of the impedance-based SHM method as it moves towards application on real structures. Specifically, the focus is on allowing for quick installation of a small self-contained monitoring device that uses the impedance method as its basis. This necessitates the development of models, both analytical and statistical, that can be incorporated into the device and used to quickly determine the parameters of the system.

The objectives are summarized as the following:

- *Development of a model based on circuit analysis of the previously proposed low-cost circuit for impedance-based SHM.* Models for the impedance method already exist as presented by Liang et al (1994a, 1994b), however, these may be difficult to implement. Additionally, Aglietti et al. (2002) have pointed out that the
impedance method of modeling may not always be the most accurate. Modeling becomes necessary when more information is needed for more complex functions of the SHM system, such as estimation of remaining life. It is desired that ultimately, the model could be small enough in size to be included on a chip for local processing of impedance data.

- **Use of the model at high frequencies that are typical of impedance-based SHM.** The spectral element method (SEM) is used in combination with the electric circuit analysis for modeling of piezoceramic (PZT) electrical impedances. Timoshenko beam elements are used to model the base structure and three-layer elements including adhesive and PZT layers are used to incorporate resonances of the PZT that occur at high frequencies. Throw-off elements are used to approximate very large structures.

- **Investigation of various configurations of possible systems.** For a distributed sensor system, the connection of the many transducers and processing of each may become overwhelming if they are all treated individually. Different types of connections are possible, primarily parallel and series with in phase and out of phase actuation, and are investigated. Experimental tests are performed to determine if a preference towards one type of connection over the other exists.

- **Investigation and generalization of preferred system parameters such as frequency ranges and sensing areas.** Several experimental and analytical investigations are used to test these parameters including testing on a long highly damped beam and composite boom for use in inflatable space structures. Regions including a PZT resonance are analyzed to assess their effect on sensing ability.

- **Statistical investigation of analysis procedures and system parameters.** The distribution of baseline damage metrics is determined to help assess how the large quantities of data produced by a monitoring system can be handled statistically.
1.3 Research Contributions

The research presented in this dissertation aims to extend the body of knowledge regarding impedance-based structural health monitoring to aid in the transition from a laboratory technique to eventually a developed system actively observing a real world structure. This involves many areas including high frequency structural dynamics, smart material behavior and statistics. While some issues have been addressed others have been raised. This document will also help to guide further efforts to bring impedance-based SHM into use.

The primary contributions of this work are, first, showing that high frequencies, including resonances of the PZT sensor/actuator, can be modeled using a three-layer spectral element. Second, resonances of piezo-sensors were statistically analyzed showing that in some cases frequency ranges with such a resonance provide better monitoring capabilities. Third, structures that would not typically exhibit resonant peaks in the impedance response were analyzed to show that resonances are not necessary for impedance-based SHM. Fourth, a statistical assessment of sensing area as related to frequency of excitation was performed. Finally evaluation of baseline distributions of damage metrics providing for improved damage detection capabilities.

1.4 Impedance Method

The impedance-based health monitoring method is made possible through the use of piezoelectric patches (most often lead zirconate titanate, PZT) bonded to the structure that act as both sensors and actuators on the system. When a piezoelectric is stressed it produces an electric charge. Conversely when an electric field is applied the piezoelectric produces a mechanical strain. The patch is driven by a sinusoidal voltage sweep. Since the patch is bonded to the structure, the structure is deformed along with it and produces a local dynamic response to the vibration. The area that one patch can excite depends on the structure configuration and material. The response of the system is transferred back
from the piezoelectric patch as an electrical response. The electrical response is then analyzed where, since the presence of damage causes the response of the system to change, damage is shown as a phase shift or magnitude change in the impedance. A more detailed explanation of the technique can be found in the references by Liang et al. (1994a, 1994b), Sun et al. (1995b) and Park et al. (2003), who were leading developers of the impedance method at Virginia Tech.

Impedance-based SHM has been tested successfully on many laboratory structures (Sun et al., 1995b, Park et al., 2000b, Giurgiutiu et al., 2002, Park et al., 2003), as well as a few *in situ* applications (Berman et al., 1999, Peairs et al., 2003). The impedance method has many advantages compared to global vibration based and other damage detection methods. The principal advantages of the impedance approach compared to other techniques are the following (Park et al., 2003):

- The technique is not based on any model, and thus can be easily applied to complex structures.
- The technique uses small non-intrusive actuators to monitor inaccessible locations.
- The sensor (PZT) exhibits excellent features under normal working conditions, has a large range of linearity, fast response, light weight, high conversion efficiency and long term stability.
- The technique, because of high frequency, is very sensitive to local minor changes.
- The measured data can be easily interpreted.
- The technique can be implemented for on-line health monitoring.
- Low excitation forces (usually less than 1 V) produce power requirements in the range of microwatts, making the method an ideal candidate to be run by a self powered system.

Typically, the impedance method uses a damage metric to summarize and quantify the comparison of the frequency response functions from the impedance measurements.
Common damage metrics include the root mean square deviation (RMSD) and 1minus the cross correlation coefficient.

Much of the current work in impedance based SHM is focused on preparing the method for inclusion on real world structures. This involves miniaturization of the data acquisition hardware, combination with wireless telemetry and addressing powering issues including eventually adding power harvesting techniques. Additionally, the robustness of the system must be validated. Robust systems will be able to withstand the extreme environments where SHM is needed and be able to determine not only the structure’s health, but the monitoring system’s health over long periods of time. This may even involve steps such as flight qualification.

Though the impedance-based method is not model based, a model of the piezoelectric device and structure interaction becomes necessary for the goal of damage prognosis with SHM systems. Further benefits of a model are optimization of frequency range and sensor location when designing the SHM system and allowing better insight into sensor self-diagnostics, transfer impedance and multiplexing issues.

Several techniques have already been used to develop models of impedance-based SHM and piezoelectric device/structure interaction in general. Models have been classified as either the static type or the impedance type. In the static type, such as the model developed by Crawley and de Luis (1987), the actuator force is independent of the actuation frequency, whereas in the impedance approach, the dynamic piezoelectric properties are included in the dynamic equilibrium. The impedance approach, first proposed by Liang et al. (1994a), includes the effects of actuator damping and inertia. For the static approach, the model is based on the force and displacement relation
\[
F = K_a(x-x_m)
\]
for a single degree-of-freedom system where \(x\) is the displacement, \(F\) is the force exerted by the actuator and \(K_a\) is the static stiffness of the piezoelectric device and
\[
F = -K_s(x)
\]
where \(K_s\) is the spring constant of the single degree-of-freedom structure. For the impedance approach, as seen in the representative diagram (Figure 1.1), the
structure’s relation is instead \( F = -Z_S(\dot{x}) \) where \( Z_S \) is the structure’s mechanical impedance given by

\[
Z_S = c + m \frac{\omega^2 - \omega_n^2}{\omega} i
\]  

(1.1)

for \( c, m, \omega, \) and \( \omega_n \), the damping coefficient, mass, excitation frequency and resonant frequency (given by \( \omega_n = \sqrt{K_s/m} \)) respectively. The model assumes harmonic steady state excitation so that

\[
\dot{x} = \omega i x.
\]  

(1.2)

The single degree-of-freedom model of a structure then becomes

\[
F = -[c \omega i - m(\omega^2 - \omega_n^2)]x. 
\]  

(1.3)

This impedance approach accounts for the dynamic characteristics of the PZT and structure interaction and is represented by the diagram in Figure 1.1.

---

**Figure 1.1. Liang impedance model diagram**

When the PZT dynamics are included in the system, the mechanical admittance (inverse of impedance), \( Y \), is derived for a beam as (Liang et al.)

\[
Y(\omega) = i\omega (\bar{e}_{33}^T \hat{\epsilon} + Z_s(\omega) d_{33}^2 \hat{y}_{ss} \varepsilon)
\]  

(1.4)
where $Z_a$ and $Z_s$ are the PZT material’s and the structure’s mechanical impedances, respectively, $Y_{xx}^E$ is the complex Young’s modulus of the PZT with zero electric field, $d_{3x}$ is the piezoelectric coupling constant in the arbitrary $x$ direction at zero stress, $\varepsilon_{33}^T$ is the dielectric constant at zero stress and $a$ is a geometric constant of the PZT. This equation indicates that the electrical impedance of the PZT bonded to the structure is directly related to the mechanical impedance of a host structure.

Several researchers have improved on the model by Liang et al. Zhou et al. (1996) expanded it to include two dimensional structures. Fairweather (1998) and Littlefield (2000) have described a method of determining the mechanical impedance of a structure from a finite element model. Giurgiutiu and Zagrai (2000) developed models for free, fully constrained and elastically constrained piezoelectric patches excited at very high frequencies (up to 1500 kHz). However, the impedance response of the sensors attached to a beam structure was modeled and experimentally verified only up to 30 kHz and later results on plates were only available up to 40 kHz (Zagrai, 2001). Cheng and Wang (2001) applied the impedance model to multiple PZT actuator driven systems in the bimorph configuration. Xu and Liu (2002) included the effect of bonding layers into the impedance model to more accurately detect damage in composite repair patches. Bhalla (2004) created the concept of effective mechanical impedance to more effectively account for the two dimensional interaction of the PZT patch with the host structure. Using this concept, the structural impedance using FEM software can be determined without including electric degrees of freedom.

Two models of particular note are those of Esteban (1996) and Ritdumrongkul et al. (2004). Both of these studies utilized spectral elements in their formulation. Esteban’s model incorporated Liang’s method and was used in an attempt to quantify the sensing area of the structure. Damage was not accounted for in this model, however. Ritdumrongkul et al. modeled bolted joints at a relatively low frequency range (for the impedance method) of 0-500 Hz. Their model of the PZT structure interaction was a modification of active constrained layer SEM model by Lee and Kim (2001) that includes
the effects of the bonding layer. A combined impedance method and SEM model based
wave propagation technique has been presented by Park et al. (2000a), although the
modeling of the impedance was not performed in that study.

1.5 Outline of Dissertation

This dissertation consists of a collection of work on impedance-based structural health
monitoring, presented or to be presented in conferences or journals with a strong interest
in health monitoring. Because of this, some information may be repeated in different
sections throughout the document. It is hoped that such repetitions may be helpful for a
reader, who may not be interested in the entire document, in that he or she will have more
background information in the sections used. The chapters of this dissertation are all
related in that they address issues regarding moving impedance-based SHM towards
implementation on real world structures.

Chapter 2 discusses modeling of impedance based SHM based on the spectral element
method (SEM) and circuit analysis of the piezoelectric sensor/actuator and data
acquisition system. SEM is a method that combines spectral analysis techniques with
conventional finite element techniques and is especially suited for high frequency
modeling. A derivation of a Timoshenko beam spectral element is described as an
example. The output of the SEM is combined with the circuit model of the data
acquisition technique to produce impedance signatures. Results are shown for an
aluminum beam. In Chapter 3, the model of impedance in Chapter 2 is demonstrated in
various multi sensor configurations and compared to experimental results. Multiple
sensor configurations were difficult to handle with previous impedance models. The
differences in actuation and sensing voltages of several possible electric circuits are
analyzed. Chapter 4 shows the ability of a three layer spectral element presented by Lee
and Kim to model resonances of the PZT sensor, which occur in the high frequency
ranges typical of impedance-based SHM. Chapter 5 presents an analytical and
experimental investigation of the sensing area on a fiberglass beam. A method of
combining the results from the experiment with the model results is shown. Chapter 6
describes statistical modeling of damage metrics from impedance measurements and compares the ability of common damage metrics to ignore changes in the impedance signatures that would cause false damage identification. Chapter 7 presents a detailed statistical analysis on frequency range determination. Specifically, whether to use frequency ranges containing resonances of the piezoelectric sensors is addressed. Chapter 8 summarizes the findings of the dissertation and concludes with recommendations for areas of further research.
Chapter 2

Circuit and Spectral Element Impedance Model

As mentioned in the introduction, in general, when using the impedance-based method, a model is not needed for the basic problem of recognizing that a structure has been damaged. The method relies on a set of baseline impedance data to which further sets of data are compared. When the structure is damaged, changes in the data can be seen when compared to the baseline, thus indicating damage. However, modeling becomes necessary when more information is needed for more complex functions of the SHM system, such as estimation of remaining life. It is desired that ultimately, the model could be small enough in size to be included on a chip for local processing of impedance data. In addition, suitable models would aid in identifying and locating damage more accurately as well as designing the SHM system itself.

2.1 Impedance Modeling Background

Models for the impedance method such as those developed by Liang et al. and the derivatives discussed in the first chapter already exist. However, these may be difficult to implement. Additionally, Aglietti et al. (2002) have pointed out that the impedance method of modeling may not always be the most accurate. In comparisons of several modeling techniques of a piezoelectric patch bonded to a simply supported panel, the Zhou impedance model predicted resonant frequencies furthest from the experimental results. This was only investigated at low frequencies, but indicates that the impedance modeling method is not the exclusively effective way to model PZT/structure interactions. A new model developed here is based on the low-cost impedance method previously reported in Peairs et al. (2004).
Since impedance-based SHM relies on high frequency excitation of the structure with the complicated electro-mechanical coupling property of piezoelectric patches, finite element modeling may not be easily applicable or computationally efficient. Therefore, in this study, the spectral element method (SEM) is used in combination with electric circuit analysis for accurate modeling of piezoceramic (PZT) electrical impedances. SEM more accurately models higher frequency vibrations since the mass is modeled “exactly” and incorporates higher order models more easily (Doyle, 1997). SEM is based on a dynamic stiffness matrix that is evaluated at each frequency of interest. The method allows elements to span the entire distance between discontinuities such as a change in geometry or material properties, whereby only a few elements are needed to model an entire structure. Frequency dependent forces are discretized using the fast Fourier transform (FFT) to be input into the model.

This chapter describes the incorporation of the spectral element method with piezoelectric patch models and electric circuit analysis. Experimental results are compared to the model at various frequency ranges and for the formulation of the transfer impedance between multiple sensors. The aim of this study is to develop a simplified model of the impedance measured for structural health monitoring applications using the spectral element method to enable high frequency analysis, in combination with electric circuit analysis.

### 2.2 Circuit Analysis for Impedance-base SHM

The circuit analysis is based on the low-cost impedance device previously presented (Peairs, 2004). In this study, the original low-cost circuit is modified by adding a capacitor in series with the sensing resistor to reduce the impedance mismatch at low frequency ranges (Figure 2.1).
If the sensing voltage is considered as another voltage source, the circuit in Figure 2.1 can be divided into two parts, one with $V_i$ (input) and the other with $V_p$. In Figure 2.2, $Z_{cp}$ is the electrical impedance of the piezoelectric sensor and $Z_{eq}$ is the electrical impedance of the capacitor and sensing resistor. $V_p$ is the sensing voltage of the piezoelectric sensor caused by the actuating voltage, $V_i$ and $V_o$ is the measured output voltage. $V_p$ can be easily obtained from a structural model following the self-sensing actuation concept (Dosch et al., 1992) and the superposition principle for linear circuits.

Analyzing the left hand side of Figure 2.2,

$$Z_{cp} = \frac{V}{I} \quad (2.1)$$
where $V$ is the voltage across the piezoelectric device and $I$ is the current through it. Furthermore,

$$V = V_i - V_o$$  \hspace{1cm} (2.2)

and

$$I = \frac{V_o}{Z_{eq}}$$  \hspace{1cm} (2.3)

then

$$Z_{cp} = Z_{eq} \left( \frac{V_i}{V_o} - 1 \right).$$  \hspace{1cm} (2.4)

So, for the circuit in Figure 2.1,

$$Z_{cp} = \left( R_s + \frac{1}{j\omega C} \right) \left( \frac{V_i}{V_o} - 1 \right).$$  \hspace{1cm} (2.5)

If the circuit is used without a capacitor, the capacitance, $C$, is infinite. Experimentally, both $V_i$ and $V_o$ are simultaneously measured to obtain $Z_{cp}$.

Analytically, the output voltage $V_o$ can be solved for when the actuating circuit and sensing circuit in Figure 2.2 are added as follows,

$$V_o = Z_{eq} I$$  \hspace{1cm} (2.6)

where,
\[ I = \frac{V_i + V_p}{Z_{cp} + Z_{eq}}. \]  \hspace{1cm} (2.7)

If \( Z_{cp} \) is treated as a capacitor with impedance \( 1/(j\omega C_p) \), then

\[ V_o = \left(V_i + V_p\right) \frac{(1 + j\omega R_s C_p)C_p}{C + C_p + j\omega R_s C_p}. \]  \hspace{1cm} (2.8)

Note that this is different than having the added capacitor parallel to the sensing resistor. This is due to the fact that in the series configuration, the variation of \( C_p \) (such as against temperature changes) affects both the real and imaginary part of the impedance. If no capacitor is added, the sensitivity of the real part of impedance to variations in \( C_p \) is removed, as seen in the following equation

\[ \lim_{C \to \infty} V_o = \left(V_i + V_p\right) \frac{j\omega R_s C_p}{1 + j\omega R_s C_p}. \]  \hspace{1cm} (2.9)

If the solution for \( V_o \) is substituted into the equation for \( Z_{cp} \), the impedance can be written as a function of the applied voltage, \( V_i \), and sensing voltage, \( V_p \).

The addition of the sensing resistor (in comparison to a measurement with an impedance analyzer) causes the voltage drop across to the piezoelectric sensor to gain an imaginary component. If the frequency is low (below the cut off frequency, \( 1/R_s C_p \)), the effect of the imaginary part will be small. Analysis of the circuit on the left side of Figure 2.2 shows that if the PZT is treated as a capacitor, the voltage across the PZT will be

\[ V = \frac{j\omega C_p V_i}{jC_p + C(j\omega R_s C_p)}. \]  \hspace{1cm} (2.10)

Again, if no capacitor is added, the voltage applied across the PZT becomes
\[
\lim_{C \to \infty} V = \frac{jV_i}{j - \omega R_s C_p}
\]

If \( C \) is equal to \( C_p \), then the voltage drop across the PZT is approximately halved with the addition of the capacitor to the sensing circuit. Since the current through the PZT is proportional to the voltage, the current would also approximately halve with the addition of a capacitor of the same size. Consequently, the power being applied by the PZT is reduced by one fourth the amount of the power without an added capacitor. On the other hand, in the case where the added capacitor is equal to the capacitance of the PZT, the magnitude of the measured output voltage is increased by approximately four times the output voltage without the capacitor. This can significantly help the acquisition of impedance measurements since \( V_o \) can be very small, especially when relatively low frequencies are measured. In order to maintain a more effective actuation of the PZT, a capacitance should not be added to the low-cost circuit if possible. For low frequency excitation it is assumed that the voltage across the PZT is the input voltage. If a laboratory impedance analyzer is used, the voltage across the PZT is always the input voltage.

Other approaches used for modeling, such as those by Liang et al. or Bhalla (2004), require the mechanical impedance of a structure to be calculated. One of the significant advantages of the circuit analysis modeling approach is that only the sensing voltage at each frequency is needed, which can be obtained from a structural model. This has various benefits such as allowing the analysis of transfer impedance (transfer function between multiple impedance sensors) and the ability to account for the high frequency resistance of the PZT. Using a structural model paired with the circuit analysis presented, many design variables such as sensor spacing, actuation level and frequency, and sensor connection can be investigated.
2.3 Spectral Element Method

With conventional finite elements, the accuracy of the solution depends on the number of divisions of a structure. For an accurate representation of a response, 10-20 elements per wavelength of the highest frequency of interest must be used (Alford, 1974). This causes high frequency modeling using FEM to be prohibitively expensive. Some researchers have used FEM as the basis for modeling the impedance method, however, to obtain acceptable results, only relatively low frequencies could be modeled (Zaigrai and Giurgiutiu, 2001, Zu et al., 2002).

SEM, on the other hand, relies on a frequency dependent dynamic stiffness matrix. According to Lee (2004), the dynamic stiffness matrix was first developed by Kolousek in 1941 and the formulation of spectral elements was accomplished by Narayanan and Beskos in 1978. The spectral element’s development corresponds with the prevalence of computing and finite element methods in solving engineering problems. It should be noted that the dynamic stiffness matrix used for a spectral element is not the same as a conventional dynamic stiffness matrix formed by assuming harmonic motion and subtracting the mass matrix multiplied by the circular frequency, \( \omega \), squared from the conventional stiffness. A dynamic stiffness computed in such a fashion will be monotonic whereas the dynamic stiffness computed for the spectral element will have many zeroes.

A different stiffness matrix is developed at each frequency of interest. With the spectral element method, the mass of the structure is often referred to as being treated exactly. The shape functions are the exact solutions to the governing differential equations of each element (compared to the shape functions of traditional finite elements, which are polynomials). This means that each element can exactly represent the structural dynamics rather than approximate them as with FEM. One element can be used to model an entire section of a structure, provided that there are no discontinuities, thus greatly reducing the computational cost of a model. Elements can be combined to form a global stiffness matrix, the same way as the traditional FEM approaches. The forces are applied to the structure and the solution is obtained in the frequency domain. The results at each
frequency can then be combined using the computationally efficient inverse fast Fourier transform (IFFT) to obtain the time response.

An extensive description of SEM and applications, including examples showing SEM’s increased accuracy over the finite element method, can be found in the references by Doyle (1997) and Lee et al. (2000).

The derivation of the spectral element for a Timoshenko beam is shown here as an example. This derivation more closely follows Lee’s method of derivation rather than Doyle’s since Lee’s is more easily implemented in computer programs, which is helpful when elements for more complex situations are to be generated. The equation of motion of a free uniform beam with transverse displacement, $v$, and slope due to bending $\phi$, as shown in Figure 1, can be found using Hamilton’s principle as

$$E I \frac{\partial^2 \phi}{\partial x^2} + G A K_1 \left[ \frac{\partial v}{\partial x} - \phi \right] = \rho l K_2 \ddot{\phi}$$  \hspace{1cm} (2.12)

where $\rho$ is the mass per unit volume, $A$ is the cross-sectional area, $E$ is the elastic modulus, $I$ is the moment of inertia about the neutral axis, $v(x,t)$ is the transverse displacement at a time $t$ and distance $x$ along the beam, $\phi(x,t)$ is the transverse displacement at a time $t$ and distance $x$ along the beam, $q$ is the applied load and $K_1$ and $K_2$ are the numerical factors adjusting the effect of shear deformations and rotational inertia respectively. If $K_1$ is infinite and $K_2$ is zero the equation of motion for the Euler Bernoulli beam is obtained.

In addition, the internal bending moment, $M$, and transverse shear force, $Q$, are given from Hamilton’s principle as
The spectral representations of \( v \) and \( \phi \) are

\[
v(x, t) = \sum_n \hat{v}_n(x, \omega_n) e^{i\omega_n t}
\]

\[
\phi(x, t) = \sum_n \hat{\phi}_n(x, \omega_n) e^{i\omega_n t}
\]

If these relations are substituted into the equations of motion, the coefficients \( \hat{v}_n \) and \( \hat{\phi}_n \) must satisfy the following equations for every frequency point, \( n \)

\[
GAK_1 (\hat{v}^* - \hat{\phi}^*) + \omega^2 \rho A \hat{v} = 0
\]

\[
EI \hat{\phi}^* + GAK_1 (\hat{v}' - \phi) + \omega^2 \rho I K_2 \phi = 0
\]

where the subscript \( n \) has been omitted. Assuming solutions of

\[
\hat{v} = a e^{-ikx}
\]

\[
\hat{\phi} = b e^{-ikx}
\]

and substituting into the equation of motion gives the characteristic equation

\[
\left[ GAK_1 EI \right] k^4 - \left[ GAK_1 \rho I K_2 \omega^2 + EI \rho A \omega^2 \right] k^2 + \left[ \rho I K_2 \omega^2 - GAK_1 \rho A \omega^2 \right] = 0.
\]
Solving for $k$ gives four roots, two positive and negative pairs. The formulas for the wave numbers are not explicitly defined here because of their length, but may be found in the references by Doyle or Lee. In these references they are generally referred to as $k_1$ and $k_2$, since they occur in pairs. However, it is easier to refer to them independently as $k_1$ through $k_4$ if the formulation of the spectral element is being done in an automated fashion by a computer.

The amplitude ratio, or relation between coefficients $a$ and $b$, may be found from the first equations of motion as

$$ R_n = -i \frac{GAK_1 k_n^2 - \rho A \omega^2}{k_n GAK_1} \quad (n = 1,2,3,4) \quad (2.18) $$

Where in this case the subscript $n$ refers to which wavenumber the amplitude ratio is being calculated for. Note that this is the negative inverse of the amplitude ratio found in Doyle. This assumes that the coefficients on the rotational displacement are scaled rather than the coefficients on the transverse displacement. The general solutions of the equations of motion in the frequency domain are then

$$ \hat{v}(x) = a_1 e^{-ik_1x} + a_2 e^{+ik_1x} + a_3 e^{-ik_2x} + a_4 e^{+ik_2x} \quad (2.19) $$
$$ \hat{\phi}(x) = R_1 a_1 e^{-ik_1x} + R_2 a_2 e^{+ik_1x} + R_3 a_3 e^{-ik_2x} + R_4 a_4 e^{+ik_2x} $$

Inserting the end conditions determines nodal relation

$$ \hat{v}(0) = \hat{v}_1 = a_1 + a_2 + a_3 + a_4 $$
$$ \hat{\phi}(0) = \hat{\phi}_1 = R_1 a_1 + R_2 a_2 + R_3 a_3 + R_4 a_4 $$
$$ \hat{v}(L) = \hat{v}_2 = a_1 e^{-ik_1L} + a_2 e^{+ik_1L} + a_3 e^{-ik_2L} + a_4 e^{+ik_2L} $$
$$ \hat{\phi}(L) = \hat{\phi}_2 = R_1 a_1 e^{-ik_1L} + R_2 a_2 e^{+ik_1L} + R_3 a_3 e^{-ik_2L} + R_4 a_4 e^{+ik_2L} \quad (2.20) $$

where $L$ is the length of the beam. It is helpful to state this in matrix form as
\[ d = H(\omega)A \quad (2.21) \]

Where \( d \) is a vector of nodal displacements

\[ d = \{ \hat{\psi}_1 \hat{\phi}_1 \hat{\psi}_2 \hat{\phi}_2 \}^T, \quad (2.22) \]

with \( T \) indicating the transpose of the vector, \( H \) is the matrix

\[
H = \begin{bmatrix}
1 & 1 & 1 & 1 \\
R_1 & R_2 & R_3 & R_4 \\
e^{-ik_1L} & e^{-ik_2L} & e^{-ik_3L} & e^{-ik_4L} \\
R_1e^{-ik_1L} & R_2e^{-ik_2L} & R_3e^{-ik_3L} & R_4e^{-ik_4L}
\end{bmatrix}
\quad (2.23)
\]

and \( A \) is a vector of the coefficients

\[ A = \{a_1, a_2, a_3, a_4\}^T. \quad (2.24) \]

Next, the nodal loads can be related to nodal displacements from the equations for internal bending moment and transverse shear force. These in spectral form are

\[
Q(x) = GAK \left[ \hat{\nu}' - \hat{\phi} \right]
\]

\[ M(x) = EI \hat{\phi}' \quad (2.25) \]

Substituting the end conditions into this gives

\[
\hat{Q}_1 = -Q(0) = iGAK \left( k_1R_1a_1 + k_2R_2a_2 + k_3R_3a_3 + k_4R_4a_4 \right);
\]

\[
\hat{M}_1 = -M(0) = iEI \left( R_1k_1a_1 + R_2k_2a_2 + R_3k_3a_3 + R_4k_4a_4 \right);
\]

\[
\hat{Q}_2 = Q(L) = iGAK \left( k_1R_1a_1e^{-ik_1L} + k_2R_2a_2e^{-ik_2L} + k_3R_3a_3e^{-ik_3L} + k_4R_4a_4e^{-ik_4L} \right);
\]

\[
\hat{M}_2 = M(L) = iEI \left( R_1k_1a_1e^{-ik_1L} + R_2k_2a_2e^{-ik_2L} + R_3k_3a_3e^{-ik_3L} + R_4k_4a_4e^{-ik_4L} \right). \quad (2.26)
\]
Again, this can be shown in matrix form as

$$f = G(\omega)A$$  \hspace{1cm} (2.27)

where $f$ is the vector of forces on the nodes of the element, $A$ is again the vector of the coefficients and $G$ is given as

$$G = \begin{bmatrix}
iGAK_1k_1R_1 & iGAK_1k_2R_2 & iGAK_1k_3R_3 & iGAK_1k_4R_4 \\
iEIR_1k_1 & iEIR_2k_2 & iEIR_3k_3 & iEIR_4k_4 \\
iGAK_2k_1e^{-ik_1L} & iGAK_2k_2e^{-ik_2L} & iGAK_2k_3e^{-ik_3L} & iGAK_2k_4e^{-ik_4L} \\
iEIR_1e^{-ik_1L} & iEIR_2e^{-ik_2L} & iEIR_3e^{-ik_3L} & iEIR_4e^{-ik_4L}
\end{bmatrix}. \hspace{1cm} (2.28)

Now the coefficient vector $A$ can be eliminated since

$$A = H(\omega)^{-1}d. \hspace{1cm} (2.29)$$

So the force-displacement relation is

$$f = G(\omega)H(\omega)^{-1}d \hspace{1cm} (2.30)$$

where $G(\omega)H(\omega)^{-1}$ is the spectral element dynamic stiffness matrix often referred to as $S(\omega)$ by Lee or $\hat{k}$ by Doyle.

The variational method and state vector approach to formulating the spectral element have also been described in Lee.

Once the spectral element dynamic stiffness matrix has been formulated, it can be assembled the same as conventional finite element matrices. After assembling, boundary conditions are applied in the same manner as conventional finite element boundary
conditions, however, an additional infinite boundary condition exists. This generates what is sometimes referred to as a “throw-off” element since it takes energy out of the system. For a Timoshenko beam element, the infinite boundary conditions are

\[
\begin{align*}
\dot{Q}_1 &= -Q(0) = -EI \frac{d^2 \phi}{dx^2} - \rho I \omega^2 \dot{\phi} \\
\dot{M}_1 &= -M(0) = -EI \frac{d \phi}{dx} 
\end{align*}
\tag{2.31}
\]

In Figure 2.3, these boundary conditions would indicate that waves could travel to the right indefinitely.

After applying the boundary conditions, the Fourier transform of the applied force is input into equation 32. Then the nodal displacements can be solved for by simply multiplying the inverse of the dynamic stiffness matrix by the nodal forcing vector. Although the dynamic stiffness matrix must be evaluated at each frequency of interest, it is still relatively faster than traditional FEM approaches since it is much smaller in size. The Timoshenko beam element is the primary spectral element used in the research presented here. However, many other spectral elements have been formulated. Most are focused on one-dimensional structures. In addition to reduced processing times, there are many other benefits of SEM

- For analysis of impedance based SHM, the results are in the frequency domain. It is possible, however to convert the results to the time domain using the inverse Fourier transform.
- Damping can be included as complex modulus as \( E = E_0 (1 - i \eta) \) where \( \eta \) is the loss factor.
- Frequency or rate dependent parameters such as viscoelasticity are easily incorporated into spectral element models.
- Higher order theories can be added without adding DOF’s.
• The method temporarily removes time from problem leaving a pseudo-static problem.
• Only frequency ranges of interest need to be modeled.

As with every modeling technique, there are drawbacks to using SEM. One of the most practical limitations is that no commercial spectral element codes currently exist. This requires increased model development time compared to the more common finite element method for which many programs have been developed. Also, though many new spectral elements have been formulated, there are still many limits to what they can model. Generally, the governing equations of motion must be able to be solved analytically. This limits the characteristic equation to a fourth order polynomial or eighth order if the roots are repeated. Most spectral elements are limited to one-dimensional structures. Spectral elements for two dimensional structures exist; however, they generally are very limited by their assumptions. One example is Doyle’s plate element, which is limited to only Levy-type plates. Doyle also concludes that spectral elements will have difficulty handling localized discontinuities. The current approach is to combine spectral elements with a small area of conventional finite elements where discontinuities exist which he calls a super element. Regardless of the limitations, Doyle predicts that any problem solved by finite elements will eventually be able to be solved by spectral elements.

2.4 Spectral Element Model Development

The initial spectral element model developed of a cantilever beam follows the procedure presented by Doyle for spectral beam elements for a cantilever frame element. The frame element combines the axial and transverse dynamics of an element. Because of the high frequency nature of the analysis, a Timoshenko beam model is used that accounts for transverse shear and rotational inertial as well as the Love’s rod theory which includes a transverse component of velocity in axial vibration. The model contains seven elements, four aluminum beam sections and three aluminum beam sections with PZT attached to one side as shown in Figure 2.4.
Table 2-1 lists the geometric and material properties used in the model of the beam. The remaining properties of the beam are calculated from those given in the table. Specifically, shear modulus, $G$, given by $G = E_o/(2(1 + \nu))$, complex elastic modulus, $E$, given by $E = E_o + iE_o\eta$, area moment of inertia, $I$, given by $I = bh^3/12$, polar moment of area, $J$, given by $J = bh^3(1 - 0.58h/b)/3$ (Ashby, 1999) and Timoshenko shear coefficient, $K$, given by $K = (10(1 + \nu))/(12 + 11\nu)$ (Cowper, 1966). The PZT’s are
material type PSI-5H-S4-ENH manufactured by Piezo Systems, Inc. The PZT properties are shown in Table 2-2.

The spectral element model produces the displacement and rotation at each node for each frequency of interest. The displacement and rotation at points in between can also be determined, using the element shape functions, however, only the nodal information was used in this analysis. A more detailed description on the spectral Timoshenko model can again be found in the references by Doyle, Lee et al. and Krawczuk et al. (2003).

For transverse vibration, the model is forced with two opposing moments at either end of the PZT where the alternating voltage is being applied. The magnitude of the moment is the same at each frequency since the actuation in general is a swept sine. Assuming low excitation levels, the moment is proportional to the applied voltage and can be determined using the following relation (Fanson and Caughey, 1987, Crawley and Anderson, 1990).

\[
Mpzt = \frac{Ea \cdot bd_{31} \cdot V}{2 \cdot ta} \left( y_c^2 - \left( \frac{t_b}{2} \right)^2 \right) \quad y_c = \frac{t_a + t_b}{2},
\]

where \(Ea\) is the actuator elastic modulus, \(b\) is the beam (and actuator) width, \(ta\) is the actuator thickness, \(d_{31}\) is the piezoelectric strain coefficient in the 1 direction due to an applied electric field and \(V\) is the voltage applied to the actuator. It should be noted that this is considered a static method of modeling the PZT/structure interaction when compared to the impedance method of modeling.

At frequencies below the first resonance of the PZT, the moment produced by the PZT is relatively frequency independent. The results in this chapter are in this frequency range. In the case where the frequency range of interest is above the first resonance frequency of the PZT, the moment applied can be adjusted to reflect the changing dynamics. In addition, the model presented in this chapter is meant to be a simple, easily executable
model, which does not require expensive processing power, as is normally required for high frequency modeling.

From the resulting displacements the output voltage of the PZT, \( V_p \), can be calculated as (Fanson and Caughey, Crawley and Anderson).

\[
V_p = \frac{E_s t_a g_{31} t_p}{2L_{pzt}} (\phi_1 - \phi_2) \tag{2.33}
\]

where \( g_{31} \) is the piezoelectric voltage coefficient, \( L_{pzt} \) is the length of the PZT and \( \phi_1 \) and \( \phi_2 \) are the rotational displacements at either end of the PZT. For axial vibration, opposing forces are similarly applied at the nodes at the end of the PZT element and the resulting output voltage is calculated from the displacements. The voltages resulting from the axial and rotational displacements are summed to calculate the total output voltage and from that, impedance.

Due to the unavoidable uncertainty in measurements of the beam dimensions as well as variations in material properties at high frequencies, the model was tuned by adjusting the beam stiffness, the length of individual elements and Timoshenko shear coefficient. The optimization toolbox in MATLAB was used to optimize these parameters with two objective functions: the sum of the squared differences between resonance frequencies of the model and experimental data and the sum of the squared differences between all data points of the model and experimental data. Additionally, a small mass of 1 gram was added to the location of each PZT. This represents the effect of otherwise unmodeled features of the PZT sensor including, wiring and solder, copper tape for connection to the back side electrode and excess glue. Finally, a small internal resistance is included since the PZT is not purely capacitive.
2.5 Initial SEM/Circuit Analysis Modeling Results

Though the low-cost impedance method has already been shown to be equivalent to impedance data taken with analyzers such as the HP 4194A and HP 4192A, measurements were taken with both methods to verify the circuit analysis with SEM impedance model.

Initial results are only reported below 20 kHz because of limitations of the SigLab analyzer used for the low-cost method. A 100 ohm sensing resistor (with no capacitor) was used for the low-cost method. First a single measurement was made using the middle PZT mounted on the beam (PZT 2). Results, in the frequency range of 10-20 kHz, are shown in Figure 2.5.

![Figure 2.5. Initial model result without optimization.](image)

The circuit analysis with SEM model does a reasonable job of predicting both the real and imaginary impedance measured by the low cost method, especially given the unavoidable errors in measurement of geometric properties, and tabular nature of the
material properties. It appears that almost all of the resonances in the experimental data
are reflected in the model results. In addition, the relative magnitudes of the peaks in the
model results are the same as those in the experimental data. This is due in part to the
selection of the loss factor, which was chosen to match the damping seen in the
experimental data as represented by peak heights. Discrepancies in peak height can also
be partially attributed to the discrete nature of the experimental data and model results.

After optimization as described in Section 2.4 the results are improved dramatically
(Figure 2.6). No significant trend in error of the resonance location indicating that what
error is present is random. The effect of the value selected for the capacitance of the PZT
is most noticeable in the imaginary portion of the impedance. The measured capacitance
of the PZT followed an overall increasing trend in this frequency range. The capacitance
of the PZT in the model was selected so that the imaginary impedance would match that
of the low-cost model.

![Graph](image)

Figure 2.6. PZT 2 model and experimental impedance
2.6 Damage Modeling

Progressive increases in damage levels were introduced to the experimental beam by making a 1 mm wide saw cut completely through the thickness (h) of the beam between PZT 2 and 3, 39 cm from PZT 2. The length of the cut was initially approximately 1/4 the width of the beam and subsequently increased to 1/2 and then 3/4 the width of the beam. Figure 2.7 shows the maximum damage level introduced. Impedance measurements were made at each damage case and compared to the model. Damage was simulated in the model by decreasing the width of the beam the same amount as in the experiment and decreasing the stiffness in the location of the cut to attempt to account for the increase in stress intensity factor in the location of the cut. A similar approach has been used by McGuire, et al. (1995) and Lakshmanan and Pines (1998) to model damage. Also, it should be noted that the damage simulation is not intended to exactly represent damage, but to show that damage can easily be applied in the model. The model does an acceptable job of modeling the response to damage at the lower damage levels as seen in Figure 2.8. However, as damage increases towards a level closer to failure, effects not included in the damage model appear.

Figure 2.7. Beam showing PZT 2, PZT 3 and damage
Unfortunately, the type of damage applied will have minimal effects at this relatively low frequency range. This can be seen in the wavenumber for a Timoshenko beam element which, at low frequencies, reduces to

\[ k_{1,2,3,4} = \pm \left( \frac{\rho A \omega^2}{EI} \right)^{1/4}, \pm i \left( \frac{\rho A \omega^2}{EI} \right)^{1/4}. \] (2.34)
A reduction in the width of the beam will change the moment of inertia of the beam in that section, but it will be canceled out by the reduction in area. At higher frequencies the changes in the numerator and denominator cancel as well. Only a small change due to the polar moment of area, $J$, in the formulation of the wavenumber for the Love rod element, given by

$$ k_{1,2} = \pm \left( \frac{\rho A \omega^2}{EA - v^2 \rho J \omega^2} \right)^{1/2} \tag{2.35} $$

will cause change in the model’s response due to a reduction in width. The changes that are seen are most likely due to nonlinear effects such as higher stress at the cut location due the weight of the end piece. Additional peaks could be due to modes of vibration including torsional modes that are not included in the model, yet more easily excited when the width of the beam is decreased.

In a separate analysis to estimate the sensitivity of the PZT patches at varying distances from the damage, a 2 cm section of the beam was reduced to one half of its original stiffness and damping was doubled. This damage could be similar to the effects seen from a delamination in a composite beam. The damage was simulated in the same location as before, between PZT’s 2 and 3, 39 cm from PZT 2, 13 cm from PZT 3 and 92.35 cm from PZT 1. Only the real part of the impedance is monitored for damage since it has been shown to be more responsive to damage than the imaginary part. The results for each PZT are shown in Figure 2.9.
The resonant peaks in the impedance response did shift to the left as is expected for a loss of stiffness. Additionally, the cross correlation coefficient between the damaged and undamaged response was calculated as a damage metric for each PZT to determine if the proximity to the damage location had a noticeable effect on the change in impedance response. The coefficients were subtracted from 1 so that higher numbers would represent more damage as is traditional. The calculated metrics were 0.492, 0.378 and 0.486, indicating the presence of damage as in the traditional impedance method. However, these damage metric values do not seem to be related to the proximity of the sensor to the simulated damage. It is somewhat expected due to the low damping present in the beam. It is believed that an analysis at higher frequency or a more damped structure would not have this issue since local modes rather than global modes may be excited.
2.7 Summary

A model based on the circuit analysis of the previously presented low-cost impedance measuring technique has been developed. The model calculates the response of the structure using the spectral element method. Spectral elements are particularly well suited to modeling for impedance-based SHM. Specific advantages over traditional FEM methods are that they can model high frequencies more efficiently, only require calculation of the results in the frequency range of interest, can include higher order theory without significantly more computational requirements and the results are in the frequency domain.

The development of specific spectral elements has been described extensively by several authors including Doyle and Lee. Derivation of a Timoshenko beam element is included as an example. The Timoshenko element was chosen for the example since it is required to accurately model the higher frequencies on which impedance based SHM is based. Also, the methods necessary for developing the Timoshenko beam element are also necessary for developing more complex elements discussed in later chapters. The derivation is presented in a form so that it may be automated to a large degree using symbolic computing programs.

Results are shown for a PZT bonded to an aluminum beam at relatively low frequencies for the impedance method. The results capture the main features of the experimental impedance measurement, however, are improved by accounting for details such as added mass due to the electrical connections and excess glue. This model is considered “static” compared to the impedance modeling method developed by Liang et al. and its derivatives since the force input into the structure does not vary with frequency.

A simple simulation of damage was performed with both the model and experimental beam. The damage introduced into the beam had minimal effects on the impedance response until it was severe. At that point, effects not included in the model such as drooping due to gravity became apparent.
The small size of models using SEM with circuit analysis for impedance-based SHM may allow placement of the algorithm on a computer chip integrated into the sensor in a SHM system such as in Grisso et al. (2005) for advanced functions such as damage prognosis (estimation of remaining life) of a structure. It can also allow the modeling of infinite length beams which will aid in the modeling of local modes of the structure in question. The next chapter will focus on application of this model to multiple sensors.
Chapter 3

Initial Model Demonstrations

For a distributed sensor system, the connection of the many transducers and processing of each may become overwhelming if they are all treated individually. Here it is shown that the circuit model of impedance can easily represent the different types of connections possible, primarily parallel and series with in phase and out of phase actuation. Experimental tests are performed to see if a preference towards one type of connection over the other exists. Through the circuit analysis, it is shown that a parallel connection will excite the beam more. The energy applied to the beam in the series configuration is also examined to see if a damage feature that acts in a nonlinear fashion may provide more benefits than the increase in output voltage of the PZT’s.

3.1 Multiple active sensor modeling

Since the impedance method uses structural vibrations at a very high frequency, the sensing area of one sensor/actuator is fairly small. This is a benefit in that measurements are relatively unaffected by boundary condition changes and damage location is inherently determined to be near the sensor. However, in many instances it is desirable to be able to determine the condition of a much larger area than one sensor/actuator can monitor, or examine several areas not mechanically connected. This can be accomplished using an array of sensor/actuators. In some cases, such as when installation hardware and/or system power is limited, the sensor/actuators are multiplexed. This involves electrically connecting the sensors together, so that all can be excited, and measuring the response, with one frequency sweep. This type of measurement is difficult to handle with Liang’s impedance model, however, is easily accounted for in the circuit analysis model.
Several studies have previously investigated multiplexing. Vinod (1997) simultaneously measured multiple PZT’s that interrogated different areas of an aluminum truss structure. The results showed that multiplexing is effective at reducing the interrogation time to assess the health of the structure but reduces the sensitivity of the measurements to damage. Additionally, Park et al. (2001) wired multiple PZT sensors together to improve the interrogation time of many joints in a pipeline structure and also reported that sensitivity may be decreased by multiplexing. Bhalla et al. (2002) connected two PZT’s bonded to an aluminum plate to a commercially available multiplexer and switching system. This allowed the impedance analyzer to simultaneously scan both PZT’s, but then once damage is indicated, switch to single PZT sensing and actuation to determine the location of the damage. The details of the connection were not found for any of these studies: specifically, whether the PZT’s were connected in series or in parallel.

This chapter presents an investigation of multiplexing both experimentally and with the impedance model presented in the previous chapter. First a detailed investigation of multiplexing is carried out with a highly damped beam. Then, transfer impedance measurements, which depend on multiplexing the sensor/actuators is modeled for the aluminum beam presented in the previous chapter.

### 3.2 Multiplexing

Since both the imaginary and real impedance data can be accurately represented, the circuit model can correctly predict combinations of sensors. Analysis of a highly damped polymer beam was performed experimentally and with the circuit analysis model presented in the previous chapter. Three PZT’s (material type PSI-5H-S4-ENH manufactured by Piezo Systems, Inc.) were bonded to a 0.64 cm thick PMMA (Plexiglas) beam as shown in Figure 3.1. Connections to the bottom electrode of each PZT were made with copper tape partially inserted underneath the PZT. The beam was suspended to approximate a free-free boundary condition.
Measurements of impedance were made in steps of 25 Hz from 0.1 kHz to 100.1 kHz. A 15 kHz section from 5 khz to 20 kHz of the measurement made with PZT is shown in Figure 3.2. Each PZT was measured separately and in connection with the other PZT’s as follows:

1. PZT 1 independently
2. PZT 2 independently
3. PZT 3 independently
4. PZT 1 in parallel with PZT 2, top electrodes of each PZT connected
5. PZT 1 in parallel with PZT 2, top of PZT 1 connected to bottom of PZT 2 and vice-versa
6. PZT 1 in series with PZT 2, bottom of PZT 1 connected to top of PZT 2
7. PZT 1 in series with PZT 2, bottom of PZT 1 connected to bottom of PZT 2
8. PZT 1 in series with PZT 2, bottom electrode of PZT 2 connected to top of PZT 1
9. PZT 1 in parallel with PZT 3, top electrodes connected, bottom electrodes connected
10. PZT 1 in parallel with PZT 3, top of PZT 1 connected to bottom of PZT 3 and vice-versa
11. PZT 1 in series with PZT 3, bottom of PZT 1 connected to top of PZT 3
12. PZT 1 in series with PZT 3, bottom of PZT 1 connected to bottom of PZT 3
13. PZT 2 in parallel with PZT 3, top electrodes connected, bottom electrodes connected
14. PZT 2 in parallel with PZT 3, top of PZT 2 connected to bottom of PZT 3 and vice-versa
15. PZT 1, PZT 2, PZT 3 in series, bottom of PZT 1 connected to top of PZT 2, bottom of PZT 2 connected to top of PZT 3
16. PZT 1, PZT 2, PZT 3 in parallel, tops connected together, bottoms connected together
17. PZT 1, PZT 2, PZT 3 in parallel, tops of PZT 1 and 3 connected to bottom of PZT 2, top of PZT 2 connected to bottoms of PZT 1 and 3.

Results indicate that at relatively low frequencies, connection order significantly changes the measured response due to wave interaction (Figure 3.3). However comparing Figure 3.3 to Figure 3.4, for two PZT’s connected together, despite changes in the scale of the measurements, there are only two possible shapes of the response. These correspond to in-phase actuation and out-of-phase actuation of the PZT’s.
Next, damage was inflicted on the beam in the form a drilled through-hole. The cross correlation damage metric was calculated for 10 kHz wide frequency ranges. Results are shown in Figure 3.5 for each connection case. The results indicate that even though the impedance measurement shapes and scales may be different for different connection types of a set of actuators, the sensitivity to damage is the same.

Figure 3.3. Two possible parallel connections for PZT 1 and 2.
Figure 3.4. Several possible series connections for PZT 1 and 2.

Figure 3.5. Damage metric for each frequency range and connection type.
3.3 Modeling of Multiple PZT Circuits

As mentioned, the SEM combined with circuit analysis model of impedance measurements should be able to easily handle the combination of sensor/actuators in either series or parallel. Circuit diagrams for the two configurations with two piezoelectric elements each are shown in Figure 3.6.

Figure 3.6. Configuration of multiplexed circuits with two sensors: a) Parallel b) Series.

For the circuit analysis the PZT’s are again simply treated as capacitors.

Parallel Actuation

In parallel the capacitances are added resulting in an increase in total current through the circuit. For two PZT’s, the current through the low-cost circuit becomes

\[ I = \frac{\omega C (C_{p1} + C_{p2}) V_i}{j (C_{p1} + C_{p2}) + C (j - \omega R_i (C_{p1} + C_{p2}))} \quad (3.1) \]

From this, the voltage drop across both capacitors can be found to be
\[ V = \frac{jCV_i}{j(C_{p1} + C_{p2}) + C\left(j - \omega R_s\left(C_{p1} + C_{p2}\right)\right)}. \quad (3.2) \]

This is applied to each PZT, though the current through the PZT will depend on the capacitance of the individual PZT. If a capacitor is not added for impedance matching, the applied voltage simplifies to

\[ V = \frac{-jV_i}{-j + \omega R_s\left(C_{p1} + C_{p2}\right)}. \quad (3.3) \]

If the capacitances of the PZT’s are equal size, then the beam will be excited by approximately twice as much energy with the addition of the second PZT in parallel. In the case where no sensing resistor or capacitor is used (measurement with a laboratory impedance analyzer), the voltage applied to the PZT’s is obviously \( V_i \). The current through the total PZT elements is simply \( I = j\alpha(C_{p1} + C_{p2})V_i \) and will exactly double for two capacitances of equal size. The total power being transmitted to the beam for two PZT’s with equal capacitance connected in parallel is twice what it would be for a single PZT. Unfortunately since the total capacitance has increased, the cutoff frequency is decreased proportionally.

**Series Actuation**

For the series configuration the impedance of the capacitors is added. For the circuit in Figure 3.6, the voltage applied to PZT 1 is

\[ V = \frac{-j(C_{p1} C_{p2})V_i}{-j(C_{p1} C_{p2}) + C\left(C_{p1} + C_{p2}\right) + \omega R_s CC_{p1} C_{p2}}. \quad (3.4) \]

This reduces to
\[ V = \frac{C_{p2}V_i}{C_{p1} + C_{p2} + j\omega R_i C_{p1} C_{p2}} \]  

(3.5)

if no capacitor is added to the sensing impedance. If no sensing resistor or capacitor is used (laboratory analyzer) the voltage across PZT 1 is

\[ V = \frac{C_{p2}V_i}{C_{p1} + C_{p2}}. \]  

(3.6)

From the last equation it is apparent that for the case of PZT’s with equal capacitances, the voltage applied to one PZT is half what would be applied if it were the only PZT. The current through one of two PZT’s with equal capacitance is also one half what it would be if it were applied to only one PZT. Therefore, the total power being transmitted to the beam for two PZT’s with equal capacitance connected in series to be reduced to one half what it would be for a single PZT or one quarter what is transmitted with the PZT’s connected in parallel (for the same applied voltage).

**Parallel Sensing**

A similar effect is seen when combined PZT’s are acting as sensors. If the PZT’s are connected in parallel, the voltage out of the low-cost circuit is

\[ V_o = (V_i + V_p) \frac{(1 + j\omega R_i C)(C_{p1} + C_{p2})}{C + (C_{p1} + C_{p2}) + j\omega R_i C(C_{p1} + C_{p2})}. \]  

(3.7)

where \( V_p \) is the sum of the output voltage from the PZT’s. \( V_p \) can either be increased or decreased compared to \( V_p \) of a single PZT at each frequency, depending on whether or not the voltages in the PZT’s are being produced in phase or out of phase. \( V_p \) is small compared to \( V_i \), so the overall level of \( V_o \) will depend mostly on \( V_i \). \( V_p \) can be found by calculating the Thevenin equivalent of the sensing circuit with the PZT’s represented as
two voltage sources. \( V_p \) will be the weighted values of the individual voltages divided by the total capacitance of the PZT’s as follows

\[
V_{p\,parallel} = \left( \frac{C_{p1}V_{p1} + C_{p2}V_{p2}}{C_{p1} + C_{p2}} \right).
\]  

(3.8)

If a capacitor is not added to the sensing impedance, then the voltage out will be

\[
V_o = \left( V_i + V_p \right) \frac{\omega R_s \left( C_{p1} + C_{p2} \right)}{\omega R_s \left( C_{p1} + C_{p2} \right) - j}. \]

(3.9)

Compared to \( V_o \) of a circuit with just one PZT and only a resistor for the sensing impedance, for PZT’s with equal capacitance, the ratio of \( V_o \) for two PZT’s to one PZT in the parallel configuration is

\[
\frac{V_{o\,2\,\text{PZT}s}}{V_{o\,1\,\text{PZT}}} = \frac{2j - 2C_p R_s \omega}{j - 2C_p R_s \omega}. \]

(3.10)

This indicates that the voltage out will also approximately double for the addition of another PZT in parallel. The impedance of two equal PZT’s in parallel will reduce by one half (capacitance doubles).

**Series sensing**

For sensing in a series configuration the voltage out will be

\[
V_o = \left( V_i + V_{p1} + V_{p2} \right) \frac{C_{p1}C_{p2} \left( -j + CR_o \omega \right)}{-j \left( C_{p1}C_{p2} + C \left( C_{p1} + C_{p2} \right) \right) + CC_{p1}C_{p2}R_o \omega}. \]

(3.11)

In this case the voltage due to the PZT will add. When the low-cost circuit is used without an added capacitor, the voltage out becomes
\[ V_o = \left( V_i + V_{p1} + V_{p2} \right) - j\left( \frac{C_{p1}C_{p2}R_s\omega}{C_{p1} + C_{p2}} \right) + C_{p1}C_{p2}R_s\omega \]  

(3.12)

Again, compared to a single PZT, if a second equal PZT is added in series, the ratio of output voltages \( V_o \) will be

\[ \frac{V_{o2PZT}}{V_{o1PZT}} = \left( \frac{V_i + V_{p1} + V_{p2}}{V_i + V_{p1}} \right) - j\left( \frac{2C_pR_s\omega}{-2j + 2C_pR_s\omega} \right) \]  

(3.13)

indicating that since the voltage produced by the PZT’s is relatively small, the output voltage will be approximately half that of a single PZT.

### 3.4 Modeling of PMMA Beam

Static material properties of PMMA are given as an elastic modulus of 2.24-3.34 GPa and density of 1170-1200 kg/m³ by Callister (2000) and Poisson’s ratio of 0.35-0.4 by Goodfellow (2006). However, PMMA exhibits viscoelastic behavior as reported by Ferry (1961). In the frequency range of 1-10 kHz, the reported elastic modulus for PMMA is approximately 4 GPa, and increasing with frequency, shear modulus, 2 GPa and increasing with frequency and loss factor 0.04 and decreasing with frequency. Since the exact properties of the PMMA beam are not known exactly and vary with temperature, constant values of 4 GPa, 2 GPa and 0.37 were selected for the elastic modulus, shear modulus and Poisson’s ratio as inputs to the analysis. The loss factor was set at 0.03 since this value seems to split the difference between the high and low parts of the frequency range. The capacitance of the bonded PZT’s was measured as 51 nF, 53 nF and 50 nF for PZT 1, 2 and 3 respectively. The density was calculated as 1258 kg/m³ from the calculated volume and mass measurements. Point masses of two grams were added to the transverse coordinate to account for PZT, adhesive and connections. It should also be noted that the material parameters were not optimized to provide an exact fit to the experimental data, but the model results do predict the general trend of the data.
The initial results of the model significantly over predicted the amount the beam was excited. This was due to the fact the applied moment calculated from equation 14 in chapter one (Crawley and Anderson, 1990), does not include any effects of the beam stiffness. Instead, the enhanced pin-force model (Chaudry and Rogers, 1994) was used to calculate the moment produced on the beam. For the enhanced pin-force model, the moment produced by a PZT in the unimorph configuration is given as

\[
M = -\frac{E_a b t_a^2 (\Psi T^2 + 1) I_{31}}{2T (3 + \Psi + \frac{1}{T^2})} t_a V, \text{ where } \Psi = \frac{E_b t_b}{E_a t_a}, \quad T = \frac{t_b}{t_a} \quad (3.14)
\]

The enhanced pin-force model, though still relatively inaccurate, provides sufficient results for this demonstration since the thickness of the beam is more than 20 times the thickness of the PZT. Initial results for the excitation of PZT 2 on the PMMA beam are shown in Figure 3.7 and for PZT 1 in Figure 3.8.

![Figure 3.7. Non-optimized model and experimental impedance for PZT 2 on PMMA beam.](image-url)
Figure 3.8. Non-optimized model and experimental impedance for PZT 1 on PMMA beam.

Next the two PZT’s were both “activated” in the model and connected in parallel using the circuit analysis given above. Both in phase and out of phase parallel connections were modeled as shown in Figure 3.9. Compared to Figure 3.10, the model has the same reduction in peak height and shift in imaginary part as seen in the experimental data. Also, the same general pattern of peaks is seen in both the experimental and modeled impedance response.
Figure 3.9. PZT 1 and PZT 2 possible connections in parallel (model).

Figure 3.10. PZT 1 and PZT 2 possible connections in parallel (experiment).
Next the model was configured to handle the series configuration. In the series case the voltages produced by the PZT’s simply add. However, the voltage applied to the PZT’s is split between the two. The impedance in the circuit doubles for PZT’s of equal size. The model results are as seen in Figure 3.11 and can be compared to the experimental results in Figure 3.12. Again, the model easily handles the new connection types.

Figure 3.11. PZT 1 and PZT 2 possible connections in series (model).
3.5 Transfer Admittance

The measurement of transfer admittance (sometimes referred to as cross-admittance) depends on the ability to multiplex signals. The transfer admittance (inverse of impedance) can be compared to the FRF’s generated in the modal analysis and can be used to generate the mode shapes of the vibrating structure (Castanien and Liang, 1996, Sun et al., 1995a). If it is assumed that the PZT sensors are identical, the following formula describes the formulation of the transfer admittance ($Y_{xy}$)

$$Y_{xy} = \frac{Y - (Y_{xx} + Y_{yy})}{2}$$  \hspace{1cm} (3.15)

where $Y_{xx}$ and $Y_{yy}$ are the admittances of the sensors measured separately, and $Y$ is the admittance of the sensors measured in parallel with each other.
The transfer impedance is calculated between the second and third PZT of the aluminum beam presented in the preceding chapter using measurements from both the HP analyzer and low-cost impedance measuring method. To calculate the impedance of the sensors measured in parallel, the sensor capacitance, $C_p$, doubles in the equation for $V_o$ and the rotational displacement of both PZT’s is averaged for in the equation of $V_p$. Figure 3.13 shows the remaining components needed for the computation of the transfer impedance and Figure 3.14 shows the results.

Figure 3.13. a) PZT 3 SEM model and low-cost response b) PZT 2 and 3 SEM model and low-cost response.
Figure 3.14. Transfer admittance comparison

The model response is shaped more like the response obtained with the HP4192A than the low cost response. This can be attributed to the greater amount of noise in the low cost response compared to the HP analyzer, which has relatively low noise, and model, which has zero noise. The noise masks the information in region where the structural response is low (valleys in the FRF). However, the response is accurate at the peaks and antiresonances of the FRF’s.

3.6 Conclusions

For a distributed sensor system, the connection of the many transducers and processing of each may become overwhelming if they are all treated individually. The circuit model of impedance can easily represent the different types of connections possible, primarily parallel and series with in phase and out of phase actuation. Experimental results did not
show a preference towards one type of connection over the other in terms of the damage metric. However, the damage introduced was relatively large and near-field. In the case of damage that is more difficult to detect, such as very small or far-field damage, the connection type may affect whether the damage can be “seen” or not. Through the circuit analysis, it was determined that a parallel connection will excite the beam more. The series connection may produce a larger effective total voltage by the PZT’s, however, since the energy applied to the beam in the series configuration may not be enough to excite a damage feature that acts in a nonlinear fashion, the greater actuation of the parallel connection may provide more benefits than the increase in output voltage of the PZT’s.

Other considerations need to be taken into account, however. If the connections are made in series, damage to one sensor or one section of wiring may cause the sensor/actuators in that circuit not to work. This is avoided with the parallel connection, however, more wiring is required for the same size network. Furthermore, it would be easier to switch to individual monitoring of the PZT’s in the parallel mode.
Chapter 4

Modeling PZT Resonances with Constrained Layer Spectral Element

Very few researchers have attempted to model the impedance response of a structure excited by a PZT above the lower order global modes. For example, Liang et al. (1994a) only report results at frequencies less than 2 kHz. Naidu (2003) reports that the research done by Winston et al. (2001) shows that finite element modeling can be used to predict the response at high frequencies, however, Winston et al.’s research still only involves the lower order global modes. In addition, Winston et al. neglects any effect of the actuator. Giurgiutiu and Zaigrai (2000) report a modeled impedance measurement of a structure with PZT up to 30 kHz, but select a structure so that this range only includes five resonant peaks. They also show results up to the 2 MHz range, but this is only for the PZT actuator. Even Ritdumrongkul (2004) who uses a slightly modified version of the element presented in this chapter, only reports results up to 2.5 kHz and do not include any regions that would possibly include a resonance of the PZT.

Unfortunately, the basic Timoshenko beam based SEM model still has difficulty predicting the response at the higher frequencies typically used in the impedance method. For the aluminum beam considered in the previous chapters, at frequencies above 30-40 kHz, there are large discrepancies in the calculated and experimental impedance measurements as seen in Figure 4.1.
The primary discrepancy is the large increase in response at approximately 44 kHz that is not seen in the modeled impedance. This is due to the resonance of the PZT, which occurs where the impedance of the bonded PZT actuator is the complex conjugate of the host structure’s impedance (Liang, 1994b). The purpose of this chapter is to incorporate dynamics of the PZT and beam that produce the resonant peaks of the PZT in the impedance response at high frequencies.

4.1 Modeling with the Impedance Method

As a comparison, the impedance response is modeled using Liang’s method by calculating the structural impedance using the SEM model. The impedance of the PZT actuator, $Z_a$, is given as

$$Z_a = \frac{K_A(1 + \eta)}{\omega} \frac{kl_a}{\tan(kl_a)} i$$

(4.1)

where $K_A$ is the static stiffness of the PZT in the axial direction. The wavenumber, $k$, given by $k^2 = \frac{\omega^2 \rho}{Y_{11}^E}$, where $\rho$ and $Y_{11}^E$ are the density and complex modulus at zero electric field of the PZT actuator, along with the length of the actuator, $l_a$, determine the
resonant frequency of the PZT. The mass of the PZT actuator is not included in $Z_a$, so would need to be added into $Z_s$. The rotational structural impedance given by

$$Z_r = \frac{M}{\omega \left( \theta_2 - \theta_1 \right)},$$

(4.2)

where $M$ is the moment applied to a structure at the ends of the PZT and $\theta_1$ and $\theta_2$ are the resulting displacements at the ends of the PZT. Liang’s method continues for a pair of actuators in the bimorph configuration. For a single PZT bonded to the structure, to convert the rotational structural impedance to the in-plane structural impedance for use in Liang’s relation, Sun et al. (1995b) gives the relationship between rotational mobility and in-plane mobility, which in terms of impedance is

$$F = \left( \frac{Z_r}{\left( t_b/2 + t_a/2 \right)^2} \right) \ddot{x} = Z_s \ddot{x}.$$  

(4.3)

The two impedances are shown in Figure 4.2.
Figure 4.2. Impedance of structure and actuator from Liang's model.

As referenced in the first chapter, the resulting impedance can be found from the following equation for admittance of the PZT bonded to the structure.

\[
Y(\omega) = i\omega(\bar{\epsilon}_{33})^T - \frac{Z_s(\omega)}{Z_s(\omega) + Z_a(\omega)} d_{33}^2 \hat{Y}_{ss}^E
\]  

(4.4)

The results show the inclusion of the PZT resonance as seen in Figure 4.3.
4.2 Lee’s Active Constrained Layer Element

Another method of modeling the PZT resonance is the incorporation of Lee and Kim’s three layer spectral element of a bonded piezoelectric to an elastic beam into the model. The element development is discussed in detail in by Lee and Kim (2001). The element models the coupling between the piezoelectric element and the beam based on axial-bending-shear coupled equations of motion developed from piezoelectric constitutive equations. An additional degree of freedom is introduced that is the rotation of the adhesive layer between the piezoelectric and base beam as seen in Figure 4.4.
The angle of the adhesive layer, \( \psi \), depends on the difference between the axial motion of the base beam, \( u_b \), axial motion of the piezoelectric, \( u_p \), the rotation of the base beam, \( \phi \), and the thickness of each layer as follows

\[
\psi = \frac{u_p - u_b + \frac{1}{2} (h_b + 2h_v + h_p)\phi}{h_v}. \tag{4.5}
\]

An additional kinematic relation for the axial displacement of the viscoelastic layer is given as

\[
u_v = u_b - \frac{1}{2} (h_b + h_v)\phi + \frac{1}{2} h_v \psi. \tag{4.6}
\]

Shear deformations and rotary inertia are ignored for the base and piezoelectric layers, but are considered for the adhesive layer. The axial motions of each layer are also considered independently, though eliminated from the final equations of motion through the kinematic relationships shown above, involving the shear angle of the adhesive layer and rotational angle of the base beam. The later is also simply the derivative with respect to axial distance, \( x \), since shear deformation of the base beam is ignored.

One issue of note is that Ritdumrongkul, who later used Lee’s three-layer spectral element with an axial tension applied, reports a different boundary condition than Lee for

![Figure 4.4. Lee three layer active constrained layer element.](image-url)
the degree of freedom corresponding to the rotation of the base beam. Lee states the boundary conditions derived along with the equations of motion using Hamilton’s principle as

\[
\begin{align*}
N &= EAu_b' - \beta \omega'' + \epsilon_4 y' \\
M &= EI_w \omega'' - \beta u_b' - \epsilon_2 y' \\
Q &= -M' + \gamma \ddot{\psi} - \epsilon_1 \ddot{y} + \epsilon_2 y'' \\
R &= EI_y \psi' - \epsilon_3 w'' + \epsilon_4 u_b'
\end{align*}
\] (4.7)

where \( N \) is the internal force in the axial direction, \( M \) is the internal bending moment, \( Q \) is the internal transverse force and \( R \) is the internal moment on the shear angle of the adhesive. The terms on the right side of the equations are defined in Lee’s paper and book. When the moment is substituted into the equation for the transverse force the result is

\[
Q = -EI_w \omega'' + \beta u_b'' + 2\epsilon_2 y'' + \gamma \ddot{\psi} - \epsilon_1 \ddot{y} \] (4.8)

However, Ritdumrongkul reports that the transverse force as

\[
Q = -EI_w \omega'' - \alpha \dddot{u}_b + \beta \dddot{u}_b' + \gamma \ddot{\psi} - \epsilon_1 \ddot{y} + \epsilon_2 y'' \] (4.9)

The internal transverse force provided by Ritumrongkul was used to develop the spectral elements used in all analyses with the three layer spectral element since use of Lee’s boundary condition did not provide realistic results.

The moments and forces applied to the element by the PZT are as follows:

\[
\begin{align*}
N_p &= bd_1 E_p V \\
M_p &= \frac{1}{2} bd_1 E_p (h_b + 2h_v + h_p) V \\
R_p &= bd_3 E_p h_v V
\end{align*}
\] (4.10)
where $N_p$ is the force in the axial directions, $M_p$ is the bending moment and $R_p$ is the moment on the shear angle of the adhesive. The moment applied to the shear angle degree of freedom is equal to the force applied to the axial degree of freedom, multiplied by the thickness of the adhesive layer. The resulting output voltage is again calculated from the difference in rotational and axial displacements at the ends of the element.

### 4.3 High Frequency Aluminum Beam Results

Initially, the model was evaluated with nominal values for the adhesive modulus. The properties of the cyanoacrylate adhesive were not reported by manufacturers, however, for values for the elastic and shear modulus of cyanoacrylate were found in the literature as 6 GPa (Volinsky et al., 1999) and 2.8 GPa (Park, J.-M. et al., 2000), 1.5 GPa (Van Vliet et al., 2004) or 0.1 GPa (Adhesive Design Toolkit, 2006) respectively. It is also apparent that there is significant variation in the performance of the adhesive as evident from the three values of shear modulus reported for cyanoacrylate. The model using Lee’s three layer element did not predict the large increase in response seen in the experimental data when an adhesive thickness of $2.02 \times 10^{-5}$ m and nominal values of adhesive modulus were used as seen in Figure 4.5.
Unfortunately, the reported values of adhesive modulus do not seem to be representative of the interaction between the PZT and the structure. It is believed this is due in large part to variations in the bond layer. The variations include to voids and the copper tape used to allow electrical connection to the side of the PZT bonded to the structure. The copper tape not only has it’s own elastic and shear properties, but also includes a layer of conductive adhesive, which is not as nearly stiff as the cyanoacrylate. To complicate matters even more, the copper tape only covers a portion of the undersurface of the PZT. Hence, in the model, the thickness, elastic modulus and shear modulus were adjusted in a model updating approach, to position the peak from the piezoelectric at the same frequency as the peak in the experimental response. This is similar to the adjustment of a representative stiffness for the bonding layer as in Xu and Liu’s (2002) impedance model that includes bonding layer effects. For the plot shown in Figure 4.7, the values used for the adhesive properties are $2.02 \times 10^{-5}$ m, $1.80 \times 10^8 + i \times 4.72 \times 10^5$ N/m$^2$ and $4.80 \times 10^8 + i \times 1.26 \times 10^5$ N/m$^2$ for adhesive layer thickness, complex shear modulus and elastic modulus respectively. Again, these values are not the actual values reported by the adhesive manufacturer, but representative values that take into account the quality of the bond between the PZT and base material. The recalculated response for the frequency range 20
kHz to 90 kHz of the aluminum beam using Lee’s three layer element at the location of the PZT’s can be seen in Figure 4.6.

The model including Lee and Kim’s element shows the addition of large peaks due to the bonded piezoelectric resonance. As seen in Figure 4.7, which shows the individual contributions of the axial, rotational and shear angle degrees of freedom to the overall impedance, the large peaks at 45 and 63 kHz are due to the vibration of the PZT out of phase with the base beam, causing a large increase in the impedance response. The location and size of the peak depends on the geometric and material properties of both the piezoelectric and adhesive.

Figure 4.6. Model with Lee and Kim’s piezoelectric-adhesive-base three layer beam
A significant difference between the model and experimental data at higher frequencies is the greater peak density in the experimental data. There are several possible explanations for the cause of the difference. First, the model only accounts for transverse and axial vibration of the beam. At lower frequencies (less than 30 kHz), these make up the primary modes of vibration. At higher frequencies, torsional modes may appear, as well as modes of vibration in the stiffer bending direction. This can be somewhat addressed by adding degrees of freedom to include shaft dynamics in the model. Also, modes of vibration of the base structure that provides the cantilever boundary condition may be activated at those frequencies. In addition, the effect of the bonding layer plays an important role in modifying structural responses at higher frequency ranges. Finally, at higher frequencies, the modes of vibration shift from global modes to local modes. This could possibly be addressed by modeling the beam as having an infinite length by including throw off elements. Throw off elements are elements with only one node that can be included in SEM. For more details see Doyle (1997).
4.4 High Frequency PMMA Beam Results

To further assess the performance of the model in predicting the location of the PZT resonances, the three layer element was also used to model the PMMA beam used in the multiplexing analysis. The same method was used to bond the PZT to the beam as with the aluminum beam, though the adhesive type and applied pressure while bonding may be slightly different. One benefit of the PMMA beam is that the bond surface is visible through the beam. As seen in Figure 4.8, with a normal amount of pressure applied via a 2.5 kg mass and rubber block only slightly larger than the PZT, there are still numerous voids in the adhesive layer. In addition, the copper tape introduces a large area of unknown bonding conditions.

![Figure 4.8. Underside of bonded PZT viewed through PMMA beam.](image)

The measured bonded response of the PZT is shown in Figure 4.9.
Figure 4.9. Bonded response of PZT 1 and PZT 2 attached to PMMA beam.

It is impossible to distinguish large peaks in the response as due to the PZT or just characteristics of the vibration response of the beam. The response of a PZT of approximately the same size as the bonded PZT’s was measured and is shown in Figure 4.10.

Figure 4.10. Unbonded impedance of PZT of same size as PZT’s bonded to PMMA beam.
Comparing the free to bonded response of the PZT’s, it appears that most of the peaks in the response are due to the vibration characteristics of the beam structure, rather than effects of the PZT. Also, this provides an example of the difficulties in modeling the resonances of PZTs and uncertainties from bonding. The peaks in the unbonded response shift to lower frequencies upon bonding (53 kHz to 49 kHz and 89 to 82 kHz) assuming they were in the same location for the two PZT’s before bonding. The second peak in the unbonded response becomes are significantly less pronounced when bonded though the first is nearly the same height after bonding. It is not immediately clear which direction the PZT is vibrating in for each of the large peaks. However, when the longer dimension of the PZT was reduced to the length of the shorter dimension, to form a square PZT, the lower frequency peak disappears, indicating that the 90 kHz peak is the peak that should be represented in the model.

If the value for the shear modulus of the bonding layer is entered into the model as approximately 4/5 (to account for poorly bonded areas) the average of the reported value in the literature (resulting in 1.6 GPa), the model predicts a peak due to the shear rotation of the adhesive at 90 kHz as seen in Figure 4.11.

![Figure 4.11. Model of PMMA beam with resonant peak of shear angle of adhesive.](image)

The response of the rest of the frequency range does not completely represent response of the experimental beam. This is believed to be due to complications from viscoelastic
properties and possibly imperfections such as misalignment of the PZT on the beam. In addition, it is very difficult to estimate how the variations in the bonding layer (including the copper tape and copper tape’s adhesive) affect the value of the loss factor. This will have a significant effect on the height and width of the resonant peaks due to PZT resonances.

The values used for the adhesive properties in the case of the PMMA beam are not nearly as far from the values reported in the literature as the values needed to represent the response of the aluminum beam. This indicates that the aluminum beam may have been poorly bonded. Possible causes of the poor bond include not using enough adhesive, allowing the adhesive to partially cure before mounting the PZT or possibly abuse of the beam such as large deformations.

4.5 Conclusions

The effect of the active sensor resonance on the high-frequency impedance response typically used in impedance-based SHM has not been considered before. The results shown in this chapter indicate that the active constrained layer element presented by Lee could be used to model the interaction between the PZT and structure at high frequencies. Specifically, the shear angle of the PZT can represent the resonance of the PZT since it is a function of independent bending and axial extension of all three layers. Many factors complicate the modeling at high frequencies including nonconstant or not fully known material properties and sensitivity to small imperfections in the structure. These factors may cause varying degrees of shifts and reductions in magnitude of the PZT resonance frequencies. This necessitates a model updating approach, especially to determine representative values for the bonding condition.

Future impedance-based SHM systems will increasingly rely on models for improved functionality. Since the impedance response measure depends significantly on the bonding conditions, an effort should be made to simplify the bond condition for monitoring. This could include not using copper tape on metallic structures, or perhaps
using very thin electrodes, such as gold leaf, for nonmetallic structures. In addition, the results highlight the importance of developing sensor diagnostic techniques such as the methods proposed by Park et al. (2005a) and Chan et al. (2005).

Although resonance of the PZT is one of the main features of the impedance response at high frequencies, it may be beneficial, however, to attempt to avoid frequency regions that include PZT resonances since they dominate the response and may mask changes in the structural response. This is investigated in a later chapter.
Chapter 5

Impedance of Long Beams and Rails

The damage identification problem for structural health monitoring (SHM) is inherently best solved using local techniques. This is due to the fact that failure in a local area caused by local defects or damage can take the entire structure out of service (failure). Impedance-based structural health monitoring relies on the local and active interrogation of a structure by piezoelectric sensor/actuators to be able to detect changes in a structure. An entire structure could be monitored using an array of local sensors that will also facilitate the localization of damage. However, on structures of significant size the number of sensors required to monitor the structure, and thus sensor density becomes important. This is limited by the sensing area of each sensor placed on the structure, which varies with the material and configuration of the structure, as well as the size of the damage to be identified and design of SHM system. The research presented in this chapter describes an investigation of sensing area as related to frequency range of excitation using impedance based SHM. The damage detection ability determined though the experimental investigation is compared to the results predicted by the spectral element based model of impedance-based SHM. Additionally, the impedance of a long rail is compared to a short section of rail both experimentally and analytically. The results show that impedance-based SHM is possible even in the absence of distinct modes or peaks in the impedance response.

5.1 Experimental Investigation of a Long Beam

The test specimen consists of a 2.44 x 0.051 x 0.0065 m pultruded fiberglass beam hung to approximate free-free boundary conditions. The beam was manufactured by
Strongwell Corporation and was part of their EXTREN series 625 fiberglass plates. A square piezoceramic patch (Piezo Systems, Inc. material type PSI-5H-S4-ENH), covering the entire width of the beam (0.051 m), and 0.267 mm thick is bonded to one side at the end of the beam as seen in A copper electrode was attached to the bonded side of the patch and protrudes from under the PZT to allow for electrical connection.

![Figure 5.1. PZT bonded to end of composite beam.](image)

Twenty-two baseline impedance measurements were made with an HP 4194A impedance analyzer. These tests were made over the period of 1 week in order to capture the variability during the testing in the timeframe that the remaining tests with damage would take. Each baseline measurement consists of 63, 400 point measurements at 1 V excitation. Each 400 point measurement has a frequency range of 4 kHz giving a total frequency range from 0.1 to 252.1 kHz. A mean baseline measurement was calculated and is shown in Figure 5.2. From that, the root mean squared deviation (RMSD) values for the real part of each individual baseline compared to the mean baseline were calculated as a damage metric.
Figure 5.2. Baseline impedance measurements and average.

Figure 5.3. Average baseline damage metric.
The mean and standard deviation of the damage metric was then calculated for each frequency range as shown in Figure 5.3. Since all the measurements are for an undamaged beam, for frequency ranges with larger damage metrics the variations are greater.

Damage to the beam was simulated by adding 50 g of mass (25 g on each side) to a point on the beam. This method of simulating damage was used so that damage could be relocated on the beam or completely removed. Measurements were made at 16 intervals along the beam. The first mass location was 0.099 m from the edge of the PZT. For the second damage location the mass was moved to 0.251 m from the edge of the PZT and additional damage locations were spaced approximately every 0.15 m. The RMSD value was calculated for each frequency range at each damage location. These damage metrics compared to the average baseline damage metric are shown in Figure 5.4.

Figure 5.4. Damage metric with decreasing distance to damage location.
In Figure 5.4, the damage metric for the first impedance measurement corresponds to the average baseline damage metric with the subsequent measures showing the damage metric as the distance to the PZT decreased. It is clear that at the lower frequencies the PZT can “see” the mass even when it is at the other end of the beam. Increase in the damage metric for the measurements made with the mass at frequency ranges where there are peaks in the impedance measurements may indicate that those frequencies are sensitive to damage. However, closer investigation shows that the baseline damage metric also increases at those frequencies as well as seen in Figure 5.5.

A better approach is to utilize hypothesis testing to calculate if the impedance measurement for that frequency range is statistically significantly different based on a given confidence interval. For this, the damage metric values for each frequency range are assumed to be normally distributed with a mean and standard deviation calculated from the baseline measurements. A one sided $t$-statistic is calculated as

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s}$$

where $\bar{x}$ is the damage metric being tested, $\mu_0$ and $s$ are the mean and standard deviation respectively of the baseline damage metrics and $n$ is one since only one measurement was made of that impedance with mass at that location. If the $t$-statistic is greater than 2.508 (for 22 baseline measurements), then damage is indicated with 99% confidence at that frequency range for the mass location in question. When this approach is used, only the measurement with mass located 0.099 m from the PZT predicts damage at most frequency ranges greater than 20 kHz. The rate at which the higher frequency ranges detect the mass reduces quickly and by the time the mass is moved approximately 0.86 meters away no frequencies greater than 20 kHz can indicate that the mass has been attached to the beam. A typical damage metric for the beam is shown in Figure 5.6. The trend indicates that frequencies with large peaks in the impedance measurement may not be as capable of identifying damage as those that are have a more level profile. These
peaks are caused by the resonance of the PZT. The structural response of the beam is masked by the PZT resonance in these areas of the impedance response.

Figure 5.5. Average baseline damage metric and real impedance.

Figure 5.6. $t$-statistic of impedance measurement with mass at 1.45 m from PZT.
5.2 Modeling with Spectral Elements and Circuit Analysis

Many factors will affect the sensor density in the design of a SHM system. It may be impossible to test all of the possible sensor placements. Additionally, power restrictions may limit the level of actuation allowed for active sensors, which would affect their sensing region. Hence, prediction of the sensing region with a model becomes important. Using the spectral element combined with circuit analysis based model the sensing area of the impedance method is addressed. The material parameters were estimated from the manufacturer’s data sheets, though these values were reported simply as an estimate for design. Also, the properties of the individual components of the composite were not reported. The material was assumed to be isotropic to simplify the analysis for this experiment. A flexural modulus of 14 GPa and tensile modulus of 12 GPa was reported by the manufacturer, however, a modulus of 19 GPa was found to be more representative of the actual dynamics. A density of 1870 kg/m$^3$ was calculated and used in the model. The loss factor was estimated to be 0.032 for all frequencies.

An initial investigation was made by modeling the added mass to the beam as a point mass. The effect of the point mass on the dynamic stiffness matrix was calculated as

$$\hat{F}_i = -m\ddot{u}_i$$

where $\hat{F}_i$ is the force at the node where the mass is added, $m$ is the added mass and $\ddot{u}_i$ is the second time derivative of the transverse or axial displacement and the node where the mass is added. This simplifies to $\hat{F}_i = m\omega^2$, where $\omega$ is the excitation frequency, in the spectral element formulation, and is subtracted from the corresponding element in the dynamic stiffness matrix. Results from the first frequency range are shown in Figure 5.7. Similar trends are seen in the experimental and model responses, indicating that the model may be used to predict the sensing area of the structure.
Figure 5.7. Comparison of impedance measurements as added mass is moved further from PZT. The top plot shows the model’s response and bottom plot shows the experimental result.

Results at higher frequencies again show the possibility of modeling the PZT resonances using Lee’s spectral element (Figure 5.8). As with the aluminum beam, the shear modulus of the adhesive layer used in the model (6MPa) was much less than the reported material parameter. This indicates poor adhesion, which is not unexpected due to the relatively large dimension of the PZT and reiterates the importance of developing accurate sensor diagnostic procedures. The elastic modulus used in the model was 0.7 GPa and loss factor assumed to be twice that of the base material. Although all resonances are not as clear in the total response as in the experimental data (Figure 5.2), when the response of the different modes of vibration is shown individually (Figure 5.9), the PZT resonances are evident and at approximately the same number as the experimental data. Due to the fact that the PZT’s are square, one can expect the PZT
resonances in the length and width direction to be at the same frequency. This is one possible reason that the experimental PZT resonances may be more pronounced.

Figure 5.8. High frequency results for composite beam.
The response of the model up to 100 kHz with added mass was compared to the baseline results using the cross correlation damage metric. The $t$-statistic was calculated assuming a baseline damage metric of zero and variance equal to that found in the experimental model. Initial values computed for the $t$-statistic shown in Figure 5.10 over-predict the sensing ability of the PZT. However, increasing the loss factor, as seen in Figure 5.11, produces results similar to the experimental results. Two possible explanations for this are: 1) That this difference could be due to an inaccurate estimate of damping in the original model or 2) That modeling the mass as a point mass without including the joint between the added mass and structure neglects to take into account the damping between the surfaces. Figure 5.12 shows the effect of doubling the mass added. The model predicts a relatively small increase in the $t$-statistic compared to the effect seen with an increase in damping. This indicates that the damage metric is either close to saturated and would not be able to easily distinguish between the two damage levels, or the PZT still can not “see” the added mass at that frequency range.

Figure 5.9. Impedance response of individual modes of vibration of PZT bonded to fiberglass beam.
Figure 5.10. Initial sensing ability results on composite beam.

Figure 5.11. Sensing ability of PZT on composite beam with increased loss factor.

Figure 5.12. Sensitivity to increased mass added.
5.3 Monitoring of Rails

Rail lines are subject to many types of damage that can, in the worst cases, cause train derailments. Current rail inspection techniques require train traffic to be interrupted while workers and equipment move along the track. Moreover, a technician with rail testing experience is required to analyze the results. Simple proof of concept experiments to determine if impedance based structural health monitoring could be used to detect anomalies in rail tracks, and in particular broken rails was conducted and is reported in Bouteiller et al. (2006). This section is based on an extension of that study with an investigation of a very long rail.

The monitoring device is envisioned to be powered by ambient vibration and thermal gradients provided by passing trains and daily thermal cycles. The device would store the energy and utilize the stored energy periodically to inspect the track (according to the track usage schedule). If damage occurs or starts to occur, a warning signal would be transmitted to a substation, and then broadcast to the appropriate operator listing the location and extent of the damage.

Railroads are relied on throughout the U.S and the world to transport a variety of cargos. The costs of a rail accident have the potential be enormous, especially in the cases of passenger trains or trains carrying hazardous materials. One of the most damaging rail accidents is a derailment. The majority of derailment causes can be grouped into three major categories: track-related, equipment-related and train operations-related (Transportation Safety Board of Canada, 2004, Federal Railroad Administration, 2002). Detecting problems with the rail, such as track buckling, broken rails and track geometry issues, before they cause an accident will improve the safety and reliability of rail transportation by preventing derailments due to track failure. This section focuses on proof of concept experiments to determine if impedance based structural health monitoring may be used to detect anomalies in rail tracks, and in particular broken rails.
The costs of rail failures include not only inspection costs, but also train delays (when penalties are payable by the track authority and its contractors to train operating companies), remedial treatments (rail replacement, weld repair), pre-emptive treatments (rail grinding), costs due to derailment and loss of business confidence and customer support. Some of these costs can be effectively established, but, for example, it is difficult to estimate the cost of a derailment. Moreover, derailments are more frequent in low-speed freight systems where they are less devastating than high-speed passenger networks. A high-speed derailment can be overwhelming. The cost of rail failure in the US, for direct property damage only, has been estimated at $111 million in the year 2000 and has been increasing. As the reported damage figures only include property damage, the full cost of these accidents is estimated to be about 2 to 3 times higher (Federal Railroad Administration).

Today, the most common non-destructive testing (NDT) technique used to detect cracks in rails is ultrasonic testing. Some research has been devoted to improving the inspection speed and the software necessary for interpretation of the signals to give better information to the technician (SGS X-PER-X, Inc.). Techniques using eddy current or magnetic induction have also been studied (Oukhellou and Aknin, 1997). These methods are time consuming inspections, requiring the traffic to be interrupted during the testing. In addition, a technician with rail testing experience is required to analyze the results.

The main advantage of using the impedance method for rail inspections would be to ensure a real-time damage detection method. This avoids the need for stopping traffic on the track and tedious examinations. Thus, the impedance-based structural health monitoring method could prove to be quite useful for the rail industry. The following sections first describe some generalities about railroad tracks, derailments, and rail defects. Experimental results that examine the impedance method used on a rail under laboratory conditions from Bouteiller et al. are discussed. Finally, comparisons between a short sample rail used in the laboratory and an infinite length rail are made using both experimental results from a portable impedance analyzer and the SEM with circuit analysis model.
5.4 Rail and Damage Description

The steel rail is responsible for transmitting wheel forces to the track bed and guiding vehicles. The rail is exposed to harsh environments and endures large cyclic loads due to the rolling wheel contact forces. An increase in the forces due to the wheel is anticipated, as the trend is moving toward heavier cars with greater axle loads and higher train speeds. These changes increase potential accident severity (Federal Railroad Administration). The rail is susceptible to metal fatigue which can lead to its partial or complete failure. There is no redundancy if a rail fails. For this reason, initial cracks must be detected and rectified immediately. If the damage is not detected in time, catastrophic derailment of vehicles may result. (Cannon et al., 2003)

The standard form of rail used around the world is the "flat bottom" or "Vignole-type" rail, named after the British engineer who designed it. This rail has a wide base or "foot", a narrower top or "head", and in between is the "web" as seen in Figure 5.13.

![Figure 5.13. Shape of the rail.](image)

The rails are connected to sleepers, (also known as ties) made from concrete or wood. This keeps the rails aligned and supports them vertically. The sleepers are in turn supported by the ballast (usually rock or gravel), which is supported by subballast and...
finally the substructure of the railway. Rails are connected to each other by welding and fishplates, which bolt onto each rail to provide a reinforcing lap joint.

Defects in the rail can either be due to initial manufacturing imperfections of the rail (ex: residual stresses) or the consequence of train traffic. In general, they are of a macroscopic and geometric nature and usually reversible by track maintenance. The replacement or repair of the rail depends on the severity of the damage present. Several defects in the vicinity of each other increase their severity and require quicker replacement or repair. If there is a threat of a rail breaking, the speed limit will be reduced for that section, and the rail is replaced as soon as possible. The following is a summary of common defects found in rails as provided by Cannon et al.

A classic example of a rail manufacturing defect is the tache ovale or kidney defect caused by thermal effects during rail manufacturing. This defect may be the origin of very serious problems and even reach epidemic proportions in rails if there is a problem with the manufacturing process. The kidney defect is detected either by visual inspection or with aid from ultrasonic equipment.

![Figure 5.14. Rolling surface disintegration.](image)

Horizontal cracking of the rolling surface also originates in the manufacturing stage. Cracking may cause peeling of the rolling surface and is also detected by either visual inspection or with the aid of ultrasonic equipment. Longitudinal vertical cracking may expand and split the rail head in two. Detected by ultrasonic equipment, rails containing longitudinal cracking should be immediately replaced. Another defect due to the rail manufacturing process is a gradual disintegration of the rolling surface of the rail (Figure
Some damage, like the wheelburn defect caused by spinning wheels, is the consequence of inappropriate handling, installation and use. There are also defects caused by the exhaustion of the rail steel’s inherent resistance to fatigue damage. For example, surface gauge-corner head shelling first shows long dark spots. These spots are early signs of underlying metal disintegration which can worsen and lead to the formation of lips on the side face. Of course, this represents a non-exhaustive list of potential defects.

Preventive maintenance and inspection tasks of the rail are a large expense for railroads companies. To ensure the structural integrity of the rail at all times, annual ultrasonic inspections are required. Most of the time, no damage is found during these expensive inspections. However, inspections cannot be eliminated because the security of the users cannot be put in jeopardy, and the repercussions of non-detected damage may be so large. Thus, in the engineering community, there is a growing interest in developing new techniques based on real-time damage detection methods. The impedance-based health monitoring technique is highly suitable for this task. Many traditional NDE methods must be performed during out of service periods or can only be applied at certain intervals, whereas the impedance-based method provides continuous, high speed, on-line monitoring with potential autonomous use. By using a qualitative technique such as the impedance-based method, tracks can be continuously monitored. Moreover, a better understanding of the current status of the rail would give a better maintenance plan and reduce the frequency and cost of service inspections. The impedance-based method can access remote areas, and the bulky equipment used for conventional NDE would be replaced by a portable and remote-controlled device. The ultimate goal of this method is to be able to remotely monitor the structure integrity with a computer and warn the user when a rail failure occurs.
As mentioned, proof-of-concept experiments were performed in Bouteiller et al. (2006). These experiments were performed on a 1.42 m long section of sample railroad provided by the Association of American Railroads (AAR) seen in Figure 5.15. PZT sensors were bonded to the structure and the effects on the RMSD damage metric of various types, locations and levels of simulated damage were investigated. Significant results from these tests include:

- Added mass (magnets) can be detected on the web of the rail, however, the RMSD values when the magnet is attached on the head or on the foot are not easily distinguishable with RMSD values from the baseline.
- Higher frequencies are not sensitive to added mass since it acts more as a boundary condition change than damage to the structure.
- Higher frequencies can better distinguish between different levels of damage than low frequencies
- The impedance method was not able to localize the damage along the relatively short length of rail.

### 5.5 Long Rail Testing with AD5933

Based on the results in Bouteiller et al., further testing was performed on actual track. An Analog Devices AD5933 evaluation board was used to make impedance measurements of a PZT bonded to an *in situ* rail.
The AD5933 is a chip sized impedance converter. It is provided on an evaluation board (designated AD5933EB), which includes the necessary circuitry to make an impedance measurement. The chip and board are not customized for health monitoring, however, can potentially be powered solely by the USP port of a laptop computer. (though in the testing presented here, an external power source was used for more consistent results.) Hence, the AD5933EB is useful for procuring one-off in the field measurements.

The AD5933 primarily records the magnitude of impedance. The real and imaginary parts of impedance can be calculated, however, the AD5933 does not record the phase data with enough precision near the limits of its measurement range to make the transformation meaningful. The nominal range for the AD5933 is 100 ohm to $10 \times 10^6$ ohm. The nominal frequency range is 0-100 kHz. The magnitude data still contains approximately the same information as the real data, though, as seen in Figure 5.16, which show the real part of impedance measured by the HP4194A and the magnitude of impedance measured by the AD5933 for a PZT bonded to a bolted joint structure. In order to force the measured impedance of the PZT into the measurable range of the AD5933, a 500 ohm resistor was connected in series with the PZT. The AD5933 was also calibrated using a 500 ohm resistor. A 2 V peak to peak excitation and 40k ohm feedback resistor were used for all AD5933 impedance measurements.

![Figure 5.16](image-url)

Figure 5.16. a) Real part of impedance measured by HP4194a. b) Magnitude of Impedance measured by AD5933 evaluation board.
The impedance of a 2.54 x 2.54 cm PZT bonded to the short rail sample was measured using the AD5933 with the same settings as was used with the bolted joint to confirm that the settings would work for PZT’s with slightly different values of impedance. Next a PZT the same size as was used on sample rail was bonded to a long straight section of track. The surface of the rail web was first sanded to provide a good bond surface. Cyanoacrylate was used as the adhesive. The PZT was held in place by hand with firm pressure during the first several minutes after placement.

After allowing the cyanocrylate adhesive to dry for over 4 hours, the impedance was measured with the AD5933. Two baseline measurements were made at both 10-20 kHz and 30-40 kHz. The vibrations produced by the PZT could be heard in the rail in the audible range of excitation. This indicates that the bond was sufficient.

The rail could not actually be damaged for this testing, so a series of magnets, as were used in the study on the short section of rail (one and two 25 g magnets and a 2.6kg magnet) were attached to the web of the rail 10-20 cm from the PZT location. An impedance measurement in the 10-20 kHz range was made after each mass addition. Only one “damage” measurement with all the magnets attached was made for the 30-40 kHz range. The measurements are shown in Figure 5.17 and Figure 5.18. After testing the PZT was removed.
The measured response contains significant levels of noise. A few peaks in the data seem repeatable from one measurement to the next, however, it is impossible to discern whether they are artifacts of the AD5933 or actual structural peaks. The measurements were compared with the correlation coefficient damage metric to determine if the
impedance measurements changed in response to the added mass. The results are shown in Figure 5.19 a and b.

![Figure 5.19](image)

**Figure 5.19.** a) Damage metric for 10-20 kHz frequency range on in situ rail. b) Damage metric for frequency range 30-40 kHz on in situ rail.

Although all cases with an added mass show an increase in the damage metric, it is doubtful that the impedance method could reliably detect this change since the change in the damage metric is small. Unfortunately, it was not possible to make enough measurements to quantify the variability of the baseline. However, the same tests were repeated on the short rail in the lab with the AD5933. The results are shown in Figure 5.20. In this case, the damage metric increases significantly for all of the damage cases. In addition clear peaks are seen in the response.

![Figure 5.20](image)

**Figure 5.20.** a) Damage metric on sample rail with added mass. b) Impedance (magnitude) of PZT bonded to sample rail from 10-20 kHz.
It should also be noted that the height of the correlation coefficient damage metric is approximately 0.015 for all of the damage cases on the short rail, frequency range 10-20 kHz. On the in situ rail, the damage metric is approximately 0.015 for one of the damage cases, however, is close to .01 for the two others. As will be discussed in more detail in the next chapter, correlation coefficient damage metrics can be directly compared. This indicates that the in situ beam measurements are not reacting to the damage. Since the bond to the rail appeared to be sufficient, it is likely that this is due to the lack of reflected waves on the near infinitely long rail. This may not have allowed the attached mass to be fully excited and thus would produce no reflected waves.

5.6 Modeling of Infinite Beams

To more fully examine the possibility of using impedance-based SHM on long one-dimensional structures, the spectral element combined with circuit analysis model is used for further analysis. The goal was not to exactly replicate the impedance measured by a PZT attached to the rail, but to assess the effect of the infinite boundary condition on the ability to detect damage.

In this investigation, for simplicity, only transverse vibrations were considered. The rail was treated as a rectangular beam with a width and thickness of 18.6 cm and 1.8 cm, respectively. The width of the model beam is the height of the rail in its normal orientation. The thickness of the model beam is the thickness of the web at the location of the bonded PZT. The second area moment of inertia and area of the model beam is input as 6.1186 x 10^{-6} m^{4} and 8.9 x 10^{-3} m^{2}, respectively. These are the moment of inertia and area given by Jeong et al. (1998) for a 140 RE rail, as designated by the Federal Railroad Administration. Initially, five elements were used to model the short sample section of rail as seen in Figure 5.21.

![Figure 5.21. Short rail section beam model. (Dimensions are in cm.)](image)
The SEM model is only a one dimensional model, so it can not accurately handle the bonded PZT, which in this case does not cover the entire width of the rail. Thus, the Lee active constrained layer element is not used. The Timoshenko beam SEM with circuit analysis model is used to represent the response. Again, in this case, the model will not exactly predict the impedance of the PZT patch bonded to the rail. It will, however, provide insight into the effects of the infinite boundary conditions. Damage was simulated in the model by reducing the area, thickness and moment of inertia values of a 10 cm section of beam, 20 cm from the PZT. The damaged value for area was reduced to one half the original value, for thickness one half the original value and for moment of inertia, one quarter the original value. The size of the damage section was chosen to be the size of damage observed on actual rails. The amount of the reduction in values was chosen so that a change in the impedance signature was clearly visible. Results are shown in Figure 5.22.

![Graph showing impedance vs. frequency for model and model with simulated damage.](image)

**Figure 5.22.** Beam model that is the length of the short sample rail.

To evaluate an infinite length beam, a one-noded or throw-off element is used. The formulation of the throw-off element for a Timoshenko beam is given in Doyle as

\[
\begin{bmatrix}
\dot{V}_1 \\
\dot{M}_1
\end{bmatrix} =
\begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
\begin{bmatrix}
\dot{v}_1 \\
\dot{\phi}_1
\end{bmatrix}.
\]  

(5.3)

The individual stiffness terms are given as
\[ \hat{k}_{11} = \frac{k_2^2 - k_1^2}{R_1 - R_2}, \quad \hat{k}_{12} = \hat{k}_{21} = \frac{i(k_1 - k_2)}{R_1 - R_2}, \quad k_{22} = \frac{i(R_1k_2 - R_2k_1)}{R_1 - R_2}, \quad (5.4) \]

where \( k_1 \) and \( k_2 \) are the wave numbers and \( R_1 \) and \( R_2 \) are the amplitude ratios defined in Chapter 2. The throw-off element is formulated by neglecting the terms in the equation of motion that correspond to waves moving from the infinite end.

Initially, the throw-off element is evaluated with a four element model that includes an element where the moments are added for the PZT, an element of normal rail between the PZT element and the damage, the damage element and finally the throw-off element. First, the model was evaluated with no damage. As can be seen the results shown in Figure 5.23 a, the impedance response of the semi-infinite beam exhibits no peaks. However, when damage is introduced, waves are reflected back to the PZT. Therefore, peaks appear in the response (Figure 5.23 b).

Figure 5.23. a) Impedance response of semi-infinite beam. b) Impedance response of semi-infinite beam with damage.

Similar results were obtained by Ahmida and Arruda (2002), though only low frequencies and a single point force were considered. At higher frequencies or for larger PZT’s, the waves produced at one end of the PZT begin to have an effect at the other end of the PZT and the free end. The effects of several PZT sizes and an increase in stiffness are shown in Figure 5.24. The cusp seen in all responses at 17.75 kHz, except the reduced beam stiffness response, is the cut-on frequency where the imaginary wavenumbers of
the Timoshenko beam become real. Doyle gives the expression for this cut on frequency as

\[ \omega_c = \sqrt{\frac{GAK}{\rho l K_2}}. \]  

(5.5)

Figure 5.24. Impedance response of semi-infinite beam with various configurations.

If a second throw-off element is added to the opposite end of the beam, the impedance response is close to a flat line. However, when damage is introduced the response changes significantly as seen in Figure 5.25 a and b.

Figure 5.25. a) Infinite beam response with and without damage. b) Close up of response from 20-50 kHz.
This indicates that impedance based SHM can detect damage even in the absence of peaks. The techniques shown in this study are not only applicable to rails, but also to any large, one-dimensional structure such as a cable on a suspension bridge or long boom on a satellite. Another possibility for impedance-based monitoring on rails not investigated here is monitoring with multiple sensors so that not only reflected waves will have an effect, but modulation of the waves (in the presence of damage) will be revealed if no wave is reflected back to the originating PZT with enough energy to be reflected. Also, the support structure of the rail was not considered in this investigation.

5.7 Conclusions

Many of the laboratory studies used to evaluate impedance-based SHM have only considered relatively small parts compared to the actual structures on which the technique may be used. In this chapter the impedance method is tested on two structures with one dimension much larger than what the PZT can excite: a long, highly damped beam, and an infinite length rail. These structures were used to investigate several aspects of the impedance response. It was found that the sensing area varies greatly with excitation frequency of the PZT as expected. Additionally, it is influenced by the peaks in the response due to the resonances of the bonded PZT. The damping of the structure is critical to the size sensing area and thus it is also essential that the loss factor be correctly identified for modeling. The spectral element combined with circuit analysis model developed is useful for determination of sensing area. The spectral element model was also modified to represent an infinite length rail. A very long or infinite length rail will have no resonance peaks in the impedance response. In the experimental studies, it was impossible to induce realistic damage to an actual rail to determine if impedance-based SHM would be effective. However, the model shows that impedance-based SHM will still be able to detect damage even in the absence of peaks normally seen in the response.
Chapter 6

Statistical Analysis of Damage Metric

At very high frequencies it becomes increasingly difficult to predict the exact impedance signature of a piezo sensor exciting a real structure. Many factors complicate any attempt at modeling in the frequency ranges typically used for impedance based structural health monitoring. At such frequencies the measured signal can easily appear like a random signal, except that it is repeatable upon subsequent measurements. Hence, it is convenient to view the impedance response in a statistical framework using a damage metric.

In order to reliably set a threshold for a damage metric, it is necessary to know the underlying distribution of the metric with no damage. In addition, this allows comparison of one set of data with associated baselines to another set with its own baseline measurements. Park et al. (2005b) have suggested using extreme value statistics with an autoregressive model with exogenous inputs. Knowing the base distribution of the damage metric could help to determine which extreme value distribution would be most applicable in their outlier analysis framework. The general goal of a damage metric is to determine how similar an impedance measurement is to an undamaged impedance measurement. The most common damage metrics include the root mean square deviation (RMSD) and correlation coefficient damage metrics. Other damage metrics, such as the sum of the square differences or error and covariance, are closely related to these.

Some authors have shown limitations with the statistical approach and suggested a parametric approach relying on frequency shifts of resonances instead (Naidu, 2003).
However, such approaches are impractical at frequencies high enough to see small amounts of damage.

### 6.1 Statistical Modeling of Damage Metrics

The RMSD damage metric is defined as

\[
\text{RMSD} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{Z_{1,i} - Z_{2,i}}{Z_{1,i}} \right)^2}
\]

(6.1)

where \(Z_1\) and \(Z_2\) are the real parts of the baseline measurement and measurement used for comparison respectively, \(i\) is the frequency interval and \(n\) is the total number of frequency points used in the comparison. Alternatively, the RMSD is sometimes scaled by the baseline values instead of the number of points as defined as

\[
\text{RMSD} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{Z_{1,i} - Z_{2,i}}{Z_{2,i}} \right)^2}
\]

(6.2)

This is an attempt to make the damage metric unaffected by the level of the impedance measurements. However, it makes the damage metric vary according to number of points contained in the measurements compared.

Often impedance measurements shift slightly in the vertical plane, as seen in the plot of two healthy baseline measurements (Figure 6.1a).
Figure 6.1. a) Example of vertical shifting in undamaged baselines. b) Baselines with vertical shift removed.

While this would cause an increase in each of the above damage metrics (eq. 1 and 2), it is generally a result of slight changes in temperature or possibly resistance of the connections to the analyzer, not a result of damage. To make the damage metric insensitive to these shifts, the mean of each measurement is subtracted from the measurements producing the result seen in Figure 6.1 b. The RMSD damage metric then becomes

$$\text{RMSD} = \sqrt{\frac{\sum_{i=1}^{n} (Z_{1,i} - \bar{Z}_1 - (Z_{2,i} - \bar{Z}_2))^2}{n}}, \quad (6.3)$$

where the overbar indicates the mean of the measurement. This is similar to the average squared difference damage metric presented by Raju (1997).

The correlation coefficient damage metric is closely related to the RMSD damage metric and is given by

$$1 - \rho = 1 - \frac{\text{cov}(Z_{1},Z_{2})}{S_{Z_{1}} S_{Z_{2}}} = 1 - \frac{\sum_{i=1}^{n} (Z_{1,i} - \bar{Z}_1)(Z_{2,i} - \bar{Z}_2)}{n - 1 S_{Z_{1}} S_{Z_{2}}}, \quad (6.4)$$
where $\rho$ is the correlation coefficient, $\text{cov}$ is the expected value of the product of $Z_1$-mean($Z_1$) and $Z_2$-mean($Z_2$) (covariance) and $S$ represents the sample standard deviation. Again, $Z$ is generally just the real part of impedance. The correlation coefficient is an indicator of how well the baseline and measurement being tested are linearly related. The formulation is generally based on an ordinary least squares regression. The baseline and test measurement are plotted against each other and the linear regression of the test data of the baseline data is calculated. Equations for the regression line are found in most statistics texts. This method considers the vertical distance between the data plotted against each other and the linear regression of the test data on the baseline data. The regression line minimizes the sum of the square of this distance, known as the residual. The sum of the residuals squared is related to the correlation coefficient as

$$
(1 - \rho^2)^\frac{1}{2} S_{Z_2} = \sum_{i=1}^{n} (Z_{i,2} - Z_2) - \frac{\text{cov}(Z_1, Z_2)}{S_{Z_1}^2} (Z_{i,1} - Z_1) \quad (6.5)
$$

Since the covariance of $Z_1$ and $Z_2$ divided by the variance of $Z_1$ is very close to 1 for a measurement with no damage, this is very nearly the same as the square of the difference in $Z_1$-mean($Z_1$) and $Z_2$-mean($Z_2$) as seen in Figure 6.2. The difference between the residuals becomes more pronounced as the test measurement becomes less like the baseline (as with the introduction of damage). This is evident for even a small amount of change (Figure 6.3). This indicates that the RMSD and correlation coefficient damage metrics will react similarly to damage and that the variation in the baseline measurements will have related distributions.
Figure 6.2. Residuals squared and difference squared for undamaged data.

Figure 6.3. a) Baseline (undamaged) and damaged data. b) Residuals squared and difference squared for damaged data. The undamaged residuals and differences are shown for reference.

Only considering the vertical distance between the regression line and data assumes that the measurement of the baseline has no error. Alternatively, considering the distance of the data orthogonally from the regression line treats both variables equally. However, if the baseline is an average of several baseline measurements, it should have less error than a single test measurement. Hence, the ordinary least squares method is a good approximation.
Assumptions of the regression analysis are that the residuals are normally distributed with an expected value of zero and constant variance and that the residuals are independent of each other. The first two of these assumptions appear to be acceptable for the undamaged measurement, as shown in the scatter plot and histogram of the residuals (Figure 6.4). However, the residuals do not appear to be independent of each other as seen in the plot of residuals versus frequency (Figure 6.5). If less frequency points were used over the same frequency range or frequency points were spaced further apart, the residuals would become more independent. However, this may cause loss of sensitivity since it is not known in advance which data points will be affected by damage.

Figure 6.4. a) Regression of undamaged test data on baseline data. b) Histogram of residuals from undamaged test data versus baseline data.

Figure 6.5. Regression error versus frequency. Impedance data is shown for reference.
The distribution of the residuals and differences is normal and provides the basis for the analysis of the distribution of the damage metrics.

For the RMSD damage metric, if it is assumed that \( Z_1 - \text{mean}(Z_1) \) minus \( Z_2 - \text{mean}(Z_2) \) is normally distributed, with a mean of zero and variance of \( \sigma^2 \) then the probability distribution function of this difference is given by

\[
f_X(x|0, \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}, \quad -\infty < x < \infty \tag{6.6}
\]

where \( X \) is the random variable given by \( Z_1 - \text{mean}(Z_1) \) minus \( Z_2 - \text{mean}(Z_2) \). If \( X \) is squared the distribution of a new random variable, \( Y = g(x) = X^2 \), is given by

\[
f_Y(y) = \sum_{i=0}^{k} f_X(g_i^{-1}(y)) \left| \frac{d}{dy} g_i^{-1}(y) \right|
\]  

where \( g_i(y) \) is monotone on the interval \( i \). So for the given \( g \) and \( f_x \) the new distribution becomes

\[
f_Y(y|\sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{y}{2\sigma^2}} \left( \frac{1}{2} y \right) - \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{y}{2\sigma^2}} \left( -\frac{1}{2} y \right) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{y}{2\sigma^2}} \left( \frac{1}{2} y \right)
\]  

\( 0 < x < \infty \)

since \( g(y) \) can equal \( \pm y^{1/2} \). If \( \sigma^2 \) is equal to one, then this result is a chi square distribution with one degree of freedom. Otherwise this is a Gamma distribution with shape parameter, \( \alpha \), and scale parameter, \( \beta \), where \( \beta \) equals \( 2\sigma^2 \) and \( \alpha \) equals \( p/2 \) where \( p \) is the degrees of freedom (one in this case). Since the sum of independent random variables with a gamma distribution and common scale parameter is a gamma distribution with the shape parameters summed, the probability density function (pdf) of the sum of the squared differences (minus the means) is
\[ f_w(w|\sum \alpha, \beta) = \frac{1}{\Gamma(\sum \alpha)\beta^{\sum \alpha}} w^{(\sum \alpha)-1} e^{-\frac{w}{\beta}} = \frac{1}{\Gamma(n/2)(2\sigma^2)^{n/2}} e^{-\frac{\sigma^2}{2\sigma^2} \left(\frac{n-1}{n}\right)} \]  \hspace{1cm} (6.9) \]

where \(\Gamma(\cdot)\) is the gamma function and \(n\) is the number of frequency points.

Following the same methods as before, the distribution of the RMSD damage metric (scaled by the number of points) is

\[ f_m(m) = \sqrt{\frac{W}{n}} = \frac{2}{m\Gamma(n/2)(2\sigma^2)^{n/2}} e^{-\frac{nm^2}{2\sigma^2}} \left(\frac{nm^2}{2}\right)^{\frac{n}{2}} \]  \hspace{1cm} (6.10) \]

where \(m\) is the RMSD damage metric. While not a named distribution, this is similar to a chi distribution. It is skewed, though less than a chi square distribution. As the number of frequency points, \(n\), increases towards infinity, the distribution will become more centered. With a large value of \(n\) it is reasonable to assume that the distribution is normal.

For a SHM system, the baseline should be based on many measurements. The RMSD damage metric with subtracted means does not assume this is the case because only the mean of the frequency points is subtracted from the measurements. The Central Limit Theorem provides that, if good sampling techniques are used, the average of the means of each sample will have a normal distribution around the mean of the parent distribution. Otherwise, the data will have a distribution approximating the parent distribution. To simplify the parent distribution of the RMSD the difference in measurements may be scaled by the standard deviation of the multiple measurements. Unlike subtracting the mean of the measurement, this requires calculation across a set of measurements. This modified RMSD damage metric is formulated as
\[ \text{RMSD} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{\left( \left( Z_{1,i} - Z_{1} \right) - \left( Z_{2,i} - Z_{2} \right) \right)}{S_{Z_{1,i}}} \right)^2} \] (6.11)

where \( S_{Z_{1,i}} \) is the standard deviation of each frequency point after the vertical shift of each measurement has been removed. Since multiple baseline measurements have been made the average of the baseline measurements, \( Z_{\text{avg}} \), can be substituted for \( Z_1 \). It should be noted that the \( Z_{\text{avg}} \) differs from \( \overline{Z} \) in that the former is the mean of different sweeps where as the latter is the mean of different frequency points of the same sweep. If it is assumed that each sweep has already been shifted, then, with \( Z_{\text{avg}} \), substituted for \( Z_1 \), the damage metric becomes

\[ \text{RMSD} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{\left( Z_{\text{avg},i} - Z_{2,i} \right)}{S_{Z_{1,i}}} \right)^2}. \] (6.12)

This is recognized as the \( Z \)-statistic (as used for hypothesis testing) for each frequency point, squared and summed. This causes the variance, \( \sigma^2 \), in the equation to be one. Unfortunately, one can not simply look up values from the chi square distribution, which is commonly reported in statistics texts or programs, and then take the square root. In addition, dividing the difference between the measurements by the number of points, while causing the mean of the metric to be insensitive to the length of the measurements, also can not be accounted for by simply dividing the values from tables of the chi square distribution by the number of points. To compare this new damage metric to the RMSD without incorporating the standard deviation into the formulation, a set of 12 baseline measurements plus 18 measurements with varying levels of damage from an MFC actuator on a composite structure are analyzed. All analyses are performed with the measurements shifted so that their means are all zero. The baseline measurements after vertical shifting are shown in Figure 6.6. The expected value of this damage metric with no damage is one.
6.2 Comparison of Common Damage Metrics

Previous researchers have investigated how sensitive various damage metrics used with the impedance method are to damage and how well the metrics correlate to the amount of damage present in the structure (Zagrai and Giurgiutiu, 2001, Grisso, 2004). Here a brief examination of the sensitivity to non-damage changes in the damage metric is presented for the two most common forms of the damage metric, the root mean square deviation and cross correlation coefficient.

A section of the real part of the impedance response of a structure was selected and compared to a measurement of the damaged response. The two responses are seen in Figure 6.7. The units are unimportant since analysis with a damage metric works at any frequency range with any signal.
Various forms of the RMSD damage metric as well as the correlation coefficient damage metric were computed. Next the measurements were modified in ways that would not generally indicate an increase in damage. The first modification involved doubling the length of the measurement by adding each measurement end to end. This would be equivalent to comparing two damage metrics calculated from pairs of measurements that were different lengths. The second modification was doubling the height of each measurement. If pairs of measurements were made at different frequency ranges and the heights of the peaks were different in the two frequency ranges, it would not be desirable for the difference in peak size to affect the comparison of damage metrics. Finally, one measurement was vertically shifted. This occurs frequently in cracked PZT’s due to different areas of the PZT being electrically connected. However, it does not necessarily indicate damage. Also, the primary effect of a small temperature change on the real part of the impedance measurement is often a vertical shift. The results are shown in Table 6-1.

Table 6-1. Damage metric values for non-damage changes in measurements.

<table>
<thead>
<tr>
<th>Damage Metric</th>
<th>original comparison</th>
<th>double length</th>
<th>double height</th>
<th>vertically shift one measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSD</td>
<td>2.554</td>
<td>2.554</td>
<td>5.108</td>
<td>50.065</td>
</tr>
<tr>
<td>RMSD with difference in data normalized by average baseline value</td>
<td>0.304</td>
<td>0.304</td>
<td>0.304</td>
<td>5.952</td>
</tr>
<tr>
<td>RMSD with means shifted to zero</td>
<td>0.304</td>
<td>0.304</td>
<td>0.304</td>
<td>11.891</td>
</tr>
<tr>
<td>RMSD with means shifted to zero and normalized by average baseline value</td>
<td>2.554</td>
<td>2.554</td>
<td>5.108</td>
<td>2.554</td>
</tr>
<tr>
<td>scaled RMSD (divide by baseline)</td>
<td>15.730</td>
<td>22.246</td>
<td>15.730</td>
<td>1047.700</td>
</tr>
<tr>
<td>1-ρxy</td>
<td>0.130</td>
<td>0.130</td>
<td>0.130</td>
<td>0.130</td>
</tr>
</tbody>
</table>

Cases where the damage metric indicates a change where ideally there would be none are indicated with a line through the resulting value. It is evident that the correlation
coefficient damage metric is the least sensitive to non-damage changes in the impedance response. Less common DM’s such as \((1-\rho^2)^3\) and \((1-\rho)^2\), reported by Zagrai and Giurgiutiu, would generally just change scaling of the above damage metrics. A similar analysis was performed by Naidu, however an insensitivity to any change was regarded as a negative effect, which may not always be the case.

### 6.3 Conclusions

Since it is difficult to predict the exact impedance signature of a piezo sensor exciting a real structure, the technique is typically used in a statistical framework. To apply statistical techniques it is necessary to have knowledge of the underlying distribution. It has been shown that even though the baseline damage metric may not be symmetrically distributed, it is sufficient, due to the large number of points typical of an impedance measurement, and can be approximated by the normal or t distribution. This can greatly simplify simple damage detection analysis. A modification of the RMSD damage metric has also been proposed that is essentially the \(Z\)-statistic for each frequency point squared and summed. This uses many baseline measurements to create a more statistically correct damage metric. Additionally, common damage metrics were compared for insensitivity to non-damage changes in the impedance response. The cross correlation coefficient damage metric was shown to be unaffected by non-damage changes in the impedance measurements.
Although, a significant body of research has been developed about impedance-based SHM, the technique is still not developed enough to be a “plug-n-play”, “slap-on” or automatically installed, monitoring system. The method still generally requires personnel familiar with impedance-based structural health monitoring to select parameters of the system to be able to reliably detect small amounts of damage on a structure. The research presented in this chapter addresses the frequency range selection of impedance measurements used for SHM. Two experimental structures are used to examine frequency range selection. The frequency ranges with resonances of the actuator are compared to frequency ranges without. The results indicate that the ranges with the sensor/actuator resonances are better for monitoring with the impedance-based method.

In a monitoring framework, if the damage metric is larger than a user specified level, then the structure is said to be damaged. This level can be set using the method of outlier detection. If the data comes from a normal distribution and the number of measurements is large, a simple \(Z\)-statistic may be used. Also, if the data comes from a non-skewed distribution, the \(t\)-statistic can be used even for small data sets (population variance unknown). Alternatively, some researchers have concluded that the damage sensitive features in SHM systems will follow an extreme value distribution (Worden, 2002). In this case, several different procedures may be used, including: the Box-Cox transformation for transforming Weibull data to a normal distribution, the Wilcoxon test, or tests on the median. The probability (p-value) of a measurement may also be calculated if the population distribution is known. Todd et al. (2001) concluded that
outlier detection methods, or more specifically, the frequency of outliers, while good for detecting damage, are not a good method of classifying levels of damage since the percentage of outliers saturates quickly once the structure is damaged.

Several past studies have examined the selection of appropriate frequency ranges for impedance-based SHM. A suitable frequency range depends on the structure being used and type of damage to be detected. The frequency range selected should have many peaks, since this indicates that there is a large amount of structural information. (Sun et al., 1995b, Park et al., 2003). Often, several frequency ranges that show many peaks are monitored. Historically, the frequency range has been determined through trial and error. Gyekenyesi et al. (2005) performed a good study on frequency range selection for impedance measurements that identified frequency ranges with low variability. Unfortunately, the sensitivity to damage of each frequency range was not taken into account. They also did not associate the regions with the best response to features of the impedance measurement so that future researchers would not need to repeat the same study as theirs. Simmers (2005) has reported results of sensitivity tests for frequency range selection. In several studies, corrosion damage was simulated on structures using small amounts of wax and the frequency ranges with the highest sensitivity were selected for monitoring. In one study of induced corrosion, the variation of the frequency range was taken into account as well by subtracting a confidence interval from the damage metric. Resulting positive values deemed the frequency range acceptable for monitoring. The detection trend was not consistent for different sensors and proximities to the damage nor was detectability related to features of the impedance measurement.

In addition to having high peak density and low noise, in order to ensure that damage can be “seen”, the wavelength of the excitation signal must be smaller than the characteristic length of the damage (Nokes and Cloud, 1993). However, at frequency ranges high enough to produce wavelengths that can detect small amounts of damage, the characteristics of the sensor/actuator itself may have an effect on the ability to detect damage. Specifically, resonances of the piezoelectric sensor/actuator often cause large
peaks in the impedance signature, as well as an increase in modal density near the resonance, which may affect the sensing abilities at those frequencies.

In order to investigate the effect of the active sensor resonances on sensing ability, experiments were performed on two structures, an aluminum beam, and a composite boom. The composite was expected to exhibit higher damping and thus a more localized sensing area. PZT’s were used as actuators in the aluminum beam experiment and macro-fiber composites (MFC), which could conform to a curved shape, were used in the composite boom experiments.

### 7.1 Aluminum Beam Experiment

In the first experiment, two different sized PZT patches were attached to opposing sides of a 121.92 x 3.175 x .635 cm aluminum beam as seen in Figure 7.1. The dimensions of PZT 1 and PZT 2 measure 1.27 x 2.54 cm and 3.83 x 2.54 cm respectively. Both PZT’s are Piezo Systems, Inc. material PSI - 5H4E, 0.267 mm thick, bonded to the base structure with cyanoacrylate, with copper tape protruding from underneath to provide for electrical connection. Before bonding, the impedance of each PZT was measured in its free condition from 0.1 to 200 kHz in 20 Hz intervals. Electrical connection was made by attaching copper tape with conductive adhesive to both electrodes of the PZT. The resulting impedances are shown in Figure 7.2.

![Figure 7.1. a. PZT 1 b. PZT 2 (attached to same end, opposite side of beam).](image-url)
Figure 7.2. Impedance response of unbonded PZTs.

Once the PZTs were bonded, a series of baseline measurements were made over a period of several days with each PZT again from 0.1 to 200 kHz in 20 Hz steps. A 12 gram magnet was then attached to each side of the beam (25 g total), as a simulation of damage, approximately 70.5 cm from the end with the PZTs and the impedance measured in the damage state.

In order to determine how well different frequency ranges for each PZT can detect the damage, the baseline data was averaged to form a single baseline impedance measurement. The data was then analyzed in 8 kHz intervals. Each baseline measurement was compared to the average baseline using the maximum cross correlation between the baseline and damaged measurement for that frequency range as a damage indicator. The correlation was subtracted from 1 to make the comparison consistent with other damage metrics (increasing damage causes an increase in the metric). This damage metric is insensitive to shifting and scaling of the measurements, making it ideal
for comparing measurements between two PZTs and at different frequency ranges. The damage metrics for all baseline measurements were then averaged and the standard deviation calculated for each frequency range. For this test the damage metric values were assumed to be normally distributed. A plot of the average baseline damage metric and standard deviations for each PZT is shown in Figure 7.3 and Figure 7.4.

The metrics for the damaged case were then calculated and compared to the baseline metrics using hypothesis testing for outlier analysis. To test if the mean of the new
sample is larger than the mean of the baseline the following hypothesis test is constructed:

\[ H_0 : \mu = \mu_0 \]
\[ H_1 : \mu \geq \mu_0 \]  \hspace{1cm} (7.1)

where \( \mu \) is the sample mean and \( \mu_0 \) is the population mean, in this case, the mean of the baseline damage metric. The test statistic, \( Z_0 \), was calculated for each frequency range using the following equation,

\[ Z_0 = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \]  \hspace{1cm} (7.2)

where \( \bar{X} \) is the sample mean and \( n \) is the number of samples. In this case, \( \bar{X} \) is the value of the damaged metric, \( n \) is 1 since only one sample is being compared, and \( \mu_0 \) and \( \sigma \) are the baseline mean and standard deviation. This tests whether the mean of the new measurement is the same or greater (using a one sided test) than the mean of the baseline measurements based on a given confidence level. If \( H_0 \) is true the probability is at least \( 1-\alpha \) that the test statistic, \( Z_0 \), is less than \( Z_\alpha \). In this example \( \alpha \) is chosen to be 0.01 so that if \( Z_0 \) is less than 2.33, there is a 99% probability that the mean of population has not changed. Again, in this study, the distribution of damage metrics for a PZT at a given frequency range is assumed to be normal and the baseline measurements are assumed to accurately represent the mean and deviation of the population. For testing on an actual structure, extreme value distributions (Weibull, Gumbel or Frechet) may be a better candidate distribution. However, since this study only aims to compare the ability of two PZT’s to sense damage on the same structure, the more common normal distribution was used.

The resulting \( Z_0 \) values, along with the average baseline measurements and unbonded measurements for PZTs 1 and 2 are shown in Figure 7.5. As can be seen in the first two plots of the baseline impedance signatures with unbonded impedance signatures, the
resonance frequencies of the PZT, in this case, do not change significantly when bonded. Focusing on plot c, as expected, it is shown that the impedance signatures are much more sensitive at low frequencies to the added mass than at high frequencies. In addition, the large PZT does a better job at detecting the damage than the smaller, since it can more effectively excite the beam. Also, comparing the height of the $Z$-statistic at different frequencies seems to indicate the PZT resonances have a diminishing effect on the sensing ability of the large PZT. Plot d compares the sensing ability of the PZTs at each frequency range. Although, the larger PZT better senses the damage than the smaller PZT, at a few frequencies, the small PZT performs equally well, or even better than the large PZT. This is indicated by the ratio of the $Z$-statistic being equal to or larger than 1.0. These frequencies seem to occur in most cases immediately following the peaks in the small PZT unbonded response. The PZT resonances are not clearly visible in the bonded response, however, a small shift to higher frequencies is expected due to the stiffening effects of bonding. The optimum testing frequencies occur where the PZT resonance is expected to be.
Figure 7.5. a) Baseline and unbonded impedance of PZT 2. b) Baseline and unbonded impedance of PZT 1. c) Z statistic for PZT 1 and 2 for added mass. d) Ratio of Z-statistics for added mass.
A second test was carried out to further investigate the sensing ability of the two PZTs. In order to reduce the low frequency sensing ability of each PZT, an additional baseline measurement was made with increased mass variability. A small mass of approximately 2 g was attached to the beam for one of the baseline measurements. In addition, the damage was changed from an added mass to a 1 mm wide quarter width cut. The average damage metric for PZT 1 with the increased mass variability is shown in Figure 7.6. The increase in variability is mostly seen in the lower frequency ranges. This highlights one of the benefits of high frequency methods, that they are less sensitive to boundary condition changes.

![Figure 7.6. Average baseline for PZT 1 (small) with increased mass variability.](image)

Results from this experiment are shown in Figure 7.7. Again, baseline and unbonded measurements are included for reference. It can be seen again that lower frequencies are not as responsive to the damage in this case due to the added mass variability. In addition, the small PZT is relatively more sensitive than in the previous case. The higher frequencies, however, react similarly to the damage. Again, the data indicates that the regions just after the unbonded PZT resonances should have the best sensing ability as test frequency ranges. This emphasizes the importance of investigating the entire frequency range of impedance-based SHM as opposed to simply randomly choosing a frequency range. However, frequencies with a PZT resonance can still detect damage,
and may be more efficient at detecting far away damage because of the greater excitation ability at resonance.

Figure 7.7. a) Baseline and unbonded impedance of PZT 2. b) Baseline and unbonded impedance of PZT 1. c) Z-statistic for PZT 1 for quarter width cut and magnet. d) Ratio of Z-statistics.
7.2 Composite Boom Experiment

The second structure examined was a composite boom for use in a space reflector. It is envisioned that such structures could be made by inflating and then rigidizing them once they have reached orbit. The specific boom used in this study was a thermoplastic resin system rigidizeable boom provided by ILC Dover Inc.

Previous work has investigated the impedance method’s ability to monitor composites. Pohl et al. (2001) induced damage on carbon fiber reinforced polymers and detected it by monitoring changes in the impedance peaks. Identification of delamination in composites was done by Bois and Hochard (2002) using the impedance method. Research by Grisso et al. (2004a, 2004b) shows the feasibility of detecting cracks in cross-ply graphite/epoxy composites using PZT and the impedance method. Some preliminary work has also been presented in Tarazaga et al. (2006a) concerning an inflatable rigidizable boom where damage was simulated by mass loading and the impedance method was used to detect changes.

Commercially available Macro-Fiber Composite (MFC) (Wilkie et al., 2000) piezocomposite devices are used as transducer elements in this study. Piezoelectric composite devices, such as active fiber composites (AFC) (Harrah et al., 2004) or MFC, are flexible and conformable. This permits them to be integrated easily into or on curved structures. They also have high actuation performance and durability, compared with monolithic piezoceramic plates or wafers, and possesses orthotropic mechanical and piezoelectric properties, which can be useful in optimizing their structural actuation and sensing performance (Sodano et al. 2004). MFC piezoelectric composite actuators have also been successfully integrated within space rigidized composite booms in several previous studies at NASA Langley Research Center (Bent et al. 2006, Jenkins et al. 2001, Tarazaga et al., 2006c).

Two sizes of MFC devices were used. The ability of the impedance method to detect actual perforations in the structure, such as those caused by impacting meteorites or
orbital debris (MMOD), is assessed. These perforations potentially could occur in pairs, i.e., with an initial entry hole and a path-dependent exit hole. Previous investigations have looked at detecting and assessing the size of hole damage and location from the collocated sensors (Tarazaga et al. 2006b), where as, the current investigation focuses on frequency range selection.

The experiments were conducted using an untapered, cylindrical, inflatable-rigidizable composite boom. The boom structure consisted of two 0/90 carbon fiber plies consolidated with a proprietary space-rigidizeable thermoplastic resin system. All testing was performed at room temperature where the thermoplastic resin system is below its designed glass transition temperature, and the structure is in its rigid phase. The boom was 1.73 m long by 97.8 mm in outer diameter. Net wall thickness was 0.61 mm though this varied significantly. Holes in the boom already existed in some locations where the fiber tows did not form a close mesh.

The boom was suspended from the ceiling to provide a free-free mechanical boundary configuration. Two commercially manufactured MFC actuators (Smart Material Corporation, Sarasota, FL) were installed near one end of the boom on opposite sides of the structure using epoxy. All electrical impedance measurements were taken using an HP 4194A impedance analyzer. The experimental setup, including the arrangement and dimensions of the MFC devices, is shown in Figure 7.8.

Simulated micrometeorite damage was applied to the structure by hand-drilling holes of various diameters into the boom wall. The holes were applied in pairs to simulate MMOD strikes with entry and possible exit perforations. Hole diameters ranged from 0.79 mm to 4.76 mm, and were drilled in three regions of the structure: at the midpoint of the beam, at ¼ the length of the beam away from the sensors, and at the end of the beam farthest from the sensors. Figure 7.9 shows the regions chosen for the test.
Figure 7.8. a) Suspended boom configuration and HP 4194A Impedance analyzer used for testing and b) Close up of large MFC (109 mm x 73.7 mm) c) Close up of small MFC (31.8 mm x 25.4 mm).

A series of at least 12 baselines consisting of 12400 frequency points each were made with each MFC over a frequency range of 0.1 kHz to 248.08 kHz (20 Hz steps). A sample impedance measurement for each sensor is shown in Figure 7.10. The baselines were taken over several days to account for the possibility of variations caused by fluctuations in the laboratory temperature and relative humidity. The measurements were all taken in sets of three consecutive measurements by an automated control program for the impedance analyzer.

The data is divided into 31 frequency ranges consisting of 401 data points for analysis. Each frequency range is 8 kHz wide. Throughout the analysis each frequency range is
treated independently. In addition, a new set of baseline data was measured before
damage was introduced at a new location. This generates 186 data sets for the current
testing. The baseline measurements are averaged at each frequency range. The damage
metric is then calculated for each baseline measurement by comparing it to the average
baseline. The mean and standard deviation of the damage metrics for each frequency
range is calculated. Examples of the average baseline damage metric for the large and
small MFC’s are shown in Figure 7.11 and Figure 7.12. The level of the average baseline
damage metric was consistent for each set of baseline measurements made for each
damage location. Additionally, higher baseline metrics also have higher variability.

As simulated MMOD damage is applied to the boom, the damage metric for each
succeeding measurement is calculated and then compared to the average baseline
measurement. Example trends in damage metrics with increasing hole diameter are
shown for the large MFCs response in Figure 7.13. Metrics shown are for the 128.1 to
136.1 kHz frequency range and the 192.1 to 200.1 kHz frequency range.

![Figure 7.10. Sample baseline impedance measurements (real part) for large and small
MFC on composite boom.](image)
Figure 7.11. Average baseline damage metric for large MFC, with baseline measurement shown for comparison.

Figure 7.12. Average baseline damage metric for small MFC, with baseline measurement shown for comparison.
Figure 7.13. Damage metric of large MFC at 128-136 kHz and 192-200 kHz, for increasing levels of damage at the midpoint of the boom.

The damage metrics were analyzed using the same method as was used on the aluminum beam testing. In this case though, since multiple measurements were made for one damage level, the number of degrees of freedom in the test statistic was increased to three. This may be too large of an increase because the assumption of independent data is questionable since the measurements were made consecutively in groups of three. Taking the data consecutively causes each measurement in a group of three to be acquired with approximately the same environmental conditions (temperature and humidity) compared to the overall environmental variability. This causes clustering which causes the confidence intervals of the data to be too narrow (Wears, 2002) and ultimately the test statistic to indicate damage more strongly than is actually measured. These effects can be corrected by reducing the degrees of freedom in the calculated test statistic. Since the actual degrees of freedom is not known the number of degrees of freedom are conservatively kept at 1 for the calculation of the test statistic. Variance of the sample being tested is assumed to be the same as the variance of the baseline measurements. This allows inferences to be made using a single population’s characteristics rather than
two different populations. Results for each frequency range for the large MFC with the closest damage location are shown in Figure 7.14.

It can be seen that the frequency ranges that contain resonances of the MFC sensor are more sensitive to the damage than frequency ranges that are flat. Also, the largest increases in damage metric from one damage level to the next are for the largest holes. This may indicate that the damage metric is sensitive to the total area of the damage rather than just diameter of the hole. It should also be noted that the frequency ranges that increase more for the large damage also increase more for the smaller damage. In general, the higher frequencies that are sensitive to damage have a greater distinction between the levels of damage. Finally, several frequency ranges indicate damage when in fact there is none. If the mean value of the test statistic for the data with no damage is larger than the test statistic for 99 percent of the baseline data (99 percent, one-sided confidence interval) then that frequency range is assumed to be unusable for damage detection in this study. The undamaged measurements indicate such an interval has too high a likelihood of false positives. The results for the large MFC with damage at the closest location are shown again without the frequency ranges with false positive indications in Figure 7.14c.

This study had relatively high rates of false positives, indicating that the variance of the data was estimated too narrowly. A primary cause of this is most likely again the assumption of independence of each data point. However, since the same assumptions were made for each frequency range, the results can still be compared across frequency ranges. If waiting for a relatively larger damage size is acceptable, the threshold level may be increased, thus reducing the number of false positives, and the more sensitive frequency ranges may be used. Complete results for each frequency range and damage location are shown in Figure 7.15 and Figure 7.16 for the small and large MFC’s respectively.

It is interesting to observe that for the large MFC the test statistic actually increases on average as the damage location moves further away from the sensor location. This is due to differences in the variability of the baseline damage metric. Since separate sets of
baseline measurements were used for each damage location, the variability changed for each set of damage location. This indicates that the data is non-stationary. Since stationarity is a relative term, this is not an indication of any problem with the impedance method; only that in the future data should be taken for a longer period of time. Additional variability also impacts the analysis since the amounts of damage for different nominal values of damage can not be exactly reproduced. Regardless, the frequency range to frequency range comparisons within each damage location still remain valid, which is the primary concern of this study.
Figure 7.14. Damage metrics of each frequency range for the large MFC with close damage.
Figure 7.15. Damage metrics of each frequency range for the small MFC at each damage location.
Figure 7.16. Damage metrics of each frequency range for the large MFC at each damage location.

Since it is already more clear which frequency ranges are better predictors of damage, comparing the large MFC to the small MFC is not as useful for determining frequency ranges in this experiment compared to the aluminum beam experiment. One interesting observation though is that at the frequency ranges where the small MFC could identify damage, the test statistic was significantly higher for near field damage than the test statistic of the large MFC. The large MFC did a better job of identifying far field damage.
7.3 Conclusions

The aim of this chapter is to address the question of which frequency ranges are best for structural health monitoring using the impedance method. In the past, the ranges are generally selected by trial and error, which often involves inducing a removable amount of simulated damage. Sensitivity analysis of frequency ranges has been performed by few previous researchers; however, since the results have not been compared to features in the impedance measurement, a study would need to be performed for each new structure tested.

Several experiments on two very different types of structures were used to examine the frequency range selection for impedance-based SHM. The frequency ranges that include the resonances of the sensor/actuators are, in general, more suitable frequency ranges to monitor for damage identification. This corroborates with testing at frequency ranges with high modal density. Their variation is less than frequency ranges without a PZT or MFC resonance, but they are more responsive to damage. The actuator resonances can be predicted through models such as the Liang’s impedance model, or by measuring the response before bonding. The large MFC attached to the composite boom had the clearest results in this series of tests. The resonant peaks in the unbonded condition are spaced at a regular interval and also show up clearly in the bonded response. In cases where the actuator resonances are not as evident once bonded, the results indicate the PZT resonance still affect the sensing ability, though to a lesser extent.

The experiments also highlight the advantages of monitoring at high frequency ranges. In the aluminum beam experiment the damage metric is sensitive to damage at high frequency ranges even with changing conditions, in this case mass variability. Also with the composite boom, the higher frequency ranges were more effective at distinguishing various levels of damage.

Frequency range selection was not studied in the presence of large temperature range variations. Unlike adding a mass to induce increased variability, a temperature change
would change properties not only of the structure, but also of the sensor/actuator. The changing sensor properties potentially could cause a shift in the PZT or MFC resonant peak. If the resonant peak location changed, then although the tests presented in this chapter suggest that frequency ranges with peaks due to the sensor/actuator do have better sensing ability, these ranges may need to be avoided. However, the sensitivity of the sensor/actuator resonance peaks to temperature would need additional testing than what was performed in this study to be verified. Other potential investigations could include size of frequency range and number of points in a frequency range.

In summary, the results indicate that frequency ranges that contain resonances of the active sensor are preferable for monitoring. However, in cases where analytical modeling is to be used, it may be preferable to avoid such frequency ranges because the location of such resonances may be difficult to predict if the bonding condition is unknown.
Chapter 8

Conclusions and Recommendations

Structural health monitoring has become a heavily pursued area of research prompted by societal demands and technology providing increasing capability for gathering, transfer and analysis of large quantities of data. Impedance-based SHM shows promise for development into a system to apply SHM to real world structures. An in service system is envisioned to include on board processing and perhaps wireless transfer of data. Ideally, a system could be produced as a slap-on or automatically installed addition to a structure. The research presented in this dissertation addresses issues that will help make such a system a reality.

8.1 Summary and Conclusions

The conclusions and contributions of the work presented can be summarized as follows:

- A model was developed based on circuit analysis of the previously proposed low-cost circuit for impedance-based SHM. It is desired that ultimately, the model could be small enough in size to be included on a chip for local processing of impedance data. Results are shown for a PZT bonded to an aluminum beam. The results capture the main features of the experimental impedance measurement, however, are improved by accounting for details such as added mass due to the electrical connections and excess glue.

- The spectral element method (SEM) is used in combination with the electric circuit analysis for modeling of piezoceramic (PZT) electrical impedances. Timoshenko beam elements are used to model the base structure and three-layer elements including adhesive and PZT layers are used to incorporate resonances of the PZT that occur at high frequencies. A strong dependence of PZT resonance
location on the adhesive layer properties was found, highlighting the importance of sensor diagnostic procedures.

- Investigation of various configurations of possible systems. For a distributed sensor system, the connection of the many transducers and processing of each may become overwhelming if they are all treated individually. Different types of connections are possible, primarily parallel and series with in phase and out of phase actuation, and are investigated. Experimental tests have been performed to determine if a preference towards one type of connection over the other exists and none was apparent. The increase in energy applied to the beam in the parallel versus series configuration indicates that this method of multiplexing is preferable since nonlinear effects of damage may be exercised due to the increased excitation.

- Experimental and analytical investigations have been used to test the variation in sensing area with frequency range. Frequency ranges including a PZT resonance are compared to ranges without to assess their effect on sensing ability.

- Impedance-based SHM was shown, through an analytical investigation, to be able to detect damage on structures that do not exhibit resonances, such as a very long rail.

- The distribution of baseline damage metrics was determined to help assess how the large quantities of data produced by a monitoring system can be handled statistically. Impedance-based SHM is still based on the change in measurements that inherently include statistical uncertainties. Rigorous statistical analysis and signal processing procedures need to be used to evaluate the measurements. It was shown that even though the baseline damage metric might not be symmetrically distributed due to the large number of points typical of an impedance measurement, it can be approximated by the normal or t distribution. This can greatly simplify simple damage detection analysis and allows results to presented with statistical confidence. A modification of the RMSD damage metric has also been proposed that is essentially the squared sum of the Z-statistic for each frequency point.
- Preferred excitation frequencies for micro-fiber composite (MFC) active sensors were determined for a long composite boom under development for use in rigidizable inflatable space structures. The frequencies with resonant peaks were found to have better sensing capabilities.

### 8.2 Recommendations for Future Work

While some issues have been addressed others have been raised. Future investigations should continue to improve the impedance method and characterize its limitations so that it may be used to improve the operation and safety of many types of structures. In this research the model reflected the experimental response at high frequencies only after large adjustments to the bonding layer parameters. The actual values of these parameters are not well known. To help avoid this difficulty, methods for attaching the active sensor to the structure should be investigated and standardized. The bond layer should be repeatable and uncomplicated to allow for modeling.

The establishment of test beds on real structures for evaluating impedance-based and other types of monitoring systems should be strongly pursued. Such test beds should allow long term installation of systems under realistic environmental conditions (as opposed to the laboratory) and possibly allow the introduction of damage or an equivalent simulation.

Future research should continue to develop algorithms that can be incorporated on chip-based SHM systems with limited processing, memory and data transfer capabilities. Additionally hardware improvements to reduce the size and increase the capabilities of impedance based SHM systems, are needed.

In the area of modeling high frequency dynamics, creation of a spectral element code containing an element library tested for accuracy and pre and post processing capabilities similar to commercial finite element programs would be very useful. One possibility would be to incorporate SEM into an existing finite element code. Additionally, two
dimensional spectral elements that are not restricted by a limited set of boundary conditions should be developed. Subsequently, a two-dimensional active constrained layer element could be formulated.
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Vita

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