Exploiting Cyclostationarity for Radio
Environmental Awareness in Cognitive Radios

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(ABSTRACT)

The tremendous ongoing growth of wireless digital communications has raised spectrum shortage and security issues. In particular, the need for new spectrum is the main obstacle in continuing this growth. Recent studies on radio spectrum usage have shown that pre-allocation of spectrum bands to specific wireless communication applications leads to poor utilization of those allocated bands. Therefore, research into new techniques for efficient spectrum utilization is being aggressively pursued by academia, industry, and government. Such research efforts have given birth to two concepts: Cognitive Radio (CR) and Dynamic Spectrum Access (DSA) network.

CR is believed to be the key enabling technology for DSA network implementation. CR based DSA (cDSA) networks utilizes white spectrum for its operational frequency bands. White spectrum is the set of frequency bands which are unoccupied temporarily by the users having first rights to the spectrum (called primary users). The main goal of cDSA networks is to access of white spectrum. For proper access, CR nodes must identify the right cDSA network and the absence of primary users before initiating radio transmission.

To solve the cDSA network access problem, methods are proposed to design unique second-order cyclic features using Orthogonal Frequency Division Multiplexing (OFDM) pilots. By generating distinct OFDM pilot patterns and measuring spectral correlation characteristics of the cyclostationary OFDM signal, CR nodes can detect and uniquely identify cDSA networks. For this purpose, the second-order cyclic features of OFDM pilots are investigated analytically and through computer simulation. Based on analysis results, a general formula for estimating the dominant cycle frequencies is developed. This general formula is used extensively in cDSA network identification and OFDM signal detection, as well as pilot pattern estimation.

CR spectrum awareness capability can be enhanced when it can classify the modulation type of incoming signals at low and varying signal-to-noise ratio. Signal classification allows CR to select a suitable demodulation process at the receiver and to establish a communication link. For this purpose, a threshold-based technique is proposed which utilizes cycle-frequency domain profile for signal detection and feature extraction. Hidden Markov Models (HMMs) are proposed for the signal classifier.

The spectrum awareness capability of CR can be undermined by spoofing radio nodes. Automatic identification of malicious or malfunctioning radio signal transmitters is a major concern for CR information assurance. To minimize the threat from spoofing radio devices, radio signal fingerprinting using second-order cyclic features is proposed as an approach for Specific Emitter Identification (SEI). The feasibility of this approach is demonstrated through the identification of IEEE 802.11a/g OFDM signals from different Wireless Local Area Network (WLAN) card manufactures using HMMs.
Dedication

This dissertation is dedicated to my heavenly Father, God, my wonderful wife HyunKyung Lee, my two lovely daughters, SooYoung and BoYoung, my parents, and my parents in law.
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Chapter

1 Introduction

The tremendous ongoing growth of wireless digital communication has been raised the spectrum shortage and security issues. In particular, the need for new spectrum is the main hurdle in continuing this growth. Recent studies on radio spectrum usage [1, 2] indicates the main reason of spectrum scarcity is that pre-allocation of spectrum bands to specific wireless communication applications leads to poor utilization of those allocated bands. Cognitive radio (CR) and dynamic spectrum access (DSA) are enabling technologies for solving the spectrum scarcity problems.

Realizing the fact that the licensed spectrum remains unused most of the time, the Federal Communication Commission (FCC) is considering a paradigm shift on spectrum allocation policy towards the adoption of unlicensed, rule-based strategies for certain frequency bands. On May 19, 2003, the FCC convened a workshop to examine the impact that CR could have on spectrum utilization and to study the practical regulatory issues that it raises. As a result, the FCC expressed its desire to improve access to radio spectrum through better use of time, frequency, power, bandwidth, and geographic space and its strong belief that CR holds the potential to accomplish these goals [1, 2]

In addition to CR, there is another concept for increasing spectrum usage efficiency. DSA has been proposed by DARPA (Defense Advanced Research Program Agency) xG (next
Generation) program to access all available spectrums dynamically for military and eventually commercial use. This program aims for demonstrating factor of 10 increase spectrum utilization. DARPA xG program believes that CR is one of key enabling technologies for DSA network implementation.

A cDSA (CR based DSA) network can access the available spectrum through the sensing of unoccupied spectrum or “white spectrum”. Then cDSA network uses the white spectrum as long as it doesn’t interfere with “primary users”, those users that have first rights to the spectrum. In a cDSA network, the CR nodes called “secondary users” have to evacuate the spectrum immediately when a primary user occupies the channel and have to move next available white spectrum. This channel migration entails identifying primary users and secondary users. It is challenging for secondary users to determine if a detected signal belongs to a primary user or other CR node. A static common control channel has been suggested to address the primary user and CR node identification problem [3]. However, this approach requires a complex and dedicated common control channel to handle dynamically changing white spectrum. Bureaucratic intervention is needed to establish regulations for the common control channel which would limit the evolution of the CR technologies. Furthermore, the common control channel becomes a single-point-of-failure. To solve this problem, the cyclic features of OFDM (Orthogonal Frequency Division Multiplexing) pilots of the OFDM signal are proposed in this research. The distinct cyclostationary signatures of the OFDM pilots are exploited to identify transmissions that belong to the same cDSA network. The advantage of this technique is that the signal does not need to be demodulated to determine if it belongs to the same radio network and thus no common control channels are required.

The cyclic spectral analysis for cDSA identification is performed using the incoming signal. Each CR node maintains a table containing the cyclic spectral analysis result with time information and setup a plan for moving to the next available white spectrum. The time information indicates when the cyclic spectrum information is obtained. This temporal information can be used to predict the spectrum usage pattern at specific location.
The unique advantages of OFDM based systems make it well suited for cDSA network implementations. By controlling the inputs of IFFT (Inverse Fast Fourier Transformation) during OFDM symbol generation, the spectrum shape can be controlled dynamically to take advantage of white spectrum. In addition, OFDM can effectively remove inter symbol interference and is thus robust to frequency selective fading.

This research provides substantial investigation of OFDM pilot cyclic features using the second-order cyclic spectral analysis and computer simulation. A general formula is derived to predict cyclic features of OFDM pilots. Using this general formula, CR nodes in cDSA network are able to identify other CR nodes. This general formula also facilitates estimating an incoming signal’s pilot patterns.

An example of to cDSA network in commercial sector is IEEE 802.22 WRAN (Wireless Regional Area Network)[4] which is the first worldwide effort to define a novel common air interface standard based on CR. The IEEE 802.22 WRAN uses the unoccupied TV spectrum for its operational band. TV spectrum is attractive because of its deterministic channel allocation and excellent radio propagation characteristics [5]. In addition, the TV bands are heavily underutilized; most TV channels are unoccupied most of the time. To utilize the fallow TV spectrum efficiently and effectively, IEEE 802.22 focuses on developing physical and medium access control (MAC) layers so that WRAN will not interfere with incumbent users, for instance, TV receivers and wireless microphones. Thus, detecting the existence of incumbents, usually at a low SNR, is of paramount importance in making a viable WRAN.

For incumbent signal sensing, energy detection and matched filtering are usually used for CR but they have significant disadvantages. The energy detector shows poor performance in detecting low and varying SNR signals. In the case of the matched filter, although it provides optimal performance in the sense of maximizing received signal-to-noise ratio, a matched filter requires demodulation of a primary user’s signal. This means that cognitive radio must have a priori knowledge of primary user signal at both PHY and MAC layers,
such as modulation type, pulse shaping filter type, and signal frame structure. In addition, those conventional signal detection technologies cannot recognize the modulation type of incoming signal. Modulation classification which tells the modulation type of incoming signal allows CR to select appropriate demodulation method at the receiver.

To achieve signal detection and classification at low SNR signals, the cycle frequency domain profile, its crest factor, and HMMs (Hidden Markov Models) based pattern matching algorithm are suggested in this research. In many scenarios where a conventional energy detector or a matched filter fails to detect a signal, the suggested approach can detect and classify, even when the incoming signal has a very low and varying SNR.

The ideal cDSA network assumes all CRs are working cooperatively toward the common goal of increasing spectral efficiency. However, such an ideal situation may not be guaranteed. Malfunctioning CR devices or intentional malicious devices can spoof the CR network. If a spoofing user disseminates false information to other CRs, the results could be catastrophic. Therefore, CR must have the ability to track and block a network intruder that generates false radio environmental information. This dissertation proposes to use cyclic features to authenticate a specific radio device. To prove the feasibility of this concept, cyclic signatures of IEEE 802.11a/g OFDM signal are measured to identify specific cyclic feature variations caused by manufacturer specific circuit characteristics. The cyclic feature variations provide a device “fingerprint” that can be useful to help identify a particular radio transmitter’s manufacturer. Such fingerprinting of WLAN transmitted signal is similar to SEI (Specific Emitter Identification), a real-time measurement used in military applications to distinguish friendly and enemy radar signals for threat evaluation. If a distinctive cyclostationary signature can be extracted from the device, this can aid in authenticating a user. In CR application, this RF fingerprint can be
used to help distinguish malicious radio device producing false radio environment information\textsuperscript{1}.

This dissertation is organized as follows-

- Chapter 2 reviews the concept of environmental awareness for CR and recent advances in this research area.
- Chapter 3 provides a theoretical background of second-order cyclostationary signal analysis and presents cyclic spectral analysis results for basic modulation techniques based on linear periodically time varying (LPTV) transformations. This chapter describes how pulse shaping impacts cyclic features.
- Chapter 4 provides an analytical investigation of OFDM second-order cyclic features. The impact of guard interval and cyclic prefix of OFDM on the cyclic features is investigated and a general formula is presented to predict the cycle frequencies of dominant OFDM cyclic features. The OFDM pilot-feature detection performance is investigated using the single-cycle detector to show how the guard interval impacts on the detection performance. It is also shown that how to use this formula cDSA network identification.
- Chapter 5 proposes a signal detection and feature extraction algorithm using crest factor of cycle frequency domain profile (CDP).
- Chapter 6 provides a signal classification method using a Hidden Markov Model (HMM) to exploit cyclostationary signal features.
- Chapter 7 presents detail procedures for SEI of IEEE 802.11a/g OFDM signals. The CDP feature variations from five different WLAN cards are measured and compared using HMMs. This chapter demonstrates the feasibility of discrimination of WLAN cards from five manufactures using the CDP features.
- Chapter 8 presents a summary and conclusions of the research.

\textsuperscript{1} In general SEI implies identification of a unique individual unit from its RF fingerprint. Although we believe the techniques presented here can eventually accomplish this, the limited amount of data collected allows us only to claim identification to the manufacturer.
1.1 Contributions

To date, the original contributions of this research include:

1. Providing a comprehensive mathematical analysis and computer simulation of second-order OFDM cyclic features, which includes:
   a. The spectral correlation from OFDM QAM,
   b. The spectral correlation from OFDM pilots,
2. Developing general formula for estimating dominant OFDM pilot cyclic features.
4. Proposing and analyzing a detection and classification algorithm using a HMM and the crest factor of cycle frequency domain profile (CDP) for low and varying SNR.
5. Developing a SEI technique for WLANs based on second-order cyclic features and HMMs.
Chapter

2 Cognitive Radios

2.1 Introduction

A cognitive radio, as defined by Mitola [6], is a software radio [7] that is aware of its environment and its capabilities, it can alter its physical layer behavior, and is capable of following complex adaptation strategies. An addendum for this definition is that a cognitive radio learns from previous experience and can deal with situations that were not planned at the radio's initial design time.

However, this CR definition is not unique. As CR gains more attention, many different definitions emerge. Although the definitions vary, the purpose of CR can be summarized twofold: increase spectrum efficiency and improve radio communication performance.

Conventional adaptive radios can improve radio communication performance by adapting transmission power level, modulation, and error control coding scheme. However, CR’s situational awareness and learn from previous experience, allows it to change its power, modulation, and error control in advance by predicting situations where performance degradation may occur.
One of the major technical challenges for CR implementation is the awareness of surrounding spectrum and radio communication environment. In addition, CR depends highly on the information provided by other CRs. CR requires a method to block or track down a malicious user disseminating wrong radio environmental information.

This work concentrates on spectrum awareness through radio signal identification, signal detection, and classification using cyclostationary properties of radio signals and investigates specific emitter identification to detect the spoofing radio nodes.

Before we delve into the details of spectrum awareness for CR, the fundamental concepts, applications, available implementation, standards, and major challenges of CR are discussed.

### 2.2 Cognitive Radio Fundamental Concepts

CR has drawn great attention due to its potential for solving current spectrum shortage problems and enhancing radio communication performance. These result in many academic institutions and industries generating various definitions for CR, according to their own needs, based on the original definition by Mitola in 1999 [6, 8]. In this section, the CR definitions are reviewed to describe the commonalties and the differences. The most popular and frequently cited CR definitions which are made by Simon Haykin and Joseph Mitola are discussed first.

- **By Simon Haykin**

  By quoting the computational definition of cognition mentioned in [8], we can understand that cognition can be viewed as mental states and processes that intervene between input stimuli and output responses. These processes are described by various algorithms and they lend themselves to scientific investigations. Also, taking into consideration that one of the
purposes of using a cognitive radio is for improving spectrum utilization, S. Haykin in [8] defines CR as:

“An intelligent wireless communication system that is aware of surrounding environment, and uses methodology of understanding-by-building to learn from the environment and adapt its internal states to corresponding changes in certain operating parameters (e.g. transmit power, carrier frequency, and modulation strategy) in real-time, with two primary objectives in mind:

- Highly reliable communications whenever and wherever needed;
- Efficient utilization of the radio spectrum.”

• **By Joseph Mitola**

Another popular CR definition can be found in the 1999 paper by Joseph Mitola III [6, 9]. He coined originally the term of cognitive radio. He defined CR as “A radio that employs model based reasoning to achieve a specified level of competence in radio-related domains.”

The model based reasoning can be explained through an example. A user commutes everyday using a deterministic route and passes through several cell sites. Then, the mobile device equipped with CR can negotiate the best channels for each cell site and call handover plans to provide optimized communication quality. If the user takes a new route due to a traffic jam, the CR device can negotiate possible cell sites on the way by predicting new route using GPS (Global Positioning System) and a local map.

Given the previous two definitions, we can note six very important key words which describe the ideal characteristics of a cognitive radio: (1) intelligence, (2) awareness, (3)
learning, (4) adaptability, (5) reliability and (6) efficiency. Let’s discuss these key words in more detail.

Intelligence gives the radio the ability to make autonomous decisions. Awareness lets the radio be familiar with its capabilities as well as the surrounding environment. If the CR learns from previous experiences, it will be able to deal with situations that were not anticipated at the initial time of deployment, allowing the CR to evolve according to its surroundings, new policies or agreements. Also, as new regulations are enforced, the CR can adapt accordingly. The CR needs to be reliable and efficient, and if not, the success of spectrum sharing can be jeopardized.

Furthermore, we can discuss the CR's functionalities in terms of cognition cycle as presented by Mitola in [6, 10]. Figure 2.1 shows the cognition cycle.

![Figure 2.1: Cognition cycle of a radio](image)

Figure 2.1: Cognition cycle of a radio^2 [6, 10]

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^2 (c) Dr. Joseph Mitola III, used with permission.
The cognition cycle depicts how a CR may interact with its surroundings. The CR receives stimuli from the outside world, which are processed in the cognition cycle to formulate a response. If we relate the radio's cognition cycle to the tasks performed for spectrum efficiency, we can describe the following scenario: a cognitive radio observes the current spectrum conditions in a given band, orients itself, identifies available spectrum, plans the possible uses for it, decides on the best plan, and acts (allocating resources, negotiating protocols, etc.). While engaging in transmission the radio may observe the signal of an incumbent spectrum user. The radio's cognition cycle performs another analysis and jumps out of the spectrum to minimize interference to the user. One of the major challenges in this situation is to detect the user and switch channels without disrupting its own or others' communications.

- **By FCC**

The FCC (Federal communications commission) defines the CR in [11] as “A CR is a radio that can change its transmitter parameters based on interaction with the environment in which it operates.” In addition, the FCC expresses the relationship between CR and SDR (Software Defined Radio) in [12] as “The majority of cognitive radios will probably be SDRs, but neither having software nor being field reprogrammable are requirements of a cognitive radio.” Applying CR concepts to the transmitter, the FCC is trying to simplify regulatory procedures by allowing flexible spectrum sharing, provided that the interference level caused by CR to other radios is kept to a minimum. Conventionally, the FCC controls maximum allowed transmission power at the predetermined frequency bands for regulation. The FCC views that cognitive radio features include:

- Frequency agility,
- Listen before transmit,
- Dynamic frequency selection,
• Adaptive modulation,
• Transmit power control,
• Location awareness,
• Negotiated use.

• **By IEEE**

The growth of interest for dynamic spectrum access network resulted in the establishment of the IEEE P1900 standard committee in the first quarter 2005 jointly by the IEEE Communications Society (ComSoc) and the IEEE Electromagnetic Compatibility (EMC) Society. The objective of this effort is to establish supporting standards dealing with new technologies and techniques being developed for next generation radio and advanced spectrum management. On March 22, 2007 the IEEE Standards Board approved the reorganization of the IEEE 1900 effort as Standards Coordinating Committee 41 (SCC41), Dynamic Spectrum Access Networks (DySPAN). Although P1900 is renamed to SCC41, the working group under SCC41 still uses the term P1900. There are four working groups in SCC41.

• IEEE P1900.1: Standard terms, definitions and concepts for spectrum management, policy defined radio, adaptive radio and software defined radio.
• IEEE P1900.2: Recommended practice for interference and coexistence analysis.
• IEEE P1900.3: Recommended practice for testing and analysis to be used during regulatory compliance and stakeholder\(^3\) evaluation of radio systems with DSA capability. The standard also specifies radio system design features that simplify the evaluation challenge.
• IEEE P1900.4: Architectural building blocks enabling network-device distributed decision making for optimized radio resource usage in heterogeneous wireless access networks

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\(^3\) People or entities who believe their system could be affected by the recommended practices.
The IEEE P1900.1 defines CR in [13] as “A cognitive radio is able to sense its operational environment and on this basis adjusts its radio operating parameters. Radio control functionality can be performed through software as in software controlled radio.”

In the FCC spectrum policy task force report [14], cognition is defined as “the act or process of knowing including both awareness and judgment.” However, IEEE P1900 takes a more limited view of the term cognition than that in either the artificial intelligence (AI) community or in the classical dictionary definition of “cognition.” In this regards, IEEE P1900 thinks that the “judgment” aspect of cognition is associated with intelligent radios but not with cognitive radios. The separation of learning capability from cognitive radio is shown in Figure 2.2.

![Figure 2.2: Cognitive radio in the intelligent radio system components by IEEE P1900 [13]](image-url)
• **By ITU**

The working party 8A (WP 8A: Land mobile service excluding IMT-2000; amateur and amateur-satellite service) and working party 8F (WP8F: IMT-2000 and systems beyond IMT-2000) under study group 8 (SG8: Working on Mobile, radiodetermination, amateur and related satellite services) at international telecommunication union radio communication section (ITU-R) has working definition for CR as “A radio or system that senses and is aware of its operational environment and can dynamically, autonomously, and intelligently adjust its radio operating parameters so as to make best use of available spectrum and radio technology.”

• **By SDRF**

Recently, the SDR Forum (SDRF) approved CR definition in [15]. The approved document was developed from the collaboration between the SDR Forum and the IEEE P1900.1 working group. The SDRF has following CR definition:

“(A) Radio in which communication systems are aware of their environment and internal state and can make decisions about their radio operating behavior based on that information and predefined objectives. The environmental information may or may not include location information related to communication systems.

(B) Cognitive Radio as defined in (A) that utilizes Software Defined Radio, Adaptive Radio, and other technologies to automatically adjust its behavior or operations to achieve desired objectives”

• **By MPRG at Virginia Tech**

Virginia Tech Cognitive Radio Working Group has the following CR working definition [16] “An adaptive radio that is capable of the following: a) awareness of its environment and its own capabilities; b) goal driven autonomous operation; c) understanding or
learning how its actions impact its goal; and d) recalling and correlating past actions, environments and performance."

• **Common factors of CR**

There may other definitions for CR in addition to the definitions presented in the previous paragraphs, but we can extract some commonalities that can be identified among these definitions. The general capabilities commonly expected from those definitions are:

1. *Observation* - the radio is capable of collecting information about its operating environment. This is the main topic of this work,

2. *Adaptability* - the radio is capable of changing its waveform[^4], and

3. *Intelligence* - the radio is capable of understanding its environment and how its actions influence its own performance.

In addition to these common capabilities of CR, security features should be incorporated into CR. It is usually assumed that CR devices are cooperative with each other to achieve common goals. However, it is expected that intentional or accidental radio operation may occasionally disrupt cooperation among CRs. Such situation can be caused by intentional or unintentional spoofing radio devices and results in significant degradation of the spectrum awareness performance of CR. Therefore, it is required to permit only authorized use and prevent unauthorized modifications to radio parameters and false information distribution regarding surrounding spectrum usage. To track down such spoofing devices and associated users, specific emitter identification (SEI) must be considered for enhancing the spectrum awareness capability and level of security of CR.

[^4]: The definition of waveform can be found in JTRS. A waveform is all of the functionality between the antenna and the user. Formally, JTRS Operational Requirements Document (ORD) defines the waveform as “In JTRS system, the term waveform is used to describe the entire set of radio functions that occur from the user input to the RF output and vice versa”.

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The controlling mechanism for a CR with the above capabilities can be implemented as an independent entity, a CE (cognitive engine), which interacts with radio devices (adaptable radio transceiver implemented with software defined radio). A CE can be defined as the “intelligent” agent that manages the cognition tasks in a cognitive radio, in other words, an agent which can be considered as a control process. Given the input from its environments or user, the CE analyzes the situation, performs the necessary calculations, and proceeds by responding or reacting to the stimuli. As an example, modulation scheme, frequency of transmission, and frequency of reception can be the responses to the user requirements and the current environment conditions. The CE is the entity with “intelligence” which provides “awareness” of its environment and its own capabilities, goal driven “autonomous operation”, “understanding” the impact of its actions, and “learning” from past actions and environments.

2.3 Level of Cognition

The CR definitions discussed in the previous section vary from institution to institution due to different expectations of how CR improves or solves their problems. In other words, CR functionalities can be divided by the complexity of problems to be solved with CR. This division also helps assessing the capabilities of CR.

Joseph Mitola suggests nine levels of CR functionality division in [10, 17].

<table>
<thead>
<tr>
<th>Level</th>
<th>Capability</th>
<th>Task Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Pre-programmed</td>
<td>The radio has no model-based reasoning capability</td>
</tr>
<tr>
<td>1</td>
<td>Goal-Driven</td>
<td>Goal-driven choice of RF band, air interface, and protocol</td>
</tr>
<tr>
<td>2</td>
<td>Context-Awareness</td>
<td>Infers external communications context (minimum user involvement)</td>
</tr>
<tr>
<td>3</td>
<td>Radio Aware</td>
<td>Flexible reasoning about internal and network architectures</td>
</tr>
</tbody>
</table>
Another interesting division of the term “cognition” can be found in [18], called integrated cognition (INCOG). INCOG is a research program conceived by the Defense Advanced Research Projects Agency (DARPA) that develops architectural strategies (including principal components, interfaces, and principles of interrelationship) that enable computation based systems to achieve a reasonable imitation of human cognition. In INCOG, the cognition system should have following capabilities: the ability to learn using different approaches; the ability to reason in many domains; the ability to acquire and build knowledge; and the ability to connect, combine, integrate, collaborate, and unify knowledge across many domains. The framework for “ingredients of INCOG” is shown in Figure 2.3. The cognition level in cognitive radio can also be assessed using the ingredients of INCOG.

5 Fluent represents things that does not change, or that change in highly regular, predictable ways. Fluents explicitly represent events that have duration [18]. In cognitive radio, a fluent can be the impulse response of a particular set of RF channels.
2.4 Major Applications of Cognitive Radios

Originally, the CR concept was foreseen by Joseph Mitola [6] to provide a solution for the spectrum shortage problem in 3G (3rd Generation) radio systems in Europe. However, as the investigations of CR progress, the applications of CR have gone beyond radio spectrum pooling, such as improving radio communication performance. But, CR applications are not limited to the above two areas [19]. In this section, spectrum utilization and link reliability improvements are explained in more detail.

- **Improving spectrum utilization efficiency**

Spectrum scarcity is driven mainly by regulatory and licensing practices for spectrum allocation and not by the fundamental lack of spectrum. This fact is revealed through massive spectrum measurements by Shared Spectrum Company (SSC) [20].
SSC performed spectrum occupancy measurements in each band (30 MHz-3,000 MHz) at seven locations from January 2004 until August 2005 for the National Science Foundation (NSF) under subcontract to the University of Kansas.

The studies found significant available spectrum in most bands [20] as shown in Figure 2.4 and Figure 2.5:

- The average occupancy over all of the locations is 5.2%
- The maximum total spectrum occupancy is 13.1% (New York City)
- The minimum total spectrum occupancy is 1% (National Radio Astronomy Observatory)

Realizing the fact that the licensed spectrum remains unused most of the time, the Federal Communication Commission (FCC) is considering a paradigm shift on spectrum allocation policy towards the adoption of unlicensed, rule-based strategies for certain frequency bands. By establishing spectrum use etiquette which is not a complete and strict regulation, CR can take advantage of the spectrum awareness and goal-driven behavior to assure conformance to etiquette and to identify and report offenders to human authorities.
Figure 2.4: Spectrum occupancy in each band averaged over seven locations [20]

Figure 2.5: Spectrum occupancy at each location [20]
• **Improving link reliability**

The second most popular application of CR is improving link reliability [21]. This application clearly shows the difference between CR and conventional adaptive radio. Many adaptive radios can improve link reliability by changing communication parameters such as channel, modulation type, forward error control code scheme, and transmission power according to the deterministic and permanent internal program. However, CR can react to unexpected situations through its ability to learn. The difference becomes clear with the following example of signal quality on the commute path shown in Figure 2.6.

A user commutes over his/her favorite path with a CR. The CR can measure the signal quality associated with the location and temporal information. If signal quality on a specific path or driving specific time frame degrades severely (red mark in Figure 2.6) due to radio signal shadowing or communication traffic overflow (or RF crowding), the CR eventually learns the situation. When the user enters the poor signal region, CR can anticipate the poor signal and set up several plans for that situation and choose the best action such as change frequency, transmission power, channel coding, signal bandwidth, or register to the idlest base station ‘*prior*’ to the actual signal degradation.

Figure 2.6: Path and associated signal quality for a CR [21]
2.5 Major Challenges in Cognitive Radio

CR is broadly accepted as the most plausible technology to solve the spectrum shortage problem and to improve communication performance. However, to achieve such goals, CR has to overcome the following major challenges (first group is investigated in this work).

**CR challenges investigated in this work:**

- CR based Dynamic Spectrum Access Network (cDSA) Identification: How can CR identify CR node in cDSA network and initiate communication in dynamically changing white spectrum?
- Weak Primary/Secondary User Signal Detection: How can CR detect weak primary/secondary user signals?
- Signal Classification/Protocol Identification: How can CR identify modulation type of incoming signal and its protocols?
- CR network security: How can CR protect against an attack from a CR network intruder generating spoofing radio information? How can CR identify spoofing radios and associated users?

**CR challenges beyond this work:**

- Policy restriction: How can CR know what frequencies are potentially available for use under specific policy given space and time?
- Interference level and duration: How long can CR potentially cause interference to the incumbent radios? How can CR radio estimate the interference level at the incumbent user due to its radio transmission?
- Frequency Selection: How can CR select which frequency and signal format to use?
• Transmission power: How can CR know how much power for transmission at a given frequency?
• Radio knowledge exchange: How can CR exchange radio knowledge with others and negotiate with neighbors?
• Release recognition: How can CR recognize when to release spectrum in the presence of other signals?
• Frequency Agility: How can CR change frequency and maintain connectivity?
• Reasoning: How does CR establish a plan and choose the best action given radio environment?

CR challenges beyond this work are not limited to the items listed above. Any technical challenge for CR implementation should be added to the above list.

2.6 Cognitive Radio Activities and Initiatives

In this section, the most renowned activities and initiatives to develop CR systems at four major entities are described. Those entities are IEEE 802.22, DARPA xG, End-to-End Reconfiguration, and Virginia Tech CR testbed.

2.6.1 IEEE802.22 - Wireless Regional Area Network

The world’s first standardization effort on CR is the IEEE 802.22, established in November 2004. IEEE 802.22 is designed to provide alternative to wired line broadband access in the form of fixed point-to-multipoint wireless regional area networks in rural areas using fallow VHF/UHF TV broadcast bands (54 MHz - 862 MHz) [4]. The goal of IEEE 802.22 is to define an international standard that may operate in any regulatory regime. Therefore, the current 802.22 project identifies the U.S. frequency range of
operation from 54-862 MHz, while there is an ongoing investigation to extend the operational range to 41-910 MHz to meet additional international regulatory requirements. Also, the standard accommodates the various international TV channel bandwidths of 6, 7, and 8 MHz.

The 802.22 system also specifies a fixed point-to-multipoint wireless air interface whereby a base station (BS) manages its own cell and all associated Consumer Premise Equipments (CPEs), as shown in Figure 2.7.

![Figure 2.7: Service topology of IEEE 802.22 WRAN](image)

The BS controls the medium access in its cell and transmits in the downlink to the various CPEs, which respond back to the BS in the uplink. In addition to the traditional role of a BS, it also manages a unique feature of distributed sensing. This is needed to ensure proper incumbent protection and is managed by the BS, which instructs the various CPEs to
perform distributed measurement of different TV channels. Based on the feedback from CPEs, the BS decides which steps should be taken for proper system operation.

The 802.22 system specifies spectral efficiencies in the range of 0.5 bit/(sec/Hz) up to 5 bit/(sec/Hz). The physical layer of 802.22 supports the average data rate of 18 Mbps in a 6 MHz TV channel when the average spectral efficient is 3 bits/sec/Hz assumed. In order to obtain the minimum data rate per CPE, a total of 12 simultaneous users have been considered which leads to a required minimum peak throughput rate at edge of coverage of 1.5 Mbps per CPE in the downlink. In the uplink, a peak throughput of 384 kbps is specified. Another distinctive feature of 802.22 WRAN as compared to the existing IEEE 802 standards is the BS coverage range due to the good propagation characteristic of VHF/UHF bands [5], current specified coverage range is 33 Km at 4 Watts CPE EIRP.

The most challenging issues are sensing incumbent users such as TV receivers and wireless microphone.

### 2.6.1.1 Incumbent User Sensing in 802.22

802.22 devices have to minimize the interference to the two major primary devices, TV receiver and wireless microphones. The protection of TV signal has paramount importance in the design of 802.22 systems. The commercial deployment of 802.22 systems will be leveraged by demonstrating effective TV signal protection while maintaining secondary-user communications. In the future, it may be necessary to co-exist with other unlicensed devices in this band. The detection of wireless microphones is more difficult than TV signal detection. The nominal communication range of wireless microphone is around 200 meters and its usage is sporadic. Thus, 802.22 WG is considering two methods to aid in the detection of wireless microphone signals at CPEs. The two methods are using beacon signal and changing modulation scheme for wireless microphone. This standardization effort for wireless microphones is still an ongoing process. CPEs and BS have to detect TV
signals to find the best channel in the sense of achieving user requested quality of service (QoS) and minimizing interference to TV receivers. For these purposes, 802.22 working group (WG) is considering a stop (or quiet) period in the physical layer to enhance primary signal detection. Figure 2.8 shows the quiet period within an 802.22 superframe. The superframe is comprised of three major parts: a physical preamble (PA), a superframe control head (SCH), and a number of frames. The details of these parts are out of scope and can be found in [4].

This quiet period is synchronized between BS and CPEs to maximize the primary user signal detection capability. However, this synchronized stop period degrades the system throughput significantly due to the reduction of effective communication time. The synchronized stop requires a global clock such as GPS and it results in additional complexity at BS and CPE. In addition to the synchronized quiet period channel sensing, 802.22 requires TV signal detection at low SNRs to increase primary user signal detection performance and to mitigate the hidden node problem. Fading, in particular shadowing
results in the hidden node problem, where one node in the network may not be able to sense a TV signal within TV signal protection region (Figure 2.9). If a CPE initiates transmission using the channel, it will cause interference to the TV receiver. To overcome the hidden node problem, the network may utilize the results of spectrum sensing from multiple CPEs in order to make a reliable decision as to whether the WRAN network is inside or outside the TV protection contour. However, multiple CPEs may not detect TV signal when they are shadowed simultaneously. Thus, multiple high-sensitivity sensors would be more effective in solving the hidden node problem. Signal detection and classification utilizing cyclic feature of the incoming signal may provide possible solution for high sensitivity sensors. Cyclostationary signal detection can remove the background noise effectively and thus can detect signals at very low SNR.

Figure 2.9: Hidden node problem. CPE can cause interference to TV receiver within TV service contour when CPE fails to detect TV signal due to shadowing.
2.6.2 DARPA xG Program

The Defense Advanced Research Programs Agency (DARPA) next generation (xG) communication program aims for dynamic access in all available frequencies for military applications. However, DARPA xG program can also impact the commercial sector. By developing key technologies and architectures, DARPA xG plans to demonstrate a factor of 10 increase spectrum access [22, 23]. The DSA concept is created within this program.

The key technologies needed to enable this capability boil down to three areas: adaptive frequency agile waveforms, wide band spectrum sensing, and policy reasoning technology. The major players for the key technologies are

- Shared Spectrum Company (SSC): Making prototype xG radios including spectrum sensing and adaptive frequency agile waveform. SSC is known for the initial work of massive spectrum measurement and findings of low spectrum usage efficiency,
- Rockwell Collins: Developing low power and wide band spectrum sensing,
- SRI International: Developing a machine readable spectrum policy language and policy conformance reasoner technology [24, 25].

Upon successful development of the key technology and architecture, DARPA xG expects autonomous dynamic spectrum access in the future through a cognition cycle as shown in Figure 2.10.

![Figure 2.10: Autonomous dynamic spectrum access through DARAP xG Program [22]](image-url)
The xG operation can be also explained below Figure 2.11. This figure shows two reasoning entities, policy reasoner and system strategy reasoner. The policy reasoner accepts waveform transmission requests from the system strategy reasoner and check policy conformance by consulting the policy database. The system strategy reasoner controls the radio’s transmissions. It builds transmission requests based on sensor data and its current strategies. Replies to its requests from the policy reasoner may affect existing strategy.

In addition to these key technologies, DARPA xG considerers cognitive radio as another key enabling technology to their vision of advanced networking by allowing individually radios to perform complex operations needed make better use of spectrum and support high data rate applications.

The most functionalities of system strategy reasoner overlap with CR. By separating the controlling functionality from system strategy reasoner and merging the capability of strategy into CR, the strategy can be improved with the help of CR. In addition, the radio platform can be software defined radio with various waveform libraries. One important aspect of developing xG radio is the HMI (Human Machine Interface). HMI should provide a method of human intervention for the internal processing of xG radio. Finally, xG radio will take advantage of JTRS SCA (Software Communication Architecture).
Therefore, the functional blocks shown in Figure 2.11 can be viewed as SCA components and controlled by main controller as usually done in the SCA framework. The conceptual view of modified xG radio is depicted in Figure 2.12.

![Conceptual xG radio block diagram incorporated with CR](image)

**Figure 2.12:** Conceptual xG radio block diagram incorporated with CR

### 2.6.3 E2R Project and Wireless World Research Forum in European Union

End-to-end reconfigurability (E2R) is a research program started in 2004 within the program of research and technological development of the European Commission (EC) [26]. E2R’s goal is to develop key enabling technologies to support heterogeneous and generalized wireless access, in a flexible and intelligent manner to realize ubiquitous access, pervasive services, and dynamic resources provision. E2R eventually allows seamless communication experience to the end-user and the operators in heterogeneous networks and radio access technologies.
E2R project has three phases.

- **Phase 1 (2004-2005)** - A definition phase that identifies available technologies and assess the feasibility for reconfigurability,

- **Phase 2 (2006-2007)** - This phase concentrates on the most promising solutions identified in Phase 1 and assesses any emerging new technologies, while in parallel evolving towards an integrated framework. CR and SDR are considered the main technologies for E2R,

- **Phase 3 (2008-2009)** - This phase will complete the proof of concept by demonstrating that reconfigurability.

E2R considers that CR is a necessary technology to support the sharing of wide frequency spectrum using single reconfigurable RF transceiver leveraged by SDR in a network comprised of heterogeneous radio access technology (RAT). In other words, SDR is considered as a technology for supporting multiple RAT and CR is a technology on top of SDR that enables dynamic detection unoccupied spectrum resource and choose appropriate RATs.

Wireless world research forum (WWRF) is the main entity of providing technical guidance for E2R. In WWRF, there are six work groups (WG) and WG6 (Cognitive Wireless Networks and Systems) focuses on CR [27]. The WWRF’s view of CR and reconfigurability is depicted in Figure 2.13. As this figure shows, CR is mainly considered for efficient spectrum sharing rather than enhancement of communication performance.

![Diagram of CR and Reconfigurability](image)

**Figure 2.13:** WWRF’s view for CR and reconfigurability [27]
2.6.4 CORTEKS – Cognitive Radio Tektronix System

The CORTEKS radio is a procedural cognitive radio implemented at Virginia Tech using a PC that leverages Virginia Tech’s OSSIE SCA implementation [28] and the following commercial off-the-shelf (COTS) test equipment from Tektronix

- **Arbitrary Waveform Generator AWG430** – used to create a multi-mode transmitter
- **Real Time Spectrum Analyzer (RSA3408A)** – used to perform signal detection and demodulation.

The overall architecture of CORTEKS is depicted in Figure 2.14. The arbitrary waveform generator AWG710 is used for carrier frequency generation. Thus, the AWG710 can be replaced with any signal generator such as HP8648C.

The concept of CORTEKS can be considered as a primitive xG radio. Governed by software defined policy, the CORTEKS radio acts as a secondary user and adapts its frequency, modulation, data rate, and transmission power to maximize throughput while avoiding interference with primary users. The SDR part is comprised of the OSSIE SCA, real time spectrum analyzer (RSA), and arbitrary waveform generator (AWG). The CR part is a computer program installed on a personal computer. An artificial neural network (ANN) is used for radio parameter optimization to maximize the usage of available radio resources. The RSA monitors surrounding spectrum usage and demodulates incoming signals. If the received signal quality does not meet the desired QoS (Quality of Service), CR tries to formulate best action based on available spectrum, required QoS, given spectrum policy, and previous knowledge. Finding the best action is a multiple parameter optimization problem to achieve several goals simultaneously such as minimizing transmission power and bandwidth and maximizing data throughput. This optimization is performed using the ANN by maximizing a utility function. The overall communication parameter optimization process is shown in Figure 2.15.
Figure 2.14: CORTEKS components and interface

Figure 2.15: Radio parameter optimization using artificial neural networks
2.7 Chapter Summary

In this chapter, the concept of cognitive radio and how it can solve serious problems in wireless communications are discussed. In addition, major challenges for CR implementation are addressed. As this chapter explains, CR is an interdisciplinary research area due to the requirement of a broad range of knowledge to implement major CR functionalities. Despite the complex functionalities of CR, the major CR functionalities can be divided two parts according to the level of abstraction: learning and spectrum awareness. The learning capability of CR can be considered as a human brain which requires high-level knowledge abstraction. In contrary, spectrum awareness can be considered as signal processing which comes from direct physical observation. Those capabilities are usually interconnected to achieve specific goals. However, the sensing capabilities of spectral awareness can be treated independently at low level CRs.
Chapter

3 Cyclostationarity and Cyclic Spectral Analysis for Basic Modulated Signals

3.1 Introduction

A very important approach to designing signal sensing and classification algorithms is to exploit cyclostationarity of modulated signals. Most modulated signals have cyclostationary features due to periodicities embedded by the modulation process, such as carrier frequency, symbol rate, and pilot pattern. These parameters determine the cyclic features, including cycle frequency locations and spectral correlation amplitudes. Furthermore, these cyclic features can be extracted at very low SNR with uncertain noise levels whereas conventional energy detector shows poor detection performance. Another advantage of cyclostationary approach for spectrum sensing is that it can detect and extract signal features without demodulating the incoming signal. Furthermore, a distinctive cyclic feature can be built into modulated signal to help with CR device identification. However, conventional signal analysis intentionally eliminates cyclostationary features through phase randomization to compute the power spectral density (PSD) [29] or ignores the issue altogether.
Cyclic spectral analysis is taken as the main analysis tool in this work. Spectral correlation and coherence are the main tools for CR network identification, signal detection and classification, and specific emitter identification. In this chapter, the fundamentals of cyclic spectral analysis are reviewed. Spectral correlation and spectral coherence of BPSK and QPSK are evaluated to understand basic statistical cyclic analysis. In addition, the impact of pulse shaping on the spectral correlation and spectral coherence functions is investigated.

3.2 Motivation

Physical phenomena that involve periodicities give rise to random data for which appropriate probabilistic models exhibit periodically time-variant parameters. For instance, the periodicity in modern digital communication signals arises from embedded periodic elements such as carrier frequency and symbol rate. A process \( x(t) \) is said to be cyclostationary in the wide sense if its mean and autocorrelation are periodic [30, 31]:

\[
M_x(t + T_o) = M_x(t)
\]  

(3.1)

\[
R_x(t + T_o, u + T_o) = R_x(t, u)
\]  

(3.2)

The cyclostationarity in Eq. (3.1) and (3.2) can be found in modulated signals. The example shown below represents a special case of modulated signal. Other modulated signal having cyclostationarity can be investigated similar approach applied following example.

Let’s consider a digital waveform \( y(t) \) that can be expressed by

\[
y(t) = \sum_{k=-\infty}^{\infty} b_k s_i(t - kT_b) + (1 - b_k) s_o(t - kT_b)
\]  

(3.3)
where \( b_k \) is a stationary random bit sequence having values of one and zero, \( s_0(t) \) and \( s_1(t) \) are predetermined different waveforms with period \( T_b \) (bit rate \( R_b = 1/T_b \)).

In communication signal design, the binary input data sequence can be mapped to an antipodal waveform in which \( s_0(t) = -s_1(t) \). Then, the Eq. (3.3) can be written by

\[
y(t) = \sum_{k=-\infty}^{\infty} d_k s(t - kT_b)
\]

(3.4)

where \( d_k \) is the zero mean stationary random sequence having values +1 and -1.

In communication systems, transmitted signal bandwidth and power must conform to radiation regulations. PSD is a fundamental measure of the bandwidth and the power of a signal.

To get the PSD of Eq. (3.4), the Weiner-Kinchin theorem, which states that the PSD of a stationary signal can be obtained by Fourier transformation of autocorrelation function, is used. Thus, we have to derive the autocorrelation function which is given by

\[
E \left\{ y(t_1) y^*(t_2) \right\} = E \left\{ \sum_{m=-\infty}^{\infty} d_m s(t_1 - mT_b) \sum_{n=-\infty}^{\infty} d_n s(t_2 - nT_b) \right\}
\]

\[
= E \left\{ \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} d_m d_n \times s(t_1 - mT_b) s(t_2 - nT_b) \right\}
\]

\[
= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} E \left\{ d_m d_n \right\} \times s(t_1 - mT_b) s(t_2 - nT_b)
\]

(3.5)

We assume a real signal and the conjugation is ignored in Eq.(3.5).

Now, we need to evaluate the autocorrelation of the input data sequence, \( E\{d_i d_j\} \). Let’s consider a zero mean, independent, and identically distributed random bit sequence having the following probabilities,

\[
Pr\{d_m = +1\} = P \quad \text{and} \quad Pr\{d_n = -1\} = 1 - P
\]

(3.6)
Then, the autocorrelation function of the random input sequence is

\[
\mu(m-n) \triangleq E\{d_m d_n^*\} = \begin{cases} 
E\{d_m^2\} = 1^2 P + (-1)^2 (1-P) = 1 & \text{for } m = n \\
E\{d_m\}E\{d_n\} = (E\{d_m\})^2 = (2P - 1)^2 & \text{for } m \neq n
\end{cases}
\]  

(3.7)

Eq.(3.7) can be written as

\[
\mu(q) = \begin{cases} 
1 & \text{for } q = 0 \\
(2P - 1)^2 & \text{for } q \neq 0
\end{cases}
\]  

(3.8)

Therefore, Eq.(3.5) can be simplified further

\[
E \left\{ y(t + \tau) y(t) \right\} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \mu(m-n) \times s(t_1 - mT_b) s(t_2 - nT_b) \\
\triangleq R_y(t_1, t_2) = R_y(t_1 + mT_b, t_2 + nT_b) \\
= R_y(t + \tau, t)
\]  

(3.9)

where \( t_1 - t_2 \triangleq \tau \).

Eq.(3.9) is clearly periodic in \( t_1 \) and \( t_2 \) with period \( T_b \) and thus is not a stationary process. Here \( y(t) \) is called second-order cyclostationary random signal (the mean of \( y(t) \) is zero and can be consider periodic in \( t \)). In this example, the symbol generation at every symbol rate results in cyclostationarity. For other types of modulation, such as carrier frequency hopping and spreading sequence in CDMA, may produce cyclostationarity as well.

One approach to calculating the PSD of a cyclostationary signal is to randomize its phase so that it becomes stationary [29]. This conversion does not change the PSD. However, we will lose the additional benefits from cyclostationarity with this phase randomization.

Let’s define a stationary process from a cyclostationary one by introducing a random phase into the signal.

\[
y(t) \triangleq x(t - \theta)
\]  

(3.10)
The random phase variable \( \theta \) is assumed to be uniformly distributed over the period \( T \) and independent of the cyclostationary process \( x(t) \). Then \( y(t) \) can be transformed to a stationary process by averaging over the random phase. That is,

\[
m_y = E_{\theta}\{m_x(t, \theta)\} \quad \text{and} \quad R_y(\tau) = E_{\theta}\{R_x(t, \theta, \tau)\} \quad (3.11)
\]

where \( E_{\theta}\{\cdot\} \) is the expectation in the probability theory over random variable \( \theta \).

With the phase randomization concept, we can rewrite the Eq.(3.5) like as,

\[
R_y(\tau) = \frac{1}{T_b} \int_0^{T_b} R_y(t, \tau) dt \\
= \frac{1}{T_b} \int_0^{T_b} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \mu(m-n) \times s(t + \tau - mT_b)s(t - nT_b) dt \\
= \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \int_{-nT_b}^{(1-n)T_b} \sum_{q=-\infty}^{\infty} \mu(q) \times s(u + \tau - qT_b)s(u) du \\
= \frac{1}{T_b} \sum_{q=-\infty}^{\infty} \mu(q) f_s(\tau - qT_b) \quad (3.12)
\]

where a change of variable \( u \triangleq t - nT_b \) is performed and we assume

\[
f_s(\tau) \triangleq \frac{1}{T_b} \int_{-\infty}^{\infty} s(u)s(u+\tau) du.
\]

Therefore, the cyclostationary signal becomes stationary signal by averaging out the time dependency. The PSD of the stationary signal can be obtained using the Wiener-Kinchin theorem and the sampling identity for Fourier transforms and is given by

\[
S_y(f) = \frac{4P(1-P)}{T_b} \left| F_x(f) \right|^2 + \frac{(2P-1)^2}{T_b} \sum_n \left| F_x(n/T_b) \right|^2 \delta(f - n/T_b) \quad (3.13)
\]

The input signal sequence is usually modeled as an equally likely random sequence. Thus, with \( P=1/2 \), the PSD will not have any delta function and is given by
\[ S_x(f) = \frac{1}{T_0} |F_x(f)|^2 \] (3.14)

This implies that the spectral shape is only determined by the PSD of user information.

### 3.3 Cyclic Autocorrelation

Usually, autocorrelation functions are considered in communication signal processing because they allow PSD generation. Eq. (3.2) can be rewritten as:

\[ R_x(t + T_0 + \tau / 2, t + T_0 - \tau / 2) = R_x(t + \tau / 2, t - \tau / 2). \] (3.15)

Then \( R_x(t + \tau / 2, t - \tau / 2) \), a function of two independent variables, \( t \) and \( \tau \), is periodic in \( t \) with period \( T_0 \) for all values of \( \tau \). Thus, we can express Eq.(3.15) in Fourier series:

\[ R_x^\alpha(t + \tau / 2, t - \tau / 2) = \sum_\alpha R_x^\alpha(\tau)e^{j2\pi\alpha t}, \] (3.16)

where \( \{R_x^\alpha(\tau)\} \) are the Fourier coefficients given by,

\[ R_x^\alpha(\tau) \triangleq \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} R_x(t + \tau / 2, t - \tau / 2)e^{-j2\pi\alpha t}dt, \] (3.17)

and \( \alpha \) ranges over all integer multiples of the fundamental frequency \( 1/T_0 \). For polyperiodic or almost cyclostationary signals which have more than one fundamental frequencies such as \( \{1/T_1, 1/T_2, \ldots, 1/T_n\} \), the previous equations need to be generalized. This can be accomplished by having \( \alpha \) in Eq. (3.16) range over all integer multiples of all fundamental frequencies of interest like \( \alpha \in \{k_1/T_1, k_2/T_2, \ldots, k_n/T_n\} \) with any integer number \( k_i \) such that \( R_x^\alpha(\tau) \) is not identically zero. For Eq.(3.17), the following
modification required in order to include the effect of signal components having different fundamental periodicities.

\[ R_x^\alpha (\tau) \triangleq \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} R_x(t + \tau / 2, t - \tau / 2) e^{-j2\pi \alpha \tau} dt \]  
(3.18)

The \( R_x^\alpha (\tau) \) is called cyclic auto-correlation function (CAF) and is the starting point of cyclic spectral analysis. In the following sections, the key equations used in this work are investigated briefly.

### 3.4 Cycloergodic Cyclostationary Process

A random process is said ergodic if the long time averages of a realization (sample function or sample path) of a random process is equal to the corresponding statistical average (ensemble average or expectation operation) with probability one. This can be explained mathematically as

\[
\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt = E\{X(t)\} \quad \text{and} \quad \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x^*(t + \tau) dt = E\{X(t)X^*(t + \tau)\} = R_x(\tau)
\]  
(3.19)

where \( X(t) \) is a random process, \( x(t) \) is a realization of random process \( X(t) \), and \( \overset{p}{=} \) indicates the equality holds with probability one.

Ergodicity simplifies the analysis of random data measurement by allowing the exchange of statistical average and time average. In general, stationary signals are assumed ergodic.

The ergodic concept can be extended to the cyclostationary signal. The cyclostationary signal that holds ergodicity property is called cycloergodic cyclostationary process or
simply cycloergodicity [32]. If we assume that a cyclostationary signal is cycloergodic, then we can express the Eq.(3.18) as

\[R^\alpha_x(\tau) \triangleq \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} R_x(t + \tau/2, t - \tau/2) e^{-j2\pi \alpha t} \, dt\]

\[= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} E\{X(t + \tau/2) X^*(t - \tau/2)\} e^{-j2\pi \alpha t} \, dt\]  \hspace{1cm} (3.20)

\[= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t + \tau/2) x^* (t - \tau/2) e^{-j2\pi \alpha t} \, dt\]

Thus, the CAF can be expressed as

\[R^\alpha_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} E\{X(t + \tau/2) X^*(t - \tau/2)\} e^{-j2\pi \alpha t} \, dt\]

\[p \hspace{1cm} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t + \tau/2) x^* (t - \tau/2) e^{-j2\pi \alpha t} \, dt\]  \hspace{1cm} (3.21)

### 3.5 Spectral Correlation Function

The dual of the CAF is the spectral correlation function. If we define two time functions as \(u(t) \triangleq x(t) e^{-j\pi \alpha t}\) and \(v(t) \triangleq x(t) e^{j\pi \alpha t}\), then the generalized cross correlation for \(u(t)\) and \(v(t)\) can be written as:

\[R^\alpha_{uv}(\tau) \triangleq \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u(t + \tau/2) v^*(t - \tau/2) \, dt\]

\[= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t + \tau/2) e^{-j\pi \alpha (t + \tau/2)} x^* (t - \tau/2) e^{-j\pi \alpha (t - \tau/2)} \, dt\]

\[= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t + \tau/2) x^* (t - \tau/2) e^{-j2\pi \alpha t} \, dt\]

Therefore, the CAF is simply time-averaged cross-correlation between frequency-shifted versions of the process \(x(t)\). This implies that a process exhibits cyclostationarity only if correlation exists between some frequency-shifted versions of the process. \(R^\alpha_{uv}(\tau)\) is the
conventional cross correlation of the two complex valued frequency shifted signals. Consequently, the $R_s(\tau)$ can be considered as a inverse Fourier transform of the limit cross spectrum $S_s(\alpha \cdot f)$ of $u(t)$ and $v(t)$. From this, the following notation is introduced

$$R_s(\tau) \triangleq R_s(\tau) = FT^{-1}\{S_s(\alpha \cdot f)\} = FT^{-1}\{S_s(\alpha \cdot f)\} \quad (3.23)$$

where $S_s(\alpha \cdot f)$ is called spectral correlation function (SCF).

Eq. (3.23) can be verified different way. Let’s take the time-variant finite-time Fourier transformation of $u(t) \triangleq x(t)e^{-j\omega t}$ and $v(t) \triangleq x(t)e^{j\omega t}$. Then they will yield

$$U_T(t, f) \triangleq \int_{t-T/2}^{t+T/2} u(w)e^{-j2\pi f w} dw = \int_{t-T/2}^{t+T/2} x(w)e^{-j2\pi f w} e^{-j2\pi f w} dw = X_T(t, f + \frac{\alpha}{2})$$

and

$$V_T(t, f) \triangleq \int_{t-T/2}^{t+T/2} v(w)e^{-j2\pi f w} dw = \int_{t-T/2}^{t+T/2} x(w)e^{j2\pi f w} e^{-j2\pi f w} dw = X_T(t, f - \frac{\alpha}{2}) \quad (3.24)$$

where time-variant finite-time Fourier transformation is defined as

$$X_T(t, f) \triangleq \int_{t-T/2}^{t+T/2} x(u)e^{-j2\pi f u} du \quad (3.25)$$

To mitigate random effect due to limited observation, time-domain smoothing is required. Let’s define time-domain smoothed cross correlation as

$$S_{xy}(t, f) \triangleq \frac{1}{\Delta t} \int_{t-\Delta t/2}^{t+\Delta t/2} S_{uv}(u, f) du \quad (3.26)$$

where the time-variant cross periodogram can be expressed as

$$S_{uv}(t, f) \triangleq \frac{1}{T} U_T(t, f)V_T^*(t, f)$$

$$= \frac{1}{T} X_T(t, f + \frac{\alpha}{2}) X_T^*(t, f - \frac{\alpha}{2}) \triangleq S_{xy}(t, f) \quad (3.27)$$
The second line of Eq.(3.27) is known as cyclic periodogram. It is well known that the periodogram can be converted into the cross correlogram [33],

\[ S_{xy}(\cdot) = FT\{R_{xy}(\cdot)\} \quad (3.28) \]

where the cross correlogram is defined as

\[ R_{xy}(\tau) \triangleq \frac{1}{T} \int_{-|\tau|/2}^{t+|\tau|/2} x_{\tau}(t + \frac{\tau}{2}) \overline{x}_{\tau}(t - \frac{\tau}{2}) dt \quad (3.29) \]

If the periodogram-correlogram relation is applied to the Eq.(3.27), the time-variant cross periodogram will be

\[ S_{uv}(t, \cdot) = FT\{R_{uv}(t, \cdot)\} \quad (3.30) \]

where

\[ R_{uv}(t, \tau) \triangleq \frac{1}{T} \int_{-|\tau|/2}^{t+|\tau|/2} u(t + \frac{\tau}{2}) \overline{v}(w - \frac{\tau}{2}) dw \quad (3.31) \]

From Eq.(3.27), Eq.(3.30) can be written as,

\[ S_{uv}(t, \cdot) = FT\{R_{uv}(t, \cdot)\} = S_{xy}(t, \cdot) = FT\{R_{xy}^{\alpha}(t, \cdot)\} \quad (3.32) \]

where

\[ R_{xy}^{\alpha}(t, \tau) \triangleq \frac{1}{T} \int_{-|\tau|/2}^{t+|\tau|/2} x(t + \frac{\tau}{2}) \overline{x}(w - \frac{\tau}{2}) e^{-j2\pi\omega w} dw \quad (3.33) \]

In the statistical point of view, the time-series needs to be infinite, \( T \rightarrow \infty \) to suppression any random effect. Therefore,
\[
\lim_{T \to \infty} R_x^\alpha(t, \tau) = R_x^\alpha(\tau) 
\]  
(3.34)

From Eq.(3.22), Eq.(3.33) can be rewritten as

\[
R_x^\alpha(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(w + \frac{\tau}{2}) x^*\left(w - \frac{\tau}{2}\right) e^{-j2\pi w \tau} dw 
\]  
(3.35)

Another important aspect in statistical cyclic spectrum is that the SCF from temporal smoothing method can also be derived using spectral smoothing when the following condition is met [33],

\[
\Delta t >> T 
\]  
(3.36)

This result can be written mathematically

\[
S_{xy}^\alpha(t, f)_{Af} \approx S_{xy}^\alpha(t, f)_{1/T} \quad \text{when } \Delta t >> T 
\]  
(3.37)

It is clear that the observation duration \(T\) is the spectral resolution \((1/\Delta f)\). Thus, the spectrally smoothed spectral correlation can be reinterpreted as

\[
S_{xy}^\alpha(t, f)_{Af} \triangleq \frac{1}{\Delta f} \int_{-\Delta f/2}^{\Delta f/2} \frac{1}{\Delta t} X_{Af}(t, w + \frac{\alpha}{2}) X_{Af}^*(t, w - \frac{\alpha}{2}) dw 
\]  
(3.38)

Therefore, to obtain reliable statistical spectral correlation, following condition should be met

\[
\Delta t \Delta f >> 1 
\]  
(3.39)

The ideal spectral correlation can be obtained by
\[
S_x^\alpha (f) = \lim_{T \to \infty} \lim_{\Delta \to \infty} S_x^\alpha (t, f)_{\Delta T}
\]

and

\[
S_x^\alpha (f) = \lim_{\Delta \to 0} \lim_{\Delta_T \to \infty} S_x^\alpha (t, f)_{\Delta_T}
\]  

(3.40)

For ideal spectral correlation and cyclic autocorrelation functions, the following relation can be established:

\[
S_x^\alpha (f) = \int_{-\infty}^{\infty} R_x^\alpha (\tau)e^{j2\pi f \tau} d\tau
\]  

(3.41)

This relation is called cyclic Wiener relation as the counterpart of Wiener-Kinchin theorem in probabilistic theory.

### 3.6 Spectral Coherence and Cycle Frequency Domain Profile

The spectral correlation function is a cross-correlation function between frequency components at \((f + \alpha / 2)\) and \((f - \alpha / 2)\). It measures the temporal correlation between frequency components separated from each other by \(\alpha\). The correlation coefficient can be obtained by normalizing with the geometric mean of the variances of the two quantities involved in the correlation, which are \(X(f + \alpha / 2)\) and \(X^*(f + \alpha / 2)\). That is, for any two random variables \(X\) and \(Y\), the correlation coefficient is

\[
C_{XY} \triangleq \frac{E[XY^*]}{[\sigma_X^2 \sigma_Y^2]^{1/2}}
\]

(3.42)

For the spectral correlation function, it can be identified \(X\) as \(X(f + \alpha / 2)\) and \(Y\) as \(X^*(f - \alpha / 2)\). The variances of these two quantities are the power spectral density (PSD)
value $S(f + \alpha/2)$ and $S(f - \alpha/2)$. Therefore, the spectral coherence (SC) is mathematically defined by,

$$C_\alpha^a(f) \triangleq \frac{S_\alpha^a(f)}{[S_\alpha^a(f + \alpha/2)S_\alpha^a(f - \alpha/2)]^{1/2}}$$

(3.43)

The amplitude of the SC ranges from 0 to 1, but the SC itself is on the (closed) unit circle due to its complex form with magnitude one. The main advantage of the coherence is that it is a normalized measure of the strength of cyclostationarity. By looking at the features relative to the coherence for $\alpha = 0$, which is perfectly coherent, we can tell what is strong and what is weak independent of the signal's signal to noise ratio (SNR). Therefore, a single threshold to the entire coherence function can be applied and all the significant cycle frequencies can be obtained. In addition, the SC is invariant to the linear transformation of the incoming signal if it does not eliminate the cyclic features at cycle frequency domain.

The spectral correlation and coherence are three dimensional (3D) information. Usually, 3D information analysis is cumbersome due to the relatively large data size comparing to 2D data. Thus, it is desirable to have smaller data size for spectral correlation and coherence. It is convenient to visually assess the cycle-frequency parameter by evaluating the maximum over spectral frequency $f$,

$$I(\alpha) \triangleq \max_f |C_\alpha^a(f)|$$

(3.44)

This results in the spectral coherence Cycle-frequency Domain Profile (CDP).

The CDP can also be applicable to the spectral correlation function. The spectral correlation CDP is defined as

$$J(\alpha) \triangleq \max_f |S_\alpha^a(f)|$$

(3.45)
### 3.7 Conjugated Version of Spectral Analysis

If the target cyclostationary signal is complex, then the normal (or non-conjugate) cyclic spectral analysis used for real signals is not sufficient. Therefore, it is required to perform conjugate cyclic spectral analysis. Usually, non-conjugated (or normal) cyclic spectrum is related to symbol rate, chip rate, and bit rate. On the other hand, the conjugated cyclic spectrum shows carrier frequency related cyclic features. In this regard, normal cyclic feature is called lower region of cyclic spectrum and conjugated cyclic feature called upper region of cyclic spectrum.

The conjugate (or upper region) cyclic periodogram can be obtained by conjugating the conjugated time-varying finite time Fourier transformation in the normal cyclic periodogram. That is

\[
S_{xx}^\alpha (t, f) \triangleq \frac{1}{T} X_T(t, f + \alpha / 2) \left( X_T(t, \alpha / 2 - f) \right)^* 
\]  

(3.46)

where

\[
\int_{t-T/2}^{t+T/2} x^* (u) e^{-j2\pi fu} du = (X_T(t, -f))^* 
\]  

(3.47)

The conjugate spectral correlation is obtained by frequency smoothing as

\[
S_{xx}^\alpha (t, f) \gammaf = \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} S_{xx}^\alpha (t, w)dw 
\]  

(3.48)

If we assume that the frequency resolution is sufficiently high and the observation duration is sufficiently long, then \( S^\alpha_{xx} (t, f) \gammaf \rightarrow S^\alpha_{xx} (t, f) \).

Therefore, the conjugate spectral coherence and the conjugate spectral coherence CDP are expressed as
\[ C_x^\alpha(f) \triangleq \frac{S_x^\alpha(f)}{[S_x(f + \alpha / 2)S_x(\alpha / 2 - f)]^{1/2}} \]  

(3.49)

and

\[ I_c(\alpha) \triangleq \max_f \left| C_x^\alpha(f) \right| \]  

(3.50)

### 3.8 Evaluation of Second Order Cyclostationarity using LPTV Transformation

Evaluation of cyclic auto-correlation function (CAF) and spectral correlation function (SCF) can be simplified when we separate the target signal into input excitation and periodic component. Such separation is valid for most digitally modulated signals which are usually generated with a periodic carrier frequency excited with a stationary data input. Such modulation is a form of linear periodically time varying (LPTV) transformation [30, 33-35]. LPTV system cannot be analyzed with linear time invariant (LTI) method. However, LPTV systems can be transformed into a sum of frequency shifted LTI systems.

#### 3.8.1 LPTV System

A second-order cyclostationary signal can be obtained by a LPTV transformation of stationary input signal with period \(T_0\)

\[ y(t) = \int_{-\infty}^{\infty} h(t,u)x(u)du \]  

(3.51)

where the time variant impulse response is \( h(t,u) = h(t + T_0, u + T_0) \).

There is a general formula [30, 33] for the CAF and SCF for the LPTV system input and output. The detail derivation the input-output CAF and SCF relation is explained in Appendix A.
If the LPTV input is comprised of two or more stationary signals and the LPTV system have two or more periodic components, we have to use vector notation. This vector notation for LPTV systems is very useful in analyzing CAF and SCF for modern digital communication system.

Let’s consider a LPTV transformation having scalar output from the input column vector and the LPTV row vector system which is given by

\[ y(t) = \int_{-\infty}^{\infty} h(t,u)x(u)du \]  \hspace{1cm} (3.52)

Eq. (3.52) can be expressed in terms of sum of frequency shifted LTI systems as (See appendix A.)

\[ y(t) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} g_n(t-u)e^{j2\pi nu/T}x(u)du \]  \hspace{1cm} (3.53)

where

\[ h(t,u) = \sum_{n=-\infty}^{\infty} g_n(t-u)e^{j2\pi nu/T_0} = \sum_{\beta} g_{\beta}(t-u)e^{j2\pi n\beta u} \]  \hspace{1cm} (3.54)

where \( n/T_0 \equiv \beta \).

Then, the general formula for the CAF and SCF are given by [30, 33]

\[ \hat{R}_y^\alpha(\tau) = \sum_{\beta} \sum_{\nu} \text{trace} \left[ \hat{R}_x^{\alpha-(\beta+\nu)}(\tau)e^{-j\pi(\beta+\nu)\tau} \right] \otimes r^\alpha_{\beta\nu}(-\tau) \]  \hspace{1cm} (3.55)

and

\[ \hat{S}_y^\alpha(f) = \sum_{\beta} \sum_{\nu} G^\nu_{\beta}(f+\alpha/2) \hat{S}_x^{\alpha-(\beta+\nu)} \left( f - \frac{\beta+\nu}{2} \right) G^\nu_{\beta}(f-\alpha/2) \]  \hspace{1cm} (3.56)
in which $\hat{R}_x^\alpha$ is the matrix of cyclic cross correlation of the element of vector $x(t)$, and $r_{nm}^\alpha$ is the matrix of finite cyclic cross correlation of the Fourier coefficient function $g_n$, which is defined by,

$$r_{nm}^\alpha(\tau) \triangleq \int_{-\infty}^{\infty} g_n^\dagger(t + \tau / 2) g_m^* (t - \tau / 2) e^{-j2\pi\tau t} dt \quad (3.57)$$

Most modulated signal has a product modulation form (or sum of production modulation) as

$$y(t) = x(t) p(t) \quad (3.58)$$

where $x(t)$ determines amplitude and $p(t)$ is a periodic carrier.

The modulated signal $y(t)$ can also be expressed using Fourier series as

$$y(t) = x(t) \sum_{n=-\infty}^{\infty} P_n e^{-j2\pi n t / T_0} \quad (3.59)$$

where $T_0$ is the fundamental period of the carrier.

Let’s compare Eq.(3.53) and Eq.(3.59). The two equations will be identical when $g_n(\tau) \triangleq \delta(\tau)P_n$. Therefore, if we know Fourier series coefficient of a periodic signal component, we can use LPTV transformation for CAF and SCF evaluation.

### 3.9 Output of Linear Time Invariant System for Cyclostationary Input Signal

The output of the linear time invariant (LTI) system with cyclostationary input is also cyclostationary when the LTI system does not remove the statistically periodic components embedded into the input signal. The cyclostationary signal input and output relation of LTI
system can be easily derived using the characteristic of the power spectral density (PSD) input and output relation of LTI system in probabilistic theory [36].

Let’s consider an LTI transformation

\[ y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau \]  \hspace{1cm} (3.60)

where \( h(t) \) is the impulse response of the LTI system.

To investigate the relationship between cyclostationary signal input and output at LTI system, two LTI systems are considered

\[ w(t) = g_1(t) \otimes u(t) \]
\[ z(t) = g_2(t) \otimes v(t) \]  \hspace{1cm} (3.61)

The cross spectra for the two LTI systems can be given by [36]

\[ S_{wz}(f) = G_1(f)G_2^*(f)S_{uv}(f) \]  \hspace{1cm} (3.62)

If we define

\[ u(t) \triangleq x(t)e^{-j\pi f t} \] \hspace{0.5cm} and \hspace{0.5cm} \[ v(t) \triangleq x(t)e^{j\pi f t} \]
\[ g_1(t) \triangleq h(t)e^{-j\pi f \tau} \] \hspace{0.5cm} and \hspace{0.5cm} \[ g_2(t) \triangleq h(t)e^{j\pi f \tau} \]

then, the outputs of the two LTI systems will be

\[ w(t) = \int_{-\infty}^{\infty} x(\tau)e^{-j\pi f \tau}h(t-\tau)e^{-j\pi f (t-\tau)}d\tau = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau e^{-j\pi f \tau} \]
\[ = y(t)e^{-j\pi f \tau} \]  \hspace{1cm} \hspace{1cm} (3.63)

\[ z(t) = \int_{-\infty}^{\infty} x(\tau)e^{j\pi f \tau}h(t-\tau)e^{j\pi f (t-\tau)}d\tau = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau e^{j\pi f \tau} \]
\[ = y(t)e^{j\pi f \tau} \]  \hspace{1cm} (3.64)

We can get the following relations from Eq.(3.63) and Eq. (3.64).
\[ S_x^\alpha(f) = S_w(f) \quad \text{and} \quad S_y^\alpha(f) = S_w(f) \]
\[ G_i(f) = H\left(f + \frac{\alpha}{2}\right) \quad \text{and} \quad G_z(f) = H\left(f - \frac{\alpha}{2}\right) \]  

(3.65)

Substituting Eq.(3.65) into Eq.(3.62) yields

\[ S_y^\alpha(f) = H\left(f + \frac{\alpha}{2}\right)H^*\left(f - \frac{\alpha}{2}\right)S_x^\alpha(f) = e^{-j2\pi(f+\alpha/2)t_0}e^{j2\pi(f-\alpha/2)t_0}S_y^\alpha(f) \]
\[ = e^{-j2\pi\alpha t_0}S_y^\alpha(f) \]  

(3.66)

If time delay is introduced to the input signal as \( y(t) = x(t-t_0) \), then this is equivalent to the output of LTI system with system impulse response \( h(t) = \delta(t-t_0) \) for the input \( x(t) \).

Using Eq.(3.66), we have

\[ S_y^\alpha(f) = H\left(f + \frac{\alpha}{2}\right)H^*\left(f - \frac{\alpha}{2}\right)S_x^\alpha(f) = e^{-j2\pi(f+\alpha/2)t_0}e^{j2\pi(f-\alpha/2)t_0}S_y^\alpha(f) \]
\[ = e^{-j2\pi\alpha t_0}S_y^\alpha(f) \]  

(3.67)

Therefore, an output with time delayed input \( x(t-t_0) \) to an LTI system \( q(t) \) can be written as

\[ S_y^\alpha(f) = Q\left(f + \frac{\alpha}{2}\right)Q^*\left(f - \frac{\alpha}{2}\right)S_x^\alpha(f)e^{-j2\pi\alpha t_0} \]  

(3.68)

3.10 SCF of Periodically Sampled Signal

Let’s consider the following periodic sampling of the cyclostationary signal

\[ y(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t-nT_0) = \sum_{n=-\infty}^{\infty} x(nT_0)\delta(t-nT_0) = \sum_{n=-\infty}^{\infty} x_n\delta(t-nT_0) \]  

(3.69)

The sampling will produce replicas over bi-frequency plane; cycle frequency and spectral frequency. This repetition is explained mathematically as
\[
\tilde{S}^\alpha_x(f) = \tilde{S}^\alpha_x(f + \frac{k}{T_0}) = \tilde{S}^{\alpha \pm k/T_0}_x \left( f \mp \frac{k}{2T_0} \right) = \tilde{S}^{\alpha \pm 2k/T_0}_x(f)
\]  

(3.70)

The detailed derivation of (3.70) is shown in [30, 33] and Appendix B.

The relation between continuous SCF and discrete SCF is given by

\[
\hat{S}_y^\alpha(f) = \frac{1}{T_0} \tilde{S}^\alpha_x(f)
\]  

(3.71)

where \( \hat{S}_y^\alpha(f) \) and \( \tilde{S}^\alpha_x(f) \) indicate continuous and discrete SCFs.

### 3.11 Overall Procedure of Cyclic Spectral Analysis

The overall procedure for evaluating spectral coherence using frequency-domain averaging is described below.

The received signal is converted to intermediate frequency (IF) signal \( x(t)_{IF} \). For digital signal processing, the IF signal is digitized using analog-to-digital conversion (ADC). An \( M \)-point Fast-Fourier Transform (FFT) is applied to the digital data and the cyclic periodogram is evaluated by correlating frequency shifted versions of FFT output. To estimate the spectral correlation function, frequency smoothing method is applied. Then, the spectral coherence and CDP are obtained from the spectral correlation function estimation as shown in Eq.(3.38). The observation length \( N \) implies the number of \( M \)-points FFT operation. Therefore, the total incoming signal samples taken become \( (N \times M) / 2 \).
3.12 Bottleneck in Cyclic Spectrum Calculation

The computational complexity of spectral correlation function (SCF) is relatively high to be implemented in a mobile communication device. The main bottleneck for the evaluation of SCF is the frequency smoothing.

A general method for implementing the smoothing is filtering. However, filtering carries a heavy computational burden. For instance, the output of \( N \)-th order FIR filter with impulse response \( h[n] \) for the input \( x[n] \) will be

\[
y[n] = \sum_{k=0}^{N} h[k] x[n-k]
\] (3.72)

If input data size is \( M \), then the total computation will be approximately \((M \times N)\) additions and \((M \times N)\) multiplications. For \( N=128 \) and \( M=1024 \), about \(131 \times 10^3\) additions and \(131 \times 10^3\) multiplications are required. However, this operation can be dramatically reduced if the impulse response has a rectangular shape [37]. The moving average can be computed by adding the scaled new input value while removing the oldest scaled input value. If an
M-point FFT is used for the time-varying finite Fourier transform with \( N \)-th order moving average, then the SCF at \( f_1 \) will be: (Note that \( f \) is not normalized. The range of \( f \) is from 0 to \( M-1 \)):

\[
S_x^\alpha (f_1) = \frac{1}{N} \sum_{n=-N/2}^{N/2} \left[ \frac{1}{M} X_M \left( f_1 - n + \frac{\alpha}{2} \right) X_M^* \left( f_1 - n - \frac{\alpha}{2} \right) \right] \\
= \frac{1}{N} \sum_{n=-N/2}^{N/2} S_{xu}^\alpha (f_1 - n)
\]

(3.73)

And SCF at the next frequency bin \( f_2 = f_1 + 1 \) will be

\[
S_x^\alpha (f_2) = \frac{1}{N} \sum_{n=-N/2}^{N/2} S_{xu}^\alpha (f_2 - n) \\
= \frac{1}{N} \sum_{n=-N/2}^{N/2} S_{xu}^\alpha (f_1 + 1 - n) \\
= S_x^\alpha (f_1) + \left( S_{xu}^\alpha (f_1 + 1 - N/2) - S_{xu}^\alpha (f_1 + 1 + N/2) \right) / N
\]

(3.74)

This implies that the SCF at next (digital) frequency bin only requires two additions and one division. The overall computation requires \( (N + (M-1) \times 2) \approx 2 \times 10^3 \) additions and \( M = 1024 \) multiplications. Thus, this method reduces the smoothing processing time significantly. In sequential programming such as C and C++, this method requires recursive operation such as “for-loop” for SCF evaluation.

However, the looping operation is not suitable for MATLAB, which is optimized for vector operation. Therefore, the looping operation should be translated to a vector operation. The vector form of MATLAB code outlined below.

- \( X=\text{fft}(x,2^\times M) \); % Spectral resolution is enhanced by applying 2 time FFT size
- \( \text{CPgram\_Alpha}=X(\text{Alpha}\:\text{end})*\text{conj}(X(1:\text{end}-\text{Alpha}+1)) \); % For normal case
- \( \text{SCF} = \text{cumsum}(\text{CPgram\_Alpha})/N \);

The above 3 lines MATLAB codes evaluate can be executed quickly to calculate the SCF for the specific cycle frequency \( \alpha \).
3.13 Cyclic Spectral Analysis of Modulated Signals

In this section, the SCF for binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) in additive white Gaussian noise environment is investigated.

The simulation parameters for generating spectral correlation and coherence are as follows:

- Sampling frequency $F_s = 1$, every frequency is normalized to $F_s$,
- Intermediate carrier frequency (IF) $F_c = 0.25$,
- Symbol rates is $F_{sym} = 0.1$,
- Data samples per observation at IF, $T=256$ samples,
- FFT size is 512,
- Number of observations is $N=100$ (= 25600 samples),
- Signal-to-noise ratio, $SNR=3$ dB which is defined as

$$SNR = \frac{E_s}{N_0}$$

(3.75)

where $E_s$ is symbol energy, $B$ is the signal bandwidth, and $N_0$ is the noise power spectral density.

3.13.1 Spectral Correlation and Coherence Function of BPSK

The binary phase shift keying (BPSK) can be expressed mathematically as [38, 39]

$$s(t) = \sum_{n=-\infty}^{\infty} \sqrt{\frac{2E_b}{T_b}} m(n) q(nT_b + nT_c) \cos(2\pi f_c t)$$

(3.76)

- $E_b$ is the bit energy,
• Stationary random binary sequence of $m(n)$ has +1 and -1 equally likely per symbol duration $T_b$.

• Bit rate is $F_d = 1/T_b = 0.1$.

• $q(t)$ is normalized SQuare-root Raised Cosine (SQRC) filter with roll of factor $\beta = 0.5$ (or excess bandwidth 50%).

Using LPTV and periodic sampling property, the BPSK SCF is evaluated as,

$$\hat{S}_s^a(f) = \frac{E_b}{2T_b^2} \left[ Q\left(f + f_c + \frac{\alpha}{2}\right)Q^*\left(f + f_c - \frac{\alpha}{2}\right)\tilde{S}_m(f + f_c) + 
\quad Q\left(f - f_c + \frac{\alpha}{2}\right)Q^*\left(f - f_c - \frac{\alpha}{2}\right)\tilde{S}_m(f - f_c) \right] + 
\quad Q\left(f + f_c + \frac{\alpha}{2}\right)Q^*\left(f - f_c - \frac{\alpha}{2}\right)\tilde{S}_m^{a + 2f_c}(f) + 
\quad Q\left(f - f_c + \frac{\alpha}{2}\right)Q^*\left(f + f_c - \frac{\alpha}{2}\right)\tilde{S}_m^{a - 2f_c}(f) \right\}$$

(3.77)

where $Q(f)$ is the Fourier transform of the pulse shaping function $q(t)$ and $\tilde{S}_m(f_c)$ implies discrete signal.

Therefore, the cyclic feature of BPSK will be

$$\hat{S}_s^a(f) = \begin{cases} 
\frac{E_b}{2T_b^2} \left[ Q(f + f_c)Q^*(f + f_c)\tilde{S}_m(f + f_c) \right], & \alpha = 0 \\
\frac{E_b}{2T_b^2} \left[ Q(f)Q^*(f)\tilde{S}_m(f) \right], & \alpha = -2f_c \\
\frac{E_b}{2T_b^2} \left[ Q(f)Q^*(f)\tilde{S}_m(f) \right], & \alpha = 2f_c \\
0, & \text{otherwise}
\end{cases}$$

(3.78)
The stationary random binary sequence $m(n)$ is treated as periodically sampled from continuous signal. Then Eq. (3.70) can be used to explain such sampling. Due to the sampling, BPSK cyclic feature will appear at every symbol rate. However, the repeated cyclic feature at $k=1$ will be dominant when SQRC pulse shaping is applied. The magnitude of this repeating cyclic feature will diminish quickly for $k>1$. This is explained in the section 3.14.

Therefore, the dominant cyclic feature will appear at cycle frequencies $2f_c, 2f_c \pm f_{symbol}$ and $\pm f_{symbol}$. The actual positive cycle frequencies for dominant BPSK cyclic features given the simulation parameters above are

$$\begin{align*}
2f_c &= 2 \times 0.25 = 0.5, \\
2f_c \pm f_{sym} &= 0.5 \pm 0.1, \\
f_{sym} &= 0.1.
\end{align*}$$

(3.79)

The dominant cyclic features are identified in Figure 3.2 and Figure 3.3.

![Figure 3.2: Spectral coherence of BPSK.](image)
3.13.2 Spectral Correlation and Coherence Function of QPSK

The quadrature phase shift keying (QPSK) modulation can be expressed mathematically [38, 39]

\[
s(t) = \sqrt{\frac{2E_b}{T_s}} \left[ \sum_{n=-\infty}^{\infty} m_i(n)q(t+nT_s)\cos(2\pi F_s t) \right. \\
\left. - \sum_{n=-\infty}^{\infty} m_Q(n)q(t+nT_s)\sin(2\pi F_s t) \right]
\]  

(3.80)

- \(E_b\) is the symbol energy,
- Symbol rate is \(F_d = 1/T_s = 0.1\),
- \(q(t)\) is normalized SQuare-root Raised Cosine (SQRC) filter with Roll of factor \(\beta = 0.5\),
- \(m_i(n)\) is a stationary random binary sequence having +1 and -1 equally likely per symbol duration \(T_s\),
• $m_q(n)$ is a stationary random binary sequence having +1 and -1 equally likely per symbol duration $T_s$,

• $m_t(n)$ and $m_q(n)$ are identically distributed and independent (I.I.D).

QPSK SCF can be evaluated using LPTV transformation and given by

$$
\hat{S}_c^a(f) = \frac{E_s}{2T_s^2} \left[ Q \left(f + f_c + \frac{\alpha}{2}\right) Q^* \left(f + f_c - \frac{\alpha}{2}\right) \left(\tilde{S}_m^a(f + f_c) + \tilde{S}_m^a(f - f_c)\right) + Q \left(f - f_c + \frac{\alpha}{2}\right) Q^* \left(f - f_c - \frac{\alpha}{2}\right) \left(\tilde{S}_m^a(f - f_c) + \tilde{S}_m^a(f + f_c)\right)\right] + \frac{E_s}{2T_s^2} Q \left(f + f_c + \frac{\alpha}{2}\right) Q^* \left(f - f_c - \frac{\alpha}{2}\right) \left[\tilde{S}_m^{a+2f_c}(f) + \tilde{S}_m^{a+2f_c}(f)e^{j\pi}\right] + \frac{E_s}{2T_s^2} Q \left(f - f_c + \frac{\alpha}{2}\right) Q^* \left(f + f_c - \frac{\alpha}{2}\right) \left[\tilde{S}_m^{a-2f_c}(f) + \tilde{S}_m^{a-2f_c}(f)e^{-j\pi}\right] (3.81)
$$

Since $m_t(n)$ and $m_q(n)$ are I.I.D, then their cyclic spectrums are identical in the sense that

$$
\tilde{S}_m^a(f) = \tilde{S}_m^a(f) = \begin{cases} \tilde{R}_m(0), & \text{for } \alpha = k/T_0 \\ 0, & \text{for } \alpha \neq k/T_0 \end{cases} (3.82)
$$

Then, Eq.(3.81) can be further simplified to

$$
\hat{S}_c^a(f) = \frac{E_s}{T_s^2} \left[ Q \left(f + f_c + \frac{\alpha}{2}\right) Q^* \left(f + f_c - \frac{\alpha}{2}\right) \tilde{S}_m^a(f + f_c) + Q \left(f - f_c + \frac{\alpha}{2}\right) Q^* \left(f - f_c - \frac{\alpha}{2}\right) \tilde{S}_m^a(f - f_c)\right] (3.83)
$$

QPSK has only one peak regarding symbol rate on CDP with SQRC pulse shape filtering. The balanced signal constellation of QPSK cancels the carrier frequency related features as explained in Eq.(3.81). This fact applies to other balanced modulation schemes, such as
16QAM and 64 QAM which will not have the carrier frequency related cyclic feature due to the balanced signal constellations.

Usually, the maximum spectral coherence of balanced modulate signal which corresponds to symbol rate is not as strong as the peak related carrier frequency component. On this ground, the detection performance of QPSK signal is degraded than the BPSK signal.

The QPSK SCF and SCF CDP are shown in Figure 3.4 and Figure 3.5. In these figures, we can identify that carrier frequency related cyclic features are cancelled out.

![Figure 3.4: Spectral coherence of QPSK.](image-url)
3.14 Pulse Shaping Impact on SCF

It is common practice to limit the modulated signal bandwidth by applying pulse shaping such as squared root raised cosine filter (SQRC). However, the cyclic features from the modulated signal without band limiting pulse shaping are worth investigating as a baseline.

3.14.1 SCF with Rectangular Pulse Shaping

In this section, the impact of rectangular pulse shaping on the SCF is investigated. For this purpose, PSD and its phase of rectangular window are also investigated. The phase information is one of the main parameters in determining SCF. Thus, the phase for SCF is plotted together.

Let the impulse response \( q(n) \) be the window determining a symbol which is obtained by periodic sampling of \( q(t)|_{t=nT_s} \). If \( N \) samples are in a symbol, then \( q(n) \) can be express by
\[ q(n) = \begin{cases} 1/N, & 0 \leq n < N \\ 0, & \text{otherwise} \end{cases} \] (3.84)

The frequency response of Eq. (3.84) can be obtained by discrete Fourier transform

\[
Q(f) = \sum_{n=-\infty}^{\infty} q(n)e^{-j2\pi fn} = \frac{1}{N} \sum_{n=0}^{N-1} e^{-j2\pi fn} = \frac{1}{N} \frac{1-e^{-j2\pi fN}}{1-e^{-j2\pi f}} \\
= \frac{e^{-j\pi f(N-1)}}{N} \frac{\sin(\pi fN)}{\sin(\pi f)}
\] (3.85)

The magnitude and phase are

\[
|Q(f)| = \frac{1}{N} \left| \frac{\sin(\pi fN)}{\sin(\pi f)} \right| \quad \text{and} \quad \angle Q(f) = -(N-1)\pi f
\] (3.86)

The magnitude and phase plots for the quasi-ideal case (that is relatively long observations) and \(N=16\) are shown in Figure 3.6.

Let’s investigate the symbol rate related cyclic features for BPSK at positive cycle frequencies related to carrier frequency. The positive SCF related to the carrier frequency can be written as

\[
\hat{S}_{\alpha>0}^\alpha(f) = \frac{E_s}{2T_s} Q\left( f - f_c + \frac{\alpha}{2} \right) Q^\dagger\left( f + f_c - \frac{\alpha}{2} \right) \hat{S}_{m}^{\alpha - 2f_c}(f)
\] (3.87)
Due to the periodicity property of discrete SCF, we have following replicas

$$
\hat{S}_s^{\alpha>0}(f) = \frac{E_s}{2T_s^2} Q \left( f - f_c + \frac{\alpha}{2} \right) Q^* \left( f + f_c - \frac{\alpha}{2} \right) \tilde{S}_m \left( f - \frac{k}{2T_s} \right)
$$

(3.88)

When $\alpha = 2f_c - k / T_s$, we will have

$$
\hat{S}_s^{\alpha=2f_c-k/T_s}(f) = \frac{E_s}{2T_s^2} Q \left( f - f_c + \frac{2f_c - k / T_s}{2} \right) Q^* \left( f + f_c - \frac{2f_c - k / T_s}{2} \right) \tilde{S}_m \left( f - \frac{k}{2T_s} \right)
$$

$$
= \frac{E_s}{2T_s^2} Q \left( f - \frac{k}{2T_s} \right) Q^* \left( f + \frac{k}{2T_s} \right) \tilde{S}_m \left( f - \frac{k}{2T_s} \right)
$$

$$
= \frac{E_s}{2T_s^2} Q \left( f - \frac{k}{2T_s} \right) Q^* \left( f + \frac{k}{2T_s} \right) \tilde{S}_m (f)
$$

(3.89)

Let’s look at the Eq.(3.89) graphically for $k = 1$ and 2.
Figure 3.7: Magnitude and phase response of frequency shifted rectangular windows.
Shift of half symbol rate. 16 Samples per symbol.
The maximum value of product is around 0.4.
The phase is positive and constant with 180 degree wrapping.

Figure 3.8: Magnitude and phase response of frequency shifted rectangular windows.
Shift of one symbol rate. 16 samples per symbol.
The maximum value of product is around 0.15.
The Phase is negative and constant with 180 degree wrapping.
As Figure 3.7 and Figure 3.8 indicate, the magnitude for the product of frequency shifted rectangular pulse shaping functions is decreasing as the $k$ increases. However, it is not approaching to zero. The maximum magnitude of SCF replicas for various frequency shifts are presented in Figure 3.9.

![Graph showing magnitude variations of pulse shaping function product with various shifts]

**Figure 3.9: Magnitude variations of pulse shaping function product with various shifts**

### 3.14.2 SCF with SQRC Pulse Shaping

The SQRC filter impulse response is obtained by applying square root with raised cosine (RC) filter impulse response. The RC filter impulse response in frequency domain is given by
\[ P(f) = \begin{cases} 
1 & , \quad 0 \leq |f| \leq \frac{(1 - \beta)}{2T_0} \\
\frac{1}{2} & , \quad \frac{(1 - \beta)}{2T_0} < |f| \leq \frac{(1 + \beta)}{2T_0} \\
0 & , \quad |f| > \frac{(1 - \beta)}{2T_0}
\end{cases} \] (3.90)

Then, the SQRC filter impulse response will be

\[ Q(f) = \sqrt{P(f)} \] (3.91)

The ideal RC filter response in the frequency domain is shown in Figure 3.10 and Figure 3.11.

Figure 3.10: Magnitude of ideal RC filter with three roll-off factors. Symbol rate is normalized to 1.
As we did in rectangular pulse shaping, let’s only look at symbol rate related cyclic features at positive cycle frequencies. The result is shown graphically below.
Figure 3.12: Pulse shaping effect on SCF. The roll-off of SQRC filter is 0.5. 50% excess bandwidth red line indicates the linear phase. Blue line is reversed phase due to conjugation. The dotted line is the aggregated phase due to multiplication.

As Figure 3.12 indicates, the symbol rate features only occur at the cycle frequency with ± one symbol rate shifts. As the SQRC roll-off factor approaches zero, the symbol rate related cyclic feature disappears.

When we compare Figure 3.7 and Figure 3.12, the maximum value of symbol rate feature with the rectangular pulse shaping is greater than the one with the SQRC pulse shaping. Therefore, the symbol rate related cyclic features using rectangular pulse shaping exhibit relatively stronger features than SQRC pulse shaping.

3.14.3 The Impacts of SQRC Pulse Shaping on BPSK SC CDP

The amplitude of symbol rate related cyclic features is investigated as a function of roll-off factors or excess bandwidths of SQRC pulse shaping filters in BPSK signals.

Figure 3.13 and Figure 3.14 clearly show that the symbol rate related cyclic features are suppressed with small SQRC roll-off factors.
Figure 3.13: BPSK CDP with SQRC roll-off factor $\beta = 0.01$.

Figure 3.14: BPSK CDP with SQRC roll-off factor $\beta = 0.99$. 
3.15 Chapter Summary

This chapter reviews the fundamental concepts of second-order cyclostationarity: the cyclic autocorrelation function (CAF) and cyclic spectral correlation function (SCF). The CAF and SCF of BPSK and QPSK signals are evaluated using linear periodically time varying (LPTV) transformation.

In addition, the cyclic feature variations according to the pulse shaping functions are investigated. It is shown that the cyclic feature diminishes quickly for larger values of cycle frequencies due to pulse shaping. However, the dominant cyclic features for the squared root raised cosine (SQRC) filtered signal appears at cycle frequencies of $2f_c \pm kf_{sym}$. However, for $k > 1$, the symbol rate related cyclic features diminishes substantially due to band limited characteristic of SQRC filtering.

One interesting result in this chapter is the cyclic feature variations for different modulations. A QPSK signal does not have the carrier frequency related cyclic features due to its balanced signal constellation. However, the BPSK which has 0 and 180 degrees phase shift clearly shows the carrier frequency related cyclic features at $2f_c$.

The results and analysis techniques shown in this chapter provides a theoretical foundation for following chapters.
Chapter

4 Second-order Cyclic Spectral Analysis of OFDM Signal and its Application to cDSA Networks

4.1 Introduction

High throughput in conventional single carrier systems is difficult to achieve. An increase in data rate results in a decrease in the symbol duration which thus increases the intersymbol interference (ISI). To eliminate the ISI, computationally demanding equalizers are required, and even the equalizers cannot eliminate ISI perfectly. Although, code division multiple access (CDMA) systems are dominant in third generation (3G) systems due to their relative high data rate, CDMA performance also degrades with delay spread and frequency selective fading.

On the other hand, orthogonal frequency division multiplexing (OFDM) can effectively remove ISI and thus be robust to frequency selective fading with the help of a guard interval (GI), cyclic prefix, and a relatively low input data rate. OFDM has yet another advantage of spectrum sharing in cognitive radio applications. By controlling the inputs of
the IFFT during OFDM symbol generation, the spectrum shape can be controlled dynamically to take advantage of spectral holes or “white” spectrum. Practical and important milestones in using OFDM are the IEEE 802.22 standard [4], the world’s first standardization effort for CR to use the underutilized TV spectrum, and the IEEE 802.16h standard [40], the license-exempt operation of WiMAX (Worldwide Interoperability for Microwave Access).

OFDM gains more attention for the implementation of cDSA network [41, 42]. Cyclic feature analyses for signal detection, classification, and CR based dynamic spectrum access (cDSA) network management have become very important. However, the lack of analytic investigation of OFDM pilot cyclic features has limited the utilization of OFDM cyclic features. Thus, this chapter provides a substantial investigation on OFDM cyclic features using the second-order statistical cyclic analysis and computer simulation.

In this chapter, the investigation approach for OFDM second-order cyclic features is divided into two parts to contrast the impact on the OFDM GI;

- LPTV transformation based analysis for the OFDM signal without GI,
- Second-order statistical cyclic analysis for the OFDM signal with GI.

A cyclic signature is the unique combination of dominant peak on the CDP. Therefore, it is important to estimate cycle frequencies of dominant cyclic features. The purpose of this analysis is to determine the dominant cyclic features of OFDM pilots for cDSA network identification.

First, we investigate the CAF or SCF of OFDM pilots without the GI. This simplified version has all zero inputs for the subcarriers except for a pair of subcarriers for the pilots. The pilot sign sequence is assumed to be a zero mean equal probable for + and -. However, a real system, such as IEEE 802.11a/g, uses a very long periodic pseudo-random sequence. These pseudo-random sequences are designed to have a near zero cross-correlation. The cross correlation property of pseudo-random sequences allows white random sequences to be used for the evaluation of dominant pilot cycle frequencies.
To understand the impact of the GI on the dominant pilot cycle frequencies, the CAF of OFDM pilots is evaluated using the second-order statistical cyclic spectral analysis instead of using LPTV transformation. The LPTV method provides a compact and ideal expression for CAF and SCF for some modulated signal. However, GI size and pilot subcarrier location in OFDM can produce a phase discontinuity between OFDM symbols. This phase discontinuity is not modeled using the combination of quadrature and direct components of a modulated signal, such as QPSK, and it is not suitable for LPTV transformation. Therefore, a general formula for the dominant OFDM pilot cycle frequencies is derived using a different approach. The general formula is verified with computer simulation and measured data.

Finally, this chapter concludes by proposing a method for the identification of CR nodes for frequency coordination in cDSA networks using OFDM pilot cyclic features without common control channels. In addition to the CR node identification, a method for pilot pattern recognition using the dominant OFDM pilot cyclic is proposed.

4.2 Overview of OFDM Systems

Orthogonal frequency-division multiplexing (OFDM) is gaining in popularity as a means to meet the ever-increasing demands arising from the explosive growth of Internet, multimedia, and broadband services. OFDM-based systems are able to deliver high data rate, operate in the hostile multipath radio environments, and allow efficient sharing of limited resources such as spectrum and transmit power. Those benefits of OFDM technology lead CR systems to use OFDM as the main radio access technology. This trend is found in IEEE 802.22 and IEEE 802.16h systems.

The advantages of an OFDM system can be summarized as:

- Reduces the effect of delay spread due to multipath with simple channel equalizer.
- Mitigates frequency selective fading which affects only small portion of subcarriers.
- Improves spectrum efficiency by using orthogonal multicarrier modulation.
- Allows efficient sharing of limited spectrum resource by dynamically controlling the number of subcarriers and gains of each subcarrier.
- Improves data rate by controlling the subcarrier modulation type and the number of subcarriers.

The spectrum efficient characteristic of OFDM, due to orthogonal multicarrier modulation, is depicted in Figure 4.1. Single carrier systems require guard bands to minimize the interference or crosstalk from adjacent channels and results in less efficient spectrum usage than OFDM.

![Spectrum of Individual Subcarrier](image)

Figure 4.1: Orthogonal multicarrier modulation with 5 subcarriers.
## 4.3 OFDM Signal Generation

The generic OFDM signal can be expressed mathematically as,

\[
r(t) = \sum_{n=-\infty}^{\infty} w(t-nT_s) \left\{ \sum_{k=-N/2}^{N/2-1} d_{k,n} e^{j2\pi k\Delta_f (t-T_{GI}-nT_s)} + s_n \sum_{k=-N/2}^{N/2-1} c_k e^{j2\pi k\Delta_f (t-T_{GI}-nT_s)} \right\} \tag{4.1}
\]

where

- \( n \) is the OFDM symbol index,
- \( N \) is the number of subcarriers. This is an even number,
- \( T_{GI} \) is the guard interval duration. \( T_{GI} \) is the small portion of \( T_{FFT} \) which is the IFFT/FFT period. Usually, \( T_{GI} = (T_{FFT}/n) \) with \( n = 4, 8, 16, \) or \( 32 \),
- \( d_{k,n} \) is the complex symbol of subcarrier \( k \). These complex values are usually BPSK, QPSK, 16QAM, and 64QAM symbols. The \( d_{k,n} \) is designed not to overlap with pilot locations,
- \( \Delta_f \) is the subcarrier spacing. This is determined by dividing signal bandwidth by the number of subcarriers, \( N \),
- \( w(t) \) is the OFDM symbol waveform shaping function. It is assumed to be rectangular pulse of one OFDM symbol duration, \( T_{sym} = T_{FFT} + T_{GI} \),
- \( c_k \) is the predetermined BPSK pilot symbol. \( c_k \) is -1 or +1. The pilot symbol location is designed not to overlap with the data symbol subcarriers.
- \( s_n \) is a white random sequence that determines the sign of the BPSK pilot symbol. This prevents spectral peak due to a deterministic pilot pattern.

The unique feature that makes OFDM robust in fading channel is the guard interval (GI) and cyclic prefix. The cyclic prefix is generated by copying a small portion (usually 1/4, 1/8, 1/16, or 1/32) of the IFFT result to the beginning of the IFFT output, as shown in Figure 4.2 and Figure 4.3. This ensures the orthogonal property among the subcarriers [43]. However, GI usually introduces a phase discontinuity between OFDM symbols as shown in Figure 4.3. This phase discontinuity will cause cyclic feature variation. These cyclic feature variations are investigated in more detail later.
Before the digital-to-analog conversion of the OFDM symbol, the symbol goes through OFDM waveform shaping procedure by applying a time domain windowing function. Through the windowing procedure, the out-of-band spectrum can be reduced. However, the OFDM band-limiting waveform shaping is not considered in this work. The overall process of OFDM signal generation is shown in Figure 4.5.
4.4 Cyclic Spectral Analysis for the Pilot Only OFDM without Guard Interval

In this section, CAF and SCF for pilot only OFDM signals without the GI are derived. This analysis reveals the basic CAF and SCF characteristics of OFDM signals and sets the state for analyzing OFDM with the GI. This section is comprised of two sub sections:

- Cyclic feature analysis of continuous OFDM signals,
- Cyclic feature analysis of periodically sampled OFDM signal.

This two step analysis makes the initial analysis simple. The cyclic feature of the discrete OFDM signal is derived from continuous version using the periodic sampling property discussed in chapter 3.

4.4.1 Cyclic Features of Continuous Pilot Only OFDM without Guard Interval

If we consider only the pilot parts of the OFDM signal, then we have the following continuous form of the OFDM signal,
\[ r(t) = s(t) \sum_{k=-M}^{M} c_k e^{j2\pi k \Delta_f t} \] (4.2)

where the amplitude time-series is a PAM (Pulse Amplitude Modulation) signal.

The effect of the OFDM symbol windowing function \( w(t) \) is investigated later with the discrete signal analysis. For the simplicity of analysis, two different pilot frequencies (except the frequency sign difference) are investigated. The general case of having multiple pilots can be extended from the two pilot OFDM signal analysis result.

The resulting four pilot OFDM signal is given by

\[
\begin{align*}
    r(t) &= s(t) \left( c_a e^{j2\pi f_1 t} + c_{-a} e^{-j2\pi f_1 t} + c_b e^{j2\pi f_2 t} + c_{-b} e^{-j2\pi f_2 t} \right) \\
    &= s(t) c(t)
\end{align*}
\] (4.3)

where

\[
f_1 \triangleq k_a \Delta_f \quad \text{and} \quad f_2 \triangleq k_b \Delta_f \] (4.4)

\[
c(t) = c_a e^{j2\pi f_1 t} + c_{-a} e^{-j2\pi f_1 t} + c_b e^{j2\pi f_2 t} + c_{-b} e^{-j2\pi f_2 t}
\] (4.5)

and \( k_a \) and \( k_b \) are the subcarrier index for the pilots and Eq. (4.5) is a periodic carrier.

Eq.(4.3) is an LPTV transformation [34] of \( s(t) \) for which the Fourier coefficient functions \( g_n(\tau) \) of the impulse response function are given by

\[
g_n(\tau) = \left[ P_n Q_n \right] \times \delta(\tau),
\] (4.6)

where

\[
P_n \triangleq \lim_{T \to \infty} \int_{-T/2}^{T/2} c(t) e^{-j2\pi n \Delta_f t} \, dt \quad \text{and} \quad Q_n \triangleq \lim_{T \to \infty} \int_{-T/2}^{T/2} c(t) e^{-j2\pi n \Delta_f t} \, dt
\] (4.7)

---

\(^6\) LPTV transformation shown in [6] is summarized in appendix A.
By the definition of LPTV transformation in [34], Eq. (4.3) can be written as

\[ r(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g_n(t-u)x(u)e^{j2\pi nu/T_s} du \]  

(4.8)

where

\[ \tau \triangleq t-u \]  

(4.9)

\[ x(t) \triangleq [s(t) \quad s(t)]^T \]  

(4.10)

Substituting Eq. (4.6) into (4.8) yields

\[ r(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} [P_n \quad Q_n] \times \delta(t-u)[s(u) \quad s(u)]^T e^{j2\pi nu/T_s} du \]

\[ = \sum_{n=-\infty}^{\infty} e^{j2\pi nu/T_s} \int_{-\infty}^{\infty} (P_n s(u) + Q_n s(u)) \delta(t-u) du \]

\[ = s(t) \sum_{n=-\infty}^{\infty} e^{j2\pi nu/T_s} P_n + e^{j2\pi nu/T_s} Q_n \]

(4.11)

From Eq. (4.3) and Eq. (4.11), the \( P_n \) and \( Q_n \) must be zero except

\[ P_{-k_a} = c_{-a}, \quad P_{k_a} = c_a \quad \text{and} \quad Q_{-k_b} = c_{-b}, \quad Q_{k_b} = c_b \]  

(4.12)

The CAF and SCF of the LPTV output can be evaluated using the general formula in [34]. To use the general formula, the cyclic crosscorrelation matrix for the \( x(t) \) and \( h_n(\tau) \) are required. They are given by

\[ \hat{R}_x^\alpha(\tau) \triangleq \int_{\alpha}^{\infty} x^T(t+\tau/2)x^*(t-\tau/2)e^{-j2\pi nm} dt = \begin{bmatrix} \hat{R}_j^\alpha(\tau) & \hat{R}_i^\alpha(\tau) \\ \hat{R}_i^\alpha(\tau) & \hat{R}_j^\alpha(\tau) \end{bmatrix} \]  

(4.13)

and the \( r_{nm}^\alpha \) will be
\[
\hat{R}_n^\alpha (\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g_n^*(t + \tau/2) g_m^*(t - \tau/2) e^{-j2\pi\alpha t} dt \\
= \left[ \prod_{n=m}^{\infty} \left[ \prod_{n=m}^{\infty} \delta(t + \tau/2) \left[ P_m \ Q_n^* \right] \times \delta(t - \tau/2) e^{-j2\pi\alpha t} dt \right] \\
= \left[ \prod_{n=m}^{\infty} \left[ \prod_{n=m}^{\infty} P_n P_m^* \right] \right] \left[ \prod_{n=m}^{\infty} \left[ Q_n Q_m^* \right] \right] \delta(\tau) e^{-j\pi\alpha}. \quad (4.14)
\]

Substituting Eq. (4.13) and (4.14) into the CAF general formula yields

\[
\hat{R}_n^\alpha (\tau) = \sum_{n=m}^{\infty} \sum_{m=n}^{\infty} \left[ \frac{\hat{R}_n^{\alpha-(n-m)/T_0} (\tau) P_n P_m^* + \hat{R}_n^{\alpha-(n-m)/T_0} (\tau) Q_n Q_m^*}{\prod_{n=m}^{\infty} \left[ \prod_{n=m}^{\infty} P_n P_m^* \right] \prod_{n=m}^{\infty} \left[ Q_n Q_m^* \right]} e^{-j\pi(\alpha+m)/T_0} \right]. \quad (4.15)
\]

If we expand the double summation of Eq.(4.15) , then we will have

\[
\hat{R}_n^\alpha (\tau) = \left[ \frac{c_a c_a^* \hat{R}_n^\alpha (\tau) e^{-j2\pi f_1 \tau} + e^{j2\pi f_1 \tau} c_a c_a^* \hat{R}_n^{\alpha-2f_1} (\tau)}{P_n P_m^* \prod_{n=m}^{\infty} \left[ \prod_{n=m}^{\infty} P_n P_m^* \right]} \right] \left[ \frac{e^{-j2\pi f_2 \tau} c_a c_a^* \hat{R}_n^\alpha (\tau) + e^{j2\pi f_2 \tau} c_a c_a^* \hat{R}_n^{\alpha-2f_2} (\tau)}{Q_n Q_m^* \prod_{n=m}^{\infty} \left[ \prod_{n=m}^{\infty} Q_n Q_m^* \right]} \right] \\
\left[ \frac{e^{-j\pi(\alpha+f_1)/T_0} \hat{R}_n^{\alpha-(f_1+f_2)} (\tau) e^{-j\pi(f_2+f_1)/T_0}}{P_n P_m^* \prod_{n=m}^{\infty} \left[ \prod_{n=m}^{\infty} P_n P_m^* \right]} \right] + \left[ \frac{e^{-j\pi(f_2+f_1)/T_0} \hat{R}_n^{\alpha+(f_2+f_1)} (\tau) e^{-j\pi(f_2+f_1)/T_0}}{Q_n Q_m^* \prod_{n=m}^{\infty} \left[ \prod_{n=m}^{\infty} Q_n Q_m^* \right]} \right] \right]. \quad (4.16)
\]

The corresponding ideal SCF can be obtained by Fourier transformation of the CAF.
Using the fact that stationary signals do not have cyclic features, the non-zero SCF is found only at the following cycle frequencies:

\[ \{\alpha\} = \{0, \pm 2f_1, \pm 2f_2, \pm (f_2 + f_1), \pm (f_2 - f_1)\} \]  \hspace{1cm} (4.18)

### 4.5 Periodic Sampling of Pilot Only OFDM without GI

Discrete OFDM signals can be generated using the periodic sampling of continuous signal and band limiting pulse shaping. This can be expressed mathematically as

\[ y(t) = \sum_{n=-\infty}^{\infty} s_n w(t - nT_s) \]  \hspace{1cm} (4.19)

where \( s_n \) is the complex sequence at the output of parallel to serial conversion in Figure 4.4, \( w(t) \) is the OFDM symbol waveform shaping window function, and \( T_s \) is the sampling period. \( w(t) \) is assumed to be rectangular window function.

Eq.(4.19) can be written in terms of convolution as
\[ y(t) = \{s(t)p(t)\} \otimes w(t), \quad (4.20) \]

where \( \otimes \) is the convolution operator and \( p(t) \) is an impulse train which is given by

\[ p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) . \quad (4.21) \]

The output \( y(t) \) in Eq.(4.20) is the result of an LTI transformation of the input, which is periodically sampled. Then, \( w(t) \) can be considered as the impulse response of the LTI system. As we discussed in Chapter 3, the SCF of Eq. (4.20) can be written in terms of the input SCF, and the LTI system frequency response as

\[ \hat{S}_y^a(f) = \frac{1}{T_s} W\left(f + \frac{\alpha}{2}\right) W^*\left(f - \frac{\alpha}{2}\right) \hat{S}_s^a(f) , \quad (4.22) \]

where \( W(f) \) is the Fourier transformation of the rectangular window and has the form of a sync function.

Using Eq.(4.22), the ideal SCF of the OFDM pilot in Eq.(4.17) can be rewritten as (for only positive cycle frequencies due to symmetry)
\[
\hat{S}^{a>0}(f) = \frac{1}{T_s} \left[ c_a c_{-a}^* W \left( f + \frac{\alpha}{2} - f_1 \right) W^* \left( f - \frac{\alpha}{2} + f_1 \right) \hat{S}_s^{-\alpha-2f_1} (f) + 
\right]
\]

\[
\left. + c_b c_{-b}^* W \left( f + \frac{\alpha}{2} - f_2 \right) W^* \left( f - \frac{\alpha}{2} + f_2 \right) \hat{S}_s^{-\alpha-2f_2} (f) \right]
\]

\[
\left. + \left[ c_b c_{-a}^* W \left( f + \frac{f_2 + f_1}{2} + \frac{\alpha}{2} \right) W^* \left( f + \frac{f_2 + f_1}{2} - \frac{\alpha}{2} \right) \hat{S}_s^{-\alpha-(f_2+f_1)} (f + f_2 + f_1) \right] \right) (f_2 + f_1) \right)
\]

\[
\left. + c_b c_{-a}^* W \left( f + \frac{f_2 - f_1}{2} + \frac{\alpha}{2} \right) W^* \left( f + \frac{f_2 - f_1}{2} - \frac{\alpha}{2} \right) \hat{S}_s^{-\alpha-(f_2-f_1)} (f + f_2 - f_1) \right) \right)
\]

\[
\left. + c_a c_{-b}^* W \left( f - \frac{f_1 + f_2}{2} + \frac{\alpha}{2} \right) W^* \left( f - \frac{f_1 + f_2}{2} - \frac{\alpha}{2} \right) \hat{S}_s^{-\alpha-(f_1+f_2)} (f - f_1 + f_2) \right) \right)
\]

where it is assumed \( f_2 > f_1 \).

Next, the sampling effects on Eq. (4.23) are needed to be investigated. Using the sampling identity of cyclostationary signals discussed in chapter 3, the Eq. (4.23) can be written in terms of integer multiples of sampling frequency as

\[
\hat{S}^{a>0}(f) = \frac{1}{T_s} \left( SCF1 + SCF2 + SCF3 \right) \] (4.24)

where

\[
SCF1 = c_a c_{-a}^* W \left( f + \frac{\alpha}{2} - f_1 \right) W^* \left( f - \frac{\alpha}{2} + f_1 \right) \hat{S}_s^{-\alpha+kf/T_s-2f_1} \left( f - \frac{k}{2T_s} \right) \] (4.25)

\[
+ c_b c_{-b}^* W \left( f + \frac{\alpha}{2} - f_2 \right) W^* \left( f - \frac{\alpha}{2} + f_2 \right) \hat{S}_s^{-\alpha+kf/T_s-2f_1} \left( f - \frac{k}{2T_s} \right),
\]
\[ SCF_2 = c_a c_{-a}^* W \left( f - \frac{f_2 - f_1}{2} + \alpha \right) W^* \left( f - \frac{f_2 - f_1}{2} - \alpha \right) \]
\[ \times S_{s k T}^{a+k/T_s / (f_2 + f_1)} \left( f - \frac{f_2 - f_1}{2} - \frac{k}{2T_s} \right) \]
\[ + c_b c_{-b}^* W \left( f + \frac{f_2 - f_1}{2} + \alpha \right) W^* \left( f + \frac{f_2 - f_1}{2} - \alpha \right) \]
\[ \times S_{s k T}^{a+k/T_s / (f_2 + f_1)} \left( f + \frac{f_2 - f_1}{2} - \frac{k}{2T_s} \right) \]
\]

and

\[ SCF_3 = c_a c_{-a}^* W \left( f - \frac{f_2 + f_1}{2} + \alpha \right) W^* \left( f - \frac{f_2 + f_1}{2} - \alpha \right) \]
\[ \times S_{s k T}^{a+k/T_s / (f_2 - f_1)} \left( f - \frac{f_2 + f_1}{2} - \frac{k}{2T_s} \right) \]
\[ + c_b c_{-b}^* W \left( f + \frac{f_2 + f_1}{2} + \alpha \right) W^* \left( f + \frac{f_2 + f_1}{2} - \alpha \right) \]
\[ \times S_{s k T}^{a+k/T_s / (f_2 - f_1)} \left( f + \frac{f_2 + f_1}{2} - \frac{k}{2T_s} \right) \]
\]

where \(1/T_s\) is the sampling frequency and \(k\) is an integer number.

The largest valid value for \(T_s\) is the OFDM symbol duration \(T_{sym}\). Consequently, the smallest frequency shift for \(W(f)\) due to the argument \(k/2T_s\) is \(1/2T_{sym}\). Note that the dominant OFDM pilot features are obtained when \(k = 0\). The dominant cyclic features diminish as \(k\) increases.

The magnitude of the SCF shown in Eq. (4.25) will have maximum values at \(\alpha = 2f_1 - k/T_s\) and \(\alpha = 2f_2 - k/T_s\) when \(k = 0\).

\[ |SCF_1| = |W(f)|^2 S_{s 0}^0 (f)_{\alpha=2f_1} + |W(f)|^2 S_{s 0}^0 (f)_{\alpha=2f_2} \]
\]

where \(c_a = c_{-a} = c_b = c_{-b} = 1\) is assumed.
Due to periodic sampling of the cyclostationary signal, the $SCF1$ will have duplicates at $\alpha = 2f_1 - k/T_s$ and $\alpha = 2f_2 - k/T_s$ when $k \neq 0$. Let’s investigate the smallest frequency shift for $k/T_s = 1/T_{sym}$.

$$|SCF1| =$$

$$W\left(f + \frac{1}{2T_{sym}}\right)W^*\left(f - \frac{1}{2T_{sym}}\right)\hat{S}_s^0\left(f - \frac{1}{2T_{sym}}\right)_{\alpha = 2f_2 - 1/T_{sym}} +$$

$$W\left(f + \frac{1}{2T_{sym}}\right)W^*\left(f - \frac{1}{2T_{sym}}\right)\hat{S}_s^0\left(f - \frac{k}{2T_s}\right)_{\alpha = 2f_2 - 1/T_{sym}}$$

The OFDM symbol windowing function $W(f)$ does not overlap perfectly, as in the case of $k = 1$. Therefore, the maximum value of magnitude of the $SCF1$ will be reduced. This implies that there will be copies of cyclic features that are weighted with frequency-shifted OFDM windowing functions at every OFDM symbol rate. The same argument is also applied to $SCF2$ and $SCF3$.

Up to now, we have focused on the normal (or non-conjugated) CAF and SCF. Conjugated CAF and SCF are also useful for investigating upper region cyclic features of complex baseband signals. In this analysis, we focus on normal CAF and SCF first which are symmetric to cyclic frequency zero. Therefore, CAF and SCF at positive cycle frequency provide sufficient information for the cyclic features of the input signal. In this regard, Eq.(4.23) only shows the positive cycle frequencies.

The SCF is three dimensional (3D) data and the plot usually requires a lot of memory. Therefore, we resort to 2D plots called cycle frequency domain profiles (CDP) [44], where CDP is defined as

$$J(\alpha) = \max_f \left| S_{\alpha}^s(f) \right|. \quad (4.30)$$
Depending on the argument of maximization in Eq.(4.30), there are three types of CDP, CDP for cyclic auto correlation function, CDP for spectral correlation and CDP for spectral coherence. We call them CAF CDP, SCF CDP, and SC CDP.

4.6 Two QAM Input OFDM Models without Pilot and Guard Interval

In the previous section, the cyclic features of the pilot are investigated when no GI is inserted. Next, the cyclic features of the subcarriers having QAM (QPSK, 16QAM, and 64QAM) inputs without guard interval are investigated. Using the same approach used in pilot only OFDM analysis, first continuous QAM inputs having two arbitrary subcarriers are investigated. This analysis can be extended to the multiple QAM input OFDM signal using the method used for two QAM input OFDM signals.

Let’s investigate continuous QAM OFDM with arbitrary subcarrier frequencies \( f_0 \) and \( f_i \) \(( f_0 \neq f_i \)). It can be expressed as

\[
y(t) = z(t)e^{j2\pi f_0 t} + g(t)e^{j2\pi f_i t}
\]

(4.31)

where the QPSK input signals are given by

\[
z(t) = (z_r(t) + jz_i(t))/\sqrt{K} \quad \text{and} \quad g(t) = (g_r(t) + jg_i(t))/\sqrt{K}
\]

(4.32)

in which \( z_r, z_i, g_r, \) and \( g_i \) are independent and identically distributed (I.I.D) and have the following values based on modulation type.

- QPSK: -1 or 1
- 16QAM:-3, -1, 1, or -3
- 64QAM:-7, -5, -3, -1, 1, 3, 5, or 7
The value $K$ is the modulation dependent power normalization factor and is $K = 2$ for QPSK, $K = 10$ for 16QAM, and $K = 42$ for 64QAM.

Thus, the input signal vector for QAM input OFDM is defined as

$$x(t) = [z(t) \ g(t)]^T.$$  \hspace{1cm} (4.33)

To use the LPTV transformation, the cross correlation matrix, $R_x^a(\tau)$, of the input signal elements, as well as the cross correlation of Fourier coefficients for the LPTV system, $r_{nm}^a$, are required. Then, the $R_x^a(\tau)$ is given by

$$R_x^a(\tau) = \begin{bmatrix} R_{zz}^a(\tau) & R_{zg}^a(\tau) \\ R_{gz}^a(\tau) & R_{gg}^a(\tau) \end{bmatrix}.$$  \hspace{1cm} (4.34)

and the $r_{nm}^a$ is

$$r_{nm}^a(\tau) = \begin{bmatrix} P_{n}^m & P_{n}^m \\ Q_{n}^m & Q_{n}^m \end{bmatrix} \delta(\tau)e^{-j\pi\alpha}.$$  \hspace{1cm} (4.35)

Plugging Eq.(4.34) and Eq.(4.35) into the CAF general formula in [34] yields

$$\hat{R}_y^a(\tau) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left[ \hat{R}_{zz}^{a-(nf_0+mf_0)}(\tau)P_{n}^mP_{n}^m + \hat{R}_{zg}^{a-(nf_0+mf_0)}(\tau)Q_{n}^mP_{n}^m + \hat{R}_{gz}^{a-(nf_0+mf_0)}(\tau)P_{n}^mQ_{n}^m + \hat{R}_{gg}^{a-(nf_0+mf_0)}(\tau)Q_{n}^mQ_{n}^m \right] e^{-j\pi(nf_0-mf_0)\tau}.$$  \hspace{1cm} (4.36)

This result can be simplified further as

$$\hat{R}_y^a(\tau) = \hat{R}_{zz}^{a-(f_0+f_0)}(\tau)e^{-j2\pi\left(\frac{f_0-f_0}{2}\right)\tau} + \hat{R}_{zg}^{a-(f_0-f_0)}(\tau)e^{j2\pi\left(\frac{f_0+f_0}{2}\right)\tau} + \hat{R}_{gz}^{a-(f_0+f_0)}(\tau)e^{j2\pi\left(\frac{f_0-f_0}{2}\right)\tau} + \hat{R}_{gg}^{a-(f_0-f_0)}(\tau)e^{-j2\pi\left(\frac{f_0+f_0}{2}\right)\tau}.$$  \hspace{1cm} (4.37)

The corresponding SCF is obtained using the cyclic Wiener relation and is given by
\begin{equation}
\hat{S}^a_y(f) = \hat{S}^{a-(f_0+f_1)}_{zz} \left( f + \left( \frac{f_0-f_1}{2} \right) \right) + \hat{S}^{a+(f_0+f_1)}_{gg} \left( f - \left( \frac{f_0-f_1}{2} \right) \right) + \hat{S}^{a+(f_0-f_1)}_{eg} \left( f - \left( \frac{f_0+f_1}{2} \right) \right) + \hat{S}^{a-(f_0-f_1)}_{ge} \left( f + \left( \frac{f_0+f_1}{2} \right) \right).
\end{equation}

The input QAM signal is assumed to be an I.I.D random process. Therefore, in the limit, the cross correlation between the random processes \( z(t) \) and \( g(t) \) approaches zero. Thus, Eq.(4.38), in the limit, can be simplified to

\begin{equation}
\hat{S}^a_y(f) = \hat{S}^{a-(f_0+f_1)}_z \left( f + \left( \frac{f_0-f_1}{2} \right) \right) + \hat{S}^{a+(f_0+f_1)}_g \left( f - \left( \frac{f_0-f_1}{2} \right) \right).
\end{equation}

Using the periodic sampling property used in cyclic feature evaluations for the discrete OFDM pilot, Eq.(4.39) can be written in terms of the sampling period \( T_s \) and the rectangular OFDM waveform shaping function \( W(f) \) and is given by

\begin{equation}
\hat{S}^a_y(f) = \frac{1}{T_s} W \left( f + \left( \frac{f_0-f_1}{2} \right) + \frac{\alpha}{2} \right) W^* \left( f + \left( \frac{f_0-f_1}{2} \right) - \frac{\alpha}{2} \right) + \frac{1}{T_s} W \left( f - \left( \frac{f_0-f_1}{2} \right) + \frac{\alpha}{2} \right) W^* \left( f - \left( \frac{f_0-f_1}{2} \right) - \frac{\alpha}{2} \right) \times S_{\alpha+k/2T_s}^{a-(f_0+f_1)} \left( f + \left( \frac{f_0-f_1}{2} \right) + \frac{k}{2T_s} \right) + \frac{1}{T_s} W \left( f + \frac{f_0-f_1}{2} \right) W^* \left( f - \frac{f_0-f_1}{2} \right) \times S_{\alpha+k/2T_s}^{a+k/2T_s} \left( f - \frac{f_0-f_1}{2} + \frac{k}{2T_s} \right),
\end{equation}

where \( k \) is an integer number and the OFDM symbol duration \( T_{sym} \) is the integer multiples of the sampling period \( T_s \).

\( \hat{S}^a_z \) and \( \hat{S}^a_g \) are the SCFs for the QAM signal. Therefore, the SCFs only show symbol rate features around cycle frequency zero. As we discussed in chapter 3, balanced modulation schemes such as QPSK, 16 QPSM, and 64 QAM cancel the cyclic feature for carrier frequency. Thus, \( \hat{S}^a_z \) and \( \hat{S}^a_g \) do not have the cyclic features related to their subcarrier
frequencies. However, if observation symbols are not large, then the statistical characteristics for the input signals \( z(t) \) and \( g(t) \) do not have the white random process. This limited observation results in non-zero cyclic features related to the QAM OFDM subcarriers. However, the observation limitation of OFDM signals is common.

Usually OFDM signals have large subcarriers. For instance, IEEE 802.11a/g has 64 subcarriers. For the proper cyclic spectral analysis, the sampling frequency must be greater than twice the signal bandwidth. The minimum sampling frequency of IEEE 802.11a/g is \( 2 \times 20 \text{MHz} \). This implies that there are 128 samples per IEEE 802.11a/g OFDM symbol. If we observe 100 symbols, then the FFT bin size for SCF evaluation must be at least 32768. This large FFT size makes the frequency smoothing time long. The longest IEEE 802.11a/g burst size is 152 OFDM symbols (See chapter 7). This small symbol size results in non-zero cyclic features for the QAM inputs.

### 4.7 Simulation Result for Pilot Only and No GI OFDM Signals

Here, Eq. (4.23) is verified through computer simulation using an IEEE 802.11a/g OFDM signal. IEEE 802.11a/g OFDM signals have the following parameters:

- 64 subcarriers (use a 64 point IFFT),
- 20 \( \text{MHz} \) bandwidth
- Subcarrier spacing, \( \Delta_f = 20 \text{MHz} / 64 = 312.5 \text{KHz} \),
- 4 pilot subcarriers at subcarrier indexes \( \{-21, -7, 7, 21\} \),
- \( c_a = c_2 = 1, \; c_{-a} = c_{-2} = 1, \; c_b = c_{21} = -1, \) and \( c_{-b} = c_{-21} = 1 \),
- A 127 bit pseudo random sequence determines pilot sign. This sequence repeats every 127 OFDM symbols. If we only focus on the dominant cycle frequencies for the OFDM pilots, then the periodic pseudo random sequence can be treated as a white random sequence when sufficiently long OFDM symbols are available.
However, the exact shape of CAF and SCF for the periodic pseudo random sequence will be different from the one with the white random sequence,

- 16 point (1/4 of 64 subcarriers) guard interval (GI). Temporarily, the GI is not considered to make the pilot only OFDM signal.

Figure 4.5 shows the IEEE 802.11a/g subcarrier mapping for the IFFT.

![Figure 4.5: IEEE 802.11a/g OFDM 64 points IFFT input and output mapping [45].](image)

Therefore we have the following two subcarrier frequencies,

\[ f_1 \triangleq 7\Delta_f = 2.1875 \text{ MHz} \quad \text{and} \quad f_2 \triangleq 21\Delta_f = 6.5625 \text{ MHz}. \]  

(4.41)

To make the pilot only signal, all of the subcarriers are set to zero except the pilots, and the GI is not generated. For the simulation, 110 OFDM symbols are generated and a 16384 point IFFT is used. Thus, the spectral resolution is approximately 1.221 KHz.

The resulting positive cycle frequencies are found using Eq.(4.18) and the corresponding SCF CDP is presented in Figure 4.6.

\[
\alpha = \begin{cases} 
2f_1 = 4.375 \text{MHz} & \text{at } f = 0 \text{MHz} \\
2f_2 = 13.125 \text{MHz} & \text{at } f = 0 \text{MHz} \\
f_2 - f_1 = 4.375 \text{MHz} & \text{at } f = \pm 4.375 \text{MHz} \\
f_2 + f_1 = 8.75 \text{MHz} & \text{at } f = \pm 2.1875 \text{MHz}
\end{cases}
\]  

(4.42)
In order to identify the major spectral features of the SCF and their relation to Eq. (4.23), the spectral shapes are plotted over spectral frequencies at specific cycle frequencies using computer simulation. As Eq. (4.42) indicates, two cyclic features overlap at the cycle frequency $\alpha = 4.375$. The spectral shape at a cycle frequency of $\alpha = 4.375$ will have the following form (note that the SCF equations below are approximations for the white random pilot sign sequences.)

$$
\left| \hat{S}_s^{\alpha=2\hat{f}_1 \text{ and } \hat{f}_2-\hat{f}_1} \right| = (f) \frac{1}{T_s} \left| W(f) \right|^2 \left| \hat{S}_s(f) \right| 
+ \frac{1}{T_s} \left| W\left(f + \frac{\hat{f}_2 + \hat{f}_1}{2}\right) \right|^2 \left| \hat{S}_s\left(f + \frac{\hat{f}_2 + \hat{f}_1}{2}\right) \right| 
+ \frac{1}{T_s} \left| W\left(f - \frac{\hat{f}_1 + \hat{f}_2}{2}\right) \right|^2 \left| \hat{S}_s\left(f - \frac{\hat{f}_1 + \hat{f}_2}{2}\right) \right|
$$

(4.43)
The spectral shape at cycle frequency $\alpha = 8.75 \, MHz$ is depicted below.

$$
\left| \hat{S}_y^{\alpha=f_2+f_1}(f) \right| = \frac{1}{T_s} \left| W \left( f - \frac{f_2-f_1}{2} \right) \right|^2 \left| \hat{S}_y \left( f - \frac{f_2-f_1}{2} \right) \right| + \frac{1}{T_s} \left| W \left( f + \frac{f_2-f_1}{2} \right) \right|^2 \left| \hat{S}_y \left( f + \frac{f_2-f_1}{2} \right) \right|
$$

(4.44)
The spectral shape at cycle frequency $\alpha = 13.125 MHz$ is depicted below.

$$\left| \hat{S}_{y}^{\alpha=2f_2}(f) \right| = \frac{1}{T_s} \left| W(f) \right|^2 \left| \hat{S}_s(f) \right|$$  \hspace{1cm} (4.45)

Next, the CDP for ordinary IEEE 802.11a/g signal is simulated. QPSK data is used for non-pilot subcarriers. The result is depicted in Figure 4.10. The cyclic features related to the pilots are still dominant and can be identified.
4.8 Evaluation of OFDM Pilot Signals with Guard Interval

OFDM combats multipath delay spread efficiently with the help of a guard interval and cyclic prefix. The delay spread causes inter-symbol interference and deteriorates receiver performance. To eliminate the multipath delay spread completely, a guard interval longer than the multipath delay spread and cyclic prefix is required. Therefore, the cyclic spectral analysis for OFDM signals with guard interval is needed to represent a real system. In this section, the cyclic prefix concept is reviewed briefly and the SCF for OFDM signal having GI and a cyclic prefix is investigated using second-order statistical cyclic analysis.

The guard interval’s duration is usually an integer fraction of the IFFT output, such as 1/4, 1/8, 1/16, and 1/32 based on the maximum delay spread. The cyclic prefix is created by copying the last part of the IFFT output with the length equal to the guard interval duration.
and placing it at the beginning of the IFFT output. Through this procedure, the orthogonal property among the OFDM subcarriers is maintained and phase continuity between the cyclic prefix and the IFFT output is maintained. However, the GI can cause various phase shifts between OFDM symbols other than zero or 180 degrees according to the GI length and the subcarrier index. These phase shifts make the cyclic spectral analysis using LPTV transformation difficult. Therefore, the cyclic features of the OFDM pilots affected by GI are estimated using cyclic statistical analysis instead of LPTV transformation. The SCF is predicted using the CAF.

4.8.1 Investigation of Phase Shift between OFDM Symbols

As we mentioned earlier, the GI can cause phase discontinuity between OFDM symbols. Such phase discontinuity results in cyclic feature variations. The possible phase shifts for OFDM with \( N \)-point IFFT are summarized in the table below.

<table>
<thead>
<tr>
<th>Phase Shift</th>
<th>Subcarrier Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>All (No pilot sign change)</td>
</tr>
<tr>
<td>( \pi )</td>
<td>All (Pilot sign change)</td>
</tr>
</tbody>
</table>

If the OFDM signal does not have GI, then the pilots have two possible phases, zero and 180 degrees. Thus, the strong cyclic features related to the pilots are obtained. This is confirmed in the previous section for pilot only OFDM signal analysis.

The following two tables show possible phase shifts at OFDM symbol boundaries when GI sizes are 1/4 and 1/8 of the IFFT size.
Table 4-2: Phase shifts between OFDM symbols when the guard interval duration is $T_{FFT}/4$

<table>
<thead>
<tr>
<th>Phase Shift</th>
<th>Remainder of Subcarrier Index divided by 4, mod($k,4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi/2$</td>
<td>1</td>
</tr>
<tr>
<td>-$\pi/2$</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\pi$</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4-3: Phase shifts between OFDM symbols when the guard interval duration is $T_{FFT}/8$

<table>
<thead>
<tr>
<th>Phase Shift</th>
<th>Remainder of Subcarrier Index divided by 8, mod($k,8$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi/4$</td>
<td>1</td>
</tr>
<tr>
<td>-$\pi/4$</td>
<td>7</td>
</tr>
<tr>
<td>$\pi/2$</td>
<td>2</td>
</tr>
<tr>
<td>-$\pi/2$</td>
<td>6</td>
</tr>
<tr>
<td>$3\pi/4$</td>
<td>3</td>
</tr>
<tr>
<td>-$3\pi/4$</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-$\pi$</td>
<td>4</td>
</tr>
</tbody>
</table>

where mod($k,g$) is the function of evaluating modulus after dividing $k$ with $g$.

The above tables show the phase change when the pilot signs are the same. If the pilot sign is changed at the next OFDM symbol, then the sign of the phase shift must be changed.

$^7$ $T_{FFT}$ is the IFFT duration. The OFDM symbol period is $T_s = T_{GI} + T_{FFT}$, where $T_{GI}$ is the GI duration.
Let’s investigate the phase change using the case of 64 subcarrier OFDM with 1/4 GI duration. The time domain plot for the subcarrier index for the pilot [7,-7] is shown below.

![Time Domain Plot of 64 Subcarriers Pilot Only OFDM. Pilot Index = 7,-7. Same Pilot Sign.](image)

Figure 4.11: Time domain plot for 64 subcarrier pilot only OFDM. Pilot index is 7 and -7. Two OFDM symbols are divided by vertical line. The 90 degrees phase shift occurs at the symbol boundary. If the pilot sign is not the same between the two OFDM symbols, then a -90 degree phase shift will occur.

4.8.2 CAF Estimation of the OFDM Pilots with Guard Interval

To investigate the impacts of GI on the pilot cyclic features, consider the pilot only OFDM signal which includes GI,

\[
x(t) = \sum_{n=-\infty}^{\infty} s_n \times \left(e^{j2\pi f_1 \beta} + e^{-j2\pi f_1 \beta} - e^{j2\pi f_2 \beta} + e^{-j2\pi f_2 \beta}\right) w(t - nT_s)
\]  

(4.46)

where

\[
\beta = (t - nT_s - T_{GI})
\]  

(4.47)

\[
f_1 = k_1 \Delta_f \quad \text{and} \quad f_2 = k_2 \Delta_f
\]  

(4.48)
Let’s define GI index ‘G’ as the ratio of IFFT to GI duration

\[ G \triangleq \frac{T_{\text{IFFT}}}{T_{\text{GI}}} \quad (4.49) \]

Then, the relation between the GI and OFDM symbol duration is given by

\[ T_s = T_{\text{IFFT}} + T_{\text{GI}} = \left(1 + \frac{T_{\text{GI}}}{T_{\text{IFFT}}} \right)T_{\text{IFFT}} = (1 + 1/G)T_{\text{IFFT}} = (1 + 1/G)/\Delta_f \quad (4.50) \]

It is assumed that \( E\{s_n s_m\} = \sigma_s^2 \delta(n-m) \), where \( \delta(\cdot) \) is the Dirac delta function.

The correlation function of Eq.(4.46) is

\[ R_x(t + \tau/2, t - \tau/2) = E\{x(t + \tau/2)x^*(t - \tau/2)\} \quad (4.51) \]

Thus, we have the following correlation function

\[
R_x\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) = \sigma_s^2 \sum_{n=-\infty}^{\infty} \left(e^{j2\pi f_1(\beta + \tau/2)} + e^{-j2\pi f_1(\beta + \tau/2)} - e^{j2\pi f_2(\beta + \tau/2)} - e^{-j2\pi f_2(\beta + \tau/2)} \right)
\times \left(e^{-j2\pi f_1(\beta - \tau/2)} + e^{j2\pi f_1(\beta - \tau/2)} - e^{-j2\pi f_2(\beta - \tau/2)} + e^{j2\pi f_2(\beta - \tau/2)} \right)
\times w(t + \tau/2 - nT_s)w^*(t - \tau/2 - nT_s) \quad (4.52)
\]

The time delayed product of the sinusoidal component is expanded as

\[
PS \triangleq \left(e^{j2\pi f_1(\beta + \tau/2)} + e^{-j2\pi f_1(\beta + \tau/2)} - e^{j2\pi f_2(\beta + \tau/2)} - e^{-j2\pi f_2(\beta + \tau/2)} \right)
\times \left(e^{-j2\pi f_1(\beta - \tau/2)} + e^{j2\pi f_1(\beta - \tau/2)} - e^{-j2\pi f_2(\beta - \tau/2)} + e^{j2\pi f_2(\beta - \tau/2)} \right)
\times \left(e^{j2\pi f_1(\beta - \tau/2)} + e^{-j2\pi f_1(\beta - \tau/2)} - e^{j2\pi f_2(\beta - \tau/2)} + e^{-j2\pi f_2(\beta - \tau/2)} \right)
\times \left(e^{-j2\pi f_1(\beta + \tau/2)} + e^{j2\pi f_1(\beta + \tau/2)} - e^{-j2\pi f_2(\beta + \tau/2)} - e^{j2\pi f_2(\beta + \tau/2)} \right)
\times \left(e^{j2\pi f_1(\beta - \tau/2)} + e^{-j2\pi f_1(\beta - \tau/2)} - e^{j2\pi f_2(\beta - \tau/2)} - e^{-j2\pi f_2(\beta - \tau/2)} \right)
\times \left(e^{-j2\pi f_1(\beta + \tau/2)} - e^{j2\pi f_1(\beta + \tau/2)} - e^{-j2\pi f_2(\beta + \tau/2)} + e^{j2\pi f_2(\beta + \tau/2)} \right)
\times \left(e^{j2\pi f_1(\beta - \tau/2)} - e^{-j2\pi f_1(\beta - \tau/2)} + e^{j2\pi f_2(\beta - \tau/2)} - e^{-j2\pi f_2(\beta - \tau/2)} \right)
\times \left(e^{-j2\pi f_1(\beta + \tau/2)} - e^{j2\pi f_1(\beta + \tau/2)} - e^{-j2\pi f_2(\beta + \tau/2)} + e^{j2\pi f_2(\beta + \tau/2)} \right)
\times \left(e^{j2\pi f_1(\beta - \tau/2)} - e^{-j2\pi f_1(\beta - \tau/2)} + e^{j2\pi f_2(\beta - \tau/2)} - e^{-j2\pi f_2(\beta - \tau/2)} \right)
\times \left(e^{-j2\pi f_1(\beta + \tau/2)} - e^{j2\pi f_1(\beta + \tau/2)} - e^{-j2\pi f_2(\beta + \tau/2)} + e^{j2\pi f_2(\beta + \tau/2)} \right)
\times \left(e^{j2\pi f_1(\beta - \tau/2)} - e^{-j2\pi f_1(\beta - \tau/2)} + e^{j2\pi f_2(\beta - \tau/2)} - e^{-j2\pi f_2(\beta - \tau/2)} \right) \quad (4.53)
\]
We know that normal (or non-conjugate) CAF is symmetric over the cycle frequency zero. Therefore, the CAF of positive cycle frequencies will provide necessary information for the CAF of OFDM pilots. If we assume that the frequency $f_2$ is greater than $f_1$, the CAF at positive cycle frequencies, except zero, is expressed by

$$PS^+ = \left\{ e^{i2\pi f_1 2\beta} - e^{i2\pi f_2 2\beta} + e^{i2\pi (f_2(\beta-\tau/2)+f_1(\beta+\tau/2))} - e^{i2\pi (f_2(\beta-\tau/2)-f_1(\beta+\tau/2))} \right\} \times w(t + \tau / 2 - nT_s)w^* (t - \tau / 2 - nT_s)$$

where the superscript + indicates positive cycle frequencies.

The positive autocorrelation function for Eq.(4.54) is given by

$$R_x\left( t + \frac{\tau}{2}, t - \frac{\tau}{2} \right)^+ = \sigma_s^2 \sum_{n=-\infty}^{\infty} \left\{ e^{i2\pi f_1 2\beta} - e^{i2\pi f_2 2\beta} + e^{i2\pi (f_2(\beta-\tau/2)+f_1(\beta+\tau/2))} - e^{i2\pi (f_2(\beta-\tau/2)-f_1(\beta+\tau/2))} \right\} \times w(t + \tau / 2 - nT_s)w^* (t - \tau / 2 - nT_s)$$

Eq.(4.55) needs to be checked to make sure that it is periodic with period $T_s$. In other words, the following identify needs to be proved

$$R_x\left( t + T_s + \frac{\tau}{2}, t + T_s - \frac{\tau}{2} \right)^+ = R_x\left( t + \frac{\tau}{2}, t - \frac{\tau}{2} \right)^+$$
This periodic characteristic is identified as

\[ R_s \left( t + T_s + \frac{\tau}{2}, t + T_s - \frac{\tau}{2} \right)^+ = \sigma_s^2 \sum_{n=-\infty}^{\infty} \left\{ e^{j2\pi f_1 (t + T_s - nT_s - T_ga)} - e^{j2\pi f_1 (t + T_s - nT_s - T_ga)} + e^{j2\pi (f_2 - f_1) (t + T_s - nT_s - T_ga)} \times \left( e^{-j\pi (f_2 - f_1) \tau} - e^{j\pi (f_2 - f_1) \tau} \right) \right\} \]

\[ \times w(t + T_s + \tau / 2 - nT_s) w^*(t + T_s - \tau / 2 - nT_s) \]  

(4.57)

Therefore, Eq. (4.57) is a periodic signal with period \( T_s \).

This periodicity allows the CAF evaluation using \( \alpha = k / T_s \), where \( k \) is an integer.

Namely,

\[ R_s^{\alpha+} (\tau) = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} R_s \left( t + \frac{\tau}{2}, t - \frac{\tau}{2} \right)^+ e^{-j2\pi \alpha t} dt \]

\[ = \sigma_s^2 \sum_{n=-\infty}^{\infty} \left\{ e^{-j4\pi f_1 T_ga} e^{j2\pi f_1 T_ga} e^{j2\pi 2t} + e^{-j2\pi (f_2 - f_1) T_ga} \times \left( e^{-j\pi (f_2 - f_1) \tau} - e^{j\pi (f_2 - f_1) \tau} \right) \times e^{j2\pi (f_2 - f_1) \tau} \right\} \]

(4.58)

\[ \times \left\{ e^{-j4\pi f_1 T_ga} e^{-j2\pi (\alpha + 2 f_1) \tau} - e^{-j4\pi f_1 T_ga} e^{-j2\pi (\alpha - 2 f_1) \tau} \right\} \]

\[ + e^{-j2\pi (f_2 - f_1) T_ga} \times \left( e^{-j\pi (f_2 - f_1) \tau} - e^{j\pi (f_2 - f_1) \tau} \right) \times e^{-j2\pi (f_2 - f_1) \tau} \]

\[ \times \left\{ e^{-j4\pi f_1 T_ga} e^{-j2\pi (\alpha - f_1) \tau} - e^{-j4\pi f_1 T_ga} e^{-j2\pi (\alpha + f_1) \tau} \right\} \]

(4.58)

where \( |\tau| \leq T_s \) and \( w(t) \) is assumed to be a rectangular window.

Eq. (4.58) can be divided into four parts based on the distinctive cycle frequencies of the dominant cyclic features.

\[ R_s^{\alpha+} (\tau) \triangleq CAF = CAF1 - CAF2 + CAF3 + CAF4 \]  

(4.59)

where

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\[ \text{CAF} 1 \triangleq \sigma^2_s \left\{ e^{-j4\pi f_1 T_s} \sin \left[ \frac{\pi (\alpha - 2 f_1) (T_s - |\tau|)}{\pi (\alpha - 2 f_1) / T_s} \right] \right\} \] (4.60)

\[ \text{CAF} 2 \triangleq \sigma^2_s \left\{ e^{-j4\pi f_2 T_s} \sin \left[ \frac{\pi (\alpha - 2 f_2) (T_s - |\tau|)}{\pi (\alpha - 2 f_2) / T_s} \right] \right\} \] (4.61)

\[ \text{CAF} 3 \triangleq \sigma^2_s e^{-j2\pi (f_2 + f_1) T_s} \times \left\{ e^{-j\pi (f_2 - f_1) \tau} - e^{j\pi (f_2 - f_1) \tau} \right\} \frac{\sin \left[ \pi (\alpha - (f_2 + f_1)) (T_s - |\tau|) \right]}{\pi (\alpha - (f_2 + f_1)) / T_s} \] (4.62)

\[ \text{CAF} 4 \triangleq \sigma^2_s e^{-j2\pi (f_2 - f_1) T_s} \times \left\{ e^{-j\pi (f_2 + f_1) \tau} - e^{j\pi (f_2 + f_1) \tau} \right\} \frac{\sin \left[ \pi (\alpha - (f_2 - f_1)) (T_s - |\tau|) \right]}{\pi (\alpha - (f_2 - f_1)) / T_s} \] (4.63)

The purpose of this analysis is to find the cycle frequencies of the dominant OFDM pilot features that can be used to constitute a CDP signature. Therefore, the cycle frequencies that maximize the individual terms of Eq.(4.59) are required. This is equivalent to maximizing each term in Eq.(4.59) over the variable \( \alpha \). Note that the \( k \) in \( \alpha = k / T_s \) is an integer number. Then we have the following cycle frequencies:

- \( \alpha = \frac{k}{T_s} = 2 f_1 = 2 k_1 \Delta_f = 2 k_1 \frac{1+1/G}{T_s} \Leftrightarrow k = \left[ 2 k_1 (1+1/G) \right]_{\text{close}} \)

- \( \alpha = \frac{k}{T_s} = 2 f_2 = 2 k_2 \Delta_f = 2 k_2 \frac{1+1/G}{T_s} \Leftrightarrow k = \left[ 2 k_2 (1+1/G) \right]_{\text{close}} \)

- \( \alpha = \frac{k}{T_s} = f_2 + f_1 = (k_2 + k_1) \Delta_f = (k_2 + k_1) \frac{1+1/G}{T_s} \Leftrightarrow k = \left[ (k_2 + k_1) (1+1/G) \right]_{\text{close}} \)

- \( \alpha = \frac{k}{T_s} = f_2 - f_1 = (k_2 - k_1) \Delta_f = (k_2 - k_1) \frac{1+1/G}{T_s} \Leftrightarrow k = \left[ (k_2 - k_1) (1+1/G) \right]_{\text{close}} \)

where \( \left[ . \right]_{\text{close}} \) takes the nearest two integer values for the non-integer argument.
In summary, the cycle dominant cycle frequencies for OFDM pilot are given by

\[ \{\alpha\} = \left\{ \frac{2k_1 (1+1/G)}{G}, \frac{2k_2 (1+1/G)}{G}, \frac{(k_2 + k_1) (1+1/G)}{G}, \frac{(k_2 - k_1) (1+1/G)}{G} \right\} \times f_s, \quad (4.64) \]

Let’s verify Eq. (4.64) using the IEEE 802.11a/g signal parameters.

- Pilot indexes are \( k_1 = \pm 7 \) and \( k_2 = \pm 21 \),
- GI index is \( G = 4 \),
- OFDM symbol rate is \( f_s = 250 KHz \).

Then, the dominant positive cycle frequencies of IEEE 802.11a/g pilots are

\[ \{\alpha\} = \left\{ \frac{2 \times 7 \times (1+1/4)}{G}, \frac{2 \times 21 \times (1+1/4)}{G}, \frac{(7 + 21) \times (1+1/4)}{G}, \frac{(21 - 7) \times (1+1/4)}{G} \right\} \times 250 KHz
\]

\[ = \left\{ \frac{17.5}{G}, \frac{52.5}{G}, \frac{35}{G}, \frac{17.5}{G} \right\} \times 250 KHz \]

\[ = \left\{ \{17.5\}, \{52.5\}, \{35\}, \{17.5\} \right\} \times 250 KHz \]

\[ = \{\{4.25, 4.50\}, \{13.0, 13.25\}, \{8.75\}, \{4.25, 4.50\}\} \text{ MHz}. \quad (4.65) \]

The cyclic features at cycle frequencies 4.25 MHz and 4.50 MHz overlap.

To verify the results shown in Eq.(4.65), CAF (3D plot), CAF at specific time delay \( \tau \) (2D plot), and CAF at specific cycle frequency \( \alpha \) (2D plot) of IEEE 802.11a/g pilots are plotted in Figure 4.12, Figure 4.13, Figure 4.14, Figure 4.15, Figure 4.16, and Figure 4.17. In these plots, the time delay \( \tau \) is normalized with the OFDM symbol duration \( T_s \).

In Figure 4.14, the cyclic feature at cycle frequency \( \alpha = 8.75 \text{ MHz} \) becomes dominant with time delay \( \tau / T_s = 0.0283 \). This time delay is equivalent to 113.2 ns in IEEE 802.11a/g.
Figure 4.12: The 3D magnitude plot of IEEE 802.11 a/g pilot only CAF. Note that the cycle frequency is discrete. There are cyclic features at cycle frequencies (4.25, 4.5) MHz and (13.0, 13.25) MHz. These features are thicker than the cyclic feature found at a cycle frequency of 8.75 MHz.

Figure 4.13: IEEE 802.11a/g pilot cyclic feature with zero normalized time delay. The cycle feature for cycle frequency of 8.75 MHz is almost zero.
Figure 4.14: IEEE 802.11a/g pilot cyclic feature with 0.0282 normalized time delay. The cycle feature for cycle frequency of 8.75 MHz is the strongest.

Figure 4.15: IEEE 802.11a/g pilot cyclic feature with cycle frequency $\alpha=4.25$ MHz. There are two dominant features related to CAF1 and CAF4. The corresponding SCF will be shown at DC for CAF1 and frequency shifted for CAF4.
Figure 4.16: IEEE 802.11a/g pilot cyclic feature with cycle frequency $\alpha=8.75$ MHz. The corresponding SCF will be shown at frequency shifted for CAF3.

Figure 4.17: IEEE 802.11a/g pilot cyclic feature with cycle frequency $\alpha=13.25$ MHz. The corresponding SCF will be shown at DC for CAF2.
4.8.3 General Formula for the Cycle Frequencies of Dominant OFDM Pilot Cyclic Features

Based on the second-order cyclic spectral analysis of the OFDM pilots, the impacts of GI size on the OFDM pilot features are identified.

To formulate the GI impact on the dominant pilot cyclic features, the analysis results shown in Eq. (4.18) and Eq. (4.64) are summarized.

- Dominant positive cycle frequencies of OFDM pilot without GI.

\[
\{\alpha\} = \{2k_1, 2k_2, (k_2 + k_1), (k_2 - k_1)\} \times \Delta_f
\]

(4.66)

- Dominant positive cycle frequencies of OFDM pilot with GI.

\[
\{\alpha\} = \left\{ \left[2k_1 \left(1+1/G\right)_{close} \right]_{close}, \left[2k_2 \left(1+1/G\right)_{close} \right]_{close}, \left[(k_2 + k_1)(1+1/G)\right]_{close}, \left[(k_2 - k_1)(1+1/G)\right]_{close} \right\} \times f_s
\]

(4.67)

We call the cycle frequencies in Eq.(4.66) as base cycle frequencies or just ‘base’.

These results are also applicable for three OFDM pilots. Thus, Eq. (4.66) is modified to predict dominant pilot cyclic feature from three pilots as:

\[
\{\alpha\} = \left\{ 2k_1, 2k_2, 2k_3, (k_3 + k_1), (k_3 + k_2), (k_2 + k_1), (k_3 - k_1), (k_3 - k_2), (k_2 - k_1) \right\} \times \Delta_f
\]

(4.68)

where \(0 < k_1 < k_2 < k_3\).

Similar modifications apply to Eq.(4.67) for three OFDM pilots. In this way, Eq.(4.66) and Eq.(4.67) are generalized for a larger number of pilots.
Eq.(4.66) and Eq.(4.67) can be divided two parts: doubled subcarrier index and combined subcarrier indexes. We call them ‘non-combined’ carrier and ‘combined’ carrier, respectively.

If the divisions \( (2k / G) \) and \( ((k_1 + k_2) / G) \) in Eq.(4.67) produce an integer number, then the pilot cycle frequency from Eq.(4.67) will be the same as Eq.(4.66). However, if the divisions do not generate an integer number, then the pilot cycle frequency deviates from the base. We call the deviation a split of Eq.(4.66) or just ‘split’. The split can be expressed mathematically:

- For non-combined carrier

\[
\alpha_n = \pm \left( 2 \times k \times \Delta_F + \left\{ - \left( \frac{m}{G/2} \right), \left( 1 - \frac{m}{G/2} \right) \right\} \times f_s \times \bar{\delta}(m) \right)
\]

(4.69)

- For combined carrier

\[
\alpha_c = \pm \left( (k_2 \pm k_1) \times \Delta_F + \left\{ - \left( \frac{n}{G} \right), \left( 1 - \frac{n}{G} \right) \right\} \times f_s \times \bar{\delta}(n) \right)
\]

(4.70)

where \( \text{mod}(k, G) \) is the modulus after the division of \( k \) by \( G \), \( m = \text{mod}(k, G/2) \) and \( k \) is the positive pilot subcarrier index, \( n = \text{mod}(k_2 \pm k_1, G) \) with \( 0 < k_1 < k_2 < k_3 \), and \( \bar{\delta}(\cdot) \) is zero if the argument is zero, and one otherwise.

Note that the split reduces the strength of the pilot cyclic feature due to the cycle frequency deviation from the base which has the maximum strength.
- (Example A)- IEEE 802.11a/g with pilot indexes 10 and 22. No split case.

The dominant pilot cycle frequencies are given by

- For non-combined carrier

\[
\text{mod}(22, 2) = 0 \quad \Rightarrow \quad (2 \times 22) \times \Delta_f = 13.75 \text{MHz}
\]

\[
\text{mod}(10, 2) = 0 \quad \Rightarrow \quad (2 \times 10) \times \Delta_f = 6.25 \text{MHz}
\] (4.71)

- For combined carrier

\[
\text{mod}([22 \pm 10], 4) = 0 \quad \Rightarrow \quad (22 \pm 10) \times \Delta_f = 10 \text{ and } 3.75 \text{MHz}
\] (4.72)

Those cyclic features will have the strongest amplitudes and they are the same as the PSD feature (\( \alpha = 0 \)). The cycle frequencies and their amplitudes are shown in Figure 4.18.

![Figure 4.18: SCF magnitude of the SCF CDP for pilot only IEEE 802.11a/g OFDM signals with zero inputs for non-pilot subcarriers. The subcarrier indexes for the pilots are [10, 22, -10, -22].](image-url)
(Example B)- IEEE 802.11a/g with pilot indexes 11 and 20. Split case.

The dominant pilot cycle frequencies are given by

- For non-combined carrier

\[
\begin{align*}
\text{mod}([20], 2) &= 0 \\
\Rightarrow (2 \times 20) \times \Delta_f &= 12.5 \text{MHz} \\
\text{mod}([11], 2) &= 1 \\
\Rightarrow (2 \times 11) \times \Delta_f + \left\{-(f_{sym} / 2), (f_{sym} / 2)\right\} &= 6.75 \text{ and } 7.0 \text{MHz}
\end{align*}
\]

(4.73)

- For combined carrier

\[
\begin{align*}
\text{mod}([20+11], 4) &= 3 \\
\Rightarrow (20+11) \times \Delta_f + \left\{-(3f_{sym} / 4), (f_{sym} / 4)\right\} &= 9.5 \text{ and } 9.75 \text{MHz} \\
\text{mod}([20-11], 4) &= 1 \\
\Rightarrow (20-11) \times \Delta_f + \left\{-(f_{sym} / 4), (3f_{sym} / 4)\right\} &= 2.75 \text{ and } 3.0 \text{MHz}
\end{align*}
\]

(4.74)

The cyclic feature strength at \( \alpha = 12.5 \) is the strongest due to no cyclic feature split. The cyclic feature strength at \( \alpha = 9.75 \) is stronger than the cyclic feature at \( \alpha = 9.5 \). Because the cyclic feature at \( \alpha = 9.5 \) is close to the base. The dominant cycle frequencies and their amplitudes are shown in Figure 4.19.

Note that the cyclic feature strengths at \( \alpha = 9.5 \) and \( \alpha = 3.0 \) are the smallest. These cyclic features are the most far away from their bases given GI size.
4.9 SCF of Ideally Generated Ordinary OFDM Signals

Up to now, the pilot only OFDM signal is investigated. However, the overall impact from other data subcarriers on cyclic features has not been investigated. Using computer simulation, how the data subcarriers with balanced modulation (e.g. QPSK or 16QAM) affect the pilot cyclic features is investigated.

For this investigation, ordinary OFDM signal is generated using computer simulations that conform to the IEEE 802.11a/g standard. For the simulation, data subcarriers are modulated with a QPSK and 64QAM, a balanced modulation. As we showed in section 4.6, the cyclic features related QPSK subcarriers are ideally zero except symbol rate features. The QPSK and 64 QAM related cyclic features appear as a noise-like floor due to the limited observation length. The computer simulation results are shown in Figure 4.20 and Figure 4.21.
Figure 4.20: SCF magnitude of the ordinary SCF CDP for IEEE 802.11a/g OFDM signal with QPSK inputs. The cycle frequencies at 4.25 and 4.6MHz overlap.

Figure 4.21: SCF magnitude of the ordinary SCF CDP for IEEE 802.11a/g OFDM signal with 64QAM inputs.

In these plots, the dominant pilot cyclic features are found at

\[ \alpha = \{4.25, 4.50, 8.5, 13, 13.25\} \text{MHz} \quad (4.75) \]
4.10 SCF of Measured IEEE 802.11a/g Signal

We developed an IEEE 802.11a/g WLAN signal interceptor to digitize this OFDM signal. We then captured the WLAN uplink signal from the LinSys WLAN card (WPC55AG), installed in a laptop computer, transmitted to the Motorola (WR850G) access point (AP). We configured the WLAN and AP to transmit a large file containing random data at the maximum speed (54Mbps with 64QAM modulation) continuously. We captured seven uplink bursts from the LinkSys WLAN card. Each burst is filtered with a 22MHz low-pass filter and digitized using a digital oscilloscope with 50MHz sampling frequency. To plot the SCF of the captured signal, seven SCFs are averaged to minimize random effects due to a noise and interference. The measurement setup is explained in more detail in Chapter 7. The measured SCF CDP from the WLAN card is plotted in Figure 4.22. From this figure, the identified dominant positive cycle frequencies are

\[
\alpha = \{4.251, 4.501, 8.749, 13, 13.25\} MHz
\]  (4.76)

The result shown in Eq. (4.76) is consistent with the analysis and the ideally generated SCF CDP results shown in Eq.(4.65) and Eq.(4.75).

Figure 4.22: SCF CDP plot of the measured IEEE 802.11g OFDM signal from LinkSys WLAN card.
4.11 OFDM Pilot Cyclic Feature Detection Performance Analysis

The detection performance variations for OFDM pilot cyclic features due to GI are investigated in terms of the detection rate \( P_D \), the false alarm rate \( P_{FA} \), and the signal to noise ratio (SNR). The two most popular methods for the evaluation of cyclic feature detection performance are the Giannakis & Dandawate (G&D) method from [46] and the Gardner & Spooner (G&S) method from [47]. The G&D method uses the asymptotic statistics under a restrictive assumption of an AWGN channel to create the detector. However, the G&S method directly finds the signal detector and parameter estimator. The test statistics for the G&S method for single cyclic features at cycle frequency \( \alpha \) is given by

\[
y(t)_{sc} = \left| \int_{-\infty}^{\infty} S_i^\alpha(f)^* S_{\alpha}(t,f) df \right| \tag{4.77}
\]

where \( S_i^\alpha(f) \) is the ideal SCF and \( S_{\alpha}(t,f) \) is the cyclic periodogram which is discussed in Chapter 3.

The single cycle detector compares the statistics in Eq.(4.77) to a threshold. The threshold can be set up by measuring the statistics for various types of interference and does not depend on the signal of interest.

The detection performance for IEEE802.11a/g pilot cyclic features are evaluated using the G&S method with AWGN noise. To compare the cyclic feature detection performance at cycle frequency \( 2f_c \) with the pilot indexes \( k = 20 \) and \( k = 21 \) and 1/4 GI, the receiver operating characteristic (ROC) is evaluated using sub-optimal single cycle detector and is presented in Figure 4.23. The corresponding cyclic features can be found using the general formula for the pilot in (4.69) and is given by
As Eq. (4.78) indicates, there is a cyclic feature split for pilot index $k = 21$ and the cyclic feature strength is expected to be smaller than the cyclic feature for pilot index $k = 20$. For the pilot index $k = 21$, the cycle frequency 13.0MHz is taken for performance evaluation.

As Figure 4.23 indicates, the detection performance of the cyclic feature for a signal having 0 and 180 degrees phase shift is better than the cyclic feature for a signal having other than 180 degrees phase shifts.

As this result indicates, the pilot pattern and GI size affects the cyclic feature detection performance. Therefore, the OFDM pilot cyclic signatures need to be chosen in order to have 0 or 180 phase shifts at the OFDM symbol boundary. This is equivalent to having the

$$
\{\alpha\}_{k=20} = 2 \times k \times \Delta_f + \left\{ \frac{m}{G/2} \times f_{sym}, 1 - \frac{m}{G/2} \times f_{sym} \right\} \times \delta (m) = 12.5 \text{MHz}
$$

$$
\{\alpha\}_{k=21} = 2 \times k \times \Delta_f + \left\{ \frac{m}{G/2} \times f_{sym}, 1 - \frac{m}{G/2} \times f_{sym} \right\} \times \delta (m) = \{13.00, 13.25\} \text{MHz}
$$

\[(4.78)\]
zero value for the modulo operation given the pilot index and GI size in Eq.(4.69) and Eq.(4.70).

### 4.12 OFDM Pilot Cyclic Signature for cDSA Network Identification

The 4G and the beyond 4G system is expected to converge to an OFDM based technology. In addition, the 4GB system is expected to include cognitive radio technology in order to enhance radio transmission performance and increase spectrum efficiency. One of the most promising properties of OFDM is that it can tailor its spectral shape to fit into the available white spectrum by controlling the IFFT inputs. This flexible spectral shaping property of OFDM is very useful for adapting to the dynamic change in the bandwidth and channel of ‘white’ spectrum as shown in Figure 4.24.

**Figure 4.24:** Concept of dynamic spectrum access by exploiting white spectrum in [48].

DSA # indicates the time mark of the frequency migration.
As an example, consider the situation in which primary and secondary users coexist. The secondary users are CR enabled and can establish dynamic spectrum access (cDSA) networks to utilize white spectrum. A cDSA network can be comprised of two CR or multiple CR devices or CR nodes. The primary user can be a 6 MHz digital TV signal and the secondary user can be an OFDM based signal. In this case, the minimum bandwidth of white spectrum can be set to 6 MHz and spectrum sensing is performed by scanning TV channels (F1, F2, ..., F5 in Figure 4.24). If two contiguous TV channels are unoccupied, the cDSA network can concatenate the channels for its operational bands (DSA 4 uses both F2 and F3 in Figure 4.24).

The efficient usage of white spectrum through dynamic spectrum access and CR technologies requires spectrum awareness, frequency coordination, and adaptation [48]. The spectrum awareness includes spectrum sensing and spectrum policy knowledge. The frequency coordination includes finding each of the CR devices in time and frequency. This is also known as FR (Frequency Rendezvous) [48]. This adaptation is a method of minimizing harmful interference to other radio nodes by controlling transmission power and signal bandwidth. The adaptation is beyond the scope of this work.

An energy detector is often used for white spectrum sensing due to its simple structure. However, the performance of the energy detector degrades in low and varying SNR signals. Conventional methods for frequency coordination use a common control channel. However, the common control channel is not spectrally efficient. A common control channel also needs to find white spectrum which is dynamically changing in time and frequency. This implies that CR devices have to keep track of the dynamic common control channels.

To solve the technical challenges in spectrum sensing and frequency coordination for the cDSA network, OFDM pilot cyclic features are proposed for spectrum sensing and frequency coordination in this work. A similar approach using OFDM cyclic features for a DSA network can be found in Sutton’s work [49]. Sutton generates unique OFDM cyclic
signatures by controlling the specially generated QAM inputs. However, Sutton’s work misses two important points in generating OFDM cyclic signatures. One is the cyclic feature split and thus the decrease of cyclic feature strength. Another is the combined cyclic features from the multiple QAM inputs.

In wireless communications using OFDM, pilots are usually included in signal design to minimize channel fading effect. Although, the GI and cyclic prefix can remove the effects of multipath channel fading, this is only possible when the GI is always greater than maximum delay spread of the multipath channel fading. If pilots are added to the OFDM signal having QAM cyclic signatures, then the pilots and the special QAM inputs will generate additional features. Thus, the OFDM cyclic signature is obscured and identification of the DSA network can be unsuccessful.

In this work, the spectrum sensing and frequency coordination are performed by detecting the pilot cyclic features and generating unique OFDM pilot cyclic signatures. The unique OFDM pilot cyclic feature serves as the identifier of the cDSA network. If the cDSA network changes its operational frequency band to other white spectrum, the CR nodes associated with the cDSA network move to the next available white spectrum and identify the right cDSA network by verifying the pilot cyclic signature before initiating radio transmission. For proper operation of the cyclic feature based cDSA network identification, the cyclic signature for each cDSA network should be unique. One simple method of obtaining unique pilot cyclic signatures from other cDSA networks is scanning cyclic signatures in the whole spectrum bands. The allowed spectrum bands for cDSA operation should be determined by local spectrum policy. For instance, the DSA spectrum for IEEE 802.22 is the 6 MHz TV channels in the U.S. The spectrum policy can be obtained from a local policy server or radio environment map (REM) [50].

The OFDM pilot cyclic signature should be unique. This implies that each cDSA network has a different pilot pattern. Therefore, a CR node which tries to join the cDSA network has to have knowledge of the pilot pattern of the target cDSA network to obtain correct
information from the cDSA network, as well as proper channel equalization. To solve this problem, the general formula for the dominant OFDM pilot cycle frequencies is proposed to estimate the pilot pattern of incoming signal.

4.12.1 Pilot Cyclic Signature Generation Method

To increase the detection performance of OFDM pilot cyclic signatures at the receiver of the CR node, the pilot patterns need to be carefully determined in order to not have cyclic feature split. In other words, the strong cyclic features can be obtained by making the modulo operation zero in the general formulas of Eq. (4.69) and Eq. (4.70). This is equivalent to making every pilot subcarrier index integer multiples of the GI index $G$.

To illustrate the difference of cyclic feature strength according to the pilot pattern, the best and the worst cases are evaluated using the IEEE 802.11a/g signal. The GI index is 4.

- The best case: All cyclic features are the strongest.
  - Pilot index: $k_1 = 8$ and $k_2 = 16$.
  - Modulo operations: $\text{mod}(k_1, G/2) = \text{mod}(8, 2) = 0$, $\text{mod}(k_2, G/2) = \text{mod}(16, 2) = 0$, $\text{mod}(k_1 + k_2, G) = \text{mod}(24, 4) = 0$, $\text{mod}(k_2 - k_1, G) = \text{mod}(8, 4) = 0$.
  - Expected dominant cycle frequencies from the general formula:
    $$\{\alpha\} = \pm\{0, 2.5, 5.0, 7.5, 10.0\} MHz$$
    (4.79)

- Simulation result:
• The worst case: Some cyclic features are the weakest
  o Pilot index: $k_1 = 7$ and $k_2 = 20$.
  o Modulo operations: $\text{mod}(k_1, G/2) = \text{mod}(7, 2) = 1$, $\text{mod}(k_2, G/2) = \text{mod}(20, 2) = 0$, $\text{mod}(k_1 + k_2, G) = \text{mod}(27, 4) = 3$, $\text{mod}(k_2 - k_1, G) = \text{mod}(13, 4) = 1$.
  o Expected dominant cycle frequencies from the general formula:
  
  $$\{\alpha\} = \pm\{0, 4.0, 4.25, 4.5, 8.25, 8.5, 12.5\} MHz$$  \hspace{1cm} (4.80)

  o Simulation result:
As Figure 4.26 indicates, the cyclic feature at cyclic frequency $\alpha = 8.25\, MHz$ is the weakest. This cyclic feature may not be detected with strong noise present, while other features may be detected. Therefore, the pilot patterns should avoid having the weakest features. The weak features of one should correspond to strong features to minimize false alarm rate.

Note that the pilot patterns for strong OFDM pilot cyclic features are limited, but certainly the removal of the pilot pattern combinations which generates the weakest cyclic feature would still help.

### 4.12.2 Frequency Rendezvous using Pilot Cyclic Signature

The cDSA network uses the white spectrum opportunistically. Therefore, a cDSA network has to evacuate the channel ‘immediately’ and has to move to the next available white spectrum if a primary user is detected. Conventionally, a common control channel is used to coordinate white spectrum among the DSA radio nodes. However, this common control channel is spectrum inefficient and requires a global protocol for the control message, and
perhaps the physical layer, to assure correct information transfer to each CR node. To solve this problem, the dominant OFDM pilot cyclic feature is proposed for FR (Frequency Rendezvous).

For FR, each cDSA network has to maintain an FR table that contains the frequency migration plan. This allows rapid cDSA network formation without the help of the common control channel. The FR plan can be established by finding and prioritizing white spectrum. One method of prioritizing white spectrum is to order white spectrum randomly. This random prioritizing can reduce the chance of collision. cDSA network searches white spectrum from the highest priority to the lowest in the case of FR. However, the prioritizing method of white spectrum can be determined adaptively and intelligently by cDSA network to minimize collisions between the cDSA networks during FR. The collision is discussed later.

To illustrate the role of the OFDM pilot cyclic feature in the FR process, let’s consider two cDSA networks sharing white spectrum, as shown in Figure 4.27. It is assumed that the available frequency bands for white spectrum, primary user signal type, and excluded spectrum from the DSA are obtained from the local policy server.

At time frame T1, the operational frequencies for cDSA1 and cDSA2 have settled on the frequency bands F1 and F2. The CR nodes in cDSA network perform spectrum sensing and build the FR table. In Figure 4.27, the highest priority white spectrum for cDSA1 and cDSA2 are F3 and F5 at time frame T1.

At time frame T2, a primary user initiates radio transmission over the frequency band F1. CR nodes in cDSA1 detect the primary user and evacuate the channel F1 immediately and move to F3, the next available white spectrum as indicated in the FR table. Before using the white spectrum F3, cDSA1 has to verify that the spectrum is still available using spectrum sensing. If the spectrum F3 is confirmed as white spectrum, the CR node switches to band F3 and resumes radio transmission. Other CR nodes of cDSA1 resume radio transmission after identification of cDSA1 using the OFDM pilot cyclic feature. Next,
cDSA1 and cDSA2 update their FR table to reflect the white spectrum changes and establish a new FR plan based on the spectrum sensing results. The new FR plans for cDSA1 and cDSA2 are shown in time frame T2 in Figure 4.27.

At time frame T3, primary users occupy the channel F3 and F6. Therefore, cDSA1 and cDSA2 have to evacuate their frequency bands immediately and move to frequency band F5, which is the highest priority white spectrum for both cDSA networks. However, there will be a collision at frequency band F5. To resolve this collision, the random backup method can be used. Each cDSA network retries spectrum access to F5 after random delay. If cDSA1 gains access to the white spectrum F5, then it resumes radio transmission over the channel. Eventually, cDSA2 identifies that F5 is occupied by another cDSA network by comparing the OFDM pilot cyclic feature and thus changes its channel to the next available frequency band, as indicated in the FR table. The cDSA2 network resumes radio transmission using frequency band F8. At time frame 5, cDSA1 and cDSA2 find stable white spectrum and settle on frequency bands F5 and F8. The FR plans for cDSA1 and cDSA2 at time frame T5 reflect the new white spectrum at F1 and F3.
4.12.3 Pilot Pattern Identification

Each cDSA network has different pilot patterns using unique OFDM pilot cyclic features. A CR node that wants to join a cDSA network has to know the pilot pattern of the target cDSA network. In this work, the pilot patterns of cDSA networks are identified using the general formula. To demonstrate the pilot pattern identification, let’s consider that a CR node wants to join the cDSA network having the OFDM pilot cyclic signatures shown in Figure 4.26. The CR node obtains OFDM pilot cyclic signature through cyclic spectral
analysis and records the dominant cycle frequencies. The obtained dominant cycle frequencies from Figure 4.26 are:

\[
\{\alpha\} = \pm\{0, 3.999, 4.25, 4.5, 8.25, 8.501, 12.5\}MHz
\]

(4.81)

It is evident from the general formulas in Eq. (4.69) and Eq. (4.70) that the highest dominant cycle frequency comes from the non-combined carrier. Thus, one of the pilot subcarrier indexes can be obtained using Eq. (4.69). Additionally, the absence of any cyclic features at plus or minus the OFDM symbol rate, relative to the cyclic feature at \(\alpha = 12.5MHz\), indicates that this is not the split. Therefore, the pilot subcarrier index is \(k_2 = 20\).

\[
k_2 = \text{round}\left(\frac{12.5MHz}{(2\times\Delta_f)}\right) = 20
\]

(4.82)

where \(\text{round}(\cdot)\) rounds the argument to the nearest integer. This operation is required to make the integer subcarrier index.

Next, the other subcarrier index can be evaluated using the second highest cycle frequency pair at \(\alpha = 8.25MHz\) and \(8.501MHz\). To generate these cycle frequencies, the largest two pilot indexes need to be additively combined. In addition, these cycle frequencies are separated by 251KHz away. This separation is close to the OFDM symbol rate. Therefore, it can be the result of the split and the base can be the middle of these cycle frequencies, \(\alpha = 8.3755MHz\). Therefore, the next pilot subcarrier index is obtained as

\[
k_1 = \text{round}\left(\frac{8.3755MHz}{\Delta_f} - k_2\right) = 27 - 20 = 7
\]

(4.83)

Therefore, the pilot subcarrier indexes are estimated as \([7, 20]\) to provide the correct identification.
4.13 Chapter Summary

In this chapter, the cyclic features for OFDM signals are investigated using LPTV transformation and second-order statistical cyclic analysis. The GI introduces a phase discontinuity between OFDM symbols and results in a cyclic feature deviation from the cyclic feature of non-GI OFDM signals. This is analyzed using second-order cyclic spectral analysis of OFDM pilot signals. Based on the analytical results, a general formula is generated to predict the cycle frequencies of the dominant OFDM pilot cyclic features. The analytical results are verified with both computer simulation and actual measurement data.

The OFDM pilot cyclic feature is proposed to address the technical challenges of white spectrum coordination in cDSA networks. In cDSA networks, the most challenging technical problem is maintaining the common control channel that coordinates white spectrum among cDSA networks. Usually, this entails very complex algorithms, as well as standardized protocols for the common control channel. And the common control channel itself is wasting spectrum resource. This technical challenge is handled with OFDM radio access technology and OFDM pilot cyclic features. By evaluating the OFDM pilot’s cyclic features, CR nodes can detect primary users and identify other CR nodes in cDSA network. The methods of generating strong OFDM pilot cyclic features using the general formula are also proposed to increase the cyclic feature detection performance at the receiver of the CR node. Using the OFDM pilot cyclic features, the white spectrum coordination problem of the cDSA network is investigated. Each cDSA network has a unique cyclic pilot signature to differentiate itself. Therefore, the pilot pattern of the cDSA network should be known to the CR node which wants to join the specific cDSA network. The pilot pattern of the cDSA network is identified using the general formulas, Eq. (4.69) and Eq. (4.70).
Chapter 5

Signal Detection and Feature Extraction

5.1 Introduction

Cognitive radios, in spectrum sharing framework, are the devices that have low priority access to the spectrum. These devices or networks should offer minimum interference to the primary or licensed network. On the other hand, primary user networks have no requirement to alter their mode of operation because of spectrum sharing by the secondary users. Therefore, cognitive radios should be able to independently detect the presence of primary users continuously. This is especially true due to the hidden node problem, as discussed in Chapter 2. Because of this key issue the spectrum sensing has become a very challenging problem.

Spectrum sensing at low SNR conditions is critical to mitigate the hidden node problem and to enhance spectrum awareness. Cyclostationary signal detection is considered to be one of the most viable candidates for the detection of signals with varying SNR. However, this approach usually requires long observation time and often prohibitive amount of computational overhead.
If the form a target signal is known, then the matched filter is the optimal detector in the sense of maximizing the SNR at the output of signal detector. However, it is almost impossible to know all signal formats and even if all signal formats are known, the CR would need to evaluate all of the possible formats in order to make a decision. In this chapter, the energy detector and optimal cycle detector [47, 51] are compared. The optimal cycle detector requires exact information of cycle frequencies and it also requires true power spectral density of the received signals to operate optimally. However, this optimal cycle detector needs to search the whole bi-frequency plane of SCF for signal detection. To overcome this problem of the optimal cyclic detector, the CDP based signal detection is proposed. This method exploits the distinct peaks in the CDP for signal detection. To extract these distinctive cyclic features from the erratic CDP floor, the crest factor (CF) of the CDP is utilized. In addition, these extracted cyclic features obtained using the CF serve as a feature vector for signal classification. Through Mote Carlo simulations of the optimum cycle detector, the energy detector, and our proposed method, their signal detection performance are compared.

### 5.2 Energy Detector

The energy detector has been used extensively in radiometry due to its non-coherent detection capability and simplicity. Its statistical behavior is also well developed in literature [52, 53]. However, the energy detector has some drawbacks. Energy detector performance is highly susceptible to noise variability or noise uncertainty, which is defined in Eq. (5.1) as

$$\rho_n \triangleq \frac{\sigma_x^2}{(\mu_x)^2}$$  \hspace{1cm} (5.1)
where $X$ is uniform random variable (RV) with width $[A, B]$ and $\sigma^2_X$ and $\mu_X$ are the variance and mean of the RV. Noise variability models the situations in which either the detector only has an approximate knowledge of the true noise floor or the noise floor can actually change over time. In a real situation, the noise is only approximation of Gaussian random variable and its variance is uncertain. Other factors of noise variability are sensor calibration error, changes in thermal noise, changes in low noise amplifier (LNA) gain due to thermal variation, and error in the estimate due to the interference.

Another disadvantage of the energy detector is that it does not differentiate between modulated signal, interference, and noise. Therefore, it is not appropriate for signal sensing in varying noise environments and cannot perform signal classification satisfactorily. In this dissertation, the energy detector is considered for signal sensing as the baseline technique for signal detection. The block diagram of the energy detector is depicted in Figure 5.1.

\[
x(t) \xrightarrow{\text{ADC, } x(n)} \text{N-Points FFT, } X(k) \xrightarrow{\text{Squaring, } |X(k)|^2} \text{Block Average, } \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2 \xrightarrow{\text{Hypothesis Testing with Threshold}}
\]

**Figure 5.1: Energy detection block diagram [54].**

### 5.3 Backgrounds of Optimum Cycle Detector

For the cyclostationary signal detection, it is well known that a quadratic transformation instead of a linear filter (such as matched filter) is appropriate for a signal detector [33, 47].

First, let’s consider stationary signal detection using the quadratic transformation. Then, a detection problem can be expressed mathematically as
\[ x(t) = \begin{cases} s(t) + n(t) & : \text{when signal is present} \\ n(t) & : \text{when signal is absent} \end{cases}, \quad |t| \leq \frac{T}{2} \] (5.2)

where \( T \) is the signal observation duration.

A non-linear device, such as quadratic transformation, can create a strong signal feature even with strong noise. The transformed signal is then used to obtain the decision statistics

\[ y_{\text{detector}} = \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} k(u, v)x(u)x(v)dudv \] (5.3)

where \( k(u, v) \) is a quadratic transformation kernel or weighting function [30]. Then, signal detection is performed by comparing the decision statistics with a threshold \( \eta \) and this is expressed as

\[ y_{\text{detector}} \begin{cases} > \eta & \Rightarrow \text{Decide signal } s(t) \text{ is preset.} \\ < \eta & \Rightarrow \text{Decide signal } s(t) \text{ is absent.} \end{cases} \] (5.4)

A performance metric for this decision statistic is

\[ d \triangleq \frac{\left| E\{y_{\text{detector}} \mid s(t) \text{ is preset}\} - E\{y_{\text{detector}} \mid s(t) \text{ is absent}\}\right|}{\sqrt{\text{var}\{y_{\text{detector}} \mid s(t) \text{ is absent}\}}} \] (5.5)

where \( d \) is known as deflection coefficient and \( E[\cdot] \) is the normal expectation operator.

When we assume that the random signal is wide sense stationary (WSS) and the noise is white Gaussian with two sided noise power \( N_0/2 \) (Watt/Hz) and is independent. Then the deflection coefficient \( d \) reduces to

\[ d = \frac{\left| \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} k(u, v)R_s(u - v)dudv \right|}{2N_0^2 \left| \int_{-T/2}^{T/2} k^2(u, v)dudv \right|^{1/2}} \] (5.6)
where $R_s(u-v)$ is the result of $E\{x(u)x(v)\}$.

Deflection coefficient is also known as noise distance for the signal detection problem [53]. To find optimal quadratic transformation kernel in the sense of maximizing signal detection probability, we have to understand the characteristic of deflection coefficient in the signal detection problem.

When noise power is significant (i.e., in low SNR condition), the deflection coefficient can be further simplified as [55]

$$Q^{-1}[P_{FA}]-Q^{-1}[P_D] = \frac{m_x-m_n}{\sigma_n} \triangleq d$$

(5.7)

Where $m_x$, $m_n$, and $\sigma_n$ are the means and variance of received signal under the hypothesis in Eq. (5.2). $P_D$ and $P_{FA}$ denote the probability of detection and probability of false alarm. And $Q[\bullet]^{-1}$ is the inverse function of $Q[\bullet]$ defined as,

$$Q(x) \triangleq \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{v^2}{2}\right)dv$$

(5.8)

Now, the Eq. (5.7) can be re-written as

$$P_D = Q[Q^{-1}(P_{FA})-d]$$

(5.9)

Thus, maximizing deflection coefficient maximizes the probability of detection due to monotonic increasing function with decreasing argument of $Q[\bullet]$.

Therefore, we have to find the quadratic transformation kernel that maximizes the deflection coefficient. Using Cauchy-Schwartz inequality in [56] for Eq. (5.6), the maximum deflection coefficient is obtained when the quadratic transformation kernel is given by

$$k(u,v) = cR_s(u-v)$$

(5.10)
for any constant $c$. For convenience $c = 1/(2N_0^2T)$ is used. Thus, resulting detection statistic is written as

$$y_{\text{detector}} = \frac{1}{N_0^2T} \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} R_i(u-v)x(u)x(v)dudv$$

(5.11)

By introducing the change of variables $u = t + \tau / 2$ and $v = t - \tau / 2$, the above detection statistic can be expressed as

$$y_{\text{detector}} = \frac{1}{N_0^2T} \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} R_i(\tau)R_s(\tau) d\tau$$

(5.12)

where $R_i(\tau)$ is

$$R_s(\tau) \triangleq \begin{cases} 
\frac{1}{2} \int_{-|\tau|/2}^{(|\tau|/2)} s(t + \tau / 2)x(t - \tau / 2)dt, & |\tau| \leq T \\
0, & |\tau| > T
\end{cases}$$

(5.13)

Eq. (5.13) is the correlogram of $x(t)$ that we discussed in Chapter 3. Applying Parseval’s relation to Eq. (5.12) yields

$$y_{\text{detector}} = \frac{1}{N_0^2} \int_{-\infty}^{\infty} S_s(f)P_T(f)df$$

(5.14)

where $P_T(f)$ is the periodogram of $x(t)$. Thus, this optimum detector calculates the periodogram of the noise measurement and correlates it with the spectrum of the random signal that needs to be detected. If we assume the signal power spectrum $S_s(f)$ is flat over the passband, then we can define sub-optimal detector as

$$y_{\text{sub}} = \frac{S_0}{N_0^2} \int_{-B}^{B} P_T(f)df$$

(5.15)
This is simply the scaled average power in the measurements within the signal pass-band. The device that implements Eq. (5.15) is typically called a radiometer. Thus, the more general optimum detector Eq. (5.14) is actually an optimum radiometer.

The optimum radiometer result can be applied to the cyclostationary signal detection problem. The optimal detector statistic for the cyclostationary signal is obtained by replacing the correlogram, the power spectral density, and the periodogram with the cyclic correlogram, the spectral correlation, and the cyclic periodogram respectively. Therefore, resulting optimum cyclostationary signal detector statistics can expressed as

$$y_{a,mc}(t) = \sum_{\alpha} \frac{1}{N_0} \int_{-\infty}^{\infty} S_{xy}^\alpha(f)^* S_{xy}^\alpha(t, f) df$$  \hspace{1cm} (5.16)$$

where the sum is over all $\alpha$ for which the SCF $S_i^\alpha(f)$ is not identically zero.

The $y_{a,mc}(t)$ is called multi-cycle detector and also called optimal cycle detector in the sense of maximizing deflection coefficient. Sometimes the optimal cycle detector just takes single cycle frequency and is called single cycle detector

$$y_{a,sc}(t) = \frac{1}{N_0} \int_{-\infty}^{\infty} S_{xy}^\alpha(f)^* S_{xy}^\alpha(t, f) df$$  \hspace{1cm} (5.17)$$

5.4 Signal Detection with CDP

In signal sensing using SCF, it is not practical to search the bi-frequency plane of SCF. To mitigate this difficulty, we propose to use the crest factor (CF) of CDP to determine signal existence, when signal sensor does not have a priori knowledge of the incoming signals.

The CDP-based signal detection is considered in the dimension reduction of the optimal cycle detector. For the optimal cycle detector, the detection is a two dimensional problem of evaluating the energy over all spectral frequencies given the exact cycle frequency information. This procedure repeats over all available cycle frequencies. To reduce the
computational complexity, we propose to use the maximum value of the spectrum at each cycle frequency for signal detection. The result of this process generates the CDP and it reduces the three-dimensional detection problem to two dimensions. This reduction in dimension allows us to simplify the search of modulation parameters and carrier frequency. This can be easily visualized by introducing an imaginary cyclic-spectrum analyzer. Assume that cyclic spectrum analyzer can display CDP. The frequency range of interest is set by a spectral frequency span and the cyclic frequency range of interest is set by a cycle frequency knob. The cyclic spectrum analyzer will scan available cycle frequencies and displays the maximum value over the spectral frequency.

To use the CDP for signal detection, we must select the peaks related to the incoming signal. We introduce the crest factor to collect the peaks. We call these peaks the incoming signal’s dominant peaks.

Modulated signal produces non-zero spectral correlation for some cycle frequencies. On the other hand, a stationary signal, for instance AWGN, does not show spectral correlation. However, the spectral correlation of the stationary component can have non-zero value due to limited observation length. The non-zero spectral correlation from stationary signals forms a non-zero CDP floor. Thus, it is required to find dominant peaks from the CDP floor. To find dominant peaks related to the incoming signal, CF of CDP is used.

The CF is the peak amplitude of a waveform divided by the root-mean-squared (RMS) value [57] and can be written as

$$ CF_n \triangleq \frac{\text{Maximum}}{\text{RMS}} = \frac{\max(I_n(\alpha))}{\sqrt{\sum_{\alpha=0}^{N-1} I_n^2(\alpha)}/N} \quad (5.18) $$

---

8 Some use the standard deviation when the mean of a target waveform is zero.
where the subscript \( n \) stands for a stationary signal (typically AWGN) component of received signal, \( I_n(\alpha) \) is the CDP, and \( N \) is the total number of discrete cycle frequencies. Note that CF is a dimensionless quantity.

In general, the purpose of the crest factor calculation is to give an analyst a quick idea of how much impact it can cause in a waveform. For instance, the CF of sine wave is 1.414. If a glitch in the sine wave generator causes a spike greater than maximum value of sine wave amplitude, then the CF will be greater than 1.414. In that case, the deviation of CF from the target value shows an abnormality of sine wave generator. For CDP-based signal detection, the impact is defined as a cyclic feature caused by cyclostationary signal and is independent of the type of the signal under consideration. Figure 5.2 shows the steps to calculate the CF from the CDP of the stationary signal.

To test a signal existence, a binary hypothesis testing is performed [53, 58]. The \( H_0 \) is the null hypothesis and \( H_1 \) represents detection hypothesis.

\[
\begin{align*}
H_0 &: x(t) = n(t) \\
H_1 &: x(t) = s(t) + n(t)
\end{align*}
\]  

Then, the CDP-based detector is defined when the signal is present as

\[
y_{CDP} = CF \left( \max_{f} \left[ \frac{S^\alpha_x(f)}{\left[ S_x(f + \alpha/2)S_x(f - \alpha/2) \right]^{1/2}} \right] \right) \triangleq CF^\alpha_x 
\]  

where \( CF(\bullet) \) is the crest factor of any input argument, \( S^\alpha_x(f) \) is spectral correlation of incoming signal \( x(t) \), and \( S^\alpha_x(f) \) is power spectral density (PSD) of \( x(t) \) with \( \alpha/2 \) frequency shift. The detector, \( y_{CDP} \), is a random quantity due to stationary noise \( n(t) \) and random data.
\( y_{CDP} \) is compared with a threshold \( \eta \) to test the presence of the signal

\[
\begin{align*}
\{ y_{CDP} < \eta : & \text{ Declare signal is absent} \\
\{ y_{CDP} > \eta : & \text{ Declare signal is present}
\end{align*}
\]  

(5.21)

Let us compare the CDP based signal detection with the optimal detector maximizing deflection coefficient in Eq.(5.17). This deflection coefficient can be written in terms of mean and standard deviation of the CDP.

\[
d = \frac{m_s - m_n}{\sigma_s} / \sigma_n
\]  

(5.22)

when \( m_s \) and \( m_n \) are the mean of CDP of the received signal and stationary component, and \( \sigma_n \) is the standard deviation of CDP of the stationary component.

However, the impact of cyclic features due to cyclostationary signal component on the mean value \( m_s \) is trivial when the number of cycle frequencies is large and only a few cyclic features exist. In this case, the deflection coefficient \( d \) in Eq. (5.22) is almost zero. This will not be an effective measure for signal detection. Let's now consider a modified
version of the deflection coefficient $d$ in Eq. (5.22) for CDP-based signal detection where $CF_x$ is the $CF$ of received signal's CDP and $CF_n$ is the $CF$ of stationary signal's CDP.

$$d_{CDP} = \frac{Max(I_x(\alpha))}{RMS(n)} - \frac{Max(I_n(\alpha))}{RMS(n)} = CF_x - CF_n$$ (5.23)

The CDP-based detector maximizes the Eq.(5.23). This implies that the CDP based signal detection maximizes the difference between CFs for received signal and stationary components and it leads to maximizing signal detection probability of the detector in Eq.(5.20).

The overall detection problem using CF of CDP boils down to a Neyman-Pearson (NP) approach [53]. Therefore, we need the probability density function (PDF) of $CF_n$ or $y_{CDP}$ of the stationary signal to establish the threshold that satisfies a required detection performance metric. For the NP approach, the detection performance metrics are probability of false alarm ($P_{FA}$) and the probability of detection ($P_D$) [53]. The statistics of $y_{CDP}$ are governed by the stationary signal component. Thus, the statistics of $y_{CDP}$ when signal is not present is sufficient to describe the probability of false alarm. The statistics of $y_{CDP}$ of a stationary signal is obtained by Monte Carlo simulation and the PDF of $y_{CDP}$ is approximated using its histogram [59].

Now, we can estimate the probability of false alarm ($P_{FA}$) using the CF histogram of the stationary component and the threshold $\eta$.

$$P_D \triangleq \Pr\{y_{CDP} \mid H_1 > \eta\}$$ (5.24)

Also, the detection probability is obtained using the threshold that satisfies the required false alarm rate,

$$P_{FA} \triangleq \Pr\{y_{CDP} \mid H_0 > \eta\}$$ (5.25)
Figure 5.3: Crest factor distribution for stationary component at the output of CDP detector. Two thresholds (THs) for $P_{FA}=0.1$ and $P_{FA}=0.01$ are shown as two dotted vertical lines.

5.5 Extracting the Modulation Feature from CDP

The CDP is also used for a pattern-matching for signal classification. In CDP, the modulation features are defined as the dominant peaks’ amplitude and cycle frequencies. For pattern matching, hidden Markov model (HMM) is employed in this work due to its robust pattern recognition capability and scalability.

The input sequence to HMM based signal classifier is a pattern to be compared with target pattern stored in a database. In other words, HMM compares an input sequence with the reference sequences stored in database and produce a likelihood value indicating the closeness between the input sequence and reference sequences. This is discussed further in the subsequent signal classification chapter in Chapter 6. The input sequence of the CDP is obtained by using the detection threshold. The procedure of making the binary input sequence from the CDP is described in Figure 5.4.
The step-by-step explanation of Figure 5.4 is described here:

1. Setup testing sequence buffer with size $N$ and initialize it with all zero. The buffer size $N$ reflects the length of the CDP.

2. Find maximum value and RMS value of the CDP. Set the cycle frequency of the maximum value to $n$.

3. Evaluate the CF of the CDP.

4. If the CF is greater than the threshold value, then the testing buffer with cycle frequency $n$ is set to one. If the CF is less than the threshold, the procedure of finding the dominant peak stops.

5. To find the next dominant peak, the maximum of CDP is replaced with the mean value of the CDP. This allows us to find the next dominant peak of the CDP.

6. The procedure (1)-(5) repeats until all dominant peaks are found.
Find maximum peak and its cycle frequency $n$

Evaluate crest factor ($CF$)

If $CF > \text{Threshold}$

Replace the peak with the mean value of CDP

START

testing_sequence = zeros(1, N)

Find maximum peak and its cycle frequency $n$

Evaluate crest factor ($CF$)

CF > Threshold

YES

testing_sequence($n$) = 1

NO

Replace the peak with the mean value of CDP

END

Figure 5.4: Flowchart of signal feature extraction using crest factor.

Figure 5.5 shows the CPD of a BPSK signal and its input sequence obtained using the procedure shown in Figure 5.4. The solid line indicates the CDP of the BPSK CDP and dotted line indicates the input sequence. The CDP length is set to be 1024. Note that the triangular peaks are replaced with rectangular peaks to simplify the computation.

Figure 5.5: BPSK CDP (solid line) and its testing sequence (red dotted line).
5.6 Simulation Results of CDP-based Signal Detection

We simulate the CDP-based detection using threshold that produces a 10% probability of false alarm. The optimal cycle detector and energy detector are tested together to compare the detection performance with the CDP-based detector. For CDP-based detection, we simulated BSPK and QPSK signal with SNR from 0dB to -14dB with observation lengths of 50, 75, and 100 blocks (one block is 32 symbols or 256 samples), and noise variability is $\rho_N = 0.2$.

For energy detection, the observation length is fixed to 100 blocks. We assume that the signal bandwidth is unknown. We also assume that modulation parameter and carrier frequency are only known to the optimal cycle detector. The observation block of optimal cycle detector is fixed to 50. To obtain the detection performance, 1000 trials are performed for a given SNR and observation length. The results are shown in Figure 5.6 and Figure 5.7.

![Figure 5.6: Probability of detection for BPSK signal with 10% false alarm, $\rho_N = 0.2$. CD is the cycle detector, CDP is the CDP based detector, and ED is the energy detector. 50 obs. implies the observation length is 50.](image)

---

9 This implies the single cycle detector with cycle frequency at twice the carrier frequency.
From Figure 5.6 and Figure 5.7, we deduce that the optimal cycle detector shows the best detection performance but it needs the prior knowledge of the cycle frequency and true PSD. The performance of the CDP based detector is comparable to optimal cycle detector down at SNR=-10dB in BPSK and SNR=-6dB in QPSK with noise variability $\rho_N = 0.2$. However, the CDP-based detection performance degrades faster than optimal detector beyond -10dB and -6dB, for BPSK and QPSK respectively. For large observation, the performance of energy detector is found to be better than the CDP based detector for SNR < -10dB. Therefore, the energy detector can be used to supplement the decision process of CR at very low SNR when relatively long observations are possible.
5.7 Chapter Summary

This chapter provides a method for detecting a signal and cyclic feature extraction using the CDP and its crest factor. This method does not require exact information of the detecting signal. The CDP crest factor can be easily obtained to setup appropriate threshold which is used for declaring the existence of cyclic feature at specific cycle frequency. The CDP crest factor based detection performance is compared with optimal single cycle detector and energy detector. It is shown that the detection performance of CDP crest factor based detector is similar to the optimal single cycle detector at SNR>-10dB for BPSK and SNR>-6dB for QPSK.

In addition, the CDP crest factor and an appropriate threshold will be used to generate the cyclic feature or signature for signal classification. In the next chapter, signal classification is investigated using hidden Markov models (HMMs) and CDP features.
Chapter

6 Hidden Markov Model Based Signal Classification

6.1 Background

The classification of unknown signals using second-order cyclostationary approach allows us to identify the incoming signal in low SNR conditions [60, 61]. The second-order cyclostationarity reveals distinct features in the cyclic domain based on the modulation of the transmitted signal. The ability to detect and classify the incoming signal at low SNR with noise uncertainty is very important for the satisfactory operation of primary and secondary networks. The successful signal classification facilitates CR networks in choosing appropriate demodulation process at the receiver. We observe that the CDPs of differently modulated signals exhibit unique peaks at certain cycle frequencies. We select hidden Markov model, a widely used tool for pattern recognition, for the recognition of these unique patterns.

\[^{10}\] This chapter’s work was performed with the help of Dr. Ihsan Akbar at Tyco Electronics.
To investigate the performance of our signal classifier in AWGN channel and in a varying noise environment, we use computer based simulation and evaluate the probability of correct classification in various SNRs and different observation lengths.

In this chapter, we first present the fundamentals of HMM and discuss the algorithm used for training and recognition. We then evaluate the performance of the HMM-based signal classifier by using four basic signal types (BPSK, FSK, MSK, and QPSK).

### 6.2 Fundamentals of HMMs

Hidden Markov model is basically a Markov chain in which the state sequence is hidden and can not be directly observed. It is a doubly stochastic model in which both state transitions as well as output observations are governed by probabilistic distributions. Let us look at a simple example to understand this process. Suppose that there is an urn that contains balls having three different colors distributed in different proportions. Suppose also that these colors are white, gray, and black.

![Figure 6.1: Urn with balls having different colors.](image)

We pick a ball at random and observe its color. The ball is put back in the urn and then a new ball is picked. Upon the completion of this experiment we have a sequence of balls that may have different colors. This is the simplest case of a random process. Now we
extend this concept to a more complex case. Now suppose that instead of a single urn we have \( N \) urns.

These urns contain balls having three colors with different proportions. In Figure 6.2, \( N=3 \), and there are three colored balls, i.e., white, gray and black. We randomly select an urn according to some probability distribution, and pick a ball from that urn. After noting the color of the ball, we place the ball in the same urn and then select another urn at random. The urn might be the same urn selected before or it can be a different urn.

![Three urns with balls having different colors.](image)

**Figure 6.2: Three urns with balls having different colors.**

We pick a ball from the newly selected urn and observe its color. Hence there are two random processes occurring one after another, i.e., the selection of an urn and the selection of a ball from that particular urn.

![Sequence of balls drawn from the three urns.](image)

**Figure 6.3: Sequence of balls drawn from the three urns.**

Figure 6.3 shows one possible sequence that can be obtained from this procedure. Note that every urn has a specific distribution for different colored balls. The urns are represented by states, and looking at the sequence shown in Figure 6.3, the sequence in which different urns are selected cannot be determined. We can only observe the color of
the balls picked from the urns. We have no way of knowing the urns from which the balls were picked. This is an example of a hidden Markov model (HMM). The output probability distribution of each state may be different from each other and the probability of picking a ball depends upon the probability distribution of the balls in that particular urn (state of a Markov model). To describe HMM mathematically, we need to define three variables; a state transition matrix (probability of state transition from urn to urn), an output probability distribution matrix (probability of choosing grey or white or black ball at a specific urn), and an initial state probability vector (probability of taking one of three urns from the beginning of ball choosing).

First, state transition matrix is explained. At each observation instant, a state transition is assumed to occur with certain probability. The likelihoods of these transitions are governed by the state transition probabilities. Let us denote the probability of making the transition from state $j$ to state $i$ by $a(i \mid j)$. The state transition matrix can be given by

$$
A = \begin{bmatrix}
    a(1 \mid 1) & a(1 \mid 1) & \cdots & a(1 \mid 1) \\
    \vdots & & & \vdots \\
    a(i \mid j) & & & \vdots \\
    \vdots & & & \vdots \\
    a(N \mid 1) & a(N \mid 2) & \cdots & a(N \mid N)
\end{bmatrix}
$$

(5.26)

where $N$ is the total number of states in the model.

The state transition probabilities are assumed to be stationary in time so that $a(i \mid j)$ does not depend upon the time $t$ at which the transition occurs. If the random variable takes on a discrete value and does not depend on time $t$, then it is called a homogeneous Markov chain. Every column of matrix $A$ must sum up to unity, since it is assumed that a transition takes place with certainty each time including the transition to the origin. The sequence of states that corresponds to generating a given observation sequence is the first of two
random processes associated with an HMM. We call this a state random process $x$. The associated random variables are $x(t)$. Therefore, the $a(i | j)$ can be expressed as

$$a(i | j) \triangleq \Pr(x(t) = i | x(t-1) = j)$$

(5.27)

for any arbitrary time $t$ and first order Markov process.

It is convenient to define the state probability vector,

$$\pi(t) \triangleq \begin{bmatrix}
\Pr(x(t) = 1) \\
\Pr(x(t) = 2) \\
\vdots \\
\Pr(x(t) = N)
\end{bmatrix}$$

(5.28)

The state probability vector can be expressed in terms of state transition matrix $A$,

$$\pi(t) = A\pi(t-1)$$

(5.29)

In fact, given initial state probability vector $\pi(1)$, it can be shown by recursive way that

$$\pi(t) = A^{t-1}\pi(1)$$

(5.30)

The observation sequence is assumed to be a discrete-time stochastic process here denoted by $y(t)$. The observation sequence can be expressed as $Y = \{y^T_t\} = \{y_t\}$, $t \in \{1, 2, \cdots, T\}$. If we define observation probability density function (PDF) for state $i$ as $f_{y(t)|x(t)}(\xi | i)$, the generation of a particular observation sequence is governed by this PDF. In case of stationary process, the time index can be removed in the PDF.

If the observation random process $y(t)$ takes $K$ possible observations, then it is sufficient to assign an observation a single integer, say $k$, where $1 \leq k \leq K$. In this case it is sufficient to know the probability distribution over the outputs for each state which can be denoted by $b(k | i) \triangleq \Pr(y(t) = k | x(t) = i)$. Thus, we can define output probability distribution (or simply observation probability) matrix $B$
For completeness, let's define the observation probability vector,

\[
p(t) = \begin{bmatrix}
    \Pr(y(t) = 1) \\
    \Pr(y(t) = 2) \\
    \vdots \\
    \Pr(y(t) = K)
\end{bmatrix}
\]  \hspace{1cm} (5.32)

Then the observation probability vector can be expressed in terms of observation probability matrix and initial state probability,

\[
p(t) = BA^{-1} \pi(1)
\]  \hspace{1cm} (5.33)

Finally, an HMM, \( \lambda \), can be written in the following form,

\[
\lambda = \{A, B, \pi(1)\} \hspace{1cm} (5.34)
\]

### 6.3 The Three Problems in HMM

There are three key problems in HMMs that are of immense interest in real world applications. Usually the two problems, i.e., the recognition and training are enough to explain the application at hand [62, 63].
1. Evaluation or Recognition Problem:

Given an observation sequence \( Y = \{y_t\}, t = 1,\ldots,T \), and the model \( \lambda = \{A, B, \pi(1)\} \), how can we compute \( \Pr(Y | \lambda) \), the probability of generating the observation sequence using HMM \( \lambda \) ?

2. Decoding Problem:

Given the observation sequence \( Y = \{y_t\}, t = 1,\ldots,T \), how can we choose a state sequence \( Q = \{q_t\} \) which is optimal in some meaningful sense? The \( q_t \) is the state of the system at time \( t \).

3. Training problem or learning problem:

How can we adjust the model parameters \( \lambda = \{A, B, \pi(1)\} \) to maximize \( \Pr(Y | \lambda) \) ?

In most applications, only two problems, recognition and learning, are enough to solve a given problem. For our signal classification, the recognition and training problems of the HMM are considered.

It should be noted that the initial condition greatly affects the likelihood of the model during the training phase. It is almost impossible to obtain the optimal initial condition that can lead us to the global maximum of the log likelihood. One of the practices for finding better model from the training sequence is to test various initial conditions and choose the one that gives the maximum value of the likelihood. However, the HMM obtained though this process is not guaranteed to be the best model.

Obtaining a HMM from a training sequence is a tedious and time taking process. The recognition, however, is relatively simpler and quicker than training. The cognition procedure uses the HMM obtained from the training phase and the likelihood value is obtained by processing of input sequence with HMM once. There are several ways to train a HMM and among them the Baum-Welch algorithm (BWA) is one of the most popular
way of training. The BWA is used extensively in sciences and engineering to train a HMM from a given sequence.

### 6.4 HMM Learning and Recognition of the Discrete Observation

The discrete Hidden Markov Model is described by an \((N \times N)\) state transition matrix \(A\) and a \((K \times N)\) observation probability matrix \(B\). An iterative procedure for estimation these parameters from a given observation sequence, obtained through simulation or measurement is based on the Baum-Welch algorithm or simply the BWA [59, 64, 65]. This iteration algorithm is designed to converge to the maximum likelihood estimator of \(\lambda = \{A, B, \pi(1)\}\) that maximizes the likelihood function for the observation given the model [59, 64, 65]. The goal is to compute estimator of the elements of the transition matrix given by,

\[
\bar{a}(j|i) = \frac{\text{Expected number of transitions from } i \text{ to } j}{\text{Expected number of transitions from } i} \tag{5.35}
\]

and the estimation of the element of the symbol observation probability matrix, which is given by,

\[
\bar{b}(k|j) = \frac{\text{Expected number of observations } k \text{ at state } j}{\text{Expected number of visit to state } j} \tag{5.36}
\]

The detail derivations for \(\bar{a}(j|i)\) and \(\bar{b}(k|j)\) using BWA are provided in [59, 64, 65].

Once we have obtained the model \(\lambda = \{A, B, \pi(1)\}\) through BWA, the cognition process is the evaluation of \(\Pr(Y|\lambda)\). For the case of binary sequences, the probability of generating the observation sequence given the model can be written mathematically as
\[
\Pr(y^T_1 \mid \lambda) = \pi(1)' B(y_1) AB(y_2) A \ldots AB(y_T) 1
\] 

(5.37)

where \( \pi(1)' \) is the initial state probability row vector, \( B(y_k) \) with \( k \in \{1, \ldots, T\} \) is \((N \times N)\) square matrix having all zero entries except diagonal denoting the probability of generating symbol from different states, and \( 1 \) is \((N \times 1)\) all one column vector.

As an example, let us consider simple HMM for the binary sequence and three hidden states. If we assume that the output probability distribution matrix \( B \) is,

\[
B = \begin{bmatrix}
0.8 & 0.2 \\
0.6 & 0.4 \\
0.3 & 0.7
\end{bmatrix}
\]

(5.38)

then, the \( B(0) \) and \( B(1) \) for binary input for ‘0’ and ‘1’ will be

\[
B(0) = \begin{bmatrix}
0.8 & 0 & 0 \\
0 & 0.6 & 0 \\
0 & 0 & 0.3
\end{bmatrix}
\quad \text{and} \quad
B(1) = \begin{bmatrix}
0.2 & 0 & 0 \\
0 & 0.4 & 0 \\
0 & 0 & 0.7
\end{bmatrix}
\]

(5.39)

6.5 Signal Classification using HMMs

The CR is required to have the ability to sense and characterize its RF environment and adapt accordingly. A typical goal of the adaptation process is to achieve the maximum throughput while minimizing the interference to other operating radios. Spectrum sensing itself does not provide enough flexibility in maximizing throughput and minimizing interference to others. Identification of incoming signal helps CR to adjust its modulation parameters for the situation at hand. For example, primary and secondary user classification may impact the masking of transmission power. Usually, the primary-user protection requires a tighter RF transmission mask than for secondary-users. Therefore, CR
must include signal classification in order to maximize the system throughput. In addition, the signal classification allows the CR to choose appropriate demodulation method for incoming signal. In this chapter, our focus is on HMM for pattern matching based signal classification using cycle frequency domain profile (CDP).

Signal classification using pattern matching method is divided into four major processing blocks: RF front end, signal detection, feature extraction, and pattern matching. The main purpose of the RF front end is to translate the incoming signal into a baseband digital signal. The CR RF front end must process a wide frequency range. Usually, the CR RF front end requires very high linearity and a fast analog-to-digital (A/D) conversion. To manage in-band interference appropriately, CR may employ combinations of various technologies such as adaptive notch filtering, RF filtering using MEMS (Micro-Electro-Mechanical Systems), and spatial filtering using an adaptive antenna. The CR RF front-end usually takes advantage of software defined radio (SDR) technology to meet such strict requirements [7, 54]. In IEEE 802.22, a primitive CR technology, the wideband and interference rejection requirements of the CR RF front-end can be relaxed due to the deterministic frequency allocation of the TV channels and known possible interferences.

The signal detection block determines whether the incoming signal needs to be classified or not. This signal detection stage ignores the signal that hasn’t shown any inherent cyclic feature. The exclusion of weak signals from signal classification helps in reducing the false alarm rate.

The feature extraction block in CR maps the baseband signal into a feature vector which is then used by the signal classifier. The performance of signal classification block depends on how well the feature extraction block collects distinctive features from the incoming signal.

Finally, the pattern matching block assigns the feature vector to the specific modulation type. The feature vector of the incoming signal is generated by converting the cycle frequency domain profile into a binary sequence using the CF.
As we discussed earlier, HMM must be trained or taught about the target to be recognized. This learning method is known as supervised learning. To investigate the performance of HMM-based signal classification, classification of four modulation types is demonstrated. These four modulation types are BPSK, FSK, MSK, and QPSK. The HMM training procedure determines the model parameter, $\lambda = \{A, B, \pi(1)\}$. We assume that the model has three hidden states. The determination of the optimal number of hidden Markov states is an open problem and an estimator that can consistently estimate the optimum number of hidden states is not known yet [66-68]. For simplicity, we consider binary training sequence for HMM. The size of the state transition matrix and the dimension of the output probability distribution matrix become (3 by 3) and (3 by 2) respectively. The binary training sequence or binary feature vector is obtained by converting CDP into a binary vector using the CDP crest factor. For signal classification, we have to train four HMMs, one for each modulation type. The Baum-Welch algorithm (BWA) is used to train four HMMs with 500 different initial conditions. The number of initial conditions is determined empirically based on by iterative training and recognition. If the feature vectors for different modulation type are similar to each other, then using large initial conditions in HMM training improves the performance of the HMM-based classification block. We assume that the model, $\lambda = \{A, B, \pi(1)\}$ for a modulation type is evaluated and stored in a central database such as the Radio Environment Map (REM) in [50] or at the base station for the IEEE 802.22 applications. The CR must obtain HMMs from the database (or REM) for all possible modulation types available in a specific geographical location at that particular time. The CR is now ready to classify the incoming signal. To extract signal features from digital baseband signal, the CDP is evaluated and is converted into a binary feature vector. If any feature is not detected, then the signal is classified as no signal and disqualified for signal classification. The feature vector is fed into four HMMs and each HMM produces a likelihood value for the input feature vector. This likelihood value indicates the probability of generating the feature vector with the given HMM. A higher likelihood value implies more similarity between the input feature vector and the vector...
associated the HMM. Finally, the signal classification is achieved by choosing maximum likelihood value from the four HMMs. The overall signal classification procedure is depicted in Figure 6.4.

![Diagram](image-url)

**Figure 6.4: HMM signal classification block diagram.**

### 6.6 Simulation Results

Monte Carlo simulations are performed to evaluate the performance of the HMM-based signal classifier. The HMMs (in Figure 6.4) are trained with an SNR of -6dB and 200 observation blocks. One observation block is equivalent to 32 symbol duration. Different incoming signals (BPSK, FSK, MSK, and QPSK) with SNR of -6dB and -9dB with noise variability $\rho_n = 0.2$ are tested with varying observation lengths.
The percentage of successful classification is measured for various observation lengths. The results are summarized in Figure 6.5 and Figure 6.6. Note that the signals having a SNR = -9 dB usually require more observation blocks to achieve a given performance than when the signals have a SNR = -6 dB. The signal classification for FSK shows better result than for other signal types. The reason for better classification is due to two pure sinusoidal components related to the two carrier frequency components. FSK results in a high peak for the carrier frequency component in the CDP. The next best signal classification performance is found for the MSK. The MSK can be treated as a staggered QPSK with half sine wave pulse shaping. Thus the feature vectors generated at feature extraction state are more distinct and are more robust to noise compared to the square raised cosine pulse shaped signals such as BPSK and QPSK. The QPSK has less strong cyclic feature only related to the symbol rate. However, this feature is unique in that no carrier frequency components. Therefore, HMM signal classifier can classify QPSK better than BPSK.

For the 90% correct classification of SNR = -6 dB, BPSK and QPSK signals requires about 65 observations \((16640 = 65 \times 256 \text{ samples})\) and for SNR = -6 dB, FSK and MSK signals require about 45 observations \((11520 = 45 \times 256 \text{ samples})\). In SNR = -9 dB, 90% correct classification requires observation length of more than 100 \((25600 = 100 \times 256 \text{ samples})\) for FSK and MSK and 200 \((51200 = 200 \times 256 \text{ samples})\) for BPSK and QPSK.
Figure 6.5: Successful classification rate of digital signal when incoming signals have SNR of -6dB.
Training signal has -6dB SNR and 200 blocks.

Figure 6.6: Successful classification rate of digital signal when incoming signals have SNR of -9dB.
Training signal has -6dB SNR and 200 observation length.
6.7 Performance Comparison

We compared our result with the signal classification method which uses pattern matching method used in [69]. Note that it is difficult to compare two methods directly due to different assumptions. Generally, our CDP and HMM based signal classification requires longer observation time than conventional pattern matching based signal classification. In addition, often conventional classification methods do not consider noise variability. Thus, for comparison we focus on the classification performance at low SNR without considering observation time and noise variability.

The target classification system uses the standard deviation of the instantaneous amplitude, phase and phase change of the instantaneous signal to extract a feature vector from time-domain signal. The classification is performed using Artificial Neural Network (ANN). The overall system block diagram of the system is shown in Figure 6.7. The decoder in Figure 6.7 maps the ANN output into modulation type\textsuperscript{11}.

![Figure 6.7: Information flow for ANN based signal classifier [69].](image)

As shown in Table 6-1, the 90% classification rate requires SNR=100dB. The main reason of using high SNR is the inherent noise susceptibility of temporal feature extraction method. This method cannot suppress noise effectively to enhance signal feature. However, the proposed method in this chapter can classify an incoming signal with a successful classification rate of more than 90% at SNR=-9dB.

\textsuperscript{11} Observation interval is not provided in [69]
Table 6-1: Success rate of classification using non-cyclic features [69].

<table>
<thead>
<tr>
<th>Modulation Type</th>
<th>Probability of successful classification at a given SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10dB</td>
</tr>
<tr>
<td>BPSK</td>
<td>100%</td>
</tr>
<tr>
<td>QPSK</td>
<td>64%</td>
</tr>
<tr>
<td>FSK</td>
<td>43%</td>
</tr>
</tbody>
</table>
6.8 Chapter Summary

In this chapter, a technique for classification of radio signals using crest factor of CDP and hidden Markov model in a cognitive radio (CR) environment is proposed and investigated. The simulation results show that the incoming signals can be successfully classified, even at very low SNR provided the number of observation blocks is sufficiently large. In the CR application, the training sequence can be retrieved from a database which is maintained in the sensor itself, in the local spectrum management agency, or in base stations. In this work, the signal classification was performed in stationary noise environment such as an AWGN channel.
Chapter

7 Specific Emitter Identification in CR\textsuperscript{12}

7.1 Introduction and Motivations

The tremendous ongoing growth of wireless communication technology and new wireless services demand stronger security. In particular, the heavy dependence on knowledge regarding spectrum measurement that originates from peer cognitive radios (CRs) requires the identification of spoofing devices. Therefore, automatic identification of a malicious or malfunctioning signal transmitter is one of the major concerns in CR applications. To minimize the threat from spoofing radio devices, the specific emitter identification (SEI) concept is introduced to provide for CR devices’ electromagnetic fingerprinting. This fingerprinting of CR devices allows not only tracking the spoofing radio devices but also enhancing physical layer security. Although most radio systems have network-level protocols to secure the exchanging of data, physical-level security has advantages over network-level security. The most vulnerable point of network-level security is that it cannot differentiate cloned security related data such as user identification or authentication keys. However, the duplication of physical-level security measures is more

\textsuperscript{12} This chapter’s work was performed with the help of Dr. Chad M. Spooner at North West Research Association
challenging than that for network-level security. The forgery of a device-specific electromagnetic fingerprint by a network intruder is thought to be as difficult as duplication of a human fingerprint.

The idea of SEI\(^\text{13}\) originated in military communications to classify an enemy’s radar signal for threat evaluation. It is also called radar-emitter recognition. SEI utilizes emitter-specific non-intentional transmissions due to non-ideal electrical component characteristics of the transmitter caused by degraded or low quality components. The interpulse information, such as pulse repetition interval and pulse width, is investigated in [70] for simple radar signal identification. For complex radar signals, the intrapulse [71] information, such as pulse rising and falling times, pulse rising and falling angles, angle of pulse, and pulse point, is utilized for SEI [71, 72]. The SEI technology used for radar signal identification usually relies on high SNR and good channel propagation conditions. However, such inter and intra pulse information gets easily obscured at low and fluctuating SNR. In addition, modern communication usually practices high data rate and irregular burst transmission and is deployed in indoor or urban areas where multipath fading is common. Therefore, the inherent features caused by the advanced transmitter are usually not available for emitter-specific feature extraction in low, varying SNR, and in a fading channel. For this reason, the recognition features need to have some tolerance to the varying noise and channel conditions. This leads us to consider statistical signal features instead of time-domain features as in the radar problem.

In this chapter, the emitter specific second-order cyclic feature variations are investigated for SEI. For this purpose, IEEE 802.11a/g WLAN signals from five different manufacturers are measured. The five manufacturers are Motorola, LinkSys, DLink, Netgear, and IBM. Through the measurement and evaluation of second-order cyclic

\(^{13}\) The term of SEI can be confusing. This work shows how to distinguish different manufacturers. SEI maybe possible to use the concepts presented here to identify individual unit, but this requires massive data collection.
spectrum of the measured data, the potential of second-order cyclic features for SEI is demonstrated\textsuperscript{14}.

The structure of this chapter is as follows. In section 2, brief review of conventional technologies as well as the new approach for SEI is provided. In section 3, the IEEE 802.11a/g WLAN signal characteristics are discussed. In section 4, the measurement setup of WLAN signal is illustrated. In section 5, the second-order cyclic analysis of IEEE 802.11a/g signal is presented. The radio device specific second order cyclic features are also discussed. In section 6, specific emitter classifier using hidden Markov models (HMMs) is presented for identification of the IEEE 802.11a/g signal emitters. Finally, conclusions and future research direction are discussed in section 7.

7.2 Conventional and New Technologies for SEI

Successful SEI depends on the radio-specific feature extraction of the transmitted electromagnetic waveform. Any features which represent distinctive characteristics of a specific radio can be a candidate for SEI. Some techniques found in the previous work and cyclostationary signal analysis method which is newly introduced for SEI in this work are discussed in the following list.

7.2.1 Conventional SEI Technologies

7.2.1.1 Time-Domain Features

Conventional SEI for radar system heavily depends on the time domain feature caused by on and off transient of transmitter [71-73]. Some well known time domain features are pulse duration, pulse repetition rate, pulse rising and falling time, pulse rising and falling

\textsuperscript{14} IEEE 802.11n signal from Belkin card is also tested. Although, IEEE 802.11n has backward compatibility with IEEE802.11a/g, its exploitable features are quite unique. For the sake of uniform comparison among devices, the Belkin result is not shown in this work.
angle, angle of pulse, and pulse point [71-73]. When line-of-sight propagation and high power radar signal are provided, those features can be extracted by SEI module mounted on an airplane which is monitoring the radar signal originating from ground or ocean.

The time-domain features can also be extracted from modern communication signal. Most modern communication signal possesses the highly recurrent anomalies in nominally known portion of its signal, such as preambles or synchronization sequences. However such time domain features are only available for limited time duration and can be extracted in high SNR and good channel condition. In addition, the time domain feature extraction needs very high sampling rate [74] (5GHz sampling rate for WLAN turn-on transient measurement) and this may not feasible in an ordinary mobile radio device. The major difficulty of using time domain features is that they can not be improved by averaging due to relatively small observation length in low and varying SNR.

Nevertheless the limited time-domain features at high SNR and good channel environment, SEI has to consider every possibility of extracting radio device specific features.

### 7.2.1.2 Frequency-Domain Features

The second most common feature used for SEI is the power spectrum of the transmitted signal. The power spectral density (PSD) provides signal power as well as the spectral shape for the signal of interest and may provide radio device specific features. The spectral features include the symmetry of main and side-lobe shape, roll-off rate of pass-band, bandwidth, unintended tone in pass-band, pass-band shape, or out-of-band shape. Those frequency-domain features are also easily corrupted by high and varying noise and interferences. The frequency-domain features could not also be improved by increasing observation length and averaging.
7.2.1.3 Phase Space Features

One of common methods for nonlinear system analysis is the phase space investigation. The phase space analysis can provide distinctive features of nonlinear power amplifier of a radio signal transmitter [75]. In phase space analysis, the nonlinear system analysis begins by reconstructing the phase space trajectory of a target system by utilizing time-delayed version of an observed scalar quantity which is known as embedding [76] a scalar time series signal and these delayed quantities serve as coordinates for the phase space. For instance, for a scalar signal \( x(t) \), a vector \( w(t) = [x(t), x(t-\tau), \ldots, x(t-(D-1)\tau)] \) is constructed. The determination of the delay \( \tau \) and phase space dimension \( D \) depends on how accurately model the nonlinear system [77]. As a simple instance, a two-dimensional phase space trajectory can be plotted using the delayed signal component such as \( x(t) \) versus \( x(t-n\tau) \) with an integer \( n \) and may produce a unique feature in terms of two-dimensional shape for the specific non-linear system such as nonlinear power amplifier.

7.2.2 New SEI Technologies

7.2.2.1 Second-order Cyclic Features

As we discussed, most man made signals have second-order cyclostationarity. Cyclostationary signal reveals the existence of correlation between widely separated spectral components. This is called spectral redundancy. The objective of all cyclic spectral analysis applications is to exploit the redundancy of a signal to increase signal reliability or extract additional feature information. The most favorable characteristic of exploiting spectral redundancy is that the spectral redundancy can be extracted in high and varying noise or interference.

The spectral correlation function (SCF) [30, 33] is the second-order cyclic spectrum measurement. And the spectral coherence (SC) [30, 33] is the normalized measurement of
SCF. The SCF and SC are functions of two frequency variables: the spectral frequency $f$ and the cycle frequency $\alpha$. The former is the usual continuous frequency parameter encountered in conventional spectral analysis, whereas the latter is a discrete parameter that takes on values related to the signal's embedded periodicities, such as symbol rate, chip rate, and carrier frequency offset. The SCF and SC can be accurately measured even when the signal is subjected to strong and varying noise or co-channel interference, and the cycle frequencies can be accurately measured for these impairments and also when the signal undergoes linear distortion such as multipath.

### 7.2.2.2 High-order Cyclic Features

The signal characteristics of nonlinear system output for the cyclostationary signal input can be analyzed with the high-order cumulants or high-order cyclic spectral analysis [78-80]. In addition, QAM signals can also be classified with the high-order cumulants[81]. In general, nonlinear systems are expressed by power series of input signal and this leads to evaluate the cumulants of the cyclostationary signal input. The Fourier coefficients of the cumulants characterize the periodicity of the cumulants and are called cyclic cumulants. Both the cyclic cumulant Fourier frequencies (cycle frequencies) and the cyclic cumulant functions can be affected by transmitter-specific nonlinear properties, such as the nonlinear power amplifier. However, the high-order cyclic features are not discussed in this work and are left for future research.

SEI for modern communication signals should combine all available features discussed above or any other features from other technologies with data fusion technology based on a given situation or application to improve classification performance. The second-order cyclic feature extraction approach for SEI in this work can be considered as one of the input elements for SEI feature sets.
7.3 IEEE 802.11a/g WLAN Signal Characteristics

The construction of data acquisition systems is dependent on the physical layer structure of the target signal. Therefore, this section examines the IEEE 802.11a/g signaling format including Medium Access Control (MAC) and physical layers. The underlying reason for using IEEE 802.11a/g signal for SEI is the Orthogonal Frequency Division Multiplexing (OFDM) radio access technology. In addition, IEEE 802.11a/g WLAN signal is popular and easily obtainable everywhere.

IEEE 802.11 a/g standard describes only the physical layer behavior of how to transmit/receive MAC data using OFDM technology. To minimize the dependence of MAC layer to the physical layer, there is a Physical Layer Convergence Protocol (PLCP) sublayer between MAC and physical layer. This sublayer simplifies the physical layer service interface to the IEEE 802.11 MAC services. Therefore, the MAC Protocol Data Unit (PDU) is delivered to PLCP sublayer to generate Physical layer Protocol Data Unit (PPDU) for radio transmission using OFDM. In PLCP sublayer, the MAC PDU is converted into PPDU by adding appropriate header and control bits. The PPDU is transmitted into the air through OFDM signal format shown in Figure 7.1.

![Figure 7.1: Physical layer frame structure of IEEE 802.11a/g](image)

The first 16 $\mu$s duration in Figure 7.1 indicates OFDM training structure (or physical layer convergence layer (PLCP) preamble), where $t_1$ to $t_{10}$ denote short training symbols and $T_1$ and $T_2$ denote long training symbols. The purposes of training sequence are signal detection, automatic gain control (AGC), frequency offset estimation, and time
synchronization. The training symbols are highly periodic and its radio characteristics can be easily identified using conventional signal processing methods. In addition, the limited opportunity to collect the training sequence may result in unreliable cyclic features estimation. Therefore, the 16 $\mu$s training part of the physical frame is not considered for subsequent second-order cyclic spectral analysis. These training symbols are automatically removed during signal capture process.

The PLCP preamble (or training sequences) is followed by the ‘SIGNAL’ and ‘DATA’ fields. The ‘SIGNAL’ field contains the data rate and the 12 bit ‘LENGTH’ field. The ‘LENGTH’ field contains the number of octets in the MAC PDU currently requesting the physical layer to transmit. The maximum value for ‘LENGTH’ is 4095 octets (or bytes). The supported data rates associated with modulation type and convolutional coding rate are summarized in Table 7-1. As the table indicates, the maximum data rate of 54Mbps is achieved with the 64-QAM modulation and the 3/4 convolutional coding rate.

<table>
<thead>
<tr>
<th>Data rate (Mbits/s)</th>
<th>Modulation</th>
<th>Coding rate (R)</th>
<th>Coded bits per subcarrier ($N_{BPSC}$)</th>
<th>Coded bits per OFDM symbol ($N_{CBPS}$)</th>
<th>Data bits per OFDM symbol ($N_{DBPS}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>BPSK</td>
<td>1/2</td>
<td>1</td>
<td>48</td>
<td>24</td>
</tr>
<tr>
<td>9</td>
<td>BPSK</td>
<td>3/4</td>
<td>1</td>
<td>48</td>
<td>36</td>
</tr>
<tr>
<td>12</td>
<td>QPSK</td>
<td>1/2</td>
<td>2</td>
<td>96</td>
<td>48</td>
</tr>
<tr>
<td>18</td>
<td>QPSK</td>
<td>3/4</td>
<td>2</td>
<td>96</td>
<td>72</td>
</tr>
<tr>
<td>24</td>
<td>16-QAM</td>
<td>1/2</td>
<td>4</td>
<td>192</td>
<td>96</td>
</tr>
<tr>
<td>36</td>
<td>16-QAM</td>
<td>3/4</td>
<td>4</td>
<td>192</td>
<td>144</td>
</tr>
<tr>
<td>48</td>
<td>64-QAM</td>
<td>2/3</td>
<td>6</td>
<td>288</td>
<td>192</td>
</tr>
<tr>
<td>54</td>
<td>64-QAM</td>
<td>3/4</td>
<td>6</td>
<td>288</td>
<td>216</td>
</tr>
</tbody>
</table>

Table 7-1: IEEE 802.11a data rates [45]
The ‘DATA’ field contains actual user data. The ‘DATA’ is comprised of MAC PDU, 16 bit ‘SERVICE’ filed, 6 tail bits, and optional padding bits. The 16 bit ‘SERVICE’ field contains all zeros to help scrambler synchronization of the receiver. The 6 tail bits reset the convolutional coder. The optional padding bits make ‘DATA’ field as the multiple of the N_{DBPS} (Data bits per OFDM symbol) shown in the last column of Table 7-1. Therefore, we can predict the longest OFDM radio signal burst using ‘DATA’ related information.

As we discussed in Chapter 3, longer signal observation length results in more reliable cyclic spectrum estimation. Therefore, design of signal interceptor should take into account for capturing the longest OFDM radio burst. Let’s evaluate the longest OFDM burst size when data rate is 64 Mbps. The longest OFDM burst is related to the maximum number OFDM symbols. The maximum number of OFDM symbol can be attained when the ‘LENGTH’ field is the maximum value, 4095 bytes. Then, we can have following relation,

\[ N_{SYM} = \left\lceil \frac{16 + 8 \times 4095 + 6}{N_{DBPS}} \right\rceil = 152 \text{ OFDM Symbols} \]  \hspace{1cm} (7.1)

Thus, the theoretical longest OFDM radio burst can last

\[ 16\mu s + 152 \times 4.0 \mu s = 0.62 \text{ msec} \]  \hspace{1cm} (7.2)

WLAN signals shares 2.4GHz unlicensed bands (called ISM (Industrial, Scientific and Medical) bands) or 5 U-NII (Unlicensed-National Information Infrastructure) bands with other unlicensed device. There are 11 WLAN channels at the 2.4GHz ISM bands. The 11 WLAN channels are summarized in Table 7-2. Each channel has 22MHz bandwidth and only 3 out of 11 channels are not overlapped as depicted in Figure 7.2. This channel overlap implies that each unlicensed device has to check the channel occupancy before use.
Table 7-2: IEEE 802.11b/g channel numbers and frequencies [45]

<table>
<thead>
<tr>
<th>Channel Number</th>
<th>Frequency (MHz)</th>
<th>Channel Number</th>
<th>Frequency (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>2412</td>
<td>#7</td>
<td>2442</td>
</tr>
<tr>
<td>#2</td>
<td>2417</td>
<td>#8</td>
<td>2447</td>
</tr>
<tr>
<td>#3</td>
<td>2422</td>
<td>#9</td>
<td>2452</td>
</tr>
<tr>
<td>#4</td>
<td>2427</td>
<td>#10</td>
<td>2457</td>
</tr>
<tr>
<td>#5</td>
<td>2432</td>
<td>#11</td>
<td>2462</td>
</tr>
<tr>
<td>#6</td>
<td>2437</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 7.2: Another view of IEEE 802.11b/g frequency allocation plan

To achieve the fair access of the unlicensed spectrum among the unlicensed users, IEEE 802.11 MAC and physical layers practice burst transmission and random backoffs. The increased unlicensed users within the same unlicensed band results in reduced radio burst size in order to minimize the radio signal collision. The maximum radio burst size can be
controlled with the value of the fragmentation threshold. This threshold is a user controllable value. Usually, the default value for the fragmentation threshold is set to the maximum value of 2048 octets. However, this fragmentation is only applicable for unicast (one-to-one communication) and not applicable for broadcast or multicast. Therefore, most radio signal burst size is less than the theoretical maximum burst size of 0.62 msec.

The fragmentation occurs at the MAC layer. If the MAC service data unit (SDU) is greater than the fragmentation threshold, the MAC SDU is fragmented. The fragmented MAC SDU is transformed into MAC PDU by adding 30 octets MAC header and 4 octets frame error check bit (i.e. 32 bits CRC). In this fragmentation setup, the maximum value for the ‘LENGTH’ field is 30+2048+4 (=MAC header + the maximum fragmentation threshold + CRC) octets. Therefore, the maximum number of OFDM symbols in fragmentation mode is given by

\[ N_{SYM} = \left\lceil \frac{(16 + 8 \times 2082 + 6)}{N_{DBPS}} \right\rceil = 78 \text{ OFDM Symbols} \quad (7.3) \]

where \( \lceil . \rceil \) is the round operation that rounds the input argument to the nearest integer.

Thus, we have the following OFDM radio burst duration

\[ 16 \mu s + 78 \times 4.0 \mu s = 0.328 \text{ msec} \quad (7.4) \]

The signal interceptor explained in the next section needs to be designed to capture the longest OFDM signal burst in fragmentation mode.

### 7.4 Measurement Setup of IEEE 802.11a/g Signal

A signal interceptor is built to capture IEEE 802.11a/g signals in the 2.4 GHz ISM bands. The WLAN signal interceptor is shown in Figure 5.1. The signal interceptor is located around 40cm away from the laptop computer equipped with WLAN card under test.
measurement setup minimizes the chance of signal corruption through multipath fading and ensures a strong signal with a data rate 54 Mbps to be captured.

In this measurement, we focus on the uplink signal, which is transmitted from WLAN card to Access Point (AP). Because the security of WLAN stations is more vulnerable than AP’s. The closer location of WLAN station than the AP to the signal interceptor results in higher energy signal for uplink signal. This high energy uplink signal allows the signal interceptor to capture only uplink signal by controlling the trigger level of the digital oscilloscope.

Five different 802.11a/g WLAN cards from Motorola, LinkSys, DLink, Netgear, and IBM are tested. The detail information of WLAN cards and AP are summarized in the Table 7-3 and Table 7-4.

### Table 7-3: Wireless access point used for measurement

<table>
<thead>
<tr>
<th>AP Number</th>
<th>Manufacturer</th>
<th>Model Number</th>
<th>Serial Number</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP #1</td>
<td>Motorola</td>
<td>WR850G</td>
<td>20030132364103800100</td>
<td>802.11b/g Wireless Router</td>
</tr>
</tbody>
</table>

### Table 7-4: Wireless LAN PC adapters used for measurement

<table>
<thead>
<tr>
<th>STA Number</th>
<th>Manufacturer</th>
<th>Model Number</th>
<th>Serial Number</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>STA #1</td>
<td>Motorola</td>
<td>WN825G</td>
<td>20010133423421600100</td>
<td>802.11b/g PCMCIA Card</td>
</tr>
<tr>
<td>STA #2</td>
<td>LinkSys</td>
<td>WPC55AG</td>
<td>MDL003904420</td>
<td>802.11a/b/g PCMCIA Card</td>
</tr>
<tr>
<td>STA #3</td>
<td>D-Link</td>
<td>DWL-G650</td>
<td>BN2K441015059</td>
<td>802.11b/g PCMCIA Card</td>
</tr>
<tr>
<td>STA #4</td>
<td>Netgear</td>
<td>WAG511</td>
<td>WG53A34ZC003286</td>
<td>802.11a/b/g PCMCIA Card</td>
</tr>
<tr>
<td>STA #5</td>
<td>IBM</td>
<td>802CAG</td>
<td>31P9119ZB174F3AL09R</td>
<td>802.11a/b/g CardBus Adapter</td>
</tr>
</tbody>
</table>
The RF front end of signal interceptor is built using standard RF components such as Low Noise Amplifier (LNA), Surface Acoustic Wave (SAW) Band Pass Filter (BPF), Low Pass Filter (LPF), signal mixer, and omni-directional antenna. The detail specification of the components is summarized in the Table 7-5.
Table 7-5: WLAN signal interceptor components and equipments

<table>
<thead>
<tr>
<th>Component name</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4GHz Omni-antenna</td>
<td>Omni Directional Antenna</td>
</tr>
<tr>
<td>BPF (MSS-08-2450-200)</td>
<td>Passband Loss (@2450MHz) = 0.9dB; Center freq.: 2450MHz; Bandwidth: 200MHz</td>
</tr>
<tr>
<td>LNA (ZQL-2700)</td>
<td>Gain: 25dB; Noise Figure: 1.3dB; Max. Input Power: 3dBm</td>
</tr>
<tr>
<td>Mixer (ZEM-4300)</td>
<td>Conversion Loss: 9.5dB; 7dBm LO, up to 1dBm RF</td>
</tr>
<tr>
<td>IF Amp. (ZFL-1000H)</td>
<td>Gain: 28 dB</td>
</tr>
<tr>
<td>LPF (SLP-21.4)</td>
<td>Passband (DC-22MHz) Passband Loss: &lt;1dB</td>
</tr>
<tr>
<td>Signal Generator (HP 8684C)</td>
<td>100KHz~3200MHz</td>
</tr>
<tr>
<td>Digital oscilloscope (Tektronix TDS 694C or TDS 580D)</td>
<td>Real time digital oscilloscope; max. snap shot record length: Up to 30K samples/channel with special option. (Maximum record length is 120K /channel with special option.) A maximum sample rate of up to 10 GS/s simultaneous on 4 channels. ADC resolution: 8 bits</td>
</tr>
</tbody>
</table>

The omni-directional antenna collects RF signals in the air. The captured RF signal goes through BPF SAW Filter to pick only the 2.4GHz band WLAN signals and is amplified using LNA for limiting noise and providing enough power to the subsequent signal processing. Then, the 2.4GHz RF signal is down converted to the 11MHz intermediate frequency (IF) signal. The conversion to the IF signal is achieved by controlling the difference between the carrier frequency, \( f_c \) and local oscillator (or signal generator) frequency, \( f_L \) to 11MHz like as \( f_c = f_L - 11 \text{ MHz} \). Then, the IF signal is further amplified before analog-to-digital conversion. To remove the unnecessary image signal caused by the IF conversion process, the IF signal is fed into LPF to remove higher frequency component greater than 22 MHz. Finally, the low-pass filtered IF signal is digitized using digital oscilloscope with sampling rate 50MHz. This ensures Nyquist sampling rate for the band.
limited IF signal and thus alias-free data sampling. The maximum number of samples per measurement is set to 30,000 and thus the digital oscilloscope can capture up to 0.6 msec WLAN signal burst. These sampling rate and data record size allow capturing the longest OFDM radio burst in fragmentation mode. The digitized signal is transferred to the host computer through general purpose interface bus (GPIB or IEEE 488) cable for digital signal processing.

The transferred signal is checked whether the number of samples of the longest uplink signal burst without 16 μs training sequence is at least 10,000 samples (or 2 msec). The underlying reason for limiting minimum sample size of uplink burst is to attain more accurate result for cyclic spectral analysis. If the minimum burst size of the uplink is less than 10,000 samples, this captured signal is removed automatically. The test process of checking the uplink sample size is shown in Figure 7.4

![Time Domain Plot of AS15GU54C11M038.](image)

**Figure 7.4:** Captured WLAN uplink signal. Sampling frequency is 50MHz. The green colored line indicates automatic filtering of time-domain signal. Only filtered time domain data is used for CDP generation.

If the transferred signal is greater than the minimum uplink sample size, then the first 16 μs is removed and normalized to have unit power. Then, the normalized signal is converted to a complex baseband signal using Hilbert transformation and down conversion
of positive spectral component to DC. The complex baseband signals are stored for the later second-order cyclic spectral analysis. For future reference of measurement data, the file name of complex baseband signals conform to the file naming rules depicted in Figure 7.5.

<table>
<thead>
<tr>
<th>AS</th>
<th>#</th>
<th>#</th>
<th>G/B</th>
<th>U/D</th>
<th>#</th>
<th>C</th>
<th>#</th>
<th>M</th>
<th>#</th>
<th>#</th>
<th>#</th>
<th>.dat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP &amp; STA</td>
<td>STA Number</td>
<td>AP Number</td>
<td>G:802.11G B:802.11B</td>
<td>U:Uplink D:Downlink</td>
<td>Data Rate in Mbps</td>
<td>Channel Number</td>
<td>Measurement Number</td>
<td>3 Digit Measurement Number</td>
<td>File Extension</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 7.5: File naming rule for captured signal

The station and AP numbers follow the indexes in the Table 7-3 and Table 7-4. For instance, a captured signal from station Motorola is named as:

“AS11GU54C11M031.dat” : Uplink signal from Motorola station to Motorola AP with the channel number 11 and 54Mbps data rate.

Finally, ninety uplink signals are collected per WLAN card and stored.

The spectral correlation function (SCF) and spectral coherence (SC) for the complex based band signal are evaluated. The radio device specific fingerprint is generated by evaluating the cycle frequency domain profiles (CDPs) for SCF and SC.
7.5 Second-order Cyclic Spectral Analysis of IEEE 802.11a/g Signals

To extract WLAN device specific electromagnetic features, the second order cyclic features (SCF and SC) are investigated. Whatever the source of emitter-specific statistical variation, the exploitable statistical features must be reliably estimated from the captured waveform for proper SEI operation. Therefore, underlying causes of radio device specific features are not investigated. However, there are some possible conjectures for the causes of emitter specific features that include the aging and degraded low quality electric components of a transmitter, the nonlinear characteristics of the power amplifier, and fading channels.

7.5.1 Second-order Cyclic Features

The averaged maximum spectral correlation and spectral coherence magnitude (normal and conjugate) are evaluated for the measurement data and are plotted from Figure 7.6 to Figure 7.15.

The dominant cyclic features of normal and conjugate CDPs are related to the OFDM four pilots which are correlated perfectly within one OFDM symbol. The cycle frequencies of dominant OFDM pilot cyclic features consist of two parts: normal (or lower region) and conjugate (or upper region) cyclic features. The cycle frequencies for dominant cyclic features for lower and upper regions are given by:

\[ \alpha_{\text{lower}} = \pm\{0.0, 4.25, 4.5, 8.75, 13.0, 13.25\}MHz \]
\[ \alpha_{\text{upper}} = 2f_c + \alpha_{\text{lower}} \]  \hspace{1cm} (7.5)

where \( f_c \) is the center frequency of the complex baseband signal. The center frequency of complex baseband signal is -1.5MHz. Therefore, the cycle frequencies for upper cyclic features are shifted by -3MHz. This is identified at the conjugate spectral SCF or SC plots.
Those cycle frequencies can be obtained from the general formula discussed in chapter 4 and is given by

\[
\alpha_n = \pm \left[ 2 \times k \times \Delta_f + \left\{ - \left( \frac{m}{G/2} \right), \left( 1 - \frac{m}{G/2} \right) \right\} \times f_s \times \delta(m) \right] \\
\alpha_c = \pm \left( (k_i \pm k_1) \times \Delta_f + \left\{ - \left( \frac{n}{G} \right), \left( 1 - \frac{n}{G} \right) \right\} \times f_s \times \delta(n) \right)
\]

(7.6)

where \( \text{mod}(k, G) \) is the modulus after the division of \( k \) by \( G \), \( m = \text{mod}(k, G/2) \) and \( k \) is the positive pilot subcarrier index, \( n = \text{mod}(k_i \pm k_1, G) \) with \( 0 < k_1 < k_2 < k_3 \), and \( \delta() \) is zero if the argument is zero, and one otherwise. And \( \alpha_n \) is the cycle frequency for the non-combined subcarrier, \( \alpha_c \) is the cycle frequency for the combined subcarriers, \( \Delta_f \) is the subcarrier spacing, \( f_s \) is the OFDM symbol rate, \( G \) is the ratio of IFFT to guard interval duration, and \( k_i \) are pilot subcarrier index.

The magnitude of dominant cyclic features related to OFDM pilots varies from measurement to measurement. In addition, the nominal cycle frequencies for the OFDM pilot cyclic features are also varying, but the deviation from the nominal values is very small. However, the small changes in the cycle frequency bring substantial variation of cyclic feature strength due to the discrete nature of cycle frequency. Therefore, the cycle frequencies for the dominant OFDM pilot features are used as a guide for finding the exact cycle frequencies for each collected signal. This is achieved by finding the actual dominant cyclic features in a small frequency band around the nominal dominant cycle frequencies. One of the causes for cycle frequency shift is the local oscillator drift at the transmitter side; another could be time variations in the interceptor.

To investigate statistical characteristics of pilot related dominant features, their mean and standard deviation evaluated over 90 records per WLAN station. Those results are plotted in Figure 7.16 and Figure 7.17. For the statistics of normal CDP, only positive cycle frequencies are shown due to its symmetry characteristic. From these plots, the spectral correlation function’s dominant features have distinctive radio device specific attributes for
SEI except for IBM and Netgear, which give rise to features that are quite similar, but there are still potentially exploitable differences in the sequence of dominant features taken as a whole.

Figure 7.6: Averaged Normal/Conjugate SCF CDP for Motorola AP linked with Motorola STA
Figure 7.7: Averaged Normal/Conjugate SCF CDP for Motorola AP linked with LinkSys STA

Figure 7.8: Averaged Normal/Conjugate SCF CDP for Motorola AP linked with DLink STA
Figure 7.9: Averaged Normal/Conjugate SC CDP for Motorola AP linked with Netgear STA

Figure 7.10: Averaged Normal/Conjugate SC F CDP for Motorola AP linked with IBM STA
Figure 7.11: Averaged Normal/Conjugate SC CDP for Motorola AP linked with Motorola STA

Figure 7.12: Averaged Normal/Conjugate SC CDP for Motorola AP linked with LinkSys STA
Figure 7.13: Averaged Normal/Conjugate SC CDP for Motorola AP linked with DLink STA

Figure 7.14: Averaged Normal/Conjugate SC CDP for Motorola AP linked with Netgear STA
Figure 7.15: Averaged Normal/Conjugate SC CDP for Motorola AP linked with IBM STA

Mean and Standard Deviation for Dominant Normal SCF Cyclic Features.
Figure 7.16: Mean and standard deviation for dominant normal cyclic features.  
Top: Normal SCF, Down: Conjugate SCF
7.6 Specific Emitter Classification using HMM

The WLAN card identification is performed using hidden Markov models (HMM) [82] that are widely used in pattern matching. The dominant cyclic features of the captured signal show different patterns for different WLAN card manufacturers and HMM can take advantage of these patterns to the WLAN card manufacturer. Through HMM-based pattern matching method, the potential of OFDM cyclic features for SEI is demonstrated. Note that the SNR of captured signal is very high.
7.6.1 Feature Extraction for HMM Identifier

Feature extraction is the process of producing application specific distinctive features from input data. This stage must extract distinctive features to make HMM identifier work appropriately. For HMM training and recognition, the following features are extracted\(^{15}\).

\[
F_{1,i} = \max_f \left| \hat{S}^\alpha_x (f) \right|, \quad \alpha_j \in A_1
\]
\[
F_{2,j} = \max_f \left| \hat{C}^\alpha_x (f) \right|, \quad \alpha_j \in A_2
\]
\[
F_{3,k} = \max_f \left| \hat{S}^\alpha_x (f) \right| \quad \text{and} \quad F_{4,k} = \max_f \left| \hat{C}^\alpha_x (f) \right|, \quad \alpha_k \in A_3
\]

where the cycle frequency sets \(A_1, A_2, \) and \(A_3\) are defined by

\[
A_1 = \alpha_{lower\_pos}
\]
\[
A_2 = \alpha_{lower\_pos} \cup \left\{ k \times f_{sym} \right\}_{k=1}^{15}
\]
\[
A_3 = \alpha_{upper}
\]

where the \(\alpha_{upper\_pos}\) is the positive upper region cycle frequency including cycle frequency zero.

Finally, the feature vector for HMM training and recognition is generated by combining the features in Eq.(7.7) as

\[
FV = \left\{ \left\{ F_{1,i} \right\}, \left\{ F_{2,j} \right\}, \left\{ F_{3,k} \right\}, \left\{ F_{4,k} \right\} \right\}
\]

(7.9)

7.6.2 Classification using HMM

To train the HMM, ten feature vectors are selected and used per WLAN card. This method can simulate random data input using limited measurement data set. This feature vector is

\(^{15}\) Features were suggested by Dr. Chad M. Spooner in [83] Chad M. Spooner, "Specific Emitter Identification for Modern Communication Signals," White Paper, NWRA, Mar. 17, 2008.
averaged to reduce feature variance and is used for training the HMM for specific manufacturer of WLAN card. Instead of just using one training vector for a particular WLAN card, four training sequences are used to obtain better identification. The Baum-Welch algorithm [84] is used to train HMMs with three hidden states. The overall training process is depicted in Figure 7.18.

In the identification stage, ten feature vectors are selected randomly to make various test signals from the specific WLAN card manufacturer data and are averaged before the identification. The identification is performed by choosing maximum log-likelihood value generated by the WLAN card specific HMMs for the given averaged feature vector input. The overall classification process is explained in Figure 7.19. The confusion matrix of HMM identification for 200 trials is shown Table 7-6.
Ten WiFi Signal Measurements

Evaluate CDPs

Average CDP

Generate a Feature Vector

HMMs for Motorola

p(FV | λ_{1,k})

HMMs for LinkSys

p(FV | λ_{2,k})

HMMs for DLink

p(FV | λ_{3,k})

HMMs for Netgear

p(FV | λ_{4,k})

HMMs for IBM

p(FV | λ_{5,k})

Select Maximum

p(FV | λ_{a,k})

Figure 7.19: HMM classification process.

Table 7-6: Confusion matrix for identification of five WLAN cards. High SNR

<table>
<thead>
<tr>
<th>True card</th>
<th>Identified card</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLink</td>
<td>IBM</td>
</tr>
<tr>
<td>DLink</td>
<td>192</td>
</tr>
<tr>
<td>IBM</td>
<td>13</td>
</tr>
<tr>
<td>LinkSys</td>
<td>0</td>
</tr>
<tr>
<td>Motorola</td>
<td>0</td>
</tr>
<tr>
<td>Netgear</td>
<td>37</td>
</tr>
</tbody>
</table>
To investigate the SEI performance at low SNR, AWGN is added to the normalized complex baseband signal to make SNR=0dB. The test procedure for SNR=0dB is same as the high SNR case. The SEI performance of 0dB signal is evaluated through 200 trials and the confusion matrix is shown in Table 7-7.

Table 7-7: Confusion matrix for identification of five WLAN cards, SNR=0dB

<table>
<thead>
<tr>
<th>True card</th>
<th>Identified card</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DLink</td>
</tr>
<tr>
<td>DLink</td>
<td>169</td>
</tr>
<tr>
<td>IBM</td>
<td>0</td>
</tr>
<tr>
<td>LinkSys</td>
<td>0</td>
</tr>
<tr>
<td>Motorola</td>
<td>0</td>
</tr>
<tr>
<td>Netgear</td>
<td>0</td>
</tr>
</tbody>
</table>

As Table 7-7 shows, the HMM classifier with OFDM cyclic feature vector can classify WLAN manufacture with correct classification rate about 65% except IBM and Netgear which have similar features.

7.7 Chapter Summary

The potential for specific emitter identification using second-order OFDM cyclic features is demonstrated. For this purpose, we developed an automated collection system to gather IEEE 802.11a/g OFDM WLAN signals. The SCF and SC CDPs of the collected signals are evaluated and their distinctive features are compared. Using the second-order cyclic features and a HMM classifier, five different WLAN cards are identified successfully. Based on the results, the second-order cyclic statistics show potential as features for performing SEI for 802.11a/g OFDM radios.
Future work should examine conditions of low and varying SNR and fading channel environment to justify practically. To bolster the results presented in this exploratory work, additional measurements with increased number of WLAN cards should follow. This will provide better information of the feature vector extraction. In addition, as a future possible work, higher order cyclic spectral analysis may also help to reveal the transmitter specific features which are related to the nonlinear effects caused by non-linear power amplifier at the transmitter.
Chapter

8 Conclusions

In this dissertation, radio environmental awareness of CR (Cognitive Radio) is investigated using the second-order cyclic spectral analysis. The second-order cyclic features of wireless communication signal are utilized to address the challenges in radio environmental awareness for CR such as cDSA (CR-based Dynamic Spectrum Access) network identification, signal detection and classification at low and varying SNR (Signal-to-Noise Radio), and enhancement of CR security.

The first category of research results focus on characterizing the cyclic spectrum of OFDM pilot and applying it to cDSA network identification. The second-order cyclic signature of OFDM (Orthogonal Frequency Division Multiplexing) pilots is proposed for the cDSA network identification. To generate strong OFDM pilot cyclic features, the behavior of OFDM pilot features associated with the GI (Guard Interval) are investigated. A general formula for estimating dominant OFDM pilot cyclic features is developed. Using this general formula, it is shown how to identify primary users and CR nodes in cDSA networks without a dedicated common control channel. The followings are contributions resulting from OFDM cyclic spectral analysis:

- Second order cyclic spectral expressions for OFDM pilots,
- General formula for estimation of dominant OFDM pilot cyclic features,
• Detection performance using dominant pilot cyclic feature associated with the GI, and
• A solution for cDSA network identification using OFDM pilot cyclic features.

The second category of research results focus on cyclic feature extraction for signal sensing and classification. Spectrum sensing and signal classification schemes are developed. The signal detection uses the CDP (Cycle-frequency Domain Profile) and crest factor of the CDP, and HMMs (Hidden Markov Models) to classify signals with low and varying SNR. In addition, we can also determine signal parameters such as the modulation type and symbol rate. The following are contributions related to spectrum sensing for CR:

• Feature extraction method by exploiting crest factor of CDP and evaluation of its detection capability,
• A HMM-based signal classification method that can correctly classify different types of incoming signals even at low and varying SNR, and
• Performance superiority of the developed cyclostationary signal detector over the energy detector at low and varying SNR.

The third category of research shows the feasibility of SEI (Specific Emitter Identification) using OFDM cyclic features. There is evidence that second-order statistics can be used to perform specific emitter identification for IEEE 802.11a/g compliant radios. For this purpose, an automated signal collection system was developed to capture IEEE 802.11a/g WLAN (Wireless Local Area Network) signals from five different WLAN cards. The CDPs for SCF (Spectral Correlation Function) and SC (Spectral Coherence) are evaluated to extract device specific features. It is shown that five WLAN cards can be identified successfully with greater than 90% accuracy using HMM base pattern matching algorithm. The following are contributions related to the SEI for CR:

• Feature extraction technique,
• A HMM-based signal classification architecture, and
• Results showing effectiveness of approach to classify five different WLAN manufacturers.

Several extensions to this research are recommended for future investigation. These extensions include:

• Detection performance evaluation of dominant OFDM pilot cyclic features in fading channels,
• SEI techniques for identical models of WLAN cards, and
• High-order cyclic spectral analysis to extract exploitable cyclic features for SEI due to non-linear power amplifier.
Appendix

A Cyclostationarity from Linear Time Varying Transformation

A.1 Introduction

In this appendix, the underlying concept of LPTV transformation is explained, which is used extensively to evaluate cyclic features of modulated signal in this work. The detail derivation of the general formula for the CAF and SCF of the LPTV output is found in Gardner’s work [30, 33].

A.2 LPTV Transformation

A second-order cyclostationary signal can be obtained from a LPTV transformation

\[ y(t) = \int_{-\infty}^{\infty} h(t, u)x(u)du \]  \hspace{1cm} (A.1)

where the \( h(t, u) = h(t + T_0, u + T_0) \) is the LTPV impulse response and \( x(t) \) is the input excitation.
The transfer function of LPTV system can be explained using $g(t, \tau) \triangleq h(t, t-\tau)$ and $\tau \triangleq t-u$. This LPTV system become periodic in its first argument

$$g(t+T_0, \tau) = g(t, \tau) \quad (A.2)$$

We can express the LPTV system $g(t, u)$ in terms of Fourier series for periodic argument $t$ like as

$$g(t, \tau) = \sum_{n=-\infty}^{n} g_n(\tau)e^{j2\pi nt/T_0} \quad (A.3)$$

where

$$g_n(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g(t, \tau)e^{-j2\pi nt/T_0} dt \quad (A.4)$$

or we can have

$$g(t, t-u) = h(t, u) = \sum_{n=-\infty}^{n} g_n(t-u)e^{j2\pi nt/T_0} \quad (A.5)$$

The Eq.(A.3) can also be expressed in terms of Fourier transform

$$G(t, f) = \int_{-\infty}^{\infty} g(t, \tau)e^{-j2\pi f \tau} d\tau \quad (A.6)$$

The Eq.(A.6) is also periodic in $t$ and so we can represent it in terms of Fourier series,

$$G(t, f) = \sum_{n=-\infty}^{\infty} G_n(f)e^{j2\pi nt/T_0} \quad (A.7)$$

where
In addition, the Fourier transform of Eq. (A.4) can be written by

$$G_n(f) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} G(t, f) e^{-j2\pi n f T_0} dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \int_{-\infty}^{\infty} g(t, \tau) e^{-j2\pi n \tau T_0} e^{-j2\pi f \tau} d\tau dt$$

(A.8)

Note that Eq. (A.8) and Eq. (A.9) are the same.

Finally we have very important results by substitution Eq. (A.5) into Eq. (A.3) given by

$$y(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g_n(t-u) e^{j2\pi n T_0} x(u) du$$

$$= \sum_{n=-\infty}^{\infty} y_n(t) e^{j2\pi n T_0}$$

(A.10)

where

$$y_n(t) = \int_{-\infty}^{\infty} g_n(t-u) x(u) du$$

(A.11)

Consequently, $x(t)$ is LTI transformed with impulse response function $g_n(t)$ for each $n$. Thus, the cyclostationary signal $y(t)$ can be characterized by LTI transformation and frequency shifts.

Let’s investigate how Eq. (A.10) is related to a modulation scheme. Consider generalized amplitude modulation (AM) signal

$$y(t) = x(t) p(t)$$

(A.12)

where $x(t)$ is input excitation and $p(t)$ is a periodic carrier with Fourier series
\[ p(t) = \sum_{n=-\infty}^{\infty} P_n e^{j2\pi nt/T_0} \]  

(A.13)

where \( T_0 \) is the fundamental period of the carrier frequency.

Therefore, we have

\[ y(t) = \sum_{n=-\infty}^{\infty} P_n x(t)e^{j2\pi nt/T_0} \]  

(A.14)

If we establish following relation for the impulse response function \( g_n(t) \),

\[ g_n(\tau) = \beta_n \delta(\tau) \]  

(A.15)

then

\[ y_n(t) = \int_{-\infty}^{\infty} g_n(t-u)x(u)du = \int_{-\infty}^{\infty} \beta_n \delta(t-u)x(u)du \]

\[ = \beta_n x(t) \]  

(A.16)

and

\[ y(t) = \sum_{n=-\infty}^{\infty} \beta_n x(t)e^{j2\pi nt/T_0} \]  

(A.17)

Eq. (A.14) is equivalent to Eq. (A.17) when \( \beta_n = P_n \). Therefore, we can apply LPTV transformation method for CAF and SCF of a generalized modulation waveform.

A.3 General Formula of CAF and SCF for the Cylostationary Signal from LPTV Transformation
The CAF of the output of LPTV system for an input excitation is derived. By definition of CAF, we can have following relation for the Eq.(A.1).

\[
\hat{R}_y(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} y(t + \tau / 2) y^*(t - \tau / 2) e^{-j2\pi\alpha t} dt
\]

\[
= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left( \int_{-\infty}^{\infty} h(t, u)x(u)du \right) \left( \int_{-\infty}^{\infty} h(t, v)x(v)dv \right)^* e^{-j2\pi\alpha t} dt
\]

By substitution Eq. (A.10) into Eq.(A.18), the CAF can be written by

\[
\hat{R}_y(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left( \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g_n(t - u)e^{j2\pi nT_0}x(u)du \right) \times
\]

\[
\left( \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} g_m(t - v)e^{j2\pi mT_0}x(v)dv \right)^* e^{-j2\pi\alpha t} dt
\]

To make the whole calculation simple, following change of variables are performed.

\[
u = t - t' - \tau' / 2 \quad \text{and} \quad v = t - t' + \tau' / 2
\]

\[
t' = t - (u + v) / 2 \quad \text{and} \quad \tau' = v - u
\]

Then Eq.(A.19) becomes
\[ \hat{R}_x^{\alpha} (\tau) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_n \left( t + \frac{\tau + \tau'}{2} \right) g^*_m \left( t - \frac{\tau + \tau'}{2} \right) e^{-j\pi(n+m)\tau' T} e^{-j2\pi n\tau'} \times \]
\[
\left( \lim_{T \to \infty} \frac{1}{T^{1/2}} \int_{-T/2}^{T/2} x(t - t' - \tau'/2)x^*(t - t' + \tau'/2)e^{j2\pi n\tau t/T} e^{-j2\pi m\tau' t/T} e^{-j2\pi nt} dt \right) dt' d\tau' \]
\[
= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} g_n \left( t + \frac{\tau + \tau'}{2} \right) g^*_m \left( t - \frac{\tau + \tau'}{2} \right) e^{-j\pi(n+m)\tau' T} e^{-j2\pi n\tau'} \times \]
\[
\hat{R}_x^{\alpha-n-m} (\tau') dt' d\tau' \]
\[
= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} g_n \left( t + \frac{\tau + \tau'}{2} \right) g^*_m \left( t - \frac{\tau + \tau'}{2} \right) e^{-j2\pi n\tau'} dt' \times \]
\[
\hat{R}_x^{\alpha-n-m} (\tau') dt' d\tau' \]
\[
= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} g_n \left( t + \frac{\tau + \tau'}{2} \right) g^*_m \left( t - \frac{\tau + \tau'}{2} \right) e^{-j2\pi n\tau'} dt' \times \]
\[
\hat{R}_x^{\alpha-n-m} (\tau') dt' d\tau' \]
\[
= \sum_{\beta} \sum_{\nu} \hat{R}_x^{\alpha-\beta-\nu} (\tau') e^{-j\pi(\beta+\nu)T} \otimes r_{\beta\nu}^{\alpha} (\tau) \]
\[ (A.21) \]

where
\[ \beta \triangleq n / T \quad \text{and} \quad \nu \triangleq m / T \]  
\[ (A.22) \]

and
\[ r_{nm}^{\alpha} (\tau) \triangleq \int_{-\infty}^{\infty} g_n \left( t + \frac{\tau}{2} \right) g^*_m \left( t - \frac{\tau}{2} \right) e^{-j2\pi n\tau} dt \]  
\[ (A.23) \]

Spectral correlation can be obtained using convolution property like as
\[ S^{\alpha} (f) = \sum_{\beta} \sum_{\nu} G_\beta (f + \alpha / 2) S^{\alpha-\beta-\nu} (f - (\beta + \nu) / 2) G^{*}_\nu (f - \alpha / 2) \]  
\[ (A.24) \]
Appendix

B  Cyclostationarity of Periodically Sampled Signal

B.1  Introduction

To analyze the discrete version of CAF and SCF, an investigation of the relationship between continuous and discrete CAF and SCF is required. For this purpose, the CAF and SCF of periodic sampling function are summarized based on [30, 33].

B.2  Discrete Signal from Continuous Signal using Periodic Sampling

A digital signal can be obtained using periodic sampling of a continuous signal as shown below equations.

\[ y(t) = \sum_{n=-\infty}^{\infty} x(t) \delta(t - nT_0) = \sum_{n=-\infty}^{\infty} x(nT_0) \delta(t - nT_0) \]  \hspace{1cm} (B.1)
The output signal can be considered as a product of input signal with periodic sampling function given by,

\[ y(t) = x(t) \cdot p(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = x(t) \frac{1}{T_0} \sum_{n=-\infty}^{\infty} e^{-j2\pi nT_0} \quad (B.2) \]

The SCF of Eq.(B.2) can be written using the result shown in Eq. (A.24)

\[ \hat{S}_y^\alpha(f) = \frac{1}{T_0^2} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \hat{S}_x^{\alpha-nT_0} \left( f - \frac{m}{2} + \frac{n}{2T_0} \right) \quad (B.3) \]

We can see the repetition of the output SCF due to periodic sampling of input signal from Eq.(B.3).

Now consider the discrete time series \( x(nT_0) \) shown in Eq.(B.1) and evaluate its CAF. However, the shift of \((\tau/2)\) in continuous CAF definition is not possible for the discrete sequence. Thus, the asymmetric CAF definition which introduces \( e^{-j\pi \alpha \tau} \) is used for CAF [30, 33, 35]. The resulting discrete CAF is given by,

\[ \tilde{R}_x^\alpha(kT_0) \triangleq \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(nT_0 + kT_0) x^*(nT_0) e^{-j2\pi nT_0} e^{-j\pi \alpha kT_0} \quad (B.4) \]

This can be converted into integration form using synchronized averaging identity in [33, 35]. Synchronized averaging is the oldest known technique for extracting periodicity from random data. Synchronized averaging identity is given by,

\[ \hat{M}_x(t) = \lim_{x \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(t + nT_0) = \sum_{n=-\infty}^{\infty} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t + u)e^{-j2\pi nT_0} du \quad (B.5) \]

Therefore, the Eq.(B.4) can be rewritten by,
\[
\tilde{R}_x^\alpha (kT_0) = \sum_{n=-\infty}^{\infty} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(kT_0 + u) x^*(u) e^{-j2\pi n u} e^{-j\pi n kT_0} e^{-j2\pi n u/kT_0} du \\
= \sum_{n=-\infty}^{\infty} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(kT_0 + u) x^*(u) e^{-j2\pi (u+kT_0) / 2} e^{-j\pi n u/kT_0} du
\] (B.6)

Now we can make symmetric ACF for Eq. (B.6) due to continuous parameter \( u \). Let’s make following substitution.

\[
u = v - kT_0 / 2 \quad \text{and} \quad kT_0 + u = v + kT_0 / 2
\] (B.7)

Then, the Eq. (B.6) can be expressed by,

\[
\tilde{R}_x^\alpha (kT_0) = \sum_{n=-\infty}^{\infty} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(v + kT_0 / 2) x^*(v - kT_0 / 2) e^{-j2\pi (v-kT_0/2+\tau) / \tau} e^{-j2\pi (v-kT_0/2)} dv \\
= \sum_{n=-\infty}^{\infty} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(v + kT_0 / 2) x^*(v - kT_0 / 2) e^{-j2\pi v (\alpha+n/kT_0)} e^{-j\pi n k} dv \\
= \sum_{n=-\infty}^{\infty} \tilde{R}_{x^{\alpha+n/kT_0}} (kT_0) e^{-j\pi nk}
\] (B.8)

Using Wiener relation, discrete SCF is attained by Fourier transform of discrete CAF in Eq.(B.8).

\[
\tilde{S}_x^\alpha (f) = \sum_{k=-\infty}^{\infty} \tilde{R}_x^\alpha (kT_0) e^{-j2\pi kT_0 f} \\
= \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \tilde{R}_{x^{\alpha+n/kT_0}} (kT_0) e^{-j\pi nk} e^{-j2\pi kT_0 f} \\
= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \tilde{R}_{x^{\alpha+n/kT_0}} (kT_0) e^{-j2\pi kT_0 (f-n/2T_0)}
\] (B.9)

The \( \tilde{R}_{x^{\alpha+n/kT_0}} (kT_0) \) term in Eq.(B.9) is the sampled version of \( \tilde{R}_{x^{\alpha+n/kT_0}} (\tau) \). Therefore, the Fourier transform will be repeated in frequency domain. The relation of the Fourier
transformation between sampled signal and the continuous signal being periodic sampled is well known as [85],

\[ X_d(f) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} X_c \left( f - \frac{m}{T_s} \right) \]  

(B.10)

where the subscript \( d \) and \( c \) indicate discrete and continuous signal and \( T_s \) is the sampling period.

Thus, the Eq. (B.9) is simplified like as,

\[
\tilde{S}_x^\alpha(f) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \hat{R}_x^{\alpha+n/T_0}(kT_0)e^{-j2\pi kT_0(f-n/2T_0)} \\
= \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \hat{S}_x^{\alpha+n/T_0} \left( f - \frac{m}{T_0} - \frac{n}{2T_0} \right) 
\]  

(B.11)

Eq.(B.11) reveals the periodic characteristics of discrete SCF which is periodically sampled from the continuous SCF and this periodicity can be explained below

\[
\tilde{S}_x^\alpha \left( f + \frac{k}{T_0} \right) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \hat{S}_x^{\alpha+n/T_0} \left( f + \frac{k}{T_0} - \frac{m}{T_0} - \frac{n}{2T_0} \right) \\
= \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \hat{S}_x^{\alpha+n/T_0} \left( f - \frac{m}{T_0} - \frac{n}{2T_0} \right) = \tilde{S}_x^\alpha(f) 
\]  

(B.12)

\[
\tilde{S}_x^{\alpha \pm k/T_0} \left( f \pm \frac{k}{2T_0} \right) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \hat{S}_x^{\alpha+(n \pm k)/T_0} \left( f - \frac{n \mp k}{2T_0} - \frac{m}{T_0} \right) \\
= \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \hat{S}_x^{\alpha+n/T_0} \left( f - \frac{m}{T_0} - \frac{n}{2T_0} \right) = \tilde{S}_x^\alpha(f) 
\]  

(B.13)

where \( k \in \mathbb{Z} \).
Thus,

\[ S_x^{a (\alpha \pm s \frac{1}{T_0})} (f) \]

\[ = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \tilde{S}_x^{(\alpha \pm s \frac{1}{T_0})} \left( f - \frac{n}{2T_0} - \frac{m}{T_0} \right) \]

\[ = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \tilde{S}_x^{\alpha (n \pm s \frac{1}{T_0})} \left( f - \frac{n + s}{2T_0} - \frac{m}{T_0} \right) \]

\[ = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \tilde{S}_x^{\alpha (n \pm s \frac{1}{T_0})} \left( f - \frac{n \pm s}{2T_0} - \frac{m}{T_0} \right) \]

\[ = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \tilde{S}_x^{\alpha (n \pm s \frac{1}{T_0})} \left( f - \frac{n \pm s}{2T_0} - \frac{m}{T_0} \right) \]

\[ = \tilde{S}_x^{\alpha} (f) \]

where \( s \) is even integer.

In summary, there is following periodicities are found for discrete SCF

\[ \tilde{S}_x^{\alpha} (f) = \tilde{S}_x^{\alpha} \left( f + \frac{k}{T_0} \right) = \tilde{S}_x^{\alpha (\alpha \pm k \frac{1}{T_0})} \left( f \pm \frac{s}{2T_0} \right) = \tilde{S}_x^{\alpha \pm 2k \frac{1}{T_0}} (f) \]  \hspace{1cm} (B.15)

Finally, we have following relationship between the output continuous SCF and input discrete SCF like as,

\[ \tilde{S}_y^{\alpha} (f) = \frac{1}{T_0^2} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \tilde{S}_x^{\alpha-n \frac{1}{T_0}} \left( f - \frac{m}{2T_0} + \frac{n}{T_0} \right) \]

\[ = \frac{1}{T_0} \left( \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \tilde{S}_x^{\alpha-n \frac{1}{T_0}} \left( f - \frac{m}{2T_0} + \frac{n}{T_0} \right) \right) \]  \hspace{1cm} (B.16)

\[ = \frac{1}{T_0} \tilde{S}_x^{\alpha} (f) \]

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