Modeling of Herschel/Quincke-Liner Systems for the Control of Aft Fan Radiation in Turbofan Engines

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Commercial aviation transportation has experienced an overwhelming growth over the years. However, this expansion has encountered an important barrier: noise. Several studies have shown that residents in these areas experience problems such as stress and sleep disturbance. These problems have translated into demands for a better quality of life from airport residents which in turn have translated into more stringent aircraft noise regulations. As a result, large amounts of resources have been diverted towards the improvement of existing noise attenuation technologies and the development of more effective ones. In terms of turbofan generated noise, the most widely used technology is that of absorbent materials or liners. In recent investigations Alonso et al. have combined Herschel/Quincke (HQ) tubes with liners. This combination has the potential of effectively controlling pure tones and broadband noise in inlet sections of modern turbofan engines. Since a comprehensive approach for engine noise reduction will involve both inlet and aft HQ-Liner systems, additional research efforts were needed to evaluate their performance at reducing aft fan radiation.

In the present work, a combination of traditional liners and Herschel/Quincke waveguide resonators for aft fan radiation control is proposed. A theoretical model is developed in order to predict noise reduction due to such systems. The newly developed tool was then utilized to design an HQ-liner that was installed and tested in the aft section of the NASA Active Noise Control Fan (ANCF) rig. This experimental data was utilized to prove the potential of these systems and to validate the mathematical model. Analytical predictions correlate well with experiments.

The NASA ANCF rig is not representative of a real turbofan engine. In order to assess the behavior of HQ-Liners in a more realistic environment a new system was specifically designed for a generic turbofan engine and its performance analyzed. The sound field inside HQ tubes has been described assuming plane waves only. This assumption limits the model to frequencies below the tube first resonance. In order to overcome this limitation a new model accounting for higher order modes inside the tubes has been developed.
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1. INTRODUCTION

Commercial aviation transportation has experienced an overwhelming growth over the years. It has become a key factor in the economic development of a nation by contributing not only to business activities but also to leisure travel. It is estimated that 1.6 billion passengers worldwide use air transportation, figure that is expected to rise to 2.3 billion in 2010. The air transportation industry is also responsible for the generation of a massive number of jobs directly or indirectly related to it [1, 2]. Obviously, this growth has had a direct impact on airports. Airports have had and will have to expand their facilities in order to meet the increasing demand of passengers. However, this expansion has encountered an important barrier: noise. Aircraft noise is considered to be one of the main obstacles for the ever-expanding air transportation industry. Precisely, the communities around airports as seen in Figure 1.1 suffer from more than just annoyance from aircraft traffic. Several studies have shown that residents in these areas experience stress, sleep disturbance, and reduction in work performance.

![Figure 1.1: Population concentrated around Itami Airport, Osaka Japan. (Picture obtained from www.airliners.net, Copyright holder: Danny H. Masson)](image)

Demands for a better quality of life from airport residents have translated into more stringent aircraft noise regulations. Such regulations impose severe restrictions that affect not only the aircraft and aircraft engine industry but also airports and airline companies. These
restrictions, therefore, reduce the potential development of the entire air transportation industry. From this perspective, noise has turned into a commercial issue.

In an effort to overcome this barrier, aircraft and engine companies have diverted large amounts of resources towards the improvement of existing noise attenuation technologies and the development of new and more effective ones.

In an aircraft, many sources are responsible for noise generation, some of them more relevant than the others depending on the flight condition. Regulations are more concerned with noise generation during airport transit, which involves take-off and landing. During landing the two major noise sources are the engine and the airframe, i.e. flaps and landing gear, while the main source during take-off is the engine. The engine noise is the focus of this dissertation.

1.1. Noise Sources in Turbofan Engines

Over the years and with the evolution of early jet engines into modern and more efficient high-bypass (HBP) turbofan engines, the noise pattern has changed. In the beginning, jet noise, radiated back through the exhaust, was the dominant source of overall noise levels. As jets and early turbofans developed into the modern HBP engines the large diameter fan became the main noise source; the compressor, the jet and the turbine being relegated now to a second place. The different noise sources in a turbofan engine can be summarized as indicated in Figure 1.2: i) fan and compressor noise propagating to the front through the inlet, ii) fan noise radiating to the rear through the bypass duct and iii) jet and turbine noise radiating to the rear through the nozzle.

Fan generated noise can be divided into three main components [3]: i) pure tones, i.e. blade passage frequency (BPF) tone and its harmonics, ii) shaft order tones, and iii) broadband.

i) **BPF tone and its harmonics**: two generating mechanisms fall into this category: rotor-alone and rotor-stator interaction tones. In the case of rotor-alone tones, noise generation is greatly related to blade loading and thickness [36, 5]. The sound field created by the rotor will rotate at the same speed as the fan’s. In case of the rotor-stator interaction, noise generation is due to the interaction between wakes shed by the rotor blades and the stator vanes located
downstream. The sound field will also rotate, however, in this case the speed will be a function of the ratio given by the number of blades and stators.

**ii) Shaft order tones:** The two most important mechanisms here are the “Buzz saw” noise and the “sum and difference of tones”. The later is exclusively related to compressor noise. The former occurs when the blade tip reaches supersonic speed. Since Buzz-saw tones are generated by the fan tips, they should happen only at the BPF and harmonics, however due to non-uniformities in the blades this type of noise ends up being a function of the shaft frequency as well. Buzz saw tones only propagate forward into the inlet.

**iii) Broadband component:** is strictly related to the random process of turbulence [6].

![Diagram of noise sources in a modern high bypass ratio turbofan engine.](image)

**Figure 1.2:** Major noise sources in a modern high bypass ratio turbofan engine.

Figure 1.3 shows a typical power spectrum of a modern turbofan engine [33]. Power levels corresponding to the BPF tone and its harmonics are between 10.0 and 20.0 dB higher than the broadband component. Effective Perceived Noise Level or EPNL dB is the metric used to quantify noise from aircraft flyovers. So, even though BPF tone noise can be over 20.0 dB higher than the broadband component, it has been proven that, in terms of EPNL dB, controlling the
BPF tone alone does not produce a significant change in noise level. Thus, it is imperative to attenuate both BPF noise and broadband. Different approaches have been tried to reduce fan radiation. The next section presents some of these approaches.

![Typical noise spectrum of a modern turbofan engine](image)

**Figure 1.3:** Typical noise spectrum of a modern turbofan engine.

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### 1.2. Control of Noise Radiation from Turbofan Engines

Several techniques have been investigated to control noise radiation in turbofan engines. These methods can be divided into two main groups: i) control of the source and ii) attenuation of the noise field. Source control involves the modification of the noise generation mechanism directly. The second approach concentrates on attenuating noise already generated by the source along the propagation path. Among the different technologies in the first group, probably the most radical is the rotor-stator design. The key component responsible for tone noise generation is the rotor-stator interaction which is the impingement of the wake shed by the rotor blades onto the stator vanes. In order to weaken the effect of these wakes, stator vanes are leaned back and swept off the radial direction [7-10]. The separation between rotor and stator is also increased. This approach produces and outstanding attenuation but it has the disadvantage of generating
high stress concentrations at the vanes. Another approach consists of working with the rotor and stator blade count [7]. By properly selecting the number of blades, the cut-off of rotor-stator interaction tones can be achieved. However, this is a feasible solution only for the BPF tone. In order to cut-off the harmonics of the BPF, the number of blades and vanes would be impractical. Another effective method to attenuate tone noise is by trailing edge blowing. The idea is to reduce rotor-stator interaction by adding mass flow to the wake and therefore decrease the fan wake deficit. Such approach has been investigated among others by Leitch et al.[12], Sutliff et al.[13], Waitz et al. [14], Brookfield et al. [15], and Borgoltz et al. [17]. Unlike the previous technologies, this one has the capability to adjust to different flight conditions by changing the blowing rate.

Active noise control is one of the sound propagation path control methods proposed. The idea behind active noise control is to generate a sound field with the same intensity and out of phase as the one to be cancelled. Given the complexities associated with this technology, active noise control only concentrates on attenuating pure tones. Several methods have been investigated. Among these, arrays of resonating and electromagnetic drivers [7, 59], active airfoils [4, 16] and wake generators [11] can be cited. A potential disadvantage of these methods, if not properly applied, is “spill-over” which is the amplification of sound by the secondary sources. Due to the complexity and cost of active control methods, they have not been implemented in practice.

Inlet shaping is another noise control option investigated. This includes inlet duct cross section design to minimize noise radiation and scarf intakes, both of them with aerodynamic constrains imposed by the fan performance [18,19]. A scarf is basically an extension of the lower lip of the fan intake so that sound waves reflect upwards minimizing the amount of radiation received by an observer on the ground [20-23].

The most widely used noise attenuation technology is that of absorbent materials, i.e. liners. As shown in figure 4.1, a liner is a passive attenuation device formed by a back plate and a perforated screen separated by a resonating cavity. This resonating cavity is divided into several compartments by a honeycomb. Liners as just described provide good attenuation over a narrow frequency band defined, mainly by the core depth, i.e. honeycomb depth. In order to
1. Introduction

increase the frequency range, a high resistance wire mesh is generally bonded to the perforated screen. This wire mesh adds damping to the resonating cavity reducing the amount of attenuation provided by the liner at its tuning frequency but increasing the liner effective frequency range. Noise attenuation due to liners is directly proportional to the liner surface. In 1960, liners started to be used in inlet sections of turbofan engines. With the arrival of modern HBP turbofan engines and their shorter inlets, liner effectiveness was adversely affected due to the reduction of effective surface treatment. In addition, the large diameter of the fan lowers the BPF tone to frequencies outside of the liner most effective frequency range. As already mentioned EPNL dB reduction depends on the attenuation of both broadband and BPF component, hence the need to enhance the liner performance at lower frequencies.

Recent investigations by Hallez et. al. [25, 26] and Smith et. al. [27, 28] suggested the application of Herschel/Quincke (HQ) tubes for pure tone reduction in inlet sections of turbofan engines. An HQ tube is basically a resonator device that reduces noise by both reflecting [29] and scattering [24] energy. In his work, Hallez implemented a circumferential array of HQ tubes to reduce noise in a hard-walled circular duct. Results were then extended to control noise radiation from turbofan engine inlets as indicated in Figure 1.5. Initially HQ resonators were utilized to attenuate plane waves only [30, 31, 32]. In Hallez’ work HQ tubes were implemented in a three dimensional sound field defined by the spinning higher order modes present at the BPF.
In an attempt to improve the liner performance at low frequencies Alonso [33, 34], investigated the performance of the HQ tubes combined with a state-of-the-art single degree of freedom liner. To this end, an inlet HQ-Liner system was designed and tested at the Rolls Royce 649 turbofan inlet rig simulator [33, 34]. Through these experiments, it was successfully shown the potential of combining HQ tubes with liners. HQ tubes provided as much as 2.0 dB over the liner at the BPF tone and around ~1.0 dB to overall broadband attenuation.

Since a comprehensive approach for engine noise reduction will involve both inlet and aft HQ-Liner systems, additional research efforts were needed to evaluate their performance at controlling aft fan radiation. Thus, one of the goals of this dissertation is to extend the application of HQ-Liner systems to the control of aft fan noise.

In order to gain a better understanding of noise cancellation mechanisms taking place in aft ducts due to HQ-Liner systems, an analytical tool is needed. It is, therefore, the another objective of this dissertation, to develop a model to predict noise attenuation in aft sections due to HQ-Liner combinations.
1. Introduction

The next section presents a literature review concerning the modeling of propagation and attenuation of noise inside ducts.

1.3. Previous Work on Modeling of Sound Propagation and Attenuation in Lined Ducts

Among the first attempts to model sound propagation and attenuation in ducts was Morse’s [35] who predicted the reduction of a sound field propagating down a lined rectangular duct without flow. However, it was Tyler and Sofrin [36] in 1961 that made a significant step forward by giving a clear mathematical and physical explanation of the sound propagation phenomena taking place in circular and annular ducts. It was Tyler’s and Sofrin’s milestone research the one that provided duct acoustic research with the basis for further significant development. One of the most common duct sections investigated was the rectangular one. Among different research efforts, there is Ko [37] who in 1971 modeled sound attenuation in lined rectangular ducts with mean uniform axial flow. Several other researchers such Mariano [38], Unruh and Eversman [39], Hersh and Catton [40] and Tester [41] investigated the effects of shear flow on sound attenuation. A mean flow velocity gradient such as the one encountered in shear flow produces refraction of the sound waves. According to Yurkovich who investigated the problem in circular [42] and annular ducts [43], shear flows have the effect of degrading the attenuating capabilities of a liner. Additional work on circular and annular ducts was carried out by Eversman [44] and Ko [45, 58] that studied the attenuation of sound convected with uniform and shear flow in circular ducts. In case of annular ducts, previous analytical work was done by Benzakein [46] and Scott [47] who investigated the case of sound attenuation in the presence of a liner without flow. At the same time, Munger and Plumblee [48] and Shankar [49] modeled the case of attenuation with shear flow.

In general, the above mentioned work makes certain assumptions intended to simplify the mathematical development. Such assumptions consisted of a duct with a constant cross section area, locally reactive acoustic lining, and constant wall acoustic properties. In a real turbofan engine such conditions may not be valid. Thus, several authors analyzed the sound propagation
and attenuation phenomena in ducts without restricting themselves to those assumptions. Among them is Scott [50] who investigated the case of sound propagation in a duct with a non-locally reacting liner. Vaidya [51] solved the acoustic equations in ducts lined with a non-locally reacting and non-uniform absorbent material. In 1977 Eisenberg [52] analyzed the effect of a varying cross section. Nayfeh et al. [53] combined the effect of shear flow and annular varying cross sections. In 1998 Rienstra went one step further by combining variations in the cross section, mean flow and wall impedance [54]. This work is particularly useful when analyzing sound propagation in inlet sections of turbofan engines and the presence of the rotor spinner needs to be accounted for.

The effects of swirling flow in sound transmission have been recently studied by Golubev and Atassi [55]. They concluded that acoustic wave propagation is dominated by convection and refraction.

In general after Tyler and Sofrin [36], the sound field has been represented as a superposition of modes. In order to find the mode shapes the eigen-values of the characteristic equation has to be solved. This characteristic equation is different for each problem and it is strictly related to the problem’s boundary conditions. Finding the eigen-values in a lined duct is a task that can be particularly challenging, especially when sound propagation in annular ducts is being investigated. The reason why finding the eigen-values can be extremely complicated is because the characteristic equation does not have a closed form solution and the eigen-values are complex. In an attempt to overcome some of the issues related to solving the characteristic equation Alonso [33] proposed the use of the simplex method [56]. This simplex method is also utilized in the present dissertation.

As mentioned in the previous section, recent investigations suggested the application of HQ tubes for pure tone reduction in inlet sections of turbofan engines. In the presence of HQ tubes, the sound field inside the duct can be represented as the superposition of the disturbance and the effect of HQ tubes. This effect has been modeled as piston sources vibrating with unknown velocities. This assumption limits the model capabilities to frequencies below the tube first resonance. The coupling between the two systems is performed by matching pressure and particle velocity at the HQ tube/duct interface. This coupling results in a linear system of
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equations that are solved for the piston velocities which in turn are used to uniquely define the sound field in the system.

Such analytical model has been implemented by Alonso in lined circular inlets [33]. Since the aft model is an extension of the inlet model, the main tasks to accomplish this objective are:

• Modify the inlet HQ-liner model to account for the boundary condition imposed by the annular duct inner wall.
• Make the proper modifications so that the effect of HQ tubes placed on the inner wall can be estimated.
• Use experimental data to validate the new model.

1.4. Dissertation Objectives

This section describes the main objectives and contributions of the present research effort. As already mentioned, the need for a comprehensive approach dictates the study of HQ-liner systems in aft sections. The difficulties found in previous works suggested the need for investigating aft HQ-liner systems in a much simpler and well controlled experimental setup. As a consequence, the NASA Active Noise Control Fan or ANCF rig [7, 57], which features an aft annular cross section, was selected as the test facility in this research effort. Thus, the first objective of this dissertation is the investigation of the performance of an HQ-Liner system in a well controlled laboratory setup, e.g. aft section of the ANCF NASA rig.

To facilitate the design of HQ-Liner systems and in order to have an analytical tool to help understand the physics behind annular duct noise attenuation, a mathematical model was needed. The second goal of this dissertation is the development and validation of a model that can predict noise attenuation due to HQ-liner systems in annular ducts.

Due to lack of data, the inlet HQ-liner model developed by Alonso was only been validated on attenuation of the broadband component. Therefore the third objective of this research effort consists of validating the inlet model for tone reduction.
Herschel-Quincke tubes are resonator devices. This means that damping, as the one introduced by the liner wire mesh, can potentially degrade their performance. As a consequence, it is another objective of the present work to assess the influence of the liner wire mesh on the attenuation capabilities of HQ tubes when combined with liners.

The NASA ANCF rig provided the well controlled setup needed for technology assessment and model validation. However, the environment in the ANCF rig isn’t representative of real turbofan engine. Significantly larger Mach numbers, temperatures, noise levels, and hub-to-tip ratios in modern turbofan engines are characteristics that may affect the performance of the HQ-liner systems. Therefore, another goal is to investigate the performance of an HQ-Liner system in a realistic turbofan engine environment.

Both aft and inlet analytical models have represented the effect of the HQ tubes as that of a single vibrating piston assuming that only plane waves were present inside the tubes. However, the single-piston/plane-wave approach fails when the frequency range is high enough that higher order modes are present inside the HQ-tube. In order to overcome this limitation, the last goal of this dissertation is the modeling of HQ tubes including the effect of higher order modes.

The next section describes the organization of the dissertation.

1.5. Dissertation Organization

This dissertation is divided in seven chapters and five appendices as illustrated in Figure 1.6. The first chapter is the introduction. In Chapter 2, the necessary modifications to extend the circular model to annular ducts are presented. The formulation to include the effects of higher order modes inside the tubes is also derived here. Finally, the last section of Chapter 2 presents the modeling of transmissions and reflections effects at the ends of the liner, e.g. modeling of the sudden change in wall impedance at the hard wall/liner interfaces.

Chapter 3 analyzes data collected at the NASA ANCF rig. This data was utilized not only to address the feasibility of aft HQ-Liner systems but also to validate the model developed in Chapter 2. A complete data set was collected on both inlet and aft systems. Inlet systems also featured two HQ-Liners spool pieces placed one after the other. This configuration will be referred to as “multiple HQ-Liner spool piece”.
In chapter 4 the validation of the aft and inlet HQ-liner models is carried out. Predictions on the “multiple spool piece configuration” are also included. This configuration was located right next to the fan, where the presence of the spinner makes the duct cross section transition from annular to circular half way through the first spool piece. Two different approaches with different level of success were used in order to predict noise attenuation due to this configuration working in such environment.

Flow conditions in the ANCF rig and flow conditions in a real turbofan engine are not the same. Larger Mach numbers, flow temperatures and hub-to-tip ratios impose different conditions that may affect the performance of the systems. Thus, in chapter 5, the new aft model is utilized to design an HQ-Liner for a modern turbofan engine and to analyze its performance.

Finally, Chapter 6 presents the most important conclusions of this dissertation along with a few recommendations for future work.
Figure 1.6: Dissertation diagram.
Chapter 2. Mathematical Models

2. MATHEMATICAL MODELS

Chapter 2 presents the mathematical derivations that are the cornerstone of the present research effort. This chapter is organized as follows; Section 2.1 concentrates on the model that describes a sound pressure field propagating in an annular duct. A similar model has already been developed for circular ducts and explained in detail in [33]. Since the derivation is similar, only those modifications necessary to extend the circular model to annular ducts will be included here. As shown in [33], the HQ tube model is limited to frequencies below the first higher order mode is cut-on, i.e. only plane waves are assumed present inside the tubes. In order to overcome the limitation, an alternative model that handles higher order modes has been developed and it is presented in Section 2.2. Finally, section 2.3 shows the mathematical formulation that describes the influence on the sound field of a sudden change of wall impedance.

2.1. Model for HQ-Liners Installed in Annular Ducts

In this section, the model for the HQ devices in conjunction with liners is developed for the aft duct of turbofan engines. The model is based on considering annular constant cross sectioned ducts. As seen in Figure 2.1a, the last \( \sim 20 \text{in} \) of the aft duct of the NASA Glenn ANCF fan rig is exactly an annular duct of constant cross section and thus it is ideal to validate this model. It is important to note that real turbofan engines often have the bypass duct bifurcated as shown in Figure 2.1b. The bifurcations render the aft duct into two c-duct sections. However, the early stage of this work called for a much simpler environment than that of a real turbofan engine, therefore, this geometry will not be considered in this dissertation.

The mathematical model for inlet HQ-liner systems has already been described in detail by Alonso [33]. Since the general approach is similar for inlet and aft ducts, this section will concentrate mainly on stating the differences and modifications necessary to extend the circular duct model to annular ducts. To this end, section 0 describes the general modeling approach while section 2.1.2 focuses on the new solution to the Helmholtz’s equation and the associated
set of boundary conditions that accounts for the presence of the annular duct inner wall. Section 2.1.3 concentrates on the computation of the sound field propagating through the duct, i.e. due to the disturbance and the effect of the HQ tubes. Once the sound field has been defined, the performance of the HQ-liner system is measured in terms of power attenuation which is shown in section 2.1.4. The formulation to calculate power is the same for both inlet and aft models; however it will be included in this chapter only for the sake of completeness.

Figure 2.1: (a) Schematic of the NASA Glenn ANCF rig showing an HQ-liner system placed on the last 16” of the aft sector and (b) Aft section of a real turbofan engine. Note the splitters dividing the annular duct into two C-sectioned ducts (http://www.aerospaceweb.org/question/conspiracy/q0265.shtml).

2.1.1 General Modeling Approach

The sound energy generated by the fan propagates towards the duct opening. As the sound reaches the exit plane, the sudden change in cross section makes part of the energy to be reflected back towards the fan. Like in the inlet case, these reflections are neglected. Since any potential reflection off the fan is also neglected, ducts are modeled as an infinite duct. Furthermore, since in large turbofan engines the cross section changes smoothly, thus minimizing reflection effects, the cross sections are assumed to be constant. Figure 2.2a and b illustrate the geometries used to solve for the propagating sound field in the inlet and aft ducts, respectively.
In addition, the flow is considered uniform and non viscous whereas the wall lining is assumed to be locally reactive. In the case of the aft section, acoustic treatment can be applied on both the inner and the outer walls (inner treatment not shown in the figures). Moreover, liners and HQ-tubes on the inner and outer walls may have different properties.

**Figure 2.2:** Infinite constant cross section lined duct models for (a) inlet and (b) aft ducts.

The modeling approach is depicted in Figure 2.3. The entire sound field can be thought as the result of two acoustic systems acting together. The first system is given by the sound field inside the duct whereas the second is given by the sound field inside the tubes (see Figure 2.3b). The effect of the HQ-waveguides on the duct sound field is modeled as piston sources vibrating with unknown velocities. This assumption limits the model capabilities to frequencies below the tube first higher order mode, i.e. only plane waves are present inside the tubes. The coupling between the two systems is performed by matching pressure and particle velocity at the HQ tube/duct interface. This coupling results in a linear system of equations that are solved for the piston velocities which in turn are used to uniquely define the sound field in the system. The first step in this direction is given by solving the eigen-value problem. Once the eigen-values have been found and since the problem is linear, both the disturbance and the sound field generated by the piston sources can be expressed as a superposition of acoustic modes [26]. The solution to the eigen-value problem is described in the next section.
2.1.2 The eigen-value problem

This section describes the solution to the eigen-value problem. Once the eigen-values have been found, the sound field propagating through an annular duct can be defined.
Figure 2.4 shows an infinite annular duct along with the cylindrical coordinate system used in the derivations, e.g. \( \vec{r}(r, \theta, z) \). Outer and inner wall radius are defined by \( a \) and \( b \), respectively. The walls are assumed to be treated with locally reactive liners where \( \beta_{wa} \) and \( \beta_{wb} \) are normalized liner admittances for the outer and inner walls respectively. Note that \( \beta_{wa} \) and \( \beta_{wb} \) do not have to be the same.

\[\text{Figure 2.4:} \text{ Schematic of an annular duct with liners on the outer and inner walls.}\]

The homogeneous acoustic wave equation that describes a convected sound field in a moving media with uniform, non-viscous flow and assuming harmonic motion in cylindrical coordinates is given as [63]

\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} = -k_0^2 p + 2iMk_0 \frac{\partial p}{\partial z} + M^2 \frac{\partial^2 p}{\partial z^2}
\]

where \( p(\vec{r}, t) \) is the pressure and it is a function of the position \( \vec{r}(r, \theta, z) \) and time \( t \), \( M \) is the mean flow Mach number, and \( k_0 \) is the acoustic free field wave-number. Note that the flow in this equation is assumed to be in the \( z \)-direction.

Euler’s equation in the radial, circumferential, and axial directions are given as

\[
-\frac{\partial p}{\partial r} = i\omega p r + \rho c M \frac{\partial v_r}{\partial z}
\]
\[ -\frac{1}{r} \frac{\partial p}{\partial \theta} = i \omega \rho v_\theta + \rho c M \frac{\partial v_\theta}{\partial z} \quad 2.3 \]

\[ -\frac{\partial p}{\partial z} = i \omega \rho v_z + \rho c M \frac{\partial v_z}{\partial z} \quad 2.4 \]

where \( \vec{v}(r,t) \) is the particle velocity vector whose components are \( (v_r, v_\theta, v_z) \). These equations will be used when deriving an expression for the boundary conditions.

The assumed solution to equation 2.1 is given by

\[ p(r,\theta,z,t) = \Phi(r,\theta)e^{-ik_zz}e^{iot} = R(r)\Theta(\theta)e^{-ik_zz}e^{iot} \quad 2.5 \]

In this equation, \( R(r) \) represents pressure distribution in the radial direction and \( \Theta(\theta) \) pressure distribution in the circumferential direction. This function satisfies the periodic condition \( \Theta(\theta) = \Theta(\theta + 2m^* \pi) \) with \( m^* = 0,1,2,\ldots \). The function \( e^{-ik_zz} \) represents the pressure axial variation, i.e. \( k_z \) is the axial wavenumber of the sound field. The last term is the time term and it will be omitted from the rest of the derivations.

Replacing solution 2.5 into equation 2.1 and reordering leads to

\[ \frac{r^2}{R} \left( \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) + \frac{1}{\Theta} \frac{d^2 \Theta}{d\theta^2} + r^2 k_{mn}^2 = 0 \quad 2.6 \]

where

\[ k_{mn}^2 = k_0^2 - k_z^2 (1 - M^2) - 2k_\theta k_z M \quad 2.7 \]

with \( k_{mn} \) been the eigen-wavenumber (or eigen-value).

The solution of the partial differential equation in 2.6 in the circumferential direction is given by

\[ \Theta(\theta) = \cos(m\theta + \alpha) \quad 2.8 \]

for stationary modes or by

\[ \Theta(\theta) = e^{im\theta} + e^{-im\theta} \quad 2.9 \]

for rotating modes, with \( m = 0,1,2,3,\ldots \).

In the radial direction, the solution is represented by

\[ R(r) = C_{mn}J_m(k_{mn}r) + D_{mn}Y_m(k_{mn}r) \quad 2.10 \]
where \( J_m(.) \) and \( Y_m(.) \) are the Bessel’s function of the first and second kind, respectively; and \( C_{mn} \) and \( D_{mn} \) are unknown constants to be determined from the boundary conditions.

As compared to the circular duct case, the presence of the inner wall in an annular duct imposes an additional boundary condition which significantly complicates the finding of the eigen-values. The approach to obtain an expression for the boundary conditions has been thoroughly explained by Alonso [33], however and for completeness it will be briefly repeated next.

The first step consists of assuming a radial velocity distribution similar to that of the pressure, i.e. \( v_r = v(r)\Theta(\theta)e^{-ikz}e^{i\omega t} \). The second step implies matching particle displacement at the liner surface, as explained by Ko [37, 58]. According to Ko, the presence of flow introduces a plane vortex sheet located an infinitely small distance off the liner within which the mean axial velocity is zero (see Figure 2.5). Since outside the vortex sheet the particle is convected with the flow there is a discontinuity in the radial component of the particle velocity that needs to be accounted for. Matching particle displacement across the vortex sheet leads to

\[
\frac{v'}{i\omega} = \frac{v}{i\omega(k_0 - Mk_z')/k_0}
\]

2.11

In this expression \( v' \) is the particle velocity at the wall on the liner side and \( v \) is the particle velocity on the duct side (see Figure 2.5). Note that when \( M = 0, v' = v \).

Figure 2.5: Schematic of particle velocity affected by the presence of shear flow.
Since the particle velocity inside the liner and within the vortex sheet is the same [37, 58], the particle velocity on the liner side $v'$ in 2.11 can be expressed in terms of the pressure and the liner locally reactive specific admittance, e.g. $\beta_{wa}$ or $\beta_{wb}$ depending on the surface being considered. This leads to an expression for the particle velocity on the duct side

$$v(b, \theta) = -\frac{\beta_{wa}}{\rho c} \left( \frac{k_0 - k_z M}{k_0} \right) p(b, \theta)$$ 2.12

for the outer wall ($r = a$), and

$$v(b, \theta) = \frac{\beta_{wb}}{\rho c} \left( \frac{k_0 - k_z M}{k_0} \right) p(b, \theta)$$ 2.13

for the inner wall, i.e. $r = b$. Note that equation 2.13 is only valid for the annular duct case. By substituting 2.12 and 2.13 into the radial component of Euler’s equation and replacing pressure $p$ by its assumed solution the following set of boundary conditions are obtained for the circular and annular ducts

<table>
<thead>
<tr>
<th>Circular Duct (Inlet)</th>
<th>Annular Duct (Aft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$- \frac{\partial R}{\partial n} = \frac{\partial R}{\partial r} \bigg</td>
<td><em>{r=a} = -i\beta</em>{wa} \left( \frac{k_0 - k_z M}{k_0} \right)^2 R \bigg</td>
</tr>
<tr>
<td>Solution bounded at $r=0$</td>
<td>$\frac{\partial R}{\partial n} = \frac{\partial R}{\partial r} \bigg</td>
</tr>
<tr>
<td>2.14</td>
<td>2.15</td>
</tr>
<tr>
<td>2.16</td>
<td></td>
</tr>
</tbody>
</table>

where $n$ is the direction perpendicular to the duct walls, defined positive into the duct. Since the radial direction is defined positive outwards, derivation in the normal and radial directions have the same sign along the inner wall and different signs along the outer wall, hence, the sign difference observed in equations 2.15 and 2.16. If any wall is not acoustically treated then the admittance is set to zero and the right hand side of the equations above vanishes given the boundary conditions for the hard wall case.
The characteristic equation that allows solving for the mode eigen-wavenumber, \( k_{mn} \), is obtained by replacing the solution \( R(r) \) from equation 2.10 into the boundary conditions in 2.14 (for circular ducts) or in 2.15 and 2.16 (for annular ducts). For the particular case of the inlet, the Bessel function of the second kind, \( Y_m(.) \), has a singularity at \( r = 0 \) and it requires to set \( D_{mn} = 0 \). For the annular duct case, the characteristic equation is given by the determinant of a homogeneous system of two equations with three unknowns, i.e. \( k_{mn} \), \( C_{mn} \) and \( D_{mn} \). Thus the characteristic equation for both cases is given by

\[
\begin{align*}
\text{Circular Duct (Inlet)} & \quad \text{Annular Duct (Aft)} \\
\frac{i\beta_{wa}}{k_0} (k_0 - Mk_z)^2 J_m(k_{mn}a) = -k_{mn} J'_m(k_{mn}a) & \quad a_{11}a_{22} - a_{21}a_{12} = 0 \quad (2.17) & \quad (2.18)
\end{align*}
\]

where

\[
\begin{align*}
a_{11} &= k_{mn} J'_m(ak_{mn}) + B_{za} J_m(ak_{mn}) \\
a_{12} &= k_{mn} Y'_m(ak_{mn}) + B_{za} Y_m(ak_{mn}) \\
a_{21} &= -k_{mn} J'_m(bak_{mn}) + B_{zb} J_m(bak_{mn}) \\
a_{22} &= -k_{mn} Y'_m(bak_{mn}) + B_{zb} Y_m(bak_{mn})
\end{align*}
\]

with

\[
\begin{align*}
B_{za} &= i\beta_{wa} \frac{(k_0 - Mk_z)^2}{k_0} \\
B_{zb} &= i\beta_{wb} \frac{(k_0 - Mk_z)^2}{k_0} \quad (2.20)
\end{align*}
\]

The relative value between the constants \( C_{mn} \) and \( D_{mn} \), i.e. eigenvectors, for the \( mn^{th} \) mode is obtained as \( (D/C)_{mn} = -a_{11}/a_{12} \).

In the presence of a liner, the solution to the eigen-problem is a function of the axial wavenumber, \( k_z \), which in turn is a function of \( k_{mn} \) through equation 2.7. This fact implies that the eigenvalue solution depends on the direction of propagation of the modes, and therefore the
Chapter 2. Mathematical Models

characteristic equation has to be solved for the positive and negative propagating waves independently. Thus, the positive and negative propagating mode axial wavenumber are given by

\[
k_{z}^{(+)} = -k_{0}M + \frac{\sqrt{k_{0}^{2} - (1 - M^2)(k_{mn}^{(+)})^2}}{(1 - M^2)}
\]

\[
k_{z}^{(-)} = -k_{0}M - \frac{\sqrt{k_{0}^{2} - (1 - M^2)(k_{mn}^{(-)})^2}}{(1 - M^2)}
\]

Since the characteristic equation does not have a close form solution, a numeric root finding approach has been used. The approach used is the simplex method [56], which allows finding the minimum of a multivariable real function without taking derivatives. In this end, the absolute value of equations 2.17 and 2.18 is calculated so that the roots of the original characteristic equations will be found now as the minima of a real valued function. The numeric approach requires an initial guess for the roots \( k_{mn}^{(+)} \) and \( k_{mn}^{(-)} \). Since at low frequencies the liner admittance goes to zero for all liners, resembling a hard wall case, this initial guess is the solution to the hard wall problem. For the case of the circular duct, the hard wall eigen-wavenumbers are tabulated. For an annular duct, the eigen-value problem has to be solved for every particular hub-to-tip ratio \( b/a \). They are found numerically by solving for the roots of the following expression

\[
\begin{vmatrix}
    k_{mn}J_{m}(ak_{mn}) & k_{mn}Y_{m}(ak_{mn}) \\
    k_{mn}J_{m}(bk_{mn}) & k_{mn}Y_{m}(bk_{mn})
\end{vmatrix} = 0
\]

Since the eigen-values for the hard wall case are all real, a standard bisection approach is utilized. Note that equation 2.22 can be obtained from 2.18 by setting \( \beta_{wa} \) and \( \beta_{wb} \) to zero.

Once the eigen-wave-numbers \( k_{mn}^{(+)} \) and \( k_{mn}^{(-)} \) have been found, the associated mode-eigen-functions can be obtained as
for positive (+) and negative (-) propagating waves in circular and annular ducts, respectively.

At this point the sound field in the duct can be written in terms of these eigen-functions or mode shapes. The next section presents the expressions defining the sound field propagating in the duct due to both, the disturbance and the piston sources accounting for the effect of the HQ tubes.

### 2.1.3 Duct sound field

The sound field propagating through a duct, can be written as

\[
p(\vec{r}) = p_d(\vec{r}) + P(\vec{r})
\]

where \( p_d(\vec{r}) \) is the pressure at a certain point \( \vec{r}(r, \theta, z) \) due to the disturbance and \( P(\vec{r}) \) is the pressure at the same point due to the piston sources accounting for the effect of the HQ tubes.

The sound field due to the disturbance can be written as a linear combination of positive and negative spinning modes propagating in the positive \( z \)-direction. That is

\[
p_d(\vec{r}) = \sum_j \sum_m \left( A_{mn}^{(+)\,d} \Phi_{mn}^{(+)\,d}(r, \theta) \right)^{\text{pos}} e^{-ik^{(+)\,d}z} e^{i\alpha z} + \sum_j \sum_m \left( A_{mn}^{(+)\,d} \Phi_{mn}^{(+)\,d}(r, \theta) \right)^{\text{neg}} e^{-ik^{(+)\,d}z} e^{i\alpha z}
\]

where superscripts “\text{pos}” and “\text{neg}” indicate the direction of rotation of the disturbance modes; \( A_{mn}^{(+)\,d} \) and \( A_{mn}^{(+)\,d} \) are the known modal amplitudes for the positive and negative rotating modes; and \( \Phi_{mn}^{(+)\,d}(r, \theta) \) takes the form of equations 2.23a and 2.24b for circular and annular ducts respectively. In these equations, the circumferential variation \( \Theta(\theta) \) of the modes is given by \( e^{-im\theta} \) for positive rotating modes and by \( e^{+im\theta} \) for negative rotating ones.
The effect of the HQ tubes is modeled as radiating pistons located at the HQ tube/duct interface. Provided that the higher order modes are cut-off inside the tubes, the openings can be represented by a single piston vibrating with a certain velocity $V_n$ (one degree of freedom model - 1DOF). Therefore, the sound field due to a piston source is expressed as

$$p^{(+)}(r, \theta, z|\rho_n, \theta_n, z_n) = -i\rho c \sum_{m} \sum_{n} \frac{(k_0 - k_z^{(+)} M)^2}{k_0} V_n \int \int \mathcal{G}_{m,n}^{(+)}(r, \theta, z|\rho_n, \theta_n, z_n) r \, d\theta \, dz$$

$$p^{(-)}(r, \theta, z|\rho_n, \theta_n, z_n) = -i\rho c \sum_{m} \sum_{n} \frac{(k_0 - k_z^{(-)} M)^2}{k_0} V_n \int \int \mathcal{G}_{m,n}^{(-)}(r, \theta, z|\rho_n, \theta_n, z_n) r \, d\theta \, dz$$

where $\mathcal{G}_{m,n}$ is the Green’s function and it takes the form of

$$\mathcal{G}_{m,n}^{(+)}(\rho, \theta, z|\rho_0, \theta_0, z_0) = A_{m,n}^{(+)} \Phi_{m,n}^{(+)} e^{-k_0^{(+)} (z-z_0)}$$

$$\mathcal{G}_{m,n}^{(-)}(\rho, \theta, z|\rho_0, \theta_0, z_0) = A_{m,n}^{(-)} \Phi_{m,n}^{(-)} e^{-k_0^{(-)} (z-z_0)}$$

for positive and negative propagating waves, respectively. The limits of integration in 2.27 and 2.28 are given by the size of the source as indicated in Figure 2.6. The problem with this formulation is that neither the amplitudes of the Green’s functions, $A_{m,n}^{(+)}$ and $A_{m,n}^{(-)}$, nor the velocities $V_n$ of the vibrating pistons are known. The process of finding these amplitudes as well as the piston velocities has been developed by Alonso [33] for the circular duct case. In the case of an annular duct, the presence of the inner most wall calls for additional mathematical manipulations. The extension of this formulation is a contribution of the present work and is presented in the next two sections.
Figure 2.6: Schematic of the HQ tube/duct interface modeled as a square-sectioned vibrating piston.

2.1.3.1 Amplitudes of the Green’s functions

Equation

\[ \frac{\partial^2 g}{\partial r^2} + \frac{1}{r} \frac{\partial g}{\partial r} + \frac{1}{r^2} \frac{\partial^2 g}{\partial \theta^2} + \frac{\partial^2 g}{\partial z^2} + k_0^2 g - 2iMk_0 \frac{\partial g}{\partial z} - M^2 \frac{\partial^2 g}{\partial z^2} = \delta(r-r_0)\delta(\theta-\theta_0)\delta(z-z_0) \]

2.31

This point source radiates sound in both positive and negative directions. The solution to equation

is expressed in terms of the Green’s function as

\[ g(r|\vec{r}_0) = g^{(+)}(r|\vec{r}_0)H(z-z_0) + g^{(-)}(r|\vec{r}_0)\left[1 - H(z-z_0)\right] \]

2.32

where \( H(z-z_0) \) is the Heaviside function and its values are \( H(z-z_0) = 1 \) if \( z > z_0 \), \( H(z-z_0) = 0 \) if \( z < z_0 \) and \( H(z-z_0) = 1/2 \) if \( z = z_0 \); \( g^{(+)}(r|\vec{r}_0) \) and \( g^{(-)}(r|\vec{r}_0) \) are the Green’s functions for positive and negative propagating waves and they can be written as (same as equation 2.29 and 2.30).
\[
\begin{align*}
\mathcal{g}^{(+)}(\bar{r}|z_0) &= \sum_{m_g=0}^{M_g} \sum_{n_g=0}^{N_g} A_{m_g,n_g}^{(+)} \Phi_{m_g,n_g}^{(+)} e^{-k_z^{(+)}(z-z_0)} \\
\mathcal{g}^{(-)}(\bar{r}|z_0) &= \sum_{m_g=0}^{M_g} \sum_{n_g=0}^{N_g} A_{m_g,n_g}^{(-)} \Phi_{m_g,n_g}^{(-)} e^{-k_z^{(-)}(z-z_0)}
\end{align*}
\]

As shown, there are \((N_g+1) \times (M_g+1)\) unknown \(A_{m_g,n_g}^{(+)}\) and \((N_g+1) \times (M_g+1)\) unknown \(A_{m_g,n_g}^{(-)}\). In order to find the necessary equations to obtain the amplitudes, the first step consists of replacing equation 2.32 into equation 2.31. The resulting equation should be multiplied by \(\Phi_{\varphi_0} = \Phi_{\varphi_0}^{(+)} H(z-z_0) + \Phi_{\varphi_0}^{(-)} [1-H(z-z_0)]\) and then integrated over a small volume and letting the axial dimension of the volume to vanish. The reason to integrate over this small volume is to remove the Kronecker’s delta on the right hand side in equation 2.31. In the presence of a liner, the orthogonality condition of the eigenfunctions or modes is only satisfied in the circumferential direction, which means that the volume integral of the product \(\Phi_{\varphi_0} \Phi_{m_g,n_g}\) vanishes when \(\varphi \neq m_g\).

In other words, circumferential orders are decoupled and consequently can be solved independently from each other. This leads to the following expression

\[
\sum_{n_g=0}^{N_g} k_z^{(+)} A_{m_g,n_g}^{(+)} \int_0^{2\pi} \left( \frac{\Phi_{m_g,n_g}^{(+)} + \Phi_{m_g,n_g}^{(-)}}{2} \right) \Phi_{m_g,n_g}^{(+)} I m_{m_g} r d\varphi - k_z^{(-)} A_{m_g,n_g}^{(-)} \int_0^{2\pi} \left( \frac{\Phi_{m_g,n_g}^{(+)} + \Phi_{m_g,n_g}^{(-)}}{2} \right) \Phi_{m_g,n_g}^{(-)} r d\varphi = \frac{\Phi_{m_g,n_g}^{(+)}(r_0, \theta_0) + \Phi_{m_g,n_g}^{(-)}(r_0, \theta_0)}{2(1-M_g^2)}
\]

and \(m_g = 0, 1, 2, \ldots M_g\). Equation 2.34 unfolds into a system of \((N_g+1)\) equations with \(2(N_g+1)\) unknowns for every circumferential order \(m_g\). At this point an important remark should be made regarding the right hand side of equation 2.34. The mode shapes \(\Phi_{m_g,n_g}\) have the form \(\cos[m_g(\theta-\theta_0)] \left( J_{m_g}(k_{mn} r) + \frac{D}{C_{mn}} Y_{m_g}(k_{mn} r) \right)\), where \(\cos[m_g(\theta-\theta_0)]\) indicates that the sound field due to the point source is non-rotating. Equation 2.34 shows that the right hand side has to be evaluated at \((r_0, \theta_0)\) leading to \(\left( J_{m}(k_{mn} r_0) + \frac{D}{C_{mn}} Y_{m}(k_{mn} r_0) \right)\) where \(r_0\) take the value of \(a\) or \(b\) for the inner and outer duct walls, respectively. Therefore, the amplitudes of the
Green’s function in an annular duct have to be found independently for sources located on the outer and inner walls.

A second set of \((N_g + 1)\) equations can be obtained by applying continuity of the Green’s function, i.e. continuity of the sound field at the location of the source, i.e. \(g^{(+)} = g^{(-)}\).

Substituting equations 2.33 into the equation above, multiplying again by \(\Phi_{cr} = \Phi_{cr}^{(+)} H(z - z_0) + \Phi_{cr}^{(-)} [1 - H(z - z_0)]\) and integrating over the same small volume as before leads to

\[
\sum_{n_r=0}^{N_g} \int_0^a \int_0^b \left( \frac{\Phi^{(+)}_{m_r n_r} + \Phi^{(-)}_{m_r n_r}}{2} \right) \Phi^{(+)}_{m_r n_r} r dr dl - \int_0^a \int_0^b \left( \frac{\Phi^{(+)}_{m_r n_r} + \Phi^{(-)}_{m_r n_r}}{2} \right) \Phi^{(-)}_{m_r n_r} r dr dl = 0 \tag{2.35}
\]

Then, \(A^{(+)}_{m_r n_r}\) and \(A^{(-)}_{m_r n_r}\) are determined by solving an \(2(N_g + 1) \times 2(N_g + 1)\) linear system of equations, where \((N_g + 1)\) is the total number of radial modes used in the Green’s functions. As already mentioned, this linear system has to be solved for every circumferential order mode and will be a function of the radial location of the source. The mode shapes \(\Phi^{(+)}_{mn}\) and \(\Phi^{(-)}_{mn}\) are obtained using equations 2.23 and 2.24 for circular and annular ducts respectively. In this particular case, \(\Theta(\theta) = \cos(m(\theta - \theta_o))\), meaning that the sound field generated by the point source located at \(\theta_o\) is symmetric around \(\theta_o\) and it is non-spinning.

A key step in order to get the amplitudes of the Green’s functions is to solve the integrals

\[
\int_0^a \int_0^b \left( \frac{\Phi^{(+)}_{m_r n_r} + \Phi^{(-)}_{m_r n_r}}{2} \right) \Phi^{(+)}_{mn} r dr dl \quad \text{(for positive propagating waves)}
\]

\[
\int_0^a \int_0^b \left( \frac{\Phi^{(+)}_{m_r n_r} + \Phi^{(-)}_{m_r n_r}}{2} \right) \Phi^{(-)}_{mn} r dr dl \quad \text{(for negative propagating waves)}
\]

By considering equations 2.24 it will be noticed that the solution of the integrals in the circumferential direction for both positive and negative propagating waves is given by

\[
\int_0^{2\pi} \cos[m(\theta - \theta_o)] d\theta = \begin{cases} 2\pi; m = 0 \\ \pi; m \neq 0 \end{cases}
\]

The solution in the radial direction, i.e. the orthogonalization factor gets significantly more complicated due to the presence of the inner wall.
**Orthogonalization factors**

The derivation will concentrate only on the orthogonalization factor for the positive propagating waves. The negative propagating wave case is very similar. By considering equation 2.24, the solution to the integral in the radial direction for positive propagating waves is given by

\[
\Lambda_{m,n}^{(+)ar} = \frac{1}{2} \int_a^b (H_{mr}^{(+)} + H_{mr}^{(-)}) H_{mn}^{(+)r} dr
\]

In case of the negative propagating sound field, the derivation is the same provided \(H_{mn}^{(+)}\) is changed to \(H_{mn}^{(-)}\), where \(H_{mn} = J_m(k_{mn} r) + \left(\frac{D}{C}\right)_{mn} Y_m(k_{mn} r)\).

Expanding 2.36 gives

\[
\Lambda_{m,n}^{(+)ar} = \frac{1}{2} \left( \hat{\Lambda}_{m,n}^{(+)ar} + \Lambda_{m,n}^{(-)ar} \right) = \frac{1}{2} \left\{ \int_b^a H_{mr}^{(+)} H_{mn}^{(+)r} dr + \int_b^a H_{mr}^{(-)} H_{mn}^{(+)r} dr \right\}
\]

Replacing equations 2.24 in the first term of the expression above leads to

\[
\hat{\Lambda}_{m,n}^{(+)ar} = \int_b^a \left( J_m(k_{mn} r) + \left(\frac{D}{C}\right)_{mn} Y_m(k_{mn} r) \right) \left( J_m(k_{mr} r) + \left(\frac{D}{C}\right)_{mn} Y_m(k_{mr} r) \right) r dr
\]

Expanding the equation above gives

\[
\int_b^a \left[ J_m(k_{mn} r) J_m(k_{mn} r) \right] dr + \left(\frac{D}{C}\right)_{mn} \int_b^a \left[ J_m(k_{mn} r) Y_m(k_{mn} r) \right] dr + \left(\frac{D}{C}\right)_{mn} \int_b^a \left[ Y_m(k_{mn} r) Y_m(k_{mn} r) \right] dr
\]

At this point, there are two possibilities for the solution of the integral depending on the values of the Bessel functions: either \(k_{mn} r \neq k_{mr} r\) or \(k_{mn} r = k_{mr} r\).

1. **Case 1: \(k_{mn} \neq k_{mr}\)**

   If the arguments of the Bessel function are not the same, then the solution to the first term in 2.39 is given by
The second integral in 2.39 is given by

\[
\frac{1}{(-ik_{mn})^2 + (k_{mr})^2} \left\{ ak_{mn} \left[ J_{m-1}(ak_{mn}) J_m(bk_{mn}) - \left( \frac{b}{a} \right) J_{m-1}(bk_{mn}) J_m(bk_{mn}) \right] + ak_{mr} \left[ -J_{m-1}(ak_{mr}) J_m(ak_{mn}) + \left( \frac{b}{a} \right) J_{m-1}(bk_{mr}) J_m(bk_{mn}) \right] \right\} 
\]

2.40

The third integral has the following form

\[
\frac{1}{(-ik_{mn})^2 + (k_{mr})^2} \left\{ ak_{mn} \left[ J_{m}(ak_{mn}) Y_{m-1}(ak_{mn}) - \left( \frac{b}{a} \right) J_{m}(bk_{mn}) Y_{m}(bk_{mn}) \right] + ak_{mr} \left[ -J_{m-1}(ak_{mr}) Y_m(ak_{mn}) + \left( \frac{b}{a} \right) J_{m-1}(bk_{mr}) Y_m(bk_{mn}) \right] \right\} 
\]

2.41

and, finally, the last term

\[
\frac{1}{(-ik_{mn})^2 + (k_{mr})^2} \left\{ ak_{mn} \left[ J_{m-1}(ak_{mn}) Y_m(ak_{mn}) - \left( \frac{b}{a} \right) J_{m-1}(bk_{mn}) Y_m(bk_{mn}) \right] + ak_{mr} \left[ -J_{m}(ak_{mr}) Y_{m-1}(ak_{mr}) + \left( \frac{b}{a} \right) J_{m}(bk_{mr}) Y_{m-1}(bk_{mr}) \right] \right\} 
\]

2.42

2. Case 2: \( k_{mn} = k_{mr} \)

If the arguments of the Bessel functions are the same, then the following expressions give the solution to equation 2.39.

In case of the first term the solution is given by

\[
\frac{1}{4} a^2 \left\{ J_{m}^2(ak_{mn}) - J_{m-1}(ak_{mn}) J_{m+1}(ak_{mn}) \right\} + \left( \frac{b}{a} \right)^2 \left\{ J_{m}^2(bk_{mn}) - J_{m-1}(bk_{mn}) J_{m+1}(bk_{mn}) \right\} 
\]

2.43
In case of the second integral, the solution is given by

\[
\frac{1}{4} a^2 \left\{ Y_m^2 (a k_{mn}) - Y_{m-1} (a k_{mn}) Y_{m+1} (a k_{mn}) \right\} \left[ \frac{b}{a} \right]^2 \left[ - Y_m^2 (b k_{mn}) + Y_{m-1} (b k_{mn}) Y_{m+1} (b k_{mn}) \right] \times \left( \frac{D}{C} \right)_{mn} \left( \frac{D}{C} \right)_{mn}^{(\ast)}
\]

2.45

The analytical solution to the last two terms, i.e. cross terms, is much more complicated requiring numeric integration using the simple trapezoidal rule.

Finally, to find the second term \( \Lambda_{m,nn}^{(-)} \) in 2.37, the approach is repeated in the same manner as described by equations 2.39 to 2.45.

Once the amplitudes of the Green’s functions have been obtained, the pressure due to the piston sources can be computed using equations 2.27 and 2.28. The next step consists of determining the vibrating velocities of the piston sources.

**2.1.3.2 Piston velocity**

The piston velocities necessary to define the effect of the HQ tubes on the duct sound field is computed from the solution of a linear system of equations. This system of equation is obtained from coupling the sound field on the HQ tubes with the sound field in the duct at the HQ tube/duct interfaces. To this end, both the acoustic pressure and radial component of the particle velocity at the piston/duct interfaces are matched as indicated in equations 2.46 and 2.47, respectively.

\[
\left\langle p \right\rangle_{\hat{a}/o} = p_{\hat{a}/o}' \quad \text{2.46}
\]

\[
\left\langle v \right\rangle_{\hat{a}/o} = v_{\hat{a}/o}' \quad \text{2.47}
\]
where \( \langle p \rangle_{i/o} \) and \( \langle v \rangle_{i/o} \) are the average pressure and particle velocity at the interface inside the duct and \( p'_{i/o} \) and \( v'_{i/o} \) are the pressure and particle velocity at the interface inside the HQ tubes. The subscripts \( i/o \) identify the two ends of the tubes.

The average pressure at the piston surface on the duct side is given by

\[
\left\{ \begin{array}{l}
\langle p \rangle_{ow-li} \\
\vdots \\
\langle p \rangle_{ow-No}
\end{array} \right\} = \begin{bmatrix}
Z_{os} & \begin{bmatrix}
V_{P_{ow-li}} \\
\vdots \\
V_{P_{ow-Mo}}
\end{bmatrix} \end{bmatrix} + \begin{bmatrix}
\langle p \rangle_{dist-ow-li} \\
\vdots \\
\langle p \rangle_{dist-ow-Mo}
\end{bmatrix}
\]

where the first term represents the average pressure on the surface of a piston observer “o” due to a piston source (“s”) vibrating with unknown piston velocity; \( \{V_p\} = \{V_{P_{ow-li}}, \ldots, V_{P_{ow-No}}, V_{P_{iw-li}}, \ldots, V_{P_{iw-Mo}}\}^T \) is a \( 2(N+M) \times 1 \) array containing the unknown piston velocities; \( N \) and \( M \) indicate the total number of HQ-tubes located on the outer and inner wall respectively; and subscripts \( ow/iw \) indicate outer and inner wall. Matrix \( [Z_{os}] \) is a \( 2(N+M) \times 2(N+M) \) array and it accounts for the influence of the pistons upon each other. Each one of its coefficients is calculated (see Appendix A for details) as the average pressure over the piston observer “o” divided by the velocity of the piston source “s”. Thus, the array has units of impedance and it will be referred to as cross impedance matrix. The second term on the right hand side is a \( 2(N+M) \times 1 \) array containing the average pressure on the piston surface due to the disturbance. It is important to note that in the case of an annular duct without tubes on the inner wall, \( M \) vanishes and equation 2.48 reduces to the formulation found in reference [6] for circular ducts.

The uniform pressure on the piston surface inside the HQ-tubes is given in matrix form by
In this equation, the zeroes off the main diagonal indicate that the sound field in a particular tube is independent of the others. The main diagonal in the array are \(2 \times 2\) matrices that contain information on the tube dynamics. The components of this matrix is directly related to

\[
\begin{pmatrix}
p'_{\text{oi}} \\
p'_{\text{oi-Mo}} \\
p'_{\text{oi-1i}} \\
p'_{\text{oi-Mo}} \end{pmatrix}_{2(N+M)\times 1} = \begin{bmatrix} Z'_{1-\text{ow}} & 0 & 0 & 0 \\ 0 & \ddots & \ddots & \ddots \\ 0 & \ddots & \ddots & \ddots \\ 0 & \ddots & \ddots & \ddots \end{bmatrix} \begin{pmatrix}
p'_{\text{io}} \\
p'_{\text{io-Mo}} \\
p'_{\text{io-1i}} \\
p'_{\text{io-Mo}} \end{pmatrix}_{2(N+M)\times 1}
\]

which defines the relationship between pressure and velocity at the tubes ends. This expression was derived assuming a straight open-open waveguide, with no flow and considering only plain waves, i.e. 1DOF model previously investigated by Hallez [25, 26]. The equation includes a correction due to the impedance of the perforate screen (with or without a wire mesh) at the HQ-tube/duct interface.

In matching average particle velocities, the formulation for the annular duct differs from that of the circular case. The difference arises in that the sign of the liner induced velocity \(v'_{\text{liner-ow/iw}}\) is defined in terms of the duct reference system, i.e. positive in the radial outward direction whereas the piston velocity \(V'_{\text{pl}}\) is defined positive into the control volume used by the Green’s divergence theorem. The Green’s divergence theorem is used when deriving an expression for the sound field radiated by the pistons [33]. In other words, when the source is placed on the outer wall \(V'_{\text{pl}}\) is positive downwards as shown in Figure 2.7. On the other hand, when the source is placed on the inner wall \(V'_{\text{pl}}\) is positive upwards.
The sign of the average particle velocity $\langle v \rangle_{li/o}$ will be such that $V_{pl}$ equals $\langle v \rangle_{li/o}$ for the hard wall case, i.e. $v_{liner-ow/ivw} = 0$. Therefore, on the outer wall the average particle velocity is defined as

$$\langle v \rangle_{li/o} = V_{pl} - v_{liner-ow}$$  \hspace{1cm} 2.51$$

For sources placed on the inner wall the average particle velocity is defined as
\[
\langle v \rangle_{li/o} = V_{pl} + v_{\text{liner-ow/iw}} \quad 2.52
\]

where \( v_{\text{liner-ow/iw}} \) is the liner induced velocity and it is defined in terms of the average pressure at the piston face and the liner acoustic properties as

\[
\langle v \rangle_{\text{liner-ow}} = \frac{\beta_{wa}}{\rho c} \langle p \rangle_{li/o} \quad 2.53
\]

for the outer wall, and

\[
\langle v \rangle_{\text{liner-iw}} = \frac{\beta_{wb}}{\rho c} \langle p \rangle_{li/o} \quad 2.54
\]

for the inner wall.

The velocities in equation 2.51 and 2.52 can be written in vector form as

\[
\begin{bmatrix}
\langle v \rangle_{\text{ow-1i}} \\
\vdots \\
\langle v \rangle_{\text{ow-No}} \\
\langle v \rangle_{\text{iw-1i}} \\
\vdots \\
\langle v \rangle_{\text{iw-Mo}}
\end{bmatrix}
= 
\begin{bmatrix}
V_{p_{ow-1i}} \\
\vdots \\
V_{p_{ow-No}} \\
V_{p_{iw-1i}} \\
\vdots \\
V_{p_{iw-Mo}}
\end{bmatrix}
- 
\begin{bmatrix}
v_{\text{liner-ow}} \\
\vdots \\
v_{\text{liner-ow}} \\
v_{\text{liner-ow}} \\
\vdots \\
v_{\text{liner-ow}}
\end{bmatrix}
2.55
\]

In order to obtain the piston velocities \( V_{p} \), and on account of equation 2.46, the right hand side of 2.55 is replaced in the right hand side of 2.49. Due to the pressure matching at the tube/duct interface, the left hand side of this new expression is replaced by the right hand side of 2.48. The resulting equation is presented below.
Replacing equations 2.53 and 2.54 into 2.56 and rearranging yields a linear system of equations
where

\[
[B] = \begin{bmatrix}
\frac{\beta_{wu}}{\rho c} & \cdots & \frac{\beta_{wu}}{\rho c} \\
\cdots & \frac{\rho c}{\beta_{wb}} & \cdots \\
\frac{-\beta_{wb}}{\rho c} & \cdots & \frac{-\beta_{wb}}{\rho c}
\end{bmatrix}
\]

The solution of this system will give the piston velocities \( V_p \) for an annular duct with HQ-tubes distributed on both the outer and inner walls. In case of an annular duct with no HQ-tubes on the inner wall or a circular duct, the system reduces to the expression derived in reference [33]. For the sake of clarity, this final expression is repeated here.
Once the piston velocities have been determined and along with them the sound field within the duct, the transmitted power can then be estimated.

The new model for aft radiation control using HQ-liner systems has been implemented in a computer program using FORTRAN language (see Appendix C). The code was then used to design an HQ-liner system for the bypass section of the NASA ANCF rig. The system was tested and the collected data was used to validate the new model. Chapter 3 analyzes in detail the results of the experiments.

### 2.1.4 Power computation

The performance of the systems is evaluated by comparing sound power with and without HQ-liner systems. Sound power is calculated as

\[
\begin{align*}
    w(z_n) &= \int_{0}^{2\pi} \int_{0}^{a} I_z(r, \theta, z_n) r dr d\theta \\
    &\quad 2.60
\end{align*}
\]

Where the intensity component in the duct axial direction is given by

\[
I_z = \frac{1}{2} \text{Re} \left[ pv^* + \rho c |v_z|^2 M + \frac{|p|^2}{\rho c} M + v_z p^* M^2 \right] \\
&\quad 2.61
\]

In this expression the axial component of the velocity is computed with help of Euler’s equation in the duct axial direction

\[
v_z = \sum \sum \frac{k_z^{(+)}}{\rho c (k_0 - k_z^{(+)}) M} p_{mn}^{(+)} \\
&\quad 2.62
\]

The pressure is given by
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\[
p = \sum_{m=0}^{M} \sum_{n=0}^{N} (A_{mn}^{(+)})^{pos} H_m(k_{mn}^{(+)} r) e^{-im\theta} e^{-ik_{mn}^{(+)} z} + \sum_{m=0}^{M} \sum_{n=0}^{N} (A_{mn}^{(+)})^{neg} H_m(k_{mn}^{(+)} r) e^{im\theta} e^{-ik_{mn}^{(+)} z}
\]

2.63

Note that in equation 2.63 the positive propagating sound field has been divided into positive and negative rotating, i.e. \( e^{-im\theta} \) and \( e^{im\theta} \) respectively. The modal amplitudes for positive propagating and positive rotating waves \( (A_{mn}^{(+)})^{pos} \) and for positive propagating and negative rotating waves \( (A_{mn}^{(+)})^{neg} \) already include the effect of the liner and the HQ tubes upon the disturbance. The calculation of these amplitudes is the same as for the circular duct, and it is repeated for completeness in appendix B. Also note that both the disturbance and the sound field due to the HQ tubes are represented by the same number of modes therefore the subscript “\( g \)” exclusively related to the number of modes in the Green’s function will be used only to define the highest circumferential order \( M_g \) and radial order \( N_g \) used. Finally, \( H_m(k_{mn}^{(+)} r) \) is none other than

\[
\left( J_m(k_{mn}^{(+)} r) + \frac{D}{C} \right) Y_m(k_{mn}^{(+)} r)
\]

Replacing, equations 2.63 and 2.62 into 2.61 and this into 2.60 leads to

\[
W(z_m) = \frac{1}{2} \operatorname{Re} \left[ \frac{2\pi \alpha^2}{\rho c} \left( \sum_{m=0}^{M} \sum_{n=0}^{N} \sum_{r=0}^{N} \left[ (A_{mn}^{(+)})^{pos} (A_{mr}^{(+)})^{pos} + (A_{mn}^{(+)})^{neg} (A_{mr}^{(+)})^{neg} \right] e^{-ik_{mn}^{(+)} z} e^{-ik_{mr}^{(+)} z} \frac{k_{mn}^{(+)} k_{mr}^{(+)} \Lambda_{m,nr}^{(+)}}{(k_0 - k_{mn}^{(+)} M)(k_0 - k_{mr}^{(+)} M)} \right] + \right.
\]

\[
M^2 \left[ \left( (A_{mn}^{(+)})^{pos} (A_{mr}^{(+)})^{pos} + (A_{mn}^{(+)})^{neg} (A_{mr}^{(+)})^{neg} \right) e^{-ik_{mn}^{(+)} z} e^{-ik_{mr}^{(+)} z} \frac{k_{mn}^{(+)} \Lambda_{m,nr}^{(+)}}{(k_0 - k_{mn}^{(+)} M)} \right] + \right.
\]

\[
M \left[ \left( (A_{mn}^{(+)})^{pos} (A_{mr}^{(+)})^{pos} + (A_{mn}^{(+)})^{neg} (A_{mr}^{(+)})^{neg} \right) e^{-ik_{mn}^{(+)} z} e^{-ik_{mr}^{(+)} z} \frac{k_{mn}^{(+)} \Lambda_{m,nr}^{(+)}}{(k_0 - k_{mn}^{(+)} M)} \right] + \right.
\]

\[
M^2 \left[ \left( (A_{mn}^{(+)})^{pos} (A_{mr}^{(+)})^{pos} + (A_{mn}^{(+)})^{neg} (A_{mr}^{(+)})^{neg} \right) e^{-ik_{mn}^{(+)} z} e^{-ik_{mr}^{(+)} z} \frac{k_{mn}^{(+)} \Lambda_{m,nr}^{(+)}}{(k_0 - k_{mn}^{(+)} M)} \right]
\]

2.64
This equation is a general expression for the computation of power and can be used in any situation, i.e. hard wall, HQ tubes on hard wall, liner alone or HQ-Liner systems. The calculation of the orthogonalization factor \( \hat{\Lambda}_{m,nr}^{(+)} \) has already been explained in section 2.1.3.1.

### 2.2. Multi-degree-of-freedom HQ Tube Model

In this section a multi degree of freedom model is developed. As already mentioned, the 1DOF model assumes plane waves inside the tubes. This assumption justifies modeling the effect of HQ tubes on the duct sound field as that of a single piston. Despite the simplicity of the approach, the validity of this model is limited to frequencies below the tube first resonance. Thus, the objective of this section is to extend the capabilities of the HQ model to frequencies above the tube first resonance. To this end the presence of higher order modes will be accounted for by means of the Green’s function. The lack of experimental data has made the thorough validation of the new model not possible. However a first validation has been included in Appendix F. Here, prediction of power reduction using the 1DOF and the MDOF models below the tubes first resonance are compared to each other. Appendix F also includes a few results using the new MDOF model when higher order modes are present inside the tube.

**Mathematical derivation**

In this section, the model of the acoustic field inside the HQ tubes is extended to account for higher order modes. To this end the single square piston at the HQ tube opening used so far will be divided into four smaller sub-pistons as indicated in Figure 2.8. By dividing the original piston into four smaller sub-pistons the effect of the tube higher order modes (1,0), (0,1) and (1,1) can also by included in the model.

For the sake of simplicity and similarly to the previous model, this dynamic system is approximated as a straight tube (see Figure 2.9) with length equal to the tube mean length \( L \) (see Figure 2.8). In this approach, effects introduced by the curvature are neglected.
Figure 2.8: HQ tube with opening at tube/duct interface divided into four sub-pistons.

Figure 2.9: HQ tube approximated as straight waveguide.
Like in the 1DOF model, the acoustic variables at the two openings of the tube have to be related. Thus, the pressure and particle velocity at the eight sub-pistons must be related leading to a $8 \times 8$ impedance matrix as

$$\begin{bmatrix}
    p_1 \\
    \vdots \\
    \vdots \\
    p_8
\end{bmatrix} = 
\begin{bmatrix}
    z_{11} & \cdots & \cdots & z_{18} \\
    \vdots & \ddots & \ddots & \vdots \\
    \vdots & \ddots & \ddots & \vdots \\
    z_{81} & \cdots & \cdots & z_{88}
\end{bmatrix}\begin{bmatrix} V_1 \\
    \vdots \\
    \vdots \\
    V_8
\end{bmatrix}$$

where $p_1$ through $p_8$ are the uniform pressure acting over the surface of each one of the sub-pistons and $V_1$ through $V_8$ are the associated uniform velocities. The objective here is to find the components $z_{ij}$ of the new impedance matrix for each tube. The impedance of the liner wire mesh and the screen placed at the tube/duct interface will be added at the end. The mathematical derivation of the tube dynamics including higher order modes is presented next.

The hard wall Green’s function was used to describe the sound field inside the tube. This sound field has to satisfy the homogeneous linear differential equation

$$\nabla^2 p + k_0^2 p = 0$$

where $p(x,y,z,t) = P(x,y,z)e^{i\omega t}$ is the pressure at $(x,y,z)$ and $k_0$ is the free field wave number. The motion of the sub-pistons at the tube’s end is the source that generates the acoustic pressure. The pressure due to the motion of the boundaries can be obtained using the hard wall Green’s function. The Green’s function can now be expressed as [62]

$$g(x|x_0) = \sum_{l}^{L} \sum_{n}^{N} \sum_{m}^{M} \frac{\Phi_{lnm}(x,y,z)\Phi_{lnm}(x_0,y_0,z_0)}{(k_0^2 - k_{lnm}^2)ab\Lambda_{lnm}}$$

where $k_{lnm}$ is the wavenumber of the $lnm^{th}$ acoustic mode that satisfy the hard wall boundary condition; $\Phi_{lnm}(x,y,z)$ is the associated mode shape of the $lnm^{th}$ acoustic mode and it is defined as $\Phi_{lnm} = \cos(k_x x)\cos(k_y y)\cos(k_z z)$ with the eigenvalues $k_x = l\pi / L, \quad k_y = n\pi / a, \quad$ and $k_z = m\pi / b$ in the $x, y$ and $z$ directions, respectively. The point $(x_0, y_0, z_0)$ defines the location of a point source whereas $(x, y, z)$ is the location of the receiver. The tube size is defined by $a, b$ and
as shown in Figure 2.9. Finally, $\Lambda_{\text{inm}}$ is the acoustic mode orthogonalization factor with values defined as follows:

1. $n = l = m = 0$ then $\Lambda_{\text{inm}} = 1$
2. $n = l = 0$ or $n = m = 0$ or $m = l = 0$ then $\Lambda_{\text{inm}} = 0.5$
3. $n = 0$ or $m = 0$ or $l = 0$ then $\Lambda_{\text{inm}} = 0.25$
4. everything else $\Lambda_{\text{inm}} = 0.125$

The Green’s function in equation 2.67 is now used to find the pressure due to the motion of the sub-pistons as

$$p(x) = -i\omega \rho V \int g(\frac{x}{x_0}) ds$$

where $S$ is the surface area of the sub-piston as indicated in Figure 2.9, $x_0 = (x, y, z)$ is a point on the surface of the sub-piston and $x = (x, y, z)$ defines the point at which pressure is being calculated; $V$ is the unknown uniform velocity of the sub-piston. Replacing 2.67 into 2.68 gives

$$p(x, y, z / x_r, y_r, z_r) = -i\omega \rho V \sum_{l} \sum_{n} \sum_{m} \Phi_{\text{inm}}(x, y, z) \int \int \Phi_{\text{inm}}(x_r, y_r, z_r) d\hat{z} d\hat{y}$$

or

$$p(x, y, z / x_r, y_r, z_r) = -i\omega \rho V \sum_{l} \sum_{n} \sum_{m} \frac{\Phi_{\text{inm}}(x, y, z) \varphi_{\text{inm}}^{(r)}}{abL\Lambda_{\text{inm}}}$$

where $\varphi_{\text{inm}}^{(r)}$ is given by

$$\varphi_{\text{inm}}^{(r)} = \int \int \Phi_{\text{inm}}(x_r, y_r, z_r) d\hat{z} d\hat{y}$$

Note that the limits of integration correspond to the size of the $r^{th}$ sub-piston and therefore the pressure $p(x, y, z / x_r, y_r, z_r)$ in equation 2.69 and 2.70 is the pressure at $(x, y, z)$ due to the
motion of this sub-piston, the center of which is given by \((x_r, y_r, z_r)\). As seen in Figure 2.9, the pressure due to each of the eight sub-pistons must be obtained. The resulting pressure at a particular point is then a linear contribution of the motion of all the sub-pistons.

The next step consists of finding the influence that each one of the sub-pistons exercises upon each other. The average pressure over the receiver “s” due to a source “r” can be calculated as

\[
\bar{p}_s = \frac{1}{S_s} \int_{y_s-a/4}^{y_s+a/4} \int_{z_s-b/4}^{z_s+b/4} p(x_r, y_r, z_r) \, dy \, dx
\]

In equation 2.72 the limits of integration are those of the receiver whose surface area is given by \(S_s\). The location of the receiver along the \(x\) axis is defined by \(x_s\).

Dividing both sides of Equation 2.72 by \(V_r\) leads to

\[
z_{sr} = \frac{\bar{p}_s}{V_r} = \frac{1}{V_r S_s} \int_{y_s-a/4}^{y_s+a/4} \int_{z_s-b/4}^{z_s+b/4} p(x_r, y_r, z_r) \, dy \, dx
\]

where \(z_{rs}\) has impedance units. Replace now equation 2.72 into 2.73 and solve the integral to obtain

\[
z_{sr} = -i \omega \rho \sum_{l}^{L_s} \sum_{n}^{N_s} \sum_{m}^{M_s} \frac{\phi_{lm}^{(s)} \phi_{lm}^{(r)}}{(k_{lm}^2 - k_{in}^2) a_{HQ} b_{HQ} L \Lambda_{in}}
\]

where

\[
\phi_{lm}^{(s)} = \int_{y_s-a/4}^{y_s+a/4} \int_{z_s-b/4}^{z_s+b/4} \Phi_{lm}(x_r, y_r, z_r) \, dy \, dx
\]

Equation 2.74 is the expression that allows the calculation of each one of the 64 components of the HQ tube dynamics matrix.

Like in the 1DOF model the tube dynamics does not include the effect of the liner wire mesh and the screen. This correction is made as follows

\[
\{p_r\} = \{p\} + [z_{screen}]\{y\}
\]

In equation 2.76 \(\{p_r\}\) is an 8×1 vector containing the average pressure at every sub-piston surface corrected due to the presence of the liner wire mesh, \(\{p\}\) is an 8×1 vector with the average pressure at every sub-piston surface without the correction, \([z_{screen}]\) is an 8×8 diagonal
matrix, containing the screen impedance (assumed locally reacting) and \( \{ V \} \) is an \( 8 \times 1 \) vector with the sub-piston velocities. Inserting 2.65 into 2.76 leads to

\[
\{ p_i \} = [z_{sr}]\{ V \} + [z_{screen}]\{ V \} \tag{2.77}
\]

or

\[
\{ p_i \} = ([z_{sr}]+[z_{screen}]\{ V \} \tag{2.78}
\]

where \([Z\prime_{io}]=[z_{sr}]+[z_{screen}]\) is the new multi-DOF tube dynamics. This expression includes the screen and liner wire mesh impedance and it is used in equation 2.57 in section 2.1.

### 2.3. Transmission-reflection effect

When incident energy propagating inside a duct encounters a discontinuity as the one imposed by acoustic lining (see Figure 2.10), part of it gets reflected back and part of it gets transmitted. It has been shown by Zorumski [61] that reflection of energy plays an important role in duct noise attenuation and therefore can be used to reduce transmitted noise. The fact that reflections can be a significant contributor to duct noise attenuation indicated that its inclusion in the model developed in Section 2.1 could be important.

Wall impedance discontinuities are also responsible for the redistribution or scattering of acoustic energy. If energy is transferred into modes that are self-attenuating or cut-off then scattering will also be responsible for providing some extra attenuation.

The problem of transmission-reflection effects in ducts treated with several acoustic liners was first studied by Zorumsky [61]. His formulation will be re-derived here for the particular case of a duct (circular and annular) treated with a single liner. Even though the formulation is not new, its implementation and evaluation on noise reduction due to HQ-liner systems is a contribution of this dissertation.

The remainder of this section will then concentrate on the derivation of the necessary formulation to calculate transmissions and reflections of a sound field as it propagates through both circular and annular ducts. The new formulation will be added to the model developed in section 2.1. In order to assess the influence of reflections on power attenuation, predictions including and neglecting these effects will be compared to experiments in Chapter 4.
Figure 2.10: Schematic of transmission-reflection effect due to wall impedance discontinuity at HW/liner interfaces.

It is important to mention the two simplifications made based on the characteristics of the liners used in this work. Since the liners utilized had uniform impedance and presented no discontinuities in the circumferential direction, a circumferential mode $m$ scattered energy between its radial components. This means that the transmission-reflection problem had to be solved for each circumferential mode independently, i.e. the orthogonality condition is satisfied in the circumferential direction thus decoupling the modes.

The second simplification comes from assuming that the attenuation provided by the liners is such that the interaction between the two interfaces of the finite length liner can be neglected.

The first step in this derivation consists of matching pressure and particle velocity in the $z$-direction at the liner/hard wall interface as
\[ p_i + p_r = p_t \]  
\[ v_i + v_r = v_t \]

where \( p_i, p_r \) and \( p_t \) are the incident, reflected and transmitted pressure in the disturbance for a particular \( m \) order mode and can be written as

\[ p_i = \sum_n A^{i}_{mn} H^{i}_{mn} \Theta_m \]
\[ p_r = \sum_n A^{r}_{mn} H^{r}_{mn} \Theta_m \]
\[ p_t = \sum_n A^{t}_{mn} H^{t}_{mn} \Theta_m \]

where \( A^{\ell}_{mn} \) are the known incident modal amplitudes at a particular axial location, i.e. multiplied by \( e^{-ik_z z} \), \( n=0,...,N \) indicate the multiple radial modes, and \( N \) is the highest radial mode included in the calculations. The unknown amplitudes of the reflected and transmitted modes are given by \( A^{r}_{mn} \) and \( A^{t}_{mn} \), respectively. The mode shapes for the incident, reflected, and transmitted waves are given \( H^{\ell}_{mn}(r) \Theta_m(\theta) \) with \( \ell = i, r, t \), respectively. The function \( H^{\ell}_{mn}(r) \) defines the mode shapes in the radial direction and involves the Bessel functions of the first and second kind as defined in section 2.1 and it is different for all three type of waves. On the other hand, the function \( \Theta_m(\theta) \) has also been defined in section 2.1 and describes the pressure distribution for spinning modes in the circumferential direction. Since the liner is circumferentially continuous, this function is the same for all three waves. Note that in the following derivation “i” stands for incident, “r” stands for reflected and “t” stands for transmitted.

Using Euler’s equation, the particle velocity can be written as follows:
where $M$ is the duct flow Mach number. 

Replacing equation 2.81 and 2.82 into 2.79 and 2.80 leads to

$$
\sum_n A^i_{mn} e^{-ik^i_z z} + \sum_n A^r_{mn} e^{-ik^r_z z} = \sum_n A^i_{mn} H^i_{mn} e^{-ik^i_z z} \quad 2.83
$$

$$
\sum_n A^i_{mn} H^i_{mn} e^{-ik^i_z z} + \sum_n A^r_{mn} H^r_{mn} e^{-ik^r_z z} = \sum_n A^i_{mn} H^i_{mn} e^{-ik^i_z z} \quad 2.84
$$

Note that $\Theta_m$ is the same for incident, reflected and transmitted waves, therefore it cancels out. 

Let us consider first the impedance discontinuity at the liner leading edge, e.g. hard wall/lined interface. It is clear that equations 2.83 and 2.84 are not enough to solve for the $2(N+1)$ unknowns, i.e. the $N+1$ transmitted wave amplitudes and the $N+1$ reflected wave amplitudes. 

When the duct walls are not acoustically treated, i.e. hard wall case, $H^i_{mn}(r)$ and $H^r_{ml}(r)$ are orthogonal to each other, which means that $\int_a^b H^r_{ml}(r)H^i_{mn}(r) r dr$ vanishes for $l \neq n$ [61]. So multiplying 2.83 and 2.84 by $rH^r_{ml}$ and then integrating along the radial direction leads to

$$
\sum_{n=0}^N A^{(i)}_{mn} I^{(ir)}_{ml} + \sum_{n=0}^N A^{(r)}_{mn} I^{(rr)}_{ml} = \sum_{n=0}^N A^{(i)}_{mn} I^{(ir)}_{ml} \quad 2.85
$$

$$
\sum_{n=0}^N \frac{A^{(i)}_{mn}}{(k_0 / k_z^{(i)}) - M} I^{(ir)}_{ml} + \sum_{n=0}^N \frac{A^{(r)}_{mn}}{(k_0 / k_z^{(r)}) - M} I^{(rr)}_{ml} = \sum_{n=0}^N \frac{A^{(i)}_{mn}}{(k_0 / k_z^{(i)}) - M} I^{(ir)}_{ml} \quad 2.86
$$

Where
where $a$ and $b$ are the outer and inner radii respectively. Due to the orthogonality condition, $I_{mml}^{ir}$ and $I_{mml}^{rr}$ vanish when $l \neq n$. As a consequence equations 2.85 and 2.86 unfold into two systems of $(N+1)$ equations with $2(N+1)$ unknowns (as $l$ goes from 0 to $N$) where the unknowns are the transmitted and reflected amplitudes $A_{mn}^{(t)}$ and $A_{mn}^{(r)}$. After replacing $n$ by $l$ on the left hand side of 2.85 and 2.86, the systems can then be written in matrix form as

\[
\begin{bmatrix}
A_{m1}^{(t)}I_{m1}^{ir} + A_{m1}^{(r)}I_{m1}^{rr} \\
A_{m}^{(t)}I_{ml}^{ir} + A_{m}^{(r)}I_{ml}^{rr} \\
\vdots \\
A_{m}^{(t)}I_{m(N+1)}^{ir} + A_{m}^{(r)}I_{m(N+1)}^{rr}
\end{bmatrix}
\begin{bmatrix}
A_{m(N+1)}^{(t)}I_{m(N+1)}^{ir} + A_{m(N+1)}^{(r)}I_{m(N+1)}^{rr}
\end{bmatrix}
\]

\[= 2.87 \]

\[
A_{m1}^{(t)}I_{m1}^{ir} \\
\vdots \\
A_{m1}^{(t)}I_{m(N+1)}^{ir} \\
A_{m}^{(t)}I_{ml}^{ir} \\
\vdots \\
A_{m}^{(t)}I_{m(N+1)}^{ir}
\]

\[
\begin{bmatrix}
A_{m1}^{(t)}I_{m1}^{ir} \\
\vdots \\
A_{m1}^{(t)}I_{m(N+1)}^{ir} \\
A_{m}^{(t)}I_{ml}^{ir} \\
\vdots \\
A_{m}^{(t)}I_{m(N+1)}^{ir}
\end{bmatrix}
\begin{bmatrix}
A_{m1}^{(t)}I_{m1}^{ir} \\
\vdots \\
A_{m1}^{(t)}I_{m(N+1)}^{ir} \\
A_{m}^{(t)}I_{ml}^{ir} \\
\vdots \\
A_{m}^{(t)}I_{m(N+1)}^{ir}
\end{bmatrix}
\]

\[= 2.88 \]
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\[
\begin{bmatrix}
\frac{A_{m1}^{(i)}}{K_{m1}^1} I_{m11}^{ir} + \frac{A_{m1}^{(r)}}{K_{m1}^2} I_{m11}^{rr} \\
\vdots \\
\frac{A_{m1}^{(i)}}{K_{m1}^N} I_{m1(N+1)}^{ir} + \frac{A_{m1}^{(r)}}{K_{m1}^N} I_{m1(N+1)}^{rr}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{A_{m(N+1)}^{(i)}}{K_{m(N+1)}^1} I_{m(N+1)(N+1)}^{ir} + \frac{A_{m(N+1)}^{(r)}}{K_{m(N+1)}^2} I_{m(N+1)(N+1)}^{rr}
\vdots \\
\frac{A_{m(N+1)}^{(i)}}{K_{m(N+1)}^N} I_{m(N+1)(N+1)}^{ir} + \frac{A_{m(N+1)}^{(r)}}{K_{m(N+1)}^N} I_{m(N+1)(N+1)}^{rr}
\end{bmatrix}
\]

where

\[
\begin{align*}
K_{m1}^1 &= \left( k_0 / k_z^{(i)} - M \right) \\
K_{m1}^2 &= \left( k_0 / k_z^{(r)} - M \right) \\
K_{mN}^N &= \left( k_0 / k_z^{(i)} - M \right)
\end{align*}
\]

and have been introduced to simplify equation 2.89. The subscripts \( l \) and \( n \) indicate rows and columns respectively. With equations 2.88 and 2.89 the amplitudes of the transmitted and reflected waves at the liner leading edge can be calculated. In order to do this rewrite 2.88 as

\[
\begin{bmatrix}
A_{m1}^{(r)} I_{m11}^{rr} \\
\vdots \\
A_{m(N+1)}^{(r)} I_{m(N+1)1}^{rr}
\end{bmatrix}
= 
\begin{bmatrix}
\sum_{n=0}^{N} A_{mn}^{(i)} I_{mn1}^{ir} - A_{m1}^{(i)} I_{m11}^{ir} \\
\vdots \\
\sum_{n=0}^{N} A_{mn}^{(i)} I_{mn(N+1)}^{ir} - A_{m(N+1)}^{(i)} I_{m(N+1)(N+1)}^{ir}
\end{bmatrix}
\]

Replace then 2.91 into 2.89 and reorder as
In this expression the only unknowns are the transmitted amplitudes $A^{(i)}_{m}$. Once these transmitted amplitudes have been calculated, they should be replaced into equation 2.91 so that the amplitudes of the reflected waves can be determined.

The procedure to calculate transmissions and reflections at the liner trailing edge is the same. In this case $rH_{r}^{l} dr$ has to be replaced by $rH_{m}^{l} dr$ so that the orthogonality condition will be satisfied in the hard wall section after the liner. Since the derivation is similar to the leading edge case only the final equations will be presented next.
Chapter 2. Mathematical Models

\[
\begin{bmatrix}
I_{m11}^{rt}\left(\frac{1}{K_{m1}^1} - \frac{1}{K_{m1}^2}\right) & \cdots & \cdots & I_{m(N+1)1}^{rt}\left(\frac{1}{K_{m(N+1)}^3} - \frac{1}{K_{m(N+1)}^2}\right) \\
\vdots & \ddots & \ddots & \vdots \\
I_{m1l}^{rt}\left(\frac{1}{K_{ml}^1} - \frac{1}{K_{ml}^2}\right) & \cdots & I_{mnl}\left(\frac{1}{K_{ml}^3} - \frac{1}{K_{ml}^2}\right) & \cdots & I_{m(N+1)l}^{rt}\left(\frac{1}{K_{m(N+1)}^3} - \frac{1}{K_{m(N+1)}^2}\right) \\
I_{m(N+1)1}^{rt}\left(\frac{1}{K_{m(N+1)}^3} - \frac{1}{K_{m(N+1)}^2}\right) & \cdots & \cdots & I_{m(N+1)(N+1)}^{rt}\left(\frac{1}{K_{m(N+1)}^3} - \frac{1}{K_{m(N+1)}^2}\right)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sum_{n=0}^{N} A_{mn}^{(i)} I_{m11}^{it} & \cdots & \cdots & \sum_{n=0}^{N} A_{mn}^{(i)} I_{m(N+1)1}^{it} \\
\sum_{n=0}^{N} A_{mn}^{(i)} I_{m1l}^{it} & \cdots & \cdots & \sum_{n=0}^{N} A_{mn}^{(i)} I_{m(N+1)l}^{it} \\
\sum_{n=0}^{N} A_{mn}^{(i)} I_{m(N+1)1}^{it} & \cdots & \cdots & \sum_{n=0}^{N} A_{mn}^{(i)} I_{m(N+1)(N+1)}^{it}
\end{bmatrix}
\]

\[
A_{ml}^{(i)} I_{m11}^{it} = \begin{bmatrix}
\sum_{n=0}^{N} A_{mn}^{(i)} I_{m11}^{it} + \sum_{n=0}^{N} A_{mn}^{(r)} I_{m11}^{rt} \\
\sum_{n=0}^{N} A_{mn}^{(i)} I_{m1l}^{it} + \sum_{n=0}^{N} A_{mn}^{(r)} I_{m1l}^{rt} \\
\sum_{n=0}^{N} A_{mn}^{(i)} I_{m(N+1)1}^{it} + \sum_{n=0}^{N} A_{mn}^{(r)} I_{m(N+1)1}^{rt} \\
\sum_{n=0}^{N} A_{mn}^{(i)} I_{m(N+1)(N+1)}^{it} + \sum_{n=0}^{N} A_{mn}^{(r)} I_{m(N+1)(N+1)}^{rt}
\end{bmatrix}
\]

2.93

and

In equations 2.93 and 2.94 the incident amplitudes \(A^{(i)}\) are the amplitudes of the transmitted modes calculated at the liner leading edge that have propagated through the liner until they have reached the trailing edge. The values of \(K^1\), \(K^2\) and \(K^3\) are as follows

\[
K_{mn}^1 = \left(\frac{k_0}{k_m^1} - M\right) \\
K_{mn}^2 = \left(\frac{k_0}{k_m^2} - M\right) \\
K_{ml}^3 = \left(\frac{k_0}{k_m^3} - M\right)
\]

2.95

With equation 2.93 the amplitudes of the reflected waves at the second interface can be estimated. With the amplitudes of the reflected waves and equation 2.94 the amplitudes of the transmitted waves at the second interface can be defined.
3. HQ-LINER SYSTEMS ON THE ANCF RIG

Experiments were performed at the Active Noise Control Fan (ANC F) rig at the NASA Glenn facility [7, 57]. The objective of these experiments was to collect data in order to assess the performance of HQ-liner systems installed in annular ducts and to validate the aft radiation HQ-liner model developed in Chapter 2. Since data was also obtained for inlet HQ-liner systems, this data was also used to validate the inlet HQ-liner code developed by Alonso [33]. The reason for choosing the ANCF rig was due to its unique capability of measuring the modal structure of the sound field inside the rig. In-duct data not only provided disturbance information but also valuable insight that contributed to improve both inlet and aft radiation models. This improvement involves the calculation of transmissions and reflections due to a sudden change of wall impedance such as the one that occurs when the sound field propagating through a hard wall section encounters a liner (see section 2.3 for model details). In addition, far-field data was also collected and compared to model predictions.

This chapter is organized in several sections. The first section, Section 3.1, describes the NASA rig as well as the instrumentation used. Section 0 focuses on the design of the HQ-Liner systems. The different test configurations are presented in Section 3.3. Results summarizing the performance of every configuration at several engine speeds are shown and analyzed in section 3.4.

3.1. Experimental Hardware

The test facility used in this research was the ANCF Rig at NASA Glenn shown in Figure 3.1a. This is a low speed fan with 16 blades at 28° of incidence and 28 stator vanes. The spacing between the rotor and the stator was half a blade chord measured at the rotor hub. Both the inlet and aft duct sections consisted of three spool pieces 48in in diameter ranging in length from 12 to 16in (see Figure 3.1a). These spool pieces were removable and interchangeable facilitating the installation of the HQ-Liner systems as dictated by the different configurations tested. The aft duct center body has an 18in diameter at the stator vane location transition to a 24in diameter
cylinder at the duct exit with only the last 20 in having a constant hub-to-tip ratio at 0.5. The maximum operating speed for this fan is approximately 1900 rpm for a tip Mach number \( M_{\text{tip}} = 0.35 \). Though the HQ-liner systems were designed for a fan speed of 1800 rpm, the fan was operated and data taken over a range of speeds, i.e. at 1500, 1600, 1700, and 1800 rpm. At the fan speed of 1800 rpm, the inlet and aft duct flow speeds are 0.115 and 0.15 Mach, respectively. For the fan and vane count used here, the blade-passage-frequency (BPF) tone is cut-off for this range of speeds, e.g. all acoustics modes at BPF decay exponentially.

**Figure 3.1:** (a) NASA-Glenn ANCF rig and (b) cross section view of the rig showing the location of far-field and in-duct microphones.
Chapter 3. HQ-Liner Systems on the ANCF Rig

The fan is instrumented with both far-field and in-duct microphone measurements. As shown in Figure 3.1b, the far-field system consisted of two microphone arrays centered at the inlet and aft duct open planes. Each array had 15 evenly spaced microphones, i.e. inlet array over the 0°-90° sector and aft array over 90°-160°. The picture in Figure 3.1a shows the far-field microphones positioned around the rig. These microphones were used to compute far-field radiated power and radiation directivities for both broadband and tonal noise components.

The in-duct measurements consisted of two rotating microphone rakes used in both the inlet and aft ducts [57]. These rotating rake systems allow measuring the amplitude of the acoustic modes radiating outward for the tonal components only, i.e. 1BPF, 2BPF, and so forth. As indicated in the cross section view in Figure 3.1b, the inlet rotating rake was positioned between the last hard wall spool piece and the duct lip. The aft rake was located just at the duct opening. The picture in Figure 3.2 shows the aft rake. Note that this rake is very close to the edge of the liner spool section (see description in Section 3.3).

![Aft Rake](image)

**Figure 3.2:** View of the aft rotating rake.

### 3.2. HQ-Liner System Design

This section describes the design and fabrication of the HQ-liner system that were tested in the ANCF rig. The criterion to design the HQ-liner systems is based on the concept that the HQ tubes should complement the liner’s attenuation. Since liners are mostly used as broadband attenuators, it was decided that the liner should be designed to reduce the broadband component
between frequencies ranging from 2.5 and 5.5 the BPF tone. The HQ system was then designed to attenuate the 2BPF tone (the BPF is cut-off) and potentially provide additional broadband reduction in particular around and below the 2BPF tone where the liner is less effective. Figure 3.3 illustrates this concept. Unlike in previous efforts where the liner and HQ tubes were designed independently, this is the first attempt to design them concurrently.

![Figure 3.3: Illustration of design criterion for HQ-liner system.](image)

Based on the characteristics of the rig, a few design constrains were considered. Firstly, it was decided that the HQ-liner system was going to be installed on the constant cross section of the aft duct as shown in Figure 3.1b. Secondly, the HQ-liner would be installed in both the inner and outer surfaces of the annular duct, e.g. there are two aft HQ-liner systems. For simplicity, both the inner and outer systems will have the same properties and they will always be tested together. Thus, in the results section when presenting the aft HQ-liner data, it is implicit that both inner and outer treatments were installed. Other parameters that were assumed a priori are: (i) the fan design speed is 1800 rpm; (ii) the liner length is 16in (0.4 m) for both inner and outer surfaces and inlet liner as well.

From a preliminary sizing of the HQ tubes, it was determined that the number of arrays and HQ tubes per array that can be installed are: (i) outer wall of aft section a single array of 40 HQ tubes and (ii) inner wall a single array with 16 HQ tubes. As indicated before, all the tubes have the same dimensions.
As indicated in Chapter 2, the model needs accurate information of the fan disturbance. This information is defined in Section 3.2.1. The design of the Aft HQ-Liner systems is carried out in Section 3.2.2.

### 3.2.1 Fan Disturbance Modeling

To design both the liner and HQ tubes, it is critical to have the fan disturbance accurately defined in terms of the complex modal amplitudes, i.e. magnitude and phase, for the hard wall duct. This information was obtained for both the 2BPF tone and the broadband noise components using different methods. It is important to note that the complex modal amplitudes have to be known accurately since the tonal power reduction in a lined wall environment is strongly dependent on the relative phase between the disturbance modes.

**2BPF Tone:** In the case of the 2BPF tone, the measured complex modal amplitudes by the rotating rake in the hard wall condition were used as disturbance input. At the 2BPF tone, the (4,0) and (4,1) rotor-vane interaction modes are the dominant noise source at the design speed of 1800 rpm. At this fan speed, the cut-on frequency of the (4,0) and the (4,1) mode are 461.6 and 787.7 Hz, respectively. The modal amplitudes are measured at the position of the rotating rakes as shown in Figure 3.4a. For modeling purposes, these modes must be translated to the beginning of the liner which implies a correction in the phase of the modal amplitudes (see Figure 3.4b). Due to the difference in the mode axial wavenumbers, the relative phase between the two modes changes as the modes are translated. The relative phase between the modes, \( \Delta \phi \), at the position where the liner starts is computed using the mode axial wavenumbers and the translated distance \( L \), i.e. distance between the rotating rake and the liner leading edge. This relative phase is as follows

\[
\Delta \phi = \left[ \left( k_{z_{(4,0)}}^{(+)\_rake} \right) - \left( k_{z_{(4,1)}}^{(+)\_rake} \right) \right] L + \Delta \phi_{\text{rake}} \tag{3.1}
\]

where \( \left( k_{z_{(4,0)}}^{(+)\_rake} \right) \) and \( \left( k_{z_{(4,1)}}^{(+)\_rake} \right) \) are the axial wavenumber of the (4,0) and (4,1) modes for hard wall condition, and \( \Delta \phi_{\text{rake}} \) is the relative phase between the modes measured by the rake.
Figure 3.4: (a) Position of the rotating rakes for modal amplitude measurements in the ANCF rig and (b) schematics of the modal amplitude phase correction to the (4,0) and (4,1) modes due to translation of the modes to the beginning of the liners.

**Broadband:** The method used to provide the models with the broadband disturbance is a modified version of the equal modal power and random phase approach. In the standard version the broadband noise contains all modes that are cut-on in the hard wall condition. The modal amplitude of these cut-on modes is obtained by assigning the same power to each one of them.
while the phase among them is randomly assigned. The standard version of this method has been
previously used with quite satisfactory results [58, 66, 67]. In the modified version, additional
modes are further eliminated once they propagate into the liner. If a mode is strongly attenuated
by the liner, it is eliminated from the disturbance. The key reason for this technique is that the
liner attenuation is over-estimated because these strongly attenuated modes tend to hide the
effect of the other more weakly attenuated modes. The criterion used to eliminate these modes is
by using a parameter $\Gamma^*$ as proposed by Morse and Ingard [63]

$$\Gamma^* = \text{Re} \left( \frac{(k_{mn}^*)^2}{k_0^2} \right) \geq \Gamma$$  \hspace{1cm} (3.2)

where $k_{mn}^{(+)}$ is the mode eigen-wavenumber in the lined section; $k_0$ is the free field wave number;
and $\Gamma$ is the parameter which according to Morse should be set to one. If the parameter $\Gamma^*$ for a
mode is larger than $\Gamma=1$, the mode is the mode is strongly attenuated by the liner and eliminated
from the disturbance. However, it was found in the present work that the value of $\Gamma$ is problem
dependent, i.e. flow Mach number and in particular the liner acoustic properties, and it should be
adjusted for better results.

A similar method to the one just described has also been recently implemented [33]. In this
work, Alonso worked directly with the imaginary component of the modal axial wave number
$k_z^+$, e.g. the parameter that determines the attenuation of the mode in the lined duct. In this case,
the parameter $\Gamma^*$ is defined as

$$\Gamma^* = \text{Abs} \left( \text{Im}(k_z^{(+)}) \right) \geq \Gamma$$  \hspace{1cm} (3.3)

Those modes with an imaginary component larger than $\Gamma$ are highly attenuated and
therefore eliminated from the disturbance.

It is important to note that during the HQ-liner design process the standard equal power
method was used, e.g. no hard wall cut-on modes were eliminated. However, during the model
validation effort both Morse’s and Alonso’s methods were implemented and compared against
experiments.

Once the disturbance for both pure tones and broadband component has been defined, the
HQ-liner performance was analyzed using two approaches. The first one neglected transmissions
and reflections effects at the liner/hard wall interfaces also used in the HQ-liner design process. The second technique included the effects introduced by the change in wall impedance at the liner/hard wall interfaces. In order to assess the influence of these impedance discontinuities, both techniques were compared against experiments.

Table 3-1 summarizes the disturbance information provided to the inlet and aft radiation control models.

Table 3-1: Fan disturbance definition for design of an HQ-liner system for the aft section of the rig at 1800 rpm.

<table>
<thead>
<tr>
<th>Noise Component</th>
<th>Description</th>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2BPF tone</td>
<td>(4,0) – L_W = 111.2 dB (4,1) – L_W = 104.0 dB Relative phase 100°</td>
<td>Assume tone dominated by interaction tones (4,0) and (4,1). Relative phase assumed at liner’s leading edge. Transmission and reflection was not considered at any interface</td>
</tr>
<tr>
<td>Broadband</td>
<td>Include all cut-on modes on hard wall condition. Equal power distribution and random phase. At this point no discriminator was used.</td>
<td>A FORTRAN routine was used to assign each mode with a particular phase. The routine uses a seed number which was set at 1234. Repeatability due to different seed numbers is analyzed in Appendix D. Transmission and reflection was not considered at any interface</td>
</tr>
</tbody>
</table>

3.2.2 AFT HQ-Liner Design

This section describes the design of the aft HQ-Liner system. Due to cost constraints, a liner already available from previous experiments and the same HQ tubes used in the aft section were used for testing the inlet HQ-liner system. The design method consisted of three main steps which are explained next.

1) Broadband Liner Performance

The first step in the process was to investigate the liner broadband attenuation for a range of core depths and resistances, e.g. liner parametric analysis. In this study, the liner normalized
impedance was defined as \( \hat{Z} = \hat{R} - \cot(k_o d) \) where \( k_o \) is the free-field acoustic wavenumber, \( d \) is the liner core depth, and \( \hat{R} \) is the normalized resistance assumed to be constant. Note that the liner impedance definition does not account for the mass-like reactive effect of the liner wire mesh and perforate screen. Thus, the variable \( d \) does not represent the physical core depth of the liner. The liner properties can also be defined in terms of its tuning frequency \( f_{\text{liner}} \) (frequency where the reactive part of the impedance vanishes) and normalized resistance \( \hat{R} \). The tuning frequency of the liner without including the wire mesh mass-like effect is given by \( f_{\text{liner}} = \frac{c}{4d} \).

Figure 3.5 shows the overall broadband attenuation due to the liner as a function of the core depth and resistance. The results reported is the overall noise reduction over the frequency range encompassing fan orders 40-88 (i.e. from 2.5 to 5.5 BPF or 1200-2640 Hz frequency range at 1800 rpm). The results show that the liner performance is basically insensitive over a wide range of liner parameters, i.e. core depths \( 0.9'' \le d \le 1.2'' \) and resistances \( 1.0 \le \hat{R} \le 1.8 \) lead to \( \sim 8.5-10 \) dB reduction.

Figure 3.5: Overall broadband attenuation between 2.5 and 5.5 BPF as a function of liner core depth \( d \) and normalized resistance \( \hat{R} \). Note that the liner performance is rather insensitive within a range of resistances and core depths (within dashed-line box).
2) Broadband HQ-Liner Performance

The second step was to determine the impact of the HQ tubes on the broadband attenuation due to HQ-liner systems. To this end, the broadband attenuation for the HQ-liner was computed for two core depths \(d = 1.2'' \text{ & } 1.05''\) and two liner resistances \(R = 1.0 \text{ and } 1.4 \rho c\) within the range of good performance of the liner. A preliminary HQ tube design was selected to investigate the performance of the combined HQ-liner system [64]. The HQ dimensions for these study was: mean length \(L = 12.3\text{ in } (0.3124\text{ m})\), interface distance \(\ell = 7.62\text{ in } (0.1935\text{ m})\) and a cross section area of \(7.56\text{ in}^2 (0.0049 \text{ m}^2)\). Note that the HQ tube mean length \(L\) includes the liner core depth \(d\) (see Figure 3.6).

![Figure 3.6: Schematic showing HQ tube dimensions.](image-url)

Figure 3.7a and 3.6b show sound power reduction for both HQ-liner and liner systems as a function of frequency for \(d = 1.2\text{ in}\) selected resistances. From Figure 3.7a, the HQ tubes basically improved the broadband attenuation at low frequencies \((\leq 1400 \text{ Hz or fan order } \leq 46)\) by about 1-2 dB. As compared to the higher resistance liner, lowering the resistance improved the liner performance near 1400 Hz while decreasing the attenuation at higher frequencies. On the other hand, the HQ system also yielded slightly better attenuations since the HQ is a resonator device, i.e. its performance improves as the “damping” of the HQ tubes decreases which is dictated by the liner resistance. The results for the smaller core depth of 1.0” are very similar except that the frequency of optimum attenuation is shifted slightly to higher frequencies.

In order to summarize the results into a simple noise metric, the overall broadband sound power reduction for two frequency ranges for the cases investigated is presented in Figure 3.8.
The frequency ranges are (i) from 2.5-5.5 BPF and (ii) from 1.5-5.5 BPF. From this figure, it is concluded that the liner with higher resistance yields only marginally better attenuation at higher frequencies. On the other hand, the HQ system provides more noise reduction when combined with a lower resistance liner. These two effects tend to cancel each other out making the HQ-liner broadband performance relatively insensitive to the liner resistance selected (at least within the range of parameters investigated). However, it is important to note that the improvement provided by the HQ tubes is higher than the loss of liner performance. This result suggest that a liner with lower resistance \( R = 1.0 \rho c \) is better.

![Figure 3.7](image)

**Figure 3.7:** Broadband power reduction for liner and HQ-liner systems. Liner properties (a): \( d = 1.2" \) and \( R = 1.4 \rho c \) - (b) \( d = 1.2" \) and \( R = 1.0 \rho c \)
3) 2BPF Tone HQ-Liner Performance

The final step in the design of the aft HQ-liner systems focused on determining the optimum combination for maximum attenuation of the 2BPF tone component. To this end, the optimum HQ tube parameters given by the centerline length $L$ and interface distance $\ell$, i.e.

**Figure 3.8:** Broadband power attenuation due to different HQ-Liner systems for (a) 2.5-5.5 BPF and (b) 1.5-5.5 BPF ranges. HQ-Liner reduction broken down into liner reduction and HQ contribution relative to the liner.
distance between tube’s openings, must be determined. The cross sectional area is not a design variable and here it is defined by the number of tubes used in the array, e.g. 40 and 16 in the outer and inner wall arrays. The optimum HQ tube dimensions were determined for all four liners investigated.

![Image of HQ-liner system](image)

**Figure 3.9:** 2BPF tone HQ-liner power reduction as a function of tube dimensions for a liner with core depth $d = 1.2$ in and resistance $R = 1.0\rho c$.

From this optimization process, the disturbance for the 2BPF tone were the blade-vane interaction tones (4,0) and (4,1) as defined in Table 3-1. The results showed that for all four liners the optimum HQ dimensions were $L = 12.5$ in and $\ell = 9$ in. As an example of this optimization process, Figure 3.9 shows the HQ-liner attenuation for a range of tube parameters for the liner with $d = 1.2''$ and $R = 1.0\rho c$. This figure also shows that there is a range of tube dimensions that leads to basically the same performance (indicated by the dashed-line circle, i.e. $12'' \leq L \leq 13''$ and $8'' \leq \ell \leq 9.5''$), i.e. like liners the HQ is a robust attenuation device.

The design approach described was used for the HQ-Liner to be installed in the aft duct. Based on the results presented here, the designed system is defined in Table 3-2. Note that the core depth $d = 1.0$ in in this table is the actual physical liner’s core depth corrected to account for...
the mass-like effect induced by the reactance of the liner’s screen. The reason for this correction is to ensure that both the modeled liner and the actual physical liner have the same resonance frequency. Here, the design produced a liner with a resonance frequency of 2813 Hz (resulting from \( d = 1.2'' \) and assuming a speed of sound \( c = 343 \text{ m/s} \)). In order to obtain the proper core depth the sum of both the liner and the screen reactances have to be zero

\[
0 = b - \cot\left(\frac{2\pi f_{\text{liner}}}{c} d\right)
\]

where \( b \) is the non-dimensional reactance of the screen at the liner resonance frequency \( f_{\text{liner}} = 2813.0 \text{ Hz} \), \( c \) is the speed of sound and \( d \) is the physical liner core depth that satisfies equation 3.3. It is important to mention that since the liner is a new design, the screen impedance is not known, hence, it has to be assumed or taken from previous liners.

Figure 3.10 shows the normalized impedance of the liner used in the aft duct. It is also important to note that the mean length \( L \) of the tubes includes the liner core depth as indicated in Figure 3.6.

### Table 3-2: Optimum HQ-Tube and Liner physical properties for the aft section of the ANCF rig.

<table>
<thead>
<tr>
<th>HQ Tubes</th>
<th>Liner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Sec. Area [in²]</td>
<td>Centerline Length, ( L ) [in]</td>
</tr>
<tr>
<td>5.96</td>
<td>13.5</td>
</tr>
</tbody>
</table>
In case of the inlet, a liner was already available. As a consequence, only the third step of the parametric study was applied. The objective was to find an adequate set of tubes for this particular liner. The results of the analysis revealed that the tubes that would provide the best performance were similar to those used in the aft. Therefore, due to cost constraints it was decided to implement in the inlet the tubes designed for the aft with a mean length shortened by as much as twice the difference between the liners core depth, i.e. 0.4 in. The final dimensions of the resulting inlet HQ-Liner system are shown Table 3-3. Figure 3.11 shows the normalized impedance of the liner used in the inlet.

**Figure 3.10:** Normalized impedance of the liner designed for the aft section of the NASA rig.

**Table 3-3:** Optimum HQ tube and liner properties for the inlet section of the ANCF rig.

<table>
<thead>
<tr>
<th>HQ Tubes</th>
<th>Liner</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.96</td>
<td>13.1</td>
</tr>
</tbody>
</table>
Finally it is important that the array of HQ tubes represents impedance discontinuities to the disturbance sound field and the potential for scattering effects. The circumferential modes into which energy will be scattered are defined by

\[ m_s = m_d \pm k \times nHQ \]  

where \( m_d \) is the mode present in the disturbance, \( nHQ \) is the number of HQ tubes used in the array, and \( m_s \) is the mode into which energy is been scattered as a consequence of the interaction between the disturbance and the HQ tubes. As already explained, at the 2BPF at 1800 rpm the two most dominant modes in the disturbance are the \((4,0)\) and \((4,1)\) in both inlet and aft sections of the ANCF rig. Accordingly, the lowest circumferential mode into which energy will be scattered is \( m_s = -12 \), which, at the 2BPF at 1800 rpm is cut-off. The previous analysis shows that the performance of the HQ tubes will not be affected by scattering effects.

**HQ-Liner Systems Fabrication**

The picture in Figure 3.12a shows the aluminum spool section for the bypass duct of the ANCF rig that was fabricated to implement the HQ-liner system. The spool piece structure was made in two halves to allow its implementation. Similar aluminum structures were built for the
inlet and bypass inner duct liners. The integration of the liners and the supporting structure is illustrated in Figure 3.12b and c. The liner (made in two halves as well) was bonded to the two aluminum supporting rings and the beams connecting the rings. The liner seams at the junction of the two halves were around a 1/8” thick providing a nearly uniform liner free of discontinuities.

Figure 3.12: (a) Inlet/Aft aluminum rig; (b) inlet/aft liners attached to aluminum rig and (c) center body liner attached to its corresponding aluminum rig.

The HQ tubes are mounted on the back of the liners with minimum impact on the liner structure. The integration of the HQ tubes to the liner had to be carried out in such a way as to form a sealed waveguide for the system to work as intended, i.e. without acoustic leaks. In order to overcome the leakage problems detected during the previous testing, the integration scheme depicted in Figure 3.13a was implemented for the outside wall liners. The liners were built with the back plate made of a high percentage open area (POA) perforate. This back plate was blocked using aluminum tape. A 1/8 in thick plastic curved plate (vacuum formed in ABS) was
then bonded to the aluminum tape as shown in Figure 3.13b. This prevented acoustic leakage from the back of the liner. Sealant foam (closed cell foam) was bonded to the tube’s edge to prevent acoustic leakage between the tube and plastic panel as shown in Figure 3.13c. The HQ tubes were then mounted to the plastic panels using two screws at the tube ends. A 2-straps-with-thumb-screws mechanism was then used to apply pressure on the tube to seal the tube’s edge as seen in Figure 3.13d. The same strapping method, with some modifications, was used for the liner of the aft inner duct wall, i.e. center body. Figure 3.14 shows straps used in the center body.

Figure 3.13: (a) Schematic of HQ-Liner integration; (b) plastic plate bonded to the aluminum tape; (c) HQ tube with sealing foam and (d) inlet and aft HQ-Liner systems integrated.
The HQ-liner spool pieces were also designed for easy removal and replacement of the tubes with devices that blocked the back opening of the liner as shown in Figure 3.15c. In this way, the HQ-liner spool piece was able to be reconfigured as a liner. In addition, a thin tape (3M-5413 gold) bonded to the liner wire mesh (wet surface) was used to block the liner yielding a hard wall condition (Figure 3.15a). Furthermore, cutting an opening on the tape at the precise location of the HQ tubes resulted in the HQ tubes installed on a hard wall duct as shown in Figure 3.15b.

Figure 3.15: (a) Tape covering the liner to simulate hard wall; (b) HQ-tape has been attached to the aft and center body liners to simulate HQ tubes on hard wall and (c) HQ tubes in the process of being replaced by blocking devices so that the liner case can be tested.
3.3. **Test Configurations**

The HQ-liner systems developed and fabricated in the previous section were tested in the ANCF rig in various configurations. There were two main types of configurations tested. In the first one, identified as “**Single HQ-Liner Spool Configurations**” and presented in Section 3.4.1, the inlet and aft HQ-Liner systems designed in the previous section were installed in the inlet and aft sections of the rig respectively. The aft outer and inner treatments were always tested together hence “aft HQ-liner system” always refers to treatment in both walls of the aft duct.

In the second configuration, denoted as “**Multiple HQ-Liner Spool Configuration**” both HQ-Liner spool pieces were installed in the inlet and it is presented in Section 3.4.2.

**Single HQ-liner Spool Configurations**

Figure 3.11 describes the configuration when single HQ-liner spool pieces were installed. As indicated before, the aft HQ-liner system included acoustic treatment of both outer and inner (or center body) walls positioned at the same axial location, i.e. sector of constant hub-to-tip-ratio as sketched in Figure 3.16b. The inlet HQ-Liner system was placed on the second spool piece 12.0in upstream the face of the fan as indicated in Figure 3.16a.

The inlet and aft HQ-liner systems were tested independently as well as together defining three basic test configurations: (i) inlet treated and hard wall (HW) aft, (ii) hard wall inlet and aft treated, and (iii) both inlet and aft treated. These three basic acoustic treatment configurations are denoted by the acronym ATC 1 through 3, respectively. Each of the three basic test configurations (ACT-1 through 3) were configured as hard wall (HW), HQ-tubes on a hard wall duct (HQ-HW), just a liner (Liner), and HQ tubes with the liner (HQ-liner). These sub-configurations define 11 test cases as listed in Figure 3.16. Every one of them was tested at the four fan speeds of 1500, 1600, 1700, and 1800 rpm. The case of hard wall inlet and aft ducts is referred to as the “baseline configuration”.

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### Basic Configuration ATC – 1: Inlet Treated and Aft HW

<table>
<thead>
<tr>
<th>Case</th>
<th>Inlet</th>
<th>Aft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HW</td>
<td>HW</td>
</tr>
<tr>
<td>2</td>
<td>Liner</td>
<td>HW</td>
</tr>
<tr>
<td>3</td>
<td>HQ-HW</td>
<td>HW</td>
</tr>
<tr>
<td>4</td>
<td>HQ-Liner</td>
<td>HW</td>
</tr>
</tbody>
</table>

![Diagram](image)

### Basic Configuration ATC – 2: Inlet HW and Aft Treated

<table>
<thead>
<tr>
<th>Case</th>
<th>Inlet</th>
<th>Aft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HW</td>
<td>HW</td>
</tr>
<tr>
<td>5</td>
<td>HW</td>
<td>Liner</td>
</tr>
<tr>
<td>6</td>
<td>HW</td>
<td>HQ-HW</td>
</tr>
<tr>
<td>7</td>
<td>HW</td>
<td>HQ-Liner</td>
</tr>
</tbody>
</table>

![Diagram](image)

### Basic Configuration ATC – 3: Inlet Treated and Aft Treated

<table>
<thead>
<tr>
<th>Case</th>
<th>Inlet</th>
<th>Aft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HW</td>
<td>HW</td>
</tr>
<tr>
<td>8</td>
<td>Liner</td>
<td>Liner</td>
</tr>
<tr>
<td>9</td>
<td>HQ-HW</td>
<td>HQ-HW</td>
</tr>
<tr>
<td>10</td>
<td>HQ-Liner</td>
<td>HQ-Liner</td>
</tr>
</tbody>
</table>

![Diagram](image)

**Figure 3.16:** Description of test configurations; (a) Inlet treated and Aft HW (ATC-1), (b) Inlet HW and Aft treated (ATC-2) and (c) Inlet treated and Aft treated (ATC-3).
Inlet data, i.e. ATC-1, was used to validate the inlet model on pure tone attenuation. Lack of information on the disturbance had previously restricted the validation of the inlet radiation control model to only broadband attenuation [33]. Aft data, or ATC-2, was used to assess the performance of HQ-Liners installed in bypass annular sections as well as to validate the noise prediction model for annular ducts developed in Chatter 2. The testing of ATC-3 was intended to reveal any mutual interference between the inlet and aft acoustic treatments.

**Multiple HQ-liner Spool Configurations**

The two systems designed in section 3.1 were installed in the inlet section as indicated in Figure 3.17. The main objectives of this test are to assess the influence of:

1) adding a second HQ-Liner system,
2) the liner resistance on the performance of the HQ tubes.

By using the same techniques described in the previous section, the two systems were able to be quickly reconfigured into the five test cases shown in Figure 3.17. The first configuration features both liners taped, i.e. HW/HW, so that baseline case could be measured. By comparing cases 2 and 3, i.e. HW/HQ-Liner and HQ-Liner/HQ-Liner, the effects of extending the lined area as well as the number of HQ arrays can be assessed. In configuration number 4, the resistance of the 1.7$\rho c$ liner has been locally reduced by removing the liner wire mesh at the tube openings as shown in Figure 3.18. The removal of the liner wire mesh at the tube openings reduced the resistance from 1.7$\rho c$ to 0.21$\rho c$. It should be remembered that the HQ tubes are resonator devices; as such, the high damping introduced by the liner wire mesh can seriously degrade their performance. By comparing configurations 3 and 4 the influence of the liner resistance on the performance of the HQ tubes can be determined. In the last configuration, the HQ tubes have been removed from the back of the liners so that the liner alone could be tested. As explained in the previous section, the openings resulting from the tubes removal were blocked with the specially designed ribs (see Figure 3.15c). Note that in this test the 1.7$\rho c$ liner has the wire mesh removed at the tube openings. By comparing configurations 4 and 5, the contribution of the HQ tubes relative to the liner can be established.
### Chapter 3. HQ-Liner Systems on the ANCF Rig

#### Figure 3.17: Different multiple spool piece test configurations.

<table>
<thead>
<tr>
<th>Config.</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; Spool piece with 1.7pc liner</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; Spool piece with 1.0pc liner</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HW</td>
<td>HW</td>
</tr>
<tr>
<td>2</td>
<td>HW</td>
<td>HQ-Liner</td>
</tr>
<tr>
<td>3</td>
<td>HQ-Liner</td>
<td>HQ-Liner</td>
</tr>
<tr>
<td>4</td>
<td>HQ-Liner (resistance locally reduced)</td>
<td>HQ-Liner</td>
</tr>
<tr>
<td>5</td>
<td>Liner (resistance locally reduced)</td>
<td>Liner</td>
</tr>
</tbody>
</table>

#### Figure 3.18: Two HQ-liner systems placed in the inlet section of the NASA ANCF.
3.4. Experimental results

This section presents the experimental results. Results corresponding to the “Single HQ-liner spool configurations” are presented in the next section. Results corresponding to the Multi HQ-liner spool configuration” are presented in Section 3.4.2.

3.4.1 Single HQ-liner Spool Configurations

For the single HQ-liner spool configurations depicted in Figure 3.16, results in the form of figures and tables are organized in a 2-rows by 2-columns layout defining 4 cells. The results for ATC-1 and -2 are presented in rows 1 and 2, respectively; whereas inlet and aft data are presented in the left and right columns respectively. As an example of this layout, the aft data for the HW inlet and treated aft will be found in the 2nd row and right column cell. Results corresponding to ATC-3, i.e. HQ-liner systems simultaneously installed in inlet and aft sections of the rig are presented in separately in 3.4.1.3.

In order to check repeatability and to establish confidence bounds, the hard wall and liner cases in both inlet and aft ducts were measured twice, i.e. configurations 1 and 9 in Figure 3.16. The results from this repeatability study are described in detail in Appendix E. Other cases were not measured due to time and cost constrains.

3.4.1.1 Far-Field Data

The present section is divided into two main subsections; one for the 2BPF tone and the second for the broadband component. In each of these subsections, results are organized according to the acoustic treatment implemented, i.e. liner, HQ-liner and HQ-HW. Results are presented in terms of sound power level reductions at all fan speeds tested. Power levels have been estimated from far-field sound pressure spectra. In addition, power directivity patterns, i.e. $P_{rms}^2$ measured by each microphone, multiplied by the microphone associated area and divided then by $\rho c$ is also analyzed. Power spectrum for the design speed of 1800 rpm is included for the broadband component.
2BPF Tone Results

The far-field sound pressure spectra were used to estimate radiated sound power over the inlet and aft sectors independently for the 2BPF tone.

Liner Results: Figure 3.19 shows the liner sound power reduction for the 2BPF tone at 1500, 1600, 1700 and 1800 rpm. As shown in Figure 3.19a, the inlet liner did provide power reduction in the inlet far-field. At the design speed of 1800 rpm, the 2BPF was attenuated by 3.7 dB. Figure 3.19b shows the performance of the inlet liner in the aft sector. As indicated, there has been some noise increment, e.g. ~1 dB, at 1500 and 1600 rpm. Figure 3.19d shows power reduction in the aft far-field sector due to the liners. Significant sound power reductions are achieved, especially at 1500 rpm. The attenuation of the 2BPF tone at 1800 rpm was 3.9 dB. The performance of the aft liners in the inlet far-field sector shows some noise reduction, e.g. ~1.5 dB at 1600 and 1800 rpm, and some noise increment of as much as ~1.5 dB at 1700 rpm.

Figure 3.20 shows power directivity patterns for the 2BPF tone at 1800 rpm for the hard wall and liner cases. Measurements show that the inlet liner was most effective over the 10°-30° and 50°-90° far-field sectors with a very small reduction over 30°-50° sector (see Figure 3.20a). Unlike in the inlet, the aft liner provided near uniform attenuation over all aft directions (see Figure 3.20b).

The directivity patterns in Figure 3.20 provide some additional insight. The disturbance at the 2BPF tone at 1800 rpm is characterized by the rotor-stator interaction modes (4,0) and (4,1). The hard wall inlet directivity shows two clearly defined radiation lobes at about 30° and 60°, respectively. It is likely that these two radiation lobes at 30° and 60° are due to the (4,0) and (4,1) modes, respectively. The liner directivity shows a single lobe centered at ~40° suggesting that the liner mainly attenuated the (4,1) mode. On the other hand, the rather uniform attenuation shown in the aft sector is an indication that it is likely that both the (4,0) and (4,1) modes were reduced by the liner. These results are further confirmed from the in-duct modal data presented in section 3.4.1.2.
Figure 3.19: Sound power attenuation of the 2BPF tone due to the liner for different fan speeds. (a) and (b) liner positioned in the inlet and (c) and (d) liner positioned in the aft duct.
Figure 3.20: Directivity patterns of the 2BPF tone for HW and Liner configurations at 1800 rpm for the liner placed in the (a) inlet and (b) aft duct.
HQ-Liner Results: Sound power reduction due to HQ-Liner systems is presented in Figure 3.21. The performance of the HQ-Liner is divided into the contribution of the liner and HQ tubes separately, i.e. sum of the two gives the attenuation of the HQ-liner combination. The contribution of the HQ tube has been estimated by subtracting the results for the liner from the HQ-liner data. The results in Figure 3.21a and b show the inlet HQ-Liner power reduction in the inlet far-field sector. These figures show that the attenuation is mostly provided by the liner at many of the test conditions. At the design fan speed (1800 rpm), the inlet HQ-Liner produced the highest reduction with a power attenuation of 5.0 dB with a contribution from the HQ tubes of 1.4 dB. For the aft HQ-Liner system, the results are shown in Figure 3.21c and d. They clearly indicate a better performance of the HQ-tubes. The HQ tubes improved the performance of the liner for all cases. In particular, the aft HQ-liner system provided a power attenuation of 6.5 dB with a contribution from the HQ tubes of 2.6 dB at the design fan speed.

Figure 3.22 shows directivity patterns for the 2BPF tone at 1800 rpm for the hard wall and HQ-liner cases. In order to more readily observe the HQ contribution, the liner case has also been included. For the HQ-liner in the inlet (ATC-1), the maximum attenuation occurs in the 20°-30° and 50°-90° far-field sectors. The HQ tubes degrade the attenuation mainly in the 30°-60° sector. For the aft HQ-liner system (ATC-2), the far-field attenuation is achieved in all direction with the HQ tubes contributing mainly in the 130°-150° sector.
Chapter 3. HQ-Liner Systems on the ANCF Rig

**Figure 3.21:** Sound power attenuation of the 2BPF tone due to the liner and HQ tube contribution to attenuation (measured relative to the liner) for different fan speeds. (a) and (b) HQ-liner positioned in the inlet and (c) and (d) HQ-liner positioned in the aft duct.
Figure 3.22: Directivity patterns of the 2BPF tone for HW, Liner and HQ-liner configurations at 1800 rpm for acoustic treatments placed in the (a) inlet and (b) aft duct.
**HQ-HW Results**: Sound power attenuation for the HQ-HW case is shown in Figure 3.23. When the HQ tubes installed in the inlet (ATC-1) the best attenuation is achieved at 1800 rpm with 1.9 dB reduction. At the other fan speeds, attenuation oscillates around 0.5 dB except at 1700 rpm where the HQ tubes seem to slightly increase the noise (see Figure 3.23a and b). A rather surprising result is the significant noise increase towards the aft sector (see Figure 3.23b). The performance of the HQ-HW case installed in the aft duct (ATC-2) is shown in Figure 3.23c and d. It is observed that reduction was achieved at all fan speeds. At 1800 rpm the 2BPF tone has been attenuated by as much as 2.0 dB.

Figure 3.24 shows power directivity patterns for the 2BPF tone at 1800 rpm. Similar to the liner, the HQ tubes in the inlet provided the best reduction over the sideline 45°-90° far-field sector. As in the case of the liner, this radiation pattern once again indicates that the HQ tubes most likely attenuated the (4,1) mode more than the (4,0). In the aft far-field sector, the attenuation is nearly uniform at all directions, i.e. 90°-160° sector, similar to the liner case.
Chapter 3. HQ-Liner Systems on the ANCF Rig

Figure 3.23: Sound power attenuation of the 2BPF tone due to HQ tubes on hard wall for different fan speeds. (a) and (b) HQ tubes positioned in the inlet and (c) and (d) HQ tubes positioned in the aft duct.
Figure 3.24: Directivity patterns of the 2BPF tone for HW and HQ tubes on HW configurations at 1800 rpm for the HQ tubes placed in the (a) inlet and (b) aft duct.
Broadband Results

Broadband data is presented in a similar manner as the pure tone in the previous section. Broadband data has been divided in broadband centered on the 2BPF tone, targeted mainly by HQ tubes, and broadband above 2.5 BPF, targeted mainly by the liner. For convenience, broadband centered on the 2BPF is referred to as 2BB. These results are grouped according to the treatment used, i.e. liner, HQ-liner and HQ-HW cases. As a general observation, it is important to note that all broadband results are much more consistent when compared to tonal results in the previous section.

Liner Results: Liner results are presented in Figure 3.25. The inlet liner case (ATC-1) yielded good sound power reduction in the inlet for the 2BB with levels ranging between 3 and 4 dB as seen in Figure 3.25a. It is also interesting to note that there is a small attenuation in the aft sector with reductions ranging between 0.5 and 1 dB (Figure 3.25b). The aft liner (ATC-2) also yielded good attenuation in the aft sector. The 2BB reduction starts at 2.2 dB at 1500 rpm and steadily increases up to 3.2 dB at 1800 rpm. Unlike the inlet liner, the aft liner yielded a small increase in power over the inlet sector as shown in Figure 3.25c.

Figure 3.26 shows power directivity pattern for the 2BB component at 1800 rpm, i.e. design point. The results in this figure reveal that the liner yielded a nearly uniform reduction of noise over the far-field sectors for all basic treatment configurations. It should be noted that in both the inlet and aft sectors, the reductions near the rig axis are a few decibels lower than over the other directions, i.e. 0°-25° in the inlet and >150° in the aft.

In order to show in more detail the frequency dependence of the liner broadband attenuation, Figure 3.27 shows the power spectrum for the HW and Liner cases for ATC-1 and -2 at 1800 rpm. Figure 3.27a shows that the inlet liner provided an almost constant power reduction of ~5 dB in the inlet between fan orders 26 through 80 (or between 840 and 2400Hz). The attenuation levels rolls off at higher frequency range. On the other hand, the aft liner does show a clear frequency range of best performance around order 56 (1680 Hz) with attenuations ~ 8 dB. The performance quickly decays at lower frequency and no noticeable attenuation below order 32 (2BPF tone). The roll off at higher frequencies is, however, less pronounced. Since the goal of
the liners was to reduce broadband noise levels between 2.5 and 5.5BPF (or between orders 40 and 88), Table 3-4 shows overall broadband reduction levels in this range for all treatment cases. The results in this table suggest a slightly better performance of the aft liners.

Table 3-4: Overall liner sound power reduction [dB] in the 2.5 ~ 5.5 BPF range at 1800 rpm.

| ATC Inlet | Inlet: Treated - Aft HW (ATC-1) | 3.8 | 0.7 |
| Inlet: HW - Aft Treated (ATC-2) | 0.0 | 5.6 |

![Figure 3.25: Sound power attenuation of the 2BB broadband component due to the liner for different fan speeds. (a) and (b) liner positioned in the inlet and (c) and (d) liner positioned in the aft duct.](image)
Figure 3.26: Directivity patterns of the 2BB broadband component for HW and Liner configurations at 1800 rpm for the liner placed in the (a) inlet and (b) aft duct.
Figure 3.27: Power spectrum of the broadband component for HW and Liner configurations at 1800 rpm. (a) and (b) liner positioned in the inlet and (c) and (d) liner positioned in the aft duct.
HQ-Liner Results: Sound power attenuation of the 2BB component for the HQ-Liner case is presented in Figure 3.28. Just like for the 2BPF tone, the HQ-Liner performance has been broken down into liner contribution and HQ tube contribution relative to the liner. The HQ-Liner reduction is then given by the sum of the two. From Figure 3.28, it can be observed good performance of the HQ-liner system in both the inlet and aft sectors. However, there are two clear facts. The first observation is that the HQ tubes when combined with the inlet liner, degraded the performance of the liner, i.e. HQ reduces liner attenuation by 0.2-0.6 dB (see Figure 3.28a). It is rather surprising that the HQ tubes resulted in an increase of sound power in the aft sector ranging from 0.7-1.2 dB (see Figure 3.28b). The second observation is that the HQ tubes, now combined with the aft liners clearly improved their performance. Figure 3.28d shows that the HQ tubes improved the liner performance by 1.3-1.5 dB at the 2BB component. In the inlet sector (Figure 3.28c), the aft HQ also resulted in a slight liner performance improvement.

Figure 3.29 shows power directivity of the broadband component around the 2BPF at 1800 rpm for the HW and HQ-liner systems. To more easily observe the HQ contribution, the directivity for the liner case is also included. As it can be observed, The HQ-Liner system provided less attenuation than the liner through the entire inlet sector from 0° to 90°. On the other hand, in the aft sector, the HQ tubes improved the attenuation provided by the liner at every location measured.

Table 3-5 presents the overall broadband power reduction in the 2.5-5.5 BPF range (between fan orders 40 and 88) at 1800 rpm. The net impact of the HQ on the liner is included in parentheses. As it can be seen, the HQ tubes installed in the aft slightly improved the liner performance in the aft sector. On the other hand, in the inlet, HQ tubes degraded the liner reduction by 0.5 dB.

Figure 3.30 shows broadband power spectrum for HW, Liner, and HQ-Liner at 1800 rpm. This figure confirms that the HQ tubes in the aft provided the best performance around the 2BPF tone, i.e. 2BB broadband component, and slightly improved the liner performance between 2.5-5.5 BPF. In the inlet, it can be observed that the tubes degraded the liner.
Table 3-5: Overall HQ-liner Sound power reduction [dB] in the 2.5 ~ 5.5BPF range at 1800 rpm

<table>
<thead>
<tr>
<th>ATC Inlet: Treated - Aft HW (ATC-1)</th>
<th>ATC Inlet: HW - Aft Treated (ATC-2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet: HW - Aft Treated (ATC-2)</td>
<td>-0.1(-0.1)</td>
</tr>
</tbody>
</table>

Figure 3.28: Sound power attenuation of the 2BB broadband component due to the liner and HQ tube contribution (measured relative to the liner) for different fan speeds. (a) and (b) HQ-liner positioned in the inlet and (c) and (d) HQ-liner positioned in the aft duct.
Figure 3.29: Directivity patterns of the 2BB broadband component for HW, Liner and HQ-Liner configurations at 1800 rpm for acoustic treatments placed in the (a) inlet and (b) aft duct.
Inlet

![Power spectrum of the broadband component for HW, Liner and HQ-liner configurations at 1800 rpm. (a) and (b) acoustic treatments positioned in the inlet and (c) and (d) acoustic treatments positioned in the aft duct.](image)

**Figure 3.30:** Power spectrum of the broadband component for HW, Liner and HQ-liner configurations at 1800 rpm. (a) and (b) acoustic treatments positioned in the inlet and (c) and (d) acoustic treatments positioned in the aft duct.
**HQ-HW Results:** The sound power attenuation of the 2BB component for the HQ-HW case is presented in Figure 3.31. The HQ tubes yielded reduction at all fan speeds. In case of the HQ tubes installed in the inlet (Figure 3.31a) attenuation levels range from 1.1 dB at 1500 rpm to 1.6 dB at 1800 rpm. In case of the aft HQ system, attenuation levels are fairly uniform oscillating in the 1.5-1.8 dB range.

The radiation directivity for the 2BB at 1800 rpm is shown in Figure 3.32. It can be observed that the HQ tubes provided the best attenuation in the 25°-90° inlet and 90°-150° aft sectors.

The broadband attenuation as a function of the frequency is presented in Figure 3.33. Both the HW and HQ-HW broadband power spectra at 1800 rpm are included. The maximum HQ attenuation occurs around the fan order 32 (the HQ tube design point). In order to quantify this performance, the overall broadband power reduction in the 2.5-5.5 BPF range is shown in Table 3-6. The aft HQ system yielded a very good reduction at 1.5 dB.

Table 3-6: Overall HQ-HW sound power reduction [dB] in the 2.5 ~ 5.5 BPF range at 1800 rpm.

<table>
<thead>
<tr>
<th>ATC</th>
<th>Inlet</th>
<th>Exhaust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet: Treated - Aft HW (ATC-1)</td>
<td>0.8</td>
<td>-0.2</td>
</tr>
<tr>
<td>Inlet: HW - Aft Treated (ATC-2)</td>
<td>0.1</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Figure 3.31: Sound power attenuation of the 2BB broadband component due to HQ tubes placed on hard wall for different fan speeds. (a) and (b) HQ tubes positioned in the inlet and (c) and (d) HQ tubes positioned in the aft duct.
Figure 3.32: Directivity patterns of the 2BB broadband component for HW and HQ tubes on HW configurations at 1800 rpm for HQ tubes placed in the (a) inlet and (b) aft duct.
Figure 3.33: Power spectrum of the broadband component for HW and HQ tubes placed on hard wall configurations at 1800 rpm. (a) and (b) HQ tubes positioned in the inlet and (c) and (d) HQ tubes positioned in the aft duct.
3.4.1.2 In-duct data

Part of the measurements collected at NASA Glenn consisted of in-duct modal data gathered using rotating rakes in both inlet and aft ducts. The microphone rotating rakes allow for the determination of the amplitudes of the acoustic modes propagating in the inlet (upstream) and the aft (downstream) at the BPF tone and harmonics. This section presents the results from these in-duct measurements for the 2BPF tone only. The next subsection concentrates on a brief fan-stator interaction modal analysis. The second and last subsection describes modal results for the liner, HQ-liner, and HQ-HW systems.

Rotor-stator interaction analysis

The circumferential modes that are excited due to the aerodynamic interaction between the 16 fan blades and the 28 stator vanes are listed in Table 3-7. The mode cut-off frequencies are also listed in Table 3-7. In the computation of the cut-off frequency, it is assumed the flow speed corresponding to the fan operating at 1800 rpm. Thus, flow speeds of M=0.115 and M=0.154 were used for the inlet and aft ducts, respectively. Results in Table 3-7 indicate that the 1BPF tone is cut-off in both the inlet and aft ducts. The interaction modes present at 2BPF tone are the (4,0) and (4,1) in both sections of the rig. As shown, the (4,0) mode is always cut-on for the range of speed tested. The (4,1) is also cut-on for the range of speed tested with the exception of 1500 rpm in the inlet.

Table 3-8 presents the measured sound power levels for the (4,0) and (4,1) modes for the hard wall case at the four fan speeds tested. The data indicates the 2BPF tone power to be slightly higher for the (4,0) mode, i.e. power difference between the (4,0) and (4,1) modes to be ~2-4 dB. Figure 3.34 shows the modal decomposition for hard wall case measured in the inlet and aft ducts for the 2BPF tone. The left column shows the inlet results whereas the right one represents the aft results. Modes included range from circumferential order \( m = -9, \ldots, 0, \ldots 9 \) and radial orders \( n = 0,1,2 \). The last row in the figure shows total power for a particular \( m \) order. As expected, the two most important modes present in the inlet and aft ducts at the 2BPF are the (4,0) and (4,1) modes. There are some spurious modes about 12-15 dB below the \( m=4 \) modes.
However, it can be observed the presence of the (5,0) mode in the aft duct. The power of this mode relative to the (4,0) and (4,1) decreases as the fan speed increases. For example, the (5,0) mode is at least 10 dB lower than the $m=4$ modes at the design speed. The only exception occurs at 1500 rpm in the aft duct where the (5,0) is shown as the dominant mode. The noise mechanism responsible for this (5,0) mode is not known. It is also important to note that for all the other acoustic treatments, the (4,0) and (4,1) modes are dominant, i.e. no strange modes, in particular at the design speed.

Table 3-7: Fan-stator interaction spinning mode analysis at 1800 rpm. 

\[ m = n \times B \pm k \times V \quad B = 16 \quad V = 28 \]

<table>
<thead>
<tr>
<th>k</th>
<th>m</th>
<th>Inlet Radial Order</th>
<th>Aft Radial Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>1BPF ( (n=1) )</td>
<td>-2 -40</td>
<td>3815.6 4405.0 4847.5</td>
<td>3815.6 4405.0 4847.5</td>
</tr>
<tr>
<td></td>
<td>-1 -12</td>
<td>1237.9 1671.8 2018.0</td>
<td>1237.9 1667.0 1979.8</td>
</tr>
<tr>
<td></td>
<td>0 16</td>
<td>1610.7 2075.2 2439.5</td>
<td>1610.7 2074.2 2429.0</td>
</tr>
<tr>
<td>2BPF ( (n=2) )</td>
<td>-2 -24</td>
<td>2350.8 2863.9 3258.7</td>
<td>2350.8 2863.9 3257.7</td>
</tr>
<tr>
<td></td>
<td>-1 4</td>
<td>474.0† 827.8 1131.0</td>
<td>461.6 787.7 1238.8</td>
</tr>
<tr>
<td></td>
<td>0 32</td>
<td>3085.1 3638.2 4058.8</td>
<td>3085.1 3638.2 4058.8</td>
</tr>
<tr>
<td>3BPF ( (n=3) )</td>
<td>-2 -8</td>
<td>860.2 1258.8 1585.0</td>
<td>859.3 1230.2 1546.8</td>
</tr>
<tr>
<td></td>
<td>-1 20</td>
<td>1981.7 2471.9 2851.5</td>
<td>1981.7 2471.9 2850.5</td>
</tr>
<tr>
<td></td>
<td>0 48</td>
<td>4545.2 5164.1 5627.6</td>
<td>4545.2 5164.1 5627.6</td>
</tr>
</tbody>
</table>

† Red color indicates cut-on modes

Table 3-8: Modal power [dB] – 2BPF - 4,0 and 4,1 modes – Hard Wall case

<table>
<thead>
<tr>
<th>RPM</th>
<th>Inlet</th>
<th>Total</th>
<th>Aft</th>
<th>Total</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>(4,0) 101.9</td>
<td>101.9</td>
<td>(4,0) 95.9</td>
<td>93.3</td>
<td>97.8</td>
</tr>
<tr>
<td>1600</td>
<td>(4,0) 101.6</td>
<td>100.8</td>
<td>104.3</td>
<td>96.4</td>
<td>99.0</td>
</tr>
<tr>
<td>1700</td>
<td>(4,0) 103.3</td>
<td>101.1</td>
<td>105.3</td>
<td>99.9</td>
<td>100.3</td>
</tr>
<tr>
<td>1800</td>
<td>(4,0) 105.2</td>
<td>102.5</td>
<td>107</td>
<td>105.4</td>
<td>101.3</td>
</tr>
</tbody>
</table>
Chapter 3. HQ-Liner Systems on the ANCF Rig

Figure 3.34: Modal decomposition for the 2BPF tone for the inlet and aft sections of the NASA ANCF rig.
2BPF Tone Results

The present section analyzes the data collected by the inlet and aft rotating rakes. Liner, HQ-Liner and HQ-HW results are presented in the same way as far-field data. That is the left column of a figure has inlet information whereas the right one shows results for the aft. Each row corresponds to one basic acoustic configuration, i.e. ATC-1 on top and ATC-2 at the bottom. In some configurations, rake data was not collected. These cases are indicated in the figures.

Liner results: Figure 3.35 shows the attenuation for modes (4,0) and (4,1) as well as for the total, i.e. sum of the two. In the inlet, it is clear that the liner is very effective at controlling the (4,1) with modal attenuations as high as 17.1 dB at 1800 rpm. Such reduction implies that this mode is basically eliminated from the duct. Note that the (4,1) mode power drops to the same or lower levels that the spurious modes shown in Figure 3.34h. This result corroborates the conclusion in section 3.4.1.1 from observation of the far-field data that the attenuation towards the sidelines (angles ≥ 45°) is due to the elimination of the (4,1) mode. Figure 3.35a also shows a maximum total reduction of 6.2 dB at 1800 rpm. The aft rake data for this case was not collected. Figure 3.35b shows modal power reduction for the aft liner (ATC-2). Mode (4,1) still exhibits more reduction than the (4,0) mode except at 1500 rpm.

Figure 3.36 compares in-duct and far-field power reduction data for the 2BPF tone. In the case of the inlet, far-field and in-duct data present consistent trends with some substantial differences in the reduction levels in some cases. For example, in Figure 3.36a the inlet liner shows a power reduction of 6.2 dB from in-duct data and 3.7 dB from far-field data at 1800 rpm. In the case of the aft liner, there is no clear agreement between the in-duct and far-field results as seen in Figure 3.36d.
Chapter 3. HQ-Liner Systems on the ANCF Rig

Figure 3.35: Sound power attenuation of the 2BPF tone and its two most dominant modes due to the liner for different fan speeds. (a) liner positioned in the inlet and (b) liner positioned in the aft duct.
Figure 3.36: In-duct/far-field comparison of sound power attenuation of the 2BPF tone due to the liner for different fan speeds. (a) liner positioned in the inlet and (b) liner positioned in the aft duct.
HQ-Liner results: Figure 3.37 through 3.34 show results for the HQ-Liner system. Figure 3.37, once again, shows the (4,1) mode to be the most attenuated of the two. The HQ contribution in Figure 3.38 reveals that the tubes in the aft provided a small (~0.6 dB) total additional attenuation, most of it from reduction of the (4,1) mode (see Figure 3.38d). The HQ contribution in the inlet also shows an improvement with respect to the liner (Figure 3.38a). However, this time the (4,0) mode is the most attenuated one. Consequently, the total reduction due to the tubes ranges between 0.7 dB at 1800 rpm and 1.4 dB at 1500 rpm.

Figure 3.39 and 3.39 compare in-duct and far-field data for the HQ-Liner and HQ contribution relative to the liner, respectively. As it can be seen in these figures, there seems to be some degree of agreement in the general trend. However, the power reduction levels are off for the most part.
Figure 3.37: Sound power attenuation of the 2BPF tone and its two most dominant modes due to the HQ-liner for different fan speeds. (a) and (b) HQ-liner positioned in the inlet and (c) and (d) HQ-liner positioned in the aft duct.
Figure 3.38: HQ contribution (measured relative to the liner) to sound power attenuation of the 2BPF tone and its two most dominant modes for different fan speeds. (a) HQ-liner positioned in the inlet and (b) HQ-liner positioned in the aft duct.
Figure 3.39: In-duct/far-field comparison of sound power attenuation of the 2BPF tone due to the HQ-liner for different fan speeds. (a) and (b) HQ-liner positioned in the inlet and (c) and (d) HQ-liner positioned in the aft duct.
Figure 3.40: In-duct/far-field comparison of the HQ contribution (measured relative to the liner) to sound power attenuation of the 2BPF tone for different fan speeds. (a) HQ-liner positioned in the inlet and (b) HQ-liner positioned in the aft duct.
**HQ-HW results:** Figure 3.41 and 3.41 show the results for the HQ-HW case. Like the liner, the modal reduction shows that the HQ tubes mainly attenuated the (4,1) mode in all cases. For example, the HQ-HW system installed in the inlet (Figure 3.41a) shows attenuation levels as high as 18.6 dB at 1600 rpm. Because of the poor attenuation of the (4,0) mode in many cases, the total attenuation of the 2BPF tone is not significant. An exception to this case is the results at 1700 rpm where the (4,0) and (4,1) were reduced by 5.4 and 9.7 dB, respectively, leading to a 6.5 dB total attenuation. The results for the aft HQ-HW system lead to similar conclusions. As seen in Figure 3.41d, the aft HQ-HW shows total attenuation levels ranging between 2.6 dB at 1600 rpm and 6.0 dB at 1700 rpm. At the design speed of 1800 rpm, the total reduction achieved is 4.7 dB. Once again, the best attenuation is of the (4,1) mode.

Figure 3.42 compares in-duct and far-field power reduction. The conclusion is the same as the HQ-liner case. There seems to be some degree of agreement but levels are off for the most part, in some cases by as much as 6.0~7.0 dB.
Figure 3.41: Sound power attenuation of the 2BPF tone and its two most dominant modes due to HQ tubes on hard wall for different fan speeds. (a) and (b) HQ tubes positioned in the inlet and (c) and (d) HQ tubes positioned in the aft duct.
Figure 3.42: In-duct/far-field comparison of sound power attenuation of the 2BPF tone due to the HQ tubes on hard wall for different fan speeds. (a) and (b) HQ tubes positioned in the inlet and (c) and (d) HQ tubes positioned in the aft duct.
3.4.1.3 **Inlet-Aft Acoustic treatments interference**

This section concentrates on the influence that different acoustic treatments exercise upon each other when simultaneously implemented in the inlet and aft sections of the rig. Data identified in Figure 3.16 as ATC-3 is analyzed here. In order to easily visualize this influence the cases where acoustic treatment was used in the inlet and aft only, i.e. ATC-1 and 2, are also included.

The analysis of ATC-3 data shows no clear pattern as to the mutual influence of inlet and aft systems at the 2BPF tone. According to both far-field and in-duct data, there are many cases that show no clear evidence of interference. However, there are cases where the simultaneous implementation of inlet and aft treatment clearly affected the performance of the liner, particularly in the inlet. Figure 3.43 shows directivity patterns at 1800 rpm of the 2BPF tone for the baseline, liner, and HQ-liner configurations. As it can be observed, by comparing Figure 3.43a and c, the simultaneous implementation of liners on both sections of the rig enhanced the performance of the inlet liner in the 25º-90º sector increasing the attenuation at the 2BPF between 1.3-4.5 dB. In the case of the HQ-liner system, simultaneous implementation also had a beneficial effect in the inlet HQ-liner system performance in the 0º-35º sector with an increase in reduction between 1.8-5.8 dB. The 60º-90º sector also exhibits an improvement between 2.5-5.7 dB.

By comparing Figure 3.43b and c, it can be observed that simultaneous implementation also had an influence on the aft. When inlet and aft liners were tested together, the aft liner improved its performance particularly in the 120º-145º sector with a maximum increase of 3.7 dB. HQ-liners present the same pattern. Simultaneous testing enhanced the performance of the aft through out the majority of the aft sector (except between 135º-145º) with an improvement ranging 1.3-7.7 dB.

In case of the broadband component, the simultaneous implementation did not change significantly the performance of any acoustic treatment. Differences oscillate around 0.5 dB.
Figure 3.43: Directivity patterns of the 2BPF tone for HW, Liner and HQ-liner configurations at 1800 rpm for acoustic treatments placed in the (a) inlet, (b) aft and (c) inlet and aft duct simultaneously.
3.4.2 Multiple HQ-Liner spool pieces

In this section results corresponding to the multiple HQ-liner spool pieces configuration are presented. The four cases investigated are summarized in section 3.3 (see Figure 3.17). For the sake of convenience, this figure is repeated here. Due to time constrains, data was collected only in the inlet section. Like in the single spool piece case, four fan speeds were tested, i.e. 1500, 1600, 1700 and 1800 rpm. Far-field data is analyzed in the next section, whereas in-duct data is presented in Section 3.4.2.2.

<table>
<thead>
<tr>
<th>Config.</th>
<th>1st Spool piece</th>
<th>2nd Spool piece</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HW</td>
<td>HW</td>
</tr>
<tr>
<td>2</td>
<td>HW</td>
<td>HQ-Liner</td>
</tr>
<tr>
<td>3</td>
<td>HQ-Liner</td>
<td>HQ-Liner</td>
</tr>
<tr>
<td>4</td>
<td>HQ-Liner (resistance locally reduced)</td>
<td>HQ-Liner</td>
</tr>
<tr>
<td>5</td>
<td>Liner (resistance locally reduced)</td>
<td>Liner</td>
</tr>
</tbody>
</table>

Figure 3.44: Different multiple spool piece test configurations.

3.4.2.1 Far-field results

These results are presented for the 2BPF tone and the broadband component at all four speed tested. In addition, directivity patterns as well as power spectrum for the fan speed of 1700 rpm are also presented. In this particular case, the fan speed of 1700 rpm was chosen instead of the design speed of 1800 rpm for reasons that will become clear in the next section.
2BPF Tone results

Figure 3.45 shows 2BPF tone sound power attenuation due to the four cases listed in Figure 3.16. The first observation is that the results are not completely consistent between the speed tested. Figure 3.45a shows noise attenuation at 1500 rpm. The highest reduction is provided by Configuration 3, i.e. two HQ-liners combined together yields 8.4dB with the other configurations producing between 6.0-6.6 dB. Removal of the wire mesh at the tube openings on the first spool piece HQ-liner degraded the performance of the entire system, e.g. Configuration 4 produced 1.9 dB less attenuation than Configuration 3 and only 0.6 dB more than Configuration 5. Note also that the single HQ-liner system in Configuration 2 is as effective as multiple HQ-liner systems. The results at 1600 rpm shown in Figure 3.45b indicate very similar performance as for the 1500 rpm case. However, the noise reduction levels are about 2 dB lower.

The results at 1700 rpm seen in Figure 3.45c leads to different conclusions. All multiple spool pieces configurations, i.e. configurations 3, 4 and 5 produced, outperforms the single HQ-Liner. The best multiple HQ-Liners system was the one with the wire mesh removed in the 1st spool section, e.g. 1.5 dB more than Configuration 3 and 3.3 dB more than the liners (HQ-contribution). This result alone confirms the benefits of reducing the liner resistance at the tubes openings. At 1800 rpm, the results follow the same pattern as observed at 1500 rpm.

The above results suggest that the number of arrays, liner properties, and impedance values at the tube openings are parameters that need all to be considered simultaneously in the design process. Note that the multiple HQ-liner systems tested were not design specifically for this test but rather just combined from HQ-Liners systems had been designed to meet different requirements. Thus, this is probably the reason the best attenuation levels were obtained at 1700 rpm instead of 1800 rpm.
Figure 3.45: Attenuation of the 2BPF tone due to different configurations at (a) 1500 rpm, (b) 1600 rpm, (c) 1700 rpm and (d) 1800 rpm, measured in the inlet sector of the far-field.

Figure 3.46 shows directivity patterns at 1700 rpm. This case was selected instead of 1800 rpm because of the better performance of Configuration 4. In order to more easily assess the behavior of each configuration the hard wall case has also been included in this figure. The hard wall case repeated the pattern observed in section 3.4.1.1. The inlet sector presents two radiation lobes at 30º and 70º corresponding to the (4,0) and (4,1) modes, respectively. As it can be observed, configurations 3, 4, and 5 control the (4,1) mode (the least dominant of the two) in similar fashion. On the other hand, the (4,0) mode is best controlled by Configuration 4 (HQ-liner systems with wire mesh removed) confirming what was observed in Figure 3.45c.
3.46 also indicates that the removal of the wire mesh seemed to have an unwanted side effect on the aft sector. As it can be seen sound power levels have been increased by as much as 12.0 dB at 130°.

![Figure 3.46](image.png)

**Figure 3.46:** Directivity patterns of the 2BPF tone for different configurations at 1700 rpm measured in the inlet sector of the far-field.

**Broadband results**

In this section attenuation of the broadband results are presented. Just like before, broadband data has been divided into power reduction between 1.5-2.5 BPF (referred to as 2BB), targeted mainly by the HQ tubes and attenuation within 2.5-5.5 BPF targeted mainly by the liner.

Figure 3.47 shows reduction of the 2BB component at every fan speed for configurations 2 through 5. As it can be seen, results are much more uniform than in the 2BPF tone case. These configurations produced reduction levels between 3.7-5.0 dB depending on the fan speed, 1.0-2.0 dB more than the single HQ-Liner (*Configuration 2*). In every case the removal of the liner wire mesh degraded by as much as 1.5 dB the performance of the HQ tubes to the point that this configuration produced 0.75 dB less reduction than the liners at 1700 rpm (see Figure 3.47c). In general, the contribution from the HQ tubes when two HQ-liner spool pieces were used is less than 1.0 dB when compared to the attenuation provided by the two liners.

Figure 3.48 presents directivity patterns of the 2BB component at 1700 rpm. Reduction levels are rather uniform through most of the inlet sector of the far-field, i.e. 10°-70°. The worst
performance is provided by the single HQ-Liner system. The other three configurations produced similar results. The power reduction provided by *Configuration 4* drops slightly towards the side line (>70°).

**Figure 3.47:** Attenuation of the 2BB tone in the inlet due to different configurations at (a) 1500 rpm, (b) 1600 rpm, (c) 1700 rpm and (d) 1800 rpm, measured in the inlet.
Figure 3.48: Directivity patterns of the 2BB broadband component for different configurations at 1700 rpm measured in the inlet sector of the far-field.

Figure 3.49 shows the attenuation of the broadband component between 2.5-5.5 BPF at every fan speed and for all the configurations investigated. As it can be seen, every configuration produced similar results regardless of the fan speed. As in the 2BB case, Configuration 2 produced the worst attenuation at ~3.5 dB. As for the other three cases, all of them reached same levels of reduction, i.e. between 5.0-6.0 dB. If configurations 3, 4 and 5 are compared, it will be observed that the contribution of the HQ tubes relative to the liner is not significant. Furthermore, when the liner wire mesh is removed from the tube openings the performance of the system drops by ~1.0 dB (compared to Configuration 5) independently of the fan speed.

Such bad effects of the wire mesh removal on both 2BB and broadband between 2.5-5.5 BPF components are rather surprising considering previous results. An indication as to the reason why this happened is given in Figure 3.50. This figure shows that when Configuration 4 was tested there was an unknown source introducing energy to the system as indicated by the increments in power level observed every 16 engine orders.
Figure 3.49: Overall attenuation of the broadband component between 2.5 BPF and 5.5 BPF due to different configurations at (a) 1500 rpm, (b) 1600 rpm, (c) 1700 rpm and (d) 1800 rpm measured in the inlet.
Figure 3.50: Power spectrum of the broadband component for different configurations at 1700 rpm measured in the inlet sector of the far-field.

### 3.4.2.2 In-duct results

Figure 3.51 shows in-duct data collected by the rotating rake at all four fan speeds and for every configuration tested. Note that at 1500 rpm the (4,1) mode is cut-off. Every chart in the figure shows attenuation of each one of the most dominant modes in the disturbance, i.e. the (4,0) and the (4,1) and the sum of the two. Note that at 1500 rpm the (4,1) mode is cut-off. As it can be seen every configuration controls mainly the (4,0) mode with values as high as 17.0 dB (Configuration 4 at 1800 rpm - see Figure 3.51d). The (4,1) mode presents good levels of attenuation at ~8.0 dB (Configuration 3 at 1800 rpm - see Figure 3.51d). If in-duct and far-field data in Figure 3.45 are compared, it will be seen that the general tendency is followed by both data sets, especially at 1700 and 1800 rpm where the difference in attenuation levels is at the most 1.5 dB. At 1500 rpm, far-field and in-duct data present the same pattern but reduction levels are off by as much as 7.0 dB. At 1600 rpm significant differences, i.e. 5.0 dB, are observed in configurations 3, 4 and 5. In addition, far-field data shows that when the liner wire
mess is removed (Configuration 4) the performance of the system drops by 1.2 dB. In-duct data, on the other hand, exhibits the opposite behavior. When the wire mesh is removed the attenuation increases by 0.5 dB.

**Figure 3.51:** Attenuation of the 2BPF tone and its dominant modes due to different configurations at (a) 1500 rpm, (b) 1600 rpm, (c) 1700 rpm and (d) 1800 rpm, measured in the inlet section of the rig.
4. MODEL VALIDATION

This chapter presents comparison between the experimental results and prediction with the aim of validating the developed models. As already mentioned, the main focus of the dissertation is the aft fan radiation control model. However, since the inlet has been previously validated only in terms of broadband attenuation, 2BPF tone predictions will also be included here and compared to experiments. For the sake of completeness, inlet broadband predictions will be included as well. Since the models can predict the attenuation of both the HQ-liner and the liner independently, the validation is performed for the following treatment options: liner, HQ-liner, and HQ tubes on a hard wall duct (HQ-HW). To determine the importance of accounting for the transmission and reflections effects at the liner/hard wall interfaces (presented in section 2.2), predictions both including and neglecting these effects are analyzed.

This chapter is organized in three sections. Section 4.1 presents the comparison between experimental and predicted results for the Single HQ-Liner Spool Configurations (experimental results presented in section 3.4.1). Both the inlet and aft HQ-liner codes are validated in this section. The Multiple HQ-Liner Spool Configurations (experimental results presented in section 3.4.2) were tested only in the inlet of the ANCF rig. Thus, section 4.2 present the validation of the inlet HQ-Liner code using these tests. Finally, section 4.3 summarized the main conclusions from this model validation studies.

4.1. Validation using Single HQ-Liner Spool Configurations

In this section, predictions are compared to experimental data obtained on HQ-liner systems installed in the inlet and aft sections of the NASA ANCF rig. Since both HQ-liner systems were designed to control noise radiation at 1800 rpm, the validation concentrates mainly on this fan speed. However, results at 1700 rpm are also included in the study. Section 4.1.1 presents 2BPF tone results whereas section 4.1.2 concentrates on the broadband component.
4. Model Validation

4.1.1 2BPF Tone

This section analyzes the capabilities of inlet and aft models at predicting 2BPF tone reduction. To this end, the sound power reduction for the 2BPF tone is compared to both in-duct and far-field experimental data. Since the model is based on the prediction of individual modes, power reduction comparison for the two dominant rotor-vane interaction modes is also performed. In this case, the predicted attenuation is compared to in-duct experimental data obtained with the rotating rakes (see section 3.1).

Liner Results

Figure 4.1 compares measured and predicted attenuation of the 2BPF tone and the (4,0) and (4,1) modes due to the liner at 1800 rpm in the inlet and aft sections of the rig. Transmission-reflection effects are not included in these results. It can be observed that the aft model predicted attenuations are accurate not only for the individual modes but also for the total tone (Figure 4.1b). The maximum difference between experiments and predictions is around 2.0 dB for the (4,1) mode. In case of the inlet, good results were also achieved at estimating the total tone attenuation. The mode attenuation prediction is not very accurate. As it can be observed, predictions and experiments are off by more than 12.0 dB for the (4,1) mode. In case of the (4,0) mode, the predicted attenuation is only 2.5 dB off with respect to measurements. It is important to note that the reason for the good prediction of the total reduction in lieu of the poor estimation of the (4,1) mode reduction is that the (4,0) mode was the most dominant. From Table 3.8, it can be observed that the sound power level of the (4,0) mode is 2.7 dB higher than the (4,1) mode at 105.2 dB.

Figure 4.2 compares experiments to predictions including transmission-reflection effects. The improvement in the predictions is striking especially in the inlet. The 12.0 dB difference observed in the (4,1) mode has been reduced to less than 4.0 dB while prediction of the (4,0) mode is also better than in Figure 4.1. In case of the aft section, the inclusion of transmission-reflection effects has also enhanced the predicted reductions.
4. Model Validation

Figure 4.1: 2BPF tone liner power reduction - Comparison between predictions and experiments for the (4,0) and (4,1) modes and the total at 1800 rpm. (a) Inlet and (b) aft liners.

Figure 4.2: 2BPF tone liner power reduction - Comparison between predictions and experiments for the (4,0) and (4,1) modes and the total at 1800 rpm including transmissions and reflections. (a) Inlet and (b) aft liners.
Figure 4.3 compares predictions to experiments at 1700 rpm. Given the improved performance of the model including transmission-reflection effects, the predictions in the figure include these effects. As observed, inlet predictions are very accurate with a maximum error of around 2.0 dB. The aft model also shows good accuracy specially for the (4,0) mode and the total attenuation.

![Graph](image)

**Figure 4.3:** 2BPF tone liner power reduction - Comparison between predictions and experiments for the (4,0) and (4,1) modes and the total at 1700 rpm including transmission and reflection effects. (a) Inlet and (b) aft liners.

**HQ-Liner Results**

Results for the HQ-Liner model validation are presented in terms of HQ contribution relative to the liner. HQ contribution is estimated by subtracting liner attenuation from HQ-liner attenuation. Figure 4.4 shows the modal and total 2BPF tone reduction produced by the HQ tubes over the liner. In these results, the transmission and reflections effects are not included. It is generally observed that the models predict well the total tone reduction and its modal components. The only exception is the (4,1) mode in the inlet which is off by as much as 9.0 dB. As in the 1800 rpm case in Figure 4.1, the total tone reduction is still predicted well because the (4,0) dominates, e.g. power of the (4,0) mode is 2.2 dB higher than the (4,1) at 103.3 dB.

Figure 4.5 presents HQ attenuation over the liner when transmission and reflection effects are included. The inlet model again shows a notorious improvement in the prediction of the (4,1) mode with differences reduced now to 4.0 dB. The modal predictions in the aft are degraded relative to the results in Figure 4.4. In spite of this, the total tone attenuation is still predicted
reasonably well. It is important to mention, that in the aft measured in-duct attenuation is around 0.5 dB. The repeatability analysis (see Appendix E) shows that for measured in-duct data to be reliable attenuation levels should be higher than 1.3 dB. When predictions are compared to the far-field value, the inclusion of transmission and reflections in the model improved the predictions by 0.6 dB.

**Figure 4.4:** 2BPF tone power reduction due to HQ tubes relative to the liner - Comparison between predictions and experiments for the (4,0) and (4,1) modes and the total at 1800 rpm. (a) Inlet and (b) aft HQ-liners.

**Figure 4.5:** 2BPF tone power reduction due to HQ tubes relative to the liner - Comparison between predictions and experiments for the (4,0) and (4,1) modes and the total at 1800 rpm including transmissions and reflections. (a) Inlet and (b) aft HQ-liners
Figure 4.6 shows sound power reduction at 1700 rpm. In this case, inlet and aft models worked very well, in particular in the inlet where the difference between predictions and experiments is less than 0.5 dB. In the aft, the attenuation of the (4,0) mode is still off by 2.7 dB. However, for the (4,1) mode and the total attenuation differences have decreased to less than 2.0 dB.

![Power Reduction Diagram](image)

**Figure 4.6:** 2BPF tone power reduction due to HQ tubes relative to the liner - Comparison between predictions and experiments for the (4,0) and (4,1) modes and the total of the two at 1700 rpm including transmissions and reflections. (a) Inlet and (b) aft HQ-liners.

**HQ Tubes on a Hard Wall**

Figure 4.7 shows power attenuation due to the HQ tubes only. Since there are no sudden changes in wall impedance, transmission and reflections effects have not been considered. As it can be seen the model produced good results in the inlet with differences between predictions and measurements around 2.0-3.0 dB. In the aft, good results were achieved when predicting the (4,0) and the total reduction. However, the attenuation of the (4,1) mode was not predicted correctly indicating an error of 9.5 dB.

Results for the 1700 rpm condition are shown in Figure 4.8. The in-duct data shows that neither the inlet nor the aft model worked well at predicting the attenuation of the 2BPF tone and its components. Differences range between 4.0-8.0 dB. However, the far-field data shows that the inlet predictions are off by 2.5 dB whereas in the aft that difference dropped down to less than 1.0 dB.
4. Model Validation

4.1.2 Broadband

This section concentrates on broadband validation. Even though the main focus of this work is aft fan radiation control broadband inlet comparison is also included. Like in section 4.1.1, predictions of broadband attenuation have been performed including and neglecting
transmissions and reflections at the liner/hard wall interfaces. Again, both sets of results are compared to experiments.

**Liner results**

Figure 4.9 shows power reduction due to the liner over the 300-3100 Hz range at the design fan speed of 1800 rpm. In this case, transmissions and reflections have not been considered. In addition, the factor $\Gamma$ that discriminates between strongly attenuated modes and weakly attenuated ones as defined by Alonso was used [33]. This value of $\Gamma$ was set at 3 for the aft and 5 for the inlet. Results in Figure 4.9 show good agreement between predicted and measured data in both inlet and aft sections. In the inlet, predictions match experiments very well between 1000-2300 Hz under predicting the liner reduction by as much as 2.0 dB below 1000 Hz. Predictions also differ from measured data at higher frequencies, i.e. over predicts attenuation by as much as 3.0 dB at 3100 Hz. In the aft, predictions follow experiments well for most of the frequency range investigated. However, there are discrepancies below 1500 Hz.

![Figure 4.9: Broadband liner power reduction - Comparison between predictions and experiments at 1800 rpm. (a) Inlet and (b) aft liners.](image-url)
Figure 4.10 compares predictions to experiments also at 1800 rpm when transmissions and reflections are considered and when the discriminating factor $I'$ as defined by Morse and Ingard [63] was used. The value of $I'$ was set at 0.42 for the aft and 1.0 for the inlet. Both inlet and aft models worked much better. In the inlet, the agreement at higher frequencies (>2000 Hz) is much better. The aft model shows a dramatic improvement below 1500 Hz and about the same performance at higher frequencies. The aft model is not capable of reproducing the two peaks observed in the measurements at ~2250 and ~2550 Hz. These two peaks may be the result of just cut-on modes which are strongly attenuated by the liner. It is possible that these just cut-on modes are not included in the prediction due to the discriminating factor $I'$ used in the calculations.

Figure 4.11 compares predictions to experiments at 1700 rpm. Predictions accounted for transmissions and reflections and used Morse’s and Ingard’s definition of $I'$ which was set at 0.42. Once again results show very good agreement between the predictions and experiments, in particular for the inlet.

**Figure 4.10:** Broadband liner power reduction - Comparison between predictions and experiments at 1800 rpm including transmissions and reflections. (a) Inlet and (b) aft liners.
4. Model Validation

**Figure 4.11**: Broadband liner power reduction - Comparison between predictions and experiments at 1700 rpm including transmissions and reflections. (a) Inlet and (b) aft liners.

**HQ-Liner**

Figure 4.12 shows the predicted HQ contribution calculated neglecting transmissions and reflections at the liner/hard-wall interfaces and using the $\Gamma$ discriminator as defined by Alonso [33]. In the inlet, the predicted HQ contribution matches remarkably well the experimental results. In the inlet, the predicted HQ contribution matches remarkably well the experimental results. In the inlet, the predicted HQ contribution matches remarkably well the experimental results. In the inlet, the predicted HQ contribution matches remarkably well the experimental results. In the low frequency range, i.e. below 1500 Hz differences are around 1.0 dB. However, the predictions above 1500 Hz indicate a slight increase in noise (HQ degrade the performance of the liner) while the results indicate a small reduction of about 0.5 dB.

Figure 4.13 shows the same case but now including the transmissions and reflection effects and Morse’s $\Gamma$ discriminator. In the inlet, there is no difference with the previous approach which was already extremely good. In the aft sector, predictions have improved throughout the entire frequency range. However, at high frequencies a few negative reduction peaks are observed in the predictions. The reason for this is that at high frequencies the simplex method is not accurate at finding the correct eigenvalues particularly of high circumferential modes [33]. These higher order modes are used by the Green’s function to represent the sound field due to the HQ tubes. When transmissions and reflections are calculated at the second interface, the effects of having the wrong eigenvalues are intensified due to the radial mode coupling induced by the sudden
change in wall impedance. Hence, the negative peaks observed at high frequencies. When transmissions and reflections are not included (see Figure 4.12) predictions look much cleaner.

Figure 4.12: Broadband HQ tube power reduction relative to the liner- Comparison between predictions and experiments at 1800 rpm. (a) Inlet and (b) aft HQ-liners.

Figure 4.13: Broadband HQ tube power reduction relative to the liner- Comparison between predictions and experiments at 1800 rpm including transmissions and reflections. (a) Inlet and (b) aft HQ-liners.
Figure 4.14 compares predictions with experiments at 1700 rpm. In this case, predictions include transmissions and reflections and Morse’s discriminating factor $\Gamma$ set at 0.42. As it can be observed, both inlet and aft models produced very accurate predictions similar to those at 1800 rpm. This result is not unexpected since, as far as the broadband noise component, there should not be significant differences in the fan disturbance between 1700 and 1800 rpm speeds.

**Figure 4.14:** Broadband HQ tube power reduction relative to the liner- Comparison between predictions and experiments at 1700 rpm including transmissions and reflections. (a) Inlet and (b) aft HQ-liners.

**HQ-HW results**

Finally, Figure 4.15 shows broadband sound power reduction for the HQ systems on a hard wall duct, i.e. HQ-HW case. This figure shows that the predicted results agree very well over most of the frequency range investigated especially in the inlet. In the aft, the main difference between predictions and experiments appears at both ends of the frequency range investigated, i.e. below 700 Hz and above 2300 Hz.

Figure 4.16 shows measured and predicted power attenuation due to HQ tubes on hard wall at 1700 rpm. Again, inlet predictions match experiments extremely well.
4. Model Validation

Figure 4.15: Broadband HQ-HW power reduction - Comparison between predictions and experiments at 1800 rpm. (a) Inlet and (b) aft HQ tubes on hard wall.

Figure 4.16: Broadband HQ-HW power reduction - Comparison between predictions and experiments at 1700 rpm. (a) Inlet and (b) aft HQ tubes on hard wall.
4.2. Multiple HQ-Liner Spool Configurations

In this section, experimental data on the multiple HQ-liner spool configurations described in Chapter 3 is compared to predictions. These configurations were only installed in the inlet. As indicated in the schematics in Figure 4.17 the spinner, with a 9.0\text{in} radius at the fan face, extends roughly half way through the 1\text{st} spool piece. This means that an annular cross section of hub-to-tip-ratio 0.375 at the fan face smoothly transitions into a circular cross section. The variation in the cross section also introduces changes in the flow speed as the flow propagates through the duct. Such a problem has been investigated by Rienstra in [54] where an exact solution was obtained. This solution also accounts for variations in the wall impedance. At the moment, neither the inlet nor the aft models has the capabilities to handle the annular-to-circular-duct transition problem. As a consequence two modeling alternatives of variable success have been utilized to predict these configurations.

The first one consists of assuming a circular duct of uniform cross section neglecting the presence of the spinner. Therefore, since the resulting cross section is circular, the model for circular ducts was utilized. Results using this approach will be referred to as “Circular model”. The second approach consists of accounting for the presence of the spinner. To this end, reduction due to the HQ-system placed on the first spool piece (see Figure 4.17) was calculated using the annular duct model. The second HQ-Liner system is assumed to be on a circular cross section hence the model for circular ducts was used again. This approach will be referred to as “Annular/circular model”.

4. Model Validation

Figure 4.17: Schematics of inlet multiple HQ-liner spool piece configuration.

Since some of the broadband measurements were not reliable (see Section 3.4.2.1), only 2BPF tone attenuation for the design fan speed of 1800 and 1700 rpm are included in this section. Predictions include the effect of transmissions and reflections at every interface, including the thin hard-walled strip 1.3" long between liners (see Figure 4.17). This hard-walled strip is the result of the aluminum frames to which the liners are attached and that were described in section 3.2.2. In case of the "Annular/circular model" reflections due to cross section change, e.g. from annular to circular were not considered. The total attenuation due to the multiple spool pieces configuration is calculated as follows: for the first spool piece, the circular model (or the aft if the spinner is accounted for) is run using measured modal information as input. The output of the model provides not only power attenuation but also the modal amplitudes used in the computation of this power attenuation. These amplitudes are used as input to the circular model in order to estimate power reduction due to the second spool piece. Total power reduction due to the entire configuration is obtained simply by adding the results of the two runs. As it can be seen, this approach also allows determining the contribution from each spool piece.
Configuration 3 Results

In Configuration 3 two HQ-Liner systems were utilized. The first one placed on the first spool piece featured HQ tubes on the $1.7\rho c$ liner, whereas the second one placed on the second spool piece had HQ tubes on the $1.0\rho c$ liner. Predictions of the performance of this configuration using the Circular model and the Annular/circular model are compared with in-duct and far-field data in Figure 4.18 and 4.19. At 1800 rpm (see Figure 4.18), both methods produced good results with the Circular model perfectly matching in-duct data. For the Annular/circular model differences with experiments are less than 1.0 dB in case of modal attenuation and 2.1 dB or less in case of total attenuation. At 1700 rpm (see Figure 4.19), none of the methods predicted well the attenuation of (4,0) mode, both of them overestimated the attenuation by as much as 6.2 dB. In case of the (4,1) mode, predictions show a little improvement with smaller differences at around 1.5-2.5 dB. In case of the total reduction predictions are off by as much as 2.8-6.0 dB. In general, the Annular/circular model produced slightly better results than the Circular model.

As already explained the contribution from each spool piece can also be estimated. Figure 4.20 and 4.21 show the contribution of each one of the two HQ-Liner systems to the attenuation of the 2BPF tone and its two most dominant modes. Since the Annular/circular model produced better results, the analysis was performed using this approach. At 1800 rpm (see Figure 4.20), both systems contribute equally to the attenuation of the 2BPF tone with 4.7 dB each. In case of the 1st HQ-liner system most of its contribution comes from the attenuation of the (4,0) modes whereas the second system contributes by mainly attenuating the (4,1). As indicated in Figure 4.21, at 1700 rpm most of the attenuation comes from the first HQ-liner spool piece.

![Figure 4.18](image-url)
4. Model Validation

**Figure 4.19:** Comparison between predictions and experiments of the attenuation for the (4,0) and the (4,1) modes and the total at 2BPF due to Configuration 3 at 1700 rpm.

**Figure 4.20:** Contribution of each one of the two HQ-Liners used in Configuration 3 to attenuation for the (4,0) and the (4,1) modes and the total at 1800 rpm. Predictions performed accounting for the presence of the spinner.
4. Model Validation

![Figure 4.21](image)

**Figure 4.21:** Contribution of each one of the two HQ-Liners used in Configuration 3 to attenuation for the (4,0) and the (4,1) modes and the total at 1700 rpm. Predictions performed accounting for the presence of the spinner.

**Configuration 4 Results**

*Configuration 4* also features two HQ-Liner systems. Like in the previous case, the first one placed on the first spool piece combines HQ tubes on a 1.7ρc liner, whereas the second one placed on the second spool piece had tubes on a 1.0ρc liner. However, in order to assess the influence of liner resistance on the HQ tubes performance, the wire mesh of the 1.7ρc liner was removed at the tube openings. In-duct and far-field data collected for this configuration is compared to predictions in Figure 4.22 and 4.23.

At 1800 rpm (see Figure 4.22), modal reduction predictions are within 3.0 dB from in-duct data with the *Annular/circular model* producing slightly better results than the *Circular model*. In case of the total attenuation of the 2BPF tone, both approaches are less than 3.0 dB off the in-duct value but around 7.0 dB off the far-field measurement. The 1700 rpm case is presented in Figure 4.23. As it can be observed, none of the methods produced good results with differences above 4.0 dB at least in all cases. Again the *Annular/circular model* produced slightly better results.
4. Model Validation

Figure 4.22: Comparison between predictions and experiments of the attenuation for the (4,0) and the (4,1) modes and the total at 2BPF due to Configuration 4 at 1800 rpm.

Figure 4.23: Comparison between predictions and experiments of the attenuation for the (4,0) and the (4,1) modes and the total at 2BPF due to Configuration 4 at 1700 rpm.

Figure 4.24 and 4.25 show the contribution to attenuation due to both HQ-Liner systems. The Annular/circular model was used in the calculation of the contribution due to each system. At 1800 rpm (see Figure 4.24), both HQ-liners provided 5.1 dB to the 10.2 dB of total attenuation of the 2BPF tone. In this case, it is however the second system the one providing most of the reduction of the (4,0) mode with 12.6 dB and the first one mostly attenuating of the (4,1) mode with 4.9 dB. The 1700 rpm case presented in Figure 4.25 exhibits the same pattern observed in Configuration 3. Most of the attenuation of the 2BPF tone and its components is provided by the HQ-Liner system placed on the first spool piece.
4. Model Validation

Figure 4.24: Contribution of each one of the two HQ-Liners used in Configuration 4 to attenuation for the (4,0) and the (4,1) modes and the total at 1800 rpm. Predictions performed accounting for the presence of the spinner.

Figure 4.25: Contribution of each one of the two HQ-Liners used in Configuration 4 to attenuation for the (4,0) and the (4,1) modes and the total at 1700 rpm. Predictions performed accounting for the presence of the spinner.

Configuration 5 Results

In Configuration 5 only the liners were tested. As indicated in Figure 4.17, the first spool piece (the one closest to the fan) contained the $1.7\rho c$ whereas the second spool piece contained
the 1.0pc. Since Configuration 4 was tested before Configuration 5 the 1.7pc liner had the wire mesh removed at the tube openings. In order to predict reduction due to a liner with such discontinuities a slight modification to the impedance $Z'$ in equation 2.49 was introduced. Such a modification had been previously used by Alonso in [60] and is repeated here for the sake of completeness. In equation 2.49, $Z'$ was defined as

$$Z' = Z^{td} + Z_{sc/POA}$$

where $Z^{td}$ is the tube dynamics and $Z_{sc/POA}$ is a diagonal matrix with the impedance of the wire mesh or, if this has been removed at the tube openings, the perforate screen. In the present case where the tubes are no longer attached to the liner, $Z'$ is defined as

$$[Z'] = \begin{bmatrix}
    z_{POA1} - i \cot(k_0d) & \cdots & 0 & \cdots & 0 \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    0 & \cdots & z_{POAJ} - i \cot(k_0d) & \cdots & 0 \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    0 & \cdots & 0 & \cdots & z_{POANN} - i \cot(k_0d)
\end{bmatrix}$$

where $z_{POAJ}$ is the impedance of the perforate screen of the $jj^{th}$ discontinuity modeled as a radiating piston and $i \cot(k_0d)$ accounts for the liner core. Recall $k_0$ is the free field wave number and $d$ is the physical liner core depth. The total number of radiating pistons is given by $N$. Note that $Z'$ is now a diagonal matrix as well. Once these modifications were added to the aft and inlet models the following results were obtained for the present configuration.

Figure 4.26 and 4.27 compare measured and predicted attenuation of the 2BPF tone and its components for Configuration 5. Again, predictions at 1800 rpm Figure 4.26 produced better results than at 1700 rpm in Figure 4.27. In both cases the Annular/circular model generated slightly better results than the Circular model except when predicting the attenuation of the (4,1) mode. At the design fan speed, the difference between in-duct measurements and predictions is 2.2 dB at the most. At 1700 rpm, results are not as good as at 1800 rpm. The worst case is the (4,0) mode where predicted results are 4.6 dB off in-duct data.

Figure 4.28 and 4.29 show the contribution to attenuation due to each one of the two liners used. Like in previous cases the contribution was estimated using the Annular/circular model. The most unexpected result is observed at 1800 rpm where the second liner degraded the
performance of the entire system. As it can be observed, the second liner increased the sound power of the (4,0) mode by 3.6 dB. At 1700 rpm, there were not unexpected results with most of the attenuation provided by the first system.

**Figure 4.26:** Comparison between predictions and experiments of the attenuation for the (4,0) and the (4,1) modes and the total at 2BPF due to *Configuration 5* at 1800 rpm.

**Figure 4.27:** Comparison between predictions and experiments of the attenuation for the (4,0) and the (4,1) modes and the total at 2BPF due to *Configuration 5* at 1700 rpm.
Figure 4.28: Contribution of each one of the two Liners used in Configuration 5 to attenuation for the (4,0) and the (4,1) modes and the total at 1800 rpm. Predictions performed accounting for the presence of the spinner.

Figure 4.29: Contribution of each one of the two HQ-Liners used in Configuration 5 to attenuation for the (4,0) and the (4,1) modes and the total at 1700 rpm. Predictions performed accounting for the presence of the spinner.
**HQ Contribution**

In this section, the contribution from the HQ tubes is investigated. It should be recalled that in *Configuration 4* two HQ-liner spool pieces were combined together. Of these two the one placed closer to the fan face had the liner wire mesh removed at the tubes openings. In *Configuration 5*, the tubes had been removed so that the liner attenuation could be measured. By comparing these two cases the contribution from the HQ tubes relative to the liner can be assessed.

Figure 4.30 and 4.31 compare the measured and predicted contribution of the HQ tubes to the attenuation of the 2BPF tone and its components. Of the two models available the *Annular/circular* one generated the best predictions. At 1800 rpm (see Figure 4.30), the modal component prediction is extremely good with differences of 1.1 dB. In case of the 2BPF tone predictions are also good if compared to in-duct data. In this case the difference increased up to 1.8 dB. As observed in Figure 4.31, at 1700 rpm differences between predictions and experiments are in the neighborhood of 3.0 dB.

![Figure 4.30](image)

**Figure 4.30:** Comparison between predictions and experiments of the Contribution of HQ tubes used in *Configuration 4* to power attenuation of the (4,0) and the (4,1) modes and the total at 1800 rpm.
4. Model Validation

Figure 4.31: Comparison between predictions and experiments of the Contribution of HQ tubes used in Configuration 4 to power attenuation of the (4,0) and the (4,1) modes and the total at 1700 rpm.

If Figure 4.30 and 4.5 are compared, it will be noticed that at 1800 rpm both single and multiple spool pieces configurations produced virtually the same contribution to the attenuation of the 2BPF tone. When 40 HQ tubes were used (single configuration), their contribution was around 1.3 dB. After the number of HQ tubes was doubled (multiple configuration), they generated only 0.3 dB more. On the other hand, at 1700 rpm HQ tubes in Configuration 4 produced as much as 3.9 dB (see Figure 4.31); this is 2.7 dB more than the single spool piece case at 1.2 dB in Figure 4.6.

It should be remembered that the systems utilized in the multiple spool piece configuration were specifically designed one for the inlet and the other for the aft for flow conditions corresponding to 1800 rpm. When the two HQ-liner systems were combined together as indicated in Figure 4.17 the frequency of optimum performance combination changed. This is a clear indication that the number of arrays as well as the impedance at the tube openings should be considered as parameters in the design process.

Figure 4.32 and 4.33 show contributions to power attenuation of each one of the tubes arrays utilized at 1800 and 1700 rpm, respectively. As it can be observed, the 1st spool piece, having tubes with liner resistance locally reduced at their openings, degraded the attenuation of the (4,0) mode, i.e. the most dominant of the two. On the other hand, the second spool piece degraded the attenuation of the (4,1) mode. This way the total predicted attenuation was only 3.9 dB. At 1700 rpm in Figure 4.33, the total predicted attenuation of the 2BPF tone was 5.9 dB or 2.0 dB more than at 1800 rpm. The reason is that in this case both systems contributed to the
attenuation of the (4,0) mode producing a total of 10.1 dB as opposed to the 8.3 dB at 1800 rpm.

**Figure 4.32:** Contribution of each one of the two HQ tubes array used in *Configuration 4* to attenuation for the (4,0) and the (4,1) modes and the total at 1800 rpm. Predictions performed accounting for the presence of the spinner.

**Figure 4.33:** Contribution of each one of the two HQ tubes array used in *Configuration 4* to attenuation for the (4,0) and the (4,1) modes and the total at 1700 rpm. Predictions performed accounting for the presence of the spinner.
4.3. **Overall Conclusions for Model Validation**

This section summarizes the most important conclusions of the validation of the inlet and aft radiation control models, for the configurations presented in the sections 4.1 and 4.2.

**Single HQ-liner spool piece:**

*Liner:* a very good level of accuracy has been obtained at predicting the 2BPF tone and its two most dominant modes in the inlet and in the aft, particularly when transmission and reflection effects were considered. At the design fan speed of 1800 rpm, differences between predictions and experiments were only 0.7 dB or less. In case of the inlet, the inclusion of transmission and reflections improved the predictions significantly.

In case of the broadband component, combining transmission and reflection effects with Morse’s and Ingard’s discriminating parameter has notoriously improved predictions in both inlet and aft through out the entire frequency range, i.e. between 300-3100 Hz.

**HQ-liner:** Both inlet and aft model produced very good results at predicting the attenuation of the 2BPF tone and its components. In case of the aft model at 1800 rpm, predictions are compared to far-field data only since the repeatability analysis proved in-duct data unreliable. The aft model predicted 2BPF attenuation at 1800 and 1700 rpm quite well with differences below 3.0 dB. In case of the inlet differences are below 4.0 dB.

Like in the liner case, the prediction of HQ contribution to broadband attenuation has been improved through out the entire frequency range by using Morse’s discriminating factor and including transmission and reflection effects at the liner/hard wall interfaces.

**HQ-HW:** In the 2BPF tone case, inlet and aft predictions are close to the experimental data. However, for this case differences between prediction and experiments are higher than the previous cases, reaching levels of ~8.0 dB. In case of the broadband components, both inlet and aft models produced good results within the 2.5-5.5 BPF frequency range.
**Multiple HQ-liner spool pieces:**

Even though the inlet and aft models do not have the capabilities of contemplating the annular-to-circular-duct-transition problem they have been used to predict the performance of two HQ-liner systems operating in such environment.

Two different approaches have been considered, the first one identified as *Circular model* neglects the presence of the spinner; the second one, identified as *Annular/circular model* accounts for the spinner in a rudimentary way, i.e. it assumes constant hub-to-tip ratio and it does not include transmissions and reflections due to the change in cross section.

Even though predictions are not as accurate as in the single spool piece case, the *Annular/circular model* produced better results. This indicates the importance of including in the model the effects induced by the annular-to-circular-duct transition, i.e. changes in flow Mach number and transmissions and reflections of the sound field.
5. Application: HQ-Liner in a Real Turbofan Engine

5. APPLICATION: HQ-LINER IN A REAL TURBOFAN ENGINE

A comprehensive approach to engine noise reduction involves both treatment of the inlet and aft. As a consequence additional research efforts have been performed in order to develop a model capable of predicting aft fan noise reduction due to HQ-liners. Such a model has been developed and validated in previous chapters. As part of the development phase, the NASA ANCF rig was used in the validation. However, the conditions in the ANCF rig are not representative of real turbofan engines. For instance, the sound power levels in turbofan engines can exceed 140 dB as compared to the 120 dB produced by the ANCF fan. In addition, maximum fan tip Mach number ($M_{tip}$) of the ANCF, i.e. ~1800 rpm, is 0.33 whereas in a real turbofan engine $M_{tip}$ becomes supersonic. Other differences are in the flow velocity and hub-to-tip ratios. These differences may affect the performance of the HQ-liner systems. Thus, the objective of the present chapter is to design an HQ-Liner for a real turbofan engine and analyze its performance in a more realistic environment.

Chapter 5 is organized such that section 5.1 concentrates on describing a generic turbofan engine used in the modeling. Information on the disturbance required by the model to predict noise attenuation is provided in section 5.2. The design of the HQ-liner system for the generic turbofan engine and results summarizing its performance are presented in section 5.3. Finally, section 5.4 presents the most important conclusions of this chapter.

5.1. Generic Turbofan Engine Parameters

This section defines the characteristics of a “generic” modern turbofan engine for which an HQ-Liner system will be designed. Review of engine parameters has shown a striking resemblance among different turbofan engines, i.e. modern turbofan engines are essentially alike through a scaling factor. The scaling factor is the fan diameter. The reason for this is that these modern turbofan engines have in common a fan tip Mach number of ~1.0 near cut-back
5. Application: HQ-Liner in a Real Turbofan Engine

condition, e.g. ~70-75% of full power. For instance, Figure 5.1 shows two turbofan engines. Figure 5.1a presents the Allison AE3007 with a fan diameter of 43.5 in while Figure 5.1b shows the RR RB211-535 with a fan diameter of 74.5 in. The fan diameter ratio is roughly 1.71 and for a constant tip Mach number at cut-back, the fan speed for the Allison engine is 1.71 times larger than for the RR engine. This also ensures that the flow Mach number inside the duct does not change significantly between the two engines.

It has also been observed that in general the fan blade count of these modern engines is relatively low in the 20-26 range. Figure 5.1 shows the Allison and RR engines having 24 and 22 blades, respectively. The similar blade count and blade tip Mach number of the different engines implies that the normalized BPF tone frequency is again very similar. This can be observed by non-dimensionalizing the free acoustic wavenumber at the BPF using the fan radius as

\[
(k_0)_{BPF} a = \frac{2\pi f_{BPF} a}{c} = \frac{2\pi B \times rpm}{60c} a = BM_{tip} \tag{5.1}
\]

where \(a\) is the fan radius, \(B\) is the blade count, \(rpm\) is the fan speed in revolutions per minute, and \(M_{tip}\) is the fan tip Mach number.

Equation 5.1 shows that given the similar blade count and fan tip Mach number of modern turbofan engines, the normalized BPF tone frequency is basically alike for all engines. It is this normalized frequency that dictates the acoustic characteristics of the BPF tone and the harmonics.

Furthermore, the noise attenuation characteristics of the HQ tubes and liner are determined by their resonance frequencies. They are defined by the HQ tube mean length \(L\) and liner core depth \(d\), respectively. Since the engine speed and thus the BPF tone is inversely proportional to the fan size, the HQ tube length and liner core depth will change directly proportional with the engine size, e.g. an engine three times larger will have a liner and HQ tubes also three time larger since the BPF tone frequency will be three time smaller.

Therefore, the noise attenuation levels by an HQ-liner system should be basically the same for two different engines if everything is properly scaled. Thus, it was decided to evaluate the performance of an aft HQ-liner system for a “generic” turbofan engine that has the typical parameters of modern engines. The results will then be presented in non-dimensional form.
Engine dimensions will be normalized by the fan diameter whereas frequencies will be normalized by the BPF tone frequency. This approach allows avoiding the use of proprietary data of a specific engine. The parameters of the “generic” turbofan engine are listed below in Table 5-1.

Figure 5.1: Example of real turbofan engines. (a) Allison AE3007 (http://www.aerospaceweb.org/question/conspiracy/q0265.shtml) and (b) Rolls-Royce RB211-535 (http://www.aerospaceweb.org/question/conspiracy/q0265.shtml)
Table 5-1: Generic turbofan engine parameters at ~70% power, e.g. cut-back.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aft duct Hub-to-tip ratio(^1)</td>
<td>~ 0.7</td>
</tr>
<tr>
<td>Aft duct flow speed (M) at cut-back</td>
<td>~ 0.4</td>
</tr>
<tr>
<td>Number of rotor blades</td>
<td>22</td>
</tr>
<tr>
<td>Number of struts holding the fan</td>
<td>8</td>
</tr>
<tr>
<td>Normalized BPF Tone frequency at cut-back(^2): (f_{BPF}^*)</td>
<td>20.3</td>
</tr>
</tbody>
</table>

1- Estimated from Figure 5.1a.
2- Estimated using equation 5.1

Figure 5.1a shows the nacelle flow path of a small turbofan engine. As it can be observed, the hub-to-tip ratio in the aft section of the engine remains virtually unchanged for most of its length resembling an annular duct. Thus, the annular duct HQ-liner model developed in previous chapters is appropriate to model this generic engine.

The aft HQ-Liner system will be designed to target the BPF tone and broadband at cut-back condition. The system will be designed such that the HQ tubes will focus on the BPF tone as well as on the broadband component below and around the BPF (between 0.77-1.17 BPF). The liner will be targeted to the BPF tone and higher frequencies.

5.2. Disturbance

The noise attenuation prediction model needs information on the disturbance. The objective of the present section is to provide such information for both the BPF tone and the broadband component.

It is assumed that the BPF tone is dominated by the rotor-stator and rotor-strut interaction modes. However, the stator count in turbofan engines is selected to cut-off all the interactions modes at the BPF tone. This leaves the rotor-strut interaction modes as the only ones present in the disturbance at the BPF tone. The disturbance for the broadband case will be provided by using the same version of the equal-power and random-phase model utilized in Section 4.1.2 with a \(I^* = 0.49\).
5. Application: HQ-Liner in a Real Turbofan Engine

Table 5-2 shows the cut-on frequencies of the rotor-strut interaction modes. In this table, $m$ and $n$ indicate circumferential and radial orders, respectively. For the rotor and strut count selected here, there are 10 modes propagating in the aft duct at the BPF tone. It was decided to include all these modes in the disturbance with the same amplitude. The relative phase among them was randomly assigned. This approach produced the power levels listed in Table 5-3 for every circumferential mode. Note that only the total circumferential power is shown. For instance, the 130.1 dB of the $m = -2$ mode accounts for the power of the (-2,0), (-2,1) and (-2,2) modes.

Table 5-2: Normalized cut-off frequencies for rotor-strut interaction modes (22 Blades and 8 struts)

<table>
<thead>
<tr>
<th>Radial Modes</th>
<th>$n = 0$</th>
<th>$n = 1$</th>
<th>$n = 2$</th>
<th>$n = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-26</td>
<td>1.34</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>-18</td>
<td>0.95</td>
<td>1.18</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>-10</td>
<td>0.55</td>
<td>0.77</td>
<td>1.14</td>
<td>--</td>
</tr>
<tr>
<td>-2</td>
<td>0.11</td>
<td>0.51</td>
<td>1.00</td>
<td>1.49</td>
</tr>
<tr>
<td>6</td>
<td>0.33</td>
<td>0.61</td>
<td>1.05</td>
<td>--</td>
</tr>
<tr>
<td>14</td>
<td>0.75</td>
<td>0.96</td>
<td>1.27</td>
<td>--</td>
</tr>
<tr>
<td>22</td>
<td>1.15</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

¥ - Cut-off frequency normalized by the BPF tone frequency.

Table 5-3: Total power of each circumferential mode present in the disturbance.

<table>
<thead>
<tr>
<th>Circumferential mode $m$</th>
<th>Power [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-18</td>
<td>103.5</td>
</tr>
<tr>
<td>-10</td>
<td>114.9</td>
</tr>
<tr>
<td>-2</td>
<td>130.1</td>
</tr>
<tr>
<td>6</td>
<td>119.5</td>
</tr>
<tr>
<td>14</td>
<td>111.1</td>
</tr>
</tbody>
</table>
5. Application: HQ-Liner in a Real Turbofan Engine

5.3. Design and Performance of Aft HQ-Liner System

The design of the HQ-Liner system for the aft section of the generic turbofan engine is presented in this section. The design process is divided into the following steps: The first step is Liner design, explained in section 5.3.1. Here, attenuation of the broadband component due to several liners has been analyzed in order to identify the liner with best overall performance. The second step, HQ tube design, is contemplated in section 5.3.2; here, the preliminary dimensions of the HQ tubes to be combined with the optimum liner are established. The third and final step, HQ-Liner results, is presented in section 5.3.3. In this section, the performance of the HQ-liner defined in the previous two sections is investigated. In addition, results are presented for variations in the HQ tube parameters such as mean length, interface distance, number of tubes in the arrays, and number of arrays. In all cases, the liner designed in section 5.3.1 is used.

Since it is more common to only have the outer surface of the bypass lined, it was decided that a combination of HQ tubes and liner will be used on the outer wall and only HQ tubes in the inner wall. The HQ-tube interface with the aft duct is assumed to be covered by a 28% POA perforated plate.

5.3.1 Liner Design

The first step in the design process consisted of defining the liners that best attenuate the broadband component within the normalized frequency range 0.78-1.55 BPF. The liner is assumed to be a single-degree-of-freedom linear liner. The length of the acoustic treatment is assumed to be 35% of the duct outside radius and only the outer wall of the aft duct is treated. In order to select the liners, three liner resistances $R$, i.e. 1.0, 1.4, and 1.8\(\rho c\), were combined with five liner reactances $X$ calculated as

$$X = b - \cot(k_0 d)$$  \hspace{1cm} 5.2

where $k_0$ is the free field wavenumber at cut-back condition, $d$ is the liner core depth, and $b$ is the non-dimensional reactive component of the impedance of the liner wire-mesh, i.e. mass like effect. The effect of the wire-mesh is small and the value for the wire-mesh used on the ANCF liners was implemented here. A total of five different $d$ were considered. These different core depths defined the liners resonant frequencies. These resonant frequencies ranged between...
1.0BPF and 1.85 BPF. Recall that the liner resonant frequency $f_{\text{liner}}$ is the one that makes $X = 0$ in 5.2.

Broadband power attenuation due to the various liners considered was calculated. The overall attenuation within the non-dimensional frequency range of 0.78-1.55 BPF is plotted in Figure 5.2 as a function of the liner resistance and $f_{\text{liner}}$. Maximum attenuation of 8.8 dB is produced by a liner with a resistance of $1.4\rho c$ and a non-dimensional resonance frequency of 1.45 BPF. However, Figure 5.2 shows the overall attenuation to be rather insensitive to the liner properties for the ranges $1.0\rho c < R < 1.7\rho c$ and $1.35 \text{ BPF} < f_{\text{liner}} < 1.85 \text{ BPF}$. The liner with $1.0\rho c$ and $f_{\text{liner}} = 1.65 \text{ BPF}$ was selected to be combined with HQ-tubes.

For completeness, Figure 5.3 shows broadband sound power attenuation spectra for the liner investigated. Note that the peak attenuation for the liners occurs at frequencies below the BPF. In addition, increasing the liner resistance flattens the attenuation spectra.

**Figure 5.2:** Overall sound power attenuation of the broadband component between 0.78-1.56 BPF at cut-back condition.
Figure 5.3: Broadband power reduction at cut-back condition due to liners with a resistance of (a) 1.0, (b) 1.4, and (c) 1.8\(\rho_c\).
5.3.2 HQ Tube Design

This part of the design process involves the definition of the normalized tube dimensions, i.e. mean length $L^*$, interface distance $\ell^*$ and cross section diameter $d_{HQ}^*$. Once the tube dimensions have been defined, the next step is to establish the number of HQ tubes per array. The dimensions of the HQ tubes were obtained based on previous research efforts. The HQ diameter was selected such as to have plane waves inside the HQ tube over the frequency range of interest, i.e. 0.77-1.17 BPF. Thus, the mean length, interface distance, and tube diameter normalized by the fan diameter $D$ are found to be:

1. $L^* = L/D = 0.1618$
2. $\ell^* = \ell/D = 0.1029$
3. $d_{HQ}^* = d_{HQ}/D = 0.038$

Once the mean length, interface distance, and tube cross section area have been defined, the next step is to determine the number of HQ tubes per array. The maximum number of tubes that can be fitted in the inner wall of the aft duct is given as $0.7\pi D / d_{HQ} = 0.7\pi / d_{HQ}^* = 57$ where 0.7 is the hub-to-tip ratio. To allow for some separation between the tubes, a maximum of 45 tubes are assumed to be placed on the inner wall. Similarly, a total of 45 HQ tubes was assumed in the HQ array position on the outer wall, e.g. maximum number of tubes that could be installed is $\pi D / d_{HQ} = \pi / d_{HQ}^* = 82$. Finally, it was decided to use two HQ arrays on both outer and inner walls. Figure 5.5 shows a drawing of the aft HQ tubes and their location relative to the liner described in the previous section. Table 5-4 summarizes the preliminary dimensions of the HQ tubes. It is important to note that the impedance of the screens at the HQ tube/duct interfaces is defined by the liner properties in the outer wall and the 28%POA perforate in the inner wall. The resistance of these screens are listed in Table 5-3.
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Figure 5.4: HQ-liner system installed in an annular duct that represents the aft section of a generic turbofan engine with a hub-to-tip ratio 0.7.

Table 5-4: HQ tubes dimensions for generic turbofan engine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Outer wall</th>
<th>Inner wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L^*$</td>
<td>0.1618</td>
<td>0.1618</td>
</tr>
<tr>
<td>$\ell^*$</td>
<td>0.1029</td>
<td>0.1029</td>
</tr>
<tr>
<td>$d^H_{HQ}$</td>
<td>0.038</td>
<td>0.038</td>
</tr>
<tr>
<td>Screen resistance at HQ-tube openings [(\rho_c)]</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Number of tubes</td>
<td>45</td>
<td>45</td>
</tr>
</tbody>
</table>

5.3.3 HQ-Liner Results

This section presents the results for the attenuation due to the HQ-liner system designed in the previous sections. As already mentioned, results are also presented for HQ-liner systems where tube parameters, e.g. mean length, interface distance, number of tubes, have been changed. In all cases, the liner designed in section 5.3.1 is always used. For the sake of completeness, the performance of HQ tubes on hard wall is also included. Result for different HQ-liner configurations are presented next.
HQ-liner configurations

Table 5-5 shows five different HQ tube designs used in conjunction with the liner. Note that design 2 corresponds to the initial dimensions presented in section 5.3.2. In addition, each of the HQ tube designs in Table 5-5 was tested in the three configurations shown in Table 5-6. Most of these configurations are self-explanatory except for the second. This configuration is the same arrangement as the first one but the liner resistance has been locally reduced at the tube/liner interface in an attempt to improve tube performance by reducing the damping. To this end, the liner wire-mesh was replaced by a 28% POA perforated plate, e.g. the same as the one used on the inner wall. Configuration 3 when combined with HQ tube design 2 will be referred to as the baseline configuration.

Table 5-7 shows the six configurations used to investigate the influence on power reduction of the number of arrays and the relative influence of inner and outer arrays. For these particular cases only HQ tube design 2 was utilized.

Table 5-5: Parameters of different HQ tube designs investigated in different configurations

<table>
<thead>
<tr>
<th>Design</th>
<th>(L^*)</th>
<th>(L^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1765</td>
<td>0.0882</td>
</tr>
<tr>
<td>2</td>
<td>0.1618</td>
<td>0.1029</td>
</tr>
<tr>
<td>3</td>
<td>0.1912</td>
<td>0.1029</td>
</tr>
<tr>
<td>4</td>
<td>0.2206</td>
<td>0.1029</td>
</tr>
<tr>
<td>5</td>
<td>0.2059</td>
<td>0.1176</td>
</tr>
</tbody>
</table>

Table 5-6: Configurations used to assess the influence of the number of HQ tubes and liner wire mesh on HQ contribution to sound power reduction.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>Outer wall</th>
<th>Inner wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 arrays of 35 tubes combined with liner</td>
<td>2 arrays of 26 tubes on hard wall</td>
</tr>
<tr>
<td>2</td>
<td>2 arrays of 35 tubes combined with liner (liner resistance locally reduced at tube opening)</td>
<td>2 arrays of 26 tubes on hard wall</td>
</tr>
<tr>
<td>3</td>
<td>2 arrays of 45 tubes combined with liner</td>
<td>2 arrays of 45 tubes on hard wall</td>
</tr>
</tbody>
</table>
Table 5-7: Configurations used to investigate the effect on sound power reduction due to a change in the number of HQ tube arrays and to investigate the relative contribution of inner and outer wall arrays.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>Outer wall</th>
<th>Inner wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (baseline)</td>
<td>2 arrays of 45 tubes combined with liner</td>
<td>2 arrays of 45 tubes on hard wall</td>
</tr>
<tr>
<td>5</td>
<td>2 arrays of 45 tubes combined with liner</td>
<td>0 arrays</td>
</tr>
<tr>
<td>6</td>
<td>0 arrays</td>
<td>2 arrays of 45 tubes on a hard wall</td>
</tr>
<tr>
<td>7</td>
<td>1 array of 45 tubes combined with a liner</td>
<td>1 arrays of 45 tubes on hard wall</td>
</tr>
<tr>
<td>8</td>
<td>4 arrays of 45 tubes combined with liner</td>
<td>2 arrays of 45 tubes on hard wall</td>
</tr>
<tr>
<td>9</td>
<td>4 arrays of 45 tubes combined with liner</td>
<td>1 array of 45 tubes on hard wall</td>
</tr>
</tbody>
</table>

Sound power attenuation of the BPF tone as well as the broadband component due to every configuration mentioned above was calculated. Results for the BPF tone are presented next.

Figure 5.5a-b compare BPF tone reduction due to the HQ tube designs listed in Table 5-5 for configurations 1 and 3 in Table 5-6. The BPF attenuation due to the liner is 12.2 dB. Results in these figures allow determining the effect of changing the number of HQ tubes in the arrays. The reduction in the number of tubes resulted in significant less attenuation due to the HQ for virtually all tube designs. In case of the tube with an $L^* = 0.1618$ and $\ell^* = 0.1029$, the reduction provided by configuration 3 is 4.7 dB as compared to the 2.3 dB provided by configuration 1.

Figure 5.5: HQ contribution to BPF tone sound power reduction for configurations (a) 1, and (b) 3, for different HQ tube designs.
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Figure 5.6a presents HQ tube contribution when the liner resistance was not locally reduced and Figure 5.6b shows results when the liner wire mesh was replaced by a 28% POA at the tube/duct interface. As observed, locally reducing the resistance at the duct tube interfaces improves the HQ performance. The optimum attenuation for the case with uniform liner resistance is given by the $L^* = 0.1618$ and $\ell^* = 0.1029$ tube with a 2.3 dB reduction. For the case of the liner resistance locally reduced, the reduction increases to 2.8 dB obtained with the same tubes. This indicates that the removal of the wire mesh did not result in a significant improvement. The reason may reside in the fact that only the tubes on the outer wall are affected by the liner wire mesh. This also may be indicative of the considerable importance of the contribution of the HQ tubes located on the inner wall.

Figure 5.6: HQ contribution to BPF tone sound power reduction for configurations (a) 1, and (b) 2, for different HQ tube designs.

The attenuation of individual modes allows gaining further insight into the liner, HQ tube, and combined system performances. Note that the modes present in the duct are the 10 propagating modes listed in Table 5-2 and all of them have the same amplitude. In presenting the modal data, the negative spinning modes in Table 5-2, e.g. $m$=-2, -10, and -18, were modeled as positive propagating modes. Since HQ tubes are parallel to the duct axis, a negative propagating mode and its positive counter part will be affected in the same exact way by the HQ tubes. In addition, the absence of swirling flow implies that the eigen-wavenumbers of positive and negative rotating modes are the same [55].

Figure 5.7 shows modal attenuation due to the liner (Figure 5.7a) and due to the HQ tubes with respect to the liner (Figure 5.7b-f) for the configuration 1 in Table 5-6. The last row in these
figures presents the total power reduction in each circumferential. Note that the Figure 5.7a shows that the liner attenuation increases with the circumferential order of the mode, e.g. liner strongly attenuated the (18,0) mode with over 50.0 dB reduction. In case of the HQ tube contribution in Figures 5.7b-f, it is evident the scattering of the incident disturbance by the low number of HQ tubes in the arrays. These figures show scattering into the $m=1, 3, 6, 7, 11, 12, 15, 16$ circumferential modes as indicated by the red bars with negative attenuation, e.g. increase in levels. For the sake of clarity, only the scattered total circumferential power is shown in these figures. The HQ array responsible for these scattering effects can be identified by using the expression

$$m_s = m_d \pm k N_{HQ}$$

5.3

where $m_s$ is the circumferential order of the scattered modes, $m_d$ is the circumferential order of the disturbance mode, $N_{HQ}$ is the number of tubes in the array, and $k$ is an integer. For example, the incident mode $m_d=18$ scatters energy into the $m=-8$ (shown as positive) mode due to the first array of 26 HQ tubes on the inner wall. As this scattered mode propagates down the duct, the downstream set of HQ tubes will produce additional scattering effects, e.g. $m_s=-8+26=18$. The results in Figure 5.7 clearly show a degradation in the HQ system performance due to scattering.

Results in this figure also reveal that the HQ tube design directly impacts the modes more effectively attenuated. For instance, HQ tube design 5 in Table 5-5 ($L^*=0.2059$ and $\ell^*=0.1176$) is very effective at controlling the $m=14$ modes while design 4 in Table 5-5 ($L^*=0.1765$ and $\ell^*=0.0882$) mainly attenuates the $m=2$ modes. Moreover for a fixed $\ell^*$, increasing $L^*$ leads the HQ attenuation to shift from the lower to higher circumferential modes (see Figure 5.7c, e and f).

The same type of results is shown in Figure 5.8 for configuration 3 in Table 5-6. As expected, this configuration not only does not lead to scattering effects but also yields higher modal attenuation levels. These effects explain the better performance of a system with a larger number of HQ tubes per array observed in Figure 5.5b.
Figure 5.7: BPF tone modal sound power reduction: (a) due to the liner and (b), (c), (d), (e), and (f) due to the different HQ tubes designs listed in Table 5-5. HQ contribution is given relative to the liner. The liner was combined with 2 arrays of 35 HQ tubes on the outer wall and 2 arrays of 26 HQ tubes on the inner wall, i.e. configuration 1 in Table 5-6.
Figure 5.8: BPF tone modal sound power reduction: (a) due to the liner and (b), (c), (d), (e), and (f) due to the different HQ tubes designs listed in Table 5-5. HQ contribution is given relative to the liner. The liner was combined with 2 arrays of 45 HQ tubes on the outer and inner walls, i.e. configuration 3 in Table 5-6.

In order to determine the influence on BPF tone power reduction due to the number of arrays and the relative influence of the outer and inner arrays, results for cases in Table 5-7 are analyzed next. As already mention only HQ tube design 2, e.g. $L^* = 0.1618; \ell^* = 0.1029$, will be investigated in combination with the liner.
The BPF tone reduction for all configurations in Table 5-7 are summarized in Table 5-8. It is very interesting to find that the most effective HQ arrays are the ones positioned on the inner surface of the aft duct on a hard wall, e.g. configuration 6 in Table 5-8. These inner surface arrays provide 3.9 dB reduction out of the 4.7 dB obtained using the baseline configuration that has arrays in both inner and outer surfaces.

**Table 5-8:** BPF tone power reduction at cut-back due to several HQ-liner configurations. HQ tube design 2 was used.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>Configuration Description (from Table 5-7)</th>
<th>Power Reduction [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (Baseline)</td>
<td>2 array of 45 tubes on the outer wall and 2 arrays of 45 on the inner wall</td>
<td>4.7</td>
</tr>
<tr>
<td>5</td>
<td>2 arrays of 45 tubes on the outer wall</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>2 arrays of 45 tubes on the inner wall</td>
<td>3.9</td>
</tr>
<tr>
<td>7</td>
<td>1 array of 45 tubes on the inner and outer walls</td>
<td>2.8</td>
</tr>
<tr>
<td>8</td>
<td>4 array of 45 tubes on the outer wall and 2 arrays of 45 on the inner wall</td>
<td>3.2</td>
</tr>
<tr>
<td>9</td>
<td>4 array of 45 tubes on the outer wall and 1 array of 45 on the inner wall</td>
<td>1.5</td>
</tr>
</tbody>
</table>

A very surprising result is that configurations with larger number of arrays such as configurations 8 and 9 in Table 5-8 produced less attenuation. The explanation for these results is found in the modal analysis. Figure 5.9b-d compare modal attenuation due to the HQ tubes relative to the liner for configurations 5, 6 and 7 in Table 5-8. As it can be observed when tubes are located on the inner and outer walls (Figure 5.9d) attenuation is achieved for all the modes in the disturbance, mainly the $m = 2$, i.e. the most dominant mode in the disturbance (see Table 5-3). If tubes are located on the outer wall only (see Figure 5.9b) attenuation concentrates on higher circumferential modes. For instance, power reduction of the $m = 14$, e.g. one of the least dominant modes in the disturbance, is around 2.1 dB. If tubes are placed on the inner wall (see Figure 5.9c), reduction focuses on the lower circumferential modes, mainly the $m = 2$ at roughly 4.1 dB.

Figure 5.9e presents configuration 8. As indicated, increasing the number of tubes on the outside increases the attenuation of the higher circumferential modes, i.e. reduction of $m = 14$ mode is 2.6dB as opposed to 2.0dB produced by the baseline configuration (Figure 5.9a). At the
same time increasing the number of arrays on the outer wall reduces attenuation levels on the m = 2 mode by as much as 1.0dB which explains why the total attenuation of the BPF tone provided by this configuration is less than the baseline (see Table 5-8). Finally, Figure 5.9f shows configuration 9. This configuration features five arrays of 45 tubes; four are on the outer wall whereas only one is placed on the inner wall. As it can be observed, the attenuation of the m = 2 mode is dramatically reduced to 1.9 dB.

Figure 5.9: BPF tone modal attenuation for: configuration (a) 4; (b) 5; (c) 6; (d) 7; (e) 8; and (f) 9
Since HQ tubes are also intended to contribute to broadband attenuation, their broadband performance has also been investigated. Configurations 1, 2, and 3 from Table 5-6 and configurations 5 through 9 from Table 5-7 are analyzed. Since the broadband attenuation is not very sensitive to the HQ-parameters, only the tubes with dimensions $L^* = 0.1618$ and $\ell^* = 0.1029$ are presented (design 2 in Table 5-5).

Figure 5.10 shows the broadband attenuation due to the HQ tubes for configurations 1, 2 and 4 (baseline). For reference, the liner performance was presented in Figure 5.3. Table 5-9 presents the overall broadband power reduction over two frequency ranges of interest. In all cases, the maximum attenuation provided by the tubes occurs around the tube 2nd resonance frequency (0.95 BPF). As expected, the baseline configuration which has more tubes in the arrays yields better results, i.e. overall attenuation within the non-dimensional frequency range 0.78-1.17 BPF is 3.5dB as opposed to 2.3dB of the other two configurations (see Table 5-9). In addition, it can be noticed that the reduction of the liner resistance at the tube/liner interface improved the contribution from the HQ tubes only around their 1st resonance frequency, e.g. 0.475 BPF.

![Figure 5.10](image)

**Figure 5.10:** HQ tube contribution to broadband attenuation. In every configuration the optimum liner was combined with HQ tube design 2.
5. Application: HQ-Liner in a Real Turbofan Engine

Table 5-9: Overall sound power reduction of the broadband component due to HQ tubes relative to the liner for different configurations. In every case HQ tube design 2 was used.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>Configuration Description (from Table 5-6)</th>
<th>Overall sound power reduction [dB] within the two non-dimensional frequency ranges of interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.78-1.17 BPF</td>
</tr>
<tr>
<td></td>
<td>Liner</td>
<td>9.9</td>
</tr>
<tr>
<td>1</td>
<td>2 arrays of 35 tubes combined with liner on the outer wall and 2 arrays of 26 on the inner wall</td>
<td>2.3</td>
</tr>
<tr>
<td>2</td>
<td>2 arrays of 35 tubes combined with liner (liner resistance locally reduced at tube opening) 2 arrays of 26 on the inner wall</td>
<td>2.3</td>
</tr>
<tr>
<td>4</td>
<td>2 arrays of 45 tubes combined with liner on the outer wall and 2 arrays of 45 on the inner wall</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Figure 5.11 and 5.12 show the contribution of the HQ tubes for configurations 5 through 9 and the baseline. Overall attenuations for the frequency ranges of interest are presented in Table 5-10.

Figure 5.11 compares configurations 5, 6 and 7 to the baseline. As in the BPF tone case, the most important result is that most of the attenuation is provided by the tubes located on the inner surface, i.e. hard wall. In fact when tubes are placed on the outer wall only, the contribution is almost negligible at 0.4~0.3dB depending on the frequency range (see Table 5-10). When the same number of tubes is used on the inner wall attenuation levels are 2.8~1.6dB. Note that maximum attenuation occurs around the BPF except for configuration 5, i.e. it shifted towards 0.8 BPF.

Figure 5.12 compares the last two configurations in Table 5-10. The baseline has been included as a reference. As indicated, when 6 arrays of tubes are used (4 on the outer wall; 2 on the inner wall) overall reduction increased, e.g. to 3.9 and 2.1dB for the respective frequency ranges (see Table 5-10). Also note that when the number of HQ tubes is increased on the outer wall the frequency of maximum attenuation shifts towards the low frequency range. If number of arrays on the inner wall is reduced (configuration 9) the overall attenuation decreased respect to
the base line. This confirms the importance of the contribution of the tubes located on the inner hard wall.

**Figure 5.11:** HQ tube contribution to broadband attenuation. In every configuration the optimum liner was combined with HQ tube design 2.

**Figure 5.12:** HQ tube contribution to broadband attenuation. In every configuration the optimum liner was combined with HQ tube design 2.
Table 5-10: Overall sound power reduction of the broadband component due to HQ tubes relative to the liner for different configurations. In every case HQ tube design 2 was used.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>Configuration Description (from Table 5-7)</th>
<th>Overall sound power reduction [dB] within the two non-dimensional frequency ranges of interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.78-1.17 BPF</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.78-1.56 BPF</td>
</tr>
<tr>
<td>Liner</td>
<td></td>
<td>9.9</td>
</tr>
<tr>
<td>Baseline</td>
<td>2 arrays of 45 tubes combined with liner on the outer wall and 2 arrays of 45 on the inner wall</td>
<td>3.5</td>
</tr>
<tr>
<td>5</td>
<td>2 arrays of 45 tubes on the outer wall</td>
<td>0.4</td>
</tr>
<tr>
<td>6</td>
<td>2 arrays of 45 tubes on the inner wall</td>
<td>2.8</td>
</tr>
<tr>
<td>7</td>
<td>1 array of 45 tubes on the inner and outer walls</td>
<td>2.0</td>
</tr>
<tr>
<td>8</td>
<td>4 array of 45 tubes on the outer wall and 2 arrays of 45 on the inner wall</td>
<td>3.9</td>
</tr>
<tr>
<td>9</td>
<td>4 array of 45 tubes on the outer wall and 1 array of 45 on the inner wall</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**HQ-tubes on hard wall configurations**

For the sake of completeness, the performance of the HQ tubes on hard wall is also included in this section. Both BPF tone and broadband sound power attenuation are studied. As already mentioned, a 28% POA with a resistance of 0.5ρc are assumed at the tube/duct openings.

The configuration investigated corresponds to that of 2 arrays of 45 tubes on both inner and outer walls. This configuration will be referred to as configuration 10 and it was combined with the five HQ tube designs listed in Table 5-5. Note that configuration 10 is the same as configuration 3 in Table 5-6 but without the liner.

Figure 5.13 compares BPF tone reduction due to the HQ tube designs for configuration 10. The best attenuation is obtained with tube design 3 ($L^* = 0.1912$ and $\ell^* = 0.1029$) at 9.0 dB.

Even though the HQ-liner combination produces more attenuation of the BPF than the liner or HQ tubes alone, HQ tubes are more efficient when placed on a hard wall. For instance, the HQ-liner combination attenuates the BPF in 16.9 dB, of which 4.7 dB come from the HQ tubes.
When HQ tubes are on hard wall, they attenuated the BPF in 9.0 dB, this is 4.3 dB more than the 4.7 dB when combined with a liner (see Figure 5.5b). There are two reasons for this behavior. The first one is related to the lower resistance of the screen placed at the opening of the tubes on the inner wall (see Table 5-4). Since tubes are resonating devices a high resistance can significantly degrade their performance. The second reason has to do with the nature of the tubes. HQ tubes are passive devices; which means that their performance depends on the excitation. When tubes are combined with a liner the excitation has already been significantly attenuated by the liner.

Figure 5.14 shows BPF tone modal attenuation for the configuration considered. Like in the HQ-liner case, changing the number of tubes can change the mode on which the system concentrates.

![Figure 5.13: BPF tone sound power reduction due to HQ tubes on hard wall for configuration 10 for different HQ tube designs.](image)

Figure 5.15 shows broadband attenuation due to HQ tubes for configuration 10. The HQ tube design corresponds to the one that yielded the best BPF attenuation i.e. $L^* = 0.1912$ and $\ell^* = 0.1029$. The overall reduction within the frequency ranges of interest are 2.9 dB for 0.78-1.17 BPF and 0.78-1.56 BPF.
Figure 5.14: BPF tone modal sound power reduction: due to (a), (b), (c), (d), and (e) the different HQ tubes designs listed in Table 5-5, for configuration 10.
5. Application: HQ-Liner in a Real Turbofan Engine

5.4. Overall Conclusions

The objective of the present chapter was to specifically design an HQ-liner system for general turbofan engine and assess its potential when working in a more realistic environment. This section summarizes the most important conclusions of the design and analysis of the different HQ-liner configurations installed in a general turbofan engine.

- Attenuation levels produced by the HQ-Liner combination confirmed the potential of the concept at controlling both the BPF tone and the broadband component. The baseline HQ-liner configuration attenuated the BPF tone by 16.9 dB with 4.7 dB provided by the HQ tubes. In case of the broadband component, the overall sound power reduction due to the HQ tubes was significant at 3.9 dB around the BPF and 1.9 dB between 0.78-1.56 BPF. The liner provided 9.9 dB and 8.4 dB, respectively.

- The effectiveness of the tubes at controlling the BPF tone depends on the tubes focusing on the most dominant modes. This can be obtained not only by changing the tube dimensions but also by changing the number of tubes per array and the number of arrays. This implies...
that, unlike so far, the number of HQ tubes should be included as a parameter in the design process.

- The HQ arrays located on inner hard wall contributed more to the attenuation of the BPF tone and the broadband component than the tubes placed on the outer lined wall.
6. CONCLUSIONS AND FUTURE WORK

A model to predict fan radiation attenuation in annular ducts due to the combination of HQ tubes and a liner has been developed. This model is based on a Green’s function formulation for lined ducts with convective flow. The model is an extension of the circular duct model developed previously by Alonso [33]. Specific modifications have been introduced to this formulation in order to account for the presence of noise treatment on the inner wall of the aft duct. Both the inlet and aft HQ-liner models assume that the opening of the sound field inside the HQ-tubes is described in terms of plane waves. This modeling assumption implies that the effect of the HQ tubes can be modeled as uniform pistons sources radiating into the duct. This plane-wave/single-piston model (1DOF) model imposes an upper frequency limit in the analysis. In order to overcome this limitation, a new multi-degree-of-freedom model for the HQ tube dynamics has been developed. The sound field inside the HQ tube is expressed in terms of the plane wave and higher order modes (MDOF).

To validate the accuracy of the HQ-liner models, a series of experiments were performed using the NASA ANCF rig. To this end, an aft HQ-liner system was designed and tested. Both in-duct and far-field data was recorded and compared to the predicted results to validate the codes.

The ANCF rig is an excellent facility to make accurate measurements and thus it is ideal for code validation. However, this fan is not representative of real turbofan engines. In order to investigate the performance of HQ-liner systems in a more realistic environment, a study was undertaken to estimate the potential noise reduction of an aft HQ-liner system in a “generic” modern turbofan engine.

The main conclusions of this research effort are summarized next along with a few recommendations for future work.
6. Conclusions and Future Work

6.1. Conclusions

The most important conclusions obtained throughout the present research effort are listed below:

1) A model to predict aft HQ-liner fan radiation attenuation has been developed. This model was utilized to design an HQ-liner system to be tested at the NASA Glenn ANCF rig. The experimental data collected confirmed the potential of HQ-Liner systems at controlling both pure tones and broadband noise components. At the design condition of 1800 rpm, the HQ tubes contributed 2.6 dB over the 3.9 dB provided by the liner alone to the attenuation of the 2BPF tone. For the broadband, the HQ tubes slightly improved the overall performance.

2) The aft radiation control model has been successfully validated. Predicted and measured attenuation of the 2BPF tone and the broadband component agreed well. For instance, at design speed of 1800 rpm, differences between predicted and measured attenuation of the 2BPF tone were below 0.8 dB. In case of the broadband, also at 1800 rpm and within the 2.5-5.5 BPF range, differences were less than 1.0 dB. The only exception is the attenuation of the broadband due to the liner at around 5.5 BPF, where differences were between 4.0-6.0 dB.

3) An additional HQ-liner system was also tested in the inlet of the ANCF rig. This experimental data was used to complete the validation of the inlet model developed by Alonso [33] at predicting pure tone reduction. This validation was also successful. The error in the predicted 2BPF tone reductions were in the 0.5-4.0 dB range.

4) During the validation process it was found that reflections at the HW/liner interfaces played an important role in the reduction of sound and therefore had to be included in the prediction model.

5) The effect of the high resistance wire mesh of typical liners on the performance of the HQ-tubes was also investigated experimentally. To this end, a test where the wire mesh was removed at the HQ tube/duct openings was performed and compared to the unchanged wire mesh case. As predicted by the models, the experimental results clearly showed better attenuation. The HQ tube attenuation of the 2BPF tone increased by 1.5 dB at the design
6. Conclusions and Future Work

speed of 1800 rpm. This implied that the local reduction of the liner resistance at the HQ tube openings is beneficial and it has to be included as a parameter in the design process.

6) A MDOF model to describe the HQ tube dynamics has been developed using Green’s functions. Predictions using the MDOF model have been compared to those using the 1DOF model in Appendix F. The analysis has shown that a MDOF model would be justified when the frequency of interest exceeds the tube’s first resonance. For this to be the case, the tube cross section has to be large compared to the disturbance wave length. However, given the tight room constrain existing in real turbofan engines, this situation is highly unlikely.

With the objective of investigating the performance of HQ-Liner systems in a more realistic environment, an HQ-liner system for the aft sector of a generic turbofan engine was designed and evaluated. This effort has lead to the following conclusions:

7) Predicted attenuation for an aft HQ-Liner system installed in a “realistic” turbofan engine again showed the potential of the concept at controlling the BPF tone and the broadband component. The HQ contribution to BPF tone reduction was as high as 4.7 dB over the 12.2 dB provided by the liner. In case of the broadband component the overall HQ contribution was also significant particularly around the BPF tone at with an additional reduction of 3.9 dB over the 9.9 dB provided by the liner.

8) Arrays located on the inner hard wall of the engine contributed with most of the attenuation provided by the HQ tube system. For instance, the total HQ contribution to the attenuation of the BPF tone was 4.7 dB relative to the liner. Of this 4.7 dB, around 3.9 dB were provided by the tubes on the inner hard wall.

6.2. Future work

Based on the results from this research effort and its conclusions, the following research topics are recommended for future work:
1) **C-sectioned duct model:** In many cases, the aft section of real turbofan engines feature two c-sectioned ducts instead of the annular sectioned duct modeled in this dissertation. Thus, it might be important to modify the model developed here to handle c-sections.

2) **Duct with varying cross section:** Neither the inlet nor the aft duct in turbofan engines are perfect constant circular annular cross sections. For example, in the inlet the presence of the spinner poses the problem of an annular duct that transitions into a circular one. The model developed in this dissertation assumes a constant cross section. It is, therefore, suggested to extend the model to handle ducts with varying cross sections. However, the relaxation of this constant cross section limitation might be very challenging for a closed formulation.

3) **Alternative applications of HQ-liner systems:** The concept of HQ-liner systems have been investigated so far in connection with noise radiation control in turbofan engines. However its application can be extended to any problem involving sound propagation in ducts, for instance, gas turbines. Gas turbines are widely used and its applications include as different environments as tanks, helicopters and power plants. Thus it is recommended to investigate the potential of this new technology in controlling noise radiation from gas turbines.
Appendix A: Cross Impedance Matrix $Z_{os}$

In this section, the impedance matrix $Z_{os}$, first used in equation 2.48, is derived. This matrix accounts for the influence of the pistons upon each other. Each one of its coefficients is calculated as the average pressure over the piston observer “o” divided by the velocity of the piston source “s”. Or in mathematical form $\frac{\overline{p}_{os}}{V_s} = Z_{os}$, where $V_s$ is the unknown piston source velocity and $\overline{p}_{os}$ is the average pressure over the piston observer due to the piston source. This way, in order to compute the impedance function, the average pressure over a piston observer “o” due to a piston source “s” has to be calculated first.

The first step in this derivation is to determine the sound field generated by a finite piston of surface area $S$ and unknown velocity $V$ as illustrated in Figure A.1. This sound field is obtained using the Green's function. The entire derivation will be shown for a pistons positioned over the outer wall ($r = a$), however in case of an annular duct, they can also be located over the inner wall, i.e. $r = b$.

![Figure A.1: Piston source on an annular duct.](image)
Appendix A: Cross Impedance Matrix $Z_{os}$

A.1. Finite piston source radiation

The Green's function for a point source positioned on the outer wall at $a, \bar{\theta}, z$ is given as

$$g^{(+)}(r, \theta, z / a, \bar{\theta}, z) = \sum_{m_x} \sum_{n_y} A_{m_x n_y}^{(+)} \Phi_{m_x n_y}^{(+)} e^{-k_{m_x n_y}^+ (z-z)} \quad A.1$$

$$g^{(-)}(r, \theta, z / a, \bar{\theta}, z) = \sum_{m_x} \sum_{n_y} A_{m_x n_y}^{(-)} \Phi_{m_x n_y}^{(-)} e^{-k_{m_x n_y}^- (z-z)} \quad A.2$$

For $z \geq \bar{z}$ and $z \leq \bar{z}$ respectively, where the eigen-function for a non rotating sound field is given by

$$\Phi_{m_x n_y}^{(+/-)} = \cos[m_x (\theta - \bar{\theta})] J_{m_x} \left( k_{m_x n_y}^{(+/-)} r \right) + \left( \frac{D}{C} \right)^{(+/-)} Y_{m_x} \left( k_{m_x n_y}^{(+/-)} r \right) \quad A.3$$

In this equation, (+) and (-) indicate positive and negative propagating waves; $M_x$ and $N_x$ are the highest circumferential and radial mode included in the Green's function. $A_{m_x n_y}^{(+)}$ and $A_{m_x n_y}^{(-)}$ are the amplitudes of the Green’s function for positive and negative propagating waves for a source located on the outer wall. The pressure at $(r, \theta, z)$ due to the $n^{th}$ finite piston source with source velocity $V_n$ is then obtained by integrating the Green's function over the surface of the source as

$$p(r, \theta, z / a, \theta_n, z_n) = -i \omega \rho V_n \sum_{m_x=0}^{M_x} \sum_{n_y=0}^{N_y} \left( k_0 - k_{m_x n_y} \right)^2 \int_{z_{n-x}}^{z_{n+x}} \int_{z_{n-y}}^{z_{n+y}} g_{m_x n_y}^{(-)} (r, \theta, z / a, \theta_n, z_n) d\theta d\bar{\theta} d\bar{z} \quad A.4$$

where the dimension $d$ and $\alpha$ of the source are given in terms of the source area as $d = \sqrt{S}/2$ and $\alpha = \sqrt{S}/2a$, respectively, and $(a, \theta_n, z_n)$ is the location of the source center as defined in the Figure A.2.

By replacing A.3 into either A.1 or A.2 and these into A.4 it becomes clear that there are two integrals that need to be solved. The first one is with respect to the coordinate $\theta$, which is easily obtained as

$$\int_{\theta-n}^{\theta+n} \cos[m(\theta - \bar{\theta})] d\theta = \frac{2a \alpha \sin(m\alpha)}{m\alpha} \cos[m(\theta - \theta_n)] = K_0(\alpha) \cos[m(\theta - \theta_n)] \quad A.5$$
Appendix A: Cross Impedance Matrix $Z_{os}$

where $K_\alpha(\alpha) = \frac{2a\alpha\sin(m\alpha)}{ma}$, note that every subscript “g” in the last two expressions has been omitted for simplicity. For the special case of $m = 0$, equation A.5 gives $2a\alpha$. The solution of the second integral depends on the position of the observation point $z$ relative to the position of the piston source $z_n$. The three possible cases are illustrated in Figure A.2.

**Case 1**

The observer point is downstream of the source, i.e. $z > z_n + d$. This case is illustrated in Figure A.2a and implies that $z > z$. Thus, the integral $\int_{z_n-d}^{z_n+d} e^{-ik^{(+)}_{z}(z-z')} d\bar{z}$ for $z \geq z$ needs to be solved. This leads to

$$\int_{z_n-d}^{z_n+d} e^{-ik^{(+)}_{z}(z-z')} d\bar{z} = e^{-ik^{(+)}_{z}(z-z_n)} \frac{\sin(k^{(+)}_{z}d)}{k^{(+)}_{z}d} 2d$$

A.6

The pressure due to the source positioned at $(a, \theta_n, z_n)$ is then obtained by replacing Equation A.1 into Equation A.4 and solving the integrals in A.5 and A.6. This is

$$p(r, \theta, z / a, \theta_n, z_n) = -i\omega p V \sum_{m_z=0}^{M_z} \sum_{n_z=0}^{N_z} \left( k_0 - k^{(+)}_{z} M \right)^2 A^{(+)}_{m_zn_z} \cos[m_n(\theta - \theta_n)] H_{m_n} (k^{(+)}_{z} r)$$

A.7

$$K_\alpha(\alpha)e^{-ik^{(+)}_{z}(z-z_n)} \frac{\sin(k^{(+)}_{z}d)}{k^{(+)}_{z}d} 2d$$

where $H_{m_n} = J_{m_n} (k^{(+)}_{z} r) + \left( \frac{D}{C} \right)_{m_n} Y_{m_n} (k^{(+)}_{z} r)$.

**Case 2**

The observer point is upstream of the source, i.e. $z < z_n - d$. This case is illustrated in Figure A.2b and implies that $z < z$. Thus, the integral $\int_{z_n-d}^{z_n+d} e^{-ik^{(+)}_{z}(z-z')} d\bar{z}$ for $z \leq z$ needs to be solved. This leads to
Appendix A: Cross Impedance Matrix $Z_{os}$

$$\int_{z_n-d}^{z_n+d} e^{-ik_z^{(-)}(z-z_n)} d\bar{z} = e^{-ik_z^{(-)}(z-z_n)} \frac{\sin(k_z^{(-)}d)}{k_z^{(-)}d} 2d$$  \hspace{1cm} A.8

Pressure due to the source is now obtained by replacing $A.2$ into $A.4$ and solving the integrals in $A.5$ and $A.8$

$$p(r, \theta, z / a, \theta_n, z_n) = -i\omega p V_n \sum_{m_g=0}^{M_g} \sum_{n_g=0}^{N_g} \left( k_0 - k_z^{(-)} M \right)^2 \frac{k_z^{(-)}d}{k_0} A_{m_g n_g}^{(-)} \cos[m_g (\theta - \theta_n)] H_{m_g} (k_{m_g n_g}^{(-)} r)$$

$$K_\theta(\alpha) e^{-ik_z^{(-)}(z-z_n)} \frac{\sin(k_z^{(-)}d)}{k_z^{(-)}d} 2d$$  \hspace{1cm} A.9

with $H_{m_g} = J_{m_g} (k_{m_g n_g}^{(-)} r) + \left( \frac{D}{C} \right)^{(0)} Y_{m_g} (k_{m_g n_g}^{(-)} r)$

**Case 3**

The observer point is on the surface of the source, i.e., $z_n - d < z < z_n + d$. This case is illustrated in Figure A.2c and it requires separating the integral in two parts as

$$\int_{z_n-d}^{z_n+d} e^{-ik_z^{(-)}(z-z_n)} d\bar{z} = \int_{z}^{z_n+d} e^{-ik_z^{(-)}(z-z_n)} d\bar{z} + \int_{z}^{z_n+d} e^{-ik_z^{(-)}(z-z_n)} d\bar{z}.$$  \hspace{1cm} A.10

The solution of these two integrals leads to

$$\int_{z_n-d}^{z_n+d} e^{-ik_z^{(-)}(z-z_n)} d\bar{z} = \frac{1 - e^{-ik_z^{(-)}(z-z_n + d)}}{ik_z^{(-)}} - \frac{1 - e^{-ik_z^{(-)}(z-z_n - d)}}{ik_z^{(-)}}$$

An expression for the pressure then is obtained by inserting $A.1$ and $A.2$ into $A.4$. After solving the integrals in $A.5$ and $A.10$ the following expression for the pressure is obtained

$$p(r, \theta, z / a, \theta_n, z_n) = -i\omega p V_n \sum_{m_g=0}^{M_g} \sum_{n_g=0}^{N_g} \left( k_0 - k_z^{(+)} M \right)^2 \frac{k_z^{(+)}d}{k_0} A_{m_g n_g}^{(+)} \cos[m_g (\theta - \theta_n)] H_{m_g} (k_{m_g n_g}^{(+)} r) \times$$

$$K_\theta(\alpha) \frac{1 - e^{-ik_z^{(+)}(z-z_n + d)}}{ik_z^{(+)}} - \frac{1 - e^{-ik_z^{(+)}(z-z_n - d)}}{ik_z^{(+)}}$$

$$1 - e^{-ik_z^{(+)}(z-z_n - d)}$$

$$1 - e^{-ik_z^{(+)}(z-z_n - d)}$$  \hspace{1cm} A.11
Appendix A: Cross Impedance Matrix $Z_{os}$

Figure A.2: Integration cases (a) case 1: observer is upstream of the source, (b) case 2: observer is downstream of the source, (c) case 3: observer is at the same axial location of the source.
Appendix A: Cross Impedance Matrix $Z_{os}$

### A.2. Impedance matrix $Z_{os}$

Once expressions for the average pressure have been obtained the HQ system impedance matrix can be calculated as

$$Z_{os} = \frac{p_{os}}{V_s} = \frac{1}{V_s S_o} \int_{-d_s}^{z_s+d_s} \int_{\theta_s-\alpha_o}^{\theta_s+\alpha_o} p(a, \theta, z / a, \theta_s, z_s) ad\theta dz$$  \hspace{1cm} (A.12)

This equation represents the influence of a source vibrating with unknown velocity $V_s$ located on the outer wall over an observer placed on the outer wall as well. However, in case of an annular duct sources can also be located on the inner wall. In any case the derivation is similar, being the only change the radial location which will appear in the Bessel’s function $H_m(k_{mn}r)$.

The solution of A.12 once again requires the solution of two integrals. The first one is

$$\sin(k_o^{(+)}d_o) \frac{\sin(k_z^{(+)}d_s)}{k_z^{(+)}d_s} \int_{\theta_s-\alpha_o}^{\theta_s+\alpha_o} \left[ \cos(m(\theta - \theta_s)) \frac{2a \alpha_o \sin(m \alpha_o)}{m \alpha_o} \cos(m(\theta_o - \theta_s)) - K_\theta(\alpha_o) \cos(m(\theta_o - \theta_s)) \right] \ d\theta$$  \hspace{1cm} (A.13)

where the subscripts “g” have been omitted for simplicity. The second integral is with respect to the $z$-coordinate which depends on the location of the piston observer “o” relative to the source “s”. As in the previous section, three cases are possible (see Figure A.3a-c).

#### Case 1

The piston observer “o” is downstream of the source “s”, i.e. $z_o - d_o \geq z_s - d_s$. In this case, when equation A.7 is replaced into A.12 and the integral

$$\int_{z_s-d_s}^{z_s+d_s} e^{-ik_z^{(+)}(z-z_s)} \frac{\sin(k_z^{(+)}d_s)}{k_z^{(+)}d_s} 2dz$$

is solved, the following expression for the cross impedance function is obtained

$$Z_{os} = -i \omega \rho \frac{M}{S_o} \sum_{m_o=0}^{M} \sum_{n_o=0}^{N_o} \left[ \frac{k_0 - k_z^{(+)M}}{k_0} \right]^2 A_{m_o n_o}^{(+)} \cos(m G(\theta_o - \theta_s)) \cos(m G(\theta_o - \theta_s)) H_{m_o n_o}(k_z^{(+)}d_o) K_\theta(\alpha_o)$$  \hspace{1cm} (A.14)
Appendix A: Cross Impedance Matrix $Z_{os}$

**Case 2**

The piston observer “o” is upstream of source “s”, i.e. $z_o - d_o < z_s - d_s$. In this case, Equation A.9 is replaced into A.12. After solving the integral

$$
\int_{z_o - d_o}^{z_s - d_s} e^{-ik_z^{(-)}(z - z_s)} \frac{\sin(k_z^{(-)}d_s)}{k_z^{(-)}d_s} 2d_z dz
$$

the cross impedance function becomes

$$
Z_{os} = -\frac{i\omega \rho}{S_o} \sum_{m_g=0}^{M_y} \sum_{n_g=0}^{N_y} \frac{(k_0 - k_z^{(-)}M)}{k_0} A_{m_g,n_g}^{(-)} \cos[m_g(\theta_o - \theta_s)] H_{m_s}(k_m^{(-)}a_o) K_o(\alpha_s)
$$

$$
e^{-ik_z^{(-)}(z_o - z_s)} \frac{\sin(k_z^{(-)}d_o)}{k_z^{(-)}d_o} 2d_o \frac{\sin(k_z^{(-)}d_s)}{k_z^{(-)}d_s} 2d_s
$$

**Case 3**

The piston observer “o” is at the same location as the source “s”. In addition, it is assumed that they have same dimensions, i.e. $d_o = d_s$. In this case, Equation A.11 is replaced into A.12 and the solution of

$$
\int_{z_o - d_o}^{z_s + d_s} \left[ \frac{1 - e^{-ik_z^{(-)}(z - z_s + d_s)}}{ik_z^{(-)}} - \frac{1 - e^{-ik_z^{(-)}(z - z_s - d_s)}}{ik_z^{(-)}} \right] dz
$$

yields

$$
Z_{os} = -\frac{i\omega \rho}{S_o} \sum_{m_g=0}^{M_y} \sum_{n_g=0}^{N_y} \left\{ \frac{(k_0 - k_z^{(-)}M)}{k_0} A_{m_g,n_g}^{(+)} \cos[m_g(\theta_o - \theta_s)] H_{m_s}(k_m^{(+)}a_o) K_o(\alpha_s) \times 
\left[ \frac{2d_o}{ik_z^{(+)}} + \frac{1 - e^{-ik_z^{(+)}2d_o}}{(k_z^{(+)})^2} \right] 
\frac{(k_0 - k_z^{(+)}M)}{k_0} A_{m_g,n_g}^{(-)} \cos[m_g(\theta_o - \theta_s)] H_{m_s}(k_m^{(-)}a_o) K_o(\alpha_s) \times \right. 
\left\{ \frac{2d_o}{ik_z^{(-)}} + \frac{1 - e^{-ik_z^{(-)}2d_o}}{(k_z^{(-)})^2} \right\} \right\}
$$

A.16
Figure A.3: (a) case 1: piston observer “o” is downstream of the source “s”
(b) case 2: piston observer “o” is upstream of the source “s”
(c) case 3: piston observer “o” and source “s” are at same axial location
Appendix B: Sound Field Modal Amplitudes

Section 2.1.4 presents the formulation used to compute power in order to evaluate the performance of an HQ-liner system. In these formulations it is assumed that the disturbance sound field and the sound field due to the HQ tubes are represented by the same number of modes therefore the subscript “g” exclusively related to the number of modes in the Green’s function will be used only to define the highest circumferential mode order $M_g$ and radial mode order $N_g$ used.

The performance evaluation is calculated in terms of the transmitted pressure. The equation for the transmitted pressure 2.63 is repeated next

$$ p = \sum_{m=0}^{M_g} \sum_{n=0}^{N_g} (A_{mn}^{(+)} \text{pos}) H_m(k_{mn}^{(+)} r)e^{i\theta} e^{-ik_z^{(+)} z} + \sum_{m=0}^{M_g} \sum_{n=0}^{N_g} (A_{mn}^{(+)} \text{neg}) H_m(k_{mn}^{(+)} r)e^{i\theta} e^{-ik_z^{(+)} z} \quad B.1 $$

In this equation, (+) indicates positive propagating waves; “pos” refers to positive spinning modes whereas “neg” identifies negative spinning ones; $M_g$ and $N_g$ are the highest circumferential and radial modes included in the Green’s function. The modal amplitudes $(A_{mn}^{(+)} \text{pos})$ and $(A_{mn}^{(+)} \text{neg})$ have the following form

$$ (A_{mn}^{(+)} \text{pos}) = (A_{mn}^{(+)} d) \text{pos} + \sum_{r=1}^{N_r} (A_{mn}^{(+)} r) \text{pos} V_r \quad B.2 $$

$$ (A_{mn}^{(+)} \text{neg}) = (A_{mn}^{(+)} d) \text{neg} + \sum_{r=1}^{N_r} (A_{mn}^{(+)} r) \text{neg} V_r \quad B.3 $$

Where $(A_{mn}^{(+)} d) \text{pos}$ and $(A_{mn}^{(+)} d) \text{neg}$ are the known modal amplitudes present in the disturbance; $V_r$ is the vibrating velocity of the $r^{th}$ piston source calculated using Equation 2.57; $(A_{mn}^{(+)} r) \text{pos}$ and $(A_{mn}^{(+)} r) \text{neg}$ are the $mn$ modal amplitudes of the sound field due to the piston sources. The next step is to find expressions for both of them.

Equation B.4 defines the pressure at a point $(r, \theta, z)$ due to the $r^{th}$ piston source, located at $(r, \theta_r, z_r)$
Appendix B: Sound Field Modal Amplitudes

\[ p(r, \theta, z / a, \theta_r, z_r) = -i \omega p V_r \sum_m \sum_n \left( A^{(+)\_m}_n \right)_r H_m(k^{(+)\_m}_r) e^{-i k^{(+)\_m}_r z} \]

or

\[ p(r, \theta, z / a, \theta_r, z_r) = -i \omega p V_r \sum_m \sum_n \left( k^{(+)\_m}_0 - k^{(+)\_m}_z M \right) A^{(+)\_m}_n \cos[m(\theta - \theta_r)]H_m(k^{(+)}_{rn}) K_\theta(\alpha_r) e^{-i k^{(+)\_m}_r z} \]

\[ e^{-ik^{(+)\_m}_r (z - z_r)} \sin(k^{(+)\_m}_z d) \frac{k^{(+)\_m}_z d}{2d} \]

Where \( \left( A^{(+)\_m}_n \right)_r = \left( k^{(+)\_m}_0 - k^{(+)\_m}_z M \right) A^{(+)\_m}_n \cos[m(\theta - \theta_r)]K_\theta(\alpha_r) e^{-i k^{(+)\_m}_r z} \sin(k^{(+)\_m}_z d_r) \frac{k^{(+)\_m}_z d_r}{2d_r} \). This expression has been derived in Appendix A and used in Equation A.7. The sound field generated by the piston is non-spinning and symmetric with respect to \( \theta = \theta_r \), as indicated by \( \cos[m(\theta - \theta_r)] \). However, it can be expressed as a set of positive and negative spinning modes as

\[ \cos[m(\theta - \theta_r)] = e^{-im\theta} \left( \frac{e^{im\theta}}{2} + e^{im\theta} \left( \frac{e^{-im\theta}}{2} \right) \right) \]

By inserting the last expression into B.5 and this into B.4, the following expression for the pressure due to a finite source is obtained

\[ p(r, \theta, z / a, \theta_r, z_r) = -i \omega p V_r \sum_m \sum_n \left( A^{(+)\_m}_n \right)_r^{(pos)} H_m(k^{(+)\_m}_r) e^{-im\theta} e^{-i k^{(+)\_m}_r z} + \sum_m \sum_n \left( A^{(+)\_m}_n \right)_r^{(neg)} H_m(k^{(+)\_m}_r) e^{im\theta} e^{-i k^{(+)\_m}_r z} \]

Where \( \left( \left( A^{(+)\_m}_n \right)_r \right)^{(pos)} \) and \( \left( \left( A^{(+)\_m}_n \right)_r \right)^{(neg)} \) are given by

\[ \left( A^{(+)\_m}_n \right)_r^{(pos)} = \left( k^{(+)\_m}_0 - k^{(+)\_m}_z M \right) A^{(+)\_m}_n H_m(k^{(+)}_{rn}) K_\theta(\alpha_r) \sin(k^{(+)\_m}_z d_r) \frac{k^{(+)\_m}_z d_r}{2d_r} e^{-i k^{(+)\_m}_r z} \]

\[ \left( A^{(+)\_m}_n \right)_r^{(neg)} = \left( k^{(+)\_m}_0 - k^{(+)\_m}_z M \right) A^{(+)\_m}_n H_m(k^{(+)}_{rn}) K_\theta(\alpha_r) \sin(k^{(+)\_m}_z d_r) \frac{k^{(+)\_m}_z d_r}{2d_r} e^{-i k^{(+)\_m}_r z} \]

It is important to remember that the amplitudes of the Green’s function \( A^{(+)\_m}_n \) depend on the radial location of the source, i.e. piston sources located on the outer wall will have different values than those located on the inner wall. Replacing equations B.7 and B.8 into equations B.2
and B.3 respectively leads to the final expression for the amplitudes of the transmitted sound field

\[
(A_{mn}^{(+)})^{\text{pos}} = (A_{mn}^{(+)})^{\text{neg}} = \sum_{r=1}^{N} V_r \left( \frac{k_0 - k_{zr}^{(+)}}{k_0} \right)^2 A_{mn}^{(+)} H_m^{(1)}(k_{mn} r) K_\theta^{(+)}(\alpha_r) \frac{\sin(k_{zr}^{(+)})}{k_{zr}^{(+)}} \frac{d_r}{k_{zr}^{(+)}} 2d_r \frac{e^{im\theta}}{2} e^{-ik_{zr}^{(+)}} z_r
\]

B.9

B.10

which are, in turn, the amplitudes used in equations 2.63 and 2.64.
Appendix C: Annular Duct Code

The theory developed in Chapter 2 has been implemented in a computer code using FORTRAN language. The code will be referred to as “Annular duct code.” The structure of the “Annular duct code” is similar to that of the one developed for circular ducts in [33]. However, even though the organization is the same, each one of the routines has been modified to handle the new geometry. In addition, a new routine to estimate transmission-reflection effects has been included.

Unlike the circular duct code, where the initial solution used to solve the eigen-problem is tabulated, the “Annular duct code” has to be provided with an initial solution which is different for every hub-to-tip ratio. In order to find this initial solution, a root-finding bisection method has also been implemented into a FORTRAN routine. This code will be referred to as “Bisection code”.

The rest of the appendix is organized as follows; the first section presents a flow chart of the “Annular duct code” including the “Bisection code”. The second section presents the input data required to estimate sound power reduction by an HQ-liner system. Finally, the last part of the appendix briefly mentions the output files and their content.

C.1. Flow chart

This section shows the major routines used to estimate power attenuation due to HQ-liner systems.
Figure C.1: Major routines constituting the “Annular duct code”.

Appendix C: Annular Duct Code
C.2. Input files

There are two input files, one for the “Annular duct code” and other for the “Bisection code”. In case of the “Annular duct code”, the input file is organized in three major groups:

1) **Annular duct information**
   - Flow: Mach number, density $[kg/m^3]$, speed sound $[m/s]$.
   - Duct geometry: Outer radius $[m]$, hub-to-tip ratio.
   - Initial solution: eigen-problem solution for hard wall case for a particular hub-to-tip ratio (provided by “Bisection Code”).

2) **Acoustic treatment information**
   - *HQ tubes*:
     - Modes included in the Green’s function.
     - HQ tube dimensions: cross section area $[m^2]$, tube mean length $[m]$ and interface distance $[m]$.
     - HQ tube location: radial location (outer or inner wall), axial location $[m]$.
     - Number of HQ tube arrays and number of tubes per array. When more than one array is been used the relative array angular location should also be included.
     - Non dimensional impedance of the screen placed at the tube openings.
   - *Liner*:
     - Non dimensional liner impedance.
     - Liner radial location: liners can be placed on the outer or the inner wall or on both at the same time.
     - Liner length.

3) **Disturbance**
   - Pure tone case: This information is usually measured and then input to the code.
   - Broadband case: A modified version of the equal-power-random-phase approach is used to provide broadband disturbance information.
Other parameters needed as input are convergence parameters for the eigenvalue finding routine, the frequency range of interest and printing controllers.

In case of the “Bisection code”, the input file reduces to the duct hub-to-tip ratio, radial and circumferential modes for which the eigenvalues are to be found and a few parameters required by the root finding bisection method.

### C.3. Output files

In case of the “Annular duct code”, there are several output files, however two of them are the most important. The first one stores power reduction due to the acoustic treatment being analyzed and for the frequency or frequency range of interest. The second one saves general information such as the solution to the eigen-problem, amplitudes of the Green’s function, orthogonalization factors, the disturbance, average pressure over pistons, piston velocity, HQ tube impedance, cross impedance matrix ($Z_{os}$) and modal power for every frequency in the selected range. Printing controllers determine how much of this information is included in the output file.

In case of the “Bisection code”, there is only one output file that stores the initial eigenvalues solution used by the “Annular duct code”.

Appendix D: Broadband Random Phase Analysis

In section 3.2.1 a modified version of the equal-power-and-random-phase approach was used to provide the model with information on broadband disturbance. This method assumes that every mode present in the disturbance has the same power while assigns a random phase to each one of them. The random process is carried out by a FORTRAN routine called “sunif.for”. Phase values are assigned according to a seed number fed into the routine each time it is called. During the design and validation of the aft radiation control model developed in Chapter 2 the seed number was set at 1234. Since the power attenuation depends on the relative phase among the modes in the disturbance, the main objective of this appendix is to assess the effects of changing the seed number on sound power reduction of the broadband component. To this end broadband attenuation in the aft section of the NASA ANCF rig at 1800 rpm due to the liner alone, HQ-Liner and HQ tubes on hard wall was calculated. Three seed numbers arbitrarily chosen, i.e. 1758, 3125, and 5214, were used to define the disturbance. Power reduction due to all these configurations was compared to the attenuation obtained using the original seed number of 1234 and already presented in Chapter 4. Results are organized according to the acoustic treatment. This way, Section D.1 presents liner results; Section D.2 shows HQ-Liner results in terms of HQ contribution relative to the liner; and section D.3 focuses on HQ tubes on hard wall.

D.1. Liner results

Broadband attenuation due to the liner calculated using a disturbance defined according to the seed numbers mentioned above are compared in Figure D.1. These results show no significant variations. Figure D.2 presents the absolute value of power reduction difference calculated between each one of the three new cases and the original one with a seed number of 1234. In general curves remain below 2.0 dB. Only three or four frequencies exhibit values reaching levels as high as 5.0 dB (Figure D.2c).
Appendix D: Broadband Random Phase Analysis

Figure D.1: Aft broadband sound power reduction due to liners at 1800 rpm using four different seed numbers.

Figure D.2: Liner case. Absolute value of power reduction difference between disturbance with seed numbers of 1234 and (a) 1758, (b) 3125 and (c) 5214.
D.2. HQ-Liner results

Figure D.3 shows the contribution to aft broadband power reduction from the HQ tubes measured relative to the liner. As it can be seen, all cases present virtually the same reduction. Again, the absolute value of the difference between the contribution calculated with a seed number of 1234 and each one of the others is plotted in Figure D.4. These plots confirm what was observed in Figure D.3, i.e. reduction levels provided by every single case are relatively the same, especially at low frequencies. At higher frequencies there are only two or three peaks in the neighborhood of ~2.0 dB with only one of them reaching ~3.5 dB (see Figure D.4a).

Figure D.3: Aft broadband sound power reduction due to HQ tubes relative to the liner at 1800 rpm using four different seed numbers.
Finally, HQ-HW results are presented in Figure D.5. Just like in the previous case, every seed number produced the same reduction. This behavior is also shown by Figure D.6 where the absolute value of the difference among these cases is presented. As indicated, levels remain below to ~0.25 dB.

Concluding this analysis, it can be said that changing the seed number does not alter significantly the performance of any of the acoustic treatments analyzed in this dissertation. Therefore, the conclusions drawn in Chapter 4 concerning the validation of the model remain the same regardless of the seed number utilized.
Appendix D: Broadband Random Phase Analysis

Figure D.5: Aft broadband sound power reduction due to HQ tubes on hard wall at 1800 rpm using four different seed numbers.

Figure D.6: HQ tubes on hard wall. Absolute value of power reduction difference between disturbance with seed numbers of 1234 and (a) 1758, (b) 3125 and (c) 5214.
Appendix E: Repeatability Analysis

In order to establish the degree of confidence in the results, two test configurations were measured twice. Additional experiments for repeatability studies were not performed due to time and cost constraints. The cases repeated are the hard wall baseline and the liners installed in both the inlet and aft ducts at the same time, i.e. ATC-3. The results of those measurements are presented in this section. In the case of the HW, only far-field data was collected. Far-field data is discussed in section E.1 whereas in-duct measurements are presented in section E.2. Finally, section E.3 summarizes the repeatability results for the design speed of 1800 rpm.

E.1. Far-field data

Far-field data is presented for the 2BPF tone and broadband component in sections E.1.1 and E.1.2, respectively. Pure tones repeatability analysis is performed in terms of power reduction at all fan speeds; it also shows directivity patterns at 1800 rpm only. In the broadband case the analysis focuses on the attenuation of the 2BB, i.e. broadband between 1.5-2.5 BPF, and the overall reduction between 2.5-5.5 BPF at all fan speeds. Directivity patterns and power spectrum at 1800 rpm are also included for the 2BB case.

E.1.1 2BPF Tone

Hard wall

Figure E.1 shows power difference between the HW measurements for the 2BPF tone at all four fan speeds tested. It is observed better repeatability in the inlet than in the aft especially at higher speeds. At the design condition of 1800 rpm, measurements differ by 0.1 dB in the inlet and by 1.0 dB in the aft.
Appendix E: Repeatability Analysis

<table>
<thead>
<tr>
<th>Fan Speed [rpm]</th>
<th>1500</th>
<th>1600</th>
<th>1700</th>
<th>1800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference [dB]</td>
<td>0.5</td>
<td>1.0</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Figure E.1:** 2BPF tone power difference between two hard wall cases measured different days in the (a) inlet and (b) aft duct.

Figure E.2 shows power directivity patterns for the 2BPF tone at 1800 rpm. Differences of ~2.0 dB are observed in the 45°-90° sector and in the 135°-145°. Also a difference of 5.5 dB appears at 100°.

**Figure E.2:** Directivity patterns of the 2BPF tone at 1800 rpm for two hard wall cases measured different days.
**Liner**

Figure E.3 shows power difference between two liner measurements at the 2BPF tone as a function of fan speed. In the case of the aft, power reduction changed by as much as 1.7 dB from one measurement to the other. At the design speed of 1800 rpm, differences between the two measurements are 0.9 dB in the inlet and 0.7 dB in the aft.

![Figure E.3: 2BPF tone power difference between two liner cases measured different days in the (a) inlet and (b) aft duct.](image)

<table>
<thead>
<tr>
<th>Fan Speed [rpm]</th>
<th>1500</th>
<th>1600</th>
<th>1700</th>
<th>1800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference [dB]</td>
<td>0.7</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fan Speed [rpm]</th>
<th>1500</th>
<th>1600</th>
<th>1700</th>
<th>1800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference [dB]</td>
<td>1.7</td>
<td>0.6</td>
<td>1.2</td>
<td>0.7</td>
</tr>
</tbody>
</table>

**Figure E.4** shows power directivity patterns. The most significant differences are observed in the 60°-90° sector at 4.5 dB and 3.0 dB in the 110°-125° sector.

![Figure E.4: Directivity patterns of the 2BPF tone at 1800 rpm for two Liner cases measured different days](image)


**E.1.2 Broadband**

The broadband analysis is organized in a similar fashion as that of the pure tone, i.e. according to the acoustic treatment. The broadband repeatability analysis includes 2BB, i.e. broadband between 1.5-2.5 BPF, and broadband component between 2.5-5.5 BPF.

**Hard wall**

Figure E.5 shows power difference between two hard wall measurements for the 2BB. Both inlet and aft sectors show differences below 0.5 \( \text{dB} \). At 1800 rpm, the inlet and aft data differs by 0.3 and 0.4 \( \text{dB} \), respectively.

![Power Difference Chart](image)

<table>
<thead>
<tr>
<th>Fan Speed [rpm]</th>
<th>1500</th>
<th>1600</th>
<th>1700</th>
<th>1800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference [dB]</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Figure E.5:** 2BB power difference between two hard wall cases measured different days in the (a) inlet and (b) aft duct

Figure E.6 shows power directivity patterns for the 2BB component at 1800 rpm. Both measurements agree extremely well with differences less than 0.2 \( \text{dB} \).

Figure E.7 shows broadband power spectrum also at 1800 rpm. Major differences (less than 1.0 \( \text{dB} \)) are found between engine orders 16 and 32. However, at higher fan orders measurements are virtually the same with an overall variation of less than 0.2 \( \text{dB} \).
Appendix E: Repeatability Analysis

Figure E.6: Directivity patterns of the 2BB at 1800 rpm for two hard wall cases measured different days.

Figure E.7: Power Spectrum of two hard wall cases at 1800 rpm measured different days in the (a) inlet and (b) aft duct.

Liner

Sound power difference of the 2BB is shown in Figure E.8. Differences in the results are similar to those of the hard wall case, e.g. repeatability around 0.6-0.8 dB in the inlet and around 0.5-0.6 in the aft. At the design fan speed, inlet and aft sectors show a variability of 0.6 dB.
Radiation directivity at 1800 rpm is shown in Figure E.9. A small difference of less than 0.5 dB is evenly distributed along the inlet and aft sectors.

Power spectrum for these two liner measurements are shown in Figure E.10. Good repeatability is observed through out the entire spectrum with differences within the fan order range targeted by the liner of less than 0.7 dB.
Appendix E: Repeatability Analysis

Figure E.10: Power Spectrum of two liner cases at 1800 rpm measured different days in the (a) inlet and (b) aft duct.

E.2. In-duct data

Only liner data for configuration ATC-3 was available to perform in-duct repeatability analysis. The repeatability study is presented in terms of power difference of the (4,0) and (4,1) modes as well as the total 2BPF tone at all four fan speeds. Figure E.11 shows these results.

As observed, differences in the inlet are very small for the individual modes and the total. At the design condition, a variation of 0.2 dB was measured for the 2BPF and its components. In the aft section, variability in sound power is significant. For individual modes, it could be as high as 3.3 dB and for the 2BPF tone it can reach 2.1 dB depending on the fan speed. At 1800 rpm, the total tone power variability drops down to 0.4 dB.
Appendix E: Repeatability Analysis

<table>
<thead>
<tr>
<th>Fan Speed [rpm]</th>
<th>1500</th>
<th>1600</th>
<th>1700</th>
<th>1800</th>
<th>1500</th>
<th>1600</th>
<th>1700</th>
<th>1800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Differ. [dB]</td>
<td>(4,0)</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>0.2</td>
<td>1.5</td>
<td>2.4</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>(4,1)</td>
<td>0.0</td>
<td>0.3</td>
<td>0.8</td>
<td>0.2</td>
<td>1.7</td>
<td>3.3</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>total</td>
<td>0.2</td>
<td>0.1</td>
<td>0.8</td>
<td>0.2</td>
<td>1.6</td>
<td>0.4</td>
<td>2.1</td>
</tr>
<tr>
<td>Aft</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Differ. [dB]</td>
<td>(4,0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure E.11: Power difference of the 2BPF tone and its most dominant modal components between two liner cases measured different days in the (a) and (b) aft duct.

E.3. Summary

This section summarizes the results of the repeatability study for the 2BPF, the 2BB and the broadband component between 2.5~5.5 BPF at the design fan speed of 1800 rpm.

Far-field data

Table E-1 shows inlet and aft power differences for two measurements of hard wall and liner configurations at 1800 rpm using far-field data. In order to have a sense of degree of confidence in the results, these ranges can be compared to the sound power reductions for the different configurations tested. To this end, Table E-2 is also included here and contains the attenuation obtained by the various noise treatments. By comparing these two tables, the following conclusions can be drawn:

(i) **HQ-HW**: The attenuation results for the HQ tubes in a HW duct can be considered reliable, in particular for the 2BB. For example, the 2BB attenuation oscillates around 1.6 dB while the maximum difference in the two HW measurements is 0.3 dB. For the 2BPF
tone, the attenuation oscillates around 1.9 dB while the maximum difference between the two HW measurements is now 1.0 dB, i.e. in the aft section of the rig.

(ii) **Liner:** The liner results *can also be considered reliable* since the liner reductions are larger than 3.2 dB in all cases.

(iii) **HQ-Liner:** Note that this case is not shown in the tables. However, adding the liner and HQ contribution yields the HQ-Liner attenuation. Reminiscent of the liner results, the HQ-liner and HQ contribution results can be considered reliable.

### Table E-1: Repeatability values for HW and Liner cases at 1800 rpm for far-field data

<table>
<thead>
<tr>
<th>Inlet Difference [dB]</th>
<th>Aft Difference [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pure tone</strong></td>
<td><strong>Broadband</strong></td>
</tr>
<tr>
<td></td>
<td>2BB</td>
</tr>
<tr>
<td>Pure tone</td>
<td>2BPF</td>
</tr>
<tr>
<td>HW</td>
<td>0.1</td>
</tr>
<tr>
<td>Liner</td>
<td>0.9</td>
</tr>
</tbody>
</table>

### Table E-2: Far-field reduction levels due to the different acoustic treatments tested.

<table>
<thead>
<tr>
<th>Inlet Reduction [dB]</th>
<th>Aft Reduction [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pure tone</strong></td>
<td><strong>Broadband</strong></td>
</tr>
<tr>
<td></td>
<td>2BB</td>
</tr>
<tr>
<td>Pure tone</td>
<td>2BPF</td>
</tr>
<tr>
<td>Liner</td>
<td>3.7</td>
</tr>
<tr>
<td>HQ Contribution</td>
<td>1.4</td>
</tr>
<tr>
<td>HQ-HW</td>
<td>1.9</td>
</tr>
</tbody>
</table>

### In-duct

In-duct

Table E-3 shows the inlet and aft sound power difference for two measurements of the liner configuration (ATC-3) at 1800 rpm using in-duct data. The results are presented for the (4,0), (4,1), and the sum of the two. The repeatability in the inlet duct is excellent. The aft duct measurements show a variability of up to 1.2 dB in the modes individually while the total shows a range of 0.4 dB. Since the rotating rakes are positioned on a hard-wall section of the ducts, it is reasonable to assume that the variability of the modal results to be independent of the treatment.
To have a sense of the degree of confidence in the modal results, these ranges can be compared to the sound power reduction due to the different configurations tested. To this end, the attenuation due to the different noise treatments are presented in Table E-4. Comparing these two tables, it is clear that the inlet results are reliable. On the other hand, in the aft duct, the HQ-contribution for the HQ-Liner configuration is not reliable.

**Table E-3:** Repeatability values for HW and Liner cases at 1800 rpm for in-duct data.

<table>
<thead>
<tr>
<th>Inlet Difference [dB]</th>
<th>Aft Difference [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pure tone (2BPF)</td>
</tr>
<tr>
<td>(4,0)</td>
<td>(4,1)</td>
</tr>
<tr>
<td>HW</td>
<td></td>
</tr>
<tr>
<td>Liner</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Table E-4:** In-duct reduction levels due to the different acoustic treatments tested

<table>
<thead>
<tr>
<th>Acoustic treatment</th>
<th>Inlet Reduction [dB]</th>
<th>Aft Reduction [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2BPF tone</td>
<td>2BPF tone</td>
</tr>
<tr>
<td></td>
<td>(4,0)</td>
<td>(4,1)</td>
</tr>
<tr>
<td>Liner</td>
<td>4.5</td>
<td>17.1</td>
</tr>
<tr>
<td>HQ Contribution</td>
<td>1.0</td>
<td>-4.2</td>
</tr>
<tr>
<td>HQ-HW</td>
<td>0.1</td>
<td>4.5</td>
</tr>
</tbody>
</table>
Appendix F: HQ Tube Multi Degree of Freedom Model

As mentioned in Chapter 2, the dynamics of the HQ tubes has been modeled so far using plane waves only. Even though the approach has the advantage of its simplicity it also has an important limitation. HQ tubes have to have a cross section small enough so that only plane waves are present inside the tubes. In an attempt to overcome the issue, a multi degree of freedom (MDOF) model has been developed in Chapter 2. In this model the single piston has been replaced by two pistons in the duct axial direction and two pistons in the duct circumferential direction (“2 by 2” disposition). Such arrangement is expected to account for the presence of the (0,0), the (0,1), the (1,0) and the (1,1) modes. The objective of this appendix is then to validate the new tube dynamics model and to assess the differences between the 1DOF and the MDOF model when higher order modes are cut-on.

The lack of experimental data renders a complete validation not feasible. However a first validation can be performed by comparing predictions using the 1DOF and the MDOF models when plane waves only are present in the tubes. To this end, a series of predictions of noise reduction in the inlet and aft sections of the NASA rig using the new model are compared to those discussed in Chapter 4. This is carried out in the next section. Once the model has been thus validated, predictions using the 1DOF and the MDOF models for tubes with a larger cross section are compared in section F.2. Finally, section F.3 summarizes the most important conclusions of this study.

F.1. Model Validation

The MDOF model was used to predict the performance of the systems used on the NASA ANCF rig. The parameters defining these systems have been given in section 3.2.2 (see Tables 3.2 and 3.3).
For the present analysis the most important parameter is the tube cross section which is used to define the cut-on frequency of higher order modes. The cross section of inlet and aft tubes was the same at 5.96 $in^2$. In order to simplify the modeling, tubes have been assumed to have a rectangular cross section as indicated in figure 2.9. The side $a$ of the cross section 2.4 $in$ or 0.06 $m$. If the speed of sound is 344.1 $m/s$ then the cut on frequency of the first two higher order modes, i.e. (0,1) and (1,0), is 2867.5 $Hz$. The upper limit of the frequency range of interest was 5.5 BPF or 2640 $Hz$ at 1800 rpm. So as it can be seen the plane wave assumption for the ANCF rig was a valid one. From this perspective both the MDOF and 1DOF model should give similar results.

The configurations investigated are: i) HQ-Liner combination and ii) HQ tubes on hard wall. Predictions include the sound power attenuation of the 2BPF tone and the broadband component at 1800 rpm in the inlet and aft sectors of the rig. These results were compared to predictions obtained using the 1DOF model already validated in Chapter 4. Results on the 2BPF tone are presented next.

**2BPF Tone Results**

Figure F.1 shows reduction of the 2BPF tone and its components for the inlet and the aft calculated using the two models mentioned above. At 1800 rpm the 2BPF tone occurs at 960 $Hz$, well below the cut-on frequency of the first higher order modes in the tubes. As it can be seen, both models produced virtually the same attenuation. The aft (see Figure F.1b) exhibits the largest difference in the prediction of the (4,1) mode at 0.5 dB.

The case of the HQ tubes placed on hard wall is shown in Figure F.2. In the inlet (Figure F.2a) the largest variations are observed in the (4,0) mode and the total 2BPF tone at only 0.25 dB. In the aft the differences in the prediction of the (4,0) and the total has decreased to around 0.1 dB. The largest discrepancy appears in the prediction of the (4,1) mode at 0.6 dB.
Appendix F: HQ Tube Multi Degree of Freedom Model

**Figure F.1:** HQ Contribution to power reduction of the 2BPF tone and its components measured relative to the liner at 1800 rpm. Predictions performed using MDOF and 1DOF models in the (a) inlet and (b) aft duct. HQ-Liner systems as described in section 3.1.3

**Figure F.2:** Power reduction of the 2BPF tone and its components due to HQ tubes on hard wall at 1800 rpm. Predictions performed using MDOF and 1DOF models in the (a) inlet and (b) aft duct. HQ-Liner systems as described in section 3.1.3

**Broadband Results**

Figure F.3 and F.4 show power reduction of the broadband component due to the HQ tubes relative to the liner and on hard wall at 1800 rpm respectively. Both inlet and aft cases are considered. In case of the HQ contribution relative to the liner the differences are negligibly small especially in the inlet (Figure F.3a). In case of the aft (Figure F.3b) note that the 4DOF model also predicts no contribution from the HQ tubes at higher frequencies.
In case of the HQ tubes placed on a hard-walled environment, small differences begin to appear in the inlet and aft sectors at frequencies around 1900 Hz (see Figure F.4a and b). These differences may be due to the higher order modes. Even though higher order modes are still cut-off, they have a pressure distribution. Given the small size of the tubes, these cut-off modes do not have time to decay affecting, thus, the tube dynamics. In case of the HQ tubes combined with a liner (Figure F.3) such a pattern is hidden by the liner. At high frequencies, the liner is entirely responsible for the attenuation leaving no room for contribution from the tubes.

![Figure F.3: HQ Contribution to broadband power reduction relative to the liner at 1800 rpm predicted using MDOF and 1DOF models in (a) the inlet and (b) the aft. HQ-Liner systems as described in section 3.1.3](image)

![Figure F.4: Broadband power reduction due to HQ tubes on hard wall at 1800 rpm predicted using MDOF and 1DOF models in the (a) inlet and (b) aft duct. HQ-Liner systems as described in section 3.1.3](image)
F.2. Application of the MDOF Model

Once the model has been validated, it was decided to increase the tube cross section. This way, higher order modes would appear within the frequency range of interest, i.e. 1.5-5.5 BPF at 1800 rpm. The new tubes have a cross section of 0.02 \( m^2 \) or 5.2 times larger than the previous model. With this value, four higher order modes are present: The (0,1) and the (1,0) mode with a cut-on frequency of 1230 Hz; the (1,1) mode with a cut-on frequency of 1730 Hz and the (2,0) mode with a cut-on frequency of 2460 Hz. Note that at 1230 Hz, the ratio between the disturbance wave length and the tube cross section side is \( \lambda_d / a = 0.28/0.14 \approx 2 \). The new cross section allowed only 20 HQ tubes in the inlet and aft outer wall and 8 HQ tubes on the inner wall of the aft section.

A brief analysis showed that at the 2BPF, such a small number of tubes did not produced scattering in the inlet. In case of the aft some energy could be transferred into the \( m = -4 \) mode. However, since the tubes are placed parallel to the rig axis, there is no difference between positive and negative rotating modes. Hence, HQ tubes will also attenuate the \( m = -4 \).

Power attenuation due to HQ tubes combined with a liner and HQ tubes on a hard wall has been calculated at 1800 rpm. Once more, results include attenuation of the 2BPF tone and the broadband component. The 2BPF tone is presented next.

2BPF Tone Results

Contribution to power attenuation of the pure tone at 960 Hz at 1800 rpm due to the HQ tubes relative to the liner is shown in Figure F.5. In both, inlet and aft sectors of the rig, significant differences are observed in the attenuation of the (4,1) mode, especially in the aft where the MDOF model predicts a reduction of around 8.0 dB higher than the 1DOF model. In case of the inlet this difference is much smaller at 2.5 dB. The total attenuation of the 2BPF as well as the (4,0) mode is the same for either model.

The attenuation of the 2BPF tone due to HQ tubes on hard wall exhibits a similar pattern, however the largest difference appear now in the attenuation of the (4,0) mode and the total tone. In case of the inlet, the 4DOF model predicts around 2.0 dB more than the 1DOF model for the
Appendix F: HQ Tube Multi Degree of Freedom Model

(4,0) mode and 1.5 dB more for the total tone. In case of the aft this difference drops to roughly 1.0 dB for each of the components and their sum.

![Inlet](image)

**Figure F.5:** HQ Contribution to power reduction of the 2BPF tone and its components measured relative to the liner at 1800 rpm. Predictions performed using MDOF and 1DOF models in (a) the inlet and (b) the aft. HQ tube cross section: 0.02 m$^2$.

![Aft](image)

**Figure F.6:** Power reduction of the 2BPF tone and its components due to HQ tubes on hard wall at 1800 rpm. Predictions performed using MDOF and 1DOF models in the (a) inlet and (b) aft duct. HQ tube cross section: 0.02 m$^2$.

**Broadband Results**

Figure F.7 shows the contribution to attenuation due to the HQ tubes measured relative to the liner. At frequencies around and below the 2BPF, differences oscillate around 0.2 dB in the inlet and 1.0 dB in the aft. As already explained, at high frequencies, the liner produces most of the attenuation leaving no room for HQ tube contribution. This situation hides the effect of the higher order modes present in side the tubes.
Figure F.8 presents broadband power reduction due to the HQ tubes on hard wall. This figure is probably the most important since it shows for the first time a notorious difference between the two models. As indicated in Figure F.8, differences are more important in the aft than in the inlet. In case of the aft the MDOF model predicted between 1.0 and 2.0 dB more reduction above 1200 Hz. In case of the inlet the MDOF model predicted around 1.0 dB more.

**Figure F.7:** HQ Contribution to broadband power reduction relative to the liner at 1800 rpm predicted using MDOF and 1DOF models in the (a) inlet and (b) aft duct. HQ tube cross section: 0.02 m²

**Figure F.8:** Broadband power reduction due to HQ tubes on hard wall at 1800 rpm predicted using MDOF and 1DOF models in the (a) inlet and (b) aft duct. HQ tube cross section: 0.02 m²
F.3. Conclusions

A model to describe the HQ tube dynamics in the presence of higher order modes has been developed and validated against the 1DOF model. The most important conclusions of this analysis are summarized next.

The validation process showed, as expected, that when only plane waves are present in the tubes the 1DOF and the MDOF models produce virtually the same result. Differences between the two models start to appear as the frequency nears the cut-on frequency of the first higher order mode, i.e. 2867.5 Hz. The reason for this, may be because higher order modes, even though cut-off, still have a pressure distribution that may slightly affect the tube dynamics.

When the cross section was increased 5.2 times, four higher order modes were cut-on within the frequency range of interest. In this case, differences between the two models were only 1.0-2.0 dB. The only exception is the HQ contribution to attenuation of the (4,1) mode (see figure F-5b) at ~8.0 dB measured relative to the liner. In other words, the analysis seems to indicate that even with large tubes the 1DOF model produce acceptable predictions. This result is encouraging since, given the tight room constrains existing in real turbofan engines, large tubes are unlikely to be used.
References


References


Vita

Diego de la Riva was born in Cordoba, Argentina on September 8, 1973. After graduating from The Colegio Nacional de Montserrat he decided to become an Aerospace Engineer. After six years of studies he obtained his degree in Aeronautical Engineering from the Universidad Catolica de Cordoba/Instituto Universotario Aeronautico as part of a joint program between the two universities. On July 1999, he decided to come to the United States of America in order to start his graduate studies in the Aerospace and Ocean Engineering department at the Virginia Polytechnic Institute and State University. Under the guidance of Dr. William J. Devenport he began a Masters of Science program. The main objective of this research effort was to develop an inviscid model to analyze the evolution of turbulence through a compressor cascade. In April 2001 he obtained his degree. In an attempt to broaden his knowledge and expand his fields of expertise he became part of Ph.D. program in the Mechanical Engineering department also at Virginia Tech with Dr. Ricardo Burdisso as his advisor. The research areas of interest were fluid mechanic and aeroacoustic. On July 1, 2006 he completed the program with a dissertation on Turbofan noise control.