Effects of Thermoacoustic Oscillations on Spray Combustion Dynamics with Implications for Lean Direct Injection Systems

by

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Abstract

Thermoacoustic instabilities in modern high-performance, low-emission gas turbine engines are often observable as large amplitude pressure oscillations and can result in serious performance and structural degradations. These acoustic oscillations can cause oscillations in combustor through-flows and given the right phase conditions, can also drive unsteady heat release. To curb the potential harms caused by the existence of thermoacoustic instabilities, recent efforts have focused on the active suppression of these instabilities. Intuitively, development of effective active combustion control methodologies is strongly dependent on the knowledge of the onset and sustenance of thermoacoustic instabilities. Specially, non-premixed spray combustion environment pose additional challenges due to the inherent unstable dynamics of sprays. The understanding of the manner in which the combustor acoustics affect the spray characteristics, which in turn result in heat release oscillation, is therefore, of paramount importance. The experimental investigations and the modeling studies conducted towards achieving this knowledge have been presented in this dissertation.

Experimental efforts comprise both reacting and non-reacting flow studies. Reacting flow experiments were conducted on a overall lean direct injection, swirl-stabilized combustor rig. The investigations spanned combustor characterization and stability mapping over the operating regime. The onset of thermoacoustic instability and the transition of the combustor to two unstable regimes were investigated via phase-locked chemiluminescence imaging and measurement and phase-locked acoustic characterization. It was found that the onset of the thermoacoustic instability is a function
of the energy gain of the system, while the sustenance of instability is due to the in-phase relationship between combustor acoustics and unsteady heat release driven by acoustic oscillations. The presence of non-linearities in the system between combustor acoustic and heat release and also between combustor acoustics and air through-flow were found to exist. The impact of high amplitude limit-cycle pressure on droplet breakdown under very low mean airflow and the localized effects of forced primary fuel modulations on heat release were also investigated.

The non-reacting flow experiments were conducted to study the spray behavior under the presence of an acoustic field. An isothermal acoustic rig was specially fabricated, where the pressure oscillations were generated using an acoustic driver. Phase Doppler Anemometry was used to measure the droplet velocities and sizes under varying acoustic forcing conditions and spray feed pressures. Measurements made at different locations in the spray were related to these variations in mean and unsteady inputs. The droplet velocities were found to show a second order response to acoustic forcing with the cut-off frequency equal to the relaxation time corresponding to mean droplet size. It was also found that under acoustic forcing the droplets migrate radially away from the spray centerline and show oscillatory excursions in their movement.

Modeling efforts were undertaken to gain physical insights of spray dynamics under the influence of acoustic forcing and to explain the experimental findings. The radial migration of droplets and their oscillatory movement were validated. The flame characteristics in the two unstable regimes and the transition between them were explained. It was found that under certain acoustic and mean air-flow condition, bands of high droplet densities were formed which resulted in diffusion type group burning of droplets. It was also shown that very high acoustic amplitudes cause secondary breakup of droplets.
“Read: In the name of thy Lord Who createth,
Createth man from a clot,
Read: And thy Lord is the Most Bounteous,
Who teacheth by the pen,
Teacheth man that which he knew not.”

_Quran (20:114)_

“My Lord! Advance me in knowledge.”

_Quran (96:1-5)_)
Dedicated to the loving memory of my daughter,
Ayesha Chishty

...may you rest in peace. Ameen
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Chapter 1

Introduction

The process of combustion is composed of interacting processes in thermodynamics, fluid mechanics, chemical kinetics and transport. Combustion therefore, is a dynamic process even when one may attempt to solve the involved conservation equations independent of the time variable. The dynamics in combustion comes from its dependence on the interacting chemical and physical processes and the variations therein.

Some of the elementary interactions, which indicate the prevalence of dynamics in combustion are: unsteady strain effects; flame/vortex interaction; flame/acoustic interaction; vortex and/or acoustic enhanced mixing and subsequent combustion; flame response to incident composition non-uniformity; flame lift off and blow off; and flame acceleration.

Ultimately, the dynamics of the combustion processes is the origin of more serious combustion instabilities (to be discussed later in the Chapter). Coupling between the burning of fuel and the unsteady motions, which cause combustion instabilities, is due to the sensitivity of combustion processes to macroscopic flow variables like pressure, velocity, temperature and fuel-air ratio.

Combustion dynamics, although studied for over a century, still remains the central issue of technological developments in gas turbine engines, used both in power generation and propulsion [1].
1.1 Spray Combustion

Liquid spray combustion offers a major contribution to the world’s total energy needs. Its applications range from household burners to industrial furnaces and from power generation to propulsion. In high energy density applications, Faeth [2] has classified the spray combustion systems under five categories: Pre-vaporizing systems like afterburners and ramjets; liquid rocket motors; gas turbine combustors; industrial furnaces and diesel engines. This categorization has been done on three essential features of spray combustion, which are: the method of introduction of fuel and oxidizer in the chamber, the flow conditions inside the chamber and the interaction of spray fuel with the chamber surfaces. In addition, not only is spray combustion an inherently unstable mechanism because of the unsteadiness involved in the evolution of sprays, but also unlike gaseous combustion it cannot be truly designated as premixed or as diffusion. Spray combustion is almost always a dual mode mechanism, showing both premixed and diffusion characteristics at any one time in different regions of the flow field. Thus, spray combustion poses challenges, which are either non-existent or considered insignificant in gaseous or solid combustion.

Because of the considerable practical importance of spray combustion and the complexities involved in the process, it had remained an area of active research for almost a century. Excellent reviews on the progress of research may be found in Faeth [2] and Law [3]. A wealth of knowledge is also available in literature on the different aspects and features of spray combustion, which include injection processes, spray formation and drop size distribution; single drop evaporation and burning, drop-to-drop interaction in dense sprays and effects of external conditions on drop behavior; and pollutant formation in spray combustion.

Formation of spray involves first the disintegration of liquid jet into sheets and ligaments and then into drops, followed by the secondary breakup of the parent drops into smaller droplets. Extensive literature is available on both the primary and secondary breakup. Character of liquid jet disintegration depends on the type of liquid, the liquid discharge velocity and the ambient conditions. Earlier work by Rayleigh [4] suggested that the jet breakup was due
CHAPTER 1. INTRODUCTION

the development of axisymmetric waves in the jet because of surface tension forces. This theory was extended to include the effects of asymmetric waves and aerodynamics forces with the increase in discharge velocity. However, these explanations were limited to low speed laminar jets only. For high-speed turbulent jets, a lot of work has been done in recent times by Faeth and coworkers [5, 6, 7, 8]. It was determined that aerodynamic effects were small for liquids injected into light gases at standard conditions and the main cause of breakup was the turbulence developed in the injector passage. Differences in turbulent primary breakup between round free jets and plane wall jets were also highlighted. It was also concluded that the dominant cause of primary drop formation was Rayleigh breakup of the ligaments. However, it was also shown that about 10% of the times the formation was due to breakup of entire ligaments caused by the velocity fluctuation in the liquid core. The final phase of spray formation is the deformation of primary drops and their subsequent breakup. The major experimental contribution in this area is attributed once again to Faeth and coworkers [9, 10, 11], while a number of notable references may be cited for the analytical and numerical efforts [12, 13, 14, 15]. It was shown that two non-dimensional parameters, Weber number (We) and Ohnesorge number (Oh) completely characterize the deformation and secondary breakup mechanism. The first parameter determines the intensity of aerodynamic forces relative to the stabilizing surface tension forces, while the second quantifies the effects of internal viscous forces. These investigations revealed the presence of three breakup regimes based on the values of these parameters, namely, bag, multimode and shear breakup.

The overall combustion system behavior is dependent on the bulk spray characteristics, which are quantified in terms of drop size distribution (DSD). DSD is typically defined in terms of distribution functions by Nukiyama and Tanasawa or Rosin and Rammler [16] and has been shown to depend in turn on the injector type, quality of atomization and feed line pressure (see for example the classical works by Chigier [17] and Lefebvre [18]). More recent work [19] has shown that narrower DSD enhances both combustion efficiency and heat release. The spray characteristics in terms of DSD and mean drop size therefore, characterize
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the temperature distribution and pollutant formation in a combustion system. A wider DSD can produce a range of local compositions spanning a wide range of stoichiometries.

For combustion to occur in systems using liquid fuel the condensed phase first needs to gasify and since liquid fuel is typically introduced as a spray of droplets, it is quite reasonable to expect that the collective behavior of individual drops characterize the bulk spray properties. For the same reason, a great deal of research has been conducted towards droplet vaporization/oxidation and droplet dynamics. Detailed reviews can be found in Williams [20] and Law [3]. A major contribution in this area, especially in the numerical modeling of droplet combustion, is attributed to Sirignano and coworkers. This specific topic will be addressed in greater detail later in Chapter 4. However, it may be mentioned here that individual droplets not only interact with the continuous medium surrounding them but also influence other droplets in their vicinity. Thus, any study on spray combustion cannot be considered comprehensive unless the multi droplet interaction effects are accounted for [21].

Depending on the type of injection system, a fuel spray may be comprised of regions of different droplet number densities. While single droplet combustion models may be quite practical in dilute spray regions, models involving group combustion of droplet clouds are more realistic in dense spray regions. Extensive research in this area has been actively pursued by Chiu and coworkers (see for example Chiu and Liu [22]). Four modes and regions of group burning in spray flames were defined based on a group-combustion number (G-number), which was defined as the ratio of total heat transfer rate between the two phases to the rate of energy transport by convection (Ref. Kuo [23]). For high values of G-number \( G > 10^2 \), external sheath burning occurs, with a core of non-vaporizing droplets, surrounded by a layer of vaporizing droplets and a flame at a stand-off distance outside the spray boundary. For \( 10^2 > G > 1 \), external group combustion occurs with an inner vaporizing cloud and diffusion flame at a standoff distance from the boundary of the droplets. For marginally low values of G-number \( 10^{-2} < G < 1 \), the combustion modes was defined as internal group combustion, with the main flame inside the spray boundary while individual droplet burning occurs in the outer region of the spray. The last mode identified
was for \( G < 10^{-2} \), where only individual droplet combustion occurs.

## 1.2 Interest in Lean Direct Injection Combustion

The demand for high efficiency gas turbine engines for use in both power generation and propulsion has resulted in high compression ratios and high turbine inlet temperatures. The advancement in the turbine blade technologies has further motivated the endeavor for higher temperatures. In turn, the modern combustors are required to be designed to accommodate the rising operating pressures and temperatures. This requirement on combustors often conflicts with the desire for environmentally benign low emissions, imposed by international regulations and increasing environmental awareness.

Although higher combustor pressures and temperatures result in lowering the emissions of unburned hydrocarbons (UHC) and carbon monoxide (CO) by speeding the oxidation of fuel, they however, result in the formation of higher levels of nitrogen oxides (NOx) [24]. NOx are the main elements responsible for the formation of ground-level ozone and smog. They also contribute towards global warming and the formation of acid rain. Studies conducted by the US Environment Protection Agency (EPA) and International Civil Aviation Organization (ICAO) indicate that 34% of the total NOx production is attributed to gas turbine engines.

The primary NOx in combustion systems is Nitric Oxide (NO). It is formed mainly by two different mechanisms: thermal and prompt. Thermal NO production is temperature dependent and is maximum at flame temperatures in excess of 1800 K [25]. The thermal NO formation becomes extremely large at higher operating pressures. On the other hand, the prompt NO mechanism is the dominant NO formation process in the flame zone. It takes effect as a result of formation of intermediate radicals like HCN, CN or \( \text{N}_2\text{O} \). The NO yield from prompt mechanism is consistently always prominent. Studies conducted to determine the effect of equivalence ratio on NO formation [26], indicate higher prompt mechanism induced production rates near stoichiometric conditions, which is where the temperature is
also maximum. Because of the consistency of prompt NO formation, the overall reduction in NO emissions can only be achieved by reducing the thermal NO.

Motivated by the environmental concern and guided by the formation mechanisms of NO, low NOx combustor design have been investigated by gas turbine manufacturers mainly to reduce thermal NO. A review on the development of low NOx combustors is presented by Solt and Tuzson [27]. Water/steam injection and catalytic treatment of exhaust gases were two NOx control strategies that were earlier adopted. These methods were however, found to be both cost ineffective and hardware intensive. This led to the investigation of dry lean combustion concepts to achieve low NOx levels using much simplified configurations. From the point of view of liquid-fueled gas turbines, several concepts have so far been studied namely: Lean Pre-mixed Pre-vaporized (LPP), Rich-burn/Quick-quench/Lean-burn (RQL) and Lean Direct Injection (LDI).

LPP combustors have been found to give the lowest levels of NOx as they are designed to maintain low and uniform flame temperature. Liquid fuel is first atomized and vaporized and then homogeneously mixed with combustion air before introduction in the combustion section. LPP combustors are successfully being used in ground-based power generation application. However, their use in aircraft propulsion applications is limited because of additional hardware required by the pre-mixing chamber and more so because these combustors are prone to auto-ignition and flash back.

In RQL combustors reduction in NOx is achieved by preventing stoichiometric combustion. This is done via a three-stage process, where fuel is burned in a controlled fuel rich and fuel lean regions separated by air quenching. RQL combustors have excellent operability range. The potential utilization of RQL concept are limited by the ability of the quench process to rapidly and uniformly dilute the fuel rich mixture and to transport it to the lean zone. The complexity of RQL design is another factor responsible of the not-so-wide use of this concept.

The limitations imposed by auto-ignition and flash back and the design complexities en-
counter in the applications of LPP and RQL concepts have led to the development of LDI combustors. In such combustors, fuel is directly introduced in the combustion section, where it is simultaneously vaporized, mixed and burned with the combustion air. Because of reduced dimensions, simplicity and no unwanted flash back issues even at elevated pressures and temperatures, LDI concept has a great potential for aircraft applications. Studies conducted to compare LPP and LDI concepts [28] under same operating conditions have shown comparable NOx levels when LDI combustor was operated leaner than the LPP combustor. In addition, the LDI combustor showed a higher combustion efficiency. The biggest challenges in LDI combustors are to achieve effective atomization and rapid mixing of fuel and air to avoid formation of local high temperature zones, which give rise to thermal NO formation. A number of investigations have therefore, been conducted to address the mixing issue in LDI combustors (for example refer [29]). The application prospects of LDI concept have lately focused the research efforts on the development of genuine LDI injectors [30]. The objective of these efforts, once again, is to ensure effective atomization leading to abrupt vaporization and uniform vapor-air mixing to avoid high temperature pockets in the combustor.

1.3 Thermoacoustic Instabilities

Combustion instabilities are often regarded as oscillations in the compressible gas medium within the combustion chamber. They may be initiated and/or driven by perturbations in burning and mean flows. The geometry of the combustor and the differential response of the injector system also have dominant effects on the combustion instabilities [31]. In spray combustion, the inherent spray dynamics also affect combustion instabilities. According to McManus et al. [32] the causes of combustion instabilities can be grouped under three categories:

1. Intrinsic instabilities, which depend on the nature and properties of the reactants. They occur in gas-fueled combustors due to chemical and thermo-diffusive effects that
modify the flame propagation rate, while in liquid-fueled systems these instabilities may be a result of spray unsteadiness or variations in vaporization rate.

2. Hydrodynamics instabilities, which may occur as a result of fluctuations in mean flow or because of the formation of large scale coherent structures. The periodic formation of these turbulent structures lead to unsteady heat release.

3. Combustor system instabilities that may be caused due to the formation of shock waves or may be due to the combustor acoustics. These may also result due to acoustic coupling of the combustor with the injection system.

Thermoacoustic instabilities fall under the third category and result because of a coupling between combustor resonance modes and the unsteady heat source, where the unsteadiness in the heat release rate is a direct consequence of oscillating acoustic pressure field. As shown in Figure 1.1, a driving process generates a perturbation in the flow/heat release and a feedback process couples this perturbation to the driving mechanism and produces the

![Diagram](image)

**Figure 1.1:** Elementary processes leading to self-sustained thermoacoustic instability.
resonant interaction, which may lead to oscillatory combustion. The driving mechanism, as shown, may involve a wide variety of elementary (or rate controlling) processes. Studies have shown that the feedback provided by the combustor acoustics affect these processes via acoustic velocity oscillations, which: affect the surface density of the flame and the reaction rate [33]; cause formation of periodic vortical structures [34]; cause oscillations in fuel spray vaporization [35]; and prompt droplet breakdown [36]. The intensity of these interactions was found to depend on the magnitude of flow oscillations [37]. The result thus, is oscillatory combustion.

Most practical combustors are prone to thermoacoustic instability. This is because combustors are high energy density systems with weak acoustically damped geometries. Thus very small fraction of energy is sufficient to cause oscillations in pressure field. Also, when combustors are operated under lean conditions, even small fluctuations in the elementary processes may cause large oscillations in heat release rate [38].

The significance of thermal-acoustic coupling was documented more than a century ago by Lord Rayleigh [39]. Known after him as the Rayleigh criterion, it gives a general qualitative explanation of the phenomenon. According to the criterion, the effects of periodic heat communication to a vibrating mass of gas depends upon the phase of the vibration at which the heat transfer takes place. If the heat is received by the gas at moment of greatest compression or rejected at the moment of greatest rarefaction, the vibratory motion is amplified and there is a production of acoustic power. On the contrary, if heat is given at the moment of greatest rarefaction or removed at the moment of greatest compression an attenuation of vibration takes place and there is absorption of acoustic power. For combustion instabilities to exit and grow, the overall acoustic energy gains in the combustor must exceed the overall energy losses. It has been shown [40] that the self-excited instabilities once established would continue to grow until limited by non-linear mechanisms into a periodic finite-amplitude oscillation (Figure 1.2).
Figure 1.2: Growth of small perturbation into finite-amplitude limit-cycle oscillations.

Quantification of the Rayleigh criterion is obtained in the literature [32] by defining a parameter called Rayleigh Index $G$ as,

$$G = \frac{1}{V} \frac{1}{T} \int_V \int_T \dot{q}(x,t) \dot{p}(x,t) dT dV$$

(1.1)

where, $T$ is the time for an oscillation cycle and $V$ is the combustor volume. Also, $\dot{q}$ and $\dot{p}$ are the local unsteady heat release rate and local pressure fluctuations respectively. In terms of Rayleigh criterion amplification of instability occurs if $G > 0$ and damping occurs if $G < 0$.

The overall framework of the mechanism that drives thermoacoustic instabilities was developed and first presented in depth by Crocco and Cheng [41] in 1956. Their efforts were directed towards rocket motors and it was shown that perturbations in the conditions of the combustion coordinating processes cause a variation in the rate of heat release. The deviation drives acoustic waves that travel upstream in the feed lines causing fluctuations in pressures and velocities. This creates a periodic variation in the composition and flow of
the reactants, thus changing the flame structure and causing a variation in the heat release. Depending on the phase relationship between this heat release and pressure oscillations the acoustic perturbation would either be amplified or damped. In their effort to interpret the instability mechanism a time-lag model was presented, which even now is one of the standard methods of explaining combustion instabilities. Here the time-lag was defined as the time elapsed between injection of fuel and the sudden conversion into hot gases, shown as combustion delay in Figure 1.3.

![Figure 1.3: Time lag concept as defined by Crocco and Cheng.](image)

Now, about fifty years later, a greater knowledge base has been established. Elaborate experimental and modeling work has been accomplished, for premixed [42, 43, 44, 38, 45, 46] and non-premixed [47, 48] gaseous combustion, to identify the major parameters effecting combustion driven instabilities and the mechanism through which they influence these instabilities. Even in spray combustion, where most of the work has been restricted to liquid
rocket engine studies [41, 49, 50] and very little experimental details are available for confined direct injection combustion [51, 52], the effects of the major processes are clear in general. The major candidate factors controlling the temporal and spatial dynamics of the flame (and thus the thermoacoustic pulsations) as identified in these studies are: liquid spray atomization and vaporization, flow-generated structures, chemical kinetics, combustor geometry, flame anchoring techniques and/or any combination of these processes. The mechanisms through which each of these parameters affect the instability characteristics are discussed in the following sections, from the perspective of the research work presented in this dissertation.

1.3.1 Liquid Spray Atomization and Vaporization

Work done in the area of acoustic interactions with liquid spray combustion, with a few exceptions, has been restricted to analytical studies. These theoretical studies have suggested that the development of acoustic oscillations is dependent on atomization, vaporization and location of spray injection inside the combustor [53, 54, 55]. It was found that oscillatory ambient properties have a significant effect on the heat and mass transfer processes as well as on the relative velocity between the gas and the liquid phases. This results in oscillatory vaporization, which as a rate-controlling process can allow the pulsating chemical energy conversion to be in phase with pressure fluctuations and drive the combustion instability. The concepts of three characteristic time scales were also introduced namely: acoustic time, which was proportional to the inverse of acoustic frequency; slip (relaxation) time, which was defined as the time taken by the droplet to respond to the changes in its environment; and droplet residence time or evaporation time, which is the same as fuel lag-time introduced by Crocco [41]. It was shown that all three time scales must be comparable to excite vaporization induced thermoacoustic instabilities.

Experimental works [52, 56] have shown that the presence of acoustic field enhances the combustion process by the presence of an alternating flow and results in shorter droplet
burnout distance. This effect is more pronounced when the spray is located in regions of acoustic velocity anti-nodes.

Since vaporization depends on size of droplets exiting the injector and on the evaporation characteristics of the fuel, the atomization and fuel type play important roles in the vaporization driven combustion instability [50]. It has been shown that pressure and velocity variations result in variation of droplet size and distribution, which in turn effects the characteristics time of the vaporization process. It must be noted however, that atomization does not take part directly in the feedback of oscillations but contributes indirectly via vaporization towards combustion instabilities. It also affects the effectiveness of the active control via fuel modulation.

1.3.2 Flow Structures

Large-scale flow structure also called coherent structures have been found to play an important role in causing combustion instabilities by influencing combustion and heat release processes via control of fuel-air mixing [57]. Flow-dynamics generated unstable modes associated with flow instabilities were related to recirculation, occurring either in the wake of bluff-bodies or at the sudden expansion in a backward-step dump of a combustor. This and similar works [48, 58] have emphasized vortex shedding as a driving mechanism of combustion instabilities. Vortices are formed in the shear layer between high speed stream of unburned reactants and low speed stream composed of hot combustion gases. When these vortices interact, a large interface is formed between the reactants and the combustion products leading to fine scale turbulent mixing and sudden release of heat. This is repeated each cycle of pressure oscillations and therefore, cause periodicity in heat release, which if in right phase with the pressure fluctuations may cause high amplitude instabilities. The inner recirculation zone along the combustor axis was found to support helical instability modes while the external recirculation zone was associated with both axisymmetric and helical modes.
1.3.3 Flame Anchoring

The significance of recirculation towards combustion instability was emphasized above. The relative size of the recirculation zone has a strong bearing on the stability and confinement of the flame. These two parameters in turn influence combustion instability [52, 25]. Experimental studies have shown the dominating influence of the external recirculation zone on flame stabilization [59]. In practice, the recirculation can be induced in two ways [58]. First, by imparting strong swirl to the combustion air, it is possible to create a radial pressure gradient, sufficient to cause a reverse flow along the axis of rotation. And second by employing bluff bodies and rearward-facing steps to create wake zones. Swirl stabilization is particularly more effective in liquid phase direct injection combustion. Moreover, the strength of the recirculation zone can be managed by the direct control of imparted swirl, which is defined in terms of Swirl Number. Swirl has also been found to enhance mixing of fuel and air, an important prerequisite for low emission and high efficiency non-premixed combustion [60]. Similarly, other laboratory test studies have demonstrated that swirl affects the combustion dynamics via changes it can cause in the flame size and structure [61, 62] and via its influence on mixing [63]. Since the swirl stretches the flame circumferentially, it has been found to enhance the azimuthal instability mode. A recent analytical study into swirl, heat release and their dynamic interaction in premixed combustion has shown increase in combustion instability with increase in swirl beyond a critical value [64].

1.3.4 Chemical Kinetics

Experimental, analytical and numerical studies on premixed combustion have shown that equivalence ratio fluctuations have a considerable effect on combustion instability [65, 66, 67, 68]. Fluctuations in reaction rate and heat release are caused by oscillations in equivalence ratio, which may be induced by fluctuation in fuel flow rate, or by velocity fluctuations in the vicinity of fuel injector, or by fluctuations in acoustic pressure. Oscillations in equivalence ratio have in fact been suggested as the main driving force in unsteady premixed combustion
especially at lean conditions. This is attributed to increased chemical time scales associated with lower equivalence ratios, which result in a better coupling between acoustic and kinetic processes. The equivalence ratio variation has also been advocated as the factor, which closes the feedback loop necessary for thermoacoustic instability [65].

Research on liquid non-premixed diffusion combustion, however highlights that variations in chemical reaction time as a result of fuel-to-air ratio fluctuations do not have a significant effect on oscillations in burning and heat release rates as compared to vaporization effects [50, 69]. This is shown to be true because vaporization characteristic time is expected to be at least one order of magnitude higher than the chemical characteristic time, and thus the rate-controlling factor. Chemical kinetics are therefore, not regarded by most researchers as a driving mechanism for combustion instability in spray combustion.

1.3.5 Combustion System Geometry

Size, shape and location of various components in the combustion system play a vital role in enhancement or attenuation of combustion instabilities as has been shown theoretically and experimentally [66, 70, 71, 72]. Successful application of flame holders and other passive control devices manifest the importance of this relationship. Changes in the length of the combustor have been found to modify the acoustic mode structure and it’s coupling to dynamic heat release. Similarly, shape of the combustor has its own consequences. In rectangular geometries, the transverse modes are all basically similar, whereas in cylindrical chambers the transverse modes can either be radial or tangential. Flow visualization experiments and simulations conducted in reacting flows have shown that a large vortex forms behind a backward facing step in dump combustors and oscillates violently in phase with the pressure oscillations in the chamber. In the like manner, location of injectors, axial swirlers and other components inside the combustor system has been found to alter the acoustics and combustion instability [61, 73]. Depending upon the acoustic impedances at the boundaries the combustor oscillations can interact with the reactant feed lines, which as already
discussed, give rise to thermoacoustic instabilities.

1.4 Control of Thermoacoustic Instabilities

Thermoacoustic instabilities have been a continuous source of concern in the development of high-performance, low-emission gas turbine systems. These instabilities manifest themselves as high amplitude pressure oscillations in the combustion systems and result in both performance and structural degradation. As reported by Lieuwen and McManus [74], the cost of repair and replacement of structural components damaged mainly due to combustion instability related problems exceed $1 billion annually. In addition, these damages result in increased downtime of the systems.

Thermoacoustic instabilities, as highlighted earlier, have been extensively examined for over 50 years. However, there has been a surge of interest in the last one decade or so. The key driver behind these recent efforts has been the desire to control or completely attenuate these instabilities. Both passive and active control techniques have been employed for this purpose. The key essence of all control methods is to disrupt the coupling between the unsteady heat release rate and the combustor acoustic oscillations.

Passive control methods involve hardware and design modifications. Some of these measures include installation of baffles, resonators and/or acoustic liners that suppress the amplification of acoustic waves in the combustor [49]. The objective of these installations is to force the resonance to occur at frequency ranges where the driving mechanisms are inadequate to sustain oscillations. Other passive techniques, that have been successfully applied to industrial gas turbines involve designing of premixing ducts, which are insensitive to acoustic pressure oscillations [75]; thus, avoiding the fluctuations in fuel-air ratios and subsequent unsteadiness in heat release rates. Passive control methods have been found to be quite effective for controlling high frequency oscillation encountered in rocket motors, afterburners and certain gas turbines. However, the effectiveness of these methods is often limited
Active control techniques, on the other hand, have shown effectiveness over a wide range of frequencies (especially at lower frequencies) without requiring major hardware redesigning. The stabilization method involves actively injecting perturbations into the combustor using active control actuators to decouple the processes responsible for thermoacoustic instability namely, unsteady heat release rate and acoustics. This is normally done by introducing a closed-loop control around the thermoacoustic instability loop, as shown in Figure 1.4. Although the concept involved in active control methods is quite old (introduced in 1952 by Tsein [76]), the recent and renewed interest stems from the advancements in actuator and sensor technologies.

Figure 1.4: Overview: (a) Thermoacoustic instability coupled with acoustic feedback. (b) Application of active control loop to the system.

Development and application of active control techniques involve three steps: modeling of the combustion system; designing the controller; and developing the actuation system. The dynamic combustor system modeling, from the perspective of designing a control strategy, falls under two classes namely, physically based and system-identification based models. A detailed review is presented by Annaswamy and Ghoniem [77]. Since combustion is inherently
a non-linear and complex process with number of variables affecting its behavior, extensive efforts have been devoted in developing reduced-order combustion models, ranging from models for simpler flames [78, 79, 80] to those for more complex flames [81, 82, 83]. As for control design, different strategies have been adopted over the years [84, 77]. These are phase-shift control, adaptive time-delay control, self-tuning control, observer-based control, system-identification-based control, and open-loop control. Active control actuators used in the studies conducted to date include speakers, pulsing jets, spark plugs, oscillating shakers and fuel injectors. An overview is presented by Zinn and Neumeier [85]. Actuation has been tried both in air and fuel supply lines to achieve control over instability. Air side modulation was primarily done to modify the coherent flow structures, while purpose of fuel modulation was to modify heat release by changing equivalence ratios. However, over the years it has been generally agreed that modulating the injection rate of total fuel flow offers the most practical and promising approach for combustion instability suppression. Liquid fuel modulation has been successfully applied, to varying degrees, by many researchers [86, 87, 88, 89, 90].

Though promising, the utilization of effective active control techniques however, requires exact prediction of onset of instability and corresponding frequencies, amplitudes and growth rates. For this reason, understanding of acoustics-thermal coupling is essential, which in turn demands better knowledge of the ways the rate-controlling parameters (mentioned in last section) affect the plant and flame characteristics. Also, it is essential to understand how an effective control approach modifies the flame dynamics and controls the instability growth.

1.5 Motivation for Current Research and Specific Objectives

The principles of spray combustion are to a large extent well understood and have achieved a maturity level where they can be reliably applied towards heating, power generation and propulsion. The challenges that are currently faced in this field reflect the changes in the
The growing awareness about environmental safety and economic impact has led the technology towards innovating environmentally benign and cost effective combustion systems. The identification of the basic chemical and physical mechanisms and how they couple under these new developments is the challenge of today.

The need for lean combustion concepts and the impact of lean burning on the overall performance and life of gas turbine systems was highlighted in the preceding sections. It was also pointed out that combustion systems with high energy release rate, burning liquid fuels, are prone to thermoacoustic instabilities. These instabilities cause perturbations in the combustor through-flows and the spray behavior. The combined effect of these disturbances is to cause unsteadiness in the heat release, which in turn has the potential to enhance the amplitude of pressure oscillations.

The motivation for the work presented in this dissertation comes from the realization that the development of effective active methodologies to control thermoacoustic instabilities is presently marred due to lack of understanding of the cause-and-effect relationship between combustor acoustics and spray combustion dynamics. In a lean direct injection environment where the spray vaporization, mixing and burning are taking place simultaneously, it is imperative to determine where and when the spray droplets will burn under the influence of an oscillating flow field. Thus there is a need to: experimentally determine the onset of thermoacoustic instabilities in a particular combustion system and its effects on spray combustion; and analytically or numerically explain the physics of the experimental observations.

The overall purpose of this research is to gain understanding of the effects of combustor high amplitude pressure oscillations on sprays and spray combustion dynamics and to demonstrate the implications of these effects on LDI combustor operation. The specific objectives are:

1. Design and commission a direct injection liquid-fueled experimental combustion facility that has the capability to operate both in stable and unstable modes.

2. Map the characteristics of the combustor under stable and unstable operation.
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3. Study the influence of acoustic excitation on spray behavior in terms of droplet size and velocity distribution.

4. Model spatial and temporal tracking of spray droplets as they traverse in a reacting and oscillating flow field to determine when and where they burn.

5. Study the frequency dependence of sprays to aid in formulating fuel transfer function.

1.6 Organization of the Dissertation

The chapters that follow in this dissertation have been organized to independently address each of the research objectives. In addition to the methodologies employed in each study and discussion of the results, literature surveys pertaining to these particular efforts are also presented in each chapter.

Chapter 2 explains in detail the design and the experimental setup of the 75 kW overall lean direct injection, kerosene-fueled combustor. Since the combustor can be operated in stable and unstable modes, stability mapping and characterization of the combustor under these operating conditions is also discussed. The characterization involves flow field measurements, dynamic pressure, heat release and acoustic velocity measurements and emission measurements. The transition of the combustor from stable to unstable operation is also tracked and explained. The changes that occur in the combustor performance under the influence of active instability control via fuel modulation are also shown.

Chapter 3 discusses the non-reacting flow experiments conducted to study the effects of acoustic excitation on spray behavior using Phase Doppler Anemometry (PDA). The experimental setup and the design of an acoustic rig, designed for the very purpose are also explained. A novel methodology to process the phase-locked measurements of droplet diameters and velocities is discussed together with the results of the different measurements that were conducted.
Modeling efforts for tracking the droplet trajectories in an acoustically excited flow field are discussed in Chapter 4. Details of the currently available models and the justification of the approach adopted in the present effort are also shown. Results from both reacting and non-reacting flow simulations are presented. Also discussed in the chapter are the results of an analytical study, which was conducted to explain the impact of acoustic pressure on secondary droplet breakup.

Chapter 5 discusses the setup, methodology and results of a novel application of Time-Resolved Digital Particle Image Velocimetry (TRDPIV). These measurements were conducted to investigate the frequency response of modulated sprays. These pilot experiments were conducted to demonstrate the ability of TRDPIV to capture the spray dynamics. Validation of the data by comparison with PDA data is also given.

Although conclusions drawn from each study are given at the end of corresponding chapters, the overall conclusions from the research that explains the combustor behavior over the operating range is given in Chapter 6. Also discussed in the chapter are the recommendations for future work.
Chapter 2

Thermoacoustic Characterization of Combustor

To meet the research objective, a direct injection combustor rig was constructed, which is capable of operating in distinctive stable and unstable modes. Characterization of the rig was conducted to map the combustor’s performance over its operating range. The details of the experimental work along with the explanation of the onset and sustenance of thermoacoustic instability in the combustor are presented in the succeeding sections.

2.1 Design and Construction of Liquid-Fueled Combustor Rig

All reacting flow experiments were performed on a vertically mounted 75 kW atmospheric combustor rig. The combustor is swirl-stabilized, with a backward step and cylindrical geometry. Kerosene was used as fuel for the experiments. A photograph of the rig is given in Figure 2.1 and a schematic is shown in Figure 2.2. The rig is comprised of three main sections: the combustor; the burner; and the flow conditioning section, which is connected to the air
and fuel supply lines. The combustor is made up of hot and cold sections and separated by 6.35 mm thick insulation packing made from Fiberfrax cloth. The hot section is equipped with two 202mm x 50mm and one 202mm x 76mm rectangular viewing windows made up of 6.35 mm thick GE Type-124 fused silica quartz plates. The windows possess required optical, thermal and physical properties and were installed for imaging and chemiluminescence/other non-intrusive measurements. The two smaller windows are placed 150° to each other for scattering measurements in 1st refraction mode, while the larger window is oriented at 90° to one of the smaller windows for imaging purposes. The cold section of the combustor is wrapped with 6.35 mm copper tubing to provide water-cooling. The effective combustor section’s length of 1219.2 mm was selected after a series of tests to give the combustor the self-excitation capability in a frequency range of interest. The water-cooling was provided to cool down the gas temperature in the downstream half of the combustor. The relative lengths of the two sections were determined based on combustor’s resemblance to a Sondhauss tube.
Figure 2.2: Schematic of the combustor rig (all measurements are in mm).
with quarter wave acoustic mode shape as its fundamental frequency. Such a tube has been shown to exhibit pronounced acoustic oscillations, when the hot and cold sections are equal in length and there is step change in the two sections’ temperatures, thus the use of insulator packing. The combustor is provided with adequate number of taps along the length and the circumference for acoustic pressure, temperature and emission measurements.

The burner section, as shown in Figure 2.3, consists of an air plenum, a centrally located fuel injection nozzle and a mechanical swirler, which sits coaxially around the injector. A 63.5 mm overlap is provided between the combustor and the burner section so that the complete flame is captured by the viewing windows. The plenum has been fabricated to accommodate various sizes of swirlers and injector nozzles. It is also equipped with two measurement taps, and two water tapings to provide cooling to the face plate on which the combustor sits. The faceplate has a diverging quarl and is replaceable. Both pressure and air assist type of atomizers can be accommodated. The fuel (and also air in case of air assist atomizer) is fed

![Figure 2.3: The burner section of the rig.](image)
to the injector nozzle from the bottom of the rig through a 6.35 mm stainless steel tubing. Axial location of the injector nozzle can be changed by adjusting the position of the lance, which houses the nozzle.

The flow conditioning section is made up of 76 mm ID steel pipe with a plate welded at the bottom and a tee section to house an acoustic driver (Figure 2.2). Air is introduced in the rig via four air injectors equally spaced around the circumference at the bottom of the section. The injectors are 12.7 mm stainless steel tubes, each with three 90° apart rows of equally spaced 1.58 mm diameter holes for air entry. This air inlet arrangement ensures that the rig is acoustically decoupled from the air supply plumbing. The flow conditioning section is also equipped with an optional aerodynamic swirling arrangement, which consists of 12.7 mm copper tubing connected tangentially to the section in two sets. Air entering through these tangential ports mixes with the axial air and provides a swirling flow. The degree of swirl can be controlled by varying the ratio of air split between the axial and tangential ports.

2.2 Experimental Setup

The overall experimental setup is shown in Figure 2.4. Kerosene is delivered from the non-pressurized tank to the combustor by an Anderson Bowen gear pump (details of equipment are given in Appendix B), at a maximum pressure of 4.14 MPa. The pump is driven by a 745.7 W, 3450 rpm 3-phase electric motor from World Wide Corporation. The fuel feed line is comprised of: a cartridge filter from Parker, to prevent the feed line from foreign object contamination; a pressure regulator, to maintain a maximum specified feed pressure; accumulator, to avoid pressure pulsation in the feed line; flow control valves, for both coarse and fine metering of the fuel; analog pressure gauges, to monitor feed line pressure at various locations; a flow meter from AW Company with a proportional frequency sensor, to monitor the fuel volume flow rate; and two dynamic pressure transducers from Omega, to monitor and record mean fuel pressure in the feed line as well as pressure oscillations during fuel
module. The air is fed to the system from a 306.75 kg/h, 0.86 MPa, Ingersol Rand compressor, via an in-line process heater from Omega. With the use of the heater a temperature of 294K was maintain during all reacting flow experiments reported in the dissertation. Two Eldridge air flow meters, one each in axial and tangential air supply lines, are provided to monitor air flow rate to the combustor. Pressure regulators and flow control valves are also provided in the air feed line to regulate and meter the air flow rate. The system is also equipped with a rotameter from Matheson to monitor the air flow rate to the air assist injector. Data acquisition is provided using a data acquisition system from National Instruments, while front-end interface is provided using LABVIEW software.

Figure 2.4: Experimental setup of the combustor rig.
2.3 Measurement Methodology

The measurements conducted to characterize the thermoacoustic behavior of the combustor included: acoustic pressure/velocity measurements; heat release rate measurements; and temperature and emission measurements. Figure 2.5 shows the locations on the combustor rig where the measurements were made.

2.3.1 Acoustic Pressure and Velocity Measurements

Acoustic pressure measurements were made using button-type differential dynamic pressure sensors from Honeywell/Sensym ICT. The particular pressure transducers were chosen because of their low cost, low noise and robust qualities. A strain gauge amplifier/power unit from Vishay Corporation was used to amplify the pressure signals from the sensors and also to provide the excitation voltage to the sensors. The sensors are capable of measuring acoustic pressures of up to 34.5 kPa. The sensors were mounted external to the combustor rig using fabricated pressure probes.

The pressure signals from the sensors after amplification were collected using a data acquisition system from National Instruments, equipped with a 8-channel simultaneous sample and hold card and an anti-aliasing filter. The data collection interface was built using LABVIEW software also from National Instruments. The collected data was also verified by simultaneously measuring the pressure signals on a digital signal analyzer from Hewlett Packard. The sound pressure levels (SPL) were then calculated from the recorded pressure data using a sensor sensitivity of 2.297 kPa/V, which was based on the manufacturer’s provided typical sensitivity value and the settings on the amplifier.

The acoustic velocities in the combustor were calculated using acoustic pressure signals acquired from two closely placed pressure transducers at P3 and P4 locations (Refer Figure 2.5). These locations were selected for the determination of acoustic velocities after ensuring that no velocity node existed at these locations in the frequency bandwidth of interest. This was
Figure 2.5: Measurement locations on the combustor.
done using an acoustic velocity circuit (for details on the circuit, refer [83]) and exciting the combustor rig with a white noise signal from a speaker, under no flow conditions.

For a standing acoustic wave pattern in the combustor, the acoustic velocity is 90° out of phase with the acoustic pressure. The acoustic velocity can then be determined from the knowledge of acoustic pressures by applying the momentum equation from classical acoustics [91]. For an inviscid homogeneous medium, this equation is of the form,

\[
\frac{\partial \hat{u}}{\partial t} + \frac{1}{\rho_o} \nabla \hat{p} = 0
\]  

(2.1)

Here \(\rho_o\) is the density of the medium and \(\hat{u}\) and \(\hat{p}\) are the acoustic velocity and pressure respectively. Since for the combustor used in the experiments, the length to diameter ratio is much larger than unity, only longitudinal acoustic modes are assumed to dominate and thus Eqn. 2.1 takes a 1-D form, which after rearranging and integrating over one acoustic time period \(T\) can be written as,

\[
\hat{u}(t) = \int_T \frac{1}{\rho_o} \frac{\partial \hat{p}(t)}{\partial x} dt
\]  

(2.2)

or in the frequency domain,

\[
\hat{u}(\omega) = \frac{\Delta \hat{p}(\omega)}{2\pi f_o \rho_o \Delta x}
\]  

(2.3)

where \(f_o\) is the frequency of longitudinal acoustic mode. Eqn. 2.3 was used to calculate the acoustic velocities in the combustor.

### 2.3.2 Dynamic Heat Release Rate Measurements

The method of qualitatively measuring the unsteady heat release rate by optically capturing the chemiluminescence of combustion radicals is widely accepted and have been used by many researchers. Some of the chemiluminescent species investigated are \(CH^*, C_2^*, CO_2^*\) and \(OH^*\). Haber and Vandsburger [92] have recently advocated the use of \(OH^*\) chemiluminescence as
Figure 2.6: Optical train for OH* chemiluminescence measurements.

the major indicator of heat release rate over a wide range of equivalence ratios. Especially in ultra lean combustion environment OH* is an attractive chemiluminescence specie, since CH* and C2* are virtually absent at these low equivalence ratios. To account for the global OH* chemiluminescence, the following single reaction was suggested,

\[ HCO + O \rightarrow CO + OH^* \] (2.4)

Since the formyl radical HCO is a major intermediate species in hydrocarbon fuel combustion, the dynamic OH* chemiluminescence signal was used as a measure of unsteady heat release rate.

The optical setup to capture the chemiluminescence is shown in Figure 2.6. The optical system was mounted on a 3-D traverse mechanism and accessed the combustor chemiluminescence through the optical window provide on the combustor. The system was designed based on thin-lens approximation. Additionally, it was assumed that the flame is thin (with no depth) and that the chemiluminescence was diffused. A 75 mm focal length, fused-silica lens with a 50.4 mm diameter was used to focus the chemiluminescence on a photo multiplier tube (PMT) from Hamamatsu Photonics. Fused-silica lens was selected since it has very good transmission capability in the ultra violet (UV) range of wavelengths. For the dimensions shown in the figure, a 202mm x 76mm flame area was projected on the PMT window.
Figure 2.7: Circuit diagram for current to voltage conversion and amplification of \( \text{OH}^* \) signal.

having a dimension of 8mm x 2mm. A 307.81 nm optical filter was used between the lens and the PMT to capture only the \( \text{OH}^* \) emissions. The PMT converted the chemiluminescence signal to a proportional current, which was then converted to voltage and amplified using the circuit shown in Figure 2.7. The data was recorded using the data acquisition system described earlier.

Quantification of the chemiluminescence signal in terms of physical power units was accom-

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>4.25</td>
</tr>
<tr>
<td>R</td>
<td>4kΩ</td>
</tr>
<tr>
<td>( k_{\text{PMT}} )</td>
<td>( 1.35e^{-6}) Watt/Amp</td>
</tr>
<tr>
<td>( e_{\text{optics}} )</td>
<td>( 2e^5 )</td>
</tr>
</tbody>
</table>

Table 2.1: **Values of constants used in Equation 2.5**
plished utilizing the measured voltage signal and adopting a relationship given by Haber and Vandsburger [92],

$$I_{OH} (\text{Watt}) = e_{optics} k_{PMT} \left[ \frac{I_{OH} (\text{Volt})}{(G)(R)} \right]$$  \hspace{1cm} (2.5)

where $G$ and $R$ are the gain and the input resistance of the circuit used to measure the chemiluminescence voltage respectively. $k_{PMT}$ denotes the conversion factor from PMT output current to PMT input light power, while $e_{optics}$ accounts for the efficiency of the geometric optics. For the circuit of Figure 2.7, the PMT specifications and the optics used in the experiments, the values of these parameters are given in Table 2.1.

### 2.3.3 Temperature and Emission Measurements

Temperature measurements were collected in the combustor and in the air and fuel feed lines. On the combustor, both hot gas and combustor surface temperatures were recorded using a 16-channel, cold-junction-compensated data acquisition card from CyberResearch Incorporation. Type K thermocouples with 0.076 mm wire size were used at all locations except in the flame area where Type B thermocouple with 0.203 mm wire size were used.

NOx emission measurements were made during all operating modes of the combustor. An analyzer from Thermo Electron Corporation was used for this purpose. Gas samples were collected 150 mm upstream of the combustor exit. A probe assembly was specially fabricated to ensure radial uniformity in sample collection.

### 2.4 Acoustic Characterization

Acoustic characterization of the combustor rig was done prior to the reacting flow experiments to determine the resonance modes of the rig. Figure 2.8 shows the power spectrum of the acoustic pressure measured at P3 location on the combustor under no flow conditions.
Figure 2.8: Noise floor in the combustor due to power electronics.

Figure 2.9: Noise floor in the combustor due to mean flow generated turbulence.
CHAPTER 2. THERMOACOUSTIC CHARACTERIZATION OF COMBUSTOR

The spectrum indicates the noise floor solely due to power electronic used on the combustor. Combustion air was next introduced in the rig and acoustic pressure profiles were noted at the same location for a range of air flow rates. Figure 2.9 shows the power spectra for four air flow rates, which specify the noise floor under mean flow generated turbulence.

The resonance modes of the rig were then determined by exciting the combustor with both white noise and random chirp signals under no flow conditions. A speaker was used as the acoustic driver, which was placed at the tee-section at the bottom of the combustor (Refer Figure 2.2). The resonance frequency was found to be around 72 Hz as shown by the fundamental peaks in Figure 2.10. A good coherence between the perturbation and acoustic response signals was observed, as shown in Figure 2.11. This experimentally determined value of the resonance frequency was found in good agreement to the value calculated by an equation given in Hassa et al. [93],

Figure 2.10: Experimentally determined resonance modes of the combustor.
$f_o = \frac{c}{2\pi} \sqrt{\frac{A_e (V_1 + V_2)}{\pi (V_1 V_2)}}$  \hspace{1cm} (2.6)

The above equation was deduced by applying an analogy between the flow conditioner-burner-combustor arrangement of the rig with two hollow spaces connected with a narrow passage, as shown in Figure 2.12(a). In the equation $V_1$ and $V_2$ are the volume of flow conditioning and combustor sections respectively, $A_e$ is the effective area of the burner section incorporated with a mechanical swirler, while $c$ is the speed of sound in the burner section. Using the dimensions shown in Figure 2.2 and using room temperature as the representative temperature in the burner, the calculated resonance frequency of the rig is found to be 75 Hz.

It was however, found that under reacting flow condition and when the combustor was operating in normal stable mode the resonance frequency measured at the same P3 location was around 123 Hz (combustor acoustic behavior under the complete operating range and
all stability modes is discussed later in the chapter in more detail). This frequency of 123 Hz corresponds to the resonance frequency of the combustor section alone, after taking into account the hot gas temperature in the combustor. The shift in frequency of the rig from 72 Hz to 123 Hz is attributed to the presence of flame in the later case. The flame provides a closed (reflective) boundary condition for the acoustic waves at the combustor-burner interface. This is because of the high acoustic impedance created at the flame location due to sudden rise in gas temperature. The flame therefore, isolates the combustor from the rest of the system and the combutor acts like a closed-open tube, as shown in Figure 2.12(b). Under this condition the resonance frequency can then be computing using,

\[ f_o = \frac{c}{\lambda} \]  \hspace{1cm} (2.7)

where \( \lambda \) is the acoustic wavelength. Since for a closed-open tube the fundamental acoustic mode is one-quarter wave mode (with pressure anti-node at the closed end and pressure node at the open end), the value of \( \lambda \) equals four times the length of the combustor section.
Eisinger [94] has shown that combustion systems in a close-open configuration behave like Sondhauss tube, where Rayleigh criterion driven thermoacoustic oscillations may be generated if heat is supplied to the gas at the closed end (greatest compression) or extracted at the open end (greatest rarefaction) of the tube. Based on this very reasoning, the downstream half of the combustor section was provided with external cooling (Refer Figure 2.2) to aid the onset of thermoacoustic instability when the combustor was required to be operated in an unstable mode.

### 2.5 Flow Field Characterization

Effective mixing of fuel and air is an important prerequisite in direct injection combustors to ensure uniform burning and low emissions. This can be achieved in such combustors by imparting swirl to the combustion air. Additionally, swirl flows also result in compact flames and aid flame stabilization. Typically, two methods are employed to induce swirl to the flow [95]: aerodynamic swirling, where a part of fluid stream may be introduced tangentially in a cylindrical chamber; or mechanical swirling, where guide vanes are used in the axial flow.

For the combustor used in the present research, the choice of mechanical swirling over aerodynamic type was made after conducting a series of tests using both the methods and visually observing the flame appearance. Selected results of these tests are summarized in Appendix A. For aerodynamic swirling, the varying degree of swirl was obtained by varying the split between tangential and axial air flow rates. On the other hand, for the mechanical swirling the variation in swirl intensity was achieved by using swirlers with different vane angles. It was found that the optimum mixing quality over the combustor operating conditions was achieved by using a mechanical swirler with $45^{\circ}$ vane angle. A schematic of the swirler is shown in Figure 2.13. All subsequent measurements on the combustor were conducted using this swirl arrangement. Pressure loss across the swirler was calculated using the method.
Figure 2.13: Schematic of 45° mechanical swirler (measurements in mm).

given by Lefebvre [96] and was found to be 4.89 kPa. The geometric Swirl Number, as defined by Beer and Chigier [95] and as calculated from Equ. 2.8, was found to be 0.81,

\[ S_g = \frac{2}{3} \left(1 - \left(\frac{D_h}{D_s}\right)^3\right) \frac{1}{1 - \left(\frac{D_h}{D_s}\right)^2 \tan \theta} \]  

(2.8)

Here \( D_h \) and \( D_s \) are the hub and swirler diameters respectively and \( \theta \) is the vane angle.

Velocity field measurements were conducted using Hot-Wire Anemometry. Equipment manufactured by AA Labs was used for this purpose. A four-sensor hot-wire probe was used. The 3-dimensional measurements were taken over a range of airflow rates and at two locations along the combustor longitudinal axis, one at the inlet to the quarl and the other at the inlet of the backward step (exit of the quarl, refer Figure 2.3). The recorded data was processed using the method given by Wittmer et al. [97]. The profiles of axial and tangential velocity components at the quarl inlet are shown in Figure 2.14 and Figure 2.15 respectively. Due to physical restrictions the measurements could not be taken closer to the walls, where the velocities approach zero due to no-slip conditions. The axial velocity profile shows an
CHAPTER 2. THERMOACOUSTIC CHARACTERIZATION OF COMBUSTOR

Figure 2.14: Axial velocity profile at quarl inlet with 45° axial swirler.

Figure 2.15: Tangential velocity profile at quarl inlet with 45° axial swirler.
Figure 2.16: Axial velocity profile at dump inlet with 45° axial swirler.

Figure 2.17: Tangential velocity profile at dump inlet with 45° axial swirler.
axisymmetric distribution. The drop in velocities at, and in the vicinity of the centerline is due to the presence of fuel injector and also due the presence of a recirculation zone, which the hot-wire anemometry was not capable to detect. At lower airflow rates magnitude of the velocities shows an increase proportional to the increase in airflow rate. However, the same proportionality is not reflected beyond the flow rate of 52.2 kg/h. This was attributed to higher boundary layer losses associated with higher air flow rates. Measurements for the velocity profiles at the inlet of the backward step, as shown in Figure 2.16 and Figure 2.17, gave similar results.

From the hotwire measurements, the actual Swirl Number (for the maximum flow rate used in these experiments) at the two locations were also calculated using Equ. 2.9 given by Chen [98]. These were found to be 0.253 and 0.125 at the inlets to the quarl and the backward step respectively,

![Figure 2.18: Turbulent Velocity Profile at Quarl Inlet.](image-url)
Here $U_z$ and $U_\theta$ are mean axial and tangential velocities in polar coordinates and $R$ is the outer radius of the flow passage.

The turbulence level in the air flow at various air mass flow rates were also determined from the recorded velocity measurements. Radial distributions of the turbulent velocities are shown in Figure 2.18 and Figure 2.19 at the quarl inlet and the inlet to the backward step.

### 2.6 Stability Mapping

Combustor performance was mapped over its complete operating range. The operating range was defined by the limitations imposed by the fuel injector and the fuel pump capacities,
CHAPTER 2. THERMOACOUSTIC CHARACTERIZATION OF COMBUSTOR

Table 2.2: Specification of injectors used in the experiments.

<table>
<thead>
<tr>
<th>Atomizer</th>
<th>Mass flow rate (kg/h)</th>
<th>Spray angle</th>
<th>Orifice size (mm)</th>
<th>FN  @ 0.862 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>WDB-0.40</td>
<td>1.269</td>
<td>60°</td>
<td>0.201</td>
<td>1.37</td>
</tr>
<tr>
<td>WDB-0.50</td>
<td>1.587</td>
<td>60°</td>
<td>0.211</td>
<td>1.71</td>
</tr>
<tr>
<td>WDB-0.75</td>
<td>2.380</td>
<td>60°</td>
<td>0.234</td>
<td>2.56</td>
</tr>
<tr>
<td>WDB-1.00</td>
<td>3.174</td>
<td>60°</td>
<td>0.280</td>
<td>3.42</td>
</tr>
<tr>
<td>WDB-1.50</td>
<td>4.761</td>
<td>60°</td>
<td>0.330</td>
<td>5.13</td>
</tr>
</tbody>
</table>

air supply capability and the thermal loading of the combustor. The injectors used in the investigations were solid-cone, pressure swirl atomizers from Delavan Corporation. Various capacity atomizers were used in the experiments. For the purpose of further discussions, these atomizers will be referred by their Flow Number (FN). The FN was calculated using the definition by Mellor [99], which is,

\[
FN = \frac{\dot{m}_F}{\sqrt{\Delta P}}
\]

where \(\dot{m}_F\) is the mass flow rate of fuel (in kg/h) and \(\Delta P\) is the differential pressure across the atomizer orifice (in MPa). The atomizers used in the experiments and their corresponding FN are given in Table 2.2. WDB-1.00 atomizer (FN 3.42) was used as the base line injector.

Figure 2.20 depicts a typical stability map over the range of tested conditions using FN 3.42 atomizer. The abscissa of the plot indicates the fuel mass flow rate and the ordinate denotes the Global Equivalence Ratio (GER), which is defined as the fuel-to-air ratio normalized by its stoichiometric value. The lines running across the plot represent the corresponding mass flow rate of combustion air. At low GER the combustor was found to operate in a stable mode, characterized by a well-mixed and compact flame as shown in Figure 2.21 (a). As the GER was increased the combustor behavior became thermoacoustically unstable entering
Figure 2.20: Stability map of combustor’s operating range using FN 3.42 injector. Blue and red O represent the transition to 1\textsuperscript{st} and 2\textsuperscript{nd} unstable regimes respectively when only air flow rate was varied. Blue and red Δ indicate the same transitions when only fuel flow rate was varied.

what is labeled in Figure 2.20 as the 1\textsuperscript{st} unstable regime. This operationally unstable regime was characterized by a poorly-mixed, luminous flame surrounding diffusion burning of individual droplets or droplet clouds (Figure 2.21 (b)). An abrupt transition to the 2\textsuperscript{nd} unstable regime was encountered when the GER was further increased towards stoichiometric. In this operating regime the flame appeared highly stretched and resembled a lean pre-vaporized pre-mixed flame, as shown in Figure 2.21 (c).

This GER dependent behavior of the combustor, transitioning between stable to 1\textsuperscript{st} unstable to 2\textsuperscript{nd} unstable modes, was found independent of the manner in which GER was varied. That is, either via changes in air flow rate with constant fuel flow rate (shown in Figure 2.20 by blue and red O for the two transitions respectively); or changes in fuel flow rate keeping air flow rate constant (shown by blue and red Δ for the two transitions respectively); or changes in both air and fuel flow rates.
Stability mapping was also conducted using atomizers of various capacity, as listed in Table 2.2. This was done to investigate the effect of changes in mean drop size on the combustor’s stability behavior. For simplex type injectors spraying hydrocarbon fuels, Bayvel and Orzechowski [16] have given an empirically determined relationship between the Sauter Mean Diameter ($D_{32}$), produced by an atomizer and the atomizer FN, which is

$$D_{32} \propto (FN)^{0.0975} \quad (2.11)$$

According to the relationship smaller FN atomizers will generate smaller droplets, keeping all other operating parameters constant.

The results of these experiments are summarized in Figure 2.22, which indicates the transition boundaries for the various FN injectors. As can be noted, the 1\textsuperscript{st} unstable boundary was found independent of the FN, while a distinct difference was observed at the 2\textsuperscript{nd} unstable boundary. The transition to the 2\textsuperscript{nd} unstable regime was found delayed when lower FN atomizers were used, while for higher FN atomizers a shrinkage in the 1\textsuperscript{st} unstable region was observed. Keeping in view the FN dependent behavior of the 2\textsuperscript{nd} unstable boundary and the flame appearance in 2\textsuperscript{nd} unstable regime, it was hypothesized that the transition to this unstable regime was due to secondary breakup of droplets by the presence of high acoustic pressure forces prevailing under those conditions. The testing of this hypothesis will be discussed in Chapter 4.
Figure 2.22: Stability map of combustor’s operating range using different FN injectors. Blue and red symbols represent the transition to 1st and 2nd unstable regimes respectively. O for FN 2.56, ◆ for FN 3.42 and ▽ for FN 5.13.

2.7 Characterization of Stability Regimes

The interesting observations made during the stability mapping, i.e., the presence of three distinct, GER dependent stability regimes were also found to be very unique. A survey of the available literature did not reveal any evidence of such a behavior ever been observed before. The task of characterizing each one of these stability regimes was therefore, taken up. Unless otherwise stated, the FN 3.42 atomizer was used in all the reported experiments.

The distinctive appearance of the flame in the three stability regimes was shown in Figure 2.21. Similarly, distinct behavior was noted in acoustic pressure profiles measured at P3 location. The power spectral densities of acoustic pressure in the three regimes is shown in Figure 2.23 for an experiment that was conducted with fixed fuel mass flow rate at 3.48 kg/h and varying GER by varying the air mass flow rate. As can be observed, the sound pressure level (SPL) at the fundamental frequency increases from 80 dB in the stable operation to
Figure 2.23: Power spectral densities of combustor acoustic pressure in the three stability regimes.

Figure 2.24: Time signals of combustor acoustic pressure in the three stability regimes.
161dB in the 1st unstable regime with resonance frequency shifting from 123 Hz to the limit cycle value of 118 Hz. The limit cycling is evident from the appearance of non-linear even harmonic peaks in the pressure profile. Higher acoustic modes also get excited as evident from the more prominent higher harmonic peaks. In the 2nd unstable regime the limit cycle frequency further shifts to a lower value of 100 Hz with a slight increase (to 167dB) in the sound amplitude. The even harmonic peaks are much more prominent indicating strong presence of non-linear processes. Corresponding time traces of the pressure signals in the three regimes are shown in Figure 2.24. Limit cycle oscillation can be noted from the sinusoidal behavior of the signals in the two unstable regimes.

The changes in limit cycle frequency and amplitude as GER increases from a lean limit towards stoichiometric are summarized in Figure 2.25. The corresponding acoustic velocities over the range of GER are shown in Figure 2.26. Also shown in Figure 2.26 are the calculated velocities of the spray droplets relative to the convective co-flowing gas velocities. The droplet velocities were experimentally determined using Phase Doppler Anemometry in a non-reacting setup (to be discussed in Chapter 3). The gas velocities were calculated using combustor dimensions and combustion air mass flow rates at the corresponding GER with fuel mass flow rate fixed at 3.48kg/h. As can be noted, at higher GER, the acoustic velocities in the combustor reach very high amplitudes in comparison to the droplet relative velocities. As it was discussed in the literature survey of Chapter 1, acoustic velocity oscillations affect spray combustion dynamics either via formation of periodic vortical structures or via oscillations in fuel spray vaporization. They may also prompt droplet secondary breakdown. The extend of their domination depends on their amplitude relative to the magnitude of other convective flows. The effects of high amplitude acoustic velocity oscillations are further discussed under modeling investigations in Chapter 4.

The temperature, NOx emissions and the mean heat release rate profiles are shown in Figures 2.27, 2.28 and 2.29 respectively. The temperature profiles were measured in the flame zone at T3 location and in the burner section, just upstream of the swirler at TS2 location. The flame zone temperatures shown in Figure 2.27 are the radiation corrected values, where
Figure 2.25: Changes in limit cycle frequencies and limit cycle amplitudes at different GER values.

Figure 2.26: Comparison between acoustic velocities and droplet relative velocities at different GER values.
Figure 2.27: Temperature profiles in the combustor and burner sections.

Figure 2.28: NOx emissions at different GER values.
Figure 2.29: **Mean heat release rate at different GER values.**

The correction was calculated using Eqn. 2.12 \[100\].

\[
\Delta T = \frac{1.25 D_b \varepsilon \sigma T_{tc}^4}{\lambda_g R_{ed}^{0.25}}
\]

where

\[
R_{ed} = \frac{\rho_g U_g D_b}{\mu_g}
\]

Here, \(\varepsilon\) is the emissivity of thermocouple wire, \(\sigma\) the Stefan-Boltzmann constant, \(D_b\) the thermocouple bead diameter, and \(\lambda_g, \rho_g\) and \(\mu_g\) the gas properties. Also, \(U_g\) is the gas velocity, while \(T_{tc}\) is the thermocouple measured temperature.

As shown, a sudden increase of about 200K in upstream temperature was observed when the combustor was operated in the 2\(^{nd}\) unstable regime, indicating that the flame elongation observed in this regime (Figure 2.21 (c)) was also happening in the upstream direction. The upstream movement of the flame front causes the acoustically closed boundary to move upstream, which in turn results in an increase of the effective length of the combustor. Thus, the lowering of the limit cycle frequency observed in the 2\(^{nd}\) unstable operational
mode. Also shown in Figure 2.27 is a distinct drop in the flame zone temperature (measured in the thermal boundary layer near the wall) for the 2\textsuperscript{nd} unstable regime. Corresponding drop in the NOx emission and the mean heat release rate were also observed as shown in Figure 2.28 and Figure 2.29. The drop in NOx levels during the transition of the combustor from 1\textsuperscript{st} to 2\textsuperscript{nd} unstable regime, was found proportional to the injector size. FN 5.13 atomizer showed a ppm decrease of 50\% while FN 3.42 and FN 2.56 injectors showed 29\% and 26\% decrease respectively. The observation on the decrease of mean heat release rate qualitatively varify the presence of distributed and strained reaction zone in the 2\textsuperscript{nd} unstable regime. It may be reminded that the heat release rate measurements are based on global \(OH^*\) chemiluminescence signal averaged over a 202mm x 76mm flame area. A drop seen in the heat release rate value in Figure 2.29 therefore, indicates the decrease in average heat release rate over the viewing area.

Figures 2.30 through 2.32 show the evolution of thermoacoustic instability in the combustor with increasing GER. Power spectral densities of acoustic pressures in the combustor

![Power spectral densities of acoustic pressures in the combustor](image)

\textbf{Figure 2.30: Evolution of instability in the combustor.}
Figure 2.31: Evolution of non-linear coupling in the flow conditioning section.

Figure 2.32: Evolution of acoustically driven heat release rate.
and flow conditioning sections and of heat release rate are shown in these figures. The pressure signals of the combustor and flow conditioning sections were measured at P3 and P2 locations respectively. As can be seen, that at a GER value of 0.4, although there is no preferred oscillations in the heat release rate (Figure 2.32), nevertheless the mean thermal energy in the combustor was high enough to excite the combustor at its quarter wave resonance frequency (Figure 2.30, some energy contents are also observed at the 1\textsuperscript{st} harmonic frequency). A coupled oscillation in the upstream pressure can also be observed in Figure 2.31. At this operating condition, the acoustic losses in the combustor prevent the combustor from going unstable. These acoustic losses, as highlighted by Clanet et al. [101], occur due to acoustic radiation at the open end, due to heat diffusion at the combustor walls and due to the presence of spray fuel in the combustor. As the GER value is raised to 0.45, the amplitude of acoustic oscillations in the combustor also increases and the first sign of acoustic coupling with heat release rate is observed (spike indicated in Figure 2.32) at a preferred frequency equal to the combustor resonance frequency. Any further increase in GER causes the acoustically driven heat release rate oscillations to get in phase with the combustor pressure oscillations. Thus, making the combustor thermoacoustically unstable and exhibiting even larger pressure oscillations. With reference to the combustor stability map (Figure 2.20), this GER limit signifies the boundary of 1\textsuperscript{st} unstable regime. Measurements taken well inside the 1\textsuperscript{st} and 2\textsuperscript{nd} unstable modes of operation (at GER of 0.5 and 0.7) show the sharp peaks in pressure and acoustically driven heat release rate spectra. The presence of distinct even harmonics indicate the presence of strong non-linear effects [43]. It may also be noted from Figure 2.31 that there also exists a strong non-linear coupling between the combustor and the flow conditioning sections, as evident from the presence of high amplitude even harmonics in the power spectrum of upstream pressure.

The power spectral densities of the acoustic pressures in the combustor and flow conditioning sections at a GER of 0.8 (combustor operating in the 2\textsuperscript{nd} unstable regime) are shown in Figure 2.33. Two interesting observations are made. First is the appearance of high frequency energy at 335 Hz in the combustor pressure profile. A corresponding but to a lesser degree
Figure 2.33: Power spectral densities of acoustic pressures in the combustor and flow conditioning sections while operating in 2\textsuperscript{nd} unstable regime.

Figure 2.34: Time traces of acoustic pressures in the combustor and flow conditioning sections while operating in 2\textsuperscript{nd} unstable regime.
effect is also seen in the flow conditioning section pressure (no related effect is observed in the heat release rate). The second observation is regarding the strength of the even harmonics in the upstream pressure profile. The sound pressure levels of these even harmonics are noted to be even higher than the fundamental values. The time signals corresponding to the above power spectra are shown in Figure 2.34. Two prominent periodic oscillations per combustor acoustic cycle can be noted in the upstream pressure signal. It can be concluded from the second observation that the strong acoustic presence in the combustor and its coupling with the upstream flow conditioning sections causes the upstream section to limit cycle itself at a frequency close to its own resonance frequency of 198 Hz.

2.8 Sensitivity of Limit Cycle Behavior

Sensitivity of the limit cycle to variations in GER, fuel feed line pressure and FN was also investigated. Figure 2.35 shows the effect of changes in GER on limit cycle amplitude and frequency. FN 3.42 atomizer was used in these experiments. The air mass flow rate was kept constant at 126.78 kg/h and thus the variations in GER resulted due to variations in fuel injection pressure (i.e., fuel mass flow rate). No specific trend in amplitude variations is noted, with the values within 4 dB. This is expected since the oscillations are at limit cycle. However, it is noted that with the increase in GER, the limit cycle frequency also increases. The reason for this increase is the reduction in time lag between fuel injection and burning, which happens because of two reasons: the rise in gas temperature due to increase in GER and reduction in mean droplet sizes because of the increase in injection pressure. Both effects cause increase in fuel evaporation rate and thus lower the time lag.

The effect of change in atomizers on limit cycle behavior is shown in Figure 2.36. FN 2.56, 3.42 and 5.13 injectors were used. The GER and the fuel mass flow rate were kept the same at 0.58 and 4 kg/h respectively. Once again, no specific trend in amplitude is observed with the variations within 4 dB. On the other hand, the limit cycle frequency shows an increase
Figure 2.35: Effects of changes in GER on limit cycle behavior.

Figure 2.36: Effects of changes in atomizer size on limit cycle behavior.
with the increase in atomizer FN. This is attributable to the lower spray velocities associated with higher FN injectors, when the flow rate is kept the same. This causes longer residence time of the droplets in the combustion zone, which enhances their evaporation rate and in turn results in lowering time lag between injection and burning.

2.9 Effects of Geometry Change

The effects of change in combustor geometry on the characteristics of the stability regimes were studied by reducing the length of the combustor by half. The experiments were performed with FN 3.42 atomizer. All other operating and testing conditions were kept same as for the original full-combustor tests. The power spectra of the acoustic pressure and the heat release rate under the three stability modes are shown in Figure 2.37 and Figure 2.38.

The resonance frequency of the combustor and the consequent limit cycle frequencies under unstable operation were found to double, as can be seen in the figures. This confirms the

Figure 2.37: Power spectral densities of combustor acoustic pressures in the stable and unstable regimes for half length of the combustor.
Figure 2.38: Power spectral densities of heat release rate in the stable and unstable regimes for half length of the combustor.

argument presented earlier about the combustor acting as close-open tube under reacting flow conditions.

2.10 Characterization under Primary Fuel Modulation

Characteristics of the combustor was also studied when active control was applied to attenuate the instability. Control was achieved via proportional modulation of the primary fuel. Methodology involved the use of a phase-shift controller to control the actuation system, which comprised a 500 micron piezoelectric stack and a throttle valve. The system has an excellent bandwidth and can handle 10 times more the flow rates required for the reported experiments. The combustor pressure signal was used as feedback signal for the control algorithm and the feed line between the actuator and the injector was tuned to extract maximum authority from the actuation system. A 125% fuel pressure modulation was achieved under certain conditions. For more details on the construction, capabilities and
Figure 2.39: Power spectral densities of combustor pressure with and without control application.

For the control methodology of the active control system, readers may refer Schiller [90].

Figure 2.40: Suppression of SPL over the GER range as a result of control application.
For the control experiments both acoustic pressure and heat release rate measurements were recorded. Pressure signal were obtained in the combustor rig at P2 and P3 locations and in the fuel feed line at P1 location. The base line atomizer (FN 3.42) was used in these experiments and the variations in GER was achieved by varying the fuel mass flow rate while keeping the air mass flow rate fixed at 126.8 kg/h.

Figure 2.39 shows the level of instability attenuation, achieved when the control was applied. The plot shows a suppression of 65 dB in the fundamental peak. For this test the combustor was operated at a GER of 0.56. Control experiments were conducted over the complete 1st unstable regime. Figure 2.40 shows the magnitude of suppression in sound pressure level achieved over the range of GER. Attenuation as large as 73 dB was obtained at certain conditions.

The intent of the active fuel modulation is to decouple the unsteady heat release process from the driving acoustic oscillations by periodically injecting fuel out of phase with the pressure oscillations. This is shown in terms of power spectral densities and time traces in

![Graph showing intent of active fuel modulation in terms of pressure spectra.](image-url)
Figure 2.42: Intent of active fuel modulation in terms of time signals.

Figure 2.41 and 2.42 respectively. Pressure signals from the combustor and the fuel supply line as well as heat release rate signals are shown. Phase shift in the modulated fuel was achieved by introducing a delay in the signal to the actuation system. The almost complete attenuation of the heat release rate coupling, via fuel injection at the fundamental frequency of oscillation, is evident.

During the control experiments it was observed that at lower GER instabilities, near the 1st unstable boundary, the combustor could be conveniently brought in and out of the instability by simply turning the controller off or on, without altering the gain and delay settings of the control signal. These gain and delay values were however found dependent on the operating GER values [102]. This is depicted in Figure 2.43, where the phase relationship between the combustor pressure oscillations and the oscillation in heat release rate and phase difference between injector pressure modulation and heat release rate oscillations are shown for a GER of 0.5. Trials 1, 3, 5 and 7 are with the controller on and Trials 2, 4 and 6 are with the controller off. Turning the controller on made the acoustic pressure and heat release rate oscillations out of phase by more than 140°. The combustor thus demonstrated a stable
Figure 2.43: Phase difference of heat release rate with combustor acoustic oscillations and injection fuel modulations at GER=0.50. Trials 1, 3, 5 and 7 with controller on and trials 2, 4 and 6 with controller off.

behavior in these trials. Whereas, when the controller was turned off, the phase difference between the oscillating process was under 90°, which was sufficient to cause the coupling and the combustor showed an unstable behavior in these trials. Also, it can be noted from Figure 2.43 that the phase difference between injector pressure modulation and heat release rate oscillations show the same two values (about 180° and 150°) for the two sets of trials.

The response of the combustor to active control at higher GER values was however, found to be quite different from what was observed at lower GER values. Besides higher actuator gains needed at higher GER to get control, it was also found that once a controlled combustor was made unstable by turning the controller off, it could not be brought back to the same stable operation by simply turning the controller back on. To stabilize the combustor, the delay counts to the actuation signal had to be first raised till a partial attenuation in sound amplitude was reached and then brought back to the original setting to regain the original stability level. Pressure and heat release measurements were made during this sequence
to determine the phase relationship of the heat release rate with the combustor acoustic oscillations (to be referred as combustor phase from here on) and the injector fuel pressure modulation (to be called injection phase from here on). The results are shown in Figure 2.44 for a GER of 0.57. Trial 1 represents the beginning of the sequence when the combustor was stabilized by the controller after appropriate values of gain and delays were selected. The selected delay values is seen to cause an injection phase of about $180^\circ$. The combustor stability is evident by the large combustor phase difference. Turning the controller off (Trial 2) makes the combustor unstable, apparent by the $90^\circ$ combustor phase value. When the controller is turned back on (Trial 3) the combustor still shows an unstable behavior since the injection phase difference, as can be observed, is lower than the required value (of about

Figure 2.44: Phase difference of heat release rate with combustor acoustic oscillations and injection fuel modulations at GER=0.57. Trial 1: Stable combustor with control on, Trial 2: Unstable combustor with control off, Trial 3: Unstable combustor with control on, Trial 4: Marginally stable combustor with control on and higher delay setting and Trial 5: Stable combustor with control on and original delay setting.
180° at this GER. The delay counts therefore, are required to be raised (Trial 4) to bring the injector phase closer to the required value. This makes the combustor marginally stable, as can be noted by the increase in combustor phase. The controller delay can then be adjusted back to original setting (Trial 5) to regain the required injector phase difference and achieve the original level of combustor stability. This behavior of the control sequence, as explained by Hines [102], occurs because of the saturation of the actuation system with higher gains required at higher GER. It can also be concluded from the above discussed observations that at higher GER, the required delay to the actuation signal is dependent upon the degree of instability in the combustor. Combustor under high acoustic amplitudes require a higher delay to be induced in the control signal and vice versa.

The frequency response function (FRF) between the combustor acoustic pressure and heat release rate was determined from measurements recorded during system identification experiments conducted by the VACCG researchers on the combustor. The identification was

![Frequency response magnitude and phase between the heat release rate and combustor pressure oscillations.](image)

**Figure 2.45:** Frequency response magnitude and phase between the heat release rate and combustor pressure oscillations.
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Figure 2.46: Pole-Zero plot for the curve fit of Figure 2.45.

conducted via open-loop forcing of a stabilized combustor at GER of 0.44 and fuel mass flow rate of 4kg/h. The fuel injection was modulated by keeping the gain to the actuation signal fixed (11% feed pressure modulation) and making sine-sweep around the resonance frequency of the combustor. The results are shown in Figure 2.45. Also shown in the figure is a second order curve fit with 2-poles and 2-zeros, which shows a reasonable agreement with the experimental data. The corresponding poles and zeros of the transfer function are shown in Figure 2.46. The plot shows the presence of two marginally stable poles and the presence of zeros indicate a time lag in the transfer function.

2.11 Conclusions from Combustor Characterization Studies

The main conclusions drawn from the experimental investigations conducted to characterize the combustor are enumerated below:
1. Presence of flame decouples the combustor section from rest of the rig and provides an acoustically closed boundary for the acoustic wave. The fundamental standing wave mode in the combustor is therefore, a one-quarter wave mode, with pressure anti-node at the combustor-burner interface and pressure node at the combustor exit.

2. Mechanical swirling with $45^\circ$ vane angle is found to be more effective fuel-air mixing method. The actual swirl numbers obtained at the quarl inlet and inlet to the backward step are 0.253 and 0.125 respectively.

3. The maximum flow generated turbulence levels in the combustor under the tested air flow rates are 40% and 50% relative to the mean flow rates, at the inlets to the quarl and backward step respectively.

4. Depending on the GER and the relative level of acoustic dominance, the combustor demonstrates three very distinct and unique stability and operational regimes.

5. The regions of unstable operation under the 1$^{st}$ and the 2$^{nd}$ unstable modes depend on the size capacity of the fuel injectors. In other words, the range of GER under any unstable operation is dependent on the mean droplet size. Smaller mean droplets delay the transition to 2$^{nd}$ unstable regimes, while larger mean droplets cause a shrinkage in 1$^{st}$ unstable regime.

6. About 80dB rise in SPL is observed when the combustor is unstable. The maximum pressure oscillations observed at any GER are 4.5% relative to the mean pressure in the combustor.

7. The combustor acoustics is the driving mechanism of the thermoacoustic instability in the combustor. It drives the heat release rate oscillations once the acoustic energy losses are overcome by the energy gains due to increasing GER.

8. Under transition to unstable conditions, the limit cycle frequencies are found to shift to lower values compared to the resonance frequency of the combustor.
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9. Limit cycle frequency is sensitive to changes in GER, fuel feed line pressure and atomizer size. The sensitivity is proportional to the increase in these parameters and manifest because of the associated decrease in time lag between the injection and heat release rate.

10. Magnitude of acoustic velocities in the combustor is comparable to the relative convective flow velocities in the 1st unstable regime and are four times higher in the 2nd unstable regime.

11. A probable cause of transition to 2nd unstable regime is secondary breakup of droplets, which may happen because of the presence of high acoustic forces relative to the drag forces due to convective flow.

12. The flame elongation observed in the 2nd unstable regime occurs both in upstream and downstream directions relative to the plane of backward step.

13. A sudden decrease in NOx emissions occurs when the combustor transitions from the 1st to 2nd unstable mode. This decrease is proportional to the size of the injector used. As high as 50% decrease is achieved with FN 5.13 atomizer.

14. A sudden decrease in mean heat release rate also occurs during the transition, which qualitatively indicates the presence of distributed and strained reaction zone in the 2nd unstable regime.

15. Strong non-linear acoustic coupling is found between the combustor and the upstream flow conditioning section.

16. At higher GER, when the acoustic dominance is also high relative to convective flows, the upstream flow conditioning section also exhibit limit cycle oscillations at a frequency close to the resonance frequency of the section.

17. Application of active control via proportional primary fuel modulation causes a suppression of as high as 73 dB SPL in the fundamental peak of instability.
18. At lower GER, combustor can be brought in and out of the instability by simply turning the controller off or on at a particular value of gain and delay to the actuation signal.

19. At higher GER, the on-off control is ineffective and the delay counts to the actuation signal are required to be adjusted to regain stability.

20. An effective phase lag value of $180^\circ$ between the injector pressure modulation and heat release rate oscillations is required to control the combustor.

21. Frequency response function between the heat release rate and combustor pressure oscillations under open-loop excitation of the combustor indicates the presence of two marginally stable poles and two zeros indicative of a time lag in the transfer function.
Chapter 3

Investigation of Spray-Acoustic Interaction

The influence of acoustic oscillations on the dynamics of spray combustion were discussed in Chapter 1, while the experimental observations made on the liquid-fueled combustor regarding the degree of acoustic influence were presented in Chapter 2. The influence manifests itself not only on vaporization and combustion of spray droplets but also affect the size and velocity distribution of the droplets in the spray. To quantify the effects of acoustic excitation on droplet size and velocity dynamics, non-reacting flow experiments were conducted using Phase Doppler Anemometry and are discussed in this chapter.

3.1 Phase Doppler Anemometry

Phase Doppler Anemometry (PDA) is the extension of the widely used non-intrusive particle velocity measurement technique known as Laser Doppler Anemometry (LDA). The method falls into non-imaging, single particle counter category [103]. It uses light scattering interferometry and simultaneously measures size and velocity of spherical droplets. The method can also be used to provide volume flux, trajectory drop angle and time-resolved diagnostics.
of sprays. The measurements are dependent upon the wavelength of the scattered light from the droplets. The method was originally developed by Bachalo [104] and is now considered a benchmark technique for drop sizing and velocity measurement needs for most two-phase flow research. Drop sizing measurements are based on the finding that relative phase shift of the light scattered by the mechanism of reflection or refraction is proportional to the drop size. While, the velocity measurements are based on fringe anemometry [16], where the drop velocity is proportional to the Doppler frequency.

Figure 3.1: A typical Phase Doppler Anemometry setup.

A typical PDA setup is shown in Figure 3.1. A splitter is used to split the beam from a single laser source into two coherent laser beams. The beams are then made to intersect, thus forming an interference pattern in the volume of intersection known as ‘probe volume’. If the beams are made to cross at the cross section where the respective beam diameters are minimum, then the interference planes or fringes formed will be nearly parallel. Normally the probe volume is aligned in such a way that the fringes are oriented perpendicular to the spray centerline axis. With such an arrangement, the intensity of light scattered from a droplet moving through the probe volume will vary with a frequency (Doppler frequency),
which is proportional to the velocity component in the direction of the axis. The light intensity is captured by the use of a photo-multiplier tube (PMT) detector and converted into a current pulse, which contains the frequency information relating to the velocity to be measured \[105\]. However, the detector have no way to distinguish between the negative and positive frequencies corresponding to negative and positive droplet velocities. This directional ambiguity is eliminated in the laser anemometry measurements by the use of a Bragg cell, placed in the path of one of the laser beams. The Bragg cell adds a fixed known frequency shift to the beam and till the time the droplet velocity does not introduce a negative frequency larger than this shift, the detector is able to measure the directionally correct velocity.

Velocity measurements in laser anemometry are achieved by the use of only one PMT, however, for sizing measurement pair or pairs of detectors are needed. When placed at appropriate locations, separations and scattering angles, the detectors in the pair pick up the light intensity from the same droplet at different phases. The phase difference is then linearly related to the droplet size. Normally three PMT detectors are used. Two phase values from two detectors combinations are used to determine and validate the droplet diameter. The use of two phase difference values also eliminate the ‘\(2\pi\) ambiguity’ arising from the arrival of unusually large droplets in the probe volume. Since ideally the sum of all three phase differences from the three detectors pairs should cancel out for a spherical droplet, the third phase difference value is used to validate the sphericity of the droplets passing through the probe volume.

Placement of the receiving optics along with the PMTs, as discussed, is very crucial for size measurements in PDA. Basically three different optical configurations, based on dominant light scattering modes, can be used: forward refraction (1\(^{st}\) order), reflection and backward refraction (2\(^{nd}\) order). Off axis forward refraction pose significant advantages as suggested by Drain \[106\]: Measurements are relatively unaffected by random beam attenuation; the response is linear over the entire working range; and the instrument potentially has a large size measurement range. In forward refraction, two type of configurations are typically
used with scattering angle of 70° or 30°. Because of its proximity to Brewster angle, the 70° configuration has the least influence of reflected light [106] and is therefore, 1st order refraction dominant. For the 30° scattering the forward refraction mode is not as dominant, nevertheless the intensity from 1st order refraction is almost twenty times larger than that from reflection. The advantage with 30° compared to 70° scattering configuration is that the absolute scattering intensity is much larger, which results in high data rate [107].

The size and velocity data captured by the optics is recorded by the analyzer in the form of number count per class size. An analysis code, using the relationship between Doppler frequency and velocity and some standard drop size distribution function, then converts this raw drop count into meaningful velocity and drop size distributions respectively.

### 3.2 Experimental Setup

To study the influence of one-dimensional acoustic field on spray behavior, both static time-averaged and dynamic phase-locked measurements were conducted. An isothermal acoustic rig was specifically constructed from Poly-Vinyl Chloride (PVC) material. Water was used as the working fluid for these non-reacting experiments. The photograph of the rig and its schematic diagram are shown in Figure 3.2 and Figure 3.3 respectively. The rig consists of a flow conditioning section and an acoustic chamber (test section). The top part of the flow conditioning section (dome assembly) is made up of two concentric cylinders. Air is let inside the flow conditioning section through four 12.7mm tubes placed at 90° around the circumference of the outer cylinder of the dome assembly. The air is first directed towards the back wall and then through the inner cylinder, where it passes through a couple of honeycomb screens. Finally, the air is passed through a 45° mechanical swirler, similar to one used on the combustor rig, before being introduced in the test section. Water is fed to a FN 3.42 pressure-swirl simplex atomizer (once again, similar to the one used in reacting flow experiments) through a 6.35mm tube housed inside a lance assembly. A piston pump
from Hydra-Cell driven at 1750rpm was used to supply water to the rig.

The experimental setup for the PDA measurements is shown in Figure 3.4. A one-dimensional Phase Dynamic Analyzer from DANTEC Measurement Technology was used to measure the spray droplet size and axial velocity distributions. The system comprised of a 750 mW ($\lambda$ = 514.5 nm) Argon Ion laser, 57x40 FiberPDA receiving probe, 58n70 FiberPDA detector unit and a 58n10 PDA signal processor, together with transmitting and receiving optics. Three PMTs were used to capture the scattered light intensity. The focal length of the transmitting and the receiving probe lenses was 310mm with an initial beam separation of 27.44mm. The measurement configuration was first-order refraction with a scattering angle of $30^\circ$. Given this arrangement, the fringe spacing was 5.83 micron, with probe volume
Figure 3.3: Schematic of the isothermal acoustic rig (all dimensions in mm).
dimensions of approximately 0.023 mm$^3$. A refractive index of $n = 1.334$ was taken for the water spray droplets. Sampling at a point was performed for 10,000 particle counts at a rate of 4000 Hz. The validation rate for all experiments was at least 95%. The location of the PDA probe volume was fixed in space while the spray was traversed relative to the fixed reference location so that measurements can be obtained over several points in the spray cone.

For the frequency response analysis, swept-sine measurements for 80-500 Hz were performed. Excitation below 80 Hz was considered unreliable because of the limitation of the acoustic driver. A digital signal analyzer from Hewlett Packard was used to generate the source signal and also to record the pressure signals. The button-type differential dynamic pressure sensors from Honeywell/Sensym ICT were used for the pressure measurements. Dynamic phase-
locked measurements were ensured via the encoder function of the PDA signal processor [105]. The encoder and the acoustic driver were triggered simultaneously from the same source signal.

### 3.3 Data Processing Methodology for Dynamic Measurements

The primary outcome of any laser anemometry measurement is the production of a photo current pulse from the PMT detector. The pulse, known as Doppler burst, is produced each time a droplet passes through the probe volume. The amplitude of the burst depends upon the size, velocity and trajectory of the droplet. These burst signals are then used to compute velocity and size information. Usually a burst detector with 3-level detection scheme is used to control the gate signal to the processor, processing the emission signals. The 3-level scheme help reduce the sensitivity to unwanted noise in the signals. Only bursts with a preset amplitude levels are accepted and thus not all detected bursts are used for processing the required information.

Because of the reasons that the arrival of droplets in the probe volume is random and that not all detected bursts are processed, the data sampling in any laser anemometry is randomly uneven. This therefore, creates a problem in the dynamics analysis of the data, such as spectral analysis or frequency response analysis. Many techniques have been investigated and applied to process the unevenly sampled data for the purpose of acquiring dynamic information. The most common of these methods are: auto regression analysis, Lomb-Scargle periodogram and interpolation.

Auto regression techniques have been successfully applied in resolving both PDA and LDA data for spectral analysis [108]. They however, assume that the raw data has been sampled with a Poisson distribution, which may not always be the case.
The periodogram method, which was originally developed by Lomb [109] and later modified by Scargle [110] is a powerful method for the spectral analysis of both evenly and unevenly sampled times series. The limitation of the method is that the resulting spectrum is very inconsistent and not a good estimator of the true spectrum. The method therefore, requires a pre-conditioning technique like Welch-Overlapped-Segment-Averaging (WOSA) to be applied on the raw time data [111].

Interpolation methods are the most widely used methods due to the simplicity of their application. In this method the unevenly sampled data is first curve fitted (either linearly or with higher orders) and then resampled. Choice of a pertinent resampling frequency is important to avoid significant biases in the processed data. The method has however, been found to underestimate high frequency components in the spectrum.

In the present effort, a completely different approach was adopted for the dynamic analysis of the raw unevenly sampled PDA data. The processed data was also later validated using interpolation technique with appropriate resampling frequency. The applied strategy uses Discrete Fourier Transform (DFT), which however, is an evenly sampled based technique and additionally requires that the data be periodic. The procedure followed to resolve the original PDA measurements into DFT favored evenly sampled data is shown in Figure 3.5.

Using the encoder function of the Particle Dynamic Analyzer, the measurements (velocity and diameter) were phased-locked with the start of the acoustic excitation cycle. With this setup, the PDA processor sends out a reset pulse at the start of each cycle and thus a corresponding ‘zero-crossing’ time stamp was obtained on the raw data (Figure 3.5 (a)). The time stamp was used to rearrange the data in order to determine the arrival time of the bursts within each cycle (Figure 3.5 (b)). Next, the cycle time period was split into N equal time bins corresponding to every $360^\circ/N$ phase in the cycle. The final step was to time average the raw data for each time bin over all the cycles as shown in Figure 3.5 (c). As can be noted the final form of the data is not only evenly spaced but is also periodic. Thus, the two requirements to conduct a DFT for the spectral analysis were met. However,
Figure 3.5: Procedural steps in the transformation of unevenly sampled PDA time series measurements into evenly spaced data and the application of DFT.
the resolved evenly spaced data was over just one period of oscillation and therefore, was
not sufficient to obtain the pertinent resolution in the frequency domain. This limitation
was resolved by first determining the Fourier coefficients from the one period of data using
Equ. 3.1 through Equ. 3.3, and then resampling using Equ. 3.4 over one second of time
(Figure 3.5 (d)). A 1Hz frequency resolution was thus obtained for the spectral analysis.

\[
a_o = \frac{1}{N} \sum_{n=1}^{N} X(n)
\]  
\[
a_m = \frac{2}{N} \sum_{n=1}^{N} X(n) \cos(m\omega n\Delta t)
\]  
\[
b_m = \frac{2}{N} \sum_{n=1}^{N} X(n) \sin(m\omega n\Delta t)
\]

where

\[N = \frac{T}{\Delta t}\]

and

\[X(t) = a_o + \sum_{m=1}^{M} a_m \cos(m\omega t) + \sum_{m=1}^{M} b_m \sin(m\omega t)\]

In the above equations, \(a_o\), \(a_m\), \(b_m\) are the Fourier coefficients; \(N\) is the number of evenly spaced samples; \(T\) is the time period of oscillation; \(\Delta t\) is the time interval between two evenly spaced samples; \(\omega = 2\pi f\) with \(f\) being the frequency of oscillation; \(n\) is the sample index; \(m\) is the frequency index; and \(M\) is the maximum frequency index used to perform the Fourier approximation and resampling. It was found during the data processing that for best fits, \(M = N - 2\) and that the resampling frequency should be more than double the frequency with which the raw data (unevenly sampled) was collected.

The final step in the spectral analysis was to perform DFT on the resampled, evenly spaced and periodic data (Figure 3.5 (e)).

The validation of the results obtained via the above discussed procedure was conducted by comparing with results obtained by cubic interpolation. A very fine grid was used for the interpolation curve fit and the resampling frequency chosen was in the same order as that used for resampling in DFT method.
3.4 Characterization of the Rig

Acoustic characterization of the iso-thermal rig was first conducted to determine its resonance frequency. This was accomplished by exciting the speaker with white noise under no-flow conditions and measuring acoustic pressure 25mm downstream of the spray injection plane. The acoustic pressure instrumentation was the same as described in Chapter 2. Figure 3.6 shows the power spectral densities at three excitation amplitudes. The resonance frequency of the rig was found to be around 435 Hz.

Since a piston pump was used for the PDA experiments, pressure fluctuations were expected in the water feed line. Calibration of the feed line was conducted to quantify the percentage fluctuations due to the pump. Calibration was conducted at different mean supply pressures and the results are shown in Figure 3.7. High amplitude fluctuations at 30 Hz (corresponding to 1750 rpm) and its harmonics can be observed. To ensure that the spray characteristics measured by the PDA are independent of supply line effects, it was considered essential
CHAPTER 3. INVESTIGATION OF SPRAY-AcouSTIC INTERACTION

Figure 3.7: Percentage of fluctuations in the water supply line in the original setup.

Figure 3.8: Percentage of fluctuations in the water supply line with the addition of muffler assembly.
to dampen the pump generated oscillations in the supply line. Thus, a muffler assembly was added in the supply line just upstream of the injector. Remarkable suppressions in the amplitude of pulsations were obtained as shown in Figure 3.8.

3.5 Spray Characterization under Static Conditions

Static characterization of the atomizer was conducted to understand the structure and axial velocity/size distribution of the spray. The measurements also served as a reference for comparison with the dynamic measurements (to be discussed later in the chapter). FN 3.42 atomizer was used in almost all the reported experiments.

Figure 3.9 show the radial distribution of droplet velocities and diameters at an axial location of 25 mm downstream of the injector. For these measurements the water injection pressure was kept at 1.03 MPa. Here and elsewhere, unless otherwise stated, all mean diameters are expressed in terms of Sauter Mean Diameter (SMD), which is defined as,

\[
SMD = D_{32} = \frac{\sum_{i=1}^{N} D_i^3 n_i}{\sum_{i=1}^{N} D_i^2 n_i}
\]

where \(n_i\) are the number of droplets with diameter \(D_i\) and \(N\) is the total number of droplets.

Two distinctive regions can be identified from Figure 3.9. The inner core with smaller droplets and higher velocities and an outer core where the SMD monotonically increases to its maximum value. In the outer shell the droplet velocities decrease as SMD increases. The decrease in velocity at the spray outer boundary is 43% from its centerline value of 6.44 m/s, while the SMD of the droplets at this outmost location is 173% higher than 9.14 micron value that is noted at the centerline. This spray characteristics is typical of a pressure swirl atomizer where due to centrifugal action of the swirl, the larger droplets are thrown towards the periphery of the spray. Also, due to their larger inertia, the larger droplets have lower velocities and vice versa. From the experiments the effective spray cone angle was determined to be 65° at the tested injection pressure.
Figure 3.9: Radial distribution of droplet axial velocity and SMD at an axial location of 25mm downstream of the injector and injection pressure of 1.03MPa.

Figure 3.10: Diameter distribution at fixed axial location of 25mm and four radial location, (a) centerline, (b) 6mm radial, (c) 12mm radial and (d) 16mm radial.
Figure 3.10 shows the corresponding histograms of diameter at various radial locations. In the inner core, from centerline to 6mm radial location (Figure 3.10 (a) and (b)), the size distribution and the average values are almost the same. The distribution is skewed towards the lower diameter values. At the inner boundary of the outer shell, 12mm radial location (Figure 3.10 (c)), the droplet diameters are more normally distributed with a two-fold increase in the average value. At the periphery of the outer core, 16mm radial location (Figure 3.10 (d)), the distribution favors the higher values and there is a further increase in the mean diameter.

In spray combustion, SMD (also known as $D_{32}$) is most commonly used as the representative mean diameter of the spray because it signifies the volume-to-surface ratio, which is an important parameter in determining heat and mass transfer. Other essential indicators of spray quality are the maximum and minimum drop diameters, which are normally defined on the basis of volume fraction percentiles as $D_{0.9}$ and $D_{0.1}$ respectively. $D_{0.9}$ finds its application in the design of combustors as it determines the completeness of the combustion process within

![Figure 3.11: Comparison between the radial distributions of various representative diameters.](image)
the available combustor length. On the other hand, since the smaller droplets are more easily influenced by the convective gas medium, $D_{0.1}$ is an important indicator in determining the location of burning in the combustor. A comparison of different experimentally determined diameters is therefore, shown in Figure 3.11. For comparison purpose, also shown in the figure is the arithmetic mean diameter (average diameter).

Radial variations in the spray characteristics were also measured at two other axial locations: 30mm and 50mm downstream of the injector and the results are shown in Figure 3.12. The injection pressure was kept the same as in the previous experiments. Deeper penetration for larger droplets is observed. Also, at the periphery of the spray, where the larger droplets exist, the axial velocities are noted to be almost zero. This indicates that besides the inertial reasons for lower velocities for larger droplets, another reason why the axial velocity components show a decline especially at the spray boundary, is the dominance of swirling velocity component associated with larger droplets.

![Figure 3.12: Radial profiles of droplet axial velocity and diameter at three axial locations.](image)
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Figure 3.13: Effects of variation in injection pressure on droplet axial velocity and diameter measured at three radial locations.

Experiments were also conducted with the axial measurement location fixed at 25mm and varying injection pressures. Various capacity injectors were used and results for FN 2.56 injector at three radial locations: centerline, 10mm and 15mm, are shown in Figure 3.13. The injection pressures were varied from 0.69 MPa to 2.07 MPa. As expected from a pressure swirl atomizer, the SMD shows a decrease with the increase in injection pressure. However, it is observed that the decrease in size levels-off after a certain pressure, i.e., no further decrease occurs. This behavior is more pronounced at centerline as compared to the outward locations. Also, it is noted that at higher injection pressures, the radial variation in SMD decreases, showing a more uniform (with respect to diameter) spray at higher pressures.

The velocity profiles in Figure 3.13 show that the droplet axial velocities initially increase with the increasing pressures, but at higher injection pressures show an opposite trend. The pressures at which the axial velocities show a decline depends upon the radial location in the spray. At the centerline the decrease occurs at a higher injection pressure as compared
Table 3.1: Experimental conditions for dynamic PDA measurements.
(*All cases except Case 9 were performed under quiescent environment.
Co-air flow of 10.25kg/h was used in Case 9).

<table>
<thead>
<tr>
<th>Case</th>
<th>Feed line Pressure (MPa)</th>
<th>Acoustic Amplitude (Watt)</th>
<th>Measurement location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.03</td>
<td>40</td>
<td>25 axial, 0 radial</td>
</tr>
<tr>
<td>2</td>
<td>1.03</td>
<td>30</td>
<td>25 axial, 0 radial</td>
</tr>
<tr>
<td>3</td>
<td>1.03</td>
<td>20</td>
<td>25 axial, 0 radial</td>
</tr>
<tr>
<td>4</td>
<td>1.55</td>
<td>40</td>
<td>25 axial, 0 radial</td>
</tr>
<tr>
<td>5</td>
<td>2.07</td>
<td>40</td>
<td>25 axial, 0 radial</td>
</tr>
<tr>
<td>6</td>
<td>1.03</td>
<td>40</td>
<td>25 axial, 10 radial</td>
</tr>
<tr>
<td>7</td>
<td>1.03</td>
<td>40</td>
<td>40 axial, 0 radial</td>
</tr>
<tr>
<td>8</td>
<td>1.03</td>
<td>40</td>
<td>40 axial, 16 radial</td>
</tr>
<tr>
<td>9*</td>
<td>1.03</td>
<td>40</td>
<td>25 axial, 0 radial</td>
</tr>
</tbody>
</table>

3.6 Spray Response to Acoustic Forcing

Dynamic phase-locked measurements from 80-500 Hz were conducted to determine the spray response to acoustic forcing. The experiments were conducted over a range of feed line pressures and flow rates, both in quiescent environment and with co-flowing swirled air. Measurements were made at four locations in the spray cone and were related to the variations in mean and unsteady inputs. The test matrix for dynamic phase-locked experiments is given in Table 3.1. Case 1 was selected as the base line case. Only Case 9 was performed with a
co-flow air of 10.25 kg/h.

The spray responses are presented both in terms of the mean (DC offset) as well as the oscillating (AC) components as illustrated in Figure 3.14. SMD is used to represent the mean diameter.

3.6.1 Response to Acoustic Amplitude Variations

The effects of acoustic forcing on droplet average axial velocities ($U$) and SMD ($D_{32}$) are shown in Figure 3.15. The water feed pressure was kept the same at 1.03 MPa in each case, as was the measurement location, which was 25mm from the plane of injection on the spray centerline. The three forcing cases 1 through 3 are compared in the figure with the no-excitation case. It is seen that under acoustic forcing, $U$ values decrease and $D_{32}$ values increase as compared to the no-excitation case. It can be noted that the influence of acoustic forcing is larger at lower frequencies, a trend which was found consistent in these and all other
Figure 3.15: Effects of variations in acoustic forcing amplitude on droplet $U$ and $D_{32}$.

Figure 3.16: Percent change in $U$ and $D_{32}$ with variation in acoustic forcing amplitude.
measurements. As the frequency of excitation increases, the variation in mean values level off to the no-excitation value. Similar trends are seen in the other two forcing amplitude cases. However, as the acoustic forcing amplitude decreases, so does its influence. The changes in mean spray parameters in response to the variations in acoustic forcing amplitude (at lower excitation frequencies) is shown in Figure 3.16. At the maximum forcing amplitude of 40 watts, the maximum decrease in $U$ is 46% and the maximum increase in $D_{32}$ is 38% from their respective no-excitation case values. The variations in the mean values are due to the variations in size distribution in the presence of an acoustic field. The smaller droplets, due to their smaller inertia, more closely follow the acoustic field oscillation and move away from the measurement location, thus shifting the size distribution towards a higher value. As the $D_{32}$ value in the measurement volume increases, a corresponding decrease in $U$ is seen, owing to the inertia-imposed tardiness of larger droplets.

### 3.6.2 Response to Feed Line Pressure Variations

Influence of injection pressure and thus the liquid flow rate on the acoustic-spray coupling is shown in Figure 3.17. Here Case 1 is compared with Case 4 and 5. The acoustic amplitude and the measurement location were kept the same at 40 watts and 25 mm on centerline respectively. As before both the excitation and no-excitation values of $U$ and $D_{32}$ are shown. For the no-excitation case, as the flow rate increases, $U$ also increases and the value of $D_{32}$ decreases. The $U$ values under excitation show larger variations with the increase in injection pressure, whereas the variations in $D_{32}$ show a decline. This is summarized in Figure 3.18 for three lower excitation frequencies (where the influence was maximum). The maximum variations in $U$ are 48% and 53% for 1.55 and 2.07 MPa cases respectively, as compared to 46% of the baseline case. On the contrary, the maximum variations in $D_{32}$ fall to 21% and 0.1% as the pressure increases from 1.03 MPa to 1.55 and 2.07 MPa respectively. The reason for this trend is attributable to narrower size distributions and lower $D_{32}$ values because of better atomization obtained at the elevated injection pressures for the simplex type
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atomizers.

Figure 3.17: Effects of variations in injection pressure on droplet $U$ and $D_{32}$.

Figure 3.18: Percent change in $U$ and $D_{32}$ with variation in injection pressure.
3.6.3 Response at Different Locations in the Spray Cone

In order to study spray response to acoustic excitation in different regions of the spray, measurements were made at four locations, corresponding to Cases 1, 6, 7 and 8 of Table 3.1. The acoustic forcing amplitude and feed line pressure were held constant at 40 watts and 1.03MPa respectively. The results are shown in Figures 3.19 and 3.20. It is observed that under non-oscillating flow conditions \( U \) shows higher values and \( D_{32} \) shows lower values on the spray centerline as compared to radially outward locations. This is because of the drop size distribution in pressure-swirl atomizer, whereby large number density of larger sized droplets are found in the radially outward location of the spray as compared to the spray axis. For the acoustic forcing cases it is observed that the \( U \) variations are more pronounced at the centerline locations as compared to the outward locations. The velocities fall below the no-excitation case and the maximum \( U \) variations are 46% at 25mm centerline location and 21% at 25mm x 10mm radially outward location. Similarly, at 40mm centerline location

![Figure 3.19: Spatial variations in droplet \( U \) and \( D_{32} \). Axial measurement location 25mm downstream of injector.](image)
Figure 3.20: **Spatial variations in droplet $U$ and $D_{32}$.** Axial measurement location 40mm downstream of injector.

and 40mm x 16mm outward location the variation in $U$ are 50% and 15% respectively.

The trend in $D_{32}$ centerline values, as shown in Figures 3.19 and 3.20, is found to be the same as seen in the previous experiments, i.e., the values under acoustic excitation increase from the no excitation values. The maximum variations are 38% at 25 mm and 24% at 40 mm locations. However, the $D_{32}$ values at off-centered locations show an opposite trend, where values for the excitation cases either fall below values measured under the no excitation cases or show no change from the un-excited values. The reasons for the spatial variations reported above are as follows: As mentioned earlier, for the pressure-swirl atomizers, the no-excitation droplet size distribution is such that larger numbers of small sized droplets are found at the centerline as compared to outward locations where a larger number of large sized droplets are present. As the flow field is acoustically excited, the smaller droplets respond more easily to the longitudinal acoustic velocity because their smaller inertia allows the decaying jet velocity vector to become insignificant relative to the acoustic velocity vector. This results in a migration of these smaller droplets radially outwards. As a result, the slower moving
large droplets are left behind at the centerline locations and thus an increase in $D_{32}$ and a
decrease in $U$ is observed. On the other hand, the off-center locations now have a larger
proportion of smaller droplets and thus the $D_{32}$ value decreases. However, the $U$ values do
not show a corresponding increase because of the radial movement of droplets, which do not
contribute to the axial velocity component measured by the PDA. The modeling results to
be discussed in Chapter 4, validate these reasonings.

![Figure 3.21: Effects of swirl co-air flow on $U$ and $D_{32}$. Case 1 is compared with Case 9.](image)

3.6.4 Response to Swirling Co-Airflow

Effects of swirling co-airflow on spray dynamics under acoustic field excitation were investi-
gated next. The results at low frequency acoustic oscillations (including at 70 Hz) are shown
in Figure 3.21, where Case 1 is compared with Case 9. A 10.22 kg/h co-flow air was intro-
duced in the rig. Measurements were made at the 25 mm downstream centerline location
with 40 watts acoustic excitation amplitude and 1.03 MPa injection pressure. It is seen that
swirling air increases the $D_{32}$ values with corresponding decrease in $U$ values. However, the frequency effects on spray dynamics is found attenuated by the presence of the swirl air.

### 3.6.5 Response of Oscillating Diameter and Velocity Components

The experimental results discussed so far were based on mean values of droplet velocities and diameters. In order to understand the spray dynamics under the influence of acoustic excitation, it is also necessary to study the frequency response of the oscillating (AC) components of velocity, $\dot{U}$ and diameter, $\dot{D}$. Figure 3.22 shows these values for Case 1. Oscillating components of both velocity and diameter show a maximum response around 80 Hz. The $\dot{U}$ value shows another peak at 400 Hz, which is around the resonance frequency of the chamber. The maximum value of $\dot{U}$ is 0.85 m/s, which translates to 25% modulation from the mean value ($U$) at the same frequency. As for $\dot{D}$ the maximum variation is 7.5% from the

![Graph showing frequency response of $\dot{U}$ and $\dot{D}$](image)

**Figure 3.22:** Frequency response of $\dot{U}$ and $\dot{D}$ for 40 watts acoustic excitation at 1.03 MPa injection pressure.
mean value at the same frequency of 80 Hz. This data processed via the DFT method was verified by comparing with data processed by cubic interpolation. The comparison is shown in Figure 3.23. Quite a good agreement exists between the two data sets for the velocity measurements. However, for the diameter data, although the same trends are depicted, the values are off especially at the lower end of the frequency bandwidth.

Using the \( \dot{U} \) values of Figure 3.22 the droplet peak-to-peak displacement were also calculated over the tested frequencies. This is shown in Figure 3.24. A maximum oscillatory excursion of 3.4mm is seen. This displacement is quite significant keeping in mind the acoustic velocity of only 0.125 m/s achieved at this excitation setting. Compared to this, the acoustic velocities in the combustor are in the order of 25 to 40 m/s in the unstable regimes and thus their effect on the droplet oscillatory excursions is expected to be substantial.

Frequency response of droplet \( \dot{U} \) and \( \dot{D} \) to changes in input variables were also measured. A comparison of responses at the three acoustic forcing amplitudes (corresponding to Cases
Figure 3.24: Droplet peak-to-peak excursion due to the oscillating acoustic flow field.

Figure 3.25: Comparison between the frequency response at different acoustic forcing amplitudes.
Figure 3.26: **Comparison between the frequency response at different injection pressures.**

1, 2 and 3) is shown in Figure 3.25. The response is seen to be proportional to the excitation amplitude. Also, all three profiles show similar trend. Frequency response at varying injection pressures is compared in Figure 3.26 (Cases 1, 7 and 8). With increase in injection pressure, the magnitude of both $\hat{U}$ and $\hat{D}$ decreases. This behavior is due to higher mean droplet velocities (associated with higher injection pressures) relative to the acoustic velocities. Also for the two higher pressures (1.55 and 2.07 MPa), the diameter response profiles are almost identical.

Profiles of droplet peak-to-peak displacement in response to the changes in acoustic amplitude and injection pressure are shown in Figure 3.27. Here, Cases 2 and 3 and Cases 7 and 8 are separately compared with the baseline case. The magnitude of displacement decreases with the decrease in excitation amplitude and with the increase in injection pressure. Once again, the relative dominance of acoustic velocities over the convective velocities is the reason for the observed behavior.
During the acoustic forcing experiments, the acoustic pressures were also recorded. These pressure measurements were used to normalize the droplet velocities in order to determine the velocity response relative to the acoustic pressures. Comparison of these normalized velocity responses are presented in Figures 3.28 through 3.30. The data shown has been non-dimensionalized by the ratio of average droplet velocity to mean acoustic pressure at the respective frequencies ($\frac{\bar{U}}{\bar{P}}$). The response to change in acoustic forcing amplitude is shown in Figure 3.28. The measurements were made 25 mm downstream of the injector and the injection pressure was fixed at 1.03 MPa. It is noted that the rate of roll-off of the response is proportional to the acoustic amplitude. In other words, the higher the acoustic forcing the higher is the droplet velocity response. As in the previous results, the response is more pronounced at the lower frequencies. The cut-off frequency beyond which no significant response is observed is roughly 250 Hz. This cut-off frequency commensurates with the relaxation time of a 18 micron droplet, which is almost the mean droplet diameter noted under the test conditions. The relaxation time $\tau_r$ was determined using Equ. 3.6.
Figure 3.28: **Normalized and non-dimensionalized spray velocity response at different forcing amplitudes.**

taken from Clift et al. [112],

\[
\tau_r = \frac{D^2 \left( \frac{2\rho_L}{\rho_g} + 1 \right)}{9\nu_g}
\] (3.6)

where \( D \) is the mean droplet diameter, \( \rho_L \) and \( \rho_g \) are the liquid and gas densities and \( \nu_g \) is the viscosity of gas.

The droplet oscillatory response is also dependent on its velocity amplitude relative to the acoustic velocity amplitude. Thus for a fixed acoustic velocity value, sprays with higher velocities will have lower response and vice versa. This is depicted in Figure 3.29, where non-dimensional response to different injection pressures is shown. At lower injection pressure, the droplet velocities are lower and thus the response is higher. It is also noted that the cut-off frequency has moved up to around 500 Hz, since at higher injection pressures the mean droplet diameter are lower and thus have a lower relaxation time.

As discussed earlier, under the acoustic forcing influence the smaller droplets tend to move radially outwards and thus the population of smaller droplets increase at the spray bound-
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Figure 3.29: Normalized and non-dimensionalized spray velocity response at different injection pressures.

Figure 3.30: Normalized and non-dimensionalized spray velocity response at different locations.
aries compared to the centerline. Since smaller droplets are easily influenced by the flow oscillations, a higher droplet response is expected at the spray periphery. This comparison is shown in Figure 3.30 for measurements made at three different location in the spray. It may also be noted that droplet response is much higher near the injection point. As for the cut-off frequency, it is seen from the figure that depending on the location of measurement the frequency varies from 150 Hz to 350 Hz, since the mean droplet diameter varies at these different locations.

3.6.6 Frequency Response Analysis

Frequency response analysis was conducted on the velocity response of the droplets under the influence of acoustic forcing with varying amplitudes and injection pressures. The analysis was also conducted on data collected from various locations in the spray.

Frequency response function (FRF) between oscillating droplet velocities and the excitation

![Figure 3.31](image.png)

**Figure 3.31:** FRF between droplet velocity and excitation voltage signal for various acoustic excitation amplitudes.
Figure 3.32: FRF between droplet velocity and excitation voltage signal for various injection pressures.

Figure 3.33: FRF between droplet velocity and excitation voltage signal for measurements obtained from two axial locations.
signal was determined and are presented as Bode plots in Figure 3.31 through 3.33. Ignoring the variations due to experimental errors, the plots show a linear response under various acoustic excitation amplitudes and injection pressures. The magnitude plots validate the trends observed in the velocity behavior discussed in the previous section. A high response is noted at the lower frequencies, which roll-off as the frequency of excitation is increased. Because of the rig resonance the response again shows a rise around 400 Hz. For measurements taken from off-center locations however, velocity response does not show a uniform roll-off. Nothing concrete can be concluded from this data. Because of the limitation of 1-D velocity measurement capability of the PDA system used in the experiments, the azimuthal velocity components (which are more dominant at the radially outward locations) could not be captured. Thus the data fails to show the total velocity response.

The phase plots shown in the figures indicate that there exists a lag in the velocity response. Ignoring the rig generated response, the almost -20 dB/decade roll-off in the magnitude is not matched with a corresponding 90° phase shift. Rather, a larger phase difference is noted in all the figures. This validates the inherent lag in the system due to the relaxation time associated with the droplets.

### 3.7 Conclusions Regarding Spray-Acoustic Interaction

Under the limitations of only one-dimensional velocity measurements, the following conclusions are made about the response of spray droplets to the presence of acoustic oscillations:

1. A novel technique based on DFT, adopted to carry out the dynamic frequency analysis of unevenly sampled PDA data, show results which are in good agreement with those obtained from commonly used interpolation method.

2. Static spray characteristics show two distinct regions in the spray. An inner core having smaller sized droplets with higher velocities and an outer shell where the diameter
increases monotonically and velocity shows a decline.

3. In the inner core the size distribution is uniform. At the inner boundary of the outer shell the distribution is Gaussian with a two-fold increase in mean diameter value. The largest sized droplets are found at the periphery of the spray.

4. The increase in diameter at the spray boundary is 173% from the centerline value, while the droplet velocity decreases in the outer region by 43% of its centerline value.

5. Spray cone angle is found to be 65°.

6. Higher sized droplets penetrate further downstream in the spray.

7. At the spray periphery the droplet velocities are dominated by their swirl component, especially at higher injection pressures.

8. Mean droplet diameter decreases with the increase in injector pressure. However, beyond certain pressure value (which is dependent on the injector size), the mean diameter does not show any further decrease. The behavior is more pronounced at the spray centerline.

9. At higher injection pressures the droplet radial size distribution is more uniform across the spray.

10. The droplet velocity initially increase with increasing injection pressure. It however, starts falling beyond a certain pressure, the value of which is dependent on the injector size and the location of measurement inside the spray.

11. Under acoustic forcing mean droplet diameter increases at the centerline and shows a decrease at the radially outward locations. This indicates the radial migration of smaller droplets under the acoustic influence.

12. Spray response to acoustic forcing is directly proportional to the acoustic amplitude and is inversely proportional to mean droplet size and the injection pressure.
13. The influence of acoustic forcing is prominent at lower frequencies of excitation. As the frequency increase the spray response falls off. The cut off frequency after which no substantial response is observed, is related to the relaxation time associated with the mean droplet size.

14. At the centerline location the maximum decrease in mean axial velocity under acoustic forcing is 53%, while the maximum increase in mean diameter is 38%.

15. Spray response is more pronounced at the spray centerline compared to radially outward locations.

16. Acoustic forcing causes high amplitude excursions in droplet displacement. A maximum peak-to-peak displacement of 3.4 mm at an acoustic velocity of 0.125 m/s is observed. The peak-to-peak excursions are directly proportional to acoustic amplitude. The oscillatory displacement show a decrease with increase in injection pressure.

17. Addition of swirl co-airflow tends to decrease the influence of acoustic field.

18. The dynamic droplet velocity response is linear with the excitation signal amplitude.

19. The velocity response indicate a phase lag due to the inherent relaxation times associated with the droplets.
Chapter 4

Modeling of Droplet Dynamics

In the introduction to spray combustion in Chapter 1, it was pointed out that it is considered reasonable and in many cases practical to expect that the collective behavior of individual droplets, in terms of their vaporization, burning and dynamics, can characterize the gasification and oxidation of the complete spray. This in turn reflects the combustor performance. It is therefore, quite common in spray combustion numerical studies to adopt droplet combustion as a model problem. This approach is especially useful if the intent is to track individual droplets as they are subjected to and traverse in high temperature unsteady gaseous stream. The purpose of the modeling effort presented here is to study the influence of acoustic oscillations on spray combustion. It has been shown [113, 114] that heat transport within a cylindrical resonator is enhanced by the presence of an acoustic field. Under such conditions the heat transport by convection has a major contribution in the overall heat transfer from the hot combustor gases to the spray and thus the G-number as defined by Chiu and Liu [22] is expected to be low. Since low G-number favors individual drop burning, the same approach has been adopted in the model.
4.1 Phenomenological Description

A description of droplet combustion is first presented as given by Law [3], with certain additions as applicable to the experimental scenario discussed in Chapter 2. The present effort, discussed in this chapter, to formulate a Droplet Trajectory Model (DTM) is based on this adaptation.

A spherical droplet at the fuel inlet temperature is injected into a high temperature gas flow field with a superimposed acoustically oscillating flow. The high temperature environment seen by the subject droplet, results from the combustion of the stream of droplets injected at an earlier time. Since the droplet surface temperature is lower than the surroundings, heat is conducted towards the droplet surface. Part of this heat is used up to heat the droplet interior and rest is used to vaporize the liquid at the drop surface. This creates a fuel vapor concentration gradient and the vapor diffuses radially outwards, away from the droplet surface. The transport of vapor causes a decrease in the vapor concentration at the surface from its saturation value thus prompting further gasification of the droplet.

Because of the non-radial convective flow there exist a relative velocity between the droplet and the gaseous medium. The surrounding gas flow also causes shear stresses on the droplet surface, which generate internal recirculation within the droplet. This enhances the vaporization rate of the droplet and reduces the droplet heat up time. The presence of the additional acoustic field, cause a further enhancement of the vaporization process because of the increase in heat and mass transfer rates associated with this oscillatory flow [114].

On accumulation of sufficient vapor concentration, ignition occurs either by the introduction of an external spark or by the presence of sufficiently hot surrounding gas. This results in a non-spherical and non-symmetrical diffusion flame around the droplet at a location where the outwardly-diffusing fuel vapor and inwardly-diffusing oxidizer meet in a near stoichiometric ratio. The chemical reactions occur extremely fast and thus the reaction zone collapses into an infinitely thin flame sheet [115]. The non-symmetry of the flame is caused by the presence
of convective flow, which would otherwise be symmetric around the droplet under quiescent conditions. The heat release from the reaction causes the temperature rise in the continuous gas medium, which then aids in further vaporization and burning of the droplet.

The description presented here may be extended to the combustion of sprays with certain drop size and velocity distributions. Each droplet in the spray goes through the phenomenological life cycle discussed above with vaporization, burning and dynamics as functions of its initial size and velocity. In the case of sprays, the end of the overall flame zone would then correspond to the disappearance of the largest droplet in the distribution.

4.2 Modeling in Spray Combustion

The discussion provide in the previous section highlights that droplet combustion involves heat, mass and momentum transfer between the condensed (liquid) and continuous (gas) phases with the coupling at the interface. Thus the modeling approach involves two coupled systems of equations describing the properties of the two phases. Before the two sets of equations are formulated it is imperative to obtain a deeper understanding of the terms used in these equations with respect to issues like fuel composition, multi-drop interaction, gas flow behavior and their effects on droplet vaporization and burning.

Recent increase in the use of derived fuel has prompted interest in investigating the effects of fuel composition on droplet vaporization and combustion. As a multi-component fuel droplet traverse in a reactive flow field the more volatile fuel components vaporize earlier than the less volatile ones, thus creating a non-uniform distribution of vapor concentration and gas phase equivalence ratio. Tong and Sirignano [116] studied the transient vaporization of multi-component droplet and highlighted that fuel components vaporize approximately and sequentially in the order of their relative volatility and that the rate of vaporization was independent of the mixture composition during vaporization and combustion. Similar trend was observed by Prommersberger et al. [117], during their studies on real aviation
CHAPTER 4. MODELING OF DROPLET DYNAMICS

fuel. They concluded that because the relative volatility affects the vaporization rate, a realistic formulation of thermo-physical data of the fuel droplet needs to be incorporated for realistic evaporation simulations. A multi-component droplet vaporization model utilizing continuous thermodynamics has been presented by Tamim and Hallett [118].

Droplet heating has a pronounced effect on the vaporization rate and subsequently on burning. Earlier theories suggested a quasi-steady behavior for liquid droplet, see for example Godsave [119]. The droplet temperature was assumed to be uniform and constant, both in space and time domains. The unsteady nature of droplet heating was first realized by Law [120], who assumed infinite conductivity and rapid internal mixing. The droplet temperature was considered spatially uniform but temporally varying. The infinite conductivity model was later relaxed by a conduction heating model [121], which catered for the interior spatial variation of temperature. However, experimental observations of El Wakil et al. [122] suggested that under vigorous internal circulation, perpetual temperature uniformity is maintained inside the droplet. Subsequent investigations by Prakash and Sirignano [123], Tong and Sirignano [124] and others, considered both transient heating and internal circulation effects to model droplet vaporization and their results are in good agreement with experimental observations. A summary of these model and their results can be found in Sirignano [125].

As was discussed in Chapter 1, the individual droplet models are quite authentic in predicting the bulk spray combustion behavior in regions of the spray where the droplet number density is low. For dense spray regions where the inter droplet spacing is small, the neighboring droplets have a profound influence on the vaporization and dynamics of the individual droplets by modifying the ambient conditions in the near field of any given droplet. This effect therefore, needs to be accounted for in the droplet burning models. Silverman and Sirignano [21] have studied the multi-droplet interaction. Their findings show that the presence of the neighboring droplets decreases the evaporation rate of a droplet because the heat transport to an interacting droplet is slower as compared to an isolated droplet. Also the relative velocity of an interacting droplet is higher for most of its lifetime. It was also shown that the interaction effects were more pronounced for small droplets.
The properties and nature of the convective gas flow including the influence of acoustic oscillations have considerable effects on the vaporization of the droplet. Models accommodating these effects have been investigated. Tong and Sirignano [126, 35] showed that droplet vaporization shows an oscillatory behavior in response to an oscillatory flow field. The unsteadiness in the vaporization was found to be strong enough to cause unsteady combustion, which under certain conditions were sufficient to initiate self-sustained thermoacoustic oscillations. Maghami et al. [127] conducted both theoretical and experimental analysis and showed that under the influence of an acoustic field the evaporation time of the droplets decrease by 30%.

4.3 Classification of Modeling Methods

As highlighted earlier, spray combustion modeling in general and individual droplet burning modeling in particular involves solving two sets of coupled conservation equations, one for the gas phase and the other for the liquid phase. Three types of formulation are in practice: Eulerian-gas/Eulerian-liquid approach, Eulerian-gas/Lagrangian-liquid technique and Probabilistic formulation [128]. In the Eulerian-Eulerian formulation, both phases are treated as continuous. This approach has been shown to be extremely useful in modeling the homogeneous regions of the spray burning in premixed mode and also in droplet modeling when the droplet sizes are in the order of 10 microns or less. On the other hand, in the Eulerian-Lagrangian modeling approach the gas and the liquid are treated as continuous and condensed separated phases respectively. This technique is preferred when the intent is to track the history of individual droplets in the dilute regions of the spray. The probabilistic approach finds its usefulness in situations when very fine resolution is required. In such cases at any instant in time most cells in a grid will not contain any droplet and thus a probability density function is needed to locate the droplet.

Classification of models in spray combustion is typically done on the basis of Eulerian-
Eulerian and Eulerian-Lagrangian formulations. Locally Homogeneous Flow (LHF) models fall under the Eulerian-Eulerian category while the Separated Flow (SF) models follow the Eulerian-Lagrangian approach. A summarized classification of the prevailing models is given by Kuo [23]. The LHF models represent the simplest treatment of a multi-phase flow [129]. The basic assumption in these models is that the rates of mass, momentum and energy transport between the two phases are very fast in comparison to the rate of development of the flow. This implies that both phases are in complete dynamic and thermodynamic equilibrium at each point in the flow. LHF models also neglect slip effects between the two phases and thus can be used only as a limiting case of spray combustion by assuming infinitely small droplets. The LHF assumption is also quite reasonable when studying spray evaporation at supercritical conditions. LHF models give good first approximation and are simple to formulate and since the initial droplet size and velocity does not play a role, these models do not require any information about the injector. Another advantage of LHF models is that they are less computationally expensive.

The SF models overcome the basic limitations of the LHF models in that both slip and inter phase transport are considered. These models have therefore, been used widely to study spray combustion. From a numerical point of view, they belong to a class of models called Particle-Source-in-Cell Models (PSICM) or Discrete-Droplet Models (DDM) [23] and rely heavily on empirical correlations to account for the exchange processes. In SF approaches, the entire spray is divided into finite groups of discrete droplets and each group is assigned a representative initial size, velocity, direction and history. The characteristic droplets are tracked through the flow using a Lagrangian formulation. The gas phase conservation equations are solved using a Eulerian formulation with a source term to account for the inter phase coupling. Depending upon the level and significance of turbulence considered in the model, the SF models are further divided into Deterministic Separated Flow (DSF) and Stochastic Separated Flow (SSF) Models.

DSF models are generally applied to dilute regions of the spray and it is assumed that they do not interact with neighboring droplets. These models are therefore considered quite reliable.
for individual droplet vaporization and combustion studies. They have been found to be less computationally expensive as compared to SSF models [130]. However, these models ignore the droplet dispersion due to turbulence and the influence of turbulence on inter phase transport rates. This is considered appropriate when the characteristic droplet relaxation times (defined as the time taken by the droplet to adjust to the changes in the gas flow) are much larger than the characteristic times of turbulent fluctuations. For flows with high void fraction and high liquid-to-gas density ratio, the DSF models also ignore the influence of virtual mass and Bassett forces.

The SSF models are perhaps the most comprehensive models developed to study not only discrete droplet dynamics but also particle dispersion in turbulent liquid jets. In SSF models the instantaneous velocity in the liquid phase momentum equation is replaced with a time averaged and a fluctuating fluid velocities. They have been found to agree well with the experimental data in a wide variety of flows [129]. However, the SSF models require extended computer time since a large number of trajectories are required to determine a statistically stable solution [130].

Models that incorporate the advantages of both DSF and SSF models have also been developed to improve both accuracy and efficiency (refer Chan [131]).

### 4.4 Formulation of Droplet Trajectory Model

The basic methodology behind any thermoacoustic instability control approach is to disrupt the phase relationship between the combustor acoustic oscillations and the unsteady heat release rate. Effectiveness of an active control technique, like for example primary fuel modulation, depends on the fundamental knowledge of when and where the droplets burn both before and after the application of fuel modulation. Tracking the spatial and temporal histories of the droplets is thus imperative.

The objective behind the current modeling effort was to develop an analytical tool, which:
1. Is computationally less intensive than the commercially available software packages.

2. Can be used to provide physical explanations for the experimental observations.

3. Can be used to conduct sensitivity/parametric analysis for the fuel modulation investigations.

A Deterministic Separated Flow (DSF) approach is employed to model the dynamics of droplets as they traverse in a reacting flow field superimposed with high amplitude limit-cycle acoustic oscillations. The choice of DSF over Stochastic Separated Flow (SSF) method is made because the droplets sizes considered in the model have relaxation times much larger than the stochastic turbulent eddy time scales.

The computational domain consists of a cylindrical combustor, shown in Figure 4.1. Combustion air is introduced in the combustor through a $45^\circ$ axial swirler with negligible radial

![Figure 4.1: Computational domain for the Droplet Trajectory Model. Temperatures are specified at the inlet, wall and exit of the combustor (Dirichlet boundary conditions).](image)
velocity component. Kerosene fuel spray is injected at the combustor centerline. Presence of the acoustic field in the combustor is simulated by an oscillating axial velocity super-imposed on top of the mean combustion air velocity. Spray is considered an ensemble of groups of spherical droplets. Each group is assigned a representative size, a velocity and a direction. The droplet environment is shown in Figure 4.2.

The description of the model follows the phenomenological depiction presented in section 4.1. The following assumptions are made to achieve the objective of computational efficiency:

1. Spherical symmetry of the droplet is maintained. However, due to the consideration of a convective flow, the spherical symmetry of the flame around the droplet is relaxed.

2. Multi-droplet interaction and effects of neighboring droplets on the vaporization and combustion of individual droplets is ignored. This assumption is considered valid.
as most experimental measurements (for example PDA measurements discussed in Chapter 3) are made in relatively dilute regions of the spray.

3. Droplet deformation and secondary breakup is ignored. This topic is independently investigated in Section 4.8.

4. Constant pressure process is assumed.

5. The combustor and the air/fuel feed lines are assumed acoustically decoupled. This amounts to assuming that an acoustic velocity node exists at the combustor inlet and that there is no effect of combustor acoustics on the mass flow rates of the incoming air or fuel.

6. Turbulent effects on droplet dispersion and inter phase transport are neglected. This is in accordance with the justification for choosing the DSF approach in the model.

7. Buoyancy and gravitational effects are ignored since the amplitude of acoustic and convective flow forces in the model exceed the magnitude of buoyancy and gravitational forces.

8. Virtual mass and Bassett forces are also neglected. The assumption is realistic since the liquid-to-gas density ratio is high in the model.

9. Reaction zone thickness is considered infinitely small (flame sheet approximation). This means that the rates of chemical reactions are considered much faster than the diffusion rates in the gas phase.

10. The gas phase mass and thermal diffusion rates are considered equal (unity Lewis Number).

11. For the purpose of estimating gas temperature distribution, quasi-steadiness in gas phase is considered. Because of considerable difference in liquid and gas phase densities, the gas phase transport rates are much faster compared to that of the liquid phase and thus the assumption.
12. Droplet vaporization and burning is assumed simultaneous. Thus the variations in the vapor concentration between the droplet surface and the flame sheet caused by the continuous regression in the droplet size is ignored. Simply stated vapor accumulation in this region is ignored.

13. Fuel is considered to constitute a single component. Justification for this assumption is presented in section 4.5.2.

14. Gas phase species diffusion due to temperature and pressure gradients are neglected (no Soret or Dufour effects).

15. The droplet internal temperature is assumed to be governed by the “onion-skin model”. Accordingly only two temperatures are associated with the droplet. One for the interior of the droplet and one at the surface. The internal temperature is considered spatially uniform because of the rapid internal mixing assumption inside the droplet due to internal circulation. The reasons for internal circulations have been stated in section 4.1. The temporal variation in internal temperature is however, maintained. For the surface temperature, the liquid boiling point temperature is assumed.

The conservation equations for the liquid and the gas phases, under the above assumptions are presented next.

### 4.4.1 Liquid Phase Equations

For the DSF modeling the Lagrangian formulation requires ordinary differential equations for the droplet surface regression rate, droplet velocity and temperature of the flame surrounding the droplet.

The droplet surface regression rate is obtained by applying conservation of mass and species equations at the droplet surface. These equations with the spherical symmetry assumption
take the form,

$$\frac{d}{dr} \left( \rho_s U_s R^2 \right) = 0 \quad (4.1)$$

$$\rho_s U_s R^2 \frac{dY_s}{dr} = \frac{d}{dr} \left( R^2 D_s \rho_s \frac{dY_s}{dr} \right) + R^2 \dot{\omega}_F \quad (4.2)$$

where, $\rho_s U_s$ is the gaseous mass flux of fuel leaving the droplet surface, $R$ is the droplet radius, $D_s$ is the mass diffusivity, $Y_F$ is the fuel vapor mass fraction and $\dot{\omega}_F$ is the reaction rate.

From Equ. 4.1,

$$\dot{m}(r) = \dot{m}_F = 4\pi R^2 \rho_s U_s = \text{constant}$$

Applying Fick’s Law, an expression for fuel flow rate, $\dot{m}_F$ is obtained [25],

$$\dot{m}_F = -4\pi R^2 \rho_s \frac{D_s}{1 - Y_F} \frac{dY_F}{dr} \quad (4.3)$$

For unity Lewis Number $D_s$ can be replaced by $\alpha_s$, the thermal diffusivity. Using Equ. 4.3 together with Equ. 4.2 and applying boundary conditions at the droplet surface, Equ. 4.3 can be written in the form,

$$\dot{m}_F = 4\pi R^2 \rho_s \alpha_s ln \left( 1 + B_o \right) \quad (4.4)$$

where $B_o$ is the Spalding Transfer Number and is expressed as [25],

$$B_o = \frac{C_{p_g} (T_g - T_s)}{h_{fg} + C_{p_L} (T_s - T_{D_i})} = \frac{\Delta h_c / \nu_{st}}{h_{fg} + C_{p_L} (T_s - T_{D_i})} \quad (4.5)$$

The Spalding Transfer Number is a ratio of available-to-consumed energy in droplet evaporation or droplet burning. In Equ. 4.5, $C_{p_g}$ and $C_{p_L}$ are the gas and liquid specific heats; $T_g$, $T_s$, $T_{D_i}$ are the gas, droplet surface and droplet internal temperatures; $h_{fg}$ and $\Delta h_c$ are the heat of vaporization and heat of reaction respectively. Also, $\nu_{st}$ is the stoichiometric air-to-fuel ratio.
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With the assumption that $T_s = T_{boil}$, $Y_F$ at the surface is unity and in Equ. 4.4 $\rho_s$ and $\alpha_s$ may be replaced by $\rho_g$ and $\alpha_g$ respectively.

To obtain an explicit expression for the droplet regression rate or rate of change in diameter, a mass balance at the droplet surface can be written as,

$$ \frac{dm_D}{dt} = -\dot{m}_F $$

(4.6)

This implies that the rate of decrease of droplet mass equals the rate at which the liquid is vaporized. In Equ. 4.6, $m_D = \pi \rho_L D^3 / 6$. Writing Equ. 4.4 in terms of droplet diameter and substituting in Equ. 4.6,

$$ \frac{dD}{dt} = \frac{-4\alpha_g}{D} \left( \frac{\rho_g}{\rho_L} \right) ln \left( 1 + B_o \right) $$

or

$$ \frac{dD^2}{dt} = -8\alpha_g \left( \frac{\rho_g}{\rho_L} \right) ln \left( 1 + B_o \right) $$

(4.7)

Equ. 4.7 implies that the regression rate of droplet is a constant, namely evaporation constant $K_o$. Thus,

$$ \frac{dD^2}{dt} = -K_o $$

(4.8)

Equ. 4.8 can be integrated to get the well known $D^2$-Law expression for the instantaneous droplet diameter D in terms of evaporation constant and droplet initial diameter $D_o$.

$$ D^2 = D_o^2 - K_o t $$

(4.9)

The evaporation constant strongly depends on the gas flow properties surrounding the droplet, as can be observed from Equ. 4.7. For a convective flow, it takes the form [132],

$$ K_{oc} = K_o N_{uD} $$

(4.10)

Where the Nussult Number $N_{uD}$ is given as,

$$ N_{uD} = 1 + 0.276 R_{eD}^{0.5} Pr^{0.33} $$
Here, $R_{eD}$ is the liquid Reynold Number based on droplet diameter and relative velocity $\vec{U}_r$ and $P_r$ is the Prandtl Number.

Okai et al. [133] further modified the expression for the evaporation constant to account for the presence of oscillating acoustic flow. Thus, the expression used in the current model takes the form,

$$K = K_{oc} + AD_{ac} (1 - D_{ac}/2D_c)$$  \hspace{1cm} (4.11)

Where $A$ is a non-dimensional constant, $D_{ac}$ is the acoustic diffusivity and $D_c$ is the value of $D_{ac}$ at the maximum value of $K$. The acoustic diffusivity can be expressed in terms of acoustic excursion $X_{ac}$, acoustic velocity $U_{ac}$ and acoustic frequency $f_{ac}$ as,

$$D_{ac} = X_{ac}^2 f_{ac} = \frac{U_{ac}}{2\pi f_{ac}}$$

The value of $D_c$ is obtained from the experimental work of Kumagai and Isoda [114], who determined that under the influence of acoustic oscillations $K_{oc}$ may increase as much as 15%. Thus using Equ. 4.11, $D_c = 0.3K_{oc}/A$

The second ordinary differential equation in the Lagrangian formulation of DSF, for the droplet velocity, is obtained from the momentum conservation of an accelerating droplet,

$$\sum \vec{F} = m_D \frac{d\vec{U}_D}{dt}$$  \hspace{1cm} (4.12)

From here the droplet trajectory is obtained from,

$$\frac{d\vec{X}_D}{dt} = \vec{U}_D$$  \hspace{1cm} (4.13)

In Equ. 4.12 and 4.13 $\vec{U}_D$, $\vec{F}$ and $\vec{X}_D$ are the droplet velocity, external forces on droplet and the droplet displacement respectively. Due to assumptions 7 and 8, the only force on the droplet is the drag force due to the motion of the droplet relative to the gas flow field. Thus,

$$\vec{F} = \frac{\pi}{4} C_d \rho_g |\vec{U}_r| \vec{U}_r D^2$$  \hspace{1cm} (4.14)
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In Eqn. 4.14, $\vec{U}_r$ is the droplet relative velocity ($\vec{U}_g - \vec{U}_D$). The relative velocity terms are written in this particular form to preserve the velocity direction. The gas velocity $\vec{U}_g$ is made up of two components, the mean convective gas velocity and an oscillating acoustic velocity. Combining Equs. 4.12 and 4.14 and rearranging, the following expression is obtained,

$$\frac{d\vec{U}_D}{dt} = \frac{3}{4} C_d \left( \frac{\rho_g}{\rho_L} \right) \frac{\vec{U}_r}{D}$$

Equ. 4.15 can also be written in a convenient form in terms of the droplet relaxation time $\tau_D$,

$$\frac{d\vec{U}_D}{dt} = \frac{\vec{U}_r}{\tau_D}$$

where

$$\tau_D = \frac{4D\rho_L}{3C_d\rho_g|\vec{U}_r|}$$

The coefficient of drag $C_d$ introduced in Eqn. 4.14 is defined for a case with evaporation/combustion as,

$$C_d = \frac{C_{do}}{1 + B_o}$$

where

$$C_{do} = \frac{24}{R_{e_D}} \left( 1 + 0.15R_{e_D}^{0.687} \right)$$

Finally, the temperature of the flame surrounding the droplet is obtained from the energy conservation equation expressed in the Shvab-Zeldovich form, which for spherically symmetric situation is,

$$\frac{1}{R^2} \frac{d}{dr} \left[ R^2 \left( \rho_s U_s \int C_{p_s} dT - \rho_s D_v \frac{d}{dr} \int C_{p_s} dT \right) \right] = -\sum_{j=1}^{N} h_{f_j} \dot{\omega}_j$$

With flame-sheet approximation and applying boundary conditions at the droplet surface and flame sheet, an expression for the flame temperature is obtained in terms of the Transfer Number,

$$T_f = \left[ \frac{C_{p_L}}{C_{p_s}} \left( T_s - T_{D_s} \right) + h_{f_d} \right] \left( \nu_{st} B_o - 1 \right) + T_s$$
4.4.2 Gas Phase Equations

In the DSF model the Eulerian formulation for the gas phase require conservation of mass and energy equations with the source/sink terms at the liquid gas inter phase.

The gas phase mass conservation equation is required since mass in the form of vapors is continuously transferred from the liquid to the gas phase during the spray evaporation process. The mass balance helps in monitoring the gas phase equivalence ratio during the droplet lifetime. The equation with the quasi-steady assumption can be written as [128],

$$\frac{\partial}{\partial x_i} (\rho_g U_{g_i}) = n \dot{m}_F$$

(4.19)

Here $i$ represents the coordinate axis and $n$ is the droplet number density, which can be obtained from the knowledge of injected liquid fuel flow rate $\dot{m}_L$ and the number rate of droplets $\dot{N}$. The gas phase equivalence ratio at any location is obtained from,

$$\Phi (x_i) = \frac{\nu_{st} \dot{m}_F (x_i)}{\dot{m}_{air}(0)}$$

(4.20)

The mean convective gas phase velocity $U_g(x_i)$ can be determined from the knowledge of gas mass flow rate $\dot{m}_g(x_i)$, obtained from Equ. 4.19, and the characteristic unit area $A_c$ of the domain. Thus,

$$U_g(x_i) = \frac{\dot{m}_g(x_i)}{\rho_g(x_i) A_c}$$

(4.21)

The density $\rho_g$ and all other gas phase properties are dependent on the gas temperature distribution and therefore, a gas phase energy conservation is required. In the current formulation, a form given by Poinssot and Veynate [134] is used where pressure and viscous term have been ignored,

$$\frac{\partial}{\partial x_i} \left( \rho_g U_{g_i} \int C_{p_g} dT_g \right) = S_T + \dot{Q} - \frac{\partial}{\partial x_i} \left( -\lambda \frac{\partial T_g}{\partial x_i} \right)$$

(4.22)

The net volumetric radiation heat source $\dot{Q}$ in terms of emissivity $\epsilon$, Stephan-Boltzman constant $\sigma$ and wall temperature $T_w$ is expressed as,

$$\dot{Q} = \epsilon \sigma \frac{(T^4_g - T^4_w)}{L_c}$$

(4.23)
Here, $L_c$ is the length of the combustor. The net source term $S_T$ is expressed as,

$$S_T = Q_{comb} - n\dot{m}_F (Q_{\text{drop heating}} + h_{fg} + Q_{\text{vapor heating}})$$ (4.24)

The individual terms in Equ. 4.24 are defined according to Bhatia and Sirignano [135] as,

$$Q_{comb} = \sum_{j=1}^{N} \dot{\omega}_j h_{f_j}$$ (4.25)

Where $\dot{\omega}_j$ and $h_{f_j}$ are the reaction rate and heat of reaction of species $j$ respectively.

$$Q_{\text{drop heating}} = 4\pi R^2 \lambda_L \frac{dT_d}{dr} + C_{p_L} (T_s - T_{D_i})$$ (4.26)

$$Q_{\text{vapor heating}} = C_{p_g} (T_g - T_s)$$ (4.27)

If rapid droplet internal mixing is assumed then the first term on the right hand side of Equ. 4.26 can be ignored.

### 4.5 Description of Sub-Models

The formulation presented in the above section is incorporated in a Droplet Tracking Model (DTM). During the tracking process this main model interacts with certain thermodynamic and chemical kinetics sub-models, to obtain iteratively updated values of equilibrium combustion products and reaction rates, at each time step. Schematically, this is shown in Figure 4.3. These values are then used to determine the gas phase temperature distribution.

In addition, the temperature dependent liquid and gas phase properties are obtained by DTM via interpolation of look-up tables. Description of the two sub-models used to obtain mole fraction and reaction rates of the species are presented next.
4.5.1 Chemical Equilibrium Model for Calculating Mole Fraction

Mole fraction of the equilibrium combustion products are determined by using the model by Olikara and Borman [136]. The technique provides a rapid means to calculate the mole fractions as a function of temperature, pressure and equivalence ratio.

The model assumes that kerosene fuel and air at a given equivalence ratio (Φ) react and the products, subject to temperature and pressure, achieve equilibrium. A one-formula surrogate
fuel representation of kerosene is used after Wang [137]. Thus,

$$C_{12}H_{24} \frac{18}{\Phi} (O_2 + 3.76N_2) = n_1H + n_2O + n_3N + n_4H_2 + n_5O_2 + n_6N_2 + n_7OH +$$

$$n_8NO + n_9CO + n_{10}H_2O + n_{11}CO_2 \quad (4.28)$$

Where $n_j$ are the mole of product species. If

$$N = \sum_{j=1}^{11} n_j$$

Then, Equ. 4.28 can also be written in term of mole fractions $x_j$ by introducing,

$$\chi = \frac{1}{N}$$

Thus,

$$\chi \left[ C_{12}H_{24} \frac{18}{\Phi} (O_2 + 3.76N_2) \right] = x_1H + x_2O + x_3N + x_4H_2 + x_5O_2 + x_6N_2 + x_7OH +$$

$$x_8NO + x_9CO + x_{10}H_2O + x_{11}CO_2 \quad (4.29)$$

In order to solve for the 12 unknowns, five equations come from the definition of mole fraction and the atomic balance for C, H, O and N, thus,

$$\sum_{j=1}^{11} x_j = 1 \quad (4.30)$$

$$12 \chi = x_9 + x_{11} \quad (4.31)$$

$$24 \chi = x_1 + 2x_4 + x_7 + 2x_{10} \quad (4.32)$$

$$\frac{36}{\Phi} \chi = x_2 + 2x_5 + x_7 + x_8 + x_9 + x_{10} + 2x_{11} \quad (4.33)$$

$$\frac{135.36}{\Phi} \chi = x_3 + 2x_6 + x_8 \quad (4.34)$$

The seven additional equations are obtained from the knowledge of equilibrium constants for the non-redundant hypothetical reactions at a given normalized pressure $P$. These are,

$$\frac{1}{2}H_2 \rightleftharpoons H : \quad K_{P_3} = \frac{x_1P^{0.5}}{x_4^{0.5}} \quad (4.35)$$
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\[
\begin{align*}
\frac{1}{2}O_2 &\rightleftharpoons O : \quad K_{P_2} = \frac{x_2 P^{0.5}}{x_5^{0.5}} \\
\frac{1}{2}N_2 &\rightleftharpoons N : \quad K_{P_3} = \frac{x_3 P^{0.5}}{x_6^{0.5}} \\
\frac{1}{2}O_2 + \frac{1}{2}H_2 &\rightleftharpoons OH : \quad K_{P_4} = \frac{x_7}{x_4^{0.5} x_5^{0.5}} \\
\frac{1}{2}N_2 + \frac{1}{2}O_2 &\rightleftharpoons NO : \quad K_{P_5} = \frac{x_8}{x_5^{0.5} x_6} \\
\frac{1}{2}O_2 + H_2 &\rightleftharpoons H_2O : \quad K_{P_6} = \frac{x_{10}}{x_4 x_5^{0.5} P^{0.5}} \\
\frac{1}{2}O_2 + CO &\rightleftharpoons CO_2 : \quad K_{P_7} = \frac{x_{11}}{x_9 x_5^{0.5} P^{0.5}}
\end{align*}
\]

After cross substitution, the number of equations are reduced to four with four unknowns \( x_4, x_5, x_6 \) and \( x_9 \). These equations, which are non-linear are of the form,

\[
f_i (x_4, x_5, x_6, x_9) = f_i (\vec{x}^i) = 0 \quad \text{with} \ i=1,2,3,4
\]

The set of non-linear equations are solved using Newton-Raphson iterative method. A Jacobian matrix is first constructed which is of the form,

\[
J (\vec{x}) = \begin{bmatrix}
\frac{\partial f_1 (\vec{x})}{\partial x_4} & \frac{\partial f_1 (\vec{x})}{\partial x_5} & \frac{\partial f_1 (\vec{x})}{\partial x_6} & \frac{\partial f_1 (\vec{x})}{\partial x_9} \\
\frac{\partial f_2 (\vec{x})}{\partial x_4} & \frac{\partial f_2 (\vec{x})}{\partial x_5} & \frac{\partial f_2 (\vec{x})}{\partial x_6} & \frac{\partial f_2 (\vec{x})}{\partial x_9} \\
\frac{\partial f_3 (\vec{x})}{\partial x_4} & \frac{\partial f_3 (\vec{x})}{\partial x_5} & \frac{\partial f_3 (\vec{x})}{\partial x_6} & \frac{\partial f_3 (\vec{x})}{\partial x_9} \\
\frac{\partial f_4 (\vec{x})}{\partial x_4} & \frac{\partial f_4 (\vec{x})}{\partial x_5} & \frac{\partial f_4 (\vec{x})}{\partial x_6} & \frac{\partial f_4 (\vec{x})}{\partial x_9}
\end{bmatrix}
\]

On the basis of some initial estimate for \( \vec{x} \) the system is then solved using,

\[
\vec{y} = J (\vec{x})^{-1} \left[ -\vec{F} (\vec{x}) \right]
\]

where

\[
\vec{F} (\vec{x}) = [f_1 (\vec{x}) , f_2 (\vec{x}) , f_3 (\vec{x}) , f_4 (\vec{x})]^T
\]

From this value of \( \vec{y} \), the improved value of \( \vec{x} \) is obtained,

\[
\vec{x}^{(k+1)} = \vec{x}^{(k)} + \vec{y}^{(k)}
\]
The iterative process is terminated when \( \| \vec{y} \| \leq \) a specified tolerance. The mole fraction values obtained from Newton-Raphson methods are then used to calculate the rest of the unknowns by substituting them in Equ. 4.30 thru 4.41.

The Newton-Raphson technique is quite sensitive to the initial values and must therefore, be determined through a judicial estimation. A method of achieving this is given in Ref [136].

### 4.5.2 Chemical Kinetics Model for Determining Reaction Rates

Realistic numerical studies on spray combustion rely heavily on the accurate modeling of the fuel thermophysics, including representation of fuel formula, thermodynamics and finite-rate chemical kinetics.

Hydrocarbon fuels like kerosene are a complex blend of many components. For example, kerosene is typically considered [138] to consists of 10 hydrocarbons species. Each constituent specie requires multiple step reaction mechanism. Extensive computational resources are therefore, required to completely model the chemical kinetics of such fuels. The first step to simplify the calculations is to represent such fuel mixtures by a one-formula surrogate model, while ensuring that the important thermodynamic properties are retained. The second step in the efficient modeling of multi-component fuels is to develop a quasi-global finite-rate kinetic mechanism. This is typically done by representing the decomposition of the straight-chain and cyclic hydrocarbons by two irreversible global steps and adding a wet CO mechanism to convert the intermediate species into the final combustion products [139].

A thermophysical characterization of kerosene, based on the above two steps, has been presented in detail by Wang [137]. The results of Wang’s work are adapted in the current modeling effort. Kerosene is represented with a single-formula fuel \( C_{12}H_{24} \). The thermophysical properties are given in Table 4.1. The specific heat \( C_{pf} \) is determined making use of the standard fourth-order polynomial form,

\[
\frac{C_{pf}}{R_u} = a_1 + a_2T + a_3T^2 + a_4T^3 + a_5T^4
\]
Table 4.1: **Thermodynamic data for kerosene.**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molecular formula</td>
<td>C_{12}H_{24}</td>
</tr>
<tr>
<td>Molecular weight</td>
<td>168</td>
</tr>
<tr>
<td>Heat of combustion (kJ/kg)</td>
<td>-42707.4</td>
</tr>
<tr>
<td>Heat of formation (kJ/kg)</td>
<td>-2297.8</td>
</tr>
</tbody>
</table>

Where \( R_u \) is the universal gas constant and \( a_i \) are the thermodynamic coefficients. The values of these coefficients based on least-square fit for two temperature ranges is shown in Table 4.2.

The quasi-global finite-rate chemical kinetics model of Wang is used with the following modifications:

1. Soot formation is ignored and thus the cyclic global step for naphthene is not considered.

2. The wet CO mechanism is replaced by the one presented by Gardiner [140].

3. A 3-step Zeldovich thermal NOx mechanism [25] is added to account for the remaining species of Equ. 4.29.

Table 4.2: **Thermodynamic coefficients for kerosene.**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>300-1000 K</th>
<th>1000-5000 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>0.36508691e+01</td>
<td>0.36440206e+02</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.10207987e+00</td>
<td>0.54614801e-01</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>0.13124466e-04</td>
<td>-0.16091151e-04</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>-0.76649284e-07</td>
<td>0.21478497e-08</td>
</tr>
<tr>
<td>( a_5 )</td>
<td>0.34503763e-10</td>
<td>-0.10131180e-12</td>
</tr>
</tbody>
</table>
The 12 species-12 reaction mechanism used, is shown below:

\[ C_{12}H_{24} + 6 (O_2 + 3.76N_2) \rightarrow 12CO + 12H_2 + 22.56N_2 \]  \hspace{1cm} (4.46) \\
\[ H_2 + O_2 \leftrightarrow 2OH \]  \hspace{1cm} (4.47) \\
\[ OH + H_2 \leftrightarrow H_2O + H \]  \hspace{1cm} (4.48) \\
\[ OH + OH \leftrightarrow H_2O + O \]  \hspace{1cm} (4.49) \\
\[ H_2 + O \leftrightarrow H + OH \]  \hspace{1cm} (4.50) \\
\[ H + O_2 \leftrightarrow O + OH \]  \hspace{1cm} (4.51) \\
\[ H_2O + O \leftrightarrow 2OH \]  \hspace{1cm} (4.52) \\
\[ CO + OH \leftrightarrow H + CO_2 \]  \hspace{1cm} (4.53) \\
\[ CO + O_2 \leftrightarrow O + CO_2 \]  \hspace{1cm} (4.54) \\
\[ O + N_2 \leftrightarrow NO + N \]  \hspace{1cm} (4.55) \\
\[ N + O_2 \leftrightarrow NO + O \]  \hspace{1cm} (4.56) \\
\[ N + OH \leftrightarrow NO + H \]  \hspace{1cm} (4.57) \\

The rate of reactions of chemical species \( j \) are calculated using,

\[
\dot{\omega}_j = \frac{d}{dt}[C_j] = \sum_{i=1}^{M} \left( v''_{ji} - v'_{ji} \right) q_i \quad \text{with} \quad j=1,2,\ldots,N
\]  \hspace{1cm} (4.58) \\

Here, \( v'_{ji} \) and \( v''_{ji} \) are the stoichiometric coefficients of the reactants and the products respectively for specie \( j \) in reaction \( i \); \([C_j]\) is the concentration of specie \( j \); and \( q_i \) is the rate-of-progress variable for the \( i \)th reaction, which in the standard form is expressed as,

\[
q_i = k_{fi} \prod_{j=1}^{N} [C_j]^{v'_{ji}} - k_{ri} \prod_{j=1}^{N} [C_j]^{v''_{ji}}
\]  \hspace{1cm} (4.59) \\

Equ. 4.59 is used to calculate the rate-of-progress variable for all reactions in the mechanism except Equ. 4.46, which is determined from the form given by Wang, i.e.,

\[
q_1 = k_{f1} P^{0.3} [C_{12}H_{24}]^{0.5} [O_2]
\]  \hspace{1cm} (4.60)
Table 4.3: Rate constants for kerosene reaction mechanism.

<table>
<thead>
<tr>
<th>Equation</th>
<th>(A_r)</th>
<th>(B_r)</th>
<th>(E/R)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.46</td>
<td>3.888e04</td>
<td>1.0</td>
<td>1.220e04</td>
<td>[137]</td>
</tr>
<tr>
<td>4.47</td>
<td>1.700e13</td>
<td>0</td>
<td>2.407e04</td>
<td>[140]</td>
</tr>
<tr>
<td>4.48</td>
<td>2.190e13</td>
<td>0</td>
<td>2.590e03</td>
<td>[140]</td>
</tr>
<tr>
<td>4.49</td>
<td>6.023e12</td>
<td>0</td>
<td>5.500e02</td>
<td>[140]</td>
</tr>
<tr>
<td>4.50</td>
<td>1.800e10</td>
<td>1.0</td>
<td>4.480e03</td>
<td>[140]</td>
</tr>
<tr>
<td>4.51</td>
<td>1.220e17</td>
<td>-0.91</td>
<td>8.369e03</td>
<td>[140]</td>
</tr>
<tr>
<td>4.52</td>
<td>1.500e10</td>
<td>1.14</td>
<td>8.683e03</td>
<td>[140]</td>
</tr>
<tr>
<td>4.53</td>
<td>4.000e12</td>
<td>0</td>
<td>4.030e03</td>
<td>[140]</td>
</tr>
<tr>
<td>4.54</td>
<td>3.000e12</td>
<td>0</td>
<td>2.500e04</td>
<td>[140]</td>
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<tr>
<td>4.55</td>
<td>1.800e11</td>
<td>0</td>
<td>3.837e04</td>
<td>[25]</td>
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<tr>
<td>4.56</td>
<td>1.800e07</td>
<td>1.0</td>
<td>4.680e03</td>
<td>[25]</td>
</tr>
<tr>
<td>4.57</td>
<td>7.100e10</td>
<td>0</td>
<td>4.500e02</td>
<td>[25]</td>
</tr>
</tbody>
</table>

The forward reaction rate constant \(k_{fi}\) is expressed in the standard Arrhenius form, while the backward reaction rate constant \(k_{ri}\) is calculated from \(k_{fi}\) and the knowledge of equilibrium constant \(K_P\). Thus,

\[
k_{fi} = A_r T_f^{B_r} \exp \left[ -\frac{(E/R)_i}{T_f} \right] \tag{4.61}
\]

\[
k_{ri} = \frac{k_{fi}}{K_P} \tag{4.62}
\]

The empirical values of the Arrhenius rate equation constants \(A_r\) and \(B_r\), and the normalized activation energy \(E/R\) for all 12 reactions are given in Table 4.3.
CHAPTER 4. MODELING OF DROPLET DYNAMICS

4.6 Solution Methodology

The Lagrangian formulated set of equations, Equ. 4.7, Equ. 4.13 and Equ. 4.15, in the Droplet Trajectory Model constitute an initial value problem. It is solved using Forth Order Runge-Kutta method. The initial values of droplet diameter and velocity is obtained from the non-reacting PDA experiments (discussed in Chapter 3), while the values for acoustic velocity and acoustic frequency are obtained from the experimental reacting-flow measurements (discussed in Chapter 2). The gas phase velocity is calculated at each time step using Equ. 4.21 and the droplet life time is determined from Equ. 4.9. The initial gas and liquid mass flow rates that appear in Equs. 4.20 and 4.21 are calculated from the knowledge of injector properties and the Global Equivalence Ratio (GER). This information is also used to determine the droplet number density (used in Equ. 4.19).

The numerical solution to the Eulerian formulated 2-D form of energy equation (Equ. 4.22) is obtained using a Finite Difference scheme. A central difference approximation is used for the second derivative terms, while a backward difference approximation (upwinding) is used for the first derivative terms. Dirichlet boundary conditions are applied (refer Figure 4.1), specifying the temperature profiles at the inlet, outlet and wall of the combustor and using symmetry condition at the combustor centerline. A uniform temperature profile is assumed at the inlet, while experimentally determined profiles are used at the outlet and the wall. The finite difference form of the 2-D energy equations with square meshing of length $h$ are,

$$
\begin{align*}
\left[-4\alpha - hU_g x - hU_g y + \left( \frac{h^2 \varepsilon \sigma}{L_c \rho_g C_p g} \right) T_{m,n}^3 - \frac{h^2 n \dot{m}_F}{\rho_g} \right] T_{m,n} + \\
\alpha T_{m+1,n} + (\alpha + hU_g x) T_{m-1,n} + \\
\alpha T_{m,n+1} + (\alpha + hU_g y) T_{m,n-1} &= S_n
\end{align*}
$$

(4.63)

for all interior nodes, and

$$
\begin{align*}
\left[-2\alpha - hU_g x + \left( \frac{h^2 \varepsilon \sigma}{L_c \rho_g C_p g} \right) T_{m,n}^3 - \frac{h^2 n \dot{m}_F}{\rho_g} \right] T_{m,n} + \alpha T_{m+1,n} + \\
(\alpha + hU_g x) T_{m-1,n} &= S_n
\end{align*}
$$

(4.64)
for the symmetric boundary nodes. Here, $m$ and $n$ are the node index in $x$ and $y$ directions. In Eqs. 4.63 and 4.64 the right hand term $S_n$ is,

$$S_n = \left( -\frac{h^2}{\rho_g C_{p_g}} \right) \left\{ \sum_{j=1}^{N} \dot{\omega}_j h_{f_j} - m_{F} \left[ C_{p_L} (T_s - T_{D_i}) + h_{fg} - C_{p_g} T_s \right] \right\} + \left( \frac{h^2 \epsilon \sigma}{L_c \rho_g C_{p_g}} \right) T_w^4$$

The chemical source term $\sum_{j=1}^{N} \dot{\omega}_j h_{f_j}$ comes from the combustion of individual droplets and is considered a point-source in the calculations. A point-source approximation gives more accurate results if the grid size used in the calculations is much larger than the diameter of the droplet [141]. On the other hand, a refined grid is required to improve the resolution of gas phase calculations, which also increases the computer time. A trade off is therefore made and a grid size of 100 micron is considered in the DTM model. Droplets of less than 100 micron can be conveniently handled by the model.

The set of approximate finite-difference equations for the unknown gas phase temperatures at each node in the domain is obtained by applying Eqs. 4.63 and 4.64. This set is then solved by using Gauss-Seidel iterative scheme with a suitable termination tolerance. To accelerate the convergence, Successive-Over-Relaxation (SOR) method is also incorporated in the model.

### 4.7 Results and Discussion

Keeping in view the objectives of the present modeling efforts highlighted in section 4.4, three types of simulations were conducted: non-reacting flows without evaporation, non-reacting flow in uniform high temperature bath with evaporation and reacting flows. The results are presented and discussed in this section.
4.7.1 Non-Reacting Flow without Evaporation

The non-reacting flow results are discussed first and compared with the PDA experimental results. For the purpose of this study, evaporation of water droplets was ignored. The results discussed here are based on a spray taken as an ensemble of three groups of droplets, with mean diameters of: 10, 15 and 20 microns. These values were reasonably close to the droplet size distribution observed in the PDA experiments. To investigate the influence of acoustic field on droplet trajectories, the initial velocity of each droplet was assumed 3m/s. This is once again consistent with the values observed in PDA experiments for the 1.03 MPa feed line pressure. The initial droplet distribution was assumed uniform over the complete cone angle.

Figure 4.4 shows the droplet trajectories upon injection into a quiescent ambient pressure medium without any acoustic presence. The initial half-cone angle is 30°. As the spray travels downstream, it starts to spread radially outwards. The smallest sized droplets, because of their smaller inertial resistance to the aerodynamic drag forces, are the first to show the radial deviation, followed by the next higher size and so on. The outward movement is because the axial deceleration of the droplets is much faster than their radial deceleration.

The result of the acoustic field excitation on the spray evolution is shown in Figure 4.5. A 80 Hz cosine wave was used to introduce the acoustic velocity oscillation with peak amplitude of 0.125 m/s, which was the same as that obtained during PDA measurements at the same frequency. For this simulation, the injection of the droplets was phase-locked with the start of the acoustic cycle. As shown, the smaller droplets are the first to get affected. The droplets are forced to migrate radially outwards and their axial propagation is halted by the acoustic field. The maximum axial distance that the droplets can travel is proportional to their size. The affect of this radial movement is to modify the drop size distribution in the spray. The behavior of the spray as seen in Figure 4.5, qualitatively validates the experimental findings shown in Figures 3.19 and 3.20. The radial movement of smaller droplets under the acoustic field influence is the reason for increase in mean diameters measured at the
Figure 4.4: Droplet trajectories in the absence of unsteady acoustic field.

Figure 4.5: Droplet trajectories under 80Hz, 0.125 m/s acoustic velocity oscillations.
Figure 4.6: Effect of acoustic forcing on droplet oscillatory displacement. Simulation performed at 80Hz, 0.75 m/s acoustic velocity.

Figure 4.7: Effect of increase in acoustic forcing on droplet oscillatory displacement. Simulation performed at 80Hz, 1.50 m/s acoustic velocity.
centerline locations and decrease in these values measured at radially outward locations.

The effects of acoustic amplitude are shown in Figure 4.6 and 4.7 where droplet trajectories for an injection angle of 30° are plotted. As may be noted higher acoustic amplitudes result in larger oscillatory excursions of the droplets and cause the radial migration to occur earlier.

Simulations were also performed for the case when 2 m/s swirling co-airflow was introduced, through a 45° axial swirler, to the spray under acoustic forcing. The results are shown in Figure 4.8. Values of acoustic velocity amplitude and frequency and droplet initial velocities were 0.125 m/s, 80 Hz and 3 m/s respectively. It is observed that the swirl air cancels part of the acoustic field dominance observed in Figure 4.5. Under the influence of the swirl velocity, the spray is found to be more radially compact and more uniformly distributed.

Figure 4.8: Effect of swirling 2 m/s co-airflow on droplet trajectories, when spray is also subjected to excitation of 80 Hz, 0.125 m/s acoustic velocity.

Figure 4.9 shows the spray behavior when the acoustic velocity amplitude and swirl co-airflow velocity are of the same order of magnitude. Acoustic velocity amplitude of 3 m/s was selected. As can be observed, this results in spatial bands of high droplet densities. It
Figure 4.9: Spray behavior when acoustic velocity and swirl co-airflow velocity are comparable. Airflow velocity is 2 m/s and acoustic velocity is 3 m/s at 80 Hz.

may also be noted from the figure that the dense pockets of droplets appear in intervals, which correspond to the acoustic wavelength (approximately 19mm) at an excitation frequency of 80 Hz.

The results discussed in this section show the same trends observed in the non-reacting PDA experiments and validate the conclusions drawn from the experiments, that is: under the influence of an acoustic field the droplets migrate radially outward; acoustic excitation causes modifications in droplet trajectories and size and density distribution; the larger size droplets offer higher resistance to acoustic forcing, whereas the smaller size droplets flow the acoustic field oscillations more closely; and acoustic oscillations cause oscillations in droplet velocities and displacements.
4.7.2 Non-Reacting Flow in Uniform High Temperature Bath with Evaporation

Droplet dynamics with full combustor conditions and evaporation are discussed next. In the simulations flow was still considered non-reacting and the droplets are assumed to be injected in a bath of uniform temperature. The temperature values were taken from the combustor tests, discussed in Chapter 2.

Figure 4.10: Trajectory modeling results for a drop size of 50 micron injected in a high temperature environment at four values of phase lag relative to the acoustic cycle. Droplet trajectories under stable, $1^{st}$ and $2^{nd}$ unstable operational modes are represented by blue O, red Δ and green ∇ respectively.

Figure 4.10 shows the modeling results for the high temperature case. Once again, a cosine wave was used to introduce the acoustic velocity oscillations, with amplitudes corresponding to the limit-cycle amplitudes in the combustor. Monodispersed spray was assumed and 50
micron droplets was injected in a uniform temperature bath of combustion gases, at phase lags of 0, 90, 180 and 270 degrees relative to the start of the acoustic cycle. Results for three GER values: 0.40, 0.60 and 0.90, corresponding to the three stability regimes of the combustor are shown in the figure. The figure depicts the flame boundary over one acoustic period. Under stable condition (blue O) the flame is tulip-shaped and there is no change in its shape over the cycle. Under intermediate acoustic levels (red ∆ representing the 1st unstable regime), the flame has a larger spread and is seen to change locations over the acoustic cycle. At high levels of acoustic amplitudes (green ∇ representing the 2nd unstable regime), it is seen that for half of the acoustic cycle, the axial traverse of the droplets is reversed from the injection point. This retardation of the droplets in a preheated environment may lead to a high density of fuel that gets pre-vaporized, well-mixed and possibly dilute before it is convected into the reaction zone during the other half of the acoustic cycle. This extreme condition explains the flame behaviors observed in the 2nd unstable regime of the combustor where rise in temperature was observed upstream of the injection point.

With the model qualitatively depicting the behavior of the combustor, a parametric analysis was conducted to study the effects of changes in mean drop injection velocity and fuel modulation on the droplet trajectories over one acoustic period. Such an analysis is helpful in determining how the fuel modulation can be utilized to suppress the acoustic influence in the combustor. The combustor operating conditions in the 1st unstable regime (at GER of 0.6) were selected for the simulations. Figure 4.11 shows the changes in droplet trajectories and thus the flame location when the mean droplet velocity is increased from 5 to 12.5 m/s. As the relative magnitude of the droplet velocity increases compared to the acoustic velocity, a larger radial spread is noted. As for the axial travel of the droplet, the droplets burn completely as close as 20 mm to as far as 120 mm from the injection plane, depending on their injection instant with respect to the start of the acoustic cycle.

The changes in droplet trajectories as a result of fuel modulation in a combustor with acoustically excited flow field were studied next. The fuel modulation was simulated by adding an oscillating component to the mean droplet injection velocity. The frequency of
Figure 4.11: Effects on droplet trajectories due to increase in injection velocity.

Figure 4.12: Effects of fuel modulation without phase lag on droplet trajectories.
oscillation was kept the same as the acoustic frequency and the phase of the fuel modulation with respect to the acoustic oscillations was varied. As before the operating conditions at GER of 0.6 were used. Figure 4.12 shows the results of fuel modulation without any phase lag. The droplet velocity was kept at 5 m/s. The modulation was varied from 10 to 100%. For comparison, the no modulation case is also shown. No significant difference in trajectories is noted even with 100% modulation. Keeping the modulation at 100%, an injection phase lag was introduced and the results of this simulation is shown in Figure 4.13. Substantial difference in the trajectories are noted with droplets showing deeper axial penetration as compared to no phase delay case.

The discussion based on Figure 4.11 through Figure 4.13 has shown that the droplet trajectories and thus the location of their burning can be significantly modified by varying the relative velocity of the droplets and by modulating fuel with some phase delay. As shown in Figure 4.14 with an optimum selection of these input parameters, same droplet trajectories may be maintained (to a considerable extent) over the acoustic cycle and thus the influence
Figure 4.14: Droplet trajectories over an acoustic period with injection velocity of 9 m/s, 200% modulation and a phase delay of 15°.

of acoustic oscillations may be suppressed. The values for droplet velocity, modulation and phase lag chosen for this simulation were 9 m/s, 200% and 15° respectively.

4.7.3 Chemically Reacting Flow

The droplet dynamics under reacting flows were next studied using Droplet Trajectory Model. Monodisperse spray was assumed. Results from two of the tested cases with droplet diameter of 50 and 100 micron are presented here. Uniform temperature profiles were assumed at the boundaries. Although swirling combustion air flow was considered, the radial variations in axial and tangential velocity components were ignored. The acoustic velocity, as before was simulated by a cosine waveform with amplitude and frequency values of 25 m/s and 114 Hz, as observed in combustor experiments at GER of 0.6. Similarly all other operating conditions were also taken from the reacting flow experiments at this GER (refer
Figure 2.25 through Figure ??). The results are presented to show the droplet response when it is injected at four different instances in the acoustic cycle at an half-cone angle of 30° with reference to the axis of symmetry.

The results for 50 micron droplet are discussed first. The changes in droplet diameter over its lifetime are shown in Figure 4.15. Although the droplet injected at the start of the acoustic cycle shows a lower lifetime, the overall difference in lifetimes is not substantial. Droplet lifetime remains independent of the injection instant. This is because the droplet lifetimes are less than a quarter of the acoustic period at 114 Hz (between 1.5 and 1.8 msec). Thus the droplets completely evaporate and burn before the occurrence of any acoustic effects.

Figure 4.15: Life histories of 50 micron droplet when injected at different phase delays.

The droplet trajectories are shown in Figure 4.16. Since the trajectories are good indicators of the flame location, the axial and radial movement of the flame may be noted over the lifetime of the droplet in an acoustic cycle. The difference in trajectories is once again not significant as the droplets burn quickly within 20 mm axially and 3.25 mm radially of the
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Figure 4.16: Trajectories of 50 micron droplet over one acoustic cycle.

Figure 4.17: Mass burning rate of 50 micron droplet when injected with different phase delays.
injection location.

Figure 4.17 shows the overall mass burning rate of the spray calculated using the mass burning rate of a single droplet and the number density of the droplets in the spray, which was determined from the global fuel mass flow rate. Since the model ignores any accumulation of vapors between the droplet surface and the flame front, this burning rate is also equivalent to the droplet evaporation rate. The burning rate shows an initial increase and later fall off, almost at the end of the droplet lifetime. The variations in burning rate over time is negligible, which again is due to the fact that the droplet lifetime is quite short compared to the acoustic period. However, difference is noted in the axial location where maximum burn rate occurs.

Keeping all operating conditions same, simulation was also performed for droplets with diameter of 100 micron. Figure 4.18 indicates a rise in droplet lifetimes compared to the 50 micron case. The lifetimes are now comparable to the acoustic period and thus a substantial

![Life histories of 100 micron droplet with different phase delays](image)

**Figure 4.18:** Life histories of 100 micron droplet when injected with different phase delays.
Figure 4.19: Evolution of evaporation rate constant of a 100 micron droplet.

difference in their life histories is noted depending on their injection instant in the acoustic cycle. Over one cycle, the lifetime varies as much as 60%. It is also noted that the rate at which the droplet diameter decreases is not constant over its lifetime, even for any one given instant of injection. The reason as shown in Figure 4.19, is due to the changes that occur in the evaporation rate constant as a result of oscillating flow field.

Droplet trajectories for the four different injection instances are shown in Figure 4.20. Both radial and axial oscillations in the flame location are noted. For the results shown, the radial and axial excursions are 52% and 47% respectively. The radial and axial penetration is also larger compared to the 50 micron case due to the larger survival time of the droplets. The corresponding droplet axial velocity profiles are shown in Figure 4.21. The initial step response and the oscillating nature of the profiles are noted, which result due to the acoustically influence convective flow field.

The mass burning rates of the 100 micron droplet over the acoustic cycle, as a function of time and axial location are shown in Figure 4.22 and 4.23 respectively. Although the trends
CHAPTER 4. MODELING OF DROPLET DYNAMICS

Figure 4.20: Trajectories of 100 micron droplet over one acoustic cycle.

Figure 4.21: Axial velocity profiles of 100 micron droplet injected with different phase delays.
Figure 4.22: Evolution of mass burning rate of 100 micron droplet.

Figure 4.23: Axial variations in mass burning rate of a 100 micron droplet when injected at different phase lags.
are similar, variations with respect to maximum values, location and time are noted. Useful information with regards to implication on thermoacoustic instabilities may be deduced from the results presented in these figures. First, from the figures it is noted that the peak-to-peak fluctuations in maximum burning rate, due to acoustic influence, is 0.31 kg/h. This amounts to 14% of the value determined under no acoustic presence with all other conditions remaining same. With appropriate scaling between burning rate and heat release rate, this implies the existence of acoustically driven oscillating heat release rate.

The variations in mass burning rate in the radial direction is shown in Figure 4.24. From the profiles shown here and in Figure 4.23, the regions of maximum burning rate (or alternately heat release rate) can be identified in the overall flame domain. These regions are highlighted in Figure 4.25 with red boundaries. The length and the radial thickness of the maximum burning rate regions are 41% and 30% of the overall flame length and radial spread respectively. The maximum burning rate region in comparison to the overall dimensions of the experimental combustor is shown in Figure 4.26. All dimensions have been respectively

![Figure 4.24: Radial variations in mass burning rate of a 100 micron droplet when injected at different phase lags.](image-url)
Figure 4.25: Regions of maximum mass burning rate in the flame.

Figure 4.26: Regions of maximum mass burning rate in comparison to the resonance modes in the combustor.
normalized by the length and the diameter of the combustor. Also shown in the figure are the acoustic modes for the fundamental resonance frequency and its next two higher harmonics. As can be noted, the maximum heat release associated with maximum burning takes place near the location of maximum compression. Such a situation is susceptible to thermoacoustic instabilities, as was highlighted in Chapter 1.

Figure 4.27: Phase relationship between acoustic velocity, acoustic pressure and burning rate.

The information obtained from Figure 4.22 regarding the timing of the maximum burning rate may be used to determine the phase difference between the heat release rate and acoustic oscillations. As can be noted from the figure, the occurrence of the maximum burning varies in time and depends on the injection phase. Taking into account both these time delays, it is found that the average time lag between the start of the acoustic cycle and maximum burning rate is 6.63 msec, which is almost three quarters of the acoustic period at the excitation frequency of 114 Hz used in the simulation. Using this time lag value the burning rate oscillation is plotted in Figure 4.27. Also shown are the simulated acoustic velocity and the corresponding acoustic pressure. As can be noted, the acoustic pressure and the
burning rate are in phase, which is a favorable condition for the self-excited thermoacoustic instability.

4.8 Investigation of Secondary Drop Breakup due to Acoustics

Secondary atomization or drop breakup and its effects on spray combustion characteristics were briefly outlined in Chapter 1. The influence of aerodynamic forces with respect to the surface tension and viscous forces in the droplet, dictate the mechanism and timing of the secondary breakup. Studies show that secondary breakup is almost always preceded by deformation and oscillation of the droplet [142]. As the aerodynamic forces (measured in terms of droplet relative velocities) intensify, the droplet is no longer able to maintain its spherical shape. This results in oscillatory deformation, which is followed by breakup.

A number of models have been developed to simulate droplet deformation and secondary breakup. The two widely used models that have been successfully used in comprehensive spray combustion codes are the Taylor Analogy Breakup (TAB) model and Droplet Deformation and Breakup (DDM) model proposed by O’Rourke and Amsden [12] and Ibrahim et al. [143] respectively. Certain modifications have also been made in these models to account for the original limitations (see for example Park and Yoon [144] and Liu et al. [145]).

The TAB model is formulated based on the assumption that the droplet oscillatory deformation is analogous to a damped forced harmonic oscillator like a spring-mass-damper system. The spring restoring force and the damping force are replaced by the surface tension and viscous forces in the droplet respectively, while the external force on the mass is substituted by the gas aerodynamic force. Spherical symmetry of the droplet is also assumed. The model is simple to implement and yet have been found to predict sizes and velocities of the resulting drops that are consistent with the experimental measurements. TAB model however, has
two main limitations. One, it can only track the fundamental mode of oscillations and two, the changes in drag due to instantaneous changes in droplet shape are not catered. The first limitation is nonetheless justified by realizing that the fundamental mode is the longest lived and most important mode of oscillation. The droplet breakup criterion in TAB model is when the amplitude of droplet oscillations at the north and south poles is equal to droplet radius.

The DDM model accounts for some of the limitations of TAB model. Drop dynamics is formulated in terms of center of mass of half droplet and it is assumed that the droplet deforms from an initial spherical shape to an oblate spheroid and the drag effects associated to this shape change is taken into consideration. Unlike the TAB, the DDB model suggests that the breakup occurs at different droplet sizes for different Weber number $W_e$. Thus the breakup criterion is Weber number dependent. The biggest limitation of DDB model is that it is only applicable for $W_e > 20$.

### 4.8.1 Model Formulation

During the discussion on combustor characterization (Chapter 2) it was shown that the acoustic levels reach very high amplitudes. Review of experimental work by Anilkumar et al [36], indicates that such high acoustic levels may cause a breakup of droplets. Based on this observation it can be hypothized that the combustor’s transition from the 1st to the 2nd unstable regime is due to acoustically caused secondary breakup. An analytical study is therefore, conducted to investigate this hypothesis. Since the Weber numbers encountered are less than 20, a TAB model is adapted in this study. In addition, instantaneous variations in droplet drag due to drop shape changes is also considered, as suggested by Park and Yoon [144].

The model formulation is initiated from a 1-D equation for droplet dynamics of the form,

\[
m_D \ddot{y} + \mu_L D_o \dot{y} + \sigma_L y = F_{ext}
\]  

(4.65)
Figure 4.28: Spherical droplet treated as a harmonic oscillator.

Here, $m_D$ is the droplet mass, $D_o$ is the droplet initial diameter, and $\mu_L$ and $\sigma_L$ are the liquid viscosity and surface tension respectively. In the equation, $y$ represents the oscillatory deformation at the north and south poles of the droplet, as shown in Figure 4.28. The external force $F_{ext}$ is made up of an aerodynamic drag force $F_d$ on the droplet due to its velocity relative to the gas phase velocity, and a pressure force $F_P$ due to acoustic pressure of magnitude $P'$. Thus,

$$F_{ext} = F_P - F_d = P' A_c - \frac{1}{2} \rho_g |U_r|^2 C_d A_c$$

(4.66)

Where $A_c$ is the droplet frontal area. Substituting Equ. 4.66 and $m_D = \frac{\pi D_o^3 \rho_L}{6}$ in Equ. 4.65 and rearranging,

$$\ddot{y} + \left( \frac{6\mu_L}{\pi D_o^2 \rho_L} \right) \dot{y} + \left( \frac{6\sigma_L}{\pi D_o^3 \rho_L} \right) y = \left( P' - \frac{1}{2} \rho_g |U_r|^2 C_d \right) \left( \frac{3}{2D_o \rho_L} \right)$$

(4.67)

Introducing a non-dimensional deformation $z = y/D_o$, Equ. 4.67 can be written as,

$$\ddot{z} + \left( \frac{6\mu_L}{\pi D_o^2 \rho_L} \right) \dot{z} + \left( \frac{6\sigma_L}{\pi D_o^3 \rho_L} \right) z = \left( P' - \frac{1}{2} \rho_g |U_r|^2 C_d \right) \left( \frac{3}{2D_o \rho_L} \right)$$

(4.68)
The $C_d$ term is the drag coefficient for a solid sphere, corrected to account for the changes in droplet deformation [144]. Thus,

$$C_d = C_{dsphere} (1 + 2.632z) \quad (4.69)$$

Here,

$$C_{dsphere} = \frac{64}{R_e D}$$

A close form solution to Equ. 4.68 is readily available for the specified initial values of $z$ and $\dot{z}$. This is of the form,

$$z(t) = z_1 e^{r_1 t} + z_2 e^{r_2 t} + \frac{F}{k} \quad (4.70)$$

where,

$$z_1 = z(0) - z_2 - \frac{F}{k}$$

$$z_2 = \frac{(z(0) - F/k) r_1 - \dot{z}(0)}{r_1 - r_2}$$

$$F = \left( P' - \frac{1}{2} \rho g |U_r|^2 C_d \right) \left( \frac{3}{2D_o^2 \rho L} \right)$$

In Equ. 4.70 $r_1$ and $r_2$ are the roots of the characteristic equation,

$$mr^2 + br + k = 0 \quad (4.71)$$

with

$$m = 1$$

$$b = \left( \frac{6\mu \rho L}{\pi D_o^2 \rho L} \right)$$

$$k = \left( \frac{6\sigma \rho L}{\pi D_o^3 \rho L} \right)$$

The breakup criterion in terms of the non-dimensional deformation $z$ is,

$$z_{ss} \geq 0.5 \quad (4.72)$$

where, $z_{ss}$ is the steady state value of $z$. 
4.8.2 Results

Under the influence of a 1-dimensional acoustic field, a droplet behaves like a harmonic
oscillator and demonstrates an oscillatory deformation. If the influence of acoustic pressure
force is dominant relative to the external drag force and the internal restoring surface tension
force, droplet breakup occurs.

Time traces of oscillatory deformation of kerosene droplets under the above scenario are
shown in Figure 4.29. Non-dimensional deformations of four different droplet sizes are shown
under fixed acoustic pressure and convective drag forces. For the presented results, droplet
velocity of 5 m/s was assumed. The temperature at which the liquid properties were deter-
mined as well the acoustic pressure and the convective flow conditions were taken from the
combustor experiments at a GER of 0.6 (refer Figure 2.25 through Figure ??). It is noted
from Figure 4.29 that larger droplets demonstrate higher oscillations in their deformation
with lesser degree of damping as compared to the smaller droplets. As a result their steady

![Figure 4.29: Oscillatory deformation of different size droplets under the same external acoustic and other operating conditions.](image-url)
state deformation may exceed the breakup criterion limit of 0.5. Thus larger droplets are more susceptible to secondary deformation due to acoustic pressures.

For the same four droplet sizes, a number of such simulations were performed over the entire range of tested combustor conditions (as were discussed in Chapter 2). The results of the analysis are summarized in Figure 4.30. The non-dimensional deformations are plotted versus the GER. The GER values here, only serve as indicators for the corresponding values of the sound pressure level, acoustic velocity and relative droplet convective velocities. The droplet breakup criterion, as described earlier, is the non-dimensional deformation value of 0.5. The influence of combustor acoustics on the droplet breakup, relative to the convective flow is evident. The plot shows that the smaller the mean size of the droplets, the higher the critical value of GER at which the breakup occurs. These results thus, qualitatively verify the experimental findings shown in Figure 2.22, where, for a smaller FN injector (which produce smaller mean size droplets) the 2nd unstable regime (characterized by droplet secondary breakup) is reached at a higher value of GER.
4.9 Conclusions from Droplet Modeling Studies

The modeling effort was undertaken to investigate the dynamics of sprays under the influence of acoustic flow field and also to find plausible physics based explanations for the experimental observations. Based on the results and discussion presented in the preceding sections, the main conclusions from the investigation are as follows:

1. Most of the observations noted during the reacting flow combustor experiments and the non-reacting flow PDA experiments have been qualitatively validated.

2. Smaller droplets follow the flow field oscillations more closely as compared to larger droplets, which demonstrate inertia imposed tardiness.

3. Under the influence of oscillating acoustic field, the droplets migrate radially outwards and their axial traverse is halted. The maximum axial distance that the droplet can travel is proportional to their size.

4. Acoustic excitation thus, cause modifications in droplet trajectories and size and density distributions.

5. Acoustic oscillations cause high amplitude excursions in droplet velocity and displacement, which is proportional to the amplitude of the applied acoustic field.

6. Swirl co-airflow tends to cancel part of acoustic field dominance. Under the impact of the swirling velocity component, the spray is more radially compact.

7. When the swirl velocities and the acoustic velocities are of the same order of magnitude, droplets tend to form dense bands in intervals that correspond to the acoustic wavelength.

8. High levels of acoustic amplitudes cause the droplet to travel, evaporate and burn even upstream of the injection location.
9. Higher injection velocities cause radial spreading of the spray and cause substantial variations in the flame shape and location.

10. Axial and radial travel of the droplets depends on the injection lag with respect to the start of the acoustic cycle.

11. Fuel flow modulation alone without phase delay from the acoustic cycle has insignificant effect on the droplet dynamics. However, addition of phase delay profoundly affect the flame structure.

12. With the appropriate choice of injection velocity and modulation phase delay, the droplet trajectories and thus the location of their burning can be tailored to suppress the acoustic dominance.

13. Acoustically dominated flows affect the droplet dynamics only when droplet lifetimes are comparable to the acoustic period.

14. Over an acoustic cycle, the oscillating flow field cause variations in: droplet lifetime, radial and axial flame location, magnitude of the mass burning rate as well as the location and timing of the maximum heat release.

15. Acoustic oscillations cause oscillations in droplet burning rates.

16. Acoustic pressure forces cause oscillatory deformation of droplets. If the influence of acoustic pressure force is dominant relative to the external drag force and the internal restoring surface tension force, droplet secondary breakup occurs.

17. Larger droplets demonstrate higher oscillations in their deformation with lesser degree of damping as compared to smaller droplets. Thus larger droplets are more susceptible to acoustically driven secondary breakup.
Chapter 5

Characterization of Pulsed Sprays

As discussed in Chapter 1, high-bandwidth fuel modulation has recently been promoted as the preferred method for active control of spray combustion instabilities. This technique targets the very essence of heat-acoustic coupling to successfully attenuate the high amplitude instabilities. Achieving optimum control authority over the unstable operating range of combustors is a challenge and requires detailed understanding of spray behavior during modulation.

The limitations of more traditional spray sizing techniques like Phase Doppler Anemometry (PDA) in performing dynamic analysis was highlighted in Chapter 3. Because of the randomness of sampling, such methods require post processing algorithms to extract meaningful information from the acquired data, which in turn is likely to induce bias in the analysis. Thus, there is a need to explore the feasibility of better techniques for quantifying dynamic spray behaviors. A pilot effort was undertaken in collaboration with Fluid Systems Laboratory to demonstrate the ability of a novel experimental, high frequency and time-resolved, global optical flow diagnostics to characterize pulsed spray flows. Time-Resolved Digital Particle Image Velocimetry (TRDPIV) implementation employed for the dynamic investigation of the modulated spray is discussed in this chapter.
5.1 Time Resolved Digital Particle Image Velocimetry

Experimental efforts for spray characterization have thus far mainly been restricted to steady sprays and very little has been done to capture the dynamics of temporally modulated sprays. The past decade has seen the widespread use of Digital Particle Image Velocimetry (DPIV) in polydispersed multi-phase flows, to provide global measurements of both particle size and velocity. These efforts combine size measurements with DPIV using either fluorescence-Mie scattering ratio [146] or interferometry principles [147] to carry out the sizing tasks. Boedec and Simoens [146] were able to accurately measure the velocity and size distribution in a high-pressure spray. However, their implementations required complicated experimental setups, multiple cameras, optical filters and calibration for the fluorescence intensity quantification. Damaschke et al. [147] used Global Phase Doppler (GPD), which is an interferometry based methodology, where the particle/droplet diameter is proportional to the number of fringes formed by out of focus particle images. Despite the high accuracy and the global character of their method, the optical resolution of the system was a limiting parameter, because of the out of focus effects.

Pereira et al. [148] introduced an out-of-focus method for size measurements based on a defocusing DPIV principle applied on liquid bubble flows, which enhanced the capabilities of conventional DPIV systems by allowing bubble/droplet size information in two-phase flows. An off-axis aperture, collecting light from a point source (bubble) was used to generate an out-of-focus image. By collecting the information of the light intensity, the particle pattern, and the blurriness for each image pair, the method resolved both the velocity and the size of the bubble. In addition, by analyzing the intensity of the defocused particle, the Airy disks that form, provided information for the out-of-plane velocity component. This method too, required multiple cameras in order to overcome the measurement ambiguities.

Since the above methods are limited to sampling rates in the order of 15-30Hz, they do not resolve the time depended characteristics of the flow. This constitute a major shortcoming; especially in the case of spray atomization, where the inherent unsteadiness of the flow is
the dominant parameter that governs the break-up dynamics. In such cases, sampling rates in the order of kHz are necessary.

Development of a Time-Resolved DPIV (TRDPIV) system with kilohertz sampling rate was reported by Abiven and Vlachos [149]. The capabilities of the DPIV system was extended to allow simultaneous velocity and size quantification of poly-dispersed multi-phase flows using direct imaging and a single camera configuration delivering a system as simple and robust as a planar two component TRDPIV. The complicated experimental approaches based on interferometer or fluorescence-Mie ratio are thus eliminated. The method provided planar image based droplet sizing using Mie scattering from DPIV measurements, with greater than 5KHz sampling rate.

In the effort discussed here, the TRDPIV was deployed to quantify the structure of a pulsed spray. The objective of the pilot experiment was to demonstrate the ability of the system to characterize the time depended characteristics of a polydispersed distribution of droplets by resolving their apparent size. However, the method is claimed to measure both the droplet size distribution as well as the droplet velocities, from the same recorded images.

Figure 5.1: Experimental setup for modulated spray characterization using TRDPIV.
5.2 Experimental Setup and Measurement Methodology

The experiments are conducted in a non-reacting quiescent environment. The experimental and diagnostic setups are shown in Figure 5.1 and 5.2 respectively. A 10 kHz pulsing Copper Vapor Laser was used to deliver a sheet of light with illumination energy of 5 mJ. The laser sheet was passed through a cylindrical lens and focused into a vertical plane parallel to the axis of the spray. A Phantom-IV CMOS Camera placed normal of the laser sheet captured the scattering intensity of water spray droplets with pixel resolution of 128x128, a frame rate of 10 kHz, and a total sampling time of 1 sec. A major limitation of any digital imaging implementation of a sizing method stems from the leakage (blooming) effect inherent in CCD technology sensors. Leakage is caused due to overexposed pixels, which subsequently

Figure 5.2: Schematic representation of the TRDPIV diagnostic setup.
overflow the excess charge to their neighboring pixels. As a result, the apparent image
diameter of a particle droplet or bubble does not correspond to the true image diameter.
CMOS technology eliminates the blooming effect, thus allowing resolution of a multi-phase
flow. More specifically, the camera used in this study delivers 100,000:1 blooming ratio with
no pixel-to-pixel spill over. In other words, the saturation of the CCD sensor is eliminated.
This feature is of great importance since it simplifies the experimental setup alleviating the
need for optical filters and/or fluorescence based imaging, enhances the signal-to-noise ratio
and more importantly, allows accurate shape and size quantification of droplets or bubbles
present in the flow. In addition, this camera is more sensitive than conventional CMOS
sensors and has comparable sensitivity to CCD’s. Specifically, the sensitivity of the sensor
with zero contrast and zero illumination is quantified with an equivalent ASA rating of 800.
Employment of CMOS imaging technology represents an essential element of this effort.

The laser and the camera were synchronized in order to operate in a single pulse per frame
mode. The camera is equipped with an internal shutter, which was set at 10 $\mu$sec in order to
reduce the ambient background light, thus increasing the signal to noise ratio of the images.
The laser pulse and the shutter trigger to the camera were synchronized and controlled with
a PC/workstation. A multifunction National Instruments 6025E data acquisition card was
used to collect the data. Two 20 MHz digital counters with timing resolution of 50 nsec
were used to generate the sequence of pulses necessary to accurately trigger the laser and
the camera. Once the laser and the camera received the trigger pulses, a feedback signal was
recorded in order to assure the accurate timing of the process. The overall timing uncertainly
of the control system, also accounting for the laser pulse jitter, was estimated in the order
of 1 nsec, which was negligible as compared to the spray velocity ranges encountered during
the experiments. The collected image information were processed to determine the droplet
diameters.

A full-cone simplex atomizer with FN 13.68, from Delavan Spray Technologies, was used in
the experiments with water as the working fluid. The water feed line pressure and the flow
rates were 0.69 MPa and 15.56 kg/h respectively. Spray modulations were obtained using
the actuation system described in Chapter 2. Measurements were recorded at 16% and 9% pressure modulations, relative to the mean pressure of 0.69 MPa, and at a pulsing frequency of 464 Hz. In order to determine the effect of frequency on spray dynamics, measurement were also conducted at 100 Hz with 9% pressure modulation. The test matrix is given in Table 5.1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Injection Pressure</th>
<th>Modulation</th>
<th>Modulation Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.69</td>
<td>16</td>
<td>464</td>
</tr>
<tr>
<td>2</td>
<td>0.69</td>
<td>9</td>
<td>464</td>
</tr>
<tr>
<td>3</td>
<td>0.69</td>
<td>9</td>
<td>100</td>
</tr>
</tbody>
</table>

The actual droplet diameters were calculated from the apparent, measured diameters. A dynamically adaptive threshold filter [149] and a local maximum intensity filter [150] served to identify the individual droplet images. Subsequently, a recursive nearest neighborhood search identified all the pixels that form each droplet, thus defining the droplet region of interest (ROI). For simplicity, the area was treated as a circle and thus the apparent diameter was estimated. Nevertheless, the method is not limited by spherical assumptions. An elliptical droplet shape can be determined and the ellipse axis can be quantified. It is worth mentioning that this apparent diameter corresponds to the cross sectional area of the droplet that intersects the laser sheet. Therefore, as the droplets move outside of the illuminated region, their apparent image changes. Once the apparent droplet image diameter is known, Lorenz-Mie theory provides the exact solution to the problem of scattering of light by a single homogeneous sphere of arbitrary size, and relates the light scattering intensity to the true particle/droplet diameter. However, since Mie solution is difficult to apply for practical applications, the diameter was thus approximated by geometrical-optics principles. Assuming illumination of a particle of diameter $d_p$ by a Gaussian intensity profile laser beam, the
recorded image of the particle $d_e$ is given by,

$$d_e^2 = M^2 d_p^2 + d_s^2$$  \hspace{1cm} (5.1)

Here $M$ is the magnification and $d_s$ is the diffraction limited spot diameter, which depends on the focal length number $f$, and wavelength of a coherent monochromatic laser light source $\lambda$ as,

$$d_s = 2.44 (M + 1) f \lambda$$  \hspace{1cm} (5.2)

In the discussed setup $\lambda$ was 512 nm.

### 5.3 Validation of Results by Comparison with PDA Data

Validation of the TRDPIV results was performed with steady flows in order to gain confidence on the data collection and processing methodology, prior to the dynamic modulated spray measurements. For comparison, independent experiments were performed using both TRDPIV and PDA under the same experimental setup and flow conditions. Two sets of measurements were conducted at the mean injection pressures of 0.55 and 0.69 MPa. The PDA equipment used was the same as described in Chapter 3. All measurements, for both the systems, were made in a 6.35mm x 6.35mm region with center located 48 mm downstream of the injection plane on the spray centerline. The corresponding image window for TRDPIV measurements was 512 x 512 pixels with the magnification $M$ and focal length number $f$ of 1.29 and 2.8 respectively. The results are shown in Figure 5.3 and 5.4 for the two injection pressures respectively. As noted, excellent agreement is obtained in the diameter distributions and number densities. The TRDPIV measured mean diameter shows a 10% and 14% higher value than the PDA data for the two injection pressure cases respectively.
Figure 5.3: Comparison between TRDPIV and PDA acquired spray sizing data at an injection pressure of 0.55 MPa.

Figure 5.4: Comparison between TRDPIV and PDA acquired spray sizing data at an injection pressure of 0.69 MPa.
Figure 5.5: Time averaged drop diameter distribution for the modulated spray cases. Dotted line represents the weighted mean value.
CHAPTER 5. CHARACTERIZATION OF PULSED SPRAYS

5.4 Results and Discussion

Using TRDPIV, all dynamic measurements under modulated sprays were made in a 128x128 pixel window located 70 mm downstream of the injection plane and 40 mm radially outward from the spray centerline. The magnification $M$ and focal length number $f$ used for these measurements were 1.3 and 1.8 respectively. The recorded particle diameter $d_e$ was obtained using 2000 frames acquired at a rate of 10 kHz. The statistical method employed was the least square volume fit scheme. Figure 5.5 shows the time averaged particle diameter distribution for cases 1 through 3. For the two 464 Hz modulation cases there is no distinguishable difference in either the distribution or the weighted average diameter. In comparison, for the 100 Hz modulation the size distribution is spread out towards the higher value and thus the average diameter shows an increase of about 8%. The distribution difference between the 464 and 100 Hz cases is because of the spray interaction with the quiescent gas-phase. The lower frequency modulation induces comparatively higher scale vortices, which tend to cluster the smaller sized particles towards spray centerline [151], leaving the larger droplets in the region of measurement.

The power spectra of diameter variations for the cases 1 to 3 are shown Figure 5.6. As can be noted the TRDPIV system is able to capture the dynamic response of the spray at the frequency of modulation. The magnitude of rms fluctuations is not very significant, since the amplitude of applied modulation was small. The differences in rms amplitude at the preferred frequencies can once again be attributed to the comparative scale of the coherent structures formed by the spray-gas coupling. It is also noted that the diameter dynamics is a function of both the amplitude and frequency of modulation. As the modulation amplitude is decreased from 16% to 9% mean injection pressure (between case 1 and case 2 at a constant frequency of 464 Hz), the response reduces by 15%. On the other hand, if the modulation frequency is reduced from 464 Hz to 100 Hz (between case 1 and case 3 at a constant pressure modulation of 9%), the rms diameter increases by 35%.

Variations in the drop size distribution over a modulation cycle are shown in Figure 5.7.
Figure 5.6: Power spectra of droplet diameter dynamics for the modulated spray cases.
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Figure 5.7: Cyclic variation in spray size distribution for Case 2 with 9% modulation at 464 Hz.

Figure 5.8: Cyclic variation in spray size distribution for Case 3 with 9% modulation at 100 Hz.
and Figure 5.8 for the two modulation cases 2 and 3 respectively. The distributions were phase averaged over 100 cycles. The results are consistent with the observations from the power spectra of Figure 5.6. At 464 Hz modulation, (Figure 5.7) there is negligible change in the droplet size distribution over the modulation cycle. In comparison, for the 100 Hz modulation the size distribution shows distinct variation over the cycle, as seen in Figure 5.8. The weighted mean size variation over the cycle is however, found to be the same in both cases.

5.5 Conclusions from Modulated Spray Study

The objective of the experimental efforts was to test the application of Time-Resolved DPIV method in quantifying the size dynamics of modulated sprays. The main conclusions drawn from the results presented in the chapter are summarized below:

1. TRDPIV is able to capture the size dynamics of modulated sprays in the tested range of modulation amplitudes and frequencies. It therefore, has the potential to be a valuable tool in investigating dynamic response of modulated sprays.

2. The diameter distribution, mean value and number density determined from TRDPIV agree to an excellent level with the PDA acquired data under same steady flow conditions.

3. Spray response in terms of drop size and size distribution is a function of both modulation amplitude and frequency.

4. The response is more prominent at higher modulation amplitudes and at lower frequencies.

5. From the results presented here and the explanation given by Chang et al. [151], it is found that the size distribution is also dependent on the degree of coupling between the
modulated spray and the entraining air, which manifest in the formation of coherent structures.
Chapter 6

Conclusions and Recommendations

Forman Williams narrates in one of his publications [152] that the first time he was exposed to the full equations describing the chemically reacting flows, he had said to himself,

Surely there is enough there to occupy me for a lifetime.

Spray combustion in an acoustically induced oscillating flow field is perhaps one of the toughest problems in thermo-fluids. It therefore, warrants a thorough investigation of each and every process involved and the ‘lifetime’ observation by Williams certainly appears to be true.

The objective of my research efforts conducted in the past three-and-half years of ‘my lifetime’ was to demonstrate the influence of acoustically dominated flows on spray and spray combustion dynamics. Both experimental and numerical modeling studies were conducted. The main findings from these investigative efforts are summarized here, followed by some recommendations for any future work in these areas.
6.1 Summary of New Knowledge and Consequences

The findings from each individual study conducted during the course of the research were presented at the end of the respective chapters. The highlights of these findings are recapitulated here from the perspective of explaining the overall characteristics of the combustor under the tested operating conditions.

The presence of an acoustic field influences the spatial and temporal dynamics of sprays, thus affecting the spray combustion characteristics and possibly leading to self-excited thermoacoustic instabilities in the combustor. As was shown in Chapter 3, the acoustic excitation modifies the droplet size and velocity distributions inside the spray. It also cause oscillations in droplet velocities, which result in oscillatory excursions of the droplets. The response of the droplets is linear to the acoustic forcing. Higher amplitudes of acoustic forcing cause higher variations in droplet diameter and velocity distributions. However, the response of the droplets is dependent on the frequency of excitation and their location in the spray. The response is higher at lower excitation frequencies. The cut-off frequency, beyond which the response is negligible, depends on the relaxation times associated with the droplet diameters. The cut-off frequency is lower for larger droplets and vice versa. Similarly, different regions of spray respond by various degrees to the acoustic forcing depending, once again, on the average droplet size in that region. The response of the droplets is also dependent on the spray injection pressure and thus on the droplet velocities relative to the acoustic velocities. For higher relative velocity droplets the response is lower and vice versa. Also the presence of swirling co-flow air suppresses the influence of longitudinal acoustic field.

The modeling results of Chapter 4 indicate the physics governing the spray response to acoustic forcing. The smaller droplets, because of their lesser inertia, follow the acoustic oscillations more closely as compared to larger droplets. Under the acoustic influence the droplets migrate radially outward and their axial propagation is halted. The penetration of the larger droplets is more than the smaller ones. The distribution of droplets in the spray also depend on the level of acoustic dominance compared to other relative convective flows.
Periodic dense regions of higher droplet concentrations occur when the swirling convective flow velocities and the acoustic velocities are of comparable magnitudes. Under extremely high acoustic levels, the axial movement of the droplets is even reversed. As a result, they either burn upstream of the injection location or get pre-vaporized before being accelerated downstream in the combustion zone. The high level of acoustic oscillations also cause secondary droplet breakdown, especially when the relative convective velocities are low compared to the acoustic velocities. Modeling results also indicate that the acoustic oscillations in addition to modifying the droplet trajectories, also induce oscillations in the burning rate of droplets. The associated oscillatory heat release rate couples with the acoustic oscillations to cause favorable conditions for self-excitation in the combustor.

In summary, the dynamic non-reacting measurements via Phase Doppler Anemometry (PDA) and separated flow Droplet Trajectory Modeling (DTM) show that the presence of an oscillatory acoustic field influences the temporal and spatial behavior of sprays. Not only the size and density distributions are modified, but so are the drop trajectories. Mean values of drop velocity and diameter both change in ways that are dependent on their spatial location from the spray centerline. More importantly, the acoustic oscillations induce oscillations to droplet velocities, axial displacements and burning rates and also cause drop breakup. The droplets’ initial velocities and size distributions, spatial location, and presence of external aerodynamic forces (swirling co-flow effect) also have a significant effect on the expected spray response.

In light of the above conclusions and the measurements of Chapter 2, the thermoacoustic behavior of the experimental combustor and the unique differences in the flame structures are explained next. At lower Global Equivalence Ratio (GER) values although there is no preferred oscillations in the heat release rate, nevertheless the mean thermal energy density in the combustor is high enough to excite the combustor at its quarter wave resonance frequency. At this operating condition, the acoustic losses in the combustor (due to acoustic radiation at the open end, due to heat diffusion at the combustor walls and due to the presence of spray fuel in the combustor) prevent the combustor from going unstable. The
acoustic amplitudes are negligible and thus, in this operating mode, the flame stabilization and mixing is dominated by the swirling co-airflow.

As the GER value is raised, the amplitude of acoustic oscillations in the combustor also increase and its effects on the heat release rate oscillations start appearing at a preferred frequency equal to the combustor resonance frequency. Any further increase in GER, causes the acoustically driven heat release rate oscillations to couple with the combustor pressure oscillations. Here, the combustor enters the 1st unstable regime and the combustor exhibits even larger limit-cycle acoustic oscillations. Under this unstable condition, the acoustic velocity amplitudes are comparable to the convective flow velocities. The acoustic dominance is therefore, just enough to adversely effect the fuel-air mixing and cause the formation of high droplet concentration pockets. This results in highly luminous flame due to diffusion burning of these droplet pockets.

Further increase in GER results in further increase of acoustic amplitudes and decrease in relative convective velocities. This situation causes secondary breakup of droplets. The smaller droplets thus formed, get swept upstream of the injection plane under the high acoustic effects for part of the acoustic cycle. Here at the upstream location, some of the droplets burn and the rest are pre-vaporized before being swept back downstream in the combustion zone during the remaining half of the acoustic cycle. The high acoustic amplitudes not only enhance the pre-vaporization, but also improve vapor-air mixing to give the flame the pale-blue, pre-vaporized, well-mixed appearance observed in the 2nd unstable operating mode of the combustor. This also results in reduction of NO emissions as observed in reported experiments.

In conclusion, the net dominance of acoustic oscillations relative to the other convective combustor flows is the primary reason for the onset and sustenance of the thermoacoustic instability and for the differences in the unique flame response over the tested operating conditions.

The research effort was able to demonstrate the effects of the influence of thermoacoustic
oscillations on the dynamics of sprays and spray combustion. These effects have major implications on the operation of Lean Direct Injection (LDI) combustors where the intention is to achieve effective atomization leading to abrupt vaporization and uniform vapor-air mixing to avoid high temperature pockets in the combustor. As established from the research, presence of larger droplets cause acoustic driven oscillations in burning rate, which in turn have the potential to cause self-sustained thermoacoustic instabilities in LDI combustors. The experimental investigations also demonstrated the exclusive occurrence of thermoacoustic instabilities on the richer side of the stable operation, which is very untypical in lean combustors.

During the course of the research efforts, two novel measurement and analysis techniques were successfully developed/applied. The first is the use of Discrete Fourier Transform in a method developed to carryout dynamic frequency analysis of unevenly sampled data acquired via Phase Doppler Anemometry (PDA). The second is the application of high frequency Time-Resolved Digital Particle Image Velocimetry (TRDPIV) to characterize the dynamics of modulated sprays.

6.2 Recommendations for Future Work

In pursuance of the research objectives, both experimental and modeling investigations were conducted to study the spray combustion behavior under thermoacoustic oscillations and primary fuel modulation. During this course certain areas were identified, which may be explored in future endeavors to supplement the current findings. The recommendations are as follows:

1. The differences in flame response in the three identified stability modes of the combustor were attributed to the relative dominance of the acoustic oscillations. One of the ways in which the effects of these oscillations manifest themselves is in the degree of vapor-air mixing. Planar Laser-Induced Fluorescence (PLIF) may be deployed to
identify vapor concentration regions and the quality of mixing in the three stability regimes of the combustor.

2. The spray response to acoustic forcing and injection pressure modulations were studied under non-reacting conditions. Such studies may be extended to full combustor conditions to investigate the spray dynamics under reacting flows. TRDPIV is recommended since particle tracking, velocity measurements and drop sizing can all be performed from the same collected images.

3. Analytical justification were given for the secondary breakup of droplets under high amplitudes of acoustic oscillations. Experimental verification may be conducted using either PDA or TRDPIV. Development of a test facility with acoustic levels as high as those experienced in the reacting flow experiments is impractical. It is therefore, recommended that the upstream flow-conditioning section of the combustor, which has been found to demonstrate high amplitude acoustic oscillations during the unstable combustor operation, may be utilized for this purpose after necessary modifications.

4. The effects of acoustic oscillations on the spatial and temporal burning of droplets were numerically demonstrated using a single droplet burning model and the overall spray response was postulated. It is recommended that a spray model based on the Rosin-Rammler or Nukiyama-Tanasawa spray distribution functions may be incorporated in the trajectory model.

5. Open-loop forcing was adopted in the Droplet Trajectory Model (DTM) to investigate the effects of the acoustic oscillations on droplet burning rate. It is recommended that an acoustic model may be incorporated in DTM to study the closed-loop effects of acoustic-burning rate coupling.

6. It was shown that TRDPIV is a valuable tool to characterize the dynamic response of modulated sprays. It is recommended that such measurements may be performed for the purpose of frequency response analysis. The analysis may be extended to
velocity measurements as well. From these analysis useful correlations for predicting the gain and phase of the fuel transfer function may be obtained. Such a knowledge is considered most essential in improving the effectiveness of active combustion control methodologies.
Appendix A

Flame Visualization to Determine Quality of Mixing

In order to determine the air and fuel mixing and its effects on the flame appearance, visual imaging was conducted using a commercial digital camera. This was done on a 190.5mm quartz combustor using Delavan WDB-0.40 (FN 1.37) solid-cone pressure swirl atomizer, under aerodynamic as well as mechanical swirling. For mechanical swirling, 30°, 45° and 60° axial swirlers were used. The effect of axial location of the swirlers was also studied. The results are shown in Figure A.1. During the experiments the fuel flow rate and the Global Equivalence Ratio were kept constant at 1.728 kg/h and 0.35 respectively. The appearance of the flame under maximum possible aerodynamic swirl of the combustor is shown in Figure A.1 (a). The flame appears quite compact but very luminous, indicating poor fuel and air mixing. Use of mechanical swirling improved the mixing as evident from the increasing blue regions in Figure A.1 (b). This image was taken using 30° axial swirler located 76.2 mm upstream of the atomizer tip. Considerable improvement in mixing was achieved, as shown in Figure A.1 (c), when the swirler was moved downstream closer to the nozzle at a distance of 6.35 mm from the tip. Use of 45° and 60° swirlers located at the downstream stream swirler position further enhanced the fuel-air mixing with optimum
results obtained with 45° swirler. This can be seen in Figure A.1 (d) and Figure A.1 (e).

Effects of the location of the injector nozzle with respect to the burner were also studied. It was found that the best fuel-air mixing, as evident by a symmetric and blue flame, is achieved when the atomizer tip was placed 10 mm below the quarl inlet and at 6.35 mm above the axial swirler.

Figure A.1: Flame Images at various configurations: (a) Aerodynamic swirling, (b) 30° mechanical swirler located 76.2 mm upstream of the atomizer, (c) 30° mechanical swirler located 6.35 mm upstream of the atomizer, (d) 45° mechanical swirler located 6.35 mm upstream of the atomizer, (e) 60° mechanical swirler located 6.35 mm upstream of the atomizer.
Appendix B

Equipment Information

Table B.1: Equipment information.

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<th>Equipment</th>
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<th>Model No.</th>
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<td>Worldwide Corp.</td>
<td>WW-1-14-31C</td>
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<td>Fuel flow meter</td>
<td>AW Company</td>
<td>JVA-10KL with HEF-1 sensor</td>
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<td>Omega</td>
<td>PX212-1KGV</td>
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<td>Fuel injectors</td>
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<td>National Instruments</td>
<td>PCI-MIO-16XE-10 card</td>
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<td>SCXI-1140 sample/hold module</td>
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<td>CYDAS TCP</td>
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<td>Vishay Measurement Group</td>
<td>2120B Amp/2110B power supply</td>
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<td>Dynamic signal analyzer</td>
<td>Hewlett Packard</td>
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<td>Photomultiplier tube</td>
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<td>Thermocouple ‘B’ type</td>
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<td>AA Lab</td>
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Bibliography


