Pilot Variability During Pilot-Induced Oscillation

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(ABSTRACT)

Pilot Induced Oscillations (PIO) are described as pilot-aircraft dynamic couplings which can lead to instability in an otherwise stable system. Previous and ongoing research has attempted to explain, predict, and avoid such oscillations. In contrast to other research, this effort backs away from pilot models and PIO avoidance and focuses on the characteristics of the pilot before, during, and after a PIO. Often, PIO’s can be explained by limit cycles occurring in a non-linear system where the non-linearities cause a sustained, constant amplitude oscillation. The primary instigators in such a PIO are usually a non-linear element (i.e. rate limit saturation) and a trigger event (i.e. pilot mode switching or increased pilot gain). By performing analysis in the frequency domain, determining such oscillations becomes easier. Using spectrograms and power spectral density functions, the frequency content of a signal in the pilot-aircraft system can also be investigated.

An F-14 flight test was recently performed where the hydraulic system was modified to determine the feasibility of trying to recover the aircraft (land on carrier) during such an extreme hydraulic failure. During testing, a severe PIO occurred because of the tight tracking task used during aerial refueling. While performing spectrograms and power spectral analysis, an increase in power concentration at the PIO frequency was observed.

With a linear approximation of the F-14 aircraft dynamics, a closed-loop system containing the aircraft, actuator, and pilot dynamics is developed so that limit cycle analysis can be performed. With stable limit cycle solutions found possible, a pilot-in-the-loop simulation is performed to verify the pilot model used in limit cycle analysis. Using the flight test data, limit cycle analysis, and pilot-in-the-loop simulation, a connection between variation in pilot behavior and PIO predicted by the increase in power concentration is investigated.

The resulting connection showed that an increase in pilot gain along with a transition from observing pitch attitude to pitch rate are the possible trigger events causing the PIO. The use of spectrograms as a PIO predictor is shown to be possible, provided the necessary calculations can be completed in real-time.
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Chapter 1

Introduction

1.1 Introduction

Pilot-in-the-loop oscillation (PIO), also known as aircraft-pilot coupling, is the interaction between a pilot and aircraft that causes unintended excursions in aircraft attitude and flight path [1]. Although it is difficult to determine the cause of the PIO, a majority of PIO’s are thought to be caused by deficiencies in design or pilot training. This research considers both of these possible causes in an analysis of PIO experienced in the F-14 aircraft.

During the flight test of an F-14 aircraft with degraded hydraulic pressure, several PIO’s were encountered during aerial refueling. The degraded hydraulic pressure effectively reduced the maximum actuator rate from 30 deg/sec to 10 deg/sec (Cause 1). It was also noted in pilot comments that the oscillation had a rapid onset typical of a switch in pilot behavior (Cause 2). This switch in pilot behavior will be investigated by using parameter identification to characterize the pilot as a function of time.

1.2 Background

Before proceeding with PIO analysis, some background information is provided to establish the basis for this research. The background information includes a discussion of the fighter aircraft used in the analysis. Also included is a description of the flight test along with the piloting technique observed in the flight test. Finally, some general information on PIO’s
will be presented.

1.2.1 Fighter Aircraft Description

To investigate methods to predict the existence of PIO tendencies, it is first necessary to develop an aircraft model. This model should have sufficient fidelity so that PIO analysis methods can be verified before being tested in an actual aircraft. A model of the Grumman Aircraft Company (GAC) F-14 was developed for this investigation. The F-14 was chosen because of several factors; availability of aircraft aerodynamic data and control law specifications, flight test data of aerial refueling tasks that demonstrate PIO tendencies, and the fact that the F-14 is representative of today’s fighter aircraft.

The F-14 is a supersonic, twin engine, swing-wing, two seat, military fighter designed for air superiority. The F-14 is in use today by the United States Navy as a carrier based multi-role fighter. This role requires the fighter to perform aerial refueling during typical flight activities. The aerial refueling task is the focus of our investigation into PIO tendencies for this type of aircraft.

It is well known that actuator rate limiting plays a considerable role in inducing a PIO. With the high speed actuator requirements of today’s fighter control laws, actuator rate limiting occurs often during fine tracking tasks. Depending on the type of control system (reversible or irreversible), the pilot may or may not know of the existence of rate limiting on the control surface. The F-14 aircraft has an irreversible control system.

As stated in Reference 2, the F-14 aerial refueling task was tested during dual hydraulic failures. This flight test was performed to determine the feasibility of attempting to recover the aircraft after such a failure. During these aerial refueling tasks, several PIO’s were encountered during final engagement of the aerial refueling probe to the basket. The PIO is evident by the increased frequency and amplitude of the pilot stick forces and the sawtooth shape of the control surface deflection recorded in flight test data. The sawtooth shape identifies rate limiting. Although the fully operational F-14 has very limited PIO tendencies, by reducing the available hydraulic pressure to the actuators, the rate limit PIO appears during difficult or high gain tasks such as aerial refueling.

The hydraulic failure tested in flight test simulated the F-14’s BUFCM (Backup Flight Control Mode). This mode is selected when severe malfunctions occur during aircraft oper-
ations. With BUFCM selected, the F-14 stability augmentation system (SAS) is no longer operational. During normal operations, the lack of a SAS has little influence because of the favorable bare airframe dynamics. But during tight tracking tasks (i.e. aerial refueling), the reduced stability of the airframe without the SAS has caused problems in the past. The flight test was performed for this reason.

1.2.2 F-14 Flight Test

In 1994, the Navy voiced concerns about the F-14 aircraft returning to the ship or base after severe hydraulic failure. Naval Air Test Center performed several tests to determine the safety and feasibility of returning to base after severe hydraulic failure. These tests consisted of both landing and refueling tasks. The research presented in this work focuses on the aerial refueling task due to the likelihood of encountering PIO during such a maneuver.

The flight test task involved the damaged aircraft flying formation behind an A-7 tanker aircraft while the tanker flew slow S-turns. This task was formulated in an attempt to find PIO ‘cliffs’ (abrupt changes in handling characteristics). Once satisfied that no cliffs existed, the pilot attempted to refuel. This task was repeated multiple times with varying modes, configurations, and aircraft states.

During one such task, the pilot was attempting to ‘plug-in’ to the tanker when he rapidly encountered a PIO of frequency near 0.5 Hz. The pilot quickly pulled the throttle to idle and restored normal hydraulic pressure. Even with these corrective inputs, the aircraft deviated to a point above the horizontal tail of the tanker aircraft. The chase plane also had to perform evasive maneuvers to avoid a mid-air collision. The F-14 pilot stated after the flight that the PIO happened very quickly with little or no warning. This flight (Reference 2, Figure J-05) is the data used in this analysis for comparison.

1.2.3 Piloting Technique

During the flight test investigation, it was discovered that piloting technique played an important role in finding handling quality cliffs. Two piloting techniques, low and high gain, were used. The low gain technique involved the RADAR-Intercept Officer to verbally guide the pilot into position for refueling. The pilot did not visually make contact with the basket
until just before plug-in. This method kept the pilot from overdriving the system; therefore, keeping the pilot's gain low. In the high gain technique, the pilot aggressively tried to track the basket throughout the task. This technique caused a high pilot workload and therefore, high pilot gains.

Each pilot in the flight test was instructed to use both techniques in determining possible handling quality cliffs. Of course, the high gain technique was deemed responsible for a PIO during the flight test. In no configuration was the low gain technique linked to a PIO. In the flight test report, it stated that although a low gain technique did not cause a PIO, it does not preclude a fleet pilot's low gain task from causing a PIO.

1.2.4 PIO Background

The recent increase of research in the area of PIO is due to the recent accidents such as the USAF YF-22 and the Swedish Saab Gripen [3] but PIO is not a new phenomena. Since the rapid advances of aircraft after World War II, research has been done on PIO’s. This research brought about the criteria used today in MIL-STD-1797A [4]. With PIO’s continuing to occur in today’s aircraft, the research community has begin to update those criteria for the purpose of alleviating PIO occurrences.

But regardless of the criteria used in aircraft design, pilots will still be a part of the closed-loop system (unless the aircraft is uninhabited). This pilot-aircraft dynamic interaction is the basis for PIO occurrence. Also since the pilot is adaptive in nature, the prediction of PIO’s is further complicated. With the incorporation of pilot models containing this adaptive nature, future criteria should improve PIO prediction and detection.

1.3 Objectives

A considerable amount of PIO analysis has been focused on preventing a PIO from occurring either by altering the design before building the aircraft or altering the control law after the aircraft has been built. This analysis moves away from both by attempting to determine a PIO real-time predictor, the spectrogram. Using spectrograms, a time history of frequency content can be used to pinpoint a rapid increase in power concentration. The intent of this analysis is to show that this increase in power concentration is caused by a combination of
actuator rate limiting and pilot behavior variation.

To show this correlation, a combination of limit cycle analysis, pilot-in-the-loop simulation, and parameter identification are used. Limit cycle analysis is necessary to insure the existence of an oscillatory solution for a given pilot, actuator, and aircraft model. Pilot-in-the-loop simulation is required to both validate the limit cycle analysis and acquire data for the parameter identification. The parameter identification is used to verify the pilot model used in the limit cycle analysis. Parameter identification is also used to characterize pilot behavior during the oscillation.

By performing the above tasks, the objective of this research is to determine a correlation between pilot behavior and an increase of power concentration in the spectrogram function. Using this linkage with actual flight test data, this report will show the spectrogram as a possible real-time PIO predictor.
Chapter 2

Power Spectrum Density and Spectrogram Analysis

2.1 Introduction to Frequency Analysis

Frequency analysis is typically used to characterize the system response to a sinusoidal input. Frequency analysis consists of using Fourier transforms or discrete Fourier transforms (DFT) to show the frequency content of a continuous-time signal. Since the signal can be considered infinite in length, windowing is applied to form a finite length range prior to the DFT computation. For linear systems, the system can be described by the magnitude and phase relationship between the input and output signals. The advantage to using frequency domain rather than time domain lies in the capability of frequency analysis to observe broad bandwidth, wide frequency range of signals, where low frequency signals may be lost in time domain analysis due to high frequency noise [5].

If signals can be assumed sinusoidal (superposition of varying amplitude and frequency periodic signals) then the signal can be approximated by the Fourier series. The Fourier series, \( y(t) \), is a weighted sum of sine and cosine functions given by

\[
y(t) = a_o + 2 \sum_{n=1}^{\infty} [a_n \cos(2\pi n ft) + b_n \sin(2\pi n ft)]
\]

(2.1)
where

\[ f = \frac{1}{T} \]  \hspace{1cm} (2.2)

\[ a_n = \frac{1}{T} \int_0^T y(t) \cos(2\pi n ft) \, dt \]  \hspace{1cm} (2.3)

\[ b_n = \frac{1}{T} \int_0^T y(t) \sin(2\pi n ft) \, dt \]  \hspace{1cm} (2.4)

If the signal is sampled at equally spaced discrete points and has \( 2^n \) points, then the Fast Fourier Transform (FFT) can be used. The FFT exploits the knowledge of \( 2^n \) points to streamline the calculation of the Fourier transform. The Discrete Fourier Transform (DFT) performs a Fourier transform on a signal sampled at discrete points. The DFT is the frequency analysis method used in the rest of this chapter. The DFT is described by

\[ Y(k) = \sum_{n=1}^{N} y(n) W_N^{kn}, \text{ where} \]  \hspace{1cm} (2.5)

\[ W_N = e^{-\frac{2\pi}{N}} \]  \hspace{1cm} (2.6)

and \( N \) is the length of the discretized vector \( y \). By using DFT, the power spectral density (PSD) and spectrogram can be determined for a particular signal. For this analysis, pilot stick position is the DFT input signal where the PSD and spectrogram describe the frequency content of the signal. The PSD is a technique that identifies the frequency contents of a signal. The spectrogram performs several PSD's using windowing to obtain the frequency content as a function of time.

### 2.2 Power Spectral Density Function

Power spectral density (PSD) analysis is capable of showing frequency content of a signal for a discrete time period or window. The PSD is defined as the rate of change of the mean square value of the signal with respect to frequency [5]. The equation that describes the
PSD (or correlation factor) is given by

\[ G_{yy}(k\Delta f) = \frac{2h}{N} \left| \sum_{n=1}^{N} y(n)e^{-j\frac{2\pi kn}{N}} \right|^2 \]  \hspace{1cm} (2.7)

where \( \Delta f = \frac{1}{kN} \) is the frequency spacing for \( \frac{N}{2} \) points. Equation 2.7 is similar to the DFT equation (2.5) in that the absolute value of the DFT is squared, then averaged by the number of points in the input vector, \( y \).

The PSD describes the frequency content of a signal. In this analysis, the PSD will show the dominant frequency during a PIO. As will be shown in the examples, the noisier the signal (the more frequency and amplitude variation), the noisier the PSD. The bandwidth of the PSD is one-half the sample rate. In the examples below, the signals are sampled at 20 Hz giving a PSD bandwidth of 10 Hz. The purpose of using the PSD in analysis of PIO is to show that a PSD of the pilot stick position reveals the characteristics of the PIO. The advantage of using frequency analysis allows the detection of low frequency oscillations, such as a slow PIO (0.5 Hz), in high frequency noise.

### 2.3 Spectrogram Function

A spectrogram is a discrete-time Fourier transform of a signal using a sliding window (i.e Hanning Window). The window size of a spectrogram determines the length of the signal for the DFT. Unlike the PSD, the spectrogram shows the frequency content as a function of time. The spectrogram also shows the dominant frequency by using contour lines. The varying distance between contour lines depicts an increase in concentration of power during the time slice.

The spectrogram serves to give an indication of varying pilot behavior as the task becomes more difficult. The pilot behavior is evident through stick position frequency and amplitude. This changing behavior is shown in the spectrogram as a rapid increase in concentration, or color. In bode terms, the increase in concentration is described as an increase in magnitude near the resonance frequency. As discussed in Chapter 4, this concentration of power also describes a limit cycle oscillation.
2.4 DFT Window Selection

The spectrogram uses a discrete window to limit the influences on the DFT analysis. Many windowing techniques were developed for filtering methods because the spectrogram is one of the primary tools. The window can be considered as a weighting matrix that pre-multiplies the discretized input signal to effectively remove the signal content outside of the desired window.

Window length selection affects the result of the DFT through frequency and time resolution. To increase the frequency resolution of a DFT, the window length can be reduced. However, the ability to recognize changes in the signal as a function of time is also reduced. The selection of window length becomes a compromise between frequency resolution and time resolution.

The window shape also affects the results of the DFT. The typical window tapers to zero at the boundary to emphasize only a portion of the signal for analysis. Usually the shape is selected so that the Fourier transform of the window is narrow in frequency compared to the Fourier transform of the signal to be analyzed. In filtering techniques, the shape of the window is divided into two sections; the main lobe and side lobes. The bandwidth of the DFT is related to the width of the main lobe while the degree of rejection of adjacent frequencies (leakage) is affected by the relative side-lobe amplitudes [6].

Window types include rectangular, Kaiser, Hanning, and Hamming [6]. All of the DFT's in this analysis use a Hanning window with window length of 3 seconds. The DFT uses 1024 frequency bins to discretize the frequency range. With this introduction of PSD and spectrograms completed, the following sections will show examples of each. These examples will demonstrate the characteristics expected to be seen in analyzing flight test data.

2.5 PSD and Spectrogram examples

The goal of this frequency analysis is to show that a PIO is revealed by an increase of power concentration at the oscillation frequency. By performing a PSD and spectrogram of the pilot's stick position, we hope to see an increase in concentration coincident with an increase in task difficulty; specifically, as the pilot attempts to "plug-in" to aerial refueling probe. Before looking at actual flight data, some examples will be discussed to review the
characteristics of a PSD and spectrogram.

2.5.1 Sine Example

The first example, Figure 2.1, shows a simple sine function given by the equation

\[ y = \sin(2\pi t 0.5). \]  \hspace{1cm} (2.8)

where the input signal to the PSD, y(t), represents a 0.5 Hz sine wave. By using MATLAB\textsuperscript{TM}, Figure 2.1 was created with a Hanning window of length 3 seconds. The top plot shows the spectrogram. The window size is evident by the lack of information during the first 3 seconds. The solid white region along the 0.5 Hz line represents a constant concentration along the entire time history. The middle plot shows the input signal, y(t). The bottom plot shows the PSD of the input signal with the expected peak at 0.5 Hz.

2.5.2 Sine Example with Noise

By adding high frequency noise to the pure sine signal, this example analysis represents what is more common in practice. The input signal was created by summing the original sine signal (Equation 2.8) with a random noise signal. The random noise signal was created with the random number generator in MATLAB\textsuperscript{TM}. Figure 2.2 shows the results of the frequency analysis with the noisy sine signal.

Again the top plot shows a spectrogram of the input signal with a Hanning Window of 3 seconds. Rather than seeing a constant power concentration over the time period, the concentration varies randomly as a function of time. Although the input signal is noisy, the peaks for each time slice still occur at the 0.5 Hz frequency line. The points at which the spectrogram turns bright red represent the highest concentration of power. The middle plot shows the input signal. The bottom plot shows the PSD where the largest peak occurs at 0.5 Hz. Other smaller peaks do occur throughout the frequency range due to the noise in the signal. This example is more representative of flight test data where high concentration regions in the spectrogram point to PIO type behavior and the peak in the PSD represent the frequency of the PIO.
Figure 2.1: MATLAB™ PSD Example - Sine Wave
Figure 2.2: *MATLAB*\textsuperscript{TM} PSD Example - Noisy Sine Wave
2.5.3 F-14 Flight Test Data Example

An actual flight test data example, taken from Reference 2, was used to perform the spectrogram and PSD example analysis. Figure 2.3 shows three plots where the top plot is the spectrogram of the input signal. The input signal (stick position) is shown in the middle plot. The bottom plot is the PSD of the input signal. Notice in the middle plot, the PIO begins at 12.5 seconds and ends at 19 seconds as shown by the shaded region. After 19 seconds, the pilot restores normal hydraulic pressure to the aircraft in an attempt to recover the aircraft and avoid loss of the aircraft.

The PIO can be seen in the spectrogram as an increase in power concentration shown as bright red approximately 3 seconds after the PIO is first seen in the input time history. This delay occurs because of the window length used in the spectrogram. As evident in both the spectrogram and PSD (bottom plot), the approximate PIO frequency is 0.5 Hz. This frequency corresponds to the frequency of the PIO determined in flight test. The PSD plot it much noisier than the examples because of the randomness of the signal although the 0.5 Hz peak is easily the largest.

The PIO experienced during the flight test occurred as the aircraft came within 2 to 3 feet of engaging the basket. With the decreasing range to the basket, this increase in “power” could be linked to a variation of pilot behavior as the pilot performs the fine tracking task. This notion that the pilot varies his/her behavior as the distance to the target decreases is the fundamental question in this analysis. Can we show that the concentration of power at one frequency, due to a variation of the pilot technique, is a predictor of PIO?

2.5.4 Virginia Tech Simulator Example

By looking forward to the pilot-in-the-loop simulation analysis performed in Chapter 5, we can support the above statement about the connection between pilot behavior variation to the existence of a PIO. Looking at Figure 2.4, again the increase in concentration of power is evident in the top plot, a spectrogram of the input signal. The input signal, stick position, is shown in the middle plot. The bottom plot shows the PSD of the input signal where the dominant frequency occurs at approximately 0.4 Hz. As will be discussed in Chapter 5, a pilot-in-the-loop simulation analysis was performed to show the pilot variation during the tracking task. By showing that this variation in pilot behavior explains the
Figure 2.3: MATLAB™ F-14 PSD Example
Figure 2.4: MATLAB™ Virginia Tech Pilot-in-the-Loop PSD Example
increase in concentration of power in the spectrogram, the spectrogram has the capability of predicting a PIO occurrence. Of note is the limitation of the spectrogram in predicting a PIO occurrence in real-time. The 3 second delay created by using the windowing technique causes the prediction of the PIO to lag the actual PIO. Hopefully, future work will be able to bypass this problem allowing spectrograms to predict PIO in real-time.
Chapter 3

Linear Aircraft Model Description

Clearly, a PIO is caused by a fundamental change in the pilot/aircraft dynamic stability. The characteristic roots or poles of the closed-loop system indicate relative stability. To identify this movement of poles, an aircraft and pilot model must be derived. This chapter performs the necessary calculations and derivations to obtain an F-14 type aircraft model for subsequent analysis. This aircraft model contains linear aerodynamics, actuator dynamics, and a control law implementation. A $MATLAB^TM$ implementation is also developed and verified in this chapter. Figure 3.1 summarizes the linear model build-up.

![Diagram](image)

Figure 3.1: Linear Model Block Diagram
3.1 Linear Aerodynamics Model

To pursue the analysis and determination of PIO, a realistic aircraft model is desired. As mentioned in Section 1.2.1, an F-14 type model will be used. A short period approximation, taken from Reference 8, is given by

$$
\begin{bmatrix}
V_T - Z_\alpha & 0 & 0 \\
-M_\alpha & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{\alpha} \\
\dot{Q} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
Z_\alpha & V_T + Z_q & 0 \\
M_\alpha & M_q & 0 \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha \\
Q \\
\theta
\end{bmatrix} +
\begin{bmatrix}
Z_{\delta_e} \\
M_{\delta_e} \\
0
\end{bmatrix}
\delta_e
$$

(3.1)

where the states are $\alpha$ (angle of attack, AOA, in radians), $Q$ (pitch rate in radians per second), and $\theta$ (pitch attitude in radians). Pitch attitude is appended to the standard short period approximation so that it is available for feedback to the pilot model. The force and moment derivatives in Equation 3.1 are defined in Table 3.1. The non-dimensional derivatives

<table>
<thead>
<tr>
<th></th>
<th>$Z$</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$-\frac{\bar{q} S}{m} (C_{d_\alpha} + C_{L_\alpha})$</td>
<td>$\frac{\bar{q} S \bar{c}}{m} C_{m_\alpha}$</td>
</tr>
<tr>
<td>$q$</td>
<td>$-\frac{\bar{q} S \bar{c}}{2 m V_T} C_{L_q}$</td>
<td>$\frac{\bar{q} S \bar{c}^2}{2 I_{yy}} C_{m_q}$</td>
</tr>
<tr>
<td>$\delta_e$</td>
<td>$-\frac{\bar{q} S \bar{c}}{m} C_{L_{\delta_e}}$</td>
<td>$\frac{\bar{q} S \bar{c}^2}{2 I_{yy}} C_{m_{\delta_e}}$</td>
</tr>
<tr>
<td>$\dot{\alpha}$</td>
<td>$-\frac{\bar{q} S \bar{c}}{2 m V_T} C_{L_\alpha}$</td>
<td>$\frac{\bar{q} S \bar{c}^2}{2 I_{yy}} C_{m_\alpha}$</td>
</tr>
</tbody>
</table>

are $C_{d_\alpha}$ (zero alpha drag), $C_{L_\alpha}$ (lift curve slope), $C_{m_\alpha}$ (pitch moment curve slope), $C_{L_q}$ (lift due to pitch rate), $C_{m_q}$ (pitch moment due to pitch rate), $C_{L_{\delta_e}}$ (lift due to stabilator deflection), and $C_{m_{\delta_e}}$ (pitch moment due to stabilator deflection). The aircraft physical constants include $I_{yy}$ (moment of inertia about the y-axis), $\bar{c}$ (mean chord), and $S$ (wing area). Dynamic variables are defined as $V_T$ (trim velocity) and $\bar{q}$ (dynamic pressure). All the above values are given in Reference 7. A longitudinal short period model was selected because of the PIO observed in flight test. As discussed in Section 1.2.2, the PIO was characterized by a longitudinal oscillation at approximately 0.5 Hz, or 1.57 rad/sec. This oscillation frequency falls within the range of a short period approximation rather than a
phugoid approximation.

By adding an output equation to Equation 3.1, the transfer function, \( G(s) \), can be determined and incorporated into the creation of a closed-loop system. This transfer function represents the dynamics from input (stabilator deflection) to output (i.e. pitch attitude). The output equation is not described at this point because of the uncertainty of the pilot feedback structure (signals observed by pilot). As the aircraft model development continues, this uncertainty will be eliminated. The closed-loop system also includes actuator, \( A(s) \), and pilot, \( P(s) \), dynamics as shown in Figure 3.1.

## 3.2 Actuator Dynamics

The influence of actuator rate limiting to the closed-loop system during PIO has been established by previous research [9]. The basis for this influence is the fact that control surfaces have mass and inertia. The actuators are therefore limited physically by the acceleration limits of the control surfaces. With today’s high gain fly-by-wire systems, the requirement for fast control surfaces is amplified. With this conflict between actuator limitations and control law requirements, it is easy to see the importance of rate limiting on PIO incidences. Although rate limiting is evident in several recent PIO’s, it is not yet clear if rate limiting is the cause or effect of PIO’s [3].

Actuator rate limiting occurs when pilot input command error requires more rate than the actuator can provide. The lack of actuator performance is due to insufficient hydraulic flow rate or actuator design limits. In fly-by-wire aircraft, there are two types of actuator rate limits possible: software and hardware. Software limits consist of control law rate limits implemented to avoid overdriving the actuator. Hardware limits are actuator limits that vary with hinge moments and hydraulic flow rate available to the actuator.

The software (or control law) rate limiter consists of differentiating the control command signal, limiting the rate, and integrating the limited rate to obtain the rate limited command signal. This technique is shown in Figure 3.2. The limiter adds a zero and pole at the origin in a pole-zero map. This addition of a pole and zero causes the system to no longer act as a low frequency system but rather a high frequency system. In most cases, the transition to a high frequency system violates the describing function assumptions discussed in Chapter 4. Many in-flight simulators use this approach to imitate rate limiting of the actuator.
The second approach involves modeling the actuator as a first order system, shown in Figure 3.3. The first order system is separated into several parts so that the rate limit can be applied directly to the commanded rate. This method does not require the differentiation (addition of zero) that caused a violation of describing function assumptions. This model will be used in subsequent analysis because it represents the function of an actual hydraulic actuator.

The input to the actuator model is commanded stabilator deflection. Commanded stabilator deflection is a function of either pilot stick position or force. Taking the difference between the commanded stabilator and the actual stabilator deflection then creates the error signal, $e$. This error passes through a lag gain described by a time constant, $\frac{1}{r} = \frac{1}{0.05} \text{ sec}^{-1}$. At this point, the actuator rate signal is limited. The limited signal is then integrated to produce the actual stabilator deflection.

The instantaneous rate limit of the actuator is of course dependent on many variables related to the state of the aircraft as well as the hydraulic system. For this analysis, an assumed constant rate limit of 10 deg/sec is used. This rate limit was taken from the flight tests described in Reference 7. Although excessive, the rate limit corresponds to the hydraulic mode (BUFCM) during the F-14 flight test. See Section 1.2.2 for a discussion of the flight test performed on the F-14.
3.3 Aircraft Control Law

The F-14 control law is a simple mechanical stability augmentation system (SAS). This SAS supplements the bare airframe dynamics to allow the pilot to exploit the full aerodynamic capability of the aircraft [2]. For the flight test under study, the hydraulic failure (BUFACM mode) was selected to study the suitability of returning to base after severe hydraulic failure. While in BUFACM mode, the SAS is inoperative. Because the SAS is inoperative, the only control law feature incorporated into the aircraft model is the stick gearing. The stick gearing relates stick position (inches) to stabilator command (degrees).

3.3.1 Stick Gearing

The stick gearing gain ($K_{gear}$) was originally determined from Reference 7 by approximating the actual relationship (non-linear) as a linear curve fit. This approach worked well throughout the limit cycle analysis. When real-time pilot-in-the-loop simulation was attempted, it was obvious that the stick gearing was inappropriate. Pilots were unable to perform the experimental task because of over-sensitive stick characteristics. The gearing was therefore reworked so that full stick deflection corresponded to full stabilator deflection. This modification reduced the stick gearing by a factor of 4. In further discussion, it will become evident that the stick gearing changes the pilot gain but not the structure of the pilot model. The pilot model is mentioned here to complete the development of the closed-loop system, shown in Figure 3.1.

3.4 Pilot Model Structure

A vital part of PIO analysis lies in how the pilot is modeled. The pilot’s interaction with the airframe relies on the outputs observed by the pilot (i.e. pitch rate, pitch attitude, normal acceleration, etc.). A considerable amount of research has been conducted in an attempt to accurately model the pilot during a PIO event. It is not the purpose of this work to analyze the accuracy or suitability of these models. Instead, the pilot characteristics, specifically how they vary, will be investigated with the assumption that these characteristics can be applied to any pilot model. For this reason, a simplified pilot model will be used to reduce the complexity of the analysis. With that said, the focus now turns to determining the
appropriate feedback signals.

There are infinite possibilities in the selection of the feedback signals. The pilot can observe/feel a multiple of signals that can aid in determining the desired input. Reference 1, page 20 list many of these possible signals. These endless possibilities cause some confusion with regard to the signals the pilot is actually observing during a PIO occurrence. To narrow the list of possibilities, a simple root locus procedure is used to investigate the ability for the observed signal or combination of signals to cause a PIO.

By using a simple pilot gain model, \( P(s) = K_p y \), a PIO can occur with the assumed observed signal, \( y \), if the locus of roots cross the imaginary axis. Figure 3.4 shows the root locus for several individual signals using the simple pilot gain model in the closed loop system (actuator, linear aerodynamics, and pilot models). It is evident from this figure that pitch rate, \( Q \), is the only signal capable of causing an oscillation because the root locus branches cross the imaginary axis. The conclusion that pitch rate is the only individual signal possible.

\[\text{Figure 3.4: Simple Pilot Gain Root Loci - Single Feedback Signal}\]
of causing a PIO is troubling. Pilot comments reveal that several signals are generally used while performing the aerial tracking task. These signals include pitch attitude, pitch rate, and normal acceleration.

Along with the possibility of using simple pilot gain models, there is much research into dynamic (as opposed to static) pilot models. The crossover model is one such instance where the pilot is modeled as a lead-lag system with some amount of time delay [1]. During this analysis, many models were investigated and many caused the system to have oscillatory characteristics. The final pilot model was chosen based on pilot-in-the-loop simulation where pilot comment aided in the recognition of feedback signals. The final pilot model included proportional and derivative terms as in a lead compensator. It can be written as

\[
P(s) = K_p(\tau_p s + 1).
\]

The pilot model has the use of a leading zero \((\frac{-1}{\tau_p})\) to aid in increasing closed-loop bandwidth as much as possible. Figure 3.5 shows the root locus corresponding to the lead compensator pilot model using pitch attitude as the feedback signal. The lead compensator has a time constant of \(\tau_p = 40\) seconds. Notice the locus crosses the imaginary axis confirming the possibility of PIO. By choosing pitch attitude as the feedback signal, this model can also be considered a multiple loop feedback structure. The multiple loops are formed because \(Q = s\dot{\theta}\). Both pitch attitude and rate are feedback. This pilot model has some interesting features that will be explained in Chapter 5. With these root locus observations, it is clear that there are many possible feedback structures that will cause PIO.

### 3.5 Matlab Implementation

Once the linear model has been derived with bare airframe and actuator dynamics, it is important to verify the linearized model using actual flight test data. MATLAB\textsuperscript{TM}'s Simulink was the analysis tool for this comparison. Simulink performs simulation using basic block diagram objects eliminating the need for extensive development work. By developing Figure 3.1 in Simulink, an F-14 type aircraft system was formed using the actuator and linear model derived earlier. To verify the Simulink model, recorded stabilator deflection data from flight test was an input to the simulation and the outputs were compared with actual flight
Figure 3.5: Lead Compensator Pilot Model Root Loci using Pitch Attitude as Feedback Signal
test data from Reference 2. Appendix A contains the MATLAB™ code to create the F-14 linear model approximation.

Figure 3.6 contains time histories for stabilator deflection and the outputs: normal acceleration, pitch rate, and alpha. Also shown in this figure is the approximate range of PIO evident by the sawtooth portion in the stabilator deflection time history (shaded region). Unfortunately, actuator rate was not recorded during the flight test. The actuator rate limit used in analysis can be confirmed by calculating the slope during the sawtooth portion (approximately 10 deg/sec). By comparing the frequencies, amplitudes, and phase differences of the actual flight data to that of the Simulink model response, it appears that the linear model correlates quite well. In the normal acceleration plot, there is an obvious bias that is most likely due to the definition of normal acceleration during level flight (0 for the Simulink data, 1 for the flight data). The rest of the discrepancies could be due to disturbances not modeled in Simulink. Also during the flight test, pilot procedure was to revert to normal hydraulic operation if a PIO occurred. By reverting to normal hydraulic pressure, the SAS is also restored. This pilot method is likely the cause for discrepancies in the time histories after the PIO occurred. It should be noted that the angle of attack time history is adjusted because the flight test data measuring angle of attack is in AOA units rather than actual angle of attack.

Another idiosyncracy related to the flight test data occurred in the sign convention of the stabilator deflection. The flight test data used a positive stick deflection to positive stabilator deflection while the Simulink model used the conventional definition of positive stick deflection to negative stabilator deflection. While this difference does not affect the eventual limit cycle analysis, it caused many problems in confirming the correct modeling of the F-14 aircraft. Now that the linear model has been verified, the actuator model needs to be verified.

Figure 3.7 shows a comparison of Simulink model response and the flight test data by using stick position as the input to the actuator/linear aircraft model shown in Figure 3.1. Figure 3.7 contains time histories for stick position, stabilator rate, stabilator deflection, and the outputs: normal acceleration, pitch rate, and alpha. Also shown in this figure is the approximate range of PIO evident by the sawtooth time history (shaded region). As in the previous verification, the frequencies, phase, and amplitudes correspond quite well between the two data sources with similar discrepancies as discussed above.
Figure 3.6: MATLAB™ Stabilator Deflection Verification Data
As mentioned in Section 3.3.1, the stick to stabilator gearing gain had to be varied from the original value in order for the Simulink simulation to match the actual flight test data correctly. It was determined that this value was an approximation on data given by Reference 7 and therefore adjustable to insure compatibility. The gain was set originally to $-17.5 \text{ deg/in}$, which approximately matched the slope of the stick position to stabilator deflection data given in Reference 7. Eventually, the gearing gain was altered to -4.21 deg/in based on the pilot-in-the-loop simulation discussed in Chapter 5. This gain was selected to match the ratio of total stick deflection to total stabilator deflection. By looking at Figure 4.2, further basis for allowing variation in the stick gearing gain is supported in that a change in stick gearing also corresponds to a change in pilot gain.
Figure 3.7: MATLAB™ Stick Position Verification Data
Chapter 4

Limit Cycle Analysis

It was hypothesized in Chapter 2 that the concentration of power at the resonant frequency can be explained by the movement of poles toward the imaginary axis. This chapter provides further evidence of that possibility. The objective of this chapter is to show that limit cycles for the F-14 aircraft are possible. Given the existence of a limit cycle, the frequency and amplitude of the PIO can then be determined.

4.1 Introduction to Limit Cycle Analysis

A limit cycle is an unforced, sustained oscillation of a non-linear system. Limit cycle oscillations have been noted to be analogous to PIO's. Limit cycles can be shown to exist if any two complex conjugate poles lie on the imaginary axis of the complex plane. In other words, the poles have no real component. In the case of our aircraft-pilot model, the non-linear element can be isolated while the linear portions are represented in lumped parameter form. With this separation, a quasi-linear approximation, known as a describing function, can be used to represent the non-linear element. Using the notation of Reference 10, the linear portion is represented by $L(s)$ and the non-linear portion is represented by $\Delta$, as shown in Figure 4.1.
4.1.1 Describing Functions

To aid in solving for the limit cycle solutions, the non-linear elements of the system are approximated by sinusoidal input describing functions (SIDF). The SIDF technique assumes that the input to each non-linearity is sinusoidal and oscillating at the same frequency, \( \omega \), so that

\[
x(t) = A \sin(\omega t + \psi) .
\]  

(4.1)

In Equation 4.1, \( A \) represents the signal amplitude, \( \omega \) represents the frequency, and \( \psi \) represents the phase shift between the input signal, \( t \), and the output signal, \( x(s) \). But to use SIDF, the linear portion, \( L(s) \), must act like a low-pass filter so that the super harmonic signal components can be neglected during analysis. With regards to the software actuator described in Chapter 3, the zero added by differentiating violates this assumption. For this reason, the software actuator model can not be used in SIDF analysis. The hardware model can be used in SIDF analysis. The non-linear portion, \( \Delta \), can be then evaluated as \( N(A,\omega) \).

The actuator rate limiter described in Chapter 3 represents a non-linear saturation element. The SIDF for this element is given by

\[
N(A,\omega) = \frac{2}{\pi} \left[ \sin^{-1} \left( \frac{\gamma}{A} \right) + \frac{\gamma}{A} \sqrt{1 - \left( \frac{\gamma}{A} \right)^2} \right] .
\]  

(4.2)

where \( \gamma \) is the saturation limit value (10 deg/sec for the actuator) [11]. The ratio between input amplitude and saturation limit (\( \frac{\gamma}{A} \)) can be considered as an ‘equivalent gain’ because \( N(A,\omega) \) is approximately \( \frac{\gamma}{A} \). By substituting \( j\omega \) for the Laplace operator, \( s \), Figure 4.1 can
be described by the transfer function

\[ [1 + L(j\omega)N(A, \omega)] x(j\omega) = 0 \quad (4.3) \]

where \( x \) represents the input signal to the single non-linear element. By solving Equation 4.3, a limit cycle solution can be determined. To solve this equation, the unknowns \( (A, \omega, \text{and any unknowns from the linear model}) \) need to be determined using non-linear search techniques. This equation is called the harmonic balance equation.

### 4.2 Analysis Technique

This section describes a numerical method to find limit cycle solutions using the describing function technique.

#### 4.2.1 Model Review

![F-14 Longitudinal Block Diagram for Limit Cycle Analysis](image)

First, let us alter slightly the model derived in Chapter 3. The altered closed-loop system is shown in Figure 4.2. The non-linear saturation element has been replaced by an ‘equivalent gain’, \( N \), where the function of amplitude and frequency has been dropped for simplicity. The aircraft model, \( G(s) \), is given by the short period approximation with the addition of the state, pitch attitude. The aircraft model is given by Equation 4.4. The output equation
(Equation 4.5) has now been added.

\[
\begin{bmatrix}
V_T - Z_\alpha & 0 & 0 \\
-M_\alpha & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\alpha} \\
\dot{q} \\
\dot{\theta} \\
\end{bmatrix} =
\begin{bmatrix}
Z_\alpha & V_T + Z_q & 0 \\
M_\alpha & M_q & 0 \\
0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\alpha \\
q \\
\theta \\
\end{bmatrix} +
\begin{bmatrix}
Z_{\delta_e} \\
M_{\delta_e} \\
0 \\
\end{bmatrix}
\delta_e \quad (4.4)
\]

\[
\theta = \begin{bmatrix}
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\alpha \\
q \\
\theta \\
\end{bmatrix} \quad (4.5)
\]

The actuator portion of Figure 4.2, A(s), is a rate limited first-order lag (Equation 3.2). The input to the actuator is stick position error \((e_{\delta_s})\) and the output is actual stabilator position \((\delta_e)\). The gearing gain, \(K_{Gear}\) translates from stick position to commanded stabilator position, \((\delta_{ce})\). The differential equation for the actuator is given in Equation 4.6 where the time constant, \(\tau\), is 0.05 seconds.

\[
\delta_{e_{limit}} = \frac{N}{\tau}(-\delta_e + K_{gear}e_{\delta_s}) \quad (4.6)
\]

The pilot portion of Figure 4.2, P(s), represents a lead compensator. This compensator contains a gain, \(K_p\), plus first order dynamics (Equation 3.4). The pilot model uses the output, \(\theta\), from the aircraft, \(G(s)\), as the feedback signal. The output of the pilot model, stick position \((\delta_s)\), is differenced with the reference signal, \(R(s) = 0\), to determine the commanded stick position to the actuator model. The pilot model was chosen based on the arguments proposed in Section 3.4.

In transfer function form, the closed-loop system from \(R(s)\) to pitch attitude is given by

\[
T(s) = \frac{A(s)G(s)}{1 + A(s)G(s)P(s)} \quad (4.7)
\]

where \(T(s)\) is a function of \(N\), \(K_p\), and \(\tau_p\). By choosing these variables, the closed-loop characteristics of the system are known. Since Equation 4.7 is quasi-linear, the solution can be difficult to determine.
4.2.2 Limit Cycle Algorithm - Nyquist Approach

The Nyquist Approach, taken from Reference 10, is a graphical approach to solve the quasi-linear harmonic equation. From Figure 4.2, the actuator rate signal before limiting can be represented as

$$\dot{\delta}_c = \frac{1}{\tau} [\delta_{ce} - \delta_c]. \quad (4.8)$$

The input signal to the actuator is \(R(s) = 0\)

$$\delta_{ce} = -K_{gear} P(s) G(s) \delta_c. \quad (4.9)$$

Combining Equations 4.8 and 4.9 leads to the transfer function from stabilator position \(\delta_c\) to stabilator rate \(\dot{\delta}_c\).

$$\dot{\delta}_c = \frac{1}{\tau} [ -K_{gear} P(s) G(s) - 1 ] \delta_c \quad (4.10)$$

But also

$$\delta_c = \frac{1}{s} N \delta_c \quad (4.11)$$

so the complete closed-loop transfer function with the equivalent gain, \(N\), is

$$\dot{\delta}_c = \frac{1}{\tau} [ -K_{gear} P(s) G(s) - 1 ] \frac{N}{s} \delta_c. \quad (4.12)$$

Combining both sides of the equation results in

$$0 = \left\{ \frac{1}{\tau} [ -K_{gear} P(s) G(s) - 1 ] \frac{N}{s} - 1 \right\} \delta_c. \quad (4.13)$$

Assuming in Equation 4.13 that \(\dot{\delta}_c\) is non-zero, the rest of the right hand side must be zero for an oscillation to continue, or

$$\frac{-1}{N} = \frac{1}{\tau s} \left[ 1 + K_{gear} P(s) G(s) \right]. \quad (4.14)$$
Equation 4.14 can be used to determine $N$ given a value for $K_p$ and $\tau_p$. The Nyquist Approach plots the real versus imaginary frequency response of the right-hand side of Equation 4.14. By altering the pilot characteristics until the Nyquist plot crosses the real axis, a limit cycle solution is guaranteed. The limit cycle solution is found by graphically selecting the point where the imaginary part is exactly zero. This solution corresponds to a real part equal to $\frac{1}{N}$. A zero imaginary solution is selected because $N$ is defined as a real number. As will be demonstrated later, the limit cycle solution is dependent on the pilot characteristics.

Figure 4.3 demonstrates this method for the F-14 model. The F-14 model has a pilot gain of 70.8 in/rad and a pilot lead time constant of 0.3 seconds. Using the definition of $N$ (equivalent gain), solutions for $N$ must be real. Since the Nyquist contour crosses the imaginary axis twice, two limit cycle solutions exist ($N = 0.0119$ and $N = 0.0709$). Using Nyquist theory, it can be shown that the point at which the contour moves from negative to positive imaginary values ($N = 0.0119$) is stable while the other is unstable [13].

Looking again at the two limit cycle solutions, the frequency and amplitude of these limit cycles can be determined. The contour line or frequency locus depicts the frequency response of the system. The frequency increases along the contour from left to right in Figure 4.3. For a given value of $N$, the approximate amplitude of oscillation can be determined using the assumption $N = \frac{\omega}{A}$. The amplitudes and frequencies for both limit cycle solutions are shown in Table 4.1. Notice that the stable oscillation has a slower frequency but larger amplitude than the unstable oscillation.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$\omega$ (rad/sec)</th>
<th>$A$ (rad/sec)</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0119</td>
<td>1.7354</td>
<td>14.666</td>
<td>Stable</td>
</tr>
<tr>
<td>0.0709</td>
<td>3.3751</td>
<td>2.461</td>
<td>Unstable</td>
</tr>
</tbody>
</table>

### 4.3 F-14 Limit Cycle Solutions

Using the Nyquist Approach, this section discusses the analysis of the F-14 model to determine limit cycle solutions.
Figure 4.3: Limit Cycle Solutions using Nyquist Approach - $K_p = 70.8 \text{sin}/\text{rad}, \tau = 0.3\text{sec}$
4.3.1 Determining N and $\omega$

The Nyquist Algorithm was used to determine a pilot gain, pilot lead time constant, and equivalent gain, N. The objective is to verify that the limit cycle characteristics are consistent with the amplitude and frequency observed in the flight test and frequency analysis discussed earlier.

Because the pilot model is an important part of the closed-loop system, further analysis was performed to show the influence of varying pilot characteristics on limit cycle solutions. Figures 4.3 and 4.4 show this influence of pilot behavior on the limit cycle solutions. Figure 4.3 shows the possible limit cycle solutions for a pilot gain of 70.8 in/rad with a time constant of 0.3 seconds. This combination of gain and time constant shows two possible limit cycle solutions. Figure 4.4, with a pilot gain of 46.2 in/rad, shows Nyquist plots for three different time constants (0.3, 0.32, and 0.35 seconds). By comparing the two figures, it is evident that by altering the pilot gain from 70.8 in/rad to 46.2 in/rad, the Nyquist plot is shifted in the imaginary direction. By altering the pilot time constant, the Nyquist plot is rotated about the origin. By increasing the time constant by only 0.02 seconds, a limit cycle is avoided.

Using the N determined in Figure 4.4, the input amplitude and oscillation frequency were found to be $A = 4.987$ rad/sec and $\omega = 2.0$ rad/sec for a pilot gain of 46.2 inch/rad and a time constant of 0.3 seconds. The oscillation frequency, 2.0 rad/sec or 0.6369 Hz, compares nicely with the flight test analysis of 0.5 Hz [2]. These results could be improved by varying the pilot characteristics until the exact frequency was found to match the flight test frequency. However, the exact correlation with flight test data is not the primary point of limit cycle analysis, it is rather to determine if conditions exist for a stable limit cycle to occur.

4.3.2 Time History Comparison

Beyond the comparison of limit cycle amplitudes and frequencies, it is good practice to compare time histories. Again using MATLAB\textsuperscript{TM} 's 'Simulink' tool, the closed-loop system was simulated using the quasi-linear equivalent gain, N, in place of the non-linear rate limit. Figure 4.5 shows the results of the simulation. It is clear from the very lightly damped oscillations that a stable limit cycle solution has been found. The characteristic sawtooth shape does not appear in the stabilator position because the limit cycle analysis is a quasi-linear solution where the non-linear element is replaced by a linear gain. Using Figure 3.7
Figure 4.4: Limit Cycle Solution using Nyquist Approach - $K_p = 46.2 \text{m/rad}$
Figure 4.5: Limit Cycle Time Histories - $N = 0.0350$, $K_p = 46.2 \text{in/rad}$, $\tau = 0.3 \text{sec}$
as a reference, a comparison of the signal amplitudes show some discrepancies in the peak magnitudes. Of most interest is the increase in amplitude of the states, $\theta$, $\alpha$, and $q$. Although unclear from the flight test data, it is feasible that the PIO did not fully develop because the pilot quickly returned the aircraft to normal hydraulic pressure, aborting the task. As will be seen in Chapter 5, where the test pilots continued to attempt to perform the task, oscillation amplitudes approach those values found in limit cycle analysis.

By determining that the rate limited F-14 aircraft exhibits limit cycles for specific pilot characteristics ($K_p$ and $\tau_p$), we now turn our focus onto describing the variation of the pilot characteristics instigating a limit cycle, or PIO.
Chapter 5

Pilot-in-the-Loop Analysis

During the frequency analysis done in Chapter 2, the question arose as to what characteristics of the pilot led to a power concentration at a particular frequency. Using the fact that PIO’s are analogous to limit cycles, a limit cycle analysis was performed to show that the rate limited aircraft exhibited PIO tendencies. While performing this analysis, it was noted that the pilot characteristics ($K_p$ and $\tau_p$) were a key factor in the pilot-aircraft system demonstrating limit cycle behavior.

This chapter investigates the variation of pilot characteristics during a fine tracking task. The intent is to quantify the relationship between pilot task and pilot behavior. To show this relationship, a pilot-in-the-loop simulation will be performed. This simulation is required because the flight test data lacks the necessary signals to quantify task performance, such as tracking error.

5.1 Virginia Tech Simulation Facilities

The Virginia Tech Simulation Laboratory provides the opportunity for pilot-in-the-loop analysis in a realistic environment without the cost of actual flight. However, the simulation is still a simulation and cannot be expected to replicate actual flight. The simulation uses a combination of several utilities to perform pilot-in-the-loop simulation: graphical user interface, visual system, cockpit interface, data storage, and aircraft model.

The Virginia Tech Simulation uses the CASTLE (Control Analysis and Simulation Test Loop...
Environment) programming environment, developed by the NAWCAD Manned Flight Simulator, to perform the top-level interfaces with other utilities. It also performs all aircraft model calculations. CASTLE provides a platform where multiple aircraft models can be developed and flown on either a desktop simulation or in the motion cockpit. The graphical user interface allows the user to initialize the aircraft states, trim the aircraft, begin simulation, record variables, and plot time histories. This interface also controls the communication with the visual system and the cockpit. See Nichols [15] and Scalera [16] for discussions on CASTLE and the use of the simulator at Virginia Tech, respectfully.

The visual system, an Evans and Sutherland calligraphic night-time visual computer, drives a pilot cockpit view along with a side window view. The visual system has the capability of displaying several ground models (airports, carriers, and generic land) or a formation aircraft. The visual system takes viewpoint data from CASTLE and displays the corresponding visual scene to the pilot in the cockpit. At this time, the refresh rate of the visual scene is unknown although no jumpiness is visible while flying. The lack of jumpiness insures the visual update rate is on the order of 60 Hz.

The cockpit interface controls the passing of pilot inputs (stick position, rudder pedal position, throttle position, and switches) to CASTLE for use in the aircraft model. The interface also uses CASTLE data to enable switches, lights, etc. This interface performs calculations at approximately 80 Hz.

An advantage to actual flight, the CASTLE environment allows recording of any variable added to the Data Pool during compilation. This capability allows in-depth analysis and debugging of any portion of the aircraft model. For this analysis, real-time simulation using CASTLE allows storage of variables that were not available during flight test. The data storage utilities allow data to be stored in several formats (ASCII, MATLAB™ ASCII, MATLAB™ binary, etc.) at a preset rate (100 Hz).

The aircraft model developed for CASTLE was taken directly from the linear model created in Chapter 3. Additional dynamics (engine and phugoid mode) were added to create a higher fidelity simulation. No lateral-directional dynamics were added to the simulation because the piloting task was a purely longitudinal maneuver. Similar to the verification done for the Simulink aircraft model, stabilator and stick position were used as inputs to the CASTLE environment while outputs were recorded to insure that the simulation was an accurate model of the linearized F-14 aircraft.
5.2 Simulation Pilot Task

The piloting task was designed to emulate the actual flight test task (aerial refueling) as closely as possible. The simulation could not exactly duplicate the flight test task because the visual system did not contain the necessary models to perform aerial refueling. Instead, a similar task was created by flying wingman formation with another aircraft.

Because the aircraft model developed in CASTLE contained only longitudinal dynamics, the task was restricted to isolate pilot input to a pure short period longitudinal controller. This restriction was done by setting all lateral directional states to zero. The throttle input was also set so that the pilot had a constant closure rate of 2 knots.

The task consisted of starting at a trim location 200 feet directly aft of the inboard wing light on the formation aircraft. The inboard wing light is located at semi-span along the trailing edge of the formation aircraft wing. The task then required the pilot to keep the inboard light within the inner circle (dashed line) of the piper located on the windscreens. The piper, as shown in Figure 5.1, contained two ellipses around a cross hair. This piper was used so that the pilot would have a reference location, much like a fuel probe, to compare to the inboard light. The piper was drawn as ellipses so when mounted to the angled windscreens, the pilot observed circles. Because the aircraft model was limited to just longitudinal motion with a constant closure rate, the pilot used only longitudinal stick to control the error between the cross hairs of the piper and the inboard light on the formation aircraft. This task was performed numerous times by three pilots in an attempt to acquire data that used both low and high gain techniques as discussed in Section 1.2.3.

There were 3 pilots used in the study. Pilot A is a retired naval engineering test pilot with 3400 total hours predominately in fighter aircraft. Beyond normal squadron activities, Pilot
A performed carrier suitability testing at the Naval Air Warfare Center. Pilot B is an Air Force flight test engineer with 500 hours of flight test experience. Pilot B acquired many of the flight hours in a C-17 and F-16. Pilot C was a graduate student with 1 hour of actual flight time and many simulation hours.

Each pilot was allowed several flights with no actuator rate limiting so that the pilot could become accustomed to the task and aircraft. Once the pilot was comfortable with the aircraft and task, the actuator rate limits were added to the simulation. Each flight’s data was recorded and analyzed. Below is a discussion of these results.

5.3 Simulation Results

Figures 5.2 and 5.3 show data for a rate limited flight using pilot B. The time spent rate limiting the actuator in Figure 5.2 along with the increase in stick deflection and commanded stabilator deflection demonstrates a high gain technique. In this technique, the pilot is aggressively trying to track the formation aircraft. Along those same lines, Figure 5.3 shows an overall increase in amplitude and a detectable sinusoidal appearance in the recorded signal. The altitude offsets approached 100 feet during the oscillation. Comparing this flight with data taken from actual flight test shows similar amplitudes yet the simulated task has larger final magnitudes much like the time histories shown in the limit cycle analysis. This increase in magnitude towards the end of the task could be due to a difference in piloting strategy. When a PIO occurred in the actual flight, the pilot restored hydraulic pressure, effectively removing the rate limit. During the simulation task, the pilot continued to attempt to maintain the inboard light within the piper, therefore further increasing the PIO amplitudes.

Similar results to the ones above were obtained by all three subject pilots. With widely varying experience, it is peculiar to find such similar results. Perhaps this task has identified a predominant piloting behavior of PIO's that is irrelevant to training techniques. In other words, the behavior exhibited by the pilots would occur regardless of the experience of the pilot. The similar trends could also be due to the fundamental errors present in a simulation task attempting to emulate actual flight. Some of these errors are discussed in the next section.
Figure 5.2: Simulation Flight Results - Controls - Rate Limited
Figure 5.3: Simulation Flight Results - States - Rate Limited
5.4 Simulation Errors

Before analyzing the data acquired during the simulation exercise, a discussion on the validity of the data is required. By using the simulator at Virginia Tech without motion, the task performed obviously does not match the task performed during the actual flight test. The question then arises whether pilot-in-the-loop, fixed-base simulation can be used in identifying PIO occurrences. This question is still being debated by many researchers in this field [1]. The argument tends to lie in the necessity of motion cues during the task. Some researchers state that motion cues are not necessary because most flight test pilot are trained to ignore motion cues (i.e., during instrument flying). These researchers believe that a realistic visual system can replace the need for motion cues. Other researchers insist the need for motion cues because visual systems in current simulators lack the necessary refresh rate. The data presented in this paper is not intended to add to this debate. Instead, the obvious simulation errors will be listed and the analysis will continue.

Many of the simulation errors were identified by Pilots A and B who have actual flight hours. These errors included inaccurate stick dynamics, unrealistic task conditions, and aircraft modeling errors. Though pilot B had no flight time in an F-14 aircraft, the lack of accurate F-14 stick gearing and stick dynamics were concerns. Pilot A stated that the lack of lateral movement and fixed closure rate automatically made the task unrealistic. While performing the task, all pilots commented on the correctness of the linear model approximation. The aircraft model did not seem realistic. The tendency for the aircraft to oscillate was much higher than any other aircraft the subject pilots had flown. This could be due to the 10 deg/sec rate limit present on the actuator. Pilot A (the only pilot with aerial refueling experience) commented the simulation task was much more difficult to accomplish than actual flight refueling.

Other simulation errors include time delays throughout the system, lack of detail in the visual system, and lack of motion cues. Although unable to determine the actual delays present in the simulation, it can be assumed that some delays exists in updating the visual scene. These visual delays were deemed important but unavoidable because the visual system was the only cue used to judge error during the task. Along those same lines, the visual system lacked the detail required to perform aerial refueling. The pilot was instead asked to fly formation with the other aircraft. As stated above, the lack of motion cues removed the ability of the pilot to sense accelerations.
Based on these errors, it may be thought that simulations are in no way a good tool to perform PIO analysis. On the contrary, according to Reference 1, the simulator can be an effective tool as long the task is difficult enough to stress the pilot-aircraft system and the dynamics of the modeled aircraft are as close as possible to the actual aircraft.

5.5 Pilot Model Verification

By performing pilot-in-the-loop simulation, the accuracy of the pilot model proposed in the limit cycle analysis can also be examined. By using parameter identification techniques with the data recorded during the simulation, the characteristics of the pilot model can be extracted. First, pilot comment was used to determine the structure of the pilot model along with expected trends of the pilot characteristics during the task.

5.5.1 Pilot Comment

With any simulated piloting task, pilot comment is a very useful tool to locate any discrepancies between the simulation task and the actual flight test. Pilot comments can also help to determine the pilot feedback structure.

The subject pilots were asked to comment on the amount of effort required to perform the task, signals observed during the task, and any other comments related to pilot behavior. The amount of effort by the pilot directly relates to the piloting technique, low gain versus high gain. The signals observed by the pilot during the task will help to form the pilot feedback structure. The other comments will further help to determine the parameter identification approach that will demonstrate the pilot characteristics during the task.

5.5.2 Parameter Identification

According to Figure 4.2, the assumed pilot model consisted of a lead compensator model where pitch attitude was the chosen feedback signal. To verify this model, parameter identification (PID) using a least squares estimate (LSE) was attempted.
### PID Development

Given a pilot transfer function of the form,

\[ P(s) = \frac{\delta_s}{\theta} = K_p(\tau_p s + 1) \quad (5.1) \]

LSE parameter identification can be posed in the following way. By assuming \( q = s\theta \), the inverse Laplace of Equation 5.1 yields,

\[
\begin{align*}
\delta_s &= K_p\tau_p q + K_p\theta, \quad \text{or} \\
\delta_s &= K_q q + K_\theta \theta
\end{align*}
\]

where \( K_q = K_p\tau_p \) and \( K_\theta = K_p \). Separating Equation 5.3 into known and unknown parameters gives

\[
\begin{align*}
\delta_s &= \begin{bmatrix} q & \theta \end{bmatrix} \begin{bmatrix} K_q \\ K_\theta \end{bmatrix}, \quad \text{or} \\
U &= HK
\end{align*}
\]

The unknown parameters (K) represent the pilot gains. The known parameters (U and H) represent commanded stick position, pitch attitude, and pitch rate respectfully. These matrices are constructed using measured data. The matrix U will have one column and the number of rows equal to the number of data points. The matrix H will have two columns and the same number of rows as in U.

The goal of the LSE is to determine the best K for Equation 5.5. By solving for the unknown gains, Equation 5.5 becomes

\[ K = H^{-1} U \quad (5.6) \]

where the asterisk stands for the pseudo inverse. A pseudo inverse is used because H is rarely square. Using recorded data for the known parameters, the pilot gains can be determined.

Pilot comment showed a definite increase in difficulty as the pilot approached the tanker aircraft. By slightly altering the form of Equation 5.3, the LSE method can capture the characteristics of the pilot as a function of distance to the tanker aircraft. Expecting to show that the pilot gains were inversely proportional to the distance, the gains in Equation 5.3...
were redefined to be

\[ K_q(\delta x) = K_{q0} + K_{q1} \delta x, \quad \text{and} \]
\[ K_\phi(\delta x) = K_{\phi0} + K_{\phi1} \delta x \]

where \( \delta x \) is the recorded distance between the inboard light and the pilot’s eye. Reforming Equation 5.3 with the definitions in Equations 5.7 and 5.8 gives,

\[ \delta_s = \begin{bmatrix} q \\ q \delta x \\ \theta \\ \theta \delta x \end{bmatrix} \begin{bmatrix} K_{q0} \\ K_{q1} \\ K_{\phi0} \\ K_{\phi1} \end{bmatrix} \]

Equation 5.9 is of the same form as Equation 5.5 and can be solved as shown in Equation 5.6 where \( K \) is now a vector of dimension 4 rather than 2.

**PID Results**

Figure 5.4 shows one such fit for a simulation flight with actuator rate limiting. This figure corresponds to the flight data shown in Figures 5.2 and 5.3. The top plot in Figure 5.4 compares the actual stick position recorded during simulation to the lead compensator pilot model identified by the gains shown in the middle plot where pitch attitude was used as the feedback signal. Although not an exact fit, the pilot model and corresponding gains do quite well to approximate the actual pilot. The gains shown in the middle plot, \( K_q \) and \( K_\phi \), were calculated using Equations 5.7 and 5.8. The numerical values used in the gain build-up are given in Table 5.1. Examining the middle plot, it appears that the gains being inversely proportional to the distance is inconclusive. It appears inconclusive because \( K_\phi \) is decreasing in amplitude while \( K_q \) is increasing in amplitude. The bottom plot shows the ratio of \( K_\phi \) to \( K_q \) as a function of time (or distance to the target). Note that the ratio tends toward zero as the pilot approaches the tanker aircraft. This trend is an interesting phenomena that appeared in all the rate limited simulation results for all pilots.
Figure 5.4: Simulation Pilot Gain Identification
Table 5.1: PID Pilot Gains

<table>
<thead>
<tr>
<th>$K_{q0}$ $(in\delta_s - sec)$</th>
<th>$K_{q1}$ $(\frac{in\delta_s - sec}{ft})$</th>
<th>$K_{\theta0}$ $(in\delta_s)$</th>
<th>$K_{\theta1}$ $(\frac{in\delta_s}{ft})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0790</td>
<td>-0.0002</td>
<td>0.0196</td>
<td>-0.0004</td>
</tr>
</tbody>
</table>

By looking more closely at the gain definitions for $K_q$ and $K_\theta$, one sees that

\[
\frac{K_q}{K_\theta} = \tau_p \quad \text{and} \quad K_\theta = K_p. \tag{5.10}
\]

Reforming Equation 5.1 to

\[
\frac{\delta_s}{\theta} = K_p \tau_p \left( s + \frac{1}{\tau_p} \right) \quad \text{or,} \tag{5.12}
\]

\[
\frac{\delta_s}{\theta} = K_q \left( s + \frac{K_\theta}{K_q} \right), \tag{5.13}
\]

the ratio of $K_\theta$ to $K_q$ represents the zero location for the assumed pilot model dynamics. As this ratio tends to zero (or the formation aircraft gets closer), the pilot tends to prefer pitch rate rather than pitch attitude, $(K_\theta \to 0)$. This result makes intuitive sense based on pilot comment. As the formation aircraft gets closer, the pilot no longer focuses on error but rather the rate at which the error is occurring. Looking again at Figure 5.4, the gain, $K_q$, increases as the pilot approaches the tanker aircraft. By showing in Equation 5.13 that $K_q$ is the pilot gain, the theory of pilot gain being inversely proportional to distance to target seems to have merit. This analysis was performed on several simulation flights with rate limiting and the same trends appeared in all flights.

Looking again at the zero location of the pilot model dynamics, notice the negative value of the ratio. This negative value describes a non-minimum phase zero because the zero is located in the right-half plane on a pole-zero map. This non-minimum phase zero has the result of adding phase lag to the system rather than phase lead (if the zero was in the left-half plane). This lag can be shown by producing a Bode plot of the pilot model for two time slices during the flight test, as shown in Figure 5.5. Looking at the gain plot of Figure 5.5, the difference between gain magnitudes is relatively small near the oscillation frequency (1.57
Figure 5.5: Pilot Model Bode Plot of Two Time Slices
rad/sec). At lower frequencies, the magnitude tends to decrease as a function of time. The phase plot shows an increase in phase lag throughout the frequency range as a function of time. At the oscillation frequency, the phase difference between the pilot model at 5 seconds and 20 seconds is approximately 20 degrees. This increase in phase describes an increase in lag.

As stated in Reference 14, this phenomena of adding phase lag rather than phase lead, called “crossover regression”, usually occurs in acceleration plants. Rather than add lead to increase the speed of the system, the pilot attempts to slow down the system by decreasing the bandwidth frequency (adding lag). This “crossover regression” was evident in pilot comment. The subject pilots stated that at some point during the task, they focused more on slowing down the rate of error rather than minimizing error. The pilots effectively gave up on reducing the higher frequency components of error dynamics.

Overall, the pilot model describes a pilot who starts the task by tracking pitch attitude error with relatively small gain. As the task continues, the pilot’s gain increases to a point that the pilot’s bandwidth is not capable of tracking the error. The pilot then begins to focus on minimizing error rate (pitch rate) by reducing his/her bandwidth to slow the system down. Meanwhile, the pilot’s gain is increasing to a point where the combination of altering behavior and increasing gain has caused a PIO.

5.6 Spectrograms and pilot variation correspondence

To this point, the discussion has been about verifying the pilot model used in the limit cycle analysis. One step further is taken in linking the pilot model variation to the spectrograms discussed in Chapter 2. The spectrogram shown in Figure 2.4 was calculated with the simulation data shown in Figures 5.2 and 5.3.

From the discussion about spectrograms, it was proposed that the increase in concentration of power shown in the spectrogram can be described by the system poles moving to the imaginary axis. The purely imaginary poles of the system also define a limit cycle. After insuring that the linear model had limit cycle solutions for a particular pilot model, pilot-in-the-loop simulation was performed. By analyzing pilot characteristics using parameter identification, it was shown that the fine tracking task of flying formation showed both an increase in pilot gain along with altering behavior as the target distance decreased. These
pilot characteristics were much like the aerial refueling engagements seen in flight test. By linking the change in pilot characteristics to the increase in concentration shown in the spectrograms, it has been shown that the spectrogram can be an indicator of PIO.
Chapter 6

Conclusion

In conclusion, the objective of this analysis involved investigating the use of a spectrogram as a PIO predictor. The increase in power concentration seen in the spectrogram was shown to be linked to the PIO trigger (i.e. pilot behavior). The link between the two was accomplished using limit cycle analysis, pilot-in-the-loop simulation, and parameter identification.

PSD and Spectrograms

The PSD is a frequency domain analysis tool that determines the frequency content of a given time history. In this analysis, the PSD was used to determine the dominant oscillation frequency for a stick position time history. It was hypothesized that this frequency corresponded to a PIO frequency.

Taking it a step further, the spectrogram performs several discrete-time Fourier transforms to show the frequency content of a signal as a function of time. For a given time history, the spectrogram shows a concentration of power at a particular frequency. It was hypothesized that this power concentrations could be linked to a PIO occurrence. In other words, if a power concentration was detected in the spectrogram, it is feasible to state that a PIO occurrence is possible.
Limit Cycle Analysis

In Chapter 4, F-14 short period dynamics, actuator dynamics, and a pilot model formed a closed-loop system for analysis. Using describing functions for the non-linear element, it was determined that the pilot-aircraft system contained limit cycle tendencies. Limit cycle solutions were found that had characteristics similar to those seen in actual flight test.

Pilot-in-the-loop Simulation

Once limit cycle solutions were found, pilot-in-the-loop simulation was performed to both verify the pilot model used in the limit cycle analysis and characterize the pilot’s behavior during the oscillation. In Chapter 5, the simulator was used to perform a formation task in an attempt to emulate the actual flight test task. Using data recorded from the simulation, parameter identification was used to quantify the characteristics of the pilot model used in the limit cycle analysis.

It was shown that the pilot transitioned from tracking pitch attitude to pitch rate during the task while increasing his/her overall gain. It was also shown that the pilot actually added phase lag rather than lead in an attempt to slow the system down in a manner similar to the phenomenon known as “crossover regression”. Overall, the pilot model identified matched well with pilot’s comments during the simulation tasks.

Pilot Variation during PIO

The link between pilot behavior and power concentration in the spectrogram has been investigated. It was found that during the simulation task, the pilot increased gain as well as transitioned from a pitch attitude tracking task to a pitch rate tracking task. This variation of behavior was shown to create the necessary conditions for an oscillation to occur. The spectrogram of one such flight showed a concentration of power at approximately the same oscillation frequency. The spectrogram also showed the approximate time of oscillation.
Future Work

Now that the link between power concentration in the spectrogram and PIO has been shown to exist, there are several areas that need more investigation. First, for spectrograms to serve as a PIO predictor, the time delay associated with windowing during the frequency analysis needs to be reduced or eliminated. This improvement is a necessary step to allowing real-time PIO prediction. Also, the pilot models used in this analysis were kept simple to make analysis easier. More complicated pilot models need to be evaluated to insure the trends are valid globally. Along with more complicated pilot models, higher fidelity simulation (i.e. motion) is needed so that different feedback structures can be investigated. Looking at pilot comment from the flight test, normal acceleration cues are an important signal used by the pilot during fine tracking tasks. The simulation used in this analysis did not have motion capabilities.

If real-time spectrograms can be determined, investigations into the correct procedure to notify the pilot of a PIO occurrence are necessary. This procedure could include triggering a light in the cockpit, an aural cue, or control law alterations to react to a possible PIO. Of course, removing the pilot from the loop by ignoring pilot commands is an option that will definitely be deemed inconceivable by the piloting community.
Bibliography


Appendix A

$MATLAB^TM$ Linear Model Approximation
% Longitudinal F-14 Linear Model
%
% Drew Robbins, 9/22, VPI
%
%---------------------------------------------------------------------------
clear;

global A_ac B_ac C_ac D_ac Za Zq Zde Vt G

% Trim Condition
h = 20000; %ft
rho_sl = 2.3769e-3; % Sea Level
%rho = 1.2673e-3;
rho = rho_sl;
Vc=337.561; %ft/sec 200 knots - Indicated
Vt = Vc*sqrt(rho/rho_sl);
qbar = .5*rho*Vt^2;

% Constants
S = 565; %ft^2
c = 117.62/12; %ft
G = 32.167;
Weight = 55375; %lbs
m = Weight/G;
Iyy =247511; %slug-ft^2 or lb-sec^2-ft
R2D = 180/pi;

%Non-Dimensional Stability Derivatives
CDO = 0.0;
CL0 = 0.0886;
CLa = 0.1050*R2D;%/deg
CLadot = 3.44; %/rad
CLq = 8.80; %/rad
CLde = 0.0140; %/deg

CM0 = 0.0188;
CMa = -0.0101*R2D; %/deg
CMadot = -6.27; %/rad
CMq = -17.65; %/rad
CMde = -.0217; %/deg   elevator -> negative teu->nu
\texttt{\%Dimensional Derivatives}
\texttt{Ma = qbar*S*c*CMa/Iyy;}
\texttt{Za = -qbar*S*(CDO+CLA)/m;}
\texttt{Mq = qbar*S*c^2*CMq/(2*Iyy*Vt);}
\texttt{Zq = -qbar*S*c*CLq/(2*m*Vt);}
\texttt{Mde = qbar*S*c*CMde/Iyy;}
\texttt{Zde = -qbar*S*CLde/m;}
\texttt{Madot = qbar*S*c^2*CMadot/(2*Iyy*Vt);}
\texttt{Zadot = -qbar*S*c*CLadot/(2*m*Vt);}

\texttt{\% Short Period Approximation}
\texttt{M = [ Vt-Zadot \quad 0;}
\texttt{\quad -Madot \quad 1];}
\texttt{A = [Za \quad Vt+Zq;}
\texttt{\quad Ma \quad Mq];}
\texttt{B = [Zde;}
\texttt{\quad Mde];}
\texttt{Atmp = inv(M)*A;}
\texttt{Btmp = inv(M)*B;}
\texttt{Za_p = Atmp(1,1);}
\texttt{Zq_p = Atmp(1,2);}
\texttt{Ma_p = Atmp(2,1);}
\texttt{Mq_p = Atmp(2,2);}
\texttt{Zde_p = Btmp(1,1);}
\texttt{Mde_p = Btmp(2,1);}
\texttt{A_ac = [Za_p \quad Zq_p \quad 0; \% Alpha(rad), Q(rad), Theta(rad)}
\texttt{\quad Ma_p \quad Mq_p \quad 0}
\texttt{\quad 0 \quad 1 \quad 0];}
\texttt{B_ac = [Zde_p;}
\texttt{\quad Mde_p;}
\texttt{\quad 0];}
\texttt{C_ac = [-Vt/G*Za_p \quad -Vt/G*(Zq_p-1) \quad 0; \% Nz}
0 0 1 ]; \text{%Theta}

D_{ac} = [-\dot{Vt}/G*Z_{de_p};0];
Vita

Andrew C. Robbins was born on November 26, 1973 to Jackson S. Robbins and Kay G. Johnson in Little Rock, Arkansas. After finishing high school in Fort Worth, Texas, Andrew attended Texas A&M University where he received his bachelor degree in Aerospace Engineering, Magna Cum Laus, in 1997. He then attended Virginia Polytechnic Institute and State University where he received his Masters of Science in Aerospace Engineering in 1999, concentrating in dynamics and controls with an emphasis in simulation.