Digital Control of Levitation

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(Abstract)

Electromagnetic levitation has been commonly researched for the use in ground transit systems. It is ideal for high-speed applications that require low friction. The principle is simple, use electromagnetic force to balance the force imposed by gravity. However, for attractive levitation the system is unstable and nonlinear. Two dominant approaches to this problem have been to use a state feedback control system or a simple linear PID compensated control architecture. State feedback is a well-known control technique, but is complicated to implement and can rely on linearization of the system dynamics. The simple PID control structure is very easy to implement, but can have severe performance degradation in the presence of noise. This system can usually be identified by its large acoustic noise. This is primarily due to the differential term in the controller. This thesis proposes a solution that uses two concepts: Current Command Generation (CCG) and a closed velocity loop.

CCG linearizes the control structure by utilizing the known magnetic properties of the system to convert a desired force to a current for any given air gap. This removes squared command terms from the control structure. This allows for a reliable and predictable implementation of linear feedback control systems.

The PID implementation of an attractive levitation system uses two control loops. The inner loop is a current controller, which receives current commands from the outer position loop. The proposed control architecture uses three loops. The innermost loop is the current controller, which receives current commands for the CCG. The middle loop is a velocity controller, which receives commands from the position (outer most) loop and produces force command output used as inputs to the CCG. The three loops consist of two Proportional Integral (PI) controllers for the current and velocity controllers and a Proportional (P) controller. There is no derivative term, making the proposed solution’s performance far less dependent on noise.

This architecture removes the necessity of nonlinear elements in the control architectures and improves noise rejection through the use of the velocity loop. The acoustic noise performance of this system is enhanced by both of these methodologies and is shown in the experimental setup.
Dedication

I dedicate this dissertation to my father, mother, and two sisters who accepted my time away from them during the course of this work. They have always believed in my work and provided moral support.
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# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>x</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>xi</td>
</tr>
<tr>
<td>Chapter 1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Chapter 2. SISO Control Design</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Attractive Electromagnetic Levitation Modeling</td>
<td>4</td>
</tr>
<tr>
<td>2.2 State Space Formulation of SISO Levitation Modeling</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Velocity Loop Control Derivation</td>
<td>10</td>
</tr>
<tr>
<td>2.4 Position Loop Control Derivation</td>
<td>12</td>
</tr>
<tr>
<td>2.5 Force Loop Control Derivation</td>
<td>14</td>
</tr>
<tr>
<td>2.6 Current Command Generation (CCG) Implementation</td>
<td>17</td>
</tr>
<tr>
<td>Chapter 3. MIMO System Modeling and Analysis</td>
<td>18</td>
</tr>
<tr>
<td>3.1 MIMO System State Space Modeling</td>
<td>18</td>
</tr>
<tr>
<td>3.2 Linearization of the MIMO System</td>
<td>22</td>
</tr>
<tr>
<td>Chapter 4. Simulations</td>
<td>25</td>
</tr>
<tr>
<td>4.1 Simulation Structure and Gain Settings</td>
<td>25</td>
</tr>
<tr>
<td>4.2 Simulation Results</td>
<td>27</td>
</tr>
<tr>
<td>Chapter 5. Levitation Implementation and Experimental Results</td>
<td>40</td>
</tr>
<tr>
<td>5.1 Hardware Implementation</td>
<td>40</td>
</tr>
<tr>
<td>5.2 DC Link</td>
<td>43</td>
</tr>
<tr>
<td>5.3 Power Control Board</td>
<td>44</td>
</tr>
<tr>
<td>5.4 Gap Sensor Interface Board</td>
<td>46</td>
</tr>
<tr>
<td>5.5 PWM Interface</td>
<td>47</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5.6 Current Feedback and Over-Current Protection</td>
<td>48</td>
</tr>
<tr>
<td>5.7 ADMC 401 DSP and connector board</td>
<td>49</td>
</tr>
<tr>
<td>5.8 DSP Gain Conversion and Velocity Determination</td>
<td>50</td>
</tr>
<tr>
<td>5.9 Experimental Results</td>
<td>51</td>
</tr>
<tr>
<td>Chapter 6. Conclusions</td>
<td>54</td>
</tr>
<tr>
<td>APPENDIX A: MIMO LINEARIZATION (CURRENT STATE INCLUDED)</td>
<td>55</td>
</tr>
<tr>
<td>APPENDIX B: SPECIFICATIONS OF ELECTROMAGNET USED</td>
<td>66</td>
</tr>
<tr>
<td>APPENDIX C: DETAIL DRAWINGS OF MECHANICAL SYSTEM</td>
<td>67</td>
</tr>
<tr>
<td>APPENDIX D: SCHEMATICS OF ELECTRONIC HARDWARE</td>
<td>81</td>
</tr>
<tr>
<td>D.1 Power control Board Connections and Schematics</td>
<td>81</td>
</tr>
<tr>
<td>D.2 Gap Sensor Interface Board Connections and Schematics</td>
<td>86</td>
</tr>
<tr>
<td>D.3 OC Interface Board Connections and Schematics</td>
<td>90</td>
</tr>
<tr>
<td>D.4 PWM Interface Board Connections and Schematics</td>
<td>95</td>
</tr>
<tr>
<td>References</td>
<td>101</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

1.1 Inductance Profile Characteristics of the Air Gap 1
1.2 Position, Velocity, and Force Control Loops 2
1.3 PID Control Loop 3

2.1 SISO Model 4
2.2 Flow Diagram Derived from State Space Model 7
2.3 Pole – Zero Plot of State-Space Model 8
2.4 Velocity Loop of Proposed Control System 10
2.5 Pole – Zero Plot of Velocity Control Loop 11
2.6 Simplified model of Position Loop 12
2.7 Force Control Loop 15
2.8 Inductance and rate of change of inductance profiles 17

3.1 Electromagnetic Levitation Test Structure 18

4.1 Simulation Flow Diagram 25
4.2 Step Response of Proposed System (No Noise, Identical Starting Positions) 27
4.3 Current Response (No Noise, Identical Starting Positions) 27
4.4 Force Response (No Noise, Identical Starting Positions) 28
4.5 Velocity Response (No Noise, Identical Starting Positions) 28
4.6 Step Response of Proposed System (No Noise, Initial Pitch of 2 mm) 29
4.7 Current Response of Proposed System (No Noise, Initial Pitch of 2 mm) 30
4.8 Force Response of Proposed System (No Noise, Initial Pitch of 2 mm) 30
4.9 Velocity Response of Proposed System (No Noise, Initial Pitch of 2 mm) 31
4.10 Step Response of Proposed System (No Noise, Initial Roll of 2 mm) 31
4.11 Current Response of Proposed System (No Noise, Initial Roll of 2 mm) 32
4.12 Force Response of Proposed System (No Noise, Initial Roll of 2 mm) 32
4.13 Velocity Response of Proposed System (No Noise, Initial Roll of 2 mm) 33
4.14 Step Response of Proposed System (Initial Roll and Pitch of 1 mm) 33
4.15 Current Response of Proposed System (Initial Roll and Pitch of 1 mm) 34
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.16</td>
<td>Step Response of PID Controller (No Noise – Ideal Initial Cond.)</td>
<td>34</td>
</tr>
<tr>
<td>4.17</td>
<td>Current Response of PID Controller (No Noise – Ideal Initial Cond.)</td>
<td>35</td>
</tr>
<tr>
<td>4.18</td>
<td>Step Response of PID Controller (Position Feedback Noise)</td>
<td>35</td>
</tr>
<tr>
<td>4.19</td>
<td>Current Response of PID Controller (Position Feedback Noise)</td>
<td>36</td>
</tr>
<tr>
<td>4.20</td>
<td>Step Response of Proposed Controller with Noise</td>
<td>36</td>
</tr>
<tr>
<td>4.21</td>
<td>Current Response of Proposed Controller with Noise</td>
<td>37</td>
</tr>
<tr>
<td>4.22</td>
<td>Force Response of Proposed Controller with Noise</td>
<td>37</td>
</tr>
<tr>
<td>4.23</td>
<td>Velocity Response of Proposed Controller with Noise</td>
<td>38</td>
</tr>
<tr>
<td>4.24</td>
<td>Step Response of Proposed Controller with Large Noise Input</td>
<td>38</td>
</tr>
<tr>
<td>4.25</td>
<td>Current Response of Proposed Controller with Large Noise Input</td>
<td>39</td>
</tr>
<tr>
<td>5.1</td>
<td>Electronic Hardware Layout</td>
<td>40</td>
</tr>
<tr>
<td>5.2</td>
<td>Electromagnetic Levitation Test Structure</td>
<td>41</td>
</tr>
<tr>
<td>5.3</td>
<td>Mechanical Structure</td>
<td>41</td>
</tr>
<tr>
<td>5.4</td>
<td>Balluff Sensor Drive Current vs. Air Gap</td>
<td>42</td>
</tr>
<tr>
<td>5.5</td>
<td>Schematic of DC Link</td>
<td>43</td>
</tr>
<tr>
<td>5.6</td>
<td>Power Control Connection with DC Link</td>
<td>43</td>
</tr>
<tr>
<td>5.7</td>
<td>Relay Circuitry of Power Control Board</td>
<td>44</td>
</tr>
<tr>
<td>5.8</td>
<td>Alarm Circuitry of Power Control Board</td>
<td>45</td>
</tr>
<tr>
<td>5.9</td>
<td>Gap Sensor Interface Circuitry (Single Phase)</td>
<td>46</td>
</tr>
<tr>
<td>5.10</td>
<td>PWM Interface Circuitry (Single Phase)</td>
<td>47</td>
</tr>
<tr>
<td>5.11</td>
<td>Current Feedback and Over Current Protection (Single Phase)</td>
<td>48</td>
</tr>
<tr>
<td>5.12</td>
<td>Position Command vs. Position Response</td>
<td>51</td>
</tr>
<tr>
<td>5.13</td>
<td>Velocity Command vs. Velocity Feedback</td>
<td>52</td>
</tr>
<tr>
<td>5.14</td>
<td>Current Command vs. Current Feedback</td>
<td>52</td>
</tr>
<tr>
<td>C.1</td>
<td>Table Plate</td>
<td>68</td>
</tr>
<tr>
<td>C.2</td>
<td>Disturbance Rail</td>
<td>69</td>
</tr>
<tr>
<td>C.3</td>
<td>Receptacle Plate</td>
<td>70</td>
</tr>
<tr>
<td>C.4</td>
<td>Sensor Mount</td>
<td>71</td>
</tr>
<tr>
<td>C.5</td>
<td>Bearing Shaft</td>
<td>72</td>
</tr>
<tr>
<td>C.6</td>
<td>Standoff Block</td>
<td>73</td>
</tr>
<tr>
<td>C.7</td>
<td>Track Plate</td>
<td>74</td>
</tr>
<tr>
<td>C.8</td>
<td>Levitation Rail</td>
<td>75</td>
</tr>
<tr>
<td>C.9</td>
<td>Vehicle Plate</td>
<td>76</td>
</tr>
<tr>
<td>C.10</td>
<td>Vice Block</td>
<td>77</td>
</tr>
<tr>
<td>C.11</td>
<td>Vice Stop</td>
<td>78</td>
</tr>
<tr>
<td>C.12</td>
<td>Weight Guide</td>
<td>79</td>
</tr>
<tr>
<td>C.13</td>
<td>Weight Plate</td>
<td>80</td>
</tr>
<tr>
<td>D.1</td>
<td>Power Control Board Connector Layout</td>
<td>82</td>
</tr>
<tr>
<td>D.2</td>
<td>Power Control Power Line Layout</td>
<td>83</td>
</tr>
<tr>
<td>D.3</td>
<td>Power Control Relay Schematic</td>
<td>84</td>
</tr>
<tr>
<td>D.4</td>
<td>Power Control Regen and OV Schematic</td>
<td>85</td>
</tr>
<tr>
<td>D.5</td>
<td>Gap Sensor Board Interface Connector Layout</td>
<td>86</td>
</tr>
<tr>
<td>D.6</td>
<td>Gap Sensor Board Power Line Layout</td>
<td>87</td>
</tr>
<tr>
<td>D.7</td>
<td>Gap Sensor Board Sensor Feedback Schematic</td>
<td>88</td>
</tr>
<tr>
<td>D.8</td>
<td>OC Interface Board Connector Layout</td>
<td>89</td>
</tr>
<tr>
<td>D.9</td>
<td>OC Interface Board Power Line Layout</td>
<td>93</td>
</tr>
<tr>
<td>D.10</td>
<td>OC Interface Board Current Feedback Schematic</td>
<td>94</td>
</tr>
<tr>
<td>D.11</td>
<td>PWM Interface Board Connector Layout</td>
<td>96</td>
</tr>
<tr>
<td>D.12</td>
<td>PWM Interface Board PWM Signal Generation Schematic</td>
<td>97</td>
</tr>
<tr>
<td>D.13</td>
<td>PWM Interface Board PWM Signal Generation Schematic – 2</td>
<td>98</td>
</tr>
<tr>
<td>D.14</td>
<td>Power Distribution Schematic</td>
<td>99</td>
</tr>
<tr>
<td>D.15</td>
<td>Gate Driver Schematic</td>
<td>100</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

4.1 Gain Values Used in Simulation

5.1 Gap Sensor extreme Voltage outputs
5.2 ADMC 401 Connections
5.3 Input/Output Equivalencies
5.4 Implemented DSP Gain Values

C.1 Purchased Parts List – Mechanical Implementation
C.2 Machined Parts List – Mechanical Implementation
C.3 Frame World Parts List – Mechanical Implementation
# LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>Air gap referenced from CG, mm</td>
</tr>
<tr>
<td>zk</td>
<td>Air gap referenced from respective magnet k, mm</td>
</tr>
<tr>
<td>Θxz</td>
<td>Pitching angle, rad</td>
</tr>
<tr>
<td>Θyz</td>
<td>Roll angle, rad</td>
</tr>
<tr>
<td>Jxz</td>
<td>Pitching moment of inertia, m$^4$</td>
</tr>
<tr>
<td>Jyz</td>
<td>Rolling moment of inertia, m$^4$</td>
</tr>
<tr>
<td>Fzk</td>
<td>Vertical force at respective magnet k, N</td>
</tr>
<tr>
<td>Lk</td>
<td>Inductance at respective magnet k, H</td>
</tr>
<tr>
<td>tκ</td>
<td>Current at respective magnet k, A</td>
</tr>
<tr>
<td>Kpp</td>
<td>P-gain of position loop</td>
</tr>
<tr>
<td>Kpv</td>
<td>P-gain of velocity loop</td>
</tr>
<tr>
<td>Kiv</td>
<td>I-gain of velocity loop</td>
</tr>
<tr>
<td>Kpc</td>
<td>P-gain of current loop</td>
</tr>
<tr>
<td>Kic</td>
<td>I-gain of current loop</td>
</tr>
</tbody>
</table>
Chapter 1. Introduction

The concept of electromagnetic levitation systems has been under research for a long time. Many different methods have been proposed in literature and summarized in [1]. The method using DC electromagnets producing attractive forces is extremely popular. The use of the DC electromagnets in conjunction with an electric propulsion system forms the basis for a large number of magnetically levitated transportation systems [2]. The design of the electromagnets has been covered in literature [1]-[3].

Control techniques used for attractive levitation are based on two control loops. The inner loop is current control, which receives command signals from the outer loop. The outer control loop is a feedback system that controls the mechanical state, air gap, of the system. The inherit obstacles that the attractive levitation presents are the open loop instability, $i^2$ force dependence, and the nonlinear inductance dependence on air gap. The open loop instability will be covered in greater detail in Chapter 2. The $i^2$ force dependence, and the nonlinear inductance dependence on air gap are shown here in (1.1) and Figure 1.1.

\[ f_x = \frac{1}{2} \frac{\partial L_k}{\partial z_k} i^2 \]  

(1.1)

Figure 1.1 Inductance Profile Characteristics of the Air Gap

The equation in 1.1 shows the $i^2$ relationship between current and force. This presents problems because the current loop is requiring a nonlinear command from the outer loop. Linear controllers are desirable for simplicity and reliability, are not suited to produce these types command with linear feedback. This nonlinearity is coupled with the $\frac{\partial L_k}{\partial z_k}$. This term (plotted as $g_k$) in Figure 1.1 is nonlinear with air gap. A solution to this problem is to use Current Command Generation (CCG). CCG is based on using a DSP implementation to convert a force output of the outer control loop to a current command to the inner control loop. This strategy will remove nonlinearities in the control system by isolated the inner control loop from the outer control loop through mathematical conversion techniques. This principle is sound since the $g_k$ profile is known for the proposed system. The controller uses this information to
linearize the two control techniques: mechanical state control and current control. A full linearization of the system is given in Chapter 3 of this thesis.

In its simplest form, the use of a single DC electromagnet to demonstrate the principles of levitation is a single input single output (SISO) system. The use of four electromagnets to levitate a structure can be seen as a multiple input multiple output (MIMO) system where the roll, yaw and pitch of the structure affect the system dynamics [1]. The effect of the yaw angle in a magnetic levitated system is mostly accounted by the use of four electromagnets used for guidance and its effect is not considered here. Judicious assumptions can be made to reduce the system to a SISO system for the purposes of designing the force and position controllers. The controllers are selected to be of the proportional-integral (PI) form instead of the standard proportional-differential (PD) form or the PID (Figure 1.3) form. This reduces the effect of noise (sensor noise or track irregularities) on the control system. However, by removing the differential term from the position controller two an additional control loop is needed, namely, a velocity control loop. The PI controller scheme used in this thesis is shown below in Figure 1.2.

![Figure 1.2a Position and Velocity Loops](image1)

![Figure 1.2b Force control loop.](image2)

![Figure 1.3 PID control loop.](image3)
The two critical differences between this controller design and that of the PID approach are the use of a closed velocity loop, and Current Command Generation (CCG). CCG is based on the fundamental magnetic equation $$F_{zk} = \frac{1}{2} \frac{\partial L_k}{\partial z_k} i_k^2 = \frac{1}{2} g_k i_k^2.$$ The technique removes the $$i^2$$ term from the control structure through the use of a table look-up. The output of the velocity loop is a force command; this force command is converted to a current command by using tabulated values of $$g_k$$ and a square root operation. This removes squared terms from the control structure.

The closed velocity control loop is necessary to achieve position open loop stability without the use of a derivative term in the position compensator. The use of a velocity loop has trade-offs that should be considered. The introduction of a velocity loop to the system will greatly reduce the effects of noise due to absence of derivative terms in the controller, but will also slow the step response of the entire system. These control techniques will be discussed in great length in the Control Design (Chapter 3) of this thesis.

The thesis is organized as follows. Chapter 2 outlines the electromagnetic levitation system, describes the electrical and mechanical equations governing both the (SISO) system, and describes the design of the controller. Chapter 3 presents the MIMO model, linearizes the model, and draws control conclusions. Simulation results are described in Chapter 4. Chapter 5 reviews implementation and the experimental results. Conclusions are summarized in Chapter 6.
Chapter 2. SISO Control Design

2.1 Attractive Electromagnetic Levitation Modeling.

Attractive magnetic levitation is a complicated issue due to its inherent unstable and nonlinear behavior. Designing a levitation controller for a maglev system is even further complicated by the demands for controller simplicity and small steady state accelerations. Control simplicity is desirable for two reasons. First, simple controllers have predictable behavior and are well researched. This insures that the final design will be reliable. Secondly, controller simplicity minimizes DSP coding, reducing program cycle times. This will trim output update times; increasing the digital controller bandwidth. Steady state vehicle accelerations directly relates to the ride quality of the maglev system. These accelerations have been observed as audible noise in prototypes already built and running. Levitation control design should seek to squelch this undesirable mechanical behavior.

Understanding the dynamic behavior of an attractive levitation system gives insight to designing an appropriate and desirable control system. Dynamic modeling of both the electrical and mechanical systems yields the open loop properties. The open loop properties will provide information on the linearity, stability, and point to favorable control techniques to be used. Figure 2 shows a 1 Degree of Freedom (DOF) model of a single electromagnet and rail.

Dynamic modeling begins with prescribing the appropriate force equation taken from Newton’s second law. This is provided in (1).

\[ f_z - mg = -m \frac{d^2 h(t)}{dt^2} \]  

(2.1)

Equation (2.1) is a mechanics equation relating gravity, \( g \), and the input force, \( f_z \), to vehicle acceleration, \( a_z \). The corresponding electromagnetic force equation is given in (2.2). Equation (2.2) is the electromagnetic relationship between applied force, \( f_{zk} \), and the inductance, \( L_k \), airgap, \( z_k \), and applied current, \( i_k \).

\[ f_z = \frac{1}{2} \frac{\partial L_k}{\partial z_k} i_k^2 \]  

(2.2)
This equation is derived by taking the derivative of the work equation, \( \frac{\partial W}{\partial z} \), given that \( i \) is independent of gap, \( z \).

\[
W = \frac{1}{2} L_i^2 \tag{2.3}
\]

\[
\frac{\partial W}{\partial z} = \frac{1}{2} \frac{\partial L_i}{\partial z} i_z^2 = f_i \tag{2.4}
\]

The electromagnetic equation, (2.2), can be further manipulated when the electrical models of a DC magnet are included. The inductance, \( L_k \), of this system is given by (2.5).

\[
L_i(z_i) = \mu_i T_m^2 A_m/(2 z_i) = \mu_i T_m^2 w_i d_i/(2 z_i) \tag{2.5}
\]

Substituting (2.5) into (2.2) yields:

\[
f_{ik} = \frac{1}{2} \frac{\partial L_i}{\partial z_i} i_i^2 = \frac{1}{2} g_{ik} i_i^2 = \frac{\mu_i T_m^2 A_m}{4} \left( \frac{i_i}{z_i} \right)^2 = K \left( \frac{i_i^2}{z_i^2} \right) \tag{2.6}
\]

This equation indicates that the attractive levitation system is nonlinear due to the \( \left( \frac{i_i}{z_i} \right)^2 \) term. To determine the stability a pole zero plot would be desirable. To obtain a pole zero plot, linearization must be done to the model. Once the model is linearized a state space model is formed and the corresponded pole zero plot is extracted.
2.2 State Space Formulation of SISO Levitation Model

To formulate a state-space model of this system, states must be assigned using the previous dynamic models. This approach assumes that the current, $i_1$, has a control bandwidth that is much larger than the mechanical dynamics of the system. The variable $i_1$ is used as the input, $u_1$, instead of a state to simplify the model. Note that the required current controller will be modeled as a unity gain when discussing the mechanical states (position and velocity). This assumption allows for easier inspection of the mechanical transfer functions and problematic areas of this model.

The states of this system are defined as:

\begin{align*}
x_1 &= h \\
x_2 &= h
\end{align*}  \hspace{1cm} (2.7)

The output equation, $z$, is defined as:

\begin{align*}
z &= x_1
\end{align*}  \hspace{1cm} (2.8)

The system is linearized at the nominal operating point. This linearized model is used to derive a transfer functions for the velocity and position loops.

Rewriting the mechanics equations in state space form and evaluating the partial derivatives at the nominal operating point:

\begin{align*}
x &= p(x,i) = \begin{bmatrix} x_2 \\ p_1 \\ p_2 \end{bmatrix}
\end{align*} \hspace{1cm} (2.9)

outputs:

\begin{align*}
z &= q(x,i) = [v_1]
\end{align*} \hspace{1cm} (2.10)

Partial Derivatives.

\begin{align*}
\frac{\partial p}{\partial x}, \frac{\partial p}{\partial i}, \frac{\partial q}{\partial x}
\end{align*} \hspace{1cm} (2.11)

\begin{align*}
\frac{\partial p_1}{\partial x_2} &= 1, \quad \frac{\partial p_1}{\partial x_1} = 0
\end{align*} \hspace{1cm} (2.12)

\begin{align*}
\frac{\partial p_2}{\partial x_1} &= 2K \left( \frac{i_1^2}{3x_1^3} \right)
\end{align*} \hspace{1cm} (2.13)

\begin{align*}
\frac{\partial p_1}{\partial i} &= 0
\end{align*} \hspace{1cm} (2.14)
The linearized model of the attractive levitation system is shown below with dynamic currents, \( i_1 \), and dynamic states, \( x_1 \), substituted with nominal steady state values \( I_1 \) and \( X_1 \).

The flow diagram shown in Figure 2.2 is derived from this state-space model and used determine the Laplace domain function of this system.

The Laplace domain transfer function is taken from the diagram and given as:

\[
G_p(s) = \frac{s^2}{1 - \frac{2KI^2}{mX_1^3} s^3} = \frac{1}{s^2} \frac{2KI^2}{mX_1^3} \frac{X(s)}{x(s)}
\]  

(2.20)
\[ G_m(s) = \frac{2K i^2}{m X_i^2 s^2} = \frac{2K i^2}{m X_i^2 s^2} = \frac{X(s)}{u(s)} \]

\[ C_1 = \frac{-2K i^2}{m X_i} \]  

Previous Equations rewritten with \( C_1 \) and \( C_2 \).

\[ G_p(s) = \frac{s^2}{1 + \frac{C_2}{s^2}} = \frac{1}{s^2 + C_2} = \frac{X(s)}{x(s)} \]

\[ G_m(s) = \frac{C_1 s^2}{1 + \frac{C_2}{s^2}} = \frac{C_1}{s^2 + C_2} = \frac{X(s)}{u(s)} \]

A pole zero plot of the system is shown in Figure 2.3.

![Pole-Zero Plot](image)

**Figure 2.3 Pole – Zero Plot of State-Space Model**

There will be two poles placed symmetrically about the imaginary axis for any real values of \( K_i, K_{pv}, \) and \( C_2 \). Therefore, this system is open loop unstable for any real system. Traditionally this instability has been handled by using a derivative based controller (PD or PID, e.g.) in the position loop. This places a zero at the origin. This zero will pull the RHP pole to the origin, thus canceling the RHP, resulting in a stable system. This technique is theoretically adequate and simple to implement. However, the derivative term can cause undesirable control signals in the presence of noise. This is true for digital or analog implementations.
2.3 Velocity Loop Control Derivation.

The proposed approach uses velocity and position feedback control systems. The velocity loop control architecture is shown below in Figure 2.4.

\[ G_v(s) = \frac{C_1(K_p(s + K_v))}{s^2 + H_vK_pC_1s + H_vK_pK_2C_1} = \frac{1}{H_v} \cdot \frac{H_vK_pC_1s + H_vK_pK_2C_1}{s^2 + H_vK_pC_1s + H_vK_pK_2C_1} \frac{X_v(s)}{X_v'(s)} \]  

\[ G_v(s) = \frac{1}{H_v} \cdot \frac{H_vK_pC_1s + H_vK_pK_2C_1}{s^2 + H_vK_pC_1s + H_vK_pK_2C_1} \frac{X(s)}{X'(s)} \]  

Figure 2.4 Velocity Loop of Proposed Control System

The system is closed loop stable with two complex poles or a double pole (depending on the gain values) in the LHP. This was accomplished without the use of a differential term. Since, the position loop of this system encompasses the velocity loop this pole-zero plot represents the open loop poles of a proportional position controller. Open loop position stability has been achieved without the use of a PID controller or with the complexity of a state-feedback control system. A pole zero plot of this expression is given in Figure 2.5 to graphically represent \( G_v(s) \).
This velocity loop is a second order system. Gain values can be found by comparing (2.24) with (2.25). The result of this comparison is given in (2.26).

\[
G_v(s) = \frac{2\zeta_\omega \omega \, s + \omega_i^2}{s^2 + 2\zeta_\omega \omega \, s + \omega_i^2}
\]  

\hspace{1cm} (2.25) 

where

\[
2\zeta_\omega \omega_i = H_v K_p C_i \\
\omega_i = H_v K_p K_v C_i
\]  

\hspace{1cm} (2.26) 

Notice that gain determination is relatively straightforward for a second order system. However, using this gain determination technique will not guarantee the bandwidth or damping ratio assumed. This is due to the non-ideal operation of the force control loop and the inherent non-linearity of the levitation system.
2.4 Position Loop Control Derivation.

The position loop derivation is conducted using the simplified model shown in Figure 2.6.

![Figure 2.6 Simplified model of Position Loop.](image)

**K_{pp}** - Proportional Gain of Position Controller

**Hp** - Position Feedback Gain

The transfer function of this complete system is given by (2.27).

\[
G_p(s) = \frac{K_p \omega_c}{H \cdot s(s + \omega_c) + H_p K_p \omega_c} = \frac{1}{H_p} \cdot \frac{H_p K_p \omega_c}{H \cdot s(s + \omega_c) + H_p K_p \omega_c}
\]

\[
= \frac{1}{H_p} \cdot \frac{H_p K_p \omega_c}{s^2 + \omega_c^2 + H_p K_p \omega_c} = \frac{X_p(s)}{X_1(s)}
\]

This equation can be written in terms of the loop damping ratio, \( \delta_p \), and loop bandwidth, \( \omega_p \), by comparing (2.27) with (2.28). The results of this comparison are given in (2.29).

\[
G_p(s) = \frac{\omega_c^2}{s^2 + 2 \zeta_p \omega_c + \omega_c^2}
\]

\[
2 \zeta_p \omega_c = \omega_c
\]

\[
\omega_c = \frac{H_p \omega_c - K_p \omega_c}{H_p}
\]

Substitute \( \omega_c \) into \( jo \) of the transfer function to determine the relationship between the loop bandwidth, \( \omega_p \), the damping ratio, \( \zeta_p \), and frequency constant term, \( \omega_n \).
\[ |G_p(j\omega)|_{\omega_r} = \sqrt{\frac{(\omega_p^2)^2}{(-\omega_p^2 + \omega_n^2)^2 + (2\xi_p \omega_n \omega_p^2)^2}} = \frac{1}{\sqrt{2}} \]

\[ 2\omega_i = \omega_r^2 - 2\omega_r \omega_i + \omega_i^2 + 4\xi_p \omega_i \omega_r \omega_i \]

\[ 0 = \omega_r^2 - 2\omega_r \omega_i + 4\xi_p \omega_i^2 - \omega_r \omega_i - \omega_i \]

\[ 0 = \omega_r^2 + 2(2\xi_p - 1) \omega_r \omega_i - \omega_r \omega_i \]

\[ \omega_i = \frac{-2(2\xi_p - 1)\omega_i \pm \sqrt{4(2\xi_p - 1)^2 \omega_i^2 + 4\omega_i^2}}{2} \]

\[ \omega_r = \sqrt{(1 - 2\xi_p - 1)^2 + (2\xi_p - 1)^2 + 1} \cdot \omega_r \]

Solving for \( K_{pp} \).

\[ K_{pp} = \frac{\omega_r}{2\xi_p \sqrt{(1 - 2\xi_p - 1)^2 + (2\xi_p - 1)^2 + 1}} H_r \]

Solving for \( \omega_p \) in terms of \( \omega_r \).

\[ \omega_p = \frac{\sqrt{(1 - 2\xi_p - 1)^2 + (2\xi_p - 1)^2 + 1} \omega_r}{2\xi_p} \]

\[ K_{pp} = \frac{2\xi_p}{2\xi_p \sqrt{(1 - 2\xi_p - 1)^2 + (2\xi_p - 1)^2 + 1}} H_r = \frac{H_r \omega_r}{4\xi_p^2 H_r} \]
2.5 Force Loop Control Derivation.

The derivations of the velocity and position loops have been based on the assumption that the current loop bandwidth is much larger. The force loop is treated here. The derivation starts with plant modeling. The implementation of this levitation system uses a PWM switched voltage supply. Accordingly, the plant modeling begins with the fundamental voltage/current relationship given in (2.34).

\[
v_{kv} = R_s i_{kv} + \frac{d\lambda_{lev}}{dt} = R_s i_{kv} + L_{lev} \frac{di_{lev}}{dt} + g_{kv} z_{lev}
\] (2.34)

where \(v_k\) is the applied voltage, \(\lambda_k\) is the flux linkages, and \(i_k\) is the instantaneous winding current. The magnetic material is assumed to be linear and there is no leakage flux. These assumptions are expressed in equation form in (2.35).

\[
\lambda_{lev} = L_{lev} i_{lev}, \quad g_{lev} = \frac{\partial L_{lev}}{\partial x}
\] (2.35)

Simplification:

\[
\frac{di_{lev}}{dt} = -\frac{R_s}{L_{lev}} i_{lev} - \frac{g_{lev}}{L_{lev}} z_{lev} + \frac{v_{lev}}{L_{lev}} = -a_1 i_{lev} - a_2 z_{lev} + a_0 v_{lev}
\] (2.36)

where \(a_0 = \frac{1}{L_{lev}}, a_1 = \frac{R_s}{L_{lev}}, a_2 = \frac{g_{lev}}{L_{lev}}\)

For the case of magnetic levitation it assumed that the vertical velocity, \(z\), is negligible. This simplification yields (2.37).

\[
\frac{di_{lev}}{dt} = -\frac{R_s}{L_{lev}} i_{lev} + \frac{v_{lev}}{L_{lev}} = -a_1 i_{lev} + a_0 v_{lev}
\] (2.37)

The plant transfer function for the force loop is presented in (2.38).

\[
\frac{i_{lev}}{u_{lev}} = \frac{a_0}{s + a_1}
\] (2.38)

The force control loop is presented in Figure 2.7.
The critical link between the force loop and the velocity loop is the CCG. This block will be described in detail in the following sections. The simplified loop is used to derive gain values and loop parameters. The gain selections of this design were based on classical controls techniques. The design of the current controller is described in this section. A proportional plus integral controller is considered for the present implementation. The procedure followed is described in detail in [4]. The current controller was modelled as a second order transfer function. The system transfer function is given in (2.38).

\[
G_c(s) = \frac{2\zeta \omega_c s + \omega_n^2}{s^2 + 2\zeta \omega_c s + \omega_n^2} \quad G_c(s) = \frac{K_p + K_i}{s + K_p s + K_i K_p K_v} \frac{1}{H_c} \quad (2.38)
\]

\[
2\zeta, \omega_c = K_p K_v \\
\omega_n = K_i K_v K_i \\
K_i = \omega_n H_c K_v \quad (2.39)
\]

The parameters chosen to design the current controller were the damping ratio, \( \zeta_c \), and the system bandwidth, \( \omega_c \). The system bandwidth is defined as the frequency at which the magnitude of \( G_c(j\omega) \) drops to \( 1/\sqrt{2} \) of its zero-frequency value. The definition of \( \omega_c \) is given in (2.40).

\[
|G_c(j\omega)|_{\omega_c} = \left. \frac{(\omega_n^2) + (2\zeta \omega_n)^2}{(\omega_e^2 + \omega_n^2)^2 + (2\zeta \omega_n)^2} \right| = \frac{1}{\sqrt{2}} \quad (2.40)
\]

After final rearrangement the system bandwidth, \( \omega_c \), is defined in (40).

\[
\omega_c = \sqrt{(1 + 2\zeta^2) + \sqrt{(1 + 2\zeta^2)^2 + 1 \omega_e^2}} \quad (2.41)
\]

Evaluating \( K_p \) and \( K_i \) in terms of \( \zeta_c \) and \( \omega_c \) yields the following equation set.
\[
K_m = \frac{2 \zeta \omega_c}{\sqrt{(1 + 2 \zeta^2) + \sqrt{(1 + 2 \zeta^2)^2} + 1}} K_c
\]

\[
K_n = \frac{\omega_c}{2 \zeta \sqrt{(1 + 2 \zeta^2) + \sqrt{(1 + 2 \zeta^2)^2} + 1}}
\]

(2.42)
2.6 Current Command Generation (CCG) Implementation.

The equation relating current, $i_k$, to force, $F_{zk}$, is re-introduced here in (2.43).

$$F_{zk} = \frac{1}{2} \frac{dL}{dz} i_k^2 = \frac{1}{2} g_k i_k^2, \text{for } k = 1, 2, 3, 4$$

(2.43)

where the $F_{zk}$ is the applied force, the inductance of the coil is $L_k$ and the applied current to the coil is $i_k$. $g_k$ is equal to the rate of change of inductance with respect to the air gap. For the prototype electromagnet (specifications in the Appendix B), the profiles of inductance and rate of change of inductance are plotted against position in Figure 2.8. This proposed electromagnet and levitation rail do not operate in the saturation region from 0 to 6.5 A. Thus, the magnet operation was assumed to operate in the linear region yielding two-dimensional plots of $L$ and $g_k$. For the prototype system, the nominal air gap is set at 3 mm and at 2 mm the impact protection system prevents further movement of the electromagnet towards the rail. Hence the range of interest is between 2 mm to 6 mm (resting position).

![Figure 2.8 Inductance and rate of change of inductance profiles](image)

The CCG is implemented as a table look-up of $g_k$ values and math routines to solve for the current command, $i_k^*$. Current command, $i_k^*$, is expressed in equation form in (2.44).

$$i_k^* = \sqrt{\frac{2F_{zk}}{g_k}}, \text{for } k = 1, 2, 3, 4$$

(2.44)

The described control system was implemented digitally. Hence, table look-ups and math routines can be easily realized.

This chapter explored the open loop properties of attractive levitation. It was found that the characteristics of attractive levitation are unstable and highly nonlinear. Proper feedback and linearization techniques can compensate for these behaviors and yield a stable and controllable system.
Chapter 3 looks at the MIMO system to understand the coupling characteristics between the states and inputs of the MIMO system.
Chapter 3. MIMO System Modeling and Analysis

3.1 MIMO System State Space Modeling

The SISO system was linearized and feedback systems were designed to control the position of the air gap. This control strategy is to use four independent SISO control systems to control a 3 DOF MIMO system. Justifications are necessary for using four independent SISO systems on a MIMO system. Coupling between the individual states and the 4 force inputs need to be studied. If the states and inputs are decoupled, then using 4 independent SISO systems is completely justified. If this is found not to be the case, then careful considerations must be made to use this approach. Trade-offs to this approach are recognized, as well as, arguments for future research. The MIMO system strategy begins with linearization of the system, since non-linear control is out of the realm of this thesis.

The proposed levitation system structure is composed of a single aluminum plate (base plate) with four independent u-shaped electromagnets placed near the corners of the base plate. There are proximity sensors and impact protection placed in line with these magnets. At the center of the base plate are stainless steel pegs for positioning additional weights. The attraction railing is fixed to a specially designed test bench. The levitation test system attracts itself to the railing and is completely free floating during operation. Figure 3.1 shows an isometric view of the levitation test system.

Figure 3.1 Electromagnetic Levitation Test Structure
The equations for the vertical motion of the magnets are described as,

\[ z_1 = h - \frac{L}{2} \sin(\theta_{xz}) - \frac{w}{2} \sin(\theta_{yz}) \]
\[ z_2 = h - \frac{L}{2} \sin(\theta_{xz}) + \frac{w}{2} \sin(\theta_{yz}) \]
\[ z_3 = h + \frac{L}{2} \sin(\theta_{xz}) + \frac{w}{2} \sin(\theta_{yz}) \]
\[ z_4 = h + \frac{L}{2} \sin(\theta_{xz}) - \frac{w}{2} \sin(\theta_{yz}) \]

where \( z_k \) is the air gap between the top surface of the levitation core and the reaction rail, \( h \) is the vertical distance from the center of mass to the reaction rail referenced from the surface of the levitation core, \( L \) is distance between two cores in the x direction, \( w \) is the distance between two cores in the y direction, \( \theta_{xz} \) the pitch angle about the y axis and \( \theta_{yz} \) the roll angle about the x axis.

The corresponding electromagnetic force equation is given by,

\[ F_{zk} = \frac{1}{2} \frac{\partial L_k}{\partial z_k} i_k^2 = \frac{1}{2} g_k i_k^2, \text{ for } k = 1, 2, 3, 4 \]

These are the respective input and output equations to the MIMO system. A state space representation is necessary to linearize and thereby inspect the system for coupling either the states or the inputs. The system described considers \( z \) translation, \( \theta_{xz} \) pitch rotation, and \( \theta_{yz} \) the roll rotation. The states of the MIMO system have been defined accordingly.

States:
\[ x_1 = h \]
\[ x_2 = \dot{h} \]
\[ x_3 = \theta_{yz} \]
\[ x_4 = \dot{\theta}_{yz} \]
\[ x_5 = \theta_{xz} \]
\[ x_6 = \dot{\theta}_{xz} \]

The outputs are the respective gap, \( z_k \), of each of the electromagnets.

\[ z_1, z_2, z_3, z_4 \]

Because the force control loop uses CCG, the inputs are simply the forces applied at each electromagnet. This greatly simplifies the MIMO modeling and linearization.

\[ f_1, f_2, f_3, f_4 \]
The fundamental equation of motion is given by Newton’s second law and is shown applied to this model in (3.6). Where \( m \) is the total mass being levitated, \( g \) is the acceleration due to gravity, \( J_{yz} \) is the rotational moment of inertia in the yz plane, and \( J_{xz} \) is the rotational moment of inertia in the xz plane.

\[
m \frac{d^2 h(t)}{dt^2} = (f_1 + f_2 + f_3 + f_4) - mg
\]

\[
J_{xz} \frac{d^2 \Theta_{xz}(t)}{dt^2} = (f_1 + f_3) \frac{L}{2} \cos(\Theta_{xz}) - (f_3 + f_5) \frac{L}{2} \cos(\Theta_{xz})
\]

\[
J_{xz} \frac{d^2 \Theta_{xz}(t)}{dt^2} = (f_1 + f_3) \frac{w}{2} \cos(\Theta_{xz}) - (f_3 + f_5) \frac{w}{2} \cos(\Theta_{xz})
\]

Rewriting motion equations in terms of state equations.

\[
\dot{x}_1 = \ddot{h} = \frac{d^2 h(t)}{dt^2} = \frac{(f_1 + f_2 + f_3 + f_4)}{m} - g
\]

\[
\dot{x}_4 = \ddot{\Theta}_{xz} = \frac{d^2 \Theta_{xz}(t)}{dt^2} = \frac{(f_1 + f_3) \frac{L}{2} \cos(x_3)}{J_{xz}}
\]

\[
\dot{x}_6 = \ddot{\Theta}_{xz} = \frac{d^2 \Theta_{xz}(t)}{dt^2} = \frac{(f_1 + f_3) \frac{w}{2} \cos(x_5)}{J_{xz}}
\]

Setting the states as, \( x_1 = h, x_2 = \dot{h}, x_3 = \theta_{xz}, x_4 = \dot{\theta}_{xz}, x_5 = \theta_{yz}, x_6 = \dot{\theta}_{yz} \), and assuming that \( \cos(\theta_{xz}) = \cos(\theta_{yz}) = 1 \), the state equations are obtained as:

\[
\dot{x}_2 = \ddot{h} = \frac{d^2 h(t)}{dt^2} = \frac{(f_1 + f_2 + f_3 + f_4)}{m} - g
\]

\[
\dot{x}_4 = \ddot{\Theta}_{xz} = \frac{d^2 \Theta_{xz}(t)}{dt^2} = \frac{(f_1 + f_3) \frac{L}{2} \cos(x_3)}{J_{xz}}
\]

\[
\dot{x}_6 = \ddot{\Theta}_{xz} = \frac{d^2 \Theta_{xz}(t)}{dt^2} = \frac{(f_1 + f_3) \frac{w}{2} \cos(x_5)}{J_{xz}}
\]

Substituting for \( z_k \) from (1) and assuming \( \sin(\theta_{xz}) = \theta_{xz} = x_3 \) and \( \sin(\theta_{yz}) = \theta_{yz} = x_5 \), the final form of the output equation is obtained.
These simplified equations allow show the relationships between force input and the state, as well as, the influence of the states on the output gaps. This model is the basis for linearization and state space formulation presented in the next section.
3.2 Linearization of the MIMO System

The method selected for linearization assumes small perturbation around the nominal operating point. The state and output equations are redefined with the following definitions. Forces, \( f_1, f_2, f_3, f_4 \), are the inputs to the system because of the current loop bandwidth assumptions and because of the use of CCG. Appendix A contains a MIMO derivation based on current inputs. The coupling results are identical for both derivations.

State Equations

\[
x = p(x, f) = \begin{bmatrix} x_2 \\ p_x(x, f) \\ x_4 \\ p_y(x, f) \\ x_6 \\ p_z(x, f) \end{bmatrix} = \begin{bmatrix} \frac{x_2}{m} \\ \frac{(f_1 + f_2 + f_3 + f_4)}{m} - g \\ \frac{L}{2J} (f_1 + f_2) \\ \frac{W}{2J} (f_1 + f_2) \end{bmatrix}
\] (3.13)

Output Equations

\[
z = q(x, f) = \begin{bmatrix} x_1 - \frac{L}{2} x_1 - \frac{w}{2} x_3 \\ x_1 - \frac{L}{2} x_1 + \frac{w}{2} x_3 \\ x_3 + \frac{L}{2} x_3 + \frac{w}{2} x_3 \\ x_3 + \frac{L}{2} x_3 - \frac{w}{2} x_3 \end{bmatrix}
\] (3.14)

The partial derivatives of both functions are determined and the resulting equations are evaluated at the nominal operating point.

Partial Derivatives.

\[
\frac{\partial p}{\partial x}, \frac{\partial p}{\partial f}, \frac{\partial q}{\partial x}, \frac{\partial q}{\partial f}
\] (3.15)

Input Derivatives

\[
\frac{\partial p_i}{\partial x_i} = 0, k = 1, 3, 4, 5, 6
\] (3.16)

\[
\frac{\partial p_i}{\partial f_i} = 0, k = 1, 2, 3, 4
\] (3.17)
\[
\frac{\partial p_i}{\partial f_i} = \frac{1}{m}, \quad k = 1,2,3,4 \tag{3.18}
\]
\[
\frac{\partial p_i}{\partial i_k} = 0, k = 1,2,3,4 \tag{3.19}
\]
\[
\frac{\partial p_i}{\partial i_k} = \frac{L}{2J_z}, k = 1,2,\ldots, \frac{\partial p_i}{\partial i_k} = \frac{-L}{2J_z}, k = 3,4 \tag{3.20}
\]
\[
\frac{\partial p_i}{\partial i_k} = 0, k = 1,2,3,4 \tag{3.21}
\]
\[
\frac{\partial p_k}{\partial i_k} = \frac{w}{2J_z}, k = 1,4,\ldots, \frac{\partial p_k}{\partial i_k} = \frac{-w}{2J_z}, k = 2,3 \tag{3.22}
\]

Output Derivatives.
\[
\frac{\partial q_i}{\partial x_k}, \frac{\partial q_i}{\partial x_k}, \frac{\partial q_i}{\partial x_k} = 0 \quad k = 1,2,3,4,5,6 \tag{3.23}
\]
\[
\frac{\partial q_1}{\partial x_1} = 1 \tag{3.24}
\]
\[
\frac{\partial q_i}{\partial x_1}, \frac{\partial q_i}{\partial x_1} = \frac{L}{2} \tag{3.25}
\]
\[
\frac{\partial q_i}{\partial x_1}, \frac{\partial q_i}{\partial x_1} = \frac{L}{2} \tag{3.26}
\]
\[
\frac{\partial q_i}{\partial x_1}, \frac{\partial q_i}{\partial x_1} = \frac{w}{2} \tag{3.27}
\]
\[
\frac{\partial q_i}{\partial x_1}, \frac{\partial q_i}{\partial x_1} = \frac{w}{2} \tag{3.28}
\]
\[
\frac{\partial q_i}{\partial f_i} = 0 \quad k = 1,2,3,4,5,6 \tag{3.29}
\]

The state space model is formed by placing the $\frac{\partial p_i}{\partial x_k}$ terms in the A matrix, the $\frac{\partial p_i}{\partial f_i}$ terms in the B matrix, the $\frac{\partial q_i}{\partial x_k}$ terms in the C matrix, and the $\frac{\partial q_i}{\partial f_i}$ terms in the D matrix. The final matrix form of the state space model is shown below.
Final Matrix format:

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\
0 & 0 & 0 & 0 \\
\frac{L}{2J_m} & \frac{L}{2J_m} & \frac{L}{2J_m} & \frac{L}{2J_m} \\
0 & 0 & 0 & 0 \\
\frac{w}{2J_m} & \frac{w}{2J_m} & \frac{w}{2J_m} & \frac{w}{2J_m} \\
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & -\frac{L}{2} & 0 & -\frac{w}{2} & 0 \\
1 & 0 & -\frac{L}{2} & 0 & -\frac{w}{2} & 0 \\
1 & 0 & \frac{L}{2} & 0 & \frac{w}{2} & 0 \\
1 & 0 & \frac{L}{2} & 0 & -\frac{w}{2} & 0 \\
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

where A is the state or characteristic matrix of the system. B is the input/state matrix that relates the inputs, \(f_1, f_2, f_3, f_4\), to the derivative of the states, \(x_1 = \dot{h}, x_2 = \dot{\theta}_p, x_3 = \dot{\theta}_r, x_4 = \dot{x}_p, x_5 = \dot{x}_r\). C is the input/output matrix that relates the inputs, \(f_1, f_2, f_3, f_4\), to the outputs, \(z_1, z_2, z_3, z_4\). D is the disturbance matrix, which relates disturbance inputs to the output. This modeling assumed no disturbance input; this yields an empty D matrix.

The linearization of this system shows that the states are decoupled. This is shown by the empty A, characteristic, matrix (the states are also independent when the system is derived using currents, \(i_k\), are considered — see Appendix A). However the input matrices, B, is coupled. This is due to the mechanical coupling of the levitation plate. Any input force influences all the states of the system. From (3.6) – (3.8) it is given that the input matrix, B, is coupled the greatest at zero pitch and roll angles. However, for large input angles the states become coupled and control using the proposed system is not predictable. The conclusion of this analysis is that roll and pitch angles should be minimized before the gap height, h, is handled. Since the input matrix becomes highly coupled for small pitch and roll angles, the controller should inherently reject noise to avoid oscillations of the states caused by input oscillations. A possible strategy that was not employed here is to use gap command profiling to level roll and pitch angles before maintaining the nominal air gap. This is one strategy to explore in future research.

This chapter explored the coupling effects of the MIMO system. These coupling characteristics need to be understood to apply appropriate control to the system. It was shown that while the given states are decoupled about the nominal operating point, the inputs are highly coupled. Command profiling is the next logical research topic, since it can be used to lessen the input coupling effects. This proposed control system was extensively simulated to verify its performance. The simulation flow diagram, the gain values used, and comparisons between the proposed control architecture and that of a PID system are given in the next chapter.
Chapter 4. Simulations

4.1 Simulation Structure and Gain Settings

This chapter provides the gain settings used and the simulation results for both the proposed system and that of the more conventional PID controlled system. This chapter also presents the simulation flow diagram used to derive the control simulations programs. Simulation assumptions and nonlinear constraints are provided here.

The simulations used to produce the results presented used the dynamic equations described in (3.6) – (3.8). Position limiting and command limiting were also employed to produce the most accurate simulations. Position limiting was kept outputs, $z_1$-$z_4$, in the range of 2 – 6 mm. The 2 mm limit represents when the impact protection equipment would make contact with the levitation rail. The 6 mm limit represents the resting position of the levitation plate on the stand. Command limiting was applied to the velocity command to the velocity loop controller, force command to the CCG, current command to the current loop, and voltage output was controlled by PWM switching. A flow chart of the simulation is given in Figure 4.1.
The simulations were conducted on the following situations:

1. Equal initial gap (no roll or pitch angles).
2. Initial pitch angle.
3. Initial roll angle.
4. Initial roll and pitch angle.
5. Comparison with PID controller with CCG.
6. Noise Simulations of both the proposed controller and the PID controller.

Gains were tuned for both systems under the condition of no noise and equal initial gap (the ideal situation) and kept fixed for all other conditions. These gains are tabulated in Table 4.1.

Table 4.1 Gain Values Used in Simulation

<table>
<thead>
<tr>
<th></th>
<th>Proposed Controller</th>
<th>PID Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kpp</td>
<td>12</td>
<td>1000</td>
</tr>
<tr>
<td>Kpv</td>
<td>25</td>
<td>Kip</td>
</tr>
<tr>
<td>Kiv</td>
<td>150</td>
<td>Kdp (differential term) 10</td>
</tr>
<tr>
<td>Kpc</td>
<td>10</td>
<td>Kpc</td>
</tr>
<tr>
<td>Kic</td>
<td>2000</td>
<td>Kic</td>
</tr>
<tr>
<td>Hv</td>
<td>0.4</td>
<td>Hp</td>
</tr>
<tr>
<td>Hc</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Hp</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
4.2 Simulation Results

The position commands were step inputs from 6 to 3 mm. The initial gaps are plotted at time zero. The proposed system’s simulation results were then compared to the PID controller scheme to compare performance based on step response, noise rejection, and steady state error. Figure 4.2 presents the step response of the outputs z1-z4 when the initial conditions are set to 6 mm with no startup coil current.

![Graph](image1.png)

**Figure 4.2** Step Response of Proposed System (No Noise, Identical Starting Positions)

The current response of the above simulation is shown here. This was simulated using PWM generated voltage inputs to the coil. Notice the small ripple current, this is due to the relatively slow current command.

![Graph](image2.png)

**Figure 4.3** Current Response (No Noise, Identical Starting Positions)
The force outputs of the individual coils are shown below in Figure 4.4

![Figure 4.4 Force Response (No Noise, Identical Starting Positions)](image)

The velocity command and response are presented in Figure 4.5.

![Figure 4.5 Velocity Response (No Noise, Identical Starting Positions)](image)

The initial velocity command is high. This winds up the current loop quickly allowing for quick changes in the force profile. The velocity loop has a smoothing effect on the entire control system. Its
relatively quick response is set up by judiciously selecting \( H_v \) above less than unity. This is an important point. When the position error is high and the velocity is low, the position loop dominates the controller response. Conversely, when the position error is small the velocity loop dominates the controller response. This is an important tradeoff. There must be balance between the selections of \( H_v \) and \( K_{pp} \). If \( H_v \) is intuitively selected as a unity gain, the step response of the system is very slow and unresponsive. If the \( H_v \) is selected too small with respect to \( K_{pp} \) the controller acts like a bang-bang controller at the nominal point; the integrator of the velocity loop has little effect. This point will be revisited in this thesis in the implementation chapter.

The system responded well. The levitation plate reached the final gap position of 3 mm in approximately .30 seconds. The current command and current response displayed smooth profiles, which were free of jitter and noise. The next three simulations was setup using no noise input but with various starting positions. Figure 4.6 presents the gap response with an initial pitch to the levitation plate.

![Figure 4.6 Step Response of Proposed System (No Noise, Initial Pitch of 2 mm)](image)

The starting position placed gaps 3 and 4 at 4 mm and gaps 1 and 2 at 6 mm. This represents a 33 % error in initial starting positions. The controller performed well, the uneven starting position had no effect on the settling time of the levitation plate. The system still reached its final position in .30 seconds. The corresponding current, force, and velocity plots are given here.
Figure 4.7  Current Response of Proposed System  (No Noise, Initial Pitch of 2 mm)

Figure 4.8  Force Response of Proposed System  (No Noise, Initial Pitch of 2 mm)
The system responded well to the initial conditions. The settling time, .31 seconds, was nearly the same as it was for the ideal initial condition. Input coupling could not be detected from any of the given plots. The system behaves as a SISO system. The current command generation is necessary to decouple the force inputs because it reduces the position and velocity feedback loops influence on the current command by removing the square term from the output of the velocity feedback loop. The next simulations show the same system with an initial roll angle.

Figure 4.9 Velocity Response of Proposed System (No Noise, Initial Pitch of 2 mm)

Figure 4.10 Step Response of Proposed System (No Noise, Initial Roll of 2 mm)
Figure 4.11  Current Response of Proposed System  (No Noise, Initial Roll of 2 mm)

Figure 4.12  Force Response of Proposed System  (No Noise, Initial Roll of 2 mm)
Figure 4.13 Velocity Response of Proposed System (No Noise, Initial Roll of 2 mm)

The simulation shows that the system behaves as a SISO system for a step response. The next two figures show the step response and current of the proposed system with an initial roll and pitch condition.

Figure 4.14 Step Response of Proposed System (Initial Roll and Pitch of 1 mm)

Figure 4.15 Current Response of Proposed System (Initial Roll and Pitch of 1 mm)

The proposed system responded well for various initial conditions without the presence of noise. This system is compared to a implementation that uses PID controller with CCG to evaluate the performance specifications of both systems. Figure 4.16 is the step response of the PID controller.
Figure 4.16 Step Response of PID Controller (No Noise – Ideal Initial Cond.)

Figure 4.17 Current Response of PID Controller (No Noise – Ideal Initial Cond.)

The PID controller’s response was much faster than that of the proposed system. This should be expected since its operation is based on a position loop, which includes a fast acting derivative term. The responses are similar for various initial conditions. It would seem that the proposed controller has inferior performance to the PID controller. However, when noise sources are simulated the PID controller can become unstable for even small sources of noise. Hardware filtering is required for real implementation which will effectively slow the response of the controller and make performance
predictions difficult. The response of the PID controller to a white position feedback noise with peak amplitude of .05 mm is presented in figure 4.18 and 4.19.

![Graph of response](image)

**Figure 4.18** Step Response of PID Controller (Position Feedback Noise)

![Graph of response](image)

**Figure 4.19** Current Response of PID Controller (Position Feedback Noise)

The PID controller becomes unstable when subjected to the feedback noise. It can no longer track position and the currents commands oscillate between the maximum and minimum limits.

The proposed system is subjected to both position and velocity feedback noise. The noise inputs are modeled as white random noise with peaks at .05 mm and .5 mm/s, respectively. The peak levels are equivalent to the noise levels given in the PID simulation. The simulation was conducted for identical gap initial conditions. Other initial conditions show similar results.
Figure 4.20 Step Response of Proposed Controller with Noise

Figure 4.21 Current Response of Proposed Controller with Noise
of 1 mm and 10 mm/s, respectively.

4.25 provide the gap and current responses to the maximum tolerated position and velocity noise input of 1 mm and 10 mm/s, respectively.  Figures 4.24 and 4.25 provide the gap and current responses to the maximum tolerated position and velocity noise input of 1 mm and 10 mm/s, respectively.

The proposed controller has superior performance for the same noise input. The system remains stable and peak to peak current oscillation is kept to a minimum. These results show convincingly that the proposed control system is vastly superior to a PID controller in the presence of noise. Figures 4.24 and 4.25 provide the gap and current responses to the maximum tolerated position and velocity noise input of 1 mm and 10 mm/s, respectively.
The results also show that there is a performance trade-off between the two. The simulations clearly show that the proposed controller is superior to the PID controller in the presence of noise. The current output maintains the same average value as was the case with no feedback noise, but the current oscillates at a very high frequency (approx. 10 kHz). This current oscillation will produce audible noise in the prototype. This result suggests that while stability is kept, current oscillation is present with large noise inputs and will cause audible noise.

The simulations clearly show that the proposed controller is superior to the PID controller in the presence of noise. The results also show that there is a performance trade-off between the two. The proposed controller has a slower response time than the PID controller. However, careful examination of the multiple feedback loops and simulation experiments illustrate that $H_v$ is a critical response and noise parameter. $H_v$ values less than unity provide faster response times and $H_v$ values greater than unity provide better noise rejection. This experimental result is left to future research since
mathematical proof is difficult to provide in this nonlinear system. Gain scheduling of $H_v$ may provide the best of both systems, this could be a topic of future research.

This chapter provided simulations results of the proposed system. The results show that the proposed control architecture has superior performance to that of the conventional PID control architecture in the presence of noise. It should also be understood that it is possible to provide superior response performance by using gain scheduling techniques or by reducing the resiliency of the proposed system to noise. This is the general trade-off in attractive levitation systems; noise rejection vs. response performance. Chapter 5 discusses implementation issues and provides the electronic control techniques used to implement the proposed architecture. The next chapter also provides techniques used for gain conversion between simulation gains and the DSP implementation of the controller.
Chapter 5. Levitation Implementation and Experimental Results

5.1 Hardware Implementation

This chapter provides detailed hardware layouts of the electronic and mechanical systems used to implement the attractive levitation system. Simulation gain conversions are given in this chapter will the final gain values used in implementation. Experimental results of the implementation are shown and correlated with simulation results.

The hardware of the system consisting of 8 main components: 1. Mechanical test fixture. 2. DC Link. 3. Power control. 4. Gate driver (the gate driver board was previously implemented and will not be discussed in this thesis). 5. Gap sensor interface. 6. PWM interface. 7. Current feedback and over-current protection. 8. ADMC 401 DSP and connector board. A picture of the electronic hardware layout is shown in Figure 5.1.

Figure 5.1 Electronic Hardware Layout
The mechanical test structure is shown below in Figure 5.2.

Figure 5.2 Electromagnetic Levitation Test Structure

The detail drawings are presented in Appendix C. These drawings are the final revisions and the exact drawings used by the machinist. The other elements of the mechanical structure consist of gap stops, the magnet core, coils, levitation rail housing, levitation plate, and sensor and power line terminals. These elements are presented in Figure 5.3.

Figure 5.3 Mechanical Structure
The proximity sensors used were the Balluff (BAW 018 PF 1 K 03) analog inductive sensors. These sensors drive output currents which correlates linearly with air gap between 6 mm and 2 mm. The characteristic output current plot is presented in Figure 5.4. The characteristics of this sensor will be reviewed in the DSP and Gap Sensor Interface sections of this chapter.

Figure 5.4 Balluff Sensor Drive Current vs. Air Gap
5.2 DC Link

The DC link portion of the electronic hardware is comprised of the main NFB switch, the main power relay (MC1), the bypass relay (MC2), the rectifier (BD1), and the 8, 1000uF capacitors. A schematic of this system is offered here.

![Figure 5.5 Schematic of DC Link](image)

The main power and bypass relays are controlled by the power control board. The interconnections between these two systems are shown in Figure 5.6.

![Figure 5.6 Power Control Connection with DC Link](image)

The momentary switch between pins 1 and 2 latches RY1 with 120 VAC. Once RY1 is closed the solenoid of MC1 is driven by 120 VAC through the alarm relay (/Alarm) and the normally closed power off momentary switch (/Power_off). PIO 4 of the DSP connector board controls the alarm relay. In normal operation these switches are closed, allowing main power to be passed by the Power_on switch. If an alarm triggers /Alarm or if the user depresses the /Power_off switch, the path is broken opening relays, MC1 and RY1. The bypass relay, MC2, is driven by a relay on the Power Control Board, which is driven by PIO 3 of the ADMC connector board.

5.3 Power Control Board
The power control board drives the DC Link relays, measures the DC link voltage, determines over-voltage (OV) and under-voltage (UV) alarms, includes the servo_on signal generation, houses the regeneration circuitry, and contains drive relays for both the Bypass relay, and the /Alarm relay. Figure 5.7 shows the relay circuitry for the /Alarm, /Bypass, and /Servo_on signals.

![Figure 5.7 Relay Circuitry of Power Control Board](image)

The /Alarm and /Bypass signals are isolated from the relays (G5V-2-DC24) through the H11G1 optocouplers. The input diode is biased properly through the use of 220 \( \Omega \) resistors and LED’s. The outputs of the H11G1 optocouplers are connected to the solenoids of the Bypass and Alarm control relays controlling the operation of the DC Link relays, MC1 and MC2, as previously discussed. The /Servo_on signal is generated by a toggle switch mounted on the Power Control Board and the output of the PC2505 is routed directly to PIO 2 of the ADMC 401 Connector Board.

The Power Control Board determines if an over-voltage or an under-voltage has occurred. The circuitry for these two operations is presented in Figure 5.8.
Figure 5.8 Alarm Circuitry of Power Control Board

The capacitor voltage, \( V_{dc} \), is divided across the 84 k\( \Omega \) and the 100 \( \Omega \) resistors. This will reduce a nominal DC link voltage of 160 V to .2 V. The HCPL 7800 is an isolation amplifier that outputs a differential voltage with an internal gain of 8. Therefore the .2 V input will be at 8 V at the output of the TL084 op-amp. The comparator circuits (LM339) are arranged as Schmitt triggers and are provided with reference voltages from the Zener diode circuit. The reference voltages are set so that OV occurs at 220 VDC, UV occurs at 120 VDC, and the Regen Signal is activated at 200 VDC.
5.4 Gap Sensor Interface Board

The gap sensor interface board takes the current output of the proximity sensor and converts it to a 0-2.048 V input to the ADC of the ADMC 401 DSP. This conversion is done with the use of the Analog Device AD623 Instrumentation Amplifier. This amplifier contains a reference input and differential inputs with rail to rail amplification capability. This allows for easy inversion and offsetting with on chip. The interface circuitry for one phase is given in Figure 5.9.

![Figure 5.9 Gap Sensor Interface Circuitry (Single Phase)](image)

The reference chip (REF191) produces a 2.048 output given a +5 V input. This output is tied to the reference pin (PIN 5) of the AD623 chip. A voltage divider with high terminal connected to the 2.048 reference produces an additional offset of 0.234 V. This voltage is connected to the non-inverting terminal of the AD623 (PIN 3). These two reference voltages produce a combined offset of 2.282 V. The current of the sensor flows through the 235 \(\Omega\) resistor (two 470 \(\Omega\) resistors in parallel) to produce a voltage at the inverting terminal of the AD623 (PIN 2). The extreme sensor output values are evaluated and the voltage signal \(V_{gs4}\) is determined in Table 5.1.

<table>
<thead>
<tr>
<th>Air Gap (mm)</th>
<th>Current Drive (mA)</th>
<th>AD623 Pin Voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>1.400</td>
<td>0.329</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.234</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.048</td>
</tr>
<tr>
<td>8.000</td>
<td>10.000</td>
<td>2.350</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.234</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.048</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.068</td>
</tr>
</tbody>
</table>

The circuitry provides good voltage shifting. Note that for smaller air gaps the output signal is larger. This alleviates control inversion in the controller, since positive levitation movement results in both positive position and velocity signals.
5.5 PWM Interface

The PWM interface circuit receives reference signal from the DAC channels of the ADMC 401 and converts them into PWM switching signals. It does this by converting the 0 – 4V DAC signal to two ± 4 V signals. The two signals are inversions of each other. These signals are compared, simultaneously, to ± 4 V 20kHz triangle wave signals and the output of the comparator signals drive the upper and lower switches of the Gate Driver Board. Figure 5.10 provides the circuitry of the prescribed operation for one phase.

![PWM Interface Circuitry (Single Phase)](image)

The LT1208 circuitry provides the free-running ± 4 V 20kHz triangle wave. The AD623 inverts and shifts the signal so that the output (PIN) is ± 2 V. The TL084 operational amplifiers multiply the AD623 signal by two and create two signals having voltage range ± 4 V that are inverses of each other. These two signals are passed to the comparators (LM339) and digital PWM signals are sent to the buffer (PEEL18CV8). The buffer allows the PWM signals to pass to the Gate Driver Boards if the gate is on and there are no alarms.
5.6 Current Feedback and Over-Current Protection

The current feedback board converts the LA25-NP current sensor output (0-20 mA) to a ±2 V signal to be fed to the ADC of the ADMC 401. This board utilizes the AD623 to invert and shift the incoming signal. This signal is then passed through a comparator and reference to determine if an over-current has occurred. If an over-current is detected an OC alarm is triggered and all gates are closed through the PEEL buffer. Figure 5.11 shows the circuitry for one phase.

![Figure 5.11 Current Feedback and Over Current Protection (Single Phase)](image)

The reference circuitry trips the OC alarm (Pin 13 of the LM339) at 9.0 A. The current feedback $-V_{csd}$ is connected to the ADC of the ADMC 401. The feedback voltage is translated internally in the DSP code to represent currents ranging from 0 – 10 A.
### 5.7 ADMC 401 DSP and connector board

The connections to the ADMC 401 is tabulated in Table 5.2

#### Table 5.2 ADMC 401 Connections

<table>
<thead>
<tr>
<th></th>
<th>Gap Sensor Feedback</th>
<th>ADMC Connection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap</td>
<td>z1</td>
<td>ADC0</td>
</tr>
<tr>
<td></td>
<td>z2</td>
<td>ADC1</td>
</tr>
<tr>
<td></td>
<td>z3</td>
<td>ADC2</td>
</tr>
<tr>
<td></td>
<td>z4</td>
<td>ADC3</td>
</tr>
<tr>
<td>Current Sensor Feedback</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>z1</td>
<td>ADC4</td>
</tr>
<tr>
<td></td>
<td>z2</td>
<td>ADC5</td>
</tr>
<tr>
<td></td>
<td>z3</td>
<td>ADC6</td>
</tr>
<tr>
<td></td>
<td>z4</td>
<td>ADC7</td>
</tr>
<tr>
<td>PWM Interface</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>z1</td>
<td>DAC0</td>
</tr>
<tr>
<td></td>
<td>z2</td>
<td>DAC1</td>
</tr>
<tr>
<td></td>
<td>z3</td>
<td>DAC2</td>
</tr>
<tr>
<td></td>
<td>z4</td>
<td>DAC3</td>
</tr>
<tr>
<td>Power Control Board</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OV</td>
<td>PIO0</td>
<td></td>
</tr>
<tr>
<td>UV</td>
<td>PIO1</td>
<td></td>
</tr>
<tr>
<td>/Servo_on</td>
<td>PIO2</td>
<td></td>
</tr>
<tr>
<td>/Bypass</td>
<td>PIO3</td>
<td></td>
</tr>
<tr>
<td>/Alarm</td>
<td>PIO4</td>
<td></td>
</tr>
<tr>
<td>Over Current Protection</td>
<td></td>
<td></td>
</tr>
<tr>
<td>/Gate_on</td>
<td>PIO10</td>
<td></td>
</tr>
<tr>
<td>OC</td>
<td>PIO11</td>
<td></td>
</tr>
</tbody>
</table>
5.8 DSP Gain Conversion and Velocity Determination

The simulation gains were converted to DSP values. A sample computation is shown in (5.1). The conversion input and output equivalencies are listed in Table 5.3. These converted gains were then adjusted to provide optimal response and noise performance. Table 5.3 lists the implemented gain values.

\[
\frac{\text{in}_{\text{analog}} \cdot K_{iC_{\text{analog}}}}{\text{out}_{\text{analog}}} = \frac{\text{in}_{\text{digital}} \cdot K_{iC_{\text{digital}}}}{\text{out}_{\text{digital}}} \quad K_{iC_{\text{digital}}} = \frac{\text{out}_{\text{analog}} \cdot \text{in}_{\text{analog}}}{\text{out}_{\text{analog}}} \cdot K_{iC_{\text{analog}}}
\]  

(5.1)

Table 5.3 Input/Output Equivalencies

<table>
<thead>
<tr>
<th></th>
<th>Analog</th>
<th>Digital (HEX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>1 mm</td>
<td>100</td>
</tr>
<tr>
<td>Velocity</td>
<td>1 mm/s</td>
<td>100</td>
</tr>
<tr>
<td>Current</td>
<td>1 A</td>
<td>CD</td>
</tr>
<tr>
<td>Voltage</td>
<td>160 V</td>
<td>7FF</td>
</tr>
</tbody>
</table>

where \(\text{in}_{\text{analog}}\) and \(\text{in}_{\text{digital}}\) refer to physically equivalent DSP and analog input values and \(\text{out}_{\text{analog}}\) and \(\text{out}_{\text{digital}}\) refer to physically equivalent DSP and analog output values.

Table 5.4 Implemented DSP Gain Values

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_{pp})</td>
<td>2</td>
</tr>
<tr>
<td>(K_{pv})</td>
<td>1</td>
</tr>
<tr>
<td>(K_{iv})</td>
<td>3</td>
</tr>
<tr>
<td>(K_{pc})</td>
<td>4</td>
</tr>
<tr>
<td>(K_{ic})</td>
<td>40</td>
</tr>
<tr>
<td>(Hv, Hc, Hp)</td>
<td>1</td>
</tr>
</tbody>
</table>

The velocity determination method implemented is based on a M-method algorithm. This algorithm measures the position difference over a fixed time interval. The technique is applied easily but has one major tradeoff: for small time intervals the technique is very sensitive to sensor noise. To compensate for this effect, velocity determination was done at a rate of approximately 1500 Hz. This update rate is fast enough to ensure good controller stability (due to the relatively slow average speed of the system) and slow enough to negate noise effects.
5.9 Experimental Results

The following experimental results were captured on an oscilloscope: 1. Gap command and response. 2. Velocity command and response. 3. Current command and response. Note that positive velocity and position values refer to negative gap speeds and smaller position gaps, respectively. One phase is shown here. All the other phase exhibit similar results.

The position command was a step input. The output showed slight overshoot with some residual ringing, but quickly settled after .75 seconds. The position command is offset by 2.5 volts, this is because the measurement was taken from the DAC output of the ADMC 401 connector board which converts a zero input to a 2.5 V (negative inputs are less than 2.5 V and positive input values are greater than 2.5 V).

The next figure is of the velocity command versus the velocity response. The command starts at a positive spike and oscillates positive and negative until a near zero velocity command is settled at. The response is exhibits some tracking, but velocity determination discrepancies exhibit a noisy behavior. The feedback does settle to 0.

Figure 5.12 Position Command vs. Position Response

The position command was a step input. The output showed slight overshoot with some residual ringing, but quickly settled after .75 seconds. The position command is offset by 2.5 volts, this is because the measurement was taken from the DAC output of the ADMC 401 connector board which converts a zero input to a 2.5 V (negative inputs are less than 2.5 V and positive input values are greater than 2.5 V).

The next figure is of the velocity command versus the velocity response. The command starts at a positive spike and oscillates positive and negative until a near zero velocity command is settled at. The response is exhibits some tracking, but velocity determination discrepancies exhibit a noisy behavior. The feedback does settle to 0.
Figure 5.13 Velocity Command vs. Velocity Feedback

Figure 5.14 Current Command vs. Current Feedback
Figure 5.14 displays the current response of the system. The system exhibits some initial ringing but by .85 seconds the current command and current response have both stabilized to a 3.6 A value.

The experimental results look very promising for the system. Both position and current responses exhibit small levels of oscillation. This can be seen physically from steady levitation performance (position response stability) and low audible noise (current stability). One avenue of continued experimentation and research lies within the velocity determination. If improvements can be made to the velocity feedback characteristics, overall system improvement should be expected.
Chapter 6. Conclusions

The levitation experiment was successful in demonstrated an effective alternative control strategy for attractive levitation systems. By using multiple mechanical (position and velocity) control loops, the system was able to achieve quieter more stable performance. The control solution was simple to code and debug due to the phase independence and relatively simple control loop structures.

The DSP implementation of the system allows for further experimentation and research into different control structures with minimal time. The experiment did note a need to improve velocity determination. An improvement in velocity determination will improve the performance of the velocity loop. The velocity feedback issues were identified by the overshooting response of the experimental setup to a step response. They could also be identified in the oscillatory signal characteristics seen in the velocity command vs. velocity feedback plots.

The further supported the use of a CCG in this application and motor applications as well. The CCG removes necessary squared command terms from control structures. This allows for reliable and predictable use of linear compensators. This is a major simplification in control design.

This thesis presented many topics of further research. Independent position gap profiling could be a major research topic in attractive levitation systems. Research and experimentation in gap profiling may yield significant knowledge into coupling effects and which states (roll, pitch, gap) must be handled first to control systems with significant force disturbances.

Overall, this thesis was a success. The experiment yielded positive results and built a system with lower audible noise than the existing levitation system in the lab. There is still significant work to be done on attractive levitation systems, but this should be seen a positive step forward.
APPENDIX A: MIMO LINEARIZATION (CURRENT STATE INCLUDED)

States:

\[
x_1 = h \\
x_2 = \dot{h} \\
x_3 = \theta_{yz} \\
x_4 = \dot{\theta}_{yz} \\
x_5 = \theta_{xz} \\
x_6 = \dot{\theta}_{xz}
\]  
(A.1)

Outputs

\[
z_1, z_2, z_3, z_4
\]

\[
z_1 = x_1 - \frac{L}{2}\sin(\Theta_{yz}) - \frac{w}{2}\sin(\Theta_{xz})
\]
\[
z_2 = x_1 - \frac{L}{2}\sin(\Theta_{yz}) + \frac{w}{2}\sin(\Theta_{xz})
\]
\[
z_3 = x_1 + \frac{L}{2}\sin(\Theta_{yz}) + \frac{w}{2}\sin(\Theta_{xz})
\]
\[
z_4 = x_1 + \frac{L}{2}\sin(\Theta_{yz}) - \frac{w}{2}\sin(\Theta_{xz})
\]  
(A.2)

Inputs

\[
f_1, f_2, f_3, f_4
\]  
(A.3)

Equations of Motion

\[
m \frac{d^2 h(t)}{dt^2} = (f_1 + f_2 + f_3 + f_4) - mg - d(t)
\]  
(A.4)

\[
J_{yz} \frac{d^2 \Theta_{yz}(t)}{dt^2} = (f_1 + f_2) \frac{L}{2}\cos(\Theta_{yz}) - (f_3 + f_4) \frac{L}{2}\cos(\Theta_{yz}) + d(t)l_1\cos(\Theta_{yz})
\]  
(A.5)

\[
J_{xz} \frac{d^2 \Theta_{xz}(t)}{dt^2} = (f_1 + f_4) \frac{w}{2}\cos(\Theta_{xz}) - (f_2 + f_3) \frac{w}{2}\cos(\Theta_{xz}) - d(t)l_2\cos(\Theta_{xz})
\]  
(A.6)

\[
\dot{h} = \frac{d^2 h(t)}{dt^2} = \frac{(f_1 + f_2 + f_3 + f_4)}{m} - g - \frac{d(t)}{m}
\]  
(A.7)
\[
\dot{x} = \dot{\Theta}_y = \frac{d}{dt} \left( \frac{L}{2J_{yy}} \cos(\Theta_y) - (f_1 + f_2) \right) + \frac{L}{2J_{yy}} \cos(\Theta_y) + d(t) \cos(\Theta_y)
\] (A.8)

\[
\dot{x} = \dot{\Theta}_x = \frac{d}{dt} \left( \frac{L}{2J_{xx}} \cos(\Theta_x) - (f_1 + f_2) \right) + \frac{L}{2J_{xx}} \cos(\Theta_x) + d(t) \cos(\Theta_x)
\] (A.9)

where \( m \) is the total mass being levitated, \( g \) is the acceleration due to gravity, \( J_{yz} \) is the rotational moment of inertia in the yz plane and \( J_{xz} \) is the rotational moment of inertia in the xz plane. Setting the states as, \( x_1 = h, x_2 = h, x_3 = \theta_{xz}, x_4 = \theta_{xz}, x_5 = \theta_{yz}, x_6 = \theta_{yz}, \) and assuming that \( \cos(\theta_{xz}) = \cos(\theta_{yz}) = 1 \), the state equations are obtained as,

\[
\dot{x}_2 = h = \frac{d}{dt} \left( \frac{f_1 + f_2 + f_3 + f_4}{m} \right) - g \cdot \frac{d(t)}{m}
\] (A.10)

\[
\dot{x}_4 = \dot{\Theta}_{xz} = \frac{d}{dt} \left( \frac{L}{2J_{xx}} \right) \left( f_1 + f_2 - (f_3 + f_4) \right) + \frac{d(t) L}{J_{xx}}
\] (A.11)

\[
\dot{x}_6 = \dot{\Theta}_{yz} = \frac{d}{dt} \left( \frac{L}{2J_{yy}} \right) \left( f_1 + f_2 - (f_3 + f_4) \right) + \frac{d(t) L}{J_{yy}}
\] (A.12)

Substituting \( z_k \) from (1) and assuming \( \sin(\theta_{xz}) = \theta_{xz} = x_3 \) and \( \sin(\theta_{yz}) = \theta_{yz} = x_5 \),

\[
x = p(x,i) = \begin{bmatrix} x_2 \\ p_2(x,i) \\ x_4 \\ p_4(x,i) \\ x_6 \\ p_6(x,i) \end{bmatrix}
\] (A.13)

outputs:

\[
z = q(x,i,d) = \begin{bmatrix} x_1 - \frac{L}{2} x_1 + \frac{w}{2} x_5 \\ x_1 - \frac{L}{2} x_1 + \frac{w}{2} x_5 \\ x_1 + \frac{L}{2} x_1 + \frac{w}{2} x_5 \\ x_1 + \frac{L}{2} x_1 + \frac{w}{2} x_5 \end{bmatrix}
\] (A.14)

Partial Derivatives.

\[
\frac{\partial p}{\partial x} \frac{\partial p}{\partial t} \frac{\partial q}{\partial d} \frac{\partial q}{\partial d} \frac{\partial q}{\partial d}
\] (A.15)
\[
\frac{\partial p_1}{\partial x_2} = 1, \quad \frac{\partial p_1}{\partial x_k} = 0, k = 1, 3, 4, 5, 6 \quad (A.16)
\]

\[
\frac{\partial p_2}{\partial x_1} = 0 \quad (A.17)
\]

\[
\frac{\partial p_2}{\partial x_4} = 0 \quad (A.18)
\]

\[
\frac{\partial p_2}{\partial x_5} = 0 \quad (A.19)
\]

\[
\frac{\partial p_2}{\partial x_2} + \frac{\partial p_2}{\partial x_4} + \frac{\partial p_2}{\partial x_6} = 0 \quad (A.20)
\]

\[
\frac{\partial p_3}{\partial x_4} = 1, \quad \frac{\partial p_3}{\partial x_k} = 0, k = 1, 2, 3, 5, 6 \quad (A.21)
\]

\[
\frac{\partial p_4}{\partial x_1} = 2K \left( \frac{-i_1^2}{x_1^{-2}x_3^{-w}x_5^{-w}} \right) - \frac{i_2^2}{x_1^{-1}x_3^{-w}x_5^{-w}} \right) \frac{L}{2J_{yz}} + 2K \left( \frac{i_3^2}{x_1^{-1}x_3^{-w}x_5^{-w}} \right) + \frac{i_4^2}{x_1^{-1}x_3^{-w}x_5^{-w}} \right) \frac{L}{2J_{yz}} \quad (A.22)
\]

\[
\frac{\partial p_4}{\partial x_4} = 2K \left( \frac{i_1^L}{x_1^{-L}x_3^{-w}x_5^{-w}} \right) + \frac{i_2^L}{x_1^{-L}x_3^{-w}x_5^{-w}} \right) \frac{L}{2J_{x_1}} + 2K \left( \frac{i_3^L}{x_1^{-L}x_3^{-w}x_5^{-w}} \right) + \frac{i_4^L}{x_1^{-L}x_3^{-w}x_5^{-w}} \right) \frac{L}{2J_{x_1}} \quad (A.23)
\]

\[
\frac{\partial p_4}{\partial x_5} = 2K \left( \frac{i_1^w}{x_1^{-L}x_3^{-w}x_5^{-w}} \right) + \frac{i_2^w}{x_1^{-L}x_3^{-w}x_5^{-w}} \right) \frac{L}{2J_{x_1}} + 2K \left( \frac{i_3^w}{x_1^{-L}x_3^{-w}x_5^{-w}} \right) + \frac{i_4^w}{x_1^{-L}x_3^{-w}x_5^{-w}} \right) \frac{L}{2J_{x_1}} \quad (A.24)
\]

\[
\frac{\partial p_4}{\partial x_2} + \frac{\partial p_4}{\partial x_4} + \frac{\partial p_4}{\partial x_6} = 0 \quad (A.25)
\]
\[
\frac{\partial p_5}{\partial x_6} = 1, \frac{\partial p_5}{\partial x_k} = 0, k = 1, 2, 3, 4, 5 \quad (A.26)
\]

\[
\frac{\partial p_i}{\partial x_6} = 2k \left( \frac{i^i}{x_1^+ - x_3^+ - x_5^+} \right) \left( \frac{1}{L} \frac{w}{2} \right) \left( \frac{i^i}{x_1^- - x_3^- + x_5^-} \right) \left( \frac{1}{L} \frac{w}{2} \right) \frac{L}{2J_i} + 2k \left( \frac{i^i}{x_1^+ - x_3^+ - x_5^+} \right) \left( \frac{1}{L} \frac{w}{2} \right) \left( \frac{i^i}{x_1^- - x_3^- + x_5^-} \right) \left( \frac{1}{L} \frac{w}{2} \right) \frac{L}{2J_i} \quad (A.27)
\]

\[
\frac{\partial p_i}{\partial x_6} = 2k \left( \frac{i^i}{x_1^+ - x_3^+ - x_5^+} \right) \left( \frac{1}{L} \frac{w}{2} \sin x_5 \right) \left( \frac{i^i}{x_1^- - x_3^- + x_5^-} \right) \left( \frac{1}{L} \frac{w}{2} \right) \frac{w}{2J_i} - 2k \left( \frac{i^i}{x_1^+ - x_3^+ - x_5^+} \right) \left( \frac{1}{L} \frac{w}{2} \sin x_5 \right) \left( \frac{i^i}{x_1^- - x_3^- + x_5^-} \right) \left( \frac{1}{L} \frac{w}{2} \right) \frac{w}{2J_i} \quad (A.28)
\]

\[
\frac{\partial p_i}{\partial x_6} = +2k \left( \frac{i^i}{x_1^+ - x_3^+ - x_5^+} \right) \left( \frac{1}{L} \frac{w}{2} \sin x_5 \right) \left( \frac{i^i}{x_1^- - x_3^- + x_5^-} \right) \left( \frac{1}{L} \frac{w}{2} \right) \frac{w}{2J_i} - 2k \left( \frac{i^i}{x_1^+ - x_3^+ - x_5^+} \right) \left( \frac{1}{L} \frac{w}{2} \sin x_5 \right) \left( \frac{i^i}{x_1^- - x_3^- + x_5^-} \right) \left( \frac{1}{L} \frac{w}{2} \right) \frac{w}{2J_i} \quad (A.29)
\]

\[
\frac{\partial p_1}{\partial i_k} = 0, k = 1, 2, 3, 4, 5, 6 \quad (A.30)
\]
\[
\frac{\partial p_2}{\partial i_l} = \frac{K \left( \frac{2i_l}{x_l - \frac{L}{2} x_3 + \frac{w}{2} x_5} \right)^2}{m} \quad (A.31)
\]

\[
\frac{\partial p_2}{\partial i_{13}} = \frac{K \left( \frac{2i_{13}}{x_1 + \frac{L}{2} x_3 + \frac{w}{2} x_5} \right)^2}{m} \quad (A.32)
\]

\[
\frac{\partial p_2}{\partial i_{14}} = \frac{K \left( \frac{2i_{14}}{x_1 + \frac{L}{2} x_3 + \frac{w}{2} x_5} \right)^2}{m} \quad (A.33)
\]

\[\frac{\partial p_3}{\partial i_k} = 0, k = 1, 2, 3, 4, 5, 6 \quad (A.34)\]

\[
\frac{\partial p_4}{\partial i_1} = K \left( \frac{2i_1}{x_1 - \frac{L}{2} x_3 + \frac{w}{2} x_5} \right)^2 \frac{L}{2J_{yz}} \quad (A.35)
\]

\[
\frac{\partial p_4}{\partial i_2} = K \left( \frac{2i_2}{x_1 - \frac{L}{2} x_3 + \frac{w}{2} x_5} \right)^2 \frac{L}{2J_{yz}} \quad (A.36)
\]

\[
\frac{\partial p_4}{\partial i_3} = -K \left( \frac{2i_3}{x_1 + \frac{L}{2} x_3 + \frac{w}{2} x_5} \right)^2 \frac{L}{2J_{yz}} \quad (A.37)
\]
\[ \frac{\partial p_4}{\partial i_4} = -K \left( \frac{2i_4}{\left( x_1 + \frac{L}{2} x_3 - \frac{w}{2} x_5 \right)^2} \right) \frac{L}{2J_{yz}} \]  \hspace{1cm} (A.38)

\[ \frac{\partial p_5}{\partial i_k} = 0, \ k = 1,2,3,4 \]  \hspace{1cm} (A.39)

\[ \frac{\partial p_6}{\partial i_1} = K \left( \frac{2i_1}{\left( x_1 - \frac{L}{2} x_3 + \frac{w}{2} x_5 \right)^2} \right) \frac{w}{2J_{xz}} \]  \hspace{1cm} (A.40)

\[ \frac{\partial p_6}{\partial i_2} = -K \left( \frac{2i_2}{\left( x_1 - \frac{L}{2} x_3 - \frac{w}{2} x_5 \right)^2} \right) \frac{w}{2J_{xz}} \]  \hspace{1cm} (A.41)

\[ \frac{\partial p_6}{\partial i_3} = -K \left( \frac{2i_3}{\left( x_1 + \frac{L}{2} x_3 + \frac{w}{2} x_5 \right)^2} \right) \frac{w}{2J_{xz}} \]  \hspace{1cm} (A.42)

\[ \frac{\partial p_6}{\partial i_4} = K \left( \frac{2i_4}{\left( x_1 + \frac{L}{2} x_3 - \frac{w}{2} x_5 \right)^2} \right) \frac{w}{2J_{xz}} \]  \hspace{1cm} (A.43)
\[ z = q(x, i, d) = \begin{bmatrix} x_1 - \frac{L}{2} \sin x_3 - \frac{w}{2} \sin x_5 \\ x_1 - \frac{L}{2} \sin x_3 + \frac{w}{2} \sin x_5 \\ x_1 + \frac{L}{2} \sin x_3 + \frac{w}{2} \sin x_5 \\ x_1 + \frac{L}{2} \sin x_3 - \frac{w}{2} \sin x_5 \end{bmatrix} \]

(A.44)

\[
\frac{\partial q_k}{\partial x_2}, \frac{\partial q_k}{\partial x_4}, \frac{\partial q_k}{\partial x_6} = 0 \ldots k = 1, 2, 3, 4, 5, 6
\]

(A.45)

\[
\frac{\partial q_k}{\partial x_1} = 1
\]

(A.46)

\[
\frac{\partial q_1}{\partial x_3}, \frac{\partial q_2}{\partial x_3} = -\frac{L}{2} \cos x_3
\]

(A.47)

\[
\frac{\partial q_3}{\partial x_3}, \frac{\partial q_4}{\partial x_3} = \frac{L}{2} \cos x_3
\]

(A.48)

\[
\frac{\partial q_1}{\partial x_5}, \frac{\partial q_2}{\partial x_5} = -\frac{w}{2} \cos x_5
\]

(A.49)

\[
\frac{\partial q_2}{\partial x_5}, \frac{\partial q_3}{\partial x_5} = \frac{w}{2} \cos x_5
\]

(A.50)

\[
\frac{\partial q_k}{\partial i_k}, \frac{\partial q_k}{\partial d} = 0 \ldots k = 1, 2, 3, 4, 5, 6
\]

(A.51)

Simplify using

\[
\cos \Theta = 1
\]

\[
\sin \Theta = 0
\]
\[ x_i = x_i \]
\[ x_2 = 0 \]
\[ x_3 = 0 \]
\[ x_4 = 0 \]
\[ x_5 = 0 \]
\[ x_6 = 0 \]

\[ i_i = i_2 = i_3 = i_4 \]  \hspace{1cm} (A.53)

\[ \frac{\partial p_1}{\partial x_2} = \frac{1}{1}, \quad \frac{\partial p_1}{\partial x_k} = 0, \quad k = 1, 3, 4, 5, 6 \]  \hspace{1cm} (A.54)

\[ \frac{\partial p_2}{\partial x_1} = -\frac{8K_i}{m x_i^2}, \quad \frac{\partial p_2}{\partial x_k} = 0, \quad k = 2, 3, 4, 5, 6 \]  \hspace{1cm} (A.55)

\[ \frac{\partial p_3}{\partial x_4} = 1, \quad \frac{\partial p_3}{\partial x_k} = 0, \quad k = 1, 2, 3, 5, 6 \]  \hspace{1cm} (A.56)

\[ \frac{\partial p_4}{\partial x_3} = -\frac{2K_i^2 L_i^2}{J x_i^3}, \quad \frac{\partial p_4}{\partial x_k} = 0, \quad k = 1, 2, 4, 5, 6 \]  \hspace{1cm} (A.57)

\[ \frac{\partial p_5}{\partial x_6} = 1, \quad \frac{\partial p_5}{\partial x_k} = 0, \quad k = 1, 2, 3, 5 \]  \hspace{1cm} (A.58)

\[ \frac{\partial p_6}{\partial x_5} = -\frac{2K_i^2 W_i^2}{J x_i^3}, \quad \frac{\partial p_6}{\partial x_k} = 0, \quad k = 1, 2, 3, 4, 6 \]  \hspace{1cm} (A.59)

\[ \frac{\partial p_i}{\partial i_k} = 0, \quad k = 1, 2, 3, 4 \]  \hspace{1cm} (A.60)

\[ \frac{\partial p_2}{\partial i_k} = \frac{2K_i}{m x_i}, \quad k = 1, 2, 3, 4 \]  \hspace{1cm} (A.61)
\[ \frac{\partial p_3}{\partial i_k} = 0, k = 1,2,3,4 \]  \hspace{1cm} (A.62)

\[ \frac{\partial p_4}{\partial i_k} = \frac{\kappa i_k L}{J_{xy} x_1}, k = 1,2,\ldots, \frac{\partial p_4}{\partial i_k} = -\frac{\kappa i_k L}{J_{xy} x_1}, k = 3,4 \]  \hspace{1cm} (A.63)

\[ \frac{\partial p_5}{\partial i_k} = 0, k = 1,2,3,4 \]  \hspace{1cm} (A.64)

\[ \frac{\partial p_6}{\partial i_k} = \frac{\kappa i_k w}{J_{xy} x_i}, k = 1,4,\ldots, \frac{\partial p_6}{\partial i_k} = -\frac{\kappa i_k w}{J_{xy} x_i}, k = 2,3 \]  \hspace{1cm} (A.65)

\[ \frac{\partial p_1}{\partial d}, \frac{\partial p_3}{\partial d}, \frac{\partial p_5}{\partial d} = 0 \]  \hspace{1cm} (A.66)

\[ \frac{\partial p_2}{\partial d} = \frac{1}{m} \]  \hspace{1cm} (A.67)

\[ \frac{\partial p_4}{\partial d} = \frac{l_1}{J_{xy}} \]  \hspace{1cm} (A.68)

\[ \frac{\partial p_6}{\partial d} = -\frac{l_2}{J_{xy}} \]  \hspace{1cm} (A.69)

\[ \frac{\partial q_k}{\partial x_2}, \frac{\partial q_k}{\partial x_4}, \frac{\partial q_k}{\partial x_6} = 0 \ldots k = 1,2,3,4,5,6 \]  \hspace{1cm} (A.70)

\[ \frac{\partial q_k}{\partial x_1} = 1 \]  \hspace{1cm} (A.71)
\[
\frac{\partial q_1}{\partial x_3}, \frac{\partial q_2}{\partial x_3} = -\frac{L}{2} \quad (A.71)
\]

\[
\frac{\partial q_3}{\partial x_3}, \frac{\partial q_4}{\partial x_3} = \frac{L}{2} 
\]

\[
\frac{\partial q_1}{\partial x_5}, \frac{\partial q_2}{\partial x_5} = -\frac{w}{2} \quad (A.73)
\]

\[
\frac{\partial q_2}{\partial x_5}, \frac{\partial q_3}{\partial x_5} = \frac{w}{2} 
\]

\[
\frac{\partial q_k}{\partial i_k}, \frac{\partial q_k}{\partial d} = 0 \ldots k = 1, 2, 3, 4, 5, 6 
\]

Final Matrix format:

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-8K_i^2 & 0 & 0 & 0 & 0 & 0 \\
mX_1^3 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & \frac{-2K_i^2 L^2}{J_{xz} X_1^3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & \frac{-2K_i^2 w^2}{J_{xz} X_1^3} & 0
\end{bmatrix} 
\]

\[
B = \begin{bmatrix}
0 & 2K_i L & 0 & 2K_i L & 0 & 2K_i L \\
mX_1 & mX_1 & mX_1 & mX_1 & 0 & 0 \\
K_i L & K_i L & K_i L & K_i L & 0 & 0 \\
J_{xz} X_1 & J_{xz} X_1 & J_{xz} X_1 & J_{xz} X_1 & 0 & 0 \\
K_i w & K_i w & K_i w & K_i w & 0 & 0 \\
J_{xz} X_1 & J_{xz} X_1 & J_{xz} X_1 & J_{xz} X_1 & 0 & 0
\end{bmatrix} 
\]

\[
(A.75) \quad \text{Final Matrix format:}
\]

\[
(A.76) 
\]

\[
(A.77)
\]
\[
C = \begin{bmatrix}
1 & 0 & -\frac{L}{2} & 0 & -\frac{w}{2} & 0 \\
1 & 0 & \frac{L}{2} & 0 & \frac{w}{2} & 0 \\
1 & 0 & \frac{L}{2} & 0 & \frac{w}{2} & 0 \\
1 & 0 & \frac{L}{2} & 0 & -\frac{w}{2} & 0 \\
\end{bmatrix}
\]

\[D = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}\]
# APPENDIX B: SPECIFICATIONS OF ELECTROMAGNET

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal air gap</td>
<td>3 mm</td>
</tr>
<tr>
<td>Maximum air gap</td>
<td>6 mm</td>
</tr>
<tr>
<td>Nominal air gap flux density</td>
<td>0.528 T</td>
</tr>
<tr>
<td>Maximum load per electromagnet</td>
<td>15 Kg</td>
</tr>
<tr>
<td>Viscous damping coefficient</td>
<td>= 0 N/(m/s)</td>
</tr>
<tr>
<td>Rated electromagnet winding current</td>
<td>3.5 A</td>
</tr>
<tr>
<td>DC link input voltage</td>
<td>160 V</td>
</tr>
<tr>
<td>Attraction force at 3 mm, 3.5 A</td>
<td>177.5 N</td>
</tr>
<tr>
<td>Attraction force at 6 mm, 7 A</td>
<td>189.4 N</td>
</tr>
<tr>
<td>Number of turns per electromagnet</td>
<td>AWG #19, 720 Turns/magnet</td>
</tr>
<tr>
<td>Electromagnet winding current density</td>
<td>5.36 A/mm²</td>
</tr>
<tr>
<td>Electromagnet winding resistance</td>
<td>3.25 Ω/electromagnet</td>
</tr>
<tr>
<td>Rated copper loss</td>
<td>39.8 W</td>
</tr>
<tr>
<td>Winding inductance at 0.2 mm, 3.5 A</td>
<td>245.2 mH</td>
</tr>
<tr>
<td>Winding inductance at 3.0 mm, 3.5 A</td>
<td>108.0 mH</td>
</tr>
<tr>
<td>Winding inductance at 6.0 mm, 3.5 A</td>
<td>62.80 mH</td>
</tr>
<tr>
<td>Magnet pole width (Longitudinal width)</td>
<td>30 mm</td>
</tr>
<tr>
<td>Magnet pole depth (Core stack width)</td>
<td>25 mm</td>
</tr>
<tr>
<td>Magnet pole height</td>
<td>35 mm</td>
</tr>
<tr>
<td>Winding window width</td>
<td>40 mm</td>
</tr>
<tr>
<td>Magnet yoke thickness</td>
<td>25 mm</td>
</tr>
<tr>
<td>Track yoke depth</td>
<td>33 mm</td>
</tr>
<tr>
<td>Track yoke thickness</td>
<td>25 mm</td>
</tr>
</tbody>
</table>
APPENDIX C: PART LISTS AND DETAIL DRAWINGS OF MECHANICAL SYSTEM

Table C.1 (Purchased Parts List – Mechanical Implementation)

<table>
<thead>
<tr>
<th>Description</th>
<th>Manufacturer</th>
<th>Part #</th>
<th>Qty.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap Sensor</td>
<td>Balluff</td>
<td>BAW 018 PF 1 K 03</td>
<td>4</td>
</tr>
<tr>
<td>Bearings</td>
<td>McMaster-Carr</td>
<td>6262K12</td>
<td>4</td>
</tr>
<tr>
<td>Bearing Blocks</td>
<td>McMaster-Carr</td>
<td>6068K22</td>
<td>8</td>
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</table>

Table C.2 (Machined Parts List – Mechanical Implementation)

<table>
<thead>
<tr>
<th>Machined Parts</th>
<th>Qty.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 BEARING</td>
<td>4</td>
</tr>
<tr>
<td>2 DISTURBANCE RAILS</td>
<td>2</td>
</tr>
<tr>
<td>3 LEVITATION RAILS</td>
<td>2</td>
</tr>
<tr>
<td>4 RECEPTACLE_PLATE</td>
<td>1</td>
</tr>
<tr>
<td>5 SENSOR MOUNTS</td>
<td>4</td>
</tr>
<tr>
<td>6 SHAFTS</td>
<td>4</td>
</tr>
<tr>
<td>7 STAND OFF BLOCK</td>
<td>4</td>
</tr>
<tr>
<td>8 TABLE_TOP</td>
<td>1</td>
</tr>
<tr>
<td>9 TOP PLATE</td>
<td>1</td>
</tr>
<tr>
<td>10 VICE BLOCK</td>
<td>4</td>
</tr>
<tr>
<td>11 VICE STOP</td>
<td>4</td>
</tr>
<tr>
<td>12 WEIGHT</td>
<td>4</td>
</tr>
<tr>
<td>13 WEIGHT SHAFTS</td>
<td>4</td>
</tr>
</tbody>
</table>

Table C.3 (Frame World Parts List – Mechanical Implementation)

<table>
<thead>
<tr>
<th>Frame World Part #</th>
<th>Length (in.)</th>
<th>Qty.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EAL-3</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>EB-11</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>EB-7</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>EBP-7</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>EF-1</td>
<td>200</td>
</tr>
<tr>
<td>6</td>
<td>EBP-1</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>EX-2</td>
<td>22</td>
</tr>
<tr>
<td>8</td>
<td>EX-2</td>
<td>16.25</td>
</tr>
<tr>
<td>9</td>
<td>EX-1</td>
<td>42</td>
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<td>10</td>
<td>EX-1</td>
<td>29</td>
</tr>
<tr>
<td>11</td>
<td>EX-1</td>
<td>9.7</td>
</tr>
<tr>
<td>12</td>
<td>EX-1</td>
<td>32</td>
</tr>
<tr>
<td>13</td>
<td>EX-1</td>
<td>12</td>
</tr>
<tr>
<td>14</td>
<td>EX-1</td>
<td>15</td>
</tr>
</tbody>
</table>
Figure C7 Track Plate

- Ø 0.034 THRU 18X
- Ø 0.028 THRU 8X

Material: C5

Track Plate

Drawn by: PW

Drawing No: 1

Scale: FULL

Notes:
- All dimensions are in millimeters unless otherwise specified.
- Tolerances for all dimensions ± 0.030
- Tolerances on decimal and non-constant dimensions ± 0.010

Revision C: Revised 11/10/06

Virginia Tech

PERI'S CORPORATION

ORGANIZATION

DATE: 1

LEF: FIXTURE

FILE: 1

PAGE: 1
### Dimensions

<table>
<thead>
<tr>
<th>Material</th>
<th>ASME:</th>
<th>Condition</th>
<th>Precise Max (in.)</th>
<th>ASME:</th>
<th>Precise Min (in.)</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td></td>
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<td></td>
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<td>0.500</td>
<td>+0.002</td>
<td>-0.000</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure C.12 Weight Guide**

- **Part Description:** Weight Guide
- **Model:** 4
- **Scale:** 1:1

---

*Note: The drawing includes dimensions and tolerances for the weight guide, indicating precision in inches. The figure is annotated with symbols and lines for clarity.*
APPENDIX D: SCHEMATICS OF ELECTRONIC HARDWARE

D.1 Power Control Board Connections and Schematics

• **CN1**: 8 Pin 5.04 mm Terminal Block (Power Supply Connector)
  1 FG
  2 XGND
  3 +24V
  4 -15V
  5 AGND
  6 +15V
  7 DGND
  8 +5V

• **CN 2**: 5 Pin 2.54 mm Single Line Header (AMDC401 EVM I/F Connector)
  1 /ALARM  Output Signals
  2 /BYPASS
  3 /SERVO_ON  Input Signals
  4 UV
  5 OV

• **CN3**: 4 Pin Terminal Block (DC Link Connector)
  1 \( G_{DC} \)
  2 \(-R_{RG}\)
  3
  4 \( V_{DC} \)

• **CN4**: 4 Pin Terminal Block (Power Distribution Module I/F Connector)
  1 t
  2 MC2_0
  3 AMC1_4
  4 MC1_0
D.2 Gap Sensor Interface Board Connections and Schematics

- **CN1**: 8 Pin 5.04 mm Terminal Block (Power Supply Connector)
  
<table>
<thead>
<tr>
<th>Pin</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FG</td>
</tr>
<tr>
<td>2</td>
<td>-15V</td>
</tr>
<tr>
<td>3</td>
<td>AGND</td>
</tr>
<tr>
<td>4</td>
<td>+15V</td>
</tr>
<tr>
<td>5</td>
<td>DGND</td>
</tr>
<tr>
<td>6</td>
<td>+5V</td>
</tr>
</tbody>
</table>

- **CN2**: 5 Pin 2.54 mm Single Line Header (AMDC401 EVM ADC Connector)
  
<table>
<thead>
<tr>
<th>Pin</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vgs1</td>
</tr>
<tr>
<td>2</td>
<td>AGND</td>
</tr>
<tr>
<td>3</td>
<td>Vgs2</td>
</tr>
<tr>
<td>4</td>
<td>AGND</td>
</tr>
<tr>
<td>5</td>
<td>Vgs3</td>
</tr>
<tr>
<td>6</td>
<td>AGND</td>
</tr>
<tr>
<td>7</td>
<td>Vgs4</td>
</tr>
<tr>
<td>8</td>
<td>AGND</td>
</tr>
</tbody>
</table>

- **CN3**: 3 Pin 2.54 mm Single Line Header (Phase A Gap Sensor Connector)
  
<table>
<thead>
<tr>
<th>Pin</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+24V</td>
</tr>
<tr>
<td>2</td>
<td>Igs1</td>
</tr>
<tr>
<td>3</td>
<td>GND</td>
</tr>
</tbody>
</table>

- **CN4**: 3 Pin 2.54 mm Single Line Header (Phase B Gap Sensor Connector)
  
<table>
<thead>
<tr>
<th>Pin</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+24V</td>
</tr>
<tr>
<td>2</td>
<td>Igs2</td>
</tr>
<tr>
<td>3</td>
<td>GND</td>
</tr>
</tbody>
</table>

- **CN5**: 3 Pin 2.54 mm Single Line Header (Phase C Gap Sensor Connector)
  
<table>
<thead>
<tr>
<th>Pin</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+24V</td>
</tr>
<tr>
<td>2</td>
<td>Igs3</td>
</tr>
<tr>
<td>3</td>
<td>GND</td>
</tr>
</tbody>
</table>

- **CN6**: 3 Pin 2.54 mm Single Line Header (Phase D Gap Sensor Connector)
  
<table>
<thead>
<tr>
<th>Pin</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+24V</td>
</tr>
<tr>
<td>2</td>
<td>Igs4</td>
</tr>
<tr>
<td>3</td>
<td>GND</td>
</tr>
</tbody>
</table>
D.3 OC Interface Board Connections and Schematics

- CN1: 8 Pin 5.04 mm Terminal Block (Power Supply Connector)
  1  FG
  2  -15V
  3  AGND
  4  +15V
  5  DGND
  6  +5V

- CN2: 5 Pin 2.54 mm Single Line Header (AMDC401 EVM ADC Connector)
  1  -VcsA
  2  AGND
  3  -VcsB
  4  AGND
  5  -VcsC
  6  AGND
  7  -VcsD
  8  AGND

- CN3: 9 Pin D-Subminiature Connector (Female) to Converter A
  1  +5V
  2  DGND
  3  +15V
  4  AGND
  5  -15V
  6  /Tal
  7  /Tah
  8  /IsenseA
  9  FG

- CN4: 9 Pin D-Subminiature Connector (Female) to Converter B
  1  +5V
  2  DGND
  3  +15V
  4  AGND
  5  -15V
  6  /Tal
  7  /Tah
  8  /IsenseA
  9  FG

- CN5: 9 Pin D-Subminiature Connector (Female) to Converter C
  1  +5V
  2  DGND
  3  +15V
  4  AGND
  5  -15V
  6  /Tal
  7  /Tah
  8  /IsenseA
  9  FG

- CN6: 9 Pin D-Subminiature Connector (Female) to Converter D
  1  +5V
  2  DGND
  3  +15V
  4  AGND
  5  -15V
  6  /Tal
  7  /Tah
  8  /IsenseA
  9  FG
### CN7: 30 Pin Flat Cable Connector (Over Current to PWM Connector)

<table>
<thead>
<tr>
<th></th>
<th>AGND</th>
<th>16</th>
<th>-5V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>AGND</td>
<td>18</td>
<td>/Tdl</td>
</tr>
<tr>
<td>3</td>
<td>AGND</td>
<td>19</td>
<td>/Tdh</td>
</tr>
<tr>
<td>4</td>
<td>DGND</td>
<td>20</td>
<td>/Tcl</td>
</tr>
<tr>
<td>5</td>
<td>DGND</td>
<td>21</td>
<td>/Tch</td>
</tr>
<tr>
<td>6</td>
<td>DGND</td>
<td>22</td>
<td>5V</td>
</tr>
<tr>
<td>7</td>
<td>DGND</td>
<td>23</td>
<td>5V</td>
</tr>
<tr>
<td>8</td>
<td>DGND</td>
<td>24</td>
<td>OC</td>
</tr>
<tr>
<td>9</td>
<td>DGND</td>
<td>25</td>
<td>/Tbl</td>
</tr>
<tr>
<td>10</td>
<td>DGND</td>
<td>26</td>
<td>/Tbh</td>
</tr>
<tr>
<td>11</td>
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<td>27</td>
<td>/Tal</td>
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<td>12</td>
<td>AGND</td>
<td>28</td>
<td>/Tah</td>
</tr>
<tr>
<td>13</td>
<td>AGND</td>
<td>29</td>
<td>+5V</td>
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<tr>
<td>14</td>
<td>AGND</td>
<td>30</td>
<td>-5V</td>
</tr>
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</table>
### D.4 PWM Interface Board Connections and Schematics

- **CN 1**: 3 Pin 2.54 mm Single Line Header (AMDC401 EVM I/O Connector)
  
<table>
<thead>
<tr>
<th>Pin</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DGND</td>
</tr>
<tr>
<td>2</td>
<td>OC  (Output Signal)</td>
</tr>
<tr>
<td>3</td>
<td>/Gate_on (Input Signal)</td>
</tr>
</tbody>
</table>

- **CN2**: 8 Pin 2.54 mm Single Line Header (AMDC401 EVM DAC Connector)
  
<table>
<thead>
<tr>
<th>Pin</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DAC1</td>
</tr>
<tr>
<td>2</td>
<td>AGND</td>
</tr>
<tr>
<td>3</td>
<td>DAC2</td>
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<tr>
<td>4</td>
<td>AGND</td>
</tr>
<tr>
<td>5</td>
<td>DAC3</td>
</tr>
<tr>
<td>6</td>
<td>AGND</td>
</tr>
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<td>7</td>
<td>DAC4</td>
</tr>
<tr>
<td>8</td>
<td>AGND</td>
</tr>
</tbody>
</table>

- **CN3**: 30 Pin Flat Cable Connector (Over Current to PWM Connector)
  
<table>
<thead>
<tr>
<th>Pin</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AGND</td>
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<td>2</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
<td>AGND</td>
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<tr>
<td>5</td>
<td>DGND</td>
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<tr>
<td>6</td>
<td>DGND</td>
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<tr>
<td>7</td>
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<tr>
<td>14</td>
<td>AGND</td>
</tr>
<tr>
<td>15</td>
<td>AGND</td>
</tr>
<tr>
<td>16</td>
<td>-5V</td>
</tr>
<tr>
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<td>+5V</td>
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<td>/Tcl</td>
</tr>
<tr>
<td>21</td>
<td>/Tch</td>
</tr>
<tr>
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<td>5V</td>
</tr>
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<td>23</td>
<td>5V</td>
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<tr>
<td>24</td>
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<td>/Tbh</td>
</tr>
<tr>
<td>27</td>
<td>/Tal</td>
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<tr>
<td>28</td>
<td>/Tah</td>
</tr>
<tr>
<td>29</td>
<td>+5V</td>
</tr>
<tr>
<td>30</td>
<td>-5V</td>
</tr>
</tbody>
</table>
References


Phillip Vallance received Baccalaureate degrees in mechanical and electrical engineering from Virginia Polytechnic Institute and State University in 1998 and 1999, respectively. His current research is in the area of attractive electromagnetic levitation.