Exterior Penalty Approaches for Solving
Linear Programming Problems

by

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ABSTRACT

In this research effort, we study three exterior penalty function approaches for solving linear programming problems. These methods are an active set $l_2$ penalty approach (ASL2), an inequality-equality based $l_2$ penalty approach (IEL2), and an augmented Lagrangian approach (ALAG). Particular effective variants are presented for each method, along with comments and experience on alternative algorithmic strategies that were empirically investigated. Our motivation is to examine the relative performance of these different approaches based on the basic $l_2$ penalty function in order to provide insights into the viability of these methods for solving linear programs. To test the performance of these algorithms, a set of randomly generated problems as well as a set of NETLIB test problems from the public domain are used. By way of providing a benchmark for comparisons, we also solve the test problems using CPLEX 6.0, an advanced simplex implementation. While a particular variant (ALAG2) of ALAG performed the best for randomly generated test problems, ASL2 performed the best for the NETLIB test problems. Moreover, for test problems having only equality constraints, IEL2, and ASL2 (which is a finer-tuned version of IEL2 in this case) were comparable and yielded a second-best performance in comparison with ALAG2. Furthermore, a set of problems with relatively higher density parameter values, as well as a set of low-density problems were used to determine the effect of density on the relative performances of these methods. This experiment revealed that for linear programs with a high density parameter, ASL2 is the best alternative among the tested algorithms; whereas, for low-density problems ALAG2 is the fastest method. Moreover, although our implementation was rudimentary in comparison with CPLEX, all of the tested methods attained a final solution faster than CPLEX for the set of large-scale low-density problems, sometimes as fast as requiring only 16-23% of the effort consumed by CPLEX. Average rank tests based on the computational results obtained are performed using two different statistics, that assess the speed of convergence and the quality or accuracy of the solution, in order to determine the relative effectiveness of the algorithms and to validate our conclusions. Overall, the results provide insights into selecting algorithmic strategies based on problem structure and indicate that while this class of methods is viable for computing near optimal solutions, more research is needed to design robust and competitive exterior point methods for solving linear programming problems. However, the use of the proposed variant of the augmented Lagrangian method to solve large-scale low-density linear programs is promising and should be explored more extensively.
to saniye, gündüz, and burçin özdaryal…
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