Appendix A

Measurement Uncertainty

The uncertainty associated with all the laboratory measurements involves a random precision error and a fixed bias error. The errors can each be broken down further into three components. (1) calibration errors (2) data acquisition errors and (3) data reduction errors, (ANSI/ASME, 1985). Random precision errors are observed in repeated independent measurements were variations in the measurements are due to numerous error sources. The standard deviation, $\sigma$, is a measure of the distribution of the precision errors. For a normal distribution, with the true population average represented by $\mu$, the interval $\mu \pm 2\mu$ will include 95% of the total scatter in the measurements of $\mu$. The statistic, $S$, is calculated to estimate the standard deviation and is called the precision index. It is defined by the expression:

$$S = \left[ \frac{\sum_{k=1}^{N} (X_k - \bar{X})^2}{\frac{1}{N_M - 1} N_M} \right]^{\frac{1}{2}} \quad (A.1)$$

where $N_M$ is the number of measurements made and $\bar{X}$ is the average value of the measurement $X_k$ given by:

$$\bar{X} = \frac{1}{N_M} \sum_{k=1}^{N} X_k \quad (A.2)$$

The bias error $B_e$ is a systematic error which is constant for the duration of the test and is estimated by non-statistical methods. Bias errors are usually constant for the duration of a test. The bias error includes those which are known and can be calibrated out, those which are negligible and can be ignored and those which are estimated and
included in the uncertainty analysis. An example of a bias error is the sensor uncertainty as specified by the sensor manufacturer.

Once the bias and precision errors are determined, a single uncertainty number $U$ is used to express a reasonable limit of error for a given measurement. The uncertainty number is some combination of the bias and the precision errors and has a simple interpretation: the largest error reasonably expected. For example, the interval

$$X \pm U$$  \hspace{1cm} (A.3)

represents a band within which the true value of the measurement is expected to lie, (ANSI/ASME, 1985).

It is not possible to determine a rigorous confidence level on $U$, as the bias error is often based on judgment or past experience. However, it is often recommended to determine $U$ with coverage analogous to a 95% confidence interval. That is, $U$ for a symmetrical uncertainty interval is defined as

$$U = \left[ B^2 + (t, S_x)^2 \right]^{1/2} \hspace{1cm} (A.4)$$

where $B$ is the bias limit, $S$ is the precision index of the mean, and $t_s$ is the 95th percentile point for the two tailed Student t distribution.

**Amplitude Measurement Uncertainty**

Eddy current proximity probes manufactured by Bently Nevada were used to measure the amplitude of the vibrations in the rotor kits. Clements J. (unpublished work) determined the error associated with eleven Eddy current proximity probes, obtaining errors from 0.45% up to 2.33%. In the present analysis, the bias error associated with the Eddy current probes was considered as $\pm 2.33\%$. The overall precision error was estimated by taking twenty-one readings of the amplitude of the synchronous and subsynchronous vibration of the three-disk rotor, with the AMD-13 off and with the
AMD-13 on, which were substituted into Equation A.1 resulting in an estimate of the precision index. Combining these values with the bias errors in Equation A.4, using the $t_s$ value for twenty-one samples as 2.080, (ANSI/ASME, 1985), results in the range of measurement uncertainties presented in Table A.1.

Table A.1: Uncertainty in the measurements of the synchronous and subsynchronous amplitudes with the AMD-13 off and with the AMD-13 on

<table>
<thead>
<tr>
<th></th>
<th>AMD-13 Off</th>
<th>AMD-13 On</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Synchronous Amplitude</td>
<td>Subsynchronous Amplitude</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>5.27</td>
<td>2.62</td>
</tr>
<tr>
<td>$S_X$</td>
<td>0.0058</td>
<td>0.0287</td>
</tr>
<tr>
<td>$B$</td>
<td>0.1229</td>
<td>0.0611</td>
</tr>
<tr>
<td>$U$</td>
<td>0.1235</td>
<td>0.0854</td>
</tr>
</tbody>
</table>

The measurement uncertainty for the entire range of amplitude measurements will be taken as

$$U = \pm 0.1424 \text{ mils pp}$$

which represents the worst scenario of the four measurement uncertainties calculated.