Taylor Impact Test and Penetration of Reinforced Concrete Targets by Cylindrical Composite Rods

by

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(ABSTRACT)

We use the three-dimensional finite element code DYNA3D to analyze two problems: (a) the normal impact of a cylindrical monolithic or composite rod against a smooth flat rigid target, (commonly known as the Taylor impact test), and (b) the penetration of composite and monolithic steel cylindrical rods into reinforced concrete targets. The composite rod is made of either a steel or copper shell enclosing a ceramic. The ceramic and the steel are assumed to fail at a critical value of the effective plastic strain, whereas no failure is considered in the copper. The thermoviscoplastic response of steel and copper is modeled by the Johnson-Cook relation and the ceramic and concrete are assumed to be elastic-plastic. Values of material parameters in the constitutive relation for the reinforced concrete (RC) are derived by the rule of mixtures. Failure of a material is simulated by the element erosion technique for ceramic and steel, and element erosion along with stiffness reduction for the RC. The effect of the angle of obliquity of impact on the damage induced in the target is ascertained.

For the solid cylindrical copper rod impacting a smooth flat rigid target, the time history of the deformed length and the axial variation of the final diameter are found to match well with the experimental findings. For the composite rod, the diameter of the deformed impacted surface,
the shape and size of the mushroomed region and the volume fraction of the failed ceramic material strongly depend upon the impact speed, the shell wall thickness and the thickness of the solid copper rod at the front end.

Some composite cylindrical rods impacting at normal incidence RC targets were found to buckle during the penetration process in the sense that their outer diameter at a cross-section close to the impacted end increased by at least 20%. For steel penetrators, the damage experienced increased as the nose shape got blunter and the angle of obliquity became larger whereas the damage induced to the target only increased with penetrator bluntness.
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1 Introduction

In late 1930s Taylor proposed a relatively simple method to find the dynamic compressive strength of a material. However, due to the Second World War, the work was published in 1946. The method, now known as the Taylor impact test, consists of firing a solid cylinder against a massive rigid target. The dynamic flow stress of the cylinder material is estimated from the deformed shape of the cylinder. Taylor (1946, 1948) and Whiffin (1948) derived transcendental equations for the flow/yield stress in which the plastic wave speed is a free variable. They numerically determined the plastic wave speed consistent with both the measured deformation and their theory, and thus found the yield stress of the cylinder material. Jones et al. (1991, 1992) modified Taylor’s analysis of the problem by dividing it into two phases. The first high strain-rate phase ends with the attenuation of shock waves initiated by the impact. The second phase is comprised of a period of lower strain-rate plastic deformations characterized by plastic waves propagating at a uniform speed. These analyses are essentially one-dimensional and can not accurately predict the deformed shape of the cylinder. Several experimental studies (Johnson (1972), Hutchings (1983), Paprino et al. (1980)) have shown that the Taylor model provides very approximate values of the dynamic yield strength of the rod material.

These investigations have not reported the occurrence of adiabatic shear bands (ASBs), which are thin regions of intense plastic deformation, in the mushroomed region. Dick et al. (1991) conducted reverse ballistic impact tests on tungsten heavy alloy (WHA) rods comprised of ellipsoidal tungsten particles in a Fe-Ni matrix and also examined deformed rods for fracture.
No ASBs were observed in deformed rods impacted at 173 m/s. However, at the impact speed of 228 m/s, the deformation was found to have localized along a curved path extending from the transition between the mushroomed region and the undeformed rod towards the impact face. Diametrically opposite to this section, they observed ductile fracture along a similar path. Apparently, the fracture of WHA rods followed ASBs that had propagated to the mantle of the rod.

Wilkins and Guinan (1973), Batra and Stevens (1998), Stevens and Batra (1999) and Celantano (2002), amongst others, have used numerical methods to analyze thermomechanical deformations of a cylindrical rod impacting at normal incidence a smooth flat rigid target. Whereas Celantano accounted for heat conduction, the other three works did not. Stevens and Batra (1999) found that ASBs form in materials that exhibit enhanced thermal softening and their location generally agrees with that observed by Dick et al. (1991).

Here we use the FE code DYNA3D (Whirley and Hallquest (1991)) to analyze the Taylor impact test for a composite rod. That is, deformations of a cylindrical rod comprised of a copper shell filled with ceramic are analyzed; a sketch of the problem studied is shown in Fig. 1a. This was partially motivated by the earlier work of Nechitaio and Batra (1998) who showed that a kinetic energy ceramic rod can perforate reasonably thick steel and WHA. However, a ceramic rod can not be easily fired from a gun because ceramic is brittle. A possibility is to enclose a ceramic rod in a copper shell. Another motivation for this work is provided by the Department of Defense’s desire to have the projectile made of a multifunctional energetic material. In such applications,
the ceramic will be replaced by a reinforced energetic material that provides strength during launch and trajectory of the projectile but will release energy upon impacting the target. We note that Foster et al. (2001) have performed Taylor impact tests on composite rods made of copper core and AerMet-100/AF1410 sleeve.

The interest in penetration, perforation and fragmentation of reinforced concrete (RC) targets arises from their use as barriers to protect civilian buildings against blast loads and as bunkers to protect against impacts with high kinetic energy (KE) rods. Much of the earlier work through the 1800’s was experimental involving a wide range of projectile sizes and impact speeds varying from 200 to 1000 m/s. Several empirical relations to predict the depth of penetration deduced from experimental studies have been summarized by Kennedy (1976), and their range of validity has been delineated. Yankelevsky (1997) analyzed the local response of concrete slabs to low speed missile impact and compared results with those predicted by formulas proposed by Petry, the Army Corps of Engineers, the National Defense Research Committee, Kar (1978), and the UK Atomic Energy Authority (see Kennedy (1976) or Yankelevsky (1997) for these formulas). The Petry formula gives the resisting force as a function of penetrator’s initial velocity, weight, cross-sectional area, and target’s compressive strength, whereas the Army Corps of Engineers and the National Defense Research Committee formulas predict maximum depth of penetration as a function of the same parameters. The Kar formula includes stiffness of both the penetrator and the target to determine the depth of penetration, and the UK Atomic Energy Authority formula accounts for thickness and percent reinforcement of the target to predict the minimum speed of the penetrator required for perforating a target. All of these
relations are based on curve fits to experimental data, predict one parameter as a function of
others, and do not account for any local material response. Yankelevsky (1997) observed
significant variations in predictions from these relations.

The development of analytical penetration models that account for the material behavior began
in the 1940’s with equations predicting the penetration of steel targets by metallic rods. Bishop
et al. (1945) studied quasi-static expansions of cylindrical and spherical cavities and used them
to estimate forces on conical nose punches pushed slowly into metallic targets. Goodier (1965)
included the effect of inertia forces to analytically predict the penetration depth of rigid spheres
launched into metal targets. He approximated the target response by results from the dynamic,
spherically symmetric, cavity-expansion equations for an incompressible target material derived
by Hill (1948). Forrestal (1986) used the cylindrical cavity expansion equations to study the
penetration of a rigid rod into dry, porous rock and showed that they overpredict the early time
deceleration response and underpredict the later deceleration response. Forrestal and Luk (1992)
showed that the deceleration predictions from the spherical expansion approximation are in good
agreement with the experimental results. Forrestal et al. (1995) and Forrestal and Tzou (1997)
have generalized these models for penetration into metallic and concrete targets respectively. A
linear equation of state and the Mohr-Coulomb yield surface with a tension cut off were used to
represent the response of the concrete. The depth of penetration predicted from this formulation
agreed well with experimental results for impact velocities below 500 m/s. We note that an
adequate constitutive relation for a porous brittle material such as concrete should account for a
wide range of deformation behavior including dilatant shear failure, the transition from dilation
to compaction caused by pore collapse, compaction hardening, and strain-rate sensitivity.

Lixin et al. (2000) introduced an additional resistance factor, to account for the truncation effect
of the projectile nose on penetration, in the Forrestal and Tzou’s (1997) cavity expansion model.
However, their approach gives reasonable results only when the truncated part of the nose is less
than one-third the original nose length. Teland and Sjol (2004) extended the cavity expansion
 technique to include spherical as well as ogive and truncated ogive projectiles and described the
elastic-plastic behavior of concrete with a von Mises yield surface having a constant yield
strength. Penetration depths for very thick targets computed by Lixin et al. (2000) and Teland
and Sjol (2004) agreed well with the experimental values.

Numerical and experimental adaptations to the cavity expansion based methods have been given
by Warren et al. (2004) and Gomez and Shukla (2001). Warren used the finite element method
(FEM) to analyze the penetration problem and simulated concrete’s response as strain hardening
and pressure dependent yield strength and accounted for pore collapse. Gomez and Shukla
(2001) conducted experiments involving multiple impacts at the same point in the target to assess
the damage induced by impact and how the prior damage affects the penetration depth and the
crater diameter. A strength reducing scale factor was proposed to account for the damage
induced in the target by prior impacts. It was found that predictions from the spherical cavity
expansion model matched better with the corresponding experimental values than those from the
cylindrical expansion model. The incremental penetration depth increased with each impact and also with increase in the impact speed.

Li and Tong (2002) also used a cavity expansion based method but applied it to the perforation of thin concrete targets. They developed a two-phase model to predict the thickness of slabs that can be penetrated by a rigid projectile. The first phase describes material crushing and removal by Forrestal and Tzous’ (1997) method and the second phase predicts scabbing of the back panel and plug formation based on shear failure. They gave an analytical formula based on volumetric behavior and plug formation and were able to achieve more consistent agreement with experimental data.

Gold et al. (1996) have studied experimentally and analytically the penetration of concrete targets by spherical-nosed copper and tantalum cylindrical projectiles moving at speeds between 1.5 km/s and 1.9 km/s. The diameters of projectiles varied from 13 mm to 20 mm and the length/diameter from 3.9 to 14.6. The targets were 91 cm diameter and 91 cm long right circular cylinders constructed from concrete with maximum aggregate size of 1.9 cm and density of 2,240 kg/m$^3$. For the 13 mm diameter projectiles, crater entrances were approximately 250 mm in diameter which narrowed down to 50 mm at a depth of 105 mm. The deeper portion of the crater was a well rounded and slightly tapered tunnel. The depth of penetration computed with the hydrocode CALE employing a pressure-dependent yield-strength constitutive relation matched well with the experimental value.
Gold et al. (1996) also investigated effects of concrete’s constitutive properties on the penetration process by analyzing the effect of the pressure-dependent yield strength in von Mises yield criterion and also of the collapsing of pores. The hole profile computed with the concrete modeled as non-porous agreed better with that observed experimentally and the pressure dependent yield strength gave good values of both the hole profile and the depth of penetration suggesting that the constitutive properties play a significant role in predicting local effects of penetration.

Ågårdh and Laine (1999) conducted a three-dimensional (3-D) simulation of a high-speed solid steel cylinder impacting and perforating a RC slab whose thickness equaled approximately twice the penetrator length. The steel reinforcement was modeled as a single layer of reinforcing bars and the remainder of the target consisted of concrete which was described with a nonlinear shock equation of state, and whose failure due to crushing and spallation was modeled by the element erosion technique. The nearly 25% decrease in the penetration speed predicted by these simulations during the perforation of the target agreed well with that observed experimentally. Chen et al. (2004) developed a three-stage model that considers initial cratering, tunneling and shear plugging of the target and is based on the dynamic cavity expansion theory and plug formation. Their predictions of the ballistic performance of RC targets for normal and oblique impacts by rigid projectiles compare well with experimental findings.

We have reviewed above much of the work involving the penetration of RC targets. There is considerable literature on the penetration of metallic targets that has not been discussed here.
We add that Warren and Poorman (2001) have recently examined, experimentally and numerically, the effect of obliquity on the penetration of aluminum targets by steel projectiles with emphasis on the bending deformations of the projectile and its trajectory. The action of the target on the penetrator was approximated by applying a resisting force to the projectile; this force was estimated from the dynamic expansion of a spherical cavity. A good agreement between test and simulation results was obtained. Since some of the composite cylindrical rods buckled in our simulations, we mention below a few works on the dynamic buckling of rods deformed plastically.

Stowell (1948) has studied analytically the plastic buckling of columns. Pride and Heimerl (1949) conducted buckling experiments on rectangular aluminum tubes and showed that predictions from the deformation theory of plasticity were closer to the test values than those from the incremental theory of plasticity. Abramowicz and Jones (1997) performed static and dynamic axial crushing tests on 128 thin-walled mild steel columns which buckled mostly in the plastic range. The columns were dynamically loaded by masses traveling with initial impact speeds of up to 12.14 m/s and striking them axially at one end. It was observed that even relatively short columns, which enter the plastic range in a straight configuration, buckle plastically in the global inelastic buckling mode, while a transition to progressive plastic buckling was observed later in the collapse process. Karagiozova and Alves (2004) characterized the influence of impact speed, yield stress, strain-hardening and strain-rate sensitivity of the material on the dynamic buckling response of circular cylindrical shells.
subjected to axial impact loads. The impact load was applied by a moving mass striking the end faces of a shell.

We also analyze three-dimensional (3-D) deformations of a RC target impacted by either a solid-steel or a composite projectile at different angles of obliquity; a schematic sketch of this problem is shown in Fig. 2a. The penetrator is a cylindrical rod with the nose shape given by a half sine wave of base equal to the diameter of the projectile shown in Fig. 2b. The target’s impacted face is a square of side $W$, and its thickness $T$ equals two or three times the projectile diameter. Both the projectile and the target materials are modeled as elastic-plastic with the yield stress depending upon the effective plastic strain, effective plastic strain-rate and possibly temperature. The reinforcements in RC are assumed to be well mixed so that values of material parameters in its constitutive relation can be obtained from those of its constituents by the rule of mixtures. Material failure in the penetrator and the target is simulated by the element erosion technique and the target also exhibits failure in the form of stiffness reduction.

We also scrutinize the buckling of composite cylindrical rods striking at normal incidence a RC target; a schematic of this is shown in Fig. 1b. This contrasts the previous experimental and theoretical studies in which the dynamic load is applied by a moving mass. Here the kinetic energy of the rod provides the energy needed to deform it and the RC target. The rod is assumed to have buckled when its outer diameter at any cross-section is increased by 20%. We investigate buckling of the composite rod as a function of various geometric parameters and the impact speed.
2  Formulation of the Problem

We use the Lagrangian or the referential description of motion to describe three-dimensional (3-D) dynamic deformations of a cylindrical rod impacting a target. In the absence of body forces and external sources of energy, their deformations are governed by the following balance laws of mass, linear momentum, moment of momentum and internal energy; details of these balance laws may be found in several books, e.g., Truesdell and Noll (1965).

\[
(\rho J)' = 0, \quad \rho_0 \dot{\rho} = \text{Div } \mathbf{T}, \quad \mathbf{T} \mathbf{F}^T = \mathbf{F} \mathbf{T}^T, \quad \rho_0 \dot{e} = \text{tr}(\dot{\mathbf{T}} \dot{\mathbf{F}}^T),
\]

(1)

\[
\mathbf{F} = \text{Grad } \mathbf{x}.
\]

(2)

Here, \( \mathbf{x} \) gives the present position of the material particle that occupied place \( \mathbf{X} \) in the reference configuration, \( \rho \) is its present mass density, \( \rho_0 \) its mass density in the reference configuration, \( \mathbf{v} \) its velocity, \( \mathbf{T} \) the first Piola-Kirchhoff stress tensor, \( \mathbf{F} \) the deformation gradient, \( J = \det \mathbf{F} \), \( e \) the specific internal energy, \( \mathbf{F}^T \) the transpose of \( \mathbf{F} \), and a superimposed dot indicates the material time derivative. The operators \( \text{Div} \) and \( \text{Grad} \) signify, respectively, the divergence and the gradient operators with respect to referential coordinates and \( \text{tr} \) denotes the trace operator. In Eq. (1), the effect of heat conduction has been neglected. This is justified since we are interested in the short term response of the impacting bodies during which time the effects of heat conduction are likely to be negligible. This assumption facilitates the computation of the temperature rise from the incremental plastic work done without numerical integration of the balance of internal energy.
Equations (1) are supplemented with the following constitutive relations.

\[ T = J \sigma (F^{-1})^T, \quad \sigma = -p \mathbf{I} + \mathbf{S}, \quad p = K \left( \rho / \rho_0 - 1 \right), \]  
\[ \mathbf{S} = 2\mu (\mathbf{D} - \mathbf{D}^p), \quad \mathbf{S} = \mathbf{\dot{S}} + \mathbf{S} \mathbf{W} - \mathbf{W} \mathbf{S}, \]  
\[ \mathbf{D} = \frac{1}{2} (\text{grad } \mathbf{v} + (\text{grad } \mathbf{v})^T), \quad \mathbf{W} = \frac{1}{2} (\text{grad } \mathbf{v} - (\text{grad } \mathbf{v})^T), \]  
\[ \mathbf{\ddot{D}} = \mathbf{D} - \frac{1}{3} (\text{tr } \mathbf{D}) \mathbf{I}, \quad \text{tr } \mathbf{D}^p = 0, \quad \mathbf{D}^p = \mathbf{\Delta S}, \quad s_e = \sigma_y, \quad s_e^2 = \frac{3}{2} \text{tr}(\mathbf{SS}^T), \]  
\[ \dot{\mathbf{e}} = c \dot{\mathbf{\theta}} + \text{tr}(\mathbf{\sigma D})^p, \quad \sigma_y = (\mathbf{\tilde{A}} + \mathbf{\tilde{B}} (\mathbf{\epsilon}^p)^n) (1 + \mathbf{\tilde{C}} \ln(\dot{\mathbf{\epsilon}}^p / \tilde{\mathbf{\epsilon}}_0)) (1 - T^{m^p}), \]  
\[ T = (\theta - \theta_m) / (\theta - \theta_m), \quad (\epsilon^p)^2 = \frac{2}{3} \text{tr}(\mathbf{D}^p \mathbf{D}^p). \]

Here, \( \sigma \) is the Cauchy stress tensor, \( \mathbf{S} \) its deviatoric part, \( p \) the hydrostatic pressure taken to be positive in compression, \( K \) the bulk modulus, \( \mu \) the shear modulus, \( \mathbf{\dot{S}} \) the Jaumann derivative of \( \mathbf{S} \), \( \mathbf{\ddot{D}} \) the deviatoric strain-rate, \( \mathbf{D}^p \) the plastic strain-rate, \( \mathbf{D}^e \) the elastic strain-rate, \( \mathbf{D} = \mathbf{D}^e + \mathbf{D}^p \) the strain-rate tensor, \( \mathbf{W} \) the spin tensor, \( c \) the specific heat, \( \theta \) the temperature of a material particle, \( \theta_m \) its melting temperature, \( \theta_0 \) the room temperature, \( T \) the homologous temperature, \( s_e \) the effective stress, and \( \epsilon^p \) the effective plastic strain. Equation (3)_3 implies that the volumetric response of the material is elastic. Equation (4)_1 is the constitutive relation in terms of deviatoric stresses for a linear isotropic hypoelastic material, \( \text{grad } \mathbf{v} \) equals the gradient of the velocity field with respect to coordinates in the present configuration, Eq. (6)_4 signifies the von Mises yield criterion with isotropic hardening, and Eq. (7)_2 is the Johnson-Cook (1983) relation. The flow stress, \( \sigma_y \), increases with an increase in the effective plastic strain and the effective plastic strain rate but decreases with an increase in the temperature of a material particle. Truesdell and
Noll (1965) have pointed out that Eq. (4) is not invariant with respect to the choice of different objective (or material frame indifferent) time derivatives of the stress tensor. In Eq. (7), \( \tilde{A} \) equals the yield stress of the material is a quasistatic simple tension or compression test, parameters \( \tilde{B} \) and \( n \) characterize the strain hardening of the material, \( \tilde{C} \) and \( \dot{\varepsilon}_0 \) its strain-rate hardening and \( (1-T^m) \) its thermal softening. Equation (6) signifies that the plastic strain-rate is along the normal to the yield surface, and the factor of proportionality \( \Lambda \) is given by

$$\Lambda = 0 \text{ when either } s_y < \sigma_y, \text{ or } s_y = \sigma_y \text{ and } \text{tr}(SS) < 0;$$

otherwise it is a solution of

$$s_y = (\tilde{A} + \tilde{B}(\varepsilon^p)^n) \left(1 + \tilde{C} \ln \left(\frac{2}{3} \Lambda s_y / \dot{\varepsilon}_0\right)\right)(1-T^m).$$

Once \( \theta = \theta_m \) at a material point, the flow stress, \( \sigma_y \), for the material point is set equal to zero. It then behaves like a compressible, nonviscous fluid. In physical experiments, fracture in the form of a crack will ensue from the point much before it is heated up to its melting temperature. Here, we have not incorporated any fracture criterion into the problem formulation. Because a Lagrangian formulation is used and a perfect fluid cannot support shear stresses, once a material point melts, the mesh will be distorted quickly and the computations will cease. However, in our work, no material point melted.

Initially, the cylindrical rod is stress free, at room temperature \( \theta_0 \), and is moving with a uniform speed \( V_0 \). All bounding surfaces of the rod except that contacting the target are taken to be traction free. Because of the assumption of locally adiabatic deformations, no thermal boundary
conditions are needed. The target surfaces perpendicular to the plane of impact are fixed and those parallel to it are traction free except for the part in contact with the rod. The contact surface is assumed to be frictionless.

Values of material parameters for the RC are derived by the rule of mixtures:

\[ P = P_1 V_1 + P_2 V_2. \]  

Equation (11) gives exact values of the mass density \( \rho \), and the heat capacity, \( \rho c \), but approximate values of other parameters. The use of the rule of mixtures seems to be a better approximation than replacing RC by a layer of steel embedded between two layers of concrete as was done by Ågårdh and Laine (1999). The plain concrete was assumed to be non-heat-conducting material having a constant yield strength. Also, the ceramic was presumed to have a constant yield strength.

For three of the materials: steel, ceramic, and RC, a material point is assumed to fail when the effective plastic strain attains a material specific critical value whereas no failure is considered in the copper. Furthermore, a material point of RC is also assumed to be subject to a second type of failure, spallation. A RC material point fails due to spallation at a tensile hydrostatic pressure of \( p_{\text{max}} \) and after this point it can no longer support shear stress or tension. A failed element due to effective plastic strain, however, is removed from the analysis. This facilitates the analysis in that the time step size is not drastically reduced by the severely deformed element. Thus
computations can be carried farther in time than would have been the case if these elements were kept in the analysis and failure modeled by either the node-splitting technique or the use of cohesive elements.

3 Computation and Discussion of Results

The aforestated problem requires solving a system of coupled nonlinear partial differential equations. Since it is nearly impossible to solve the problem analytically, we seek its approximate solution by the FEM and employ a large scale explicit code DYNA3D (Whirley and Hallquest (1991)) to do so. It uses Lagrangian formulation of the problem, 8-noded brick elements, one-point integration rule to evaluate various domain integrals, artificial viscosity, an hour-glass control to suppress the spurious modes and a conditionally stable explicit method to march forward the solution in time. The mass matrix is evaluated only once, and is diagonalized. Stresses, strains, strain-rates and temperature are taken to be uniform but time-dependent within each element. The time step is adjusted adaptively and equals a fraction of the time taken for a dilatational wave to travel through the smallest element in the mesh. As the bodies deform, elements near the smooth target/penetrator interface become distorted severely and the time step size drops drastically. The time step is also affected by the restoring force to be applied to the interpenetrating nodes at the target/penetrator interface. In the symmetric penalty method used to enforce the non-inter-penetrating conditions, this force is proportional to the depth of interpenetration, the bulk modulus of the penetrated element, that element’s dimensions and a
user defined scale factor. Once the time step size has become too small, the intensely deformed region, if not already deleted by the element erosion technique, needs to be rezoned for computations to continue at a reasonable pace. However, no rezoning is done for the problems studied herein. Consequently penetration of very thick RC targets has not been analyzed. Values assigned to different material parameters are listed in Table 1. Here $\varepsilon_f$ equals the effective plastic strain at failure. For plain concrete the spallation failure is assumed to occur at a tensile hydrostatic pressure of 0.5 GPa. Values of the specific heat of the ceramic and the plain concrete are not listed since thermal effects in the RC and the ceramic have been neglected. Unless otherwise noted, the volume fraction of steel in the RC equals 20%. The symbol N/A in Table 1 stands for “not applicable.”

Table 1: Values of material parameters

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ GPa</th>
<th>$\nu$</th>
<th>$\tilde{A}$ MPa</th>
<th>$\tilde{B}$ MPa</th>
<th>$\tilde{C}$</th>
<th>$\tilde{\varepsilon}_0$ 1/s</th>
<th>$\theta_m$ K</th>
<th>$m$</th>
<th>$n$</th>
<th>$\rho$ kg/m$^3$</th>
<th>$c$ J/kg K</th>
<th>$\varepsilon_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>200</td>
<td>0.33</td>
<td>792.2</td>
<td>5095</td>
<td>0.014</td>
<td>1E-6</td>
<td>1793</td>
<td>1.03</td>
<td>0.26</td>
<td>7840</td>
<td>477</td>
<td>0.5</td>
</tr>
<tr>
<td>Ceramic</td>
<td>263</td>
<td>0.22</td>
<td>1930</td>
<td>0</td>
<td>0</td>
<td>1E-6</td>
<td>N/A</td>
<td>N/A</td>
<td>0</td>
<td>3420</td>
<td>N/A</td>
<td>0.05</td>
</tr>
<tr>
<td>Plain Concrete</td>
<td>37</td>
<td>0.2</td>
<td>180</td>
<td>0</td>
<td>0</td>
<td>1E-6</td>
<td>N/A</td>
<td>N/A</td>
<td>0</td>
<td>2400</td>
<td>N/A</td>
<td>0.15</td>
</tr>
<tr>
<td>Copper</td>
<td>120</td>
<td>0.33</td>
<td>89.63</td>
<td>261.64</td>
<td>0.031</td>
<td>1E-6</td>
<td>1356</td>
<td>1.09</td>
<td>0.31</td>
<td>8960</td>
<td>383</td>
<td>N/A</td>
</tr>
</tbody>
</table>

The bar wave speed in steel, ceramic, plain concrete and copper equal 5.05, 8.77, 3.93, and 3.66 km/s respectively and their respective acoustic impedances are 39.6, 30, 9.42, and 32.8 $10^6$ kg /m$^2$ s.
3.1 Comparison of computed results for the Taylor impact test with the experimental observations

In order to establish that the code is being used correctly, we simulate the Taylor impact test for a copper rod for which the time history of the deformed length and the final deformed shape have recently been observed by Ferranti et al. (2004). Values of material parameters are listed in Table 1, and those of geometric parameters are given below.

Rod length = 74.95 mm, Rod diameter = 18.86 mm, Rod speed = 205.4 m/s.

Values of material parameter for copper are taken from the literature, and other variables have values for the specimen tested. The discretization of the rod into finite elements is shown in Fig. 3a. Figure 4a-b depicts the time histories of the deformed length, and of axial velocity of the tail end of the rod. It is clear that for $t \leq 48$ $\mu$s the computed deformed length of the rod at different times matches well with that observed experimentally; test data for $t > 48$ $\mu$s is not available. The speed of the tail end remains essentially uniform for the first 45 $\mu$s, and then decreases nearly affinely to zero at 210 $\mu$s. Figure 4c evinces the comparison between the computed and the measured final shapes of the rod. The computed diameter of the impacted face is about 10% larger than the measured one and the computed final length of ~48 mm is nearly 15% smaller than the measured one. At various axial locations, the computed diameter of the final configuration is close to the corresponding test value. These differences can be attributed to incorrect values of material parameters, and the neglect of friction at the contact surface between the target and the rod.
Figure 5a-b shows fringes of the effective plastic strain at $t = 100$ and $200 \, \mu s$. Other similar experiments on copper involving large plastic deformations suggest that it develops porosity which will influence its subsequent deformations. However, effects of porosity and heat conduction have been ignored in this work. It is not clear how they will influence the final deformed shape of the rod. The effective plastic strain equals at least 100% in a small region approximately 1 mm thick and adjoining the impacted face of the rod. It drops to nearly zero at points situated more than 10 mm away from the impacted face. At $t = 48 \, \mu s$ the rod’s length has diminished from $\approx 75$ mm to $\approx 65$ mm. Animations showing effective plastic strain in the rod can be seen in the files named CopRiRT1.avi and CopRiRT2.avi. As exhibited in Fig. 5c,d, large values of the tensile hydrostatic pressure occur in small regions near the periphery of the impacted face. Thus these regions are prone to high values of porosity which will lead to their ductile failure. Note that no failure criterion is considered in this test. Large values of the compressive hydrostatic pressure (not shown in Fig. 5c,d) occur in regions surrounding the stagnation point (i.e., the centroid of the impact face) and in a wide bell-shaped region abutting points where the mushroomed region transitions into straight cylindrical portion. Whereas the axial velocity of points in the mushroomed region has dropped from $\approx 205$ m/s to $\approx 10$ m/s at $t = 48 \, \mu s$, that of material points in the top $2/3^{rd}$ of the deformed rod still equals $\approx 200$ m/s. Thus there is considerable kinetic energy available to further deform the rod plastically. All particles of the rod have slowed down to nearly zero speed at $t = 210 \, \mu s$. Material near the outer periphery of the impacted face is lifted off from the target; thus the front end of the deformed rod is not flat.
3.2 Taylor impact test for composite rods

Whereas several investigators have analyzed the Taylor impact test for a homogeneous rod, not much work has been done in studying deformations of a KE rod comprised of two materials. Here we study transient thermomechanical deformations of a ceramic rod enclosed in a copper cylindrical shell; the section of the rod by a vertical plane passing through the centroidal axis and its discretization into finite elements are shown in Fig. 3b. The ceramic is assumed to be perfectly bonded to copper. Values of parameters for the 23 simulations are shown in Table 2.

Table 2. Values of parameters in the Taylor impact test of a copper/ceramic composite rod

<table>
<thead>
<tr>
<th>Test</th>
<th>t/D</th>
<th>C/D</th>
<th>(V_0) (m/s)</th>
<th>(\varepsilon_f^{cer})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>1</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>1</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>0.09</td>
<td>1</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>0.06</td>
<td>1</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>0.03</td>
<td>1</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>6</td>
<td>0.15</td>
<td>(\frac{3}{4})</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>7</td>
<td>0.15</td>
<td>(\frac{1}{2})</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>8</td>
<td>0.15</td>
<td>(\frac{1}{4})</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>9</td>
<td>0.09</td>
<td>(\frac{3}{4})</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>10</td>
<td>0.09</td>
<td>(\frac{1}{2})</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>11</td>
<td>0.09</td>
<td>(\frac{1}{4})</td>
<td>800</td>
<td>0.05</td>
</tr>
<tr>
<td>12</td>
<td>0.15</td>
<td>1</td>
<td>600</td>
<td>0.05</td>
</tr>
<tr>
<td>13</td>
<td>0.15</td>
<td>1</td>
<td>400</td>
<td>0.05</td>
</tr>
<tr>
<td>14</td>
<td>0.15</td>
<td>1</td>
<td>200</td>
<td>0.05</td>
</tr>
<tr>
<td>15</td>
<td>0.09</td>
<td>1</td>
<td>600</td>
<td>0.05</td>
</tr>
<tr>
<td>16</td>
<td>0.09</td>
<td>1</td>
<td>400</td>
<td>0.05</td>
</tr>
<tr>
<td>17</td>
<td>0.09</td>
<td>1</td>
<td>200</td>
<td>0.05</td>
</tr>
<tr>
<td>18</td>
<td>0.15</td>
<td>1</td>
<td>800</td>
<td>0.10</td>
</tr>
<tr>
<td>19</td>
<td>0.15</td>
<td>1</td>
<td>800</td>
<td>0.15</td>
</tr>
<tr>
<td>20</td>
<td>0.15</td>
<td>1</td>
<td>800</td>
<td>0.20</td>
</tr>
<tr>
<td>21</td>
<td>0.09</td>
<td>1</td>
<td>800</td>
<td>0.10</td>
</tr>
<tr>
<td>22</td>
<td>0.09</td>
<td>1</td>
<td>800</td>
<td>0.15</td>
</tr>
<tr>
<td>23</td>
<td>0.09</td>
<td>1</td>
<td>800</td>
<td>0.20</td>
</tr>
</tbody>
</table>
The rod’s length $L$ and diameter $D$ equal 279 and 38 mm respectively. The first five tests examine the effect of the shell wall thickness, $t$. For two values of the shell wall thickness, the remaining eighteen tests delineate the effect of $C/D$, impact speed and the critical axial strain at which ceramic fails. Here $C$ and $D$ equal, respectively, the length of the solid portion of the rod at the front end and the outer diameter of the composite rod as can be seen in Fig. 1a. The design of the composite rod tested here is different from that of the sleeved rod of Foster et al. (2001).

Figure 6a-d exhibits the influence of $t/D$, $C/D$, $V_0$, and $e_f^{cr}$ upon the time history of the volume fraction of the failed ceramic. Upon impact a compressive wave propagates upwards through the bottom solid copper portion of the rod at a speed of $\sim 3.66 \text{ mm/}\mu\text{s}$. When it arrives at the copper/ceramic interface, due to unequal acoustic impedances of copper and ceramic, a part of it is reflected back and the other is transmitted into the ceramic. However, in the copper shell the compressive wave continues to propagate at $\sim 3.66 \text{ mm/}\mu\text{s}$ but in the ceramic it propagates at $\sim 8.77 \text{ mm/}\mu\text{s}$. Thus the compressive wave in the ceramic core arrives at the top surface much sooner than that in the copper shell. Results plotted in Fig. 6a evince that a decrease in $t/D$ decreases the volume fraction of the failed ceramic. Since the total volume of ceramic present increases with a decrease in $t/D$, the total amount of ceramic failed increases with a decrease in $t/D$. Recalling that a failed element is deleted from the analysis, more new free surfaces are constantly being created and their locations continue to change. From the plots of Fig. 6b we conclude that the ratio $C/D$ influences more the volume fraction of the failed ceramic than the ratio $t/D$, and the volume fraction of the failed ceramic increases with a decrease in $C/D$. For $t/D = 0.09$ and time = 75 $\mu\text{s}$, the volume fractions of the failed ceramic equal 20% and 24% for
Failure initiates later in the ceramic for \( C/D = 3/4 \) than for \( C/D = 1/4 \) since it takes longer for the compressive loading wave to arrive at the ceramic/copper interface for the larger value of \( C/D \). Recalling that the intensity of the compressive shock wave produced upon impact of the composite rod with the rigid target increases rapidly with an increase in \( V_0 \), therefore the failure initiation is considerably delayed when \( V_0 \) is decreased from 600 m/s to 200 m/s. Also at 75 \( \mu s \) following impact, volume fraction of the failed ceramic is noticeably larger for higher values of \( V_0 \). For \( V_0 = 400 \) m/s, the decrease in the wall thickness increases greatly the volume fraction of the failed ceramic. However, for \( V_0 = 200 \) m/s and 600 m/s, the volume fraction of failed ceramic for \( t/D = 0.09 \) and \( 0.15 \) are nearly the same at 75 \( \mu s \) after impact. Interestingly enough, the volume of failed ceramic is higher when \( \varepsilon_f^{ext} = 0.15 \) than that when \( \varepsilon_f^{ext} = 0.10 \).

An animation showing effective plastic strain in the deforming rod in test 8 is available in the file named ComPiRCT_08.avi and an animation showing only the deforming end of the same rod can be seen in the file ComPiRCT2_08.avi. These animations suggest that ceramic elements adjoining the copper shell wall at a height of \( C/D \) fail first. The failure may propagate both radially and axially at different speeds. A longitudinal section of a deformed rod 75 \( \mu s \) subsequent to impact is exhibited in Fig. 7. It is clear that the failure has propagated for a long distance along the copper/ceramic interface. Also, the solid copper cylinder of height \( C \) near the impact face has been virtually flattened. The time histories of the propagation of the failure along the copper/ceramic interface are depicted in Fig. 8a-d. The length of the failed ceramic along the copper/ceramic interface increases with an increase in the shell wall thickness but not
necessarily with an increase in either the impact speed or the assumed failure strain of the ceramic. For example, the length of the failed zone for $\varepsilon_{f}^{\text{cer}} = 0.15$ is between those for $\varepsilon_{f}^{\text{cer}} = 0.1$ and $\varepsilon_{f}^{\text{cer}} = 0.2$, and that for $V_0 = 400 \text{ m/s}$ is higher than that for $V_0 = 200 \text{ m/s}$ and $400 \text{ m/s}$ when $t/D = 0.09$. For $t/D = 0.15$, the length of the failed ceramic material along the copper/ceramic interface for $V_0 = 400 \text{ m/s}$ is smaller than that for $V_0 = 200 \text{ m/s}$ and $600 \text{ m/s}$.

We add that there is no correlation between the length of the failed ceramic material along the copper/ceramic interface and the total volume of failed ceramic. For $t/D = 0.03$ and 0.06 the failure does not propagate continuously along the length of the composite rod but moves in jerks in the sense that the length of the failed region stays unchanged for a certain interval and then suddenly increases. The time during which the length of the failed ceramic does not change decreases with an increase in $t/D$.

For tests 1, 5, 8 and 11 we have plotted in Fig. 9a-d longitudinal sections of the deformed rods at two instants; the left and the right pictures in each plot are at times 24 and 49 $\mu$s after impact. The deformed shape in Fig. 9b corresponding to the smallest wall thickness of 0.03D shows a kink in the wall near the top of the flattened region. It may be taken as an indication of the shell wall buckling. The left pictures in Fig. 9c,d show the lifting up from the target of the central portion of the impacted face of the rod. However, subsequently this gap is closed, and the deformed surface is flattened. Whereas the gap has closed for test 8 at 49 $\mu$s after impact (cf. Fig. 9c, right), the gap has shrunk but not completely closed yet for test 11 at 49 $\mu$s. These pictures also make it clear that neither all ceramic in a particular cross-section fails
instantaneously nor the failure zone propagates radially inwards from the ceramic/copper interface.

Figure 10a-d depicts for tests 1, 5, 8 and 11 sections by a longitudinal plane of the mushroomed regions at time 74 $\mu$s after impact. Whereas the outer surface of the mushroomed region is quite smooth for $t/D = 0.15$ that for $t/D = 0.03$ has a sharp discontinuity. Also, the central portion of the impacted surface is still lifted off from the target face. A comparison of Fig. 10a and d suggests that reducing the thickness of the front solid portion of the rod from $D$ to $D/4$ also results in a transition in the curvature of the outer surface of the mushroomed region. Keeping $C/D$ fixed at 1/4 but reducing the wall thickness from 0.15$D$ to 0.09$D$ does not change, qualitatively, the shape of the mushroomed region. Figure 11a-d show computed values of radius with axial position for the mushroomed region of the rod at 74$\mu$s for all of the tests.

Figure 12a,b shows the time history of the reduction in the length of the rod. The change in $\varepsilon_{\text{cer}}$ was found not to influence the reduction in rod’s length and hence the corresponding results are not plotted. It is clear from these plots that $t/D$ and $C/D$ have negligible effects on the reduction in rod’s length. The most significant parameter affecting rod’s length is its initial kinetic energy. With an increase in $V_0$, the rod length decreases at a faster rate and approaches its limiting value sooner.
3.3 Penetration of kinetic energy rods into RC targets

3.3.1 Dependence of penetration results upon the FE mesh

Both the penetrator and the target were discretized into coarse and fine meshes. The number of nodes in the target and the penetrator equalled 21,072 (172,210) and 1,672 (18,480), respectively, for the coarse (fine) mesh; the two meshes are shown in Fig. 13. For $D = 15\,\text{mm}$, $L/D = 2.87$, $A/D = 0.305$, $T/D = 4$, $W/D = 26.7$ and $V_0 = 800\,\text{m/s}$, Fig. 14a-d exhibits time histories of the failed target and penetrator materials, the time history of the crater diameter and the time history of the maximum temperature in the penetrator. The crater diameter equals twice the distance of the failed element in the impact face that is farthest from the point of impact. Out of the four sets of results, three match very well and only the damage induced in the penetrator is much less for the fine mesh than that for the coarse mesh. If the emphasis is on delineating the damage induced in the target, then the use of a coarse mesh will give acceptable results. We note that the CPU time required to compute results with the fine mesh is nearly 100 times that for the coarse mesh.

3.3.2 Penetration of steel rods into RC targets

Results presented in this section are with the FE mesh shown in Fig. 3c. Referring to Fig. 2, we analyze the effect of the angle of obliquity of the 38 mm diameter 279 mm long cylindrical steel rod moving at 800 m/s on the damage induced to $760\times760\times76$ mm RC target containing 20%
volume fraction of steel. The damage is defined as the volume of failed elements. Eight numerical tests summarized in the following Table 3 have been conducted.

Table 3. Geometric parameters for the eight perforation tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Angle ( \phi ) of obliquity (degrees)</th>
<th>A/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>½</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>½</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>½</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>½</td>
</tr>
</tbody>
</table>

A reduction in the value of A/D from 1 to 1/2 gives a blunter nose shape. Figure 15a,b exhibits, for two nose shapes and four angles of obliquity \( \phi \), time histories of the crater radius. For the blunt nose shape with \( A/D = 1/2 \), the angle of obliquity has virtually no effect on the crater diameter at times \( \leq 55 \mu s \). However, for the sharper nose shape having \( A/D = 1 \), the crater diameter increases with \( \phi \) partly because of the definition of the crater diameter. It takes approximately 105-117 \( \mu s \) for the penetrator to just perforate the target. We have plotted in Fig. 16a-d the cavities formed in the target and the deformed nose shape of the penetrator for four tests: \( A/D = 1, \ \phi = 0, \ 30^\circ \); \( A/D = 1/2, \ \phi = 0, \ 30^\circ \). It is evident that for each value of \( A/D \) the penetrator nose is severely deformed for \( \phi = 30^\circ \) whereas it is hardly deformed for \( \phi = 0^\circ \). For \( A/D = 1 \) and \( \phi = 30^\circ \), nearly a quarter of the penetrator nose has broken off. For \( A/D = 1/2 \) and \( \phi = 30^\circ \), virtually all of the penetrator nose has failed. Time histories of the failed target material plotted in Fig. 17a,b reveal that the angle of obliquity has no effect on the total damage
induced to the target in the sense that the volume of the failed target is unaffected. The files: SPiRCT_05.avi and SPiRCT_08.avi contain animations of tests 5 and 8 showing effective plastic strain and the erosion of elements. The volume of failed target material is larger for the blunter nose shape with $A/D = 1/2$ than that for the nose shape with $A/D = 1$; a similar result was found by Batra and Chen (1994), for a rigid rod penetrating a thermoviscoplastic target, and by Batra (1987) for steady state penetration of a viscoplastic target by a rigid rod. From the time histories of the failed penetrator material plotted in Fig. 18a,b, we conclude that the blunter nosed penetrator with $A/D = 1/2$ is more severely damaged than the one with $A/D = 1$. For each of the eight cases studied here, the maximum temperature in the penetrator equaled 500 K.

### 3.4 Penetration of composite penetrators into RC targets

Figure 1b shows a sketch of the problem studied and Table 4 lists values assigned to different variables.

<table>
<thead>
<tr>
<th>Test</th>
<th>t/D</th>
<th>A/D</th>
<th>C/D</th>
<th>%Target Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>2/3</td>
<td>1/3</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>2/3</td>
<td>1/3</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>0.09</td>
<td>2/3</td>
<td>1/3</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
<td>1/2</td>
<td>1/3</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>0.15</td>
<td>1/3</td>
<td>1/3</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>0.15</td>
<td>1/4</td>
<td>1/3</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>0.15</td>
<td>1</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>0.15</td>
<td>1</td>
<td>2/3</td>
<td>20</td>
</tr>
<tr>
<td>9</td>
<td>0.15</td>
<td>1</td>
<td>1/2</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>0.15</td>
<td>2/3</td>
<td>1/3</td>
<td>25</td>
</tr>
<tr>
<td>11</td>
<td>0.15</td>
<td>2/3</td>
<td>1/3</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>0.15</td>
<td>2/3</td>
<td>1/3</td>
<td>35</td>
</tr>
</tbody>
</table>
For each one of the twelve simulations we took

\[ D = 38 \text{ mm}, \quad L = 22D / 3, \quad V_0 = 800 \text{ m/s}, \quad T = 3D, \quad W = 15D. \]

The discretization of the penetrator and the target into finite elements is shown in Fig. 3d. Besides scrutinizing the penetration performance of the composite cylindrical rod comprised of a steel casing enclosing ceramic, we are also interested to find if it buckles during the penetration process. Furthermore, does the buckling of the penetrator affect its performance? Here the ceramic is assumed to be perfectly bonded to steel. Note that the penetrator nose is made of steel.

We hypothesize that the penetrator buckles when its outer radius at any point increases by 20%. Accordingly, we have plotted in Fig. 19a,b the time histories of the maximum increase in the outer radius for \( t/D = 0.15, 0.12 \) and \( 0.09 \), and \( A/D = 1/2, 1/3 \) and \( 1/4 \). These reveal that the nose shape plays a significant role in determining the buckling characteristics of the composite penetrator; the deformed rods for these three cases are depicted in Fig. 20a-c. Test 3 is animated in the file ComRiRCT_03.avi which shows the deforming rod and target. The percentage increases in the radius for \( C/D = 1, \ 2/3 \) and \( 1/2 \) were essentially identical to each other and were linear in time with the increase in the outer radius equaling 3% at \( t = 150 \mu s \).

Whereas an increase in the volume fraction of steel in the target from 25% to 30% enhanced the enlargement of the penetrator radius, that from 30% to 35% had virtually no effect. For 35% volume fraction of steel in the target, the maximum increase in the penetrator radius at \( t = 180 \mu s \) equaled almost 10%.
In every case the ceramic failed unevenly along the length of the penetrator in the sense that the cross-sectional area of the deformed ceramic rod varied with the axial position. Also, a considerable portion, \( \sim 2D \) in length, of the ceramic rod near the tail end failed. The volume fraction of the failed ceramic material, (cf. Fig. 21a-d) did not increase when the volume fraction of steel in the RC target was increased from 25% to 35%. Whereas the volume fraction of the failed ceramic enhanced with an increase in \( A/D \) from 1/4 to 1/3, increasing \( A/D \) from 1/3 to 1/2 had no noticeable effect on the volume fraction of the failed ceramic material. Decreasing \( t/D \) from 0.15 to 0.12 increased the amount of failed ceramic, but the subsequent decrease to 0.09 did not change the volume of failed ceramic. Recalling that the penetrator buckled for tests 3, 5 and 6, we conclude that there is no apparent correlation between the buckling of the penetrator and the volume fraction of failed ceramic.

We are also interested in how buckling of the penetrator affects its overall length. Does the bulging out of the shell wall radially displace enough material to significantly reduce it? In Fig. 22a-d we see that decreasing \( t/D \) or \( A/D \) causes a greater decrease in length as expected, but it can also been seen that only three of the twelve simulations have reductions in length greater than 3%. In these three simulations, the penetrator buckled and the overall length decreased by \( \sim 5\% \).
4 Conclusions

We have analyzed by the finite element method, finite deformations of fast moving composite and monolithic rods impacting either a smooth flat rigid target or a deformable reinforced concrete target. The composite rod is made of a circular cylindrical ceramic rod enclosed either in copper or steel shell. The target/penetrator interface is assumed to be smooth, deformations are assumed to be locally adiabatic, and failed elements are deleted from the analysis.

For the Taylor impact test with a copper rod, the computed time history of the rod length and the axial variation of the final rod diameter were found to match well with the corresponding experimental observations. None of the composite rods made of copper shell filled with ceramic buckled upon impacting a smooth flat rigid target probably because of the high ductility of copper. Sharp transitions in the curvature of the upper surface of the flattened bottom portion of the rod occurred in three out of 23 simulations. These correspond to the smallest shell wall thickness and the smallest length of the solid portion of the rod at the front end.

For the RC penetration tests, a blunt nosed penetrator caused more damage to the RC target than a sharp nosed one. However the volume of failed RC target did not increase with an increase in the angle of obliquity. Both factors, angle of obliquity and nose bluntness, increased damage to the penetrator. Nearly all of the penetrator nose with $A/D = 1/2$ and angle of obliquity $= 30^\circ$ failed. Simulations of the penetration of composite cylindrical rods comprised of steel shells filled with ceramic into RC targets revealed buckling of some of the rods; a rod was taken to
have buckled when its outer diameter increased by at least 20%. Kinetic energy rods moving at 800 m/s and having shell wall thickness equal to 0.09D and nose shapes of $A/D \leq 1/3$ buckled.
5 References


6 Figures

Figure 1. (a) Sketch of the composite copper/ceramic rod studied in the Taylor impact tests; (b) sketch of the composite steel/ceramic penetrator studied in the RC target penetration tests
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(a)

(b)
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7 Appendix

7.1 Pre-processing Programs

7.1.1 Flowcharts and Descriptions

We generate the meshes and write them into a DYNA3D input file for the Taylor Impact Tests using our own Fortran 77 programs. Below is a flowchart describing the sequence of files written and read while creating, executing, and analyzing a Taylor Impact simulation.

Flowchart 1. File generation sequence for the Taylor impact tests
The description of each file in the flowchart below.

**TaylorPen.f**
This is a Fortran 77 program that generates the node locations and mesh for the penetrators used in the Taylor Impact Tests.

**a.out**
This is the executable file written when the TaylorPen.f is compiled.

**Penetrator Info.txt**
This is a text file that lists some information about the dimensions and the mesh of the penetrator.

**D P N Cards.txt**
This file is the DYNA3D Node Cards for the penetrator. They contain the node locations and boundary conditions in the format read by DYNA3D.

**D P E Cards.txt**
This file is the DYNA3D Element Cards for the penetrator. They contain the element connectivity and material numbers in the format read by DYNA3D.

**Tecplot Penetrator.dat**
This file contains the node locations and element information in the format read by Tecplot 90. A picture of the penetrator and mesh can be created in the Tecplot for a visual check.

**TaylorInput.f**
This is a Fortran 77 program that reads the Node Cards and the Element Cards and writes them into a DYNA3D input file.

**a.out**
This is the executable file written when the TaylorInput.f is compiled.

**__input**
This is the input file for the Taylor Impact problems. Prior to being supplemented with the node and element information, the input file written by TaylorInput.f has the material properties, initial conditions, termination time, and all other parameters for the Taylor Impact Problems.
We generate the meshes and write them into a DYNA3D input file for the Concrete Penetration Tests using a different set of our Fortran 77 programs. Below is a flowchart describing the sequence of files written and read while creating, executing, and analyzing a Concrete Penetration simulation.

Flowchart 2. File generation sequence for the concrete penetration tests
Following are descriptions of each file in the flowchart.

**Pen.f**
This is a Fortran 77 program that generates the node locations and mesh for the penetrators used in the Concrete Penetration Tests.

**a.out**
This is the executable file written when the Pen.f is compiled.

**Penetrator Info.txt**
This is a text file that lists some information about the dimensions and the mesh of the penetrator.

**D P N Cards.txt**
This file contains the Node Cards; node locations and boundary conditions for the penetrator nodes in the format read by DYNA3D.

**D P E Cards.txt**
This file contains the Element Cards; element connectivity and material numbers for the penetrator elements in the format read by DYNA3D.

**D P SS Cards.txt**
This file contains the Slave Surface Cards. DYNA3D needs master and slave surface definitions for satisfying non-inter-penetration between contacting surfaces; the front of the penetrator is the slave surface.

**ANSYS Contact Surface Plot.txt**
This file contains slave surface information in the format for an ANSYS model with 2D, 4-noded elements representing each slave surface. A plot can be made in ANSYS to check the numbering.

**Tecplot Penetrator.dat**
This file contains the node locations and element information in the format read by Tecplot 90. A picture of the penetrator and mesh can be created in the Tecplot for a visual check.

**Target.f**
This is a Fortran 77 program that generates the node locations and mesh for the targets used in the Concrete Penetration Tests.

**a.out**
This is the executable file written when the TargetPen.f is compiled.
**Target Info.txt**
This is text file that lists some information about the dimensions and the mesh of the target.

**D T N Cards.txt**
This file contains the Node Cards; node locations and boundary conditions for the target nodes in the format read by DYNA3D.

**D T E Cards.txt**
This file contains the Element Cards; element connectivity and material numbers for the target elements in the format read by DYNA3D.

**D P S S Cards.txt**
This file contains the Slave Surface Cards. DYNA3D needs master and slave surface definitions to impose a contact penalty on certain surfaces; the top of the target is the master surface.

**ANSYS 2D Target Plot.txt**
This file contains master surface information in the format for an ANSYS model with 2D, 4-noded elements representing the each master surface. A plot can be made in ANSYS to check the numbering.

**Tecplot Target.dat**
This file contains the node locations and element information in the format read by Tecplot 90. A picture of the target and mesh can be created in the Tecplot for a visual check.

**Input.f**
This is a Fortran 77 program that reads the Node Cards, Element Cards, and the contact surface Cards for both the penetrator and the target and writes them into a DYNA3D input file.

**a.out**
This is the executable file written when the Input.f is compiled.

**__input**
This is the input file for the Concrete Penetration problems. Prior to being supplemented with the node, element, and contact surface information, the input file written by Input.f has the material properties, initial conditions, termination time, and all other parameters for the Concrete Penetration problems.
7.1.2 Diagrams for the mesh generator programs

_TaylorPen.f_

Here are two diagrams explaining the process used to create the mesh and the meaning of several variables in the mesh generator program _TaylorPen.f_. This mesh generator begins by creating arrays _NVar_ and _EN_. These arrays contain, respectively, the node locations and element connectivity for a 2D mesh of a cross section of the penetrator as shown below.

Next, this 2D mesh is extruded layer by layer, back to front, in the z direction creating the arrays _NVar3D_ and _EN3D_ which contain the node locations and element connectivity for the entire penetrator as shown below.
Appendix Diagram 2. Profile of the extruded 2-D mesh

Pen.f

Here are two diagrams continuing the mesh generator explanation to include the program Pen.f. This mesh generator begins with a process identical to TaylorPen.f resulting in a flat nosed penetrator shown in Appendix Diagram 2. The amount of curvature needed to add the tip length $LA$ is added to the section dimensioned $LC$ and the entire penetrator is tilted to the chosen angle $t2p$ which is the angle of obliquity.

Appendix Diagram 3. Curvature of the penetrator nose and angle of obliquity
The target mesh is generated in a similar manner and is created by the program Target.f. A 2-D mesh is created in two arrays NVa2Dr and ESS which contain, respectively, the node locations and element connectivity for a 2D mesh of the top (contact) surface of the target as shown below.

![Appendix Diagram 4. 2-D target mesh](image)

The generation of the 2D mesh begins with the creation of the more finely meshed inner area seen in the Figure. The size of this area is set by the variable Xfine which is its width in the y-direction and one half its width in the x-direction. The density of the elements in the inner area is set by the value of NumEx and this determines the size of the outer elements which are created in NumSLay layers around its perimeter. The 2D mesh is then extruded in the -z direction creating the arrays NVar and EN. The total z thickness is set by Zdepth and the number of layers in the thickness is set by NumLayzE.
7.2 DYNA3D Execution

We execute the simulations using only an executable file compiled with Version 4.0.7 of DYNA3D on an IRIX operating system. Below is an example of the command line used to execute a simulation along with a description of each command.

```
nohup ~/Path/Directory/D.sgi i=__input,f=hsp &
```

*nohup*  
This is a unix command that tells the computer to continue executing the file D.sgi even if the users exits. Any output that would normally be written to the screen is written to a text file “nohup.out.”

*D.sgi*  
This is the name we choose to give the DYNA3D executable file.

*/Path/Directory*  
A “~” is a unix symbol that represents your home directory. “/Path/Directory/” are just example directory names. This path represents the location of the executable file in your home directory.

*i=__input*  
“i=” tells DYNA3D the name of the input file to read to begin the simulation. “__input” is the file name we choose for our DYNA3D input file.

*f=hsp*  
This command tells DYNA3D to write the Time History Files which we use in all of our Post-processing programs.

*&*  
Is a Unix command that gives the user the command prompt back after executing D.sgi. This allows the user to continue working or execute other programs.
7.3 DYNA3D Time History Output Files

The DYNA3D Time History Files *disp_th*, *stress_th*, and *ps_temp* contain nodal and element information at user chosen time steps. They are written in Binary format so understanding their structure is necessary for reading them. Below is a summary of the contents and structure of each Time History File.

*disp_th*

Number of Nodes (N), Number of Time Steps (S)

Time

Number of Nodes (N), Number of Time Steps (S)

Time

n, ux, uy, uz, vx, vy, vz  
(n for Node 1)

n, ux, uy, uz, vx, vy, vz  
(n for Node 2)

n, ux, uy, uz, vx, vy, vz  
(n for Node 3)

.

.

n, ux, uy, uz, vx, vy, vz  
(n for Node N)

Time

n, ux, uy, uz, vx, vy, vz  
(at Time Step 1)

n, ux, uy, uz, vx, vy, vz  
(at Time Step 2)

n, ux, uy, uz, vx, vy, vz  
(at Time Step S)

n, ux, uy, uz, vx, vy, vz  
(for Node 1)

n, ux, uy, uz, vx, vy, vz  
(for Node 2)

n, ux, uy, uz, vx, vy, vz  
(for Node 3)

.

.

n, ux, uy, uz, vx, vy, vz  
(for Node N)
stress_th

Number of Elements (N), Number of Time Steps (S)

Time
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( \vdots \)
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \) (for Element N)

Time
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( \vdots \)
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \) (for Element N)

Time
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( \vdots \)
\( e, \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \)
\( \sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, \sigma_{eff} \) (for Element N)

ps_temp

Number of Elements (N), Number of Time Steps (S)

Time
\( e, \varepsilon_{eff}^{pl}, \text{temperature} \)
\( e, \varepsilon_{eff}^{pl}, \text{temperature} \)
\( e, \varepsilon_{eff}^{pl}, \text{temperature} \)
\( \vdots \)
\( e, \varepsilon_{eff}^{pl}, \text{temperature} \)
\( \varepsilon_{eff}^{pl}, \text{temperature} \) (for Element N)

Time
\( e, \varepsilon_{eff}^{pl}, \text{temperature} \)
\( e, \varepsilon_{eff}^{pl}, \text{temperature} \)
\( e, \varepsilon_{eff}^{pl}, \text{temperature} \)
\( \vdots \)
\( e, \varepsilon_{eff}^{pl}, \text{temperature} \)
\( \varepsilon_{eff}^{pl}, \text{temperature} \) (at Time Step S)
7.4 Post-processing Programs

The post processing programs read the initial node locations and element connectivity from the original input file. The nodal displacements and velocities as well as the element stresses, strains, and temperatures are read from the Time History Files. An examination of the following program should give other necessary details.

**Plot.f**

This program reads the DYNA3D Time History Output files original input file “__input” and writes a Tecplot 90 input file to create a plot of the deformed mesh with several variables as color contour options.
8 Vita

Wesley D. Ballew

I was born in Marion North Carolina, a fairly small town in the foothills of the Appalachian Mountains. I managed a pleasantly normal childhood and attended McDowell High School where I played offensive tackle and served as senior class president. During the summer after graduation I was in a very serious car accident, but recovered quickly and did not postpone my attendance at North Carolina State University.

At NCSU I indulged in socializing mixed with studying and counteracted those activities by lifting weights and running. I enjoyed my education in Mechanical Engineering, but as I finished a physics minor in my senior year I realized I had an affinity for the theoretical formulations I had encountered. With the advice of my undergraduate research advisor, I chose to pursue my graduate studies in the Engineering Science and Mechanics Department at Virginia Tech.

Only a piece of my experience at Virginia Tech culminates in this thesis. My graduate degree in mechanics and glimpse of academic research have enriched my abilities and broadened my perspective as an engineer.