This appendix contains example calculations of interface stiffness using the extended hyperbolic model described in Chapter 4 of this report. The stress path applied during multi-directional stress path Test T405_10 was used as the basis for these examples.

Figure E1a shows the stress path applied during Test T405_10, performed on the interface between concrete and dense Light Castle Sand. The interface response measured during the test is shown in Figure E1b for illustration. In this test, the interface was sheared following the sequence A-B-C-D-E-F-G-H. Table E1 contains information on some points along this stress path that is pertinent for the example calculations presented herein.

The parameter values for yield-inducing shear of the dense Light Castle Sand against concrete interface were presented in Chapter 4, and they are reproduced in Table E2.

The parameter values for unloading-reloading are calculated according to the criteria presented in Chapter 4. The unload-reload stiffness number $K_{urj}$ is calculated as follows:

$$K_{urj} = C_k \cdot K_I$$

where $C_k$ is the interface stiffness ratio and $K_I$ is the interface stiffness number for initial loading.

For Version I of the extended hyperbolic model, the following expression is used for the determination of the stiffness ratio $C_k$:

\[ C_k \]
where \( R_{fj} \) is the failure ratio.

\[
C_k = 0.5 \cdot \left(1 + R_{fj}\right)^2
\]

(Table 4-13)

<table>
<thead>
<tr>
<th>Point</th>
<th>( \sigma_n ) kPa</th>
<th>( \tau ) kPa</th>
<th>SL</th>
<th>( \tan(\theta) )</th>
<th>Origin</th>
<th>Loading Type</th>
<th>SL+</th>
<th>SL-</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>70</td>
<td>0</td>
<td>0</td>
<td>1.43</td>
<td>-</td>
<td>Virgin shear</td>
<td>0</td>
<td>0</td>
<td>+1</td>
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<tr>
<td>1</td>
<td>105</td>
<td>25</td>
<td>0.36</td>
<td>1.43</td>
<td>-</td>
<td>Virgin shear</td>
<td>0.36</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>B</td>
<td>137.5</td>
<td>47.5</td>
<td>0.52</td>
<td>-</td>
<td>-</td>
<td>Virgin shear</td>
<td>0.52</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>170</td>
<td>25</td>
<td>0.22</td>
<td>-1.48</td>
<td>B</td>
<td>Unloading</td>
<td>0.52</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>C</td>
<td>207.5</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>Unloading</td>
<td>0.52</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>T</td>
<td>232.5</td>
<td>47.5</td>
<td>0.31</td>
<td>0.54</td>
<td>C</td>
<td>Reloading</td>
<td>0.52</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>3</td>
<td>247.5</td>
<td>75</td>
<td>0.45</td>
<td>0.54</td>
<td>C</td>
<td>Transition loading</td>
<td>0.52</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>Y1</td>
<td>255</td>
<td>89</td>
<td>0.52</td>
<td>0.54</td>
<td>-</td>
<td>Virgin shear</td>
<td>0.52</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>D</td>
<td>277</td>
<td>130</td>
<td>0.70</td>
<td>-</td>
<td>-</td>
<td>Virgin shear</td>
<td>0.70</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>277</td>
<td>50</td>
<td>0.27</td>
<td>0</td>
<td>D</td>
<td>Unloading</td>
<td>0.70</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>Y2</td>
<td>277</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>Virgin shear</td>
<td>0.70</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>277</td>
<td>-25</td>
<td>-0.14</td>
<td>0</td>
<td>-</td>
<td>Virgin shear</td>
<td>0.70</td>
<td>-0.14</td>
<td>-1</td>
</tr>
<tr>
<td>E</td>
<td>277</td>
<td>-52.5</td>
<td>-0.28</td>
<td>0</td>
<td>-</td>
<td>Virgin shear</td>
<td>0.70</td>
<td>-0.28</td>
<td>-</td>
</tr>
<tr>
<td>Y2</td>
<td>277</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>E</td>
<td>Unloading-reloading</td>
<td>0.70</td>
<td>-0.28</td>
<td>+1</td>
</tr>
<tr>
<td>D</td>
<td>277</td>
<td>130</td>
<td>0.70</td>
<td>-</td>
<td>E</td>
<td>Reloading</td>
<td>0.70</td>
<td>-0.28</td>
<td>+1</td>
</tr>
<tr>
<td>6</td>
<td>253</td>
<td>137</td>
<td>0.81</td>
<td>-8.93</td>
<td>-</td>
<td>Virgin shear</td>
<td>0.81</td>
<td>-0.28</td>
<td>+1</td>
</tr>
</tbody>
</table>

Note: Definitions of parameters are presented in Chapter 4 and in the Notation (Appendix F).

<table>
<thead>
<tr>
<th>Table E2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary of Hyperbolic Parameter Values for the Dense Light Castle Sand against Concrete Interface</td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>( K_i )</td>
</tr>
<tr>
<td>( n_i )</td>
</tr>
<tr>
<td>( R_{fi} )</td>
</tr>
<tr>
<td>( \delta )</td>
</tr>
</tbody>
</table>
According to these equations, the value of $K_{urj}$ for the dense Light Castle Sand against concrete interface is determined using the values of $K_I$ and $R_{fj}$ presented in Table E2, as follows:

$$C_k = 0.5 \cdot (1 + 0.79)^2 = 1.602$$

$$K_{urj} = 1.602 \cdot 20700 = 33160$$

This value of $K_{urj}$ is used for modeling unloading-reloading of the interface in Version I of the model.

For Versions II and III, $C_s$ is calculated as follows:

$$C_k = (1 + R_{fj})^2 \quad (\text{Table 4-13})$$

For the dense Light Castle Sand against concrete interface, the following value of $K_{urj}$ is obtained:

$$C_k = (1 + 0.79)^2 = 3.204$$

$$K_{urj} = 3.204 \cdot 20700 = 66320$$

The scaling factor $\alpha$, which is used in Versions II and III of the model, is calculated according to the following expression:

$$\alpha = 1 + R_{fj} \quad (4-47 \text{ bis})$$

For the dense Light Castle Sand against concrete interface, the value of the scaling factor is determined as follows:

$$\alpha = 1 + 0.79 = 1.79$$
Table E3 summarizes the values of $K_{urj}$ and $\alpha$ for the dense Light Castle Sand against concrete interface.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value for Version I</th>
<th>Value for Versions II and III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{urj}$</td>
<td>33160</td>
<td>66320</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Not applicable</td>
<td>1.79</td>
</tr>
</tbody>
</table>

In each of the following sections, calculation of the interface stiffness for some points along the stress path is explained. In Chapter 4, the reader may find thorough explanations of the criteria and derivations of the equations used for these calculations.

### E.1 Interface Stiffness at Point 1

The first step in the determination of the interface stiffness during shear is the determination of the type of loading applied. As explained in Chapter 4, the type of loading is defined by comparing the magnitude of the stress level acting on the interface to the positions of the yield surfaces $SL_+$ and $SL_-$. The stress level is calculated as follows:

$$SL = \frac{\tau}{\sigma_n \cdot \tan(\delta)}$$  \hfill (2-10 bis)

The value of the interface friction angle $\delta$ for the dense Light Castle Sand against concrete interface is given in Table E2. For point 1, the values of $\sigma_n$ and $\tau$ given in Table E1 yield a stress level of 0.36. During shearing along $A-1-B$, the stress level increases continuously. Because the interface had not been sheared previously, the interface is subject to virgin shear. The formulation for interfaces at yield presented in Chapter 4 is then applicable. In this formulation, it is necessary to determine the inclination $\tan(\theta)$ of the stress path, which is defined as:

$$\tan(\theta) = \frac{d\sigma_n}{d\tau^l}$$  \hfill (4-19 bis)
The inclination at point 1, which is determined graphically in Figure E1, is 1.43. It is also necessary to determine the direction of shearing. The shear direction parameter $q$ is equal to +1 if the stress level increases during shear, and it is equal to -1 if the stress level decreases. For point 1, $q$ takes on a value of +1.

The interface stiffness $K_{st}$ for yield inducing shear is calculated from the following equations:

\[
K'_{st} = K_{st} \cdot l \quad (4-17 \text{bis})
\]

\[
l = \frac{l}{1 + SL \cdot \left[ n_j (q \cdot R_{fj} \cdot SL - 1) - q \cdot R_{fj} \cdot SL \right] \tan(\delta) \tan(\theta)} \quad (4-21 \text{bis})
\]

\[
K_{st} = K_I \cdot \gamma_w \left( \frac{\sigma_{n}}{p_a} \right)^{n_j} \cdot (l - q \cdot R_{fj} \cdot SL)^2 \quad (4-22 \text{bis})
\]

where $\gamma_w$ is the unit weight of water.

The values of the hyperbolic parameters $K_I$, $n_j$, and $R_{fj}$ for the dense Light Castle Sand against concrete interface are given in Table E2. The value of $K_{st}$ corresponds to the interface stiffness for a vertical stress path. For an inclined stress path, the interface stiffness is calculated by multiplying $K_{st}$ by the correction factor $I$ for stress path inclination. For point 1, the calculations are as follows:

\[
K_{st} = 20700 \cdot 9.8\frac{kN}{m^3} \cdot \left( \frac{105}{101.3} \right)^{0.79} \cdot (l - (+1) \cdot 0.79 \cdot 0.36)^2 = 106867 \frac{kPa}{m}
\]

\[
l = \frac{l}{1 + 0.36 \cdot [0.79 (l - (+1) \cdot 0.79 \cdot 0.36 - 1) - (+1) \cdot 0.79 \cdot 0.36 ] \tan(33.7^\circ) \cdot 1.43} = 1.41
\]

\[
K'_{st} = 106867 \frac{kPa}{m} \cdot 1.41 = 150886 \frac{kPa}{m}
\]
E.2 Interface Stiffness at Point 2

The stress level at point 2, which is calculated as described previously, is 0.22. During the previous shearing history of the interface, a greater value of stress level was reached at point B. Consequently, point 2 is inside the unloading-reloading zone defined in Chapter 4. The interface stiffness for unloading-reloading can be calculated using one of the three versions of the model.

E.2.1 Version I

In Version I, the interface stiffness is determined according to the following equation:

\[ K'_{st} = K_{urj} \cdot \gamma_w \cdot \left( \frac{\sigma_n}{\rho_a} \right)^{n_j} \]  
\[(4-23 \text{ bis})\]

According to this expression, the interface response is assumed linear, normal stress-dependent, and independent of the inclination of the stress path. For point 2, the interface stiffness is calculated as follows:

\[ K'_{st} = 33160 \cdot 9.8 \cdot \frac{kN}{m^3} \cdot \left( \frac{170}{101.3} \right)^{0.79} = 489174 \cdot \frac{kPa}{m} \]

E.2.2 Version II

In Version II, the interface response is assumed hyperbolic, normal stress dependent, and independent of the inclination of the stress path. The following equation is used for the determination of the interface stiffness:

\[ K'_{st} = K_{urj} \cdot \gamma_w \cdot \left( \frac{\sigma_n}{\rho_a} \right)^{n_j} \cdot \left[ 1 - \frac{q}{R_{fj}} \cdot (S_L - S_{L_0}) \right]^2 \]  
\[(4-25 \text{ bis})\]

The values of the hyperbolic parameters \( K_{urj}, n_j, \alpha, \) and \( R_{fj} \) for the dense Light Castle Sand against concrete interface are presented in Tables E2 and E3. As defined in Chapter 4, the origin of unloading-reloading corresponds to the
last point where the direction of shear was reversed. The stress level at the origin $SL_o$ is calculated using the following equation:

$$SL_o = \frac{\tau_o}{\sigma_{no} \cdot tan(\delta)}$$  \hspace{1cm} (4-26 bis)

where $\tau_o$ and $\sigma_{no}$ are, respectively, the shear stress and normal stress at the origin. For point 2, the origin is at point $B$, as indicated in Table E1. The stress level at the origin corresponds to the stress level at point $B$, which is calculated as follows:

$$SL_o = \frac{47.5}{137.5 \cdot tan(337^0)} = 0.52$$

The shear direction parameter $q$ at point 2 is -1 because the stress level decreases continuously along segment $B-C$ of the stress path. The interface stiffness $K'_{st}$ at point 2 then becomes

$$K'_{st} = 66320 \cdot 9.8 \cdot \left( \frac{170}{101.3} \right)^{0.79} \left[ 1 - \left( -\frac{1}{1.79} \right)^{0.79} \cdot (0.22 - 0.52) \right]^{2} = 736429 \frac{kPa}{m}$$

### E.2.3 Version III

In Version III, it is necessary to distinguish between unloading-reloading and transition loading. As indicated in Chapter 4, transition loading occurs if $(SL_- < SL < SL_+)$ and $(\tau = \tau_- \text{ or } \tau = \tau_+)$. The transition surfaces $\tau_-$ and $\tau_+$ are defined respectively as the minimum and maximum values of shear stress reached during interface shear. For point 2, the upper transition surface $\tau_+$ is defined by the shear stress at point $B$. The lower transition surface $\tau_-$ is defined by the shear stress at point $A$. It can be seen that the value of shear stress at point 2 does not meet the criteria presented previously. Therefore, point 2 corresponds to unloading-reloading, and a formulation identical to that for Version II applies. In Version III, the interface stiffness at point 2 is then identical to that calculated previously using Version II:

$$K'_{st} = 66320 \cdot 9.8 \cdot \left( \frac{170}{101.3} \right)^{0.79} \left[ 1 - \left( -\frac{1}{1.79} \right)^{0.79} \cdot (0.22 - 0.52) \right]^{2} = 736429 \frac{kPa}{m}$$
E.3 Interface Stiffness at Point T

As indicated in Table E1, the stress level at point $T$ is lower than the past maximum stress level $SL^+$ reached during shear along $A-B-C-T$. Consequently, any of the three versions of the model for unloading-reloading can be used for the determination of the interface stiffness.

E.3.1 Version I

As indicated previously, the interface stiffness is determined according to the following equation:

$$K'_{st} = K_{urj} \cdot \gamma_w \cdot \left(\frac{\sigma_n}{p_{ja}}\right)^n_j$$  \hspace{1cm} (4-23 bis)

For point $T$, the interface stiffness is calculated as follows:

$$K'_{st} = 33160 \cdot 9.8 \cdot \frac{kN}{m^2} \cdot \left(\frac{232.5}{101.3}\right)^{0.79} \cdot \frac{kPa}{m} = 626445 \frac{kPa}{m}$$

E.3.2 Version II

During shear along segment $C-D$ of the stress path, the origin, or last point of change in shearing direction, is $C$. The value of the shear direction parameter $q$ is +1 because the stress level increases during shear along $C-D$. As indicated previously, the interface stiffness is calculated using

$$K'_{st} = K_{urj} \cdot \gamma_w \cdot \left(\frac{\sigma_n}{p_{ja}}\right)^n_j \cdot \left[1 - \frac{q}{\alpha} \cdot R_f \cdot (SL - SL_0)\right]^2$$  \hspace{1cm} (4-25 bis)

The interface stiffness $K'_{st}$ at point $T$ then becomes
During shear along segment $C-D$ of the stress path, the past maximum shear stress $\tau^+$, which is given by point $B$, is reached at point $T$. The past maximum stress level $SL^+$ determined by the stress level at point $B$, is reached at point $Y1$. Consequently, all the points belonging to segment $T-Y1$ of the stress path are inside the transition region. In Version III, the normalized interface stiffness $K_{sn}$ during transition loading is calculated using the following equation:

$$K_{sn} = K_{sn}^{ts} \cdot m_k \left( SL - SL^{ts} \right)$$  \hspace{1cm} \text{(4-27 bis)}$$

where

- $K_{sn}^{ts}$ = transition stiffness number
- $m_k$ = stiffness degradation parameter
- $SL^{ts}$ = transition stress level

The transition stiffness number is the value of normalized interface stiffness at the point where the stress path crosses a transition surface, $\tau^+$ or $\tau^-$. For any point on segment $T-Y1$ of the stress path, $K_{sn}^{ts}$ is the normalized stiffness at point $T$. Similarly, the transition stress level $SL^{ts}$ is the stress level at the point where the stress path crosses a transition surface, $\tau^+$ or $\tau^-$. For any point on segment $T-Y1$ of the stress path, $SL^{ts}$ is equal to the stress level at point $T$. It can be seen that by substituting these values of $K_{sn}^{ts}$ and $SL^{ts}$ in the equation, the following normalized interface stiffness is obtained for point $T$:

$$K_{sn} = K_{sn}^{ts}$$

The value of $K_{sn}^{ts}$ depends on the previous shearing history of the interface. For the case under analysis, it may be assumed that point $T$ corresponds to unloading-reloading (stress path $C-T$). The value of $K_{sn}^{ts}$ can then be calculated using the values of the state variables of point $T$ and the formulation for unloading-reloading.
A calculation of $K'_{st}$ at point $T$, identical to that presented for Version II in the previous section, yields the following:

$$K'_{st} = 66320 \cdot 9.8 \frac{kN}{m^2} \cdot \left( \frac{232.5}{101.3} \right)^{0.79} \cdot \left[ 1 - \frac{1 + 1}{1.79} \cdot 0.79 - 0.31 \right]^{2} = 933513 \frac{kPa}{m}$$

This is the value of interface stiffness at point T. It is also convenient to determine the value of normalized stiffness $K_{sn}$. The value of $K_{sn}$ is calculated from the following equation:

$$K_{sn} = \frac{K'_{st}}{\gamma_{w} \cdot \left( \frac{\sigma_{m}}{\sigma_{a}} \right)^{n_j}} = \frac{933513}{9.8 \cdot \left( \frac{232.5}{101.3} \right)^{0.79}} = 49414$$

This value will be used for the calculation of the interface stiffness at point 3.

### E.4 Interface Stiffness at Point 3

As indicated in Table E1, the stress level at point 3 is lower than the past maximum stress level $SL_{+}$ reached during shear along A-B-C-3. Consequently, any of the three versions of the model for unloading-reloading can be used for the determination of the interface stiffness.

#### E.4.1 Version I

For point 3, the interface stiffness is calculated as follows:

$$K'_{st} = 33160 \cdot 9.8 \frac{kN}{m^2} \cdot \left( \frac{247.5}{101.3} \right)^{0.79} = 658163 \frac{kPa}{m}$$

#### E.4.2 Version II

During shear along segment C-D of the stress path, the origin, or last point of change in shearing direction, is C. The value of the shear direction parameter $q$ is $+1$ because the stress level increases during shear along C-D. The interface stiffness $K'_{st}$ at point 3 then becomes
E.4.3 Version III

As indicated previously, all the points located on segment $T-Y$ of the stress path correspond to transition loading. In Version III, the normalized interface stiffness $K_{sn}$ during transition loading is calculated using the following equation:

$$K_{sn} = K_{sn}^{ts} \cdot 10^{m_k \cdot (SL- SL_{ts})}$$  \hspace{1cm} (4-27 bis)

where

- $K_{sn}^{ts}$ = transition stiffness number
- $m_k$ = stiffness degradation parameter
- $SL_{ts}$ = transition stress level

The transition stiffness number is the value of interface stiffness at the point where the stress path crosses the transition surface $\tau +$ or $\tau -$. For point 3, $K_{sn}^{ts}$ is the normalized interface stiffness at point $T$, which was calculated in the previous section as:

$$K_{sn}^{ts} = K_{sn} at \ point \ T = 49414$$

The transition stress level is the stress level of the point where the stress path crosses the transition surface $\tau +$ or $\tau -$. For point 3, $SL_{ts}$ is the stress level at point $T$:

$$SL_{ts} = 0.31$$

The stiffness degradation parameter is calculated from the following equation:

$$K'_{st} = 66320 \cdot 9.8 \frac{kN}{m^3} \left( \frac{247.5}{101.3} \right)^{0.79} \left[ 1 - \frac{(+1)}{1.79} \cdot 0.79 \cdot (0.45 - 0) \right]^2 = 845393 \frac{kPa}{m}$$
where

\[ K_{sn}^{ys} = \text{yield stiffness number} \]

\[ SL^{ys} = \text{stress level for the current position of the yield surface} \]

The yield stiffness number \( K_{sn}^{ys} \) is the value of normalized stiffness corresponding to the current position of the yield surface \( SL^{ys} \). It is assumed that the inclination of the stress path does not influence the value of \( K_{sn}^{ys} \) \((I = 1)\). The following expression is used to calculate the yield stiffness:

\[ K_{sn}^{ys} = K_I \cdot \left(1 - q \cdot R_f \cdot SL^{ys}\right)^2 \]  \hspace{1cm} (4-32 bis)

The current position of the yield surface \( SL^{ys} \) is defined mathematically as

\[ SL^{ys} = SL^+ \text{ for shear inside the transition zone of the first quadrant} \]

\[ SL^{ys} = SL^- \text{ for shear inside the transition zone of the fourth quadrant} \]

Point 3 is located inside the first quadrant; therefore:

\[ SL^{ys} = SL^+ = 0.52 \]

During shear along segment \( C-D \) of the stress path, the stress level increases continuously. Therefore, the value of the shear direction parameter \( q \) is equal to \(+1\). The yield stiffness number \( K_{sn}^{ys} \) can now be calculated:

\[ K_{sn}^{ys} = 20700 \cdot (1 - (+1) \cdot 0.79 \cdot 0.52)^2 = 7186 \]

The value of the stiffness degradation parameter, \( m \), then becomes:
Finally, the normalized stiffness at point 3 can be calculated:

\[ K_{sn} = 49414 \cdot 10^{-3.987(0.45-0.31)} = 13667 \]

Once the normalized stiffness is calculated as described, the interface stiffness can be calculated using the following expression:

\[ K'_{st} = K_{sn} \cdot \gamma \cdot \left( \frac{\sigma_n}{\rho_a} \right)^{n_j} \]

(4-28 bis)

For point 3, the following value of \( K'_{st} \) is obtained:

\[ K'_{st} = 13667 \cdot 0.27 \cdot \left( \frac{247.5}{101.3} \right)^{0.79} = 271264 \, kPa/m \]

**E.5 Interface Stiffness at Point 4**

The stress level at point 4 is 0.27. During the previous shearing history of the interface, a greater value of stress level was reached at point D. Consequently, point 4 is inside the unloading-reloading zone, and the interface stiffness is calculated following identical procedures to those described for point 2.

**E.5.1 Version I**

For point 4, the interface stiffness is calculated as follows:

\[ K'_{st} = 33160 \cdot 0.27 \cdot \left( \frac{277}{101.3} \right)^{0.79} = 719396 \, kPa/m \]
E.5.2 Version II

The origin of unloading-reloading for point 4 is point D. Therefore, the stress level at the origin $S_{L_0}$ is 0.7. The shear direction parameter $q$ at point 4 is -1 because the stress level decreases continuously along segment $D-E$ of the stress path. The interface stiffness $K'_{st}$ at point 4 then becomes:

$$K'_{st} = 66320 \cdot 9.8 \frac{kN}{m^3} \left( \frac{277}{101.3} \right)^{0.79} \left[ 1 - \left( -\frac{1}{179} \right)0.79 \cdot (0.27 - 0.70) \right]^2 = 944513 \frac{kPa}{m}$$

E.5.3 Version III

Point 4 is not located within any of the transition regions. Therefore, the interface stiffness at point 4, according to Version III, is identical to that calculated previously using Version II:

$$K'_{st} = 66320 \cdot 9.8 \frac{kN}{m^3} \left( \frac{277}{101.3} \right)^{0.79} \left[ 1 - \left( -\frac{1}{179} \right)0.79 \cdot (0.27 - 0.70) \right]^2 = 944513 \frac{kPa}{m}$$

E.6 Interface Stiffness at Point 5

Point 5 is located inside the fourth quadrant and has a stress level of -0.14. During shearing along $Y2-E$, the stress level decreases continuously. Because the interface had not been sheared previously into the fourth quadrant, the interface is subject to virgin shear, as indicated in Table E1. The formulation for interfaces at yield presented in Chapter 4 is then applicable.

The inclination of the stress path at point 5 is zero, i.e., the stress path is vertical. The shear direction parameter $q$ is equal to -1 because the stress level decreases continuously along $Y2-E$.

The interface stiffness at point 5 is calculated following an identical procedure as for point 4. The calculations are as follows:
It must be noted that after point $E$ of the stress path is reached and the shear direction is reversed, subsequent shear along segment $E-D$ induces unloading-reloading of the interface. Consequently, the interface stiffness at all points of this segment, including point 5, must be calculated using procedures identical to those described for points 2 and 4. If Versions II or III are used, the origin for unloading-reloading along $E-D$ corresponds to point $E$.

**E.7 Interface Stiffness at Point 6**

Point 6 is located inside the first quadrant and has a stress level of 0.81. During shearing along $D-F$, the stress level increases continuously and the interface undergoes yielding. The formulation for interfaces at yield presented in Chapter 4 is then applicable.

The inclination of the stress path at point 6 is -8.93 (decreasing normal stress and increasing shear stress). The shear direction parameter $q$ is equal to +1.

The interface stiffness at point 6 is calculated following an identical procedure as for points 1 and 5. The calculations are as follows:

$$K_{st} = 20700 \cdot 9.8 \frac{kN}{m^2} \cdot \left( \frac{277}{101.3} \right)^{0.79} \cdot (1 - (-1) \cdot 0.79 \cdot (-0.14))^2 = 355237 \frac{kPa}{m}$$

$$I = \frac{1}{1 + (-0.14) \cdot \left[0.79((-1) \cdot 0.79 \cdot (-0.14) - 1) - (-1) \cdot 0.79 \cdot (-0.14)\right] \cdot \tan(33.7^\circ)} = 1$$

$$K'_{st} = 355237 \frac{kPa}{m} \cdot I = 355237 \frac{kPa}{m}$$

.$$K_{st} = 20700 \cdot 9.8 \frac{kN}{m^2} \cdot \left( \frac{253}{101.3} \right)^{0.79} \cdot (1 - (+1) \cdot 0.79 \cdot 0.81)^2 = 54210 \frac{kPa}{m}$$
It can be seen that the inclination of the stress path may have a substantial effect on the value of the interface stiffness, as evidenced by the low value of the correction factor $I$ at point 6.

\[
I = \frac{l}{1 + 0.81 \cdot [0.79((+1) \cdot 0.79 \cdot 0.81 - 1) - (+1) \cdot 0.79 \cdot 0.81 \tan(33.7^\circ) \cdot (-8.93)]} = 0.183 \\
\]

\[
K_{st}' = \frac{54210 \text{kPa}}{m} \cdot 0.183 = 9920 \text{kPa/m} \\
\]

It can be seen that the inclination of the stress path may have a substantial effect on the value of the interface stiffness, as evidenced by the low value of the correction factor $I$ at point 6.
a. Stress path applied during test

Figure E1. Multi-directional stress path test T405_10 on dense Light Castle Sand-to-concrete interface (Continued)
b. Shear stress vs. interface displacement data

Figure E1. (Concluded)