FINITE ELEMENT ANALYSIS OF FAILURE MODES IN
DYNAMICALLY LOADED PRE-CRACKED STEEL PLATES

by

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(ABSTRACT)

Finite element simulations are carried out to study transient stresses and
strains in the pre-cracked (pre-notched) 4340 steel plates impacted by a projectile
in the direction of the notch ligament. The computations employed Johnson-
Cook model which takes into account strain hardening, strain-rate hardening and
thermal softening. An approximate solution of the governing equations is sought
by using an explicit finite element code DYNA2D. We analyzed the evolution of
the shear and hoop stresses considered to be responsible for two modes of
failure: opening crack inclined at 70°, and shear crack inclined at -50 to the notch
ligament. At small impact speeds and large notch tip radii failure in the 70°
direction is due to the high tensile hoop stress. At high impact speeds and small
notch tip radii a failure develops predominantly in the (-5°) -- (-15°) direction,
within a zone of the maximum shear stress and compressive hoop stress.
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TABLE OF CONTENTS

1. List of figures ................................................................. v
2. Introduction ................................................................. 1
3. Review of experimental results ........................................... 7
4. Numerical models ......................................................... 25
5. Numerical results and discussion ....................................... 28
6. Conclusions ................................................................. 73
7. References ................................................................. 73
8. Vita ........................................................................... 79
1. LIST OF FIGURES

Fig. 1. Failure ahead of a notch tip under symmetric stress fields.

Fig. 2. Impact loading for generation of mode II at the notch tip: (a) a double-notch plate in experiments of Kalthoff (1988); d is diameter of the projectile and l is the notch length; (b) a semi-infinite plate with an edge notch used by Lee and Freund (1990); (c) experiments of Ravi-Chandar (1995).

Fig. 3. Experimental set-up of loading device and a shadow optical arrangement of Kalthoff (1987).

Fig. 4. Caustic curves (a) and light intensity patterns (b). Kalthoff (1987).

Fig. 5. High speed photographs of a double-notch polymethyl-methacrylat plate. Picture interval 3 µs. Kalthoff (1987).

Fig. 6. High speed photographs of a single-notch polymethyl-methacrylat plate. Picture interval is 2 µs. Kalthoff (1987).

Fig. 7. Evolution of stress intensity factor $K_{II}$ after impulsive in-plane-shear loading. Kalthoff (1987).

Fig. 8. Dependence of damage path on the notch tip radius $\rho$ and impact velocity $V_0$ in X2 NiCoMo 18 9 5 steel. Kalthoff (1988).

Fig. 9. Two modes of failure: opening crack (a) and shear crack (b). Kalthoff (1987).

Fig. 10. Microphotographs of failure surfaces in X2 NiCoMo 18 9 5: opening crack (a), and shear crack (b). Kalthoff (1987).

Fig. 11. Microphotographs of shear damages in X2 NiCoMo 18 9 5 (a), and 42 CrMo 4 (b). Kalthoff and Winkler (1988).

Fig. 12. Failure behavior of pre-notched impulsively loaded X2 NiCoMo 18 9 5 plates. Kalthoff and Winkler (1988).

Fig. 13. Asymmetric impact and dimensions of pre-notched plates in experiments of Zhou, Rosakis and Ravichandran (1995).

Fig. 14. Shear and opening cracks in C-300 steel, $V_0 = 23.4$ m/s. Experiments of Zhou, Rosakis and Ravichandran (1995).

Fig. 15. Crack surfaces in impulsively loaded plates. Experiments of Zhou, Rosakis and Ravichandran (1995).
Fig. 16. Shear band length as a function of impact velocity. Experiments of Zhou, Rosakis and Ravichandran (1995).

Fig. 17. Shear band length histories for C-300 steel plates at different impact velocities. Experiments of Zhou, Rosakis and Ravichandran (1995).

Fig. 18. Propagation of shear crack in C-300 steel. Impact velocity 25 m/s. Experiments of Zhou, Rosakis and Ravichandran (1995).

Fig. 19. Propagation of shear crack in C-300 steel. Impact velocity 30 m/s. Experiments of Zhou, Rosakis and Ravichandran (1995).

Fig. 20. Failure mode transitions in a polycarbonate as a function of projectile speed. Experiments of Ravi-Chandar (1995).

Fig. 21. A sketch of the problem studied and the finite element mesh used.

Fig. 22. Lagrangian - Eulerian mesh at the notch tip. Notch tip radius = 0.5 mm.

Fig. 23. Distribution of maximum shear stress (GPa) at 4.0 µs after impact. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 24. Distribution of maximum shear stress (GPa) at 10.0 µs after impact. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 25. Distribution of maximum shear stress (GPa) in the notch tip vicinity at 12.0 µs after impact. Notch tip radius 0.15 mm, impact speed 25 m/s.

Fig. 26. Distribution of effective plastic strain in the notch tip vicinity at 12.0 µs after impact. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 27. Velocity (µµ / ms) distribution within the plate and the projectile at 14.0 µs. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 28. Distribution of maximum shear stress (GPa) in the plate and projectile. Time = 12.0 µs, notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 29. Time history of maximum shear stress (GPa) in four elements on the notch tip surface oriented at ±50° and ±70° to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 30. Contours of maximum shear stress in the notch tip vicinity. Time = 39.2 µs, notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 31. Contours of effective plastic strain in the notch tip vicinity. Time = 39.2 µs, notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 32. Time history of hoop stress (GPa) in four elements on the notch tip surface oriented at 50° (a), 70° (b), -50° (c), and -70° (d).
to the notch ligament. Notch tip radius 0.15 mm, impact speed 25 m/s.

Fig. 33. Distribution of hoop stress in the notch tip vicinity. Time = 39.2 μs, notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 34. Deformations of the projectile (1), and the plate below the notch tip (2) and above the notch tip (3). Notch tip radius = 0.15 mm, impact speed ≈ 25 m/s.

Fig. 35. Deformations of the notch tip at different times. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 36. Time history of effective plastic strain in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius 0.15 mm, impact speed 25 m/s.

Fig. 37. Time histories of the length of the plastic zone (dashed line) and the length of the zone where the maximum shear stress exceeds 0.4 GPa (solid line), in the direction of -5° to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 38. Contours of maximum shear stress (GPa) in the notch tip vicinity. Time = 80 μs, notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 39. Time history of the horizontal components of the velocity at three points on the impacted plate surface. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

Fig. 40. Distribution maximum shear stress (GPa) at 48.4 ms. Curve A is a possible path for a shear crack. Notch tip radius = 0.15 mm, impact speed = 30 m/s.

Fig. 41. Distribution of effective plastic strain at 48.4 μs. Curve A is a possible path for a shear crack. Notch tip radius = 0.15 mm, impact speed = 30 m/s.

Fig. 42. Time history of effective plastic strain in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 35 m/s.

Fig. 43. Distribution of maximum shear stress (GPa) at 50.4 ms. Notch tip radius = 0.15 mm, impact speed = 35 m/s.

Fig. 44. Distribution of effective plastic strain at the notch tip. Time = 50.4 μs, notch tip radius = 0.15 mm, impact speed = 35 m/s.
Fig. 45. Contours of hoop stress (GPa) in the notch tip vicinity. Time = 50.4 μs, notch tip radius ≈ 0.15 mm, impact speed = 35 m/s.

Fig. 46. Time history of hoop stress (GPa) in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 40 m/s.

Fig. 47. Time history of maximum shear stress (GPa) in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius 0.15 mm, impact speed 40 m/s.

Fig. 48. Time history of maximum shear stress (GPa) in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius 0.80 mm, impact speed 35 m/s.

Fig. 49. Time history of hoop stress (GPa) in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius 0.80 mm, impact speed 35 m/s.

Fig. 50. Time history of effective plastic strain in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius 0.80 mm, impact speed 35 m/s.

Fig. 51. Distribution of maximum shear stress (GPa) at 42.0 μs. Notch tip radius = 0.80 mm. Impact speed = 35 m/s.

Fig. 52. Distribution of hoop stress (GPa) at 42.0 μs. Notch tip radius = 0.80 mm. Impact speed = 35 m/s.

Fig. 53. Distribution of effective plastic strain at 42.0 μs. Notch tip radius = 0.80 mm. Impact speed = 35 m/s.

Fig. 54. Distribution of maximum shear stress (GPa) at 103.6 ms. Notch tip radius = 0.80 mm. Impact speed = 35 m/s.

Fig. 55. Deformation and translation of the notch tip. Notch tip radius = 0.80 mm. Impact speed = 35 m/s.

Fig. 56. Time history of effective plastic strain in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius = 0.60 mm, impact speed = 35 m/s.

Fig. 57. Time history of effective plastic strain in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius = 0.40 mm, impact speed = 35 m/s.
Fig. 58. Distribution of maximum shear stress (GPa) at 47.6 μs. Notch tip radius = 0.40 mm. Impact speed = 35 m/s.

Fig. 59. Vectors of displacement (mm) at 61.6 μs. Notch tip radius = 0.40 mm, impact speed = 35 m/s.

Fig. 60. Influence of the notch tip radius on maximum tensile hoop stress in the 70° direction during 10 - 60 μs, and in the -70° direction during 70 - 100 μs after impact. Impact speed = 35 m/s.

Fig. 61. Influence of impact speed on effective plastic strain in the -50° direction (solid lines), and in the 70° direction (dashed lines). Notch tip radius = 0.15 mm.

Fig. 62. Influence of the notch tip radius on the growth of effective plastic strain in the -50° direction (solid lines) and in the 70° direction (dashed lines). Impact speed = 35 m/s.

Fig. 63. Three directions of crack propagation: (A) due to tension and shear, (B) due to shear and compression, (C) due to shear and compression/tension. 1 - zone of maximum tensile hoop stress, 2 - zone of maximum compressive hoop stress, 3 - line of zero hoop stress, 4 - zone of maximum shear stress.
2. INTRODUCTION

Dynamic failure analyses over the past two decades have concentrated on initiation and propagation of a crack under mode I loading conditions. According to Anderson (1991) most materials are more susceptible to fracture by normal tensile stresses than by shear stresses; mode II and mode III loadings usually do not lead to fracture. Consequently in the vast majority of practical problems studied, only mode I failure is analyzed.

The focus on mode I loading is usually justified by considering that under mixed-mode or even under pure mode II loading, the crack will quickly find the path along which the local conditions are of mode I type. While the mixed-mode loading may play a role in determining the crack path and crack speed, the fracture surface morphology is similar to that observed under pure mode I crack propagation. This is due to the principle of minimal energy required for crack growth.

Broberg (1983) showed that if the in-plane compression is sufficiently large, the only available path for crack extension might be along the direction of maximum shear and that while experimentally difficult, mode II growth should be obtained for virtually all materials. The energy required for mode II crack propagation is expected to be larger than that for mode I. We assume that under the shear dominated loading conditions materials might resist external loads more efficiently thus absorbing higher energies for shear banding.

We describe below two types of damage caused by impact loads: brittle cracking and ductile shear banding.

Brittle cracking. Subjected to pure tensile quasi-static mode I loading, cracks usually propagate along their ligament through the specimen. Subjected to quasi-static in-plane-shear mode II loading, cracks deviate from their original direction and propagate at an angle of about 63 - 70° with respect to the ligament. These types of behavior are predicted by various crack propagation criteria, e.g., the maximum tensile stress criterion of Erdogan and Sih (1963), and are verified by experimental results of Broek (1986). Lee and Freund (1990), using an elasto-dynamic analysis, showed that this angle coincides with the direction of the
maximum circumferential tensile stress at the notch tip; the crack propagates along the direction of local symmetry at a velocity of about 600 m/s. Taudou et al. (1992) showed that this velocity is in the same range as that observed under pure mode I loading conditions.

We assume that the quasi-static mode II loading conditions lead to the initiation of a shear band at the notch tip, with a short shear crack inside. The shear crack however, quickly changes direction to that of maximum circumferential tensile stress, which is the direction of minimal energy required for crack propagation as an opening crack. Shear cracks may not transform in opening cracks if external impulsive load is large enough to provide energy for a higher failure mode, i.e., more intense shear banding. Geometry of specimen and loading conditions may minimize the influence of tensile stress on crack path.

**Shear banding.** Under high speed loading, the failure of specimens made of cold-rolled steel, vacuum arc remelted steel, titanium alloys, is usually preceded by the formation of a shear band, which is a narrow region, a few micrometers (μm) wide, of intense shear deformation. Depending on material and loading rate the shear band can be elastic, elastic-plastic, or with a shear crack inside. The localization of deformation into narrow bands under high rates of straining have been observed in industrial and military applications: during shock loading, ballistic penetration, metal forming, machining, grinding, high speed fabrication, explosive fragmentation, and in fast cutting tools.

Even though Tresca (1878) reported the development of shear bands during the hot forging of platinum more than a century ago, the active research in this field started approximately 50 years ago when Zener and Hollomon (1944) observed 32 μm wide shear bands during the punching of a hole in a low carbon steel plate. They postulated that the heat generated due to plastic work made the material softer, and the material became unstable when the thermal softening equaled or exceeded the hardening of the material due to strain and strain-rate effects. Moss (1981) conducted tests similar to those of Zener and Hollomon (1944) and reported that strain rates of the order of $10^5$ sec$^{-1}$ occur within the shear band. Using high speed photography, Marchand and Duffy
(1988) observed the initiation and propagation of shear bands in torsionally loaded thin-walled specimens. They estimated the speed of the shear band tip to be approximately 500 m/s. According to Batra and Peng (1995) the study of shear bands is important because once these bands have developed, subsequent deformations of a solid body are concentrated in these narrow regions and the strength properties of the rest of the body are not fully utilized. Also, shear bands precede shear cracks in dynamically loaded ductile materials.


Experimental studies have given a better understanding of the role of many factors in the formation of shear bands. The shear banding is a highly nonlinear problem, due to both geometric and material nonlinearities, and therefore difficult to solve analytically. Modeling of the crack propagation is even more complex problem since it requires the consideration of severe changes in geometry, material properties, energy transformation, redistribution of the mechanical and thermal loads resulting from damage, and other often unanticipated micro-features of dynamic failure processes.


We consider shear band as a localized instability leading to failure of a structural element. With a shear band there is a direction of intense shear
deformation with a sliding-type shear instability of the material, due to a "self-feeding" mechanism: deformation - heating - deformation. The duration of this mechanism and consequently the length of shear damage depends on the energy input into the deformed body.

**Crack problem and symmetry.** Prediction of crack or shear band propagation requires consideration of both geometrical and physical nonlinearities, including inhomogeneities. Crack problems usually involve a singularity. Models of purely elastic materials give infinitely large stresses as well as internal energies at the notch tip. These can be improved upon by using elastic-plastic material models in which the effective stress can not be higher than the yield stress of the material. The internal energy concentrated at the notch tip is expected to be spent on the growth of a crack surface. The length and direction of the propagating damage depends on the magnitude of the energy, as well as on the stress field near the notch tip.

A number of relatively simple numerical techniques have been developed to remove excessively distorted elements and their internal energies. This provides erosion of material and a change of geometry ahead of the notch tip. We mention here the following approaches: J and T methods, the crack-tip opening displacement, and the crack-tip opening angle criteria. Bakuckas and Newman (1993) used the crack-tip opening angle approach with critical angle of 6.1° to predict stable tearing of a quasistatically loaded aluminum plate.

The curved path followed by a crack in a **problem with a symmetric stress field at the crack tip** may be explained as follows. Let a mode I loading be applied to a notch as shown in Figure 1. Let a crack initiate at the notch tip and propagate (step 1) at an arbitrary angle to the X-axis, which is the axis of local symmetry. Because of symmetry the stress field rotates the crack path toward the X-axis, developing step 2. Due to inertia crack may go below the X-axis. Consequently, the stress field rotates it back making step 3. The rotary inertia of crack propagation and internal energy dissipation make crack path approach the X-axis. This damped periodic change in the crack path can lead to brittle-to-ductile microtransitions. Deviations from this hypothetical path may be due to defects in the material.
Fig. 1. Failure ahead of a notch tip under symmetric stress fields

Fig. 2. Impact loading for generation of mode II at the notch tip:
(a) a double-notch plate in experiments of Kalthoff (1988); d is diameter of the projectile and l is the notch length;
(b) a semi-infinite plate with an edge notch used by Lee and Freund (1990);
(c) experiments of Ravi-Chandar (1995)
The above symmetry-based approach should be also applicable to mode II loading conditions as shown in Figure 1. The stress field is symmetric with respect to the notch ligament. In comparison with mode I crack path, the shear crack path is expected to be of bigger period, and shorter in length, due to the consumption of higher energies.

The likelihood of the crack propagation in one or another direction depends upon the intensity and duration of the stress fields in these directions, as well as on the energy required for crack growth in a specific structural material. For biaxial stretching of a metal plate, a time-dependent fracture criterion incorporating transient stresses, work required to develop a unit area of a crack in a given material, reflection of energy from crack surface, speed of sound in the material, and the initial area of a structural element covered with defects has been proposed by Galiev and Nechitailo (1984).

The coexistence of two or more axes of local symmetry of transient stress fields means several possible directions of crack propagation, and may result in chaotic micro- and macro-changes in the crack path.

Transitions in the failure mechanisms. Kalthoff (1987, 1988, 1990), and Kalthoff and Winkler (1988) experimentally studied failure of impulsively loaded pre-notched specimens. They apparently were the first to describe two possible modes of failure in the same specimen: the classical 70° inclined brittle crack and a shear crack inclined at (-5°) - (-10°) to the notch ligament.

Lee and Freund (1990) analyzed the experimental results of Kalthoff (1987, 1988, 1990), and Kalthoff and Winkler (1988). They considered an elastic half-space containing an edge notch, with a prescribed velocity field on a part of the boundary as depicted in Figure 2. They showed that along with the development of a mode II stress intensity factor $K_{II}$, a small negative mode I stress intensity factor $K_I$ also developed. According to Ravi-Chandar (1995) the presence of a negative $K_I$ indicates an interpenetration of the upper and lower surfaces of the notch that must be fixed by modifying the notch surface boundary conditions to include contact forces. Lee and Freund (1990) also showed that the maximum hoop stress occurred at an angle of about 63° to the notch ligament during the time interval $0 < t < 3l/c$, where $l$ is the length of the notch and $c$ is the
longitudinal wave speed. According to Ravi-Chandar (1995) the mathematical analysis of Lee and Freund (1990) is applicable until time $t = 1.414 \sqrt{c}$.

Needleman and Tvergaard (1994), using a finite element analysis, attributed the transition in failure behavior described by Kalthoff (1987, 1988, 1990) and Kalthoff and Winkler (1988) to the fact that high rate loading increases plastic strains and thermal softening of the material, thus reduces the influence of the maximum tensile stress, and therefore suppresses the brittle mode of failure.

Mason et al. (1995) conducted experiments similar to those done by Kalthoff (1987, 1988, 1990) and Kalthoff and Winkler (1988) using asymmetrically loaded single-notch steel and titanium specimens. They observed shear banding failure in Ti-6Al-4V titanium alloy, as well as a transition of a shear crack into an opening brittle crack propagating at approximately $30^\circ$ with respect to the notch ligament, in C-300 steel.

Ravi-Chandar (1995) experimentally studied the failure mode transitions in a polycarbonate polymer, from ductile to brittle to ductile again. These transitions were examined by considering the rate dependent inelastic behavior, hydrostatic compression and thermal softening of the material.

3. REVIEW OF EXPERIMENTAL RESULTS

We now review experimental results obtained with transparent materials and high strength steels by Kalthoff (1987, 1988, 1990) and Kalthoff and Winkler (1988), and by Zhou et al. (1995 b) for steel and titanium alloy. After that we will discuss experiments completed by Ravi-Chandar (1995) with a polycarbonate specimen, exhibiting both ductile and brittle failure behaviors.

In the experiments of Kalthoff (1987, 1988, 1990) and Kalthoff and Winkler (1988) a specimen with two parallel edge notches is impacted by a projectile of diameter $d$ equal to the distance $h$ between the two notches, as is schematically shown in Figure 3. Typical specimen dimensions are 100 mm x 200 mm. In another set of experiments they used single-notch specimens. The notch length was approximately 50 mm. A cylindrical steel projectile of 50 mm in diameter
Fig. 3. Experimental set-up of loading device and a shadow optical arrangement. Kalthoff (1987).

Fig. 4. Caustic curves (a) and light intensity patterns (b). Kalthoff (1987)
impacted the specimens. The projectile is accelerated by an air gun to speeds $V_0$ ranging from 10 m/s to approximately 100 m/s. The specimen was not held in any special fixture, its bottom side rests on a table surface, and can slide after the impact.

The impact of the projectile initiates a compressive wave in the middle part of the specimen that generates shear loading conditions at the notch tip. Mode II loading conditions were expected to apply for the first stage of deformation - from the moment when the compressive wave passes by the notch tip toward the opposite side of the specimen, then reflects as a tension wave, and arrives at the notch tip.

**PMMA and Araldite B transparent specimens.** To visualize stress intensification at the notch tip, the specimens made of transparent model materials PMMA (Polymethyl-methacrylat) and Araldite B were utilized. The stress field had been monitored by means of the shadow optical method of caustics, Kalthoff (1987). Due to local deformations of the specimen at the notch tip, parallel light rays illuminating the specimen experience deviations. These deviations result in nonuniform light pictures, called shadow distributions and caustic curves, and are shown in Figures 4a and 4b for conditions of pure tensile mode I and in-plane-shear mode II loading.

Figure 5 shows a series of high speed photographs for a symmetric specimen with a double-edge-notch configuration. The distance between the two notches was 3 cm. Figure 6 shows a high speed series of shadow optical photographs obtained for a specimen with a single notch. The comparison of these high-speed shadow patterns with the pure mode I and mode II optical images presented in Figures 4a and 4b reveal the following sequence of events: almost undisturbed mode I, then in about 10 μs transition to mode II, which remains almost undisturbed for about 24 μs, then it is disturbed by the waves reflected from free boundaries of the specimen. We can also see horizontal dark bands in Figure 5 which might be shear deformations caused by the impact.

**X2 NiCoMo 18 9 5 and 42 CrMo 4 steel specimens.** Kalthoff (1987, 1988, 1990) and Kalthoff and Winkler (1988) also performed experiments with high
Fig. 5. High speed photographs of double-notch polymethylmethacrylat plate. Picture interval 3 μs. Kalthoff (1987)

Fig. 6. High speed photographs of single-notch polymethylmethacrylat plate. Picture interval 2 μs. Kalthoff (1987)
strength maraging steel X2 NiCoMo 18 9 5, and Chromium - Molybdenum steel 42 CrMo 4 specimens. They used pre-notched specimens with tip radius $\rho = 0.25 - 0.8 \text{ mm}$ and pre-cracked specimens with $\rho \rightarrow 0$ under impact speeds ranging from 12 m/s to 72 m/s.

The stress concentration at the notch tip increases with the decrease in the tip radius and an increase in the impact speed. Quantitative data on the stress intensification factor $K_{II}$ are shown in Figure 7. These stress intensification rates are high, they are of the same order or even higher than those obtained with Hopkinson bars, and one to two orders higher than those obtained in drop-weight experiments.

**Two failure mechanisms in a high strength steel.** Figures 8 and 9 schematically summarize failure in X2 NiCoMo 18 9 5. No failure was observed if the impact speed is below the first critical speed $V_1$. When impact speed is between $V_1$ and the second critical speed $V_2$, a brittle opening crack propagates at approximately $70^\circ$ to the notch ligament throughout the specimen. When the impact speed is above $V_2$, a shear crack propagates at about $0^\circ - (-15^\circ)$ to the notch ligament. The critical speed $V_2$ becomes larger with increasing notch tip radius: for a notch tip radius of 0.8 mm, the critical speed $V_2$ was about 60 m/s; for $\rho = 0.3 \text{ mm}$, it was about 25 m/s; for a pre-crack, $\rho \rightarrow 0$, it was less than 20 m/s.

**Brittle fracture.** In low impact speed experiments with impact speed of about 10 m/s, the shadow optical analysis of the stress state at the notch tip of X2 NiCoMo 18 9 5 specimens initially exhibited a pure tensile mode I stress state. Then it transformed into almost undisturbed mode II stress state. Shadow optical images revealed notch-tip instability followed by the transition to tensile mode I crack opening. A crack propagates at an angle of about $70^\circ$ with respect to the ligament straight through the specimen up to its surface, separating the specimen into parts. Crack surface roughness and shear lips at the edges of specimens depicted in Figure 10a are typical for this steel.

**Shear banding.** Experiments performed at higher loading rates, with impact speeds exceeding 20 m/s, revealed a completely different mechanism of failure. Damage develops in a direction slightly inclined clock-wise with respect to the
Fig. 7. Evolution of stress intensity factor $K_{II}$ after impulsive in-plane-shear loading. Kalthoff (1987)

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Fig. 8. Dependence of damage path on the notch tip radius $\rho$ and impact velocity $V_o$ in X2 NiCoMo 18 9 5 steel. Kalthoff (1988)
Fig. 9. Two modes of cracking: opening crack (a) and shear crack (b). Kalthoff (1987)

Fig. 10. Microphotographs of failure surfaces in X2 NiCoMo 18 9 5: opening crack (a), and shear crack (b). Kalthoff (1987)

Fig. 11. Microphotographs of shear damages in X2 NiCoMo 18 9 5 (a), and 42 CrMo 4 (b). Kalthoff and Winkler (1988)
Fig. 12. Failure behavior of pre-notched impulsively loaded X2 NiCoMo 18 9 5 plates. Kalthoff and Winkler (1988)
initial position of the notch. This damage extends over a limited path and is arrested in almost all cases as shown in Figure 9. With a decrease of notch tip radius and an increase in impact speed (above 53 m/s) the length of the damaged zone increases, eventually reaching the free surface of the specimen and thus dividing it into parts.

Figure 10 b shows the shear crack surface that is different from the tensile tearing surface: it has smeared, shiny and mirror-like appearance. There are no edge shear lips; the damage extends straight through the specimen thickness.

Micrographs of the shear damage for high strength maraging steel X2 NiCoMo 18 9 5 and the Chromium - Molybdenum steel 42 CrMo 4 are presented in Figure 13. Specimens made of X2 NiCoMo 18 9 5 steel revealed a continuously increasing concentration of shear deformation towards its central line. The Chromium - Molybdenum steel revealed a white etching band indicating a phase transformation at the central part of the band. This suggests a local increase in the hardness of the base material. The shear band seems to have opened during the final stage of deformation by a tensile mode as shown in Figure 11 b.

According to Kalthoff and Winkler (1988) the maximum plastic strain at the notch tip is proportional to $V_0 / \sqrt{\rho}$, where $V_0$ is the impact speed, and $\rho$ is the notch (or pre-crack) tip radius; the term $V_0 / \sqrt{\rho}$ was offered as a strain-rate factor. Figure 12 represents the damage length versus $V_0 / \sqrt{\rho}$. In zone 1 strain-rates are small, the notch tip is stable and failure does not occur. Zone 2 corresponds to average values of strain rates, when failure is due to tensile fracture; the cracks observed in zone 2 are inclined at approximately 70° to the notch ligament causing complete failure of the specimen. Zone 3 represents shear cracking at higher strain rates; this type of failure is limited in length and extends in a direction that is slightly inclined to the notch axis. Despite the very different test parameters utilized in experiments (notch tip radii, impact velocities, specimen thickness) the observed damage in zone 3 follows almost the same path.

The transition from complete failure of the specimen by tensile fracture processes to damage of only a limited length by shear cracking indicates capability of metals to resist impact loads by absorbing considerably higher energies in shear stress dominated deformations.
Fig. 13. Asymmetric impact and dimensions of pre-notched plates in experiments of Zhou, Rosakis and Ravichandran (1995)
Fig. 14. Shear and opening cracks in C-300 steel, $V_o = 23.4$ m/s. Experiments of Zhou, Rosakis and Ravichandran (1995)
Fig. 15. Crack surfaces in impulsively loaded plates.
Experiments of Zhou, Rosakis and Ravichandran (1995)
Failure in C-300 steel and Ti-6Al-4V titanium specimens. Zhou et al. (1995 b) described experiments involving asymmetric impact and shown schematically in Figure 13. A shear crack propagates from the notch tip when the impact speed is greater than 20 m/s for the C-300 steel. When \( V_o \) is less than 29 m/s, the shear crack eventually arrests within the specimen, however, an opening crack extends from the tip of the arrested shear crack in a direction approximately 30° to the direction of impact. When \( V_o \) is greater than 29 m/s, the shear crack propagates throughout the whole ligament. The shear crack path initially follows closely the direction of the notch, and after propagating for approximately 30 mm, curves towards the impacted side of the specimen.

Figure 14 shows photographs of C-300 steel specimens with both failure modes. The length of the shear crack is about 16 mm, its surface is shiny and smooth. Shear deformation is uniform over the whole thickness of the specimen. The opening crack has gray and rough surface, typical for tensile fracture. The opening crack surface shows also shear lips, indicating effects of three-dimensional deformations. Microscopic details of the two types of crack surfaces are presented in Figure 15. A scanning electron micrograph of the opening crack surface revealed tensile mode of failure: coalescence of voids in a primarily tensile field of high triaxial stresses. The shearing direction is vertical. The densely populated elongated voids indicate ductile shear deformation inside the shear band. The average void size is approximately 10 \( \mu \)m. The morphology of the surface is similar to that of the shear flow of a very soft material smeared by the relative motion of the two sides of the shear band. These features signify material softening by very high temperatures inside the shear band. For C-300 steel, the intense internal heating was recorded in a narrow strip of 200 - 300 \( \mu \)m. Shear bands are more diffused in the titanium alloy. Similar voids and extensive smearing have also been observed by Giovanola (1988) in torsional experiments. The micrograph in Figure 15 c shows the morphology of the crack surface for a Ti-6Al-4V specimen, the average size of the voids is 25 - 30 \( \mu \)m. Unlike in Figure 17 b there is no evidence of extensive shear smearing. This can be due to lower temperatures inside the band formed in the titanium alloy.

Figure 16 shows the shear band length measured after the experiments. For the steel the band length is in the range of zero to 50 mm (which is the size of
Fig. 16. Shear band length as a function of impact velocity. Experiments of Zhou, Rosakis and Ravichandran (1995)

Fig. 17. Shear band length histories for C-300 steel plates at different impact velocities. Experiments of Zhou, Rosakis and Ravichandran (1995)
Fig. 18. Propagation of shear crack in C-300 steel. Impact velocity 25 m/s. Experiments of Zhou, Rosakis and Ravichandran (1995)

Fig. 19. Propagation of shear crack in C-300 steel. Impact velocity 30 m/s. Experiments of Zhou, Rosakis and Ravichandran (1995)
the ligament, or half of the specimen width) when \( V_o \) is in the range of 19 to 29 m/s. In the titanium alloy, the band extension is less than 8 mm for impact speeds up to 64.5 m/s. Higher speeds of shear band propagation in the C-300 steel are associated with higher rates of deformation inside the shear bands and, therefore, with higher temperatures. Figure 17 summarizes the time histories of shear band length for the three different impact speeds. Mason et al. (1994) showed that the shear band in C-300 steel initiates from the notch tip when the stress intensity factor \( K_{II} = 140 \text{ MPa m}^{1/2} \) and propagates with an average speed of 320 m/s. The average shear stress within the shear band decreases from 1.6 GPa at its initiation to 1.3 GPa during its growth. A shear band initiates at 20 - 28 \( \mu \text{s} \) after impact and is arrested at approximately 50 - 65 \( \mu \text{s} \).

Figure 18 shows a selected sequence of photo-images of the deformed material in the vicinity of the notch tip and shear cracking for an impact speed of 25 m/s. The diameter of the circular field of view is 50.4 mm. The grid of vertical lines with a pitch of 0.74 mm is used to measure band extension. The dark strip surrounding the shear band is a result of a large surface rotation or out-of-plane displacement gradient. The initiation of a shear crack occurs at approximately 23 \( \mu \text{s} \) after impact. The shear crack arrests at approximately 60 \( \mu \text{s} \) after impact. After this it is stationary relative to the specimen, but at approximately 140 \( \mu \text{s} \) a crack propagates in the 30° counterclockwise direction. This upturn corresponds to an opening crack at the tip of the arrested shear crack. The late photo-images reveal a small reverse shear deformation as a result of the stress wave reverberations in the specimen.

Figure 19 presents a sequence of images of a propagating shear crack when impact speed is about 29.7 m/s. Due to the higher impact speed, the shear crack initiates slightly sooner, at 22 \( \mu \text{s} \) after impact. Also, it propagates faster. By approximately 65 \( \mu \text{s} \) the shear crack separates the specimen into two parts.

For the Ti-6Al-4V alloy, the only mode of failure was shear cracking; the peak temperature was about 450° C. For the C-300 steel the highest temperature measured was about 1400° C, which is approximately 90% of its melting point. The highest shear crack speed, about 1200 m/s or 40% of the shear wave speed was recorded for the C-300 steel.
Fig. 20. Failure mode transitions in polycarbonate as a function of projectile speed. Experiments of Ravi-Chandar (1995)
Failure transitions in a polycarbonate. Ravi-Chandar (1995) experimentally studied failure mode transitions in a polycarbonate, a thermoplastic polymer with a molecular structure containing a bisphenol A and a carbonate. The bisphenol gives a high glass transition temperature of about 1500° C while the carbonate provides a high rotational mobility resulting in a capacity to yield in shear. Thus, the polymer exhibits a ductile fracture at low rates of loading and brittle fracture at high rates of loading. The 12.7 mm deep and 0.3 mm wide notch with a sharp tip was machined in a plate as shown in Figure 2 c. The plate was asymmetrically impacted by a polycarbonate projectile of 50 mm diameter and 100 mm length, at a speed between 25 m/s and 55 m/s. A high speed camera was used to record deformations in the specimen after impact.

The applied load is a compressive wave moving parallel to the notch. Furthermore, the Poisson effect induces an expansion normal to the notch. Thus the stress wave approaching the notch tip becomes a compressive biaxial wave. Ravi-Chandar (1995) studied the influence of the distance ε (see Figure 4 c) between the notch surface and the impacted boundary on the failure modes. He described a predominantly mode II square root singular field at approximately 40 μs after impact. At 80-110 μs after impact mode I became predominant. At about 200 μs the stress wave reflected from the far boundary of the specimen arrives at the crack tip and alters the stress field which does not show any resemblance to the square root singular field but reveals a very large shear stress gradient at the notch tip.

Figure 20 summarizes experimental results of Ravi-Chandar (1995) involving asymmetric impact loading. At impact speeds below 28 m/s the stresses near the notch tip were not large. As the impact speed is increased to the range of 28 - 50 m/s, a brittle crack propagates along a straight line inclined at about 66° to the notch ligament with a speed of about 600 m/s, Figure 20 a. This is approximately the same speed as that observed by Taudou et al. (1992) for pure mode I loading. The fracture surface is mirror-like (containing, however, periodic banded structure, 30 μm wide), and under high magnification is flat all across; shear lips are absent showing pure brittle fracture predominantly due to normal tension. Unfortunately, Ravi-Chandar (1995) did not explain another damage clearly visible in Figure 20 a, as a band inclined at approximately -75° to the
notch ligament. This type of damage will be considered in part 4 of the present work.

Another failure mechanism was observed at impact speeds above 50 m/s. As shown in Figure 20b the crack extends straight along the original notch line, and then arrests. Fracture surface morphology indicated that highly localized shear deformations occurred ahead of the notch tip and that a shear crack propagated through this shear band. The fact that crack arrested at higher impact speeds indicates that this failure mode consumes more energy than that prevalent for the 70° inclined brittle opening crack. According to Ravi-Chandar (1995), since the crack growth was along the original notch line and not along the direction of the maximum circumferential stress, the crack growth must be governed by the maximum shear stress. Also, the biaxial compression prevents the propagation of the crack in the 70° direction. Ravi-Chandar (1995) asked the question: whether or not the shear strain localized along the line ahead of the notch tip provides this type of fracture?

Ravi-Chandar (1995) compared results obtained using polycarbonate with those for high strength steel obtained by Kalthoff (1987, 1988, 1990) and Kalthoff and Winkler (1988), and concluded that while the micromechanisms of fracture are different, both materials exhibited similar macroscopical failure behavior.

4. NUMERICAL MODELS

We simulate the experimental set-up of Kalthoff (1987) and seek an approximate solution of equations governing thermomechanical deformations of the specimen by using DYNA2D, Whirley et al. (1992) which is an explicit finite element code for analyzing the transient response of two-dimensional solids. A Lagrangian - Eulerian mesh consisting of 4-node quadrilateral elements is employed. The code employs one-point integration rule to evaluate various integrals and an hour-glass control to suppress the spurious modes. The stability of the explicit central difference method is governed by the Courant condition.
according to which, the time step should be approximately equal to the time required for an elastic wave to propagate across the smallest dimension of the smallest element in the mesh. Equivalently, this time step may be related to the time-period of the highest mode of free vibration of the discretized structure.

Analyses of the stability of the finite difference method in problems of plate dynamics, expressions for the Courant time step depending on the plate model as well as some cases of numerical instability ("saw-type" instability, etc.) are presented in Nechitailo (1987a). Scale factors for computed time steps in the dynamics of plane structural elements, stability and convergence to static elastic-plastic solution using Courant time step and negative viscosity are described in Nechitailo (1987b) and Galiev et al. (1989). For a structural element of thickness \( h \), they suggested that the time step equal smaller of the following two values,

\[
\Delta t = a \, s_{\text{min}} \sqrt{\frac{\rho}{E}} \quad \text{and} \quad \Delta t = b \, \left( \frac{s_{\text{min}}}{h} \right)^2 \sqrt{\frac{\rho}{E}} \quad (1)
\]

where \( a \) and \( b \) are scale factors, \( s_{\text{min}} \) is the smallest distance across any element in the mesh, \( \rho \) is the mass density, and \( E \) is Young's modulus.

We use Cartesian coordinates to study dynamic plane strain thermomechanical deformations of an impulsively loaded pre-notched steel plate. Numerical simulations employed the Johnson-Cook (1983) model for which the yield stress is given by

\[
\sigma_y = \left[ A + B \, (\bar{\varepsilon}^p)^n \right] \left[ 1 + C \, \ln (\dot{\varepsilon}^*) \right] \left[ 1 - (T^*)^m \right], \quad (2)
\]

where \( A, B, C, n \) and \( m \) are material constants, \( \bar{\varepsilon}^p \) is the effective plastic strain, \( \dot{\varepsilon}^* \) is the nondimensional strain-rate, and \( T^* \) is the homologous temperature. The effective plastic strain is given by

\[
\bar{\varepsilon}^p = \int d \bar{\varepsilon}^p, \quad (3)
\]

where the incremental effective plastic strain is related to the incremental plastic
strains by

\[ d \varepsilon^p = \sqrt{\left( \frac{2}{3} \frac{d \varepsilon^p_{ij}}{d \varepsilon^p_{ij}} \right)}. \]  (4)

The nondimensional strain rate \( \dot{\varepsilon}^* \) is calculated from \( \dot{\varepsilon}^* = \dot{\varepsilon}^p / \dot{\varepsilon}_o \), where \( \dot{\varepsilon}^p \) is the effective plastic strain rate and \( \dot{\varepsilon}_o \) is the reference strain-rate, taken herein as 1/sec. The homologous temperature \( T^* \) is the ratio of the current temperature to the melting temperature when both are expressed in degrees Kelvin. The temperature change in DYNA2D is computed by assuming adiabatic conditions, i.e., there is no heat transfer between elements. This seems to be a good assumption since for the transient problem being studied, times of interest are so small that the actual heat transfer is negligible. It is assumed that all of the plastic work is converted into heat.

For the 4340 steel we assigned the following values: \( A = 792.19 \) MPa, \( B = 509.51 \) MPa, \( C = 0.014 \), \( n = 0.26 \), \( m = 1.03 \), bulk modulus \( K = 157 \) GPa, shear modulus \( G = 76 \) GPa, melting temperature \( \Theta_M = 1520 \) °C, specific heat \( C = 477 \) J/(kg °C), mass density \( \rho = 7850 \) kg/m³.

An equation of state defines the volumetric behavior of the material; the pressure is given by

\[ p = K \left( \frac{\rho - \rho_o}{\rho_o} \right), \]  (5)

where \( \rho \) is the current density, and \( \rho_o \) is the initial density.

Automatic contact option of DYNA2D was used to identify possible contact of the notch surfaces and prevent their interpenetrations. The automatic contact algorithm incorporates a rate-dependent Coulomb friction law:

\[ \mu = \mu_k + (\mu_s - \mu_k) e^{-\beta v_{rel}}, \]  (6)

where \( \mu_s \) and \( \mu_k \) are the static and kinetic friction coefficients, \( \beta \) is a transition coefficient governing the rate of change from static friction to kinetic friction, and \( v_{rel} \) is the relative speed between the two sliding surfaces. If \( \mu_k = 0 \) and \( \beta \)
= 0, a rate-independent friction model is recovered with $\mu = \mu_k$. For all of the simulations reported on herein we used $\mu_s = 0.18$, $\mu_k = 0.06$, and $\beta = 0.0055$.

Initially all plate particles are taken to be at rest, unstressed, and at the room temperature. All boundary surfaces are traction free and thermally insulated. All projectile particles move with velocity $V_o$.

**PART 5. NUMERICAL RESULTS AND DISCUSSION.**

We study the influence of the notch tip radius $\rho$ and the magnitude of the impact speed $V_o$ on the behavior of a pre-cracked (pre-notched) 4340 steel plate with the geometry of the specimen shown in Figures 3 and its discretization into finite elements in Figure 21. A projectile made of 4340 steel moves horizontally to the right at the speed $V_o$ and at time $t$ equal to zero strikes the plate. The projectile length is 125 mm, width is 50 mm. Both the plate and the projectile are modeled by the Johnson-Cook relation. Figure 22 depicts the Lagrangian-Eulerian finite element mesh near the notch tip at two instants during the deformation process. We will present stress and strain fields at the upper notch as well as at both notches. Because of symmetry, only half of the problem was analyzed.

**Influence of the impact velocity. The notch tip radius is 0.15 mm.**

$V_o = 25$ m/s. Figure 23 shows the distribution of the maximum shear stresses in both the plate and the projectile at 4.0 $\mu$s after impact. At approximately 10.0 $\mu$s shear stresses reach the notch tips as depicted in Figure 24, which is reasonable time for the wave speed $c = \sqrt{\frac{E}{\rho}} = \sqrt{K \cdot \frac{3(1 - 2\nu)}{\rho}} = \sqrt{157 \cdot 3 \cdot \frac{1 - 2 \cdot 0.29}{7.85}} = 5.02$ mm/$\mu$s, the notch length $l = 50$ mm, and $l/c = 9.96$ $\mu$s. At 12.0 $\mu$s the maximum shear stress grows up to 0.42 GPa as shown for the upper notch in Figure 25, and the effective plastic strain reaches 0.76% as shown in Figure 26. The two maximal zones are oriented at approximately (-5°) -- (-10°) to the notch ligament. Vectors of velocities in the plate and the projectile at 14.0 $\mu$s are shown in Figure 27. The distribution of the maximum shear stress in the two bodies at 12.0 $\mu$s is depicted in Figure 28.
Fig. 21. A sketch of the problem studied and the finite element mesh used.
Fig. 22. Lagrangian - Eulerian mesh at the notch tip. Notch tip radius = 0.5 mm.
Fig. 23. Distribution of the maximum shear stress (GPa) at 4.0 μs after impact. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

$0 < a < 0.05$
$0.05 < b < 0.10$
$0.10 < c < 0.16$
$0.16 < d < 0.21$
$0.21 < e < 0.26$
$0.26 < f < 0.31$
Fig. 24. Distribution of the maximum shear stress (GPa) at 10.0 \( \mu \)s after impact. Notch tip radius = 0.15 mm, impact speed = 25 m/s.
Fig. 25. Distribution of the maximum shear stress (GPa) in the notch tip vicinity at 12.0 μs after impact. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

0 < a < 0.07
0.07 < b < 0.14
0.14 < c < 0.21
0.21 < d < 0.28
0.28 < e < 0.35
0.35 < f < 0.42

Fig. 26. Distribution of the effective plastic strain in the notch tip vicinity at 12.0 μs after impact. Notch tip radius = 0.15 mm, impact speed = 25 m/s.

a = 0
0 < b < 0.0025
0.0025 < c < 0.0038
0.0038 < d < 0.0051
0.0051 < e < 0.0063
0.0063 < f < 0.0076
Fig. 27. Velocity (mm/µs) distribution within the plate and the projectile at 14.0 µs. Notch tip radius = 0.15 mm, impact speed = 25 m/s.
$0 < a < 0.08$
$0.08 < b < 0.15$
$0.15 < c < 0.23$
$0.23 < d < 0.30$
$0.30 < e < 0.38$
$0.38 < f < 0.46$

Fig. 28. Distribution of the maximum shear stress (GPa) in the plate and projectile. Time = 12.0 μs, notch tip radius = 0.15 mm, impact speed = 25 m/s.
Fig. 29. Time history of maximum shear stress (GPa) in four elements on the notch tip surface oriented at ±5° and ±70° to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 25 m/s.
Fig. 30. Contours of maximum shear stress in the notch tip vicinity. Time = 39.2 µs, notch tip radius = 0.15 mm, impact speed = 25 m/s.

c = 0.22 GPa
d = 0.30 GPa
e = 0.37 GPa

Fig. 31. Contours of effective plastic strain in the notch tip vicinity. Time = 39.2 µs, notch tip radius = 0.15 mm, impact speed = 25 m/s.

b = 3.4%
c = 5.1%
d = 6.8%
e = 8.5%
Fig. 32. Time history of hoop stress (GPa) in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 25 m/s.
Fig. 33. Distribution of the hoop stress in the notch tip vicinity. Time = 39.2 µs, notch tip radius = 0.15 mm, impact speed = 25 m/s.
We studied the evolution of the maximum shear stress, the hoop stress and the effective plastic strain at four elements located on the notch tip surface as depicted in the inset to Figure 29. As can be seen from Figures 29 - 31 the maximum shear stresses rapidly grow in the directions of 5° and -5° to the notch ligament, resulting in the deformation of the notch tip and development of plastic strains. Figure 32 depicts evolution of the hoop stress at the notch tip: they are tensile in the direction of 70° to the notch ligament, and compressive in (-5°), (-70°) directions, during the time interval 12 - 55 μs. Figure 33 depicts zones of tensile hoop stress at the upper part of the notch and compressive hoop stress at the lower part of the notch at 39.2 μs. A zero hoop stress border separates zones b and c.

Figure 34 shows deformations of the projectile and the plate near the impacted surfaces. The impacted part of the plate moves in the direction of the load and expands in the transverse direction due to Poisson's effect. At approximately 50.0 μs the lower surface of the notch tip struck the upper surface, while the expanding corner of the projectile hits the corner of the upper part of the notch. Then the projectile bounces back and moves in the opposite direction. Due to the elastic unloading the plate material below the notch moves backwards, as depicted for 78.4 μs in Figure 34. Figure 35 shows deformation of the notch tip at different instants of time.

According to Zhou et al. (1995 b) an “active” time interval for the shear crack propagation in a pre-notched C-300 steel plate, with notch tip radius equal to 0.15 mm and impact speed equal to 25 m/s, is 20 - 60 μs. Our computations with the double-notch 4340 steel plates and ρ = 0.15 mm, V₀ = 25 m/s, reveal that during this time interval, as can be seen from Figures 29 and 32, tensile hoop stresses in the 70° direction “compete” with the maximum shear stress in the -5° direction. The development of an opening crack in the 70° direction requires less energy than that for a shear crack. In the -5° direction the hoop stresses are compressive, and shear stresses are maximal. This stress state creates a localized plastic flow in the -5° direction, as shown in Figure 36, and may result in the development of a shear crack. The length of the shear crack will depend upon the maximum shear stress and the effective plastic strain. The expansion of the plastic zone and the length of the zone of the shear stresses
Fig. 34. Deformations of the projectile (1), and the plate below the notch tip (2) and above the notch tip (3). Notch tip radius = 0.15 mm, impact speed = 25 m/s.
Fig. 35. Deformations of the notch tip at different times. Notch tip radius = 0.15 mm, impact speed = 25 m/s.
Fig. 36. Time history of effective plastic strain in four elements on the notch tip surface oriented at 50° (a), 70° (b), -50° (c), and -70° (d) to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 25 m/s.
Fig. 37. Time histories of the length of the plastic zone (dashed line) and the length of the zone where the maximum shear stress exceeds 0.4 GPa (solid line), in the direction of -5° to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 25 m/s.
Fig. 38. Contours of maximum shear stress (GPa) in the notch tip vicinity. Time = 80 µs, notch tip radius = 0.15 mm, impact speed = 25 m/s.
higher than 0.4 GPa, in the -5° direction, are depicted in Figure 37. The length of the plastic zone expanding from the notch tip in the -5° direction is expected to be about 25 mm, which is 1/4 of the specimen width. As can be seen from Figure 37, the length of the zone of maximum shear stress in the -5° direction, drops to zero at about 50 µs. Then, at approximately 80 µs we observed a growth of the maximum shear stress in the directions of 5° and 70° to the notch ligament, as shown in Figures 29 and 38. This may change the shear band path as was experimentally observed by Zhou et al. (1995 b), and depicted in Figures 14 and 18.

Figure 39 depicts the evolution of the horizontal component of velocities at three points on the impacted plate surface. During the time interval from 10 to 50 µs an average magnitude of the velocity is one-half of that suggested by Zhou et al. (1995 a).

\[ V_o = 30 \text{ m/s} \]. Computations with a higher impact speed revealed a development of the hoop stress similar to that observed for \( V_o = 25 \text{ m/s} \). At 38.0 µs the tensile hoop stress in the 70° direction reached 0.69 GPa. The distribution of the maximum shear stress at 48.4 µs is depicted in Figure 40. The region where the maximum shear stress is between 0.37 and 0.41 GPa expands from the notch tip up to the rear surface of the plate. This creates conditions for shear cracking along the axis of symmetry of both the shear stress and the effective plastic strain zones, as depicted in Figures 40 and 41. Zhou et al. (1995 b) using single-notch C-300 steel specimens with the notch tip radius equal to 0.15 mm and impact speed equal to 30 m/s, found that a shear crack initiates at the notch tip at approximately 20 µs and reaches the rear surface of the plate at about 70 µs. In our computations with the double-notch specimen made of 4340 steel, \( \rho = 0.15 \text{ mm} \), and \( V_o = 30 \text{ m/s} \), the maximum shear stress at the notch tip reached 0.4 GPa at about 15 µs and in 48 µs reached the rear surface of the plate.

\[ V_o = 35 \text{ m/s} \]. During 11 - 50 µs after impact shear stresses are maximal in the direction of the notch ligament, and the maximum shear stress reaches 0.53 GPa. The tensile hoop stress at an element located on the notch surface in the 70° direction reached 0.69 GPa at 45.0 µs. The effective plastic strain propagates
Fig. 39. Time history of the horizontal components of the velocity at three points on the impacted plate surface. Notch tip radius = 0.15 mm, impact speed = 25 m/s.
Fig. 40. Distribution of the maximum shear stress (GPa) at 48.4 μs. Curve A is a possible path for a shear crack. Notch tip radius = 0.15 mm, impact speed = 30 m/s.

- $0 < a < 0.07$
- $0.07 < b < 0.15$
- $0.15 < c < 0.22$
- $0.22 < d < 0.30$
- $0.30 < e < 0.37$
- $0.37 < f < 0.44$
Fig. 41. Distribution of the effective plastic strain at 48.4 μs. Curve A is a possible path for a shear crack. Notch tip radius = 0.15 mm, impact speed = 30 m/s.
Fig. 42. Time history of effective plastic strain in four elements on the notch tip surface oriented at $5^\circ$ (a), $70^\circ$ (b), $-5^\circ$ (c), and $-70^\circ$ (d) to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 35 m/s.
Fig. 43. Distribution of the maximum shear stress (GPa) at 50.4 μs. Notch tip radius = 0.15 mm, impact speed = 35 m/s.
Fig. 44. Distribution of the effective plastic strain at the notch tip. Time = 50.4 \mu s, notch tip radius = 0.15 mm, impact speed = 35 m/s.

- $0.05 < b < 0.09$
- $0.09 < c < 0.14$
- $0.14 < d < 0.18$
- $0.18 < e < 0.22$

Fig. 45. Contours of hoop stress (GPa) in the notch tip vicinity. Time = 50.4 \mu s, notch tip radius = 0.15 mm, impact speed = 35 m/s.

- $a = -0.80$
- $b = -0.54$
- $c = -0.28$
- $d = 0.00$
- $e = 0.24$
Fig. 46. Time history of the hoop stress (GPa) at four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 40 m/s.
Fig. 47. Time history of maximum shear stress (GPa) in four elements on the notch tip surface oriented at $5^\circ$ (a), $70^\circ$ (b), $-5^\circ$ (c), and $-70^\circ$ (d) to the notch ligament. Notch tip radius = 0.15 mm, impact speed = 40 m/s.
fast in the \(-5^\circ\) direction, as depicted in Figure 42. Figures 43 - 45 depict distributions of the maximum shear stress, effective plastic strain, and hoop stress at 50.4 \(\mu\)s.

\[ V_o = 40 \text{ m/s}. \] Figure 46 presents the time-history of the development of the hoop stress at four elements on the notch tip surface. During the "active" time interval, from 12 to 55 \(\mu\)s after impact, both tensile and compressive hoop stresses have higher magnitudes than those for \(V_o = 25, 30\) and \(35 \text{ m/s}.\) However, the maximum shear stresses in the \(5^\circ, -5^\circ\) directions are "leading" compared to the \(70^\circ\) and \(-70^\circ\) directions, as shown in Figure 47. Highly localized plastic flow rapidly propagates along the \(-5^\circ\) direction, where the shear stresses are maximal, and hoop stresses are compressive or small tensile.

**Influence of the notch tip radius. The impact velocity is 35 m/s.**

We have also studied the influence of the notch tip radius ranging from 0.10 to 0.80 mm on the evolution of the stress and strain fields in the double-notch plates.

\[ \rho = 0.80 \text{ mm}. \] Figures 48 -50 depict the evolution of the maximum shear stress, effective plastic strain, and hoop stress at four elements located on the notch tip surface. Figures 51 - 53 show the distribution, in the vicinity of the notch tip, of the maximum shear stress, hoop stress and effective plastic strain at 42.0 \(\mu\)s after impact. The border between zones b and c in Figure 52 separates zones of tension and compression, maximum tensile hoop stress occurs in zone e. During the time interval 15 to 55 \(\mu\)s maximal values of the maximum shear stress, 0.45 - 0.48 GPa, were reached in the \(-70^\circ\) direction. Plastic strain rapidly grows in \(70^\circ\) and \(-70^\circ\) directions, Figure 50. Tensile hoop stresses thought to be responsible for crack opening reach 0.66 GPa in the \(70^\circ\) direction. As can be seen from Figure 49, at approximately 70.0 \(\mu\)s, the maximum tensile hoop stress reaches approximately 0.5 GPa in the \(-70^\circ\) direction (the hoop stresses in other directions at 70.0 \(\mu\)s are compressive). This sequence in the development of stress fields creates conditions for shear banding to occur in the \(-70^\circ\) and \(70^\circ\) directions starting at 15 \(\mu\)s, opening in the \(70^\circ\) direction at approximately 30 - 50 \(\mu\)s, and then opening of the shear band in the \(-70^\circ\)
Fig. 48. Time history of maximum shear stress (GPa) in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius = 0.80 mm, impact speed = 35 m/s.
Fig. 49. Time history of hoop stress (GPa) in four elements on the notch tip surface oriented at $5^\circ$ (a), $70^\circ$ (b), $-5^\circ$ (c), and $-70^\circ$ (d) to the notch ligament. Notch tip radius = 0.80 mm, impact speed = 35 m/s.
Fig. 50. Time history of effective plastic strain in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius = 0.80 mm, impact speed = 35 m/s.
Fig. 51. Distribution of the maximum shear stress (GPa) at 42.0 μs. Notch tip radius = 0.80 mm. Impact speed = 35 m/s.
Fig. 52. Distribution of the hoop stress (GPa) at 42.0 μs.
Notch tip radius = 0.80 mm. Impact speed = 35 m/s.
Fig. 53. Distribution of the effective plastic strain at 42.0 μs.
Notch tip radius = 0.80 mm. Impact speed = 35 m/s.
0 < a < 0.1
0.1 < b < 0.2
0.2 < c < 0.3
0.3 < d < 0.4
0.4 < e < 0.5

Fig. 54. Distribution of the maximum shear stress (GPa) at 103.6 μs.
Notch tip radius = 0.80 mm. Impact speed = 35 m/s.
Fig. 55. Deformation and translation of the notch tip. Notch tip radius = 0.80 mm. Impact speed = 35 m/s.
Fig. 56. Time history of effective plastic strain in four elements on the notch tip surface oriented at $5^\circ$ (a), $70^\circ$ (b), $-5^\circ$ (c), and $-70^\circ$ (d) to the notch ligament. Notch tip radius = 0.60 mm, impact speed = 35 m/s.
direction at 65 - 95 μs. The evolution of the shear and hoop stresses might be responsible for the failure modes experimentally observed by Ravi-Chandar (1995) in pre-notched polycarbonate plates impacted at 28 - 50 m/s, as depicted in Figure 20 b.

After 65 μs plastic flow grows fast in the angular sector of 50 - 700 to the notch ligament, where the shear stresses are maximal, Figure 54. At approximately 140 - 180 μs the tensile hoop stress creates opening conditions in the direction of the notch ligament, as well as in the 70° direction.

Figure 55 depicts changes in the shape of the notch tip and its translation in the direction of impact.

ρ = 0.60 mm. During 15 - 55 μs the fast growth of the effective plastic strain was observed in the directions of -50 and -700 to the notch ligament, as depicted in Figure 56. From 75 to 110 μs plastic strains grow predominantly in the direction of the notch ligament.

ρ = 0.40 mm. A decrease in the notch tip radius results not only in the growth of the maximum value of the effective plastic strain at the notch tip, but also alters the direction of plastic flow. Depressed shear stresses in the 70° direction and localization of plastic flow in the -5° direction are very likely responsible for the failure mode transition from brittle crack opening at 70° to a shear crack inclined at approximately -50 to the notch ligament. This transition in failure mode in double-notch steel plates was experimentally observed by Kalthoff (1988) and depicted in Figure 8: (a) an opening crack at 70° - when ρ = 0.75 - 0.85 mm and V0 = 32 m/s, and (b) a shear crack inclined at (-5°) - (-10°) to the notch ligament - when ρ = 0.35 mm and V0 = 32 m/s.

Computations with ρ = 0.40 mm revealed faster and "stable" growth of the maximum shear stress in the -5° direction, as compared to that for ρ = 0.80 and 0.60 mm. The maximum shear stress in the -5° direction dominates over that in other directions during the time intervals from 12 to 55 μs and from 64 to 115 μs. This results in fast growth of the effective plastic strain in the -5° direction as shown in Figure 57. Figure 58 depicts the distribution of the maximum shear stress. A shear crack is expected to propagate along the axis of symmetry of zone e, i.e. along the line starting at the notch tip and gradually approaching the
Fig. 57. Time history of effective plastic strain in four elements on the notch tip surface oriented at 5° (a), 70° (b), -5° (c), and -70° (d) to the notch ligament. Notch tip radius = 0.40 mm, impact speed = 35 m/s.
Fig. 58. Distribution of the maximum shear stress (GPa) at 47.6 μs. Notch tip radius = 0.40 mm. Impact speed = 35 m/s.
Fig. 59. Vectors of displacement (mm) at 61.6 μs.
Notch tip radius = 0.40 mm, impact speed = 35 m/s.
Fig. 60. Influence of the notch tip radius on the maximum tensile hoop stress in the 70° direction during 10 - 60 μs, and in the -70° direction during 70 - 100 μs after impact. Impact speed = 35 m/s.
Fig. 61. Influence of impact speed on the effective plastic strain in the -5° direction (solid lines) and in the 70° direction (dashed lines). Notch tip radius = 0.15 mm.

Fig. 62. Influence of the notch tip radius on the growth of the effective plastic strain in the -5° direction (solid lines) and in the 70° direction (dashed lines). Impact speed = 35 m/s.
Fig. 63. Three directions of crack propagation:

(A) due to tension and shear,
(B) due to shear and compression,
(C) due to shear and compression/tension.

1 - zone of maximum tensile hoop stress,
2 - zone of maximum compressive hoop stress,
3 - line of zero hoop stress,
4 - zone of maximum shear stress.
axis of the plate. During the second overall growth of the maximum shear stress, from 57 to 67 $\mu$s, the specimen motion consists of two components: translation in the direction of impact, and rotation about points A and B as depicted in Figure 59.

Figure 60 shows the influence of the notch tip radius on the maximum tensile hoop stress developed at elements located on the notch tip surface in the $70^\circ$ and $-70^\circ$ directions during the time intervals from 0 to 60 $\mu$s and from 70 to 100 $\mu$s respectively.

Numerical simulations revealed that the magnitude of the effective plastic strain as well as the direction of the plastic flow are sensitive to the impact speed and notch tip radius. Figure 61 depicts the dependence of the effective plastic strain at time 20, 40 and 60 $\mu$s, developed at two elements located on the notch tip surface in the $-5^\circ$ and $70^\circ$ directions where the effective plastic strain is expected to be maximal. Overall, at these times plastic strains in the $-5^\circ$ direction are 10 - 25 times higher than that in the $70^\circ$ direction. This creates conditions for localized growth of plastic flow and shear cracking in the direction of approximately $-5^\circ$ to the notch ligament.

At large notch tip radii, during the “active” time interval, the maximum shear stresses in the $70^\circ$ and $-70^\circ$ directions are higher than that in the $-5^\circ$ direction. A plastic flow rapidly grows in the $70^\circ$ and $-70^\circ$ directions. Under a tensile hoop stress it can transform into an opening crack inclined at $70^\circ$. In the $-70^\circ$ direction, material at the notch tip experiences shear and compression during the “active” time interval. Then hoop stresses there become tensile, making tensile cracking in the $-70^\circ$ direction possible. For $\rho = 0.7 - 0.8$ mm the effective plastic strain in the $-5^\circ$ direction is smaller than that in the $70^\circ$ direction. Figure 62 depicts the influence of the notch tip radius on the effective plastic strain in the two competing directions. Figure 63 schematically depicts three zones: maximum tensile hoop stress, maximum compressive hoop stress, and maximum shear stress. Line 3 separates the zones of tensile and compressive hoop stresses. Depending on the notch tip radius and the impact speed, a failure will occur in A, B or C directions.
6. CONCLUSIONS

We have considered transient stress and strain fields in the pre-cracked (pre-notched) plates made of 4340 steel, and impacted by a 4340 steel projectile in the direction of the notch ligament. The influence of the notch tip radius and impact speed on the development of plastic flow has been studied. Computations revealed that for relatively large notch tip radii the plastic strains grow in the directions of approximately 70° and -70° to the notch ligament. A tensile hoop stress may result in the transformation of the plastic zone into an opening crack propagating in the 70° direction. In the -70° direction material at the notch tip may fail due to shear and compression followed by stretching. At smaller notch tip radii, the plastic flow initiates at approximately -50° to the notch ligament in the direction of the maximum shear stress. It propagates along the zone of the maximum shear and compressive hoop stresses. The axis of symmetry of the zone of maximum shear stress monotonically deviates from the notch tip while approaching the rear surface of the plate. The compressive hoop stress in this zone suppresses an opening mode of failure, thus providing conditions for the shear mode of failure. The length of the shear - compression zone, and the maximum effective plastic strain developed increase with an increase in the impact speed.

7. REFERENCES


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8. VITA

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