An Investigation of the Clothoid Steering Model for Autonomous Vehicles

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ABSTRACT

The clothoid, also known as the Cornu spiral, is a curve generated by linearly increasing or decreasing curvature as a function of arc length. The clothoid has been widely accepted as a logical curve for transitioning from straight segments to circle arcs in roads and railways, because a vehicle following the curve at constant speed will have a constant change of centripetal acceleration. Clothoids have also been widely adopted in planning potential paths for autonomous vehicle navigation. They have been viewed as useful representations of possible trajectories that are dynamically feasible. Surprisingly, the assumptions that underlie this choice appear to be lightly treated or ignored in past literature.

This thesis will examine three key assumptions that are implicitly made when assuming that a vehicle will follow a clothoid path. The first assumption is that the vehicle’s steering mechanism will produce a linear change in turning radius for a constant rate input. This assumption is loosely referred to as the “bicycle model” and it relates directly to the kinematic parameters of the steering mechanism. The second assumption is that the steering actuator can provide a constant steering velocity. In other words, the actuator controlling the steering motion can instantaneously change from one rate to another. The third assumption is that the vehicle is traveling at a constant velocity. By definition, the clothoid is a perfect representation of a vehicle traveling at constant velocity with a constant rate of change in steering curvature. The goal of this research was to examine the accuracy of these assumptions for a typical Ackermann-steered ground vehicle. Both theoretical and experimental results are presented.

The vehicle that was used as an example in this study was a modified Club Car Pioneer XRT 1500. This Ackermann-steered vehicle was modified for autonomous navigation and was one of Virginia Tech’s entries in the DARPA 2005 Grand Challenge. As in typical operation, path planning was conducted using the classic clothoid curve model. The vehicle was then commanded to drive a selected path, but with variations in speed and steering rate that are
inherent to the real system. The validity of the three assumptions discussed above were examined by comparing the actual vehicle response to the planned clothoid.

This study determined that the actual paths driven by the vehicle were generally a close match to the originally planned theoretical clothoid path. In this study, the actual kinematics of the Ackermann vehicle steering system had only a small effect on the driven path. This indicates that the bicycle model is a reasonable simplification, at least for the case studied. The assumption of constant velocity actuation of the steering system also proved to be reasonably accurate. The greatest deviation from the planned clothoid path resulted from the nonlinear velocity of the vehicle along the path, especially when accelerating from a stop. Nevertheless, the clothoid path plan generally seems to be a good representation of actual vehicle motion, especially when the planned path is updated frequently.
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<td>Defense Advanced Research Projects Agency</td>
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<td>GC</td>
<td>Grand Challenge</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<td>HMMWV</td>
<td>High Mobility Multipurpose Wheeled Vehicle</td>
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<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
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<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
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<tr>
<td>NI</td>
<td>National Instruments</td>
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<td>NIST</td>
<td>National Institute of Standards and Technology</td>
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<td>RANGER</td>
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Chapter 1
INTRODUCTION

1.1 Purpose of Study

When using the bicycle model, a vehicle moving at constant velocity with its steering actuator turning at a constant rate will traverse a perfect clothoid. Many researchers have followed this line of thought and have developed autonomous navigation codes using clothoids. However, only a few researchers, like Coombs, Shin, and Kanayama, acknowledge that they are making three assumptions by accepting this idea:

1. the complex steering mechanism can be represented by a bicycle model
2. the steering actuator can provide constant steering velocity (step changes in velocity)
3. the vehicle is traveling at a constant velocity along the path

Only a small amount of simulation work has been done in the past to investigate if these assumptions are valid in practice. Even less experimental work has been done using physical tests on a running vehicle to back up those simulations. This thesis provides a detailed explanation of these assumptions as well as an in-depth investigation into the validity of the assumptions and the subsequent efficacy of the clothoid model. The work includes both a theoretical study and experimental results using an Ackermann-steered autonomous ground vehicle. The goal of this research is to determine if the assumptions that are made when using clothoids for ground vehicle navigation are indeed valid and supported by physical testing. The investigation is important because it will help shed light on the validity of the clothoid model, which has become nearly ubiquitous in ground vehicle modeling.

1.2 Thesis Overview

This thesis provides evidence that an Ackermann-steered vehicle’s path can be reasonably predicted by using clothoid-shaped paths. The paths are accurate provided that the vehicle is traveling at a constant speed and commanding a constant steering rate. The bicycle model was determined to provide an adequate estimation of the vehicle steering angle. The steering actuator used on the test vehicle was able to provide nearly instantaneous changes in steering rate, which was required to drive a piecewise constant steering rate. When the vehicle
was commanded to drive with a constant velocity along the path, it continuously drifted above and below the commanded velocity. Still, the actual velocity was close enough to the commanded velocity that any error caused by the drifting was minor.

1.3 Outline

Following the introduction to this thesis provided in Chapter 1, the second chapter provides background information needed to understand the scope of the thesis. It reviews the mathematical foundation of clothoid curves and how they have been used by researchers in the past. It also describes the assumptions that underlie the use of the clothoid model in path planning, what restrictions they place on the model in order to be valid, and what limitations they have in terms of navigation.

The third chapter describes the vehicle used in the experiments and the experiments that were performed for this investigation. The test platform is a four-wheeled utility vehicle that was converted for autonomous operation. The chapter describes the test platform and gives an overview of the stock Ackermann steering system used in the test vehicle. The two experiments that were performed tested the validity of the clothoid assumptions through physical experimentation. The first experiment was done to determine if the vehicle can drive a clothoid when it was commanded to drive at a constant velocity with a constant steering rate. This would demonstrate the vehicle’s ability to drive a clothoid when the assumptions were intended to be valid. The second experiment was performed to show how the vehicle drove when it was accelerating while attempting to steer at a constant rate. The goal was to determine if a single clothoid could still be used to predict the vehicle’s path even though the vehicle was accelerating.

The fourth chapter provides a detailed analysis of the vehicle’s steering assembly. It shows how the steering system was modeled and describes the relationship between the steering angles of the wheels and the rack. It provides data conversions between the different parts of the assembly. The chapter also walks through the equations that were used to calculate the predicted clothoid paths using the bicycle model.

The fifth chapter analyzes the data gathered during the two experiments. The data from the experiments was compared with the predicted clothoid path data. This comparison was for the steering rate, vehicle velocity, and vehicle position. The steering rate data was used to
analyze the constant steering rate assumption. The vehicle velocity data was used to analyze the
constant velocity assumption. The position data was used to analyze the validity of the bicycle
model assumption provided the other two assumptions were met.

The sixth chapter explains the conclusions that can be drawn from an analysis of the
experimental results and provides suggestions for future work. The conclusion summarizes the
results of the investigation and gives the verdict of this paper about whether the assumptions
were valid or not. The future work section discusses briefly what other projects could be done to
further the research in this area.
Chapter 2
BACKGROUND

There have been many approaches developed to solve the problem of unmanned vehicle path planning and path generation, including the use of circular arcs, line segments, and complex curves to represent the predicted path of the vehicle. The types of curves often used in these methods include B-splines and clothoids. This document is an investigation into the assumptions that are made when using the clothoid curve. Many autonomous vehicle navigation methods that have been or are being used by current researchers are based on clothoids.

2.1 Clothoids

2.1.1 Description of Clothoid Paths. A clothoid, also known as a Cornu spiral, is defined as a curved line whose curvature varies linearly with its arc-length [1]. This definition translates into the following equation:

\[ c(s) = ks + c_i \]  

(1)

where \( c \) is the curvature of the line, \( k \) is the rate of change of curvature which is also called the sharpness, and \( c_i \) is the initial curvature of the line [9]. This means that the sharpness of the clothoid is a constant value. Figure 1 shows an example of a clothoid curve segment with a sharpness of 0.1 being plotted for 30 units.

In the context of path planning for autonomous vehicles, a clothoid is produced by changing the steering angle of the vehicle at a constant rate while the vehicle is traveling at constant velocity [1]. However, for this to be accurate in practice, the following assumptions must be valid:

1. the vehicle’s steering mechanism can be mathematically represented by a bicycle model,
2. the steering rate of the vehicle is a constant value,
3. the vehicle is traveling at a constant velocity.
Clothoids have been often used in path planning algorithms because they have several key characteristics that make them advantageous [8]. One is that the path is continuous in position, heading, and curvature. This is recommended because a vehicle has inertia and a finite control response, which obviously prevents it from performing discontinuous motions. Also, the vehicle is not able to change steering angles instantly. If the vehicle wants to turn from driving straight ahead to a 30 degree right turn, it must smoothly transition the steering actuator from 0 degrees to 30 degrees. A clothoid takes this transition into account. If the planned path for the vehicle has discontinuous motion, the result will be large tracking errors between the vehicle’s planned and actual location. The second characteristic is that the curvature of the path varies linearly with the path’s arc length [8]. This is desirable because the linearity of the curvature of the path dictates the linearity of the steering motion along the path [9]. This is useful because it is generally assumed that an actuator requires less control effort to command a linear velocity profile than to command a random nonlinear one. Given this assumption, “the extent to which steering motions are likely to keep a vehicle on a desired path can be correlated to the linearity of curvature of the path [9].” Clothoid trajectories are theoretically able to meet all of these criteria, which is one reason why they are so often used in path planning operations [8]. Another reason they are used so regularly is because they appear to be dynamically feasible trajectories that the vehicle can follow while taking into account the initial steering angle and the maximum steering rate of the of the vehicle [1].
2.1.2 Literature Review. As previously mentioned, many groups of researchers have been using clothoids in their unmanned vehicle navigation algorithms to generate predicted paths that their vehicles could travel. The following section describes the efforts of a few of these groups in more detail to give a general idea of how clothoids have been used.

The Intelligent Systems Division of the National Institute of Standards and Technology (NIST) has used clothoids for planning the immediate path of their robotic offroad vehicle, the NIST High Mobility Multipurpose Wheeled Vehicle (HMMWV) [1]. The objective of their experiments was to be able to drive their vehicle autonomously offroad at speeds up to 35 km/h. For these experiments, the HMMWV was given a sequence of Global Positioning System (GPS) coordinates and was programmed to generate and navigate an obstacle-free path to these GPS points. As the unmanned vehicle drove, it also continuously replanned its path based on new real-time sensor data [1]. While working on the HMMWV, the researchers found that simple straight-line segments could be used for path planning, but at high speeds, they became unstable [1]. One of the central ideas for their solution to this problem was planning the first few seconds of travel using a sequence of clothoid segments. These paths were 20 meters long and, given the response characteristics of the steering mechanism, were dynamically feasible [1].

Don Hun Shin and Sanjiv Singh developed a method of path generation for autonomous vehicles using composite clothoid segments [9]. Their method generates a continuous path by connecting a sequence of objective points using clothoid curves. By design, Shin and Singh’s composite clothoid method insures the continuity of position, heading, and curvature of the created path. The method was also designed to have the steering rate of the vehicle be piecewise constant [9].

The Real-time Autonomous Navigator with a Geometric Engine (RANGER), which was developed by Alonzo Kelly, uses clothoids as part of its local navigation program [4]. The RANGER uses a high-fidelity, feedforward actuator dynamics and terrain following model to convert and solve the local navigation problem as an optimal control problem. Solving the problem using a feedforward method enabled Kelly to generate clothoid trajectories with less difficulty than previous methods [4]. The navigation program, using both a state space representation of a multi-input multi-output linear system and a terrain map, is able to “generate
the near clothoid responses of the vehicle naturally and directly with none of the algorithmic sensitivities of classical solutions [4].”

2.2 Clothoid Assumptions and Clothoid Paths

As discussed previously, to make use of clothoids in the navigation of an Ackermann-steered vehicle, there are three major assumptions that have to be valid. The first assumption is that a vehicle’s steering mechanism can be mathematically represented by a bicycle model. The second is that the steering rate of the vehicle is constant. The third assumption is that the vehicle is traveling at a constant velocity. The predicted clothoid path equations were derived using the bicycle model and support the need for the constant steering rate and velocity assumptions.

2.2.1 Bicycle Model Assumption. A bicycle model is a simplified model that is often used to represent a vehicle for navigational purposes. This model reduces the complexity of the problem by reducing the number of degrees of freedom that the vehicle has to two: steering and propulsion [9]. This is done by assuming that each pair of wheels on a four-wheel vehicle can be represented as a single wheel located along the longitudinal axis of the vehicle, as shown in Figure 2, while still maintaining the required accuracy [8]. The model also makes the assumption that the slip angles at both of the wheels are zero. This means that the angles that the velocity vectors of the wheels make with the longitudinal axis of the vehicle are equal to the angles of the wheels [7]. In this case, this means that front wheel velocity vector angle is equal to the steering angle while the rear wheel angle is zero. Using this model, a set of simplified equations of motion can be developed to represent the vehicle’s position, heading, and speed.

In order to use the model, a guide point, also called a reference point, must be chosen as a basis to develop the equations for steering and propulsion. Generally, the midpoint of the rear axle is chosen as the location of the guide point though any location will work. The guide point will be the location where the center of curvature connects with the vehicle forming the radius of curvature of the vehicle’s path. Since the radius of curvature is a key variable within the equations of motion, the complexity of the equations is dependent on its location.

For this experiment, the guide point was placed at the same location on the model as the IMU and GPS were located on the vehicle. The reason for this was to calculate the predicted path of the same part of the vehicle as the location whose position and speed were recorded.
during the physical experiments. This would help increase the accuracy of the comparison.

Figure 2 shows the model that was used. While the model shows that the rear tire has a steering angle, $\phi$, since the experimental vehicle is front-wheel-only steering, this angle was set to zero. The model was designed for this to be an option so this change will not have any other effect on the model. The circle represents the location of the IMU and GPS. It is also the location of the guide point. The steering angle of the front wheel is $\phi$. The radius of curvature is $R$. $V$ is the velocity of the vehicle at the guide point, and $\beta$ is the angle between the velocity vector and the longitudinal axis of the vehicle. $\beta$ is also called the slip angle of the vehicle. $L_r$ and $L_f$ are the distances between the rear axle and the guide point and the front axle and the guide point. The heading of the vehicle is represented by the angle $\theta$.

![Figure 2. Geometry of a bicycle model with guide point placed near middle of vehicle](image)

While the above figure is a correct representation of the vehicle, it is a bit more complicated as a result of placing the guide point on the body of the vehicle. This is why the rear axle is often selected as the location of the guide point as shown in Figure 3 [3, 5, 8]. One of the reasons for placing the guide point there is that the steering angle can be calculated
independently of the speed of the vehicle [9]. This allows the controls of these properties to be handled separately [8]. The steering angle can be solved geometrically at any point on the generated path using the following equations:

\[ \tan \phi = \frac{l}{r} \]  

where \( l \) is the wheelbase of the vehicle, \( r \) is the radius of the curvature of the path, and \( \phi \) is the steering angle of the vehicle [9]. This relates to the curvature of the path through the following relationship:

\[ c = \frac{1}{r} \]  

where \( c \) is the curvature of the path. This means that the steering angle is related to the curvature by:

\[ \phi = \tan^{-1} cl \]  

Figure 3. Geometry of a bicycle model with guide point placed at center of rear axle

Another advantage of placing the guide point on the rear axle is that it simplifies the calculations of vehicle’s velocity in the \( x \) and \( y \) directions. It completely removes the need for the angle \( \beta \). A third reason for placing the guide point at this location is that this point has the smallest possible turning radius that can be associated with a given steering angle [8]. A final
advantage of this guide point location is that the vehicle’s heading will be tangent with the
direction of the path the vehicle is traveling. For forward-mounted cameras and range finders,
this allows the instruments to have a better view of the area along the vehicle’s path [9].

While the bicycle model is a useful simplification of a complex system, it does create
some problems. One problem that surfaces when using the bicycle model to represent a four-
wheeled vehicle is that there is no way to actually measure the steering angle of the vehicle.
Ackermann steering, which is used almost universally in four-wheel vehicles, intentionally
creates different steering angles for the two front wheels. These are shown in Figure 4 as $\phi_L$ and
$\phi_R$. As a result, the steering angle of the vehicle, $\phi$, must be taken to be somewhere in between
these two angles. This issue will be discussed in more detail in the next chapter.

![Figure 4. Wheel angles and vehicle steering angle](image)

It should be noted that a steering system that used this model, NavLab, demonstrated
significant non-linearities in its responses during testing. However, the researchers left the
issues of repeatability and accuracy to be compensated by higher levels of control instead of by
developing a more detailed model [8].

### 2.2.2 Predicted Clothoid Equations

The predicted clothoid trajectories were calculated using the kinematic equations of motion of the vehicle. The key values that were
needed for calculating the clothoid path were the rate of change in the $x$ and $y$ directions ($\dot{x}$ and $\dot{y}$), the rate of change of the vehicle’s heading ($\dot{\theta}$), and the rate of change of the steering angle ($\dot{\phi}$). The kinematic equations for these variables were derived using the bicycle model, shown in Figure 5. Figure 5 is a slightly modified version of Figure 2. Since the vehicle used in the experiment is front-wheel-only steered, the steering angle of the rear wheels has been set to zero. The rate of change of the vehicle’s heading is calculated using the following equation:

$$
\dot{\theta} = \frac{v \cdot \cos(\beta)}{(L_f + L_r)} \tan(\phi)
$$

(5)

A detailed derivation of these equations is provided in Appendix B. The above equation shows that the rate of change of heading is dependent on not only the vehicle’s heading, but also on the position of the point on the vehicle that is being measured. This supports the earlier statement that great care must be taken when selecting a guide point. The velocity of the vehicle in the $x$ and $y$ directions is dependent on the vehicle’s heading and the angle $\beta$, as follows:

$$
\dot{x} = v \cdot \cos(\theta + \beta)
$$

(6)

$$
\dot{y} = v \cdot \sin(\theta + \beta)
$$

(7)

The angle $\beta$ is calculated using the following equation:

$$
\beta = \tan^{-1} \left( \frac{L_r \tan(\phi)}{L_f + L_r} \right)
$$

(8)

where $\beta$ is dependent on the location of the guide point and the steering angle of the vehicle.
Using these calculated rates, the equations of motion of the vehicle that still need to be defined are the vehicle’s position in the $x$ direction, position in the $y$ direction, heading, and the steering angle. The steering angle at interval $i$ ($\phi_i$) was determined by using the following equation:

$$\phi_i = \dot{\phi}_i \cdot dt + \phi_{i-1}$$  \hspace{1cm} (9)

where $\dot{\phi}_i$ is the steering rate at interval $i$, $\phi_{i-1}$ is the steering angle at interval $i$ minus one, and $dt$ is the change in time between intervals. The heading of the vehicle, the $x$ position, and the $y$ position are all calculated using similar equations:

$$\theta_i = \dot{\theta}_i \cdot dt + \theta_{i-1}$$ \hspace{1cm} (10)

$$x_i = \dot{x}_i \cdot dt + x_{i-1}$$ \hspace{1cm} (11)

$$y_i = \dot{y}_i \cdot dt + y_{i-1}$$ \hspace{1cm} (12)

Using the aforementioned rate equations and these equations of motion, the vehicle’s path was predicted based on an initial steering angle, constant steering rate, initial position, initial heading, and a constant velocity.
2.2.3 Steering Rate Assumption. The second assumption that is made when using clothoids for navigation planning is that the steering rate of the vehicle is a constant value. The reason for this deals with the clothoid’s definition of having a constant rate of change of curvature. The rate of change of curvature is also called sharpness whose units are 1/(inch * sec). The units are determined based on the fact that curvature is equal to the inverse of the radius of curvature which accounts for the 1/inch. Since sharpness is the rate of change of curvature, its units are 1/(inch * sec). A constant sharpness can be assumed as equivalent to the vehicle driving at a constant steering rate. Admittedly, they are not identical, which can be seen in Figure 6. The figure shows how the sharpness changed with time when a constant steering rate was input into the equations of motion. However, looking at Figure 7, it is obvious that the difference between a constant steering rate and a constant sharpness is very slight when looking at their effects on the curvature of the path. Figure 7 is a plot of the curvature whose rate was depicted in Figure 6. The constant sharpness line is a linear regression of the curvature data. The difference plot shows how curvature of a path differs when having a constant steering rate versus having a constant sharpness. The difference data shows that the average difference is actually 0 1/inch with a standard deviation of ± 9 e-005 1/inch. The slope of the linear regression was -0.00449 1/(inch*sec) with an intercept of 0.00673 1/inch. The r-squared value for the regression line and the curvature data was 0.9997. This supports the statement that the two lines were nearly equivalent. This means that a constant steering rate is equivalent to a constant sharpness while in this range of steering angles.

Figure 6. Sharpness versus time when the vehicle is steering at a constant steering rate
In order for the vehicle to drive at a constant steering rate, the steering actuator must be able to accelerate to the desired steering rate instantaneously. This equates to the actuator having an infinite steering acceleration which would allow for an immediate change of the steering rate [1]. Of course, this is physically impossible since the actuators have a finite response time [8]. The key to the accuracy of this assumption is whether the steering actuator can accelerate quickly enough that the error between the actual and the theoretical steering rate is so small as to be irrelevant.

The reason for the assumption ties into the very definition of a clothoid. When Kanayama and Miyake described their navigational method, one of the characteristics they wanted was for their paths to be piecewise linear curvature functions [2]. Basically, they wanted to be able to subdivide the paths into various segments. Each segment’s curvature would change at a constant linear rate. When relating this to a vehicle’s steering, a linear change in curvature is equivalent to the steering angle of the vehicle changing at a constant rate. A line whose curvature changes at a linear rate is the very definition of a clothoid. It is because researchers want their paths to be piecewise linear curvature functions that they choose to use clothoids to describe the vehicle’s paths [2].
2.2.4 Velocity Assumption. The third assumption that is necessary to make when using clothoid trajectories is that the vehicle is traveling at a constant velocity. This assumption is made for the same reasons as the steering rate assumption. A clothoid is defined as a line whose curvature changes linearly. If a bicycle model is moving at constant velocity with its steering actuator turning at constant rate, the vehicle path will be a perfect clothoid [1]. If the constant velocity assumption does not hold, the path will deviate from a clothoid. As a result of this, a clothoid is unable to depict a vehicle when it is accelerating or decelerating.

This can be proven by an examination of the mathematics behind a clothoid. The curvature of a clothoid is defined by the following equation:

\[ c = \frac{d\theta}{ds} \]  

(13)

where \( d\theta \) is the change in heading, and \( ds \) is the change in path length [8]. This shows how a path’s curvature is related to its heading and its length. In order to prove the assumption, the curvature needs to be related to the vehicle’s velocity, which is defined by the following equation:

\[ v = \frac{ds}{dt} \]  

(14)

where \( dt \) is the change in time [8]. Now if \( d\theta \) was replaced with the rate of change of the vehicle’s heading, \( \dot{\theta} \), which is defined as,

\[ \dot{\theta} = \frac{d\theta}{dt} \]  

(15)

the velocity and \( \dot{\theta} \) can be substituted into the curvature equation [8]. This would result in the following equation for curvature [8]:

\[ c = \frac{\dot{\theta}}{v} \]  

(16)

Now, to examine the path’s sharpness, the derivative of the curvature needs to be taken. The derivative of the path’s curvature would be

\[ \dot{c} = k = \frac{d^2\theta}{ds^2} = \left( \frac{d^2\theta}{dt^2} \right) \left( \frac{ds^2}{dt^2} \right) = \frac{\dot{\theta}}{v^2} \]  

(17)
where $\theta$ is the acceleration of the heading and $k$ is a curve’s sharpness. This proves that the vehicle’s velocity needs to be constant in order for the path’s sharpness to be constant. Any small change in the vehicle velocity will be squared, resulting in large changes in the sharpness.
Chapter 3
EXPERIMENTAL DESIGN

A series of experiments was designed to assess the validity of the assumptions underlying the use of the clothoid model for steering an autonomous ground vehicle. The vehicle, the materials, and the methods used in these experiments are described below. The importance of each experiment in assessing the clothoid assumptions is also described.

3.1 Description of Vehicle

The vehicle that was used to perform the experiments for this paper was an Ingersoll-Rand Club Car Pioneer XRT 1500 utility platform. This particular vehicle, known as “Rocky”, was one of Virginia Tech’s entries in the 2005 DARPA Grand Challenge (GC). Rocky, shown in Figure 8, is a four-wheeled, Ackermann-steered vehicle powered by a 20-horsepower Kubota diesel engine. Rocky was modified by the Virginia Tech GC team (VTGC) to be capable of fully autonomous operation. The vehicle has both a GPS and an inertial navigation system (INS), which were responsible for determining the vehicle’s position and speed.

Figure 8. Rocky
3.2 Ackermann Steering

Ackermann steering is a double-pivot steering linkage system that is used by most modern ground vehicles. This steering system was designed so that the rearward-facing side steering arms would converge at a single point on an extended centerline of the rear axle [6]. This would cause all wheels of the vehicle to have the same center of curvature. The result is that the inner wheel of the vehicle would rotate to a greater angle than the outer wheel. If the front wheels of the vehicle turn to the same angles, it would cause the wheels to scrub over the ground while the vehicle was following a curved path. The Ackermann design helps prevent the wheels from scrubbing [6]. In practice, however, the design completely prevents scrubbing at only three wheel positions: straight ahead and at one angle on either side of straight ahead [6]. Figure 9 shows a schematic of the steering system. When using the traditional layout for an Ackermann steering assembly, there is an over-Ackermann effect that occurs at small steering angles. This coupled with the large slip angles that occur during fast cornering, means that large amounts of scrubbing would occur at small angles when using a traditional layout. As a result of this, vehicles are usually designed with a reduced Ackermann effect. This is accomplished by having less divergence between the wheels on turns, and the rearward-facing side steering arms lie on lines that meet behind the vehicle’s back axle [6].

![Figure 9. Ackermann steering system](image_url)
3.3 Description of the Investigation

The investigation of the clothoid assumptions required an analysis of the vehicle’s steering system and an evaluation of the vehicle’s ability to drive clothoid-shaped paths. The analysis of the steering system assembly was done to explain how the system responds to a commanded input. The goal was to determine if the system response can be accurately represented by a bicycle model. The clothoid evaluation was done by performing two experiments and analyzing their results. The first experiment was designed to verify whether the vehicle does drive a clothoid-shaped path when a constant vehicle velocity and constant steering rate are input into the system. The second experiment was designed to demonstrate how the vehicle behaves when it is accelerating while steering at a constant rate. The intent was to determine how much the actual path deviates from the predicted path as a result of the acceleration.

The analysis of the steering system involved building a planar kinematic model of the vehicle’s steering assembly. This was done by measuring the lengths of all the links involved in the steering assembly and then creating a model of the system using Matlab. The model was able to depict how the vehicle would react to changes in the displacement of the rack, which was the controllable variable of the steering system. The analysis also required determining the accuracy of the existing conversion factor between the encoder counts and the vehicle’s steering angle, which was used in the vehicle’s computer code. This process revealed how accurately the vehicle responded to the inputs. This understanding was necessary because during the experiments, the only steering data the system was able to measure and record was the converted encoder data. The analysis of the steering system also required understanding how an input command was passed through the system. It was necessary to determine the original input units, how that input was entered into the steering system, and how the input was converted to different units by the various parts of the system until it reached the wheels in the form of an angle. Another reason for this process was to determine if there were any points where the input was distorted by errors in parts of the system.

Two experiments were performed to determine the vehicle’s ability to drive clothoid-shaped curves. The first experiment was designed to verify if the vehicle could indeed drive a clothoid-shaped curve when meeting the stated assumptions. The vehicle was commanded to drive at a constant velocity and turn with a constant steering rate. The vehicle’s steering rate,
speed, and position data were then compared with the predicted path's data. The second experiment commanded the vehicle to turn at a constant steering rate while accelerating. This experiment demonstrated how much the vehicle’s path deviates from the clothoid-shaped predicted paths. For this experiment, the vehicle’s path was compared with three predicted paths with velocities set at the actual path’s initial velocity, its final velocity, and the average of these two velocities. The objective was to see if any of these could be used to reliably predict the vehicle’s path.
Chapter 4

STEERING ASSEMBLY ANALYSIS

This section will discuss the kinematic model of the vehicle and the conversions that were necessary to interpret the data for this analysis. The conversion section walks through what the initial input of the system is and how it travels through the system to the wheels of the vehicle.

4.1 Kinematic Model

The bicycle model makes the assumption that the steering assembly can be represented by a single wheel at the center of the steering assembly. As the figure below shows, the steering assembly was much more complicated than the bicycle model would suggest even though this model only included the planar kinematic equations. The figure is a scaled schematic of the steering linkages on Rocky.

![Diagram of the steering linkage system](image)

Figure 10. Model of the steering linkage system
The only independent variable in this model is the rack displacement, $s$, which can be seen more clearly in Figure 11. The link lengths ($L_{1-6}$) of the steering assembly as well as $\theta_5$ and $\theta_6$ are constant values. $\theta_{1-4}$ are dependent on the rack displacement. This allows the changing of the displacement of the rack to cause the wheel angles to change. However, the relationship between the wheel angles and the displacement is not linear, as shown in Figure 12.

Figure 11. Model of the steering linkage system with 2.625 inches of displacement
As the steering displacement increases, the angle of the inside wheel of a turn will increase more than the angle of the outside wheel. For a right turn, the right wheel has a larger wheel angle than the left and vice versa for a left turn. This is a result of the Ackermann-steering assembly which is on most four-wheel vehicles. As discussed above, in a practical Ackermann-steering assembly, the lines coming off the tires only cross the centerline of the back axle at the same place twice, as seen in Figure 13. This means that even with Ackermann steering, at least one of the tires will slip at all steering angles except at those two angles and when the vehicle is driving straight [6]. The rest of the time the steering angle of the vehicle is somewhere between the angles made by the left and right tires. There is no way of predicting the vehicle’s overall steering angle precisely in this situation because it is impossible to know how much each wheel will slip. As a result, the bicycle model was evaluated by analyzing the position data versus the predicted clothoid path. If the vehicle was able to meet the steering rate and velocity assumptions, the error in the position data would represent the accuracy of the bicycle model assumption.
4.2 Conversions

It was critical for interpreting the results of the experiment to understand how the different components of the steering system react to an input. The steering assembly includes the rack and pinion, the steering linkages, the encoder, the 5:1 gearhead, and the roller chain coupling which are all shown in Figure 14. When an input command is made, the input travels through the system in the following order: the encoder, the gearhead, the coupling, the pinion, the rack, the steering linkages, and lastly the wheels. Every time the input travels from one part to the next it undergoes a unit conversion. The conversions that were needed for the purpose of this analysis were encoder counts to pinion shaft rotations, pinion shaft rotations to rack displacement, and rack displacement to wheel angles. Using these conversions, it was possible to convert the encoder counts, which were recorded throughout the experiment, to the wheel angles of the vehicle.
The conversion between rack displacement and the wheel angles was found by using the kinematic model described in the previous section. The measurements of the vehicle’s steering assembly link lengths were gathered and their relationships to each other were programmed into the code. The code was able to take different rack displacement inputs and adjust the links’ positions accordingly. This, in turn, would change the wheel angles since they are attached to the links. This relationship was discussed in more detail in the previous section. The equations for the links can be seen in Appendix A. The corresponding Matlab code is provided in Appendix D.
The wheel angles needed to be simplified to a single steering angle at each rack displacement increment in order to calculate the predicted clothoid path. Using the kinematic model, the steering angles were calculated as the line that is perpendicular to the line connecting the center of the front axle and the point where the lines perpendicular to the front wheels cross. This is the same approach as discussed in the Ackermann Steering section except that the intersection does not occur on the back axle as shown in Figure 15. The resulting estimated steering angles can be seen in Figure 16.

Figure 15. Method used to estimate the steering angle of the vehicle
Using a spare rack and pinion, the conversion between pinion shaft rotations and rack displacement was measured. The measurements proved that for every rotation of the shaft, the rack displacement changed by 1.375 inches. This ratio was consistent no matter the starting point or rotation direction. The total amount that the rack was able to move was 5.25 inches. This was the displacement caused by starting at one extreme and rotating to the other extreme. This translated into the pinion shaft having 3.82 total possible rotations. Appendix D includes all of the data on the relationship between pinion and rack.

When analyzing the relationship between encoder counts and pinion shaft rotations, it became apparent that there was some kind of mechanical error occurring. Table 1 shows the data from this test. For this test, the zero count position was set as 100,000 counts from a rack displacement of zero. The reason for this was to give the pinion the ability to make two full rotations. If the zero count position had been set at a rack displacement of zero, the pinion would not have been able to make the two rotations. For each trial, the pinion was spun in the opposite direction of the initial turn of 100,000 counts. It was spun in that direction twice and then spun in the opposite direction twice. This resulted in the pinion changing directions at the beginning and at the middle of each trial. Every time the pinion changed direction, the first rotation would
take an extra $9000 \pm 200$ counts to complete it when compared with the second rotation in that direction. This difference was most likely caused by a backlash error in the rollover chain coupling which is positioned between the gearhead and the pinion shaft. Taking this into account, the number of counts needed to rotate the pinion shaft one rotation without any backlash was $80,200 \pm 100$ counts on average. Given that the encoder is designed to have 16,000 counts in a rotation and that it is connected to a 5:1 gearhead, ideally it should take 80,000 counts. Considering the margin of error in the measurements, the experimental data supports this ideal conversion factor and it was used for the rest of the analysis.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Direction</th>
<th>Number of Rotations</th>
<th>Count Position</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
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<td>-89093</td>
<td>89093</td>
</tr>
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</tr>
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<td>88</td>
<td>80399</td>
</tr>
<tr>
<td>3</td>
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<td>89298</td>
</tr>
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<td>93</td>
<td>80299</td>
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<td>89298</td>
</tr>
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<td>80096</td>
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</tr>
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<td>80095</td>
</tr>
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<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>89</td>
<td>80200</td>
</tr>
</tbody>
</table>

It was necessary to determine the actual angles of the wheels when they are commanded to a set steering angle. This was done by commanding turns to the right and left of 10, 20, 30, and 40 degrees. This experiment was conducted with the wheels raised off the ground, Figure 17, and with them resting on the ground, Figure 18. The positive angles were right turns while the left turns produced negative angles. The data was gathered by first zeroing the rack displacement. Then a 10 degree turn was commanded and the wheel angles measured. Next, a -10 degree turn was commanded and wheel angles measured. A 20 degree angle turn was
commanded and measured after that. This procedure continued until all of the measurements at all of the turns were completed. The figures show that the vehicle’s suspension and the friction between the wheels and the ground do affect the wheel angles somewhat. However, the effects of the vehicle suspension on the steering system were beyond the scope of this investigation. The plots also show that the conversion used by the vehicle’s controller program was inaccurate. Also, given how the test was performed, the backlash error in the roller-chain coupling was the most likely cause of all the left turns having slightly smaller angles than the corresponding right turns. Unfortunately, these errors were not discovered until after the experiment trials were completed and the data collected. Given these errors introduced by the Motion Control’s conversion factor and the roller-chain coupling’s backlash, the steering angle measurements gathered do not accurately represent the vehicle’s actually steering angles. The measurements do, however, show how accurately the actuator performed in terms of encoder counts. Therefore, for the purposes of determining if the encoder can actuate a constant steering rate, the encoder counts were analyzed to determine if the encoder performed as commanded.

![Figure 17. Wheel angle test with the wheels lifted off the ground](image-url)
It was necessary to determine an estimated conversion for the initial steering angle, final steering angle, and the steering rate. These variables were needed to calculate a predicted clothoid-shaped path, which was subsequently used to compare against the actual paths the vehicle drove. The procedures for determining the predicted path will be discussed in the next section, but the conversions of three variables are discussed here.

Figure 18 shows what the different output wheel angles were after being commanded to turn to multiple steering angles. When the vehicle was commanded a steering angle of 30 degrees, the driver and passenger side wheels turned 25.9 ± 0.2 and 24.8 ± 0.3 degrees on average, respectively. As a result, the steering angle of the vehicle when commanded to be 30 degrees was estimated as the average of these two values or 25.35 ± 0.6 degrees. This angle equates to a rack displacement of -1.646 inches. This displacement was calculated using the kinematic model generated using MatLab, Appendix D. The final steering angle was estimated as –24.3 ± 0.4 degrees which was the average of –24.0 ± 0.4 and –24.6 ± 0.2 degrees. The necessary rack displacement needed for this angle was 1.584 inches.

To verify these calculations, the averaged encoder data from the trials were also used to determine the initial and final angle of the vehicle. The recorded steering angles were converted to encoder counts, then to pinion rotations, then to rack displacements, and then the kinematic model was used to calculate the steering angles from the rack displacements. Table 2 shows the list of conversion factors. The conversion from rack displacement to steering angle required the
use of Figure 16. The initial and final angles were calculated to be 25.4 and -25.4 degrees, respectively. The two different initial angles varied by only 0.24% which supports the approach taken for the initial angle calculations. The difference between the two final angles was a bit greater at 4.37%. This was most likely the result of the backlash error. It was decided that the initial and final angles calculated from the encoder data would be the ones used in calculating the predicted clothoid. The reason for this decision was that it would enable both the steering rate and the predicted amount of time needed to drive the clothoid path to remain the same as the ideal output of the commanded values. If -24.3 ± 0.4 degrees was selected as the final angle instead of -25.4 degrees, either the steering rate or the amount of time needed would have to change, otherwise the vehicle would not be able to reach the final angle at the correct time.

**Table 2. List of conversion factors**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>40 recorded degrees</td>
<td>128000 encoder counts</td>
</tr>
<tr>
<td>80000 encoder counts</td>
<td>1 pinion rotation</td>
</tr>
<tr>
<td>1 pinion rotation</td>
<td>1.375 inches of rack displacement</td>
</tr>
</tbody>
</table>

For the steering rate, it was necessary to perform the same conversions as the steering angle. As Figure 16 shows, the relationship between the rack displacement and the wheel angles was not completely linear. The predicted path must have a constant steering rate since that is the definition of a clothoid. This required the steering rate to be simplified to a constant value that goes from the initial angle to the final angle in 3 seconds. The reason for the 3 seconds requirement was because that is how long it would take an ideal system to turn from a 30 degree right turn to a 30 degree left turn at 20 degrees per second, which was the intent of the values commanded into the vehicle’s control system. As a result, the vehicle should have ideally taken 3 seconds to turn its wheel from the initial steering angle to the final angle. Following this line of thought, the steering rate that was used in the predicted clothoid calculation was 16.9 degrees per second.
Chapter 5
EXPERIMENTAL RESULTS

5.1 Experiment 1

This experiment, performed with the vehicle traveling at 5 mph and turning from a 25.41 degree right turn to a 25.41 degree left turn at a constant steering rate of 16.9 degrees per second (64,000 encoder counts per second), showed how the vehicle performs when commanding inputs that, ideally, produce a clothoid path. The objective of this experiment was to test the steering actuator’s ability to command a constant steering rate, and the vehicle’s ability to drive at a constant velocity. If the actuator and the vehicle were able to do this, then the result should have been a clothoid-shaped path.

5.1.1 Steering Rate. The first assumption that was evaluated was the constant steering rate assumption. Figure 19 shows that the steering rate was nearly constant through each trial. There was an average $0.072 \pm 0.009$ seconds needed for the actuator to accelerate up to the desired rate, and an average time of $0.184 \pm 0.041$ seconds needed to decelerate to 0 degrees per second. While this proves that the actuator needs some time to accelerate to the desired rate, the amount of difference this caused between the actual time and the predicted time was only $0.150 \pm 0.090$ seconds, or 5% of the total amount of the predicted time. Table 3 shows the total travel time it took the vehicle to turn from a 25.4 degree steering angle to a -25.4 degree steering angle at a 16.9 steering rate for each trial. These values were determined by using the conversion discussed in the previous section. Trial 2 took the longest to decelerate because the steering angle overshot the desired angle during this run. This caused the actuator to command a negative steering rate for 0.141 seconds. However, the overshoot error was only 0.005% of the total turn so for practical purposes the error can be ignored. The average steering rate for the experiment was $59,900 \pm 7400$ counts per second. This average has a difference of 6.4% from the desired rate of 64,000 counts per second. While this difference is a little larger than desired, it is to be expected with the acceleration and deceleration time totaling 5% of the travel time. These percent differences are still quite small and, therefore, the data validates the assumption that the steering actuator can command a nearly constant steering rate.
The data in Figure 19 is shown in encoder counts to more accurately represent the encoder performance. As stated above, the steering angle and the rack displacement did not have a perfectly linear relationship. Since there was inadequate time to develop and implement a more accurate relationship, the encoder data is displayed. The encoder data showed just how responsive the steering actuator was to its input, which was one of the key assumptions under investigation. The encoder data demonstrated that the actuator was able to command a nearly constant rate.

Table 3. Time it took the vehicle to turn in Experiment 1

<table>
<thead>
<tr>
<th>Trial number</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 1</td>
<td>3.078</td>
</tr>
<tr>
<td>Trial 2</td>
<td>3.297</td>
</tr>
<tr>
<td>Trial 3</td>
<td>3.078</td>
</tr>
<tr>
<td>Trial 4</td>
<td>3.141</td>
</tr>
<tr>
<td>Trial 5</td>
<td>3.156</td>
</tr>
<tr>
<td>Predicted</td>
<td>3</td>
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</tbody>
</table>
5.1.2 **Velocity.** The next assumption that was evaluated was the constant velocity assumption. The plot in Figure 20 shows that the velocity of the vehicle centers around 5 mph. The average speed of the vehicle throughout the experiment was 5.03 mph with a standard deviation of ± 0.21 mph. The difference between the commanded velocity of 5 mph and the average velocity was 0.6%. Given that the error was less than one percent, the assumption that the vehicle was able to travel at a constant velocity is reasonable.

![Vehicle velocity versus time for Experiment 1](image)

**Figure 20.** Vehicle velocity versus time for Experiment 1

5.1.3 **Position.** The paths the vehicle traveled during the experiment and the predicted clothoid path are shown in Figure 21. The figure reveals that actual vehicle paths were quite similar to the predicted clothoid path. Since the steering rate and velocity assumptions were already proven reasonable, this result means that the bicycle model is also a reasonable assumption. The main difference between the actual paths and the predicted path was that the actual paths took a little longer to reach the desired angle than the predicted path. The reason for this difference, as explained above, was the small amount of acceleration and deceleration time needed at the beginning and the end of the paths.
Figure 21. The actual and predicted paths the vehicle traveled during Experiment 1

A comparison of the clothoid path positions and the average position data for the experiment is shown in Figure 22. The maximum average difference between the experimental paths and the predicted paths was only 0.40 meters and occurred after the vehicle had traveled 6.73 meters in about 3 seconds. This equates to a difference of 5.9% of the total path length. Given how small the differences were between the clothoid path’s positions and the average position data for the experiment, it was concluded that the clothoid path offers a good prediction of where the vehicle will be in a few seconds.
The data gathered during this experiment proved that the stated assumptions were indeed valid and that the vehicle was able to drive a clothoid-shaped path if these assumptions are met. The steering angle rate was fairly constant throughout each trial of the experiment. The actuator’s acceleration times were short enough that they could be considered instantaneous. The vehicle was able to drive at approximately a constant velocity. The average velocity error was only 5% of the total speed and was thus determined to be close enough for the purposes of this experiment. When comparing the vehicle’s actual and predicted path, the maximum error was only 0.40 meters at the end of the path.

5.2 Experiment 2

This experiment was designed to see how accelerating affects the vehicle’s path and its ability to drive a clothoid path. For this experiment, the vehicle was commanded to accelerate from 5 to 10 mph while turning from a 25.4 degree right turn to a 25.4 degree left turn at a rate of 16.9 degrees per second. The predicted clothoid paths used for the comparison were a 5, 7.5, and 10 mph clothoid. These clothoids were chosen for the comparison since they represented the commanded starting velocity, ending velocity, and the midpoint between them.
5.2.1 Steering Rate. The steering rate was not affected by the vehicle’s acceleration, as shown in Figure 23. The figure is almost identical to Figure 19 from the first experiment. The average time needed to accelerate to the desired steering rate was 0.067 ± 0.008 seconds. The figure shows that for each trial, when decelerating, the rate actually drops below zero for a few tenths of a second. The reason being is the vehicle slightly overshot the desired angle each time. This extra time that was needed to settle the overshoot explains why all of the trial times are longer than the predicted 3 seconds, as seen in Table 4. The overshoots were very small though, on average only 0.0081% of the total turn. As a result, the extra time needed to settle the overshoots was ignored due to the overshoots’ small nature. This means that the deceleration time for the experiment was 0.141 ± 0.0005 seconds. The average steering rate in counts was 60,700 counts per second with a standard deviation of ± 5700 counts per second. This equates to a difference of 5.1% from the desired rate of 64,000 counts per second. With such small percent differences, the data validates the assumption that the steering actuator can command a nearly constant steering rate.

![Figure 23. Steering encoder rate versus time for Experiment 2](image)
Table 4. Time it took the vehicle to turn in Experiment 2

<table>
<thead>
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<tbody>
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<td>Trial 3</td>
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<td>Trial 4</td>
<td>3.219</td>
</tr>
<tr>
<td>Predicted</td>
<td>3</td>
</tr>
</tbody>
</table>

5.2.2 Velocity. The vehicle velocities, as seen in Figure 24, were very consistent throughout the experiment. The figure shows that for each trial it took almost 0.5 seconds after the vehicle applied full throttle before any change in velocity was apparent. Also, the vehicle overshot the desired 10 mph and was not able to settle on the desired speed before it had reached its desired steering angle. It is interesting to note just how consistent the speed was for each trial.

![Vehicle velocity versus time for Experiment 2](image)

Figure 24. Vehicle velocity versus time for Experiment 2

5.2.3 Position. The objective of this experiment was to determine how much difference there was between the actual path the vehicle drove and the predicted clothoid paths at its starting and stopping speeds. Figure 25 shows that the end points of both clothoid paths were far
off from the end points of the experimental trials. The difference between the end point of the 5 mph clothoid and the average end point of the trial was 4.91 meters. Given that the predicted path was only 6.7 meters, this represents a difference of 73% of its total predicted length. While the 10 mph clothoid’s end point was closer to the trial’s average end point, it was still off by 2.94 meters. These results prove that if the vehicle is accelerating, it will not closely follow a single clothoid predicted path.

![Figure 25. The actual and predicted paths the vehicle traveled during Experiment 2](image)

Figure 26 shows the average difference between the actual paths traveled and the predicted clothoid paths with speeds of 5, 7.5, and 10 mph. The graph clearly shows that while the 7.5 mph clothoid is much closer to the paths than the 5 or 10 mph clothoids, it is still off by 1.84 meters after 3 seconds. This equates to a difference of 27.5% of the total path length of the clothoid. This proves that clothoids are significantly less accurate in predicting a vehicle path while it is accelerating than when the velocity is constant.
Figure 26. The average difference between the actual paths the vehicle traveled and the three clothoid paths.
Chapter 6
CONCLUSIONS

6.1 Overall Conclusions

The curve type known as a clothoid has been a common choice for autonomous ground vehicle navigation. The three basic assumptions that are made when using this curve type are that the steering assembly can be represented by a bicycle model, the vehicle’s steering rate is constant, and the vehicle is traveling at a constant velocity. If these assumptions are reasonably met, the vehicle’s path should be able to be predicted by a clothoid.

Based on the results of this investigation, a vehicle is able to drive a clothoid-shaped path provided it is not accelerating or changing its steering rate. When the vehicle was commanded to drive at a constant velocity with a constant steering rate, its path could be accurately predicted by a clothoid curve. If the vehicle was commanded to accelerate, however, the resulting path could not be accurately predicted using clothoid paths. The end point of the driven path would be far off from the predicted end point.

While the steering assembly can be represented by a bicycle model, it is important to accurately model the relationships between the steering actuator, the pinion shaft, the rack, and the wheel angles. The results of this investigation demonstrated that the relationship between the rack displacement and wheel angles is nonlinear. To increase the accuracy of the vehicle steering system, this nonlinear relationship needs to be factored into the control algorithms. Other errors, such as backlash, can occur between the steering actuator and the wheels. It is important to analyze a vehicle to minimize the effect of these errors. It is also important to note that the bicycle model is a simplification; it will not precisely represent the vehicle’s steering angle. This potential error in the vehicle’s predicted steering position should, therefore, be taken into account.

While a clothoid path does have limitations, it is a reasonably accurate way of predicting a vehicle path provided that the assumptions made are understood and the navigation system only uses them when the stated assumptions are met.
6.2 Future Work

The results of this investigation suggest several other areas of research. Now that a planar kinematic model of the steering system has been developed, the next step would be to develop a three-dimensional model of the system. Such a model would be able to examine elements such as the wheels’ caster and camber angles, bump steering due to suspension travel, and the effects that these factors have on the overall performance of the steering system. An even more advanced model would include the dynamics involved in the steering system when the vehicle is in motion. This would be especially important when the speed of the vehicle approaches the vehicle’s limits. These higher-level models would provide a more accurate representation of the steering system and a more refined assessment of the vehicle’s ability to follow a planned path.

Another area of research that could be explored is an analysis of the necessary update rate of the planned path versus the speed of the vehicle along that planned path. The goal would be to determine the maximum safe speed that the vehicle could travel while updating its path plan at a given rate. Another topic for research would be an investigation of other curve types used for path generation and tradeoffs in their update rates. This would involve studying the difference in update rates for splines, circular arcs, clothoids, and line segments. It would also involve analyzing tradeoffs related to computational cost, path discontinuities, and other factors that arise when using these different curve types.

A final topic for research would be to develop a new navigation method that enhances the use of clothoids. The method would take full advantage of the positive characteristics of a clothoid-shaped path, but fully take into account the curve type’s limitations.


APPENDIX A - Ackermann Steering Equations

The following section explains how the model for the steering system was developed in Matlab.

R = links of the steering assembly
L = lines used to form triangles within the assembly
P = points where the R’s and L’s meet
a = angles between L’s
θ = angles between R’s
s = the amount the rack shifts from centered position

Figure A-1. Steering Assembly
Left Side of System:

\[ L_1 = \sqrt{d^2 + \left( s + \frac{R_4}{2} \right)^2} \]

\[ a_1 = \sin^{-1}\left( \frac{s + \frac{R_4}{2}}{L_1} \right) \]

\[ a_2 = 90 - a_1 \]

\[ L_2 = \sqrt{\left( \frac{R_1}{2} \right)^2 + L_1^2 - 2\left( \frac{R_1}{2} \right)L_1\cos(a_2)} \]

\[ a_3 = \cos^{-1}\left( \frac{L_2^2 + R_s^2 - R_5^2}{2L_2R_s} \right) \]

\[ a_4 = \cos^{-1}\left( \frac{L_2^2 + \left( \frac{R_1}{2} \right)^2 - L_4^2}{2L_2\left( \frac{R_1}{2} \right)} \right) \]

\[ \theta_4 = a_3 + a_4 + 180 \]

\[ \theta_2 \text{ and } \theta_3 \text{ were solved using Euler Formula.} \]

\[ d_j - \frac{R_4}{2} + R_2 e^{j\theta} - R_4 e^{j\theta_2} + s - \frac{R_4}{2} = 0 \]

\[ \frac{R_4}{2} - s + R_2 \cos(\theta) - R_3 \cos(\theta_2) - \frac{R_4}{2} = 0 \]

\[ d_j + jR_2 \sin(\theta) - jR_3 \sin(\theta_2) = 0 \]

\[ \theta_2 = 180 - \sin^{-1}\left( \frac{d + R_2 \sin(\theta)}{R_3} \right) \]

Right Side of System:

\[ L_3 = \sqrt{d^2 + \left( -\frac{R_4}{2} + s \right)} \]

\[ a_5 = \sin^{-1}\left( \frac{R_4 - s}{2L_3} \right) \]

\[ a_6 = 90 - a_5 \]

\[ L_4 = \sqrt{\left( \frac{R_1}{2} \right)^2 + L_3^2 - 2\left( \frac{R_1}{2} \right)L_3\cos(a_6)} \]

\[ a_7 = \cos^{-1}\left( \frac{L_4^2 + R_s^2 - R_3^2}{2L_4R_s} \right) \]

\[ a_8 = \cos^{-1}\left( \frac{L_4^2 + \left( \frac{R_1}{2} \right)^2 - L_3^2}{2L_4\left( \frac{R_1}{2} \right)} \right) \]

\[ \theta_1 = 360 - a_7 - a_8 \]

\[ d_j - \frac{R_4}{2} + R_2 e^{j\theta} - R_4 e^{j\theta_1} + s + \frac{R_1}{2} = 0 \]

\[ \frac{R_4}{2} + s + R_2 e^{j\theta} - R_3 e^{j\theta_1} - \frac{R_4}{2} - d_j = 0 \]

\[ \frac{R_4}{2} + s + R_2 \cos(\theta) - R_3 \cos(\theta_2) - \frac{R_4}{2} = 0 \]

\[ \frac{R_4}{2} + s + R_3 \cos(\theta_2) - R_4 \cos(\theta_2) - \frac{R_4}{2} = 0 \]

\[ jR_2 \sin(\theta) - jR_3 \sin(\theta_2) - d_j = 0 \]

\[ \theta_3 = \sin^{-1}\left( \frac{d + R_3 \sin(\theta)}{R_3} \right) \]
APPENDIX B – Kinematic Equations of Motion

The section shows the calculations that were done to derive the equations of motion of the system using Figure B-1, which is the same as Figure 5 [7].

\[
\sin(\phi - \beta) = \sin \left( \frac{\pi}{2} - \phi \right)
\]

\[
\frac{(\sin(\phi)\cos(\beta) - \sin(\beta)\cos(\phi)}{L_f} = \frac{\cos(\phi)}{R} \times \frac{L_f}{\cos(\phi)}
\]

\[
\tan(\phi)\cos(\beta) - \sin(\beta) = \frac{L_f}{R}
\]
\[
\sin(\beta - \phi_r) = \frac{\sin\left(\frac{\pi}{2} + \phi_r\right)}{R} \\
\left(\frac{\cos(\phi_r)\sin(\beta) - \cos(\beta)\sin(\phi_r)}{L_r} = \frac{\cos(\phi_r)}{R}\right) \times \frac{L_r}{\cos(\phi_r)} \\
\sin(\beta) - \tan(\phi_r)\cos(\beta) = \frac{L_r}{R}
\]

\[
\begin{align*}
\tan(\phi)\cos(\beta) - \sin(\beta) &= \frac{L_f}{R} \\
+ \sin(\beta) - \tan(\phi_r)\cos(\beta) &= \frac{L_r}{R} \\
(tan(\phi) - tan(\phi_r))\cos(\beta) &= \frac{L_f + L_r}{R}
\end{align*}
\]

\[
(tan(\phi) - tan(\phi_r))\cos(\beta) = \frac{L_f + L_r}{R}
\]

\[
\dot{\theta} = \omega \Rightarrow \dot{\theta} = \frac{v}{R} \Rightarrow R = \frac{v}{\dot{\theta}}
\]

\[
R = \frac{L_f + L_r}{(tan(\phi) - tan(\phi_r))\cos(\beta)}
\]

\[
\frac{v}{\dot{\theta}} = \frac{L_f + L_r}{(tan(\phi) - tan(\phi_r))\cos(\beta)}
\]

\[
\dot{\theta} = \frac{v \cdot \cos(\beta)}{(L_f + L_r)}(tan(\phi) - tan(\phi_r))
\]

\[
\phi_r = 0
\]

\[
\dot{\phi} = \frac{v \cdot \cos(\beta)}{(L_f + L_r)}\tan(\phi)
\]

\[
\ddot{X} = v \cdot \cos(\theta + \beta)
\]

\[
\dot{Y} = v \cdot \sin(\theta + \beta)
\]
\[ L_r (\tan(\phi) \cos(\beta) - \sin(\beta)) = \frac{L_f}{R} \]
\[ L_f \left( \sin(\beta) - \tan(\phi_r) \cos(\beta) = \frac{L_r}{R} \right) \]
\[ (L_f + L_r) \sin(\beta) = \cos(\beta) (L_f \tan(\phi_r) + L_r \tan(\phi)) \]
\[ \begin{align*}
\frac{\sin(\beta)}{\cos(\beta)} &= \frac{L_f \tan(\phi_r) + L_r \tan(\phi)}{L_f + L_r} \\
\tan(\beta) &= \frac{L_f \tan(\phi_r) + L_r \tan(\phi)}{L_f + L_r} \\
\beta &= \tan^{-1} \left( \frac{L_f \tan(\phi_r) + L_r \tan(\phi)}{L_f + L_r} \right) \\
\phi &= 0 \\
\beta &= \tan^{-1} \left( \frac{L_r \tan(\phi_r)}{L_f + L_r} \right) \]
APPENDIX C – Wheel Angle Measurements

This appendix shows all of the wheel measurement data gathered when determining the accuracy of the computer code’s conversion factor between steering and wheel angle. For the Trial 4 in Figures C-5 and C-6, an incorrect angle was commanded before the 40 degree turn was commanded. The angle was larger than the maximum angle that the vehicle could turn to so the wheels turned to their maximum position. This was the mostly cause of the error seen at the 40 and –40 degree turns. For this reason, these points were not included in the average calculations.

Figure C-1. Wheel angle measurements with the front wheels resting on the ground
Table C-1 lists the wheel angle averages and standard deviations while the wheels were resting on the ground. The averages are the same as those shown in Figure C-1.

**Table C-1.** List of the wheel angle averages and standard deviations while the wheels were resting on the ground

<table>
<thead>
<tr>
<th>Turns (R,L)</th>
<th>Turns (+,-)</th>
<th>Driver Side Angle Average (deg)</th>
<th>Passenger Side Angle Average (deg)</th>
<th>Driver Standard Deviation (deg)</th>
<th>Passenger Standard Deviation (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 R</td>
<td>10</td>
<td>8.9</td>
<td>6.3</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>10 L</td>
<td>-10</td>
<td>5.4</td>
<td>8.2</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>20 R</td>
<td>20</td>
<td>17.4</td>
<td>15.2</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>20 L</td>
<td>-20</td>
<td>14.4</td>
<td>16.3</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>30 R</td>
<td>30</td>
<td>25.9</td>
<td>24.8</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>30 L</td>
<td>-30</td>
<td>24.0</td>
<td>24.6</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>40 R</td>
<td>40</td>
<td>34.4</td>
<td>36.7</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>40 L</td>
<td>-40</td>
<td>35.6</td>
<td>33.5</td>
<td>0.2</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**Figure C-2.** Wheel angle measurements with the front wheels raised off the ground
Table C-2 lists the wheel angle averages and standard deviations while the wheels were raised off the ground. The averages are the same as those shown in Figure C-2.

Table C-2. List of the wheel angle averages and standard deviations while the wheels were raised off the ground

<table>
<thead>
<tr>
<th>Turns (R,L)</th>
<th>Turns (+,-)</th>
<th>Driver Side Angle Average (deg)</th>
<th>Passenger Side Angle Average (deg)</th>
<th>Driver Standard Deviation (deg)</th>
<th>Passenger Standard Deviation (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 R</td>
<td>10</td>
<td>10.9</td>
<td>9.4</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>10 L</td>
<td>-10</td>
<td>7</td>
<td>8.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>20 R</td>
<td>20</td>
<td>19.5</td>
<td>18.5</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>20 L</td>
<td>-20</td>
<td>16.5</td>
<td>17.2</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>30 R</td>
<td>30</td>
<td>28.3</td>
<td>30.0</td>
<td>0.4</td>
<td>0.0</td>
</tr>
<tr>
<td>30 L</td>
<td>-30</td>
<td>27.4</td>
<td>25.8</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>40 R</td>
<td>40</td>
<td>38</td>
<td>46.4</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>40 L</td>
<td>-40</td>
<td>42</td>
<td>34.9</td>
<td>0.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Figure C-3. Driver side wheel angle measurements with the front wheels on the ground
Figure C-4. Passenger side wheel angle measurements with the front wheels on the ground

Figure C-5. Driver side wheel angle measurements with the front wheels raised off the ground
Figure C-6. Passenger side wheel angle measurements with the front wheels raised off the ground
APPENDIX D – Rack and Pinion Measurements

This section presents the data gathered on the relationship between the pinion shaft and the rack displacement. The measurements show that a single rotation of the pinion shaft causes the rack to move 1.375 inches. The maximum number of possible rotations was 3.82.

![Figure D-1. Rack and pinion measurement setup](image)

<table>
<thead>
<tr>
<th>Number of Rotations &amp; Direction</th>
<th>Length (inches)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Left</td>
<td>1.375</td>
<td>1.375</td>
</tr>
<tr>
<td>2 Left</td>
<td>2.75</td>
<td>1.375</td>
</tr>
<tr>
<td>3 Left</td>
<td>4.125</td>
<td>1.375</td>
</tr>
<tr>
<td>3.82 Max to Left</td>
<td>5.25</td>
<td>1.125</td>
</tr>
<tr>
<td>1 Right</td>
<td>1.375</td>
<td>1.375</td>
</tr>
<tr>
<td>2 Right</td>
<td>2.75</td>
<td>1.375</td>
</tr>
<tr>
<td>3 Right</td>
<td>4.125</td>
<td>1.375</td>
</tr>
<tr>
<td>3.82 Max to Right</td>
<td>5.25</td>
<td>1.125</td>
</tr>
</tbody>
</table>
APPENDIX E – Matlab Code

This appendix shows the Matlab code that was used to analyze the steering system and to calculate the predicted clothoid paths.

Thesis Function

%The Main function that takes the Trial data, pulls out the data of interest, calculates and plots the predicted clothoid path, and plots the vehicle’s actual position, speed, and steering rate.

function [time,speed,steer,rate,pos,begin,finish,ave_pos,ave_speed,ave_steer,ave_time,time_end,x,y,time_cg,mark,second,pred_stop,difference,ave_diff,path_length,percent_difference,ave_diff_speed,ave_diff_steer,diff_speed,diff_steer,turn,steer_angle] = thesis(Trial1,Trial2,Trial3,Trial4,Trial5,start,stop,start1,stop1,stop2,experiment,exclude_overshoot)

%A second function is needed for the 2nd experiment when there are only 4 trials

%function [time,speed,steer,rate,pos,begin,finish,ave_pos,ave_speed,ave_steer,ave_time,time_end,x,y,time_cg,mark,second,pred_stop,difference,ave_diff,path_length,percent_difference,ave_diff_speed,ave_diff_steer,diff_speed,diff_steer,turn,steer_angle] = thesis(Trial1,Trial2,Trial3,Trial4,start,stop,start1,stop1,stop2,experiment,exclude_overshoot)

%start = 2 seconds before start of turn
%stop = when throttle stops being applied adjusted to be the same length
%start1 = when start of clothoid
%stop1 = when clothoid ends adjusted so that each trial is the same length
%stop2 = when clothoid ends unadjusted
%stop3 = to show when the speed of the 3rd experiment settles
%stop4 = when throttle ends unadjusted

close all;  %The average position of the vehicle during the trials
ave_pos = 0;  %The average speed of the vehicle during the trials
ave_speed = 0;  %The average steering angle of the vehicle during the trials
ave_steer = 0;  %The average time stamps of when the data was collected
ave_time = 0;  %Predicted clothoid time
conversion = 128000/40;  %The conversion of degrees to counts used by the MC

%Overshoot variable
overshoot_check = 1;  %0 means it includes overshoot, 1 means it excludes it

%Experiment number
if experiment == 2
trial = 4;
else
    trial = 5;
end

%Extracts the Data for the clothoid part of the path from all of the data collected
[heading(:,1), northing(:,1), easting(:,1), speed(:,1), steer(:,1), rate(:,1), minute(:,1), second(:,1)] = data_extractor(Trial1,start1(1),stop1(1));
[heading(:,2), northing(:,2), easting(:,2), speed(:,2), steer(:,2), rate(:,2), minute(:,2), second(:,2)] = data_extractor(Trial2,start1(2),stop1(2));
[heading(:,3), northing(:,3), easting(:,3), speed(:,3), steer(:,3), rate(:,3), minute(:,3), second(:,3)] = data_extractor(Trial3,start1(3),stop1(3));
[heading(:,4), northing(:,4), easting(:,4), speed(:,4), steer(:,4), rate(:,4), minute(:,4), second(:,4)] = data_extractor(Trial4,start1(4),stop1(4));
if trial == 5;
    [heading(:,5), northing(:,5), easting(:,5), speed(:,5), steer(:,5), rate(:,5), minute(:,5), second(:,5)] = data_extractor(Trial5,start1(5),stop1(5));
end

%Initializes the x position, y position, and time variables
temp_size = size(second);
xpos = zeros(temp_size(1),temp_size(2));
ypos = zeros(temp_size(1),temp_size(2));
time = zeros(temp_size(1),temp_size(2));
for index = 1:trial
    for z = 1:temp_size(1)
        ypos(z,index) = northing(z,index) - northing(1,index);
        xpos(z,index) = easting(z,index) - easting(1,index);
        time(z,index) = second(z,index) - second(1,index);
    end
end

%Limits the data to 6 significant figures
time = vpa(time,6);
time = double(time);
s = size(xpos);
for index = 1:trial
    trans = [cosd(heading(1,index)) -sind(heading(1,index));
    sind(heading(1,index)) cosd(heading(1,index))];
    for z = 1:s(1)
        pos(index,z,:) = trans * [xpos(z,index); ypos(z,index)];
    end
end

%Plots Only the Clothoid Part
begin = 1;
if experiment == 1
    finish = stop2 - start1 + 1;  %For 5mph
else
    finish = stop2 - start1 + 1;  %For 5mph
end
finish = stop1 - start1 + 1;  %For accel experiment
end

%Finds the maximum length of time needed for a trial
time_end = max(time);
time_end = max(time_end);

%Plots Steering Rate vs. Time
figure(4)
hold on;
plot(time(1:finish(1),1),conversion*rate(1:finish(1),1),'b');
plot(time(1:finish(2),2),conversion*rate(1:finish(2),2),'k');
plot(time(1:finish(3),3),conversion*rate(1:finish(3),3),'g');
plot(time(1:finish(4),4),conversion*rate(1:finish(4),4),'r');
if trial == 5
    plot(time(1:finish(5),5),conversion*rate(1:finish(5),5),'y');
end
xlabel('Time, seconds');
ylabel('Steering Encoder Rate, counts/sec');

%Overshoot Check
%Will exclude the overshoot data b/c it is so small
if overshoot_check == 0
    if experiment == 1
        finish = stop2 - start1 + 1;  %For 5mph
    else
        finish = stop1 - start1 + 1;  %For both accel experiment
    end
else
    finish = exclude_overshoot - start1 + 1;
end

%Plots the Position plot
figure(1)
hold on;
plot(pos(1,1:finish(1),1),pos(1,1:finish(1),2),'b');
plot(pos(2,1:finish(2),1),pos(2,1:finish(2),2),'k');
plot(pos(3,1:finish(3),1),pos(3,1:finish(3),2),'g');
plot(pos(4,1:finish(4),1),pos(4,1:finish(4),2),'r');
if trial == 5
    plot(pos(5,1:finish(5),1),pos(5,1:finish(5),2),'y');
end
xlabel('X Position, m');
ylabel('Y Position, m');

%Plots Steering vs. Time Plot
figure(2)
hold on;
plot(time(1:finish(1),1),steer(1:finish(1),1),'b');
plot(time(1:finish(2),2),steer(1:finish(2),2),'k');
plot(time(1:finish(3),3),steer(1:finish(3),3),'g');
plot(time(1:finish(4),4),steer(1:finish(4),4),'r');
if trial == 5
    plot(time(1:finish(5),5),steer(1:finish(5),5),'y');
end
xlabel('Time, seconds');
ylabel('Steering, degrees');

%Plots Speed vs. Time
figure(3)
hold on;
plot(time(1:finish(1),1),speed(1:finish(1),1),'b');
plot(time(1:finish(2),2),speed(1:finish(2),2),'k');
plot(time(1:finish(3),3),speed(1:finish(3),3),'g');
plot(time(1:finish(4),4),speed(1:finish(4),4),'r');
if trial == 5
    plot(time(1:finish(5),5),speed(1:finish(5),5),'y');%  
end
xlabel('Time, seconds');
ylabel('Speed, mph');

%Calculates the Averages of the Trials
[ave_pos,ave_speed,ave_steer,ave_time] =
data_average(pos,speed,steer,time,start,stop,start1,stop1,stop2);

%Calls the Ackermann_Steering function
[x_cross,y_cross,steer_angle,x,y,time_cg,pred_stop,turn] =
Ackermann_Steering(time_end,experiment);

%Limits the data to 6 significant figures
vpa(time_cg,6);
time_cg = double(time_cg);
vpa(ave_time,4);
ave_time = double(ave_time);

%Finds the predicted clothoid time (time_cg) values that correspond the
averaged time values of the trials
mark = 0;
for index = 1:trial
    three_second(index) = find(time(:,index) < 3,1,'last') + 1;
end
max_finish = max(finish);
if experiment == 1
    limit = 1;
else
    limit = 3;
end

%Finds the difference between the clothoid values and the values for each
trial
for number = 1:limit
for z = 1:max_finish
    [mark(z)] = find(time_cg(:,1) == ave_time(z));
end

for index = 1:trial
    for z = 1:three_second(1)
        difference(index,z,number) = sqrt((x(mark(z),number) - pos(index,z,1))^2 + (y(mark(z),number) - pos(index,z,2))^2);
        diff_steer(index,z,number) = sqrt((turn(mark(z),number) - steer(z,index))^2);
        if number == 1
            diff_speed(index,z,number) = sqrt((5 - speed(z,index))^2);
        elseif number == 2
            diff_speed(index,z,number) = sqrt((7.5 - speed(z,index))^2);
        elseif number == 3
            diff_speed(index,z,number) = sqrt((10 - speed(z,index))^2);
        end
    end
end

%Finds the average difference of the position, speed, and steering between
%the predicted and the experimental data
ave_diff = mean(difference,1);
temp_ave_diff = ave_diff(:,:,number);
ave_diff(:,:,number) = temp_ave_diff';
ave_diff_speed = mean(diff_speed,1);
temp_ave_diff_speed = ave_diff_speed(:,:,number);
ave_diff_speed(:,:,number) = temp_ave_diff_speed';
ave_diff_steer = mean(diff_steer,1);
temp_ave_diff_steer = ave_diff_steer(:,:,number);
ave_diff_steer(:,:,number) = temp_ave_diff_steer';

%Plots the average difference between the predicted and experimental position
figure(10)
hold on;
if number == 1
    plot(ave_time(1:three_second(1)),ave_diff(1,:,number),'b');
elseif number == 2
    plot(ave_time(1:three_second(1)),ave_diff(1,:,number),'r');
else
    plot(ave_time(1:three_second(1)),ave_diff(1,:,number),'g');
end
xlabel('Average Time, sec');
ylabel('Average Difference between Actual Position and Predicted Position, m');
legend('5 mph','7.5 mph','10 mph');

%Calculates the Path Length and the Percentage Difference of the predicted clothoid
if number == 1
    path_length(:,number) = (5 * 1609.344 / 60 / 60) .* ave_time(1:three_second(1));
    percent_difference = ave_diff./path_length(three_second(1)) .* 100;
elseif number == 2
    path_length(:,number) = (7.5 * 1609.344 / 60 / 60) .* ave_time(1:three_second(1));
percent_difference = ave_diff./path_length(three_second(1)).* 100;
else
    path_length(:,number) = (10 * 1609.344 / 60 / 60).*
        ave_time(1:three_second(1));
    percent_difference = ave_diff./path_length(three_second(1)).* 100;
end

%Plots the average difference percentage between the predicted and
experimental position
figure(11)
hold on;
if number == 1
    plot(path_length(:,number),percent_difference(1,:,number),'b');
elseif number == 2
    plot(path_length(:,number),percent_difference(1,:,number),'r');
elseif number == 3
    plot(path_length(:,number),percent_difference(1,:,number),'g');
end
xlabel('Distance traveled on Predicted Path, m');
ylabel('Averaged Error Percentage of Position compared with Total Path
Length, %');

%Plots the average difference between the predicted and experimental steering
angle
figure(12)
hold on;
if number == 1
    plot(ave_time(1:three_second(1)),ave_diff_steer(1,:,number),'b');
elseif number == 2
    plot(ave_time(1:three_second(1)),ave_diff_steer(1,:,number),'r');
else
    plot(ave_time(1:three_second(1)),ave_diff_steer(1,:,number),'g');
end
xlabel('Average Time, sec');
ylabel('Average Error of Steering Angle, deg');

%Plots the average difference between the predicted and experimental speed
figure(13)
hold on;
if number == 1
    plot(ave_time(1:three_second(1)),ave_diff_speed(1,:,number),'b');
elseif number == 2
    plot(ave_time(1:three_second(1)),ave_diff_speed(1,:,number),'r');
else
    plot(ave_time(1:three_second(1)),ave_diff_speed(1,:,number),'g');
end
xlabel('Average Time, sec');
ylabel('Average Error of Speed, mph');
end
Data Average Function

% Calculates the Average Position, Speed, Steering Angle, Time Intervals of % the Trials
function [ave_pos, ave_speed, ave_steer, ave_time] = 
data_average(pos, speed, steer, time, start, stop, start1, stop1, stop2)
% start = 2 seconds before start of turn
% stop = when throttle stops being applied
% start1 = when start of clothoid
% stop1 = when clothoid ends adjusted so that each trial is the same length
% stop2 = when clothoid ends unadjusted

ave_pos_temp = mean(pos,1);
ave_pos(:,:) = ave_pos_temp;
ave_speed = mean(speed,2);
ave_steer = mean(steer,2);
ave_time_temp = mean(time,2);
w = find(ave_time_temp < 0.1,1,'last');
z = find(ave_time_temp < 1,1,'last');
s = size(ave_time_temp);
ave_time_temp1 = ave_time_temp(1:w);
ave_time_temp2 = ave_time_temp(w+1:z);
ave_time_temp3 = ave_time_temp(z+1:s(1));
ave_time_temp1 = vpa(ave_time_temp1,2);
ave_time_temp1 = double(ave_time_temp1);
ave_time_temp2 = vpa(ave_time_temp2,3);
ave_time_temp2 = double(ave_time_temp2);
ave_time_temp3 = vpa(ave_time_temp3,4);
ave_time_temp3 = double(ave_time_temp3);
ave_time = cat(1,ave_time_temp1,ave_time_temp2,

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Data Extractor Function

%Extractes the data of interest from the Trial data logs
function [heading,northing,easting,speed,steer,rate,minute,second] = data_extractor(data,start,stop)

d = data;

north = data(:,6+3);
east = data(:,7+3);
steering = data(:,13+3);
minute = data(start,(-1+3));
second = data(start:stop,(0+3));
northing = data(start:1:stop,6+3);
easting = data(start:1:stop,7+3);
speed = data(start:1:stop,11+3);
steer = data(start:1:stop,13+3);
rate = data(start:1:stop,20+3);
heading = data(start:1:stop,8+3);
Ackermann Steering Function

% Testing a variety of rack displacements for Kinematic Model of the Ackermann Steering

function [x_cross, y_cross, steer_angle, x, y, time_cg, pred_stop, turn] = Ackermann_Steering(time_end, experiment)

figure_check = 1; % If = 0 then does the Trial calculations,
                    % If = 1 then does the Rack and Pinion calculations
s = 0;              % rack displacement
d = 4+7/8;
r1 = 35+1/4;
r2 = 3+15/16; % needs to equal r6
r3 = 10+1/4; % needs to equal r5
r4 = 14;
r5 = 10+1/4; % needs to equal r3
r6 = 3+15/16; % needs to equal r2
l1 = 0;
l5 = 0;
phi1 = 0;
phi2 = 0;
rho1 = 0;
rho2 = 0;
t1 = 24.5; % tire length
tw = 8.5; % tire width
L = 82; % vehicle length, axle to axle
x_cross = 0;
y_cross = 0;
steer_angle = 0;
theta = 0;
phi = 0;

if figure_check == 1
    close all;
end

% Calls the Ackermann_Angle function which determines the listed angles
[c, theta2_0, theta3_0, theta5_0, theta6_0] = Ackermann_Angle(s, d, r1, r2, r3, r4, r5, r6);

count = 0;
stop = 0;
theta2 = 0;
theta3 = 0;
theta5 = 0;
theta6 = 0;

s = -2.468; % Crosses the centerline of the back axle
s = -2.625:0.001:2.625; % Maximum rack displacement

% Calls the Ackermann_Angle function which determines the listed angles
for z = s
    count = count + 1;
[c(count,:),theta2(count),theta3(count),theta5(count),theta6(count)] = Ackermann_Angle(z,d,r1,r2,r3,r4,r5,r6);
for index = 1:6
  if c(index) == 0
    stop = 1;
  end
end
if stop == 1;
  break;
end
end

if figure_check == 1
  figure(1)
  plot(s,theta2);
  title('Theta 2');
  xlabel('Rack Displacement, inches');
  ylabel('Theta 2, deg');
figure(2)
  plot(s,theta3);
  title('Theta 3');
  xlabel('Rack Displacement, inches');
  ylabel('Theta 3, deg');
figure(3)
  plot(s,theta5);
  title('Theta 5');
  xlabel('Rack Displacement, inches');
  ylabel('Theta 5, deg');
figure(4)
  plot(s,theta6);
  title('Theta 6');
  xlabel('Rack Displacement, inches');
  ylabel('Theta 6, deg');
figure(5)
  hold on;
  xlabel('X Position, inches');
  ylabel('Y Position, inches');
end

%Calculates the positions of steering system linkages
z = 0;
for z = 1:count
  plot_check = 0;
  x4 = r4/2 + s(z);
  y4 = - d;
  x6 = r1/2;
  y6 = 0;
  x5 = x4 + r5*cosd(real(theta5(z)));
  y5 = y4 + r5*sind(real(theta5(z)));
  x3 = s(z) - r4/2;
  y3 = - d;
  x1 = -r1/2;
  y1 = 0;
  x2 = x3 + r3*cosd(theta3(z));
  y2 = y3 + r3*sind(theta3(z));

  x4 = r4/2 + s(z);
  y4 = - d;
  x6 = r1/2;
  y6 = 0;
  x5 = x4 + r5*cosd(real(theta5(z)));
  y5 = y4 + r5*sind(real(theta5(z)));
  x3 = s(z) - r4/2;
  y3 = - d;
  x1 = -r1/2;
  y1 = 0;
  x2 = x3 + r3*cosd(theta3(z));
  y2 = y3 + r3*sind(theta3(z));
\[
\rho_1(z) = \theta_2_0 - \theta_2(z);
\]
\[
\rho_2(z) = \theta_6_0 - \theta_6(z);
\]

% Calculates the lines perpendicular to the tires, their intersection point,
% and an estimate of the vehicle’s steering angle
[check(z,:),x_cross(z),y_cross(z),steer_angle(z),theta(z),phi(z)] =
Ackermann_Tires(rho1(z),rho2(z),x1,y1,x6,y6,tl tw,L,plot_check,figure_check);

% For the last rack displacement, the code plots the figures relate to
% the steering system
if z == count
  l1 = abs(x2 - x1);
  l5 = abs(x6 - x5);
  phi1 = asind(l1 / r2);
  phi2 = asind(l5 / r6);
  plot_check = 1;

  [check(z,:),x_cross(z),y_cross(z),steer_angle(z),theta(z),phi(z)] =
  Ackermann_Tires(rho1(z),rho2(z),x1,y1,x6,y6,tl tw,L,plot_check,figure_check);

  % Issue with if s = 0, so placed a holder value of 0 for the crosses
  tempxr = [x4 x5 x6];
  tempyr = [y4 y5 y6];
  tempxl = [x3 x2 x1];
  tempyl = [y3 y2 y1];

  if figure_check == 1
    % Plots the steering assembly in color
    plot(tempxr,tempyr);
    plot([0 x4],[-d y4],'c');
    plot([x6 0],[y6 0],'g');
    plot([0 s(z)],[ -d -d],'r');
    plot(tempxl,tempyl);
    plot([0 x3],[-d y3],'c');
    plot([x1 0],[y1 0],'g');
    plot([0 s(z)],[ -d -d],'r');

    % Plots the steering assembly in black
    plot(tempxr,tempyr,'k');
    plot([0 x4],[-d y4],'k');
    plot([x6 0],[y6 0],'k');
    plot([0 s(z)],[ -d -d],'k');
    plot(tempxl,tempyl,'k');
    plot([0 x3],[-d y3],'k');
    plot([x1 0],[y1 0],'k');
    plot([0 s(z)],[ -d -d],'k');
  end
end

display
% if figure_check == 1
%     place = find(x_cross == 0);
%     s_size = size(x_cross);
%     plot(x_cross(1:(place-1)),y_cross(1:(place-1)),'y');
%     plot(x_cross((place+1):s_size(2)),y_cross((place+1):s_size(2)),'y');
% end

% Calculates the difference between the wheel angles and the estimated steering angle
diff_1 = abs(rho1 - steer_angle);
diff_2 = abs(rho2 - steer_angle);

% Checks to see if the Ackermann Steering System figures should be displayed
if figure_check == 1
    figure(6);
    hold on;
    plot(s,rho1,'b');
    plot(s,rho2,'r');
    xlabel('Rack Displacement, inches');
    ylabel('Wheel Angle, deg');
    legend('Left Tire','Right Tire','Steering Angle');
end

% Checks to see which experiment it is and if the Ackermann Steering System figures should be displayed
if experiment == 1 & figure_check == 0
% Gets the predicted clothoid path with a speed of 5 mph
    [x(:,1),y(:,1),dot_theta_cg(:,1),turn(:,1),dot_x(:,1),dot_y(:,1),theta_cg_d(:,1),time_cg(:,1),steer(:,1),pred_stop(:,1)] = clothoid(5,time_end,experiment,L);
elseif figure_check == 0
% Gets the predicted clothoid path with a speed of 7.5 mph
    [x(:,2),y(:,2),dot_theta_cg(:,2),turn(:,2),dot_x(:,2),dot_y(:,2),theta_cg_d(:,2),time_cg(:,2),steer(:,2),pred_stop(:,2)] = clothoid(7.5,time_end,experiment,L);
% Gets the predicted clothoid path with a speed of 10 mph
    [x(:,3),y(:,3),dot_theta_cg(:,3),turn(:,3),dot_x(:,3),dot_y(:,3),theta_cg_d(:,3),time_cg(:,3),steer(:,3),pred_stop(:,3)] = clothoid(10,time_end,experiment,L);
else
    x = 0;
y = 0;
dot_theta_cg = 0;
turn = 0;
dot_x = 0;
dot_y = 0;
theta_cg = 0;
time_cg = 0;
steer = 0;
pred_stop = 0;
end
Ackermann Angle Function

% Returns the angles of the links in the steering system
function [c,theta2,theta3,theta5,theta6] = Ackermann_Angle(s,d,r1,r2,r3,r4,r5,r6)

% The check variable to make sure there were no errors in the calculations
c1 = 0;
c2 = 0;
c3 = 0;
c4 = 0;
c5 = 0;
c6 = 0;

syms theta2;
syms theta3;
syms theta5;
syms theta6;

% Right Side of Steering
l1 = sqrt(d^2 + (s + r4/2)^2);
a1 = asind((s + r4/2)*sind(90)/l1);
a2 = 90 - a1;
l2 = sqrt((r1/2)^2 + l1^2 - 2*l1*(r1/2)*cosd(a2));
a3 = acosd(((12^2 + r6^2 - r5^2)/(2*12*r6));
a4 = acosd((l2^2 + (r1/2)^2 - l1^2)/(2*l2*(r1/2)));
theta6 = a3 + a4 + 180;
theta5 = asind((r6*sind(theta6)+d)/r5);
x4 = r4/2 + s;
y4 = -d;
x6 = r1/2;
y6 = 0;
x5 = x4 + r5*cosd(real(theta5));
y5 = y4 + r5*sind(real(theta5));

x5_6 = x6 + r6*cosd(theta6);
y5_6 = y6 + r6*sind(theta6);

% Left Side of Steering
l3 = sqrt(d^2 + (r4/2 - s)^2 - 2*d*(r4/2 - s)*cosd(90));
a5 = asind(((r4/2 - s)*sind(90))/l3);
a6 = 90 - a5;
l4 = sqrt((r1/2)^2 + l3^2 - 2*(r1/2)*l3*cosd(a6));
a7 = acosd((l4^2 + r2^2 - r3^2)/(2*l4*r2));
a8 = acosd((l4^2 + (r1/2)^2 - l3^2)/(2*l4*(r1/2)));
theta2 = 360 - a7 - a8;
theta3 = 180 - asind((d + r2*sind(theta2))/r3); % asind range is (-90, 90)
% this means it can't give an angle in the 3rd quad. Hence the 180 -
x3 = s - r4/2;
y3 = -d;
x1 = -r1/2;
y1 = 0;
x2 = x3 + r3*cosd(theta3);
\begin{verbatim}
y2 = y3 + r3*sind(theta3);
x2_2 = x1 + r2*cosd(theta2);
y2_2 = y1 + r2*sind(theta2);

%Checking the Right Side
distance_6 = sqrt((x5-x6)^2 + (y5-y6)^2); %Should = r6
distance_5 = sqrt((x4-x5)^2 + (y4-y5)^2); %Should = r5

angle_6 = 180/pi * atan2((y5-y6),(x5-x6))+360; %Should = theta6
angle_5 = 180/pi * atan2((y5-y4),(x5-x4)); %Should = theta5

if abs(angle_6 - theta6) <= 0.01 && abs(distance_6 - r6) <= 0.01
c6 = 1;
end
if abs(angle_5 - theta5) <= 0.01 && abs(distance_5 - r5) <= 0.01
c5 = 1;
end

%Checking the Left Side
distance_3 = sqrt((x3-x2)^2 + (y3-y2)^2); %Should = r3
distance_2 = sqrt((x2-x1)^2 + (y2-y1)^2); %Should = r2

angle_3 = 180/pi * atan2((y2-y3),(x2-x3)); %Should = theta3
angle_2 = 180/pi * atan2((y2-y1),(x2-x1))+360; %Should = theta2

if abs(angle_3 - theta3) <= 0.01 && abs(distance_3 - r3) <= 0.01
c3 = 1;
end
if abs(angle_2 - theta2) <= 0.01 && abs(distance_2 - r2) <= 0.01
c2 = 1;
end

%Checking the r4 length
distance_4 = sqrt((x3-x4)^2 + (y3-y4)^2);
if abs(distance_4 - r4) <= 0.01
c4 = 1;
end

c1 = 1;
c = [c1 c2 c3 c4 c5 c6];
\end{verbatim}
Ackermann Tires Function

%Plots the tires on the steering system and calculates the estimated steering angle

function
[check,x_cross,y_cross,steer_angle,theta,phi]=Ackermann_Tires(rho1,rho2,x1,y1,x6,y6,t1,tw,L,plot_check,figure_check)

%Left front tire position variables
xt1 = x1 + t1/2*sind(rho1);
yt1 = y1 + t1/2*cosd(rho1);
x2 = xt1 - tw*cosd(rho1);
y2 = yt1 + tw*sind(rho1);
x4 = x1 - t1/2*sind(rho1);
y4 = y1 - t1/2*cosd(rho1);
x3 = xt4 - tw*cosd(rho1);
y3 = yt4 + tw*sind(rho1);

tirex1 = [xt1 xt2 xt3 xt4 xt1];
tirey1 = [yt1 yt2 yt3 yt4 yt1];

%Checks variables for the left front tire. The lengths should make the link lengths of the steering assembly match
check(1) = sqrt((xt1 - x1)^2 + (yt1 - y1)^2);
check(2) = sqrt((xt2 - xt1)^2 + (yt2 - yt1)^2);
check(3) = sqrt((xt3 - xt2)^2 + (yt3 - yt2)^2);
check(4) = sqrt((xt4 - xt3)^2 + (yt4 - yt3)^2);
check(5) = sqrt((xt1 - xt4)^2 + (yt1 - yt4)^2);

%Right front tire position variables
xt5 = x6 + t1/2*sind(rho2);
yt5 = y6 + t1/2*cosd(rho2);
x6 = xt5 + tw*cosd(rho2);
y6 = yt5 - tw*sind(rho2);
x8 = x6 - t1/2*sind(rho2);
y8 = y6 - t1/2*cosd(rho2);
x7 = xt8 - tw*cosd(rho2);
y7 = yt8 - tw*sind(rho2);

tirex2 = [xt5 xt6 xt7 xt8 xt5];
tirey2 = [yt5 xt6 y7 y8 xt5];

%Checks variables for the right front tire. The lengths should make the link lengths of the steering assembly match
check(6) = sqrt((xt5 - x6)^2 + (yt5 - y6)^2);
check(7) = sqrt((xt6 - xt5)^2 + (yt6 - y5)^2);
check(8) = sqrt((xt7 - xt6)^2 + (yt7 - y6)^2);
check(9) = sqrt((xt8 - xt7)^2 + (yt8 - y7)^2);
check(10) = sqrt((xt5 - xt8)^2 + (yt5 - y8)^2);

%Back tires positions
bx1 = x1;
by1 = y1 - L + t1/2;
bx2 = bx1 - tw;
by2 = by1;
bx4 = x1;
by4 = y1 - L - tl/2;
bx3 = bx4 - tw;
by3 = by4;
bx5 = x6;
by5 = y6 - L + tl/2;
bx6 = bx5 + tw;
by6 = by5;
bx8 = x6;
by8 = y6 - L - tl/2;
bx7 = bx8 + tw;
by7 = by8;

back_tirex1 = [bx1 bx2 bx3 bx4 bx1];
back_tirey1 = [by1 by2 by3 by4 by1];
back_tirex2 = [bx5 bx6 bx7 bx8 bx5];
back_tirey2 = [by5 by6 by7 by8 by5];

if xt4 ~= xt1
  % Determines the Intersection Point of the lines perpendicular to the
  % front tires
  mid_tirex1 = (xt2 + xt3) / 2;
  mid_tirey1 = (yt2 + yt3) / 2;
  mid_tirex2 = (xt6 + xt7) / 2;
  mid_tirey2 = (yt6 + yt7) / 2;

  m1 = (y1 - mid_tirey1)/(x1 - mid_tirex1); % Left Side
  b1 = y1 - x1 * m1;
  m2 = (mid_tirey2 - y6)/(mid_tirex2 - x6); % Right Side
  b2 = y6 - x6*m2;
  x_cross = (b2 - b1) / (m1 - m2);
  y_cross = (b2*m1 - b1*m2) / (m1 - m2);

  ms = (0 - y_cross)/(0 - x_cross); % Center of Front Axle

% Generates the Perpendicular Lines
  if m1 >0
    steer_x1 = -100:0.1:-20;
    steer_x2 = -100:0.1:20;
    steer_xs = -100:0.1:0;
    steer_xb = -100:0.1:20;
  else
    steer_x1 = -20:0.1:100;
    steer_x2 = 20:0.1:100;
    steer_xs = 0:0.1:100;
    steer_xb = -20:0.1:100;
  end
  steer_y1 = steer_x1 * m1 + b1;
  steer_y2 = steer_x2 * m2 + b2;
  steer_y2 = steer_xs * ms;
  steer_yb = y1 - L;

% Check Code
  ll = sqrt((steer_x2(1) - x6)^2 + (steer_y2(1) - y6)^2);
\[ l_2 = \sqrt{(x_{t5} - x_6)^2 + (y_{t5} - y_6)^2}; \]
\[ l_3 = \sqrt{(\text{steer}_x(1) - x_{t5})^2 + (\text{steer}_y(1) - y_{t5})^2}; \]
\[ \theta = \arccos \left( \frac{l_1^2 + l_2^2 - l_3^2}{2 \cdot l_1 \cdot l_2} \right); \]
\[ n_1 = \sqrt{(\text{steer}_x(1) - x_1)^2 + (\text{steer}_y(1) - y_1)^2}; \]
\[ n_2 = \sqrt{(x_t - x_1)^2 + (y_t - y_1)^2}; \]
\[ n_3 = \sqrt{(\text{steer}_x(1) - x_t)^2 + (\text{steer}_y(1) - y_t)^2}; \]
\[ \phi = \arccos \left( \frac{n_1^2 + n_2^2 - n_3^2}{2 \cdot n_1 \cdot n_2} \right); \]

%Estimated Steering angle of the Vehicle
if \( m_1 > 0 \)
    steer_angle = -90 + \arctan(x_{\text{cross}}/y_{\text{cross}}); \\
else \\
    steer_angle = 90 + \arctan(x_{\text{cross}}/y_{\text{cross}}); \\
end
else
    x_{\text{cross}} = 0; \\
    y_{\text{cross}} = 0; \\
    steer_angle = 0; \\
    \theta = 0; \\
    \phi = 0; \\
end

%Plots the tires and the lines
if plot_check == 1 && figure_check == 1
    plot(\text{steer}_x,\text{steer}_y, 'b'); \\
    plot(\text{steer}_x,\text{steer}_y, 'r'); \\
    plot(\text{steer}_x,\text{steer}_y, 'k'); \\
    plot(\text{steer}_x,\text{steer}_y, 'g'); \\
    legend('Left Tire', 'Right Tire', 'Steering Angle', 'Back Axle Centerline'); \\
    plot(\text{tire}_x,\text{tire}_y, 'k'); \\
    plot(\text{tire}_x,\text{tire}_y, 'k'); \\
    plot(\text{back_tire}_x,\text{back_tire}_y, 'k'); \\
    plot(\text{back_tire}_x,\text{back_tire}_y, 'k'); \\
end
Clothoid Function

%Calculates the predicted clothoid path
function
[x,y,dot_x,dot_y,theta_cg,dot_theta_cg,time_cg,steer,pred_stop] = clothoid(vcg,time_end,experiment,l)

lr = 18.625 * 0.0254; %distance from rear axle to IMU, inches converted to meters
lf = (l - 18.625) * 0.0254; %distance from front axle to IMU, inches converted to meters
l= l * 0.0254; %distance between axils, inches converted to meters

%The reference point for these variables is the position of the IMU
vcg = vcg * 1609.344 / 60 / 60; %mph converted to meter/sec
x(1) = 0; %Initial Position, m
y(1) = 0; %Initial Position, m
theta_cg(1) = 90; %Initial Heading, deg
steer(1) = 25.41; %Initial steering angle deg, assuming right turn positive, and left negative
final_steer = -25.41; %deg, assuming right turn positive, and left negative
conversion = 64000; %Steering rate converted to counts per second

dt = 0.001;
time = 0:dt:time_end;
stop = 0;

steer_rate = -16.9414; %Steering Rate deg/sec, assuming right turn positive, and left negative

theta_cg(1) = theta_cg(1) * pi/180; %Heading, converts to radians
steer(1) = steer(1) / 180 * pi * -1; %converts to radians in a standard coordinate system
steer_rate = steer_rate / 180 * pi * -1; %converts to radians in a standard coordinate system

s = size(time);
dot_theta_cg(1) = 0;
for z = 2:1:s(2)
    %Velocity in the x direction and the y directions
    dot_x = cos(theta_cg(z-1)) .* vcg;
    dot_y = sin(theta_cg(z-1)) .* vcg;

    %Rate of change in heading
    if tan(steer(z-1)) < 0
        %Must be in radians/sec!!!
        dot_theta_cg(z) = -1 * vcg./sqrt((l./tan(steer(z-1))).^2 + lr.^2);
    else
        %Must be in radians/sec!!!
    end
end
dot_theta_cg(z) = vcg./sqrt((l./tan(steer(z-1))).^2 + lr.^2);

% Determines when the angle has gotten to the final angle
if steer(z-1) >= (-1*final_steer * pi/180+final_steer * pi/180*0.00000001) % Remember coordinate system
    steer(z) = 0 * dt + steer(z-1);
    if stop == 0;
        stop = z-1;
    end
else
    steer(z) = steer_rate * dt + steer(z-1);
end

% Calculates the heading of the vehicle
theta_cg(z) = dot_theta_cg(z) * dt + theta_cg(z-1);

% Calculates the x and y position of the vehicle
x(z) = dot_x .* dt + x(z-1);
y(z) = dot_y .* dt + y(z-1);

% Plots Position of Predicted Clothoid
figure(1)
hold on;
v = vcg / 1609.344 * 60 * 60;
if v == 5
    plot(x(1:stop),y(1:stop),'m');
elseif v == 7.5
    plot(x(1:stop),y(1:stop),'y');
elseif v == 10
    plot(x(1:stop),y(1:stop),'c');
end
if experiment == 1
    legend('Trial 1','Trial 2','Trial 3','Trial 4','Trial 5','Predicted at 5 mph');
elseif experiment == 2
    legend('Trial 1','Trial 2','Trial 3','Trial 4','Trial 5','Predicted at 10 mph');
else
    legend('Trial 1','Trial 2','Trial 3','Trial 4','Predicted at 5 mph','Predicted at 7.5 mph','Predicted at 10 mph');
end

theta_cg_d = theta_cg * 180/pi; % Converts heading into degrees

% Plots Steering Angle of Predicted Clothoid
turn = -1 * steer*180/pi; % Converts to my coordinate frame
figure(2)
hold on;
if v == 5
    plot(time(1:stop),turn(1:stop),'m');
elseif v == 7.5
    plot(time(1:stop),turn(1:stop),'y');
elseif v == 10
    plot(time(1:stop), turn(1:stop), 'c');
end
if experiment == 1
    legend('Trial 1', 'Trial 2', 'Trial 3', 'Trial 4', 'Trial 5', 'Predicted at 5 mph');
elseif experiment == 2
    legend('Trial 1', 'Trial 2', 'Trial 3', 'Trial 4', 'Trial 5', 'Predicted at 5 mph', 'Predicted at 10 mph');
else
    legend('Trial 1', 'Trial 2', 'Trial 3', 'Trial 4', 'Predicted at 5 mph', 'Predicted at 7.5 mph', 'Predicted at 10 mph');
end

%Plots Speed of Predicted Clothoid
figure(3)
hold on;
if v == 5
    v = zeros(1, stop) + v;
    plot(time(1:stop), v, 'm');
elseif v == 7.5
    v = zeros(1, stop) + v;
    plot(time(1:stop), v, 'y');
elseif v == 10
    v = zeros(1, stop) + v;
    plot(time(1:stop), v, 'c');
end
time_cg = time';
pred_stop = time_cg(stop);

if experiment == 1
    legend('Trial 1', 'Trial 2', 'Trial 3', 'Trial 4', 'Trial 5', 'Predicted at 5 mph');
elseif experiment == 2
    legend('Trial 1', 'Trial 2', 'Trial 3', 'Trial 4', 'Trial 5', 'Predicted at 5 mph', 'Predicted at 10 mph');
else
    legend('Trial 1', 'Trial 2', 'Trial 3', 'Trial 4', 'Predicted at 5 mph', 'Predicted at 7.5 mph', 'Predicted at 10 mph');
end

%Plots Steering Rate of Predicted Clothoid
figure(4)
hold on;
steer_rate_temp(1:stop) = conversion;
if v == 5
    plot(time(1:stop), steer_rate_temp, '--m', 'LineWidth', 3);
elseif v == 7.5
    plot(time(1:stop), (steer_rate_temp), '-.y', 'LineWidth', 2.5);
elseif v == 10
plot(time(1:stop),(steer_rate_temp),'c');

if experiment == 1
    legend('Trial 1','Trial 2','Trial 3','Trial 4','Trial 5','Predicted at 5 mph');
else
    legend('Trial 1','Trial 2','Trial 3','Trial 4','Predicted at 5 mph','Predicted at 7.5 mph','Predicted at 10 mph');
end