Strip Crown Prediction: Developing a Refined Dynamic Roll-Stack Model for the Hot Rolling Process

By
Derek E. Slaughter

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Robert L. West
William T. Baumann
Jim Atkinson

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The steel industry has been producing flat plates through the process of hot rolling since the late 1600s. Hot rolling uses a series of rolls to progressively thin a strip of steel to a desired thickness. In deforming the strip, the rolling process causes variations in thickness across the width of the strip. These variations are commonly referred to as crown, which is specifically the difference in thickness between the center and edge of a strip. For most applications steel mill clients require flat products, or products with little variation in thickness. Therefore, variations represent wasted material which must be removed before the plate or sheet can be used in consumer products. Controlling the flatness of the metal strip is a high priority for the hot rolling business.

The purpose of this work was to develop a 3-D dynamic model of the rolling process to simulate the behaviour of a strip while being rolled and predict its profile. To accomplish this task, much of the rolling process needed to be modeled. The profile of the strip is a product of the deformation of the rolls and frame within a mill stand. Therefore, not only did the geometry of these components need to be modeled, but the material properties and dynamic motion were required as well. The dynamic nature of the process necessitated the modeling of the rotation of the rolls and translation of the strip, aspects of rolling which are not typically simulated.

Five models were developed during the project. The purpose of the first two models was to find the stiffnesses of the roll-stack and stand frame. The roll-stack refers to the rolls and their arrangement. The reference mill from which data was provided used a four-high roll-stack with two rolls above the strip and two below. The frame that holds the roll-stack, while massive, stretches when the strip is deformed between the rolls. This stretch changes the position of the rolls affecting the load and deformation of the strip. A lumped-mass model was created to simulate the dynamics of the roll-stack and frame. When the strip enters the gap between the rolls, there is a large impact force which causes the rolls to vibrate. The lumped-mass model was used to determine parameters to bring the system to steady state. The final two models simulated the entire rolling process with rotating rolls and moving strip. The 3-D dynamic rolling model was capable of predicting the strip profile due after exiting the rolls. Two calibrations were used to reduce model error before running a validation.

The rolling causes thickness variation across the width of the metal strips; therefore, strips are intentionally rolled thick to meet a minimum thickness. In modern steel mills, specialized control systems are used to adjust parameters as the steel strip passes through each stand of rolls. Varying the parameters allows the thickness and profile of the strip to be controlled. Each stand may have several rolls in different configurations. These rolls are either work rolls, which directly contact the strip, or backup rolls, which contact the work rolls and stiffen the roll-stack. The stand frame holds the rolls and provides a means to position them.

The validation results showed that the exit thickness, strip crown, and rolling load were less than 5% different from the values measured in the test data. The calibrated model was then used to derive strip crown sensitivities to gap, entry crown, work roll crown, and bending force. The 3-D dynamic model was able to predict the strip crown accurately when given calibrated information about the system. This model will be a useful tool for exploring the mechanics of hot rolling in ways that were not previously possible.
Dedication

This work is dedicated to those whom will learn and benefit from it.
Acknowledgements

This work would not have been possible without the assistance and dedication of many people. My advisor, Dr. Robert L. West Jr., was always available to provide encouragement, guidance, and expertise far beyond the scope of a faculty advisor. Our shared interests in modeling and simulation provided a foundation for discourse and a creative environment in which to develop the project. Jim Atkinson and Thomas Ranger of Industrial Process Support Services sponsored the project and made available equipment to develop and run simulations. Jim and Thomas were involved throughout the project, and provided me with the opportunity to learn a great deal about many subjects in which I am interested. My wife, Jenny Slaughter, provided encouragement and expert editing skills. Finally, I would like to thank my father, Michael Slaughter, who introduced me to the principles of engineering through the many projects we worked on together.
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Uppercase Symbols

\(C_{cr}\)  Critical damping matrix
\(C\)  Damping matrix
\(C_\Delta\)  Change in strip crown within a stand [mm]
\(C_E\)  Strip crown entering a stand [mm]
\(C_s\)  Strip crown [mm]
\(C_w\)  Work roll diameter crown [mm]
\(C_X\)  Strip crown exiting a stand [mm]
\(D_c\)  Roll barrel centerline diameter [mm]
\(D_e\)  Roll barrel edge diameter [mm]
\(D_n\)  Roll neck diameter [mm]
\(E\)  Elastic modulus [tonne/mm\(^2\)]
\(F\)  Stand reaction force [tonne]
\(F\)  Force [tonne]
\(F_n\)  Nodal force at time step \(n\) [tonne]
\(H_\Delta\)  Change in strip centerline thickness within a stand [mm]
\(H_c\)  Strip center thickness [mm]
\(H_e\)  Strip edge thickness [mm]
\(H_E\)  Strip centerline thickness entering a stand [mm]
\(H_f\)  Strip thickness at feather distance in from edge [mm]
\(H_g\)  Roll gap [mm]
\(H_X\)  Strip centerline thickness exiting a stand [mm]
\(J\)  Bending force [tonne]
\(J_b\)  Balance bending force [tonne]
\(K\)  Stiffness matrix [tonne/mm]
\(M\)  Lumped mass matrix
\(P\)  Rolling load or roll separation force [tonne]
\(R\)  Work roll radius undeformed [mm]
\(R'\)  Work roll radius deformed [mm]
\(R^2\)  Coefficient of determination

\(^{0}\) The tonne unit refers to metric tonnes force
List of Symbols

\( S_y \)  Yield stress [tonne/mm\(^2\)]
\( S_{y1} \) Initial yield stress [tonne/mm\(^2\)]
\( S_{y2} \) Ultimate yield stress [tonne/mm\(^2\)]
\( T \)  Period of oscillation [s]

Lowercase Symbols

\( a_n \)  Nodal acceleration at time step \( n \) [mm/s\(^2\)]
\( c \)  Damping coefficient
\( c_f \)  Frame damping coefficient
\( c_r \)  Roll-stack damping coefficient
\( d \)  Displacement at time \( t \) [mm]
\( d_t \)  Total displacement [mm]
\( k \)  Mean flow stress in pure shear \( (S_y/\sqrt{3}) \)
\( k \)  Spring stiffness [tonne/mm]
\( k_f \)  Frame stiffness [tonne/mm]
\( k_r \)  Roll-stack stiffness [tonne/mm]
\( k_s \)  Strip spring stiffness [tonne/mm]
\( k_t \)  Total stiffness of frame and roll-stack system [tonne/mm]
\( k_x \)  Strip exit modulus [tonne/mm]
\( L_p \)  Arc of contact length between work roll and strip [mm]
\( l_b \)  Roll barrel length [mm]
\( l_n \)  Roll neck length [mm]
\( l_s \)  Strip length [mm]
\( m \)  Mass
\( m_b \)  Backup roll mass [tonne-s\(^2\)/mm]
\( m_w \)  Work roll mass [tonne-s\(^2\)/mm]
\( t \)  Time [s]
\( t_d \)  Displacement time [s]
\( t_r \)  Run time [s]
\( u \)  Nodal displacement [mm]
\( \dot{u} \)  Nodal velocity [mm/s]
\( u_n \)  Nodal displacement at time step \( n \) [mm]
\( v_n \)  Nodal velocity at time step \( n \) [mm/s]
\( w \)  Strip width [mm]
\( w_f \)  Feather width [mm]
List of Symbols

**Greek Symbols**

- $\alpha$  
  Mass matrix proportional damping coefficient
- $\alpha_d$  
  Displacement time coefficient
- $\alpha_r$  
  Run time coefficient
- $\beta_t$  
  Total displacement time [s]
- $\beta$  
  Stiffness matrix proportional damping coefficient
- $\Delta t$  
  Explicit time step [s]
- $\epsilon$  
  Engineering Strain [mm/mm]
- $\epsilon_1$  
  Yield strain [mm/mm]
- $\epsilon_2$  
  Strain at end of stress-strain curve [mm/mm]
- $\epsilon_t$  
  True strain [mm/mm]
- $\epsilon$  
  Error [%]
- $\epsilon_c$  
  Crown error [%]
- $\epsilon_h$  
  Exit thickness error [%]
- $\epsilon_p$  
  Rolling load error [%]
- $\epsilon_t$  
  Total error [%]
- $\eta$  
  Isoparametric coordinate
- $\gamma$  
  yield stress coefficient
- $\nu$  
  Poisson’s ratio
- $\omega_i$  
  Modal frequency $i = 1, 2$ [rad/s]
- $\omega_n$  
  Natural frequency [rad/s]
- $\psi_i$  
  Isoparametric interpolation functions $i = 1, 2, 3$
- $\sigma$  
  Engineering Stress [tonne/mm$^2$]
- $\sigma_t$  
  True Stress [tonne/mm$^2$]
- $\xi$  
  Isoparametric coordinate
- $\zeta$  
  Damping ratio
- $\zeta_i$  
  Modal damping ratio $i = 1, 2$
1 Introduction

1.1 Motivation

Though rolling steel into flat sheets is an established manufacturing process, the steel industry is perpetually looking for ways to improve the quality of its products. The simplest way to improve quality is through better understanding and control of the rolling processes. For this reason, models of the rolling process have been developed throughout history to predict how aspects of the final product, such as thickness and flatness, may be affected based on the configuration of the mill. Recent advances in finite element analysis and computing have made it possible to simulate the complex rolling process in great detail.

All flat rolled products experience some flatness error due to the manufacturing process. This flatness error can range from small thickness variations to waves in the product itself. These variations force customers to request thicker material than necessary to ensure that they receive product of a certain thickness. As many customers need a product of a precise thickness, the excess must be machined away, wasting valuable time and energy. By reducing variability in thickness and better controlling the production process, it will be possible to reduce costs and increase manufacturing efficiency.

The development of highly accurate models of the rolling process will lead to the more efficient production and use of steel. If one is able to predict exactly how changing a mill configuration will affect the flatness of the product, then a mill could be optimized to minimize flatness error and perhaps eliminate it. This optimization impacts not only the mills, which could produce higher quality steel, but also the customers who would not need to machine flat products before use. In turn, the optimization would impact all consumers of products containing flat rolled steel.

1.2 Background

Flat rolling is the process by which a piece of metal, commonly called a strip or slab, is reduced in thickness to create flat sheets or plate. There are many processes and components used to create the final shape of a rolled product. To model the rolling process, the parts and their influence must be defined and understood. The following presents the relevant processes and components necessary to develop a model of the rolling process.
1.2.1 Flat Rolled Products

Flat rolled products can be classified into two groups: plates and sheets. The actual designation depends solely on the thickness to width ratio which is slightly higher for plate than sheet products. However, the width to thickness ratio is much less than one for all flat rolled products, and the rolling process is the same.

To create a flat rolled product, a strip of steel with a given initial thickness is successively reduced in thickness by being pulled through a series of rollers supported by stands. Each pass through a stand decreases the strip’s thickness. The amount of thickness reduction at each stand is closely controlled to determine the final thickness of the strip and thus the final dimensions of the product. The quality of the product is determined by measuring the amount of variation in its thickness and accuracy in obtaining nominal thickness [14].

1.2.2 Types of Rolling

The process by which the thickness of a strip is reduced can be executed while strip is cold or hot, called cold rolling and hot rolling respectively. Cold rolling is a type of flat rolling in which the temperature of the metal being rolled is roughly less than half its melting point. This method does not require the material to be heated and produces a strip with a better finish and dimensional accuracy than hot rolling. Cold rolled materials can be stronger as strain hardening occurs during the rolling process [14] [4].

Hot rolling is used to limit strain hardening in the strip. By plastically deforming the material while above its recrystallization temperature, the deformed grains are able to return to a stress free state. In addition, the workability of steel improves with temperature increase because ductility increases and yield stress decreases [31].

1.2.3 Rolling Process

The process of creating a finished flat rolled product from the original slab consists of several stages. These stages in a typical hot rolling mill are strip preparation, roughing, finishing, measuring, and coiling. A sample hot rolling mill layout is shown Figure 1.1 [14].

![Mill component layout](image)

**Figure 1.1:** Mill component layout.

**Strip Preparation**

In a rolling mill the strip starts out as a slab of steel in one of two ways. In one case, the strip is cast in a separate process or location, transferred to the mill, and heated to rolling temperature before entering the roughing stands. A reheating furnace is used to bring the strip up to rolling temperature, approximately 1300°C for most steels. Alternately, the strip can be cast directly as it’s
entering the mill by a process called continuous casting. In this process, the strip goes from molten steel to slab dimensions at the beginning of the mill itself and requires no additional heating. The thickness of the strip is typically between 200 and 300mm when entering the roughing stands [14].

**Roughing**

The roughing process is carried out in several stands to reduce the strip thickness to approximately 50mm as well as adjust the width before entering the finishing stage. The roughed width is approximately the width of the final product [14]. Figure 1.2 shows two roughing stands at a steel mill in Indiana.

![Figure 1.2: Roughing stands (Photograph by Uwe Niggemeier) [19].](image)

**Finishing**

The finishing process gives the strip its final dimensions. As the hot strip travels between the stands its thickness is successively reduced. Due to the strip’s thickness being relatively smaller than its width, the strip’s increase in width is negligible. Since this is the stage where the final product geometry is determined, the model was developed around the test data collected in this stage. The strip size entering the finishing stands has little bearing on the exit size other than width. Also, the strip dimensions do not change significantly after exiting the finishing stands other than through cooling effects. Figure 1.3 shows several finishing stands in a steel mill.

**Measuring**

The strip thickness and flatness is measured in real time by a X-ray gage at then end of the finishing stands as shown in Figure 1.1. Measuring the final dimensions of the strip is vital for the mill controllers. The controllers adjust mill parameters in real time with feedback from the gage to minimize strip flatness.
Coiling

With the strip rolled to nominal dimensions, it is run out on rollers and allowed to cool. Once the product cools sufficiently, it is coiled for shipping. A mill cooling bed is shown in Figure 1.4; the coiler is not shown.

Figure 1.3: Finishing stands (Photograph by Uwe Niggemeier) [19].

Figure 1.4: Cooling bed (Photograph by Uwe Niggemeier) [19].
1.2.4 Mill Stand

Mill stands are composed of the set of rolls between which the strip passes and the structure that supports those rolls. Stands have already been shown in Figures 1.3 and 1.2, but a more detailed examination is necessary. A stand like those pictured measures several stories tall. Whether it is used for roughing or finishing, the stand is the primary way to deform the strip. Since the finishing stage determines the final shape of the strip, the finishing stands are of the most importance. Figure 1.5 shows a single finishing stand and a graphical representation of all its parts. Only a portion of the stand in the photograph is visible as it extends below the ground.

![Figure 1.5: Stand configuration. (Left) Side view of a finishing stand [19]. (Right) Front and side view detail of stand components.](image)

The stand is broken down into several parts: rolls, frame, and actuators. The rolls can be categorized as work rolls and backup rolls, which have similar features though they differ in size. There are generally two actuators in each stand, one to control the gap through which the strip passes, and the other to control the bending of the rolls. The arrangement of the bending actuators can be different from those shown in Figure 1.5 as detailed in section 1.2.4.

**Roll Detail**

Rolls consist of two parts: barrel and neck. The barrel is the part of the roll that comes in contact with the strip or another roll. The neck is the thinner part of the roll, part of which rests in a bearing. The bearing is held by a chock which is connected to the stand frame. The geometry of the roll is described by the measurements in Figure 1.6.

The necks of the rolls may have a variety of shapes as suited to the bearings of a particular mill. The barrel, on the other hand, is determined during stand setup and can be different for each roll in each stand. The simplest form of roll barrel curvature is created by forming a symmetric parabola from the center of the roll barrel to its edge. The diameter crown $C_r$ of the roll barrel is defined as the difference in size between the barrel center diameter $D_c$ and barrel edge diameter $D_e$ as given in Equation 1.1.

$$C_r = D_c - D_e$$  \hspace{1cm} (1.1)
The roll crown in Figure 1.6 is positive because the center of the roll has a greater diameter than the edge. The roll crown may also be neutral or negative as shown in Figure 1.7.

![Figure 1.6: Roll geometry detail.](image)

![Figure 1.7: Types of roll crown.](image)

The curvature of the roll barrel affects the way the roll bends and interacts with the strip. By grinding different profiles on the roll barrel, it is possible to change many aspects of the rolling process.

The roll composition and material also affects the rolling process. The stiffer the roll, the less it deforms. Rolls can be either solid or composite, such that the barrel has a thin layer on the outside which has different material properties than the core. Figure 1.8 shows the division of the roll into shell and core. Rolls are commonly made of steel which can be modeled by a linear elastic isotropic material represented by an elastic modulus $E$ and Poisson’s ratio $\nu$.

**Roll Configuration**

Another stand aspect that affects the rolling process is the roll configuration. The rolls in a stand can be broken down into two categories: work rolls and backup rolls. Work rolls directly contact the strip, while backup rolls support and stiffen the work rolls. Work rolls and backup rolls may
be used in different arrangements for specific purposes. The four-high roll-stack configuration uses two work rolls and two backup rolls as shown in Figure 1.5. There also exist twenty-high roll-stack configurations that make use of two work rolls and eighteen backup rolls. Figure 1.5 details the rolls and frame in a four-high roll-stack. The mill used for model development used the four-high roll-stack arrangement [4].

**Stand Actuators**

As previously stated, stands are composed of rolls and a frame. This frame contains actuators that position and sometimes bend the rolls. The main actuators, referred to as gap actuators in Figure 1.5, change the distance between the work rolls. The vertical distance between the work rolls, known as the roll gap or gap, denoted by $H_g$, is the space through which the strip passes. By actuating the gap it is possible to control the thickness of the strip as it exits the stand.

The secondary actuators are called bending actuators. The bending actuators may be arranged differently than in Figure 1.5 where they act between the work roll necks. The bending actuators may also apply force between the work roll and backup roll necks or between the backup roll necks themselves [27]. This work uses the actuator configuration in which the actuators are between the work rolls. Bending actuators apply a force to the work roll necks, causing them to bend around the backup rolls. The force the bending actuators apply is commonly called the bending force and is denoted by $J$. The bending actuators also provide a balance force to ensure that the work rolls and backup rolls remain in contact. Most bending actuators can only apply force in one direction. To simulate a negative bending force, a default load is applied when the stand is calibrated called the balance bending force represented by $J_b$.

**Frame Stretch**

In a hot rolling mill the rolling loads necessary to deform the strip may be in excess of 2000 metric tonnes force\(^1\). Not only is this force sufficient to deform the strip, but it also deforms the stand frame. Though the frame is very large, as can be seen in Figures 1.5, 1.2, and 1.3, it stretches slightly when loaded. This stretch allows the rolls to move apart, changing the gap. As the gap increases, the load on the strip decreases. Thus, a small deformation of the stand structure can cause a large change in the deformation of the strip. This deformation must be accounted for when modeling the rolling process.

A distinction in the load causing the stretch must be made. The force that the rolls exert on the strip is not the same load that the stand must resist due to the addition of the bending force on the

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\(^1\)All subsequent references to tonnes refer to metric tonnes force.
work rolls. Figure 1.9 shows a free body diagram of the upper half of a roll-stack in a stand. The stand reaction force \( F \), the rolling load from deforming the strip \( P \), and the bending force \( J \) are also shown.

\[ F = P + J \]  
\( (1.2) \)

As shown in Figure 1.9 the rolling load, reaction force, and bending force is related as in Equation 1.2.

The stand stretch is calculated from the stiffness of the frame \( k_f \) and the stand reaction force \( F \) as shown in Equation 1.3.

\[ \text{Stand Stretch} = \frac{F}{k_f} \]  
\( (1.3) \)

### 1.2.5 Strip

The purpose of the previously described components is to control the geometry of the strip exiting each stand and the mill itself. The shape or profile of the strip can be influenced at each stand to achieve the target dimensions of the final sheet or plate product. The profile of the strip is typically described by the measurements shown in Figure 1.10.

The width of the strip is denoted by \( w \). The thickness of the strip along its vertical centerline is \( H_c \). The thickness of the strip along the edge is \( H_e \). Due to the change in width of the strip as it passes through the various stands, an accurate thickness cannot be measured exactly at the edge. A feather width \( w_f \) is defined from the edge of the strip to where the measurement can be reliably
taken. The thickness at the feather is denoted by $H_f$. Similar to the rolls, the strip profile is also measured by crown. Equation 1.4 defines the strip crown $C_s$

$$C_s = H_c - H_f$$  \hspace{1cm} (1.4)

as the difference in thickness between the center of the strip $H_c$ and the feather $H_f$. As with the roll crown, it is possible to have positive, neutral, and negative strip crown. Positive crown occurs when the feather thickness is less than the center thickness. Neutral crown is when the feather and center thicknesses are equal. Negative crown occurs when the center is thinner than the feather thickness. Figure 1.11 shows these three cases.

![Types of strip crown](image)

**Figure 1.11:** Types of strip crown.

Since the purpose of a stand is to change the shape of a strip, the model must consider the change in thickness and crown. Thickness change $H_\Delta$, called draft, is the difference between the strip’s entry thickness $H_E$ and exit thickness $H_X$ as defined in Equation 1.5 and shown in Figure 1.12.

$$H_\Delta = H_E - H_X$$  \hspace{1cm} (1.5)

![Strip centerline thickness change in stand](image)

**Figure 1.12:** Strip centerline thickness change in stand.

Similarly, the entry crown is represented by $C_E$ and the exit crown by $C_X$. Using these terms, Equation 1.6 defines the change in crown $C_\Delta$ within a single stand.

$$C_\Delta = C_X - C_E$$  \hspace{1cm} (1.6)
The profile of the strip is typically assumed to be parabolic in shape. This allows the entire profile of the strip to be described by very few numbers. For instance, given crown, centerline thickness, and width, it is possible to construct the entire shape of the strip.

Little is known about the material properties of a strip as it moves through the stands. Empirical models have been developed to describe steel properties at elevated temperatures; however, materials other than steel, such as aluminum or stainless steel, are also worked in these mills. In some rolling models the material is assumed to be rigid-plastic, while other models make more detailed assumptions. The strip material will be discussed further in the literature review.

1.2.6 Flatness Error

Ideally, a final product coming out of a mill would be perfectly flat across its width and length. Crown represents flatness error across the width of the strip, but error can also occur along the length of a strip. Equation 1.6 describes the change in strip crown that occurs between entering and leaving a stand. The strip can only tolerate so much change in crown before it develops another kind of flatness error. If the change in thickness at a stand is not nearly uniform, then it causes the strip to lengthen unevenly across its width. This change in length can cause either edge wave or center buckle as shown in Figure 1.13. Edge wave occurs when the strip crown increases too much within a stand and center buckle occurs when the strip crown decreases too much within a stand. This rule says that the strip crown can only be changed so much within a stand before creating additional flatness error besides profile flatness error. Flatness error along the width and length of the strip must be controlled to create a usable product.

![Figure 1.13: Strip flatness error: edge wave and center buckle.](image)

The only way to control the flatness error is to precisely control the strip crown. The strip crown is affected by the geometry of the rolls, the gap between the rolls, roll stiffness, roll bending, stand stiffness, and the strip material properties. While rolling the strip it is only possible to control a few of these parameters. The essence of controlling strip crown lies in determining which stand parameters have the greatest effect on crown and how they can be controlled. The purpose of models in hot rolling is to predict strip characteristics prior to rolling the strip based on information about the mill.

1.2.7 Finite Element Analysis

Finite element analysis (FEA) is a numerical process by which the solution to partial differential equations can be found over irregularly shaped domains. Developing this process began in
Chapter 1. Introduction

1.3 Test Data

The 1950s when computers became widely available. Since that time, great improvements have been made in FEA and in computers, allowing faster solutions to more complex problems. FEA has been applied to the hot rolling process before; however, with advances in computing speed, only recently has it become feasible to create models that simulate so many aspects of the rolling process. The finite element analysis software Abaqus was used extensively for the creation of models in this project. Abaqus is capable of solving the nonlinear contact and material problems presented by the hot rolling process [22, 29].

1.3 Test Data

The sponsor of this project, Industrial Process Support Services (IPSS) provided test data from a running mill being used to make flat steel sheets. This data included stand frame calibration data and information from online controllers for the stands in the finishing stage of the mill. The finishing stage was composed of seven stands with loads, gap, and roll geometry data for each. Measurements of the final product thickness and crown were taken after the last stand. The data from one strip was selected and the effect of stand seven on that strip was used for model development and verification. The strip from which the test data was generated was rolled directly after a roll change operation, so the rolls were initially at room temperature. The data from this strip was used as it was believed that the rolls would exhibit little thermal expansion due to heat transfer while rolling. The data collected from stand seven is provided in Appendix: Test Data.

1.4 Thesis Objectives

The objective of this work is to create a dynamic finite element model capable of predicting rolling load and strip profile, specifically strip exit thickness and strip crown. This model could then be used to determine how changing stand parameters affects the crown of the strip. To achieve these objectives, the model must be able to accurately simulate the rolling process in three dimensional space and predict rolling load, strip exit thickness, and strip crown within 10% error. This level of modeling requires large amounts of numerical computation; however, performing 3-D simulations is the only way to predict strip crown. In addition, the model must encompass the dynamic effects of the rolling process such as the oscillation of the rolls and determination of steady-state behaviour. Dynamic simulation is required to exactly simulate the roll-strip interaction from which the pressure distribution and resulting strip and roll deformations are found. The material of the strip must also be modeled to create a representative simulation of the process. The model will be considered a success if it can predict the exiting crown of the strip to within ten percent of measured test data. Finally, the model will be used to perform a preliminary sensitivity analysis to determine the effect of several parameters on strip exit crown.

1.5 Scope of Thesis

The work presented in this thesis will cover the development of the finite element model and the methods used to calibrate the given model such that it can predict strip crown in accordance with test data. Model development will encompass not only the construction of the final 3-D finite element model, but also all of the models leading up to it as listed below:

- Static Roll-Stack Model
- Static Frame Stiffness Model
• Dynamic Lumped-Mass Model
• Dynamic 2-D Rolling Model
• Dynamic 3-D Rolling Model

In addition to the development of the models, two calibration methods were used to remove uncertainties in the test data: yield-gap and work roll crown calibration. The results from these calibrations were used to create a simulation that validates the model. Once the model was validated, a preliminary sensitivity analysis was performed to determine how several stand parameters affect the strip crown.

1.6 Organization of Thesis

This thesis is divided into five sections. The first section deals with previous attempts to model the rolling process and provides background information which is used in this model. The derivation of rolling force and strip material models is presented along with a representation of the strip crown. A comparison of online and offline models is performed with the advantages and disadvantages of each.

The second section consists of model development, detailing the methods used and results from the models listed in section 1.5. First, the stand is broken down into component parts and the stiffnesses of the roll-stack and frame are determined via the static finite element models. This information is then used along with the rolling force model to develop the lumped-mass model to simulate the dynamics of the roll-stack. The results of the lumped-mass model are then verified by the 2-D rolling model, which is also used to give insight into the finite element modeling of the rolling process. Finally, the 3-D rolling model is developed which is the goal of this thesis.

The third section takes the fully developed 3-D rolling model and calibrates it against real world test data. Two calibrations are considered. The yield gap calibration reduces exit thickness and rolling load error due to uncertain measurements of strip yield stress and roll gap. The work roll crown calibration corrects for thermal expansion of the roll changing the roll crown which affects the strip crown. After each calibration a validation is performed to show the calibration effects. Finally, the last validation is compared to the test data to assess the model’s performance.

The fourth section uses the calibrated 3-D rolling model to determine the sensitivity of the strip crown to several stand parameters. The parameters gap, strip entry crown, work roll crown, and bending force were used for the sensitivity analysis. The parameters were then ranked based on the amount they influence the strip crown.

The fifth section discusses the results of the model development and calibration. In addition, areas of further development are proposed along with ways in which the model can be used for further research on the topic of hot rolling.
2 Modeling of the Hot Rolling Process

2.1 Introduction

Though the process of hot rolling began in 16th century, it was not modeled until 1925 [23]. Early models relied on simple mathematical solutions, while more modern approaches use complex numerical methods. Each model relies upon a representation of the strip, either with a stiffness approximation or a full constitutive model. Models of the rolling process may be broken down into two categories: rolling load models or strip profile models. Force models are used to predict rolling load and torque, whereas profile models attempt to calculate the deformed shape of the strip. Analytical solutions and finite element methods can be used in the solution of both types of models [9]. The purpose behind modeling the rolling process is to control it, so strip control and parameter sensitivity are also of importance. This chapter will discuss methods of modeling the rolling process and factors affecting the strip crown.

2.2 Strip Material Models

The success of any rolling process model is dependent on the representation of the strip properties. These representations include the following models: strip spring, rigid-plastic, elastic-plastic, and visco-plastic.

2.2.1 Strip Spring

The simplest representation of the strip is achieved by reducing the material properties to a spring stiffness. This method was developed by Vladimir Ginzburg in his book *High-Quality Steel Rolling: Theory and Practice*. Ginzburg [9] used strip stiffness in a 3-D finite element strip profile model. The strip modulus \( k_s \) is defined in Equation 2.1

\[
k_s = \frac{P}{wH_\Delta}
\]  

(2.1)

where \( P \) is the rolling load, \( w \) is the strip width, and \( H_\Delta \) is the change in thickness of the strip [9]. While the simplicity of this representation is appealing, it does not represent the strip well. The yielding of the material is not modeled; therefore the pressure distribution between the work roll and strip would be very much an approximation.
2.2.2 Rigid-Plastic

Many force models use the rigid-plastic model, as do some finite element models. The yield stress represents all of the material characteristics including all effects of strain rate and temperature. Figure 2.1 shows a stress-strain curve for a rigid-plastic material. In many cases involving large strains, the rigid-plastic model is adequate for calculating rolling loads [18, 13].

![Figure 2.1: Rigid-plastic stress-strain curve.](image)

Rigid-plastic material models of the strip are used because they are simple and efficient for calculation. However, this model does not well represent the actual material of the strip. In some cases, neglecting the elastic response of the strip can have significant effects on the outcome of the simulation. For this reason, the rigid-plastic material model was not used in this project [13].

2.2.3 Elastic-Plastic

A further development of the rigid-plastic model is the elastic-plastic model. The elastic-plastic model is defined by an elastic modulus $E$, an initial yield stress $S_{y_1}$, and a final yield stress $S_{y_2}$ as shown in Figure 2.2. This allows the material to have elastic deformation followed by plastic deformation with strain hardening. A variation of this material model was used for the strip in the development of this project’s 3-D rolling model [13].

![Figure 2.2: Elastic-plastic stress-strain curve.](image)
2.2.4 Visco-Plastic

The visco-plastic material model has the greatest ability to represent the actual strip material. The stresses in the strip are dependent on the strain rate during deformation. By assuming a constant yield stress, other material models neglect changes in strain rate during the rolling process. This model can capture these variations [13]. However, the determination of the material properties to capture these effects were deemed beyond the scope of this project.

2.3 Rolling Load Models

Rolling load models are used to represent the rolling load and torque required to deform a strip by a given amount [8]. The deformation in these models is considered to be plane strain with no deformation along the width of the strip. The assumption greatly simplifies the model and allows for fast evaluation of rolling loads by either 1-D mathematical models or 2-D dynamic finite element (FE) models.

2.3.1 1-D Mathematical Models

The first mathematical model for the rolling process was developed by von Karman [13] in 1925; however, it was the later refinement and solution of the equations by Orowan [20] in 1943 that is considered the standard for modeling the process [13]. Orowan’s model makes use of static equilibrium equations between the roll and strip as the strip is undergoing deformation. The distributed pressure along the arc of contact between the roll and strip, as well as the shear forces, are used to generate the equilibrium equations [13]. Orowan used a graphical method for the solution of these equations [13]. A simplification of the Orowan model was made by Sims [28] in 1954 by assuming that the angles in the roll gap are small [13]. A further simplification of Orowan’s model was made by Bland and Ford in 1948 by setting the roll pressure equal to the vertical stress in the strip [13]. In 1964, Ford and Alexander developed a simplified rolling model that is still used in the industry [4]. Alexander performed a study in 1972 using a FORTRAN program to numerically integrate and solve Orowan’s original equations. He then compared the results to the simplified models made by Sims and Bland and concluded that the analytic models were unable to calculate the rolling load with a high degree of accuracy [2]. In 1996, Freshwater [6, 7] presented simplified theories of rolling based on further computational study of von Karman’s and Orowan’s equations.

Limitations

The 1-D rolling load models have many limitations. The models assume a plane strain environment in which there is no deformation along the width of the strip. This assumption discounts the ability of the strip to change width or have variations in thickness across the width, making the models unable to calculate crown. In addition, effects of the rolling load on frame stretch is neglected. The material model of the strip is assumed to be simple: perfectly plastic with some elastic effects at the entrance and exit of the rolls [21]. Even with these limitations, the 1-D rolling models remain a useful tool in quickly estimating rolling loads.

Ford and Alexander Formulation

The Ford and Alexander rolling load model was used in the process of creating the lumped-mass model by calculating a pseudo strip stiffness. The model depends on the strip properties: yield
stress, entry thickness, and exit thickness and work roll properties: radius, elastic modulus, and Poisson’s ratio [4].

There exists a relationship between length of the arc of contact between the rolls and strip, the deformation of the rolls, and rolling load. The rolls are allowed to deform, which changes the arc of contact, affecting the rolling load. The length of the arc of contact between the rolls and the strip is defined in Equation 2.2

\[ L_p = \sqrt{R (H_E - H_X) - \frac{(H_E - H_X)^2}{4}} \approx \sqrt{R (H_E - H_X)} \]  

where \( L_p \) is the arc of contact, \( R \) is the undeformed radius of the work roll, and \( H_E \) and \( H_X \) are the entry and exit thicknesses of the strip respectively [4]. The effect of roll deformation is to change the roll diameter as calculated using Hitchcock’s equation

\[ R' = R \left[ 1 + \frac{16 (1 - \nu)^2 P}{\pi EH_\Delta w} \right] \]  

where \( R' \) is the deformed roll radius, \( H_\Delta \) is the change in strip thickness, \( E \) is the elastic modulus of the roll, \( \nu \) is the Poisson’s ratio of the roll, \( P \) is the total rolling load, and \( w \) is the width of the strip [13]. The rolling load \( P \) referred to in the previous Equations is developed as

\[ P = kW L_p \left( \frac{\pi}{2} + \frac{L_p}{H_E + H_X} \right) \]  

where \( k \) is the mean flow stress in pure shear, \( w \) is the width of the strip, and \( L_p \) is the length of contact. The mean flow stress is related to the yield stress of the material by Equation 2.5.

\[ k = \frac{S_y}{\sqrt{3}} \]  

An iterative solution of the force model is required due to the interactions between rolling load, arc of contact, and deformed roll radius.

**2.3.2 2-D Dynamic Finite Element Models**

The 2-D dynamic FE rolling load model typically attempts to model the physical motion of the strip and the resulting deformation. The finite element modeling of the 2-D rolling process is superior to the 1-D mathematical models because the boundary conditions and materials are represented more accurately [18]. Several 2-D finite element models of the rolling process have been created and shown to offer more accurate approximations of rolling load [18, 5].

The model by Mori et al. [18] was developed to look at strip deformation and rolling loads. The work rolls were assumed to be rigid and the thickness reduction of the strip was specified. The strip was composed of isoparametric quadrilaterals with four Gauss points for full integration. Half of the strip was modeled as it moved beneath the rotating work roll. This model also demonstrated likely deformation of the strip mesh during rolling [18].

The model by Dvorkin et al. [5] in 1997 was a 2-D finite element model developed as a step towards a 3-D model incorporating roll elasticity, thermal effects, and a strip material model. This 2-D rolling model represents the system using plane strain. A rigid-viscoelastic material was used to define the strip. Half of the strip and one work roll is modeled with a symmetric boundary condition on the strip centerline and the roll rotating about its axis. An Eulerian formulation was
used such that the mesh remained stationary with the strip flowing through it. Once the model was developed, it was used to determine process sensitivity to several rolling variables [5].

The model developed in this work is similar to the models by Mori et al. and Dvorkin et al. in certain respects and different in others. Whereas those two models only simulated the rolling of the work roll, this project’s 2-D model represents both the work roll and backup roll. Deformation of the rolls was not considered in the discussed models, though it was in the one developed for this thesis. All three models are able to predict rolling load and strip exit height.

### 2.4 Strip Profile Models

The purpose of a strip profile model is to predict the shape of a strip exiting a stand. These models attempt to represent deformation across the width of the rolls due to bending and contact. The interaction between the strip and the rolls is modeled in greater detail than is possible with rolling load models. Many different representations for the strip are used. Most strip profile models are based on beam or finite element theory.

#### 2.4.1 Beam Models

Beam models are based on beam theory and can be classified as simple beam models or slit beam models. The simple beam model represents the entire roll or rolls as a single beam and integrates the beam equation over the width of the roll using a distributed rolling load to determine deflection. The slit beam model represents the roll as a series of beam segments coupled by influence coefficients. By subdividing the rolls, the variation in the loads along the width of the strip can be better represented through equivalent nodal loads. Beam models are commonly used by online mill controllers as they are able to quickly predict strip profiles while the strip is being rolled [17].

**Simple Beam Models**

The first simple beam model was developed by Saxl in 1958. Saxl developed models for two-high and four-high mills. The two-high model uses beam theory as illustrated in Figure 2.3. The center of the beam is assumed to be a cantilever while the edge of the barrel is simply supported. The rolling load is represented as a uniform loading over the width of the strip. To obtain the deflection of the centerline of the roll, the beam equation was integrated over the width of the roll. The displacement due to bending and shear effects are superimposed to give the total deformation. Adjustments are also made for contact deformation between the strip and roll. A similar approach is taken with a four-high model using two beams with an elastic foundation. However, the backup roll is assumed to be rigid, having no centerline deflection. Neither of these models is capable of representing roll crown in their formulation [26, 9].

![Figure 2.3: Saxl's two-high roll-stack simple beam model [26].](image-url)
A more advanced simple beam model was developed in 2007 by A. Malik and R. Grandhi to represent twenty-high mills. Beam elements were developed using foundation moduli between the rolls. The beam elements could represent the barrel or neck of the roll along with variations in cross section. This model was significantly more flexible than Saxl’s. The main advantage of this model was that many rolls could be combined in various configurations to define the roll stack [17].

There are several deficiencies in the simple beam model. The pressure distribution is not uniform in the actual rolling process which leads to different bending behaviour as noted by Shohet and Townsend [27]. Secondly, the actual rolls behave as though the necks are cantilevered because the bearings do not allow them to rotate in the bending plane. This boundary condition leads to significantly different bending. Finally, 3-D finite element models are able to simulate the localized deformation in the roll-strip contact region which the simple beam model can only approximate.

### Slit Beam Models

The slit beam model created by Shohet and Townsend [27] in 1968 is a blend of the simple beam model and finite elements. The rolls and strip are broken down into elements of varying lengths depending on the roll and strip geometry. Rigid body motion of the rolls is allowed and crown is modeled. The bending of the rolls is calculated through influence coefficients and deformation in the roll-strip contact region is determined on an element by element basis. This representation allows for a more localized deformation at the edges of the strip. Compatibility conditions resolve the forces in rolling load and bending [10]. Compatibility conditions are written between the work roll and backup roll and the work roll and the strip. These conditions allow for the representation of roll crown by mathematical representing the closing of the gaps that crown creates. The equilibrium conditions of the rolls and strip allow for the solution of the equations as the sum of the forces between the rolls and strip must be zero. Figure 2.4 shows the discretization of the backup roll, work roll, and strip in a four-high slit beam model.

![Figure 2.4: Slit beam model of the four-high roll-stack [9].](image)

A more recent slit beam model was developed by Yun et al. which uses a 3-D finite element model to determine compliance coefficients between the rolls themselves and the strip. This method allows for calibration of the model against a static 3-D simulation for additional accuracy [32].

Slit beam models also have deficiencies with respect to 3-D finite element models. The influence coefficients are based on beam theory which may not be suitable for the rolls, because of the large diameter to length ratio called the slenderness ratio. The slit beam model also assumes that
the work roll and backup roll are in contact across the entire width of the rolls. Finally, the slit beam model does not completely model the roll-strip contact, which is approximation with discreet spring elements representing an elastic foundation [9]. The 3-D rolling finite element model better represents the roll geometry and deformation along with the contact and nonlinear strip properties.

2.4.2 3-D Finite Element Models

Finite element models are the most advanced models capable of predicting strip crown. The reason finite element models are so accurate is that they are capable of representing the geometry of the roll and strip in great detail through discretization. Not only is the geometry of the strip and rolls well represented, but also the material properties can be modeled in detail by choosing any of the models in section 2.2. Finite element models for predicting strip profile fall into two categories: static and dynamic. Static finite element only consider applied loads and elastic forces. Dynamic models include inertial and damping forces in addition to applied loads and elastic forces. There are advantages and disadvantages to both methods.

2.4.3 3-D Static Finite Element Models

Static 3-D FE models are similar to the simple and slit beam models except they are better able to represent the geometry and the strip material. A notable static model developed by Ginzburg was called the ROLL-FLEX™ which simulates the behaviour of one eighth of a four-high mill. A one-eighth roll-stack 3-D static model is shown in Figure 2.5. The eighth model is used due to symmetry conditions and the assumption that the rolls are not rotating. These simplifications, along with the strip spring model, allowed for a simple finite element model. Because of the simple strip material model and the lack of rolling, the pressure distribution between the work roll and strip is an approximation [9].

![Figure 2.5: Static 3-D finite element model of a four-high roll-stack [9].](image-url)

A similar eighth model of a four-high mill was developed by Malik and Grandhi for verification of their simple beam model [17]. Malik and Grandhi represented the strip using a foundation
modulus and rolls discretized with tetrahedral elements [17]. The goal of both the simple beam and 3-D finite element models was to predict the strip exit crown.

2.4.4 3-D Dynamic Finite Element Models
Dynamic 3-D FE models represent the deformation of the rolls and strip with the greatest accuracy. These models simulate the actual rotation of the rolls and translation of the strip. Therefore, the contact conditions and pressure distributions must develop in the same manner as the actual rolling process assuming the material and geometry models are accurate. However, these dynamic models must also deal with reaching steady state conditions and large runtimes. The final model developed for this project is a 3-D dynamic finite element model.

Until recently, fully developed dynamic FE simulations of the 3-D rolling process have been considered too computationally expensive [11]. This computational requirement has been used as a justification for the simplifications made by other beam models [11]. However, deficiencies in other models lead to the continued development of finite element models [11]. An early dynamic 3-D model of the rolling process was developed in 1997 by Zone-Chin Iln and Ven-Huei Len for the study of thermal expansion in the work roll [15].

The purpose of the model in this project is to use the 3-D dynamic rolling model to determine which controllable parameters most affect strip crown and how much that affect is. This work will perform analysis similar to the work of Shohet and Townsend, discussed in the following section, using a fully dynamic model to achieve highly accurate results. This level of analysis has not been previously performed due to prohibitively long computing times. By applying specific control parameters to the roll stack and using modern computer equipment, the run times have been reduced to the point where this analysis is feasible.

2.5 Strip Crown Control
The first studies to control strip crown were performed by Shohet and Townsend in 1968 using a slit beam model [27]. There are many factors that affect strip crown. Shohet and Townsend identify several parameters that affect strip crown: roll geometry, draft, rolling load, bending force, strip temperature, strip entry crown, strip entry thickness, and strip width. Roll geometry not only includes the sizes of the neck and barrel, but also the roll crown [27].

Certain stand parameters, such as roll size are determined by mill designers or material processing requirements and cannot be readily controlled to change strip crown. However, bending force, rolling load, and strip entry crown can all be influenced at run time and can have a large impact on strip crown [27].

The roll crown is determined by the superposition of several factors: ground crown, thermal crown, and wear crown. Shohet and Townsend represented all three types of crown with a parabola; thus, the combined crowns can be represented by a single parabola [27]. Therefore, by adjusting the work roll crown it is possible to simulate the effects of thermal expansion and wear. The thermal crown generated while rolling can be significant [24]. One experiment showed the roll crown to increase by 0.30 mm in 52 seconds while rolling a 1270 mm wide strip [9].

The next largest controllable influence on strip crown is the bending force. The bending force deforms the work roll around the backup roll which also effectively changes the roll crown. However, the effects of the bending on the roll crown cannot be represented as another parabolic crown on the roll.

While it is believed that entry crown has little influence on the strip exit crown, it still merits discussion [27]. The entry crown at each stand is the previous stand’s exit crown. Thus, the entry
2.6 Summary

There exist many components to rolling process modeling. Consideration must be given to modeling the strip material, roll-strip contact deformation, and roll bending. Models range from simple 1-D mathematical models to complex 3-D dynamic finite element models. Using these models to determine the effects of stand parameters on strip crown can be useful in controlling the strip crown. Parameter sensitivities can be calculated to determine which parameters have the most effect on strip crown. The final model of this work will use a 3-D dynamic finite element model to determine crown parameter sensitivities.
3 Rolling Model Development

3.1 Introduction

The final roll-stack model was the result of several models, each one building upon the last. These models were developed using the finite element method and a simplified representation of the stand. Each model was a necessary step forward and a check to verify the results of the previous model. The models are presented in the following order:

- Strip Material Model
- Static Roll-Stack Model
- Static Frame Stiffness Model
- Dynamic Lumped-Mass Model
- Dynamic 2-D Rolling Model
- Dynamic 3-D Rolling Model

3.2 Finite Element Method

The Finite Element Method (FEM) or Finite Element Analysis (FEA) was used extensively for the modeling of the rolling process in this work. FEA was used to simulate the structural deformation and dynamics of the roll-stack system. Both 2-D and 3-D FE models were created and used at different stages. Finite element modeling, as in all types of modeling, depends greatly on the choices of representation for the system being modeled. Each choice has advantages and disadvantages that can impact the quality of the solution. Many important decisions were made when the finite element models were created. The aspects of FEA that apply to the models in this project will be addressed in this section.
3.2.1 Abaqus Finite Element Analysis Software

The FEA software Abaqus, distributed by Simulia, was used for the Roll-Stack, Frame Stiffness, and Rolling models in this work [29]. Abaqus is designed to solve highly nonlinear material and contact problems in both static and dynamic models. Abaqus is composed of three products: CAE, Standard, and Explicit. Abaqus CAE is the graphical interface through which models are built and updated, as well as where output data is viewed and extracted. In short, CAE handles the preprocessing and postprocessing of the models. Abaqus Standard and Explicit are types of solvers; Standard uses the implicit finite element formulation while Explicit uses the explicit finite element formulation, both methods are discussed in section 3.2.9. Abaqus CAE provides a programming interface using the Python scripting language. The scripting ability of CAE was used extensively as a means of updating the models and exploring the effects of different stand parameters on the solution.

3.2.2 Purpose

The purpose of the finite element method is to solve partial differential and integral equations over irregularly shaped domains. In the case of structural and dynamic analysis, the type of analyses performed in this thesis, the system consists of a body which can move or deform in space and time. FEA breaks down the body into regular domains called elements through a process known as meshing. The characteristics of each element depend on the element’s size, shape, and material properties. This information can be used to calculate the elements’ stiffness and mass. The elements are then assembled to represent the entire body of the original system. To define how the body will behave, boundary conditions such as displacements and forces are prescribed on the body. Initial conditions such as velocities or accelerations are also applied. With the system fully described, it is possible to solve for element displacements and deformations. In addition, stresses, strains, velocities, accelerations, contact pressure, reaction forces, internal energy, and kinetic energy can be found. From these results, predictions about how a system behaves can be gathered and analyzed.

3.2.3 Problem Types

Finite element analysis problems consist of three main types: boundary value problems, initial value problems, and initial boundary value problems. Boundary value problems describe a spatially dependent, time independent system governed by differential equations whose solutions are based on boundary conditions. Both the Static Roll-Stack Model and Static Frame Stiffness Model are examples of boundary value problems because they solve for the time independent deformation of the rolls. An initial value problem represents a spatially independent, time dependent system which responds over time to a set of equations and initial conditions. The Dynamic Lumped-Mass Model is an example of an initial value problem because it does not represent the spatial aspects of the rolls, only the time dependent response. An initial boundary value problem is a combination of the initial value and boundary value problem resulting in both a spatially dependent, time dependent representation of a system. The Dynamic 2-D Rolling Model and Dynamic 3-D Rolling Model are initial boundary value problems because the deformation of the rolls and strip are found as a function of time.
3.2.4 Meshing

Meshing, the first step in finite element analysis, refers to the decomposition of bodies such as rolls and strip into subdomains or elements. The quality of a body’s mesh can have a large impact on the overall quality of the model. The distribution of elements and their sizes affect how well they are able to represent the mechanics of a body. The purpose of an element is to represent not only the geometry of a body, but also its stiffness.

The choice of element type and number can influence how well a body’s geometry is represented. For instance, a linear element can only represent linear geometry such as flat faces or edges. To model a curved surface such as a roll with linear elements, a large number of elements must be used. Conversely, a small number of quadratic elements can be used because they can represent geometry described by second order polynomials. However, quadratic elements have other limitations which are discussed in Section 3.2.5. The ability of an element to properly represent geometry is especially important in contact problems where element boundaries in separate bodies determine the forces acting between the bodies. If too few elements are used, the discretization of the force between the bodies cannot adequately represent the pressure distribution in the contact zone.

The shape and type of element also affects how well it is able to represent the stiffness of a body. Elements with large aspect ratios, the ratio of an element’s longest side to its shortest side, are unable to represent the stiffness of the body as well as a cube or square shaped element. The inaccurate stiffness stems from the distortion of the stiffness calculated in isoparametric space and transformed into physical space. A linear element also cannot represent a nonlinear stress distribution because the stresses within the element vary linearly. This makes large elements inappropriate in areas where stress does not vary linearly.

If an element or series of element misrepresents the stiffness in a body, the solution will be invalid. For this reason developing a good mesh is very important. To create a representative mesh for the rolls, they were divided into two regions: the core and the shell in accordance with the material composition of the roll. The shell and core sections were further partitioned into areas in 2-D and volumes in 3-D onto which specific mesh sizes were applied. The partitioning scheme for the work rolls in both the dynamic rolling and static stiffness models are shown in Figure 3.1. The lines represent partition and section boundaries.

The shaded regions of Figure 3.1 denote areas of fine and coarse meshing. Fine meshing is located where the roll was expected to be in contact with either the strip or another roll. A fine mesh is necessary in the areas of contact as an accurate representation of the geometry and deformation is especially important in these areas. For Figure 3.1a, the contact area is large because the roll is rotating and is in contact with the strip along that surface throughout the simulation. The contact area in Figure 3.1b is small as the roll is stationary.

Figure 3.2 shows a typical mesh on a work roll. The mesh is carefully constructed so that the elements in the shell and core are aligned except in the finely meshed areas. A similar technique was used with the strip though no partitioning was required and smaller elements were used.

The quality of a mesh can be determined by running models with varying meshing sizes and measuring the effect on a variable, such as stress at a specific location. As the size of the elements decrease and the number of elements increase, the change in stress should decrease and approach a constant value. This is known as a mesh convergence test in which a measurement is converged to within a given percentage. The minimum number of elements that give the required convergence are used in the final models because additional elements would not significantly contribute to the accuracy of the model. This method was employed to determine the quality of the meshes for this work.
3.2.5 Elements

There are many different types of elements that can be used to represent components of a model. The two groups of elements used in the models for this work were spring and damper elements, and continuum elements. Spring and damper elements are some of the simplest finite elements and were used to represent stand stiffness and damping. Continuum elements are more complex and were used to model the rolls and strip as shown in the meshing scheme in Section 3.2.4.

Spring & Damper Elements

Springs and dampers are special discrete lumped parameter elements for which stiffness and damping coefficients can be prescribed directly. Spring elements create a coupling between two nodes such that the relative displacement between the nodes causes a force to be generated opposing the direction of displacement. This force is generated per Hooke’s law, Equation 3.1,

\[ F = k(u_2 - u_1) \]  

where \( F \) is the internal force in the spring element. If the element is tied to the ground the internal force will also be a reaction force. The stiffness coefficient is represented by \( k \), \( u_1 \) is the displacement of node 1, and \( u_2 \) is the displacement of node 2. The spring system is illustrated in Figure

---

**Figure 3.1:** Roll partitioning. (a) Dynamic rolling model. (b) Static roll-stack model.

**Figure 3.2:** Roll meshing. (a) Dynamic rolling model. (b) Static roll-stack model.
3.2. Finite Element Method

3.3. A similar equation can be used for the damper element to relate the damper internal force to the relative velocities of the two nodes as shown in Equation 3.2.

\[ F = c(\dot{u}_2 - \dot{u}_1) \]  

(3.2)

The damping coefficient is represented by \( c \), \( \dot{u}_1 \) is the velocity of node 1, and \( \dot{u}_2 \) is the velocity of node 2. The damper element is shown in Figure 3.3. Spring elements were used in the static and dynamic models to represent the stand. Dampers were used in the dynamic models to control the oscillatory response of the roll-stack in the stand and between the rolls themselves.

![Figure 3.3: Spring and damper elements.](image)

Continuum Elements

While the mesh defines the shape of the elements in a body, the elements have many individual properties. The type of element and method by which the element stiffness is generated is as important as the quality of the mesh. If an element of the incorrect formulation is used, it will likely misrepresent the stiffness of the body. Two types of elements were used in the construction of the FE models in this work: bilinear quadrilaterals and trilinear hexahedrals or “bricks”. Bilinear quadrilaterals are 2-D plane elements with four nodes and eight displacement degrees of freedom. The 2-D FE models were comprised of hundreds or thousands of these elements arranged into the shape of the rolls and strip. Trilinear hexahedrons are the 3-D version of the bilinear rectangle with eight nodes and twenty-four displacement degrees of freedom. By looking at these two elements, it is possible to see how the complexity of the solution increases quickly from 2-D to 3-D. Not only does the number of elements and nodes increase, but so do the number of displacement degrees of freedom [3]. A diagram of these elements is shown in Figure 3.4.

Both the quadrilateral and hexahedral are linear isoparametric elements, which are only capable of representing linear variations in geometry and the displacement field and the resulting stresses and strains across the element. This limitation is due to the interpolation functions defining the element. In the case of the bilinear rectangles and trilinear bricks, the interpolation functions vary linearly in all of the directions across the element. The isoparametric interpolation functions in element parametric space or \( \xi \) and \( \eta \) space for the 2-D element in Figure 3.4 are given in Equation 3.3 [22].

\[
\psi_1 = \frac{1}{4}(1 - \xi)(1 - \eta) \\
\psi_2 = \frac{1}{4}(1 + \xi)(1 - \eta) \\
\psi_3 = \frac{1}{4}(1 - \xi)(1 + \eta) \\
\psi_4 = \frac{1}{4}(1 + \xi)(1 + \eta)
\]  

(3.3)
Elements with quadratic or cubic interpolation functions do exist; however, those elements do not typically represent distributed contact loads along element faces accurately unless they are modified for this purpose [3, 1].

The stiffness of an element is calculated by integrating the material stress-strain properties over the domain of the element to determine the resistance of an element to deformation. Deforming an element by displacing its nodes causes internal stresses and forces which react against the nodes. For a 2-D quadrilateral element, the stress-strain relationship can be formulated as plane stress or plane strain. In plane stress, the element thickness is assumed to be very thin relative to the planar area resulting in out-of-plane stresses of zero. In plane strain, the element thickness is assumed to be very thick relative to the planar area, which results in out-of-plane strains of zero. Both the plane-strain and plane-stress stress-strain relationships are approximations of the actual material state. The 3-D hexahedral element provides the most accurate representation of the stresses and strains as they are represented in all directions. For the 2-D FE models, a plane strain stress-strain relationship was used because the strip width is much larger than the strip height which results in very small displacements of the strip in the width direction. The 3-D models use the 3-D hexahedral element, providing the most accurate stiffness representations [16].

**Numerical Integration**

In FEA, the integration of an element’s material properties to calculate its stiffness is typically a form of numerical integration, or quadrature. The method in which the stiffness is integrated also impacts the stiffness of the element. The most common form of quadrature for developing element stiffness is Gauss quadrature. Like all forms of quadrature, Gauss quadrature involves sampling stiffness at several points throughout the element, multiplying by weights, and summing to find the total stiffness. Where these samples are taken, the number of samples, and their weights are determined by a set of quadrature rules. The number of sample points, or Gauss points, is determined by the order of the element’s interpolation functions. A polynomial of degree \( p \) can be exactly integrated by Gauss quadrature of order \( n \) where \( p \leq 2n - 1 \). Thus, 1-D linear element would only require one Gauss point to be integrated exactly because the interpolation functions are first degree. However, a 2-D linear quadrilateral, described by the interpolation functions in Equation 3.3, requires four Gauss points, two in each dimension, to integrate the second degree interpolation functions. Thus, element stiffness can be found exactly and effectively through numerical integration [3].
In some cases it is useful to underintegrate an element’s stiffness by using fewer Gauss points. This method, called reduced integration, occurs when fewer Gauss points are used than the number required to exactly integrate the functions. For example, the 2-D linear element which requires four Gauss points to be integrated exactly, could be integrated with one Gauss point. Reduced integration has advantages and disadvantages depending on how it is used.

The main advantage to reduced integration is a large decrease in the time required to compute an element’s stiffness. Because the element stiffness must be evaluated at each Gauss point, going from four points to one reduces the computation time by 75%. Reduced integration has an effect on the stiffness of the element. Since the element stiffness is sampled in fewer places, the element does not have stiffness related to certain modes of deformation. In some cases this lack of stiffness can cause problems; however, in bending quadrilateral elements tend to be too stiff or to lock when compared to experimental results. Reduced integration decreases the element stiffness in bending and so yields more accurate results. For reduced integration of linear elements Abaqus does not simply sample the element stiffness at a single point. Instead, the stiffness is computed from the average strain over the volume of the element based on a uniform strain formulation. The formulation Abaqus uses ensures element stability [3, 1].

Reduced integration elements were used in all of the applicable models developed for this thesis. The reduction in run time due to using reduced integration is significant as the element stiffnesses must be calculated at every step in the nonlinear analysis. Also, reduced integration elements allow the rolls to bend more accurately.

3.2.6 Boundary Conditions

A boundary condition (BC) is an external condition applied to a system such as a force, pressure, or displacement. Boundary conditions help to define how a system will behave when simulated. The two main types of boundary conditions used in the models for this work are displacement and load. Displacement boundary conditions describe how element nodes move and load boundary conditions describe forces on nodes. A displacement and load boundary condition can not be applied on the same node degree of freedom at the same time, only one or the other may be prescribed. Figure 3.5 uses beam elements to illustrate displacement and load boundary conditions.

![Figure 3.5](image_url)

Figure 3.5: (a) Fixed-fixed beam undeformed and deformed. (b) Simplified beam undeformed and deformed.
Displacement boundary conditions can be prescribed on any degree of freedom for any node in a mesh. The displacement can tell how far the node will move along a given degree of freedom, or specify that the node will remain stationary. In Figure 3.5 each node has three degrees of freedom, horizontal, vertical, and rotational. Nodes 1 and 3 are constrained in all three degrees of freedom, meaning that they cannot rotate or translate, because this beam is fixed on both ends. When a node is fixed in all degrees of freedom is also called an encastre boundary condition.

In Figure 3.5a node 2 has no prescribed displacements and is only affected by the force which is applied as a load boundary condition to the vertical degree of freedom. However, Figure 3.5a can be simplified using a symmetry boundary condition. The symmetry boundary condition is a special form of the displacement boundary condition which constrains the motion of node 2 in Figure 3.5b such that it responds in the same manner as Figure 3.5a. This is accomplished by prescribing that node 2 cannot displace horizontally, nor rotate. This results in node 2 having the same displacement in both deformed beam representations shown in Figure 3.5.

Another special displacement boundary condition is used in dynamic models, the velocity boundary condition. The velocity boundary condition prescribes the rotational or translational velocity of body, element, or node. A displaced symmetry boundary condition, a combination of the symmetry and prescribed displacement boundary conditions, is used in several models. The displaced symmetry boundary condition effectively moves the plane of symmetry along a path normal to the plane.

The previously mentioned boundary conditions are used throughout the models developed in this work. For clarity a boundary condition figure is included for each model to describe the type and location of boundary conditions used in that model. A summary of model boundary conditions is presented in Figure 3.6. The symmetry boundary conditions indicate that the surface may not move perpendicular to the plane of symmetry, but may slide freely in that plane. The encastre boundary condition is used to connect the roll or frame to ground, which does not move. The horizontal fixed boundary allows the roll to translate vertically and rotate, but not move horizontally which simulates being held in the stand frame. The velocity boundary condition is applied to the work roll in the Strip Rolling Model because the work roll is driven by a motor in the actual stand. Lastly the bending force is denoted by a load boundary condition on the end of the work roll neck.

### 3.2.7 Initial Conditions

Initial conditions (IC) are boundary conditions that describe the state of the model at the beginning of the simulation. Initial conditions allow the simulation to start with components at rest or already in motion. In a static analysis dynamic effects are ignored so initial conditions do not apply. Therefore, the Roll-Stack Stiffness Model and Frame Stiffness Model did not use initial conditions. The Lumped Mass Model could have made use of initial conditions as it is a dynamic model; however, initial conditions were not used as the rotation of rolls and the translation of the strip were not simulated. The 2-D and 3-D Rolling Models made use of initial conditions, which allowed them to start with the strip and rolls already at steady-state speeds. The initial conditions removed the need to simulate the strip and rolls reaching steady-state speeds, the simulation simply started with those conditions. Figure 3.7 shows the initial conditions for the Rolling Model.

### 3.2.8 Constraints

Constraints are similar to boundary conditions in that they describe how nodes are constrained to move relative to each other. Constraints are used in two places in the finite element models.
Figure 3.6: Model boundary conditions.

Figure 3.7: Rolling Model initial conditions.
The first place is to tie the shell of the roll to the core. This tie constraint requires that the nodes on the inside surface of the shell cannot separate from the nodes on the outside surface of the core, where the nodes touch. The second instance of constraints occurs in the 3-D finite element models where the degrees of freedom of the bearing surface were tied to the degrees of freedom of a single reference point. This allows load and boundary conditions to be applied at one node and affect an entire surface. This technique was used at the end of the roll necks in the 3-D finite element models to restrict their motion and apply bending forces.

### 3.2.9 Implicit and Explicit Methods

Dynamic finite element models must integrate the equations of motion to represent inertial effects. This integration is performed using either explicit or implicit integration methods to represent time dependent problems. The difference between the two methods lies in how they integrate the equations of motion. Explicit methods formulate the equation of motion such that the solution at a time step is only dependent on the conditions in the previous step. This method has the advantage of being very efficient as iteration is not needed. However, the solution is highly dependent on the time step, which can cause instability if it’s too large. The solution of an implicit method at a given time step is not only dependent on the conditions in the previous step, but also dependent on unknowns in the current step. For this reason, the method must use iteration to find the solution at the current step. While implicit methods do require iteration, they are unconditionally stable. Therefore, the choice of time step does not affect the solution [30].

The static problem of the roll-stack and frame stiffness models was solved using the implicit method. Since the purpose of those models was to obtain a load-displacement curve with no time-dependent effects, the implicit method was the most effective solver because it is unconditionally stable and allows the step time to be controlled to give a specific number of points in the curve.

The dynamic models used the explicit method to obtain time-dependent solutions of the rolling problem. The explicit method was able to solve the highly nonlinear contact between the strip and rolls where the implicit model would have had difficulties due to mesh deformation. In addition, the large strains seen in the deformation of the strip were easily handled by the explicit method. Lastly, the explicit method is computationally efficient which reduced the run time of the large 3-D rolling model [30].

### 3.2.10 Finite Element Analysis Results

After solving for nodal displacements via the integration methods, information can be extracted and analyzed from the results. Many types of information are returned from finite element analysis such as element stresses and strains along with contact pressures and reaction forces. These quantities provide the basis for the analysis of the rolling process.

### 3.3 Strip Model

The strip model consists of a strip material model and a simplified stiffness model. The strip used an elastic-plastic model for the finite element analysis performed in this work. The strip stiffness model was based upon the rolling load model of Ford and Alexander [4].

#### 3.3.1 Material Model

Information on the material properties of the strip as it passes through the stands is difficult to obtain. The test data provided a predicted yield stress based on the online controller’s reaction to the
material. The mill controllers use the Ford and Alexander rolling load model for this prediction. The material model used in the finite element models was based upon an elastic-plastic material. Since the final model will be applied to materials that may not be steel, such as aluminum, it was not reasonable to limit it to one material model. Ford and Alexander assumes a rigid plastic material which has no strain hardening effects.

The following curves are constructed in terms of true stress and true strain. True strain $\epsilon_T$ is defined in Equation 3.4

$$\epsilon_T = \ln(1 + \epsilon)$$

(3.4)

where $\epsilon$ is engineering strain. The true stress $\sigma_t$ is based on engineering stress and engineering strain as shown in Equation 3.5.

$$\sigma_t = \sigma(1 + \epsilon)$$

(3.5)

An elastic-plastic material model was used for the construction of the finite element rolling models. A graphical representation of the stress-strain curve for the material model is shown in Figure 3.7. The elastic region was defined by the elastic modulus $E$ of the material. For the models in this work, the elastic modulus was assumed to be 18 tonne/mm$^2$, slightly less than the actual elastic modulus of steel because of temperature effects. The yield stress $S_y$ was given by the test data as 0.02118574 tonne/mm$^2$. The initial yield strain $\epsilon_1$ was found by Equation 3.6.

$$\epsilon_1 = \frac{S_y}{E}$$

(3.6)

The end point of the stress-strain curve $\epsilon_2$ was set to 100% strain as the estimated strain of the element was less. The yield stress coefficient $\gamma$ was used to provide the value of the true stress at 100% true strain. This variable was originally going to be used in a material calibration, but it was found to have little effect on the rolling load and exit thickness of the strip. A value of two was chosen for the yield stress coefficient such that the stress at the end of the stress-strain curve was $2S_y$.

![Figure 3.8: Elastic-plastic stress-strain curve with elliptical strain hardening.](image)

An elliptical curve was used to define the plastic region of the stress-strain curve as shown in Figure 3.7. The shape of the ellipse is defined in Equation 3.7 which can then be solved for $\sigma$ as in Equation 3.8.
Chapter 3. Rolling Model Development

3.3. Strip Model

\[
\frac{(\epsilon - \epsilon_2)^2}{(\epsilon_2 - \epsilon_1)^2} + \frac{(\sigma - S_y)^2}{(\gamma S_y)^2} = 1
\]  

(3.7)

\[
\sigma = \sqrt{\left(1 - \frac{(\epsilon - \epsilon_2)^2}{(\epsilon_2 - \epsilon_1)^2}\right)(\gamma S_y)^2}
\]  

(3.8)

With these parameters defined, the stress-strain curve for the finite element models was generated as shown in Figure 3.9. The elastic region of the model appears vertical due to the large strains the model covers. These stress-strain curves represent the true-stress true-strain condition of the material as opposed to the engineering-stress engineering-strain.

![Figure 3.9: Strip stress-strain curve used in the FE rolling models.](image)

3.3.2 Strip Stiffness

For the lumped-mass model in section 3.7 the reaction force of the strip must be represented as a spring. Ginzburg provided one method of accomplishing this task as detailed in section 2.2.1. However, Ginzburg’s model is a special case of the generalized method used in the lumped-mass model. Since the lumped-mass model represents the actions of the roll-stack at steady-state, determining how changes in exit thickness of the strip affect the rolling load can give a pseudo stiffness of the strip. The strip exit modulus \( k_x \) is defined by the industry as

\[
k_x = \frac{\partial P}{\partial H_X} \]

(3.9)
where $P/w$ is the rolling load per unit width of the strip and $H_X$ is the exit thickness of the strip. While this derivative cannot be taken analytically because there is no closed form solution to the rolling load equations, it is possible to find it through finite differencing. The Python code in Appendix: Rolling Load Model was used to calculate the rolling load based on the stand parameters. The exit thickness is perturbed above and below the predicted exit thickness of 5.497 mm by 2.5%. The rolling load is then calculated for both cases. The partial of rolling load is then found with respect to exit thickness by Equation 3.10.

$$k_x = \frac{\Delta P/w}{\Delta H_X} = \frac{(P_1 - P_2)/w}{H_X1 - H_X2}$$

(3.10)

Performing this calculation based on the test data, gave a $k_x$ of 0.608 tonne/mm$^2$. Then, multiplying by the width of the strip provides the strip stiffness $k_s$ as shown in Equation 3.11

$$k_s = k_x w$$

(3.11)

which gave a $k_p$ of 927.8 tonne/mm for a strip 1589.5 mm wide. This value approximates the stiffness of the strip at steady-state which was then used in the lumped-mass model.

### 3.4 Model Simplifications

To reduce the complexity of the models in this project, the following assumptions were made: stand symmetry can reduce the amount of the stand that must be modeled, the stand frame can be represented by a spring, the roll bearings hold the neck such that it cannot rotate in bending, and the draft of the strip can be modeled by displacing the strip into the rolls.

#### 3.4.1 Stand Symmetry

Figure 3.10 shows the symmetry of the stand. With this assumption the entire stand was represented by a quarter model when the rolls were rotating, or by an eighth model when the rolls were not rotating. The roll-stack stiffness and frame stiffness models do not require the rolls to rotate and so where modeled by using an eighth stand representation. The 2-D and 3-D rolling models required the rolls to rotate and so used a quarter stand representation.

#### 3.4.2 Frame Stiffness

The stand’s roll-stack and frame were modeled as a series of springs as shown in Figure 3.11. By inspecting the stand, it was possible to reduce the roll-stack to a stiffness $k_r$ that was in series with the frame of stiffness $k_f$. The total stiffness of the system $k_t$ was related to $k_f$ and $k_r$ by Equation 3.12.

$$k_t = \left( \frac{1}{k_r} + \frac{1}{k_f} \right)^{-1}$$

(3.12)

The total stiffness $k_t$ was determined by the stand calibration performed at the mill. The roll-stack stiffness $k_r$ was determined by a static implicit FE model. Thus, it was possible to solve for the frame stiffness $k_f$ as in Equation 3.13.

$$k_f = \left( \frac{1}{k_t} - \frac{1}{k_r} \right)^{-1}$$

(3.13)
Chapter 3. Rolling Model Development

3.4. Model Simplifications

Figure 3.10: Stand Symmetry, front and side views respectively.

Figure 3.11: Stand stiffness representation. (a) Stand. (b) Spring representation of stand & roll stack. (c) Simplification of b. (d) Roll-stack and stand spring in series. (e) Total stiffness of system.
This method was used by the roll-stack model to calculate the frame stiffness \( k_f \). The total stiffness of the roll-stack and frame was verified by the frame stiffness model using Equation 3.12. The value \( k_f \) is used as a lumped-parameter stiffness of the spring in the FE models which represent stand stiffness.

### 3.4.3 Strip Displacement Curve

To shorten run times, it was essential that the strip and rolls be brought into contact in a manner that minimized impact loads. Instead of setting the rolls a given distance apart and letting the strip impact the gap, the strip was brought into contact with the rolls by displacement. The process is illustrated in Figure 3.12. When the simulation starts, the roll and strip are aligned and in contact. As the simulation runs, the centerline of the strip is displaced upward. The displacement distance \( d_t \) is calculated in Equation 3.14

\[
d_t = \frac{H_E - H_g}{2}
\]  

(3.14)

where \( H_E \) is the entry thickness of the strip and \( H_g \) is the gap between the work rolls. The divisor in the equation is due to half of the strip being modeled.

![Figure 3.12: Strip displacement. (A) Initial system setup. (B) Location of the strip after displacement.](image)

The displacement curve was used to describe the transition from Figure 3.12A to Figure 3.12B by specifying the location of the strip centerline over time. A double harmonic motion curve was used to describe the displacement as given in Equation 3.15

\[
d = \frac{d_t}{2} \left[ \left( 1 - \cos \frac{\pi t}{\beta_t} \right) - \frac{1}{4} \left( 1 - \cos \frac{2\pi t}{\beta_t} \right) \right]
\]  

(3.15)

where \( d \) is the displacement at time \( t \), \( \beta_t \) is the total time in which the displacement occurs, and \( d_t \) is the displacement [25]. The displacement curve is shown in Figure 3.13.
3.5 Static Roll-Stack Stiffness Model

The Static Roll-Stack Stiffness Model was the first finite element model used to determine data necessary to simulate the rolling process. The roll-stack in the model refers to the collection of the four rolls in the four-high roll-stack stand configuration. The model is considered static as the dynamic response of the roll stack was not included. The eighth roll-stack finite element model was implemented in both 2-D and 3-D as the 2-D and 3-D models provide slightly different stiffnesses. The variation in stiffness is due the assumption of plane strain in the 2-D model and its inability to represent the roll geometry, such as the neck, which the 3-D model can represent.

3.5.1 Purpose

The purpose of the Roll-Stack Stiffness Model was to determine the stiffness of the roll stack. This information is necessary as it allows the stiffness of the frame to be found given the total frame and roll-stack stiffness from the test data. The roll-stack stiffness was determined by modeling one eighth of the total roll stack, a simplification that was justified due to the symmetry of the stand. Figure 3.14 shows the calculation of the roll stack stiffness $k_r$ from the eighth model. Due to symmetry conditions, the stiffness of the eighth model is one half that of the total roll-stack.

3.5.2 Procedure

In the roll-stack model, the backup roll bearing was held fixed and the stack was compressed by displacing the bottom symmetry condition of the work roll upward as shown in Figure 3.14. The balance bending force was applied to the work roll bearing throughout the process. The boundary
was displaced such that the maximum reaction force at the backup roll bearing was approximately 1500 tonnes.

Compressing the 2-D and 3-D roll stack models gave a load-deflection curve from which stiffness was determined. Taking the slope of this curve gave the stiffness of the entire roll stack, which was used in Equation 3.13 to determine the frame stiffness $k_f$. This analysis is shown in section 3.5.4.

### 3.5.3 Model Setup

The Roll-Stack Stiffness Model was generated from information given in the test data. The relevant test data for both the 2-D and 3-D models is presented in Table 3.1. The roll geometry and material properties were generated from this data. Mesh sizes were set after performing a mesh convergence study based on calculated stiffness. The mesh was considered sufficiently converged when the stiffness varied by less than 5% with changes in mesh size. Figure 3.15 shows the resulting partitioning and meshing scheme for the 2-D roll-stack models. The 3-D model used a similar mesh as shown in Figure 3.16. Boundary conditions, discussed in Section 3.2.6, were applied as shown in Figure 3.17. The 2-D model used reduced integration quadrilateral elements while the 3-D model used reduced integration hexahedral elements as discussed in Section 3.2.5.

Several boundary conditions were defined in the roll-stack model. The symmetry planes of the rolls were constrained such that surfaces could move vertically, but not away from the planes of symmetry. These constraints allowed the eighth model to simulate the full roll-stack. The backup roll bearing was fixed in place as shown in Figure 3.14. The work roll bearing was constrained such that it could only move vertically.

### 3.5.4 Analysis & Results

The load-displacement data for the 2-D and 3-D models was extracted from the output databases generated during the Abaqus solutions. This data was post-processed to account for symmetry conditions to give the load-displacement curve of the entire roll-stack. Figure 3.18 and Figure 3.19 show graphs and curve fits for the load displacement data from the 2-D and 3-D models respectively. In both data sets, the linear curve fits represent the deflection of the roll-stack with little error. From the fits, the stiffness of the roll stacks were extracted giving a $k_r$ of 1701.6 tonne/mm for the 2-D model and a $k_r$ of 1865.55 tonne/mm for the 3-D model. These roll-stack stiffness $k_r$
Table 3.1: Roll-Stack Stiffness Model setup data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work Roll Barrel Length</td>
<td>$B_{lw}$</td>
<td>2080</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Work Roll Base Diameter</td>
<td>$B_{dw}$</td>
<td>655.715</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Work Roll Neck Length</td>
<td>$N_{lw}$</td>
<td>195</td>
<td>mm</td>
<td>3-D</td>
</tr>
<tr>
<td>Work Roll Neck Diameter</td>
<td>$N_{dw}$</td>
<td>510</td>
<td>mm</td>
<td>3-D</td>
</tr>
<tr>
<td>Work Roll Barrel Dia Crown</td>
<td>$C_w$</td>
<td>-0.16</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Work Roll Shell Thickness</td>
<td>$t_{sw}$</td>
<td>50</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Backup Roll Barrel Length</td>
<td>$B_{lb}$</td>
<td>1820</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Backup Roll Base Diameter</td>
<td>$B_{db}$</td>
<td>1589.505</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Backup Roll Neck Length</td>
<td>$N_{lb}$</td>
<td>360</td>
<td>mm</td>
<td>3-D</td>
</tr>
<tr>
<td>Backup Roll Neck Diameter</td>
<td>$N_{db}$</td>
<td>945</td>
<td>mm</td>
<td>3-D</td>
</tr>
<tr>
<td>Backup Roll Barrel Dia Crown</td>
<td>$C_b$</td>
<td>0</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Backup Roll Shell Thickness</td>
<td>$t_{sb}$</td>
<td>60</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Roll Core Elastic Modulus</td>
<td>$E_{cb}$</td>
<td>21.092</td>
<td>tonne/mm$^2$</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Roll Core Poisson’s ratio</td>
<td>$\nu_{cb}$</td>
<td>0.29</td>
<td></td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Roll Shell Elastic Modulus</td>
<td>$E_{sb}$</td>
<td>21.092</td>
<td>tonne/mm$^2$</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Roll Shell Poisson’s ratio</td>
<td>$\nu_{sb}$</td>
<td>0.29</td>
<td></td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Balance Bending Force</td>
<td>$J_b$</td>
<td>150</td>
<td>tonne</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Roll-Roll Friction</td>
<td>$\mu_r$</td>
<td>0.2</td>
<td></td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Total Stiffness</td>
<td>$k_t$</td>
<td>610</td>
<td>tonne/mm</td>
<td>2-D &amp; 3-D</td>
</tr>
</tbody>
</table>

Figure 3.15: 2-D roll-stack model partition and meshing scheme.
values were used with Equation 3.13 to predict frame stiffness $k_f$ values of 950.87 tonne/mm and 906.36 tonne/mm for the 2-D and 3-D models respectively given that the total stiffness $k_t$ was 610 tonne/mm.

### 3.5.5 Summary

The roll-stack model was based on a finite element modeling of one eighth of the stand. The roll-stack model was used to calculate the stiffness of the roll-stack. The stiffness of the stack was then used to predict the stiffness of the frame. Table 3.2 summarizes the data extracted from the 2-D and 3-D roll-stack models and the predicted frame stiffnesses.

<table>
<thead>
<tr>
<th>Type</th>
<th>$k_t$ [tonne/mm]</th>
<th>$k_r$ [tonne/mm]</th>
<th>$k_s$ [tonne/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D</td>
<td>610.00</td>
<td>1701.62</td>
<td>950.87</td>
</tr>
<tr>
<td>3-D</td>
<td>610.00</td>
<td>1865.55</td>
<td>906.36</td>
</tr>
</tbody>
</table>
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3.5. Static Roll-Stack Stiffness Model

Figure 3.18: 2-D roll-stack load-displacement curve.

Figure 3.19: 3-D roll-stack load-displacement curve.
3.6 Static Frame Stiffness Model

The Static Frame Stiffness Model differs from the Static Roll-Stack Stiffness Model only in boundary conditions. 2-D and 3-D variations of the finite element model were constructed to calculate the stiffness of the frame.

3.6.1 Purpose

The purpose of the Frame Stiffness model was to verify the predicted stiffness of the frame from the Roll-Stack Stiffness Model. This verification was done by inserting a spring representing the frame and loading the rolls. This method attempts to reproduce the stand calibration performed in the mill which determines the total roll stack and frame stiffness. The model was constructed as shown in Figure 3.20.

![Figure 3.20: Eighth model of the roll stack to determine frame stiffness.](image)

3.6.2 Procedure

The procedure for the Frame Stiffness Model is the same as for the Roll-Stack Stiffness Model except for the distance the bottom symmetry boundary condition is displaced. In the stand calibration performed in the mill, the roll stack is loaded until the load sensor reads 1500 tonnes. The same is applied in this model such that the boundary condition is displaced until the reaction force is approximately 1500 tonnes.

3.6.3 Model Setup

All aspects of this model are the same as the roll-stack model except for the backup roll boundary condition and the addition of the frame represented by a spring. The relevant test data is summarized in Table 3.3. The mesh developed was the same as in the Roll-Stack Stiffness Model and may be found in Figures 3.15 and 3.16. As discussed in Section 3.2.5, reduced integration quadrilateral elements were used for the 2-D model and reduced integration hexahedrals were used for the 3-D model. The boundary conditions from Section 3.2.6 were applied as shown in Figure 3.21.
### Table 3.3: Static Frame Stiffness Model setup data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work Roll Barrel Length</td>
<td>(B_{lw})</td>
<td>2080</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Work Roll Base Diameter</td>
<td>(B_{dw})</td>
<td>655.715</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Work Roll Neck Length</td>
<td>(N_{lw})</td>
<td>195</td>
<td>mm</td>
<td>3-D</td>
</tr>
<tr>
<td>Work Roll Neck Diameter</td>
<td>(N_{dw})</td>
<td>510</td>
<td>mm</td>
<td>3-D</td>
</tr>
<tr>
<td>Work Roll Barrel Dia Crown</td>
<td>(C_{w})</td>
<td>-0.16</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Work Roll Shell Thickness</td>
<td>(t_{sw})</td>
<td>50</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Backup Roll Barrel Length</td>
<td>(B_{lb})</td>
<td>1820</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Backup Roll Base Diameter</td>
<td>(B_{db})</td>
<td>1589.505</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Backup Roll Neck Length</td>
<td>(N_{lb})</td>
<td>360</td>
<td>mm</td>
<td>3-D</td>
</tr>
<tr>
<td>Backup Roll Neck Diameter</td>
<td>(N_{db})</td>
<td>945</td>
<td>mm</td>
<td>3-D</td>
</tr>
<tr>
<td>Backup Roll Barrel Dia Crown</td>
<td>(C_{b})</td>
<td>0</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Backup Roll Shell Thickness</td>
<td>(t_{sb})</td>
<td>60</td>
<td>mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Roll Core Elastic Modulus</td>
<td>(E_{cb})</td>
<td>21.092</td>
<td>tonne/mm(^2)</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Roll Core Poisson’s ratio</td>
<td>(\nu_{cb})</td>
<td>0.29</td>
<td></td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Roll Shell Elastic Modulus</td>
<td>(E_{sb})</td>
<td>21.092</td>
<td>tonne/mm(^2)</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Roll Shell Poisson’s ratio</td>
<td>(\nu_{sb})</td>
<td>0.29</td>
<td></td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Balance Bending Force</td>
<td>(J_{b})</td>
<td>150</td>
<td>tonne</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Roll-Roll Friction</td>
<td>(\mu_{r})</td>
<td>0.2</td>
<td></td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>Total Stiffness</td>
<td>(k_t)</td>
<td>610</td>
<td>tonne/mm</td>
<td>2-D &amp; 3-D</td>
</tr>
<tr>
<td>2-D Frame Stiffness</td>
<td>(k_f)</td>
<td>950.87</td>
<td>tonne/mm</td>
<td>2-D</td>
</tr>
<tr>
<td>3-D Frame Stiffness</td>
<td>(k_f)</td>
<td>906.36</td>
<td>tonne/mm</td>
<td>3-D</td>
</tr>
</tbody>
</table>

![Figure 3.21: Roll-Stack Stiffness Model boundary conditions.](image-url)
In the roll-stack model the backup roll bearing boundary condition was fixed such that the bearing could not move vertically. In this model the roll is able to move, though it is constrained by the stand spring which connects the roll to ground. The frame stiffness is represented by a spring element of stiffness $k_f$ equal to that predicted by the roll-stack model. In the 2-D model the frame stiffness was 950.87 tonne/mm and in the 3-D model it was 906.36 tonne/mm.

### 3.6.4 Analysis & Results

The Frame Stiffness Model was run for both 2-D and 3-D cases. Load-displacement curves were calculated from the reaction force at the frame encastre boundary condition and the displacement of the symmetry boundary condition which compressed the rolls and frame. Figures 3.22 and 3.23 show the load-displacement curves for the 2-D and 3-D models respectively. The slopes of the curves were extracted in the same manner as the roll-stack model and the predicted value of the total stiffness was compared to the actual total stiffness of 610 tonne/mm. The 2-D case gave a total stiffness of 611.46 tonne/mm and the 3-D case gave a total stiffness of 609.80 tonne/mm.

**Figure 3.22**: 2-D frame stiffness load-displacement curve.

### 3.6.5 Summary

The Frame Stiffness Model was used to verify the frame stiffness calculated from the data in the Roll-Stack Stiffness Model. The analysis showed that the calculated frame stiffness yielded a predicted total stiffness that was less than 1% different from the actual total stiffness. Table 3.4 summarizes these results. These values for the frame stiffness were used in all of the following models.
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3.7 Dynamic Lumped-Mass Model

The Dynamic Lumped-Mass Model is an explicit finite element model created to simulate the dynamic reaction of the rolls oscillating in the stand. This model uses a two degree-of-freedom lumped-mass representation of the four-high roll stack which is further simplified through stand symmetry. The model most closely approximates the dynamics of the 3-D Rolling Model.

3.7.1 Purpose

There are two purposes for creating the Dynamic Lumped-Mass Model. The first is to create a model capable of simulating the dynamics of the roll stack. The second is to determine methods of controlling the dynamic response such that the roll stack reaches steady-state conditions as quickly as possible thereby shortening the simulation run time for the rolling models. The top quarter of the stand was modeled as a multiple-degree-of-freedom system as shown in Figure 3.24.

Since this model represents a quarter of the roll-stack and stand, $m_w$ and $m_b$ are half the mass of the work roll and backup roll respectively. The frame stiffness is $k_f$, the roll-stack stiffness is $k_r$, and the strip stiffness is $k_s$. Dampers were included in the model to control the dynamic response.

![Figure 3.23: 3-D frame stiffness load-displacement curve.](image)

![Table 3.4: Frame stiffness model results.](table)

<table>
<thead>
<tr>
<th>Type</th>
<th>$k_t$ (Measured) [tonne/mm]</th>
<th>$k_t$ (Predicted) [tonne/mm]</th>
<th>$k_t$ % Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D</td>
<td>610.00</td>
<td>611.46</td>
<td>0.24</td>
</tr>
<tr>
<td>3-D</td>
<td>610.00</td>
<td>609.80</td>
<td>0.03</td>
</tr>
</tbody>
</table>
The roll damping coefficient is $c_r$ and the frame damping coefficient is $c_f$. The values for these parameters are given in section 3.7.6.

### 3.7.2 Single Degree-of-Freedom System

The Lumped-Mass Model was developed to take advantage of explicit dynamic finite element modeling techniques similar to those used in the later models, though much simplified due to choice of representation. The foundation for the lumped-mass method is the single degree of freedom mass-spring-damper system with no forcing function shown in Figure 3.25. The mass-spring-damper system is represented by the equation of motion,

$$m\ddot{x} + c\dot{x} + kx = F$$

where $m$ is mass, $c$ is damping coefficient, $k$ is stiffness, and $F$ is a forcing function. The natural frequency of the system is defined as $\omega_n$ in Equation 3.17 [12].

$$\omega_n = \sqrt{\frac{k}{m}}$$ (3.17)

A single degree-of-freedom system will oscillate indefinitely without damping. There are three conditions that can occur when damping is added depending on the value of the damping coefficient. The model can either be overdamped, critically damped, or underdamped. In an overdamped system, there is no oscillation, but the system can require a long time to reach steady-state. In a critically damped system there is no oscillation and the system goes directly to steady-state usually in twice the period of the natural frequency. Lastly, underdamped systems oscillate, but the amplitude of the oscillation continually decreases until the system reaches steady-state. These conditions are defined by the damping ratio $\zeta$ given in Equation 3.18 [12].
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3.7. Dynamic Lumped-Mass Model

\[ \zeta = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{k/m}} \] \hspace{1cm} (3.18)

The damping coefficient can be greater than one (overdamped), equal to one (critically damped), or less than one (underdamped). The responses of the single degree of freedom system with different damping ratios are shown in Figure 3.26. The solutions were computed using an ordinary differential equation integration technique. It can be seen that the system with \( \zeta \) equal to one reaches steady-state the quickest [12].
3.7.3 Multiple Degree-of-Freedom System

Similar techniques can be applied to the multiple degree of freedom system of the roll-stack and frame; however, the equations must be rewritten in matrix form. The equation of motion remains the same, though values are replaced by mass, damping and stiffness matrices which represent the coupling between the displacement degrees of freedom associated with each mass as in Equation 3.19.

\[ M \ddot{x} + C \dot{x} + K x = F \]  

(3.19)

The matrices in the equation of motion are assembled by direct inspection. Equation 3.20 describes the mass matrix of the system. Since there are four nodes, as shown in Figure 3.24, the matrix has four rows and four columns because there are four degrees of freedom.

The nodes in the model are numbered from one to four in Figure 3.24. The rows and columns in the mass, damping, and stiffness matrices correspond to values at those nodes. For instance, row and column one reference node one.

\[
M = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & m_b & 0 & 0 \\
0 & 0 & m_w & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]  

(3.20)

The stiffness and damping matrices, in Equations 3.21 and 3.22 respectively, are assembled by inspection as well.

\[
C = \begin{bmatrix}
c_f & -c_f & 0 & 0 \\
-c_f & c_f + c_r & -c_r & 0 \\
0 & -c_r & c_r & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]  

(3.21)

\[
K = \begin{bmatrix}
k_f & -k_f & 0 & 0 \\
-k_f & k_f + k_r & -k_r & 0 \\
0 & -k_r & k_r + k_s & -k_s \\
0 & 0 & -k_s & k_s
\end{bmatrix}
\]  

(3.22)

Nodes one and four are fixed, having a prescribed displacement of zero. Since these nodes cannot move, their displacement needs not be solved for so the corresponding rows and columns can be removed. This produces the reduced mass, damping, and stiffness matrices which represent the system as given in Equation 3.23 [12].

\[
M = \begin{bmatrix}
m_b & 0 \\
0 & m_w
\end{bmatrix} \quad C = \begin{bmatrix}
c_f + c_r & -c_r \\
-c_r & c_r
\end{bmatrix} \quad K = \begin{bmatrix}
k_f + k_r & -k_r \\
-k_r & k_r + k_s
\end{bmatrix}
\]  

(3.23)

The natural frequencies of the system were found by solving the eigenvalue problem represented by Equation 3.24.


$$\text{det} \left[ -\omega^2 M + K \right] = 0 \quad (3.24)$$

For the two degree-of-freedom system in Equation 3.23, the solution of Equation 3.24 resulted in two natural frequencies, \( \omega_1 \) and \( \omega_2 \). The in-phase vibration mode had a lower natural frequency than the out-of-phase vibration mode [12].

As shown in the single-degree-of-freedom system, critically damping the multiple degree-of-freedom system should cause the system to reach steady state in the least amount of time. Critically damping a multiple degree-of-freedom system is more difficult because the interactions between the masses may not decouple the contribution of the damping forces between the masses. To ensure decoupling, proportional damping is commonly used. Proportional damping formulates a damping matrix by combining the mass and stiffness matrices as shown in Equation 3.25 [12].

$$C_{cr} = \alpha M + \beta K \quad (3.25)$$

The coefficients \( \alpha \) and \( \beta \) are modal damping ratios related to the natural frequencies of the system and the damping ratio by Equation 3.26 where \( \omega_i \) denotes each natural frequency of the system and \( \zeta_i \) is the damping ratio of each mode.

$$\zeta_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \quad (3.26)$$

Given Equation 3.26 and the two degrees of freedom that the system is reduced to, it was possible to solve for \( \alpha \) and \( \beta \) in terms of the two natural frequencies \( \omega_1 \) and \( \omega_2 \). Equations 3.27 and 3.28 define \( \alpha \) and \( \beta \) respectively [12].

$$\alpha = \frac{2\zeta \omega_1 \omega_2}{\omega_1 + \omega_2} \quad (3.27)$$

$$\beta = \frac{2\zeta \omega_1}{\omega_1 + \omega_2} \quad (3.28)$$

With \( \zeta \) equal to one to critically damp the system, Equations 3.27 and 3.28 reduce to the values shown in Equation 3.29.

$$\alpha = \frac{2\omega_1 \omega_2}{\omega_1 + \omega_2} \quad \beta = \frac{2}{\omega_1 + \omega_2} \quad (3.29)$$

Once \( \alpha \) and \( \beta \) were solved for, the damping values \( c_r \) and \( c_f \) needed to critically damp the model were found. Equation 3.30 shows how the system’s mass, damping, and stiffness matrices can be related through proportional damping.

$$C_{cr} = \alpha M + \beta K = \alpha \begin{bmatrix} m_b & 0 \\ 0 & m_w \end{bmatrix} + \beta \begin{bmatrix} k_f + k_r & -k_r \\ -k_r & k_r + k_s \end{bmatrix} = \quad (3.30)$$

However, the matrix that Equation 3.29 yields cannot be represented by the dampers in the lumped-mass model of the physical roll stack stand model. This is because the matrices \( C \) and \( C_{cr} \) have different forms as shown in Equation 3.31.

$$C = \begin{bmatrix} c_f + c_r & -c_r \\ -c_r & c_r \end{bmatrix} \quad C_{cr} = \begin{bmatrix} \alpha m_b + \beta(k_f + k_r) & -\beta k_r \\ -\beta k_r & \alpha m_w + \beta(k_r + k_s) \end{bmatrix} \quad (3.31)$$
Since the dampers in the model cannot exactly critically damp the model, an approximate solution was used. Based on the matrices in Equation 3.31, expressions for $c_f$ and $c_r$ were found as shown in Equation 3.32.

$$c_f = \alpha m_b + \beta k_f$$
$$c_r = \beta k_r$$

These damping coefficients approximately critically damp the lumped-mass system.

### 3.7.4 Explicit Method Formulation

The explicit method as discussed in section 3.2 was used to calculate the time solution of the lumped-mass model, specifically the exit thickness $H_X$ of the strip over time. The explicit method uses a differencing approach to approximate the velocity and acceleration of masses in the equation of motion as given by the second-order central difference Equations 3.33 and 3.34 [30]. The variable $u_n$ is used to represent the displacement of a node at a given time step.

$$v_n = \frac{u_{n+1} - u_{n-1}}{2\Delta t}$$
$$a_n = \frac{u_{n+1} - 2u_n + u_{n-1}}{(\Delta t)^2}$$

Substituting $u_n$, $v_n$, and $a_n$ into Equation 3.19 yields Equation 3.35 and solving for $u_{n+1}$ gives Equation 3.36 which represents the displacement of the nodes at the next time step [30].

$$M(u_{n+1} - 2u_n + u_{n-1}) + \frac{\Delta t}{2}C(u_{n+1} - u_{n-1}) + (\Delta t)^2Ku_n = (\Delta t)^2F$$

$$M + \frac{\Delta t}{2}C = (\Delta t)^2[F_n - Ku_n] + \frac{\Delta t}{2}Cu_{n-1} + M(2u_n - u_{n-1})$$

### 3.7.5 Model Run Parameters

Parameters were set to define the length of time to simulate with the model. If not enough time is allowed, the model will not come to steady-state. If too much time is allowed, the user’s time is wasted. The criteria used for the steady-state condition is determined by the exit thickness of the strip varying by less than one percent. Many factors affect how long it will take for the system to reach steady-state, such as the stiffness of the frame and strip and the mass of the work rolls. These factors cannot be controlled as they are characteristics of the stand. However, the displacement time and total run time are controllable.

Since the frequency of vibration of the rolls is dependent on the properties of the stand and vary by stand, the model must be able to account for these differences. Thus, it is not possible to tune the model for one stand as it must be extensible to stands of various sizes and properties. To take this variation into account, the run time $t_r$ and displacement time $t_d$ were set based on multiples of the period of oscillation as calculated from the multiple degree-of-freedom system in section 3.7.3. The lowest natural frequency of the system was used to calculate the period of oscillation $T$. These coefficients are denoted as $\alpha_d$ and $\alpha_r$ for displacement time and run time, respectively. Expressions for displacement time and run time as functions of $T$ are given in Equation 3.37

$$t_d = \alpha_d T$$
$$t_r = \alpha_r T$$
The displacement time coefficient, $\alpha_d$, directly affects the time the model requires to reach steady-state. The coefficient $\alpha_r$ determines how long the model will run. Figure 3.27 shows the strip exit thickness over time for different values of $\alpha_d$. As $\alpha_d$ increases, the time required to reach steady-state increases. However, lower values of $\alpha_d$ causes the amplitude of oscillation to increase. If the amplitude is too large, it could cause problems for the finite element models. Figure 3.28 displays rolling load versus time. Decreasing $\alpha_d$ has a similar effect on the amplitude of the rolling load, causing it to peak much higher than the steady-state value. The run-time coefficient $\alpha_r$ is necessary for the finite element models as they do not yet use Abaqus’s internal approaches to test for steady-state response. If $\alpha_r$ is too large, the model will take longer than necessary to run; however, if $\alpha_r$ is too small, the model will not come to steady-state. The lumped-mass model was used to estimate $\alpha_r$ and $\alpha_d$ for use in the rolling models.

![Figure 3.27: Strip exit thickness $H_X$ versus time $t$.](image)

### 3.7.6 Lumped-Mass Model Implementation

Equation 3.36 was implemented using the Python scripting language to solve for the time-domain dynamic response of the lumped-mass roll-stack explicit model. Spring and damper elements were used as shown in Figure 3.29. The modified system of the explicit model is slightly different from the multiple degree-of-freedom system of Section 3.7.3 which was used to find the natural frequency and damping coefficients. The values for the parameters used in the Lumped-Mass Model are shown in table 3.5.

The spring representing the strip was replaced by the rolling load $P$ which was approximated at each time step using the Ford and Alexander rolling load model. This substitution better represents the nonlinear load generated by rolling the strip than a spring of stiffness $k_s$ which is only
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3.7. Dynamic Lumped-Mass Model

Figure 3.28: Rolling load $P$ versus time $t$.

Figure 3.29: Lumped-mass explicit model.
valid near the steady-state. The mass, damping, and stiffness matrices used in the explicit model are given in Equation 3.38.

\[
M = \begin{bmatrix} m_b & 0 \\ 0 & m_w \end{bmatrix}, \quad C = \begin{bmatrix} c_f + c_r & -c_r \\ -c_r & c_r \end{bmatrix}, \quad K = \begin{bmatrix} k_f + k_r & -k_r \\ -k_r & k_r \end{bmatrix} \quad (3.38)
\]

The rolling load \( P \) is calculated at each time step based on the strip exit thickness, and the stand parameters. The exit thickness \( H_X \) of the strip at each step is assumed to be equal to the gap \( H_g \) which varies according to the displacement curve and work roll displacement \( u_2 \). Since the displacement time \( t_d \) represents the time in which the strip is displaced, \( \beta t \) from Equation 3.15 can be replaced by \( t_d \) to form Equation 3.39

\[
d = \frac{d_t}{2} \left[ \left( 1 - \cos \frac{\pi t}{t_d} \right) - \frac{1}{4} \left( 1 - \cos \frac{2\pi t}{t_d} \right) \right] \quad (3.39)
\]

where \( d \) is the displacement of the strip centerline at time \( t \) and \( d_t \) is the total displacement. Thus, the exit thickness, \( H_X \) is given by Equation 3.40.

\[
H_X = H_E + u_2 - 2d \quad (3.40)
\]

This exit thickness, along with the work roll and strip properties can be used to predict the rolling load \( P \) at each time step. This rolling load is then applied to the work roll at the next time step. Eventually the rolling load and work roll displacement reaches steady-state.

### 3.7.7 Results

Figures 3.30 and 3.31 show the rolling load and exit thickness versus time from the lumped-mass model. They verify that the model comes to steady-state in 2.25 periods with a displacement time of 0.875 periods criteria given for the steady-state condition in section 3.7.5. Table 3.6 shows the natural frequencies of the system, the oscillation period, maximum and steady-state rolling loads and exit heights. These results were used in the construction of the 2-D and 3-D FE rolling models.

---

**Table 3.5: Lumped-Mass Model setup data.**

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work Roll Mass</td>
<td>( m_w )</td>
<td>0.0003112</td>
<td>tonne-s(^2)/mm</td>
</tr>
<tr>
<td>Backup Roll Mass</td>
<td>( m_b )</td>
<td>0.0016382</td>
<td>tonne-s(^2)/mm</td>
</tr>
<tr>
<td>Frame Stiffness</td>
<td>( k_f )</td>
<td>927.21</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>Roll-Stack Stiffness</td>
<td>( k_r )</td>
<td>1865.55</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>Strip Stiffness</td>
<td>( k_s )</td>
<td>906.36</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>Roll Damping Coefficient</td>
<td>( c_r )</td>
<td>0.9200</td>
<td>tonne-s/mm</td>
</tr>
<tr>
<td>Frame Damping Coefficient</td>
<td>( c_f )</td>
<td>2.7825</td>
<td>tonne-s/mm</td>
</tr>
<tr>
<td>Displacement-Time Coefficient</td>
<td>( \alpha_d )</td>
<td>0.875</td>
<td></td>
</tr>
<tr>
<td>Run-Time Coefficient</td>
<td>( \alpha_r )</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>Displacement Time</td>
<td>( t_d )</td>
<td>0.005958</td>
<td>s</td>
</tr>
<tr>
<td>Run Time</td>
<td>( t_r )</td>
<td>0.015320</td>
<td>s</td>
</tr>
</tbody>
</table>
Figure 3.30: Lumped-mass model strip exit thickness $H_X$ versus time $t$.

Figure 3.31: Rolling load $P$ versus time $t$. 
Table 3.6: Lumped-mass model results.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Natural Frequency</td>
<td>146.86 Hz</td>
</tr>
<tr>
<td>Second Natural Frequency</td>
<td>498.57 Hz</td>
</tr>
<tr>
<td>Oscillation Period</td>
<td>0.0068089 s</td>
</tr>
<tr>
<td>Maximum Rolling Load</td>
<td>1373.12 tonne</td>
</tr>
<tr>
<td>Steady-State Rolling Load</td>
<td>1130.73 tonne</td>
</tr>
<tr>
<td>Minimum Exit Height</td>
<td>5.2384 mm</td>
</tr>
<tr>
<td>Steady-State Exit Height</td>
<td>5.5010 mm</td>
</tr>
<tr>
<td>Strip Centerline Displacement</td>
<td>1.4726 mm</td>
</tr>
<tr>
<td>Time to Steady-State</td>
<td>0.01275 s</td>
</tr>
</tbody>
</table>

3.8 Dynamic 2-D Rolling Model

The Dynamic 2-D Rolling Model was created to fill the gap between the Dynamic Lumped-Mass Model and the Dynamic 3-D Rolling Model. The 2-D Rolling Model provided a quick and simple method to test methods of simulating the rolling process in Abaqus without the added complexity of the 3-D Rolling Model.

3.8.1 Purpose

The purpose of the dynamic 2-D finite element model was to validate the results of the Lumped-Mass Model and work with the partitioning and meshing before moving on to the 3-D finite element model. The rotation of the rolls and motion of the strip is described in Figure 3.32.

Figure 3.32: 2-D rolling model detail.
3.8.2 Procedure

The model was constructed based on the test data in Table 3.8 and as described in Section 3.8.3. The model was generated in Abaqus using the Python scripting language. The natural frequency of the rolls was approximated as in Section 3.7.5 with the Lumped-Mass Model. The same run-time and displacement-time coefficients, $\alpha_r$ and $\alpha_d$ respectively, as in the Lumped-Mass Model were used to calculate the displacement time and run time. The natural frequencies were also used to calculate the roll and frame damping coefficients. The values of these parameters are summarized in Table 3.7.

The model was used to generate an input file that Abaqus used to run the simulation. The state of the system was taken at 100 equally spaced time intervals throughout the solution time and all information is recorded into a database file. This data includes stress, strain, reaction forces, displacements, and deformed shapes. The deformed thickness of the strip and the reaction force at the ground node were of primary interest. Running the simulation required less than five minutes.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period of Oscillation</td>
<td>$T$</td>
<td>0.00681</td>
<td>s</td>
</tr>
<tr>
<td>Natural Frequency 1</td>
<td>$f_1$</td>
<td>146.87</td>
<td>Hz</td>
</tr>
<tr>
<td>Natural Frequency 2</td>
<td>$f_2$</td>
<td>498.57</td>
<td>Hz</td>
</tr>
<tr>
<td>Run-Time Coefficient</td>
<td>$\alpha_r$</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>Displacement-Time Coefficient</td>
<td>$\alpha_d$</td>
<td>0.875</td>
<td></td>
</tr>
<tr>
<td>Run Time</td>
<td>$t_r$</td>
<td>0.01532</td>
<td>s</td>
</tr>
<tr>
<td>Displacement Time</td>
<td>$t_d$</td>
<td>0.00596</td>
<td>s</td>
</tr>
<tr>
<td>Roll Damping Coefficient</td>
<td>$c_r$</td>
<td>0.8587</td>
<td>tonne-s/mm</td>
</tr>
<tr>
<td>Frame Damping Coefficient</td>
<td>$c_f$</td>
<td>2.816</td>
<td>tonne-s/mm</td>
</tr>
</tbody>
</table>

3.8.3 Model Setup

The 2-D Rolling Model was generated from the test data and parameters in Tables 3.8 and 3.7. The roll diameters and shell thicknesses were used to construct the rolls. The run time $t_r$, calculated from the run-time coefficient and the system’s lowest natural frequency, was used to find how far the work roll would rotate given its angular velocity. The strip length was defined by the length of the arc through which the work roll would rotate from Equation 3.41.

$$l_s = R \omega t_r + L_p$$

(3.41)

The length of the strip is $l_s$, $R$ is the radius of the work roll, $t_r$ is the run time, $\omega$ is the angular velocity of the roll in radians per second, and $L_p$ is the estimated arc of contact from Ford and Alexander. This calculation gave a strip length of 109.86 mm for a simulation run time of 0.01532 s. The partitions on the roll shells were then set so that the finely meshed area was slightly longer than the length of the strip. The resulting partitions and mesh are shown in Figure 3.33.

Boundary conditions for the 2-D Rolling Model are shown in Figure 3.34. The centers of the rolls were fixed such that they could move vertically but not horizontally. The strip was given a displacement symmetry boundary condition to move the strip into place via the strip displace-
<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip Entry Thickness</td>
<td>$H_E$</td>
<td>6.4337</td>
<td>mm</td>
</tr>
<tr>
<td>Strip Elastic Modulus</td>
<td>$E_s$</td>
<td>18</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Strip Yield Stress</td>
<td>$S_y$</td>
<td>0.021186</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Strip Poisson’s Ratio</td>
<td>$\nu_s$</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Strip Density</td>
<td>$\rho_s$</td>
<td>7.9596e-13</td>
<td>tonne-s$^2$/mm$^4$</td>
</tr>
<tr>
<td>Strip Velocity</td>
<td>$v$</td>
<td>6752.64</td>
<td>mm/s</td>
</tr>
<tr>
<td>Strip Length</td>
<td>$l_s$</td>
<td>109.86</td>
<td>mm</td>
</tr>
<tr>
<td>Work Roll Base Diameter</td>
<td>$B_{dw}$</td>
<td>655.715</td>
<td>mm</td>
</tr>
<tr>
<td>Work Roll Shell Thickness</td>
<td>$t_{sw}$</td>
<td>50</td>
<td>mm</td>
</tr>
<tr>
<td>Work Roll Core Elastic Modulus</td>
<td>$E_{cw}$</td>
<td>21.092</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Work Roll Core Poisson’s ratio</td>
<td>$\nu_{cw}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Work Roll Shell Elastic Modulus</td>
<td>$E_{sw}$</td>
<td>21.092</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Work Roll Shell Poisson’s ratio</td>
<td>$\nu_{sw}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Work Roll Density</td>
<td>$\rho_w$</td>
<td>7.9596e-13</td>
<td>tonne-s$^2$/mm$^4$</td>
</tr>
<tr>
<td>Work Roll Angular Velocity</td>
<td>$\omega_w$</td>
<td>20.596</td>
<td>rad/s</td>
</tr>
<tr>
<td>Backup Roll Base Diameter</td>
<td>$B_{db}$</td>
<td>1589.505</td>
<td>mm</td>
</tr>
<tr>
<td>Backup Roll Shell Thickness</td>
<td>$t_{sb}$</td>
<td>60</td>
<td>mm</td>
</tr>
<tr>
<td>Backup Roll Core Elastic Modulus</td>
<td>$E_{cb}$</td>
<td>21.092</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Backup Roll Core Poisson’s ratio</td>
<td>$\nu_{cb}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Backup Roll Shell Elastic Modulus</td>
<td>$E_{sb}$</td>
<td>21.092</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Backup Roll Shell Poisson’s ratio</td>
<td>$\nu_{sb}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Backup Roll Density</td>
<td>$\rho_b$</td>
<td>7.9596e-13</td>
<td>tonne-s$^2$/mm$^4$</td>
</tr>
<tr>
<td>Backup Roll Angular Velocity</td>
<td>$\omega_b$</td>
<td>-8.497</td>
<td>rad/s</td>
</tr>
<tr>
<td>Stand Bending Force</td>
<td>$J$</td>
<td>96.88</td>
<td>tonne</td>
</tr>
<tr>
<td>Stand Gap</td>
<td>$H_g$</td>
<td>3.9831</td>
<td>mm</td>
</tr>
<tr>
<td>Stand Strip-Roll Friction</td>
<td>$\mu_s$</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Stand Roll-Roll Friction</td>
<td>$\mu_r$</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Stand Total Stiffness</td>
<td>$k_t$</td>
<td>610</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>2-D Roll-Stack Stiffness</td>
<td>$k_r$</td>
<td>1701.62</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>2-D Frame Stiffness</td>
<td>$k_f$</td>
<td>950.87</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>Strip Stiffness</td>
<td>$k_s$</td>
<td>927.21</td>
<td>tonne/mm</td>
</tr>
</tbody>
</table>
ment curve discussed in Section 3.4.3. An area of the cross section of the roll representing the neck was given a rotational boundary condition which drove the work roll. The bending load was applied to the center of the work roll effectively reducing the rolling load. The backup roll center was connected to the frame spring which in turn was connected to ground with an encastre boundary condition.

Initial conditions were prescribed for the 2-D Rolling Model. The initial conditions allow the simulation to start with the rolls rotating and the strip translating at approximately steady-state velocities. The work roll rotates at 20.60 rad/s counterclockwise while the backup roll rotates at 8.50 rad/s clockwise. These values were calculated from roll diameters based on the test data indicating that the strip was moving at 6752.64 mm/s. Figure 3.35 shows the application of these initial conditions to the model.
There are two regions in the 2-D Rolling Model where contact between bodies occurs: between the work roll and strip, and between the work roll and backup roll. A coefficient of friction of 0.2 was used in both contact regions. Hard contact was used with a penalty constrain method. A coefficient between 0.2 and 0.3 is typical when the rolls are lubricated [24].

### 3.8.4 Analysis & Results

The rolling load and the exit thickness of the strip were extracted from the output database for post-processing. These two outputs were used to determine when the system reached steady-state. The system must reach steady-state before the exit thickness and rolling load can be measured and compared with the test data. The steady-state region was selected from a window of 15% of the data. This window was moved along the data and an error variance within the window was calculated at each position. The window with the least error variance was chosen as the steady-state region. Figure 3.36 shows the rolling load versus time for the simulation. The red line denotes the steady-state region of the simulation. The steady-state rolling load was found by averaging the rolling load in the steady-state region which gives a rolling load of 976.28 tonnes.

Figure 3.37 shows how the exit thickness data is extracted from the model. The profile of the strip, represented by the red line in the Figure, is used to determine the exit thickness along the length of the strip. This exit thickness along the strip length is then plotted in Figure 3.38. From this graph, it is possible to determine when the strip reaches steady-state in the same manner as the rolling load. Again, the exit height is averaged in the steady-state region to give an average exit thickness of 5.6129 mm.

### 3.8.5 Summary

The predicted rolling load of 976.28 tonnes differs from the test data rolling load of 1107.20 tonnes by 11.82%. However, the exit thickness differs from the test data value of 5.4969 mm by 2.11% with a predicted value of 5.6129 mm. The simulation steady-state region is from approximately 0.008 to 0.012 s which is similar to the results for the Lumped-Mass Model at 0.01275 s. These results are summarized in Table 3.9. The later time on the Lumped-Mass Model was likely due to a more stringent steady-state requirement. The response in rolling load and exit thickness was
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3.8. Dynamic 2-D Rolling Model

**Figure 3.36:** 2-D simulation rolling load $P$ versus time $t$.

**Figure 3.37:** Strip profile for determination of exit thickness.
also smooth enough that the meshing scheme appears to have been sufficiently refined to give a reasonable solution. With the strip exit height as the metric, the mesh was converged to within five percent.

The 2-D rolling model provided another step towards developing a complete roll-stack model. While the 2-D model cannot predict crown, it simulates the system dynamics and shows that the methods developed in the Lumped-Mass Model can be used to control and predict the response of the system. The next step was to develop the 3-D Rolling Model capable of representing the entire stand and predicted the strip exit crown.

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Unit</th>
<th>Predicted</th>
<th>Actual</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolling Load</td>
<td>$P$</td>
<td>tonne</td>
<td>976.28</td>
<td>1107.2</td>
<td>11.82</td>
</tr>
<tr>
<td>Exit Thickness</td>
<td>$H_X$</td>
<td>mm</td>
<td>5.6129</td>
<td>5.4969</td>
<td>2.11</td>
</tr>
</tbody>
</table>

### 3.9 Dynamic 3-D Rolling Model

The dynamic 3-D finite element model is the next step beyond the 2-D rolling model. The 3-D rolling model is the goal of this work. It is able to represent the rolls and strip as described in the test data. The model is able to measure the strip crown exiting the stand and predict the rolling loads and exit thickness accordingly. Many aspects of the 3-D Rolling Model are similar to the 2-D Rolling Model.
3.9.1 Purpose

The purpose of the 3-D model is the same as the purpose of this work, to create a dynamic finite element model capable of predicting strip profile. The model is able to represent the geometry of the rolls and the material properties of the strip such that by simulating the rolling strip, it is possible to predict the rolling load, exit thickness, and crown of the strip.

3.9.2 Procedure

The 3-D Rolling Model was constructed from the parameters given in the test data. In addition, the run-time and displacement-time coefficients used in the Lumped-Mass Model and 2-D Rolling Model were also used in this model. Since the Lumped-Mass Model, 2-D Rolling Model, and 3-D Rolling Model all represent the same geometry, the same natural frequencies, damping coefficients and run times were found. These run-time parameters are summarized in Table 3.10.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip Stiffness</td>
<td>$k_s$</td>
<td>924.38</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>3-D Roll-Stack Stiffness</td>
<td>$k_r$</td>
<td>1865.55</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>3-D Frame Stiffness</td>
<td>$k_f$</td>
<td>906.36</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>Period of Oscillation</td>
<td>$T$</td>
<td>0.00681</td>
<td>s</td>
</tr>
<tr>
<td>Natural Frequency 1</td>
<td>$f_1$</td>
<td>146.79</td>
<td>Hz</td>
</tr>
<tr>
<td>Natural Frequency 2</td>
<td>$f_2$</td>
<td>498.36</td>
<td>Hz</td>
</tr>
<tr>
<td>Run-Time Coefficient</td>
<td>$\alpha_r$</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>Displacement-Time Coefficient</td>
<td>$\alpha_d$</td>
<td>0.875</td>
<td></td>
</tr>
<tr>
<td>Run Time</td>
<td>$t_r$</td>
<td>0.01532</td>
<td>s</td>
</tr>
<tr>
<td>Displacement Time</td>
<td>$t_d$</td>
<td>0.00596</td>
<td>s</td>
</tr>
<tr>
<td>Roll Damping Coefficient</td>
<td>$c_r$</td>
<td>0.9204</td>
<td>tonne-s/mm</td>
</tr>
<tr>
<td>Frame Damping Coefficient</td>
<td>$c_f$</td>
<td>2.7816</td>
<td>tonne-s/mm</td>
</tr>
</tbody>
</table>

Boundary conditions and initial conditions were then applied as discussed in Section 3.9.3. The model was used to generate a simulation which was run in Abaqus. Running the simulation of the 3-D Rolling Model required significantly more time than the 2-D Rolling Model. The increase in run time is due to the increase in the number of degrees of freedom in the model. Moving from 2-D to 3-D in finite element analysis often causes a large increase in computation time. The simulation ran to completion in approximately 55 minutes. The state of the system was captured every $1.5e-4$ s, 100 times during the simulation. These states were stored in an output database and included data such as stresses, strains, displacements, deformations, and reaction forces. This data provides the information necessary to calculate the rolling load, exit thickness, and strip crown which the model predicts.

3.9.3 Model Setup

The geometry of the rolls was created from the test data described in Table 3.11. The roll crown was made by drawing a spline through points to create a curve, which was then revolved to make the roll barrels. The strip profile was defined by a spline from the center of the strip to its
edge. This profile was then extruded to form the strip. The length of the extrusion and length of the strip was calculated in the same manner as in Section 3.8.3. The partitions were also sized as described in the 2-D rolling model such that only the finely meshed regions were in contact during the simulation. The mesh is shown in Figure 3.39.

![Figure 3.39: 3-D rolling model mesh.](image)

Figure 3.40 illustrates the boundary conditions for the 3-D Rolling Model. The centers of the rolls were fixed such that they could move vertically but not horizontally. The strip was given a displacement symmetry boundary condition to move the strip into place via the strip displacement curve discussed in Section 3.4.3. The end surface of the work roll neck was given a rotational boundary condition which simulates the motor driving the work roll. The bending load was applied to the same surface as the rotational boundary condition. The backup roll center was connected to the frame spring which in turn was connected to ground with an encastre boundary condition. A symmetry boundary condition was applied at the middle of the rolls and strip along their widths.

![Figure 3.40: 3-D Rolling Model boundary conditions.](image)
<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip Width</td>
<td>$w$</td>
<td>1524.84</td>
<td>mm</td>
</tr>
<tr>
<td>Strip Feather</td>
<td>$w_f$</td>
<td>40</td>
<td>mm</td>
</tr>
<tr>
<td>Strip Entry Thickness</td>
<td>$H_E$</td>
<td>6.4337</td>
<td>mm</td>
</tr>
<tr>
<td>Strip Elastic Modulus</td>
<td>$E_s$</td>
<td>18</td>
<td>tonne/mm²</td>
</tr>
<tr>
<td>Strip Yield Stress</td>
<td>$S_y$</td>
<td>0.021186</td>
<td>tonne/mm²</td>
</tr>
<tr>
<td>Strip Poisson's Ratio</td>
<td>$\nu_s$</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Strip Density</td>
<td>$\rho_s$</td>
<td>7.9596e-13</td>
<td>tonne-s²/mm⁴</td>
</tr>
<tr>
<td>Strip Velocity</td>
<td>$v$</td>
<td>6752.64</td>
<td>mm/s</td>
</tr>
<tr>
<td>Strip Length</td>
<td>$l_s$</td>
<td>110.83</td>
<td>mm</td>
</tr>
<tr>
<td>Work Roll Base Diameter</td>
<td>$B_{dw}$</td>
<td>655.715</td>
<td>mm</td>
</tr>
<tr>
<td>Work Roll Barrel Length</td>
<td>$B_{lw}$</td>
<td>2080</td>
<td>mm</td>
</tr>
<tr>
<td>Work Roll Neck Length</td>
<td>$N_{lw}$</td>
<td>195</td>
<td>mm</td>
</tr>
<tr>
<td>Work Roll Neck Diameter</td>
<td>$N_{dw}$</td>
<td>510</td>
<td>mm</td>
</tr>
<tr>
<td>Work Roll Barrel Diameter Crown</td>
<td>$C_w$</td>
<td>-0.16</td>
<td>mm</td>
</tr>
<tr>
<td>Work Roll Shell Thickness</td>
<td>$t_{sw}$</td>
<td>50</td>
<td>mm</td>
</tr>
<tr>
<td>Work Roll Core Elastic Modulus</td>
<td>$E_{cw}$</td>
<td>21.092</td>
<td>tonne/mm²</td>
</tr>
<tr>
<td>Work Roll Core Poisson's ratio</td>
<td>$\nu_{cw}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Work Roll Shell Elastic Modulus</td>
<td>$E_{sw}$</td>
<td>21.092</td>
<td>tonne/mm²</td>
</tr>
<tr>
<td>Work Roll Shell Poisson's ratio</td>
<td>$\nu_{sw}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Work Roll Density</td>
<td>$\rho_{w}$</td>
<td>7.9596e-13</td>
<td>tonne-s²/mm⁴</td>
</tr>
<tr>
<td>Work Roll Angular Velocity</td>
<td>$\omega_w$</td>
<td>20.5963</td>
<td>rad/s</td>
</tr>
<tr>
<td>Backup Roll Base Diameter</td>
<td>$B_{db}$</td>
<td>1589.505</td>
<td>mm</td>
</tr>
<tr>
<td>Backup Roll Barrel Length</td>
<td>$B_{lb}$</td>
<td>1820</td>
<td>mm</td>
</tr>
<tr>
<td>Backup Roll Neck Length</td>
<td>$N_{lb}$</td>
<td>360</td>
<td>mm</td>
</tr>
<tr>
<td>Backup Roll Neck Diameter</td>
<td>$N_{db}$</td>
<td>945</td>
<td>mm</td>
</tr>
<tr>
<td>Backup Roll Barrel Diameter Crown</td>
<td>$C_b$</td>
<td>0</td>
<td>mm</td>
</tr>
<tr>
<td>Backup Roll Shell Thickness</td>
<td>$t_{sb}$</td>
<td>60</td>
<td>mm</td>
</tr>
<tr>
<td>Backup Roll Core Elastic Modulus</td>
<td>$E_{cb}$</td>
<td>21.092</td>
<td>tonne/mm²</td>
</tr>
<tr>
<td>Backup Roll Core Poisson's ratio</td>
<td>$\nu_{cb}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Backup Roll Shell Elastic Modulus</td>
<td>$E_{sb}$</td>
<td>21.092</td>
<td>tonne/mm²</td>
</tr>
<tr>
<td>Backup Roll Shell Poisson's ratio</td>
<td>$\nu_{sb}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Backup Roll Density</td>
<td>$\rho_b$</td>
<td>7.9596e-13</td>
<td>tonne-s²/mm⁴</td>
</tr>
<tr>
<td>Backup Roll Angular Velocity</td>
<td>$\omega_b$</td>
<td>-8.49653</td>
<td>rad/s</td>
</tr>
<tr>
<td>Stand Bending Force</td>
<td>$J$</td>
<td>96.88</td>
<td>tonne</td>
</tr>
<tr>
<td>Stand Gap</td>
<td>$H_g$</td>
<td>3.9831</td>
<td>mm</td>
</tr>
<tr>
<td>Stand Strip-Roll Friction</td>
<td>$\mu_s$</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Stand Roll-Roll Friction</td>
<td>$\mu_r$</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Stand Total Stiffness</td>
<td>$k_t$</td>
<td>610</td>
<td>tonne/mm</td>
</tr>
</tbody>
</table>
The same initial conditions were prescribed for the 3-D Rolling Model as the 2-D model though the geometry differed slightly. The initial conditions allow the simulation to start with the rolls rotating and the strip translating at approximately steady-state velocities. The work roll rotates at 20.60 rad/s counterclockwise while the backup roll rotates at 8.50 rad/s clockwise. These values were calculated from roll diameters based on the test data indicating that the strip was moving at 6752.64 mm/s. Figure 3.35 shows the application of these initial conditions to the model.

![Figure 3.41: 3-D Rolling Model initial conditions.](image)

There are two surfaces in the 3-D Rolling Model where contact between bodies occurs: between the work roll and strip, and between the work roll and backup roll. A coefficient of friction of 0.2 was used in both contact regions. Hard contact was used with a penalty constrain method.

### 3.9.4 Analysis & Results

Data extraction in the 3-D model is slightly more complex than in the 2-D model as more information is necessary to calculate crown. The strip exiting the rolls may be visualized as shown in Figure 3.42. Only the right half of the strip was modeled, from the centerline to the edge because the 3-D Rolling Model is a quarter symmetry model. The rolling load versus time, shown in Figure 3.43 was extracted from the reaction force in the frame, the same as the 2-D Rolling Model. The exit thickness versus the strip length, Figure 3.44, was taken from the strip thickness along the centerline, similar to the 2-D Rolling Model.

The steady-state rolling load extracted from Figure 3.43 was 936.70 tonnes, a 15.39% difference from the test data rolling load of 1107.20 tonnes. The exit thickness predicted by the model was 5.7053 mm giving an error of 3.79% from the test data exit thickness of 5.4969 mm.

The crown is calculated from the steady-state region of the strip as selected in Figure 3.44. That steady-state region is the same region represented in Figure 3.42. The hatching in the steady-state region represents the elements. Each element has four nodes on the surface of the strip, one at each corner. By taking all of the nodes in this region and plotting their thickness versus the strip width, it is possible to create a graph of exit thickness versus strip width, represented by Figure 3.45. Because there will always be some variation in the strip along the length due to simulation noise and discretization, the nodes will show some scatter. However, a curve can be fit through these points to smooth the data and make the crown calculation simpler. Since the assumed form
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Figure 3.42: 3-D rolling model strip detail.

Figure 3.43: 3-D rolling model load versus time.
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Figure 3.44: 3-D rolling model exit thickness versus strip length.

Figure 3.46 shows the actual strip exit thickness versus width data generated by the model. However, this Figure only covers the width from the centerline to the feather, not to the edge of the strip, as the crown is calculated between the center and feather only. There are many more data points across the width of the strip and in each cluster than in Figure 3.45; this is because there are many more elements in the actual model. A parabola is fit through the data and the graph shows the fit to have good correlation with the test data. The crown is calculated by taking the thickness at the center of the parabola and subtracting the thickness at the feather. One could also simply calculate the crown by taking the average value of the data points at the center during steady state and subtracting from the average of the feather data points during steady state. The crown calculated by the fitting method is 0.1185 mm. The predicted crown has 275% error from the actual crown of 0.0316 mm.

3.9.5 Mesh Convergence

A mesh convergence study was performed to determine how the discretization of the strip and rolls affect the solution. Five simulations were run, each with a finer mesh in the contact region of the rolls and strip. The number of elements in each simulation was recorded. The exit thickness and strip profile were used as metrics to determine convergence. The exit thickness along the strip is shown in Figure 3.47. The exit thickness versus strip width for each simulation is shown in Figure 3.48. The simulations with the smallest number of elements had the noisiest results.

The exit thickness of the strip for each simulation was extracted from the data. The quality
Figure 3.45: Sample representation of strip exit thickness versus width.

Figure 3.46: 3-D rolling model strip exit thickness versus width.
Figure 3.47: 3-D rolling model exit thickness convergence data.

Figure 3.48: 3-D rolling model crown profile convergence data.
of the parabolic curve fit found based on the coefficient of determination $R^2$. The number of elements, exit thickness, and fit quality are summarized in Table 3.12.

<table>
<thead>
<tr>
<th>Sim</th>
<th>Number of Elements</th>
<th>Exit thickness</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31014</td>
<td>5.948</td>
<td>0.7718</td>
</tr>
<tr>
<td>2</td>
<td>33192</td>
<td>5.8851</td>
<td>0.8736</td>
</tr>
<tr>
<td>3</td>
<td>35832</td>
<td>5.7996</td>
<td>0.9480</td>
</tr>
<tr>
<td>4</td>
<td>40056</td>
<td>5.7476</td>
<td>0.9822</td>
</tr>
<tr>
<td>5</td>
<td>46128</td>
<td>5.7046</td>
<td>0.9990</td>
</tr>
</tbody>
</table>

Figure 3.49 shows exit thickness versus the number of elements in the simulation. The exit thickness decreases with each refinement in the mesh; however, the change in exit thickness between the last two points is less than one percent. Figure 3.50 shows the convergence of the fit quality $R^2$. As the mesh is refined, there is less noise in the curve fit. With respect to fit quality there is less than two percent change in the last two simulations. These results show that the simulation with 46128 elements is reasonably converged and is most likely not giving erroneous results due to mesh discretization.
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3.10 Summary

Seven models were developed in the process of creating the final 3-D rolling model. The roll-stack model was used to determine the stiffness of the roll-stack. This data was then used to predict the stand stiffness and the stand stiffness model was used to validate that prediction by matching the calibration in the test data. The Lumped-Mass Model was used to explore the dynamics of the system, indicating how long to make the runs and how quickly to displace the strip. The stand stiffness from the first two models and the run time parameters of the Lumped-Mass Model were
used in the 2-D rolling model to verify the system dynamics. Lessons in meshing and rolling from the 2-D rolling model were then used to create the final 3-D rolling model. The 3-D model is capable of predicting the exit thickness, rolling load, and strip crown required by the project goals.

The error in exit thickness, rolling load, and strip crown predicted by the 3-D Rolling Model can be large, especially in the crown prediction. The next chapter documents two methods used to reduce this error and then validates the calibrated model.
4 Calibration, Validation, & Sensitivity

4.1 Introduction

The purpose of this chapter is to show how the stand parameters causing the error in rolling load, exit thickness, and crown were determined and how they were used to reduce the error to calibrate the model. Once the calibration was complete, the 3-D model was run again and final error estimates were found. Lastly, the model was used to determine strip crown sensitivity with respect to controllable stand parameters.

4.2 Calibration

With the 3-D rolling model created, it became necessary to further calibrate the model before generating the final validation. These calibrations assumed that there were slight variations in the test data that caused the prediction to not match the test data, especially with respect to crown. All of the outputs of interest: exit thickness, rolling load, and crown, were very sensitive to slight variations in model parameters. Thus, the result error could be reduced with slight adjustments to the parameters.

4.2.1 Selection of Parameters

An examination of the stand parameters, from work roll barrel diameter to strip width, revealed that most parameters were based on easily measured geometric properties or well known materials. Only a few parameters could cause the resulting errors from measurement uncertainty.

Strip yield stress was one of these parameters. Small variations in yield stress caused large changes in rolling load and exit thickness. The strip yield stress in the test data was measured by the online controllers using the Ford and Alexander rolling-load model. This method assumed that the rolling-load model accurately represented the rolling process, which may not be valid. Variations in rolling load can be due to factors other than yield stress, entry thickness, and exit thickness. For instance, changes in crown can cause a change in rolling load. In addition, the representation of the strip material in the 3-D rolling model was different from that of the rolling-load model, which may have been a source of error.

A second source of error was gap measurement. During the stand calibration at the mill, the rolls were brought together until the load sensors read 1500 tonnes. The displacement of the rolls
was then set to zero. All gaps for the controllers were calculated from this point, so it is not absolutely known when the rolls met. Thus, the gap measurement may be the source of some error.

A third source of error was work roll thermal expansion. The test data was collected on a strip that was rolled directly after a roll change operation. The work rolls in a mill are changed every eight hours due to wear. Thus, the rolls were at room temperature when the strip entered them. Initially it was assumed that the rolls would not be in contact with the strip long enough for significant heat transfer to occur. However, this assumption was reconsidered when the predicted crown was significantly different from the actual crown. Thermal expansion could cause the work roll barrel diameter and crown to change. While the change in diameter would not significantly affect the model, a very small change in work roll crown could dramatically change the strip crown.

Thus, the strip yield stress, gap, and work roll crown were selected as parameters which could be varied to reduce the error in rolling load, exit thickness, and strip crown.

### 4.2.2 Calibration Method

Given that strip yield stress, gap, and work roll crown where chosen to calibrate the exit thickness, rolling load, and exit crown, the question arose as to how to perform the calibration. Two calibrations were used to minimize the three errors. By reexamining the parameters, it was found that rolling load and exit thickness were mainly dependent on the yield stress and gap. Variations in work roll crown did not significantly affect the rolling load and exit thickness. Thus, one quadratic response surface was defined to find the values of yield stress and gap that would minimize rolling-load and exit-thickness error. Then, a quadratic fit was used to find the work roll crown that would minimize the strip crown error.

The first calibration, termed the Yield-Gap Calibration, varied the strip yield stress and gap to minimize the error in rolling load and exit thickness from the model prediction to the test data. The second calibration, Work Roll Crown Calibration, varied the work roll crown to account for thermal expansion to minimize strip crown error. Simulations were run after each calibration for validation.

### 4.2.3 Model Setup

The information used to construct the models for the calibrations was the same as that used to construct the 3-D Rolling Model in Section 3.9 except for the values that were varied during the calibrations. Tables 4.1, 4.2, 4.3, 4.4, and 4.5 contain the properties and values necessary to construct the models.

### 4.3 Yield-Gap Calibration

The yield-gap calibration seeks to minimize the error in rolling load and exit thickness by varying the strip yield stress and gap.

#### 4.3.1 Response Surface

Nine simulations were required to define the quadratic response surface shown in Figure 4.1. The surface is defined over an isoparametric space such that values of yield stress and gap may be chosen arbitrarily. The interpolation functions at each node are defined in Equation 4.1 [22].
Table 4.1: Yield-Gap Calibration run data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period of Oscillation</td>
<td>$T$</td>
<td>0.00681</td>
<td>s</td>
</tr>
<tr>
<td>Natural Frequency 1</td>
<td>$f_1$</td>
<td>146.87</td>
<td>Hz</td>
</tr>
<tr>
<td>Natural Frequency 2</td>
<td>$f_2$</td>
<td>498.57</td>
<td>Hz</td>
</tr>
<tr>
<td>Run-Time Coefficient</td>
<td>$\alpha_r$</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>Displacement-Time Coefficient</td>
<td>$\alpha_d$</td>
<td>0.875</td>
<td></td>
</tr>
<tr>
<td>Run Time</td>
<td>$t_r$</td>
<td>0.01532</td>
<td>s</td>
</tr>
<tr>
<td>Displacement Time</td>
<td>$t_d$</td>
<td>0.00596</td>
<td>s</td>
</tr>
<tr>
<td>Roll Damping Coefficient</td>
<td>$c_r$</td>
<td>0.8587</td>
<td>tonne-s/mm</td>
</tr>
<tr>
<td>Frame Damping Coefficient</td>
<td>$c_f$</td>
<td>2.816</td>
<td>tonne-s/mm</td>
</tr>
</tbody>
</table>

Table 4.2: Yield-Gap Calibration work roll data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Diameter</td>
<td>$B_{dw}$</td>
<td>655.715</td>
<td>mm</td>
</tr>
<tr>
<td>Barrel Length</td>
<td>$B_{lw}$</td>
<td>2080</td>
<td>mm</td>
</tr>
<tr>
<td>Neck Length</td>
<td>$N_{lw}$</td>
<td>195</td>
<td>mm</td>
</tr>
<tr>
<td>Neck Diameter</td>
<td>$N_{dw}$</td>
<td>510</td>
<td>mm</td>
</tr>
<tr>
<td>Barrel Diameter Crown</td>
<td>$C_{w}$</td>
<td>-0.16</td>
<td>mm</td>
</tr>
<tr>
<td>Shell Thickness</td>
<td>$t_{sw}$</td>
<td>50</td>
<td>mm</td>
</tr>
<tr>
<td>Core Elastic Modulus</td>
<td>$E_{cw}$</td>
<td>21.092</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Core Poisson's ratio</td>
<td>$\nu_{cw}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Shell Elastic Modulus</td>
<td>$E_{sw}$</td>
<td>21.092</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Shell Poisson's ratio</td>
<td>$\nu_{sw}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_{w}$</td>
<td>7.9596e-13</td>
<td>tonne-s$^2$/mm$^4$</td>
</tr>
<tr>
<td>Angular Velocity</td>
<td>$\omega_{w}$</td>
<td>20.5963</td>
<td>rad/s</td>
</tr>
</tbody>
</table>

Figure 4.1: Isoparametric transformation of error space.
### Table 4.3: Yield-Gap Calibration backup roll data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Diameter</td>
<td>(B_{db})</td>
<td>1589.505</td>
<td>mm</td>
</tr>
<tr>
<td>Barrel Length</td>
<td>(B_{lb})</td>
<td>1820</td>
<td>mm</td>
</tr>
<tr>
<td>Neck Length</td>
<td>(N_{lb})</td>
<td>360</td>
<td>mm</td>
</tr>
<tr>
<td>Neck Diameter</td>
<td>(N_{db})</td>
<td>945</td>
<td>mm</td>
</tr>
<tr>
<td>Shell Thickness</td>
<td>(t_{sb})</td>
<td>60</td>
<td>mm</td>
</tr>
<tr>
<td>Barrel Diameter Crown</td>
<td>(C_{b})</td>
<td>0</td>
<td>mm</td>
</tr>
<tr>
<td>Core Elastic Modulus</td>
<td>(E_{cb})</td>
<td>21.092</td>
<td>tonne/mm²</td>
</tr>
<tr>
<td>Core Poisson's ratio</td>
<td>(\nu_{cb})</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Shell Elastic Modulus</td>
<td>(E_{sb})</td>
<td>21.092</td>
<td>tonne/mm²</td>
</tr>
<tr>
<td>Shell Poisson's ratio</td>
<td>(\nu_{sb})</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>(\rho_{b})</td>
<td>7.9596e-13</td>
<td>tonne-s²/mm⁴</td>
</tr>
<tr>
<td>Angular Velocity</td>
<td>(\omega_{b})</td>
<td>-8.49653</td>
<td>rad/s</td>
</tr>
</tbody>
</table>

### Table 4.4: Yield-Gap Calibration strip data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>(w)</td>
<td>1524.84</td>
<td>mm</td>
</tr>
<tr>
<td>Feather</td>
<td>(w_f)</td>
<td>40</td>
<td>mm</td>
</tr>
<tr>
<td>Entry Thickness</td>
<td>(H_E)</td>
<td>6.4337</td>
<td>mm</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>(E_s)</td>
<td>18</td>
<td>tonne/mm²</td>
</tr>
<tr>
<td>Yield Stress</td>
<td>(S_y)</td>
<td>0.021186</td>
<td>tonne/mm²</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>(\nu_s)</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>(\rho_s)</td>
<td>7.9596e-13</td>
<td>tonne-s²/mm⁴</td>
</tr>
<tr>
<td>Velocity</td>
<td>(v)</td>
<td>6752.64</td>
<td>mm/s</td>
</tr>
<tr>
<td>Length</td>
<td>(l_s)</td>
<td>109.86</td>
<td>mm</td>
</tr>
</tbody>
</table>

### Table 4.5: Yield-Gap Calibration stand data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending Force</td>
<td>(J)</td>
<td>96.88</td>
<td>tonne</td>
</tr>
<tr>
<td>Gap</td>
<td>(H_{y})</td>
<td>3.9831</td>
<td>mm</td>
</tr>
<tr>
<td>Strip-Roll Friction</td>
<td>(\mu_s)</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Roll-Roll Friction</td>
<td>(\mu_r)</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Total Stiffness</td>
<td>(k_t)</td>
<td>610</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>3-D Roll-Stack Stiffness</td>
<td>(k_r)</td>
<td>1865.55</td>
<td>tonne/mm</td>
</tr>
<tr>
<td>3-D Frame Stiffness</td>
<td>(k_f)</td>
<td>906.36</td>
<td>tonne/mm</td>
</tr>
</tbody>
</table>
4.3. Yield-Gap Calibration

\[
\begin{align*}
\psi_1 &= \frac{1}{4}(\xi^2 - \xi)(\eta^2 - \eta) \\
\psi_2 &= \frac{1}{2}(1 - \xi^2)(\eta^2 - \eta) \\
\psi_3 &= \frac{1}{4}(\xi^2 + \xi)(\eta^2 - \eta) \\
\psi_4 &= \frac{1}{2}(\xi^2 - \xi)(1 - \eta^2) \\
\psi_5 &= (1 - \xi^2)(1 - \eta^2) \\
\psi_6 &= \frac{1}{2}(\xi^2 + \xi)(\eta^2 + \eta) \\
\psi_7 &= \frac{1}{4}(\xi^2 - \xi)(\eta^2 + \eta) \\
\psi_8 &= \frac{1}{2}(1 - \xi^2)(\eta^2 + \eta) \\
\psi_9 &= \frac{1}{2}(\xi^2 + \xi)(\eta^2 + \eta)
\end{align*}
\] (4.1)

The isoparametric representation allows yield stress \(S_y\), gap \(H_g\), and error \(\varepsilon\), to be represented parametrically as in Equation 4.2

\[
S_y(\xi, \eta) = \sum_{i=1}^{n} S_{yi}\psi_i \\
H_g(\xi, \eta) = \sum_{i=1}^{n} H_{gi}\psi_i \\
\varepsilon(\xi, \eta) = \sum_{i=1}^{n} \varepsilon_i\psi_i
\] (4.2)

where \(S_{yi}, H_{gi}, \text{and } \varepsilon_i\) are the yield stress, gap, and error respectively at node \(i\). Thus, when the nine simulations are run, the error can be expressed over the space defined by the values of \(S_y\) and \(H_g\).

4.3.2 Error Function

The error for the yield-gap calibration requires specific definition. The errors being referred to are percent errors between the simulation predicted values and test data values of exit thickness \(H_X\) and rolling load \(P\). Thus, the percent error \(\varepsilon_p\) and \(\varepsilon_h\) are defined in Equation 4.3.

\[
\varepsilon_p = \left( \frac{\text{Predicted } P - \text{Actual } P}{\text{Actual } P} \right) \times 100 \\
\varepsilon_h = \left( \frac{\text{Predicted } H_X - \text{Actual } H_X}{\text{Actual } H_X} \right) \times 100
\] (4.3)

This gives two errors: \(\varepsilon_p\) for rolling-load error and \(\varepsilon_h\) for exit-thickness error. The total error \(\varepsilon_t\) used in the response surface is defined by Equation 4.4;

\[
\varepsilon_t = \varepsilon_p^2 + \varepsilon_h^2
\] (4.4)

thus, the minimum total error represented by the surface will be zero.

4.3.3 Simulation Parameters

The nine simulations represent a full factorial experiment with three values of yield stress and three values of gap. The test data gave a yield stress of 0.02119 tonne/mm\(^2\) and gap of 3.98 mm. New values for yield stress and gap were chosen within approximately ten percent of the original values. The calibration values for yield stress were 0.019, 0.021, and 0.023 tonne/mm\(^2\) and the values for gap were 3.1, 3.5, and 3.9 mm. The experimental design is summarized in Table 4.6.

4.3.4 Simulation Generation & Running

The simulations were generated using the 3-D rolling model. The data used to construct that model are listed in Section 4.2.3 except for the yield stress and gap values which are given in Table 4.6. A total of nine simulations were generated and run.
### Table 4.6: Yield-Gap Calibration design.

<table>
<thead>
<tr>
<th>Node</th>
<th>$S_y$</th>
<th>$H_y$</th>
<th>Node</th>
<th>$S_y$</th>
<th>$H_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>tonne/mm²</td>
<td>mm</td>
<td>#</td>
<td>tonne/mm²</td>
<td>mm</td>
</tr>
<tr>
<td>1</td>
<td>0.019</td>
<td>3.1</td>
<td>6</td>
<td>0.023</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>0.021</td>
<td>3.1</td>
<td>7</td>
<td>0.019</td>
<td>3.9</td>
</tr>
<tr>
<td>3</td>
<td>0.023</td>
<td>3.1</td>
<td>8</td>
<td>0.021</td>
<td>3.9</td>
</tr>
<tr>
<td>4</td>
<td>0.019</td>
<td>3.5</td>
<td>9</td>
<td>0.023</td>
<td>3.9</td>
</tr>
<tr>
<td>5</td>
<td>0.021</td>
<td>3.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4.3.5 Data Extraction & Results

The data was extracted from the output databases. The following graphs are presented for observing simulation quality, so they do not reference specifically which node each simulation represents.

Figure 4.2 shows the rolling load versus time for the nine simulations. The rolling-load curves all have a similar shape and flatten out to steady-state at approximately the same time. Low error variance is also shown in the steady-state regions. The exit thickness along the length of the strip is shown in Figure 4.3. While the exit-thickness curves are not as clean as the rolling-load curves, they do all show the system reaching steady state. The exit thickness versus strip width with curve fits is shown in Figure 4.4. These curves show that the parabola fit does well represent the strip crown over a variety of conditions. The spread around each curve also indicates the quality of the steady-state data.

![Figure 4.2: Rolling load versus time for yield-gap calibration simulations](image-url)
Chapter 4. Calibration, Validation, & Sensitivity  

4.3. Yield-Gap Calibration

Figure 4.3: Exit thickness versus strip length for yield-gap calibration simulations.

The numerical data extracted from the graphs is summarized in Table 4.7. The symbols $S_y$, $H_g$, $P$, and $H_X$ represent the yield stress, gap, rolling load, and exit thickness respectively. The exit-thickness error, rolling-load error and total error are given by $\varepsilon_h$, $\varepsilon_p$, and $\varepsilon_t$.

Table 4.7: Yield-gap calibration simulation data summary.

<table>
<thead>
<tr>
<th>Node</th>
<th>$S_y$</th>
<th>$H_g$</th>
<th>$P$</th>
<th>$H_X$</th>
<th>$\varepsilon_g$</th>
<th>$\varepsilon_p$</th>
<th>$\varepsilon_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>tonne/mm²</td>
<td>mm</td>
<td>tonne</td>
<td>mm</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>1</td>
<td>0.019</td>
<td>3.1</td>
<td>1164.82</td>
<td>5.2032</td>
<td>5.20</td>
<td>-5.34</td>
<td>55.63</td>
</tr>
<tr>
<td>2</td>
<td>0.021</td>
<td>3.1</td>
<td>1227.9</td>
<td>5.306</td>
<td>10.90</td>
<td>-3.47</td>
<td>130.90</td>
</tr>
<tr>
<td>3</td>
<td>0.023</td>
<td>3.1</td>
<td>1281.87</td>
<td>5.4013</td>
<td>15.78</td>
<td>-1.74</td>
<td>251.90</td>
</tr>
<tr>
<td>4</td>
<td>0.019</td>
<td>3.5</td>
<td>1041.22</td>
<td>5.3944</td>
<td>-5.96</td>
<td>-1.86</td>
<td>38.99</td>
</tr>
<tr>
<td>5</td>
<td>0.021</td>
<td>3.5</td>
<td>1092.42</td>
<td>5.4821</td>
<td>-1.33</td>
<td>-0.27</td>
<td>1.85</td>
</tr>
<tr>
<td>6</td>
<td>0.023</td>
<td>3.5</td>
<td>1141.42</td>
<td>5.5698</td>
<td>3.09</td>
<td>1.33</td>
<td>11.31</td>
</tr>
<tr>
<td>7</td>
<td>0.019</td>
<td>3.9</td>
<td>911.43</td>
<td>5.5892</td>
<td>-17.68</td>
<td>1.68</td>
<td>315.46</td>
</tr>
<tr>
<td>8</td>
<td>0.021</td>
<td>3.9</td>
<td>961.34</td>
<td>5.6604</td>
<td>-13.17</td>
<td>2.97</td>
<td>182.40</td>
</tr>
<tr>
<td>9</td>
<td>0.023</td>
<td>3.9</td>
<td>1008.29</td>
<td>5.7384</td>
<td>-8.93</td>
<td>4.39</td>
<td>99.11</td>
</tr>
</tbody>
</table>

4.3.6 Calibration Results

The values of yield stress $S_y$, gap $H_g$, and the error ($\varepsilon_p$, $\varepsilon_h$, and $\varepsilon_t$) were used to construct response surfaces. Figure 4.5 shows the response surface represented as a contour of rolling-load
error squared $\varepsilon_p^2$ such that the minimum error was zero. A valley of zero error is shown going diagonally upward from left to right in the contour.

The strip exit-thickness error squared $\varepsilon_h^2$ is shown in Figure 4.6. Once again the contour represents the error squared such that all error is positive and the minimum error is zero. Another valley of zero error is found going diagonally downward from left to right.

The total error contour $\varepsilon_t$ represents the sum of the squares of rolling-load and strip exit-thickness error, is shown in Figure 4.7. When the contours from Figures 4.5 and 4.6 are combined, the valleys merge to create a localized minimum of error. The red marker at the center represents the minimum error in the contour. The location of this point was found using a bounded minimizer in the python scripting language given the error represented by the response surface.

Thus, the minimum error occurs when the yield stress is $0.02145$ tonne/mm$^2$ and the gap is $3.489$ mm. The calibrated yield stress and gap are $1.23\%$ and $12.40\%$ percent different from the test data respectively.

### 4.4 Work Roll Crown Calibration

The purpose of the work roll crown calibration was to adjust for the thermal expansion of the roll affecting its crown. The actual thermal expansion was not modeled, but its effects were simulated by changing the work roll crown $C_w$. Since only one variable is being varied, fewer runs are necessary to create a response surface. In fact, the response surface is one dimensional in the work roll crown calibration.
Figure 4.5: Rolling-load error contour, $\varepsilon_p^2$.

Figure 4.6: Exit-thickness error contour, $\varepsilon_h^2$. 
4.4.1 Response Surface

The response surface definition for this calibration was different than that of the yield-gap calibration. Since more data points could easily be taken and only one parameter was being varied, a curve fit of the data was used to generate the response surface function. Five simulations were run with varying work roll crowns and then the exit crown error $\varepsilon_c$ was calculated in reference to the measured strip crown as given by Equation 4.5

$$\varepsilon_c = \left( \frac{\text{Predicted } C_X - \text{Actual } C_X}{\text{Actual } C_X} \right) \times 100$$

where $C_X$ is the exit crown.

4.4.2 Data Extraction & Results

The test data gave a work roll crown of -0.16 mm. Simulations were run with the work roll crowns shown in Table 4.8. The curve fits of the strip data to determine the crowns are shown in Figure 4.8. The graph shows that changes in work roll crown have little effect on the exit thickness at the centerline of the strip. There is more variation in this data near the edge of the strip. The work roll crowns and corresponding exit crowns predicted by the simulations are summarized in Table 4.8.

4.4.3 Calibration Results

The strip crown error squared $\varepsilon_c^2$ was plotted against the work roll crown $C_w$ as shown in Figure 4.9. The data was fit with a second order polynomial described by Equation 4.6.
The curve fit yielded the constants in Equation 4.7.

\[ a_1 = 3.2524e + 06 \quad a_2 = -7.9975e + 04 \quad a_3 = 4.4250e + 02 \] (4.7)

The roots of the fit equation were found to be 0.0162 and 0.0084. These roots were averaged to give a calibrated work roll crown of 0.0123 mm. The calibrated crown is significantly different from the original crown. However, the work roll crown has been shown to increase by as much as 0.5 mm in the first minute of rolling a similarly sized strip [9].
4.5 Validation

Two validations were performed, one after each calibration. This was done to determine the effectiveness of each calibration.

4.5.1 Yield-Gap Calibration Validation

The yield-gap calibration validation was performed using 0.02145 tonne/mm$^2$ for the yield stress and 3.489 mm for the gap. The 3-D rolling model was run to verify that the calibration was able to reduce the error in rolling load and exit thickness. The results of this simulation produced a rolling load of 1108.21 tonnes and an exit thickness of 5.4984 mm. These predicted rolling loads and exit thicknesses differ from the measured test data of 1107.20 tonnes and 5.4969 mm by less than 0.01%. Thus, the calibration was successful in reducing the error in the parameters for which the error was measured. The strip crown has not improved as the validation predicts a crown of 0.1297 mm, a difference of 310% from the test data. This data is summarized in Table 4.9.

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Unit</th>
<th>Predicted</th>
<th>Actual</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolling Load</td>
<td>$P$</td>
<td>tonne</td>
<td>1108.21</td>
<td>1107.2</td>
<td>0.091</td>
</tr>
<tr>
<td>Exit Thickness</td>
<td>$H_X$</td>
<td>mm</td>
<td>5.4984</td>
<td>5.4969</td>
<td>0.027</td>
</tr>
<tr>
<td>Strip Crown</td>
<td>$C_X$</td>
<td>mm</td>
<td>0.12970</td>
<td>0.0316</td>
<td>310</td>
</tr>
</tbody>
</table>

Figure 4.9: Strip crown error versus work roll crown.
Final Calibration Validation

The calibrated work roll crown, 0.0123 mm, combined with the yield-gap calibration results, was used to generate and run a simulation. This validation returned a rolling load of 1083.26 tonnes, an exit thickness of 5.4950 mm, and an exit crown of 0.0330 mm. The validation results, test data, and percent error between them is summarized in Table 4.10. All of the predicted values are less than five percent different from the test data.

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Unit</th>
<th>Predicted</th>
<th>Actual</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolling Load</td>
<td>(P)</td>
<td>tonne</td>
<td>1083.26</td>
<td>1107.2</td>
<td>2.162</td>
</tr>
<tr>
<td>Exit Thickness</td>
<td>(H_X)</td>
<td>mm</td>
<td>5.495</td>
<td>5.4969</td>
<td>0.035</td>
</tr>
<tr>
<td>Strip Crown</td>
<td>(C_X)</td>
<td>mm</td>
<td>0.0330</td>
<td>0.0316</td>
<td>4.430</td>
</tr>
</tbody>
</table>

Sensitivity Analysis

Nearly every aspect of the rolling mill stand affects the strip exit crown. However, most of these parameters are dictated by product requirements other than flatness. Therefore, gap, strip entry crown, work roll crown, and bending force were chosen as the parameters to study. The gap is set at each stand in a finishing mill to produce a product of a given thickness, but this could be used to control strip crown as well. The strip entry crown is the exit crown from the previous stand. By controlling the successive exit crowns, it is possible to control the strip entry crown. The work roll crown can be controlled not only by grinding a certain profile, but also by the manner in which the roll is cooled. By controlling the initial shape of the work roll crown and its temperature, it becomes possible to control the work roll crown itself. Finally, the bending force is specified by the mill controller for the purpose of changing strip crown. Its effectiveness at controlling the strip crown can be found directly through sensitivity analysis.

Parameter Generation

To study the effects on strip crown, each of the selected parameters were varied above and below the original value by 20% in 5% increments. The values of the parameters are summarized in Table 4.11. Simulations were generated for each perturbation for a total of 36 simulations.

Results

The 36 simulations were run and data was extracted. Plots were generated of roll crown versus each parameter and a linear fit was made of each data set. Figures 4.10, 4.11, 4.12, and 4.13 show the data and curve fits for gap, entry crown, roll crown, and bending force respectively. Exit crown versus gap, Figure 4.10, shows a strong correlation and a good fit with a \(R^2\) value of 0.991. Exit crown versus entry crown, shown in Figure 4.11, has poor correlation due to the amount of scatter in the data. The curve fit has a \(R^2\) value of 0.570 indicating a poor fit. The data for exit crown versus work roll crown had a poor fit as well. There are more robust estimation techniques that can be applied to the entry crown and work roll crown data in the future. Lastly, the exit crown versus bending force shows a strong correlation with a \(R^2\) value of 0.970. Further analysis should be conducted to determine the source of variation in the work roll crown and entry crown results.
Table 4.11: Sensitivity analysis parameter perturbation summary.

<table>
<thead>
<tr>
<th>Sim</th>
<th>Multiplier</th>
<th>$H_g$ [mm]</th>
<th>$C_E$ [mm]</th>
<th>$C_w$ [mm]</th>
<th>$J$ [tonne]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.80</td>
<td>2.791</td>
<td>0.0389</td>
<td>0.009836</td>
<td>77.5</td>
</tr>
<tr>
<td>2</td>
<td>0.85</td>
<td>2.966</td>
<td>0.0413</td>
<td>0.010451</td>
<td>82.35</td>
</tr>
<tr>
<td>3</td>
<td>0.90</td>
<td>3.140</td>
<td>0.0437</td>
<td>0.011100</td>
<td>87.19</td>
</tr>
<tr>
<td>4</td>
<td>0.95</td>
<td>3.315</td>
<td>0.0462</td>
<td>0.011700</td>
<td>92.04</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>3.489</td>
<td>0.0486</td>
<td>0.012300</td>
<td>96.88</td>
</tr>
<tr>
<td>6</td>
<td>1.05</td>
<td>3.664</td>
<td>0.0510</td>
<td>0.012900</td>
<td>101.72</td>
</tr>
<tr>
<td>7</td>
<td>1.10</td>
<td>3.838</td>
<td>0.0535</td>
<td>0.013500</td>
<td>106.57</td>
</tr>
<tr>
<td>8</td>
<td>1.15</td>
<td>4.013</td>
<td>0.0559</td>
<td>0.014139</td>
<td>111.41</td>
</tr>
<tr>
<td>9</td>
<td>1.20</td>
<td>4.187</td>
<td>0.0583</td>
<td>0.014753</td>
<td>116.26</td>
</tr>
</tbody>
</table>

Figure 4.10: Strip exit crown versus gap.
Figure 4.11: Strip exit crown versus strip entry crown.

Figure 4.12: Strip exit crown versus work roll crown.
Chapter 4. Calibration, Validation, & Sensitivity

4.6. Sensitivity Analysis

The slope of each curve fit is summarized in Table 4.12. The sensitivities for entry crown and work roll crown may not be very accurate due to the poor quality of the curve fit for those data sets. Changes in work roll crown and work roll crown appear to have the greatest effect on the exit crown. The gap has little effect on the exit crown. The bending force, the main method by which the crown is controlled, appears to have little effect on the exit crown.

Table 4.12: Sensitivity analysis results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap</td>
<td>$H_g$</td>
<td>$-0.0185 \text{ mm/mm}$</td>
</tr>
<tr>
<td>Strip Entry Crown</td>
<td>$C_E$</td>
<td>$0.2198 \text{ mm/mm}$</td>
</tr>
<tr>
<td>Work Roll Crown</td>
<td>$C_w$</td>
<td>$-0.9735 \text{ mm/mm}$</td>
</tr>
<tr>
<td>Bending Force</td>
<td>$J$</td>
<td>$-0.0003 \text{ mm/tonne}$</td>
</tr>
</tbody>
</table>

Upon careful examination, it is possible to see that the results from varying the strip entry crown and the work roll crown may be noise. The range of strip exit crowns produced by these analyses are significantly smaller than those produced by varying the gap or the bending force. A wider range of strip entry crowns and work roll crowns may yield data with a more definite trend.
4.7 Summary

Two calibrations were performed, one to minimize the error in exit thickness and rolling load, and the other to minimize exit crown error. Validations were performed after each calibration. The yield-gap calibration reduced the rolling-load and exit-thickness errors to less than 1%. The work roll crown calibration decreased the exit crown error from 275% to less than 5%. Thus, the calibrated model predicts rolling load, exit thickness, and strip exit crown within 5% of test data. Sensitivity analysis was performed to determine how gap, entry crown, work roll crown, and bending force affect the strip exit crown.
5 Summary, Conclusions, and Recommendations

This chapter presents a summary of the work performed for this thesis, conclusions that may be drawn from it, and recommendations relating to further work.

5.1 Summary

Many models and methods were developed in the course of creating a refined dynamic roll-stack model for the hot-rolling process. The process began with model development based on the provided test data from a stand in an actual hot-rolling mill. Four types of models were constructed including the final 3-D rolling model which was the goal of this project. The next step was to calibrate the 3-D rolling model to account for possible errors in the test data. Once the final model was calibrated, a validation simulation was performed to determine the ability of the model to predict the results in the test data. Lastly, a sensitivity analysis was performed to determine the effects of four stand parameters on strip crown.

5.1.1 Model Development

Model development began by defining the strip material properties. An elastic-plastic model was used with strain hardening represented as an elliptical function of strain and the yield stress. This model was used to allow any metal material to be modeled, not just steel. This choice allows the finite element models which use this material model to be used for any hot-rolling application whether the material be aluminum, stainless steel, or any other metal.

Finite element modeling is dependent on the representation of the physical system. Therefore, the stand and roll-stack arrangement was carefully examined to find methods of simplification. Due to symmetry, the stand was decomposed such that a one-quarter-symmetry model could be used to create a dynamic rolling model and a one-eighth symmetry model could represent a static stiffness model. It was also found that the stand frame could be replaced by a spring of equivalent stiffness which greatly simplified model construction because modeling the frame geometry was not required.

The next step was to determine the stiffness of the roll-stack by creating the Roll-Stack Stiffness Model. This model was implemented in the finite element software Abaqus in both two and three dimensions. The Roll-Stack Stiffness Model compressed the roll-stack in a manner similar to the stand calibration performed at the mill. A load-displacement curve was generated and the total stiffness of the frame and roll-stack was found. The total stiffness of the frame and roll-stack was less than 0.5% different from the total stand stiffness determined in the test data.

Whereas the previous two models had dealt with the static behaviour of the roll stack, the Lumped-Mass Model was created to determine lumped parameters to be used to represent damp-
ing elements in the 2-D and 3-D dynamics finite element models. The lumped-mass model was an explicit finite element model developed in the Python scripting language to find the frequency of vibration of the rolls and damping coefficients to bring the roll-stack to steady state. The model relied on the estimates of the frame, roll-stack, and strip stiffnesses along with rolling load predictions based on the Ford and Alexander rolling load model. This model was used to determine multipliers to set run time and strip displacement time. The multipliers were then used in the 2-D and 3-D Rolling Models to set how long the finite element simulations would run.

The Dynamic 2-D Rolling Model was developed. This was a finite element model created in Abaqus to simulate the rolling of a strip. A plane strain stress-strain relationship was assumed and the previously mentioned elastic-plastic material model was used. This model was used for exploration of rolling modeling and meshing techniques for finding a reliable solution. Once this model was verified to work, the 3-D rolling model was constructed.

The dynamic 3-D rolling model was the final model developed. It was able to calculate rolling load, exit thickness, and strip crown. This model was capable of representing all of the important mechanics of the rolling process except for heat transfer. A mesh convergence study was performed to determine how the mesh affected the solution quality. Finally, a simulation was run to determine how well the 3-D rolling model replicated the measurements in the test data. The exit thickness was 3.79% different from the test data and the rolling load was 15.39% different. The strip exit crown had a percent error of 275% from the test data. A calibration procedure was later applied to reduce test data uncertainties that could have been the source of these errors.

5.1.2 Calibration

The stand parameter calibrations were used to reduce test data error and to minimize model prediction error. The test data and its collection methods were carefully examined for possible sources of error. It was found that the most likely sources of error were in the strip yield stress, gap, and work roll crown measurements. Two calibrations were performed, one to minimize exit thickness and rolling load error, the other to minimize strip exit crown error.

The yield-gap calibration varied the strip yield stress and the gap to find a combination which would minimize the rolling load and exit thickness error. Nine simulations were run representing a full-factorial two-parameter three-level expansion of three yield stresses and three gaps. From the results, a response surface of total rolling-load and exit-thickness error was constructed using quadratic isoparametric interpolation functions. The error over the sample space was minimized to give the values of yield stress and gap that would minimize the error. This calibration produced a yield stress of 0.02145 tonne/mm$^2$ and a gap of 3.489 mm. The calibrated yield stress differed from the test data yield stress of 0.02119 tonne/mm$^2$ by 1.23% and the predicted gap differed from the test data value of 3.983 mm by 12.40%. These calibrated values were used in the work roll crown calibration and in the model validation.

The work roll crown calibration attempted to account for the effects of the thermal expansion on the work roll crown, which affects the strip crown. Five simulations were run with varying work roll crowns. The error in the strip crown versus the test data was computed and graphed. The work roll crown that minimized the error was chosen. The calibration gave a work roll crown of 0.0123 mm which differed from the test data value of -0.16 mm by 107%. While the difference seems significant with respect to the percent difference, the actual change in crown was only 0.1477 mm and accounts for all thermal expansion of the work roll while rolling the strip.
5.1.3 Validation

Two validations were performed, one after each calibration, to determine their effectiveness and the final quality of the 3-D rolling model. The validation performed after the yield-gap calibration showed that the exit thickness and rolling load error had decreased below 0.01%. Therefore, the yield-gap calibration was successful at reducing error. However, those values of yield stress and gap caused the strip crown error to reach 310%. The rolling load $P$, strip exit thickness $H_X$, and strip crown $C_X$ predictions and errors for the validation performed after the yield-gap calibration are presented in Table 5.1.

Table 5.1: Yield-gap calibration validation data summary.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Predicted</th>
<th>Actual</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>tonne</td>
<td>1108.21</td>
<td>1107.2</td>
<td>0.091</td>
</tr>
<tr>
<td>$H_X$</td>
<td>mm</td>
<td>5.4984</td>
<td>5.4969</td>
<td>0.027</td>
</tr>
<tr>
<td>$C_X$</td>
<td>mm</td>
<td>0.12970</td>
<td>0.0316</td>
<td>310</td>
</tr>
</tbody>
</table>

The final validation was performed with the data taken from both the yield-gap and work roll crown calibrations. The results of the final validation showed that the work roll crown calibration had caused the strip exit crown error to have dropped from 312% to less than 5%. However, that calibration also caused an increase in the rolling load and exit thickness error, though they both errors remained at less than 3%. Therefore, the final calibrated dynamic 3-D rolling model was able to exceed the goal of matching test data measurements within 10% error. The rolling load $P$, strip exit thickness $H_X$, and strip crown $C_X$ predictions and errors for the final validation are presented in Table 5.2.

Table 5.2: Final validation data summary.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Predicted</th>
<th>Actual</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>tonne</td>
<td>1083.26</td>
<td>1107.2</td>
<td>2.162</td>
</tr>
<tr>
<td>$H_X$</td>
<td>mm</td>
<td>5.495</td>
<td>5.4969</td>
<td>0.035</td>
</tr>
<tr>
<td>$C_X$</td>
<td>mm</td>
<td>0.0330</td>
<td>0.0316</td>
<td>4.430</td>
</tr>
</tbody>
</table>

The final validation had greater rolling-load and exit-height errors than the yield-gap calibration validation. This effect was most likely due to changes in work roll crown affecting the rolling load and exit height. The work roll crown calibration does not appear to have been as successful at reducing error as the yield-gap calibration, which reduced errors to less than 0.1%. This may be due to the highly sensitive nature of the strip crown and the small values involved, less than 0.2 mm.

5.1.4 Sensitivity

The sensitivity analysis used the dynamic 3-D rolling model to determine which of several stand parameters affect the strip crown the most. The most controllable aspects of the stand were chosen as parameters for the sensitivity analysis: gap, strip entry crown, work roll crown, and bending force. Nine simulations were run for each parameter with varying values above and below the calibrated model. The exit crowns results were plotted versus the parameters and then curve fit. The slopes of the linear fits were determined and compared. While the entry crown and work
roll crown results showed little correlation and bad fit quality, the gap and bending force data showed good fit quality. The poor fits may have been due to the narrow range of variations used to perturb the system. By using a small range of work roll crowns and entry crowns, the exit crown did not vary sufficiently to well represent the solution space. Additional simulations need to be run to determine the sensitivities conclusively due to the poor quality of the curve fits and the narrow range of the data. Table 5.3 summarizes the sensitivity of strip exit crown to gap, strip entry crown, work roll crown, and bending force.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap</td>
<td>(H_g)</td>
<td>-0.0185 mm/mm</td>
</tr>
<tr>
<td>Strip Entry Crown</td>
<td>(C_E)</td>
<td>0.2198 mm/mm</td>
</tr>
<tr>
<td>Work Roll Crown</td>
<td>(C_w)</td>
<td>-0.9735 mm/mm</td>
</tr>
<tr>
<td>Bending Force</td>
<td>(J)</td>
<td>-0.0003 mm/tonne</td>
</tr>
</tbody>
</table>

5.2 Conclusions

There are many conclusions that can be drawn from the work presented in this thesis. The more important topics include the final model’s predictive ability, calibration, and performance versus effort.

5.2.1 Predictive Ability

The dynamic 3-D rolling model was very successful at predicting the parameters it set out to predict. It was capable of predicting the rolling load, exit thickness, and strip crown within 5% of the test data measurements, less than half of the error specified in the objectives. The rolling load and exit thickness predictions were within 0.5% of the test predictions. Further refinements of the model could bring the strip-crown error down to that of the exit-height and rolling-load error.

The sensitivity analysis results are also encouraging. Varying gap and bending force showed pronounced effects on the strip crown. The quality of the curve fits, \(R^2\) values of 0.991 and 0.970 for the gap and bending force sensitivities, indicate that using this analysis to predict strip crown is possible. Further model testing and refinements could reduce the noise in the strip entry crown and work roll crown sensitivities. In addition, these results clearly show how little an effect bending force has on strip crown. The information from the sensitivity analysis will be useful in determining which parameters would be most effective at controlling strip crown.

5.2.2 Calibration

The yield-gap calibration involved modifying the yield stress and gap by less than 15%. The work roll crown calibration changed the roll crown by less than 0.2 mm to bring the strip exit crown error from 310% to within 5% of the measured value. The relatively small adjustments of the calibration parameters show that not only is rolling load, exit thickness, and strip crown very sensitive, but also that the final parameters were not significantly different from the test data.
5.2.3 Performance versus Effort

The ability to predict the strip profile within 5% of test data is a step forward in modeling the rolling process; however, it does have a cost. Each of the models developed in this thesis is required to simulate a stand. The roll-stack stiffness model is needed to predict the stand frame stiffness. The frame stiffness model must be used to validate the frame stiffness. The lumped-mass model is needed to determine run time parameters and damping coefficients. The 2-D rolling model is used to verify those coefficients and confirm that the model reaches steady state. Finally, the 3-D model requires 14 runs to complete the calibration. There is much effort required to generate and calibration the 3-D rolling model; however, the effort is worthwhile.

Without all of these models and steps, it would not be possible to accurately simulate the rolling process. The results indicate predictions within 5% of test data. This predictive ability is significant considering that the rolling loads are over 1100 tonnes and the strip crowns are less than 0.05 mm.

Though the 3-D rolling model requires approximately an hour per simulation, advances in computing technology will continually reduce run times. Since optimizing run times was not a goal of this project, the focus was more on quality of solution than speed. Further studies into the model’s behaviour may yield simplifications or adjustments that significantly reduce run times, making it easier to explore the rolling process.

5.3 Recommendations

The following recommendations are based on observations and conclusions made during the course of the project. Areas where further development should occur include model development, sensitivity analysis, process exploration, and crown equation.

5.3.1 Model Development

While the dynamic 3-D rolling model represents an advancement in the modeling of rolling, there remains room for improvement. Modeling of the heat transfer between the strip and rolls could be used to increase the accuracy of the model prediction. This could also be used to develop scheduling and cooling methods to predict roll thermal expansion and introduce another method of controlling crown. Another area to explore in model development is reducing simulation solution times. At approximately an hour per simulation with the dynamic 3-D rolling model, performing a large number of runs quickly becomes prohibitive. By further studying mesh and representation tradeoffs, it may be possible to significantly reduce solution times. For example, using the 3-D Rolling Model as a baseline, the effectiveness of a static 3-D finite element model could be explored. If it were possible to achieve similar results with the simpler static model, simulation times could be reduced significantly.

5.3.2 Process Exploration

With a model available to represent so many aspects of the rolling process, it becomes possible to perform controlled experiments in the rolling process. Each stand parameter can be varied independently and the resulting effects on the strip can be measured. This could lead to a level of control not previously possible. For instance, the exact effects of changes in rolling friction on rolling load and exit crown could be determined. The effects of strip profile representation could be explored. The 3-D rolling model opens up many possibilities for the advancement of flatness control.
5.3.3 Sensitivity Analysis

The sensitivity analysis presented in this work was only a preliminary study. The principles used here could be applied to any stand or strip parameter. The strip width and roll sizes are other parameters that heavily influence strip crown and can be controlled during mill setup. The effect of frame stiffness on strip crown and flatness may be of interest to those designing the frames. In addition, developing sensitivities to parameters at each stand in a mill would allow the mill controllers to determine how much to change the crown and thickness at each stand to achieve a flat final product. With this model, it would be possible to design an entire framework to control the mill based on changing the most influential parameters to minimize the strip flatness error.

5.3.4 Crown Equation

With further developments in sensitivity analysis, it would become possible to create a crown equation based on Taylor series expansion. A set of controllable parameters which affect the strip crown could be chosen and perturbations around a base configuration could be performed to find sensitivities. This could be done for each configuration of each mill stand so that the online controllers could predict strip crown without the need to run a model.
Bibliography


Appendix: Test Data
### Table 1: Work roll data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrel Length</td>
<td>$B_{lw}$</td>
<td>2080</td>
<td>mm</td>
</tr>
<tr>
<td>Base Diameter</td>
<td>$B_{dw}$</td>
<td>655.715</td>
<td>mm</td>
</tr>
<tr>
<td>Neck Length</td>
<td>$N_{lw}$</td>
<td>195</td>
<td>mm</td>
</tr>
<tr>
<td>Neck Diameter</td>
<td>$N_{dw}$</td>
<td>510</td>
<td>mm</td>
</tr>
<tr>
<td>Barrel Diameter Crown</td>
<td>$C_{w}$</td>
<td>-0.16</td>
<td>mm</td>
</tr>
<tr>
<td>Shell Thickness</td>
<td>$t_{sw}$</td>
<td>50</td>
<td>mm</td>
</tr>
<tr>
<td>Core Elastic Modulus</td>
<td>$E_{cw}$</td>
<td>21.0920874</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Core Poisson's ratio</td>
<td>$\nu_{cw}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Shell Elastic Modulus</td>
<td>$E_{sw}$</td>
<td>21.0920874</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Shell Poisson's ratio</td>
<td>$\nu_{sw}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_{w}$</td>
<td>7.96E-13</td>
<td>tonne-s$^2$/mm$^4$</td>
</tr>
</tbody>
</table>

### Table 2: Backup roll data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrel Length</td>
<td>$B_{lb}$</td>
<td>1820</td>
<td>mm</td>
</tr>
<tr>
<td>Base Diameter</td>
<td>$B_{db}$</td>
<td>1589.505</td>
<td>mm</td>
</tr>
<tr>
<td>Neck Length</td>
<td>$N_{lb}$</td>
<td>360</td>
<td>mm</td>
</tr>
<tr>
<td>Neck Diameter</td>
<td>$N_{db}$</td>
<td>945</td>
<td>mm</td>
</tr>
<tr>
<td>Barrel Diameter Crown</td>
<td>$C_{b}$</td>
<td>0</td>
<td>mm</td>
</tr>
<tr>
<td>Shell Thickness</td>
<td>$t_{sb}$</td>
<td>60</td>
<td>mm</td>
</tr>
<tr>
<td>Core Elastic Modulus</td>
<td>$E_{cb}$</td>
<td>21.0920874</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Core Poisson's ratio</td>
<td>$\nu_{cb}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Shell Elastic Modulus</td>
<td>$E_{sb}$</td>
<td>21.0920874</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Shell Poisson's ratio</td>
<td>$\nu_{sb}$</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_{b}$</td>
<td>7.96E-13</td>
<td>tonne-s$^2$/mm$^4$</td>
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</table>
### Table 3: Strip data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Values</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>Width</td>
<td>$w$</td>
<td>1524.84</td>
<td>mm</td>
</tr>
<tr>
<td>Feather</td>
<td>$w_f$</td>
<td>40</td>
<td>mm</td>
</tr>
<tr>
<td>Entry Thickness</td>
<td>$H_E$</td>
<td>6.4337</td>
<td>mm</td>
</tr>
<tr>
<td>Exit Thickness</td>
<td>$H_X$</td>
<td>5.4969</td>
<td>mm</td>
</tr>
<tr>
<td>Entry Crown</td>
<td>$C_E$</td>
<td>0.0486</td>
<td>mm</td>
</tr>
<tr>
<td>Exit Crown</td>
<td>$C_X$</td>
<td>0.0316</td>
<td>mm</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>$E_s$</td>
<td>18</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Yield Stress</td>
<td>$S_y$</td>
<td>0.02118574</td>
<td>tonne/mm$^2$</td>
</tr>
<tr>
<td>Sy Multiplier</td>
<td>$\gamma$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>$\nu_s$</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_s$</td>
<td>7.96E-13</td>
<td>tonne-s$^{-2}$/mm$^4$</td>
</tr>
<tr>
<td>Temperature</td>
<td>$T$</td>
<td>853.35</td>
<td>Celsius</td>
</tr>
<tr>
<td>Velocity</td>
<td>$v$</td>
<td>6752.64</td>
<td>mm/s</td>
</tr>
</tbody>
</table>

### Table 4: Stand data.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolling Load</td>
<td>$P$</td>
<td>1107.2</td>
<td>tonne</td>
</tr>
<tr>
<td>Bending Force</td>
<td>$J$</td>
<td>96.88</td>
<td>tonne</td>
</tr>
<tr>
<td>Balance Bending Force</td>
<td>$J_b$</td>
<td>150</td>
<td>tonne</td>
</tr>
<tr>
<td>Gap</td>
<td>$H_g$</td>
<td>3.9831</td>
<td>mm</td>
</tr>
<tr>
<td>Strip-Roll Friction</td>
<td>$\mu_s$</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Roll-Roll Friction</td>
<td>$\mu_r$</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Total stiffness</td>
<td>$k_t$</td>
<td>610</td>
<td>tonne/mm</td>
</tr>
</tbody>
</table>
Appendix: Rolling Load Model

# Rolling Load Model
# Based on Ford and Alexander
#
# Functions:
# getLoad(k, he, hx, w, D, E, v)
# getKx(k, he, hx, D, E, v)
#
# Common Inputs:
# k – flow stress, Sy/sqrt(3)
# he – strip entry height
# hx – strip exit height
# w – strip width
# D – work roll diameter
# E – work roll elastic modulus
# v – work roll Poisson’s ratio

from numpy import pi, sqrt

def getLoad(k, he, hx, w, D, E, v):
    """Output: P – total rolling load""
    dh = he - hx  # draft
    hAvg = 0.5*(he + hx)
    Rp = D/2.       # initialize radius
    C = 16*(1 - v**2)/(pi*E)  # roll deformation constant
    print C
    for i in range(10):  # converge load and deformation
        arc = sqrt(Rp*dh)
        F = k*arc*(pi*0.5 + 0.5*arc/hAvg)  # unit rolling load
        Rp = D/2.*(1 + C+F/dh)  # deflected roll radius
    P = F*w
    return P  # return total rolling load

def getKx(k, he, hx, D, E, v):
    """Output: Kx – strip stiffness""
    hxa = hx*1.025    # exit height a
    hxb = hx*0.975    # exit height b
    Pa = getLoad(k, he, hxa, 1, D, E, v)  # rolling load a
    Pb = getLoad(k, he, hxb, 1, D, E, v)  # rolling load b
    Kx = abs((Pb - Pa)/((hxb - hxa)))
    return Kx