ROTOR/FUSELAGE UNSTEADY INTERACTIONAL AERODYNAMICS: A NEW COMPUTATIONAL MODEL

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DISSERTATION SUBMITTED TO THE FACULTY OF THE VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN AEROSPACE ENGINEERING

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July 27, 1999
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Keywords: Rotorcraft, Aerodynamics, Computational Fluid Dynamics, Unsteady Flow
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ABSTRACT

A new unsteady rotor/fuselage interactional aerodynamics model has been developed. This model loosely couples a Generalized Dynamic Wake Theory (GDWT) to a Navier-Stokes solution procedure. This coupling is achieved using a newly developed unsteady pressure jump boundary condition in the Navier-Stokes model. The new unsteady pressure jump boundary condition models each rotor blade as a moving pressure jump which travels around the rotor azimuth and is applied between two adjacent planes in a cylindrical, non-rotating grid. Comparisons are made between predictions using this new model and experiments for an isolated rotor and for a coupled rotor/fuselage configuration.
Acknowledgments

I would like to thank the personnel in the Subsonic Aerodynamics Branch and the U.S. Army Joint Research Program Office, Aeroflightdynamics Directorate, of the NASA Langley Research Center. In particular, I would like to thank Ms. Susan Gorton, and Mr. Edgar Waggoner for their interest in and support of this research effort, which was funded under Grant Number NCC-1245. I also would like to thank Dr. Richard Barnwell, Mr. Mark Chaffin, and Dr. Henry Jones, for the many useful discussions on computational fluid dynamics and its application to rotorcraft. A special thank you goes to my wife, Kim, and son, David, who have seen me through this entire experience.
Nomenclature

English symbols

\(a_\infty\) freestream speed of sound [m/sec]
\(a_n^m, b_n^m\) series expansion coefficients [see Appendix A]
\(A(\tilde{\rho})\) local area ratio between blade and computational cell
\(i\) the imaginary number, \(\sqrt{-1}\)
\(c\) local blade chord normalized by \(R\)
\(\tilde{c}\) coefficient function [see Appendix A]
\(C_f, C_m\) force and moment coefficients
\(C_p\) pressure coefficient
\(\tilde{C}_p\) modified pressure coefficient
\(\dot{C}_p\) unsteady component of modified pressure coefficient
\(C_T\) thrust coefficient
\(\bar{C}_T\) mean thrust coefficient
\(C_L\) roll moment coefficient
\(\bar{C}_L\) mean roll moment coefficient
\(C_M\) pitch moment coefficient
\(\bar{C}_M\) mean pitch moment coefficient
\(e_0\) stagnation energy per unit mass [m^2/sec^2]
\(\tilde{F}, \tilde{G}, \tilde{H}\) inviscid fluxes (see equations (4.3) to (4.5))
\(\tilde{F}_v, \tilde{G}_v, \tilde{H}_v\) viscous fluxes (see equations (4.3) to (4.5))
\(\bar{h}\) hinge offset location normalized by \(R\)
$H_n^m$ coefficient function [see Appendix A]
$[L]^c$ quasi-steady inflow matrix, cosine component
$[L]^s$ quasi-steady inflow matrix, sine component
$L$ sectional blade lift [N/m]
$M$ local Mach number
$M_\infty$ freestream Mach number
$N_T$ number of azimuthal time steps per rotor revolution
$P, p$ pressure [N/m$^2$]
$P_\infty$ freestream pressure [N/m$^2$]
$\bar{P}_n^m$ normalized, associated Legendre functions of the 1st kind
$\bar{Q}_n^m$ normalized, associated Legendre functions of the 2nd kind
$q_i$ $i^{th}$ component of perturbation velocity [m/sec]
$q_x, q_y, q_z$ cartesian components of heat flux [J/(m$^2$ sec)]
$\bar{Q}$ vector of conservative variables $= [\rho, \rho u, \rho v, \rho w, \rho e_0]^T$
$Q_i$ $i^{th}$ conservative variable
$\bar{r}$ radial location on disk, normalized by $R$
$r$ radial location on disk [m]
$R$ rotor radius [m]
$\mathcal{R}$ gas constant [J/(kg $K$)]
$t$ time [sec]
$T$ temperature [$K$]
$\bar{t}$ nondimensional time, normalized by $\Omega$
$V_\infty$ freestream velocity, normalized by $\Omega R$
$V_i$ induced inflow velocity [m/sec]
$V_n^m$ elements of mass flow matrix
$u, v, w$ cartesian velocities [m/sec]
$w$ local normal component of velocity at rotor disk, normalized by $\Omega R$
$w_l$ local normal component of $w$
Greek symbols

\( \alpha_{eff} \) effective angle of attack [rad]
\( \alpha_s \) rotor shaft tilt angle [rad]
\( \beta_0 \) blade mean coning angle [rad]
\( \beta_{1c}, \beta_{1s} \) first harmonics of rotor flapping [rad]
\( \beta_0 \) blade mean coning angle [rad]
\( \alpha'_j, \beta'_j \) induced inflow coefficients
\( \gamma \) ratio of specific heats
\( \delta_{ij} \) Dirac delta function [see Appendix A]
\( \Delta P \) pressure jump [N/m\(^2\)]
\( \varepsilon \) user specified tolerance
\( \lambda \) freestream component normal to disk, normalized by \( \Omega R \), positive down
\( \lambda_i \) induced inflow ratio = \( V_i/\Omega R \)
\( \bar{\lambda}_i \) time averaged induced inflow ratio
\( \bar{\lambda}_{i,mean} \) mean component of \( \lambda_i \)
\( \Lambda(\bar{r}, \psi) \) generic time averaged function [see Appendix A]
\( \Phi \) pressure function, normalized by \( \rho \Omega^2 R^2 \)
\( \tilde{\phi}_n^m(\bar{r}) \) radial expansion shape function
\( \nu, \eta, \Psi \) dimensionless ellipsoidal coordinates
\( \psi \, \bar{\psi} \) on rotor disk
\( \psi_1 \) phase of first harmonic of blade pitch [rad]
\( \rho \) local air density [kg/m\(^3\)]
\( \rho_\infty \) freestream air density [kg/m\(^3\)]
\( \mu \) rotor advance ratio = \( V_\infty/\Omega R \)
\( \mu_i \) induced inflow in \( \mu \) direction, normalized by \( \Omega R \), positive downstream
\( \mu_{i,mean} \) mean component of \( \mu_i \)
\( \theta \) local blade pitch [rad]
\( \theta_{tw}(\bar{r}) \) built-in blade twist function [rad]
\( \theta_0 \) blade collective pitch [rad]
\( \theta_c, \theta_s \) cosine and sine components of blade pitch [rad]
\( \theta_1 \) magnitude of first blade pitch harmonic [rad]
\( \tau_{m^c} \) cosine part of pressure expansion of \( m^{th} \) harmonic and \( n^{th} \) polynomial

\( \tau_{m^s} \) sine part of pressure expansion of \( m^{th} \) harmonic and \( n^{th} \) polynomial

\( \tau_{ij} \) stress tensor [m/sec]

\( \omega_n \) rigid flap natural frequency [cycles/revolution]

\( \Omega \) rotor rotational speed [rad/sec]

**Subscripts**

- \( add \): additional component
- \( i \): \( i^{th} \) component
- \( ,i \): derivative with respect to \( i^{th} \) direction
- \( n, j, q \): polynomial number
- \( \xi \): coordinate along freestream line, positive pointing upstream
- \( \infty \): freestream quantities

**Superscripts**

- \( m, r, p \): harmonic number
- \( u \): unsteady component
Operators & Acronyms

\[ \Delta (\cdot) \] operator used to denote an incremental value of \((\cdot)\)

\[ \mathcal{F} (\cdot) \] filter operator

\[ (\cdot)!! \] double factorial [see Appendix A]

\[ (\cdot)^{\ast} \] derivative with respect to \(\vec{t}\)

\[ (\cdot)^{\ddot{\cdot}} \] reference quantities

\textit{GDWT} Generalized Dynamic Wake Theory

\textit{RLM} rotor loading model

\textit{RF FM} rotor/fuselage flowfield model
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Chapter 1

Introduction

1.1 Motivation

It is well known that rotary wing aircraft aerodynamics are complicated. Unlike fixed wing aircraft, on which a steady-state flight condition typically implies steady-state aerodynamics, a rotary wing aircraft experiences a significant unsteady aerodynamic environment in all flight conditions, even in level, unaccelerated flight, due to the presence of the rotating wings (rotors). This aerodynamic environment includes the aerodynamic interactions, which are inherently unsteady and complex, between the rotor(s) and the fuselage. One example of the complexity associated with these interactions is the problem of flow separation phenomena. Whereas fixed wing aircraft typically have little significant flow separation in steady-state flight due to their streamlined fuselage shapes, rotary wing aircraft typically have blunt aft regions that are conducive to flow separation. Even in hover, the flow induced by the rotor(s) impinging on the fuselage tends to separate on the underside of the fuselage which is often a blunt surface. Sheridan and Smith [1] discuss many other examples and categories of rotorcraft interactional aerodynamics. They also state in their conclusions that:

“... it will be necessary to develop tractable theories and analytical methods to account for all these phenomena. Interactional aerodynamics of the airframe is not as neatly packaged as rotor aerodynamics. Many of the interactions involve viscous processes, and in some aspect semi-empirical techniques may always be needed. But a start must be made in developing the required mathematical models so that we can cope with these problems adequately in the vehicle design phase.”
In short, the above passage calls for the development of methods to address the coupled rotor/fuselage interactional aerodynamic effects, whereas previously, the rotor effects and fuselage effects were treated in isolation.

In the years since Sheridan and Smith’s paper [1], many types of analyses have been developed and used in the prediction of the unsteady interactional aerodynamic characteristics of rotorcraft. Figure 1.1 is a graphic that depicts and categorizes several of the major methodologies.

In the area of relatively low computational expense and complexity (see figure 1.1), singularity methods have been used. These methods typically use singularities, such as a lifting line to represent the rotor blade, a system of vortices to represent the wake, and source, doublet, and/or vortex panels to model the fuselage. In a relatively inexpensive and computationally efficient manner, these methods are able to capture low order effects on each component due to the other component, such as the mean downwash on fuselage due to the rotor or the mean inflow at the rotor disk due to the fuselage. But, since the fuselage is typically modeled using a panel method, calculation of some interactional aerodynamic effects, such as flow separation due to rotor downwash, is difficult. In cases where viscous effects are predominant, the viscous flow effects must be either ignored, specified \textit{a priori}, or determined by coupling the method to a boundary layer type model.

On the other end of the computational expense and complexity scale, at relatively high computational expense and complexity, are the methods involving computational fluid dynamics (CFD), in particular, Navier-Stokes methods. These types of methods have been used to calculate the entire flow field of the complete rotorcraft configuration, all in one computation. Even though these computational methods are theoretically able to capture all of the interactional aerodynamic couplings between the rotor(s) and fuselage, their computational expense is prohibitive for routine use.

There is a lack of methods available in the literature which fall between the singularity methods and the Navier-Stokes CFD methods for studying unsteady interactional aerodynamics of rotorcraft. The current research is motivated by the lack of available hybrid methods which are computationally efficient, yet are able to capture primary interactional aerodynamic effects between the rotor and fuselage. Figure 1.1 shows, with a dotted ellipse labeled “Hybrid Methods”, where the current work falls on the computational expense and complexity scale.
1.2 Literature Review

As discussed previously, unsteady rotor/fuselage interactional aerodynamics generally fall into three categories: (1) singularity methods, (2) CFD methods, and (3) hybrid methods. In the following sections, a brief review of each is given.

1.2.1 Singularity Methods

Singularity methods are typically characterized by the use of a source, doublet, and/or vortex panel representation of the fuselage, a lifting line or lifting surface representation of the rotor, and a vortex lattice model representation of the rotor wake. For rotorcraft analyses, these methods are used to compute the flowfield of the complete vehicle. Johnson [2] provides an extensive discussion of singularity methods used for rotorcraft analyses up through the year 1986. Since that time, other singularity methods have been developed as well. Egolf and Lorber [3] used a source panel description of the fuselage, a lifting line blade model, and a prescribed vortex wake description of the rotor/fuselage system to model the unsteady rotor/fuselage flowfield. The prescribed vortex wake was prevented from cutting through the fuselage by displacing, in an a priori manner, the segments of the vortices that would otherwise have been inside the fuselage. No attempts were made to model the wake of the fuselage or the flow separation from the fuselage. Only limited comparisons were made to experimental data. Mavris, et al. [4] used a doublet representation of the fuselage, a lifting line blade model, and a free vortex wake description of the rotor/fuselage system. No modeling was used for the fuselage wake or fuselage flow separation. Also, vortex wake filaments that are inside the fuselage were excluded from the computations. Comparisons with experimental pressures show good agreement along the top of the fuselage, but agreement degrades on the sides of the fuselage. Mavris, et al. [4] attribute these discrepancies to flow separation on the fuselage and to inadequate vortex-surface interaction predictions. Berry [5] combined a fuselage source panel representation, a source-dipole representation for the rotor, and a distorting vortex-lattice representation of the rotor wake to model the rotor/fuselage system. Comparisons are made to measured time averaged and unsteady inflow velocities; no fuselage pressure comparisons are made which include the rotor influence. Quackenbush, et al. [6] used a source/doublet description of the fuselage, a vortex-lattice model for the rotor blades and a novel “Constant Vorticity Contour (CVC)” free wake model; fuselage flow separation was not modeled. Close surface/vortex interactions were modeled using selective remeshing of the curved vortex elements and using an
“Analytical/Numerical Matching (ANM)” scheme. Computational efficiency was improved by using “fast vortex methods” for wake-on-wake and wake-on-body computations. Generally good agreement with measured results are demonstrated for time averaged induced velocities above the rotor disk and for time averaged and unsteady pressures on the top centerline of the fuselage. Crouse [7] used a source panel description of the fuselage, a lifting line blade model, and a free tip vortex wake model without an inboard wake model to represent the rotor/fuselage system. Vortex wake elements that cross the fuselage surface are handled by splitting them into smaller segments, and shifting the collocation points of these smaller segments such that they are at a specified minimum distance from the fuselage surface. This method is similar in concept to that used by Egolf and Lorber [3] as discussed above. Good comparisons of unsteady pressures were shown on the top centerline of the tail boom of the fuselage. Boyd, et al. [8] included the open-loop effects of a fuselage, represented by a non-lifting fuselage source panel method, on the rotorcraft trim in a comprehensive rotorcraft analysis. This effect was implemented in the comprehensive analysis as an additional rotor inflow distribution plus an additional rotor wake distortion due to the presence of the fuselage. Effects of the rotor on the fuselage were not modeled. Though some computations have proved successful and can be computationally efficient, all of these singularity methods suffer from the inability to predict some of the rotor/fuselage interactional effects. For example, methods that use a source panel description of the fuselage do not have the capability to determine the lift or the lift change on a fuselage due to the rotor. Also, quantities such as flow separation and drag must either be ignored, must be specified a priori, or must be determined by coupling the method to a boundary layer model.

1.2.2 CFD Methods

In recent years, unsteady calculations on complete rotorcraft configurations using CFD methods have become possible. In addition, there are several degrees of complexity that can be modeled with CFD. For rotorcraft applications, methods have been developed to solve the full potential equation, the Euler equations, and the Navier-Stokes equations.

Chen and Bridgeman [9] coupled the three dimensional boundary layer equations to the full potential equations in a blade-fixed coordinate system for an isolated rotor. The three dimensional boundary layer equations assumed that the surface curvature effects were negligible and included additional terms in the x- and z-momentum equations to account for centrifugal and Coriolis forces in the boundary layer due to blade rotation. These equations were coupled by using a modified
tangency boundary condition. This modified boundary condition enforces a velocity component ("transpiration velocity") normal to the surface which "deflects the inviscid flow from the body surface thus simulating the displacement of the inviscid flow due to the momentum defect in the boundary-layer" [9]. Good comparisons were shown for integrated drag quantities (torque) on a non-lifting isolated rotor in hover for a range of hover tip Mach numbers. Also, good chordwise pressure coefficient comparisons were shown for two radial stations in a non-lifting, forward flight condition. Bridgeman, et al. [10] solved the unsteady, full potential equation coupled to a three dimensional boundary layer model for isolated rotors in hover and forward flight. This method is similar to that presented in [9], with a number modeling improvements. Though it is possible conceptually to include a fuselage in these full potential computations, this would be difficult in practice due to the blade-fixed coordinate systems typically used in such analyses. Thus, interactional aerodynamic computations would be difficult to compute using these existing tools.

Recent solutions to the Euler equations for isolated rotor applications have used unstructured grid techniques [11, 12, 13] to refine the grid system efficiently to better capture wake structures such as tip vortices. These techniques may be extended easily to include a fuselage body. However, since the Euler equations do not include viscous terms, computation of any viscous effects (e.g., viscous drag on a fuselage) would, like the full potential equation examples above, still require coupling the method to a boundary layer analysis.

Navier-Stokes computations have also been developed for use in rotorcraft analysis. While time averaged Navier-Stokes methods have been developed and are quite practical to use [14, 15, 16, 17, 18], routine computations for unsteady flows on full configurations are not yet practical. Even so, several of these computations [19, 20] are found in the literature. Meakin [19] used a thin-layer Navier-Stokes method to calculate the flowfield around the Bell/Boeing V-22 Osprey tiltrotor aircraft, including the fuselage and the rotor, for a fictitious flight condition. Though this was a full aircraft simulation, the purpose of the computation was to demonstrate the feasibility of using a new domain connectivity algorithm for the moving, dynamic, overset grids for such a computation. Since the computations were performed to demonstrate a technology, no comparisons are made with experimental quantities. Srinivasan and Ahmad [20] used a Navier-Stokes scheme to calculate the quasi-steady flowfield for a hovering rotor mounted on a whirl tower. Due to the quasi-steady nature of the hovering condition, the equations were solved in the blade-fixed coordinate frame, using a momentum source term in the equations to account for the centrifugal force of the blade rotation. This simulation was also a feasibility study, so only a comparison of the predicted and measured mean thrust values were presented. For this simulation, which utilized
approximately 1.3 million grid points, a computational time of 14 Cray-YMP hours was quoted. Ahmad and Duque [21] used a thin-layer Navier-Stokes method with embedded, moving, overset grids to demonstrate the ability to calculate the unsteady flowfield of an isolated, two-bladed rotor in forward flight. Blade surface pressures at several radial and azimuthal and local normal load coefficients are compared to flight test data. These comparisons match reasonably well. Even though this computation did not include a fuselage body, the chimera grid scheme would render the task of including a body feasible, though at an additional computational cost. This isolated rotor computation required substantial computational resources; it required approximately 45 Cray C-90 hours and generated 40 Gigabytes of data.

Though most of these Navier-Stokes methods are suitable for computations over complete aircraft, including the helicopter rotor, all of the moving grid computations suffer from the requirement to re-compute the grid domain connectivity information at every time step; this technique is known as a “dynamic chimera scheme” and has two distinct disadvantages. First, regenerating the grid domain connectivity at every time step can be as computationally expensive as, or even more computationally expensive than, the actual flow solution. In addition, in some instances, the time step is restricted not by the flow or flow solver, but by the moving grid domain connectivity requirement that a “hole point” not become a “field point” at any time step [19]. This requirement can potentially limit the time step not to physical phenomena, but to grid cell size.

In addition to time step issues discussed above, other factors place limits on the current Navier-Stokes computations for rotorcraft. One issue is the numerical dissipation of concentrated vortices. It is well known that, in certain flight conditions, blade tip vortices can have a large effect on the rotor aerodynamics and that these vortices need to be computed in the flowfield over several rotor revolutions. Numerical studies discussed in the literature suggest that a 5th order scheme using 14 points across the vortex core produces satisfactory results for a vortex that is well-aligned with the grid [22]. However, in a typical rotorcraft simulation, the vortex location is not known a priori and thus the vortex in general will not be aligned well with the grid. Therefore a more strict resolution requirement is imposed on the numerical scheme [22]. For current methods either a prohibitively dense grid must be used to assure that the vortex is resolved well spatially, or grid adaption must be used to refine the grid in the regions that contain the vortices. Both methods are computationally expensive.

Another such issue is turbulence modeling. Many of the turbulence models in current use were developed for wall bounded flows. They are not well suited to the three dimensional, non-isotropic turbulence associated with rotor blade tip vortices.
1.2.3 Hybrid Methods

Considering the computational expense of current CFD methods, one possible approach to examining unsteady rotor/fuselage interactional aerodynamics is to use hybrid methods. For unsteady rotor/fuselage aerodynamics, several hybrid methods have been developed. One such hybrid method, developed by Steinhoff, et al. [23], modified the Navier-Stokes equations by adding a “vorticity confinement” term to the momentum equations. This new term is used to prevent, or counteract, numerical diffusion of concentrated vortical regions by “convecting” vorticity back toward the centroids of concentrated vorticity regions in the flowfield. This particular method is well suited for inclusion of a fuselage body. Boyd and Barnwell [24] first introduced a hybrid unsteady rotor model which weakly couples a Generalized Dynamic Wake Theory (GDWT) [25, 26, 27, 28] to a thin-layer Navier-Stokes model, OVERFLOW [29]. Extensive induced inflow comparisons were made between Laser Doppler Velocimeter (LDV) measurements and predictions. Even though the computations were for an isolated rotor, excellent agreement was found with measured quantities. Also presented was an outline of a method to couple a fuselage into the calculations using the overset grid capabilities in OVERFLOW. This new model uses the GDWT to obtain unsteady loading and unsteady induced inflow on the rotor, and then applies the unsteady loading inside OVERFLOW as a new unsteady pressure-jump, actuator disk-type, boundary condition. Another hybrid method, building on the previous literature [24], is developed in this research.

1.3 Present Approach

The objective of the current research is to develop an efficient, hybrid, unsteady computational model appropriate to the study of unsteady rotor/fuselage interactional aerodynamics.

In examining fully CFD, unsteady, moving grid methods for complete rotorcraft, it can be observed that small time steps are needed for method stability, for capturing aerodynamic effects that are on the order of the rotor blade chord size, and for proper usage of the dynamic chimera grid scheme. However, to capture the primary effects of rotor/fuselage interactional aerodynamics, chordwise aerodynamics on the rotor blade are of less importance than the gross loading on the rotor blade itself. This can be seen by the successes of some of the singularity methods which use lifting line rotor blade models (i.e., no chordwise loading distribution on the rotor blade) discussed previously in the “Literature Review” section above. In addition, fully CFD methods compute the rotor loading internally and require a number of rotor revolutions to obtain a periodic solution. The
combined requirements of needing very small time steps and of needing several rotor revolutions to obtain a periodic solution are a large contributor to the computational expense of these methods.

A hybrid method is developed here which reduces the computational expense by separating the rotor loading calculation from the CFD component of the computation. This hybrid method is depicted schematically in figure 1.2. From the figure, it can be seen that there are three components to this hybrid method:

1. Rotor Loading Model,
2. Rotor/Fuselage Flowfield Model,
3. Coupling Model.

### 1.3.1 Rotor Loading Model

The present approach separates the rotor loading model from the rotor/fuselage flowfield model. Splitting the procedure into these two separate models allows an otherwise computationally expensive element, the rotor loading computation, to be accomplished using an efficient, simplified model apart from the CFD computation. To compute the rotor loading, the GDWT [25, 26, 27, 28], which will be discussed in detail in a subsequent chapter, is used here. Previous implementations of the GDWT have focused on calculation of the unsteady inflow for an isolated rotor. As a significant advance over previous models, the unsteady rotor portion of the model uses the GDWT to calculate unsteady inflow and unsteady loading. The unsteady loading on the rotor is determined in the form of a $\Delta P$, or “pressure jump”, across the rotor disk. This $\Delta P$ is then used as a boundary condition in the rotor/fuselage flowfield model.

### 1.3.2 Rotor/Fuselage Flowfield Model

The unsteady loading, as determined by the GDWT, is then used in conjunction with a Navier-Stokes model, in this case, OVERFLOW, to compute the time dependent flow over the fuselage including effects of a helicopter rotor. The loading is used in the Navier-Stokes method as an unsteady boundary condition in the flowfield. This boundary condition is effectively an unsteady actuator disk model. Though there will be concentrated regions of vorticity near the edges of an unsteady actuator disk model used as a boundary condition in this manner, these are not true
“tip vortices” and thus internal structure of these flow features is of secondary importance. This alleviates the need to develop prohibitively dense grids, use grid adaption, or use higher order schemes to resolve these concentrated vorticity regions. As such, turbulence modeling of the inside of these vortex structures becomes less important as well. So, by modeling the rotor as an unsteady actuator disk in OVERFLOW, several computationally expensive requirements typically needed for full CFD rotorcraft modeling are diminished. Using the above model, the Navier-Stokes method is then used to compute the periodic flowfield of the rotor/fuselage combination. Further details of this component will be discussed in a subsequent chapter.

1.3.3 Coupling Model

With the completion of the Navier-Stokes method, there are two solutions which were obtained with the same $\Delta P$ distribution: the GDWT solution, which is for an isolated rotor, and the OVERFLOW solution, which contains both the rotor (as a boundary condition) and the fuselage body. The primary difference is that one solution contains a fuselage and the other does not. Since the loading in the GDWT depends on the rotor inflow, and since these inflow values are influenced by the presence of the fuselage, a method of coupling the two codes has been developed to account for the fuselage effects in the GDWT (and thus the rotor loading model). Since the fuselage effects on the rotor are assumed to consist of low frequency effects (as compared to the higher frequency blade-passage effects), the method employed here differences the time averaged inflow generated in the two successive solutions, and uses this difference as an “inflow correction” to the GDWT. This coupled process continues until only small differences are seen between successive iterations.
Figure 1.1: Analysis Types for Coupled Solutions.
Figure 1.2: Current Hybrid Method.
Chapter 2

Rotor Loading Model: Generalized Dynamic Wake Theory

2.1 Introduction

The Generalized Dynamic Wake Theory (GDWT) of Peters, Boyd, and He [26], Peters and He [25], and He [27] is used in the present approach to obtain unsteady loading that is to be used in conjunction with OVERFLOW as discussed previously. In a later chapter, the OVERFLOW and the coupling technique between OVERFLOW and the GDWT will be discussed. Even though the GDWT is spelled out in detail in the literature, the present chapter will describe the GDWT for background purposes and describe the particular manner in which the theory is implemented for the current research.

2.2 General Description

The GDWT is a theory that was originally designed to pose the issue of unsteady aerodynamics of a helicopter rotor in a state-space form. This type of state-space form is desirable for inclusion in a rotor stability analysis since stability analyses for the rotor dynamics are usually presented in a state-space form as well. With the aerodynamics and dynamics of the rotor stated in similar forms, the solution of the system of equations is simplified.
2.3 GDWT Outline

There are several aerodynamic concepts which are combined in the development of the GDWT. First, the acceleration potential derived by Kinner [30] for circular wing planforms is used with slight modification. The original acceleration potential derived by Kinner is a general form for the solution to Laplace’s equation (inviscid, linear, potential flow) in ellipsoidal coordinates. These modifications to Kinner’s acceleration potential function (or pressure function), applied by Peters, Boyd, and He [26], Peters and He [25], and He [27], are made to eliminate terms in the potential that are not compatible with boundary conditions associated with a rotor. This modified acceleration potential, $\Phi$, is then expressed using Legendre functions and transcendental functions in ellipsoidal coordinates as follows:

\[ \Phi(\nu, \eta, \bar{\psi}, \bar{t}) = -\frac{1}{2} \sum_{m=0}^{\infty} \sum_{n=m+1, m+3, \ldots} F_n^m (\nu) Q_n^m (i \eta) \left[ \tau_{mc}^n (i) \cos (m \bar{\psi}) + \tau_{ms}^n (i) \sin (m \bar{\psi}) \right] \] (2.1)

where the $\tau_{mc}^n$ and $\tau_{ms}^n$ terms are general coefficients of the pressure function and are, in general, functions of time and are determined from the loading on the rotor. The ellipsoidal coordinate system used here can be seen in figure 2.1. This figure shows a view of the xz plane with representative $\eta$ and $\nu$ values labeled. The $\bar{\phi}$ coordinate (not shown in figure 2.1) is an angular, azimuthal coordinate, measured around the z-axis. The “rotor disk” is defined by the following conditions:

\[ \eta = 0 \] (2.2)
\[ \nu = \sqrt{1 - \bar{r}^2} \] (2.3)
\[ \bar{\psi} = \psi \] (2.4)

where $\bar{r}$ is the radial coordinate on the rotor disk, measured from the axis of rotation, and $\psi$ is the angular, azimuthal coordinate measured about the axis of rotation. Equation (2.1) is effectively an expression for all admissible functions of loading on a rotor. To establish a link between loading on the rotor and induced inflow, the continuity equation and the linearized, incompressible Euler equations are used as follows:

\[ q_{i,i} = 0 \] (2.5)
where the summation convention is assumed over the index $i$, and $\xi$ is the coordinate pointing upstream along a streamline. Using equations (2.5) and (2.6), it can be shown that the pressure function, $\Phi$, satisfies Laplace’s equation. Also, since these equations are linear, the pressure function can be split into a component associated with each of the two terms on the left hand side of equation (2.6). Each of these components, in turn, also satisfies Laplace’s equation. As such, solutions can be derived for each component, then combined into a complete solution. The quantities used here are assumed to be total quantities, not perturbations. Also, it is assumed that only the velocity normal to the rotor disk is of interest and that it is of the following form:

$$w(\vec{r}, \psi, \vec{t}) = \sum_{r=0}^{\infty} \sum_{j=r+1, r+3, \ldots} \phi^r_j(\vec{r}) \left[ \alpha^r_j(\vec{r}) \cos(r\psi) + \beta^r_j(\vec{r}) \sin(r\psi) \right]$$

With these assumptions, a closed form set of first order, non-linear differential equations, in state-space form, can be established for the induced inflow coefficients $\alpha^r_j, \beta^r_j$. These equations are as follows:

$$\begin{align*}
\begin{bmatrix}
\vdots \\
\tilde{\alpha}_m \\
\vdots \\
\end{bmatrix}^* + [\tilde{L}]^{-1} \begin{bmatrix}
\vdots \\
V^m_n \\
\vdots \\
\end{bmatrix} \begin{bmatrix}
\vdots \\
\tilde{\alpha}_n \\
\vdots \\
\end{bmatrix} = \begin{bmatrix}
\vdots \\
\tilde{z}_{mc} \\
\vdots \\
\end{bmatrix} \\
\begin{bmatrix}
\vdots \\
\tilde{\beta}_m \\
\vdots \\
\end{bmatrix}^* + [\tilde{L}]^{-1} \begin{bmatrix}
\vdots \\
V^m_n \\
\vdots \\
\end{bmatrix} \begin{bmatrix}
\vdots \\
\tilde{\beta}_n \\
\vdots \\
\end{bmatrix} = \begin{bmatrix}
\vdots \\
\tilde{z}_{ms} \\
\vdots \\
\end{bmatrix}
\end{align*}$$

With these equations, the unsteady aerodynamics and induced inflow are cast in the time domain and in a state-space form. The problem has now been reduced to the computation of the states of the model, $\alpha^m_n, \tilde{\beta}_n$, given a loading on the rotor. This is a non-linear model since the loading and the inflow are coupled through mass flow parameter, $V^m_n$, in equations (2.12) and (2.13). As described
in reference [26], the mass flow parameter $V_m^n$, which accounts for the energy added to the flow by the rotor, takes the place of the $V_\infty$ term that results from equation (2.6) in order to extend the theory and to cover the case of hover, where $V_\infty$ approaches zero.

2.4 Solution Procedure

Though the details of the GDWT are discussed in the literature, few details are provided for the solution of this set of first order, non-linear differential equations. The procedure used in the current research is described here.

First, for a given set of rotor collective, lateral, and longitudinal pitch controls settings, the blade loading may be determined by any theory that can generate a loading given velocity (inflow) information. In the current research, as was done in previous literature [26, 25, 27], a two dimensional strip theory is employed. At first, this may seem to be an unnecessary restriction to a two dimensional theory. It has been shown [26, 25, 27] that this is not a restriction since the inflow and the loading are coupled. So, three dimensional effects, such as load reduction at the blade tip, are included in this theory. The equations of the strip theory used in this research are outlined below:

\[
\theta(\bar{r}, \psi) = \theta_{rw}(\bar{r}) + \theta_0 + \theta_c \cos \psi + \theta_s \sin \psi \tag{2.14}
\]

\[
\alpha_{eff} = \theta - \left( \frac{w + \mu \sin \alpha_s + \beta_0 \mu \cos \psi - 0.5c\dot{\theta} + \Delta\bar{\lambda}_i}{\bar{r} + \mu \sin \psi} \right) \tag{2.15}
\]

\[
\frac{L}{\rho \Omega^2 R^3} = \frac{\pi c(\bar{r} + \mu \sin \psi)^2 \alpha_{eff}}{\sqrt{1 - M^2}} \tag{2.16}
\]

Equation (2.16) gives the lift at a particular point on the rotor disk. Through the effective angle of attack, $\alpha_{eff}$, this loading theory includes local effects of the blade pitch ($\theta$), of the total induced inflow velocity ($w$), of the inflow due to the shaft tilt ($\mu \sin \alpha_s$), of the inflow due to the mean blade coning angle ($\beta_0 \mu \cos \psi$), of the velocity at the 3/4 chord point due to blade pitch rate ($0.5c\dot{\theta}$), and of the inflow correction determined by the coupling scheme ($\Delta\bar{\lambda}_i$), described in a later chapter. In addition, the lift curve slope is assumed to be $2\pi$ per radian, the Prandtl-Glauert compressibility correction is applied to the lift, and a simple stall model is used which limits the maximum angle of attack to 10 degrees. Now, given the blade pitch settings of the rotor and the rotor operating conditions, the local sectional loading can be determined from equations (2.14), (2.15), and
(2.16). With the lift distribution determined, equations (2.12) and (2.13) are solved using a 4-stage Jameson-style Runge-Kutta technique [31] until periodic induced inflow is obtained; this usually occurs within two rotor revolutions. No blade dynamics model is used in this study and the blade hinge offset is assumed to be at the center of rotation (i.e., the flap hinge is at the center of rotation).

With the above calculation complete, the mean thrust coefficient and mean hub moment coefficients can be determined from the following:

\[ \bar{C}_T = \frac{4}{\sqrt{3\pi}} \int_0^{2\pi} \tau_0^c d\psi \]  \hspace{1cm} (2.17)  
\[ \bar{C}_M = \frac{4}{3\sqrt{5\pi}} \int_0^{2\pi} \tau_1^c d\psi \]  \hspace{1cm} (2.18)  
\[ \bar{C}_L = \frac{4}{3\sqrt{5\pi}} \int_0^{2\pi} \tau_1^s d\psi \]  \hspace{1cm} (2.19)  

Since typically the initial pitch settings do not produce the desired thrust and moment coefficients on the rotor, they must be adjusted in subsequent iterations until the desired thrust and moment coefficients are obtained. This is known as “trimming” the isolated rotor; this accounts only for the desired loading on the isolated rotor and does not include any “feedback” forces from any other source (such as a fuselage). The trim procedure used in the current research for the isolated rotor is a modified Newton-Raphson technique adapted from the literature [32]. The following equation is used to determine the pitch setting “corrections” which are used to iteratively trim the isolated rotor:

\[
\begin{bmatrix}
\Delta \theta_0 \\
\Delta \theta_c \\
\Delta \theta_s \\
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial C_T}{\partial \theta_0} & \frac{\partial C_T}{\partial \theta_c} & \frac{\partial C_T}{\partial \theta_s} \\
\frac{\partial C_M}{\partial \theta_0} & \frac{\partial C_M}{\partial \theta_c} & \frac{\partial C_M}{\partial \theta_s} \\
\frac{\partial C_L}{\partial \theta_0} & \frac{\partial C_L}{\partial \theta_c} & \frac{\partial C_L}{\partial \theta_s} \\
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta C_T \\
\Delta C_M \\
\Delta C_L \\
\end{bmatrix}
\]  \hspace{1cm} (2.20)

where the matrix of partial derivatives is called the “derivative matrix” and each partial derivative is determined by using a one-sided forward difference formula and by using an independent perturbation to each of the initial pitch settings. Each row in the derivative matrix is computed by an independent perturbation of each corresponding pitch setting and solving equations (2.12) and (2.13). Once computed, the derivative matrix is held unchanged throughout the subsequent rotor
trimming process. The $\Delta C_T$, $\Delta C_M$, and $\Delta C_L$ are the changes in the current thrust and moment coefficients required to match the desired values. The trimming process is considered complete when value of the following is true:

$$\sqrt{(\Delta C_T)^2 + (\Delta C_M)^2 + (\Delta C_L)^2} < \varepsilon$$

(2.21)

where $\varepsilon$ is a specified tolerance. Now, with the trim task complete, the unsteady loading and unsteady induced inflow are known.

At the end of the trim process, the unsteady loading is in the form of a sectional loading (i.e., force per unit span). However, for use in OVERFLOW, the loading needs to be in the form of a pressure that will be applied to a grid point in the flowfield. The sectional loading is converted to a pressure (force per unit area) using the assumption that the force is evenly distributed over the chordwise and spanwise extent of the local blade section of interest. These pressures are now ready for use in OVERFLOW, which will be discussed in a later chapter.

Since the current research employs the GDWT in a manner in which it is not normally used, new computer coding has been developed here to compute required quantities from this theory, in combination with solution methods that have not previously been used with the GDWT. To validate that the new theory and models have been implemented correctly, a validation study is presented in the next chapter.
Figure 2.1: Ellipsoidal Coordinates.
Chapter 3

GDWT Validation

3.1 Introduction

Though there are numerous examples of the GDWT published in the literature, the present combination of the GDWT with the present solution procedures has not been explored in the past. Also, computer coding to do the actual computations with these combinations of theory and solution procedures was not available for the purposes of the current research. As such, validation is required to determine that the current model matches previously published literature on the subject. This validation effort is described in this chapter.

3.2 Experimental Setup

All of the experimental data used for the validation effort in this chapter was obtained from laser velocimeter inflow measurements made in the NASA Langley Research Center 14- by 22-Foot Subsonic Tunnel. The experimental setup and data used for this chapter is described in detail in references [33, 34, 35, 36]; for completeness, the experiments are described here in brief.

Two different planforms were used in this test, one rectangular and one tapered. A summary of the two geometries is given in table 3.1 and a picture of the configuration, installed in the tunnel, is shown in figure 3.1. In this figure, the rotor/fuselage configuration consists of a generic helicopter fuselage body, known as the ROtor Body INteraction (ROBIN) fuselage [37], and one of the two
rotor configurations, described above. Inflow data was taken on these two rotor configurations at several advance ratios (i.e., forward speed divided by the rotor tip speed) for 12 azimuthal stations located 30 degrees apart (starting from an azimuth location of 0˚, directly downstream of the rotor hub) and at a number of radial stations at each of these azimuth stations. Data samples were processed at a resolution of 128 samples per rotor revolution at each measurement location. For this chapter, the data is presented in two formats. The first format is the time averaged data, which is obtained by temporally averaging the unsteady data at each measurement location. The second format is time accurate data, which is the unsteady data before it is time averaged.

There are two advance ratios used in this chapter. The first, \( \mu = 0.15 \), can be thought of as a boundary between (1) very low speed flight, where the fuselage flowfield is completely dominated by the rotor wake, and (2) moderate forward flight, where the fuselage flowfield is no longer dominated by rotor wake interactions. The second, \( \mu = 0.23 \) can be thought of as a moderate forward flight case. At both speeds, pressure pulse effects on the fuselage from the passing rotor blades is felt.

The time averaged data will be presented in the form of contour plots of induced inflow ratio (i.e., magnitude of induced inflow velocity divided by rotor tip speed) mapped over the entire rotor disk. The time accurate induced inflow ratio will be presented as time histories which depict the induced inflow ratio at a particular fixed point over the rotor disk. For the contour plots of the measured data, it is worth noting that, even though the temporal resolution in the time accurate measurements corresponds to approximately 2.8˚ of blade travel, the spatial resolution in the measured data is limited. This spatial resolution limit is imposed by the limited number of azimuthal locations at which measurements were taken over the rotor disk. For the contour plots, the implication is that, since there were only twelve azimuthal measurement locations, the spatial frequency content of the induced inflow data in the azimuthal direction is limited by the Nyquist criterion to six harmonics per rotor revolution. In addition, there are some measurement locations at which no data is available.

For the sole purpose of making consistent contour plots of measured data, which are presented subsequently, the time averaged data for induced inflow ratio over the rotor disk has been linearly interpolated onto a grid consisting of seventeen evenly spaced radial stations from the 20% radius location to the tip location and onto twelve evenly spaced azimuth locations around the rotor disk. Interpolation onto a regular grid in this manner serves to “full-in” measured data that is not available at particular locations over the rotor disk.
Table 3.2 lists the matrix of cases, measured and predicted, that will be used for the validation effort in this chapter.

### 3.3 Time Averaged Induced Inflow

The following two sections present the time averaged, measured and predicted induced inflow ratio for several configurations. The quantities presented here are ones that have been derived from the measurements and predictions by time averaging the time accurate induced inflow ratio. Also, it should be noted here that the measured quantities include the effects of the ROBIN fuselage geometry and the predicted quantities do not include these effects (i.e., the predictions are for an isolated rotor).

#### 3.3.1 Rectangular Planform

Figure 3.2 shows a spatial contour plot comparison between the time averaged, measured and predicted induced inflow ratio for the rectangular planform at an advance ratio of 0.15, for a range of harmonics in the GDWT. Figure 3.2a is the measured data, including effects of the fuselage body. Figure 3.2b shows the GDWT predicted data using only two harmonics. Here, only the nominally linear streamwise gradient of induced inflow is predicted. Figure 3.2c shows the GDWT predicted data using four harmonics. In this figure, it can be seen that the major features of the measured data are predicted well. For example, the upwash on the forward section of the disk is predicted, though it does not cover as much of the forward region of the disk as in the measured data. The aft downwash regions, concentrated in the first and fourth rotor quadrants (see figure 3.18 for quadrant definitions), are also predicted. Figure 3.2d shows the GDWT predicted data using eight harmonics. In this figure, all of the same features are present as in figure 3.2c, but the details of the induced inflow are slightly different. Figure 3.3 shows the same data as in figure 3.2, but these are lateral and longitudinal subsets of the data and show the radial variation of measured and predicted induced inflow. Figure 3.3a shows measured and predicted lateral variation of the induced inflow ratio for 2, 4, and 8 harmonics; the advancing and retreating sides of the rotor disk are labeled. Figure 3.3b shows measured and predicted longitudinal variation of the inflow ratio for 2, 4, and 8 harmonics; the forward and aft portions of the rotor disk are labeled. Figures 3.4 and 3.5 show the same case as the previous two figures, except these predictions use 128 azimuth
steps per revolution instead of 64. No significant differences are apparent between the two different azimuthal resolutions; this shows that, for this particular case, 128 azimuth steps per revolution is a sufficient azimuthal resolution.

Figures 3.6 to 3.9 show the same contour plots and lateral/longitudinal plots as in figures 3.2 to 3.5, but for the higher advance ratio of 0.23. For this advance ratio, as with the lower advance ratio, little difference is seen between the use of 4 and 8 harmonics and little difference is seen between the use of 64 azimuthal steps and 128 azimuthal steps.

3.3.2 Tapered Planform

Figures 3.10 to 3.17 represent the same cases as in figures 3.2 to 3.9, but instead, using the tapered planform rotor. In comparison to the computations on the rectangular planform, the extent of the upwash region on the forward portion of the disk is better predicted based on the contour plots. Also, from the lateral and longitudinal plots, it can be seen that fewer harmonics are needed to predict the induced inflow behavior at the tip for the tapered planform. These findings for the contour plots and for the lateral and longitudinal plots are in agreement with findings in the published literature [25].

3.4 Time Accurate Induced Inflow

The previous sections presented the time averaged quantities derived from the experimental and predicted data. The following two sections present the unsteady induced inflow ratio components associated with the cases presented above except for the cases corresponding to two harmonics. These two harmonic cases are excluded at this point because using two harmonics does not adequately represent the time averaged induced inflow ratio distribution over the rotor disk. In order to show the time dependent quantities more clearly, the local time averaged quantities have been removed from each of the following plots. Figure 3.18 shows the measurement locations where the comparisons of the unsteady induced inflow will be made. The radial and azimuthal positions are shown for locations A through J. Locations A through I are used in this chapter. Location J will not be used in this chapter. It is, however, used in chapters 7 and 8 and is shown here for later reference.
3.4.1 Rectangular Planform

Figure 3.19 shows the unsteady component of induced inflow ratio at particular points on the rotor disk. The black dotted curves represent the measured data and the solid red lines represent the predicted data. In many of the measured data, a distinct set of four pulses per revolution can be seen. These pulses represent blade passages past the measurement location. The predictions at these measurement locations also show a periodic signature that generally matches the measurements in phase of the signals. Comparing figure 3.19 to figure 3.20 shows that for this configuration and this azimuthal resolution, the eight harmonic prediction matches the phase and amplitude better than the four harmonic prediction. Figures 3.21 and 3.22 show that the same holds for the 128 azimuth steps per revolution case. Figures 3.23 through 3.26 show that these results also hold for the higher advance ratio of 0.23. In these cases, it can be seen that, even using eight harmonics, the sharp, high frequency waveform of the measured inflow pulses is not matched well.

3.4.2 Tapered Planform

Figures 3.27 through 3.34 show the same features and results for the tapered blades as was presented for the rectangular blades above. As with the rectangular blade results, there is a typical, high frequency four per revolution pulse (waveform) indicative of blade passages seen at the measurement locations; the phase of these pulses generally matches the phase of the measured data. Also, as with the rectangular blade predictions, the eight harmonic results predict the amplitude better than the four per revolution results.

3.5 Observations

From the evidence presented in this chapter, the following observations can be made.

- Even though completely different methodology and computer coding from previous literature was used in the implementation and solution of the equations of the GDWT, the current results match well with the previously published literature [25, 27].

- The best overall choice for number of harmonics, considering both the advance ratio and configuration variations, is eight. This conclusion is influenced more by the time averaged
predictions than the time accurate predictions. This “weighting” toward the time averaged computations is influenced by the fact that the coupling model (discussed later in chapter 5) uses the time averaged quantities, not the time accurate quantities.

- Even with eight harmonics, it is not possible to simulate the sharpness of the blade passages.

- The induced inflow ratio predicted by the GDWT is relatively insensitive to the number of azimuthal steps used.

Based on the study presented in this chapter, and the observations presented in this section, it has been established that the new GDWT implementation is able to reproduce measured data in a manner comparable to previous literature. Thus, referring back to figure 1.2, the rotor loading model has been established. As such, the values of the ΔP pressure jump are now ready to be used in the rotor/fuselage model, which will be discussed in the next chapter.
Table 3.1: Rotor Geometries

<table>
<thead>
<tr>
<th>Property</th>
<th>Rectangular</th>
<th>Tapered</th>
</tr>
</thead>
<tbody>
<tr>
<td>radius</td>
<td>0.8606 meters</td>
<td>0.8255 meters</td>
</tr>
<tr>
<td>root chord</td>
<td>0.0660 meters</td>
<td>0.0800 meters</td>
</tr>
<tr>
<td>tip chord</td>
<td>0.0660 meters</td>
<td>0.0254 meters</td>
</tr>
<tr>
<td>taper ratio</td>
<td>(no taper)</td>
<td>3:1 past 0.75R</td>
</tr>
<tr>
<td>number of blades</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>root cutout location</td>
<td>0.24R</td>
<td>0.24R</td>
</tr>
<tr>
<td>flap/lag hinge location</td>
<td>0.06R</td>
<td>0.06R</td>
</tr>
<tr>
<td>sweep of quarter-chord</td>
<td>(none)</td>
<td>(none)</td>
</tr>
<tr>
<td>airfoil section</td>
<td>NACA 0012</td>
<td>NACA 0012</td>
</tr>
<tr>
<td>twist</td>
<td>-8˚</td>
<td>-13˚</td>
</tr>
<tr>
<td>precone</td>
<td>(none)</td>
<td>(none)</td>
</tr>
<tr>
<td>nominal thrust coefficient</td>
<td>0.0065</td>
<td>0.0065</td>
</tr>
<tr>
<td>solidity</td>
<td>0.0977</td>
<td>0.0977</td>
</tr>
<tr>
<td>nominal tip Mach number</td>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>approx. mean coning angle</td>
<td>1˚</td>
<td>1˚</td>
</tr>
<tr>
<td>shaft tilt</td>
<td>3˚ nose down</td>
<td>3˚ nose down</td>
</tr>
</tbody>
</table>
### Table 3.2: Test and Prediction Matrix

<table>
<thead>
<tr>
<th>Advance Ratio</th>
<th>No. Azimuths</th>
<th>No. Harmonics</th>
<th>Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>64</td>
<td>2,4,8</td>
<td>rectangular</td>
</tr>
<tr>
<td>0.15</td>
<td>128</td>
<td>2,4,8</td>
<td>rectangular</td>
</tr>
<tr>
<td>0.23</td>
<td>64</td>
<td>2,4,8</td>
<td>rectangular</td>
</tr>
<tr>
<td>0.23</td>
<td>128</td>
<td>2,4,8</td>
<td>rectangular</td>
</tr>
<tr>
<td>0.15</td>
<td>64</td>
<td>2,4,8</td>
<td>tapered</td>
</tr>
<tr>
<td>0.15</td>
<td>128</td>
<td>2,4,8</td>
<td>tapered</td>
</tr>
<tr>
<td>0.23</td>
<td>64</td>
<td>2,4,8</td>
<td>tapered</td>
</tr>
<tr>
<td>0.23</td>
<td>128</td>
<td>2,4,8</td>
<td>tapered</td>
</tr>
</tbody>
</table>
Figure 3.1: ROBIN Fuselage in the NASA Langley Research Center 14- by 22-Foot Subsonic Tunnel (1986).
Figure 3.2: Measured and GDWT Predicted Time Averaged Induced Inflow Ratio; Rectangular Planform, $\mu = 0.15$, 64 Azimuths.
Figure 3.3: Measured and GDWT Predicted Lateral and Longitudinal Time Averaged Induced Inflow Ratio; Rectangular Planform, $\mu = 0.15$, 64 Azimuths.
Figure 3.4: Measured and GDWT Predicted Time Averaged Induced Inflow Ratio; Rectangular Planform, $\mu = 0.15$, 128 Azimuths.
Figure 3.5: Measured and GDWT Predicted Lateral and Longitudinal Time Averaged Induced Inflow Ratio; Rectangular Planform, $\mu = 0.15$, 128 Azimuths.
Figure 3.6: Measured and GDWT Predicted Time Averaged Induced Inflow Ratio; Rectangular Planform, $\mu = 0.23$, 64 Azimuths.
Figure 3.7: Measured and GDWT Predicted Lateral and Longitudinal Time Averaged Induced Inflow Ratio; Rectangular Planform, $\mu = 0.23$, 64 Azimuths.
Figure 3.8: Measured and GDWT Predicted Time Averaged Induced Inflow Ratio; Rectangular Planform, $\mu = 0.23$, 128 Azimuths.
Figure 3.9: Measured and GDWT Predicted Lateral and Longitudinal Time Averaged Induced Inflow Ratio; Rectangular Planform, $\mu = 0.23$, 128 Azimuths.
Figure 3.10: Measured and GDWT Predicted Time Averaged Induced Inflow Ratio; Tapered Planform, $\mu = 0.15$, 64 Azimuths.
Figure 3.11: Measured and GDWT Predicted Lateral and Longitudinal Time Averaged Induced Inflow Ratio; Tapered Planform, $\mu = 0.15$, 64 Azimuths.
Figure 3.12: Measured and GDWT Predicted Time Averaged Induced Inflow Ratio; Tapered Planform, $\mu = 0.15$, 128 Azimuths.
Figure 3.13: Measured and GDWT Predicted Lateral and Longitudinal Time Averaged Induced Inflow Ratio; Tapered Planform, $\mu = 0.15$, 128 Azimuths.
Figure 3.14: Measured and GDWT Predicted Time Averaged Induced Inflow Ratio; Tapered Planform, $\mu = 0.23$, 64 Azimuths.
Figure 3.15: Measured and GDWT Predicted Lateral and Longitudinal Time Averaged Induced Inflow Ratio; Tapered Planform, $\mu = 0.23$, 64 Azimuths.
Figure 3.16: Measured and GDWT Predicted Time Averaged Induced Inflow Ratio; Tapered Planform, $\mu = 0.23$, 128 Azimuths.
Figure 3.17: Measured and GDWT Predicted Lateral and Longitudinal Time Averaged Induced Inflow Ratio; Tapered Planform, $\mu = 0.23$, 128 Azimuths.
Figure 3.18: Locations Used in Comparisons.
Figure 3.19: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Rectangular Planform, $\mu = 0.15$, 64 Azimuths, 4 Harmonics.
Figure 3.20: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Rectangular Planform, $\mu = 0.15$, 64 Azimuths, 8 Harmonics.
Figure 3.21: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Rectangular Planform, $\mu = 0.15$, 128 Azimuths, 4 Harmonics.
Figure 3.22: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Rectangular Planform, $\mu = 0.15$, 128 Azimuths, 8 Harmonics.
Figure 3.23: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Rectangular Planform, $\mu = 0.23$, 64 Azimuths, 4 Harmonics.
Figure 3.24: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Rectangular Planform, $\mu = 0.23$, 64 Azimuths, 8 Harmonics.
Figure 3.25: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Rectangular Planform, $\mu = 0.23$, 128 Azimuths, 4 Harmonics.
Figure 3.26: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Rectangular Planform, $\mu = 0.23$, 128 Azimuths, 8 Harmonics.
Figure 3.27: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Tapered Planform, $\mu = 0.15$, 64 Azimuths, 4 Harmonics.
Figure 3.28: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Tapered Planform, $\mu = 0.15$, 64 Azimuths, 8 Harmonics.
Figure 3.29: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Tapered Planform, $\mu = 0.15$, 128 Azimuths, 4 Harmonics.
Figure 3.30: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Tapered Planform, \( \mu = 0.15 \), 128 Azimuths, 8 Harmonics.
Figure 3.31: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Tapered Planform, $\mu = 0.23$, 64 Azimuths, 4 Harmonics.
Figure 3.32: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Tapered Planform, $\mu = 0.23$, 64 Azimuths, 8 Harmonics.
Figure 3.33: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Tapered Planform, $\mu = 0.23$, 128 Azimuths, 4 Harmonics.
Figure 3.34: Measured and GDWT Predicted Unsteady Induced Inflow Ratio With Mean Values Removed; Tapered Planform, $\mu = 0.23$, 128 Azimuths, 8 Harmonics.
Chapter 4

Rotor/Fuselage Flowfield Model: OVERFLOW

4.1 Introduction

A rotor loading model which computes a distributed $\Delta P$ pressure jump value on the isolated rotor disk has been established and discussed extensively in chapters 2 and 3. From figure 1.2, it can be seen that these $\Delta P$ values are used in the rotor/fuselage flowfield model to compute the entire flowfield of the combined rotor/fuselage system. The rotor/fuselage flowfield is computed by solving the Navier-Stokes equations in conjunction with conservation of mass and energy. The Reynolds averaged Navier-Stokes equations in Cartesian coordinates are as follows:

$$\frac{\partial \tilde{Q}}{\partial t} + \frac{\partial (\tilde{F} - \tilde{F}_u)}{\partial x} + \frac{\partial (\tilde{G} - \tilde{G}_v)}{\partial y} + \frac{\partial (\tilde{H} - \tilde{H}_w)}{\partial z} = 0$$

(4.1)

where,

$$\tilde{Q} = \begin{cases} 
\rho \\
\rho u \\
\rho v \\
\rho w \\
\rho e_0 
\end{cases}$$

(4.2)
In the above set of equations, \( \rho \) is the density, \( u, v, w \) are the velocity components, \( p \) is the pressure, \( e_0 \) is the total energy per unit volume, \( \tau_{ij} \) are components of the stress tensor, and \( q_x, q_y, q_z \) are components of the heat flux.

The quantities above are all considered to be averaged, or mean-flow, quantities. Also, Newtonian flow is assumed. Thus, the stress tensor is proportional to the velocity gradients in the flow. The constant of proportionality is composed of the sum of two components. The first component is the molecular viscosity, \( \mu_l \), which accounts for laminar or molecular viscosity, and is a property of the fluid. The second component is the turbulent eddy viscosity, \( \mu_t \), which accounts for the turbulence in the flow and is computed using a turbulence model.

The above system of equations is then closed by introducing the perfect gas law as follows:

\[
p = \rho R T
\]

where \( R \) is the gas constant and \( T \) is the temperature.
For the current rotor/flowfield model, a time accurate, Reynolds averaged Navier-Stokes (RANS) tool known as OVERFLOW [29] is used to solve the above set of equations. OVERFLOW is a computer code that uses a finite difference technique to solve the compressible, RANS equations in generalized coordinates. Even though OVERFLOW is technically a RANS solver, it is typically used in a thin-layer mode by ignoring the viscous terms that are not associated with the direction normal to surfaces in the flowfield; that approach is used here as well. The code employs a chimera (overset) grid scheme [38], which is helpful for complex geometries. Several solution procedure options are available in this code. A second or fourth order central difference scheme with a second and fourth order artificial dissipation scheme is used for the convective and viscous terms. A Pulliam-Chaussee scalar diagonal inversion [39] is typically used for the left-hand of the equations, though other options are possible, namely a Beam-Warming [40] scheme and a Lower Upper Symmetric Gauss Seidel (LU-SGS) scheme [41]. For steady state computations, a local time stepping scheme and a multigrid scheme [42] are implemented to accelerate convergence. Also, for steady state computations at low Mach numbers, a low Mach number preconditioning scheme [42] is used. For unsteady computations, a “Newton sub-iteration” scheme [43] is implemented for each time step to reduce linearization and factorization errors and to increase the time accuracy of the scheme to approximately second order. Also available are several turbulence models and an extensive set of boundary condition options. Available turbulence models include a Baldwin-Lomax model, a Baldwin-Barth model, a Spalart-Allmaras model, and a $k-\omega$ model; the Spalart-Allmaras turbulence model is used for all computations presented here. Boundary condition options include conditions for inviscid walls, viscous walls, periodic grids, symmetry planes, singular axes, inflow, outflow, characteristic conditions, etc. For both steady and unsteady computations, all boundary conditions are applied explicitly; some of these boundary conditions may also be applied to any region inside the volume of the computational grid, not just at the outer grid boundary faces.

This chapter describes the manner in which OVERFLOW is used for the rotor/fuselage flowfield model depicted in figure 1.2.

### 4.2 New Boundary Conditions

For the purposes of the current research, OVERFLOW has been extended to include two new, novel, explicit boundary conditions which use the pressure jump previously computed by the
GDWT to model a helicopter rotor. One of the boundary conditions is used for time averaged computations and the other is used for unsteady, time accurate computations. Both are applied to two planes of a non-rotating, cylindrical grid with an “iblanked plane” in between. Figure 4.1a shows a top view schematic of the rotor disk used in the new boundary conditions. This schematic represents the non-rotating, cylindrical portion of the grid that is used to represent the rotor disk. This “rotor portion” of the grid is a subset of the much larger overset volume grid set that is used to represent the entire rotor/fuselage flowfield. This grid subset is where the new boundary conditions are applied.

The shaded area in this figure represents a small region of the cylindrical grid; the rectangle outlined by the dark lines represents a rotor blade. Both of these regions are enlarged in the figure for clarity of presentation. Figure 4.1b shows an edge view of this same schematic. In this view, five planes can be seen. These planes consist of an upper rotor plane, an iblanked plane, a lower rotor plane, and two planes, labeled A and B, which are used in the formulation of the boundary conditions.

Although the focus of the current research is on unsteady interactions, it is necessary to have both the time averaged and time accurate boundary conditions discussed above. These boundary conditions are used in a complementary manner as follows. First, the steady state flowfield around the isolated fuselage is determined. Then, using this isolated fuselage computation as a starting point, the steady state flowfield of the fuselage, including the time averaged rotor model, can be determined using the time averaged rotor boundary condition. The previous two stages are used to set up the steady state flow features so that the unsteady computations can be used as a final stage, which is executed until a periodic solution is obtained. This “building block” approach of using a steady state computation as an initial condition to a time averaged computation, and in turn, using the time averaged computation as an initial condition to the time accurate computation, is used to reduce the computational resources required relative to using a completely time accurate computation from inception.

Next, the details of each of the new boundary conditions will be described.
4.2.1 Time Averaged Boundary Condition

As stated above, the boundary conditions are applied on two planes, separated by an “iblank plane”. Use of the “iblank plane” between the upper and lower rotor planes prevents the artificial dissipation in OVERFLOW from being activated by the new pressure jump imposed across the two planes.

The steps in the application of this boundary condition are as follows:

1. Identify the two planes (labeled A and B in figure 4.1b) surrounding the upper and lower rotor planes,

2. Average the conservative flow quantities in planes A and B as follows:

\[ Q_{i,\text{avg}} = \frac{(Q_{i,A} + Q_{i,B})}{2} \quad (4.7) \]

where \( Q_i \) are given in equation (4.2),

3. Replace the existing conservative flow quantities in the upper and in the lower rotor planes with these average values,

4. Add an “additional energy term” to the fifth conservative flow quantity (\( Q_5 \)).

Steps 1, 2, and 3 above are relatively straightforward and are accomplished with application of existing boundary conditions. Step 4 is the new step in this process introduced here and is described below.

As stated above, both boundary conditions use the pressures from the GDWT with a few slight modifications. The first new boundary condition time averages the unsteady pressures at each radial and azimuthal location, multiplies by the ratio of the local blade area to the local computational cell area, \( A(\bar{r}) \), to maintain the same level of thrust between the two methods, and divides by \( (\gamma - 1) \) to convert the pressures into an energy-like term that is compatible with the \( Q_5 \) variable above. This conversion to a \( Q_5 \) compatible quantity can be combined into one expression as follows:

\[ [\rho e_0]_{add}(\bar{r}, \psi) = \frac{A(\bar{r})}{N_T(\gamma - 1)} \sum_{t=1}^{N_T} \Delta P(\bar{r}, \psi, t) \quad (4.8) \]

\(^1\text{The term “iblank” refers to a technique employed in many methods that use the chimera scheme. This technique involves intentionally excluding certain points (here, a certain plane) from the computation of the flowfield.}\)
where $N_T$ is the number of time steps used in one revolution. The value of the left side of equation (4.8) is then split in half. One half of the term is added to the $\rho e_0$ equation for the lower plane of the rotor grid at the current azimuth and radial location, the other half is subtracted from the $\rho e_0$ equation for the upper plane in the rotor grid at the current azimuth and radial location; this effectively adds energy to the flowfield, to mimic an actuator disk. The splitting of the additional $\rho e_0$ term and placement on either side of an “iblank plane” prevents the artificial dissipation in OVERFLOW from acting on the effective additional pressure jump; the artificial dissipation in OVERFLOW is designed so that it will not operate across an iblank region. Without this iblank plane, the artificial dissipation would operate on the pressure jump, smoothing the pressure jump unnecessarily.

### 4.2.2 Time Accurate Boundary Condition

The new time accurate boundary condition is applied in a manner similar to that used for the time averaged boundary condition above. The differences are that (1) the pressures are not time averaged before they are converted to energy-like terms, and (2) the pressures are evenly distributed over all azimuthal grid lines that cross the blade chord at the local blade radial station. Similar to the time averaged boundary condition (with exception (1) above), the following additional energy-like term is determined:

$$
[(\rho e_0)_{add}] (\bar{r}, \psi, t) = \frac{A(\bar{r})}{(\gamma - 1)} \Delta P(\bar{r}, \psi, t)
$$

(4.9)

As in the time averaged boundary condition above, the value of the left side of equation (4.9) is then split in half. One half of the term is added to the $\rho e_0$ equation for the lower plane of the rotor grid, the other half is subtracted from the $\rho e_0$ equation for the upper plane in the rotor grid. As discussed above, splitting the additional term and applying it on either side of an iblank plane prevents the artificial dissipation term from acting on the pressure jump. Also, at each radial station on the blade, the number of grid lines that lie on the chord are computed based on the chord length and the arclength between successive azimuthal grid lines. The additional energy term above then is distributed evenly over these grid lines. This distribution allows the shape of the blade to be better modeled in the cylindrical grid.
4.3 Summary

Referring to figure 1.2, a rotor/fuselage flowfield model has been introduced in this chapter. This new model consists of a Navier-Stokes method in which several new boundary conditions have been developed. These boundary conditions use information from the GDWT to model an addition of energy to the flowfield as in an actuator disk method, but in a time accurate setting. In the next chapter, the third box in figure 1.2, “coupling model”, will be discussed.
Figure 4.1: Rotor Schematic for New Boundary Conditions.
Chapter 5

Coupling Model

5.1 Introduction

Since the pressure jump determined by the isolated rotor loading model depends on the inflow ratio distribution over the rotor disk, and since the presence of a fuselage alters this inflow ratio distribution, some type of coupling model is needed to adjust the inflow ratio and pressure jump distributions to reflect the effects of the fuselage. The current coupling method adopts an “inflow correction” model. In this type of model, the loading on the rotor is determined based on the inflow at the rotor disk, including additional inflow, or inflow corrections, generated by the presence of the fuselage. Presently, the forces and moments on the fuselage are not accounted for directly in the computation of the rotor forces; the rotor forces are, however, computed using inflow corrections to account for the presence of the fuselage. Since typically, for a model in a wind tunnel, the rotor is trimmed to a specified thrust, independent of the forces acting on the fuselage, this modeling assumption is valid here.

In previous chapters, discussions of the rotor loading model and the rotor/fuselage flowfield model were presented. In this chapter, the lower box of figure 1.2, the “coupling model” will be discussed.
5.2 Isolated Rotor Configurations

Previous literature [24] used a hybrid method similar to that presented here. The current research builds on the idea presented in reference [24], which is a subset of the current work. This subset is contained inside the dashed box region in figure 5.1. For an isolated rotor computation, no coupling model, or feedback loop, is needed to provide fuselage correction information to the rotor loading model. Therefore, an isolated rotor computation is determined by a single pass through the method, stopping at the end of the rotor/fuselage flowfield computation. This was the method employed in reference [24].

5.3 Rotor/Fuselage Configurations

When there is a fuselage present in the rotor/fuselage flowfield model, a correction scheme is needed to feed information back into the isolated rotor loading model to account for the fuselage presence. The direction of the arrows in figure 5.1 indicates that the correction scheme should reflect the effects of the fuselage on the rotor system, as opposed to the rotor effects on the fuselage.

In observing effects of a fuselage on the rotor system, it can be noted that time scales associated with the flow around the fuselage are much larger than those associated with the rotation of the rotor system. The effect of the fuselage on the surrounding fluid is one of slow displacement. However, due to its rotation, the rotor blade will pass this fluid particle at a much higher speed. Thus, for a given time increment, the rotor blade will traverse far more distance than a fluid particle traveling along the fuselage.

Based on these scale differences, it is expected that the presence of the fuselage produces effects on the rotor disk that are not spatially or temporally concentrated when viewed in the fuselage fixed, spatial frame of reference. However, when this effect is viewed in the rotor blade rotating frame of reference, the rotor blade “sees” the spatial distribution of inflow generated by the presence of the fuselage as a slowly varying (temporal) inflow quantity. This leads to the conclusion that the influence of the fuselage on the rotor system can be viewed as a time averaged perturbation\(^1\) to the isolated rotor configuration in the fuselage fixed reference frame.

With the view that the fuselage effect on the rotor system can be considered to be a time averaged

\(^1\)Note that here, a perturbation is not necessarily small.
inflow ratio correction distributed over the rotor disk, an efficient coupling method which accounts for the primary fuselage effects on the rotor inflow ratio, can be developed. The coupling method developed here is of that type.

To determine the inflow corrections to be fed back into the rotor loading model, a difference is taken between the time averaged, filtered inflow ratio from the rotor/fuselage flowfield model and the time averaged inflow ratio from the GDWT:

\[ \Delta \tilde{\lambda}_i = \mathcal{F}(\tilde{\lambda}^{RFFM}_i) - \tilde{\lambda}^{RLM}_i \]  

(5.1)

where \( \mathcal{F}(\cdot) \) is a filtering operation performed on \( \tilde{\lambda}^{RFFM}_i \). To show the reason for the filtering operation on the \( \tilde{\lambda}^{RFFM}_i \) term, it is necessary to examine the frequency limitations of each component in the method. For the rotor loading model component, the spatial frequency content of the induced inflow ratio determined by the GDWT, \( \tilde{\lambda}^{RLM}_i \), is limited to the number of harmonics chosen in the method. For example, it was shown in chapter 3 that eight harmonics of rotor inflow are sufficient for that model in this context. Therefore inflow ratio information, time averaged or time accurate, is limited to eight harmonics (in this example). Note that this limitation does not imply that the loading distribution computed by the GDWT is limited to the given number of harmonics. This is because the GDWT computes the inflow ratio (up to a specified number of harmonics) given any loading distribution. This computed inflow ratio influences the loading distribution through the \( w \) term in equation (2.15). Thus, the otherwise two dimensional loading distribution reflects inflow corrections up to the number of harmonics in the model.

While the GDWT limits the number of inflow harmonics computed to a specified value, the inflow computed by the rotor/fuselage flowfield model, \( \tilde{\lambda}^{RFFM}_i \), is only limited by the spatial resolution found in the rotor grid. For example, a typical rotor grid used in this method [24] uses 128 azimuthal grid lines per rotor revolution. Using the Nyquist cutoff concept, this means that the limit of the frequency content in the azimuthal direction is 64 harmonics. If \( \Delta \tilde{\lambda}_i \) were computed and used in the rotor loading model without the filtering operation, inflow corrections (and therefore loading distribution corrections) would be made inside the GDWT that are inconsistent with the inflow corrections made internal to the model using the \( w \) term in equation (2.15). To eliminate this inconsistency, the \( \tilde{\lambda}^{RFFM}_i \) term is filtered to match the frequency content of the \( \tilde{\lambda}^{RLM}_i \) term.

A consistent filtering operation, \( \mathcal{F}(\cdot) \), is derived from the GDWT method and is given in Appendix A. Applying this filtering, \( \mathcal{F}(\tilde{\lambda}^{RFFM}_i) \), then computing \( \Delta \tilde{\lambda}_i \) from equation (5.1) above, this term may be included in equation (2.15) to complete the coupling loop. With this coupling method,
all of the boxes of figure 1.2 have been described. The next few chapters will discuss applications of entire method.
Figure 5.1: Current Hybrid Method.
Chapter 6

Results: Isolated Fuselage

6.1 Introduction

Chapters 1, 2, 4, and 5, have described the components of the new computational model for rotor/fuselage unsteady interactional aerodynamics and how they are used. In the next three chapters, a full configuration will be constructed from basic components: (1) an isolated fuselage, (2) an isolated rotor, and (3) a rotor/fuselage combination. The subject of this chapter is the isolated fuselage component. Even though, strictly speaking, an isolated fuselage model is not a new computational model, it is necessary to establish that the computations can be compared favorably to measured data.

6.2 Experimental Setup

The experimental setup for the comparisons in this isolated fuselage chapter are described in detail in Freeman and Mineck [37]. However, for completeness, the setup will be discussed here in brief.

A test was conducted in the NASA Langley Research Center 14- by 22-Foot Subsonic Tunnel, in which steady state surface pressures were obtained at one hundred and sixty-two locations on a generic helicopter fuselage shape for a number of flight conditions with and without a rotor installed. This fuselage shape, which is derived mathematically using “super-ellipse” equations [37], is known as the ROtor Body INteraction (ROBIN) fuselage (see figure 6.1). In the photograph
shown in figure 6.1, the ROBIN is sting mounted, with the rotor installed. For the isolated fuselage portion of the test, the blades were removed, but the hub was left in place. Though test conditions included configurations with and without the rotor installed, only steady state pressures were measured on the fuselage. As such, only the isolated fuselage pressures will be used here.

### 6.3 Computational Grid System

A system of thirteen grids, combined using the chimera grid capabilities in OVERFLOW, is used to represent the ROBIN fuselage geometry and flowfield. The sting mount, rotor hub, and wind tunnel walls are not modeled. A listing of all of grid names and sizes are displayed in table 6.1. From table 6.1 it can be seen that five of these grids involve surface grids associated with the fuselage. These surface grids are shown and labeled in figure 6.2. Note that, for clarity of presentation, only every third grid line is plotted in figure 6.2. All remaining grids are field volume grids. Figure 6.3 shows a view of the grids immediately surrounding the fuselage. For clarity of presentation, every second grid line in this figure has been removed. Also seen is the positioning of the rotor grid with respect to the fuselage. For the isolated fuselage computations presented in this chapter, the rotor grid is simply another volume grid (i.e., there is no rotor boundary condition applied on the rotor grid). The far field grid extents approximately 2.5 fuselage lengths upstream of the fuselage nose, downstream of the fuselage tail, to the left and right of the fuselage, and above and below the fuselage.

These grids were developed such that, with minimal effort and minimal grid modification, all configurations used in the current research (including an isolated fuselage, an isolated rotor, and a rotor/fuselage combination) could be modeled simply by “grid replacement”. As an example, when converting from a rotor/fuselage configuration to an isolated rotor configuration, only the grids containing the fuselage need to be replaced; these are replaced by regular volume grids that maintain the outer boundary shape of the fuselage grids. All other grids remain the same.

These grids were also designed to maintain a “double fringe” overlap between all overlapping grids (except for the outer field grid) to maintain the second and fourth order artificial dissipation scheme used in OVERFLOW. In addition, grid spacings were refined to maintain viscous spacings at the fuselage surface such that there is at least one grid point in the laminar sub-layer region of the boundary layer and to maintain approximately 1.5 million total grid points in order to keep the computational expense reasonable.
One novel feature of this grid system is the manner in which the rotor grid is included. In previous isolated fuselage studies of the ROBIN fuselage [44, 45] the grid system did not include a rotor grid, thus a hyperbolic grid generator was used to “grow” the volume grids out from the surface grids without regard to the location of the outer boundaries of these grids. However, due to the geometry of the outer boundaries of grids produced by a hyperbolic grid generator, inclusion of a cylindrical rotor grid by the technique of “hole cutting” [46], while maintaining a double fringe chimera scheme, is difficult. This problem arises due to the lack of control of the outer boundary shape of grids generated by hyperbolic grid generation methods. To alleviate this problem, an elliptic grid generation method is used to control the outer boundary shape of some grids. With this technique, the overlap of the grids can be better controlled to produce a double fringe overlap and the amount of “hole cutting” can be substantially reduced.

For the current research, a hybrid set of grids generated by hyperbolic and elliptic grid generators [47, 48, 49, 50] is used. For volume grids in which no control was needed over the outer boundary shape, a hyperbolic grid generator was used. If control over the outer boundary geometry was desired, an elliptic grid generator was used.

### 6.4 Steady State Pressure Prediction

As discussed previously, the first step in the OVERFLOW computations is to execute the isolated fuselage configuration in a steady state mode. For the current computations, the default settings recommended for OVERFLOW [29] are used, except where noted. These defaults include central differencing of the right hand side of the equations, scalar diagonal inversion of the left hand side of the equations, a matrix dissipation scheme, low Mach number preconditioning, multigrid, full multigrid (mesh sequencing), local time stepping with a minimum CFL number constraint, and the Spalart-Allmaras turbulence model. The case presented here is for an angle of attack of 0° and a freestream Mach number of 0.1265.

Figure 6.4 shows the force coefficients in the lift, drag, and sideward directions as a function of iteration number in OVERFLOW. These force and moment coefficients are defined as follows [51]:

\[
C_f = \frac{\hat{f}}{\hat{Q}_{\infty} A_{ref}}
\]  

(6.1)
\[ C_m = \frac{\hat{M}}{\hat{Q}_\infty \hat{\dot{A}}_{ref} \hat{f}_{ref}} \]  
\[ \hat{Q}_\infty = \frac{1}{2} \hat{\rho}_\infty \hat{V}_\infty^2 \]

where a list and derivation of reference quantities for these is given in reference [51]. Though these do not directly correspond to the standard lift, drag, and side force coefficients, they do provide a useful insight into the convergence behavior of the OVERFLOW iteration from an engineering standpoint.

Figure 6.5 shows the pressure tap locations on the ROBIN fuselage. The locations are shown with respect to the downstream coordinate (x-direction). Along the top of the figure, the lowercase letters indicate the locations where predictions are compared to measurements in the subsequent figure.

Figure 6.6 compares measured and predicted pressure coefficients vs the vertical location on the fuselage at various places stations along the length of the fuselage. The vertical axis in figure 6.6 is plotted in the standard negative \( C_p \) format and the horizontal axis is the vertical location (z coordinate) of the current section; the solid lines are the predictions and the symbols are the measured values.

The pressure coefficient used here is defined in equation (6.4). From a practical standpoint, computation of \( C_p \) inside OVERFLOW is accomplished using equation (6.5) and equation (4.2), taking into account the nondimensionalizations used in OVERFLOW.

\[ C_p = \frac{P - P_\infty}{\frac{1}{2} \hat{\rho}_\infty \hat{V}_\infty^2} \]  
\[ C_p = \frac{2(\gamma - 1)}{M_\infty^2} \left[ Q_5 - \frac{Q_1^2 + Q_2^2 + Q_3^2}{2} - \frac{1}{\gamma(\gamma - 1)} \right] \]

In figure 6.6 it can be seen that predicted pressure coefficients match the experimental data well in most areas. At locations (f) and (g), discrepancies can be seen on the upper and lower portions of the fuselage. These differences could be due to the presence of the hub and sting mount in the experimental setup. In this figure, predicted \( C_p \) values are plotted from both sides of the fuselage for this symmetric flight condition. In this figure, predicted pressure coefficients from both sides of the fuselage are plotted since the computation included the full fuselage instead of a half body.
fuselage plus a symmetry condition. As expected, no difference is seen between the two sides. In addition to matching well the experimental data, these predictions are consistent with similar predictions in the literature [45], which used other Navier-Stokes methods (codes), for the same configuration and flight condition.

### 6.5 Observations

OVERFLOW has the capability to predict the steady state pressure coefficients on an isolated fuselage configuration in an incompressible flight condition, and these predictions are consistent with similar computations in the literature.
Table 6.1: Computational Grid System

<table>
<thead>
<tr>
<th>Grid Name/Description</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuselage Grid</td>
<td>93 x 117 x 25</td>
</tr>
<tr>
<td>Nose Grid</td>
<td>30 x 117 x 33</td>
</tr>
<tr>
<td>Tail Grid</td>
<td>28 x 117 x 33</td>
</tr>
<tr>
<td>Collar Grid</td>
<td>29 x 111 x 25</td>
</tr>
<tr>
<td>Top Grid</td>
<td>41 x 41 x 25</td>
</tr>
<tr>
<td>Lower Field Grid</td>
<td>55 x 50 x 12</td>
</tr>
<tr>
<td>Upper Field Grid</td>
<td>55 x 50 x 22</td>
</tr>
<tr>
<td>Nose Field Grid</td>
<td>42 x 70 x 43</td>
</tr>
<tr>
<td>Tail Field Grid</td>
<td>30 x 39 x 29</td>
</tr>
<tr>
<td>Left Field Grid</td>
<td>55 x 32 x 30</td>
</tr>
<tr>
<td>Right Field Grid</td>
<td>55 x 32 x 30</td>
</tr>
<tr>
<td>Outer Field Grid</td>
<td>93 x 82 x 26</td>
</tr>
<tr>
<td>Rotor Grid</td>
<td>37 x 129 x 43</td>
</tr>
</tbody>
</table>
Figure 6.1: ROBIN Fuselage in the NASA Langley Research Center 14- by 22-Foot Subsonic Tunnel.

Figure 6.2: ROBIN Fuselage Surface Grids.
Figure 6.3: Grid System Used in Computations.
Figure 6.4: Lift, Drag, and Sideward Direction Force Coefficients. Steady State Computation.
Figure 6.5: Pressure Tap Locations.
Figure 6.6: Measured and Predicted Pressure Coefficients vs Vertical Location for $\alpha = 0^\circ$ and $M_\infty = 0.1265$. 
Chapter 7

Results: Isolated Rotor

7.1 Introduction

Chapter 6 showed that the pressure distribution could be predicted on the isolated ROBIN fuselage configuration for a representative forward flight condition of a rotorcraft. This chapter will employ the isolated rotor method as described in chapter 5. Previous predictions using the current model [24] compared well with time averaged and time accurate laser velocimeter (LV) data. In that work, the predictions were for an isolated rotor, whereas the experimental data contained the effects of a fuselage. Subsequent to those predictions, new experimental data has been acquired on an isolated rotor configuration (described below). Comparisons between these data and the current method are the subject of this chapter.

7.2 Experimental Setup

For comparisons in this section, results from an isolated rotor configuration in the NASA Langley Research Center 14- by 22-Foot Subsonic Tunnel are shown. The isolated rotor test system (IRTS) [52, 53] is shown in figure 7.1. Here, it can be seen that the rotor is suspended from the ceiling of the tunnel from a tapered, cylindrical shaped sting. A three component laser velocimeter (LV) system was used to measure the three components of velocity at a point at an azimuth of 84°, a radial station of \( r/R = 0.81 \), and one blade chord above the tip path plane of the rotor. The rotor was trimmed to a nominal 0° flapping angle (e.g., the first flapping harmonics are \( \beta_{1c} = \beta_{1s} = 0° \)).
a nominal thrust coefficient of 0.0064, a nominal shaft tilt, $\alpha_s$, of 3° nose down, and a freestream Mach number of 0.1265. The LV data was processed at 128 samples per rotor revolution.

### 7.3 Computational Grid System

As discussed in chapter 6 for the isolated fuselage configuration, a grid system was developed that, among other things, had the ability to be used for an isolated rotor configuration with minimal changes to the grid system. For this isolated rotor configuration, several minor changes were made to the grids. These changes are as follows:

1. The nose grid has been eliminated,
2. The tail grid has been eliminated,
3. The collar grid has been eliminated,
4. The top grid has been eliminated,
5. The fuselage grid has been replaced by a volume field grid that maintains the outer boundary shape of the original fuselage grid.

Since the nose, tail, collar, and top grids from items 1, 2, 3, and 4 above were “overset” into other background grids, it was only required to delete these grids and leave the remaining background grids unchanged. The only new grid in the system is the volume grid that replaces the fuselage grid. All other grids in the system are unchanged. In essence, what remains, is a set of volume grids, identical to the full configuration, minus the fuselage surface. However, now, the new rotor boundary condition in OVERFLOW will be applied in the rotor grid as discussed in chapter 4. As with the isolated fuselage configuration, the sting mount and the wind tunnel walls are not modeled. This new grid system is shown in figure 7.2; every second grid line has been removed from this figure for clarity. The farfield extent of the grid is the same as that in figure 6.3.

### 7.4 Time Averaged Computation

The isolated rotor computations will be started with a prediction of the time averaged flowfield due to the rotor. For this, the time averaged rotor boundary condition discussed in chapter 4 is used.
This is done to minimize the number of time accurate computations that must be executed. Since this time averaged flow field is only an intermediate stage of the computation, and since there is no time averaged induced inflow data for the isolated rotor test stand available, no comparisons are made to experimental data.

### 7.5 Time Accurate Computation

Once the time averaged computation discussed above has been obtained, OVERFLOW is restarted in a unsteady mode and the method is continued until a periodic solution is achieved. The state of “periodicity” in this context refers to the lack of differences in the induced inflow ratio predictions for successive blade passages. Boyd and Barnwell [24] explored the sensitivities of several parameters in OVERFLOW for the time accurate computations of this type. Following the conclusions of that paper, the computations are executed with six Newton sub-iterations and no viscous terms. Since fourth order central flux differencing in space is now available in OVERFLOW, that is used as well.

Figure 7.3 compares the measured and predicted unsteady induced inflow in directions parallel and perpendicular to the rotor tip path plane at location “J” as shown in figure 3.18. This position is located radially at $r/R = 0.80$ and azimuthally at $\psi = 84^\circ$. Figure 7.3a is the induced inflow comparison in the direction parallel to the tip path plane (the $u$-velocity, or “inplane” velocity) and figure 7.3b is the induced inflow comparison in the direction perpendicular to the tip path plane (w-velocity, or “out-of-plane” velocity). For clarity, both of the comparisons show only the unsteady components of induced inflow. It can be seen that the unsteady $u$-velocity comparison is excellent in magnitude, phase, and waveform shape. The $w$-velocity predictions match the phase and waveform shape well, but slightly under-predicts the magnitude of the pulse. The slight discrepancies in the phase for both plots may be explained by realizing that the location of the entire experimental signal has an error band that is equal to $\pm \frac{1}{2}$ of the azimuthal resolution at which the data was acquired. This error band is equivalent to $\pm 1.4^\circ$ in azimuth. Here, the data has been plotted at the center of the error band. Also, the position of the measurement location in space and the position of the point in space used for the prediction comparisons do not exactly coincide since there is not a computational grid point that falls exactly on the measurement location; the closest point in the grid to the measurement location is shown. Comparing the coordinates for location J in the measurements and in the predictions, the error in radial location is less than 1%.
of the rotor radius; the error in azimuthal location is less than 0.4°, and the vertical location error is negligible ($\ll 1\%$ of the rotor radius).

Even though the distributed time averaged data was not measured in this experiment, it is worthwhile to examine this quantity from the predictions in the same manner as done by Boyd and Barnwell [24]. Figure 7.4 shows a contour plot comparison of the time averaged induced inflow (w-velocity). Figure 7.4a shows the out-of-plane component of the measured, time averaged induced inflow ratio for the rectangular planform rotor shown in chapter 3. This is the same experimental data from figure 3.8. As discussed in chapter 3, this experimental data contains a fuselage, whereas, the current prediction does not. Figure 7.4b shows the current, predicted, time averaged induced inflow ratio which as been filtered to roughly match the experimental data frequency content as discussed in Appendix A.

Comparing these two figures, it can be seen that the prediction contains the same general features of the measured data. It can be seen that there is an upwash on the front of the disk, that there is a downwash at the rear of the disk with more concentrated downwash in the first and fourth quadrants, and that the magnitudes of the downwash are similar. These features and the level of prediction shown are similar to those shown in reference [24] for a different rotor system. In addition, figure 7.4b shows a curious feature that has been seen before in the literature for both the current method and for the purely GDWT [25]. This feature, seen on the retreating side of the rotor in the fourth quadrant near the blade root, appears to be a region where the induced inflow is quite small compared to the induced inflow surrounding that region; this feature is not seen in the measured data. That feature can be explained by examining the rotor loading in that region. Since, physically, the rotor induced inflow is generated as a fluid reaction to the rotor loading distribution, and since the rotor loading is very small in that region due to the very low dynamic pressure there, the induced inflow is small in that region. Chapter 8 will show that this feature is affected by the isolated rotor assumption made in previous investigations.

### 7.6 Observations

The inplane and out-of-plane components of unsteady induced inflow at points above, but still close to, the rotor plane are well predicted and are consistent with previous literature. In addition, though time averaged results are not available for the isolated rotor configuration, the time averaged predicted results are consistent with previously published time averaged results at similar
conditions for similar rotor systems.
Figure 7.1: IRTS in the NASA Langley Research Center 14- by 22-Foot Subsonic Tunnel.
Figure 7.2: Grid System Used in Computations.
Figure 7.3: Measured and Predicted Induced Inflow in Two Directions at Location J for the IRTS.
Figure 7.4: Measured and Predicted Time Averaged Induced Inflow Ratio for the IRTS (Measured Data Includes Fuselage).
Chapter 8

Results: Rotor/Fuselage

8.1 Introduction

Chapters 6 and 7 showed various predictions made with the current computational model on the two separate components of a full configuration: an isolated fuselage and an isolated rotor. In this chapter, these two components are combined into a single unit using the entire computational model. For comparisons between measured and predicted quantities, several experimental data sets are used since there are no available data sets that contain measurements of all quantities of interest.

8.2 Experimental Setup

The experimental setup used is a combination of the IRTS discussed in chapter 7 and the 2-meter ROBIN fuselage [33, 34, 35, 36]. This configuration is shown in figure 8.1. In this particular test, no LV data were taken as was done in the IRTS test discussed previously. Instead, unsteady fuselage pressures were measured at several locations on the fuselage. The locations included a row of pressure taps near the top centerline of the fuselage. This row of taps nominally followed the top centerline, but were offset 0.25 inches to the advancing side of the fuselage. This offset was needed since the construction of the fuselage shell consisted of two halves which overlapped on the centerline of the fuselage. In addition, there were six unsteady pressure taps at fuselage station “e” (see figure 6.5) at several vertical locations on the left and right sides of the fuselage. Three of
these taps were on the left side and three on the right side of the fuselage. The unsteady pressure data was processed at 256 points per rotor revolution.

8.3 Computational Grid System

The grid system for the combined rotor/fuselage configuration is similar to that used in chapter 6, figure 6.3. Since the rotor grid was already included in the isolated fuselage configuration, no grid changes are needed to proceed directly from the isolated fuselage steady state results in chapter 6 to the time averaged rotor/fuselage configuration. However, for the time accurate computations, slight modifications were made to the grid which do not affect the grid shape or distribution of points. These modifications were made to accommodate a solution scheme in OVERFLOW that is very efficient in the unsteady mode.

For the unsteady rotor/fuselage configuration computations in OVERFLOW, the LU-SGS scheme is used. However, current implementation of this scheme does not allow for the use of spatially periodic boundary conditions. In OVERFLOW, spatially periodic grids and spatially periodic boundary conditions, have identical first and last planes. Figure 8.2(a) represents a schematic of a spatially periodic grid with \( k \) being the index in the periodic direction. It can be seen that \( k = 1 \) and \( k = k_{\text{max}} \) are actually the same point; for a three dimensional grid, these would be planes. To account for the fact that the LU-SGS scheme cannot be used with spatially periodic boundary conditions, the periodic grids in the rotor/fuselage grid system (excluding the rotor grid), are converted to grids that can still pass information to themselves through an extended overlap region. These grids then use a new boundary condition combination, created by using several existing OVERFLOW boundary conditions in succession, to pass information spatially in a manner similar to the traditional spatially periodic boundary condition. In this new boundary condition combination, the grids are slightly modified so that an overlap, or one-to-one match, of four grid planes is made in each of the computational grids’ spatially periodic direction. This overlap of four grid planes is used to replace the single overlap plane used in the spatially periodic boundary condition. A schematic of this is shown in figure 8.2(b). With these new overlapped planes, the “copy-to” and “copy-from” boundary conditions in OVERFLOW are used to pass information from one end of the computational grid to the other to simulate a spatially periodic condition. In table 8.1, the “copy-to” and “copy-from” columns show which information is passed between computational grid planes in the computational \( k \)-direction (the spatially periodic direction used in these grids).
Table 8.1: LU-SGS “Periodic” Boundary Condition

<table>
<thead>
<tr>
<th>Copy-from $(k \text{ value})$</th>
<th>Copy-to $(k \text{ value})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{\max} - 3$</td>
<td>1</td>
</tr>
<tr>
<td>$k_{\max} - 2$</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>$k_{\max} - 1$</td>
</tr>
<tr>
<td>4</td>
<td>$k_{\max}$</td>
</tr>
</tbody>
</table>

There were some configuration anomalies in the experimental setup for the rotor/fuselage configuration that were also incorporated into the grid system. First, due to offsets in the wind tunnel floor and ceiling, the rotor center of rotation was offset approximately two inches to the advancing side of the centerline of the fuselage. This placed the rotor rotation center at approximately the advancing side edge of the pylon atop the fuselage. Second, the fuselage shell was yawed at approximately 0.76° nose-left. Both of these anomalies have been accounted for in the grid system.

8.4 Time Averaged Computation

The isolated fuselage configuration results shown in chapter 6 are used as a starting point for the time averaged rotor computations with the combined configuration. These calculations involve using the time averaged rotor boundary condition discussed in chapter 4. As before, this portion of the calculation is an intermediate step between the steady state isolated fuselage computation and the unsteady rotor/fuselage configuration. It uses the same grid system defined above for the steady state computation. Since this is an intermediate stage of the computation, only the force convergence history will be examined to show that this portion of the computation is well behaved. Figure 8.3 is a continuation of figure 6.4 with the new portion beginning at iteration number 601 and ending at iteration number 1100.
8.5 Time Accurate Computation

For the time accurate computations, OVERFLOW is executed in an unsteady mode using the LU-SGS scheme to invert the left hand side of the equations using the new overlapping grid scheme to eliminate the spatially periodic boundary conditions. The unsteady rotor model boundary condition is applied as discussed in chapters 4 and 7. These computations are executed until a periodic result is obtained in the unsteady pressures on the fuselage.

8.6 Coupled Model Predictions

In subsequent sections, the new, coupled, computational model predictions will be presented for several iterations. The first subsection will show the unsteady fuselage pressure predictions for the first iteration\(^1\) vs. the corresponding measured quantities from the current rotor/fuselage configuration experiment. The second subsection will compare measured and predicted, unsteady induced inflow ratios from the first iteration. The third subsection will compare predicted, time averaged induced inflow ratio contours from the first iteration, which have been derived from the unsteady model, to the corresponding measured induced inflow ratio contours, taken from the experimental data presented in chapter 3. The fourth subsection will compare measured and predicted lateral and longitudinal induced inflow ratios. Subsequent sections and subsections will show the comparisons for the second and third iterations, including the coupling. Finally, an examination of the pressure contours on the fuselage surface will be made, as well as an examination of the convergence of the method in terms of the trim pitch settings as a function of iteration.

8.6.1 Iteration 1

Unsteady Fuselage Pressures

Figure 8.4 shows a comparison between the measured and predicted unsteady modified pressure coefficient on the top centerline of the ROBIN fuselage for various downstream locations (x-direction). Plotted on the vertical axis is the negative of the modified pressure coefficient, \(C_p\).

\(^1\)Here, an iteration refers to one pass through the rotor loading model followed by one pass through the rotor/fuselage flowfield model. Typically, only two rotor revolutions are required in each rotor/fuselage flowfield model iteration.
Along the horizontal axis is the reference blade location in the rotor azimuthal coordinates. Since the measurement location is fixed in the non-rotating frame, this horizontal axis can also be viewed as a temporal axis spanning one rotor revolution. The modified pressure coefficient used in these comparisons is defined by the following:

\[
C'_p = \frac{100(P - P_\infty)}{\frac{1}{2} \rho (\Omega R)^2}
\]  

(8.1)

where the typical velocity in the denominator of the \(C_p\) definition had been changed to \(\Omega R\). This modified \(C_p\) is designated \(C'_p\). Using a superscript \(u\) simply denotes that this is the unsteady component. This change in definition is needed since, for a rotorcraft, a freestream velocity approaching zero is possible as the hover condition is approached. If the normal definition of \(C_p\) were used, the values of \(C_p\) would approach infinity as the hover condition is approached. The factor of 100 on the \(C'_p\) simply serves as a convenient factor to scale the modified pressure coefficient function. Comparing the standard \(C_p\) definition and the \(C'_p\) definition above, the following relation holds between \(C_p\) and \(C'_p\):

\[
C'_p = 100 \mu_\infty^2 C_p
\]  

(8.2)

where \(\mu_\infty\) is the freestream advance ratio. To show the relation between these quantities more clearly, consider a perfect fluid. For a perfect fluid, stagnation occurs at \(-C_p = -1.0\). Using the current flight condition for the rotor/fuselage configuration, and using equation (8.2), this equates to \(-C'_p = -5.29\). Referring this to figure 8.4, a value of \(-C'_p = -1.0\) corresponds to \(-C_p \approx -0.19\).

Examining features in figure 8.4, it can be seen in that there is a dominant blade passage event at a frequency of four pulses per rotor revolution in the measured pressure signature. Here, the phase of the pressure signal is well matched, and the magnitude is slightly over-predicted.

Figure 8.5 shows the measured and predicted \(-C'_p u\) as a function of the reference blade location. Whereas figure 8.4 showed the predictions on the top centerline of the fuselage, figure 8.5 shows the predictions on the retreating and advancing sides of the fuselage (i.e., the left and right sides) for various vertical locations at the same downstream location. It can be seen that the magnitude and phase are predicted well for the advancing side locations; however, the retreating side amplitudes are slightly over-predicted.
**Time Accurate Induced Inflow Ratios**

Figure 8.6 shows the in-plane and out-of-plane components of unsteady induced inflow for the same location as that in figure 7.3. Figure 8.6, however, is a comparison between the unsteady induced inflow velocities for an isolated rotor (obtained from figure 7.3) and that for the first iteration of the method, including the fuselage. It can be seen that the presence of the fuselage has little effect on the unsteady induced inflow components at these particular locations.

**Time Averaged Induced Inflow Ratios**

At this point, it is worthwhile to check the validity of the time averaged results, obtained by time averaging the time accurate calculations, by comparing them to measured data.

Figure 8.7 shows a comparison between the measured and predicted induced inflow ratio perpendicular to the rotor tip path plane. For each of the predictions, the same pressure distribution is used in the rotor boundary condition. Figures 8.7b and 8.7c show the predicted induced inflow ratio for the isolated rotor and the predicted induced inflow ratio for the rotor/fuselage combination, using the same unsteady rotor boundary condition and pressure information. Several improvements can be seen in the induced inflow ratio prediction between the isolated rotor prediction and the rotor/fuselage combination. First, the inflow anomaly, seen in the isolated rotor prediction in the fourth quadrant near the blade root, is not present in the rotor/fuselage combination prediction. The induced inflow in this region now matches the measured data in that region. Second, the induced inflow near the forward portion of the rotor disk matches the experimental data well. For example, examining the contour line of “zero” induced inflow in the measured data and in the rotor/fuselage combination prediction, it can be seen that the upwash on the forward section of the disk now extends toward the center of the rotor. This upwash region is generated by the presence of the fuselage and is therefore not predicted in the isolated rotor prediction.

Figure 8.7d shows the difference between the induced inflow ratio for the rotor/fuselage combination and that of the isolated rotor. This difference shows the effects of the fuselage on the time averaged flowfield. As expected, there is an increased upwash near the nose of the fuselage and forward section of the pylon and an increased downwash behind the pylon. The regions of additional induced inflow in this figure show the source of the improved induced inflow predictions shown in figure 8.7c.

Figure 8.8 shows a comparison between the measured and predicted induced inflow ratio parallel
to the rotor tip path plane for the same conditions shown above. Figures 8.8b and 8.8c show the predicted induced inflow ratio parallel to the tip path plane for the isolated rotor and the predicted induced inflow ratio parallel to the tip path plane for the rotor/fuselage combination. Comparing the figures, it can be seen that the rotor/fuselage combination better predicts the induced inflow ratio. For example, examining the induced inflow contour line with a value of 0.015, it can be seen that the isolated rotor prediction is almost symmetric left-to-right, whereas the measured and predicted induced inflow ratios are both skewed at about 45° counterclockwise to the oncoming flow. Here, again, it can be seen that inclusion of the fuselage is necessary to correctly predict the time averaged induced inflow ratio.

Figure 8.8d shows the difference between the induced inflow ratio parallel to the tip path plane for the rotor/fuselage combination and that of the isolated rotor. This difference, again, shows the effect of the fuselage on the time averaged flowfield and the sources of the improved predictions with the fuselage included. As expected, there is a decrease in u-velocity near the nose of the fuselage and an acceleration near the rear of the fuselage pylon. It can be seen that, to capture the u-velocity distribution accurately, the fuselage must be included in the computation.

Lateral/Longitudinal Induced Inflow Ratios

Figure 8.9 compares the lateral and longitudinal subsets of the measured and predicted induced inflow ratio. The measured data comes from figure 8.7a, while the predicted data is taken from the combined rotor/fuselage case in 8.7c. Comparing the prediction here to the prediction using 8 harmonics in figure 3.9, it can be seen that the longitudinal induced inflow ratio is well predicted using the combined rotor/fuselage model. Whereas the peak-to-peak amplitude of the lateral induced inflow ratio was over-predicted by the GDWT isolated rotor model, it is well predicted for the combined rotor/fuselage model. However, there is a slight over-prediction of the inflow ratio near the advancing and retreating blade tip regions.

8.6.2 Iteration 2

Unsteady Fuselage Pressures

With the first iteration complete, induced inflow corrections are computed as described in chapter 5. With these induced inflow corrections, the GDWT is used to recompute the rotor loading.
With this new rotor loading, OVERFLOW is used to recompute the rotor/fuselage flowfield. This computation, or “iteration 2”, is presented in the current and subsequent sections.

Figure 8.10 shows the top centerline modified pressure coefficient in a manner similar to figure 8.4. Comparison of these two figures shows only very small differences.

Figure 8.11 shows the the modified pressure coefficient on the retreating and advancing sides of the fuselage in a manner similar to figure 8.5. Comparison of these two figures, again, shows only very small differences.

**Time Accurate Induced Inflow Ratios**

Figure 8.12 shows the in-plane and out-of-plane components of unsteady induced inflow for the same location as that in figure 7.3. As was seen in the first iteration, the fuselage has little influence on the unsteady components of induced inflow at this point in the flowfield.

**Time Averaged Induced Inflow Ratios**

Figure 8.13 shows the same information as figure 8.7, but this is the second iteration. Figures 8.13a and 8.13b show the identical information as that in figures 8.7a and 8.7b. Plots 8.13c and 8.13d are the new parts of this figure. Comparing figures 8.13c and 8.7c, it can be observed that there are only small changes between these figures except on the advancing side of the rotor disk in the first quadrant. In the first iteration, the time averaged induced inflow in that region was over-predicted. In the second iteration, it is seen that the time averaged induced inflow quantities are now closer to the measured values. However, overall, the shape of the contours did not change significantly. Comparing figures 8.13d and 8.7d, it can be observed that there are only small changes between these difference plots. Thus the effect of the fuselage on the time averaged induced inflow is quite similar between the first and second iterations.

Figure 8.14 shows the same information as figure 8.8, except it is for the second iteration. As before, figures 8.14a and 8.14b show the same information as figures 8.8a and 8.8b. Comparing figures 8.14c and 8.8c, it can be seen, again that there are only small differences between the first and second iterations. This time, however, the small differences that do occur are on the advancing side in the first part of the second quadrant of the rotor disk. A similar conclusion holds for figures 8.14d and 8.8d.
Lateral/Longitudinal Induced Inflow Ratios

Figure 8.15 compares the lateral and longitudinal subsets of the measured and predicted induced inflow ratio in a manner similar to figure 8.9. The measured data comes from figure 8.13a (and is the same as in figure 8.7a), while the predicted data is taken from the combined rotor/fuselage case in 8.13c. The predictions shown in this figure are similar to those shown in figure 8.9, and similar trends are present.

Even though differences are small between the first and second iterations, a third iteration is also presented to show that the iteration process has converged.

8.6.3 Iteration 3

Unsteady Fuselage Pressures

After iterating once again through the coupling technique, the rotor loading model, and the rotor/fuselage flowfield model, the third iteration is complete. Figures 8.16 and 8.17 once again show the unsteady modified pressure coefficient on both the top centerline and the sides of the fuselage. As with the comparison between the first and second iterations, there is no significant difference between this iteration and the previous iteration.

Time Accurate Induced Inflow Ratios

Again comparing two components of unsteady induced inflow ratio for the current iteration to the isolated rotor results in figure 8.18 shows no significant difference between the two results. In addition, there are no significant differences between this iteration and the previous iteration.

Time Averaged Induced Inflow Ratios

Examining the in-plane and out-of-plane time averaged induced inflow ratios in figures 8.19 and 8.20, as was done for the second iteration, it can be seen that there are no significant differences between the current iteration and the previous iteration.

Since there are no significant differences between the second and third iteration in the unsteady
modified pressure coefficient, in the time accurate inflow ratio, or in time averaged induced inflow ratio, it can be concluded that the iteration process is complete.

**Lateral/Longitudinal Induced Inflow Ratios**

Figure 8.21 compares the lateral and longitudinal subsets of the measured and predicted induced inflow ratio in a manner similar to figures 8.9 and 8.15. The measured data comes from figure 8.19a (and is the same as in figures 8.7a and 8.13a), while the predicted data is taken from the combined rotor/fuselage case in 8.19c. Again, the predictions shown in this figure are similar to those shown in figures 8.9 and 8.15, and similar trends are present.

### 8.7 Examination of Pressure Contours

In the preceding sections, a detailed examination of the unsteady component of the modified pressure coefficient, of the time averaged inflow ratio, and of the unsteady inflow ratio was presented for several iterations of the current method. For the final result (at the end of the third iteration), it is constructive to examine pressure distribution on the fuselage with and without the effects of the rotor. Figure 8.22(a) is a contour plot of surface pressure coefficient on the isolated fuselage configuration presented in chapter 6. It can be seen there is a typical stagnation region on the nose of the fuselage.

Figure 8.22(b) is a contour plot of the time averaged surface pressure coefficient for the rotor/fuselage configuration. This plots contains data from the “iteration 3” above. As with figure 8.22(a), there is a stagnation region on the fuselage nose region and behind the pylon. However, comparing figures (a) and (b), it can be seen that the gross time averaged effect of the rotor is to increase the pressure coefficient on the sides of the fuselage and downstream of the pylon. Figures 8.23 and 8.24 are top views of figures 8.22(a) and (b), respectively.

Figure 8.25 shows a set of surface pressure coefficient contour plots similar to those presented in figure 8.22. For comparison purposes, figure 8.25(a) shows the same data as presented in figure 8.22(b). Figure 8.25(b) shows the surface pressure coefficient for the instant at which the reference blade is at the $\psi = 0^\circ$ location. All four blades are plotted as rectangles extending from the blade root at $r/R = 0.24$ to the blade tip at $r/R = 1.0$ and are drawn “to-scale” in their actual position and orientation with respect to the fuselage; The blade chord is drawn “to-scale” as well. In this contour
plot from the unsteady computation, a large pressure pulse can be seen on the upper surface of the tail boom. This pressure pulse is a direct result of the rotor blade at the $\psi = 0^\circ$ location passing over the tail boom. Figure 8.26 shows this pressure pulse in a top view of figure 8.25(b).

Figures 8.27 to 8.35 present surface pressure coefficient contour plots at azimuth locations of $\psi = 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ,$ and $90^\circ$. These plots again show a pressure pulse traveling on and around the fuselage. It can be seen that this pulse motion is correlated with the motion of the blade. This point is made especially clear in the “Top View” plots presented here. Examination of this set of plots reveals that, locally, the unsteady surface pressure coefficient can be substantially higher than the time averaged surface pressure coefficient and that the pulses are typically of short temporal duration. In addition, the asymmetry of the unsteady loading can be seen clearly in the “Top View” figures.

### 8.8 Iteration Effects on Rotor Trim

The effect of the coupled computation on the rotor trim can be assessed by examining the pitch control settings required at each stage of the iteration at the end of the trim procedure in the rotor loading computation. These blade pitch settings are a function of rotor azimuth, are referenced to the 0.75R radial location on the blade, and are defined as follows:

\[
\theta(\psi) = \theta_0 + \theta_c \cos \psi + \theta_s \sin \psi
\]  
where $\theta_0$ is the collective pitch, $\theta_c$ is the longitudinal pitch, and $\theta_s$ is the lateral pitch. The first row in table 8.2 shows the measured collective, longitudinal, and lateral pitch settings, along with the magnitude and phase of the longitudinal/lateral combination. All quantities in table 8.2 are in degrees. The magnitude of the first harmonic of pitch, $\theta_1$, and phase of the first harmonic of pitch, $\psi_1$ can be defined as follows:

\[
\begin{align*}
\theta(\psi) &= \theta_0 + \theta_1 \cos(\psi - \psi_1) \\
\theta_1 &= \sqrt{\theta_c^2 + \theta_s^2} \\
\psi_1 &= \arctan \left( \frac{\theta_s}{\theta_c} \right)
\end{align*}
\]
where $\psi_1$ is placed in the range $0^\circ < \psi_1 \leq 360^\circ$.

Table 8.2: Pitch Control Settings [deg] as a Function of Iteration Number

<table>
<thead>
<tr>
<th>Type</th>
<th>Iteration</th>
<th>$\theta_0$</th>
<th>$\theta_c$</th>
<th>$\theta_s$</th>
<th>$\theta_1$</th>
<th>$\psi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>N/A</td>
<td>8.16</td>
<td>1.52</td>
<td>-4.13</td>
<td>4.40</td>
<td>290.2</td>
</tr>
<tr>
<td>Isolated Rotor</td>
<td>1</td>
<td>6.84</td>
<td>1.45</td>
<td>-3.42</td>
<td>3.71</td>
<td>293.0</td>
</tr>
<tr>
<td>Rotor+Fuselage</td>
<td>2</td>
<td>8.06</td>
<td>2.73</td>
<td>-3.69</td>
<td>4.59</td>
<td>306.5</td>
</tr>
<tr>
<td>Rotor+Fuselage</td>
<td>3</td>
<td>8.02</td>
<td>2.67</td>
<td>-3.71</td>
<td>4.57</td>
<td>305.7</td>
</tr>
</tbody>
</table>

From this table, it can be seen that the collective pitch setting matches the measured collective pitch for the rotor/fuselage combination cases, but is under-predicted for the isolated rotor case as expected. It can also be seen that the magnitude of the first harmonic of pitch is well matched for all of the iterations, especially for the iterations that include the fuselage in the computation. A plot of these quantities vs iteration number are shown in figure 8.36. From this figure, it can be seen that these settings have converged in just two iterations. For the computations that include the complete configuration, a phase difference of approximately $16^\circ$ can be seen. The explanation of this phase difference can be attributed to the assumption that the blade flap hinge offset is at the center of rotation. To show this, it is only necessary to examine the effect of the blade hinge offset on the rotor flap response. Gessow and Myers [54] showed that the rigid flapping natural frequency of a rotor is a function of blade hinge offset and is given by the following formula:

$$\omega_n = \sqrt{1 + \left(\frac{3}{2}\right) \bar{h}}$$

(8.7)

where $\omega_n$ is the flapping natural frequency in cycles per revolution and $\bar{h}$ is the hinge offset location as a fraction of rotor radius. Examination of equation (8.7) for $\bar{h} = 0$ shows that the flap natural frequency $\omega_n = 1.0$ per revolution and for $\bar{h} = 0.06$, the flap natural frequency is $\omega_n = 1.044$ per revolution. The difference in these two natural frequencies is $\Delta \omega_n = 0.044$ per revolution. Since there are $360^\circ$ in one revolution, the natural frequency difference equates to approximately $16^\circ$ of rotor azimuth. This means that in the experiment, which included the blade hinge offset, the phasing of the first harmonic of pitch should precede the predicted phasing by approximately $16^\circ$ of rotor azimuth. This is precisely the amount seen in table 8.2.
8.9 Resource Usage Summary

Table 8.3 lists the resource usage for each component of the current rotor/fuselage combination computations using the GDWT with 8 harmonics and 128 azimuth steps per revolution. The items listed are the CPU times in hours and minutes [hr:min] required for each iteration stage, main memory required in mega-words [mw], and which machine was used for each phase.

Table 8.3: Resource Usage as a Function of Iteration Number

<table>
<thead>
<tr>
<th>Type</th>
<th>CPU [hr:min]</th>
<th>Memory [mw]</th>
<th>Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDWT (with 8 harmonics, 128 azimuths)</td>
<td>&lt;0:07</td>
<td>5</td>
<td>Cray C-90</td>
</tr>
<tr>
<td>Isolated Fuselage</td>
<td>3:43</td>
<td>18</td>
<td>Cray C-90</td>
</tr>
<tr>
<td>Time Averaged, Isolated Rotor</td>
<td>2:16</td>
<td>15</td>
<td>Cray C-90</td>
</tr>
<tr>
<td>Time Accurate, Isolated Rotor (per rev.)</td>
<td>1:52</td>
<td>17</td>
<td>Cray C-90</td>
</tr>
<tr>
<td>Time Averaged, Rotor/Fuselage</td>
<td>3:38</td>
<td>18</td>
<td>Cray C-90</td>
</tr>
<tr>
<td>Time Accurate, Rotor/Fuselage (per rev.)</td>
<td>2:42</td>
<td>22</td>
<td>Cray C-90</td>
</tr>
</tbody>
</table>

Each of the rows in table 8.3 is for a particular component of the method for this particular case. Since execution times will differ for each particular case, these should only be used as reference quantities. It should be noted that OVERFLOW does not have a convergence criterion that halts execution at a particular convergence level. It is executed for a specified number of iterations or time steps. Each of the components involving time accurate computations are given as the CPU time required to execute each rotor revolution. For the case presented here, each time accurate stage of the computation was executed for two complete rotor revolutions to assure periodicity, even though periodicity may be achieved in fewer actual blade passage events. Thus, the CPU times above are conservative values for this case. Also, note that the GDWT is executed in a conservative manner at the same resolution required by OVERFLOW. Thus, CPU time for the GDWT computations is conservative as well. To compute the total execution time for the isolated rotor results and the first iteration of the rotor/fuselage configuration, the following formulae can be used:
Isolated Rotor:
CPU Time = GDWT
+ (Time Averaged, Isolated Rotor)
+ 2(Time Accurate, Isolated Rotor)
= 6:07 (Cray C-90)

Rotor/Fuselage, Iteration 1:
CPU Time = (Isolated Fuselage)
+ GDWT
+ (Time Averaged, Rotor/Fuselage)
+ 2(Time Accurate, Rotor/Fuselage)
= 12:52 (Cray C-90)

where the factor of two on the time accurate components indicates that two complete revolutions were executed for this particular case.

8.10 Observations

This chapter has presented results from the full hybrid method for a rotor/fuselage configuration. From the evidence presented above, several conclusions can be drawn as follow:

- The unsteady modified pressure coefficient on the top centerline of the fuselage and on the sides of the fuselage shown above, are insensitive to the small trim changes made by the time averaged induced inflow corrections which account for the presence of the fuselage.

- The in-plane and out-of-plane unsteady induced inflow velocity components are also insensitive to the small trim changes made by the time averaged induced inflow corrections which account for the presence of the fuselage.

- The time averaged induced inflow velocities are improved through the iteration/coupling process presented above. For the case presented here, it appears that one iteration is a good approximation to the correct solution and that two iterations is sufficient to capture the time averaged and time accurate induced inflow effects.

- The primary effect of the fuselage on the trimmed blade pitch settings is on the collective
pitch (for this particular case). The presence of the fuselage improves the pitch setting predictions over those from the isolated rotor case.

- For this flight condition, the primary effects of the rotor on the fuselage are a higher time averaged surface pressure coefficient below and downstream of the rotor and short duration surface pressure pulses imposed by the individual blade passages over the fuselage surfaces.
Figure 8.1: IRTS/Fuselage Configuration in the NASA Langley Research Center 14- by 22-Foot Subsonic Tunnel.
Figure 8.2: Schematic of a Periodic Grid and Replacement for Periodic Grid in LU-SGS Scheme.
Figure 8.3: Lift, Drag, and Sideward Direction Force Coefficients, Time Averaged Computation.
Figure 8.4: Measured and Predicted Unsteady Modified Pressure Coefficient on the Top Centerline of the ROBIN Fuselage, Iteration 1.
Figure 8.5: Measured and Predicted Unsteady Modified Pressure Coefficient on the Retreating and Advancing Sides of the ROBIN Fuselage, Iteration 1.
Figure 8.6: Measured and Predicted Induced Inflow in Two Directions for an Isolate Rotor and a Rotor/Fuselage Combination, Iteration 1.
Figure 8.7: Measured and Predicted Time Averaged Induced Inflow Ratio from Time Accurate Computations, Iteration 1.
Figure 8.8: Measured and Predicted Time Averaged Parallel Induced Inflow Ratio from Time Accurate Computations, Iteration 1.
Figure 8.9: Measured and Predicted Lateral and Longitudinal Time Averaged Induced Inflow Ratio from Time Accurate Computations, Iteration 1
Figure 8.10: Measured and Predicted Unsteady Modified Pressure Coefficient on the Top Center-line of the ROBIN Fuselage, Iteration 2.
Figure 8.11: Measured and Predicted Unsteady Modified Pressure Coefficient on the Retreating and Advancing Sides of the ROBIN Fuselage, Iteration 2.
Figure 8.12: Measured and Predicted Induced Inflow in Two Directions for an Isolate Rotor and a Rotor/Fuselage Combination, Iteration 2.
Figure 8.13: Measured and Predicted Time Averaged Induced Inflow Ratio from Time Accurate Computations, Iteration 2.
Figure 8.14: Measured and Predicted Time Averaged Parallel Induced Inflow Ratio from Time Accurate Computations, Iteration 2.
Figure 8.15: Measured and Predicted Lateral and Longitudinal Time Averaged Induced Inflow Ratio from Time Accurate Computations, Iteration 2
Figure 8.16: Measured and Predicted Unsteady Modified Pressure Coefficient on the Top Center-line of the ROBIN Fuselage, Iteration 3.
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Chapter 9

Summary

In this research, an efficient and accurate, hybrid method, combining a rotor loading model, a rotor/fuselage flowfield model, and a coupling technique, has been developed. This method uses the GDWT for the rotor loading model, OVERFLOW for the rotor/fuselage flowfield model, and a new coupling technique developed in this research. The motivations for development of such a model have been discussed and the relationships between the current model and other models in use today have been discussed.

A description of the Rotor Loading Model used for the first component of the model has been presented. In addition, the solution procedures employed in this model have been discussed, along with a validation study of the Rotor Loading Model. In this study, it was shown that predicted time averaged inflow distributions over a rotor disk matched measured time averaged inflow distributions well for a several flight conditions and for several rotor configurations. The sensitivity of the results to parameters (such as the number of azimuth steps and the number of harmonics used) was shown. In addition, predicted time accurate inflow quantities were compared to measured quantities. These comparisons showed that the predictions of the unsteady quantities do not match the experimental values well in waveform, but are comparable in magnitude and phase. It is also noted here that these predictions match well with previously published predictions in the literature for these rotor configurations.

A discussion of the method used for the second component of the current model, the Rotor/Fuselage Flowfield Model was also provided. The method used to solve the Navier-Stokes equations was discussed; theory and implementation of the new time averaged and time accurate boundary conditions is discussed as well. These new rotor boundary conditions are a unique feature of the
current model, and this is the first time this type of condition has been used for such a computation.

A coupling model was developed, which links the Rotor Loading Model and the Rotor/Fuselage Flowfield Models through the time averaged induced inflow of the rotor. This coupling model is a unique feature of the current model. The derivation of a novel inflow filtering technique has been presented also.

Computations which use the model presented in the first several chapters are compared with experimental data. These comparisons include results for an isolated fuselage, an isolated rotor, and a rotor/fuselage configuration. Predictions of the pressure coefficient on the surface of the isolated fuselage were shown to match experimental data well. Predictions of the time average and time accurate inflow above the rotor tip path plane for the isolated rotor configuration were shown also to match experimental data well.

Predictions of time averaged and time accurate inflow velocities above the rotor disk and predictions of unsteady pressure coefficients on the top centerline of the fuselage were shown to match experimental data well. A unique feature of this model is that predictions of the pressure coefficient on the sides of the fuselage match well with measured data. This is a significant accomplishment; previous methods have been unable to match unsteady pressures on the sides of the fuselage.
Bibliography


Appendix A

Filtering Operation

A consistent filtering operation can be derived from the GDWT. This appendix gives a derivation of the filtering operation, derived using various functions introduced in the GDWT development of reference [28]. This operation takes a quantity, $\Lambda(\bar{r}, \psi)$, which is a function of the radial and azimuthal coordinate, and derives the expressions for coefficients of an infinite series given below. The $\Lambda(\bar{r}, \psi)$ quantity is then filtered by truncating the infinite series to a finite number of harmonics and shape functions.

Starting with a given quantity that is a function of the radial and azimuthal coordinates, one can express that quantity as a double summation over the harmonics and shape functions in the GDWT as follows:

$$\Lambda(\bar{r}, \psi) = \sum_{m}^{\infty} \sum_{n}^{\infty} \bar{\phi}_n^{m}(\bar{r}) \left[ a_n^m \cos(m\psi) + b_n^m \sin(m\psi) \right]$$  \hspace{1cm} (A.1)

where $\Lambda$ is the given function, $m$ is the harmonic number ($m = 0, 1, 2, \ldots$), $n$ is the shape function number ($n = m + 1, m + 3, \ldots$), $\bar{r}$ and $\psi$ are the radial and azimuthal coordinates, $\bar{\phi}_n^{m}$ is a function of $\bar{r}$ from the GDWT, and $a_n^m$ and $b_n^m$ are the unknown coefficients. These coefficients could be, in general, a function of time. In that case, the process outlined below would be followed at each discrete time step of interest.

Multiply equation (A.1) by $\cos(p\psi)$ and integrate over $\psi$ from 0 to $2\pi$ to get the following:
\[
\int_0^{2\pi} \Lambda(\bar{r}, \psi) \cos(p\psi) \, d\psi \\
= \sum_m \sum_n \hat{a}_m^p \int_0^{2\pi} \cos(m\psi) \cos(p\psi) \, d\psi \\
+ b_m^p \int_0^{2\pi} \sin(m\psi) \cos(p\psi) \, d\psi
\]

(A.2)

Using the orthogonality relations for sine and cosine functions, the right hand side of equation (A.2) can be rewritten. The resulting form of equation (A.2) can be written as follows:

\[
\int_0^{2\pi} \Lambda(\bar{r}, \psi) \cos(p\psi) \, d\psi = \sum_m \sum_n \hat{a}_m^p \hat{c}(m) \delta_{mp}
\]

(A.3)

where \( m = 0, 1, 2, \ldots, \infty \), for both equations above, and \( n = m + 1, m + 3, \ldots, \infty \), for the first equation of (A.3) and \( n = p + 1, p + 3, \ldots, \infty \), for the second equation of (A.3). Also, \( \delta_{mp} \) is the Dirac delta function. The function \( \hat{c}(p) \) is given as follows:

\[
\hat{c}(p) = \begin{cases} 
\pi & \text{for } p \neq 0 \\
2\pi & \text{for } p = 0 
\end{cases}
\]

(A.4)

Multiplying equation (A.3) by \( [\hat{\phi}_q^p(\bar{r}) \cdot \bar{r} \cdot \sqrt{1 - \bar{r}^2}] \) and integrating from \( \bar{r} = 0 \) to \( \bar{r} = 1 \) gives the following:

\[
\int_0^1 \hat{\phi}_q^p(\bar{r}) \sqrt{1 - \bar{r}^2} \left[ \int_0^{2\pi} \Lambda(\bar{r}, \psi) \cos(p\psi) \, d\psi \right] d\bar{r} = \sum_n \left[ \int_0^1 \hat{\phi}_n^p(\bar{r}) \hat{\phi}_q^p(\bar{r}) \sqrt{1 - \bar{r}^2} d\bar{r} \right] a_n^p \hat{c}(p)
\]

(A.5)

Into “Integral A”, substitute the definition of \( \hat{\phi} \) in terms of Legendre functions, and perform a change of variables from \( \bar{r} \) to \( \nu \) using the fact that \( \nu = \sqrt{1 - \bar{r}^2} \). The resulting “Integral A” component of the above equations becomes:
\[
\int_0^1 \bar{\Phi}_n^p(\bar{r}) \bar{\Phi}_q^p(\bar{r}) \sqrt{1 - \bar{r}^2} \, d\bar{r} = \int_0^1 \frac{P^p_n(v)}{\pi} \frac{P^p_q(v)}{\pi} \frac{v^2}{4} \sqrt{\frac{\pi}{H_n^p}} \sqrt{\frac{\pi}{H_q^p}} \, dv \\
= \frac{\pi}{4\sqrt{H_n^p H_q^p}} \int_0^1 \bar{P}_n^p(v) \bar{P}_q^p(v) \, dv \\
= \frac{\pi}{4\sqrt{H_n^p H_q^p}} \delta_{mp} \tag{A.6}
\]

where \(H_n^m\) is defined in the GDWT as follows:

\[
H_n^m = \frac{(n + m - 1)!!(n - m - 1)!!}{(n + m)!!(n - m)!!} \tag{A.7}
\]

where the double factorial is defined in Peters, et al. [26], as follows:

\[
(n)!! = \begin{cases} 
(n)(n-2)(n-4)\ldots(2) & \text{for } n = \text{even} \\
(n)(n-2)(n-4)\ldots(1) & \text{for } n = \text{odd} \\
1 & \text{for } n = 0 \\
1 & \text{for } n = -1 \\
-1 & \text{for } n = -3
\end{cases}
\]

Substituting equation (A.6) back into equation (A.5) gives the following:

\[
\int_0^1 \bar{\Phi}_n^p(\bar{r}) r \sqrt{1 - \bar{r}^2} \left[ \int_0^{2\pi} \Lambda(\bar{r}, \psi) \cos(p\psi) \, d\psi \right] \, d\bar{r} \\
= \sum_n \left[ \frac{\pi}{4\sqrt{H_n^p H_q^p}} \delta_{mp} \right] a_n^p \bar{c}(p) = \sum_n \left[ \frac{\pi}{4\sqrt{H_n^p H_q^p}} \delta_{mp} \right] a_n^p \bar{c}(p) \tag{A.8}
\]

Solving equation (A.8) for the unknown coefficient \(a_n^p\) one gets:

\[
a_n^p = \frac{4H_q^p}{\pi \bar{c}(p)} \int_0^1 \bar{\Phi}_n^p(\bar{r}) \sqrt{1 - \bar{r}^2} \left[ \int_0^{2\pi} \Lambda(\bar{r}, \psi) \cos(p\psi) \, d\psi \right] \, d\bar{r} \tag{A.9}
\]
noting that $p = 0, 1, 2, \ldots, \infty$ and $q = p + 1, p + 3, \ldots, \infty$. Equation (A.9) gives the unknown $a^p_q$ coefficients of equation (A.1) given a function $\Lambda(\bar{r}, \psi)$. A similar derivation can be performed for the $b^p_q$ coefficients, which results in the following:

$$b^p_q = \frac{4H^p_q}{\pi \bar{c}(p)} \int_0^1 \hat{\phi}^p_q(\bar{r}) \bar{r} \sqrt{1 - \bar{r}^2} \left[ \int_0^{2\pi} \Lambda(\bar{r}, \psi) \sin(p \psi) \, d\psi \right] d\bar{r} \quad (A.10)$$

noting that $p = 1, 2, \ldots, \infty$ and $q = p + 1, p + 3, \ldots, \infty$. For this application, the integrals in equations (A.9) and (A.10) are computed using the trapezoidal rule.

In order to filter the function to a contain a particular frequency content, it is necessary to truncate the summation to limit the number of $a^p_q$ and $b^p_q$ coefficients to a specified number of harmonics such that $p = 0, 1, 2, \ldots, p_{\text{max}}$ for the $a^p_q$ coefficient and $p = 1, 2, \ldots, p_{\text{max}}$ for the $b^p_q$ coefficient. In this implementation, the above limits set the number of shape functions to $q = p + 1, p + 3, \ldots, p_{\text{max}}$. Renaming the indices, and applying truncation to the summations, the filtered value of $\Lambda(\bar{r}, \psi)$, shown below as $\tilde{\Lambda}(\bar{r}, \psi)$, is as follows:

$$\tilde{\Lambda}(\bar{r}, \psi) = \sum_m m_{\text{max}} \sum_{n=0}^{m_{\text{max}}+1} \hat{\phi}^m_n(\bar{r}) \left[ a^m_n \cos(m\psi) + b^m_n \sin(m\psi) \right] \quad (A.11)$$

where, $m$ is the harmonic number index, $n$ is the shape function index, and $m_{\text{max}}$ is the number of harmonics used. It should be noted that the number of shape functions used is equal the number of harmonics used, plus one ($n_{\text{max}} = m_{\text{max}} + 1$). Thus, compacting the above notation and referring to the terminology of chapter 5, the following filtering operation is defined:

$$\mathcal{F}(\Lambda) = \tilde{\Lambda} \quad (A.12)$$
Vita

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