3. EXAMPLE OF A ROAD VEHICLE

A major factor in the design of a road vehicle is the anticipated severity of the service usage. One source of severity in the usage is rough road. Vehicles moving on such roads are subjected to random loads. Under certain conditions, the road excitation on a vehicle is CS. Here are two examples:

(a) Figure 3.1 shows the vertical front suspension load for ten laps of an all terrain vehicle over a test track (Socie, 1999). The figure illustrates that the statistics of the load will not be a constant, but periodic in time. As explained earlier, this type of excitation is CS.

(b) The following section will show that the road excitation on a vehicle traveling on a road made of concrete slabs is also CS. This model of the road excitation is valid for the road made of slabs encountered in strip warning before a sharp turn.

3.1 Model of the Road Excitation

First, we make a few assumptions in order to develop a mathematical model of the road roughness. Then, we apply the analytical method, presented in chapter 2, to a vehicle moving on such road. We assume that the road is made of concrete slabs of constant length, \( d \), but different heights, \( H_i \). The vehicle is modeled by a single-degree-of-freedom spring-mass-damper system. The random excitation is due to the uneven surface of a road. The heights of slabs are assumed normally distributed, statistically independent random variables. The mean value of \( H_i \) is zero.
and its standard deviation is $\sigma_H$. We also assume that the surface of each slab is horizontal. Figure 3.2 shows a sample path of the road with the vehicle model. In practice, the slab length may have also randomness. This will introduce randomness in the time period of the vehicle excitation and may influence the structure’s stability and dynamics (Dimentberg, 1992). This can be particularly important if the frequency of excitation corresponding to the slab length is close to the system natural frequency. We are not taking into account the randomness in the slab length.

Let $Y(x)$ be the height of the road at a distance $x$ from origin. $Y(x)$ can be written as

$$Y(x) = \sum_{i=-\infty}^{\infty} H_i \text{pulse}(x - id, d)$$

(3.1)

where,

$$\text{pulse}(x, d) = \begin{cases} 1 & \text{if } 0 \leq x < d \\ 0 & \text{otherwise} \end{cases}$$

(3.2)

$x$ is the location where the pulse begins and $d$ is the duration of the pulse.

$Y(x)$ is a Gaussian process, so its mean and autocorrelation are sufficient to describe it. The mean of $Y(x)$ is zero. The autocorrelation of $Y(x)$ is

$$R_{yy}(x + \Delta, x) = E[Y(x + \Delta)Y(x)]$$

(3.3)

$$= E[\sum_{i=-\infty}^{\infty} H_i \text{pulse}(x - id + \Delta, d) \sum_{j=-\infty}^{\infty} H_j \text{pulse}(x - jd, d)]$$

where $\Delta$ is the distance between the two points for which autocorrelation is measured. The terms that correspond to $i \neq j$ in the expression for $R_{yy}(x + \Delta, x)$ are zero because the heights of the slabs are statistically independent with zero means.
Therefore,

\[ R_{Y,Y}(x + \Delta, x) = \sigma_{H}^{2} \sum_{i=-\infty}^{\infty} \text{pulse}(x + \Delta - id, d) \text{pulse}(x - id, d) \]  \hspace{1cm} (3.4)

Let the speed of vehicle be \( V \). Then \( x = Vt \). Hence the autocorrelation in time domain is,

\[ R_{Y,Y}(t + \tau, t) = \sigma_{H}^{2} \sum_{i=-\infty}^{\infty} \text{pulse}(t + \tau - iT, T) \text{pulse}(t - iT, T) \]  \hspace{1cm} (3.5)

where the time lag, \( \tau \), equals \( \Delta / V \) and the time period \( T \) equals \( d / V \).

The above-mentioned expression for the autocorrelation can be further simplified to

\[ R_{Y,Y}(t + \tau, t) = \begin{cases} \sigma_{H}^{2} \sum_{i=-\infty}^{\infty} \text{pulse}(t - iT + \tau, T + \tau) & \text{for } \tau < 0 \\ \sigma_{H}^{2} \sum_{i=-\infty}^{\infty} \text{pulse}(t - iT, T - \tau) & \text{for } \tau \geq 0 \end{cases} \]  \hspace{1cm} (3.6)

Figure 3.3 shows the variation of the autocorrelation of the road excitation with time, \( t \), where the first measurement is taken and the time difference, \( \tau \), between the first and second measurements. It can be seen that the autocorrelation is periodic in time. Therefore, the excitation due to the road roughness is a WSCS process.

3.2 Calculation of the Vehicle Response

From the single-degree-of-freedom model of the vehicle, we obtain the frequency response function, \( H(\omega) \). Using Eqs. (2.7) and (3.6), we obtain the cyclic autocorrelation, \( r_{Y,Y}(n, \tau) \), of the excitation, \( Y(t) \). From Eq. (2.8), we get the cyclic spectral density, \( S_{Y,Y}(n, \omega) \), of the excitation. Then, following the analytical method, Eqs. (2.19)-(2.21), we calculate the autocorrelation and spectral density function of the response. We took only eleven terms in the
Fourier series expansion of the autocorrelation functions of excitation and response. Table 1 shows the values for the parameters of the model shown in Fig. 3.2.

**Table 3.1: Values of the parameters of the road vehicle**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping coefficient $c$, Kg/s</td>
<td>6610.46</td>
</tr>
<tr>
<td>Slab length $d$, m</td>
<td>10</td>
</tr>
<tr>
<td>Spring coefficient $k$, N/m</td>
<td>11433.3</td>
</tr>
<tr>
<td>Mass $m$, Kg</td>
<td>1300</td>
</tr>
<tr>
<td>Velocity, $V$, Km/hr</td>
<td>100</td>
</tr>
<tr>
<td>Standard deviation $\sigma_H$, m</td>
<td>0.04</td>
</tr>
</tbody>
</table>

To validate the result obtained from the analytical method, we calculated the standard deviation (RMS) of the response by MC simulation. We generated 20 sample paths of the road. For each sample path of the road, we calculated the response of vehicle. Then, we calculated the RMS of the response as a function of time based upon the ensemble and compared it to its counterpart from the analytical method. Figure 3.4 shows the standard deviations of the response as a function of time calculated using the MC simulation and the analytical method. The difference in two results, which is negligible, is due to the following:

1) The error associated with numerical integration, Eq. (2.20),

2) The finite number of terms (= 11) used in the Fourier representation of the correlation function in Eq. (2.6), and

3) The finite number of samples (= 20) of the response used in the MC simulation.

It is important to assess the difference between the results of the CS model, Eqs. (2.19)-(2.21), and the approximate stationary model, Eqs. (2.20-a)-(2.21-a). Figure 3.4 compares the
standard deviation of the response calculated using the CS model and an approximate stationary model. It can be seen from this figure that the standard deviation of the response is not constant in time, but varies periodically. However, the approximate stationary model shows a constant standard deviation of the response. Figures 3.4 and 3.5 demonstrate that an approximate stationary model can underestimate the maximum value of the standard deviation of the vehicle response. At a velocity of 25 m/s, where the effect is most prominent, the maximum standard deviation, $\sigma_{Z\text{ Maximum}}$, is around 15% higher than the average standard deviation, $\sigma_{Z\text{ Average}}$. (Fig. 3.5). It demonstrates that an approximate stationary model underestimates the maximum value of the standard deviation of the vehicle response. As a result, an approximate stationary model could considerably underestimate the probability of failure because of first excursion or fatigue (Crandall and Mark, 1968). The reasons are that

(a) The first excursion probability is sensitive to the standard deviation of the stress, and

(b) Fatigue damage is proportional to a power ($\geq 5$) of the stress or strain at a point.

The vehicle traveling with velocity $V$, encounters each slab for time $d/V$. As a result, the statistical properties are periodic in time with period $d/V$. This can be also seen from Fig. 3.4, which shows that the period of the variation of the standard deviation is 0.36 sec. The inverse of this time period, scaled by $2\pi$, is equal to the fundamental frequency, $\omega_e$, of the autocorrelation of the excitation. Figure 3.6 shows the effect of $\omega_e$ on the standard deviation of the response. It is observed that when $\omega_e$ equals to the natural frequency, the standard deviation of response is maximum. It can be also seen that the standard deviation is sensitive to $\omega_e$ for values less than the natural frequency whereas it is insensitive to $\omega_e$ for values larger than the natural frequency. This information can help in vehicle design.

Dimentberg (1992) studied the effect of random variability in the period of a periodically nonstationary process on the stability of a dynamic system. He concluded that, if the period of the excitation is close to the natural period of the system, even a small variability in the period (e.g. 0.5%) can improve the stability of the system and can drastically reduce the intensity of vibration of the system. Consider the example of the coalmine cage traveling on a periodically
supported cable, mentioned in the first section. Even small fluctuations in the distance between the adjacent supports can considerably reduce the intensity of the response of the cage and improve its stability. Therefore, one may have to account for the variation in the period in these cases.

In this study, the natural frequency of the system modeling the road vehicle is 2.97 rad/sec while the frequency of excitation is 17.45 rad/sec in Fig. 3.4. Therefore, we do not expect that the amplitude of vibration of the system would be sensitive to variability in the frequency of excitation, which can arise from a small variability in the lengths of the slabs. The frequency of excitation varied from 6.28 rad/sec to 37.7 rad/sec in Fig. 3.5. Again variability in the lengths of the slabs is probably not significant. This variability can be significant however, for the results in Fig. 3.6 because the system natural frequency is close to the frequency of excitation in some case.