SCALE-UP OF COLUMN FLOTATION

by

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(ABSTRACT)

The parameters controlling column flotation have been investigated using laboratory and pilot-scale flotation cells. It was determined that column recovery is a function of flotation rate constant, axial dispersion, retention time and fractional air hold-up within the cell. Mathematical expressions have been developed which describe each of these parameters and the manner in which they respond to changing column geometry and flow conditions.

Based on the data and mathematical expressions developed in the laboratory and pilot-scale testing, a scale-up procedure has been developed for column flotation. Unlike other column scale-up approaches, this procedure incorporates the four primary parameters governing column recovery (i.e., axial dispersion, flotation rate constant, retention time and air fraction) along with carrying capacity limitations to provide a complete scale-up of the entire column. Based on test work conducted in a 2-inch laboratory column, scale-up predictions were made and validated for 30-inch and 8-foot diameter columns. The scale-up procedure has been incorporated into a computer package for making predictions from laboratory data.
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Chapter 1

INTRODUCTION

1.1 Preamble

Although the history of the world's mineral industry has been marked by many great achievements, indisputably one of the most overwhelming impacts was the development of the froth flotation process. Froth flotation provided a means for treating millions of tons of ore which were previously considered unrecoverable by conventional techniques of the time. It was, in fact, speculated that,

"Of all the early bonanzas mined, 
None yielded half the wealth derived, 
From waste which was left behind."

(Anon.)

According to Hoover, the first recorded use of froth flotation was for the upgrading of zinc ores in the years 1905-1906 at Broken Hill's Block 14 mine in Australia with a "measure of success". In 1911, froth flotation was introduced into the United States by the Butte and Superior Copper Co. for processing of a Montana zinc ore (Hines and Vincent, 1962).

Since that time, froth flotation has experienced an explosive growth in its application to the minerals industry. From the year 1913 until 1929, the total amount of copper ore treated by flotation grew from essentially zero to more than 50 million tons. This trend continued well into the latter half of the century, as illustrated by
Figure 1.1. It is also interesting to note that coal, nonmetallics and oxide minerals have followed similar trends. The increased use of froth flotation in the nonmetallic mineral market proved especially dramatic in that production rates reached more than a third of the total sulfide production over the course of twenty years.

With this increased usage came a realization that an overwhelming number of variables are associated with froth flotation. In 1938, L. W. Wark stated that so many variables influence flotation that it will be long before every one of them can be investigated and its influence on the process determined. Several of these parameters are illustrated in Figure 1.2, although the list is by no means comprehensive. The task of investigating each separately as well as the possible interactions among variables may never be entirely completed.

Flotation was originally developed as a bulk oil process for separating oil-wetted sulfide minerals from water-wetted gangue by agitating the dry ore with oil and then contacting the mixture with water. Researchers soon realized the importance of gas bubbles in enhancing the separation characteristics. This was initially accomplished by boiling the mixture to produce steam. Later, it was discovered that the reaction of acids on mineral carbonates would also produce a quantity of gas bubbles sufficient to improve separation. Further improvements were obtained in 1904 when Francis Elmore patented a flotation process which utilized a vacuum system for the removal of mineral-laden gas bubbles generated by the action of acids.
Figure 1.1 Total production rates by flotation for coal and minerals from the year 1960 through 1975 (Hines and Vincent, 1962).
Figure 1.2 Schematic diagram representing the independent and dependent variables associated with flotation.
in the pulp.

During this time frame, several similar attempts were made to separate sulfide minerals from gangue based on surface characteristics. One of these processes was known as skin flotation. This procedure consisted of running a thin layer of the mineral, often treated by oil, onto a water surface. The hydrophobic minerals would tend to remain at the surface while the wettable solids would sink to the bottom of the cell. Skin flotation was briefly used at the Morning Mine in Mullen, Idaho in 1911. Known as the Macquisten process, it experienced limited success treating feed rates as high as 15 tpd of sulfide minerals.

Today's froth flotation technology, characterized by the reduction of oil consumption to less than one percent and the use of mechanical agitation for gas dispersion, was a direct result of these earlier achievements. In the decades following the development of modern flotation practices, many substantial breakthroughs occurred which expanded the applicability of froth flotation. These breakthroughs included the use of fatty acids and esters as frothing agents, the use of depressants for certain mineral species, and eventually, differential flotation by successively depressing and activating one particular sulfide species as opposed to another.

The period from 1930 through 1960 saw the spread of froth flotation into the industrial minerals market. This trend, sparked by an improved understanding of the governing principles of oxide flotation, improved the economic viability of the non-sulfide minerals. This improved understanding included the mechanisms of
oxide collector adsorption, the concept of pH control in flotation circuits, and the proper use of modifiers, dispersants and conditioning techniques.

One of the most significant impacts on mineral flotation occurred in the 1960's with the introduction of the flotation column by Boutin and Wheeler (Wheeler, 1966; Boutin and Wheeler, 1967). The flotation column represented a radical break with traditional flotation machine design in terms of its physical dimensions, as well as basic operating principles. This is exemplified by the method in which a concentration gradient is achieved within the froth bed in a conventional flotation cell as opposed to a flotation column.

In conventional flotation product up-grading occurs by differential drainage of hydraulically entrained pulp away from the bubbles. The non-selective entrainment is a result of material being carried into the froth by the wake of a rising bubble and being trapped between the bubble lamella. This material mainly consists of fine gangue particles, since gravity generally prevents the coarser reject from being entrained. The only means of rejecting this material is by a loss of bubble surface area due to gross coalescence and pulp drainage as the fractional air content increases from the bottom to the top of the cell. Unfortunately, this also results in a loss of carrying capacity and hence recovery.

Column flotation differs from conventional flotation in terms of bubble generation and particle capture, as well as froth stability. Although the fundamental principles of collector adsorption and
mineral attachment remain the same, the physical aspects of the process are quite different. In column flotation, the feed slurry descends through a long, cylindrical tube against a countercurrent stream of pneumatically generated bubbles. The column geometry is such that length-to-diameter ratios usually vary from 5:1 to 30:1. As the bubbles rise into the base of the froth they are washed by a countercurrent stream of fresh water which is injected into the froth zone. This rinsing action prevents the physical entrapment of fine mineral matter, thus providing a higher degree of separation as opposed to conventional flotation. This phenomenon is shown in the froth profiles presented in Figure 1.3. Although conventional flotation indicates a sharper separation, the final product grade is significantly lower than that provided by the column. Additionally, the removal of an impeller system provides for more quiescent conditions in the column recovery zone. This is advantageous in terms of collection rates due to improved bubble/particle hydrodynamics as well as reduced axial dispersion.

The initial testwork involving column flotation was conducted by Boutin and Wheeler on Canadian iron and copper ores. Preliminary metallurgical results proved to be quite promising. However, plugging problems were encountered with the porous air diffusers used for bubble generation. This was due to the presence of solid particles in the slurry and formation of calcium carbonate on the surface of the bubbler. These initial results from the prototype column laid the foundation for a new generation of research and development work in froth flotation.
Figure 1.3 Typical profiles for column and conventional flotation froths (from Wheeler, 1966).
In the following decades column flotation experienced an explosive growth in research activities. Probably the most important result of this research effort was the increased understanding of the fundamental operating principles associated with column flotation. This included the effect of bubble/particle hydrodynamics, froth stability, mathematical modeling and process control.

Also, during this time many variations in the original flotation column design were introduced. Unfortunately, the advances in equipment design tended to exceed the understanding of the fundamental principles which govern column operation. This created a need for a scale-up procedure capable of meeting the necessary design and control criteria for industrial flotation columns.

At present, many of the available scale-up procedures address individual aspects of column design but often neglect many important parameters and their interactions. For instance, a typical scale-up procedure may be based on the concept of maintaining a constant residence time from the laboratory cell to a plant installation. However, this procedure neglects many of the fundamental parameters associated with column flotation, such as rate constant, axial dispersion and fractional air hold-up. The availability of comprehensive scale-up procedures which incorporate all of the significant variables associated with column flotation is limited.
1.2 Research Objectives

The objective of the work presented in this dissertation was to develop a scale-up procedure for column flotation. However, as discussed earlier, flotation is a complicated physico-chemical process which involves the interaction of numerous design and operating variables. Therefore, it is not possible to undertake the task of column scale-up without first investigating the basic principles of flotation, both column and conventional.

In order to gain a thorough understanding of the interplay among the variables associated with flotation a comprehensive pilot plant test program was developed. This program involved detailed testing of a 14-inch microbubble column, as well as comparison testing with a conventional rougher flotation bank. Column operation was characterized in terms of coarse coal flotation, high ash, refuse flotation and micronized coal flotation. As a result of the pilot studies, the parameters which most strongly influence column performance were identified. These include axial dispersion, fractional air hold-up, slurry retention time, flotation rate constant and froth carrying capacity. Chapters 3 through 6 quantitatively address each of these parameters and the effect which they have on column performance. The results of this investigation have been summarized in a computer-aided design (CAD) package for the scale-up of industrial flotation columns.
1.3 **Report Organization**

The results of this report have been organized in a series of relatively independent chapters. In Chapter 2, the development and pilot-scale testing of the microbubble column flotation process is presented. This includes the design of 6-, 8- and 14-inch columns for processing coal and industrial minerals. Details of the associated pilot-plant flowsheet are also discussed.

In chapters 3, 4 and 5, the effects of axial dispersion, fractional air hold-up and carrying capacity are analyzed and their role in column flotation scale-up is determined. Chapter 6 summarizes the above findings in a computerized column scale-up procedure. This routine takes into account the relationships determined in the preceding chapters in order to make predictions concerning recovery and throughput for larger scale columns. An experimental validation of the column scale-up procedure is also presented using data from 2-, 30- and 96-inch diameter columns.

The equations presented in Chapters 3 through 6 represent column performance under steady-state conditions. However, in order to analyze column response for the purpose of process control, a dynamic column flotation simulator was developed. The model and results of several different simulations are presented in Chapter 7.
1.4 References


Chapter 2
MICROBUBBLE COLUMN FLOTATION DEVELOPMENT

2.1 Introduction

Recently, there has been a renewed interest in the use of column flotation for concentrating all types of minerals, such as sulfide ores, industrial minerals and coal. In fact a great deal of interest has been generated in the area of fine coal recovery using column flotation. Several applications have been considered including processing of high ash reject material typically found in waste ponds, as well as thickener underflow and cyclone overflow streams. Specialty applications, such as the production of super- and ultraclean fuels (less than 2 percent ash and less than 1 percent ash, respectively) from micronized coal slurries, have also been investigated.

Of course the most significant impact from an economical standpoint would be recovery of the coal fines (typically less than 150 mesh) associated with the high ash reject material. These waste streams, which may carry as much as 40 tons or more of salable coal per hour, are currently being discarded since an adequate processing technique does not exist. This is of particular interest to coal companies since the mining costs have already been incurred for this material.

Several attempts have been made to implement conventional flotation for treatment of reject streams. Although reasonable recoveries are obtained, the final product ash is usually not
acceptable due to ash entrainment problems associated with conventional flotation froths. However, the countercurrent froth washing capabilities of flotation columns have made them a key candidate for treatment of this difficult material. Unfortunately, an adequate data base reflecting column performance with respect to coal processing does not currently exist. This work attempts to fill this void by exploring many of the fundamental parameters associated with column flotation of coal.
2.2 **Literature Review**

In order to conduct a competent evaluation of column performance, one must first investigate the current state of the art of column flotation machine design. As with any new technology, a multitude of innovative changes have been implemented with each inventor claiming to have "re-invented the wheel". Although several different commercial columns are available, the basic cell design does not significantly vary in terms of physical dimensions. The most controversial issue related to cell design appears to be the concept of bubble generation. Currently, a wide variation of column sparging systems are available from the different column manufacturers. These devices range from simple static porous bubblers to sophisticated mechanical bubble generator systems.

There are currently several different column designs which have been used for coal and mineral flotation. These include the Deister Flotaire column, the Leeds University column, the Column Flotation Company of Canada (CFCC) cell, the U.S. Bureau of Mines column, the Michigan Tech static tube flotation process and the Hydrochem cell. The design differences between each of the above cells are the subject of the following discussion. Since the basic operating principles of column flotation have already been discussed in Chapter 1, the following will serve only to discern mechanical differences in cell design.

The CFCC column is a direct offspring of the original column designed by Boutin and Wheeler (Boutin and Wheeler, 1967; Wheeler,
1967). In this design, air is injected into the flotation column through a porous diffuser. Wash water is added several inches below the froth by means of perforated piping. Feed enters the column just below the froth/pulp interface, which is maintained at a constant height by an automatic control valve. Several different types of material have been used for the porous medium with the best aeration provided by sintered ceramic. Unfortunately this material will plug in a matter of a few days in hard water. Best results have been obtained from cloth and rubber sheeting due to its ability to flex slightly and thus dislodge mineral deposits before plugging occurs. A significant drawback to this system is the need to shut down and drain the column in order to replace the spargers. This requires that a standby column be available for plant operations.

The Flotaire cell geometry is quite similar to that of the CFCC design, however, it differs on two main points. The first is the method of producing bubbles. The Deister column uses an external, water operated aspirator to produce fine bubbles in a frother solution solution. This solution is then injected into the base of the column. It is claimed that these generators are not subject to plugging as is the CFCC porous diffusers. The second major difference is the absence of a froth washing system in the first generation columns. However, it appears that the most recent cell designs have incorporated this capability (Zipperian and Svensson, 1988).

The U.S. Bureau of Mines flotation column is also quite similar to that of the CFCC cell except in its means of generating bubbles. The Bureau of Mines cell forces an air/water mixture through a packed
bed of fine beads at relatively high pressures. It is theorized that the bubbles are generated by the high shear imparted to the fluid as the emulsion passes through the packed bed. Unfortunately, this arrangement also appears to be subject to a loss in performance during long term testing due to plugging problems (Redfearn and Egan, 1989).

The Leeds cell appears to be a merger of both conventional and column flotation technologies. This design is essentially a tall conventional flotation bank which contains a series of barriers in the upper portion of each cell. The barriers consist of an arrangement of fixed and movable tubes which create a one-way valve action. The bubbles are temporarily trapped behind each barrier as they are washed countercurrently with a stream of clear water. The cleaning action which occurs as the bubbles pass from zone to zone essentially simulates that of the cell-to-cell arrangement of a conventional flotation bank (Dell, 1977). Recovery is achieved by passing the feed slurry from one partitioned cell to the next along the length of the machine. This allows the Leeds cell to be shorter than a conventional column since recovery is achieved through cell length, not height. A possible drawback, however, may be the need for excessive amounts of water since the flow rate of countercurrent wash water is directly proportional to cell cross-sectional area.

The Michigan Tech static tube flotation process is probably the most unique design in recent years. It consists of a standard column which is partially filled with packing. It is claimed that the packing provides a number of small, tortuous flow passages within the
column. Bubble generation is achieved within the cell itself by the action of the air flowing through these restricted passages. The packings produce fine bubbles of a relatively uniform size (Yang, 1984). Countercurrent wash water and level control are similar to the other cell designs. Although the plugging problems associated with other bubble generation devices do not exist for the static column, problems have been incurred with the column itself becoming plugged due to the small spacing required between the packing elements. This may prove to be a significant drawback to the static tube flotation machine.

The Hydrochem cell is essentially a mixed column. This design utilizes an agitator placed along the axis of the column up to the base of the pulp/froth interface. Air is injected at the base of the cell under the bottom of the mixer assembly. It is claimed that the additional turbulence enhances bubble/particle attachment for diverse particle sizes, as well as, providing different degrees of mixing and bubble size in different zones within the column (Suttil, 1988). Other cell designs have been tested, such as the Bahr and Jameson cells, but for the purpose of this investigation these are considered to be pneumatic versions of conventional flotation machines.
2.3 Research Objectives

The purpose of this work was to conduct a pilot-scale investigation of the parameters which most strongly influence column flotation performance. Column operation was to be characterized in terms of coarse coal flotation, high ash, refuse flotation and micronized coal flotation. Based on these results, preliminary relationships were developed in regards to column scale-up. Conventional flotation tests were conducted in order to make a comparison between the two technologies.

A 500 lb/hr pilot plant was constructed to conduct the flotation studies. A 14-inch microbubble column and two banks of conventional flotation cells were installed so as to be operated either in series or parallel arrangement. A crushing and grinding circuit was also developed which was capable of producing either coarse (28 mesh X 0) or micronized (-20 microns) feed slurries.

It should be noted that the intent of the pilot work was to explore the parameters associated with flotation. Therefore, the results presented in this chapter were not necessarily obtained using the optimum cell design and operating conditions.
2.4 Experimental

2.4.1 Samples

Coarse coal flotation tests were conducted on four eastern, run-of-mine coals. These were the Elkhorn III and Pittsburgh No. 8 seams, obtained from Consolidation Coal, the Upper Freeport seam, provided by EPRI, and the Lower Cedar Grove obtained from Island Creek Coal Company. The results from a proximate analysis of the run-of-mine coals are shown in Table 2.1. The weight percent sulfur is also given.

Two high ash reject samples were also tested. These were the Jellico seam, currently mined in eastern Tennessee by Consolidation Coal, and the Coalburg seam from Pittston's West Virginia operations. These samples were taken from the overflow stream of a water-only cyclone circuit which was being used for a size separation prior to heavy media separation. This is a typical arrangement for preparation facilities which are treating a relatively high ash, run-of-mine coal, especially if the ash is present as fine clays. These streams average 40 to 60% incombustibles.

Flotation studies were also conducted on two micronized coal slurries for the purpose of producing superclean fuels. The Elkhorn III and Lower Cedar Grove coals previously described were used in these tests. Size distributions of the micronized feed will be presented later in a detailed discussion of the comminution circuit and its performance.
Table 2.1

Sample Analysis for Pilot-Plant Coarse Coal Flotation Testing

<table>
<thead>
<tr>
<th>Sample</th>
<th>Volatile Matter (%)</th>
<th>Fixed Carbon (%)</th>
<th>Ash (%)</th>
<th>Sulfur (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elkhorn III</td>
<td>33.13</td>
<td>56.68</td>
<td>10.19</td>
<td>0.65</td>
</tr>
<tr>
<td>Pittsburgh No. 8</td>
<td>34.45</td>
<td>54.39</td>
<td>11.17</td>
<td>2.75</td>
</tr>
<tr>
<td>Upper Freeport</td>
<td>16.78</td>
<td>32.82</td>
<td>48.03</td>
<td>4.17</td>
</tr>
<tr>
<td>Cedar Grove</td>
<td>31.30</td>
<td>59.61</td>
<td>9.09</td>
<td>0.81</td>
</tr>
</tbody>
</table>
2.4.2 PDU Flowsheet Design

A complete piping and instrumentation flowsheet for the flotation pilot-plant is shown in Figure 2.1. Equipment was selected based on a maximum capacity of 500 lb/hr. As can be seen from the flowsheet, run-of-mine coal (-3"") is received in barrels and dumped via an overhead crane into the hammer mill feed bin. The hammer mill product (-1/8-inch) is transported into a vibrating bin feeder which feeds a ball mill by means of a weighbelt feeder. Since the capacity of the hammer mill far exceeds 500 lb/hr, the hammer mill is not matched to the rest of the circuit but is simply used intermittently to keep the vibrating bin filled. The ball mill is operated at approximately 30% solids in a closed circuit arrangement with a Sweco vibratory screen. A size separation is made at either 28 or 100 mesh, respectively, depending on whether coarse or micronized coal is required. For the case of micronization, the -100 mesh ball mill product is fed directly to the Drais stirred ball mill. The stirred ball mill also operates at 30% solids and produces a micronized coal product having a mean size between 5 and 10 microns. The product from the stirred ball mill discharges into the flotation feed sump. The slurry is then pumped to the flotation conditioning tanks where reagents are added and pulp percent solids is adjusted to the desired level. The slurry is then pumped to either the flotation columns or the conventional flotation banks. The clean coal and refuse streams from flotation are discharged into separate sumps for subsequent thickening and dewatering. The flotation pilot-plant has been designed in order to
Figure 2.1 Piping and instrumentation diagram for 500 lb/hr flotation pilot plant.
provide the maximum flexibility so that multiple circuit configurations can be evaluated.

A detailed flowsheet of the effluent handling and disposal plan is shown in Figure 2.2. In this figure it is assumed that both column flotation and conventional flotation are operating internally at 5% solids. Although the percent solids in the feed to the column may be higher than 5%, wash water is added which dilutes the feed. Refuse, discharged at approximately 1.3% solids, is flocculated in a thickener to produce a thickened slurry of approximately 20% solids which is discarded to land fill. The effluent is either returned to a clear water head tank or discharged. Refuse and product flows which are beyond the capacity of the thickener are temporarily stored in a sump for later processing.

2.4.3 Equipment and Apparatus

a) General Equipment Specifications

The next step in the development of the pilot plant, after completion of the flowsheet design, was sizing and selection of the appropriate equipment. Since a great deal of the PDU equipment was either pre-existing or donated from other facilities, the capacities associated with each were not an exact match to the pilot plant design specifications. This was compensated for by operating equipment intermittently and providing surge capacity for slurry storage. The performance evaluation for each individual piece of equipment in the flow sheet will be presented later. The following is a list of the
Figure 2.2  Effluent handling scheme for the microbubble flotation pilot plant.
specific equipment used in the pilot plant.

**Hammer Mill**: One Jeffrey Mini-Mill with a 20 x 12-inch breakage chamber, (12) 10 pound hammers, 1/8-inch grate discharge and 25 HP drive motor.

**Conveyor Belt**: One 12-foot Partz-veyor conveyor having a 10-inch belt with a fixed speed 1 HP drive motor.

**Ball Mill**: One Denver 14 x 40-inch pilot-scale ball mill with a 2 HP drive motor. A 45% media filling of 1-1/4-inch balls is used.

**Stirred Ball Mill**: One DRAIS Laboratory Perl Mill, Type PM 40 STS-V with pinned disc agitator, a 40-liter grinding chamber and a 75 HP drive motor. The mill charge consists of 2-millimeter stainless steel balls. Mill feed is provided by a variable speed Moyno type pump.

**Vibrating Screen Deck**: One 22-inch diameter Sweco vibrating screen deck with interchangeable screens. Two screens, 28 and 100 mesh (Tyler) were required for this project.

**Conventional Flotation Banks**: One four-cell Denver flotation bank with open-flow D-R No. 8 cells (3 cubic feet per cell/8.12 cubic feet active volume) and forced aeration system, and one four cell No. 8 (2.75 cubic feet per cell) cell-to-cell Denver flotation bank with self-induced aeration.

**Flotation Columns**: Three flotation columns were developed for this work having diameters of 6-, 8- and 14-inches. The 14-inch cell (14.6
cubic feet active volume) was used in the coarse coal comparison testing. Details of the column designs will be presented later in this section.

**Thickener:** One Denver pilot scale thickener, 32 inches in diameter and 22 inches deep.

**Vacuum Drum Filter:** One Denver pilot-scale drum filter with an 18-inch diameter drum which is 13-inches wide.

**Sumps:** A total of five sumps were required in the pilot plant including a 70-gallon stirred ball mill feed sump, a 200-gallon flotation feed sump, a 750-gallon clean coal tank, a 750-gallon refuse tank, and a 750-gallon clear water constant head tank.

**Pumps:** A total of 10 pumps were used throughout the circuit. The specification for each are given on the EDU flowsheet (Figure 2.1).

**Bins:** Two 10-cubic-foot dry coal storage bins were required in the pilot plant. One fed the hammer mill, and the second was a vibrating bin for providing the main feed source to the continuous pilot plant circuit. A vibrating screw feeder was incorporated into the second bin.

b) **Column Design**

In order to overcome some of the deficiencies common to conventional as well as column flotation circuits, a novel process
known as microbubble column flotation was developed at Virginia Tech. The development of this process originated in light of hydrodynamic studies that suggested that smaller air bubbles could be used to improve the capture rate and, hence, the recovery of fine particles. It was also discovered that these bubbles could best be utilized in a flotation column where countercurrent wash water could be used to prevent the non-selective entrainment of mineral fines.

The microbubble column flotation (MCF) process was initially tested using 2- and 4-inch columns which were typically 5 to 8 feet tall. Using these laboratory cells, the fundamental operating parameters associated with column performance, such as feed and wash water flow rate, gas flow rate, bubble diameter, fractional air hold up, residence time and cell geometry were characterized (Mankosa et al, 1990; Luttrell et al, 1988; Yoon and Luttrell, 1986; Yoon et al, 1987). Based on these results, scale-up relationships were developed for the design of larger columns used in the present work.

Three columns were constructed having diameters of 6-, 8- and 14-inches. A schematic diagram of these columns is shown in Figure 2.3. As shown, the columns are constructed as modular units so that multiple configurations can be easily tested by simply removing a particular sections. Each column typically consists of three sections: i) a bubble generation section, ii) an intermediate section, and iii) an overflow section.

The bubble generation section incorporates the discharge and recycle ports of the bubble generation circuit as well as the tailings discharge valve. The recycled stream from the bubble generation
Figure 2.3 Schematic diagram of pilot plant flotation columns.
circuit is aerated, distributed to the appropriate number of bubble generators and re-injected into the column. Bubble generation is achieved by means of an in-line static mixer. The in-line mixer introduces a high degree of shear agitation into the pulp as it is passed through the system. A fine, narrowly sized bubble size distribution is produced due to the high shear stress present in the fluid. The major advantage of this type of system is that the in-line mixer is essentially an open pipe and, hence, not susceptible to plugging as are conventional sparging mechanisms. The number of static mixer bubble generators depends upon the column geometry and application. Details of the bubble generation circuit (i.e., pumping requirements, static mixer geometries, etc.) can be found elsewhere (Davis, 1990).

Pulp level is controlled by a pneumatic tailings control valve which is driven by a 4-20 mA signal provided by a Honeywell pressure transducer located in the upper portion of the column. A typical arrangement of the bubble generation section is illustrated in Figure 2.4.

The intermediate column section is designed with multiple ports along the axial length to allow for flexibility in locating both the feed inlet and the differential pressure transducers. Multiple pressure transducers are used in order to determine fractional air hold up within the column recovery zone. This result can, in turn, be used to determine the bubble diameter within the cell. Furthermore, multiple intermediate sections can be used in order to accommodate the
Figure 2.4 Schematic diagram of column bubble generation section.
required residence time for a particular system.

The overflow section contains the support frame for the countercurrent water distribution system. A movable wash water assembly has been designed in order to control froth stability by changing the wash water addition point. In larger diameter columns, the overflow section is also equipped with a narrow viewing window so that bubble size, froth stability and the pulp/froth interface can be examined during operation.

c) Instrumentation

In order to obtain representative measurements of all process variables associated with the EDU performance, a complete instrumentation package was designed and installed. This included gamma-nuclear density gauges on the coarse (-28 mesh) and fine (-20 micron) coal sumps, as well as the flotation feed line. The density gauge on the flotation feed line was coupled with a magnetic flow meter to obtain mass flow. Paddle wheel flow meters were installed at all water addition points. A level control system based on resistivity was built and installed on the feed conditioning tanks to maintain a constant supply of feed to the column feed pump.

A Honeywell UDC 2000 programmable controller was used on the column to maintain a constant level. This system uses a 4 to 20-mA signal from a differential pressure transducer to make a decision based on PID logic to control a pneumatically actuated tailings valve. The system was tuned so that level could be maintained within ±1 inch.
An electronic weighbelt feeder was selected to feed the ball mill. This device is capable of producing a consistent feed rate from 50 to 500 lb/hr to the mill. The fully automated belt scale can be remotely controlled in order to provide a complete interface with future control instrumentation.

Details concerning operating characteristics of the PDU instrumentation are given in the following list.

Density Gauges: The Texas Nuclear E-Zcal density system was selected for density determination in the pilot plant. In this system the coal slurry is passed through a Z-section of piping which is located between a cesium 137 source and the scintillation detector head. The unit measures the bulk density of the process by passing a beam of radiation through the material to the detector. As the density increases, the detected radiation decreases. The system converts this decrease in radiation into material density. The Z-section, shown schematically in Figure 2.5, was necessary to present a large enough sample to the system since pipe diameters were relatively small.

(Texas Nuclear, 1987)

Magnetic Flow Meters: Omega Engineering magnetic flow meters were used to measure slurry flow rates. Operation is based on Faraday's Law of Induction, which states that when a conductor is moving in a magnetic field, a voltage is induced. This voltage is proportional to the velocity of the fluid passing through the pipe. These instruments can handle slurries containing as high as 30 percent solids at a velocity range of 0.5 to 30 feet per second with a linearity of ±1%
Figure 2.5 Schematic diagram of Z-section required for Texas Nuclear density gauge.
full scale (Omega Engineering, 1987).

Water Flow Meters: Omega Engineering Paddlewheel type flowmeters were used to monitor water flow rates throughout the plant. These sensors contain small magnets in each of four paddles attached to a central rotor. As the paddle rotates, the magnets pass a coil in the sensor. The transducer generates a linear frequency output that is proportional to the flow output. The operating range of velocities is 1 to 50 feet per second with a linearity of ±1% full scale (Omega Engineering, 1987).

Differential Pressure Gauge: A Honeywell ST 3000 flange mounted pressure to current transducer was selected for monitoring column level. This instrument can measure process pressures ranging from 0–400 inches of water with an accuracy of ±0.10% of calibrated span. Sensor output is 4–20 mA.

Control Valve: A Red Valve Series 5200 pneumatic control valve was used for tailings discharge control. Position of the normally-closed valve actuator is determined from a pneumatic control signal generated by a current to pressure transducer as a result of a 4–20 mA input from the PID controller.

PID Controller: Level control was maintained by a Honeywell UDC 2000 Mini-Pro Universal Digital Controller. Based on the input signal from the differential pressure gauge, the controller provides a 4–20 mA output signal determined by PID logic within ±0.5% of span.
Weigh Belt Feeder: An AutoWeigh weigh belt feeder was selected to provide a consistent feed rate to the ball mill circuit. The conveyer has an accuracy of ±0.5% of full scale and can accommodate input/output 4-20 mA control signals for the purpose of remote monitoring and control by means of a computer interface.
2.5 Results

2.5.1 Grinding Circuit Characterization

The primary goal of the grinding circuit evaluation was to determine the appropriate range of operating conditions for grinding and micronization of coal. Tests were conducted in order to characterize the product size distributions from both the conventional and stirred ball mill as a function of throughput for a specified feed percent solids.

The side elevation shown in Figure 2.6 illustrates the positioning of the hammer mill, ball mill, stirred ball mill and screen deck within the pilot plant. The vibrating screen was equipped with either a 28 or 100 mesh screen, depending on whether coarse or micronized feed was required. The run-of-mine Elkhorn seam coal described in section 2.4.1 was used for all PDU grinding tests. Three series of tests were conducted to determine: (i) performance during open-circuit grinding; (ii) performance during close-circuit grinding; and (iii) performance of the stirred ball mill.

Open-circuit ball mill tests were conducted at feed rates of 42, 88, 124 and 220 lb/hr. Steady-state conditions were established by taking samples at appropriate intervals to determine the mass throughput and percent solids as a function of time. The results from initial open-circuit grinding tests using a minus 3/8-inch feed are given in Table 2.2. In an attempt to improve grinding performance, a 1/8-inch grate was installed in the hammer mill in order to provide a smaller feed size distribution to the ball mill. The feed size
Figure 2.6 Side elevation of pilot plant grinding circuit.
Table 2.2

Pilot-Plant Data for Open-Circuit Grinding of Elkhorn Seam Coal

<table>
<thead>
<tr>
<th>Mesh Size</th>
<th>Feed (%)</th>
<th>Feed Rate (lb/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tyler</td>
<td>Microns</td>
<td>42</td>
</tr>
<tr>
<td>+10</td>
<td>+1680</td>
<td>2.9</td>
</tr>
<tr>
<td>10 x 14</td>
<td>1680 x 1190</td>
<td>7.6</td>
</tr>
<tr>
<td>14 x 20</td>
<td>1190 x 841</td>
<td>15.9</td>
</tr>
<tr>
<td>20 x 28</td>
<td>841 x 595</td>
<td>14.6</td>
</tr>
<tr>
<td>28 x 35</td>
<td>595 x 420</td>
<td>16.9</td>
</tr>
<tr>
<td>35 x 48</td>
<td>420 x 297</td>
<td>11.4</td>
</tr>
<tr>
<td>48 x 65</td>
<td>297 x 212</td>
<td>10.4</td>
</tr>
<tr>
<td>65 x 100</td>
<td>212 x 150</td>
<td>6.1</td>
</tr>
<tr>
<td>-100</td>
<td>-150</td>
<td>14.2</td>
</tr>
<tr>
<td>Composite</td>
<td></td>
<td>100.0</td>
</tr>
</tbody>
</table>

Mill Data: length = 40 inches
diameter = 14 inches
ball diameter = 1.25 inches
mill power = 2 HP
distributions corresponding to 1/8- and 3/8-inch grates in the hammer mill are shown in Figure 2.7. The ball mill product size distributions are shown in Figure 2.8 as a function of feed rate. Figure 2.9 shows that the mean product size decreased with a decrease in feed rate from 290 microns at 220 lb/hr to 145 microns at 42 lb/hr. Steady-state conditions for these tests were determined from the mass throughput and percent solids data shown in Figures 2.10 and 2.11.

The product size distribution obtained at the maximum possible throughput for closed-circuit grinding using 28 and 100 mesh screens are shown in Figures 2.12 and 2.13, respectively. These results correspond to feed rates of 300 lb/hr for the 28-mesh screen and 100 lb/hr for the 100-mesh screen. At higher feed rates the circulating load exceeded the capacity of the vibrating screen. Therefore, in order to test the flotation circuit at higher feed rates the grinding circuit was allowed to run for several hours in advance and the slurry stored in the flotation feed sump. In this manner feed rates as high as 500 lb/hr could be achieved.

Stirred ball mill tests were conducted in the Drais 40 liter machine described in section 2.4.3 (a). Feed rates of 75, 150 and 300 lb/hr were tested using a feed percent solids of 30% by weight. The resulting size distributions are shown in Figure 2.14. Figure 2.15 shows that as the solids feed rate increases from 75 to 300 lb/hr the median particle diameter increases from 5 to 8 microns. Since the 14-inch PDU column has a maximum feed rate of approximately 100 lb/hr of micronized coal, the stirred ball mill had adequate capacity.
Figure 2.7 Hammer mill product size distributions for 1/8- and 3/8-inch grates.
Figure 2.8  Ball mill product size distributions as a function of solids feed rate for open-circuit grinding of an Elkhorn III seam coal.
Figure 2.9 Ball mill product d(50) as a function of feed rate for open-circuit grinding of an Elkhorn III seam coal.
Figure 2.10  Ball mill product flow rate versus time for open-circuit grinding of an Elkhorn III seam coal.
Figure 2.11 Ball mill product percent solids versus time for open-circuit grinding of an Elkhorn III seam coal.
Figure 2.12  Ball mill product size distribution for grinding an Elkhorn III seam coal in closed-circuit arrangement with a 28 mesh vibratory screen.
Figure 2.13 Ball mill product size distribution for grinding an Elkhorn III seam coal in closed-circuit arrangement with a 100 mesh vibratory screen.
Figure 2.14 Stirred ball mill product size distribution as a function of feed rate for attrition milling of an Elkhorn III seam coal.
Figure 2.15  Stirred ball mill product $d(50)$ as a function of feed rate for attrition milling of an Elkhorn III seam coal.
2.5.2 Pilot-Plant Flotation Results

a) Flotation of High-Ash Refuse Material

In order to determine the feasibility of microbubble column flotation for processing high-ash, reject material, a series of tests were conducted on an eastern Kentucky, Jellico seam coal. The sample was obtained from the classifying cyclone overflow of an operating preparation plant. This material is considered to be difficult to treat by conventional methods because of its fine particle size (80% passing 325 mesh) and relatively high ash content (45-50%), derived primarily from fine clay minerals, and is normally sent to the plant thickener as reject.

The sample was processed using a 6-inch diameter microbubble column which was approximately 20 feet high. Table 2.3 summarizes the operating conditions used for the column during the course of this test work. As shown, a total of ten different sets of operating conditions were employed. The major variable examined in these tests was feed rate, which was varied between 40 and 189 lb/hr in order to generate a grade versus recovery curve for this material.

Countercurrent wash water was added at a flow rate of about 0.9 GPM in an attempt to reduce the entrainment of fine clay particles in the froth product. The aeration rate to the column was maintained at approximately 0.5 SCFM. Dowfroth M150 was used as the frothing agent in all tests at an average addition of approximately 1.5 lbs/ton, although this consumption level could have been reduced with the addition of an appropriate collector (i.e., kerosene or fuel oil). It
Table 2.3

Summary of Operating Conditions for Jellico Seam Tests

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Feed (lb/hr)</th>
<th>Wash Water (GPM)</th>
<th>Air (SCFM)</th>
<th>Frother (lb/ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0.92</td>
<td>0.53</td>
<td>1.62</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>0.92</td>
<td>0.51</td>
<td>1.79</td>
</tr>
<tr>
<td>3</td>
<td>92</td>
<td>0.92</td>
<td>0.50</td>
<td>1.34</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>0.92</td>
<td>0.46</td>
<td>1.78</td>
</tr>
<tr>
<td>5</td>
<td>85</td>
<td>0.90</td>
<td>0.46</td>
<td>1.10</td>
</tr>
<tr>
<td>6</td>
<td>85</td>
<td>0.90</td>
<td>0.50</td>
<td>1.29</td>
</tr>
<tr>
<td>7</td>
<td>107</td>
<td>0.90</td>
<td>0.50</td>
<td>1.77</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>0.90</td>
<td>0.42</td>
<td>1.10</td>
</tr>
<tr>
<td>9</td>
<td>189</td>
<td>0.90</td>
<td>0.46</td>
<td>3.71</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>0.90</td>
<td>0.50</td>
<td>2.44</td>
</tr>
</tbody>
</table>
should be noted that since this material was from a classifying
cyclone overflow, the solids content of the slurry was approximately
2-3%. This is normally considered low for column flotation which
makes it necessary to operate the column for very short retention
times (i.e., less than 5 minutes) in order to get sufficient
throughput.

Figure 2.16 shows the product grade versus recovery relationship
for this series of tests. As shown, a dramatic reduction in ash
content is achieved with a minimal loss in recovery. For example, the
ash content of this material was reduced to less than 6% while
maintaining a combustible recovery in excess of 80%. This reflects
the ability of the microbubble column to utilize countercurrent wash
water in order to remove the fine, liberated mineral matter which is
present in the form of clay minerals. The high recoveries maintained
while obtaining large reductions in product ash content are a function
of the high flotation rate constants associated with the use of
microbubbles. The corresponding grade-recovery relationship for the
reject stream, Figure 2.17, indicates a sharp increase in reject ash
content with combustible recovery. These results suggest that over
the range of values examined, there exists a nearly linear
relationship between reject ash content and combustible recovery.

Combustible recovery, product ash and reject ash contents are
shown as a function of feed rate in Figures 2.18 through 2.20,
respectively. The combustible recovery decreased from approximately
85% at a feed rate of 40 lb/hr to 55% at 190 lb/hr. Over this same
Figure 2.16 Combustible recovery as a function of product ash content for column flotation of a Jellico seam coal.
Figure 2.17 Combustible recovery as a function of reject ash content for column flotation of a Jellico seam coal.
Figure 2.18  Combustible recovery as a function of feed rate for column flotation of a Jellico seam coal.
Figure 2.19  Product ash content as a function of feed rate for column flotation of a Jellico seam coal.
Figure 2.20  Reject ash content as a function of feed rate for column flotation of a Jellico seam coal.
increase in feed rate, the product ash content improved from approximately 9% to just over 4.5% while the percent ash in the reject stream decreased from approximately 83% to less than 70%. The results from these tests are summarized in Table 2.4. It should be noted that the observed scatter in the data is partly due to slight changes in the aeration and frother addition rates employed during testing, as well as scatter in the feed ash and percent solids.

Similar tests were conducted on a Coalburg seam reject sample obtained from the Pittston Rum Creek mine in West Virginia. This was a minus 100 mesh cyclone overflow material which averaged 53% ash. The sample was initially tested in a 2-inch laboratory cell having a height of 5 feet. These results were validated in the 8-inch diameter by 15-feet high pilot-plant column. A constant frother addition of 0.5 lb/ton was maintained for each test. Aeration and wash water superficial rates were also held constant at 100 cm/sec and 20 cm/sec, respectively.

The results of both the 2- and 8-inch column tests are presented in Figure 2.21. It can be seen from these results that both columns reduced the overall product ash content from 53% to approximately 8% while maintaining a combustible recovery of greater than 90%. Furthermore, comparison of these results indicate that both columns are operating on the same grade-recovery curve. This implies that throughput predictions for larger size columns could possibly be made from 2-inch column results. The results from all of the Coalburg seam flotation tests are summarized in Table 2.5.
Table 2.4

Summary of Results for Jellico Seam Tests

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Feed Rate (lb/hr)</th>
<th>Combustible Recovery</th>
<th>Product Yield</th>
<th>Product Ash %</th>
<th>Reject Ash %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>78.1</td>
<td>50.5</td>
<td>4.62</td>
<td>72.7</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>81.8</td>
<td>48.0</td>
<td>7.32</td>
<td>80.9</td>
</tr>
<tr>
<td>3</td>
<td>92</td>
<td>80.2</td>
<td>46.7</td>
<td>6.58</td>
<td>79.8</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>85.3</td>
<td>52.5</td>
<td>10.98</td>
<td>83.0</td>
</tr>
<tr>
<td>5</td>
<td>85</td>
<td>80.4</td>
<td>46.4</td>
<td>6.10</td>
<td>80.1</td>
</tr>
<tr>
<td>6</td>
<td>85</td>
<td>71.9</td>
<td>41.2</td>
<td>4.56</td>
<td>73.8</td>
</tr>
<tr>
<td>7</td>
<td>107</td>
<td>82.3</td>
<td>52.7</td>
<td>5.68</td>
<td>77.4</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>89.0</td>
<td>60.5</td>
<td>9.14</td>
<td>82.9</td>
</tr>
<tr>
<td>9</td>
<td>189</td>
<td>56.9</td>
<td>30.1</td>
<td>4.57</td>
<td>69.1</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>84.5</td>
<td>53.3</td>
<td>7.98</td>
<td>80.7</td>
</tr>
</tbody>
</table>
Figure 2.21  Combustible recovery as a function of product ash content for column flotation of a Coalburg seam coal.
Table 2.5

Column Flotation Results for High Ash Coalburg Seam Reject

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Column</th>
<th>Product Ash (%)</th>
<th>Combustible Recovery (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2&quot;</td>
<td>13.40</td>
<td>83.87</td>
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<tr>
<td>2</td>
<td>2&quot;</td>
<td>8.43</td>
<td>72.63</td>
</tr>
<tr>
<td>3</td>
<td>2&quot;</td>
<td>8.95</td>
<td>71.58</td>
</tr>
<tr>
<td>4</td>
<td>8&quot;</td>
<td>8.65</td>
<td>69.83</td>
</tr>
<tr>
<td>5</td>
<td>8&quot;</td>
<td>6.99</td>
<td>64.76</td>
</tr>
<tr>
<td>6</td>
<td>8&quot;</td>
<td>8.10</td>
<td>73.75</td>
</tr>
</tbody>
</table>
b) Superclean Coal Flotation

In order to produce super- and ultraclean coals by any physical cleaning process, it is necessary to liberate the mineral matter from the coal prior to separation. It has been shown that microbubble flotation can produce coals containing less than 1 or 2% ash and very little sulfur provided that the mineral matter contained in the coal has been properly liberated (Yoon and Miller, 1982; Luttrell et al., 1985; Yoon and Luttrell, 1986). Several tests were conducted on micronized Elkhorn III and Cedar Grove seam coals in order to validate this result in a pilot-scale microbubble column.

The Elkhorn III sample was a minus 200-mesh, pre-cleaned blend which contained approximately 2.5 percent ash. This material was slurried at 30 percent solids and fed to the Drais stirred ball mill. Particle size analysis indicated that a median product size of 7 microns was achieved at a feed rate of 150 lb/hr, as shown in Figure 2.22. The slurry was diluted to 20 percent solids and fed to the 8-inch, microbubble column. The superficial gas and wash water flow rates were maintained at 100 and 20 cm/sec, respectively. Frother addition was maintained at 0.7 lb/ton of Dowfroth M150. A range of feed rates was examined in order to produce the grade-recovery relationship shown in Figure 2.23. From this information the residence time necessary to produce a clean coal product having less than 1% ash was calculated. It was determined that a final product containing 0.89 percent ash could be produced at a solids feed rate of 40 lb/hr in an 8-inch microbubble column.
Figure 2.22 Particle size distribution for an attrition ground Elkhorn seam coal.
Figure 2.23  Combustible recovery as a function of product ash content for column flotation of an Elkhorn seam coal.
Similar tests were conducted on a run-of-mine Cedar Grove sample which was crushed to minus 1/8-inch in the pilot plant hammer mill and wet ground at 30 percent solids in closed circuit with a 100-mesh vibratory screen. The minus 100-mesh product from the ball mill circuit was fed to the Drais stirred ball mill at a rate of 100 lbs/hr. This provided a flotation feed with a median particle size of 6.55 microns. Several tests were conducted on both the 14-inch column and conventional flotation cells in order to determine the performance characteristics of each when processing micronized feed.

Product combustible recovery is presented as a function of retention time in Figure 2.24. In all instances, retention time was based on the tailings volumetric flow rate, \( Q_t \), according to the following relationship:

\[
t = \frac{V_C}{Q_t},
\]

where \( V_C \) represents the volume of the cell. It can be seen from Figure 2.24 that the MCF cell is capable of obtaining a higher level of recovery as opposed to conventional flotation. However, the performance deteriorates quickly at retention times below approximately 25 minutes. The conventional cells consistently operated at an 80 to 85 percent recovery level over the range of retention times which were considered. It should be noted, however, that conventional flotation of micronized particles presents several problems. The most noticeable is the severe difficulties in froth
Figure 2.24 Combustible recovery as a function of retention time for column and conventional flotation of a Cedar Grove seam coal.
Figure 2.25 Combustible recovery as a function of feed ash rejection for column and conventional flotation of a Cedar Grove seam coal.
handling. Since conventional flotation cells are not equipped to handle such extremely stable froths, many problems are encountered in dealing with froth removal, thus making data collection difficult.

The advantage of column flotation is illustrated in Figure 2.25, which shows combustible recovery as a function of percent ash rejection. Ash rejection, in this application, has been defined as follows:

\[
\text{Ash Rejection (\%)} = \frac{(1 - Y_f) \times \%A_t}{\%A_f}, \quad [2.2]
\]

where,

- \(Y_f\) = fractional yield,
- \(\%A_f\) = feed ash percent, and
- \(\%A_t\) = tailings ash percent.

In this manner, an ash rejection of 100 percent indicates that all of the ash associated with the feed reported to the reject stream. Conversely, a rejection of zero would indicate that all of the ash reported to the product stream. It can be seen from Figure 2.25 that the column is capable of obtaining a higher level of ash rejection than conventional flotation for micronized coal.

In order to relate the advantages and disadvantages of each system, the cells were compared on the basis of the efficiency of separation. The separation efficiency is defined as follows:
\[ E_s = R_C - R_a, \]  \hspace{1cm} [2.3] \]

where,

\[ R_C = \text{combustible recovery}, \] and

\[ R_a = \text{ash recovery to product}. \]

Separation efficiency is shown in Figure 2.26 as a function of retention time. It can be seen from these results that the column is capable of operating at a higher separation efficiency than the conventional cells. This is a result of the higher rate of recovery provided by the microbubbles, as well as the increased ash rejection provided by the wash water in the column.

Figure 2.27 shows the product grade versus combustible recovery results for this series of tests. As indicated in previous results, the MCF cell is capable of producing a lower ash product at higher levels of recovery than the conventional cells. These results indicate that a clean coal product containing 2 percent ash can be produced at recovery levels in excess of 90 percent. Conventional flotation, however, was never able to produce superclean coal.

c) Coarse Coal Flotation

In order to compare the performance of microbubble column and conventional flotation techniques, two series of tests were conducted on a minus 28-mesh, run-of-mine Elkhorn seam coal. Similar
Figure 2.26 Ash separation efficiency as a function of retention time for column and conventional flotation of a Cedar Grove seam coal.
Figure 2.27  Combustible recovery as a function of product ash content for column and conventional flotation of a Cedar Grove seam coal.
investigations were also conducted on the Pittsburgh No. 8, Upper Freeport and Cedar Grove seam coals. The feed analyses for each coal are presented in Section 2.4.1. For each test, the run-of-mine coal was initially hammer milled to reduce the top size to minus 1/8-inch. This material was then wet ground at 30% solids in a 14 X 40-inch ball mill (described earlier) in closed circuit arrangement with a 28-mesh vibrating Sweco screen. This was necessary to ensure the proper top size of the feed slurry for the flotation tests.

Prior to flotation, the slurry was diluted to 20% solids for column flotation and 5% solids for conventional flotation. With the additional wash water added in the column, the result was that both processes were operated at approximately 5% solids within each unit. Column tests were conducted in a 14-inch diameter by 16.5-foot tall column, while conventional flotation was carried out in a bank of four, 3-cubic-foot Denver flotation cells (DR No. 8 flow through type). Aeration rates and pulp levels were adjusted independently in order to determine the optimum performance for each device. Dowfroth M150 was added at a rate of 1 lb/ton in all tests. Samples were taken of the feed, product and reject to determine the assay and percent solids of each stream. Best estimates of the data obtained for each test were determined using the BILMAT material balancing routine. The results from all four series of tests are summarized in Appendix VI.

It must be emphasized at this point that reliable pilot scale data are difficult to obtain. Laboratory and in-plant test results are usually more reliable due to the easy accessibility of large
quantities of feed slurry. In this instance, the limited capacity of the grinding circuit required that large volumes of feed slurry be ground prior to flotation and stored in large sumps in order to provide enough run time to achieve steady-state conditions in the flotation circuit. This presented problems in maintaining the integrity of the feed in terms of proper mixing and regulation of the percent solids when re-initiating start-up of the grinding circuit. For this reason, the results of the comparison testing will be discussed with more emphasis on trends in results rather than absolute values of any given result.

i) Elkhorn III

Figure 7.28 shows the column/conventional cell flotation recovery for the Elkhorn III coal seam. It appears from this result that the 14-inch MCF cell is capable of maintaining a high level of recovery at feed rates surpassing the conventional flotation bank. Unfortunately, this comparison is not appropriate due to differences in cell volumes. The MCF cell has an active volume (foth phase not included) of about 12 cubic feet. If the volumetric air fraction is also considered, this number is reduced to 9.6 cubic feet. The active volume of the conventional flotation bank (not including impeller volume) is approximately 8.1 cubic feet (7.3 cubic feet corrected for air hold up. This indicates that the column volume is approximately 24% larger than that of the conventional flotation bank. Therefore, if only the solids feed rate is considered, and not the
Figure 2.28  Combustible recovery as a function of solids feed rate for column and conventional flotation of an Elkhorn III seam coal.
Figure 2.29  Combustible recovery as a function of retention time for column and conventional flotation of an Elkhorn III seam coal.
retention time of material within the cell, the column appears to have a higher throughput. This, of course, is simply due to the excess volume of the MCF cell which provides a longer residence time for equivalent volumetric flow rates.

These results are replotted in Figure 2.29 as a function of retention time rather than solids feed rate. In this case it can be seen that the conventional cell is capable of maintaining high levels of recovery at shorter residence times than those required for the column. This response is typical of column flotation cells due to the reduced cross-sectional area of the cell. The reduction in cell surface area impedes the removal of froth, thus limiting the carrying capacity of the column. This effect will be discussed in greater detail in Chapter 5. The scatter observed for the column data is a result of a drift in the feed percent solids. Figure 2.30 shows the same data broken down on a size-by-size basis. It is interesting to note that when the recovery drop-off occurs the coarse particles are the first to be lost. This may well be an effect of froth stability, since fine particles typically form a more stable froth.

The true benefit of column flotation is the ability to remove entrained mineral matter from the froth by utilizing a counter current stream of wash water. This provides an increased ash rejection beyond what is obtainable by conventional froth upgrading. The improvement in ash rejection is illustrated by Figure 2.31, which shows product recovery as a function of percent ash rejection. It can be seen that the column provides as much as a 10 percent increase in ash rejection over conventional flotation at the same level of recovery.
Figure 2.30 Size-by-size combustible recovery as a function of retention time for column and conventional flotation of an Elkhorn III seam coal.
Figure 2.31 Combustible recovery as a function of feed ash rejection for column and conventional flotation of an Elkhorn III seam coal.
Figure 2.32  Size-by-size combustible recovery as a function of feed ash rejection for column and conventional flotation of an Elkhorn III seam coal.
Similar results are observed on a size by size basis, as shown in Figure 2.32. In this instance, both size fractions demonstrate an increased level of ash rejection. Although, it is interesting to note that the minus 100 mesh fraction demonstrates a higher level of ash rejection than does the coarser material. This is attributed to the increased liberation typically associated with fine particles. The coarser middlings particles, however, may still retain a large enough hydrophobic component to be readily floated. Fine hydrophylllic particles, although recovered by entrainment, are rejected by the wash water.

In order to relate the advantages and disadvantages of each system, the cells were compared on the basis of the efficiency of the separation. The separation efficiency for each cell is shown in Figure 2.33 as a function of the cell retention time. This incorporates the retention time advantages associated with the conventional cells as well as the increased ash rejection provided by the column. It can be seen from this plot that the column ultimately operates at a higher efficiency, although it requires a slightly longer residence time. This concept is again further validated on a size-by-size basis as shown in Figure 2.34. Once again, a higher separation efficiency is obtained for the minus 100 mesh fraction due to the higher ash rejection associated with this material, as described above.

The overall grade recovery curves for both series of tests conducted on the Elkhorn III seam coal are shown in Figure 2.35.
Figure 2.33  Ash separation efficiency as a function of retention time for column and conventional flotation of an Elkhorn III seam coal.
Figure 2.34 Size-by-size ash separation efficiency as a function of retention time for column and conventional flotation of an Elkhorn III seam coal.
Figure 2.35  Combustible recovery as a function of product ash content for column and conventional flotation of an Elkhorn III seam coal.
These results reflect the ability of the column to produce the highest possible grade for a given particle size. This is further supported by the release analysis data for an Elkhorn III coal which is shown in Figure 2.31. The release curve represents the best possible result by eliminating any entrainment effects which may have resulted in a lower quality product (Forrest, 1990). It can be seen that the column grade-versus-recovery curve coincides with the release curve, indicating that the column is obtaining the maximum possible ash rejection when processing a minus 28 mesh feed.

ii) Pittsburgh No. 8

The recovery-versus-retention time curves for the Pittsburgh No. 8 coal seam are shown in Figure 2.36. Results indicate that the conventional bank is capable of achieving the same level of recovery at shorter residence times as compared to the column. The drop off in size-by-size recovery, Figure 2.37, also illustrates that the column preferentially loses the coarser particles at shorter residence times. However, this behavior is reversed in the case of the conventional cells. In this instance the feed percent solids drifted slightly higher than expected, as compared to the initial conventional flotation tests. Therefore, it is not clear whether the decreased recovery was a result of retention time or carrying capacity.

The increased ash rejection associated with the column is illustrated in Figures 2.38 and 2.39. Once again, the minus 100 mesh fraction obtained a higher degree of ash rejection. However, it
Figure 2.36 Combustible recovery as a function of retention time for column and conventional flotation of a Pittsburgh No. 8 seam coal.
Figure 2.37  Size-by-size combustible recovery as a function of retention time for column and conventional flotation of a Pittsburgh No. 8 seam coal.
Figure 2.38 Combustible recovery as a function of feed ash rejection for column and conventional flotation of a Pittsburgh No. 8 seam coal.
Figure 2.39 Size-by-size combustible recovery as a function of feed ash rejection for column and conventional flotation of a Pittsburgh No. 8 seam coal.
should be noted that the Pittsburgh No. 8 sample did not achieve as high a level of ash rejection as the Elkhorn III coal. This is due to the fact that the majority of the ash associated with the Elkhorn III seam is predominantly a fine, liberated clay which is easily removed by flotation, even at coarser sizes. The Pittsburgh No. 8 coal, on the other hand, apparently has an ash component which is more tightly bound with the coal matrix and, thus, is not as well liberated at minus 28 mesh.

The reduced level of ash rejection also results in a lower ash separation efficiency as illustrated by Figures 2.40 and 2.41. In this instance, the overall separation efficiency is significantly reduced as compared to the Elkhorn III sample. This is partially due to the poor efficiencies associated with the 28 x 100 mesh fraction due to inadequate liberation. Once again the column required a longer retention time, however, the separation efficiency was still superior to that of the conventional cells. It is interesting to note that the conventional flotation cells do not experience the same trend in separation efficiency on the +100 mesh size fraction. This is a result of the unusually high recovery for the +100 mesh fraction.

This result tends to contradict the expected shape for the ash separation efficiency versus retention time curve. Therefore, it is most likely that this result is an anomaly resulting from experimental error.

An additional consideration associated with the Pittsburgh No. 8 seam is the performance in terms of sulfur rejection. This was not addressed for the Elkhorn III coal, since the overall sulfur content
Figure 2.40 Ash separation efficiency as a function of retention time for column and conventional flotation of a Pittsburgh No. 8 seam coal.
Figure 2.41 Size-by-size ash separation efficiency as a function of retention time for column and conventional flotation of a Pittsburgh No. 8 seam coal.
of the Elkhorn III seam is quite low (i.e., less than 0.7%). In the case of the Pittsburgh No. 8 coal, however, the feed percent sulfur can run as high as 4 percent. Therefore, an additional comparison of the different flotation processes was conducted which was based on sulfur rejection. Equation [2.1] was used to determine the sulfur rejection by substituting the sulfur assay for the ash percent.

The overall sulfur rejection is plotted against product recovery in Figure 2.42. This result indicates that the column is slightly more selective in terms of sulfur rejection than the conventional cells. This phenomenon is attributed to the use of counter current wash water, similar to the findings regarding ash rejection. Since froth flotation is a physical separation process, a reduction in the total sulfur content can be obtained only by rejecting the pyritic component of the sample. This is possible if the sample has been ground fine enough to achieve a significant degree of pyrite liberation. As shown in Figure 2.43, nearly 50% of the pyrite present in the -100 mesh fraction of the Pittsburgh No. 8 coal is greater than 50 microns in size. This agrees with the size-by-size analyses presented in Figure 2.44 which suggest that the minor improvement in selectivity was a result of a small up-grade in the finer size fraction. As expected, the 28 x 100 mesh fraction was unaffected due to inadequate liberation.

The overall and size-by-size sulfur based separation efficiencies are shown in Figures 2.45 and 2.46, respectively. Although the column has a slightly better performance, the overall efficiencies are quite
Figure 2.42 Combustible recovery versus feed sulfur rejection for column and conventional flotation of a Pittsburgh No. 8 seam coal.
Figure 2.43 Pyrite size distribution of 100-mesh x 0 samples of the four run-of-mine samples used for coarse coal flotation testing (Wang, 1990).
Figure 2.44 Size-by-size combustible recovery as a function of feed sulfur rejection for column and conventional flotation of a Pittsburgh No. 8 seam coal.
Figure 2.45 Sulfur separation efficiency as a function of retention time for column and conventional flotation of a Pittsburgh No. 8 seam coal.
Figure 2.46  Size-by-size sulfur separation efficiency as a function of retention time for column and conventional flotation of a Pittsburgh No. 8 seam coal.
low in each case. As previously mentioned, this is simply a result of the coarse particle size distribution which was used in this investigation. In order to increase the efficiency of either process it would be necessary to produce a finer feed particle size distribution.

The overall grade versus recovery curves for both ash and sulfur are shown in Figures 2.47 and 2.48, respectively. It can be seen from these results that the column outperforms the conventional cells in terms of final product ash and sulfur. The MCF cell produced a 4 percent ash product containing 2.2 percent sulfur at a 95 percent recovery level.

iii) Upper Freeport

Figure 2.49 shows the recovery versus retention time curves for the Upper Freeport seam. As with previous coals, the conventional cells are capable of operating at a shorter residence time than the column. It appears, however, that the column is able to operate at a slightly higher level of recovery. This is indicated by the high recovery associated with the column for long retention times, although additional data would be needed at residence times exceeding 20 minutes in order to further validate this assumption. Size-by-size evaluation of the column performance is shown in Figure 2.50. Once again results indicate that the +28 mesh material is the first to be lost with a decrease in retention time. The conventional cells, in this instance, were not operated at short enough residence times to
Figure 2.47  Combustible recovery as a function of product ash content for column and conventional flotation of a Pittsburgh No. 8 seam coal.
Figure 2.48 Combustible recovery as a function of product sulfur content for column and conventional flotation of a Pittsburgh No. 8 seam coal.
Figure 2.49 Combustible recovery as a function of retention time for column and conventional flotation of an Upper Freeport seam coal.
Figure 2.50  Size-by-size combustible recovery as a function of retention time for column and conventional flotation of an Upper Freeport seam coal.
Figure 2.51 Combustible recovery as a function of feed ash rejection for column and conventional flotation of an Upper Freeport seam coal.
Figure 2.52  Size-by-size combustible recovery as a function of feed ash rejection for column and conventional flotation of an Upper Freeport seam coal.
experience a decline in recovery.

The MCF column again operated at a higher level of ash rejection than the conventional cells, as observed in Figure 2.51. Although, in this case, the increase is not as significant as shown for previous coals. This is primarily due to the consistency of the Upper Freeport coal. The sample, as received from EPRI (Electric Power Research Institute), consisted of approximately 50% (by weight) of coarse rock. Therefore, the ~28 mesh feed prepared for flotation contained a disproportionate amount of coarse ash. In this type of application, the conventional cells will typically operate as efficiently as a column since coarse ash bearing minerals are typically not entrained. The small improvement in ash rejection is probably a result of rejecting the fines generated during crushing. This result is exemplified by the size-by-size ash rejection shown in Figure 2.52. It can be seen that the minus 100 mesh fraction exhibits a slight improvement while the 28 by 100 mesh fraction remains unchanged.

The ash separation, Figures 2.53 and 2.54, indicate that the column performed better on the minus 100 mesh component, although the overall efficiencies are lower due to the losses incurred by the coarse material. It becomes difficult to make reasonable comparisons regarding efficiency in this case since the column was not operating at a sufficient residence time.

As shown in Figures 2.55 and 2.56, neither type of flotation system can affect a separation in terms of sulfur. This is due to the nature of the pyrite distribution within the matrix of the Upper Freeport coal. Past studies have shown that pyrite is very finely
Figure 2.53 Ash separation efficiency as a function of retention time for column and conventional flotation of an Upper Freeport seam coal.
Figure 2.54  Size-by-size ash separation efficiency as a function of retention time for column and conventional flotation of an Upper Freeport seam coal.
Figure 2.55 Combustible recovery as a function of feed sulfur rejection for column and conventional flotation of an Upper Freeport seam coal.
Figure 2.56  Size-by-size combustible recovery as a function of feed sulfur rejection for column and conventional flotation of an Upper Freeport seam coal.
disseminated within an Upper Freeport coal, as opposed to the Pittsburgh No. 8 seam which contains a substantial portion of coarse, liberated pyrite. Therefore, in each case, any observed improvement in sulfur rejection is most likely a result of lost middlings particles. As a result, the sulfur based separation efficiency does not indicate an advantage for column or conventional flotation, as shown in Figure 2.57.

The product ash versus recovery curve, Figure 2.58, again illustrates the advantages associated with column flotation. As can be seen, the column is capable of removing an additional 50% of the product ash at a 90% recovery level.

iv) Cedar Grove

The final sample considered in the coarse coal flotation test program was the Cedar Grove seam. Initial results from this test are shown in Figure 2.59 in terms of combustible recovery as a function of cell retention time. As indicated by results from the previous coarse coal tests, the conventional bank once again is capable of producing the same level of recovery at a shorter retention time. This can be seen on a size-by-size basis as well, as indicated by Figure 2.60. In this instance, as with the Elkhorn III seam, the column tends to preferentially lose the coarser material. This, again, is attributed to froth stability problems associated with conditions of heavy froth loading. The conventional tests, however, were not operated at low enough recovery levels in order to draw a conclusion based on the
Figure 2.57 Sulfur separation efficiency as a function of retention time for column and conventional flotation of an Upper Freeport seam coal.
Figure 2.58  Combustible recovery as a function of product ash content for column and conventional flotation of an Upper Freeport seam coal.
Figure 2.59 Combustible recovery as a function of retention time for column and conventional flotation of a Cedar Grove seam coal.
Figure 2.60  Size-by-size combustible recovery as a function of retention time for column and conventional flotation of a Cedar Grove seam coal.
Figure 2.61  Combustible recovery as a function of feed ash rejection for column and conventional flotation of a Cedar Grove seam coal.
size-by-size performance.

The combustible recovery versus ash rejection curves, Figure 2.61, indicate that the column was capable of achieving a higher level of ash rejection as compared to the conventional cells. This is further illustrated by the size by size data shown in Figure 2.62, which indicates a higher level of ash rejection in both the +100 and -100 mesh size fractions.

The combined effect of recovery and ash rejection is illustrated by the separation efficiencies shown in Figure 2.63. It can be seen that the column operates at a slightly higher separation efficiency, although it requires a longer residence time. In this application, however, very little difference is observed between the column and conventional curves. This is the result of low ash content (5.5% ash) of the run-of-mine, Cedar Grove coal. In this case, a significant level of improvement relative to ash rejection is not possible without further liberation. This is indicated by the size by size results shown in Figure 2.64. It can be seen that the -100 mesh fraction performed more efficiently than the coarser fraction. This is a result of the higher recoveries associated with the -100 mesh material as well as the increased liberation characteristic of the finer sizes. Further evidence of the effect which liberation has on the separation efficiency was presented in the micronized coal flotation results presented in section 2.5.2 b.

The combustible recovery versus product grade curves for both conventional and column flotation are shown in Figure 2.65. It can be
Figure 2.62 Size-by-size combustible recovery as a function of feed ash rejection for column and conventional flotation of a Cedar Grove seam coal.
Figure 2.63  Ash separation efficiency as a function of retention time for column and conventional flotation of a Cedar Grove seam coal.
Figure 2.64 Size-by-size ash separation efficiency as a function of retention time for column and conventional flotation of a Cedar Grove seam coal.
Figure 2.65 Combustible recovery as a function of product ash content for column and conventional flotation of a Cedar Grove seam coal.
seen from this result that the column is capable of providing further reductions in ash content at the same recovery level as compared to conventional flotation. This is a result of the increased ash rejection provided by the countercurrent wash water in the column froth.

d) Scale-Up Testing

In order to develop preliminary scale-up relationships for the MCF column, a series of tests were conducted using 6-, 8- and 14-inch diameter columns to process minus 28-mesh run-of-mine Elkhorn seam coal (approximately 10% ash). In each test, the frother addition was held constant at approximately 1 lb/ton of Dowfroth M150. The appropriate feed range was then swept for each column and samples taken of the feed, product and reject to determine the assay and percent solids of each stream. Wash water and air flow rates were metered and the fractional air content was determined by differential pressure. The ash rejection versus recovery curves for each column are shown in Figure 2.66. As expected, these results indicate that a constant level of performance is achieved regardless of the column diameter. Figure 2.67 shows combustible recovery as a function of feed rate for each column diameter. As expected, combustible recovery decreases as feed rate increases due to the decrease in retention time. Similarly, as column diameter increases, the drop-off in recovery occurs at higher feed rates. Thus, Figure 2.67 serves to illustrate the applicable throughput range for each column. For
Figure 2.66  Combustible recovery as a function of product ash content for column flotation of an Elkhorn III seam coal.
Figure 2.67 Combustible recovery as a function of solids feed rate for column flotation of an Elkhorn III seam coal.
example, at a combustible recovery of 80%, the 6-inch column has a throughput of about 120 lb/hr, the 8-inch column has a throughput of about 175 lb/hr and the 14-inch column has a throughput of about 630 lb/hr. This is mainly a result of the increase in column cross-sectional area as diameter increases, although other factors may influence this result (i.e., froth stability and removal). These effects will be discussed in greater detail in the following section.
2.6 Discussion

2.6.1 Comparison of Column and Conventional Flotation

In order to compare the flotation response between various coals, the data for each of the coarse coal tests has been replotted as recovery versus retention time for both the column and the conventional flotation bank. These results are shown in Figures 2.68 and 2.69, respectively. It can be seen that all of the coals tested can be included in a relatively narrow operating band which, within reasonable limits of retention time, appears to be independent of the coal seam. This indicates that each of the four coals has relatively the same floatability. As one might expect a similar result is found for conventional flotation, as shown in Figure 2.69. Although, in this instance, the operating region occurs at a somewhat lower retention time. This result can be attributed to differences in froth carrying capacity limitations for each system. As will be explained in greater detail in Chapter 5, in many instances the rate limiting step for column flotation appears to be the froth carrying capacity. This is especially true in the case of coal flotation where the natural hydrophobicity and coarse particle size usually provide high flotation rate constants. This, coupled with the fact that 80 to 90 percent of the feed solids are usually removed as product, often necessitate that the column operate in a carrying capacity limited regime. Under these conditions, the solids cannot be removed from the top of the cell as quickly as they are collected and carried into the base of the froth. The conventional cells, however, have a larger
Figure 2.68 Combustible recovery as a function of retention time for column flotation of 28 mesh X 0, run-of-mine coal.
Figure 2.69 Combustible recovery as a function of retention time for conventional flotation of 28 mesh X O, run-of-mine coal.
Figure 2.70 Compustible recovery as a function of feed ash rejection for column flotation of 28 mesh X 0, run-of-mine coal.
Figure 2.71  Combustible recovery as a function of feed ash rejection for conventional flotation of 28 mesh X 0, run-of-mine coal.
surface area available for froth removal as compared to the column, therefore the conventional cells are usually not inhibited by carrying capacity limitations.

Although the recovery versus retention time curves indicate the magnitude of combustible recovery, they do not provide any information regarding the performance of the system in terms of separation. In many cases the high recovery levels associated with long retention times result in an increased recovery of middlings particles which leads to a deterioration of product grade. In order to incorporate product grade into an analysis of the cell performance the above results have been plotted as a function of feed ash rejection, as shown in Figures 2.70 and 2.71. It can be seen from these diagrams that the Elkhorn III coal provided the highest degree of ash rejection in both column and conventional flotation, whereas the Cedar Grove coal resulted in the lowest level of ash rejection. This is basically a result of the ash characteristics of each sample. The Elkhorn III seam has a higher feed ash which is more readily liberated than the Cedar Grove seam, therefore a higher level of ash rejection can be achieved. The Pittsburgh No. 8 and Upper Freeport coals both responded at a level intermediate to the previous examples.

In the case of conventional flotation it can be seen that the ash rejection curves for all coal seams shift further to the left indicating a lesser extent of ash rejection. This is caused by the recovery of liberated ash which is entrained with the clean coal froth product. The degree of entrainment in a conventional flotation system
is a function of cell operating conditions, such as gas flow rate and frother dosage. The maximum ash rejection, therefore, will occur under the conditions which minimize the ash entrainment.

In order to determine the point of optimum column performance, separation efficiency has been plotted as a function of ash rejection. These results, shown in Figure 2.72, clearly indicate the point at which the maximum column performance occurs in terms of recovery and ash rejection. Knowing the level of recovery at which the optimum performance is achieved, the required residence time can easily be determined from Figure 2.68. The optimum retention time for each sample, in both column and conventional flotation, is presented in Table 2.6. It can be seen from these results that although the column requires a slightly longer retention time, conventional flotation is never able to achieve the same level of ash rejection as column flotation. In fact, for the Elkhorn III seam the column produces an equivalent recovery and a higher separation efficiency at the same retention time.

The product grade versus recovery curves for column and conventional flotation are presented in Figures 2.73 and 2.74, respectively. These results provide an indication of the ultimate cleanability of the four coals which were tested. As with ash rejection, it can be seen from these results that the column ultimately provides a lower final product ash than can be produced by conventional flotation. This result is most clearly illustrated by the Upper Freeport coal which shows that column flotation reduced the product ash content by approximately 50% over that obtained using
Figure 2.72 Combustible recovery and separation efficiency as a function of percent ash rejection for column flotation of a 28 mesh X 0, run-of-mine Elkhorn III seam coal.
Summary of Flotation Performance for Coarse Coal

<table>
<thead>
<tr>
<th>Sample</th>
<th>Combustible Recovery (%)</th>
<th>Ash Rejection (%)</th>
<th>Separation Efficiency</th>
<th>Retention Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elkhorn III</td>
<td>98</td>
<td>84</td>
<td>82</td>
<td>14</td>
</tr>
<tr>
<td>Pittsburgh #8</td>
<td>97</td>
<td>65</td>
<td>62</td>
<td>18</td>
</tr>
<tr>
<td>Upper Freeport</td>
<td>97</td>
<td>47</td>
<td>44</td>
<td>20</td>
</tr>
<tr>
<td>Cedar Grove</td>
<td>90</td>
<td>77</td>
<td>67</td>
<td>20</td>
</tr>
<tr>
<td>Conventional</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elkhorn III</td>
<td>97</td>
<td>74</td>
<td>71</td>
<td>15</td>
</tr>
<tr>
<td>Pittsburgh #8</td>
<td>96</td>
<td>50</td>
<td>46</td>
<td>7</td>
</tr>
<tr>
<td>Upper Freeport</td>
<td>98</td>
<td>37</td>
<td>35</td>
<td>12</td>
</tr>
<tr>
<td>Cedar Grove</td>
<td>85</td>
<td>76</td>
<td>61</td>
<td>12</td>
</tr>
</tbody>
</table>
Figure 2.73 Combustible recovery as a function of product ash content for column flotation of 28 mesh X 0, run-of-mine coal.
Figure 2.74 Combustible recovery as a function of product ash content for conventional flotation of 28 mesh X 0, run-of-mine coal.
conventional flotation. In the case of the Elkhorn III coal the increased ash rejection provided by the counter current water enabled the column to produce a superclean coal product (< 2% ash) at a 28 x 0 mesh grind. This is due to the fact that the ash associated with the Elkhorn III sample consists mainly of a fine, liberated clay.

An additional advantage of the MCF cell is the use of fine bubbles for particle collection. The improved collection efficiency associated with fine "microbubbles" has been well documented by numerous researchers (Yoon and Luttrell, 1985; Luttrell, 1986). In fact, it has been shown that the flotation rate constant is inversely proportional to the cubic power of bubble diameter. The flotation rate constant (k) increases according to the following relationship:

\[
k = \frac{3P}{2D_b} \cdot V_g \quad [2.4]
\]

where,

- \( P \) = the probability of particle collection (\( = 1/D_b^3 \)),
- \( V_g \) = the superficial gas velocity, and
- \( D_b \) = the bubble diameter.

This relationship is illustrated in Figure 2.75 which shows the flotation rate constant as a function of bubble size for column flotation of an Elkhorn III seam coal. It can be seen from this result that the slope of this relationship closely approaches the theoretical line also shown in Figure 2.75. The true advantage of
Figure 2.75  Flotation rate constant as a function of bubble diameter for 6-inch column flotation tests of an Elkhorn III seam coal.
micronized flotation, therefore, lies in the increased flotation rates available for fine particle flotation (i.e., 5 micron mean particle size).

In order to investigate the response of both column and conventional flotation, a series of comparison tests were also conducted for micronized coal flotation. A procedure similar to that used for coarse coal flotation was utilized to evaluate cell performance for micronized coal flotation. These results are shown in Figures 2.76 and 2.77 for column and conventional flotation, respectively. For each case it can be seen that the optimum separation efficiency occurs at the peak of the ash rejection versus combustible recovery curve.

The results of the micronized flotation tests are summarized in Table 2.7. It can be seen that the column outperforms conventional flotation in terms of combustible recovery, ash rejection and separation efficiency. In fact, conventional flotation was never able to achieve recovery levels of 90 percent or greater. In this instance, the column and conventional cells were both operating at the same retention time. Therefore, there does not appear to be any advantages associated with conventional flotation in regards to micronized coal flotation.

As one would expect, micronization should provide an additional increase in ash rejection as compared to coarse coal flotation of the same seam. This is a result of the increased liberation associated with the reduction in particle size. This can be seen in Figure 2.78 which shows combustible recovery as a function of product grade for
Figure 2.76 Combustible recovery and separation efficiency as a function of percent ash rejection for column flotation of a Cedar Grove seam coal.
Figure 2.77 Combustible recovery and separation efficiency as a function of percent ash rejection for conventional flotation of a Cedar Grove seam coal.
Table 2.7

Summary of Flotation Performance for Micronized Coal

<table>
<thead>
<tr>
<th></th>
<th>Combustible Recovery (%)</th>
<th>Ash Rejection (%)</th>
<th>Separation Efficiency</th>
<th>Retention Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
<td>96</td>
<td>70</td>
<td>66</td>
<td>26</td>
</tr>
<tr>
<td>Conventional</td>
<td>87</td>
<td>68</td>
<td>55</td>
<td>26</td>
</tr>
</tbody>
</table>
Figure 2.78  Combustible recovery as a function of product ash content for column and conventional flotation of micronized and coarse Cedar Grove seam coal.
both the 20 mesh X 0 and micronized Cedar Grove coal. In this case, micronization decreased the final product ash by 0.5 to 1 percent. Similar results were obtained for conventional flotation.

According to the results obtained in this investigation, neither of the flotation technologies should be considered exclusively for all situations. It is recommended that the selection of either column or conventional flotation be determined on a case-by-case basis. The choice is most strongly influenced by the type of sample to be processed. For the case of a high ash cyclone overflow material, the column is clearly most advantageous due to the increased ash rejection provided by the wash water. In most cases conventional flotation is not capable of making grade on this type of sample due to the high degree of ash entrainment associated with water recovery.

However, conventional flotation may provide enough ash reduction for lower ash, coarse coals to meet the specified product grade. In this case, certain advantages in throughput exist with conventional flotation. Although the column can provide a higher degree of ash rejection, the flow rate of wash water through the column detracts from the available retention time. Therefore, the retention time required for a given feed flow rate will always be slightly longer in the column. Ideally, this effect can be minimized by operating the column at superficial bias flow rates very close to zero. The bias flowrate, \( V_b \), is defined as follows:

\[
V_b = V_t - V_f. \tag{2.5}
\]
It can be seen from equation [2.5] that the feed and tailings flow rates are equal as \( V_b \) approaches zero. By minimizing the bias flow rate the losses in retention time can be reduced. However, this concept is difficult to achieve on an industrial level since there is not an accurate means of measuring bias flowrate at this time. It should be stressed, however, that even for this type of feed the column provides a higher degree of ash rejection.

Since entrainment is a nonselective mechanism of particle recovery, conventional flotation can also achieve additional recovery by this means. The downward flow of wash water in a column, however, restricts the entrainment of coal particles as well as ash. Therefore, the recovery of valuables due to entrainment is eliminated in a column.

For very fine particle flotation the increased rates associated with microbubbles will most likely dictate the use of column flotation rather than conventional. This was clearly illustrated by the results presented in Table 2.7 for micronized coal flotation. This is especially true in the case of kaolin which is a reverse flotation process. In this instance, particle entrainment into the froth results in a loss of product recovery. Therefore, the counter current water actually provides a higher product recovery.

In general, column flotation can provide a higher level of ash rejection while requiring only a fraction of the floor space and approximately one-third of the energy input. However, slightly longer retention times may be required depending on how closely the column is
operated to a zero bias flow rate.

2.6.2 Preliminary Scale-Up Relationships

One of the primary goals of this project was to provide a basis for developing initial scale-up relationships for column flotation. Based on these relationships, projections can be made for larger size columns. In an attempt to quantify column scale-up, the results presented in Figure 2.67 have been replotted. Figure 2.79 shows these results normalized according to the cross-sectional area of each column. Although a fair amount of scatter exists among the data, there appears to be a discernible trend. This indicates that column throughput is a strong function of cell diameter, as expected, however other factors apparently have an influence on throughput as well. Although the throughputs for each column are different, the product grade versus combustible recovery data, shown in Figure 2.80, all lie on the same curve. This seems to indicate that scale-up to larger column diameters is possible without any detrimental effects on product grade.

The scatter in the data presented in Figure 2.79, however, tend to indicate that column scale-up is not simply a function of cell cross-sectional area. Consideration must also be given to the fact that all of the columns used in this investigation had nearly the same height, therefore the effect of column height becomes difficult to interpret. Past work has indicated that many factors, other than cell cross-sectional area, influence column performance. The controlling
Figure 2.79 Combustible recovery as a function of superficial solids feed rate for column flotation of an Elkhorn III seam coal.
Figure 2.80  Combustible recovery as a function of product ash content for column flotation of an Elkhorn III seam coal.
parameters appear to be axial mixing, residence time, fractional air
hold-up and carry capacity. Therefore, in order to draw specific
conclusions regarding scale-up, each of these parameters need to be
addressed independently. In this manner, the impact of each variable
on flotation performance can be quantified. The findings presented in
Chapters 3 through 5 represent an attempt to more clearly define
mixing, air hold-up and carrying capacity in order to provide an
accurate procedure for scale-up of column flotation.
2.7 Summary and Conclusions

1) A proof-of-concept pilot plant was developed for the purpose of demonstrating the microbubble column flotation technology. The facility was designed such that a direct comparison could be conducted between column and conventional flotation for either coarse (28 mesh X 0) or micronized feeds (-20 microns). Feed rates as high as 500 lb/hr were tested.

2) A series of tests were conducted on two different high ash reject streams (40 to 50% feed ash) obtained from operating coal preparation plants. Results indicate that a clean coal product containing less that 10 percent ash can be produced at combustible recovery levels in excess of 80 percent.

3) Comparison tests have been conducted between microbubble column flotation and conventional flotation. It has been shown that both technologies produce the same level of combustible recovery, although the column provides a higher degree of ash rejection. However, throughput is somewhat lower for the case of columns due to the loss in retention time resulting from the wash water flow rate.

4) Micronized coal flotation tests have also been conducted. Results indicate that superclean coals (less than 2 percent ash) can be produced at a 90 percent combustible recovery from feed coals containing as much as 12 percent ash.
5) Preliminary scale-up data have been collected in order to determine the relationship which exists between throughput and column diameter. Scatter in the data indicates that additional factors, as of yet unquantified, probably influence column scale-up.
2.7 References

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C.C. Dell, U.S. Patent No. 4,028,229


Signet Industrial, "Instruction Manual," Signet Industrial, P.O. Box 5770, El Monte, California.


D. Wang, 1990, personal communication.


Chapter 3

SCALE-UP - MIXING

3.1 Introduction

With the continuing depletion of high-grade mineral deposits, a greater demand will be placed on froth flotation circuits in general, both column and conventional. More recently, columns have gained a great deal of popularity, especially in the area of sulfide mineral flotation. In addition, columns are also beginning to slowly make their way in the coal industry. This can be credited to several reasons. The foremost is the increased separation efficiency associated with columns due to their use of wash water, especially on very fine particles. The second is the increased understanding of the column's fundamental operating parameters. The latter providing the operator with a greater ability to analyze column performance, as well as adapt a measure of process control.

Columns have two advantages in the area of coal preparation: i) increased recovery of fines which are typically discarded, and ii) reduced sulfur content due to the flotation characteristics of pyrite. Since future legislation regarding $SO_X$ emissions from power plants appears imminent, companies must be prepared to accommodate advanced physical cleaning techniques into their current flowsheet. Column flotation is one of the few proven technologies capable of meeting both of the aforementioned requirements. This is especially true if grinding enters the picture as a means of increasing pyritic sulfur
liberation from the coal.

Since column flotation provides a higher separation efficiency, while requiring less energy input and plant floorspace, it is a reasonable expectation that column flotation technology will be incorporated. For this reason it is necessary that a fundamental understanding of column flotation performance and scale-up characteristics be developed which is based on first principles. This will provide plant personnel and design engineers with the proper tools for efficiently incorporating column flotation into their current operation.
3.2 Literature Review

Unlike conventional flotation, columns have only been in existence since the early 1960's. Nonetheless, they have been the subject of numerous experimental investigations. In fact, a great deal of technical literature regarding the application of columns has been published. Unfortunately, not all researchers agree as to which of the numerous parameters are most significant. This is especially true in the area of column flotation scale-up. A thorough examination of the literature indicates that column design can be based on a number of variables. Among the most highly recognized of these are residence time and axial dispersion (mixing).

Much of the previous work regarding axial dispersion has been conducted on bubble columns, commonly used in chemical engineering applications as gas/liquid contactors. The bulk of this work has consisted of determining relationships for the axial dispersion coefficient, $D_a$, as a function of column flows and geometry. For instance, Reith's work, which measured fractional air hold-up and mixing as a function of superficial gas velocity, related $D_a$ to the superficial fluid velocities and column geometry (Reith, Renken and Israel, 1968). This expression was found to hold true as long as the superficial gas velocity was in excess of 10 cm/sec. Unfortunately, this limit is considerably higher than the typical conditions found in column flotation, although the basic groundwork developed by Reith may prove helpful.

Several expression were subsequently developed which related the
dispersion coefficient to various different parameters, such as the energy dissipation rate of the discontinuous phase (Baird and Rice, 1985), the superficial gas velocity (Eissa and Schugerl, 1975; Field and Davidson, 1980) liquid phase viscosity and surface tension characteristics (Alexander and Shah, 1976).

The most prolific work in this area appears to have been that of Joshi and Sharma. They linked the axial dispersion coefficient to vessel diameter and liquid circulation velocities (Joshi and Sharma, 1978) and used this information to develop design equations for column baffles (Joshi and Sharma, 1979). The conclusion of their work was that ring-type baffles should be located at an axial distance of 0.81D and that baffle hole diameters should be 0.72D, where D is the cell diameter. Improvements were also made to energy balance predictions regarding circulation velocity and fractional air hold-up by incorporating a multiple circulation cell model (Joshi and Sharma, 1979; Joshi, 1980).

The bulk of the work in this area has been summarized in a design parameter review paper for bubble column reactors (Shah et al., 1982). This paper evaluates the present state-of-the-art for estimation of parameters in bubble column reactors. It entails all of the pertinent literature and comments on inadequacies in certain areas, as well as outlining recommendations for future work.

Although a wealth of information is available regarding bubble column design and operation, the validity of the relationships have not been proven over a range which is applicable to column flotation. For instance, it is not unusual for bubble column reactors to operate
at superficial gas velocities as high as 45 cm/sec, whereas flotation columns rarely exceed 2-5 cm/sec. Flotation columns also typically have larger length to diameter ratios (>5:1) as compared to bubble column reactors. It was therefore necessary to more closely examine the above described relationships, relative to the operating characteristics associated with column flotation.

One of the first direct applications of axial dispersion concepts to column flotation was directed at reducing column mixing by means of baffling (Rice et al., 1974). Mixing was characterized for both baffled and unbaffled conditions using a salt tracer technique in an operating laboratory column. The axial dispersion coefficient was determined from the resulting residence time distribution. The validity of the tracer pulse technique for determining axial dispersion has been mathematically documented (Michelsen and Ostergaard, 1969, 1970; Anderssen and White, 1971; Turner, 1970; Levenspiel and Turner, 1970).

In this study the effect of convex and concave baffle arrangements on the axial dispersion coefficient was investigated. The results were compared to typical unbaffled conditions. It was determined that a circular, convex, center-mounted baffle system was effective in reducing axial dispersion in flotation columns. The presence of baffles typically provided reductions in dispersion of about 50 percent as opposed to the same operating conditions in an unbaffled column.

A follow-up of this initial work was conducted by Dobby and Finch
in their attempt to develop a scale-up methodology for column flotation (Dobby and Finch, 1985). They utilized residence time distribution measurements in order to determine the dispersion coefficient and mean residence time for both the liquid and solid components within the column. It was determined that the liquid axial dispersion coefficient was linearly dependent upon column diameter, and that the axial dispersion coefficient of the solids is the same as that of the liquid. However, the validity of the latter statement was not quantified for a full range of particle sizes and densities. More recent work has indicated that the axial dispersion coefficient for the liquid phase is not always indicative of that for the solids (Goodall and O'Connor, 1990).

The mean residence time of the solids within the cell was also characterized by correcting the liquid residence time for the particle settling velocity. It was assumed that particles settled under hindered conditions and the appropriate corrections were made for this condition (Richardson and Zaki, 1954). Furthermore, it was found that the residence time of solid particles within the cell was not affected by bubble swarms.

More recently, a study has been conducted which measured the effect of liquid and gas flow rates on axial dispersion (Mavros et al., 1989). It was found that mixing increased with gas flow rate up to superficial velocities of 0.1 cm/sec, beyond which there was no effect on axial dispersion. Conversely, increased liquid flow rates resulted in a decrease in column mixing. Unfortunately, the range of superficial velocities over which these tests were conducted were an
order of magnitude below typical column operating conditions.

Therefore, one must be careful when extrapolating the results of this work for different column applications. It also appears that the range of Peclet numbers which were evaluated do not reflect those which are typically found in column flotation. Therefore, the effect of mixing on column recovery, as predicted by the Levenspiel relationship, would be minimal. This concept will be addressed in greater detail later in this chapter.
3.3 Research Objectives

It is apparent from the available literature that further clarification of column mixing is necessary in regards to the degree of axial dispersion, as well as its significance relative to column performance. Furthermore, the dependence of axial dispersion on the liquid and gas flow rates needs to be quantified over a reasonable range of column operating conditions.

The purpose of the work presented in this chapter was to investigate the effect of axial dispersion on column performance. In order to add to the data base currently available from the literature, a series of tests were conducted over a typical range of liquid and gas flow rates for a number of different column geometries. The mean residence time and axial dispersion coefficients were determined in each test using a tracer pulse technique.

The effect of axial dispersion on column flotation scale-up was also characterized. This was accomplished by developing an expression which describes the change in Peclet number as a function of column geometry and flow conditions. Therefore, the difference in Peclet number between small and larger scale columns, due to changes in operating conditions, could be quantified. The impact of the Peclet number on column recovery was determined by means of the Levenspiel relationship, and this result was incorporated into the overall column scale-up procedure.
3.4 Experimental

3.4.1 Sample Preparation

A run-of-mine coal sample was obtained from the Elkhorn III seam, Kentucky, for use in all of the column flotation tests. Analysis of the raw coal indicated that the feed ash and sulfur content were 9.1% and 0.81%, respectively. Upon receipt of the sample, it was crushed to -1/4-inch in a laboratory jaw crusher and split into representative 1500-gram lots. These small lots were placed in air-tight bags and stored at -20°C in order to minimize oxidation.

Prior to each flotation test, the coal was pulverized to -65 mesh in a laboratory hammer mill. This material was then micronized at 30% solids using a 9-inch Union Process stirred ball mill. A 30-minute grinding time, utilizing 1/8-inch stainless steel balls, provided a final mass median product size of approximately 5 microns, as determined by an Elzone 80XY particle size analyzer. The slurry was then diluted to the proper feed percent solids (5-15%) and conditioned with 1.5 lb/ton of kerosene.

3.4.2 Experimental Apparatus

All of the flotation tests were conducted in 2- and 4-inch diameter, plexiglas columns in which the length-to-diameter ratio was varied from 11:1 to 20:1. The slurry was fed to the column at a point 18 inches below the froth overflow lip. A conductivity probe in the tailings discharge was used to measure the residence time distribution
(RTD) in the collection zone. The collection zone is defined as the section of the column between the froth/pulp interface and the point of bubble injection. The sensor was placed immediately adjacent to the column in order to minimize errors associated with flows through the tailings line. The probe was connected to a YSI Model 32 conductance meter. The 0-2 VDC analog recorder output signal was connected to an A/D converter which was interfaced with an IBM PC. This system provided a means of automatic data logging for the duration of the test at rates up to 10 Hz. The data for each test was stored on diskette for analysis at a later time. A schematic diagram of the system employed in these tests is shown in Figure 3.1.

An electrolyte solution containing 20% KCl by weight was used as a tracer. This solution was added in 30-cc aliquots to the feed line immediately adjacent to the column. The conductance of the tailings discharge was monitored as a function of time until the tracer solution was completely removed from the column.

In order to determine the interstitial liquid velocity $u_l$, it was also necessary to measure fractional air hold-up for each test. This was determined by means of differential pressure measurements along the length of the column. A series of electronic pressure transducers was incorporated into the data logging system in order to provide on-line measurement of fractional air hold-up. The details of this system will be presented in Chapter 4.
Figure 3.1 Schematic diagram of experimental apparatus used for measuring on-line residence time distributions in a flotation column.
3.5 Data Analysis

A typical normalized residence time distribution (RTD) is shown in Figure 3.2. The data have been corrected for background noise and normalized with respect to the initial tracer concentration, \( C_0 \), which is determined from the total area under the initial response curve. The mean residence time, \( \tau_m \), is calculated from the normalized RTD using the following equation:

\[
\tau_m = \frac{\sum C_i t_i \Delta t}{\sum C_i \Delta t},
\]

[3.1]

where \( C_i \) represents the concentration (or conductivity) at time \( t_i \).

It was found in this study that by first determining the mean residence time from equation [1] and then fitting a functional form to an integrated version of Figure 3.2, a more consistent representation of the data could be obtained. The integrated version of Figure 3.2 is known as the F curve (Levenspiel, 1972), and is shown in Figure 3.3. The functional form which is used to fit the F curve is given by:

\[
\frac{C}{C_0} = 1 - \frac{1}{2} \left( 1 - \text{erf} \left( \frac{1 - t/\tau_m}{\sqrt{Pe \tau_m}} \right) \right),
\]

[3.2]

where \( Pe \) represents the Peclet number, a measure of liquid dispersion in the column. The Peclet number is defined as:
Figure 3.2 Example of a measured residence time distribution for a flotation column.
Figure 3.3 Normalized residence time distribution ($F$ curve).
\[ \text{Pe} = \frac{u^*_L}{D^*_L} \]  

where \( u^*_L \) is the interstitial liquid velocity and \( D^*_L \) is the liquid axial dispersion coefficient. Thus, \( \text{Pe} \) represents an adjustable parameter in the fitting process as opposed to the two-parameter curve fits used by other investigators (Rice et al., 1974; Dobby and Finch, 1985). It is important to note that only the liquid dispersion number is being considered here because of the fine particle sizes involved in this analysis. It has been suggested by several investigators that the axial dispersion coefficient of fine solids in a bubble column is the same as that of the liquid (Rice et al., 1974; Imafuku et al., 1968; Argo and Cova, 1965; Dobby and Finch, 1984). Once the \( F \) curve has been obtained, \( \text{Pe} \) is determined by a sum of squares fit of equation [3.2] to the data.
3.6 Discussion

As mentioned previously, one of the main reasons for conducting this work was to establish a relationship between axial dispersion and the geometric and flow characteristics associated with flotation columns. Several empirical relationships currently exist which relate Peclet number to column diameter and superficial gas velocity. However, other significant parameters associated with column performance have not been included in previous investigations. Among these are superficial liquid velocity, fractional air hold-up and bubble diameter. Since it is apparent that all of these variables will have an impact on axial dispersion, it is imperative that they be included in any thorough analysis of column mixing.

The effect of column geometry on mixing is illustrated by the F-curves shown in Figure 3.4. In this case, all of the column flow conditions were held constant while varying the overall column height. It can be seen that an increase in the length-to-diameter ratio from 11:1 to 20:1 moved the response curve towards a more plug flow condition. However, it appears that in order to achieve conditions which approach plug flow an unreasonably high length-to-diameter ratio would be required. Since plant installations typically have L/D ratios ranging from 5:1 to 10:1, it may not be practical to consider conditions beyond this limit.

The results of the above mentioned tracer dispersion tests, along with those of other investigators are shown in Figure 3.5. This plot illustrates the change in Pe as a function of the column length-to-
Figure 3.4 Normalized residence time distributions (F-curve) as a function of column length for flotation of an Elkhorn III seam coal.
Figure 3.5 Experimentally determined Peclet number as a function of column length-to-diameter (L/D) ratio.
diameter ratio, where an increasing Peclet number indicates conditions approaching plug flow. According to these results, it seems apparent that geometry has a definite effect on column performance. This is supported by past work of Dobby and Finch which indicates that dispersion is a function of column diameter. However, their work did not consider the importance of column length, since it was conducted on columns of equal height. The wide scatter in the data shown in Figure 3.5 indicate that conditions other than geometry influence mixing within column. It has, in fact, been demonstrated that axial dispersion is also a function of the gas and liquid flow rates within the column (Mavros et al, 1989; Dobby and Finch, 1984). However, an independent relationship which incorporates all of these variables has not yet been established.

In an attempt to quantify axial mixing, a dimensional analysis was conducted which incorporated all of the variables thought to influence dispersion. These included column length and diameter, gas and liquid flow rates, fractional air hold-up and bubble diameter. As a result of the dimensional analysis procedure, it was determined that the following expression is a possible functional form which relates the controlling variables:

\[ Pe = \Lambda \left( \frac{L \ u_g}{D \ V_g} \right)^m \]  \[ 3.4 \]

where \( L \) is the length of the column recovery zone, \( D \) is the column diameter, \( V_g \) is the superficial gas velocity and \( \Lambda \) and \( m \) are
constants. The interstitial liquid velocity, \( u_\theta \), can be expressed as follows:

\[
u_\theta = \frac{V_1}{(1-\varepsilon)} \tag{3.5}\]

where \( V_1 \) is the superficial liquid velocity and \( \varepsilon \) is the fractional air hold-up. Since bubble diameter has a direct affect on the fractional air hold-up within the column, the effect of bubble size on mixing is inherently included. This point is further illustrated by the independent relationships presented in Chapter 4 which describe fractional air hold-up within the column as a function of bubble diameter and gas and liquid flow rates.

Figure 3.6 shows the change in Peclet number, for various geometries and flow conditions, plotted according to the dimensionless relationship given in equation [3.4]. It can be seen that a definite power relationship exists between the column operating parameters and the measured Peclet number. The normalized, linear version of the mixing data is shown in Figure 3.7. A least squares analysis indicates that the following expression best describes the data over the given range:

\[
Pe = 0.698 \left[ \frac{L u_\theta}{D V_g} \right]^{0.622} \tag{3.6}
\]
Figure 3.6 Experimentally determined Peclet number as a function of dimensionless flows and column geometry.
Figure 3.7  Linearized version of experimentally determined Peclet number as a function of dimensionless flows and column geometry.
The impact of a change in Peclet number on column performance was examined relative to product recovery. Figure 3.8 shows the change in recovery as a function of the dimensionless quantity $kt_m$ for a variety of mixing conditions, where $k$ represents the flotation rate constant. For the case of plug flow and perfectly mixed reactors, recovery is expressed as follows:

Plug Flow: $R = \exp(-kt_m)$ \[3.7\]

Perfectly Mixed: $R = \frac{kt_m}{1+kt_m}$ \[3.8\]

For intermediate cases, the Peclet number can be used to determine the recovery by (Levenspiel, 1962):

$$R = 1 - \frac{4a \exp(Pe/2)}{(1+a)^2 \exp((a/2)Pe) - (1-a)^2 \exp((-a/2)Pe)}$$ \[3.9\]

where,

$$a = \sqrt{1+4kt/Pe}.$$ \[3.10\]

If one considers a practical range of Peclet numbers (1 to 5), as illustrated by the cross-hatched zone in Figure 3.5, it can be seen from Figure 3.8 that the overall change in recovery is minimal. In fact, only in extreme cases, such as the large L/D ratios used in
Figure 3.8 Recovery versus $kT$ for various mixing conditions in a flotation column. The cross-hatched area represents Pe between 1 and 5.
laboratory columns, does the effect of mixing become significant. This implies that care should be taken when using results obtained with laboratory columns to make predictions concerning performance of larger columns.
3.7 Summary and Conclusions

1) Salt tracer studies have been conducted in order to quantify mixing in a laboratory flotation column.

2) Axial dispersion coefficients have been determined as a function of cell geometry and flow rates.

3) A single parameter estimation technique has been developed, utilizing the F-Curve, which provides a more convenient means of determining the Peclet number, as opposed to the two parameter search routine typically used for the interpretation of residence time distribution results.

4) It has been determined that axial dispersion increases with both gas flow rate and column diameter, while a decrease is observed with an increase in either liquid flow rate or column length.

5) An expression has been developed, by means of dimensional analysis, which describes the Peclet number as a function of column flow rates and geometry.

6) The effect of mixing on column performance has been quantified in terms of flotation recovery. It has been determined that axial dispersion has a minor effect on performance within reasonable limits of column operation. However, this influence may become more significant under extreme geometric limitations as often found in laboratory installations.
3.8 Nomenclature

A = coefficient for power relationship (A=0.698)

C_1 = tracer concentration at time (i)

C_0 = initial tracer concentration

D = column diameter

D_g = liquid axial dispersion coefficient

k = flotation rate constant

L = length of column recovery zone

m = exponent for power relationship (m=0.622)

P_e = Péclet number

R = recovery

t = time

u_g = interstitial liquid velocity

V_g = superficial gas velocity

V_1 = interstitial liquid velocity

\( \xi \) = fractional air hold-up

\( \tau_m \) = mean residence time
3.9 References


Chapter 4

SCALE-UP - AIR FRACTION

4.1 Introduction

Of the various parameters which control the overall performance of column flotation one of the most important is bubble size within the column recovery zone. The effect of bubble diameter on particle collection has been well documented by past researchers. However, its significance in maintaining a constant level of performance, relative to column scale-up considerations, needs to be further evaluated.

The effect of fractional air hold-up on column performance is equally as important. Past investigations have shown that fractional air hold-up can affect column mixing, retention time and recovery. In order to adequately represent column performance upon scale-up, these inter-relationships must be considered. Unfortunately, due to a lack of information regarding behavior of the gas phase, these effects tend to be neglected in many analyses (Villar, 1989). Therefore, it is the purpose of the work presented in this chapter to establish an expression which relates the fractional air hold-up to various parameters within the column. These include volumetric flows through the column, column geometry and the bubble size in the column recovery zone. Therefore, the combined effect of axial dispersion and fractional air hold-up on column performance can be quantified upon scale-up.
4.2 Literature Review

In order to properly address the problem of column flotation scale-up, the factors which most strongly influence performance must be considered. This entails accounting for variations in both the dependent and independent parameters associated with column operation. Parameters such as feed pulp density, flow rate, size consist, mineral floatability, pulp level and reagent dosage all directly influence column performance. The intricate relationship between bubble diameter and fractional air hold-up within the column recovery zone appears to have one of the strongest effects on column response. Therefore, the volumetric air fraction within the column has been the subject of numerous investigations.

Past work has indicated that several parameters influence the fractional air hold-up within a column. These include the volumetric flow rate of gas and liquid and the bubble diameter within the column collection zone. The effect of bubble size should be emphasized due to its strong influence on recovery through changes in the flotation rate constant. This effect has been well documented for both column and conventional cells by numerous researchers (Anfruns and Kitchener, 1977; Weber and Paddock, 1983; Yoon and Luttrell, 1986; Dobby and Finch, 1985; Luttrell, 1986; Jordan, 1988; Schulze et al, 1989). Yoon and Luttrell have shown, through extensive hydrodynamic calculations, that the flotation rate constant is indirectly proportional to the cube of the bubble diameter. This implies that even minor changes in bubble size will have a drastic effect on recovery rates.
Dobby and Finch have extended this work to include the effects of gas flow rate and air fraction on the flotation rate constant (Dobby and Finch, 1985). By incorporating a hindered rise relationship into the standard equation for slip velocity, they concluded that a maximum gas rate exists for a given bubble diameter. Beyond this limit flooding conditions will occur within the column. Flooding is defined as putting gas into the column more quickly than it can rise out due to the natural buoyancy of the bubbles. At this point gross coalescence can occur resulting in the formation of large bubbles. The increased rise velocity associated with the larger bubbles can result in a high degree of turbulence (mixing) within the column. This effect, commonly referred to as "burping", can cause increased mixing in the expanded bubble zone. This can contribute to a deterioration in column performance. It was further determined that the maximum collection rate constant is obtained while operating at or close to the maximum gas velocity.

The strong link between bubble diameter and column performance has generated much interest in developing a means of measuring bubble size, either directly or indirectly. Several direct measurements have been employed ranging from simple photographic measurement to a unique on-line size detection system (Randall et al, 1988; O'Conner et al, 1990). The later utilizes a vacuum system to draw the bubbles through a capillary which passes between two pairs of photo-transistor-LED detectors. This arrangement provides a measurement of bubble velocity and length. The data is then used to calculate the total volume in order to normalize the bubble size distribution.
Other studies have been conducted which correlate bubble rise velocity and flow conditions within the column to bubble diameter (Yianatos et al, 1986, 1988; Dobby et al, 1988). In each instance, a hindered bubble rise velocity was equated to the standard relationship for slip velocity. Bubble diameter was then determined from flow conditions within the column. This provided a reasonable estimation of bubble diameter, although it did not account for bubbles lost with the tailings flow or the effect of bubble size distribution.

Recent work has indicated that as gas rate is increased three phenomena occur which limit the maximum gas rate. The first is a loss of positive bias. Bias flow rate is defined as tailings flow rate minus the feed flow rate. A positive bias flow indicates that the net flow through the column is down. When this condition is violated the net flow shifts toward the direction of the column overflow. This results in a loss of selectivity due to recovery of a portion of the feed water. A further increase in the gas flow rate results in a loss of bubbly flow followed by deterioration of the pulp/froth interface (Finch, 1990). Using a drift flux model, theoretical estimates were compared with experimentally determined air fractions. Experimental measurements were determined by means of a conductivity probe. This apparatus has been successfully used to measure the volume fraction of air in both the bubbly and froth zones in a column (Yianatos, 1985). Good agreement has been observed between estimated and measured values of air fraction using this technique. Froth profiling, using the concept of conductivity, has also been used in locating the pulp/froth
interface for level control purposes (Gomez et al, 1989b; Huls et al, 1990)

As mentioned in Chapter 3, an extensive amount of literature exists which describes fractional air hold-up in gas/liquid contactors. Unfortunately, these types of reactors typically operate at gas flow rates which are an order of magnitude beyond flotation columns. These conditions would drive the column operation away from bubbly flow and would approach the churn/turbulent regime. In this instance, a severe degradation in performance would occur due to excessive mixing and loss of bubble/particle collection efficiency.
4.3 Research Objectives

Although relationships have been developed which describe fractional air hold-up in the flotation process, an overall solution to the problem has not been presented. A few empirical expressions have been put forth for bubble column reactors, however these are usually not applicable to column flotation. Therefore, it is the purpose of this work to develop a definitive expression for the volumetric air fraction within a column.

Using a population balance model approach, new relationships were developed which describe the fractional air hold-up within a column. The effect of geometry and flow conditions were considered. This model is capable of making allowances for bubble size distributions, as well as internal transport of bubbles due to flow conditions. These considerations have often been neglected in past studies.

Laboratory tests were conducted in order to validate the model over a typical range of column operating conditions. A series of simulations were conducted to evaluate the effect of bubble size, as well as gas and liquid flow rates, on air hold-up within the column.
4.4 Model Development

In order to properly characterize the behavior of the air phase in a flotation column, a mathematical model has been developed which describes the change in air fraction with respect to the flows associated with the column. In this model, the column is divided into two zones as shown in Figure 4.1. Zone 1 represents the low air density portion of the column below the air inlet and accounts for bubbles which are lost with the tailings flow. Zone 2 represents the remainder of the column up to the froth zone. The net change in the air content with time is determined by the difference of the volume of air per unit of time entering a zone and the volume of air per unit of time leaving a zone. Quantitatively, this balance may be given as:

\[
\text{Accumulation} = \text{Input} - \text{Output} + \text{Generation.} \quad [4.1]
\]

In the case of air, the volumetric flow which enters zone [2] consists of the aeration rate \(Q_a\), the air carried into the zone from above by the downward flow of pulp in the column \(Q_{d2}\) and the flow of bubbles into zone [2] from zone [1] due to buoyancy \(U_1A_1c_1\). Flows leaving the zone include the volume flow of air to the reject zone \(Q_{t2}\) and again the flow due to the buoyancy of the bubbles leaving zone [2] \(U_2A_2c_2\). This is shown schematically in Figure 4.2. Each flow can then be converted to fractional air content by dividing by the total volume of the zone \(V_z\). Using the above terms, the flow balance for zone [2] can be derived as follows:
Figure 4.1 Schematic diagram showing the physical representation of the zones used in the population balance model.
Figure 4.2 Schematic diagram of flows in the air phase model.
\[ c = \frac{V_a}{V_z}, \quad [4.2] \]

where by rearranging and differentiating the following functional form can be developed:

\[
\frac{dV_a}{dt} = \frac{d(eV_z)}{dt}. \quad [4.3]
\]

Upon substitution of the appropriate terms, the following expression for the air hold-up in zone [2] can be derived:

\[
\frac{dc_2}{dt} = \frac{[(Q_g + Q_d e_2 + U_1 A e_1) - (Q_t e_2 + U_2 A e_2)]/V_z}{V_z}. \quad [4.4]
\]

where

- \(Q_g\) = aeration rate,
- \(Q_t\) = downward flow rate of liquid in the column,
- \(e_1\) = fractional air hold-up in zone \([i]\),
- \(U_1\) = bubble hindered rise velocity in zone \([i]\),
- \(A\) = column cross-sectional area,
- \(V_a\) = volume of air in zone \([i]\),
- \(V_z\) = volume of zone \([i]\),

and, from the overall volume balance around the system:
\[ Q_d = Q_t - Q_g. \]  \[ 4.5 \]

A similar volume balance for zone [1] is given by:

\[ \frac{dc_1}{dt} = \frac{(Q_t c_2 - (Q_t c_1 + u_t c_1))}{V_z} \]  \[ 4.6 \]

Equations [4.5] and [4.6] can be solved numerically to yield a dynamic solution provided an expression for the hindered rise velocity of a bubble can be determined. In this analysis, the expression,

\[ U_i = U_b (1 - c_i)^m \]  \[ 4.7 \]

was used to represent the hindered rise velocity of the bubble, where \( U_b \) is the terminal rise velocity of a bubble given by the following (Luttrell, 1986):

\[ U_b = 148 D_b^{1.14}. \]  \[ 4.8 \]

where \( D_b \) is bubble diameter in centimeters and \( U_b \) in units of centimeters per second. This approach is similar to that of Richardson and Zaki for the hindered settling of particles and is applicable up to a bubble size of approximately 2 mm (Richardson and Zaki, 1954). The exponent \( m \) is an experimentally determined power factor for hindered rise.
4.5 Experimental

A 5-cm diameter by 150-cm high flotation column was constructed in order to validate the fractional air hold-up predictions from the model equations. The column, shown schematically in Figure 4.3, was equipped with two electronic pressure transducers which were interfaced with an IBM portable PC. The transducers were used to measure the differential pressure in the recovery zone which was converted to fractional air hold-up in the column using the following expression:

$$\varepsilon = 1 - \Delta P / \Delta P_w.$$  \quad [4.9]

In this expression, $\Delta P_w$ represents the differential pressure in the presence of water and $\Delta P$ is the differential pressure in the presence of an air/water suspension. The fractional air hold-up was then recorded as a function of gas and liquid flow rates.

Bubbles were generated using a 6-μm porous frit in the bottom of the column, while the tailings flow rate was controlled using a variable speed peristaltic pump. Since a certain amount of air can be lost with the tailings, the entire flow was passed through a deaeration sump before it was recycled back into the column. In this manner, the air fraction in the column was only influenced by air introduced in the porous frit and not by recycled air. Coalescence of bubbles in the column was minimized by maintaining a high concentration of frother during the experiments. Dowfroth 1012 at a concentration of 25μg/l was used as the frother in all tests.
Figure 4.3 Schematic diagram of experimental apparatus used for measuring air fraction in a flotation column.
4.6 Results and Discussion

The validation of the air phase model is shown in Figure 4.4 in terms of air fraction in the column versus superficial liquid velocity down the column. The superficial gas velocity was maintained at 1 cm/s for all tests. The bubble size in these tests was determined by fitting the air fraction at a superficial liquid velocity of 0.25 cm/s to the model equations. The best fit was obtained using a monosized bubble distribution of 535 μm. This size was then used to predict the air fraction in the column under all other flow conditions. As shown, the model appears to fit the experimental data quite well with only a slight deviation as superficial liquid velocity is increased.

In order to validate the model over a wide range of superficial gas velocities, several simulations were conducted in an attempt to fit existing experimental data (Yianatos et al, 1989). Results of these simulations are shown in Figure 4.5. It can be seen that a good correlation exists between the experimental data and the model predictions over a range of superficial gas velocities from 0 to 6 cm/s. As previously mentioned, at these gas flow rates column operation tends to leave the bubbly flow regime. This is indicated by the scatter in the air fraction data for superficial gas rates beyond 5 cm/sec. Therefore, it can be concluded that this is a reasonable upper limit for the simulator.

The model was then used to study the effect of liquid and gas flows, bubble size and bubble size distribution on the air fraction in the column. Figure 4.6 shows the effect of superficial liquid and gas
Figure 4.4 Experimental validation of the air fraction model.
Figure 4.5 Experimental validation of the air fraction model using data from external investigators (Yianatos et al, 1989).
Figure 4.6 Air fraction as a function of superficial gas velocity for superficial liquid velocities of 0.0 - 2.0 cm/s and 1000-μm bubbles.
velocity on the air fraction in the column for 1000 \( \mu m \) monosized bubbles. It can be seen that the gas velocity has a much more pronounced effect on the air fraction in the column than does the liquid velocity although both affect air fraction in the same manner (i.e., as either flow increases, air fraction increases). The dashed line indicates a boundary where the model does not produce a stable solution. In other words, the operating conditions beyond this boundary cannot exist according to the model predictions. This can be interpreted as having exceeded the maximum gas flow rate under which bubbly flow conditions can exist.

When 600 \( \mu m \) bubbles are used, as shown in Figure 4.7, the same trends are observed for the superficial velocities; however, the region of operating conditions which cannot exist increases. In general, it appears that the superficial gas velocity must be decreased in order to operate at high liquid velocities as bubble size is decreased. It should be noted, however, that while the flotation rate constant is directly proportional to the superficial gas velocity \( (V_g) \), it is inversely proportional to \( D_b^3 \) (Luttrell et al., 1988). Thus, there is still a greater incentive to use small bubbles even if the gas velocity must be decreased.

Figure 4.8 shows the effect of a bubble size distribution on the air fraction and flow conditions which can exist in a column. In this case, a uniform size distribution of bubbles having a mean size of 600 \( \mu m \) was used. As shown, the addition of a bubble size distribution tends to further reduce the flow conditions which can exist in a column as compared to the results shown for the monosized bubbles. It
Figure 4.7  Air fraction as a function of superficial gas velocity for superficial liquid velocities of 0.0 - 2.0 cm/s and 600-μm bubbles.
Figure 4.8 Air fraction as a function of superficial gas velocity for superficial liquid velocities of 0.0 - 2.0 cm/s and a uniform bubble size distribution having a volume mean of 600 μm.
appears that the presence of the fine bubbles in the distribution
tends to increase the air fraction in the column. Thus, the limiting
flow conditions are reached sooner.

The effect of bubble size distribution on air fraction was
further evaluated by conducting a series of simulations utilizing
artificially generated bubble size distributions. These are shown in
Figure 4.9 as a function of the size distribution modulus. All of the
size distributions were chosen such that the volume mean bubble size
was 1000 microns. The widest distribution covers a range of 300 to
1700 microns. These distributions were converted to a number mean
basis within the model in order to conduct the simulations. Figure
4.10 illustrates the results of these simulations. In this instance,
it can be seen that the fractional air hold-up increases dramatically
as the distribution modulus deviates further from a monosize bubble.
In general, the results of the model simulations indicate that for the
floitation of fine particles it is best to have a narrow size
distribution of fine bubbles at the highest superficial gas velocity
attainable. It is important to note, however, that the gas velocity
must be decreased as bubble size is reduced so that flooding
conditions are not exceeded.

Having established a relationship for \( \varepsilon \) as a function of flow
conditions and bubble size, a scale-up procedure can be developed
which incorporates the changes in \( \varepsilon \) which occur as column dimensions
and flows change. However, the complexity of the relationships
dictate that an analytical solution is not possible, therefore an
Figure 4.9 Cumulative bubble size distribution as a function of the size distribution modulus.
Figure 4.10  Air fraction as a function of superficial gas velocity for a superficial liquid velocity of 1.0 cm/sec and bubble size distributions ranging from monosize to uniform having a volume mean of 1000 μm.
iterative solution methodology is required. This procedure could utilize either a numerical solution approach or a steady-state solution for the above mentioned differential equations. Furthermore, by incorporating this model into a microprocessor along with inputs from differential pressure and flow sensors, it is possible to obtain real-time bubble size measurements. This information could be used as part of a control scheme to adjust frother addition in order to maintain a constant bubble size in the column as feed conditions change. It should also be noted that this simulator is not limited to a specific type of bubble generation system. Therefore, the model can be used to evaluate different systems in order to determine which bubble generation configuration provides the best aeration under a given set of flow conditions.
4.7 Summary and Conclusions

1) A mathematical model has been developed which describes the volume fraction of air as a function of bubble size distribution and flow conditions within the column.

2) The population balance model has been experimentally validated over a typical range of operating conditions using data produced in a 2-inch diameter laboratory column.

3) Simulations have been conducted which indicate that the volume fraction of air in the column increases with an increase in either superficial liquid or gas velocities.

4) The range of flow conditions which can exist in the column is reduced as bubble size is reduced. For example, for a given liquid flow rate, superficial gas velocity must be decreased as bubble size is decreased.

5) The range of flow conditions which can exist in the column is reduced when going from monosized bubbles to a bubble size distribution.

6) In general, the model indicates that narrow size distributions of fine bubbles produce the optimum conditions for treating fine particles.
4.8 Nomenclature

A = column cross-sectional area

$D_b$ = bubble diameter

$m$ = experimentally determined power factor for hindered rise

$Q_d$ = volumetric downward flow rate in the column recovery zone

$Q_g$ = aeration rate

$Q_t$ = volumetric tailings flow rate

$t$ = time

$U_b$ = bubble terminal rise velocity

$U_i$ = bubble hindered rise velocity in zone (i)

$V_a$ = volume of air in zone (i)

$V_z$ = zone volume

$\Delta P$ = differential pressure in the presence of an air/water suspension

$\Delta P_w$ = differential pressure in the presence of water

$c_i$ = volumetric air fraction in zone (i)
4.9 References


5.1 Introduction

In recent years, column flotation technology has gained a deal of popularity. Applications have spread from the base metal industry to the non-metallic minerals, oxide minerals and even coal flotation. The increased usage has generated a great deal of interest in the development of scale-up relationships for the purpose of column design. Unfortunately, many of the design practices typically implemented for conventional flotation machines are not applicable to the design of flotation columns. Therefore, new relationships are required which are capable of predicting column throughput and performance for a variety of applications.

Although a number of scale-up relationships have been put forth in regards to column scale-up, many of these have addressed only the column recovery zone. Recent work, however, has indicated that throughput and grade limitations are more strongly governed by froth zone characteristics rather than those of the recovery zone. For this reason it is necessary that the froth zone performance be included into any scale-up procedure.
5.2 Literature Review

A great deal of work has been generated over the past five years in regards to column flotation scale-up. Particular attention has been given to the controlling interest of the froth phase in column performance. Fast scale-up strategies predominantly addressed the column recovery zone, while tending to neglect contributions made by the froth. Recent work has indicated that changes in aeration and wash water rate, as well as frother addition, strongly influence the characteristics which control froth stability (Yianatos, Finch and LaPlante; 1986). These characteristics include froth depth, bubble size, bubble coalescence and fractional air hold.

The froth zone associated with flotation columns is commonly recognized as having three distinct regions: an expanded bubble zone, a packed bubble bed and a conventional draining froth. The expanded bubble zone, which originates at the froth/pulp interface, consists of small, narrowly distributed bubbles. It has been theorized that the majority of the froth up-grading and particle detachment occur at this point due to bubble coalescence. It is believed that bubble coalescence is a result of shock pressure waves generated by the impact of bubbles rising out of the pulp phase and colliding with the froth interface (Finch, Yianatos and Dobby, 1989). Bubble coalescence may also be enhanced by the low frother concentrations present in this zone. This is a result of dilution caused by the addition of large amounts of counter current wash water.

The packed bubble bed extends to the wash water inlet and is
characterized by plug flow movement and a lower degree of bubble coalescence. Bubbles are typically uniform in size and maintain their spherical shape. A conventional draining froth exists between the wash water addition point and the top of the cell. Bubbles in this region are packed to the point of deformation resulting in fractional air contents which usually exceed 80 percent. An increase in coalescence is observed due to the increased bubble residence time and high solids loading. This is a result of the slow transfer rates associated with froth removal from the top of the cell.

The overall increase in bubble diameter from the pulp zone to the froth zone results in a corresponding reduction in bubble surface area. The decrease in surface area creates a limit in regards to the rate at which solids can be delivered into the overflow. The "carrying capacity" limit is one of the factors which determine the maximum throughput achieved by a given column.

In an attempt to gain a better understanding of the column froth zone, several models have been developed which attempt to describe particle concentration, bubble diameter and air content as a function of froth height. Unfortunately, the complexities associated with a three phase froth structure result in unknown parameters which are not easily obtainable. In general, it is concluded that deep froths are necessary in order to ensure proper cleaning action. Shorter froths are ineffective due to increased mixing caused by the turbulence associated with the addition of wash water (Yianatos et al, 1987; Moys, 1984).

One of the first attempts at developing an understanding of the
carrying capacity phenomenon stemmed from an investigation of the gas phase behavior in the pulp (Xu, Finch and Yianatos, 1987). Results of this work indicated that for a given bubble diameter and liquid velocity a maximum gas flow rate exists. This conclusion was based on principles which equated a hindered bubble rise velocity to the standard definition of slip velocity in a countercurrent flow system (Masliyah, 1979). It was concluded that the maximum carrying capacity exists at the maximum superficial gas rate. This analysis was later modified to account for the bubble/particle bulk density in order to develop an expression which represents the mass of solids carried out of the cell per unit volume of gas.

Later work by Espinosa-Gomez et al. modified the aforementioned expression in order to eliminate its dependence on gas flow rate (Espinosa-Gomez et al., 1988). The carrying capacity per unit volume of gas, \( C_g \), was expressed as follows:

\[
C_g = K \frac{\pi d_p \rho_p}{d_b}
\]  

[5.1]

where \( d_p \) and \( d_b \) are the particle and bubble diameter, respectively, \( \rho_p \) is the particle density and \( K \) is a constant which incorporates phenomena such as mass transport over the cell lip and shared particle loading. The latter occurring when a single particle interconnects two adjoining bubbles.

In order to derive a functional form of the gas carrying capacity
which is independent of gas flow rate, the following modifications were incorporated into equation [5.1]:

\[ C_a = 60C_g J_g \frac{60Knd_p \rho_p J_g}{d_b} \]  

[5.2]

where \( C_a \), the mass of product per unit of time per unit of cell cross-sectional area, represents a carrying capacity which is independent of gas flow rate, \( J_g \).

It has been shown that for a porous bubbler the bubble diameter can be expressed as a function of the gas flow rate as follows (Dobby and Finch, 1986):

\[ D_b = c J_g^n \]  

[5.3]

where the constants \( c \) and \( n \) are a function of frit pore diameter and frother concentration. It can be seen from equation [5.3] that bubble diameter increases with an increase in gas flow rate. This results in a decrease in bubble surface area. Therefore, although an increase in superficial gas rate would indicate a higher carrying capacity, this effect is offset by the increase in bubble size described by equation [5.3]. In which case the carrying capacity, \( C_a \), would be independent of bubble diameter and gas flow rate. The limited available data from which experimental carrying capacities can be determined indicate that the following relationship describes \( C_a \) reasonably well for columns up
to 1 meter in diameter (Espinosa-Gomez et al, 1988):

\[ C_a = 0.0682 \, \bar{d} \rho_p \]  

[5.4]

where \( \bar{d} \) is in units of microns and \( \rho_p \) is density in grams per cubic centimeter.

The presence of a carrying capacity limit in a flotation column also implies that a certain degree of "dropback" exists. Dropback is defined as the percentage of solids entering the froth that are rejected back into the collection zone as a result of bubble coalescence. It has been shown that in some cases this circulating load can approach two or three hundred percent of the column feed rate (Luttrell et al., 1990). Recirculation of this higher grade material results in a further up-grading of the column product beyond that provided by wash water alone.

Direct measurements of froth drop back rates have been conducted on a laboratory basis. These results indicate that the froth zone recovery is independent of many of the pulp characteristics, such as the length of the column collection zone (Falutsu and Dobby, 1989). It has been speculated that the circulating load may be a result of the shock wave generated at the pulp/froth interface, as mentioned previously, and not due to bubble coalescence. It is apparent that more detailed experimental results are required in order to support either conclusion. Results also indicate that the turbulence associated with high wash water flow rates can cause bubble/particle detachment; therefore, only slight positive bias rates are recommended.
5.3 Research Objectives

It appears obvious from the previous discussion that column flotation performance can be strongly governed by the existence of a carrying capacity limitation. Therefore, this phenomenon should be incorporated into any scale-up procedure in order to properly represent column throughput and recovery. However, this concept is case specific and may not be relevant in all instances. The purpose of this work was to characterize rate limited or carrying capacity limited conditions in column flotation and to incorporate this behavior into column scale-up.

Rate limited applications are considered to be regulated by pulp interactions. These are typically interpreted as a low rate of bubble particle collision/adhesion (i.e., a low flotation rate constant or a low concentration of floatable species in the pulp). In this instance, bubble loading in the flotation pulp may never exceed the limitations specified by the froth carrying capacity, regardless of the column height. A typical example would be flotation of kaolin where particle kinetics are severely hampered by the extremely fine particle sizes (less than 10 microns).

In order to incorporate froth carrying capacity into the existing scale-up procedure, an expression was derived in terms of the column superficial feed velocity. This provided an easy way of incorporating froth bubble loading into the scale-up procedure as a limiting case to rate governed applications.
5.4 Model Development

In order to incorporate the froth carrying capacity limitations into the scale-up procedure, it was first necessary to develop a model which provided an adequate representation of the real system. This required that several assumptions be made regarding the physical transport of solids through the froth zone. In this application it was assumed that particles can only be removed by means of bubble/particle attachment. In this manner, it is possible to monitor solids removal by estimating the available bubble surface area. A second possible means of particle transfer is by entrapment between bubble lamella, although it is believed that this effect is minimal in the froth zone due to the high volumetric air fraction. Therefore, the latter effect is not considered in this approach.

5.4.1 Bubble Loading

The bubble surface area per unit time can be determined from the number of bubbles which pass through the froth zone. The total number of bubbles per unit time, \( \psi_b \), can be determined from the gas flow rate and bubble diameter as follows:

\[
\psi_b = \frac{6Q_g}{\pi D_b z^3}.
\]  

[5.5]

where
\( Q_g \) = the volumetric gas flow rate \( \left[ (\pi D_c^2/4) \times V_g \right] \),

\( D_c \) = the cell diameter, and

\( D_{b,f} \) = the average bubble diameter within the froth zone.

The surface area per unit time can then be determined by multiplying \( \psi_b \) by the surface area per bubble. Simplifying terms, an expression for surface area rate, \( S_{Ab} \), can be developed as follows:

\[
S_{Ab} = \psi_b \times \pi D_{b,f}^2, \tag{5.6}
\]

Substituting equation [5.5] into [5.6] yields:

\[
S_{Ab} = \frac{3\pi V_g D_c^2}{2D_{b,f}} \tag{5.7}
\]

The carrying capacity limitation, or maximum carrying capacity, can be determined from the surface area rate and the maximum particle packing density on a bubble. The particle packing density is determined from the particle cross-sectional area according to the following:

\[
\Gamma_p = S_{Ab} \times \frac{4\phi}{\pi D_p^2}, \tag{5.8}
\]

Where
$D_p$ = the average particle diameter within the froth zone, and
$\phi$ = the particle packing coefficient on the bubble surface.

Since carrying capacity due to bubble loading, $C_{bl}$, is defined as the mass of solids removed from the cell overflow per unit of time, it is necessary to convert equation [5.8] to a mass basis. This is accomplished by converting the particle number rate, $\Gamma_p$, to mass as follows:

$$C_{bl} = \Gamma_p \times \frac{\pi D_p^3}{6} \times \rho_{sp}$$  \[5.9\]

where

$\rho_{sp}$ = the average particle density within the froth zone.

Substituting terms into equation [5.9] and simplifying yields the following expression for the maximum solids carrying capacity due to bubble loading restrictions in the froth zone:

$$C_{bl} = \frac{\pi \Phi V_g D_p D_c^2 \rho_{sp}}{D_{b,f}}$$  \[5.10\]

5.4.2 Scale-Up Limitations

Having established a relationship which describes the froth carrying capacity limitation, it was necessary to develop a convenient
means of incorporating this term into the scale-up procedure. Since the scale-up procedure is used to determine the maximum superficial feed rate, it was decided to express the carrying capacity limitation as a function of superficial feed rate as well. This requires that several of the basic relationships describing the solid/liquid ratios and flow rates be defined.

The ratio between the product and feed solids flow rate, the fractional yield, is defined as:

\[
Y_f = \frac{C}{F} \quad [5.11]
\]

where

- \( C \) = the mass flow rate of solids to the concentrate, and
- \( F \) = the mass flow rate of solids in the feed.

Similarly, the fractional feed solids is represented as follows:

\[
S_f = \frac{F}{F+Y} \quad [5.12]
\]

where

- \( Y \) = the mass flow rate of water in the feed.

Finally, the total volumetric flow rate of feed to the cell, \( Q_f \),
can be represented as follows:

\[ Q_f = \frac{F}{\rho_{sf}} + \frac{Y}{\rho_{lf}} \]  \[5.13\]

where

\[ \rho_{sf} = \text{density of solids in the feed, and} \]
\[ \rho_{lf} = \text{density of liquid in the feed} \ (\rho_{SL} = 1). \]

By combining equations [5.11], [5.12] and [5.13], the following relationship can be established:

\[ C = \frac{Y_f Q_f}{1 + \frac{1}{\rho_{sf}} + \frac{1}{S_f} - 1} \]  \[5.14\]

This expression represents the mass flow rate of solids to the concentrate as a function of the column feed conditions and product yield. It can be seen that an increase in either the feed percent solids or feed flow rate will result in an increase in the product solids mass flowrate. However, as described above, restrictions apply to the total mass of solids which can be removed from the cell due to carrying capacity limitations. The mass flow of solids to the concentrate, \( C \), can approach the carrying capacity limitation, \( C_{BL} \), but cannot exceed this value. Therefore, equations [5.10] and [5.11] can be combined to represent the carrying capacity limitation in terms
of the volume feed rate of slurry.

Equating equations [5.10] and [5.11] yields the following:

\[
\frac{Y_f Q_f}{1 + \frac{1}{\rho_{sf} S_f}} = \frac{\pi \phi V_g D_p D_c^2 \rho_{sp}}{D_b, f}, \tag{5.15}
\]

and rearranging terms provides the following expression:

\[
V_{f,m} = \frac{\frac{\phi V_g D_p \rho_{sp}}{Y_f D_b, f}}{1 + \frac{1}{\rho_{sf} S_f}} - 1 \tag{5.16}
\]

\(V_{f,m}\) represents the maximum superficial feed rate for the column under the specified conditions. Any further increase in feed flow rate will result in a loss of product recovery due to the inability of the froth structure to remove the solids.
5.5 Results and Discussion

For any properly operated column, the bubble diameter in the froth is always significantly larger than that of the pulp. Therefore, as previously mentioned, a certain degree of solids recirculation will exist between the pulp and froth zones of the column. This indicates that if only the pulp zone is considered upon scale-up, the predicted throughput may exceed that which is allowable according to the froth carrying capacity limitations. According to several investigators, the average bubble diameter in the pulp zone of a flotation column ranges from 0.3 to 2 millimeters, depending upon the type of bubble generation mechanism (Dobby and Finch, 1986; Yoon et al, 1989; Mankosa et al, 1990). The froth bubble diameter, $D_{b,f}$ however, can range from 3 to 6 millimeters in diameter. This indicates that 5 to 10 times more material can be carried into the froth zone than can be removed from the cell. Table 5.1 shows a typical range of bubble diameters, for superficial gas rates greater than 1 cm/s, which have been back-calculated using Equation [5.10]. These tests were conducted in a 14-inch pilot plant column on 28 mesh x 0, Elkhorn III seam coal. Results indicate that the average bubble diameter in the column froth zone is $5.54 \pm 0.74$ microns.

These data tend to indicate that the froth bubble diameter is constant for $V_g$'s greater than 1 cm/s. However, the results shown in Figure 5.1 indicate that the froth bubble diameter is dependent upon the superficial gas velocity for rates below approximately 1 cm/s. Beyond this value it appears that $D_{f,b}$ remains constant.
Table 5.1

Summary of Froth Bubble Diameter Calculations

<table>
<thead>
<tr>
<th>Test #</th>
<th>$D_{b,\epsilon}$ (mm)</th>
<th>$(D_{b,\epsilon} - \bar{D}_{b,\epsilon})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.47</td>
<td>0.0041</td>
</tr>
<tr>
<td>2</td>
<td>6.51</td>
<td>0.9450</td>
</tr>
<tr>
<td>3</td>
<td>4.80</td>
<td>0.5489</td>
</tr>
<tr>
<td>4</td>
<td>5.44</td>
<td>0.0098</td>
</tr>
<tr>
<td>5</td>
<td>4.51</td>
<td>1.0463</td>
</tr>
<tr>
<td>6</td>
<td>6.74</td>
<td>1.4571</td>
</tr>
<tr>
<td>7</td>
<td>5.01</td>
<td>0.2745</td>
</tr>
<tr>
<td>8</td>
<td>5.88</td>
<td>0.1163</td>
</tr>
<tr>
<td>9</td>
<td>5.47</td>
<td>0.0049</td>
</tr>
</tbody>
</table>

Mean Bubble Diameter ($\bar{D}_{b,\epsilon}$) = 5.53 mm

Variance = 0.551 mm$^2$
Figure 5.1  Froth bubble diameter as a function of superficial gas velocity for column flotation of an Elkhorn III seam coal.
According to equation [5.6], since the froth bubble size is essentially constant, an increase in the carrying capacity can only be achieved by increasing the superficial gas flow rate. However, the effect of an increase in gas flow rate on the pulp behavior must be considered. It has been shown that an increase in the superficial gas flow rate will result in a larger pulp bubble diameter (Yianatos et al., 1988) and, hence, a decrease in the flotation rate constant. In this instance, although the froth carrying capacity is increased the amount of solids collected in the column recovery zone will decrease. If the decrease in rate constant is significantly large, the collection properties in the pulp zone may drop below the froth carrying capacity. In this case, a longer residence time would be required in order to ensure the same level of recovery.

The presence of a froth carrying capacity limitation indicates that a decline in the product yield should be observed as the feed rate to the column is increased. Figure 5.2 shows the product solids flow rate as a function of the column feed rate for three different column diameters and coal seams. The slope of this plot represents the average column yield. It can be seen that a constant yield is obtained for the Elkhorn III seam coal regardless of the column diameter up to a product flow rate of 9 lb/min/ft². Beyond this limit the column performance declines sharply. Similar results are observed for the Pittsburgh No. 8 and Cedar Grove seams. However, it is interesting to note that the decline in yield occurs at a lower feed flow rate for each sample. This indicates that the froth product flow rate is apparently coal specific. According to the strict definition
Figure 5.2 Product solids flow rate as a function of solids feed rate for column flotation of several different 28 mesh \times 0 coals and three different column diameters.
of carrying capacity, however, the transport of solids from the cell is a only a function of the physical characteristics associated with the froth (i.e., bubble surface area rate and particle loading on the bubble surface). Therefore, this observation implies that an additional phenomenon exists which regulates the stability of the froth.

Recent work has indicated that froth overloading may also control the rate of froth product removal (Luttrell and Yoon, 1990). Froth overloading is defined as the onset of instability in the froth which ultimately leads to collapse of the froth structure and loss of product flow. Results indicate that froth overloading appears to be governed by numerous variables, among these are froth percent solids, product particle size distribution, reagent dosage and fractional air hold-up. For the case of coal flotation, the high concentration of floatable material can lead to froth immobility and high retention times of particles in the froth.

Flotation columns are particularly susceptible to froth overloading conditions due to the deep froth zone that is typically used (i.e., 3 to 4 feet). This provides an opportunity for excessive drainage resulting in an immobile froth and the onset of froth overloading. Column froth stability, unlike conventional flotation cells, can be controlled by adjusting both the rate and location of the counter current wash water. Therefore, in the case of extremely stable froths, the wash water addition point can be raised completely out of the cell. This minimizes the froth percent solids and provides
for greater mobility. Conversely, for excessively wet, stable froths, such as fine particle flotation, the wash water addition point can be located deep within the froth in order to increase drainage. This, however, still provides the washing capabilities necessary to remove entrained particles.

The concept of froth overloading in conventional flotation of both coal and sulfide minerals has been addressed in detail by previous investigators (Lynch et al, 1981). It was found that for coal the froth percent solids reaches a maximum of 37.5 percent under overloading conditions. It should be noted that this result was consistent for different coals, different circuit configurations and a variety of operating conditions. This signifies the importance of froth percent solids as an indication of froth overloading.

Therefore, it can be concluded that both froth overloading and carrying capacity govern the rate of froth product removal. In fact, in many instances the froth overloading limit may occur prior to the carrying capacity limitation. Although it is recognized that froth overloading plays a significant role in determining column throughput, sufficient data has not yet been collected regarding this phenomenon. Therefore, a fundamental based approach to this problem is not currently available. Preliminary data indicates that froth overloading is coal specific and that it is controlled by froth percent solids, particle size distribution and collector concentration.
5.6 Conclusions

1) An expression has been developed which describes the maximum solids flow rate "carrying capacity" from the column froth. The carrying capacity is based on the physical parameters associated with the froth structure.

2) The froth carrying capacity has been expressed in terms of superficial feed flow rate. This provides a limit to the maximum feed flow rate which would be determined from standard column scale-up procedures.

3) Experimental results indicate that froth overloading conditions may exist which contribute to froth instabilities. Preliminary results show that froth air hold-up, percent solids, particle size distribution and collector addition affect this condition.

4) It is speculated that under certain operating conditions the onset of froth overloading may occur prior to carrying capacity limitations.
5.7 **Nomenclature**

- \(c\) = fitting parameter for equation [5.3]
- \(C\) = concentrate solids flow rate
- \(C_a\) = mass per unit of time per unit of cell cross-sectional area
- \(C_{bl}\) = bubble loading carrying capacity
- \(C_g\) = carrying capacity per unit volume of gas as define by (Espinosa-Gomez et al, 1988)
- \(d_{80}\) = particle 80 percent passing size
- \(D_b\) = bubble diameter (\(d_b\))
- \(D_{b,f}\) = bubble diameter in column froth zone
- \(D_c\) = column diameter
- \(D_p\) = particle diameter
- \(F\) = feed solids flow rate
- \(J_g\) = superficial gas flow rate
- \(K\) = constant in equation [5.1] accounting for mass transport, etc.
- \(n\) = fitting parameter for equation [5.3]
- \(Q_f\) = volumetric feed flow rate
- \(Q_g\) = volumetric gas flow rate
- \(S_{f}\) = feed percent solids
- \(S_{A_b}\) = surface area per unit time
- \(V_g\) = superficial gas flow rate
- \(V_f\) = superficial feed flow rate
- \(Y\) = feed flow rate of water
- \(Y_f\) = fractional yield
- \(\Gamma_p\) = particle packing density
\( \rho_{lf} \) = density of liquid in the feed
\( \rho_p \) = particle density
\( \rho_{sf} \) = density of solids in the feed
\( \rho_{sp} \) = density of solids in the froth product
\( \phi \) = particle packing coefficient
\( \dot{\psi}_b \) = number of bubble per unit time
5.8 References


A SCALE-UP PROCEDURE FOR COLUMN FLOTATION

6.1 Introduction

The growing interest in column flotation has created a need for a fundamental scale-up procedure which is capable of meeting the necessary design and control criteria for industrial columns. Unfortunately, most of the available scale-up procedures deal with column diameter only and not overall column volume. However, past work has indicated that scale-up is a function of both column diameter and length. Furthermore, several critical conditions must be satisfied which relate column flow conditions to the fractional gas hold-up, the mean residence time and axial dispersion. For this reason, care must be taken when extrapolating laboratory test results with the intent to make predictions for larger scale columns.

In order to attempt to overcome many of the problems associated with column scale-up, a generic scale-up procedure has been developed. Based on this analysis, throughput estimations can be made for larger diameter columns. Predictions are based on test results obtained from a laboratory flotation column and validated with results obtained from operating 30-inch and 8-foot diameter columns.
6.2 Literature Review

Column flotation is rapidly becoming an accepted part of the standard flowsheet in many mineral processing plants largely due to savings in capital and operating costs as well as improved metallurgical performance over conventional flotation. The latter factor is primarily true in fine particle flotation where the addition of wash water in the column effectively eliminates nonselective recovery due to hydraulic entrainment. Furthermore, the application of microbubbles in certain flotation columns has been found to significantly improve the recovery of very fine particles (Yoon et al., 1984).

As with most mineral processing unit operations, the recent influx of flotation columns into the industry has been far in advance of the development of fundamental knowledge related to their performance, operation and design. Column scale-up is one area in particular that has received only limited attention. There are basically three approaches being used in the scale-up of flotation columns. These can be classified as scale-up on the basis of column area, scale-up on the basis of column volume and scale-up on the basis of maintaining a constant recovery.

In the first approach, throughput is assumed to be proportional to the cross-sectional area \(D^2\) of the column. This can be expressed mathematically as:

\[
D_2 = D_1 \sqrt{\frac{Q_2}{Q_1}} \tag{6.1}
\]
where $Q_{f1}$ and $Q_{f2}$ represent the volumetric flow rates of feed slurry in a bench-scale test and in the desired full-scale column, respectively, and $D_1$ and $D_2$ represent the diameters of each of these columns. This approach may be valid provided that the preliminary data for scale-up is obtained in a bench-scale column with a very large aspect ratio, such that the height of the bench-scale column is very nearly equal to the height of the full-scale column. This approach also assumes that the change in mixing in the column resulting from the change in the aspect ratio on scale-up does not greatly affect the performance of the column. This latter assumption appears to be valid based on data reported in the literature (Mankosa et al., 1987). Scale-up on the basis of cross-sectional area generally provides a conservative estimate of column diameter. It also appears to be the approach used for scale-up of the Canadian flotation column (Boutin and Wheeler, 1967; Mathieu, 1972; Moon, 1982; Moon and Sirois, 1987).

Scale-up on the basis of volume can be represented mathematically as:

$$V_2 = V_1 \left(\frac{Q_{f2}}{Q_{f1}}\right)$$  \hspace{1cm} [6.2]

or, if the aspect ratio of the column remains constant on scale-up:

$$D_2 = D_1 \sqrt[3]{\frac{Q_{f2}}{Q_{f1}}}$$  \hspace{1cm} [6.3]

where $V_1$ and $V_2$ represent the volumes of the bench-scale column and
the full-scale column, respectively. This approach assumes that the air holdup in the column does not change on scale-up and that all of the volume resulting from an increased column height is effective volume. In other words, this type of scale-up is based on maintaining a constant slurry residence time in the column. Unfortunately, since superficial flows of liquid and air are allowed to change in this type of scale-up, the air holdup in the column may not necessarily remain constant. Furthermore, an increase in column height does not always provide additional useful volume. This is particularly true for coal flotation or any type of cleaner operation where the column is floating a major constituent of the feed material. In this case, bubble loading may become a problem and carrying capacity limits may be reached before the bubbles reach the top of the column (Yianatos et al., 1987). Thus, scale-up based on column volume can often result in an overestimation of column capacity. This approach has been reported for scale-up of the Deister Flotaire column (Deister Concentrator Company, Inc., 1988).

A more recent and fundamentally-based approach to column scale-up is scale-up on the basis of maintaining constant recovery (Dobby and Finch, 1986; Luttrell et al., 1988). In this approach, the column is represented as an axially dispersed system and recovery from the column is given by (Levenspiel, 1972):
\[ R = 1 - \frac{4a \exp(\text{Pe}/2)}{(1+a)^2 \exp(\frac{a}{2}\text{Pe}) - (1-a)^2 \exp\left(-\frac{a}{2}\text{Pe}\right)} \]  

where

\[ a = \sqrt{1+4kt/\text{Pe}}. \]  

As can be seen from Equation [6.4], recovery in a flotation column is a function of the flotation rate constant, \( k \), the mean residence time, \( t \), and a measure of mixing in the column as represented by the Peclet number, \( \text{Pe} \). The Peclet number is further defined as:

\[ \text{Pe} = \frac{u_g L}{D_g} \]  

where \( u_g \) is the interstitial liquid velocity, \( D_g \) is the liquid axial dispersion coefficient and \( L \) is the column length. While the product of \( k \) and \( t \) is given by (Luttrell et al., 1988):

\[ k_c = \frac{3L^2}{2D_b} \frac{1-c}{V_g} \frac{V_f + V_w}{V_f + V_w} \]  

where \( P \) is the probability of particle collection, \( D_b \) is the bubble diameter, \( c \) is the air fraction in the column, \( V_g \) is the superficial gas velocity, \( V_f \) is the superficial feed velocity and \( V_w \) is the superficial wash water velocity. It can be seen from Equations (6.4 -
that scale-up using this approach requires an iterative procedure since the column dimensions must be known in order to determine the superficial velocities. These dimensions must, in turn, be manipulated in order to adjust \( k \), \( \tau \) and \( \text{Pe} \) to maintain a constant recovery on scale-up. The procedure can be simplified somewhat if a constant value for \( \text{Pe} \) is assumed. It has been shown that \( \text{Pe} = 4 \) will predict column recovery within \( \pm 4\% \) for any practical column geometry (Luttrell et al., 1988). In this case, the iterative scale-up procedure can be based purely on equation [6.6], i.e., maintaining a constant \( k \tau \) product. Alternatively, an empirical relationship can be used to determine mixing in the column (Dobby and Finch, 1986).

The procedure described by Equations [6.4 - 6.8] represents a very fundamentally sound approach for column flotation scale-up; however, there are some additional constraints which must be applied when using this approach. The maximum bubble carrying capacity, for example, tends to limit column height. When bubbles become fully loaded, any additional column height is useless. Maximum carrying capacity can be estimated by assuming monolayer coverage of particles around a bubble (Dobby and Finch, 1986). Unfortunately, the appropriate bubble size to use in this calculation is difficult to estimate since it is a function of coalescence in the pulp and froth phases, and coalescence, in turn, is a function of bubble loading. Limits on column dimensions also arise as a result of changes in the air holdup, \( \varepsilon \), during scale-up. For example, air holdup increases as the superficial velocity of gas or liquid increases. Without
constraints on the column geometry during scale-up calculations, a column may be designed in which the air flooding limit is exceeded. Dobby and Finch (1986) have shown that, for monosized bubbles, the flooding limit can be determined using drift-flux analysis which results in a relationship between air holdup and the superficial velocities in the column. However, this analysis does not account for the effect of the bubble size distribution on air holdup, nor does it allow for the loss of the fine portion of the distribution with the tailings flow. This latter effect may be even more significant when very fine bubble size distributions are considered. The net result is that both the air holdup and the mean bubble size in the column may change during scale-up.
6.3 Research Objectives

Numerous investigations have been conducted in the area of column flotation. These have included flotation of sulfides, oxides, industrial minerals and coal. As a result of this work a number of procedures have been developed regarding scale-up of column flotation. Unfortunately, the majority of these techniques have not considered all of the fundamental parameters relative to scale-up. For this reason it was determined that a comprehensive procedure for the design and scale-up of column flotation should be developed.

The purpose of the work presented in this chapter was to develop a procedure for column flotation scale-up. The relationships presented in Chapters 3 through 5 were incorporated into a comprehensive routine for column scale-up. This procedure accounts for the changes which occur in fractional air hold-up, mixing and retention time as variations in the column geometry and flow conditions are encountered due to scale-up.

Laboratory column flotation tests were conducted on a high-ash reject sample obtained from an operating coal preparation plant in West Virginia. Results from the 2-inch column tests were used to make throughput predictions for a 30-inch and an 8-foot diameter microbubble column. These predictions were experimentally validated in operating columns of the same diameter.
6.4 Scale-Up Theory

6.4.1 Relevant Expressions

For the purpose of this procedure, flotation scale-up will be conducted such that a constant level of recovery is maintained. However, the relationships will allow for predictions to be made for different levels of recovery. Recovery is a function of the flotation rate constant (k), mean residence time (t) and axial dispersion (Dₗ) within the reactor. The relationship between these parameters has been described by Levenspiel as follows (Levenspiel; 1972):

\[
R = 1 - \frac{4a \exp(\text{Pe}/2)}{(1+a)^2 \exp[(a/2)\text{Pe}] - (1-a)^2 \exp[(-a/2)/\text{Pe}]}, \quad [6.8]
\]

where

\[
a = \sqrt{1+4kt/\text{Pe}}. \quad [6.9]
\]

The Peclet number is further defined as:

\[
\text{Pe} = \frac{\mu_L L}{D_L}, \quad [6.10]
\]

where \(\mu_L\) is the interstitial liquid velocity, \(D_L\) is the liquid axial dispersion coefficient and \(L\) is the length of the column recovery zone.
As shown in Figure 6.1, the dependence of flotation recovery on rate, residence time and mixing is further complicated since none of these variables are completely independent. For instance, the flotation rate constant has been shown to be a function of bubble diameter, $D_b$, and superficial gas flow rate, $V_g$, as follows:

$$k = \frac{3P}{2D_b} V_g,$$ [6.11]

where $P$ is the probability of particle collection.

The mean liquid residence time is defined as the cell volume, $V$, divided by the flow rate through the reactor, $Q_t$, (tailings volumetric flow rate for the case of flotation) according to the following:

$$\tau_L = \frac{V}{Q_t}.$$ [6.12]

In order to eliminate the area dependence of Equation [6.12] the mean residence time can be represented in terms of superficial velocities, $V_t$, and cell length, $L$, by dividing by the cell cross-sectional area. It should be noted that in order to determine the true cell residence time the volume should be corrected for the volume of the cell which is occupied by air. Making the necessary corrections yields the following expression:
Figure 6.1 Schematic diagram illustrating the interdependence of the variables which control column operation.
\[ \tau_2 = \frac{L(1-\varepsilon)}{V_L} \] 

[6.13]

where \( \varepsilon \) is the fractional air hold-up within the cell.

Column flotation, however, deals with the separation of solids. Therefore, it is necessary to consider the mean residence time of the solids within the cell. The particle residence time, \( \tau_p \), has been shown to be a function of the interstitial liquid velocity, \( u_\theta \), and the particle hindered settling velocity, \( u_p \), as follows (Dobby and Finch, 1985):

\[ \tau_p = \tau_0 \left[ \frac{u_\theta}{u_\theta + u_p} \right] \] 

[6.14]

For scale-up applications dealing with low density solids, such as coal, the particulate residence time can be approximated by the liquid residence time. In this case, the complexities of dealing with hindered particle settling in an air/water system need not be considered. This approximation also applies to very fine particle flotation. An example of this would be kaolin flotation where the top size is typically less than 5 or 10 microns. Specialty situations, such as the production of premium, coal-based fuels from micronized feed stock, would also fall into this category. In general, residence time calculations should be considered on a case-by-case basis taking into account the particle size and density of the material.

The Peclet number for a given system, as described by Equation
[6.10], is difficult to determine since it is necessary to know an axial dispersion coefficient for each set of flow conditions and column geometries. Therefore, the empirical relationship developed in Chapter 3 will be used to describe the change in mixing conditions within the column during scale-up. This relationship also depends on the flow conditions and fractional air hold-up within the column and is expressed as follows:

\[
F_{e} = 0.698 \left( \frac{L u_{e}}{D v_{g}} \right)^{0.622}
\]  

[6.15]

The relationships established for the flotation rate constant (Equation [6.11]), mean residence time (Equations [6.13] & [6.14]) and axial dispersion (Equation [6.15]) provide a means of calculating recovery for any number of conditions, provided that the fractional air hold-up within the cell can be adequately described. The fractional air hold-up is a function of bubble diameter and gas and liquid flow rates in the column. The following system of simultaneous differential equations, developed in Chapter 4, are utilized to describe the change in fractional air hold-up during scale-up:

\[
\frac{d\epsilon_{1}}{dt} = \frac{[Q_{t} \epsilon_{2} - (Q_{t} \epsilon_{1} + U_{j} \epsilon_{1})]}{V_{z}}.
\]  

[6.16]
\[\frac{dc_2}{dt} = \frac{[(Q_g + Q_d\epsilon_2 + U_1A_1\epsilon_1) - (Q_t\epsilon_2 + U_2A_2\epsilon_2)]}{V_z}, \quad [6.17]\]

where

\[Q_d = Q_t - Q_g. \quad [6.18]\]

Steady-state solutions to these relationships have been developed. However, an explicit solution for \(\epsilon\) is not available since the relationships are nonlinear. This is a result of the power dependence of the hindered rise velocity, \(U_1\), on the fractional air hold-up.
6.4.2 General Application

Scale-up based on the concept of maintaining a constant level of recovery requires that the product of \( k \) and \( \tau_0 \) be held constant. Furthermore, since the bubble diameter is considered to be constant on scale-up, the flotation rate constant will only be affected by the superficial gas rate. Therefore, scale-up is most strongly influenced by the cell retention time. The effect of a change in bubble diameter upon scale-up will be addressed later in this section. The product of \( k \) and \( \tau_0 \), as described above, can be expressed as follows:

\[
k\tau_0 = \frac{3P V_g L (1 - \epsilon)}{2D_b (V_f + \alpha V_w)}, \tag{6.19}
\]

where

\[
V_c = V_f + \alpha V_w. \tag{6.20}
\]

It is assumed that under positive bias conditions \( Q_t > Q_f \), all of the flow associated with the feed reports to tailings, as well as a percentage, \( \alpha \), of the wash water flow rate. Equation \( 6.19 \) can be solved for superficial feed flow rate as follows:

\[
V_f = \frac{3P}{2D_b k \tau_0} V_g L (1 - \epsilon) - \alpha V_w. \tag{6.21}
\]
Equation [6.21] can be expressed as a straight line with slope, \( m \),
according to the following:

\[
V_f = m L(1 - \epsilon) - \omega V_w, \quad [6.22]
\]

where

\[
m = \frac{3 PV_g}{2D_b k \tau_b}. \quad [6.23]
\]

Expressed in this manner, \( k \tau_b \) represents the results obtained in a
laboratory scale test. The superficial gas flow rate, \( V_g \), and bubble
diameter, \( D_b \), represent the conditions in the full size column.

An additional consideration related to column scale-up is the
carrying capacity restriction imposed due to the limited bubble
surface area available to remove particles. This relationship has
been discussed in detail in Chapter 5. In terms of the superficial
feed flow rate the carrying capacity limit, \( V_{f,c} \), can be expressed as
follows:

\[
V_{f,c} = \frac{4P V_g D_p \rho_s \rho_f}{P_{y_f D_b, f}} \left[ \frac{1}{\rho_f} + \frac{1}{\rho_{sf}} \right] \left[ \frac{1}{S_f} - 1 \right] \quad [6.24]
\]

phenomena associated with scale-up, it quickly becomes difficult to
mentally assess the behavior of these complex interactions as they
relate to column performance during scale-up. Therefore, Equations
[6.22] and [6.24] have been illustrated graphically in Figure 6.2.
This "Scale-Up" diagram represents all of the aforementioned
relationships which affect column performance, such as retention time,
flotation rate constant, axial dispersion and fractional air hold-up.

Figure 6.2 indicates two different regions of column operation.
The first region, described by Equation [6.22], represents column
operation under rate limited conditions. It can be seen from Equation
[6.23] that if $k_r^g$ and $P$ remain constant, the slope of this line can
only be affected by a change in $V_g$ or $D_b$ in the full size column. A
change in either of these parameters results in a change in the slope
of the line described by Equation 6.22. It can be seen that an
increase in superficial gas flow rate or a decrease in bubble diameter
in the full size column results in an increased slope, thus requiring
a shorter retention time (i.e., shorter column length). This is a
result of the increased flotation rate constant associated with either
change. Conversely, a decrease in $V_g$ or an increase in $D_b$ results in
a lower flotation rate and, hence, a longer column retention time. In
this manner, a change in either bubble diameter or superficial gas
rate during scale-up can be represented.

A common problem encountered in the design of flotation columns
is underestimation of the bubble generation capacity of the large
scale unit. This typically results in excessively large bubble
diameters when trying to maintain a constant superficial gas velocity.
Therefore, the gas flow rate must be decreased to a level where proper
Figure 6.2 Graphic illustration of the relationships which control column scale-up.
bubble generation occurs or increased significantly in order to compensate for the effect of bubble size on the flotation rate constant. Based on the rate limiting case shown in Figure 6.2, the effect of either of these changes on the required retention time can be established.

A similar effect can be observed for a change in mixing conditions. It can be seen from Equation (6.15) that an increase in either column length or interstitial liquid velocity results in more plug-flow conditions (increasing Pe). As the column approaches plug-flow operation, a shorter retention time is required in order to maintain the same recovery level. This effect can be determined from Equation (6.8). Therefore, a decrease in column length would be indicated by an increase in slope for the rate limited case in Figure 6.2. Likewise, an increase in either column diameter or gas flow rate will drive the column towards perfectly mixed conditions (decreasing Pe), thus requiring a longer retention time.

The carrying capacity limitation is also illustrated in Figure 6.2. As shown, this restriction is not a function of column length or fractional air hold-up and therefore can be represented by a horizontal line. This limit represents the maximum amount of feed which can be removed from the column due to bubble/particle attachment for the given operating conditions.

According to Figure 6.2, under rate limited conditions column throughput can be continually increased with length while maintaining the same retention time. Beyond a certain critical length, as defined by the intersection with the carrying capacity limitation, any
additional increase in feed rate can not be recovered. Therefore, the optimum operating point for a flotation column would be at the intersection of the rate limiting condition (Equation 6.22) and the carrying capacity limitation (Equation 6.24). This intersection defines the column length and retention time which is required in order to achieve the desired recovery level.
6.5 Detailed Scale-Up Procedure

It can be seen from the equations presented above that a great deal of inter-dependence exists between the variables associated with scale-up and the relationships which describe them. For instance, a change in column length will affect the residence time, axial dispersion and fractional air hold-up within the cell. Therefore, an iterative, resubstitution procedure was developed to solve all of the equations simultaneously. A detailed flowsheet of this procedure is shown in Figure 6.3.

The following is the step-by-step procedure for scale-up of column flotation. The bracketed letter at the end of each step locates the current point on the flowsheet shown in Figure 6.3.

1) Conduct a laboratory column flotation test as a function of volumetric feed flow rate using a sample of the material in question. Care should be taken in order to insure the integrity of the sample prior to flotation. Ideally, the sample should be obtained directly from an operating plant at the point in the process where flotation is being considered. It should be noted that the test must be conducted under rate limiting conditions. Otherwise, the flotation rate constants calculated from Equation [6.8] will be masked by carrying capacity effects. [Step (A)]

2) Determine the following parameters from the laboratory flotation test for each feed rate:

a) fractional air hold-up (from differential pressure),
Figure 6.3 Flowsheet illustrating the solution methodology for column flotation scale-up.
b) superficial gas, feed, wash water and tailings flow rate,
c) bias factor, $\alpha$, from equation [6.20],
d) product recovery,
e) particle retention time ($\tau_p = L/\nu_t$), and
f) Peclet number (from equation [6.15]).

Column length, $L$, is the length of the recovery zone as defined by the distance between the air injection point and the pulp/froth interface. [Step (A)]

3) Consider particle size and density and make necessary corrections to the mean particle residence time according to equation [6.14] if necessary. [Step (A)]

4) Plot product recovery as a function of particle retention time and determine the residence time required to achieve the desired recovery level. [Step (A)]

5) Using equation [6.8] determine the value of $k_{tp}$ required to match the experimental recovery. [Step (A)]

6) From the calculated value of $k_{tp}$ and the experimentally determined residence time, solve for the flotation rate constant. [Step (A)]

7) From the experimentally determined air hold-up and the measured gas and tailings flow rates, determine the experimental bubble size from equations [6.16] through [6.18]. [Step (A)]
8) Knowing $V_g$, $D_b$ and $k$, determine the probability of particle collection, $P$, using equation [6.11]. This value is only necessary if either $D_b$ or $V_g$ is allowed to change upon scale-up. [Step (A)]

9) Plot $V_f$ versus $L(1-\epsilon)$ for the chosen recovery level. Determine $\alpha V_w$ (superficial bias flow) and draw the rate limiting line between these points. [Step (A)]

10) Determine the carrying capacity limitation according to equation [6.24] and add to the plot constructed in step [9]. The intersection of these lines provide a preliminary estimate of the optimum column geometry and volumetric feed flow rate based on the initial laboratory conditions. [Step (A)]

11) Make an initial increase in column length. Previous work has indicated that an increase of no more than 5% of the initial column length is adequate. Increases greater than this tend to complicate the iterative solution. [Step (B)]

12) Determine the $V_t$ required to maintain a constant retention time from equation [6.13]. [Step (C)]

13) Calculate the new value of $\epsilon$ as a result of the change in $V_t$ from the air hold-up model (equations [6.16] through [6.18]). If flooding conditions exist, as described in Chapter 4, a further increase in column length is not allowable due to the current flow conditions in the column. [Steps (D & E)]
14) If flooding conditions do not exist continue iterations on steps [12] and [13] until a constant value of air fraction is achieved. [Step (F)]

15) Determine the new Peclet number based on the most recent flows and geometries from equation [6.15]. [Step (G)]

16) Using the new Peclet number and maintaining $k$ as a constant, determine the value of $t_p$ required to maintain the desired level of recovery. [Step (H)]

17) Repeat steps [12] through [16] until $t_p$ stabilizes. [Step (I)]

18) Determine the new value of $V_f$ from equation [6.20] and plot as a function of the new length and air hold-up.

19) Return to step [11] and continue to increase the column length until either flooding conditions or the carrying capacity limit is reached. At this point, throughput can only be increased by an increase in column diameter. [Step (E)]

20) Increase column diameter and repeat steps [11] through [18] until the desired level of throughput is achieved ($Q_f = V_f A$). [Steps J & K]

21) At this point, a change in either gas flow rate or bubble diameter for the large scale column can be considered by changing either $D_b$ or $V_g$. This requires that the entire procedure be re-
iterated until all of the necessary parameters have stabilized.

22) Determine the feed flow rate and geometry for the new cell design based on the information provided by the scale-up diagram generated from the above procedure.
6.6 Scale-Up for Coal Flotation

In order to validate the scale-up procedure, a sample was obtained from a classifying cyclone overflow in an operating coal preparation plant in West Virginia. This material typically contains 55 to 60 percent combustibles, mostly liberated clays, and has a top size of approximately 100 mesh. The high ash content associated with this stream makes it an ideal candidate for column flotation. The sample used in the 2-inch column tests was taken from a sample port installed directly in the main discharge past the collection boxes from three classifying cyclone banks. The sample was obtained prior to addition of flocculating agents.

A laboratory column flotation test was conducted as a function of feed rate. When steady-state conditions were achieved (typically 3 residence times) samples were taken of the product, reject and feed for the purpose of determining the mass flow rate of solids and liquid. Fractional air hold-up was determined for each feed rate by means of differential pressure as described in Chapter 4. Gas and wash water flow rates were also recorded for each test. Table 6.1 summarizes the results from all three tests.

Based on the fine particle size and low solids density, it was concluded that the particle retention time could be adequately determined from the liquid flow rates through the column. Retention time was determined from equation [6.13] and plotted according to its respective recovery. These results are shown in Figure 6.4. Based on the grade-recovery curve determined from the laboratory results,
Table 6.1

Summary of Operating Conditions for Scale-Up Testing

<table>
<thead>
<tr>
<th>Test #</th>
<th>$V_f$</th>
<th>$V_t$</th>
<th>$\sigma V_w$</th>
<th>$V_g$</th>
<th>$V_w$</th>
<th>$\epsilon$</th>
<th>$\tau_p$ (sec)</th>
<th>$Re$</th>
<th>Rec. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.83</td>
<td>0.98</td>
<td>0.15</td>
<td>2.0</td>
<td>0.31</td>
<td>0.1819</td>
<td>95.47</td>
<td>3.53</td>
<td>68.66</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>1.44</td>
<td>0.18</td>
<td>2.0</td>
<td>0.31</td>
<td>0.2000</td>
<td>62.36</td>
<td>4.49</td>
<td>62.18</td>
</tr>
<tr>
<td>3</td>
<td>1.77</td>
<td>1.66</td>
<td>0.16</td>
<td>2.0</td>
<td>0.31</td>
<td>0.2414</td>
<td>53.86</td>
<td>5.04</td>
<td>45.38</td>
</tr>
</tbody>
</table>
Figure 6.4  Combustible recovery as a function of retention time for column flotation of a Coalburg seam coal.
Figure 6.6, it was determined that a combustible recovery of 60 percent was required in order to produce the required product ash content of 10 percent. Therefore, results from the second feed rate test were chosen as a starting point for the scale-up procedure.

Based on the experimental results, the Peclet number was calculated and the $kt_p$ required to achieve a 60 percent recovery level was determined from equation [6.8]. The flotation rate constant was then determined according to the required retention time. Using the air fraction model, the bubble size in the recovery zone was determined from the experimental flows and fractional air hold-up. Knowing the superficial gas velocity and the average bubble diameter, the probability of particle collection was determined. This value was maintained as a constant for scale-up. The results of this analysis are summarized in Table 6.2.

The determined starting point and the bias flow rate ($\alpha V_w$) are illustrated in the scale-up diagram shown in Figure 6.5. The carrying capacity limit, estimated from equation [6.24], is also shown in this diagram. It can be seen that a feed flow rate in excess of 1.7 cm/sec is not allowable due to carrying capacity limitations.

The scale-up procedure described in the previous section was then utilized to determine the maximum column height and throughput for a 30-inch and an 8-foot diameter column. The scale-up calculations are presented in Table 6.3. These results are also included in Figure 6.5. Each calculated point shown in Figure 6.5 represents one complete pass through the entire iterative procedure outlined in Figure 6.3.
Table 6.2

Summary of Calculated Results from Scale-Up Testing

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_c$</td>
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</tr>
<tr>
<td>$V_t$</td>
<td>1.44 cm/s</td>
</tr>
<tr>
<td>$V_g$</td>
<td>2.00 cm/s</td>
</tr>
<tr>
<td>$V_w$</td>
<td>0.31 cm/s</td>
</tr>
<tr>
<td>$\sigma V_w$</td>
<td>0.60 cm/s</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>62.36 sec</td>
</tr>
<tr>
<td>$c$</td>
<td>0.2000</td>
</tr>
<tr>
<td>$D_b$</td>
<td>983 microns</td>
</tr>
<tr>
<td>$k t_p$</td>
<td>1.168</td>
</tr>
<tr>
<td>$k$</td>
<td>0.0187 sec$^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.00061</td>
</tr>
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</table>
Table 6.3

Scale-Up Calculations

<table>
<thead>
<tr>
<th>L(cm)</th>
<th>D(cm)</th>
<th>$V_p$(cm/s)</th>
<th>$c$</th>
<th>$\tau$(sec)</th>
<th>$Fe$</th>
</tr>
</thead>
<tbody>
<tr>
<td>112.0</td>
<td>5.08 (2&quot;)</td>
<td>1.25</td>
<td>0.2000</td>
<td>62.36</td>
<td>4.49</td>
</tr>
<tr>
<td>120.0</td>
<td>5.08</td>
<td>1.32</td>
<td>0.2016</td>
<td>61.75</td>
<td>4.89</td>
</tr>
<tr>
<td>125.0</td>
<td>5.08</td>
<td>1.38</td>
<td>0.2072</td>
<td>61.28</td>
<td>5.17</td>
</tr>
<tr>
<td>130.0</td>
<td>5.08</td>
<td>1.43</td>
<td>0.2133</td>
<td>60.86</td>
<td>5.45</td>
</tr>
<tr>
<td>135.0</td>
<td>5.08</td>
<td>1.49</td>
<td>0.2197</td>
<td>60.47</td>
<td>5.74</td>
</tr>
<tr>
<td>140.0</td>
<td>5.08</td>
<td>1.54</td>
<td>0.2265</td>
<td>60.13</td>
<td>6.03</td>
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<td>1.58</td>
<td>0.2341</td>
<td>59.79</td>
<td>6.32</td>
</tr>
<tr>
<td>150.0</td>
<td>5.08</td>
<td>1.63</td>
<td>0.2423</td>
<td>59.47</td>
<td>6.61</td>
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<tr>
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<td>1.70</td>
<td>0.2618</td>
<td>58.91</td>
<td>7.21</td>
</tr>
</tbody>
</table>

165.0  5.08  no solution - flooding conditions
160.0  10.00  1.65  0.2466  62.29  4.57
160.0  20.00  1.57  0.2317  66.79  2.84
160.0  30.00  1.52  0.2239  69.83  2.15
160.0  40.00  1.48  0.2188  72.14  1.76
160.0  50.00  1.42  0.2123  74.15  1.49
160.0  60.00  1.39  0.2094  75.64  1.31
160.0  70.00  1.38  0.2073  76.88  1.18
160.0  76.20 (30")  1.36  0.2061  77.59  1.11
170.0  76.20  1.45  0.2161  76.58  1.21
180.0  76.20  1.53  0.2272  75.64  1.31
190.0  76.20  1.61  0.2401  74.77  1.41
200.0  76.20  1.68  0.2557  73.91  1.52
200.0  244.00 (8")  1.55  0.2294  83.06  0.69
220.0  244.00  1.67  0.2544  81.67  0.78
240.0  244.00  no solution - flooding conditions
Figure 6.5 Scale-up predictions for a 30-inch and 8-foot diameter column based on results obtained in a 2-inch laboratory column.
It can be seen from the change in Peclet number in Table 6.3 that as column length is increased the retention time required to maintain a constant level of recovery decreases. This is a result of a decrease in mixing (i.e., increased Peclet number) associated with an increased length-to-diameter ratio. Beyond a column length of 160 cm, however, flooding conditions are encountered due to the high liquid flow rates. Therefore, the column diameter was increased while maintaining a constant height. The increase in column diameter had an opposite effect on retention time. In this instance, a longer residence time was necessary in order to achieve the same level of recovery. Having decreased the retention time, it was then possible to further increase the column length until flooding condition were again encountered. This procedure was repeated until the length and throughput for both the 30-inch and 8-foot columns was determined. The predicted results are shown in Table 6.4 in comparison with the experimental results obtained from both the 30-inch and 8-foot columns.

As a final verification of the scale-up procedure, the grade versus product recovery curve is shown in Figure 6.7. The experimental results from tests conducted on 2-, 6-, and 30-inch, as well as the 8-foot unit, are presented. It can be seen that all four columns are operating on the same grade/recovery curve. This indicates that the froth washing action for each column is consistent and that the column is operating on the release curve. The release curve, previously described in Chapter 2, represents the best possible
Table 6.4

Comparison of Predicted and Experimental Results

<table>
<thead>
<tr>
<th></th>
<th>2-Inch Measured</th>
<th>30-Inch Pred. Measured</th>
<th>8-Foot Pred. Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_f$ (cm/s)</td>
<td>1.25</td>
<td>1.68 1.73</td>
<td>1.67 1.45</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>20.00</td>
<td>25.6 ----</td>
<td>25.40 20.00</td>
</tr>
<tr>
<td>% Ash</td>
<td>9.81</td>
<td>10.0 10.33</td>
<td>10.0 12.03</td>
</tr>
<tr>
<td>% Rec</td>
<td>62.18</td>
<td>60.0 66.90</td>
<td>60.0 64.07</td>
</tr>
</tbody>
</table>
Figure 6.6 Combustible recovery as a function of product ash percent for column flotation of a Coalburg seam coal.
flotation performance for a given sample.
6.7 Discussion

The results of the scale-up procedure indicate that valid predictions can be made regarding throughput for larger diameter columns. These predictions can be made based on small scale laboratory testing. However, several important considerations need to be addressed during scale-up. One of the first priorities is to insure that the integrity of the bubble size distribution is maintained at the higher gas flow rates required for full scale columns. Although this analysis can accommodate changes in the bubble size distribution, it is not valid if the flow characteristics exceed the bubbly flow regime. This point has been previously addressed in Chapter 4. A common error upon scale-up is an underestimation of the required bubble generation capacity. This results in an insufficient number of bubble generators in the full scale column. As a result, severe slugging of the bubble generators occur as they are pushed to higher gas flow rates. This results in a drastic decrease in the fractional air hold-up and a loss of the particle collection characteristics determined in the laboratory test. In this case, the feed flow rate will have to be significantly decreased in order to maintain the desired level of recovery.

Another consideration is the use of additional water for bubble generation. This water is simply another flow into the column and therefore detracts from the residence time available for particle collection. This point is further illustrated by the scale-up diagram shown in Figure 6.2. It can be seen that the addition of bubble
generation water slides the point of origin further down the ordinate, thus signifying the need for a longer column to maintain the current value of retention time.

It should also be noted that the carrying capacity restriction is at best a fuzzy boundary, since a degree of uncertainty exists as to the true bubble size used to determine the limit. Therefore it is recommended that a column be designed such that it will operate slightly beyond the intersection of the rate limiting and carrying capacity curves. This provides a small buffer zone in regards to carrying capacity, as well as allowing for surges in feed flow rates which typically occur in any beneficiation process.

It has also been shown that additional advantages can be achieved in regards to mixing and grade by the use of longer columns. This is a result of the circulating load which exists between the pulp phase and the froth (Yan et al, 1990). The extra length also allows for dispersion of the feed slurry and bubble distribution within the cell. Although the Peclet number represents the dispersion within the cell due to flow characteristics, it does not clearly represent the distribution of feed slurry and bubbles. This is a result of the underlying assumption associated with an axially dispersed plug flow mixing model. This type of model assumes that the reactor is perfectly mixed in the radial direction. The true dispersion characteristics, however, are a function of the actual design associated with the feed inlet and bubble injectors.

Finally, it should be noted that the initial laboratory experiments must be conducted under rate limiting conditions in order
to insure that the true flotation rate constant is determined. When operating under carrying capacity restrictions collected material is rejected back into the cell due to loading restrictions. Therefore, the apparent rate constant will be lower than the actual rate of collection.
6.8 Summary and Conclusions

1) A scale-up procedure has been developed for column flotation which incorporates the flotation rate constant, retention time, axial dispersion and fractional air hold-up within the cell.

2) The procedure has been formalized into a generic approach which utilizes a computerized, iterative solution for predictions related to changing column geometries.

3) Predictions can be made for large scale industrial columns based on laboratory column flotation test results.

4) The analysis is capable of making predictions for any coal or mineral system regardless of the particle size, density or flotation characteristics.

5) Predictions have been made for full scale columns based on laboratory experiments conducted on a high ash, cyclone overflow sample obtained from an operating coal preparation plant.

6) The scale-up procedure has been validated using a 30-inch test unit, as well as an 8-foot, production column. Predictions have been shown to compare favorably with the experimentally obtained throughputs.
6.9 **Nomenclature**

A = column cross-sectional area

$D_b$ = bubble diameter

$D_i$ = diameter of column (i)

$D_a$ = liquid axial dispersion coefficient

$D_p$ = particle diameter

$k$ = flotation rate constant

$L$ = length of column recovery zone

$m$ = slope for rate limited scale-up conditions

$P$ = probability of particle collection

$Pe$ = Peclet number

$Q_d$ = volumetric downward flowrate in the column recovery zone

$Q_{f,i}$ = volumetric feed flow rate

$Q_g$ = aeration rate

$Q_{tc}$ = volumetric tailings flow rate

$R$ = recovery

$S$ = fractional percent solids

$t$ = time

$U_b$ = bubble terminal rise velocity

$U_i$ = bubble hindered rise velocity in zone (i)

$u_g$ = interstitial liquid velocity

$u_p$ = particle settling velocity

$V_f$ = superficial feed velocity

$V_g$ = superficial gas velocity

$V_i$ = volume of column i
\( V_t \) = superficial tailings velocity
\( V_w \) = superficial wash water velocity
\( V_z \) = zone volume
\( Y_f \) = fractional yield
\( \sigma \) = bias flow rate coefficient
\( \Delta P \) = differential pressure in the presence of an air/water suspension
\( \Delta P_w \) = differential pressure in the presence of water
\( \varepsilon_i \) = volumetric air fraction in zone (i)
\( \rho_{sf} \) = feed solids density
\( \rho_{sp} \) = product solids density
\( \tau \) = mean residence time
\( \tau_p \) = mean particle residence time
\( \tau_l \) = mean liquid residence time
\( \phi \) = bubble loading packing coefficient
6.10 References


"Deister Flotaire Column Flotation Cells," 1989, The Deister Concentrator Company, INC., P.O. Box 1, 901 Glasgow Avenue, Fortwayne, Indiana 46801.


7.1 Introduction

Froth flotation is generally recognized as the most practical and cost-effective method of processing fine particles. Unfortunately, the efficiency of conventional froth flotation is severely diminished when the feed stream contains a high degree of fine particles (i.e., less than 50 microns). In some instances, such as for coal cleaning, reports suggest that very fine grinding may be necessary to liberate finely disseminated mineral matter (Zitterbart et al., 1985; Mathieu and Mainwaring, 1986; Adel et al., 1989). Therefore, it is unlikely that conventional flotation equipment will fulfill many of the future requirements of the coal industry in terms of deep cleaning.

The microbubble flotation column, described in detail in Chapter 2, has been an attempt to overcome some of the deficiencies associated with conventional flotation. This process originated from hydrodynamic analyses which suggested that smaller air bubbles could be used to improve the recovery of micronized particles (Flint and Howarth, 1971; Reay and Ratcliff, 1973; Collins and Jameson, 1976; Anfruns and Kitchener, 1977). This concept was further validated by the development of extensive models describing fundamental bubble/particle interaction, as well as a population balance model of the entire column flotation process (Luttrell, 1986). These models provided the flexibility to examine process variables independently so
that the effects of changes in these variables could be evaluated.
7.2 Literature Review

The development of exact models of particulate systems has proven to be quite cumbersome due to the overwhelming number of fundamental subprocesses which describe most unit operations. Therefore it is virtually impossible for a model of a complex process, such as flotation, to accurately represent the actual system. However, information regarding the interaction of fundamental operating variables associated with a particular process can be obtained. This usually provides a great deal of insight relative to design, scale-up, control and optimization of the system under consideration. Historically, three approaches have been used for process modeling. These include empirical, probability and kinetic type models. Each has its own particular advantages and disadvantages depending on the application.

Empirical models are obtained from statistical correlations between dependent and independent variables. They can be developed from normal plant performance data or from a pre-conceived data collection program. Results can be determined off-line by means of sampling and laboratory analysis, or on-line through the use of in-stream instrumentation. An advantage of the latter method is the ability to continuously update model parameters. Empirical models are typically cheaper in terms of time and personnel and also provide a useful training tool for operators. However, care must be taken in applying these models so that predictions are not made outside of the range of the model data base. Changes in ore characteristics, as well
as different circuit configurations, could not be analyzed with an empirical type model.

Probability models are perhaps the most difficult to develop due to a lack of understanding of the fundamental concepts which contribute to the overall process. These models usually predict performance based on the probability of success of a sequence of events. For example, the probability of a floatable particle reporting to the froth launder is a function of the probabilities of bubble/particle collision, particle attachment, detachment, etc. This type of model has been used to investigate the behavior of very fine and very coarse particles in flotation systems (Kelsall et al; 1961, 1971, 1974; Davis, 1964).

Perhaps the most useful tool is the kinetic model. Kinetic models summarize the unknown fundamental sub-processes into general relationships which describe the rate of transfer within a process. Reasonable approximations of the rate constants can be determined experimentally. An example of a typical rate equation for a semi-batch flotation process is as follows:

\[-\frac{dC}{dt} = kC\]  \[\text{[7.1]}\]

where \(k\) is the rate constant, \(t\) is time and \(C\) is the concentration of particles within the cell. It follows from equation [7.1] that the integrated version of this relationship can be written as,

\[C = C_0 \exp(-kt).\]  \[\text{[7.2]}\]
Ideally, one would prefer to use a continuous distribution of flotation rate constants in equation [7.2], thus representing the entire spectrum of particle floatability. Unfortunately, this is not possible for most practical applications. Kelsall (1961) has suggested using a two component system utilizing rate constants for slow ($k_s$) and fast ($k_f$) floating fractions. In this instance, equation [7.2] can be expressed as follows:

$$C = C_0 \left[ \%F \exp(-k_s t) + (1-\%F) \exp(-k_f t) \right],$$  \hspace{1cm} [7.3]$$

where $\%F$ represents the slow floating fraction.

This concept has since been expanded upon to include multiple species systems. Luttrell et al. (1990) has shown that systems such as coal can be adequately represented using three species: clean coal (large $k$), middlings particles (intermediate $k$) and a non-floatable species ($k=0$). The weight distribution of each species was determined from a modified $m$-curve, typically used to characterize coal cleaning operations.

One of the earliest attempts at applying modeling techniques to column flotation was that of Sastry and Fuerstenau (1970). They developed an axially dispersed plug flow (ADPF) model for flotation in a countercurrent column. The ADPF model allowed the entire range of mixing conditions within the column (perfectly mixed to plug flow) to be examined by changing the Peclet number. The Peclet number is a
dimensionless indicator of the dispersion characteristics of solid particles within the column. Larger values of Pe indicate plug flow conditions and zero represents perfectly mixed conditions.

Modeling results indicated that an increase in Peclet number provided an increased recovery as well as a larger concentration gradient in the axial direction of the column. Furthermore, it was determined that for larger rate constants the majority of the collection was achieved in the upper region of the cell. Contributions from the bottom section only became significant when bubble/particle detachment was considered or for the case of low flotation rate constants. Although this approach proved useful for examining the cause/effect relationship between various parameters, it was not capable of making realistic predictions concerning actual systems due to the large number of unknown parameters associated with the model.

For systems which can be considered to be perfectly mixed or which contain complex flow patterns, macroscopic population balance models have been applied quite successfully. This approach was used by Bascur and Herbst (1982) to develop a somewhat more practical model of a conventional flotation cell. The overall goal of the work was to develop a realistic model of the system to be used for control purposes. They divided the flotation process into a number of sub-processes which were described by either rate or transport terms. Rate terms were used to describe bubble/particle attachment and detachment, while transport terms were used for representing particle entrainment and froth drainage. The model was applied quite
successfully in analyzing the dynamic response of conventional flotation to changes in aeration rate, frother addition, agitation speed and pulp level.

More recently, a similar approach was used by Luttrell (1986) for modeling of column flotation. Analogous to Bascur and Herbst, the column flotation process was described by a series of rate and transport terms. However, in this application the column was divided into a series of perfectly mixed zones in order to account for changes in flow conditions along the length of the column. Since flotation columns typically lie in a range which is intermediate to plug flow and perfectly mixed conditions, mixing was simulated by including an additional flow term within the column which non-selectively transports material from zone to zone. Results indicate that the model was quite useful in examining the effect of different operating parameters on column performance (Luttrell et al, 1987).
7.3 Research Objectives

The previous chapters have illustrated valid models which can be used for the design and scale-up of column flotation. Having accomplished this task the next logical step was to develop on-line control strategies for column flotation. Unfortunately, the scale-up relationships are not dynamic in nature and therefore cannot be used for transient control purposes. As a result, it was necessary to develop dynamic model equations which are capable of making predictions regarding air and solid phase behavior in a flotation column.

With this in mind, a population balance model was developed for the purpose of dynamic simulation and control. This model expanded on the previous mixed zone method developed by Luttrell. An increased understanding of the circulation patterns within the column allowed the characteristic flows to be redefined in order to directly incorporate mixing into the model. This reduced the complexity of the model equations and eliminated one unknown parameter. The flotation rate constant was also corrected for bubble loading and the particulate phase expanded to incorporate multi-species distributions. A series of simulations were conducted using a steady-state solution to the model equations in order to determine how the simulator would respond to changes in column operating conditions.
7.4 Mixed Zone Model

In a first attempt to describe the flow patterns within a flotation column, the column was represented as a series of perfectly-mixed zones. The number of zones depended on the desired height of the column. For each zone, a mass (or volume) balance was carried out for each particulate phase present in the column. These particulate phases included the air bubbles, unattached particles and bubble-particle aggregates. Particulate solids were further classified as either valuable, gangue or middlings material. Each species utilized a different flotation rate constant and settling velocity. Perfect liberation from the mineral matter was assumed for the valuable constituent of the feed; however, middlings particles with any degree of mineral matter could be incorporated. It was assumed that the size of particles and bubbles could be reasonably represented by a single mean value. Other factors, such as particle agglomeration, bubble coalescence and bubble-particle detachment were not included at this time. A summary of the model derivation is discussed below.
7.5 Model Development

7.5.1 Volumetric Flow Balance

The schematic shown in Figure 7.1 illustrates the volumetric flow balance around each zone of the column. The five flows considered here include feed (\( Q_f \)), tailings (\( Q_t \)), gas (\( Q_g \)), wash water (\( Q_w \)) and product (\( Q_p \)). Of these flows, \( Q_f \), \( Q_g \) and \( Q_w \) are input parameters, while \( Q_t \) and \( Q_p \) must be calculated. The feed flow (\( Q_f \)) is assumed to split into \( Q_p \) and \( Q_d \) as it enters the feed zone, after which these flows are translated throughout the upper and lower parts of the column, respectively. Likewise, \( Q_w \) is translated throughout the upper part of the column and \( Q_g \) throughout the lower part of the column. By using this approach it is possible to approximate mixing within the column.

The unknown flows of \( Q_p \) and \( Q_d \) are approximated by assuming a steady-state model of the froth and conducting an overall volume flow balance. From a steady-state air balance around the froth zone (Figure 7.2), an expression for \( Q_p \) is given as:

\[
Q_p = \frac{(U_{i-1} A c_{i-1} + Q_p c_{i-1})}{\varepsilon_F} \tag{7.4}
\]

where \( U_{i-1} \) represents the bubble hindered rise velocity, \( A \) the cross-sectional area of the column, \( c_{i-1} \) the volume fraction of air in the transition zone and \( \varepsilon_F \) the volume fraction of air in the froth. At present, the value of \( \varepsilon_F \) is obtained by considering that each bubble carries a "sheath" of slurry into the froth. The average thickness of
Figure 7.1 Schematic representation of the volumetric flow balance around the column.
Figure 7.2 Schematic representation of the volumetric air balance around the column froth zone.
the sheath \( T \) depends on parameters such as surface tension, superficial gas velocity, average bubble diameter, etc. From a simple geometric analysis, it can be shown that:

\[
\epsilon_F = \frac{D_b^3}{D_b^3 + 6D_b^2T + 12D_bT^2 + 8T^3} \quad [7.5]
\]

In the present work, \( T \) has been determined experimentally (Luttrell et al, 1987).

Once \( Q_p \) is known, \( Q_t \) is obtained from the overall volumetric flow balance given as:

\[
Q_t = Q_g + Q_w + Q_f - Q_p. \quad [7.6]
\]

Thus, equations [7.5] and [7.6] are used as supplementary equations to solve the air and solids balances discussed in the following sections.

7.5.2 Volumetric Air Balance

The terms necessary to describe the air volume balance between zones are shown schematically in Figure 7.3. The procedure is identical to that presented in Chapter 4, but is repeated here for the sake of continuity.

The net change in the air content with time is determined by the difference of the volume of air per unit time entering a zone and the volume of air per unit of time leaving a zone. For example, the
Figure 7.3 Schematic representation of the volumetric air balance around the column.
volumetric flow of air which enters zone [2] consists of the aeration rate \( Q_g \), the air carried into the zone from above by the downward flow of pulp in the column \( Q_{d3} \) and the flow of bubbles into zone [2] from zone [1] due to buoyancy \( Q_{e1} \). Flows leaving the zone include the volume flow of air to the reject zone \( Q_{e2} \) and again the flow due to the buoyancy of the bubbles leaving zone [2] \( Q_{e2} \). Each flow can then be converted to fractional air content by dividing by the total volume of the zone \( V_2 \). Upon substitution of the appropriate terms, the following expression is derived for the air hold-up in zone [2]:

\[
\frac{d\varepsilon_2}{dt} = \left[ (Q_g + Q_{d3} + U_1\varepsilon_1) - (Q_{e1} + Q_{e2} + U_2\varepsilon_2) \right] V_2 \tag{7.7}
\]

7.5.3 Unattached Solids Phase

The terms considered in the unattached solids balance are represented schematically in Figure 7.4. Three flow terms are responsible for the movement of unattached particles throughout the column. The first is the flow which carries free particles into the froth product, \( Q_p \), which is defined as the total volume flow of all phases to the product. \( Q_p \), therefore, must be corrected to account for slurry flow only. Since the fractional air hold-up in the froth zone is represented by \( \varepsilon_f \), the term \( 1-\varepsilon_f \) represents the amount of slurry leaving the froth zone. Therefore, the transport of unattached solids with the product flow can be corrected by multiplying \( Q_p \) by \( 1-\)
Figure 7.4 Schematic representation of the free solids balance for the column.
The resulting term describes the transport of unattached particles into the froth and is represented as follows:

\[ Q_p(1-\varepsilon_f)C_{f,i} \]

where

- \( Q_p \) = volumetric product flow rate,
- \( \varepsilon_f \) = froth percent air, and
- \( C_{f,i} \) = concentration of unattached particles in zone (i).

A second type of unattached solids flow which must be considered is that contribution which is carried down the column directly by \( Q_w \) and \( Q_d \). These flows must be considered separately since they can be present even if \( Q_g \) and \( Q_p \) are zero. The mass per unit time carried by either of these flows is determined simply by multiplying the flow by the concentration of unattached solids in the appropriate zone.

In addition to the flow terms, there is also a settling term and a rate term which must be considered. The settling term is very similar to the rise term used in the air balance. In this case, the Stokes equation for particle settling has been used. The rate term accounts for particles which suddenly disappear from the unattached phase and appear in the attached phase. It is defined as the product of the attachment rate constant, \( k \), and the mass of unattached particles in a given zone, \( M_{f,(i,j)} \). The attachment rate constant is determined from:
\[ k = \frac{3P}{2D_b} V_g, \]  

where \( P \) is the probability of particle collection, \( V_g \) is the superficial gas flow rate and \( D_b \) is the diameter of the bubble. In the present work, \( P \) has been evaluated as a function of particle size, bubble size and hydrophobicity using a fundamental hydrodynamic analysis detailed elsewhere (Luttrell, 1986).

Upon converting particle concentration to mass (i.e., \( C_{x,(i,j)} = M_{f,(i,j)} / V_z \)), a mass balance around the zone \([2]\) yields:

\[
\frac{dM_{f,(2,j)}}{dt} = (Q_{d(1-\epsilon_3)} + U_p A) M_{f,(3,j)} - (Q_{d(1-\epsilon_2)} + U_p A) M_{f,(2,j)} - 2M_{f,(2,j)} V_z
\]  

[7.9]

where the subscripts \((i)\) and \((j)\) represent zone number and particle type (i.e., weight percent middlings of the particle), respectively. A similar procedure can be used for handling other zones.

### 7.5.4 Attached Solids Balance

Figure 7.5 is a schematic representation of terms necessary to describe the movement of particles attached to air bubbles. Once again, consider the balance of terms around zone \([2]\). The mass flow of attached particles leaving zone \([2]\) consists of those carried out by the tailings flow \( (Q_t C_{a,(2,j)}) \) and those carried upward by the
Figure 7.5 Schematic representation of the attached particles balance for the column.
bubbles and gas flow rate \((Q_g + U_iA)C_{a, (2, j)}\). Attached particles enter the zone by either the downward flow of pulp in the column \((Q_dC_{a, (2, j)}\) or by bubble/particle attachment \((kM_f, (2, j))\). A balance of terms around zone \([2]\) yields:

\[
\frac{dM_{a, (2, j)}}{dt} = kM_f, (2, j)Y_f + (Q_g + U_iA)M_{a, (1, j)} + Q_dM_{a, (3, j)}
\]

\[
- [(Q_g + U_2)M_{a, (2, j)} + Q_tM_{a, (2, j)}]
\]

[7.10]

In the present work, particle detachment has not been considered since fine particles have a very low probability of detachment due to their low inertial force. Expressions for the other zones are obtained in a similar manner.
7.6 Derivation of Corrected Flotation Rate Constant

The flotation rate constant is an indication of how quickly particles are being collected on the bubble surface. Unfortunately, as the degree of particle loading on the bubble surface increases, area available for particle loading decreases. When the bubble becomes completely covered with particles it can no longer perform any useful work in terms of collection. In this case, it follows that the flotation rate constant approaches zero at the point of total coverage. Therefore, a technique has been developed which modifies the flotation rate constant in order to accommodate bubble loading.

From Figure 7.6 it can be seen that the surface area available for particle loading can be represented as follows:

\[ S_a = \pi(D_b + D_p)^2, \]  \[ 7.11 \]

where \( D_b \) and \( D_p \) represent the diameter of the bubble and particle, respectively. Assuming simple cubic packing, the number of particles that a single bubble can hold is then expressed as,

\[ N_p = \frac{\pi(D_b + D_p)^2}{D_p^2}. \]  \[ 7.12 \]

The total number of possible attachments in a zone, \( N_p,_{\text{max}} \) can then be determined by multiplying Equation [7.12] by the number of bubbles in that zone, \( N_b \). Knowing the volume fraction of air in any zone from
Figure 7.6 Schematic diagram representing bubble loading.
the population balance equations, the volume of air in that zone is simply the zone volume, \( V_Z \), multiplied by the fractional air hold-up, \( c \). Dividing the air volume by the volume per bubble yields the total number of bubbles per zone. This relationship can be derived as follows:

\[
N_b = \frac{V_a}{V_b}, \quad [7.13]
\]

where,

\[
V_a = cV_Z, \quad [7.14]
\]

and

\[
V_b = \frac{\pi D_b^3}{6}. \quad [7.15]
\]

Upon substitution of equations [7.14] and [7.15] into [7.13] the following relationship can be derived for the total number of bubbles in zone [i]:

\[
N_b = \frac{6c_i V_Z}{\pi D_b^3}. \quad [7.14]
\]

Therefore, the maximum number of attachments in any zone can be
represented as,

\[ N_{p, \text{max}} = N_p N_b \]  \[ 7.15 \]

where, upon substitution of the appropriate terms, the following expression for \( N_{p, \text{max}} \) is derived:

\[ N_{p, \text{max}} = \frac{6 \varepsilon V_z (D_b + D_p)^2}{D_p^2 D_b^3}. \]  \[ 7.16 \]

The fractional bubble loading is represented as the number of attached particles in any zone \([i]\) divided by the total number of attachments which are possible. The number of attached particles of type \((j)\) in any zone, \(N_{a, j}\), can be represented as follows:

\[ N_{a, j} = \frac{M_{a, j}}{\rho_j \pi D_p^3} = \frac{6 M_{a, j}}{\pi \rho_j D_p^3}, \]  \[ 7.17 \]

where,

\( M_{a, j} = \) the mass of attached particles of type \((j)\), and

\( \rho_j = \) the density of type \((j)\) particles.

The total number of attached particles of all classes, \(N_{a, \text{tot}}\), can
then be determined by:

$$N_{a, \text{tot}} = \sum_{j=1}^{J} N_{a,j},$$ \hspace{1cm} [7.18]

where $J=3$ for the case of coal flotation (i.e., pure coal=1, middlings=2, and pure ash=3).

Therefore, the fractional bubble loading, $f_b$, is represented as follows:

$$f_b = \frac{N_{a, \text{tot}}}{N_{p, \text{max}}} = \frac{D_b^3 \frac{M_{a,j}}{\rho_j}}{\pi \varepsilon V_p (D_b + D_p)^2}.$$ \hspace{1cm} [7.19]

The flotation rate constant, $k$, can now be corrected for the effect of bubble loading by multiplying by the fractional bubble loading as follows:

$$k = \frac{3P}{2D_b} \varepsilon V_g [2(1 - f_b)].$$ \hspace{1cm} [7.20]

The above functional form for fractional loading assumes that the rate constant remains at its full value until the bubble becomes fifty percent loaded. At this point, the rate constant will begin to decrease until it reaches zero when the bubble is completely covered.
7.7 Solution Methodology

Before solving the mixed zone model, it is necessary to obtain values for two unknown model parameters, probability of collection (P) and froth film thickness (T). The experimental methods for determining each of these values have been discussed elsewhere (Luttrell, Adel and Yoon; 1987).

The dynamic solution to the mixed zone model was obtained by applying the numerical Euler method. This technique has the advantage that it is simple to program; however, it can lead to large discretization errors if the step size is not sufficiently small. On the other hand, as the step size is decreased, the cumulative round-off error tends to increase. The cumulative error, however, did not appear to be substantial when the steady-state results of a dynamic simulation were compared to the true steady-state solution.

Since laboratory data were only available for steady-state conditions, a steady-state solution to the model equations was determined. The steady-state solution to the mixed zone model was obtained by setting the left-hand side of each differential equation to zero, and solving the resulting set of simultaneous algebraic equations. A resubstitution technique was used for this purpose. Using this approach, a solution could be obtained in less than one minute with compiled BASIC on an IBM-PC. The trends predicted by the model for various parameter configurations could be directly compared with those found experimentally. However, in order to use the simulator as a dynamic control model, the aforementioned differential
equations would have to be solved on-line as a function of time. In this manner, the model could be used to track time dependent changes within the column.

In presenting the simulation results, the total mass flow of mineral is determined by summing the mass of unattached and attached particles reporting to either the product or reject streams. Combining this solution with the assay of each species allowed both grade and recovery of the product and reject streams to be calculated. Other values, such as percent solids in the product and reject streams, air and liquid flow rates, etc. are also determined.
7.8 Results and Discussion

7.8.1 Dynamic Simulations

The dynamic solution to the mixed zone model was used to determine the time required to achieve steady-state from a start-up condition. This can also be used as an indication of the time needed for the column to stabilize when it encounters a disturbance, thus illustrating the potential of using the simulator as a dynamic algorithm for on-line, model-based control.

The conditions under which all simulations were conducted are given as follows:

- Superficial Feed Velocity = 15.4 cm/min
- Superficial Gas Velocity = 19.7 cm/min
- Feed Percent Solids = 5.3%
- Feed Percent Ash = 36.4%
- Mean Particle Size = 5.5 μm.

The unknown parameters of froth film thickness (F) and particle collection probability for coal (P), as determined from experimental data, were 4.9 microns and 0.00028, respectively (Luttrell, Adel and Yoon; 1987). The particle collection probability for mineral matter was assumed to be zero.

As shown in Figure 7.7, the recovery appears to reach steady-state quite rapidly (i.e. < 5 min.). However, the product ash content requires nearly 20 minutes to achieve steady-state. The simulated
Figure 7.7  Comparison of dynamic model simulation to experimental results for the flotation of an Elkhorn III seam coal (data from Luttrell, 1986).
results were found to be in good agreement with experimental results produced in a 1-inch laboratory column. The implication of this finding is that if the column is operated for a short time, biased results can be produced which will show a much cleaner product than can be achieved under steady-state conditions. Thus, it is important to monitor the product grade rather than recovery when determining steady-state conditions for flotation column testing.

7.8.2 Steady-State Simulations

a) Effect of Bubble Size

Using the steady-state solution to the mixed zone model, a series of simulations were conducted to determine the effects of various operating and design parameters on the product recovery and ash obtained during the flotation of fine coal. As a result of several simulations, it was determined that column length-to-diameter ratio, bubble diameter and wash water addition rate had the largest influence on product grade and recovery. The effects of these three parameters are illustrated in Figures 7.8 - 7.11. Figure 7.8 shows the relationship between recovery and bubble diameter ($D_b$) for various column length-to-diameter ratios (L/D) in the absence of wash water. As can be seen, recovery increases significantly as bubble size is reduced. This is a direct result of the increase in the number of bubbles and the probability of collection (P) with decreasing bubble size. The latter effect has been discussed in detail elsewhere (Luttrell, 1986). An increase in recovery is also observed as the L/D
Figure 7.8  Percent recovery as a function of bubble diameter for various length-to-diameter ratios in the absence of wash water.
ratio increases, although this increase is not as significant as that produced by decreasing the bubble size. This effect is primarily a result of the increased residence time which provides more opportunity for bubble/particle collision.

The product ash results corresponding to the simulations presented above are illustrated in Figure 7.9. As shown, the ash content tends to decrease with decreasing bubble size down to a diameter of approximately 250 microns, after which, the ash content tends to increase. The initial decrease in ash content with decreasing bubble diameter is largely due to the increase in P discussed previously. Since the mineral matter in this simulation has been considered to have no floatability, this should result in an increase in selectivity. Below a bubble size of approximately 250 microns, however, the increased water recovery due to the large number of bubbles increases the non-selective entrainment of mineral matter. This results in an increase in the product ash content. Thus, the simulations predict that operating at a bubble size between 200 and 300 microns should give the best conditions for obtaining a low ash product at a relatively high recovery. The results shown in Figure 7.9 also suggest that increasing the column L/D ratio has a significant effect on the product ash. This is a result of the increased product recovery associated with the high L/D ratios.

When countercurrent wash water is added just below the froth/pulp interface, little change is seen in the shape of the recovery curve as shown in Figure 7.10, although the recovery values are
Figure 7.9 Product ash percent as a function of bubble diameter for various length-to-diameter ratios in the absence of wash water.
Figure 7.10 Percent recovery as a function of bubble diameter for various length-to-diameter ratios in the presence of counter current wash water.
somewhat lower. In this case, wash water was added at a superficial velocity of 2 cm/min which lowered the recovery values approximately 5%. The effect of wash water on product ash, however, is considerably more significant. As shown in Figure 7.11, the use of countercurrent wash water reduces the product ash content for larger values of $D_b$ and $L/D$ ratio to less than 1%. If a typical superficial wash water velocity of approximately 20 cm/min is used, nearly all ash entrainment can be suppressed. Further evidence of this finding is shown in the experimental results shown in Figure 7.20.

b) Effect of Middlings

In order to be more representative of an actual coal flotation system, it was necessary to incorporate a third particulate phase into the model. These middlings particles represent the material which is not completely liberated. In some instances the degree of middlings can be quite substantial, thus creating difficulties in making sharp separations. Regardless of the amount of countercurrent water used, the non-liberated incombustible material cannot be removed from the flotation product.

The effect of the percentage of middlings on flotation recovery is shown in Figure 7.12 as a function of bubble diameter. It can be seen that a slight decrease in recovery occurs as the percentage of middlings is increased from 10 to 70 percent. It is interesting to note that the presence of middlings has little affect on product recovery when compared to results obtained in the absence of middlings (Figure 7.8). The presence of middlings does, however, have a
Figure 7.11 Product ash percent as a function of bubble diameter for various length-to-diameter ratios in the presence of counter current water.
Figure 7.12 Percent recovery as a function of bubble diameter for various percent middlings in the absence of counter current water.
dramatic affect on the product ash percent. Figure 7.13 shows that the product ash percent increases by more than 20% for all bubble sizes as the percentage of middlings is increased from 10 to 70%. This quite clearly illustrates the loss of selectivity which can occur due to the presence of nonliberated material.

The addition of countercurrent water has little affect on either product recovery or grade, as shown in Figures 7.14 and 7.15 respectively. The slight improvement in ash percentage is due to the removal of the small degree of free ash material which remains in the system. As the percentage of middlings increases to 70 percent the amount of free ash in the system is decreased to virtually zero. In this case, the grade versus bubble diameter curve approaches the same result as that shown in Figure 7.13 for no wash water.

c) Effect of Gas Flow Rate

The gas flow rate through the column \(Q_g\) directly affects column performance in several ways. The first, and most significant, is the change in the flotation rate constant which occurs with a change in \(Q_g\). An increase in the gas flow rate results in a linear improvement in flotation rate constant, as illustrated in equation 7.20. Unfortunately, an increase in gas flow rate also results in an increased fractional air hold-up. This relationship has been previously described in Chapter 4. The increased volumetric air fraction detracts from the cell volume and, as a result, shortens the liquid residence time within the cell. However, the results shown in
Figure 7.13  Product ash percent as a function of bubble diameter for various percent middlings in the absence of counter current water.
Figure 7.14  Percent recovery as a function of bubble diameter for various percent middlings in the presence of counter current water.
Figure 7.15  Product ash percent as a function of bubble diameter for various percent middlings in the presence of counter current water.
Figure 7.16 indicate that the improved kinetics dominate the residence time effect. In this case, the recovery is shown to increase from 40 to nearly 100 percent as the gas flow rate is increased from 0.06 to 0.5 LPM. Figure 7.16 also shows the decline in product grade with an increase in $Q_g$. This is predominantly due to the increased water recovery which occurs as the gas flow rate increases. Similar results have been found experimentally for the flotation of an Elkhorn III seam coal (Weber, 1988).

d) Effect of Feed Percent Solids

As shown in Figure 7.17, simulations indicate no change in either product grade or recovery as a function of feed percent solids for the case of no bubble loading. However, when the bubble loading correction to the flotation rate constant is included a decline in product recovery is observed. This decline is due to the decrease in flotation rate which occurs at higher percent solids due to increased particle coverage. The corresponding increase in product ash percent is simply a result of the lost recovery. This is in agreement with experimental findings, although the drop off in recovery may be more substantial due to additional changes which alter flotation conditions such as reagent concentrations.

e) Effect of Feed Rate

Figure 7.18 shows the effect of an increase in feed rate on product grade and recovery. As expected, the recovery decreases from 80 to 20 percent as the feed rate is increased from 0.06 to 0.3 LPM.
Figure 7.16 Simulated product recovery and ash percent as a function of gas flow rate.
Figure 7.17 Simulated product recovery and ash percent as a function of feed percent solids.
Figure 7.18 Simulated product recovery and ash percent as a function of feed rate.
This is predominantly an effect of residence time within the cell rather than a bubble loading effect. The product ash percent also decreased from 30 to approximately 7 percent over the same range of feed rates. This is most likely a result of a higher concentration of coal within the cell due to the lost recovery. This implies that the recovery associated with water recovery would be of a higher grade material and thus, the product ash percent would be improved. This also agrees with experimental findings (Luttrell et al, 1988) although in this case the improved product grade could also be a result of selectively rejecting middlings type particles at the shorter residence times.

f) Effect of Wash Water

The effect of countercurrent wash water on product grade and recovery is shown in Figure 7.19. It can be seen that the effect of wash water on recovery is minimal over a range of flow rates from 0 to 0.1 LPM. This is similar to previous results shown as a function of bubble diameter in Figure 7.10, however, in this case a superficial wash water velocity of only 2 cm/min was used. This was necessary in order to illustrate the effect of wash water without entirely removing all of the entrained ash material. However, it can be seen from Figure 7.19 that the removal of entrained ash can be significantly improved at higher wash water flow rates. In fact, at 0.1 LPM (19.7 cm/s) virtually all ash entrainment is eliminated. This corresponds quite well to experimental results obtained in a 1-inch column which
Figure 7.19  Simulated product recovery and ash percent as a function of wash water flow rate.
indicate that a superficial velocity of 20 cm/min is sufficient for rejection of entrained material. The recovery/grade versus wash water superficial velocity curve is shown in Figure 7.20. It can be seen that the optimum ash rejection occurs as the superficial velocity approaches 20 cm/min.

g) Effect of Column Length

One of the most significant parameters affecting column performance is height. In order to more closely examine this variable a series of simulations were conducted as a function of column height. The results, shown in Figure 7.21, illustrate the impact of a change in column height on performance. An increase in column height from 0.2 to 2 meters increased combustible recovery from 30 to nearly 90 percent. This is a result of the increased residence time associated with an increase in column height. Since particle recovery is a function of residence time, a longer column provided higher recoveries. The decrease in product ash percent is a result of the increased coal recovery. It appears that a column height of approximately 2 meters would have been optimal for these feed conditions.
Figure 7.20  Combustible recovery and product ash percent as a function of wash water flow rate for column flotation of a micronized Elkhorn III seam coal
Figure 7.21  Simulated product recovery and ash percent as a function of column length.
7.9 **Application to Scale-Up Methodology**

The population balance approach clearly would be the best method to use for column scale-up. This would incorporate all of the concepts illustrated in Chapters 3 through 5 into one model. As can be seen from Figure 7.22, the simulation results for this work follow the recovery versus $kt$ trend described earlier (Chapter 3). This indicates the model predictions are similar to those made by the empirical scale-up relationships. In this case, however, the model $kt$ curve more closely approximates a plug flow condition. This is due to the high $L/D$ ratio used for the simulations (i.e., 60:1). Under these conditions an extremely large Peclet number would exist, thus providing more plug flow conditions as represented by Figure 7.22.

Unfortunately, too many unknown parameters exist at this time in order to make accurate predictions regarding product recovery and throughput. The most noticeable being the absence of a reasonable froth model. It has been clearly illustrated that the froth carrying capacity is the largest controlling factor in regards to recovery and throughput. Until a model is developed which can provide reliable estimates of froth bubble size and fractional air hold-up, these problems will not be solved.

The population balance model is well suited to monitor the change in product grade due to the recirculation effect discussed in Chapter 5, since it monitors the concentration of each particle species within the cell. In this manner, up-grading due to the effect of circulating loads would be easily accommodated.
Figure 7.22 Simulated product recovery as a function of $kt$ in comparison to the standard plug flow and perfectly mixed recovery models.
Although the simulator does have a few deficiencies, many of the trend analyses are helpful in examining column response to changes in operating conditions. This provides a useful tool for training as well as providing a certain degree of insight into scale-up relationships. For instance, the simulations involving bubble diameter, column length, gas flow rate and wash water flowrate are quite representative of the results observed experimentally. The greatest advantage in using model-based scale-up relationship, as previously mentioned, is the ability to make dynamic predictions to changing operating conditions.
7.10 Summary and Conclusions

1) A dynamic population balance model of a column flotation cell has been developed. A steady-state solution has been generated which is capable of making predictions with regards to the influence of operating parameters.

2) The model directly incorporates both middlings particles as well as bubble size distributions. This provides a more accurate representation of the physical processes present in an actual operating column.

3) The flow balance used in this model has provided a means of directly incorporating mixing into the model, thus eliminating unknown parameters associated with previous models.

4) The simulator has the potential for use as a dynamic algorithm for the control of column flotation. This would provide an on-line procedure for the dynamic control of a flotation column.

5) The population balance model appears to have the potential for use in scale-up. This would provide a method for incorporating all of the scale-up considerations into one simulator, thus eliminating the iterative approach which is currently required.
7.11 Nomenclature

A  = column cross-sectional area
C  = concentration
Ca,i  = concentration of attached particles in zone i
Cf,i  = concentration of free particles in zone i
Co  = concentration at time zero
Db  = bubble diameter
Dp  = particle diameter
fb  = fractional bubble loading
ka  = flotation rate constant
ks  = slow floating rate constant
kf  = fast floating rate constant
Mf,i  = mass of free particles in zone i
Ma,i  = mass of attached particles in zone i
Na,j  = number of attached particles of type j
Na,tot  = total number of attached particles
Nb  = number of bubbles in zone i
Np  = maximum single bubble loading
Np,max  = maximum bubble loading for all bubbles in a zone
P  = probability of particle collection
Qd  = Qt - Qg
Qf  = volumetric feed flow rate
Qp  = volumetric product flow rate
Qt  = volumetric tailings flow rate
Qw  = volumetric wash water flow rate
\( t \) = time

\( T \) = froth film thickness

\( U_i \) = bubble hindered rise velocity in zone \( i \)

\( V_a \) = volume of air in zone \( i \)

\( V_b \) = volume of a single bubble

\( V_g \) = superficial gas flow rate

\( V_z \) = zone volume

\( c_f \) = fractional air hold-up in froth zone

\( c_i \) = fractional air hold-up in zone \( i \)

\( \rho_j \) = density of particle type \( j \)
7.12 References


Chapter 8

GENERAL SUMMARY AND CONCLUSIONS

The results of the present investigation may be summarized as follows:

1. Pilot-scale flotation studies of column and conventional flotation have been conducted at feed rates up to 500 lb/hr. Three series of tests were conducted in order to evaluate column performance on i) processing of a high ash, cyclone overflow material, ii) micronized coal flotation (-20 microns), and iii) coarse coal flotation (28 mesh X 0). The latter two tests being conducted in comparison with conventional flotation. When processing the cyclone overflow samples, the column was able to produce a clean coal product containing less than 10 percent ash at combustible recovery levels in excess of 80 percent. For micronized coal, the column provided a higher level of recovery and feed ash rejection as compared to conventional flotation, while operating at an equivalent retention time. Coarse coal flotation required that the column operate at a slightly longer retention time. However, the conventional cells were not able to produce as high a level of ash rejection as the column.

2. The effect of mixing on column flotation performance was investigated. Residence time distribution studies were conducted in 2- and 4-inch laboratory flotation columns. The classical
two parameter fitting approach was modified using the F-curve in
order to determine the liquid axial dispersion coefficient as a
function of column flow rates and geometries. Using additional
data available from the literature, a dimensionless relationship
was established which describes the Peclet number as a function
of column geometry, liquid and gas flow rates.

3. A population balance model was developed in order to determine
the effect of changing column geometries and flow conditions on
the fractional air hold-up within the column. The model is
capable of making predictions regarding the air hold-up in the
column recovery zone, as well as the extent of air which is lost
with the tailings flow rate. The model has been constructed such
that the effect of a bubble size distribution on column
performance can be examined. The model can calculate the bubble
diameter in an operating flotation column from differential
pressure measurements within the column recovery zone.

4. The concept of a carrying capacity limitation in the column
froth has been addressed. An expression has been developed which
characterizes the carrying capacity in terms of column flows and
percent solids. This equation has been quantified in terms of a
maximum superficial feed velocity which can exist for a given set
of column operating conditions.

5. A scale-up procedure has been developed for column flotation.
This routine incorporates the relationships which were derived
for axial dispersion, fractional air hold-up and froth carrying capacity. Based on the concept of maintaining a constant product recovery, this procedure can determine the required column geometry in order to match a desired level of throughput. A graphic representation of the scale-up procedure has been presented which is capable of illustrating the interaction among the various parameters associated with column operation. The effect of a change in bubble size, gas flow rate, mixing and retention time on column performance can be quantified based on this diagram.

6. Scale-up calculations have been made for 30- and 96-inch flotation columns. These predictions were based on experimental results obtained in a 2-inch diameter by 5-foot high laboratory flotation column. The scale-up predictions have been experimentally validated in operating 30- and 96-inch flotation columns processing a high ash, cyclone overflow stream. Predictions for feed flow rate and product recovery were within 5 percent of actual column operation.
Chapter 9

RECOMMENDATIONS FOR FURTHER STUDY

Based on the information gathered in the present work, further investigation into the following areas is suggested.

1. The pilot-plant flotation studies indicated that the conventional flotation banks were capable of producing comparable levels of recovery in comparison to the column. However, the degree of ash rejection was typically much lower than that of the column. As discussed in Chapter 2, this is a result of the hydraulic entrainment associated with conventional flotation. Since counter current wash water is capable of increasing ash rejection and stabilizing the column froth, it is recommended that an attempt be made to develop a froth sprinkling system for conventional flotation cells. If this could be accomplished without any detrimental effects to the froth (i.e., froth collapse) the ash rejection of the conventional cells could be increased significantly.

2. Although mixing has been quantified for the column recovery zone, the axially dispersed plug flow model used to represent the mixing conditions had the underlying assumption that the column was perfectly mixed in the radial direction. This may not necessarily be the case, especially in the immediate region of the feed inlet and bubble injectors. In this region, the high
superficial flow rates associated with the incoming fluid may cause a significant degree of mixing at the point of entry. The distribution of feed and air across the column cross-sectional area also needs to be considered. A distributor which instantly spreads the incoming fluid across the cell area would be ideal. Practically speaking, however, one should investigate different types of distributor and baffling arrangements in order to get the maximum recovery out of a given retention time. Measurements of radial dispersion may prove useful in selecting the proper distributor.

3. A further evaluation of the effect of bubble size distribution on column performance could be conducted. The air fraction simulator presented in Chapter 4 could be used to examine the impact of different sparging mechanisms on column performance by characterizing the bubble size distribution produced by each system. In this manner, conclusions could be drawn in regards to which type of bubble generator provides the optimum aeration of the column.

4. The strong influence of froth overloading and bubble carrying capacity on column performance requires that additional investigations be conducted in this area. Although several quantitative approaches have been developed, they are by no means comprehensive. The difficulty in obtaining reliable data in regards to froth behavior has also limited the information which is available to verify the current models. A large data base
needs to be developed for multiple column geometries and operating conditions, as well as type of solids processed (i.e., coal, sulfides, industrial minerals, etc.). Based on this information a more detailed understanding of the froth behavior could be developed.

5. Additional data needs to be collected in order to further validate the column scale-up procedure, especially in light of the froth characteristics discussed above. Modifications may to be necessary to the scale-up procedure in order differentiate column carrying capacity from froth overloading effects.
Appendix I: Elkhorn III
### PROXIMATE ANALYSIS - CONVENTIONAL FLOTATION

**Weight Percent - Dry Basis**

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<th>Ash Percent</th>
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FR 1 = 138.0 lb/hr
FR 2 = 256.8 lb/hr
FR 3 = 346.8 lb/hr
FR 4 = 519.6 lb/hr
FR 5 = 522.6 lb/hr
### PROXIMATE ANALYSIS - 6-INCH COLUMN FLOTATION

**Weight Percent - Dry Basis**

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<th>Fixed Carbon</th>
<th>Sulfur</th>
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FR 1 = 40.8 lb/hr  
FR 2 = 53.4 lb/hr  
FR 3 = 82.2 lb/hr  
FR 4 = 148.2 lb/hr
PROXIMATE ANALYSIS - 8-INCH COLUMN FLOTATION

Weight Percent - Dry Basis

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<th>Sulfur</th>
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FR 1 = 51.0 lb/hr
FR 2 = 93.6 lb/hr
FR 3 = 137.4 lb/hr
FR 4 = 198.0 lb/hr
### PROXIMATE ANALYSIS - 14-INCH COLUMN FLUCTATION

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<th>Ash Percent</th>
<th>Fixed Carbon</th>
<th>Sulfur</th>
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FR 1 = 300.0 lb/hr
FR 2 = 448.2 lb/hr
FR 3 = 204.0 lb/hr
FR 4 = 652.2 lb/hr
Appendix II: Pittsburgh No. 8
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FR 1 = 63 lb/hr
FR 2 = 116.4 lb/hr
FR 3 = 234.6 lb/hr
FR 4 = 304.8 lb/hr
PROXIMATE ANALYSIS - COLUMN FLOTATION

Weight Percent - Dry Basis

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<th>Sample I.D</th>
<th>Weight Percent</th>
<th>Volatile Matter</th>
<th>Ash Percent</th>
<th>Fixed Carbon</th>
<th>Sulfur</th>
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FR 1 = 223.8 lb/hr
FR 2 = 135 lb/hr
FR 3 = 500.4 lb/hr
FR 4 = 168.6 lb/hr
FR 5 = 207 lb/hr
FR 6 = 226.2 lb/hr
Appendix III: Upper Freeport
# PROXIMATE ANALYSIS - CONVENTIONAL FLOTATION

**Weight Percent - Dry Basis**

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<th>Volatile Matter</th>
<th>Ash Percent</th>
<th>Fixed Carbon</th>
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FR 1 = 107.4 lb/hr
FR 2 = 151.8 lb/hr
FR 3 = 561.6 lb/hr
### PROXIMATE ANALYSIS - COLUMN FLOTATION

Weight Percent - Dry Basis

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<th>Volatile Matter</th>
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FR 1 = 345.6 lb/hr  
FR 2 = 513.0 lb/hr  
FR 3 = 168.6 lb/hr  
FR 4 = 30.0 lb/hr
Appendix IV: Cedar Grove
### PROXIMATE ANALYSIS - COLUMN FLOTATION

Weight Percent - Dry Basis

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FR 1 = 115.6 lb/hr
FR 2 = 275.4 lb/hr
FR 3 = 129.6 lb/hr
## PROXIMATE ANALYSIS - CONVENTIONAL FLOTATION

### Weight Percent - Dry Basis

<table>
<thead>
<tr>
<th>Sample I.D</th>
<th>Weight Percent</th>
<th>Volatile Matter</th>
<th>Ash Percent</th>
<th>Fixed Carbon</th>
<th>Sulfur</th>
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<td>64.72</td>
<td>0.76</td>
</tr>
<tr>
<td>FR 3, FEED, +100</td>
<td>48.76</td>
<td>32.31</td>
<td>4.79</td>
<td>62.90</td>
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<td>FR 3, FEED, -100</td>
<td>51.24</td>
<td>30.84</td>
<td>6.57</td>
<td>62.59</td>
<td>0.70</td>
</tr>
<tr>
<td>FR 3, TAIL, +100</td>
<td>33.69</td>
<td>29.08</td>
<td>14.15</td>
<td>56.77</td>
<td>-----</td>
</tr>
<tr>
<td>FR 3, TAIL, -100</td>
<td>66.31</td>
<td>23.25</td>
<td>33.94</td>
<td>42.81</td>
<td>0.75</td>
</tr>
</tbody>
</table>

FR 1 = 107.4 lb/hr
FR 2 = 151.8 lb/hr
FR 3 = 561.6 lb/hr
VITA

Michael J. Mankosa was born on December 20, 1960 in Weirton, West Virginia, where he lived until graduating from high school in 1979 in the top twenty of his class. During this time he was actively involved in student and civic affairs and received scoutings highest honor, eagle scout. He declined an appointment to the United States Military Academy in favor of enrolling in the engineering department at Virginia Polytechnic Institute and State University in the fall of 1979.

As a senior, he was active in the service fraternity of Pi Kappa Phi, the Virginia Tech Student Union, the Virginia Tech Martial Arts Club, as well as, president of the Virginia Tech Judo Club and Secretary of the Student Chapter of AIME, The Burkhart Mining Society.

He enrolled as a full time graduate student in the summer of 1983 in order to pursue a Master of Science degree in Mining and Minerals Engineering. During this time he was inducted into the honorary society of Phi Kappa Phi. He completed his M.S. thesis in 1986 and immediately enrolled in the Ph.D. program at Virginia Tech. He was chosen as the Department of Mining and Minerals Engineering outstanding graduate student for the 1987-1988 academic year and was selected to receive the College of Engineering Graduate Student Research Award in the spring of 1990. To date he has nine publications and eight presentations at national meetings.

Michael J. Mankosa

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