DYNAMIC TESTING AND MODELING OF A SUPERELEVATED SKEWED HIGHWAY BRIDGE

by

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ABSTRACT

Created in response to the aging infrastructure in the United States, the Long Term Bridge Performance Program (LTBPP) under the Federal Highway Administration (FHWA) proposes to assess the long-term performance of representative bridges through nondestructive evaluation (NDE) techniques and visual inspection. For consistency, a set of guidelines is needed to define the procedures for testing each bridge. The NDE techniques involve dynamic testing, and the protocol for this testing has yet to be finalized.

To evaluate the dynamic testing guidelines, a 103 ft single-span, simply supported highway bridge was dynamically tested. The test bridge was characterized by a skew of 34° and superelevation around 4%. Forced vibration testing involved an impact hammer with accelerometers measuring the response. Resonant frequencies were identified from the data by picking peaks from the magnitudes of the frequency response functions (FRF). Eleven modes were identified with frequencies ranging from 2.75 Hz to 22.5 Hz. Mode shapes associated with each mode were constructed using the imaginary components of the FRFs. The half-power bandwidth method was used to estimate the damping for each mode, with values ranging from 1% to 5% of critical damping.

Finite element (FE) models of the bridge were constructed in the commercial FE software Abaqus. The effects of adding and removing superelevation and skew, varying mesh refinement, and changing boundary conditions on modal parameters were thoroughly investigated. FE models were compared to the experimental results by directly comparing frequencies and using the modal assurance criterion to compare mode shapes. Support conditions of the actual structure were bounded using the results of the comparison.

Much insight was gained about forced vibration testing as applied to a full-scale bridge. The spectral resolution of the data proved to limit the accuracy and confidence of detecting closely-spaced modes and calculating damping estimates. Also, a more controlled method of
exciting the structure was desired, such as using a shaker with a known input. Resonant frequencies of the FE models were sensitive to changes in boundary conditions, with some frequencies doubling. Both changes in boundary conditions and including skew and superelevation noticeably affected the mode shapes. When compared to the experimental results, the models with idealized roller and pin boundary conditions provided the best correlations based on resonant frequencies and mode shapes.
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Chapter 1. Introduction

1.1 Background

With the infrastructure in the United States aging, the need for condition assessment of highway bridges is continuing to increase. Of the 590,000 tunnels, bridges, and culverts monitored by the National Bridge Inventory, 152,136 bridges were deemed structurally deficient or functionally obsolete in 2007 (FHWA 2009). In response to this overwhelming statistic, the Federal Highway Administration (FHWA) recently initiated the Long Term Bridge Performance Program (LTBPP). The goal of the LTBPP is to evaluate the long term physical and functional aspects of bridge performance by compiling a database with quantitative results from nondestructive evaluation (NDE) techniques and visual inspection. This includes the creation of protocols defining testing guidelines. One of these NDE techniques is dynamic analysis.

Ewins (2000) used the phrase “Modal Testing” to describe “the processes involved in testing components or structures with the objective of obtaining a mathematical description of their dynamic or vibration behaviour.” As applied to large civil engineering structures, modal testing, or experimental modal analysis, involves using measurements from discrete sensor locations to derive modal parameters, such as resonant frequencies, mode shapes, and damping estimations. The modal parameters are determined by observing characteristics in the experimental data that are based on similar characteristics of a mathematical model. The mathematical model condenses a complex, continuous system of interacting steel and concrete components into a simplified, linear multiple degree-of-freedom system. In essence, experimental modal analysis enables global dynamic quantification of a structure through its response at a few discrete positions.

The first modern implementation of the finite element (FE) method is accredited to Richard Courant with his work in the early 1940’s (Cook et al. 2002). Since then, the FE method has been applied to various areas of applied science and engineering. Due to the scale of bridges, finite element models of bridges range in complexity from a single simple two-dimensional (2-D) model using 1-D elements to a full three-dimensional (3-D) model comprised of solid elements. The complexity of the model depends on the application. For global dynamic behavior, solid elements are rarely needed. Instead, shell elements suffice for representing the deck, and either shell elements or beam elements are appropriate for modeling the girders,
barriers, and diaphragms. A FE model provides insight into the behavior of a bridge to aid in constructing a testing plan before setting any instruments on the actual structure. The model also acts as a reference for comparison with the experimental results to aid in quantifying characteristics that are difficult to measure directly, such as boundary conditions and stress/strain distributions.

1.2 Objectives

As one of the NDE techniques used in the LTBPP, dynamic testing needs further investigation to develop a testing protocol applicable to all bridges in the program. The first objective of this research was to evaluate the viability of using impact force excitation with accelerometers placed on the surface of the deck. A test structure in Blacksburg, Virginia, was selected for this evaluation. The 103 ft single-span bridge is simply supported, has steel girders made composite with a concrete deck, and is currently free of traffic. The bridge also has the added complexity of a 34° skew and a superelevation of approximately 4%. These characteristics are representative of many of the other test bridges in the LTBPP, making the bridge a good candidate for this study. The second objective of this research was to investigate the effects of skew, superelevation, and varying boundary conditions on the resonant frequencies and mode shapes of this single-span bridge. FE models were constructed in Abaqus Version 6.7-3 (2007) with multiple variations of geometry, mesh refinement, and boundary conditions. FE models were later compared to the experimental results to assess the quality of the results and bound the restraint from the supports.

1.3 Thesis Organization

Chapter two presents a literature review of previous work involving experimental modal analysis and associated finite element modeling of highway bridges. Chapter three includes an overview of the testing, data processing, and finite element modeling procedures with a short discussion of the theoretical background behind modal analysis and digital signal processing. The results from experimental testing and numerical modeling as well as a comparison of both are presented in chapter four. Chapter five highlights the main conclusions and makes recommendations for further research.
Chapter 2. Literature Review

Dynamic testing is used by many facets of engineering including mechanical, aerospace, and civil. In all disciplines, instruments are placed on a structure at discrete locations to capture the structural response from an excitation source. The results are then analyzed by using the time domain signal or transforming the time domain signal into the frequency domain. From the results, modal parameters, such as natural frequencies, mode shapes, and damping values, are determined. A finite element model of the structure provides an approximation of the natural frequencies and mode shapes to aid in specifying excitation frequency bandwidths and instrumentation positions. The model is also a reference for comparing experimental results. Unlike other engineering disciplines, civil engineering involves large structures, such as buildings, bridges, and dams, tested in the field instead of under the controlled conditions of a laboratory, so special considerations are required (Green 1995). The following sections present previous work involving experimental testing, interpretation of results, and modeling of highway bridges. Many of the sources discuss both experimental testing and modeling, so they are presented in both the testing and modeling sections (2.1 and 2.2).

2.1 Dynamic Bridge Testing

Dynamic testing of bridges involves two main components: excitation and data acquisition. Excitation sources are categorized as ambient vibration, free vibration, or forced vibration. Ambient vibration sources come from the surrounding environment and can include wind, seismic activity, traffic, waves, tidal fluctuations, or ground vibrations generated by industries (Hsieh et al. 2006). The advantages of ambient vibrations include low cost, little to no disruption of traffic, long-term excitation, and their inherent presence. The disadvantages of this technique are in the characterization of the input. Amplitude, duration, direction, and frequency content are all subject to variability. Monitoring wind speed and direction, ground motion during seismic events, or traffic density and weight is sometimes used to characterize the input. Assumptions are made about the input that include stationarity, whiteness, and directionality (Hsieh et al. 2006). Another disadvantage of using traffic for ambient vibrations is that the dynamic characteristics of both the vehicles and bridge are being measured (Green 1995). Tests using ambient vibrations typically have long durations to help compensate for the disadvantages.
and aid in the assumptions. Green (1995) suggested using ambient vibrations for bridges with spans greater than 230 ft (70 m).

Free vibration excitation occurs when a bridge is displaced and released suddenly. Damping causes the resulting response to be transient. The initial displacement is provided from a large suspended mass or by using an adjacent structure as an anchor to pull on the bridge. Free vibration can only be performed under certain conditions because either a large mass or adjacent structure is needed. It is also dangerous to the people performing the test because the large amount of energy required to displace the bridge is released suddenly.

Forced vibration excitation provides an advantage over ambient and free vibrations because the form, location, amplitude, frequency content, duration, and time of the input are all controlled. Controlling the magnitude and frequency content of the force ensures that the response from the bridge is above the noise level of the system and that the bridge is responding to a known range of frequencies. Forced vibration is provided by either an impact or a shaker. Sohn et al. (2004) state that forced vibration presents an advantage over ambient vibration but conclude that in some situations, forced vibration is impractical. Ambient vibrations on a bridge which cannot be closed from traffic or is subject to large ambient forces are difficult to overcome with a controlled and quantifiable force input. The authors provided the example of a highway bridge loaded by high-speed traffic, wind, and thermal gradients.

Impact excitation is provided by a mass, such as a hammer or falling weight, suddenly coming in contact with the bridge. The magnitude and frequency content of the force is controlled by the size of the mass and the acceleration at impact. The acceleration of the mass at impact varies based on the velocity of the mass and the material between the mass and the bridge surface. Hammers typically have interchangeable tips with different stiffnesses, and different materials can be placed between a falling weight and the bridge. Agardh (1994) discussed considerations for impact excitation of a highway bridge, and Green (1995) provided recommendations for selecting the hammer tip. Force from the impact is quantified by either a force transducer between the mass and bridge or an accelerometer on the mass. However, measurements from an accelerometer contain error because the effective mass of the hammer can be difficult to quantify accurately and the accelerometer also senses vibrations from the hammer arm (Green 1995). The advantages of impact excitation are its low cost, simplicity, and
mobility. The disadvantage is the precise control over frequency bandwidth and amplitude. Green (1995) suggested using impact excitation for bridges with spans less than 100 ft (30 m).

Shakers come in the form of eccentric mass shakers or linear variable mass shakers. Eccentric mass shakers utilize masses that rotate about a central point. The force is controlled at different frequencies by the eccentricity of the masses. Linear variable mass shakers consist of a mass travelling along one axis. They are implemented vertically or horizontally and can produce steady-state sinusoidal, chirp, or other waveforms. Shakers provide much more control over the frequency bandwidth and amplitude of the excitation when compared to impact excitation, but they are more expensive, more difficult to move, and require an additional power source. Green (1995) suggested using shakers for bridges with spans less than 330 ft (100 m).

Data is acquired during dynamic testing using a variety of sensors, including accelerometers, velocity sensors, displacement sensors, strain gages, global positioning system (GPS) sensors, and optical sensors. Accelerometers and velocity sensors are common because of their low cost, availability, high sampling rates, sensitivity, and wide frequency range. Accelerometers are used to measure low to high frequencies, while velocity sensors respond well for low to medium frequencies (Hsieh et al. 2006). Green (1995) reported that accelerometers with a range of ±1.0 g are more than adequate for bridge modal testing. Displacement sensors typically measure relative displacements because of the size of bridges, and strain gages are configured to measure the variation in strain throughout the depth of the bridge. Recently, optical strain gages were used to directly calculate curvature mode shapes (Reynders et al. 2007). However, strain measurements may not represent the entire bridge because they depend on local mass and stiffness values (Green 1995). Noncontact sensors include GPS sensors and optical sensors. GPS sensors use the global positioning system of satellites to measure absolute displacements. Cameras record absolute displacements using photogrammetry, while lasers utilize interferometry to measure absolute velocity.

Other sources offer an extensive literature review of dynamic testing of large structures (Farrar et al. 1994; Sohn et al. 2004). However, since dynamic testing of highway bridges is the focus of this research the following sections present a detailed review of the literature in this area.
2.1.1 Farrar et al. (1994)

In 1993, researchers at Los Alamos National Laboratory (LANL) performed a series of dynamic tests on the eastbound I-40 bridge over the Rio Grande near Albuquerque, New Mexico, to test a series of vibration-based structural health monitoring techniques. The bridge was scheduled to be razed, so the researchers were able to inflict varying damage states on the bridge. The bridge consisted of three separate sections, each with three continuous spans. The non-composite structural system included a concrete deck transferring load through floor beams and steel stringers to two plate girders. The non-redundancy of the system made it an excellent candidate for this study. The researchers began by measuring the response of the bridge to ambient vibrations from traffic on the westbound bridge, which travelled through the ground to excite the eastbound bridge. These preliminary tests gave the researchers an initial estimate of the sensitivity and frequency range required of accelerometers to perform a complete modal analysis of the bridge. It also gave initial estimates of the natural frequencies. During the initial tests, multiple types of sensors, including integrated electronics piezoelectric (IEPE) accelerometers, magnetic accelerometers, and a microwave interferometer motion sensor, were used.

After reviewing the results of the initial tests, the authors decided to use 26 PCB Piezotronics model 336C IEPE accelerometers. A mechanical high-pass filter was devised that included a piece of double-sided tape between the sensor and mounting block. Both an ambient vibration and a forced vibration test were performed. Ambient vibrations were provided from traffic on the westbound bridge while forced vibration was provided from a hydraulic shaker. The shaker consisted of a 21,700 lb reaction mass supported by air springs. It was capable of providing 500 lb peak force sinusoidal input or 2000 lb peak random force input. The input frequency range during forced vibration testing varied between 2 and 12 Hz. The bridge was tested for one undamaged state and three damaged states, which included varying web damage to one of the plate girders. Post processing of the data included autopower spectra, cross power spectra (CPS), and frequency response functions (FRF). Because the input was not quantified for the ambient vibration tests, the resonant frequencies were identified using autopower spectra from all accelerometers, and the mode shapes were determined using CPS and FRFs between a reference accelerometer and all other sensors. Modal damping was calculated using the complex exponential curve-fitting method. Mode shapes, resonant frequencies, and modal damping
values were extracted from FRFs of the forced vibration test data using a rational-fraction polynomial global curve-fitting algorithm, available to the researchers through commercial software. Comparing the results from both the ambient and forced vibration tests indicated close correlation between all modal parameters. The modal assurance criterion for all identified modes was above 90%. Also, the average difference between frequencies was 0.62%, and the average difference between damping values was 17.6%. Natural frequencies ranged from 2 Hz to 5 Hz for the first 6 modes, and damping values ranged from 0.50% to 2.50% of critical damping.

2.1.2 Hsieh et al. (2006)

Multiple studies were performed at Utah State University involving structural health monitoring. Three case studies were presented by the authors. The first was the I-80 Flyover Bridge in Salt Lake City, Utah, which consisted of 25 spans and a concrete deck over 3 steel I-girders. The bridge was instrumented with 18 channels of strong motion sensors on the superstructure and 3 channels in the adjacent field area for a long-term study. Instrumentation was oriented in the longitudinal, transverse, and vertical directions relative to the bridge. The long-term instrumentation was intended to capture response due to ambient vibrations and large ground motion. To determine its dynamic characteristics, the bridge was instrumented with an additional 46 velocity transducers and accelerometers. Forced vibration was provided by a horizontal eccentric mass shaker driven at frequencies between 0.75 and 20 Hz. The forced vibration test data was analyzed using the frequency domain decomposition (FDD) method, while the ambient vibration data was analyzed using the eigensystem realization algorithm with observer/Kalman filter identification (ERA-OKID) technique.

The next case study was the South Temple Bridge, a nine-span bridge that carries I-15 over South Temple Street in Salt Lake City. The bridge had a concrete deck supported by eight steel girders with a 17.5 degree skew. The bridge was scheduled for demolition, so the researchers induced varying degrees of damage and tested repair techniques. Damage included laterally pushing and pulling on one of the concrete bents until cracking, spalling, and yielding of the reinforcing steel occurred. The bents were later repaired by injecting epoxy into the cracks and retrofitting with carbon fiber wraps. The repairs were evaluated by comparing the dynamic properties of the repaired structure to the dynamic properties in the undamaged state. Ten force-balanced accelerometers were placed on and around the bridge. Forced vibration tests were used
by driving a horizontal eccentric mass shaker between 0.5 to 5.5 Hz in 0.05 Hz increments. Data was processed using digital signal processing software.

The final case study involved the I-215 curved bridge overpass in Salt Lake City. The structure was a curved, three-span continuous bridge with five steel plate girders supporting a concrete deck. Testing involved varying the boundary conditions to determine the changing response characteristics. Twenty-five velocity transducers were positioned to measure the longitudinal, transverse, and rotational modes under forced vibration. A horizontal eccentric mass shaker provided the force and excited frequencies between 0.5 and 20 Hz. The first five natural frequencies and mode shapes were determined from the test data with frequencies ranging from 3 Hz to 12 Hz. The results of testing indicated that the frequencies reduced as the restraint of the bridge reduced. The largest change occurred for the first mode, with an overall reduction in frequency of 48%.

2.1.3 Patten et al. (1998)

Researchers at the University of Oklahoma performed dynamic testing on the Interstate 35 bridge over Walnut Creek in Oklahoma as part of a long-term monitoring study. The bridge had five continuous steel girders made composite with a concrete deck over four 100 ft spans. The bridge was also skewed at 45 degrees. Sensing equipment consisted of 60 piezo-resistive accelerometers and strain gages and four string potentiometers. A drop hammer capable of developing 25,000 lb of force excited the bridge at six locations. The hammer was dropped three times at each location. Data was sampled at 500 Hz and saved to a PC-based data acquisition system. Natural frequencies were determined using frequency response functions, and direct parameter identification methods were used to construct mode shapes. Eight modes were identified between 2.5 Hz and 5 Hz, and damping values ranged from 1.4 % to 2.5 % of critical damping. The main purpose of this research was to develop a reduced order modeling (ROM) technique to calibrate a simplified finite element model to the experimental results based on natural frequencies and mode shapes. The authors found that the frequencies from the modified model agreed with the experimental data within 3% and that the modal assurance criterion values for all corresponding modes were above 80%.

2.1.4 Ventura et al. (1996)

Dynamic tests were performed on the Colquitz River Bridge near Victoria, British Columbia, Canada, as part of a seismic retrofit and rehabilitation study. The five-span bridge
was 271 ft (83 m) long and had six continuous steel girders supporting a concrete deck. Researchers at the University of British Columbia used six force-balanced accelerometers to recorded vibrations, with two remaining stationary as reference sensors while the other four moved to various locations for different tests. Longitudinal, transverse, and vertical motions were recorded. Excitation came from two sources. The first sets of tests utilized ambient vibrations from traffic over the bridge. Tests were also performed using free vibration. Transverse modes were excited by anchoring a hoist to an adjacent abandoned railroad bridge pier and displacing the bridge laterally. A quick-release mechanism set the bridge into free vibration. Longitudinal free vibration tests were also performed by pulling between two of the bridge’s piers and releasing them suddenly. Ambient vibration test results provided the natural frequencies while the damping ratio was estimated from the free vibration test results using the logarithmic decrement method. Eleven modes were identified between 1.5 Hz and 15 Hz, and damping ratios around 3.5% were reported.

2.1.5 Abdel-Ghaffar and Scanlan (1985a; 1985b)

Researchers at Princeton University conducted vibrations tests on the Golden Gate Bridge in San Francisco, California, in June 1982 to determine its modal parameters, including effective damping, mode shapes, and associated. The suspended structure (Abdel-Ghaffar and Scanlan 1985a) and the tower (Abdel-Ghaffar and Scanlan 1985b) were tested separately. The bridge was characterized by a 6,450 ft suspended structure, a 4,200 ft central span, and two 702 ft main towers. A concrete deck rested on two stiffening trusses which were connected laterally on the top and bottom chords. Twelve force-balanced accelerometers were used during tests on the suspended structure, half of which were stationary and served as references. Six accelerometers were arranged such that two accelerometers were aligned with each of the three orthogonal directions of the bridge. The pairs of sensors were spaced as far apart as possible on the stiffening trusses as to capture purely vertical, lateral, and longitudinal motion as well as torsional motion in each direction. Six accelerometers were moved to 18 different longitudinal positions and attached using plaster-of-Paris. Ten minutes of data were recorded at each location. Only the south half of the bridge was instrumented, and the results were extrapolated to the north half of the bridge assuming symmetry and asymmetry. The analog signal from the accelerometers was sent through a low-pass filter to remove frequencies above 5 Hz. The data were later digitized at a sampling rate of 50 Hz. Excitation came from wind, traffic, and wave
action. Natural frequencies were determined from picking peaks out of the frequency spectra of the data. Mode shapes were determined from cross spectra with the reference accelerometers, and modal damping was determined using the half-power bandwidth method on the Fourier amplitude spectral peaks. The authors admit that this method was not the most accurate for determining damping. Purely vertical, lateral, and longitudinal modal parameters were distinguished from torsional motion by summing the response from the pair of accelerometers in each direction. Torsional motion was extracted by subtracting the responses. In all, 91 sets of modal parameters were determined with frequencies ranging between 0.0 and 5.0 Hz. The mode shapes and frequencies were compared to two- and three-dimensional finite element models. The authors did not elaborate about the models, but they concluded that the models were adequate for modeling the modal parameters of the bridge.

Only the south tower of the Golden Gate Bridge was tested. Nine force-balanced accelerometers were used with three serving as references at the deck level. Sets of three accelerometers were positioned at 10 vertical levels with two accelerometers oriented longitudinally to determine torsional modes and one accelerometer pointing in the transverse direction. To correlate data from the suspended structure and the tower, accelerometers were placed at the reference locations for each, and data was recorded for 30 minutes. Ambient vibrations from traffic, wind, and waves were again used as excitation and data was recorded in 10 minute intervals. Data processing was identical to that on the suspended structure data except that the analog signal from the sensors was filtered below 15 Hz. Forty-six mode shapes and natural frequencies were identified and compared to 2 two-dimensional models and 2 three-dimensional models. Details about the models were not provided, but the authors reported good correlation with the two-dimensional continuum and finite element models of the entire bridge. The three-dimensional analyses modeled the tower only and included the effects of the suspension cables and bridge deck. The connection with the pier was assumed to be rigid. The authors also reported good correlation from the three-dimensional models but concluded that the two-dimensional models were adequate for predicting the purely longitudinal tower modes.

2.1.6 Salawu and Williams (1995)

Dynamic testing was performed on a 340 ft, six-span continuous reinforced-concrete highway bridge by researchers at the University of Plymouth. Modal parameters were extracted before and after a repair to assess the feasibility of using modal characteristics to identify
damage. Three force-balanced linear servo accelerometers moved to 54 different locations on the deck while one accelerometer remained stationary as a reference. A hydraulic actuator, capable of providing 1,125 lb of force, provided vertical forced excitation. The actuator was driven with a random signal of frequencies between 0 Hz and 25 Hz. The random signal was repeated during tests to give the input periodicity. The authors state that the random aspect of the signal attenuated the nonlinear response of the bridge while the periodic aspect eliminated spectral leakage problems. Data was recorded for 10 minutes at each sensor location.

Processing of the data included FRFs between the response and the input. Coherence was used to assess the quality of the data. Modal parameters were extracted by associating the peaks in the FRFs with a single-degree-of-freedom system by using the following equation:

\[ A_n = \frac{P_r}{\omega_r^2 - \Omega_n^2} \frac{1}{\sqrt{1 + (2\omega_r \xi_r \Omega_n)^2}} \]

where \( A_n \) is the measured amplitude at frequency \( \Omega_n \), \( P_r \) is the modal participation factor for mode \( r \), \( \omega_r \) is the circular frequency for mode \( r \), and \( \xi_r \) is the viscous damping ratio for mode \( r \). An iterative process was adopted which decoupled modes and considered the effects of other modes, even modes outside of the analysis bandwidth, on the mode being analyzed. The modal assurance criterion (MAC) was used to check the orthogonality of mode shapes, and the coordinate modal assurance criterion (COMAC) was used to correlate mode shapes and a selected measurement point. For damage detection, the authors compared natural frequencies, mode shapes, and modal damping values before and after repairs were made. The real and imaginary components of the FRFs from each data point were squared and summed to form averaged FRF for the bridge. Natural frequencies decreased after repairs, but no clear trends were found in the damping values. Comparing mode shapes before and after repair using MAC values indicated a change in mode shapes in the vicinity of the repair. Using COMAC values on the same data resulted in irregularities in the vicinity of repair, but it also resulted in false positives. The authors concluded that a MAC value threshold of 0.8 and a natural frequency change greater than 5% should be used for damage identification to account for environmental factors, experimental errors, and inaccuracies in the data analysis.
2.1.7 Ren et al. (2004)

Dynamic testing was performed on the I-24 steel arch bridge over the Tennessee River in western Kentucky to calibrate a finite element model of the bridge. Ambient vibrations from traffic and wind provided “pink noise” for excitation, as the authors assumed no frequency in the ambient vibrations dominated in the bandwidth of frequencies encompassing the resonant frequencies of interest. The bridge consisted of nine spans with one 535 ft (163 m) central arch span. Two steel arches suspended either side of the concrete deck using 26 cables. The remaining spans consisted of two plate girders supporting a concrete deck through floor beams and stringers. Tests were only performed on the arch span with three reference sensor blocks remaining stationary while four moved to 30 locations on the deck. Each sensor block consisted of three orthogonal force-balanced accelerometers to measure vertical, transverse, and longitudinal response at each measurement location. Data was sampled at 1,000 Hz for 60 second intervals, and three sets of data were recorded at each measurement point. The authors chose to use 60 second intervals instead of 10 minute intervals, which are often used to achieve stationarity of the data, because of limited test time availability.

Natural frequencies and mode shapes were determined using both the peak picking method in the frequency domain and the stochastic subspace identification method in the time domain. The peak picking method involved selecting poles from the averaged normalized power spectral densities. The stochastic subspace identification method involved using the state space model of a linear system defined by:

\[ x_{k+1} = Ax_k + w_k \]  
\[ y_k = Cx_k + v_k \]

where \( x_k = x(k\Delta t) \) is the discrete time state vector containing the sampled displacements and velocities, \( u_k, y_k \) are the sampled input and output, \( A \) is the discrete state matrix, \( C \) is the discrete real output influence coefficient matrix, \( w_k \) is the process noise, and \( v_k \) is the measurement noise. \( A \) is the state matrix defined from physical system parameters, including mass, stiffness, and damping, relating the input of the system at one interval to the input at another interval. \( C \) is the state matrix defined by the input-output characteristics of the system and relates the output of the system to the state of the system in the same interval. Equations 1 and 2 are the state space
model for a linear dynamic model considering only output data and including noise. The authors concluded that both methods were adequate for selecting natural frequencies, but the stochastic subspace identification method yielded higher quality mode shapes.

### 2.1.8 Deger et al. (1994; 1995)

Researchers at the Swiss Federal Laboratories for Material Testing and Research conducted dynamic testing on two highway bridges. The first bridge (Deger et al. 1994) was the Aare Bridge between the two Swiss cantons of Solothurn and Aargau. The concrete bridge was originally designed by Robert Maillart as a deck stiffened arch bridge of 236 ft (72 m) in length and 31 ft (9.5 m) in width. The columns connecting the arch to the deck were later removed and prestressing was introduced. The second bridge (Deger et al. 1995) was tested in conjunction with the German Federal Laboratories for Material Testing and Research. It was a prestressed concrete box-girder bridge consisting of eight spans with an overall length of 800 ft (242 m). A servohydraulic vibration generator capable of producing 1120 lb (5 kN) of vertical force for frequencies above 2.3 Hz provided excitation for both bridges. The bandlimited burst random input signal consisted of manipulated white noise which had a flat spectrum for the frequency bandwidth of 1 Hz to 25 Hz for the arch bridge and 0 Hz to 20 Hz for the box-girder bridge. Structural response was measured using measurement units consisting of three accelerometers positioned orthogonally on a steel plate. Two measurement units were moved to 144 positions on the arch bridge, and four measurement units were moved to 247 locations on the box-girder bridge. Leuven Measurements & Systems (LMS) software captured twelve 16 seconds data sets for averaging at each position. Data analysis was performed using LMS CADA-X software by identifying peaks in the FRFs and valleys in the mode indicator function. The results for the arch bridge indicated 12 modes in the range of 3 Hz to 18 Hz and modal damping ratios between 0.75% and 1.13%. Thirteen modes were identified between 1 Hz and 12 Hz for the box-girder bridge with damping ratios between 1.25% and 3.61%.

### 2.1.9 Haritos et al. (1995)

Researchers at the University of Melbourne conducted dynamic testing on one span of the Yarramblack Creek Bridge in Warracknabeal, Australia, to calibrate a finite element model for condition assessment by changing the concrete material properties and deck thickness. The reinforced concrete bridge consisted of three 27 ft (8.2 m) spans on a 32° skew. Excitation was provided by a linear electro-hydraulic actuator capable of producing 3400 lb (150 kN) of force
over the frequency bandwidth of 0 Hz to 50 Hz moved to two different locations on the bridge span. Ten accelerometers, 6 piezoelectric and 4 servo, were used to measure response at 46 different location with the servo accelerometers remaining stationary. The data acquisition software TSPECTRA (1992b) controlled the linear hydraulic actuator, recorded data from all sensors, and calculated transfer functions (TRFs) from the response. A preliminary model constructed in the finite element software STRAND6.1 (1995) provided predictions for the natural frequencies and helped position sensors.

The modal analysis software package TMODE (1992a) was used to derive natural frequencies, mode shapes, and damping factors from the experimental data. TMODE determined the modal parameters by loading the TRF values into a matrix and using a weighted least squares technique with orthogonal polynomials to find the ‘best’ linear dynamic system to approximate the data. Six modes were located between 0 Hz and 50 Hz in the data. The authors concluded that their methods of testing in conjunction with the post-processing software provided reasonable estimates of the modal parameters of the bridge that could be mirrored in a calibrated finite element model.

2.1.10 Meng et al. (2004)

Researchers at Syracuse University and the Federal Highway Administration’s Turner-Fairbank Highway Research Center performed static and dynamic tests on a skewed highway bridge model. The approximately 1/6 scaled model had two continuous 12 ft spans, a width of 7 ft, and a skew of 36°. W6 x 12 steel sections were used as girders, and HSS4 x 2 x 3/16 sections represented diaphragms. Pipe 6 std. sections were used for the columns of the supports, and HSS8 x 6 x 3/8 members were used for cap beams. Pinned supports connected the girders to the central pier, and cast iron casters at the abutment represented roller supports. The bridge model was instrumented with 8 Unimeasure Inc. model PB-20 potentiometers to measure deflections during static testing and 15 accelerometers, PCB models 393C and 302B03, during dynamic testing. Both a PCB model 086B50 impact hammer and an APS Dynamics model 400 ELECTROSEIS linear electrodynamic shaker were used for dynamic excitation. The impact hammer was used for determining mode shapes, and the shaker was used for determining natural frequencies and damping. Data acquisition was performed by a MEGADAC data acquisition system, manufactured by OPTIM.
A preliminary static test was performed to verify the finite element model (FEM). The FEM was constructed in SAP2000 using shell elements for the deck, beam elements for the girders and diaphragms, and massless rigid bars to connect the beam and shell elements to ensure composite action. The authors did not elaborate any further about the FEM. Static testing was performed by pulling down on the model bridge. A comparison of the resulting experimental deflections with the FEM verified that the model represented the actual structure.

Dynamic testing began by placing the shaker on the bridge and varying the driving frequency between 5 Hz and 10 Hz. Eight natural frequencies were identified from the raw data, and shaker was driven around these frequencies to more closely identify their locations. Impact testing was performed by striking the bottom of the model bridge’s deck. A medium-stiff hammer tip was used. For all dynamic tests, 20 events were recorded. Natural frequencies were determined using the resonance method, which involved reviewing the response data and locating the frequencies of highest response. Mode shapes were determined using the curve-cutting FRF method, and the half-power bandwidth method was used to determine damping values. Damping values ranged from 1% to 4% of critical damping. The authors reported that the resulting modal properties agreed well with the FEM for all modes except the first mode, which was a translational mode in the plane of the deck that the authors did not attempt to excite.

2.2 Finite Element Modeling

The finite element method allows insight into the modal parameters of a bridge before placing any instrumentation. Instruments are located to capture the best response for certain modes by reviewing the mode shapes from the model, and the model natural frequencies indicate the approximate location of modes in the experimental data. Multiple methods are used for modeling highway bridges. These methods range from full three-dimensional (3D) solid modeling to simple models such as grillage analysis. The most commonly used methods usually fall between these two extremes, since models using 3D solid elements are computationally prohibitive and grillage analysis provides an over simplification of the problem. Chung and Sotelino (2006) described the main concepts associated with finite element modeling of bridges. They also presented and compared four different modeling techniques. In all methods, the deck is represented by shell elements. The girders are represented by either all shell elements, shell elements for the web and beam elements for the flanges, beam elements for the web and flanges, or a single beam element for the entire girder section. The latter modeling technique is known as
eccentric beam modeling (details of this technique are provided in section 3.4). In all techniques, composite action between the girders and deck are ensured by rigid links. The following sections present research using finite element modeling in conjunction with experimental data.

2.2.1 Farrar et al. (1996)

As part of a study of damage detection methods (Farrar et al. 1994), researchers at Los Alamos National Laboratory created a finite element model of the I-40 bridge over the Rio Grande near Albuquerque, New Mexico. The model was used to position instrumentation and to approximate the natural frequencies and mode shapes of the lower modes for the experimental part of the study (section 2.1.1). ABAQUS Version 5.4 (1994a) was used for all models. A preliminary study was performed on a W40 x 328 beam with free boundary conditions to determine the best modeling technique for modal characteristics. Five different techniques were considered and included: 3-node general section beam element from ABAQUS, 3-node I-section beam element, 8-node shell elements to model the web and flanges, 8-node shell elements to model the web with 3-node beam elements to model the flanges at the centroid of the flanges, and 8-node shell elements to model the web with 3-node beam elements to model the flanges at the edges of the shell. The authors concluded that all methods gave comparable results when compared to the closed-form solution.

Two models of the bridge were developed. The coarse model, consisting of 7032 degrees-of-freedom (DOF), gave comparable results when compared to the refined model with 35,160 DOFs and was chosen for all subsequent analyses. Only the three-spans in the first section of the bridge were considered. Eight-node shell elements were used to model the concrete deck and webs of the plate girders, while 3-node beam elements were used to model the stringers, floor beams, and flanges of the plate girders. Composite action was ensured between the girders and deck, though the method was not explicitly stated. Twenty-node continuum elements represented the concrete piers. Horizontal and vertical stiffeners, diagonal bracing, and concrete rebar were not included, but composite action was modeled. Variations in the constraints between the girders and piers and the boundary conditions at the ends of the model were considered, but the model with rollers at the abutment and pins at the piers correlated best with experimental data.

The experimental study included different levels of damage to the web of one plate girder. Modeling this damage consisted of releasing the bottom nodes of two adjacent web shell
elements and reconnecting them with a beam element, as shown in Figure 2-1. The different levels of damage were simulated by changing the properties of the beam element.

![Figure 2-1: Simulated Web Damage](image)

The resulting frequencies from the undamaged and damaged models agreed well with the experimental results, with differences less than 5%. The mode shapes also showed good agreement, with most MAC values above 80%. The authors found that the mode shapes were particularly sensitive to boundary conditions between the piers and girders, and the boundary conditions that produced the best correlation for the frequencies were not the same boundary conditions that produced the best correlation for mode shapes.

2.2.2 **Patten et al. (1998)**

For comparison with experimental results, a finite element model of the Walnut Creek Bridge was created using the commercial package I-DEAS (1998). Using as-built drawings and field measurements, the bridge was simulated using 620 steel beam elements, 410 composite thin shell elements, 316 rigid bar elements, and 74 steel and composite solid elements, which resulted in 811 nodes and 4,600 degrees-of-freedom. The model was calibrated by developing a reduced order model (ROM) of the bridge. The ROM method iterates on the equation of motion using a perturbation with the mass and stiffness matrices of the model and the mode shapes and natural frequencies from the test data until the perturbation is below a threshold. After calibrating the model, the authors reported differences between the experimental and numerical results less than 1.60% for the first eight modes.

2.2.3 **Ventura et al. (1996)**

To locate sensors for the experimental study, a preliminary finite element model of the Colquitz River Bridge was created in SAP90 (Wilson and Habibullah 1992). The model was based off the as-built drawings and consisted of 126 shell elements to model the deck and 244 beam elements to model the steel girders, concrete piers, and concrete cap beams. Some
assumptions made by the authors include using an effective thickness of 150 mm for the deck shell elements to account for cracking and construction variations, increasing the moment of inertia of the girders to account for composite action between the deck and girders, idealizing supports, and using links between the deck and bent to account for the lateral stiffness due to the girders. The details and rationale behind these assumptions were not stated.

The model was calibrated to correlate the analytical frequencies as close as possible to the experimental results from the ambient and free vibration tests. The motivation for this calibration was not explicitly provided in their report. The preliminary model was calibrated by first adjusting the vertical modes, then the transverse modes, and finally the longitudinal modes. Calibration using vertical modes included increasing the deck thickness to account for the wearing surface, including the effects of the sidewalk and parapet, including the diaphragms and their composite action, and removing the links between the deck and bent. The deck thickness was varied between 152 mm and 229 mm, and a thickness of 210 mm produced the best match of frequencies. The sidewalk and parapet were modeled by increasing the mass and moment of inertia of the external girders. The method for including the diaphragms was not specified. The authors found the refinement of the model from the vertical modes to be adequate for the transverse modes. Calibration of the model for longitudinal modes included adding linear springs to represent bolts at the boundary between the girders and bents, as the authors found that the stiffness of the springs dominated the longitudinal frequencies over the stiffness of the bents. The final calibrated model included 261 beam elements, 126 shell elements, 4 spring elements, and 1,172 DOFs. When comparing the first eight natural frequencies of the calibrated model to the experimental results, the authors reported an average difference of 3.5%.

2.2.4 Ren et al. (2004)

The authors utilized SAP2000 (Wilson and Habibullah 1998) for the three-dimensional models of the Tennessee River bridge. Two models were created, each representing the deck in a different manner. In the first model, referred to as Model-1, the concrete deck was not represented by any elements but instead simulated by concentrated joint forces for static analysis and lumped masses for dynamic analysis. The second model, referred to as Model-2, used shell elements for the deck. Otherwise, the two models were identical. Two-node beam elements were used for the arch members, girders, stringers, floor beams, and bracing members with three translational DOFs and three rotational DOFs at each node. Suspension wires were modeled by
truss elements, with released rotational DOFs at each node. Wall type piers were modeled by unspecified frame elements, and web walls were modeled by shell elements. Bearings were represented by rigid elements with different end conditions. Fixed bearings were simulated by releasing the rotational DOF in the vertical plane, while expansion bearings also released the longitudinal translational DOF. Longitudinal springs at the ends simulated the restraining action of adjacent spans. Pier foundations were modeled by fixed end supports.

The models were first compared to actual static dead load deflections. Both underpredicted the deflections, but the results from Model-1 were closer than those for Model-2. The models were then compared to the experimental results of the dynamic tests. Comparisons were made by considering natural frequencies, visually inspecting mode shapes, and comparing MAC values. The first three vertical modes, the first three transverse modes, and the first longitudinal mode were considered. The natural frequencies for the vertical and longitudinal modes were comparable for both models and agreed with the experimental results. The frequencies for all transverse modes differed between the models by over 20%, and the field results were closer to those from Model-1. Mode shapes from both models also correlated well with the first two vertical modes and the first transverse mode. The authors noted that comparing longitudinal mode shapes was difficult because ambient vibrations were not directed in the longitudinal direction during field tests. However, the authors did calibrate the longitudinal springs on the models so the first longitudinal frequency would match the experimental data.

2.2.5 Deger et al. (1994; 1995)

As part of a dynamic study of an arch bridge (Deger et al. 1994) and a box-girder bridge (Deger et al. 1995), researchers at the Swiss Federal Laboratories for Materials and Testing constructed models using MSC/NASTRAN (1994c) finite element software. A preliminary model of the arch bridge consisting of 200 quadrilateral plate elements was used to locate sensors and the driving point for excitation. The model was later refined to 304 elements with quadrilateral plate elements simulating the deck and 20-node brick elements modeling the arch for comparing to experimental results. Using the LMS “Link” (1994b) software package, the finite model was calibrated by changing the stiffness and mass characteristics of the model based on the first seven experimentally determined mode shapes and frequencies. Prestressing, railings, or detailed boundary conditions were not taken into account.
A finite element model of the box-girder bridge was constructed with 1000 quadrilateral plate elements and 28 beam elements, but details about the components of the bridge model were not stated. Springs were added to the foundations of the piers to simulate column-soil interaction. As with the arch bridge, LMS “Link” software calibrated the model based on the experimentally-determined natural frequencies and mode shapes by changing the mass and stiffness matrices. The authors reported a reasonable correlation based on MAC values between the mode shapes from the experimental results and the numerical model. With both models, the authors conclude that dynamic properties are much more sensitive to changes in boundary conditions than changes in mass or stiffness in analytical models.

2.2.6 Haritos et al. (1995)

Finite element models of the Yarriamblack Creek Bridge were constructed using STRAND6.1 (1995). Only the span tested experimentally was modeled, though the authors provide no reference to accounting for the other spans in the model. Using nominal values for material properties, a preliminary model of the bridge was constructed to aid in locating instrumentation and the driving points for excitation. Details were not provided about the preliminary model. A final finite element model of the bridge consisted of 156 nodes, 116 beam elements, and 120 plate elements. The girders, barriers, and diaphragms were modeled explicitly using beam elements, and the deck was modeled using plate elements. The additional stiffness provided by the parapets was incorporated in the outer girders. The mass of the shaker was added to the model at the forcing locations, and the effect of the shaker location on the FEM results was on the order of the observed effects from the experimental results. Calibration consisted of applying a range of material properties determined from core samples to the entire bridge and using the property that resulted in the best correlation with experimental data. The deck thickness was also altered to reflect the thickness observed in the field. The authors reported that five modes in the finite element model agreed well with the experimental results, but the sixth experimental mode did not appear in the numerical model.

2.3 Summary

The literature in this chapter highlights work performed in the area of dynamic testing and modeling of highway bridges. As demonstrated, accelerometers are commonly used for sensing response, and forced vibration is the preferred method of excitation. Forced vibration is a preferred method of excitation, and both shakers and impact hammers are commonly used. Data
processing involves calculating FRFs for locating natural frequencies and building mode shapes, and the half-power bandwidth method is commonly used to determine damping ratios. Eccentric beam modeling is frequently used for building a numerical model of a highway bridge. The full composite behavior assumed in eccentric beam modeling is supported through experimental results in previous research. Boundary conditions have a dramatic effect on the dynamic behavior of a bridge model. Also, many researchers use various methods of calibration to match their numerical model with their experimental results. Unfortunately, relatively little work has been done on superelevated skewed highway bridges with steel girders and a concrete deck.
Chapter 3. Methods

The methods used in this study incorporated various aspects of experimental modal analysis, digital signal processing, and finite element modeling. This chapter presents these methods by first discussing the experimental aspects and transitioning into the numerical procedures. A description of the bridge is given in section 3.1, followed by an overview of the experimental testing methods in section 3.2. Section 3.3 provides background on the theoretical dynamic model representing the bridge and signal processing techniques, while the chapter concludes with the finite element modeling methods in section 3.4.

3.1 Bridge Description

The bridge used in this study is located in Blacksburg, Virginia (Figure 3-1). It is a composite single-span bridge with steel girders and a concrete deck.

![Figure 3-1: Test Bridge](image)

The single span bridge has a superelevation that varies linearly from 5.44% on the south end of the bridge to 2.48% on the north end of the bridge. It is skewed at 34° counterclockwise as viewed from the top of the bridge, as shown in Figure 3-2. Seven W36 x 231 steel girders with PL12 x 1 cover plates support an 8.8 in. thick concrete deck, as shown in Figure 3-3. The girders are spaced at 7.8 ft, and the end diaphragm rows are 15 in. from the end of the bridge.
The girders rest on bearing pads directly beneath the end diaphragm rows, as shown in Figure 3-4. Except for the end diaphragm rows, all diaphragms are perpendicular to the centerline of the bridge and aligned in evenly spaced rows. All diaphragms are C15 x 33.9 sections. Figure 3-5 provides a view from beneath the bridge.

![Plan View of the Test Bridge](image)

**Figure 3-2: Plan View of the Test Bridge**

![Elevation of the Test Bridge](image)

**Figure 3-3: Elevation of the Test Bridge**
3.2 Experimental Testing Methods

3.2.1 Overview

Experimental testing of the bridge included placing accelerometers at predetermined locations and exciting the bridge using an impact hammer. The bridge currently has no traffic traveling across it, so instrumentation was placed on top of the bridge deck. This also allowed traffic to flow unhindered beneath the bridge. Testing commenced on Wednesday, July 1, 2009, at approximately 11:00 a.m. and continued until approximately 2:00 p.m. that afternoon.
3.2.2 Equipment

The instrumentation, cabling, and excitation source chosen for this study were those available to the author through the Virginia Transportation Research Council. Instrumentation included 9 PCB Piezotronics model 393C integral electronic piezoelectric (IEPE) accelerometers with a sensitivity (±15%) of 1000 mV/g, a measurement range of ±2.5 g peak, a frequency range (±5%) of 0.025 Hz to 800 Hz, a broadband resolution (1 to 10,000 Hz) of 0.0001 g rms, and a discharge time constant (DTC) greater than 20 seconds. Varying lengths of RG-58 coaxial cable with BNC connectors were used for most of the cabling. Because the 393C accelerometers used a 10-32 coaxial jack, varying lengths of coaxial Teflon cable with a 10-32 connector at one end and a BNC connector at the other were used to adapt the RG-58 cable for use with the accelerometers. Both types of cable had male BNC connectors, so a female-female BNC connector adaptor connected the lengths of cable. The impact hammer was a PCB model 086B50 with a sensitivity (±15%) of 1 mV/lbf, a measurement range of ±5000 lbf peak, and a frequency range from 0 Hz to 500 Hz. The hammer included interchangeable tips with descriptions of super soft, soft, medium, and hard.

Data acquisition hardware chosen for this study was provided by Dr. Alfred Wicks in the Mechanical Engineering Department at Virginia Tech. It included National Instruments (NI) cDAQ 9172 C-Series USB carrier, one NI 9233 C-Series module, and one NI 9234 C-Series module. Both the NI 9233 and NI 9234 modules are designed for use with IEPE accelerometers. They utilize delta-sigma analog-to-digital converters with analog prefiltering and anti-alias digital filtering. They also have 24 bit resolution and four BNC inputs each. Both use AC coupling to read the response signal from IEPE accelerometers. The AC coupling cutoff frequency (-3 dB) of the modules is 0.5 Hz. The cutoff frequency for a circuit containing two DTCs subject to a transient input is calculated using:

\[
DT{C}_{cir} = \frac{TC_{s} \cdot TC_{d}}{TC_{s} + TC_{d}}
\]

where \( DT{C}_{cir} \) is the discharge time constant of the circuit, \( TC_{s} \) is the DTC of the sensor, and \( TC_{d} \) is the DTC of the data acquisition system. Based on the DTC of the accelerometer and using the inverse of the AC coupling cutoff frequency of the data acquisition system, the cutoff frequency
for the circuit was 1.82 seconds, corresponding to a low frequency cutoff of 0.55 Hz. Based on
the frequency of the first mode determined from the finite element model (section 4.2), the cutoff
frequency did not present a problem.

The NI 9234 module is an updated version of the NI 9233 module, and some of the
specifications differ between them. Data rates for the NI 9233 vary between 2.0 kS/s and 50
kS/s when using the internal master timebase, whereas data rates for the NI 9234 vary between
1.652 kS/s and 51.2 kS/s when using the internal timebase. However, when used in conjunction,
the NI 9233 module inherits the internal timebase from the NI 9234 module. Therefore, data
rates for both modules used together vary between 2.048 kS/s and 51.2 kS/s.

Another caveat of using both the NI 9233 and NI 9234 modules together is a phase delay
between the modules. The delay is due to an incompatibility between the input delays in each
module. The input delay for the NI 9233 module is determined by:

\[
\delta t = 12.8/f_s + 3 \, \mu s \quad \text{for} \quad f_s \leq 25,650 \, S/s \quad (3)
\]

\[
\delta t = 9.8/f_s + 3 \, \mu s \quad \text{for} \quad f_s > 25,650 \, S/s \quad (4)
\]

where \(\delta t\) is in microseconds and \(f_s\) is in samples per second. The input delay for the NI 9234
module is determined by:

\[
\delta t = 38.4/f_s + 3.2 \, \mu s
\]

Because only frequencies less than 100 Hz were of interest, the lowest sampling rate, 2.048 kS/s,
was used. Therefore, the phase delay between modules was 0.0125002 seconds, which
corresponded to 26 samples. Unfortunately, the Equation 4 was used instead of Equation 3 for
calculating the delay before testing and the error was not found until after testing was performed.
However, the test data was later corrected to properly synchronize all channels (section 3.3.6).

National Instruments LabVIEW version 8.5 software was used to record and display data
during testing. A virtual instrument (VI) was programmed into LabVIEW to record the signals
from all channels and dynamically display the time-domain signal, autopower spectrum, cross
power spectrum, magnitude and phase or real and imaginary components of the Fourier series,
magnitude and phase or real and imaginary components of a single or averaged FRF, and the
coherence of an averaged FRF for any selected channel. This allowed real-time viewing of the data for noticing irregularities in the signals or peaks in the response. During testing, the coherence of the averaged FRF was monitored carefully to ensure that it remained near one. The front panel and block diagram of the VI are presented in Error! Reference source not found. Because of an incompatibility error when using both NI modules with NI-DAQmx 8.9, the NI-DAQmx 8.8 driver was used to interface between LabVIEW and the data acquisition hardware.

3.2.3 Instrumentation Test Plan

Testing of the bridge was scheduled for Wednesday, July 1, 2009. Testing was planned for the early afternoon, as the thermal gradient through the bridge was expected to stabilize during this time. After reviewing the finite element model of the bridge (section 4.2), the resulting mode shapes were analyzed for instrumentation placement. Sensors were placed at locations that directly correlated with nodes on the model that was later used for comparison. This ensured that mode shapes could be directly compared. The resulting layout was designed to distinguish the first eleven mode shapes from the experimental data. The instrumentation layout and accelerometer location numbering are presented in Figure 3-6.

![Figure 3-6: Instrumentation Layout and Accelerometer Location Numbering](image)

Because of the skew of the bridge, all mode shapes demonstrated both bending and torsional behavior. Also, the bending behavior of the modes was parallel to the skew. Therefore, accelerometers were arranged such that they formed rows parallel to the skew of the
bridge. Because the global behavior of the bridge was of interest, instrumentation was placed over the girders. This minimized the local effects of the deck flexing between girders. Figure 3-7 illustrates the arrangement of the accelerometers on the bridge deck during testing.

![Figure 3-7: Accelerometer Arrangement](image)

Accelerometers were mounted directly on the bridge deck surface, as shown in Figure 3-8. Plumber’s putty adhered the sensors to the deck. They acted as an interface between the accelerometers and the bridge deck, reducing noise due to the accelerometers rocking. The small difference between the axis of the accelerometer and a vertical reference was assumed to have a negligible effect on the resulting accelerations given that the maximum angle of rotation from the superelevation was 3.4°.

![Figure 3-8: Accelerometer Positioned on Bridge Deck](image)
The cables used in this study came in multiple lengths, so cable assignments depended on the location of the accelerometer. The cable lengths and designations are presented in Table 3-1. Any differences in attenuation among channels because of differences in cable lengths were considered negligible because the longest cable length was just over 100 ft and the highest frequency of interest was less than 100 Hz (NI 2009). Given the low sampling rate, the phase shift among channels due to differences in cable lengths was also considered negligible.

### Table 3-1: RG-58 Coaxial Cable Lengths and Designations

<table>
<thead>
<tr>
<th>Cable</th>
<th>Length (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>94.7 + 6.7 = 101.4</td>
</tr>
<tr>
<td>B</td>
<td>99.1</td>
</tr>
<tr>
<td>C</td>
<td>29.0 + 80.0 = 109</td>
</tr>
<tr>
<td>D</td>
<td>104.0</td>
</tr>
<tr>
<td>E</td>
<td>99.0</td>
</tr>
<tr>
<td>F</td>
<td>68.3</td>
</tr>
<tr>
<td>G</td>
<td>67.3</td>
</tr>
<tr>
<td>H</td>
<td>49.1</td>
</tr>
</tbody>
</table>

Note: Cables A and C are comprised of two cables spliced with a BNC female-female connector

A summary of cable assignments is presented in Table 3-2. Cables are matched to accelerometer locations for all tests. Cable A was used with the impact hammer for all tests. A reference accelerometer was placed at the middle of the bridge, as shown in Figure 3-6, and included in all tests. The reference accelerometer acted as a baseline for linking all tests. In the event of irregular responses from the instruments during one test, the response from the reference accelerometer acted as a basis of comparison with other tests for determining the source of the irregularity. Because of the presence of the reference accelerometer in all tests and because the number of tests was not an integer multiple of the number of accelerometer locations, some of the locations were measured twice. This provided additional insurance if the sensor at one of these locations quit working properly. During testing, two accelerometers began to produce an irregular response and were replaced with backup accelerometers. Unfortunately, this behavior was noticed late for location 37 during test T5, so location 37 was retested after all other testing was performed. The exact source of the erratic sensor behavior is unknown. The radiant heat
from the bridge deck was believed to contribute to this behavior, though all sensors were subject to the same temperatures and only two needed to be replaced.

<table>
<thead>
<tr>
<th>Test</th>
<th>Cable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>T1</td>
<td>32</td>
</tr>
<tr>
<td>T2</td>
<td>32</td>
</tr>
<tr>
<td>T3</td>
<td>32</td>
</tr>
<tr>
<td>T4</td>
<td>32</td>
</tr>
<tr>
<td>T5</td>
<td>32</td>
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<tr>
<td>T6</td>
<td>32</td>
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<tr>
<td>T7</td>
<td>32</td>
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<tr>
<td>T8</td>
<td>32</td>
</tr>
<tr>
<td>T9</td>
<td>32</td>
</tr>
<tr>
<td>T10</td>
<td>32</td>
</tr>
<tr>
<td>T11</td>
<td>32</td>
</tr>
</tbody>
</table>

Bridge deck temperatures were monitored twice during testing, once at the beginning of testing and once at the end. Temperatures were measured on both the top and bottom of the deck using an Omega model OS561 infrared thermometer. The procedure for taking measurements on top of the deck involved walking around on the deck and noting the average temperature given by the thermometer. Because traffic continued to flow beneath the bridge, access to the bottom of the bridge was limited to the ends. The average temperature of the bottom of the deck near the ends was noted and assumed to be similar to the temperature at midspan.

3.2.4 Excitation Test Plan

Impact excitation was planned for three different locations on the bridge deck, as shown in Figure 3-9. Locations were chosen based on the excitation of certain modes and away from the accelerometers as not to excite them beyond their linear range or magnify local vibration effects. Location I was selected near the maximum amplitude of a second or fourth pure bending mode. Location II was at the same longitudinal distance along the skew of the bridge as Location I but offset transversely to excite transverse modes. Location III was offset transversely from the reference accelerometer in the center of the bridge and positioned to excite transverse modes as well as a first or third pure bending mode. Figure 3-10 illustrates testing at Location I.
The hammer tip was selected based on the resulting force signal and frequency spectrum from an impact on the deck of the test bridge. The medium tip produced repeatable results with a peak force near the top of the hammer’s force range. Also, the resulting spectrum was relatively flat for the frequencies below 100 Hz. To reduce leakage problems during data analysis, data was recorded until the response from the impact had fully decayed. A Radix-2 Fast Fourier Transform (FFT) was used to change the time domain signal into the frequency
domain, so the number of datum points in each response needed to be a power of two. A review of the accelerometer responses indicated that 8192 datum points, or 4 seconds of data, was sufficient to allow the response from the impact to fully decay. Ten tests were performed at each forcing location for each sensor configuration. This ensured sufficient data for averaging during data processing, even if some of the datum sets needed to be removed.

3.3 Data Processing

3.3.1 Overview

Determining the modal properties of the bridge from the experimental data involved improving the quality of the data through digital signal processing techniques, deciding on a theoretical model, and applying the concepts from the model to the data. Ewins (2000) provided multiple theoretical models for dynamic systems with techniques for extracting modal parameters. A concise overview of one of these theoretical models including modal characteristics signatures in dynamic data is presented in sections 3.3.2 and 3.3.3. Bendat and Pearsol (2000) discussed many techniques for improving the quality of data containing random and uncorrelated content. The theory behind these digital signal processing techniques is presented in section 3.3.2.

3.3.2 Theoretical Model

The theoretical model selected to represent the bridge is a linear multiple degree-of-freedom (MDOF) system. The derivations and nomenclature are adapted from Ewins (2000). The equation of motion for this N degree-of-freedom system is defined by:

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t) \quad (5) \]

where \( M \) is the N x N mass matrix, \( C \) is the N x N proportional damping matrix, \( K \) is the N x N constant stiffness matrix, \( F(t) \) is the N x 1 forcing vector, and \( x(t) \) is the N x 1 displacement vector. The dot over \( x \) denotes a derivative with respect to time. If the forcing vector is comprised of sinusoidal functions, \( F(t) \) can be replaced by \( \{f\}e^{j\omega t} \) and, because the system is linear, \( x(t) \) can be replaced by \( \{x\}e^{j\omega t} \). \( \{f\} \) and \( \{x\} \) are N x 1 vectors containing time-independent complex amplitudes, and \( e \) is the base of the natural logarithm. Therefore, Equation 5 can be rewritten as:
\[-\omega^2 M + j \omega C + K \{x \} e^{j \omega t} = \{f \} e^{j \omega t} \quad (6)\]

where \(j\) is equal to \(\sqrt{-1}\). Mode shapes are defined in the \(N \times N\) matrix, \(\Phi\), mass-normalized such that \(\Phi^T M \Phi = I\) where \(I\) is the identity matrix. The \(r\)th mode of the system, \(\varphi_r\), is the \(r\)th column of \(\Phi\). If both sides of Equation 6 are pre-multiplied by \(\Phi^T\) and the bracketed term on the left side is post-multiplied by \(\Phi \Phi^{-1}\), the equation of motion becomes:

\[
[\Omega^2 - \omega^2 I + j 2 \omega Z_r \Omega_r] \Phi^{-1} \{x \} e^{j \omega t} = \Phi^T \{f \} e^{j \omega t} \quad (7)
\]

where \(\Phi^T K \Phi = \Omega_r^2\) and \(\Phi^T C \Phi = 2 Z_r \Omega_r\). \(\Omega_r^2\), \(Z_r\), and \(\Omega_r\) are \(N \times N\) diagonal matrices with diagonal terms \(\omega_r^2\), \(\zeta_r\), and \(\omega_r\). \(\omega_r\) and \(\eta_r\) are the natural frequency and structural damping loss factor for the \(r\)th mode. Moving terms to the right side of Equation 7 results in:

\[
\{x \} e^{j \omega t} = \Phi [\Omega^2 - \omega^2 I + j 2 \omega Z_r \Omega_r]^{-1} \Phi^T \{f \} e^{j \omega t} \quad (8)
\]

Equation 8 is in the form \(X(\omega) = H(\omega) F(\omega)\) where \(F(\omega)\) is the input to the linear system, \(X(\omega)\) is the output, and \(H(\omega)\) is the frequency response function (FRF). The frequency response function defines the input-output behavior of the system. The FRF matrix formed using a force input and displacement output is known as the receptance matrix. The individual elements of the receptance matrix are defined by:

\[
H_{ik}(\omega) = \frac{x_i e^{j \omega t}}{f_k e^{j \omega t}} = \sum_{r=1}^{N} \frac{\varphi_{ri} \varphi_{rk}}{\omega_r^2 - \omega^2 + j 2 \zeta_r \omega_r \omega} \quad (9)
\]

\(H_{ik}(\omega)\) is the response at position \(i\) on the structure to a force at position \(k\). Notice that the numerator of Equation 9 consists of spatial information from the mode shape components, and the denominator consists of global system values. Derivatives with respect to time are performed by simply multiplying the numerator by \(j \omega\). The FRF matrix for a force input and velocity output is known as the mobility matrix. The individual elements of the mobility matrix are defined by:
The FRF matrix for a force input and acceleration output is known as the inertance or accelerance matrix. The individual elements of the inertance matrix are defined by:

\[
H_{ik}(\omega) = \frac{d}{dt} \frac{x_i e^{j\omega t}}{f_k e^{j\omega t}} = \frac{j\omega x_i e^{j\omega t}}{f_k e^{j\omega t}} = \sum_{r=1}^{N} \frac{j\omega \varphi_{ri} \varphi_{rk}}{\omega_r^2 - \omega^2 + j2\zeta_r \omega \omega}
\]

An observation can be made about the system around one of the natural frequencies. When \( \omega^2 \) is equal to \( \omega_r^2 \), the component of the summation associated with the \( r \)th mode is imaginary, and the magnitude of this component achieves a relative maximum. Also, the response is dominated by this component at the natural frequency, though contributions from other modes are present. Therefore, the mode shape can be estimated from the imaginary components of the FRF at each location for a natural frequency. Because of the contributions from other modes, though, the resulting shape is the operating deflected shape at that frequency and not exactly the mode shape. Notice that the sign of the real component of the response function switches as \( \omega \) passes \( \omega_r \), which causes the phase of the response function to shift by 180°.

These characteristics are demonstrated by considering a 2 DOF system with natural frequencies \( \omega_1 = 2 \) Hz and \( \omega_2 = 3 \) Hz, mode shapes \( \varphi_1^T = (1, 1) \) and \( \varphi_2^T = (1, -1) \), and structural damping loss factors \( \zeta_1 = 0.02 \) and \( \zeta_2 = 0.02 \). If the system is excited at the first location and the acceleration response is measured at the second location, the resulting FRF inertance magnitude and phase plots at the second location appear as in Figure 3-12 and Figure 3-12, respectively. The resulting real and imaginary plots are shown in Figure 3-13 and Figure 3-14, respectively.
Figure 3-11: Magnitude of the FRF Inertance for 2 DOF System

Figure 3-12: Phase of the FRF Inertance for 2 DOF System
Figure 3-13: Real Component of the FRF Inertance for 2 DOF System

Figure 3-14: Imaginary Component of FRF Inertance for 2 DOF System
Peaks occur in the magnitude of the response function at the natural frequencies and the total response of the system at all frequencies is the superposition of the response from each individual mode. The phase also shifts 180° at the natural frequencies. In addition, the real component switches signs and the absolute value of the imaginary component achieves a local maximum at the natural frequencies. Because the mode shapes are out of phase at the second position, the sign of the imaginary component is different for both modes.

The assumptions inherent in this model include system linearity, fully quantifiable force and response, mass-proportional damping, and well-separated modes. Because these conditions are difficult to manage, certain considerations must be allowed during testing and analysis of results, as discussed in section 3.3.5.

3.3.3 Damping Estimation

Damping is a difficult parameter to quantify, and its form is typically assumed. For lightly damped structures, such as civil engineering structures, damping is mostly due to the friction at connections between structural elements (Chopra 2007). One commonly assumed form of damping is proportional damping in which the damping is velocity-dependent, frequency-independent, and proportional to the mass and stiffness. Also, the mode shapes from systems with proportional damping are not complex. The following two expressions are equivalent equations of motion for a single degree-of-freedom system with viscous damping subject to a sinusoidal force:

\[-\omega^2 m + j\omega c + k)x e^{j\omega t} = f e^{j\omega t}\]
\[(\omega^2 - \omega^2 + j2\zeta\omega\omega)x e^{j\omega t} = f e^{j\omega t}\]

\(m, c,\) and \(k\) are the mass, damping constant, and stiffness of the system, \(x\) and \(f\) are complex amplitudes, and \(\omega_r\) and \(\zeta_r\) are the natural frequency and critical damping ratio. Damping is difficult to determine from modal data, and one method for determining the damping ratio for a mode is the half-power bandwidth method.

The half-power bandwidth method is derived by considering the response from a SDOF system. For proportional damping, the FRF mobility should be used with this method (Ewins 2000). The FRF mobility for a SDOF system with proportional damping subject to a sinusoidal force is:
\[ H(\omega) = \frac{j\omega}{\omega_r^2 - \omega^2 + j2\zeta_r \omega_r \omega} \]

The real and imaginary components of the mobility are expressed as:

\[ Re(H) = \frac{2\zeta_r \omega_r \omega^2}{\omega_r^2 - \omega^2 + j2\zeta_r \omega_r \omega} \]

\[ Im(H) = \frac{j(\omega_r^2 \omega - \omega^3)}{\omega_r^2 - \omega^2 + j2\zeta_r \omega_r \omega} \]

By considering the response function on a Nyquist plot, as shown in Figure 3-15, the identities of Equations 10 and 11 are realized. A Nyquist plot of the FRF mobility for this system is a perfect circle (Ewins 2000).

\[ \tan\gamma = \frac{2\zeta_r \omega / \omega_r}{1 - (\omega / \omega_r)^2} \]  
\[ \tan(90^\circ - \gamma) = \tan\left(\frac{\theta}{2}\right) = \frac{1 - (\omega / \omega_r)^2}{2\zeta_r \omega / \omega_r} \]
If two frequencies are considered such that \( \omega_a < \omega_r < \omega_b \), the critical damping is found by:

\[
\zeta_r = \frac{\omega_b^2 - \omega_a^2}{2\omega_r\left(\omega_b \tan(\theta_b/2) + \omega_a \tan(\theta_a/2)\right)}
\]

If \( \theta_a = \theta_b = 90^\circ \) (corresponding to the half-power points), the equation simplifies to:

\[
\zeta_r = \frac{\omega_b - \omega_a}{2\omega_r} \quad (12)
\]

The half-power frequencies occur where the magnitude of the SDOF FRF mobility is \( 1/\sqrt{2} \) times the magnitude at resonance. Therefore, the half-power frequencies determined by equating the magnitude of the FRF mobility at resonance to the magnitude of the FRF mobility at the half-power points should satisfy Equation 12. The derivation of the half-power frequencies is as follows:

\[
\frac{1}{\sqrt{2}} \left| \frac{1}{2\zeta_r \omega_r} \right| = \left| \frac{j \omega}{\omega_r^2 - \omega^2 + j2\omega_r\omega} \right|
\]

Taking the reciprocal and squaring the magnitudes:

\[
\omega^4 - 2\omega_r^2(1 + 2\zeta_r^2)\omega^2 + \omega_r^4 = 0
\]

\[
\omega^2 = \omega_r^2 + 2\zeta_r^2\omega_r^2 \pm 2\zeta_r\omega_r \sqrt{1 + \zeta_r^2}
\]

\[
\omega = \omega_r \sqrt{1 + \zeta_r^2} \pm \zeta_r \omega_r \quad (13)
\]

Inspection of the half-power frequencies in Equation 13 indicates that they satisfy Equation 12.

However, the locations of the half-power magnitudes can be difficult to locate, especially when this technique is applied to MDOF systems. By comparing the imaginary part of the FRF mobility for a SDOF system to the magnitude in Figure 3-16, it is seen that the locations of the local maximum and local minimum of the imaginary plot around resonance provide a good estimate of the half-power frequencies (Ewins 2000).
The half-power bandwidth method is limited and only provides an estimate of damping when applied to MDOF systems. The contributions of other modes will cause the shape of the resonant peak to change, but the change is small for well separated modes. The half-power bandwidth method is also difficult to apply to lightly-damped structures where the width of the peak is narrow, as the frequency resolution of the data prevents a good estimate of the half-power frequencies (Ewins 2000).
3.3.4 Digital Signal Processing

The digital signal processing techniques applied to the experimental data used in this research were adapted from Bendat and Pearsol (2000). A bridge can be idealized as a constant parameter linear system, meaning that the characteristics do not change with time and the response characteristics are both additive and homogeneous. Because the bridge is tested over a short period of time using relatively small forces, these assumptions are applicable. The dynamic characteristics of this type of system are described by an impulse response function, \( h(\tau) \). The impulse response function defines the response of the system, \( y(t) \), due to an input, \( x(t) \), at any time \( \tau \) after the application of the input. This relationship is described for a causal system using Duhamel’s integral as follows:

\[
y(t) = \int_{0}^{\infty} h(\tau)x(t - \tau) d\tau
\]

The impulse response is difficult to determine from this integral if the input and response are known. However, using the complex form of the Fourier transform, \( \mathcal{F} \), to transform the system from the time domain to the frequency domain greatly simplifies the equation:

\[
\mathcal{F}\{y(t)\} = \mathcal{F}\left\{\int_{0}^{\infty} h(\tau)x(t - \tau)d\tau\right\}
\]

\[
Y(\omega) = \int_{0}^{\infty} \int_{0}^{\infty} h(\tau)x(t - \tau)e^{-j\omega t} d\tau dt
\]

Multiplying the integrand by \( e^{-j\omega \tau}e^{j\omega t} \) results in:

\[
Y(\omega) = \int_{0}^{\infty} h(\tau)e^{-j\omega \tau} d\tau \int_{0}^{\infty} x(t - \tau)e^{-j\omega (t-\tau)}dt
\]

Let \( z = t - \tau \) and, because \( x \) is a function of \( t \), \( dz = dt \), resulting in:

\[
Y(\omega) = \int_{0}^{\infty} h(\tau)e^{-j\omega \tau} d\tau \int_{0}^{\infty} x(z)e^{-j\omega z} dz = \mathcal{F}\{h(\tau)\}\mathcal{F}\{x(z)\}
\]
\[ Y(\omega) = H(\omega)X(\omega) \quad (14) \]

where \( X(\omega) \) and \( Y(\omega) \) are the frequency content of the input and output signals and \( H(\omega) \) is the frequency response. Therefore, by transforming the input and output time signals into the frequency domain, the system characteristics are described by simply dividing the spectral line of the output by the spectral line of the input at each frequency. This corresponds with the experimental model in section 3.3.2.

Experimental data is finite in length and recorded as discrete data points, so the discrete form of the Fourier series is used to transform the data from the time domain to the frequency domain using:

\[
c_m = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi mn}{N}} \quad \text{for } m = 0, 1, 2, \ldots, N - 1 \quad (15)\]

where \( c_m \) is the complex amplitude of a spectral line. The spectral lines are unique for \( m = 0, 1, \ldots, N/2+1 \) and half the actual amplitudes for all spectral lines except the bias term. The frequency resolution of the spectral content in Hertz is \( f_s/N \), where \( f_s \) is the sampling rate in samples per second and \( N \) is the total number of samples. The spectral content calculated using Equation 15 contains only frequencies that are periodic over the sample duration.

The digital signal processing techniques used in this study assume that the data from the system are stationary, ergodic, and deterministic. However, experimental measurements also include noise due to ambient vibrations, electrical noise in the sensors and cables, and digitization error. The noise is assumed to be white and random, meaning that the total noise energy is equally distributed among all frequencies. Averaging is used to reduce the magnitude of noise, but averaging of the time signals will attenuate the deterministic data with the noise unless the signals from different tests are properly synchronized. However, the modulus of the frequency content of the signals is independent of phase, so averaging in the frequency domain attenuates random content but not deterministic content. Averaging in the frequency domain does not completely remove the effects of noise, but deterministic data are easily distinguished for high signal-to-noise ratios. Averaging frequency domain signals from one position using only the positive frequencies yields the one-sided autopower spectrum, \( G_{xx} \), and averaging
frequency domain signals from two different positions using only the positive frequencies gives the one-sided cross power spectrum, \(G_{xy}\). The autopower spectrum always contains real spectral lines, while the cross power spectrum is usually comprised of complex spectral amplitudes. The root mean square (RMS) equations for both are provided below:

\[
G_{xx} = \frac{2}{K} \sum_{k=1}^{K} X_k^* \omega_k X_k \omega_k
\]

\[
G_{\tilde{x}\tilde{x}} = E[G_{xx}] = |\tilde{X}(\omega)|^2
\]

\[
G_{xy} = \frac{2}{K} \sum_{k=1}^{K} X_k^* \omega_k Y_k \omega_k
\]

\[
G_{\tilde{x}\tilde{y}} = E[G_{xy}] = \tilde{X}(\omega)\tilde{Y}(\omega)
\]

where \(X(\omega)\) and \(Y(\omega)\) are computed using Equation 15, the tilde (~) denotes purely deterministic data, * denotes the complex conjugate, and \(E[\cdot]\) denotes the expected value for an infinite number of data sets. The phase information contained within the cross power spectrum is the phase between \(x\) and \(y\) and is independent of the original reference of both signals.

The frequency response function, \(H(\omega)\), in Equation 14 is defined by an output that is entirely correlated with the input. However, the measured input can contain content both correlated to the output and uncorrelated to the output. Similarly, the response can contain both correlated and uncorrelated content from the input. Figure 3-18 provides a visual representation of a single-input/single-output (SISO) system, where the total measured input, \(X(\omega)\), and response, \(Y(\omega)\), include uncorrelated input, \(M(\omega)\), and output, \(N(\omega)\), content. The hat (^) denotes correlated data.

\[
\hat{X}(\omega) \rightarrow H(\omega) \rightarrow \hat{Y}(\omega)
\]

\[
M(\omega) \rightarrow \sum \rightarrow X(\omega) \rightarrow \sum \rightarrow Y(\omega)
\]

\[\text{Figure 3-18: Dynamic System Including Uncorrelated Content}\]
Modification of Equation 14 to include the uncorrelated content results in:

\[
\hat{Y}(\omega) + N(\omega) = H_{\text{est}}(\omega)\hat{X}(\omega) + M(\omega)
\]

where \(H_{\text{est}}(\omega)\) is the estimated frequency response function. The error between the estimated FRF and the FRF of the system depends on the magnitudes of the uncorrelated content. To calculate the estimated FRF from measured data, the measured input or output must be assumed to have no uncorrelated content. Assuming no uncorrelated content in the input measurement, the uncorrelated content on the output is:

\[
N(\omega) = \hat{Y}(\omega) - H_{\text{est}}(\omega)\hat{X}(\omega)
\]

The best estimate of the FRF is achieved by minimizing the uncorrelated content with respect to the frequency response function using a least squares minimization as follows:

\[
\frac{\partial}{\partial H} \sum_{k=1}^{K} \hat{N}_k^* N_k = 0 = \frac{\partial}{\partial H} \left[ \sum_{k=1}^{K} \left[ \hat{Y}_k - H_{\text{est}} \hat{X}_k \right]^* \left[ \hat{Y}_k - H_{\text{est}} \hat{X}_k \right] \right]
\]

\[
\frac{\partial}{\partial H} \sum_{k=1}^{K} \hat{N}_k^* N_k = 0 = \frac{\partial}{\partial H} \left[ \sum_{k=1}^{K} \hat{Y}_k^* \hat{X}_k - H_{\text{est}} \sum_{k=1}^{K} \hat{Y}_k^* \hat{X}_k - H_{\text{est}}^* \sum_{k=1}^{K} \hat{X}_k^* \hat{Y}_k + H_{\text{est}}^* H_{\text{est}} \sum_{k=1}^{K} \hat{X}_k^* \hat{X}_k \right]
\]

Considering that the derivative of a complex quantity, \(Q\), with respect to a complex scalar parameter, \(a = a_r + j a_i\), is defined as (Brandwood 1983; Therrien 1992):

\[
\frac{\partial Q}{\partial a} = \frac{1}{2} \left( \frac{\partial Q}{\partial a_r} - j \frac{\partial Q}{\partial a_i} \right)
\]

the minimization reduces to:

\[
\frac{1}{2} \left( -2 \sum_{k=1}^{K} \hat{Y}_k^* \hat{X}_k + 2 H_{\text{est}}^* \sum_{k=1}^{K} \hat{X}_k^* \hat{X}_k \right) = 0
\]
Therefore, solving for the estimated FRF returns:

\[
H_1 \equiv H_{est} = \frac{\sum_{k=1}^{K} \hat{X}_k^* \hat{Y}_k}{\sum_{k=1}^{K} \hat{Y}_k^* \hat{Y}_k} = \frac{G_{xy}}{G_{xx}} \quad (16)
\]

The estimated FRF in Equation 16 is known as the \( H_1 \) estimator. Assuming that the input contains no uncorrelated content, the expected value of the \( H_1 \) estimator is the exact solution of the FRF. However, this assumption is difficult to maintain. Averaging performed in the calculation of the autopower spectrum on the measured input does not remove the deterministic uncorrelated content, so the expected value of the autopower spectrum is:

\[
G_{\tilde{x}\tilde{x}} = G_{\tilde{x}\tilde{x}} + G_{mm}
\]

Because the uncorrelated content on the input is not correlated with either the correlated or uncorrelated content on the output and vice-versa, the expected value for the cross power spectrum is \( G_{\tilde{x}\tilde{y}} \). Therefore, the expected value of the \( H_1 \) estimator when the input contains uncorrelated content becomes:

\[
E[H_1] = \frac{G_{\tilde{x}\tilde{y}}}{G_{\tilde{x}\tilde{x}} + G_{mm}}
\]

The \( H_1 \) estimator is most useful when the uncorrelated content on the input is small, such as with force vibration testing. To quantify the correlation between the measured input and output, the power of the uncorrelated content on the output is considered as follows:

\[
G_{nn} = G_{yy} - \frac{G_{xy} G_{yx}}{G_{xx}} G_{xx} + \frac{G_{yx}}{G_{xx}} \cdot \frac{G_{xy}}{G_{xx}} G_{xx}
\]

\[
G_{nn} = G_{yy} [1 - \gamma_{xy}^2]
\]
\[ 0 \leq \gamma_{xy}^2 = \frac{|G_{xy}|^2}{G_{xx} G_{yy}} \leq 1 \] (17)

\( \gamma_{xy}^2 \) is known as the coherence function. Further analysis of the Equation 17 indicates that if both the input and output contain no uncorrelated content, the coherence function is equal to one, and \( G_{nn} \) is equal to zero. As uncorrelated content is introduced, the coherence function decreases. The coherence function will always be one when only considering one datum set.

### 3.3.5 Considerations for Experimental Testing

Using the data processing techniques of section 3.3.4, an estimate of the frequency response function of the physical structure is determined from experimental data. The resulting frequency response function is then analyzed using the techniques of section 3.3.2 to extract modal parameters. The results are only as good as the measurements, though, so special considerations must be taken during testing to minimize errors. Ewins (2000) provided guidelines for getting the best results with impact testing. These guidelines include using a consistent method for all tests. The data processing techniques in section 3.3.4 depend on averaging, so the testing technique must be consistent for the bridge response to be deterministic. Only dropping the hammer once over the duration of recording data and recording data until the responses from the impact at all response locations have fully diminished to the noise floor ensures that the effects from only one hammer impact are recorded at a time. Also, beginning and ending the data set at the noise floor reduces the effects of leakage (the distribution of energy from frequencies that are aperiodic over the sample duration to the entire spectral content). For impact testing, the frequency bandwidth of the input is controlled by the hammer tip material (Ewins 2000), so the hammer tip should not be changed between tests. Using a consistent impact magnitude well above the noise floor prevents variations in the response due to nonlinearities in the structure and keeps the uncorrelated content on the input low.

### 3.3.6 Phase Delay Correction

Due to an unfortunate miscalculation of the phase delay between data acquisition modules, the responses at some locations were not synchronized with the input channel. Instead of altering the time domain signals, the data was synchronized by changing the phase angle of the spectral lines using:
\[ \hat{c}_m = c_m e^{j\omega T} \]

where \( \hat{c}_m \) is the corrected complex amplitude, \( c_m \) is the uncorrected complex amplitude, \( \omega \) is the frequency in radians per second of spectral line \( m \), and \( T \) is the phase delay in seconds.

Considering Equations 3 and 4, the corrected data was lagged by 1.465 ms, or three datum points.

### 3.3.7 Modal Assurance Criterion

As a method of comparison, the modal assurance criterion, or MAC, (Ewins 2000) was applied to all experimentally determined mode shapes and numerical models with various boundary conditions. The MAC is an indication of the similarity between two mode shapes. When considering multiple real modes shapes, an \( M \times M \) MAC matrix is formed with each component calculated as:

\[
MAC_{ik} = \frac{\left[ \sum_{n=1}^{N} A_{ni} B_{nk} \right]^2}{\left[ \sum_{n=1}^{N} (A_{ni})^2 \right] \left[ \sum_{n=1}^{N} (B_{nk})^2 \right]}
\]

where \( A \) and \( B \) are \( N \times M \) matrices whose columns are comprised corresponding mode shapes. The diagonal terms of the MAC matrix compare corresponding mode shapes whereas the off-diagonal terms mode shapes that do not correspond. MAC values range from one for mode shapes that are exactly the same to zero for mode shapes that are orthogonal.

### 3.4 Finite Element Modeling Methods

#### 3.4.1 Overview

A finite element model of the bridge used in this study was created in Abaqus, Version 6.7-3 (2007). The model provided insight into the dynamic behavior of the bridge to assist in locating the sensors for maximum response. It also provided a basis of comparison for the experimental results.

#### 3.4.2 Eccentric Beam Modeling

The eccentric beam modeling technique was chosen based on the recommendations of Chung and Sotelino (2006). Though the authors recommended this technique for static loads, the method is also believed to be adequate for modeling dynamic behavior. This technique uses a beam element to stiffen a shell element through the following kinematic relationships:
\[
\begin{bmatrix}
    u_1^s \\
    u_2^s \\
    u_3^s \\
    \theta_1^s \\
    \theta_1^b
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 & 0 & 0 & e & 0 \\
    0 & 1 & 0 & -e & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
    u_1^b \\
    u_2^b \\
    u_3^b \\
    \theta_1^b \\
    \theta_1^b
\end{bmatrix}
\]

where the superscript \( s \) indicates a shell DOF, the superscript \( b \) indicates a beam DOF, \( e \) is the eccentricity between the shell and beam, the 1-axis is parallel to the axis of the beam, and the shell lies in the plane formed by the 1- and 2-axes. When applied to a bridge, these constraints simulate perfect composite action between a girder and deck. This technique was extended by placing the support nodes at the locations of the bottom of the girders and enforcing the same kinematic constraints between the support and girder nodes. A visual representation of this technique is shown in Figure 3-19.

Figure 3-19: Eccentric Beam Model

Implementation of this technique in Abaqus (2007) involved using the 2-node B31 beam element and 4-node S4 shell element. The B31 element is a shear-deformable linear beam element utilizing Timoshenko beam theory, and the S4 element is a full-integration, general-purpose linear shell element that allows transverse shear deformations and is compatible with the Timoshenko beam element. To verify that the S4 shell element was behaving as a plate when
applied to the bridge model, the maximum out-of-plane deflections from a simply-supported plate of varying thickness comprised of S4 shell elements and subject to a uniform load were compared to the analytical solutions. The outer dimensions of the plate were 1200 in. by 640 in. and selected to reflect the approximate characteristics of the bridge model. A material stiffness of 29,000,000 psi and Poisson’s ratio of 0.3 were assigned, and a pressure load of 10 psi applied. A fine mesh of 1920 elements was used. The analytical solution for the maximum out-of-plane deflection for a plate with the same properties is (Boresi and Schmidt 2003):

\[ w_{\text{max}} = C(1 - \nu^2) \frac{pb^4}{Eh^3} \]

where \( C \) is a constant dependent upon boundary conditions and outer dimensions, \( \nu \) is the Poisson’s ratio, \( p \) is the uniform pressure load, \( b \) is the shorter side dimension, \( E \) is the uniaxial material stiffness, and \( h \) is the plate thickness. For a simply supported rectangular plate under a uniformly distributed load with the outer dimensions previously described, \( C \) is equal to 0.1173. The results of comparing the deflection from the S4 element to the analytical solution are presented in Table 3-3.

### Table 3-3: Comparison of S4 Shell Element with Analytical Solution

<table>
<thead>
<tr>
<th>a/h</th>
<th>b/h</th>
<th>h (in.)</th>
<th>Analytical Maximum Deflection (in.)</th>
<th>ABAQUS Maximum Deflection (in.)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>1.28</td>
<td>500</td>
<td>4.9400E-05</td>
<td>1.5825E-04</td>
<td>-220.3%</td>
</tr>
<tr>
<td>4.8</td>
<td>2.56</td>
<td>250</td>
<td>3.9520E-04</td>
<td>6.4265E-04</td>
<td>-62.6%</td>
</tr>
<tr>
<td>12</td>
<td>6.4</td>
<td>100</td>
<td>6.1751E-03</td>
<td>6.9779E-03</td>
<td>-13.0%</td>
</tr>
<tr>
<td>24</td>
<td>12.8</td>
<td>50</td>
<td>0.0494</td>
<td>0.0513</td>
<td>-3.8%</td>
</tr>
<tr>
<td>40</td>
<td>21.33</td>
<td>30</td>
<td>0.2287</td>
<td>0.2315</td>
<td>-1.2%</td>
</tr>
<tr>
<td>60</td>
<td>32</td>
<td>20</td>
<td>0.7719</td>
<td>0.7727</td>
<td>-0.1%</td>
</tr>
<tr>
<td>120</td>
<td>64</td>
<td>10</td>
<td>6.1751</td>
<td>6.122</td>
<td>0.9%</td>
</tr>
<tr>
<td>150</td>
<td>80</td>
<td>8</td>
<td>12.0606</td>
<td>11.9363</td>
<td>1.0%</td>
</tr>
<tr>
<td>200</td>
<td>106.67</td>
<td>6</td>
<td>28.5882</td>
<td>28.2493</td>
<td>1.2%</td>
</tr>
<tr>
<td>300</td>
<td>160</td>
<td>4</td>
<td>96.4852</td>
<td>95.2172</td>
<td>1.3%</td>
</tr>
<tr>
<td>600</td>
<td>320</td>
<td>2</td>
<td>771.8815</td>
<td>761.058</td>
<td>1.4%</td>
</tr>
<tr>
<td>1200</td>
<td>640</td>
<td>1</td>
<td>6175.052</td>
<td>6087.01</td>
<td>1.4%</td>
</tr>
<tr>
<td>12000</td>
<td>6400</td>
<td>0.1</td>
<td>6175052</td>
<td>6086510</td>
<td>1.4%</td>
</tr>
</tbody>
</table>

\( a = \) longer side dimension = 1200 in.; \( b = \) shorter side dimension = 640 in.; \( h = \) thickness
The results in Table 3-3 indicate that the formulation of the S4 element in Abaqus 6.7-3 (2007) follows the analytical solution from plate theory closely for ratios of $a/h$ and $b/h$ greater than 24 and 12.8, respectively. Because the equivalent ratios for the bridge modeled in this study are approximately 136 and 73, the S4 shell elements behave as plates when applied to the bridge model.

3.4.3 Bridge Model Description

The model of the bridge was created using construction plans. Not until performing the experimental tests was it noticed that the actual dimensions of the bridge were slightly different from the dimensions stated on the plans. The actual bridge length was 1'-6" (1%) shorter than stated on the plans, and the actual girder spacing was narrower by 1.6" (2%). Also, the total width of the bridge was actually 1'-1" (2%) narrower. The variations in dimensions were assumed to have a negligible effect on the results of the numerical model. The dimensions used for the finite element model are presented in Figure 3-20 and Figure 3-21.

Figure 3-20: Plan View of Bridge Model Dimensions
As with the physical structure, the support nodes were located directly beneath the end diaphragm rows. Boundary conditions were only applied to the support nodes. Shell elements were placed at the midsurface of the deck, and beam elements were positioned at the centroid of the girder and barrier sections. No attempts were made to model the effects of the seal between the deck and abutment, and the abutments were considered rigid. Though the superelevation of the actual bridge varied along the length, the average superelevation, 3.95%, was used for the entire bridge model. The two transverse diaphragms at each end of the bridge seen in Figure 3-2 were not modeled and assumed to have a negligible effect on the modal parameters. The uniaxial elastic stiffness, shear modulus, Poisson’s ratio, and unit weight of the steel girders and diaphragms were set to 29,000,000 psi, 11,154,000 psi, 0.3, and 490 pcf, respectively. The uniaxial elastic stiffness, shear modulus, Poisson’s ratio, and unit weight of the concrete barriers and deck were set to 3,605,000 psi, 1,502,000 psi, 0.2, and 150 pcf, respectively. Geometric member properties of the girders and barriers were assigned using the GENERAL section designation for the B31 element. The relevant geometric properties are provided in Figure 3-22.
Because the diaphragms were channel sections, the ARBITRARY section designation was used to assign the geometric properties of the diaphragm section to the B31 element. This designation is most useful for sections comprised of rectangular segments. The relevant geometric properties of the diaphragm section are presented in Figure 3-23.
Constraints between different components of the model were applied using two different techniques. Multi-point constraints in Abaqus (2007) apply constraints by slaving a node to a master node. This removes the slaved node from calculations, effectively reducing the size of the model. However, neither boundary conditions nor loads can be applied to the slaved node. Multi-point constraints were used between the deck and barriers because no loads or boundary conditions were applied to the barriers. Because some girder nodes were constrained to both support nodes and deck nodes, multi-point constraints could not be used. Instead, a 2-node rigid connecting element, CONN3D2 with beam-type attributes, was used. This rigid element applied the same constraints as the multi-point constraint without removing any nodes from the model. CONN3D2 elements were used between all girder elements and the deck elements as well as the support nodes and girder elements. This allowed loads to be applied anywhere on the deck in future implementations. Diaphragm elements were located in the same plane as the girder elements and shared nodes with the girder elements at intersections.

**3.4.4 Parametric Study**

A parametric study was performed to determine the effects of changing element size, geometry, and boundary conditions on the modal characteristics of the bridge model. When
varying the mesh density, an approximate global element size was applied to the whole model. Approximate element sizes varied between 125 in. and 5 in. Four different physical configurations were investigated, including the bridge modeled with both skew and superelevation, skew only, superelevation only, and with no skew or superelevation. When modeling the bridge with skew, the shell elements forming the deck were skewed at the same angle as the skew of the bridge. Ideal boundary conditions were applied to all support nodes with similar boundary conditions on all nodes at each end of the bridge. The combinations of boundary conditions investigated included pin-roller, pin-pin, roller-roller, fixed-roller, fixed-pin, and fixed-fixed. The axes associated with the BCs were oriented such that one of the axes was parallel to the centerline of the bridge. A pin boundary condition restrained all translational DOFs without restraining any rotational degrees of freedom, while a roller boundary condition was similar to the pin except the translational DOF parallel with the centerline of the bridge was released. A fixed boundary condition restrained all translational and rotational DOFs. Upon further investigation of modes from the bridge models with skew and pin-roller boundary conditions, it was noticed that the modes were not symmetric about the centerline of the bridge. Applying the boundary conditions in reverse order produced mode shapes mirrored about the centerline from the pin-roller mode shape with slightly different frequencies, though the geometry of the model indicated no reason for any difference. Therefore, roller-pin boundary conditions were also investigated.

3.5 Summary

Experimental, data processing, and finite element modeling methods were presented, including a theoretical background. The bridge was tested using impact forced vibration and by measuring response along the girder lines at 63 locations. Four seconds of data were recorded 10 times for each forcing location and sensor configuration. The data was processed by considering the FRF inertance at each location. Natural frequencies were determined by locating peaks in the FRF magnitude plots and considering the associated characteristics in the FRF phase, real, and imaginary plots. Mode shapes were determined from the imaginary values of the FRF inertance at each selected frequency. Damping ratios were determined from the imaginary part of the FRF mobility. A finite element model of the bridge was created in Abaqus Version 6.7-3 using the eccentric beam modeling technique. Different parameters of the model, including mesh refinement, geometry, and boundary conditions, were altered to investigate the effects of
changing each and to bound the experimental results. Element sizes were varied between 125 in. and 5 in., and model geometry was varied with different combinations of skew and superelevation. Boundary conditions ranged from restraints in the transverse and vertical translational directions only at the supports to full restraint of all degrees of freedom at the support nodes. The experimental mode shapes were compared to the FEM mode shapes using the modal assurance criterion.
Chapter 4. Results

After performing the dynamic tests on the bridge, the data was processed to extract frequencies, mode shapes, and estimations of damping. Different configurations of a finite element (FE) model were considered for representing the bridge. After reviewing the behavior of the models, the best geometry and mesh refinement were chosen for comparison with the experimental results. The natural frequencies and mode shapes from FE models with varying boundary conditions were compared to the experimentally-determined frequencies and mode shapes. The comparisons indicated the boundary conditions that best represented the dynamic behavior of the actual bridge. The experimental results, including a discussion about the time domain data and extraction of modal parameters, are presented in section 4. Experimental Results

4.1.1 Overview

The experimental results constituted the characterization of the physical structure. Because testing was completed within a few hours, the only variable in the system was the thermal gradient in the structure. Any effects from thermal variations were expected to be small because the bridge was simply supported. As a measure of the thermal gradient throughout the deck, average temperatures on the top and bottom of the deck were measured at the beginning and end of testing. The average temperatures on the top and bottom of the deck were 94°F and 69°F, respectively, around 11:00 a.m. and 95°F and 70°F around 2:00 p.m. These temperature readings indicated that any effects from variations in temperature were minimal. The following sections present a discussion on the characteristics of the time domain signals as well as the modal parameters extracted from the experimental data.

4.1.2 Time Domain Data

Visual analysis of the time domain signals provided insight into the quality of the results. It gave a visual indication of the signal-to-noise ratio of the responses as well as indicated whether irregularities in the signal were due to the malfunctioning equipment or spurious ambient vibrations. The signals in Figure 4-1 are responses from two different spatial positions during test T2 with excitation at forcing location I. Sensor location 11 was much further away from the source of impact than location 15, which is reflected in the magnitude of the response.
Both indicate an increase in the noise floor toward the end of the signal, alluding to an ambient event that affected all sensors similarly.

![Spatial Position 11](image1.png)

![Spatial Position 15](image2.png)

**Figure 4-1: Time Signals with Similar Trends**

However, the signals in Figure 4-2 were also recorded simultaneously, but the response from spatial position 34 has a second spike that was not present in the signal from position 26 or any of the other responses. Therefore, this irregular behavior was attributed to equipment malfunction.
The time domain signals of the channels were frequently checked for irregularities. When all channels showed the same irregularity, the test was performed over again. If only one channel demonstrated the irregularity, the sensor was checked and replaced if necessary. In an attempt to test the system in a consistent state, the total testing duration was confined to the early afternoon when ambient temperatures were expected to stabilize. Every channel was not checked after each impact due to this time constraint, but every channel was checked at least once for every set of tests at each forcing location. During testing, only two accelerometers were
replaced due to an irregular signal. The erroneous behavior was suspected to be a consequence of the radiant heat from the deck in the afternoon sun. Ten data sets were taken for each of the three forcing locations at every sensing position. This allowed poor data to be discarded without detracting from the averaging capabilities of signal processing. Even with these precautions, some datum contained irregular signals and were discarded from the analysis. These signals are presented in Figure 4-3 through Figure 4-6.

Figure 4-3: Discarded Time Domain Signals from Test T7
Figure 4-4: Discarded Time Domain Signals from Test T9
Figure 4-5: Discarded Time Domain Signals from Test T10
4.1.3 Frequency Extraction

Natural frequencies were determined by considering peaks in the averaged FRF inertance magnitude plots and checking the characteristics of the phase, real, and imaginary plots at these locations. The finite element models indicated that the first thirteen modes had frequencies less than 25 Hz, so frequencies between 0.5 Hz and 25 Hz were considered. Investigation of frequencies higher than 25 Hz indicated that the quality of mode shapes deteriorated quickly for modes above 25 Hz. Overall, three FRFs corresponding to the three forcing locations were checked at each sensor position. Only peaks that consistently appeared in all data were chosen as natural frequencies. Figure 4-8 demonstrates the characteristics of a mode in the averaged FRF inertance from spatial position 6 excited by a force at location I. The location of mode 3, with a frequency of 5.5 Hz, is designated by a red asterisk or line in each plot. A peak occurred in the magnitude at 5.5 Hz with a corresponding phase shift of 180°. The imaginary component achieved a local maximum at the same frequency, and the real component passed through zero near this frequency. A check of the coherence indicated good correlation between the force and response at 5.5 Hz.
Mode Selection from the Magnitude, Phase, and Coherence Plots
Based on the spatial positions of the sensors and forcing location, some FRFs exhibited higher responses at the chosen frequencies than others. The averaged FRF inerterance magnitude at 5 Hz, denoted by a red asterisk, from forcing at location I was much higher at sensor position 33 than at position 50, as shown in Figure 4-9. Considering the locations of these accelerometers on the mode shape in Figure 4-12 corresponding to the third mode, this behavior is understandable. Position 33 is at a location of high displacement for this mode, while position 50 is at a location of little movement. In the same regard, forcing location I is at a location of high displacement for mode 3, indicating that this mode was well excited.

Figure 4-8: Mode Selection from the Real and Imaginary Component Plots
After selecting frequencies, the mode shapes were visually inspected for validity. The eleven modes identified from data corresponding to each forcing location are presented in Table 4-1. Note that the frequency resolution of the data was 0.25 Hz. Forcing at different locations produced slightly different results for the second and seventh modes. The actual modes most likely fell almost directly between the two frequencies, and forcing at different locations caused the results to favor one spectral line over the other.
As was indicated by the finite element models, many modes fell very close to each other, with some differences less than the frequency resolution of the results. This occurred for modes 1 and 2, modes 4, 5, and 6, and modes 9, 10, and 11. The resulting averaged FRF magnitude plots appeared to have wide peaks around these frequencies that shifted slightly among the results from sensor spatial locations. Figure 4-10 demonstrates this characteristic with the averaged FRF inerterance from spatial position 38 excited at forcing location II. Therefore, the resulting mode shapes were for visual verification to help distinguish two closely spaced modes.
4.1.4 Mode Shapes

Mode shapes were assembled from the imaginary values of the averaged FRF inertance corresponding to the chosen frequencies. Modes represented local maxima or minima on the imaginary plots depending on the phase of the component. Figure 4-11 presents the imaginary component of the averaged FRF inertance at spatial positions 5 and 32 from forcing at location I. The two points are out-of-phase with each other for the third mode, with frequency denoted by the red asterisk, as the components at these locations have opposite signs.

Figure 4-11: FRF Inertance Imaginary Components
The resulting eleven mode shapes constructed from data corresponding to the three forcing locations are presented in Figure 4-12, Figure 4-13, and Figure 4-14. Only real mode shapes were considered, so each component was either in-phase or completely out-of-phase with another component. Because of the contributions from other modes, the operating deflection shape was assembled as opposed to the actual mode shape, though these contributions were expected to be low because of the low damping in the structure. These contributions were more noticeable for closely spaced modes, such as modes 1 and 2, modes 4, 5, and 6, and modes 9, 10, and 11.

The forcing location had a profound effect on the resulting mode shapes. Mode 10 was not well defined for results from forcing locations I and II. Further inspection of mode 10 from forcing location III in Figure 4-14 and mode 12 from the finite element model in Figure 4-25 indicated that forcing locations I and II were near a node for this mode. Therefore, the mode was not well excited and the contributions from other modes had a profound effect on the resulting mode shape. The mode shapes for modes 4 and 6 were not well defined from data acquired through forcing at location III. Considering the mode shapes from the other forcing locations and the finite element model, forcing location III was near a node for both of these modes. This was compounded by the close proximity of mode 5. Surprisingly, mode 1 was not well defined for any of the data. None of the forcing locations were near nodes for this mode, and the finite element models indicated that the effective mass in the vertical direction was high for this mode, meaning that its response should have prevailed in the data. However, the contributions from the second mode due to its close proximity may have distorted the mode shape.
Figure 4-12: Mode Shapes from Forcing at Location I
Figure 4-13: Mode Shapes from Forcing at Location II
Figure 4-14: Mode Shapes from Forcing at Location III
4.1.5 Damping Estimation

The imaginary component of the averaged FRF mobility was used to estimate the damping ratio using the half-power bandwidth method. Two spectral lines representing a local maximum and a local minimum were chosen near each natural frequency. The damping values associated with the natural frequencies determined from the three forcing locations are presented in Table 4-2, Table 4-3, and Table 4-4, including the lower and upper frequencies, $\omega_a$ and $\omega_b$, used in the calculations.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>$\omega_a$ (Hz)</th>
<th>$\omega_b$ (Hz)</th>
<th>Critical Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.75</td>
<td>2.75</td>
<td>3.00</td>
<td>5%</td>
</tr>
<tr>
<td>2</td>
<td>3.25</td>
<td>2.75</td>
<td>3.00</td>
<td>4%</td>
</tr>
<tr>
<td>3</td>
<td>5.50</td>
<td>5.50</td>
<td>5.75</td>
<td>2%</td>
</tr>
<tr>
<td>4</td>
<td>10.50</td>
<td>10.50</td>
<td>11.00</td>
<td>2%</td>
</tr>
<tr>
<td>5</td>
<td>11.00</td>
<td>11.00</td>
<td>11.25</td>
<td>1%</td>
</tr>
<tr>
<td>6</td>
<td>11.25</td>
<td>11.25</td>
<td>11.50</td>
<td>1%</td>
</tr>
<tr>
<td>7</td>
<td>13.25</td>
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</tr>
<tr>
<td>11</td>
<td>22.50</td>
<td>22.25</td>
<td>22.75</td>
<td>1%</td>
</tr>
</tbody>
</table>

Table 4-2: Damping Estimations from Forcing Location I Data

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>$\omega_a$ (Hz)</th>
<th>$\omega_b$ (Hz)</th>
<th>Critical Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.00</td>
<td>2.75</td>
<td>3.00</td>
<td>4%</td>
</tr>
<tr>
<td>2</td>
<td>3.25</td>
<td>2.75</td>
<td>3.00</td>
<td>4%</td>
</tr>
<tr>
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<td>5.50</td>
<td>5.50</td>
<td>5.75</td>
<td>2%</td>
</tr>
<tr>
<td>4</td>
<td>10.50</td>
<td>10.75</td>
<td>11.00</td>
<td>1%</td>
</tr>
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<td>5</td>
<td>11.00</td>
<td>11.00</td>
<td>11.25</td>
<td>1%</td>
</tr>
<tr>
<td>6</td>
<td>11.25</td>
<td>11.25</td>
<td>11.50</td>
<td>1%</td>
</tr>
<tr>
<td>7</td>
<td>13.00</td>
<td>13.00</td>
<td>13.25</td>
<td>1%</td>
</tr>
<tr>
<td>8</td>
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<td>15.75</td>
<td>16.00</td>
<td>1%</td>
</tr>
<tr>
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<tr>
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<td>21.75</td>
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<td>1%</td>
</tr>
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<td>11</td>
<td>22.50</td>
<td>22.50</td>
<td>22.75</td>
<td>1%</td>
</tr>
</tbody>
</table>

Table 4-3: Damping Estimations from Forcing Location II Data
Given the low damping in the structure, the accuracy of the damping estimation was limited by the spectral resolution of the data. Inspection of Equation 12 indicates that the difference between the half-power frequencies increases as the natural frequency increases for the same critical damping ratio. The damping estimate for mode 1 was highest because the difference between half-power frequencies could not be smaller than the resolution of the data. However, the damping estimate for mode 11 used half-power frequencies that were not on adjacent spectral lines, so the estimate was assumed to be more accurate than the estimates for the lower modes. These characteristics are illustrated in Figure 4-15 for modes 1 and 11 from FRF mobility at sensor position 53 excited at forcing location III.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>$\omega_a$ (Hz)</th>
<th>$\omega_b$ (Hz)</th>
<th>Critical Damping Ratio</th>
</tr>
</thead>
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<td>3.00</td>
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<td>5.75</td>
<td>2%</td>
</tr>
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<td>11.00</td>
<td>2%</td>
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<td>1%</td>
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<td>11.50</td>
<td>1%</td>
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<td>13.00</td>
<td>13.25</td>
<td>1%</td>
</tr>
<tr>
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<td>15.75</td>
<td>16.00</td>
<td>1%</td>
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<tr>
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<td>21.25</td>
<td>21.00</td>
<td>21.25</td>
<td>1%</td>
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<tr>
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<td>22.00</td>
<td>21.50</td>
<td>22.00</td>
<td>1%</td>
</tr>
<tr>
<td>11</td>
<td>22.50</td>
<td>22.25</td>
<td>22.75</td>
<td>1%</td>
</tr>
</tbody>
</table>
4.2 Finite Element Modeling Results

4.2.1 Overview

Finite element modeling of the bridge was performed to help locate sensors and provide a comparison for the experimental results. Different configurations, based on geometry, mesh refinement, and boundary conditions, were considered in a parametric study to investigate the effects of each parameter on the modal characteristics. Based on the results of the study, a model was chosen for comparison with the experimental results. The model was compared based on frequencies and mode shapes. Details on the parametric study and comparison with the experimental results are presented in the following sections.

4.2.2 Parametric Study

The parametric study consisted of varying the geometry, mesh refinement, and boundary conditions. Four geometric configurations were considered, including models with skew and superelevation, skew only, superelevation only, and no skew or superelevation. The mesh refinement on each of these variations was varied with approximate element sizes of 125 in., 100 in., 75 in., 50 in., 40 in., 30 in., 20 in., 15 in., 10 in., and 5 in. Boundary conditions (BC) were varied on each combination of mesh refinement and geometry. The BCs combinations on each end of the bridge model were pin-roller, roller-pin, pin-pin, roller-roller, fixed-roller, fixed-pin, and fixed-fixed. The axes associated with the BCs were oriented such that one of the axes was
parallel to the centerline of the bridge. A pin BC restrained translation in along all primary axes without restraining any rotations, whereas a roller BC was similar except the translation along the centerline of the bridge was released. A fixed BC restrained all DOFs. The convergence curves considering mesh refinement for the first five frequencies of models with each of the four geometric configurations and roller-pin BCs are presented in Figure 4-16 through Figure 4-20.

Figure 4-16: Convergence for First Frequency
Figure 4-17: Convergence for Second Frequency

Figure 4-18: Convergence for Third Frequency
Figure 4-19: Convergence for Fourth Frequency

Figure 4-20: Convergence for Fifth Frequency
The convergence curves in Figure 4-16 through Figure 4-20 are representative of the convergence curves from all other BC combinations. Notice, though, that the scale for the plots is narrow, so the differences between frequencies for each configuration are small. Because of the similarities between the model with superelevation and skew and the model with skew only, the results in Figure 4-17, Figure 4-19, and Figure 4-20 for both appear to be superimposed. The same is true for the results from the model with superelevation only and the model with neither superelevation nor skew for the same figures. Based on the results and considering the computational cost of each refinement, the models with element sizes of approximately 20 in. were found to be sufficiently refined. The locations of these models are denoted by a yellow circle in the figures.

Analysis of the data indicates that explicitly modeling the superelevation alone has a minimal effect on the frequency of the modes, but the effect of adding skew was more profound. For the mesh refinement with element sizes of approximately 20 in., the first five frequencies of the model with skew only were within 0.25% of the same frequencies for the model with skew and superelevation for all BC combinations. The same assessment was true when comparing the frequencies from the model with superelevation only to the model without superelevation or skew. However, the differences in the frequencies between the model with superelevation and skew and the latter two models varied with the mode and boundary conditions. The approximate range of differences comparing the models with skew to the models without skew are presented in Table 4-5. The differences in frequency were calculated by comparing models with identical boundary conditions, and the differences were compiled from all BC combinations for each mode to form the ranges in Table 4-5. Overall, the differences between frequencies were small. The lower and upper limits of the ranges for the first, fourth, and fifth frequencies were formed by considering the models with roller-roller and fixed-fixed boundary conditions, respectively. For the second frequency, the situation was reversed, with the upper and lower limits controlled by the models with fixed-fixed and roller-roller BCs. The upper and lower limits for the third frequency were from the models with pin-pin and fixed-roller boundary conditions, respectively.
Table 4-5: Approximate Frequency Difference Ranges Comparing Models with and without Skew

<table>
<thead>
<tr>
<th>Mode</th>
<th>Approximate Range for Difference in Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3% to 3.4%</td>
</tr>
<tr>
<td>2</td>
<td>2.6% to 7.6%</td>
</tr>
<tr>
<td>3</td>
<td>-1.7% to 1.1%</td>
</tr>
<tr>
<td>4</td>
<td>-10.3% to -3.2%</td>
</tr>
<tr>
<td>5</td>
<td>-3.5% to 2.0%</td>
</tr>
</tbody>
</table>

Note: The models with skew are used as the reference of comparison.

Changing the boundary conditions on the model had a much more profound impact on the frequencies than adding skew or superelevation. Figure 4-21 through Figure 4-24 provide a visual comparison of the variations in frequencies for the first five modes of each geometric configuration considering the variations in boundary conditions. Generally, the models with roller-roller BCs provided the lower bound for the range of frequencies in each figure and the models with fixed-fixed BCs formed the upper bound. However, the lowest fourth frequency for the model with superelevation only was provided by roller-pin BCs, and the lowest fourth frequency for the model without superelevation or skew was given by the roller-pin and pin-roller BCs. Each model was asymmetric transversely about the centerline and longitudinally about the middle of the bridge in plan view. Despite these characteristics, the models with skew or superelevation only gave noticeably different frequencies for the second and fourth frequencies of the models with roller-pin and pin-roller BCs. The same BC combinations for the models with or without both skew and superelevation returned frequencies that were almost superimposed for all modes.
Figure 4-21: Boundary Condition Comparison for Model with Superelevation and Skew

Figure 4-22: Boundary Condition Comparison for Model with Superelevation Only
Figure 4-23: Boundary Condition Comparison for Model with Skew Only

Figure 4-24: Boundary Condition Comparison for Model without Superelevation or Skew
The modal assurance criterion (MAC) was used to quantify the effects of varying boundary conditions on the mode shapes. Because two different geometric configurations of the model did not have corresponding nodal positions, only models with identical geometries were compared. Also, only the vertical displacements of the mode shapes were considered as only the vertical components were of interest for comparison with experimental results. Using the model with roller-pin BCs as the reference, the mode shapes from each BC combination were compared to the corresponding mode shape from the reference for each mode, effectively constructing the diagonal of the MAC matrix. The columns of Table 4-6 through Table 4-9 present the diagonals of the MAC matrices for each geometric configuration. The only deviation from this procedure was for the models with at least one fixed boundary condition. Visual inspection of the 4th and 5th mode shapes from these models indicated that their shapes were much closer to the 5th and 4th mode shapes, respectively, from the model with roller-pin BCs. Therefore, the fourth row of the fixed-roller, fixed-pin, and fixed-fixed columns in the tables compares the mode shapes from the 5th modes to the mode shape from 4th mode of the reference and vice-versa for the last row.

Table 4-6: Diagonal Values of the MAC Matrix for Models with Superelevation and Skew

| Mode | Pin-Roller | Roller-Roller | Pin-Pin | Fixed-Roller
\† | Fixed-Pin
\† | Fixed-Fixed
\† |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.2411</td>
<td>0.9644</td>
<td>0.9672</td>
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</tr>
<tr>
<td>2</td>
<td>0.2400</td>
<td>0.9546</td>
<td>0.9638</td>
<td>0.9334</td>
</tr>
<tr>
<td>3</td>
<td>0.9467</td>
<td>0.9852</td>
<td>0.9836</td>
<td>0.9203</td>
</tr>
<tr>
<td>4</td>
<td>0.9768</td>
<td>0.9885</td>
<td>0.9909</td>
<td>0.8093</td>
</tr>
<tr>
<td>5</td>
<td>0.9156</td>
<td>0.9649</td>
<td>0.5907</td>
<td>0.8865</td>
</tr>
</tbody>
</table>

Note: The model with Roller-Pin BCs was used as the reference.

\†Mode shapes 5 and 4 were compared to mode shapes 4 and 5, respectively, of the reference.

Table 4-7: Diagonal Values of the MAC Matrix for Models with Superelevation Only

| Mode | Pin-Roller | Roller-Roller | Pin-Pin | Fixed-Roller
\† | Fixed-Pin
\† | Fixed-Fixed
\† |
<table>
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<th></th>
<th></th>
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</thead>
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<td>1.0000</td>
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</tr>
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<td>0.9940</td>
<td>0.9922</td>
<td>0.9454</td>
</tr>
<tr>
<td>3</td>
<td>0.9285</td>
<td>0.9804</td>
<td>0.9787</td>
<td>0.9129</td>
</tr>
<tr>
<td>4</td>
<td>0.9870</td>
<td>0.9856</td>
<td>0.9960</td>
<td>0.9238</td>
</tr>
<tr>
<td>5</td>
<td>0.5797</td>
<td>0.8659</td>
<td>0.0562</td>
<td>0.1486</td>
</tr>
</tbody>
</table>

Note: The model with Roller-Pin BCs was used as the reference.

\†Mode shapes 5 and 4 were compared to mode shapes 4 and 5, respectively, of the reference.
Overall, the correlations in the first four rows were high, demonstrating that varying boundary conditions generally had a minimal effect on the mode shape for lower modes. Inspection of the first column in each table indicated the symmetry of the mode shapes with pin and roller BCs about the centerline. All mode shapes of the models without skew exhibited good symmetry. The first mode shape for these models was exactly symmetric. The third, fourth, and fifth mode shapes of the models with skew were mostly symmetric. In either case, adding superelevation to the model decreased the symmetry of the mode shapes, with the most drastic effects demonstrated by the first and second modes of the models with skew. Because of the asymmetry of the mode shapes from the models with skew and superelevation, both the models with pin-roller and roller-pin BC combinations were compared to the experimental results. The symmetry of the mode shapes from the models with fixed-roller and fixed-pin BCs was not checked because the dynamic characteristics of these models were not similar to the measured dynamic properties of the actual bridge, though based on the results from the pin and roller BCs, the mode shapes were not expected to be symmetric. Because the actual structure was superelevated and skewed, the superelevated and skewed model with element sizes of approximately 20 in. was chosen for comparison with experimental data. Sensor positions for
experimental testing were chosen based on nodal positions of this model, which allowed direct comparison of analytical mode shapes to the experimentally-determined mode shapes. The first thirteen mode shapes for the model with 20 in. approximate element sizes and roller-pin BCs are presented in Figure 4-25. The mode shapes corresponding to the other BCs for the same model exhibited similar characteristics. Thirteen mode shapes were considered because the experimentally-determined mode shapes did not align closely with the analytical mode shapes. Some of the experimental mode shapes exhibited characteristics of more than one mode shape from the FE models and were compared to multiple analytical modes. In the same regard, some mode shapes from the FE models were poorly correlated with any of the experimental modes and were removed from the analysis. Further discussion on the comparison of the experimentally-determined mode shapes to the analytical mode shapes is presented in the next section.

Strangely, the ends of the bridge model mode shapes in Figure 4-25 exhibit rippling behavior of varying magnitudes. This behavior was also noticed for all other models. Closer investigation of the mode shapes indicates that the ripples are responses to in-plane translational movement of the deck. The restraints from eccentric beam modeling caused this deformation in the deck at the boundary conditions. Further investigation is needed to quantify the effects of this behavior.
Mode 1, Frequency = 2.7728 Hz
Mode 2, Frequency = 3.9660 Hz
Mode 3, Frequency = 6.4049 Hz
Mode 4, Frequency = 9.2444 Hz
Mode 5, Frequency = 11.477 Hz
Mode 6, Frequency = 11.789 Hz
Mode 7, Frequency = 12.132 Hz
Mode 8, Frequency = 14.574 Hz
Mode 9, Frequency = 16.540 Hz
Mode 10, Frequency = 18.300 Hz
Mode 11, Frequency = 20.221 Hz
Mode 12, Frequency = 21.304 Hz
Mode 13, Frequency = 22.017 Hz

Figure 4-25: Roller-Pin Mode Shapes
4.2.3 Results Comparison

The frequencies and mode shapes from the analytical models were compared to those experimentally determined. The analytical model chosen for comparison included skew and superelevation and had a mesh refinement with element sizes of approximately 20 in. Corresponding modes were based on both frequencies and visual inspections of the mode shapes. Analytical results were compared to the experimental results corresponding to each forcing location. Modes were compared even if the results from only one forcing location provided reasonable correlation. Because some experimental modes resembled more than one analytical mode and the frequencies were close, experimental modes were occasionally compared to more than one analytical mode. Discretion was used such that the frequencies from the analytical modes were reasonably close to the experimentally-determined frequency. Also, some analytical modes did not correspond to any experimental modes. For example, neither the mode shape nor the frequency from the tenth mode for roller-pin BCs in Figure 4-25 was close to any of the experimentally determined modes. Therefore, this analytical mode was removed from the comparison. Also, none of the experimental mode shapes corresponding to any of the forcing locations for mode 10 correlated with any of the mode shapes from the model with fixed-fixed BCs, so it was removed from Table 4-16 and Table 4-23. A comparison of corresponding modes based on frequency is presented in Table 4-10 through Table 4-16 for each of the different BC combinations.

Table 4-10: Frequency Comparison for Pin-Roller BCs

<table>
<thead>
<tr>
<th>ABAQUS Model</th>
<th>Experimental Results</th>
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</thead>
<tbody>
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<td>Mode</td>
<td>Frequency (Hz)</td>
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<tr>
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<td>9.2486</td>
</tr>
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<td>13</td>
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Table 4-11: Frequency Comparison for Roller-Pin BCs

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<td>ABAQUS Model</td>
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Table 4-12: Frequency Comparison for Pin-Pin BCs

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87
### Table 4-13: Frequency Comparison for Roller-Roller BCs

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<td>13</td>
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</tr>
</tbody>
</table>

### Table 4-14: Frequency Comparison for Fixed-Roller BCs

<table>
<thead>
<tr>
<th>ABAQUS Model</th>
<th>Experimental Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>4.1063</td>
</tr>
<tr>
<td>2</td>
<td>4.5163</td>
</tr>
<tr>
<td>3</td>
<td>6.5622</td>
</tr>
<tr>
<td>4</td>
<td>12.5540</td>
</tr>
<tr>
<td>5</td>
<td>11.6260</td>
</tr>
<tr>
<td>6</td>
<td>12.5540</td>
</tr>
<tr>
<td>7</td>
<td>13.6300</td>
</tr>
<tr>
<td>8</td>
<td>15.0050</td>
</tr>
<tr>
<td>9</td>
<td>15.6300</td>
</tr>
<tr>
<td>10</td>
<td>16.8960</td>
</tr>
<tr>
<td>12</td>
<td>25.2900</td>
</tr>
<tr>
<td>13</td>
<td>26.5390</td>
</tr>
</tbody>
</table>
Table 4-15: Frequency Comparison for Fixed-Pin BCs

<table>
<thead>
<tr>
<th>ABAQUS Model</th>
<th>Experimental Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>4.7616</td>
</tr>
<tr>
<td>2</td>
<td>5.3193</td>
</tr>
<tr>
<td>3</td>
<td>7.4203</td>
</tr>
<tr>
<td>4</td>
<td>12.2480</td>
</tr>
<tr>
<td>5</td>
<td>13.0470</td>
</tr>
<tr>
<td>6</td>
<td>14.1750</td>
</tr>
<tr>
<td>7</td>
<td>15.8090</td>
</tr>
<tr>
<td>8</td>
<td>16.3950</td>
</tr>
<tr>
<td>9</td>
<td>17.8330</td>
</tr>
<tr>
<td>11</td>
<td>23.2910</td>
</tr>
</tbody>
</table>

Table 4-16: Frequency Comparison for Fixed-Fixed BCs

<table>
<thead>
<tr>
<th>ABAQUS Model</th>
<th>Experimental Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>5.6155</td>
</tr>
<tr>
<td>2</td>
<td>5.9090</td>
</tr>
<tr>
<td>3</td>
<td>7.6712</td>
</tr>
<tr>
<td>4</td>
<td>15.0210</td>
</tr>
<tr>
<td>5</td>
<td>12.4150</td>
</tr>
<tr>
<td>6</td>
<td>15.0210</td>
</tr>
<tr>
<td>7</td>
<td>15.7070</td>
</tr>
<tr>
<td>8</td>
<td>18.1230</td>
</tr>
<tr>
<td>10</td>
<td>23.5790</td>
</tr>
</tbody>
</table>

Overall, the maximum difference between the analytical frequencies and the experimental frequencies at any forcing location was 104.2%. This occurred for the first frequency of the FE model with fixed-fixed BCs and the first frequency from forcing location I, though the difference in frequency was only 2.82 Hz. The model with frequencies closest to the experimentally-determined frequencies had roller-roller BCs. The maximum difference for this model was
14.1% for all forcing locations with average deviations of 0.79% for forcing location I and 0.33% for forcing locations II and III. The next closest results were from the models with pin-roller and roller-pin BCs. These models had a maximum difference around 22% for all forcing locations and average deviations around 3.9% for forcing location I and 3.4% for forcing locations II and III.

Mode shapes were compared using the modal assurance criterion (MAC). The same corresponding modes used in comparing frequencies were used to compare mode shapes. Table 4-17 through Table 4-23 present the diagonal values of the MAC matrices constructed by comparing the mode shapes obtained from data corresponding to each forcing location to mode shapes from models with each BC combination.

<table>
<thead>
<tr>
<th>ABAQUS Mode Number</th>
<th>Experimental Mode Number</th>
<th>Forcing Location I</th>
<th>Forcing Location II</th>
<th>Forcing Location III</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.5922</td>
<td>0.3131</td>
<td>0.1612</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.5875</td>
<td>0.7479</td>
<td>0.7250</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.9626</td>
<td>0.9513</td>
<td>0.9566</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.9391</td>
<td>0.7946</td>
<td>0.1261</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.8115</td>
<td>0.7732</td>
<td>0.8562</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.5390</td>
<td>0.5456</td>
<td>0.1009</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.0967</td>
<td>0.2567</td>
<td>0.2691</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>0.8497</td>
<td>0.7588</td>
<td>0.6888</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>0.8982</td>
<td>0.8985</td>
<td>0.8910</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>0.9482</td>
<td>0.7887</td>
<td>0.9163</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>0.0127</td>
<td>0.0020</td>
<td>0.5366</td>
</tr>
<tr>
<td>13</td>
<td>11</td>
<td>0.6087</td>
<td>0.6041</td>
<td>0.5400</td>
</tr>
</tbody>
</table>
### Table 4-18: MAC Comparison for Roller-Pin BCs

<table>
<thead>
<tr>
<th>ABAQUS Mode Number</th>
<th>Experimental Mode Number</th>
<th>Diagonal Elements of MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Forcing Location I</td>
</tr>
<tr>
<td>1</td>
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<td>0.4261</td>
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<td>2</td>
<td>2</td>
<td>0.8312</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.9854</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.9409</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.7979</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.0016</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.5770</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>0.8107</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>0.8822</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>0.8498</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>0.0015</td>
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<tr>
<td>13</td>
<td>11</td>
<td>0.5736</td>
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</table>

### Table 4-19: MAC Comparison for Pin-Pin BCs

<table>
<thead>
<tr>
<th>ABAQUS Mode Number</th>
<th>Experimental Mode Number</th>
<th>Diagonal Elements of MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Forcing Location I</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.5051</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.7489</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.9794</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.9491</td>
</tr>
<tr>
<td>5</td>
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<td>5</td>
<td>0.2632</td>
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<tr>
<td>8</td>
<td>7</td>
<td>0.8500</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>0.8989</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>0.8908</td>
</tr>
<tr>
<td>13</td>
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<tr>
<td>12</td>
<td>11</td>
<td>0.6286</td>
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</table>
Table 4-20: MAC Comparison for Roller-Roller BCs

<table>
<thead>
<tr>
<th>ABAQUS Mode Number</th>
<th>Experimental Mode Number</th>
<th>Forcing Location I</th>
<th>Forcing Location II</th>
<th>Forcing Location III</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.5240</td>
<td>0.4225</td>
<td>0.2635</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.7383</td>
<td>0.8914</td>
<td>0.8729</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.9902</td>
<td>0.9777</td>
<td>0.9863</td>
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<td>4</td>
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<td>0.8098</td>
<td>0.1034</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>0.8804</td>
<td>0.5748</td>
<td>0.4444</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.7733</td>
<td>0.7246</td>
<td>0.8342</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.4385</td>
<td>0.6418</td>
<td>0.0364</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.1893</td>
<td>0.0676</td>
<td>0.5744</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>0.8837</td>
<td>0.8231</td>
<td>0.7765</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>0.9419</td>
<td>0.9426</td>
<td>0.9352</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>0.5545</td>
<td>0.5751</td>
<td>0.5745</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>0.9528</td>
<td>0.8808</td>
<td>0.8918</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td>0.0047</td>
<td>0.0083</td>
<td>0.6512</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>0.6889</td>
<td>0.6769</td>
<td>0.5688</td>
</tr>
</tbody>
</table>

Table 4-21: MAC Comparison for Fixed-Roller BCs

<table>
<thead>
<tr>
<th>ABAQUS Mode Number</th>
<th>Experimental Mode Number</th>
<th>Forcing Location I</th>
<th>Forcing Location II</th>
<th>Forcing Location III</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.4503</td>
<td>0.4967</td>
<td>0.2949</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.7466</td>
<td>0.8845</td>
<td>0.8744</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.9410</td>
<td>0.9375</td>
<td>0.9341</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.8291</td>
<td>0.5458</td>
<td>0.2900</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.7779</td>
<td>0.7326</td>
<td>0.8355</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>0.1906</td>
<td>0.0599</td>
<td>0.5433</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.6153</td>
<td>0.7645</td>
<td>0.0022</td>
</tr>
<tr>
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<td>7</td>
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<td>0.6573</td>
<td>0.6045</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>0.4764</td>
<td>0.4828</td>
<td>0.4426</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>0.8766</td>
<td>0.8693</td>
<td>0.8586</td>
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<tr>
<td>10</td>
<td>9</td>
<td>0.9230</td>
<td>0.7953</td>
<td>0.8777</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>0.0010</td>
<td>0.0154</td>
<td>0.6318</td>
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<tr>
<td>11</td>
<td>11</td>
<td>0.5758</td>
<td>0.5646</td>
<td>0.4928</td>
</tr>
<tr>
<td>13</td>
<td>11</td>
<td>0.3670</td>
<td>0.3750</td>
<td>0.3815</td>
</tr>
</tbody>
</table>
Because corresponding analytical and experimental modes were matched even if the results from only one forcing location provided reasonable correlation with the analytical results, some modes produced poor MAC values. For example, mode 10 from forcing locations I and II did not return a MAC value greater than 0.05 in any of the comparisons. This behavior was expected as the forcing locations were chosen to excite certain modes (section 3.2.4).
Considering this and that the system did not change throughout testing, the best results from any of the forcing locations were not always from forcing at the same location but were always representative of the behavior of the bridge. Therefore, the boundary conditions that provided the best correlation with the experimental mode shapes were determined by considering the highest MAC value from each forcing location for each mode. Table 4-24 provides the maximum, minimum, and average MAC value when only considering the highest MAC value for each mode.

<table>
<thead>
<tr>
<th>BC Combination</th>
<th>Maximum MAC Value</th>
<th>Minimum MAC Value</th>
<th>Average MAC Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pin-Roller</td>
<td>0.9626</td>
<td>0.2691</td>
<td>0.7295</td>
</tr>
<tr>
<td>Roller-Pin</td>
<td>0.9854</td>
<td>0.4477</td>
<td>0.7692</td>
</tr>
<tr>
<td>Roller-Roller</td>
<td>0.9902</td>
<td>0.5240</td>
<td>0.7824</td>
</tr>
<tr>
<td>Pin-Pin</td>
<td>0.9794</td>
<td>0.4115</td>
<td>0.7564</td>
</tr>
<tr>
<td>Fixed-Roller</td>
<td>0.9410</td>
<td>0.3815</td>
<td>0.7064</td>
</tr>
<tr>
<td>Fixed-Pin</td>
<td>0.9648</td>
<td>0.5642</td>
<td>0.7504</td>
</tr>
<tr>
<td>Fixed-Fixed</td>
<td>0.9683</td>
<td>0.4967</td>
<td>0.7514</td>
</tr>
</tbody>
</table>

Based on the results in Table 4-24, the FE models with roller-roller and roller-pin boundary conditions provide the best correlation with the experimentally-determined mode shapes. Considering the results of comparing frequencies and mode shapes, the analytical models with roller-roller and roller-pin boundary conditions provide the best representation of the dynamic characteristics of the actual bridge. Therefore, the restraint provided by the bearing pads was between an idealized pin and roller when considering the vertical modes.

### 4.3 Summary

Eleven modes were identified with frequencies ranging between 2.75 Hz and 22.5 Hz. Critical damping value estimates ranged between 1% and 4%, though some values were limited based on the spectral resolution of the data. After performing a parametric study on different modeling configurations, a finite element model with superelevation, skew, and element sizes of approximately 20 in. was determined as the best model to represent the bridge. This model was compared to the experimental results by comparing frequencies and mode shapes. Multiple
variations in boundary conditions were applied to the model in the comparison, but the boundary conditions that best represented the dynamic behavior of the actual bridge were roller-roller and roller-pin BCs.
Chapter 5. Conclusions and Recommendations

5.1 Summary

A 103 ft single-span bridge in Blacksburg, Virginia, was dynamically tested to determine its modal properties. Excitation was provided by a small impact hammer equipped with a sensor capable of measuring up to 5000 lb. Accelerometers with a measurement range of ±2.5 g peak were mounted to the top of the bridge deck along the girder lines with plumber’s putty. Accelerations were measured at 63 locations, and force was applied at three locations. Averaged frequency response functions were calculated using the responses from the accelerometers and the measurements of the input forces. Resonant frequencies were identified by locating peaks in the magnitude plots of these FRF inertances and checking that the phase passed through 180°, the real component passed through zero, and the imaginary component was at a local maximum or minimum at the same frequency. Mode shapes were constructed using the imaginary components of the FRF inertances at each identified frequency. Damping was estimated for each identified resonant frequency by applying the half-power bandwidth method to the imaginary components of the FRF mobilities.

A finite element model of the bridge was constructed using the eccentric beam modeling technique. Models were varied based on geometry, mesh refinement, and boundary conditions. Four different geometries were considered, including a model with skew and superelevation, models with either skew or superelevation, and a model without skew or superelevation. Element sizes were varied from approximately 125 in. to approximately 5 in., and boundary conditions were varied from restraining all degrees-of-freedom at each support node to restraining only the transverse and vertical translational DOFs at each support node. The model incorporating skew, superelevation, and element sizes of approximately 20 in. was used to compare against the experimental results. Frequencies were compared directly, and mode shapes were compared using the modal assurance criterion. Boundary conditions were varied on the model to qualify the behavior of the end conditions of the bridge.

5.2 Conclusions

The experimental modal analysis techniques used in this research proved to be successful in producing data suitable for extracting modal characteristics the bridge. The procedure is cost effective and simple to perform when compared to other forced vibration testing procedures.
Therefore, it should be considered in the establishment of the dynamic testing protocols for LTBPP.

Eleven modes were identified from data corresponding to each of the three forcing locations with frequencies ranging from 2.75 Hz to 22.5 Hz. Resonant frequencies were identical among the results from all forcing location except for modes 1 and 7. Damping estimations ranged from 1% of critical damping for higher modes to 5% of critical damping for lower modes, though the damping estimations for the lower modes were assumed to be high because of the limitations of the half-power bandwidth method with the spectral resolution of the data. FE models with element sizes of approximately 20 in. were found to be sufficiently refined based on resonant frequencies. The addition of superelevation had little effect on the first five resonant frequencies, and adding skew changed the same frequencies by less than 10.5%. However, the effect of changing boundary conditions was much more profound. The first frequency doubled when comparing models with roller-roller and fixed-fixed BCs, and the fifth frequency varied by up to 4 Hz. Boundary conditions had less of an effect on the mode shapes. Comparing mode shapes using the modal assurance criterion, most mode shapes from models with different BCs correlated well for the first four frequencies, with MAC values greater than 0.9. However, the fifth mode shapes did not correlate as well. The symmetry of the mode shapes from the models with pin-roller BCs was checked. The effect of adding skew or superelevation was small, but the combination of the two drastically deteriorated the symmetry of the first two mode shapes.

The FE models with roller-roller, pin-roller, and roller-pin BCs returned the closest resonant frequencies to the identified frequencies in the experimental results. Based on the highest MAC value from the mode shapes associated with all three forcing locations at each mode, the models with roller-roller and roller-pin BCs produced the best correlation with the experimental results. Therefore, the behavior of the supports at each end of the bridge is between an idealized roller and pin.

The objectives of the LTBPP include tracking changes in the behavior of representative bridges and correlating these changes with changes in static live-load deflections and the physical condition of the structure. However, dynamic testing provides advantages over the latter two monitoring methods. Sensors can be mounted to the bottom of girders or bottom of the deck, reducing the impact on traffic by only closing the bridge for short periods of time during
tests. Also, dynamic testing does not require any large pieces of equipment, such as a truck for live-load testing. Many methods of damage detection involving modal data have been explored. Resonant frequencies, mode shapes, and mode shape derivatives are commonly used with damage detection indicators. The finite element models in this research demonstrated that changes in boundary conditions have a significant effect on resonant frequencies. Sohn et al. (2004) provided examples of research involving damage detection using modal parameters. Overall, dynamic testing is a valuable method for conditional assessment of bridges.

5.3 Recommendations

The scope of this research only incorporated one bridge. To fully evaluate the test procedure, multiple bridges of varying length, number of spans, and continuity need to be tested. This should also include curved bridges and bridges with varying amounts of skew and superelevation. Thermal effects should also be noted as testing the same bridge at different times of the year could give different results, especially for indeterminate structures. Velocity transducers should be explored as a method of augmenting the sensing capabilities of accelerometers. These transducers measure velocity, which is used to construct the FRF mobility directly. Many velocity transducers are DC coupled, which is advantageous for use with larger structures that have lower resonant frequencies.

Long-term evaluation through periodic dynamic testing under the LTBPP will provide insight into the relationship between changes in modal parameters and the condition of bridges. Fluctuations in modal parameters due to changes in the structure are expected to be small, possibly less than 0.25 Hz. However, the effect of closely-spaced modes and the spectral resolution of the data in this study proved to limit the accuracy of the results. Successful implementation of a dynamic testing protocol depends on the ability to distinguish small changes in the modal parameters. Increasing the spectral resolution of the data allows more accurate estimates of the modal parameters and is achieved by recording data over longer durations. The response from an impact force damps out quickly, so only more noise is recorded if the duration of the response from an impact force is lengthened. Also, impact methods of forced vibration tend to excite nonlinear behavior. Shakers provide a controllable alternative to impact hammers. A shaker is capable of precisely controlling the range of frequencies input into the system. It can also input energy into a bridge for several seconds, allowing bridge response, instead of noise, to be recorded for a long period of time. Shakers can also be oriented horizontally. Horizontal
shakers excite transverse and longitudinal modes, which further quantifies the dynamic behavior of a bridge. Shakers should be explored as an alternative to impact excitation.

This research found that modal parameters of FE models were sensitive to the addition of skew and superelevation. However, the effects of varying skew and superelevation were not investigated and need to be further researched. This research also concluded that boundary conditions have a significant effect on natural frequencies and mode shapes. The accuracy of a detailed model with boundary conditions quantified through field measurements should be explored. A more detailed model will also return transverse and longitudinal modes, which can be compared to experimental results obtained with a horizontal shaker.
References


Appendix A. **LabVIEW Virtual Instruments**

Figure A-1: VI Front Panel

Figure A-2: Input Selection for VI Front Panel
Figure A-3: VI Block Diagram

Determine the actual sampling rate (based on NI 9234 sampling rates) and calculates the phase delay in seconds between the NI 9233 and NI 9234 modules:

Note: The NI 9234 module MUST be in the first slot of the DAQ 9172 chassis and the NI 9233 module MUST be in the second slot.

See Figure A-5

See Figure A-9

Aligns the signals based on the phase delay:

Number of samples to consider
Note: This must not be greater than the number passed through the trigger.

See Figure A-7

See Figure A-6
Figure A-4: Input Selection Code on the Block Diagram

Figure A-5: DAQ Assistant Settings
Figure A-6: Plotting Options Code on the Block Diagram
Figure A-7: Averaging and Saving Code on the Block Diagram

Figure A-8: Signal Phase Adjustment VI
Figure A-9: Phase Delay Calculator VI