Chapter 5. SCIIB Sensor Pressure and Temperature Models

SCIIB sensors can be used to measure hydrostatic pressure by measuring the cavity length change of the sensor probe resulting from the applied pressure. This chapter will concentrate on the discussion of the SCIIB sensor model in response to hydrostatic pressure signals. In addition, the temperature model of the SCIIB sensor probe will be given to assist the future work of minimizing the temperature cross sensitivity.

5.1 SCIIB Pressure Sensor Model

The geometry of the SCIIB fiber optic temperature sensor can be illustrated using Figure 5-1. When a hydrostatic pressure is applied, the capillary tube will deform, and as a consequence the cavity length will change. By monitoring the sensor cavity length changes through the previously discussed SCIIB signal processing system, the applied pressure can thus be measured. Since the sensor will be immersed into an underground oil reservoir in actual application conditions, effects in both the longitudinal and the transverse directions should be considered in modeling its pressure response.

Figure 5-1. Geometry of SCIIB pressure sensor probe
Assume that the capillary tube has an outer radius of $r_o$ and an inner radius of $r_i$. The sensor cavity length change ($\Delta L$) resultant from the applied pressure $p$ can be expressed as [90,91]

$$\Delta L = \frac{L}{E} [\sigma_r - \mu(\sigma_t - \sigma_z)],$$  \hspace{1cm} (5-1)

where $L$ is the effective sensor gauge length defined as the distance between the two thermal bonding points; $E$ is the Young's Modulus; and $\mu$ is the Poisson’s ratio of the capillary tube. For the fused silica materials used, $E=74 \text{ Gpa}$, and $\mu=0.17$.

Three stresses are considered in the analysis: $\sigma_r$ is the radial stress, $\sigma_t$ is the tangential stress, and $\sigma_z$ is the longitudinal stress generated by the applied pressure. These three stresses can be calculated by the following equations [90,91]

$$\sigma_r = \frac{r_o^2}{r_o^2 - r_i^2} \left( 1 - \frac{r_i^2}{r_o^2} \right) p,$$  \hspace{1cm} (5-2)

$$\sigma_t = \frac{r_o^2}{r_o^2 - r_i^2} \left( 1 + \frac{r_i^2}{r_o^2} \right) p,$$  \hspace{1cm} (5-3)

$$\sigma_z = \frac{r_o^2}{r_o^2 - r_i^2} p.$$  \hspace{1cm} (5-4)

Combining Equations (5-2) through (5-4), the cavity length change of the sensor caused by the applied pressure can be calculated by

$$\Delta L = \frac{L r_o^2}{E (r_o^2 - r_i^2)} (1 - 2\mu) p.$$  \hspace{1cm} (5-5)

Equation (5-5) clearly shows that the change of the sensor cavity length is proportional to the applied pressure. Therefore, the sensor can be directly used as a differential pressure sensor.
gauge. If we calibrate the sensor cavity length to a known pressure, we then can measure
the absolute applied pressure by measuring the change of the sensor cavity length.

To avoid the phase ambiguity problem, the SCIIB system requires that the sensor must
work over the semi-linear portion of a half interference fringe, as defined in Chapter 2.
Equation (5-5) indicates that we have the flexibility of choosing different sensor
geometrical parameters to design the pressure sensors with different dynamic ranges of
pressure measurements.

In order to obtain good interference signals, the two fibers inside the capillary tube must
be maintained in a good alignment. This requires that the inner diameter (ID) of the
capillary tube can only be slightly larger than the diameter of fiber cladding (125 µm).
Therefore, we choose the inner diameter of the capillary tube to be 130 µm. Here, the
generic parameters that can be used to modify the sensor performances are the
effective sensor gauge \( L \) and the outer diameter (OD) of the capillary tube.

To further study the relationship between the sensor performances and the geometric
parameters, we plot the maximum pressure range versus the sensor effective gauge length
and the outer diameter of the capillary tube as shown in Figure 5-2. In Figure 5-2 (a), it is
assumed that the single-mode fiber-based SCIIB sensor is used and the wavelength of the
source is 1310 nm. The calculation results for the multimode fiber-based SCIIB sensor is
shown is Figure 5-2 (b), where a source wavelength of 850 nm is used. As indicated in
the plots, the change of the outer diameter of the tube doesn’t affect the measurement
range much. On the other hand, the choice of the effective gauge length can allow us to
effectively design the sensor to work in different dynamic ranges. For the single-mode
fiber sensor, by choosing the sensor effective gauge length from 200 µm to 1.4 mm, the
maximum range of pressure measurement can change from 2,000 psi to 14,000 psi. If a
multimode sensor is used, the maximum range of the pressure measurement can change
from 1,000 psi to 9,000 psi as the effective gauge length changes from 200 µm to 2 mm.
Figure 5-2. Predicted maximum pressure range versus the effective gauge length

(a). Single-mode SCIIB sensor

(b). Multimode SCIIB sensor
5.2 Temperature Model of the Sensor Probe

It is a great challenge to design the pressure sensor to have minimum (ideally zero) temperature dependence when used in harsh environments such as for downhole applications. The typical temperature variation of a downhole oil reservoir is from $-40^\circ C$ to $+200^\circ C$. Considering the same sensor probe geometry shown in Figure 5-1, this large temperature variation of the environment can cause the change of the physical lengths of the capillary tube and the fibers, which results in the change of the cavity length between the two fiber endfaces. The temperature induced cavity length change can cause the sensor drift from its operation point and introduce false reading to the hydrostatic pressure measurement results. In this section, the temperature model of the SCIIB sensor probe will be discussed to provide a guideline for the optimal sensor geometric and material design for minimizing the temperature cross sensitivity of the sensor probe.

As shown in Figure 5-1, assume that the effective sensor gauge length is $L$ and the air-gap length is $s$. If the capillary tube is chosen to have a coefficient of thermal expansion (CTE) of $\alpha_h$, and the CTE of the fibers is $\alpha_f$, then the temperature introduced cavity length change $\Delta s$ can be calculated by

$$\Delta s = [L\alpha_h - (L - s)\alpha_f] \Delta T$$

$$= [\alpha_f s - (\alpha_f - \alpha_h)L] \Delta T,$$

where $\Delta T$ is the temperature change.

Theoretically, when temperature increases, the thermal expansion of the tube intends to elongate the sensor cavity length. On the other hand, the thermally induced expansions of the two fibers tend to shrink the cavity length. These two different trends help to keep the sensor cavity length unchanged with respect to the temperature change, resulting in an inherent passive temperature compensation capability of the sensor probe. However, this
is true only to a certain extent because of the difference in the CTEs of the glass tube and the optical fibers.

The tube used in fabricating the SCIIB sensor is made of fused silica glass due to the commercial availability. Usually the CTE of fiber is larger than that of tube because the fiber core is doped with germanium to increase the index of refraction. The typical CTE of the fused silica glass tube (99.9\% silica) is about $5.5 \times 10^{-7}$ \degree C. However, the CTE of the fiber may vary from the manufacturers. The typical CTE of a single-mode fiber ranges from $5.6 \times 10^{-7}$ \degree C to $6 \times 10^{-7}$ \degree C, and that of a multimode fiber (62.5/125) is about $7.5 \times 10^{-7}$ \degree C.

The design of a temperature insensitive pressure sensor probe basically requires that the thermally induced sensor cavity length change becomes zero. Therefore, if we set $\Delta s$ in Equation (5-6) to be zero, we will have

$$\alpha_f s - (\alpha_f - \alpha_h) L = 0$$

$$\Rightarrow \frac{s}{L} = \frac{\alpha_f - \alpha_h}{\alpha_f}.$$

From the sensor performance point of view, the choice of the initial sensor cavity length depends on the balance between the coherence lengths of two SCIIB channels. In other words, the optimal initial sensor cavity length should be chosen to allow the signal channel to generate an interference signal with a large fringe visibility and at the same time to let the reference channel output a non-interfered signal. Experimental results indicates that the optimal initial sensor cavity length should be chosen at between 25\(\mu\)m to 40\(\mu\)m for single-mode fiber sensors, where the signal channel has a visibility around 50\% to 75\% while the interference in reference channel has a visibility less than 1\%. As for multimode fiber sensors, the initial sensor cavity length should be set between 5\(\mu\)m to 10\(\mu\)m, where the signal channel has a visibility of 30\% to 45\% and the reference channel has an interference fringe visibility less than 1\%.
With these typical values, Equation 5-7 can thus be used to estimate the optimal value of the sensor gauge length (L) with which the zero temperature cross sensitivity can be achieved. The results are shown in Table 5-1.

From the sensor fabrication point of view, the minimum sensor gauge length that can be fabricated right now is 500µm for single-mode sensor and 600µm for multimode sensor, which, according to Figure 5-2, will allow a dynamic range of pressure measurement 5000psi for single-mode fiber sensors and 3000psi for multimode fiber sensors. Therefore, compared to the results shown in Table 5-1, it is very difficult to fabricate a multimode fiber sensor with zero temperature cross sensitivity. However, the single-mode fiber sensors show a very good possibility to achieve minimal temperature dependence.

<table>
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<th>Multimode fiber sensor</th>
<th>Single-mode fiber sensor</th>
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<tr>
<td>Typical CTE of fiber (×10⁻⁷/°C)</td>
<td>7.5</td>
<td>5.75 ~ 6.0</td>
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<tr>
<td>CTE of fused silica tube (×10⁻⁷/°C)</td>
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<td>5.5</td>
</tr>
<tr>
<td>Optimal initial cavity length (µm)</td>
<td>5 ~ 10</td>
<td>25 ~ 40</td>
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<tr>
<td>Optimal gauge length (µm)</td>
<td>13.75 ~ 27.5</td>
<td>275 ~ 880</td>
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