Preliminary Specifications for an Exoskeleton for the Training of Balance in Balance Impaired Individuals

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There is a small but growing population of people who suffer from impaired balance. The causes range from old age to stroke to cerebral palsy. For those with only minor problems staying upright, a cane or walker is all that is needed. For some it is so debilitating that they are confined to a wheel chair. The precise cause of impairment can vary. In some, the vestibular, proprioceptive or visual impairments affect balance. In others, muscle weakness or brain damage is the cause. In another group, the brain never learned to balance in the first place. Relearning how to balance can be a struggle requiring months of costly physical therapy with a physical therapist. A machine that could help teach them how to balance would be a great help in the improvement of their lives.

This thesis presents a set of control models for an exoskeleton that will stabilize and restore stability to those with impaired balance. The control models are designed for an exoskeleton to initially force the wearer into a known profile for balancing and moving. There will then be a steady reduction in the authority of the exoskeleton over time, requiring the patient to assert more control over his or her own movement. As the authority of the exoskeleton is reduced, the patient will have to increase his or her own authority and develop his or her own control law or the patient will become less stable and eventually unbalanced.

We expect this treatment method will increase the stability of patients, allowing them to steadily adapt to standing and walking. This will then allow them mobility without the use of a wheel chair and decrease their risk of falling. Further, the use of this device will enable the patients to receive therapy at home and in their normal life without the need to visit a physical therapist for rehabilitation, enabling the patients to receive therapy at home and for a longer period of time than they currently do.
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Chapter 1

Introduction

There is a small but growing population of people who suffer from impaired balance. The causes range from old age to stroke to cerebral palsy. For those with only minor problems staying upright, a cane or walker is all that is needed. For some it is so debilitating that they are confined to a wheelchair. The precise cause of impairment can vary. In some, the vestibular, proprioceptive or visual impairments affect balance. In others, muscle weakness or brain damage is the cause. In another group, the brain never learned to balance in the first place. Relearning how to balance can be a struggle requiring months of costly physical therapy with a physical therapist. A machine that could help teach them how to balance would be a great help in the improvement of their lives.

This thesis presents a set of control models for an exoskeleton that will stabilize and restore stability to those with impaired balance. The control models are designed for an exoskeleton to initially force the wearer into a known profile for balancing and moving. There will then be a steady reduction in the authority of the exoskeleton over time, requiring the patient to assert more control over his or her own movement. As the authority of the exoskeleton is reduced, the patient will have to increase his or her own authority and develop his or her own control law or the patient will become less stable and eventually unbalanced.

We expect this treatment method will increase the stability of patients, allowing them to steadily adapt to standing and walking. This will then allow them mobility without the use of a wheelchair and decrease their risk of falling. Further, the use of this device will enable the patients to receive therapy at home and in their normal life without the need to visit a physical therapist for rehabilitation, enabling the patients to receive therapy at home and for a longer period of time than they currently do.

Chapter 2 gives an overview of the problem of impaired balance. The causes and areas of the brain that are affected are reviewed. Current therapies for coping with and improving balance are presented.

Chapter 3 reviews human models and control system identification. Different models for each direction of balance are presented.
Chapter 4 presents a human model along with its development and control system that will stabilize and train patients. Model and control system development are reviewed.

Chapter 5 presents system simulations of the models along with their use in therapy. The requirements for the actuators are reviewed.

Chapter 6 presents further research that needs to be done as well as lessons learned from other studies. Future model development and control system refinement are reviewed.

1.1 Exoskeleton Uses

Partial body exoskeletons have been in use for decades in rehabilitation. Knee and Leg braces are two such widely used examples. The knee brace has primarily been used for the relief of stress on the knee and to prevent undue stress due to impact in sports. The full leg brace has generally been used to stiffen the knee and allow individuals with weak muscles, such as muscular dystrophy, to remain standing by fixing the joints in the leg in place.

More recently, powered exoskeletons have been developed with the goal of power assistance or power amplification in mind. Exoskeletons targeted for power amplification have been developed with the goal of amplifying the strength of a normal human to increase the load they are able to carry and aid in manual labor. The Berkley Lower Extremity Exoskeleton (BLEEX) (see Fig# 1.1) and Hybrid Assistive Limb (HAL) (see Fig#1.2) are two notable exoskeletons currently under development. BLEEX is being designed for DARPA\(^1\) with the goal of offloading the weight carried by soldiers onto an exoskeleton. It is a pseudo-anthropomorphic\(^2\) exoskeleton that relies on the wearer to stabilize the system. It provides power assist through hydraulic actuation of the ankles, knees and hips in the sagittal\(^2\) plane to allow the wearer to carry a load on a backpack.

Exoskeletons developed for power assistance are targeted at patients with neurological and muscular degenerative diseases, such as muscular dystrophy, that lead to a weakness or lack of control in the muscles but otherwise leave the body’s control system untouched. Similar to BLEEX, the HAL-5 exoskeleton is also capable of allowing the user to lift and move material with reduced effort. However, unlike BLEEX, it is a full body anthropomorphic exoskeleton with actuation at the knees, hips, shoulders and elbows. It is also designed to assist users who’s own muscles are not capable of supplying the power required, not simply to carry a backpack. It utilizes electromyography\(^3\) (EMG) readings and a neuro-fuzzy control system derived from experience with the wearer to develop a gait and motion profile. As such, it requires an initial configuration session and cannot be worn immediately as BLEEX is designed to be. EXPOS\(^4\) is another exoskeleton designed to assist those with weak muscles. However, EXPOS is lower extremity only. It has 10 Degrees of Freedom (DOF) and is designed to assist the wearer and reduce the amount of effort needed to

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\(^1\)Defense Advanced Research Projects Research Agency
\(^2\)Forward/Backward
\(^3\)Reading of the electrical signals in muscles
walk and stand. The knees and hip joint are actuated in order to provide assistance to the patient. However, the entire system for EXPOS is heavy and bulky. The system uses a self-propelled cart to carry the actuators, computer and batteries required for operation.

A third set of exoskeletons have been developed with the goal of rehabilitating patients who have suffered a stroke or suffer from cerebral palsy and other afflictions. These exoskeletons are generally designed to aid or control the arms in movement and in physical therapy. A powered arm exoskeleton was developed by Rahman et al.[69] for the aid of motion and strength of the arms. The robotic limb is attached to a wheelchair and allows for a large portion of the natural movement of the arm to be used. There are two control systems that are utilized at different times by the exoskeleton. In passive mode, the exoskeleton uses a PD controller with a sensor at the wrist to move with the arm. In active mode, the system reads the EMG signals in the arm and a PD controller to actively assist in the movement of the arm. Similar upper limb systems have been used for rehabilitation of stroke patients[55] and spastic patients[17] as we will see in Chapter 2.

A simple robotic rehabilitation device for the ankle was developed by Sun et al.[82]. The robot consists of three torque motors that rotate the ankle in a series of movements. It was developed to replace a physical therapist in the movement of the foot about the ankle joint. However, the robot is only designed to provide repeat movements of the ankle and is not useful for teaching new movements.

The exoskeletons so far developed have been primarily for power amplification or power assist. A
few have been developed for rehabilitation in one manner or another. It is this last group that we will be examining more in Chapter 2.
Chapter 2

Problem Overview

Impaired balance is defined as the reduced ability or inability to maintain the Center of Mass (COM) over the Center of Pressure (COP). The inability to maintain balance has a variety of causes associated with it ranging from cerebral palsy to old age. The specific areas affected by each of the afflictions are the vestibular system\(^1\) (sensor), visual system (sensor), proprioceptive system\(^2\) (sensor), the brain (control system), the nervous system (wires), the muscles (actuators) or a combination thereof. The concentration of this thesis is on afflictions that impact the sensors and control area of the brain.

Those afflicted with mild impaired balance can generally remain standing with an increased likelihood of falling. Such impairment is usually developed later in life due to a degradation of the bodies muscles, vestibular, proprioceptive or visual system.[46] Vestibular rehabilitation (VR) for minor impairment of the vestibular system typically consists of repetitive movements such as walking [59] and stepping [81] and standing on tilting boards that accentuate slight differences in the center of mass. The purpose of VR is to have the brain develop alternative means of controlling balance. Typically, when the vestibular system has been compromised an increased reliance on the visual system is developed. [59]

Those afflicted with severely impaired balance are incapable of standing, or of standing for more than a short period of time. The causes of such impairment vary widely. Patients who have suffered a stroke or suffer from cerebral palsy are two instances of groups that are likely to be impacted by moderate to severely impaired balance cause by the brain. When these two impact balance, the neurons in the brain are directly damaged.[45] In order to repair the damage; new learning must take place in the brain. Due to the similarity, stroke rehabilitation protocols were also examined to see if they might provide insight into possible rehabilitation exercises.

\(^1\)The vestibular system is made up of two parts in the 'inner ear', the otoliths and the semicircular Canals. The otoliths act as three-dimensional accelerometers and the semicircular canals act as gyroscopes, giving feedback to the brain on orientation and acceleration.

\(^2\)The proprioceptive system is part of the somatosensory system and gives positional feedback to the brain.
2.1  Cerebral Palsy

There are three types of cerebral palsy, spastic, ataxic and athetoid. Spastic is generally caused by lesion in the motor area of the cortex and accounts for 80% of cerebral palsy cases. In spasticity, the muscles will elongate as normal and then involuntarily contract past a certain point. The muscles on each side of a joint attempt to remain contracted simultaneously, causing the two to compete for control over the limb. This is analogous to a game of tug of war between the muscles. This causes the patients to have stiff, jerky movements.

Ataxia is generally caused by cerebral lesions and accounts for 5-10% of cerebral palsy cases. Ataxia patients suffer from a lack of coordination in movement and from the lack of contraction in the muscles. As opposed to spasticity, the muscles are not contracting enough. The is analogous to a dead zone in the control of the limbs. This can cause tremors and a general lack of coordination during movement.

Athetoid is generally caused by damage to the extrapyramidal system and accounts for 10% of cerebral palsy cases. Athetoid patients suffer from involuntary jerky irregular movements. This is analogous to random noise being inserted into the control signals for the patient’s muscles. Because of this, athetoid patients have trouble balancing and maintaining posture.

Cerebral palsy can then be viewed as a combination of the lack of gross or fine motor control depending on the level of affliction in the patient. Fine motor control is used for small motions. In standing it is used for precise control of the muscles to maintain the body in the upright position. Gross motor control is used for large motions and is of general use while walking. Athetoid cerebral palsy patients exhibit a reduced amount of fine motor control where the patient must concentrate harder to achieve small, controlled movements. Ataxic patients exhibit similar problems with tremors in their motions.

Various therapies to assist cerebral palsy patients have been developed over the years and the specific therapy used depends on which type of Cerebral Palsy afflicts the patient. Orthotics such as knee and body braces have been developed that restrict and stiffen limb movement. Other therapies use repetitive motions to exercise a single muscle group at a time, to promote development in the brain of new neural pathways to control the muscles. The motions are can be administered by a physical therapist or parent while the patient learns the motions and attempts to move themselves. The author is also aware of one therapy underway, in which a doctor assists the patient in walking by providing physical support at the ankles and moving the patient’s feet.

2.2  Robots in Therapy

Prior to the introduction of machine assistance, physical therapists have had to manually move the patient’s limbs. This leads to two problems, repetition and strain. The physical therapist is not able to precisely duplicate the movements each time. The robots are capable of doing so, leading to a
superior rehabilitation for the patient. Additionally, the therapists experience strain from having to move the patient’s limbs. The robots negate this problem, as robots do not tire. The robots also have the additional benefit of needing only minimal supervision. The therapist is needed only for deciding the routine of the patient and for setting up the machine at the beginning of each session. This also leads to an increased number of patients each therapist may supervise.

Several groups have used moving platforms to train patients in improved balance. Two methods are used by these platforms to train the patients. In the first method, the patient stands on the platforms and an impulse is applied either at the patient’s hips or to the platform itself. Safety equipment is used to prevent the patient from falling and injuring themselves. The impulses are of random magnitude and direction. The patient is not told what direction they will come from ahead of time. In the case of cerebral palsy patients, decreased response time and sway leading to improved balance was observed after one week of training.

The second method utilizes a similar platform, where the patients are supported by the platform at the hip. Support of the patient by the platform is slowly decreased. Matjacic et al. use a platform with hip support combined with impulses for training. It has been shown that this method increases the stability of patients and the limits to which they are able to stabilize themselves. It is believed that the ability to slowly increase the range of stability without danger allows the patients to experiment with different methods of stabilizing themselves. However, all of these studies have used patients with mild impaired balance. Patients with severely impaired balance, who are incapable of standing, have not been tested. For insight into what therapies can be used for severely impaired patients, we turn to upper limb therapies that have been developed for patients who have suffered a stroke or other form of brain trauma.

Masiero et al. have successfully used repetitive motions controlled by a robot to improve motor movement in the arm in stroke patients. Initially, the patients were instructed to passively allow their arm to be moved by the robot. After a few repetitions at the beginning of each training session, the subjects were instructed to attempt to move their arm on their own while the robot assisted the motions. The control system utilized by the robot was basic and did not adapt to the patient. Each motion, whether when the patient was passive or actively trying to move their arm, was performed at the same velocity; no feedback from the performance of the patient was used to adjust the robot’s performance over the course of the experiment. Realized benefits from the robot were a decrease in the attention required from a physical therapist, as the therapist was only needed at the beginning of the training to develop the motor movements and at the beginning of each session for the set-up of the device. Other trials have also used robots to perform repetitive movements for a physical therapist.

Hogan et al. developed another robot to aid arm movement but with one crucial difference, the control system was varied for the patient over time in a manner similar to sliding mode and adaptive control. The patients were instructed to reach for a specific target in an array. A ‘slot’ in the range of movement of the limb was constructed that the patients were expected to move through. Deviation from the slot was met with resistance that pushed the arm back into the slot. As the patients moved through the slot, or if the patients failed to move quickly enough, the slot started
to collapse from the back toward the target assisting the patient by carrying the patient's arm in the desired movement. As the patient’s speed improved, the time until the robot started assisting was decreased. As the patient’s accuracy improved, the resistance that kept their arms in the slot was decreased. This therapy demonstrated the best performance for training post-stroke patients of the various protocols used. The group calls this 'performance-based progressive-therapy' (PBPT). The patients undergoing the PBPT protocol were able to demonstrate the same advancement in movement compared to other protocols with one third fewer repetitions. The active involvement and varying benefit of the robot is believed to have aided in the improved recovery of the patients.

Kahn et al.[36] have evaluated various robotic arm training methods. While no statistically significant differences were found between the control groups and those who used the robots in training, this could simply be due to the small sample sizes used. However, this also leaves the possibility of the robotic training methods providing an equivalent experience to one that is done completely by a therapist. Kahn et al. also present some control methods that have not been used by others for the training and rehabilitation of patients, such as forcefully stopping motion when the patient sufficiently deviates from a predefined path. The control systems presented in Chapter 4 are derived from the methods presented by Hogan[30] and Matjacic[58].
Chapter 3

Human Models and Control

There are several difficulties in determining the basic control system for the human body. The human body has a limited number of degrees of freedom, 12 in the lower body. However, there are a much larger number of sensors and actuators in comparison to the number of degrees of freedom that the control system utilizes. Each joint is over actuated with different muscles, each with their own dynamics and responses. Additionally, the three groups of sensor inputs provide several different and sometimes redundant sets of data on the current state of the human body, each with it’s own dynamics and delays. The central nervous system (CNS) must be able to integrate each of the different inputs together and send the control signals to the muscles to actuate the body accordingly.

Kuo[48] developed an optimal control model for the human body to see if it can provide information on how the CNS integrates the information from the three sensor groups and how it controls standing. The model uses simplified sensor inputs and control signal to reduce complexity, muscles are modeled in groups rather than individually and the sensory information from the visual, proprioceptive and vestibular is simplified to report the body’s parameters instead of raw data. The model was calibrated using experimental data from various trials that included or eliminated combinations of the three sensor systems. Analysis of the model when compared to experimental data showed that the model accurately predicted sway when faced with degraded or eliminated sensor information. Kuo believes this demonstrates that the CNS utilizes an internal dynamic model of the body and state estimation to compensate for degraded sensor inputs and to compensate for disturbances and changes in the system. This interpretation is supported by several other studies.[47][7][13][63][48] However, most models do not start with the assumption of an internal model. Instead, the CNS is modeled as a controller with delayed linear feedback. As will be shown below, this assumption is adequate for quiet standing.

Normal quiet standing is the state a human is in when standing with both legs on the ground and no large disturbances are present. Quiet standing is not motionless. The body moves in distinct low frequency motions. The three sensors (proprioceptive, vestibular system, visual) each have a different effect on steadying and reducing sway while standing. Additionally, sway and responses
to disturbances in the sagittal\(^1\) and coronal\(^2\) plane exhibit distinct differences and as such need to be modeled differently. Presented here are three different sets of models for standing. For quiet standing in the sagittal plane, a single link inverted pendulum will adequately model standing. For larger disturbances in the sagittal plane, multi link inverted pendulum models are presented. For sway in the coronal plane, a two link inverted pendulum is given.

### 3.1 Single Inverted Pendulum Model, Sagittal Plane:

![Figure 3.1: Single Link model of human standing, Sagittal Plane](image)

In quiet standing (standing with no disturbances) the knees and upper torso are kept stiff. Quiet standing can then be modeled as a single inverted pendulum with a single DOF (Degree of Freedom) at the ankle joint allowing motion in the sagittal plane. The ankle is then the sole actuator used for stabilizing the body in an upright position. This model has been extensively studied to determine how the brain maintains balance along with the contribution from each of the three sensor groups (Vestibular, Proprioceptive and Visual). The basic control system for the inverted pendulum model involves a PID controller with time delay feedback.

In quiet standing, humans do not stand perfectly vertical; the sway in the sagittal plane is about a point 1-2 degrees forward\[^47\]. There is also a noticeable sway in the absence of external perturbation. It is due to this sway that most models of human standing omit the integral portion of a PID controller and only use PD feedback as an integrative component in the feedback would generally reduce and eventually eliminate the sway.\[^54\]

Fukuaka et al.\[^18\] have broken down quiet standing by each of the three systems through various experiments. In each experiment, two of the systems were suppressed to allow only one system as feedback. Subjects were required to maintain balance with the limited senses. It was

\[^1\]Forward/Backward
\[^2\]Left/Right
found that each of the three systems (visual, vestibular, proprioceptive) was separately capable of stabilizing the individual. The somatosensory portion of the proprioceptive system and the vestibular system showed significant derivative gains and leading phase that has been noted in various experiments.[54][52] The visual system, perhaps due to the significant lag times, displayed derivative gains but not leading phase. In the various tests, the somatosensory system had the least variation among the various test subjects, leading to the theory that it is the primary system for balance.

The leading phase from the somatosensory and vestibular system has also been cited as a reason for believing the CNS uses internal models for stability. Masani et al.[54] examined this assumption and found that large proportional and derivative gains were able to stabilize the human body during quiet standing up to 185ms with proportional and derivative Gains of 750Nm/rad and 350Nms/rad respectively. These gains also produced the leading phase observed in experiments. Kooji et al.[43] have estimated the proportional gains at 742-835Nm/rad and the derivative gains at 122-194Nms/rad with delays of 55-66ms in experiments with healthy subjects. The gains that Masani et al. found are thus in the realm of plausible gains that may be generated by healthy humans.

### 3.2 Multi-link models, Sagittal Plane:

![Multi Link model of human standing, Sagittal Plane](image.png)

When the body cannot compensate for a disturbance with just an ankle response, a hip response is used. In a hip response, the hips are brought out of line with the ankles to maintain the COM over the COP. If this fails, then a step response is used. The exact form of a hip response depends on the direction and magnitude of the disturbance. Since the hip moves independently of the ankle, the standard single link model may no longer be used. Instead two or more link inverted pendulum models are used.

Multi-link models in the sagittal plane become difficult to analyze and derive due to the high non-linearity and the increased number of states of the system. Kuo[48] uses a double link inverted pendulum, using only the ankle and hip joints. The system is modeled with lag and stabilized with
a linear quadratic gaussian (LQG) controller. A state estimator is used in combination with a LQG in order to compensate for the delay and stabilize the system. Experiments show that the model closely simulates experimental data.

Xiang and He[83] use a more complicated model for the simulation of disturbed sagittal standing. A variable three or four link model with time delay is used. When the heels are on the ground, a three-link shank, thigh, trunk model is used. When ankle torques would force the heel off the floor, a four-link foot, shank, thigh, trunk model is used. Passive joint torques and muscle dynamics are used to increase the accuracy of the model. Xiang and He use a Smith Predictor to compensate for 100ms delay and a LQR to stabilize the linearized system. Experiments were used to calibrate the model. The LQR and Smith Predictor produce a similar response to the experimental data. However, in most cases the modeled LQR returned the COM to the equilibrium point faster than the CNS was able to in experiments.

### 3.3 Double Inverted Pendulum Model, Coronal Plane:

Unlike the sagittal plane, the body does not remain stiff at the hips when movement occurs. Instead, the hips move counter to the sway of the ankles, allowing the torso to remain upright in the coronal plane.[27] Thus, while the system is modeled as a double inverted pendulum, the control system may still be based around a SISO (Single Input Single Output) controller and may be derived in similar methods to the single inverted pendulum model in the sagittal plane.

Kimura and Jiang[27] use a PID model of a linear controller to approximate coronal plane control in quiet standing. Several interesting facts were discovered in experiments for deriving the coronal postural control law. Of note, there is a significant increase in sway frequency from 0.02hz to 0.12hz when the eyes were closed. Additionally, a correlation was found between age and balance.

---

3It has been found that the CNS performs an optimization problem of the muscles and movement in order to stabilize the human body. As such, a linear quadratic regulator (LQR) has been found to closely model the CNS response with appropriate calibration.[48][49][83]
where the elderly have a lower derivative gain in proportion to the product of their mass and height compared to younger adults. Proportional and integrative gains were found to be less affected by aging. This leads to the conclusion that the loss of the derivative gain as one ages leads to an increased risk of falling and impaired balance. Proportional and derivative were found to be in the range of 519Nm/rad and 72.3Nms/rad respectively.

Johansson et al.[33] have performed experiments where the eyes were closed and the use of the vestibular system was denied to healthy patients by the introduction of low current high frequency electrical noise into the inner ear. Coronal sway was significantly increased and there was a change in the sagittal sway. The noise was applied for three minutes in each trial. Of note is that the patients exhibited less sway towards the end of each trial than the beginning. The patients were adapting control strategies that relied less on the vestibular system and more on the proprioceptive system with adaptation regularly taking 40-50s.[35] However, while they were able to partially compensate for the loss of the vestibular system, sway magnitude was still larger towards the end of the trial than prior to the introduction of the noise.
Chapter 4

Training Stability

4.1 Model Development

For full control of the lower body, a 12 link model would be necessary (see Fig#4.1). The 12-link model is composed of joint approximations of the ankle, knee and hip joints. The ankle joint primarily rotates in the sagittal plane and coronal plane. While the ankle does allow some movement in the transverse plane, this movement is negligible and may be ignored. The ankle is then modeled as a two degree of freedom (DOF) joint. The first and second joints, L1 and L2, correspond to the rotation of the ankle in the coronal and sagittal plane respectively. The knee allows movement primarily in the sagittal plane. Little movement is allowed in either the coronal or transverse plane and damage may follow if the knee rotates too much in either of these planes. The knee is then modeled as a single DOF joint (L3) with rotation in the sagittal plane. The hip in the human body is a ball joint allowing for 3 DOF. The hip is capable of rotating in all three planes. The order of rotations used in the model is rotation in the transverse plane (L4), coronal plane (L5) and then the sagittal plane (L6). This will allow for the model to adequately simulate hip movement in most circumstances.

One limitation with the model is the joints are capable of moving in anatomically impossible ways. For example, the model allows the knees to bend forward. The inclusion of anatomical limits, such as the knee, would require one of two remedies that are difficult to include. The first would be a change in the model while the knee has sufficient torque to bend forward past the anatomical limits while using the standard model at all other times. The second would be to apply a compensatory torque that would prevent forward motion of the knees once the knees are sufficiently straight. The inclusion of either of these in the simulation is difficult and as such is not included.

1 The 12-link model is derived from the Dynamic Anthropomorphic Robot With Intelligence (DARWIN)
2 Anterior to Posterior
3 Lateral
4 Horizontal plane, parallel to the ground
Figure 4.1: 12-Link Lower Body Model
Cylinders represent degrees of freedom at each joint.
Rotation is about the axis of the cylinder.
Spheres represent the center of mass for each joint.
See Table #4.1
Table 4.1: Table of Links for 12-Link Lower Body Model, Figure #4.1

L1  Left Ankle Joint, Coronal Plane
L2  Left Ankle Joint, Sagital Plane
L3  Left Knee Joint
L4  Left Hip Joint, Transverse Plane
L5  Left Hip Joint, Coronal Plane
L6  Left Hip Joint, Sagital
L7  Right Hip Joint, Sagital
L8  Right Hip Joint, Coronal Plane
L9  Right Hip Joint, Transverse Plane
L10 Right Knee Joint
L11 Right Ankle Joint, Sagital Plane
L12 Right Ankle Joint, Coronal Plane
M2  Mass of Shank
M3  Mass of Thigh
M4  Mass of Upper Body
M11 Mass of Foot

For quiet standing, this 12-link model would involve redundant links in the system. In quiet standing, both feet remain on the ground and the legs move parallel to one another. To simplify, a model based on the inverted pendulum is used instead. The joints on the right leg (L12-L7) are combined with the corresponding left leg joints (L1-L6), as the angle in each joint should always be the same in the corresponding joint. Further, joints L4 and L9 are eliminated. Joints L4 and L9 are not capable of rotation while both feet remain fixed on the ground. This simplification reduces the number of links in the model from 12 to 5 (see Fig#4.2). The mass and inertia of each link are recalculated for the new model. To further simplify the model, the centers of mass are placed inline with the links and are not offset. The model is fully actuated and observable. The inverted pendulum has been shown to be a viable model for modeling human balance and as such is used here.

4.2 General Model:

The dynamics of the human body are expressed as follows (See Table #4.3)

\[ A(\theta)\ddot{\theta} + B(\theta, \dot{\theta})\dot{\theta}\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta}^2 - G(\theta) + F(\theta) = T_a + T_p + T_H \]

Solving for \( \ddot{\theta} \)

\[ \ddot{\theta} = A^{-1}(\theta) \left[ T_a + T_p + T_H - B(\theta, \dot{\theta})\dot{\theta}\ddot{\theta} - C(\theta, \dot{\theta})\dot{\theta}^2 + G(\theta) - F(\theta) \right] \]
Figure 4.2: Simplified Lower Body Model
Cylinders represent degrees of freedom at each joint.
Rotation is about the axis of the cylinder.
Spheres represent the center of mass for each joint.
See Table #4.2

Table 4.2: Table of Links for 5-Link Lower Body Model

| L1   | Combined Ankle Joint, Coronal Plane |
| L2   | Combined Ankle Joint, Sagittal Plane |
| L3   | Combined Knee Joint                 |
| L5   | Combined Hip Joint, Coronal Plane   |
| L6   | Combined Hip Joint, Sagittal        |
| M2   | Combined Mass of Shank              |
| M3   | Combined Mass of Thigh              |
| M4   | Mass of Upper Body                  |
When in quiet standing, the lean possible while maintaining balance is limited from approximately 12 degrees anterior[79] to 5 degrees posterior.[18] Lean in the coronal plane is limited by the width of the stance, but is of similar limitation. This small range is nearly linear and allows us to substitute a linear model for the non-linear model for the A and G matrices in controller design.

Torques created due to the movement of the limbs are called passive torques. These torques are caused by the movement of the ligaments, fluids, muscles that are not applying torque and other portions of the body. The passive torques created act as a friction. The torques are non-linear and vary from patient to patient. Ignoring the passive torques in the system will decrease the stability. If the system can be stabilized with their absence, it can be stabilized with their presence.[54] Thus, we may safely ignore the passive torques in the model. This further simplifies the model. Additionally, the torques applied by the patient $T_H$ are ignored to derive the controller for the exoskeleton.

Experiments show that the maximum angular velocity in quiet standing is below 0.15 rad/s.[78][27] This limitation reduces the maximum effect of the coriolis and centrifugal effects in the system. The coriolis and centrifugal parameters are ignored in controller development due to the low velocities and the small region of operation. The centrifugal and coriolis effects will instead be treated as disturbances in the system during operation. The system dynamics then become

$$\ddot{\theta} = A^{-1}(\theta) [T_a + G(\theta) - F(\theta)]$$

Using measurements, we determine the nominal matrices for $A(\theta)$ and $G(\theta)$ as $A_o(\theta)$ and $G_o(\theta)$ respectively.

$$\ddot{\theta} = A_o^{-1}(\theta) [T_a + G_o(\theta) - F(\theta)]$$

Substituting in the control law $\left(K \begin{bmatrix} r - \theta \\ \dot{r} - \dot{\theta} \end{bmatrix} - G_o(\theta)\right)$ for $T_a$ where $\begin{bmatrix} r \\ \dot{r} \end{bmatrix}$ is the desired state and...
\[ \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \] is the current state of the system

\[ \ddot{\theta} = A_o^{-1}(\theta) \left[ K \begin{bmatrix} r - \theta \\ \dot{r} - \dot{\theta} \end{bmatrix} - G_o(\theta) + G(\theta) - F(\theta) \right] \]

For an arbitrary controller K that stabilizes the system and the reference configuration \[ \begin{bmatrix} r \\ \dot{r} \end{bmatrix} \] existing in the operating range of the model, the non-linear system becomes

\[ \ddot{\theta} = A_o^{-1}(\theta) \left[ K \begin{bmatrix} r - \theta \\ \dot{r} - \dot{\theta} \end{bmatrix} - G_o(\theta) + G(\theta) + T_p - B(\theta, \dot{\theta})\dot{\theta} - C(\theta, \dot{\theta})\dot{\theta}^2 - F(\theta) \right] \]

Provided that

\[ \frac{G_o(\theta) - G(\theta)}{T_p - B(\theta, \dot{\theta})\dot{\theta} - C(\theta, \dot{\theta})\dot{\theta}^2} \]

are small, then the non-linear system is also stable.

### 4.3 Controller

#### 4.3.1 Controller Design

The mathematical model for quiet standing is a 10 variable system with non-linear elements. An LQR has been shown capable of stabilizing the system similar to the central nervous system (CNS). The LQR controller requires three parameters, the input cost matrix, the state cost weighting matrix and a linear system model. The linear system model was designed using the linearized inertia and gravity matrices, \( A_L \) and \( G_L \) respectively. The inertia and gravity matrices were linearized about the point \( \theta = 0 \), corresponding to vertical standing. The linearized system is described by

\[ \begin{align*}
\dot{\theta} &= \theta \\
\dot{\theta} &= A_L^{-1}G_L\theta + A_L^{-1}T_a
\end{align*} \]

The LQR parameters used are listed in table 4.4. The state and input cost weighting matrices were chosen based on experiments by Tian and He[83]. The resulting feedback matrix is of size 5x10 and may be separated into proportional and derivative portions \([K_p \ K_d]\) resulting in the closed loop state equations of

\[ \ddot{\theta} = A^{-1}(\theta) \left[ [K_p \ K_d] \begin{bmatrix} r - \theta \\ \dot{r} - \dot{\theta} \end{bmatrix} + G(\theta) + T_p - B(\theta, \dot{\theta})\dot{\theta} - C(\theta, \dot{\theta})\dot{\theta}^2 - F(\theta) \right] \]

where \( K_p \) and \( K_d \) are the proportional and derivative feedback gains of the LQR feedback matrix respectively.
Table 4.4: Table of LQR Parameters.

<table>
<thead>
<tr>
<th>State cost weighting matrix</th>
<th>\text{diag}([1 1 1 1 0 0 0 0 0])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input cost weighting matrix</td>
<td>\text{diag}([5 5 0.35 5.3 5.3])</td>
</tr>
</tbody>
</table>

\[
\text{System Matrix} = \begin{bmatrix} 0 & I \\ A^{-1}G_L & 0 \end{bmatrix} 
\]

\[
\text{Input Matrix} = \begin{bmatrix} 0 \\ A^{-1} \end{bmatrix} 
\]

This control law will stabilize the system without assistance. However, it does not allow for a different control law to be used with regards to gravity. The LQR derived feedback gains are then separated into two groups, gravity compensation and reference feedback using the linearized gravity matrix.

\[
\ddot{\theta} = A^{-1}(\theta) \left[ (K_p - G_L) K_d \right] \begin{bmatrix} \dot{r} - \dot{\theta} \\ r - \dot{\theta} \end{bmatrix} - G_o(\theta) + G(\theta) + T_p - B(\theta, \dot{\theta}) \dot{\theta} \dot{\theta} - C(\theta, \dot{\theta}) \dot{\theta}^2 - F(\theta)
\]

This design allows the gravity matrix \( G_o(\theta) \) to be independently adapted based on what is found to be the most appropriate for the patient. For example, patients who are capable of standing autonomously, but only need training, do not need the assistance of the exoskeleton to remain upright.

4.3.2 Controller Adaptation

The control system for the exoskeleton has three requirements, it must be robust enough to handle uncertainties in the patient’s body, it must initially stabilize the patient and it must force the patient to learn to stabilize themself.

This leads to the requirement for an adaptable controller. Examining the equations of motion, and including the torque applied by the patient \( T_H \), two sets of terms are apparent

\[
\ddot{\theta} = A^{-1}(\theta) \left[ (K_p - G_L) K_d \right] \begin{bmatrix} \dot{r} - \dot{\theta} \\ r - \dot{\theta} \end{bmatrix} + G(\theta) - G_o(\theta) + T_p - B(\theta, \dot{\theta}) \dot{\theta} \dot{\theta} - C(\theta, \dot{\theta}) \dot{\theta}^2 - F(\theta) + T_H
\]

\( [(K_p - G_L) K_d] \) and \( G_o(\theta) \) are the obvious matrices to adapt. Including the adaptive gains, the control system becomes

\[
\ddot{\theta} = A^{-1}(\theta) \left[ \alpha [(K_p - G_L) K_d] \right] \begin{bmatrix} \dot{r} - \dot{\theta} \\ r - \dot{\theta} \end{bmatrix} + G(\theta) - \beta G_o(\theta) + T_p - B(\theta, \dot{\theta}) \dot{\theta} \dot{\theta} - C(\theta, \dot{\theta}) \dot{\theta}^2 - F(\theta) + T_H
\]
where $\alpha$ and $\beta$ are $N \times N$ diagonal matrices of independent adaptable gains. Substituting the torques applied by the patient ($T_H$) for ($T_{HG} + T_{HD}$) where $T_{HG}$ is the torque the patient applies to compensate for gravity and $T_{HD}$ is the torque the patient applies to change position, the equation becomes

$$\ddot{\theta} = A^{-1}(\theta)[\alpha[(K_p - G_L) K_d] \begin{bmatrix} r - \theta \\ \dot{r} - \dot{\theta} \end{bmatrix} + G(\theta) - \beta G_o(\theta) + T_p - B(\theta, \dot{\theta})\dot{\theta}\dot{\theta} - C(\theta, \dot{\theta})\dot{\theta}^2 - F(\theta)]$$

Rearranging the parameters into groups,

$$\ddot{\theta} = A^{-1}(\theta)[(\alpha[(K_p - G_L) K_d] \begin{bmatrix} r - \theta \\ \dot{r} - \dot{\theta} \end{bmatrix} + T_{HD}) + (G(\theta) + T_{HG} - \beta G_o(\theta))]$$

$$+ T_p - B(\theta, \dot{\theta})\dot{\theta}\dot{\theta} - C(\theta, \dot{\theta})\dot{\theta}^2 - F(\theta)]$$

As the patient develops their own control system for $T_H$ then the parameters $\alpha$ and $\beta$ will reduce to maintain the same response of the system, maintaining stability. However, if the system is completely stable, the patients will not be forced to adapt and take authority over the exoskeleton. The learning ‘L’ parameter is added to the adaptive controller (see Fig#4.4). The parameter ‘L’ takes on the value from 0 to 1 to adjust the optimal control signals in the adaptable control law to sub optimal levels. The desired torque will be reduced from the nominal level. By doing this, the system will become less stable or unstable as the control authority of the exoskeleton will no longer be able to stabilize the system. This parameter will have to be chosen based on experimental data with each patient as each patients learning ability will be different.
Two controllers are presented. In the first controller, the patients are trained to stabilize themselves from disturbance applied from random directions and at random amplitudes. In the second controller, the patients attempt to move their body in slow motions and maintain balance throughout the movements.

### 4.3.3 Reactive Balance Training

**Controller Description**

In this protocol, the patients are instructed to attempt to remain upright in the presence of periodic random disturbances applied through the exoskeleton. The controller is designed to assist the patient in maintaining balance by initially simulating the CNS with full authority and then steadily ceding authority to the patient. The feedback gains are determined through a LQR in order to simulate the delayed CNS response to a disturbance. As was discussed in Chapter 3, this provides an approximation of a healthy CNS. (see Fig#4.5) In Chapter 3, the model discussed included delay in the approximation of the CNS. The delay was compensated for through the use of a Smith Predictor. Here, the delay of the controller is eliminated, as it is not necessary for the training of the patient. The goal of the controller is not to perfectly mimic a healthy CNS, but to provide a stabilizing force similar to a healthy CNS that the patient may train with.

Low impulses are provided through Exoskeleton to perturb the system from equilibrium. When the impulses are applied, the exoskeleton control law is temporarily suspended in order to allow the CNS to respond to the disturbance. After a delay suitable for the CNS to react, the controller is allowed to start assisting the patient again. The controller will thus be able allow training of the
CNS. This training is derived from the protocol used by Matjacic[58]. The control authority of the exoskeleton is reduced over time as the patient succeeds in remaining stable. Authority reduction may be controller manually by the patient or doctor. Alternatively, it may be reduced automatically as the patient succeeds in maintaining balance to the disturbances.

The theory behind this training protocol is that the perturbations applied to the patients will stimulate the brain to develop reactions that will stabilize the patient. As discussed in Chapter 2, the use of platforms to apply disturbances and support patients has proved beneficial. In this training protocol, the control system and exoskeleton do not need to be able to fully simulate a human response to the disturbance. Instead, the exoskeleton and control system are designed for the purpose of preventing injury to the patient and allow them to develop their own control system. This should allow for the development of gross in response to larger disturbances and fine motor control for smaller disturbances.

Portions of the protocol should be adaptable for fine motor control. Reduction in the disturbances combined with the addition of a stiffness in the exoskeleton and a reduction in the stiffness over time should allow the development of fine motor control. The stiffness could be applied, controlled and reduced through the use of a magneto rheological brake in substitution for actuators. As the patient is able to increase his or her own stiffness to that of a healthy individual, the stiffness would then be reduced.

---

**Figure 4.5: Diagram of Exoskeleton Perturbation Control System with Patient**

- **Controller** is a LQR generated from the linearized system
- **Adaptive Law** is the adaptive law for the controller
- \( \alpha, \beta \) are the adaptable control law parameters
- **\( L \)** is the learning parameter

---

Note: The diagram shows a flowchart of the control system with the following components and annotations:

- Disturbance
- Controller
- \( \alpha, \beta \)
- Exo-Skeleton
- Brain
- Body
- Adaptive Law
- \( L \)
4.3.4 Slot Training

Controller Description

In this protocol, patients are instructed to attempt to follow a movement pattern that shifts the COM about the base of support. The controller is designed to assist the patient in maintaining balance by initially asserting full authority over the motions and then steadily ceding authority to the patient. The movements are slow, repetitive and predictable. Unlike the previous controller, no delay is added to the system to simulate the delay inherent in the CNS. Additionally, with an internal dynamic model, the patients should be able to start anticipating what controls they will have to apply in order to maintain the proper movements.

![Diagram of Exoskeleton Slow Motion Control System with Patient](image)

Figure 4.6: Diagram of Exoskeleton Slow Motion Control System with Patient

The initial configuration of the controller is the same as for the prior controller, utilizing a LQR from the linearized state equations.

The theory behind this protocol is that the motions will allow the development of gross motor control through the repeated motions. The motions need not be performed while the patient is standing, but can be done while the patient is lying down or sitting. This protocol is based on the work be Hogan et al.[30], Colby and Trainor[10] and others that the patients are physically shown how to perform movements and they are then encouraged to work to achieve the movements themselves. At first, they are exerting little authority to control their limbs as they train to learn the movements. As they progress, the patients take more authority to themselves from the assisting exoskeleton until they are able to perform motions on their own. The $\alpha$ and $\beta$ matrices may be reduced either manually or by a control law as the patient improves.
Chapter 5

System Modeling and Actuator Requirements

5.1 Model Summary

The system is modeled using a sample 60kg 168cm patient with the model parameters derived from Hanavan[14]. The model uses the full non-linear system derived from Khalil et al[39]. The exoskeleton control authority is suspended for the duration of the impulse. A 100ms impulse is applied to the patient. The impulse is of random direction and constrained random amplitude. The impulse is applied through the exoskeleton and is designed to accelerate the entire body equally over the duration of the impulse. The values for the impulse are determined using the nominal values of the inertia matrix.

\[\ddot{\theta} = A^{-1}(\theta) \left[ (K_p - G_L) K_d \right] \begin{bmatrix} r - \theta \\ \dot{r} - \dot{\theta} \end{bmatrix} + G_o(\theta) + G(\theta) + T_p - B(\theta, \dot{\theta})\dot{\theta} - C(\theta, \dot{\theta})\dot{\theta}^2 - F(\theta)\]

where \(F(\theta) = A_o(\theta)D\) and D is the \(Nx1\) vector of the disturbance to be applied to the system. In this manner, arbitrary disturbances may be simulated, provided they are within the limits of the exoskeleton.

The subject is presumed to be capable of self-support and the exoskeleton is only necessary to prevent falling. As the patients are better able to balance themselves, the authority of the controller may be reduced and the maximum amplitude of the impulses may be increased. The system is modeled with the exoskeleton control system containing the only way to stabilize the subject from the disturbance.
5.2 Model Testing

For testing, a 72N (1/8th body weight) impulse is applied in the anterior direction. The response is of similar form to that given in Xiang and He[83], with exception that heel remains on the ground. The response time is slower than experiments in Xiang and He have performed. The response time for the patient returning their center of mass (COM) to 40% of the distance from the COM relative to the peak distance from the starting position due to the disturbance is approximately 1.5s. The response time for the exoskeleton controller returning the patient to the 40% position is approximately 2.85s (see Fig#5.1). This will be beneficial in that the control system is performing slower than the patients are expected to, which should be suitable for training. The peak change in COM in the subject in Xiang and He is approximately double the peak change in COM in the exoskeleton simulation. However, Xiang and He also use an impulse of approximately twice the magnitude used in the exoskeleton simulations. (see Figures#5.1-5.3)

5.3 Robustness Modeling

To test the controller for sensitivity to plant variation, the model parameters for Inertia, Mass and distance to the COM were varied by $\pm 10\%$ to cover each permutation. The 1/8th bodyweight impulse was applied in the anterior, posterior and lateral direction to compare the responses.(see Figures#5.4-5.15) In the trials; there is little variation in the response of the system with the exception of the hip angle. The hip repeatedly exhibits a response in the opposite direction of the impulse. This is due to the torques for the impulse being derived from the values of the nominal inertia and not the actual values. As such, the torque values are incorrect for keeping the torso in line with the lower body during the perturbation. This effect is most pronounced in Figure#5.5. The LQR control system is capable of stabilizing the patient with a variety of parameter changes in the patient. As such, it should prove suitable for patient training.

5.4 Authority Reduction

To test the system as the controller authority is reduced, the controller authority is varied from 100% to 10% of the nominal value using the same 1/8th body weight impulse as before. The controller is able to stabilize the model while maintaining the COM over the base of support. The only damping force included in the model is due to the controller. Passive torque has not been included. As such, as the controller authority is reduced, the settling time and the overshoot in returning the COM to the original position is greatly increased. The inclusion of passive torques will increase the damping coefficient in the system and reduce settling time. Several limits of the model are reached that should also be noted.
5.4.1 Anterior Impulse

In the simulations where control authority is below 30%, the knees bend forward during the simulation. This is due to a limitation in the model where the limits on the range of motion of the knees and other joints are not included. The simulations do not include the limits of the knees due to the increase in complexity in the model. In order to include the limitations of the knee in the model, a compensatory torque will need to be included to prevent the knees from bending further. Alternatively, a separate model would need to be developed and the system would need to switch models from one to the other, as the simulation requires.

5.4.2 Posterior Impulse

In simulations the COM exceeds the maximum backward lean of 4 degrees when authority is less than 20%. This would normally cause a human to fall over. Additionally, before reaching this point, a healthy human would normally take a step backward including in rearward lean reached when the controller is using more than 10% authority. The knees also bend forward in the simulation when the authority is reduced below 80%.

5.5 Torque Requirements

The maximum torque required for the actuators to exert varies with the maximum authority of the controller, the capabilities of the patient and the mass and parameters of the patient. The maximum torques listed assumes that the patient is incapable of providing any assistive torque for stabilization. The model was derived for 168cm 60kg patient. Children will weigh less and require less torque.

The maximum controller torques for the controller while at full authority are at the greatest when the controller takes authority immediately after the impulse. However, it is quickly reduced as the patient’s velocity is slowed. The maximum torque ordered by the controller is much less as the authority is reduced. However, the time for which the actuator must remain exerting torque is correspondingly longer.

If the patient is unable to support themself and the exoskeleton is required to compensate for gravity, the exoskeleton must also be able to support the patient. The gravity compensation is greatest shortly after the impulse. The compensation required varies greatly depending on the authority of the controller. As the authority of the controller is reduced, the patient sways more and the torques must increase. The difference between the maximum torques required for the controller at full authority and at 10% authority varies by 40 Nm. It may be possible to reduce the load on the actuators with the inclusion of a brake with the actuators such as a magneto rheological brake. This would allow the brake to compensate for gravity while the patient is not moving and
Table 5.1: Maximum Controller Torque in each simulation (Nm)

<table>
<thead>
<tr>
<th>Parameter Varying,Anterior</th>
<th>Ankles Sagittal</th>
<th>Ankles Coronal</th>
<th>Knee</th>
<th>Hips Sagittal</th>
<th>Hips Coronal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter Varying,Posterior</td>
<td>12.5</td>
<td>0</td>
<td>7.5</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Parameter Varying,Lateral</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>2.5</td>
</tr>
<tr>
<td>Varying Authority,Anterior%</td>
<td>11</td>
<td>0</td>
<td>7</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td>Varying Authority,Posterior%</td>
<td>11</td>
<td>0</td>
<td>7</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td>Varying Authority,Lateral%</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Values are rounded up to the nearest 0.5

Table 5.2: Maximum Gravity Compensation Torque in each simulation (Nm)

<table>
<thead>
<tr>
<th>Parameter Varying,Anterior</th>
<th>Ankles Sagittal</th>
<th>Ankles Coronal</th>
<th>Knee</th>
<th>Hips Sagittal</th>
<th>Hips Coronal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter Varying,Posterior</td>
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<td>3</td>
<td>5</td>
<td>0</td>
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<tr>
<td>Parameter Varying,Lateral</td>
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<td>5</td>
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<tr>
<td>Varying Authority,Anterior%</td>
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<td>19.5</td>
<td>18</td>
<td>0</td>
</tr>
<tr>
<td>Varying Authority,Posterior%</td>
<td>48</td>
<td>0</td>
<td>24.5</td>
<td>18</td>
<td>0</td>
</tr>
<tr>
<td>Varying Authority,Lateral%</td>
<td>4</td>
<td>53.5</td>
<td>3</td>
<td>0.5</td>
<td>19</td>
</tr>
</tbody>
</table>

Values are rounded up to the nearest 0.5

for decreasing the velocity of the joints prior to the need for an active torque from an actuator.

It is important to note that the torques given in the tables are for the sum of the torques in both legs. For instance, the 12Nm torque for the sagittal ankle joint in parameter varying simulation, anterior impulse is for the sum of the torque for both the left sagittal ankle joint and right sagittal ankle joint. The torque each produces is equal. The maximum torque required per joint is therefore 6Nm for the joint in this simulation.

5.6 Actuator Options

There are various actuators available today that will provide the torques required for the exoskeleton. However, not all are suitable for use. Stiffness, inertia and maximum torque are challenges in the exoskeleton. Human joints are not very stiff and are easy to move. If they are perturbed, they will rotate. Most actuators used are very stiff and will not move if an external force is applied. One
A way of compensating for stiffness is by using a spring attached to a motor to apply force indirectly. The spring is used to apply force to the joint while the motor is used to vary the force. The inertia of the actuators used is another problem. The force applied by the actuators and the position needs to change quickly. The inertia of the actuators will limit the rate at which the corrective torque may be applied and provide an undesirable torque while the joints are rotating. Lastly, the continuous torques required can be quite larger.

AC motors are capable of providing the torques required. However, AC motors have two limitations that make them unsuitable for use in this exoskeleton. The low velocity and low the range of motion of the joints of the exoskeleton would cause an AC motor to generally be on one phase of the motor, essentially performing as a DC motor.

Small, compact DC motors that are capable of producing the required torque are available. One motor produced by Electromate is capable of producing 28 Nm of continuous torque. These motors also have drawbacks. The first is the weight of the motors. The Electromate motor weighs 2.5 lb. While individually this is relatively lightweight, the exoskeleton will need an actuator at each joint for each degree of freedom. For quiet standing, the exoskeleton will require ten actuators, two for each of the joints in the five-link model. While it is possible to use smaller actuators for some of the joints, the sum weight of all ten actuators will still be significant and does not yet include the necessary battery packs for powering them. The second drawback is the operating voltage of the actuators. The actuators operate at 100 volts. This voltage becomes dangerous to the patients. While lower voltage DC motors are available, they are heavier.

Use of a gear drive instead of a direct drive motor would reduce the maximum torque required of the motors. Maxon produces a 48 volt motor capable of 480 mNm of continuous torque. The motor weighs 648g (1.43 lb). In order to provide the required 28 Nm torque, the gear drive will need a gear ratio of 60 to 1. However, the additional weight would eliminate any gains made by the use of the smaller motor with the gear drive over the direct drive motor.

Hydraulic actuators are capable of providing sufficient torque with low weight. Operating at a high pressure, the hydraulic actuator is capable of providing the required torque while using a small reservoir of fluid, reducing the weight of the hydraulic system. Electronically controllable pressure valves allow precise control and fast adjustment of the torques applied. Magnetorheological brakes offer some help with the exoskeleton. Using the brake as a variable clutch between the actuator and the joints, the brake is capable of producing controlled stiffness in the actuators.

### 5.7 Actuator Saturation

The largest combined torque from the simulations is 55.5 Nm for the sagittal ankle joints, compensating for both gravity and the controller torque. This produces a need for each ankle joint to produce 28 Nm of torque. While the actuators do not need to produce this torque for long, this is still a large amount of torque that can saturate the actuators. Saturation of the actuators will
have different effects depending on the specific therapy. If saturation of the actuators occurs in
motion training with a patient who is unable to support themselves, then the patient will fall, as
the exoskeleton will be unable to support the patient against the force of gravity. For saturation
in the Reactive Balance Training, where the patient provides the gravity compensation, saturation
will create a non-linearity in the controller feedback gains. The consequences of this depend on
the controller and the model.

To test the reaction training controller response to limited torque, the maximum actuator torque
available to each joint was varied from 1Nm to 13Nm in 1Nm increments. For full controller au-
thority, the system goes unstable when the 1/8th body weight impulse is applied when the actuators
saturate at 6Nm or less when an anterior or posterior impulse is applied. (see Fig#5.28-5.33). The
black are for simulations where the system became unstable and the blue lines are for simulations
that remained stable. For lateral impulses, the controller is capable of stabilizing the model. How-
ever, at 1Nm, anatomically impossible angles occur along with the center of mass moving outside
the base of support (see Fig#5.34-5.36). As such, we can infer that the patient would most likely
be unstable and fall down at this point.

The cause of the instability is the saturation distorting the feedback gain matrix. However, there is
a simple solution to regain stability. Reducing the controller authority will, as simulated in the au-
thority reduction simulations, reduces the maximum torque significantly. Reducing the controller
authority to operate within the capabilities of the actuators will allow the controller to stabilize the
system.
Figure 5.2: Joint Positions, Anterior Impulse, Ideal System

Figure 5.3: Patient Position, Anterior Impulse, Ideal System
Figure 5.4: Center of Mass, Anterior Impulse, Varying Patient Parameters

Figure 5.5: Joint Positions, Anterior Impulse, Varying Patient Parameters
Figure 5.6: Controller Torques, Anterior Impulse, Varying Patient Parameters

Figure 5.7: Gravity Torques, Anterior Impulse, Varying Patient Parameters
Figure 5.8: Center of Mass, Posterior Impulse, Varying Patient Parameters

Figure 5.9: Joint Positions, Posterior Impulse, Varying Patient Parameters
Figure 5.10: Controller Torques, Posterior Impulse, Varying Patient Parameters

Figure 5.11: Gravity Torques, Posterior Impulse, Varying Patient Parameters
Figure 5.12: Center of Mass, Lateral Impulse, Varying Patient Parameters

Figure 5.13: Joint Positions, Lateral Impulse, Varying Patient Parameters
Figure 5.14: Controller Torques, Lateral Impulse, Varying Patient Parameters

Figure 5.15: Gravity Torques, Lateral Impulse, Varying Patient Parameters
Figure 5.16: Center of Mass, Anterior Impulse, Varying controller Authority
Figure 5.17: Joint Positions, Anterior Impulse, Varying controller Authority
Figure 5.18: Controller Torques, Anterior Impulse, Varying controller Authority

Figure 5.19: Gravity Torques, Anterior Impulse, Varying controller Authority
Figure 5.20: Center of Mass, Posterior Impulse, Varying controller Authority

Figure 5.21: Joint Positions, Posterior Impulse, Varying controller Authority
Figure 5.22: Controller Torques, Posterior Impulse, Varying controller Authority

Figure 5.23: Gravity Torques, Posterior Impulse, Varying controller Authority
Figure 5.24: Center of Mass, Lateral Impulse, Varying controller Authority

Figure 5.25: Joint Positions, Lateral Impulse, Varying controller Authority
Figure 5.26: Controller Torques, Lateral Impulse, Varying controller Authority

Figure 5.27: Gravity Torques, Lateral Impulse, Varying controller Authority
Figure 5.28: Controller Torques, Anterior Impulse, Varying Actuator Saturation
Figure 5.29: Joint Positions, Anterior Impulse, Varying Actuator Saturation

Figure 5.30: Center of Mass, Anterior Impulse, Varying Actuator Saturation
Figure 5.31: Controller Torques, Posterior Impulse, Varying Actuator Saturation

Figure 5.32: Joint Positions, Posterior Impulse, Varying Actuator Saturation
Figure 5.33: Center of Mass, Posterior Impulse, Varying Actuator Saturation

Figure 5.34: Controller Torques, Lateral Impulse, Varying Actuator Saturation
Figure 5.35: Joint Positions, Lateral Impulse, Varying Actuator Saturation

Figure 5.36: Center of Mass, Lateral Impulse, Varying Actuator Saturation
Chapter 6

Future Work

This chapter is broken into three areas. The first portion reviews future development of the control system for the exoskeleton. The second portion reviews future refinements to the training program. The third portion reviews miscellaneous insights received from other studies that may be of use to the development of the exoskeleton.

Area 1: Control Systems
The two control protocols presented are representative of therapies in use today by physical therapists. The reactive balance training protocol has been found beneficial for patients who have limited impairment and are largely able to support themselves. For those with a greater impairment, the slot training protocol based on stroke patient therapy appears to provide the most promise for future work. The repetitive motions promote the development and construction of neural pathways for control of the body. The slot controller is also the protocol that appears to hold the most promise for training in walking. However, gait profiles will need to be developed and modified on a patient-by-patient basis for the transition to walking.

The model derived for this thesis uses parameters derived from Hanavan[14]. However, it has been found that the model derived is not always accurate.[15] While it may work for general testing for the control system, a more precise measurement may be necessary for patients. Hatze provides a method for computing the model parameters more accurately than allowed for by Hanavan at the expense of more measurements and computation time. Additionally, stiffness in the joints was not included or used for control and balancing of the model. The inclusion of this in the models will allow for better understanding of the motions and inputs required.

Area 2: Training
Experiments validating of the effectiveness of these training protocols will need to be performed. The experimentation should begin with those therapies closest to those already in use by physical therapists to determine the effectiveness, benefits and drawbacks of the training protocols. This will allow for the verification of the training protocols. A study of the drawbacks and complaints should allow for improvements in the physical therapy program.
For reactive balance training, experiments may start with a single degree of freedom at the ankles, with the movement of the other joints restricted prior to experimenting with full lower body training. For the slot control, motions may start with upper or lower limb development by control of the arms as used by Hogan et al.\[30\] or an equivalent movement for the legs while the patient is lying down.

Investigations into dancing, Tai Chi and other martial arts programs may be beneficial\[62\]. The use of dancing or martial arts is also beneficial, as it will allow the patients to learn something they may find entertaining while performing balance training. Routines that provide balance training only, but are less stimulating, may lead to boredom. In the case of children, this could lead to a less than desirable experience and may impact the success of a training program. Initially, the ideal motions are slow and give the patients time to think about their movements. Faster movements will provide two challenges. The first is the development of a control system that will safely take into account the centrifugal and coriolis forces\[1\]. The second is that beginning patients may not be able to be trained as well from faster motions that require faster responses. Additionally, the possibility of injury is increased with higher velocities, either due to external injury from falling, or internal injury to the developing muscles. For walking, routines that emphasize each phase of walking\[2\] will probably be the most beneficial for the training of walking. While these routines may not emulate a natural gate, they may assist in the training of balance in each phase of walking.

Area 3: Misc.
The actuators utilized by the exoskeleton should be back drivable or have a way that the patient may override the exoskeleton through their own input by default. \[72\] The patient needs to have final control over their movement, not the exoskeleton. The exoskeleton exists to guide their movement and train them, not to move their body for them. Thus, if the patient is making a mistake, they need to be able to override the exoskeleton automatically and learn from it. Alternatively, the exoskeleton may be making a mistake or not be compensating for something that the patient notices. However, paradoxically, the exoskeleton needs to be able to use sufficient torque to move them. As such, the exoskeleton should have a cutoff point at which it cedes all authority to the patient.

There are other lessons that may be learned from the WOTAS exoskeleton. While the orthosis performed as expected, several of the patients were uncomfortable with the device due to its size and weight (850g, 1.9lb), feeling that use would lead to ‘social exclusion’\[72\]. While not normally within the areas covered in engineering, both aesthetics and end user comfort will need to be considered during the development of an end user device. The public will be more receptive to a device that has the sleek look of HAL-5 compared to the utilitarian look of the earlier BLEEX models\[23\]. Rahman et al.\[71\] in preparation for an orthosis for muscular dystrophy patients interviewed several patients prior to development of second-generation prototypes. Tsargarakis and Caldwell\[84\] have outlined further design requirements that need to be taken into account for the patient to receive a comfortable experience.

\[1\]The B and C matrices in Chapters 4,5.
\[2\]starting walking, singles support, double support, stopping walking
From these, a few universal requirements for a production product can be determined, a few of which are mentioned here. It must provide a superior experience compared to existing methods of therapy. It must fit into the patients’ life without the patient needing to sizably adjust to the orthosis. It must be cost effective for the benefits provided. It must appear aesthetically pleasing; or to provide at least enough benefits to offset the appearance of the exoskeleton and any stigmas that may result from this. It must be reliable and durable. Finally, the device must be comfortable.
Bibliography


Appendix A

Autogeneration of Models Equations

File: Main.m
Function: Calculates the A,B,C and G matrices for the rotations and variables given using the algorithm given by Khalil et al. [39].

% Matrices Produced:
%A Matrix - Inertia Matrix
%B Matrix - Coriolis Force Matrix
%C Matrix - Centrifugal Force Matrix
%G Matrix - Gravity Matrix

% Angle Variables
syms t1 t2 t3 t5 t6
q=[t1 t2 t3 t5 t6];
% Link length Variables
syms Y2 Y3

% Ankle Link 1
L1=[cos(t1) sin(t1) 0 0;
    -sin(t1) cos(t1) 0 0;
    0 0 1 0;
    0 0 0 1];
% Ankle Link 2 (Shank)
L2=[1 0 0 0;
    0 cos(t2) -sin(t2) Y2*cos(t2);
    0 sin(t2) cos(t2) Y2*sin(t2);
    0 0 0 1];
% Knee Link (Thigh)
L3 = [1 0 0 0;
      0 cos(t3) -sin(t3) Y3* cos(t3);
      0 sin(t3) cos(t3) Y3*sin(t3);
      0 0 0 1];
% Hip Link 1
L5 = [cos(t5) sin(t5) 0 0;
      -sin(t5) cos(t5) 0 0;
      0 0 1 0;
      0 0 0 1];
% Hip Link 2 (Torso)
L6 = [1 0 0 0;
      0 cos(t6) -sin(t6) 0;
      0 sin(t6) cos(t6) 0;
      0 0 0 1];

% Inertia Variables
syms I1xx I2xx I3xx I5xx I6xx
syms I1yy I2yy I3yy I5yy I6yy
syms I1zz I2zz I3zz I5zz I6zz

% Six elements of the Inertia Matrix
I = [0 0 0 0 0;
    I2xx 0 0 0 0;
    I3xx 0 0 0 0;
    0 0 0 0 0;
    I6xx 0 0 0 0];

% 3 elements for the first moments
syms Ry2 Ry3 Ry6

% Moment of Inertia Element Matrix
S = [0 0 0 0 0;
    0 Ry2 Ry3 0 Ry6;
    0 0 0 0 0];

% 1 mass element
syms m2 m3 m6
M = [0 m2 m3 0 m6];

% Rotation Matrices
R = [L1 L2 L3 L5 L6];
% Axis of Rotation Array
P=[3 1 1 3 1];
% Number of links in the model
MaxLink=5;

% Program begins here
rho=[];
Rc=[];
Rot=eye(3);
L=zeros(3,1);
T=eye(4);
G=sym(zeros(1,MaxLink))';
for link=1:MaxLink
    T=T*[R(:,link*4-3) R(:,link*4-2) R(:,link*4-1) R(:,link*4)];
    Rot=[T(1,1) T(1,2) T(1,3);
         T(2,1) T(2,2) T(2,3);
         T(3,1) T(3,2) T(3,3)];
    for i=1:link
        G(i)=G(i)+diff(M(link)*[0 1 0]*(L+Rot*S(:,link)),q(i));
    end
    L=[T(1,4);T(2,4);T(3,4)];
end

A=sym(zeros(MaxLink*10,MaxLink,MaxLink));
for link=1:MaxLink
    rho=[rho P(link)];
    Rc=[Rc R(:,4*(link-1)+1) R(:,4*(link-1)+2)
        R(:,4*(link-1)+3) R(:,4*(link-1)+4)];
    J=Jacobs(link,rho,Rc);
    v=[J(1,:);J(2,:);J(3,:)];
    w=[J(4,:);J(5,:);J(6,:)];
    for l=1:link
        for m=1:link
            A((link-1)*10+1,l,m)=I((link-1)*3+1,1)*w(1,l)*w(1,m);
            A((link-1)*10+2,l,m)=I((link-1)*3+2,2)*w(2,l)*w(2,m);
            A((link-1)*10+3,l,m)=I((link-1)*3+3,3)*w(3,l)*w(3,m);
            A((link-1)*10+4,l,m)=I((link-1)*3+1,2)*w(1,l)*w(2,m)
                           +w(1,m)*w(2,1));
            A((link-1)*10+5,l,m)=I((link-1)*3+1,3)*(w(1,l)*w(3,m)
                           +w(1,m)*w(3,1));
            A((link-1)*10+6,l,m)=I((link-1)*3+2,3)*(w(2,l)*w(3,m)
                           +w(2,m)*w(3,1));
            A((link-1)*10+7,l,m)=M(link)*(v(1,l)*v(1,m)
                           +w(1,m)*w(1,1));
\begin{align*}
&+v(2, l) \cdot v(2, m) + v(3, l) \cdot v(3, m)); \\
&A((\text{link}-1) \cdot 10 + 8, l, m) = M(\text{link}) \cdot S(1, \text{link}) \cdot (v(2, l) \cdot w(3, m) \\
&\quad + v(2, m) \cdot w(3, l) - v(3, l) \cdot w(2, m) - v(3, m) \cdot w(2, l)); \\
&A((\text{link}-1) \cdot 10 + 9, l, m) = M(\text{link}) \cdot S(2, \text{link}) \cdot (v(3, l) \cdot w(1, m) \\
&\quad + v(3, m) \cdot w(1, l) - v(1, l) \cdot w(3, m) - v(1, m) \cdot w(3, l)); \\
&A((\text{link}-1) \cdot 10 + 10, l, m) = M(\text{link}) \cdot S(3, \text{link}) \cdot (v(1, l) \cdot w(2, m) \\
&\quad + v(1, m) \cdot w(2, l) - v(2, l) \cdot w(1, m) - v(2, m) \cdot w(1, l));
\end{align*}
end
end
end

\text{Ac} = \text{sym}(\text{zeros} (\text{MaxLink}, \text{MaxLink}));
\text{for} \ m = 1: (\text{MaxLink} \cdot 10)
\quad \text{for} \ i = 1: \text{MaxLink}
\quad \quad \text{for} \ j = 1: \text{MaxLink}
\quad \quad \quad \text{Ac}(i, j) = \text{Ac}(i, j) + A(m, i, j);
\quad \quad \text{end}
\quad \text{end}
\text{end}
\text{end}

\text{B} = \text{sym}(\text{zeros} (\text{MaxLink}, \text{MaxLink}, \text{MaxLink}));
\text{C} = \text{sym}(\text{zeros} (\text{MaxLink}, \text{MaxLink}));
\text{for} \ i = 1: \text{MaxLink}
\quad \text{for} \ j = 1: \text{MaxLink}
\quad \quad \text{for} \ k = 1: \text{MaxLink}
\quad \quad \quad \text{B}(i, j, k) = \text{diff}(\text{Ac}(i, j), q(k)) + \text{diff}(\text{Ac}(i, k), q(j)) \\
\quad \quad \quad \quad - \text{diff}(\text{Ac}(j, k), q(i));
\quad \quad \text{end}
\quad \text{C}(i, j) = \text{diff}(\text{Ac}(i, j), q(j)) - 0.5 \cdot \text{diff}(\text{Ac}(j, j), q(i));
\quad \text{end}
\text{end}

\text{File: Jacobs.m}
\text{Function: Calculates the Jacobian for the links and system given}

\text{function} \ J = \text{Jacobs(nlinks, rho, Ai)}

\text{[Rows, Columns]} = \text{size(Ai)};
\text{if} \ (\text{Columns} \neq (\text{nlinks} \cdot 4))
\quad \text{error('not all parameters are of equal length')} \\
\quad \text{return;}
\text{end}
if Rows™=4
erros('Ai is of incorrect dimension')
return;
end
if nlinks™=length(rho)
error('rho is of incorrect length')
return;
end

Jv=sym(zeros(3,nlinks));
Jw=sym(zeros(3,nlinks));
R=eye(3);
R2=eye(3);

Af=eye(4);
for j=1:nlinks
    Af=Af*[Ai(:,4*j-3) Ai(:,4*j-2) Ai(:,4*j-1) Ai(:,4*j)];
end

On=[Af(1,4);Af(2,4);Af(3,4)];

for i=1:nlinks
    At=eye(4);
    j=1;
    while j<i
        At=At*[Ai(:,4*j-3) Ai(:,4*j-2) Ai(:,4*j-1) Ai(:,4*j)];
        j=j+1;
    end
    At;
    Oi=On-[At(1,4);At(2,4);At(3,4)];

    switch rho(i)
    % prismatic
    case {0}
        Jv(:,i)=R*[0;0;1];
    % revolute
    case {1}
        Zi=R*[1;0;0];
        Jv(:,i)=[0 -Zi(3) Zi(2);
                 Zi(3) 0 -Zi(1);...
\[ -Z_i(2) Z_i(1) 0 \]*O_i; \\
\text{case} \{2\} \\
\quad Z_i=R*[0;1;0]; \\
\quad Jv(:,i)=[0 -Z_i(3) Z_i(2); \\
\quad \quad Z_i(3) 0 -Z_i(1); \\
\quad \quad -Z_i(2) Z_i(1) 0]*O_i; \\
\text{case} \{3\} \\
\quad Z_i=R*[0;0;1]; \\
\quad Jv(:,i)=[0 -Z_i(3) Z_i(2); \\
\quad \quad Z_i(3) 0 -Z_i(1); \\
\quad \quad -Z_i(2) Z_i(1) 0]*O_i; \\
\text{otherwise,} \\
\quad \text{error(}'Invalid number in Rho Array'\text{')} \\
\quad \text{return;} \\
end \\

\text{switch} \ \text{rho(nlinks-i+1)} \\
\quad \%\text{prismatic} \\
\quad \text{case} \{0\} \\
\quad \quad Jw(:,(nlinks-i+1))=[0;0;0];% \\
\quad \%\text{revolute} \\
\quad \text{case} \{1\} \\
\quad \quad Jw(:,(nlinks-i+1))=R2*[1;0;0];% \\
\quad \text{case} \{2\} \\
\quad \quad Jw(:,(nlinks-i+1))=R2*[0;1;0];% \\
\quad \text{case} \{3\} \\
\quad \quad Jw(:,(nlinks-i+1))=R2*[0;0;1];% \\
\quad \text{otherwise,} \\
\quad \quad \text{error(}'Invalid number in Rho Array'\text{')} \\
\quad \text{return;} \\
end \\
R=R*[A_i(1,4*i-3) A_i(1,4*i-2) A_i(1,4*i-1); \\
\quad A_i(2,4*i-3) A_i(2,4*i-2) A_i(2,4*i-1); \\
\quad A_i(3,4*i-3) A_i(3,4*i-2) A_i(3,4*i-1)]; \\
R2=R2*[A_i(1,4*(nlinks-i+1)-3) A_i(2,4*(nlinks-i+1)-3) \\
\quad A_i(3,4*(nlinks-i+1)-3); \\
\quad A_i(1,4*(nlinks-i+1)-2) A_i(2,4*(nlinks-i+1)-2) \\
\quad A_i(3,4*(nlinks-i+1)-2); \\
\quad A_i(1,4*(nlinks-i+1)-1) A_i(2,4*(nlinks-i+1)-1) \\
\quad A_i(3,4*(nlinks-i+1)-1)]; \\
end
\[ J = [J_v; J_w]; \]
Appendix B

Derivation of Model Parameters

In order to model the exoskeleton and patient, four sets of parameters need to be identified. The first set of parameters are the transformation matrices of each joint, the second are the mass of each link, third are the inertia matrices and the fourth are the location of the center of mass (CoM). As the human body is largely symmetrical, this reduces the number of parameters that need to be independently approximated.

The diagonal elements of the Inertia Matrix contains the principle moments of inertia along the three axis of rotation. The off diagonal elements contain the moment of inertia tensors, which are zero only when the joint is symmetric about all three axes. In the human body, this will not happen. However, it may be sufficiently approximated without the need to include the tensors.

$$I_i = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

c_i is the vector from the joint from of reference to the location of the center of mass for the joint. Using the work performed by Hanavan[14], the parameters may be approximated by some simple body measurements and the model simplified with the elimination of various parameters. Each joint is approximated as an elliptical cylinder or a partial cone with the center of mass along the vertical axis. This allows for a reasonable approximation to be made while discarding the inertia tensor and two of the measurements for the CoM of each joint. The foot, shank and thigh may

<table>
<thead>
<tr>
<th>Table B.1: Table of Link Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_i$</td>
</tr>
<tr>
<td>$m_i$</td>
</tr>
<tr>
<td>$L_i$</td>
</tr>
<tr>
<td>$c_i$</td>
</tr>
</tbody>
</table>
Table B.2: Table of Links for 12-Link Lower Body Model

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>Ankle, Coronal Plane</td>
</tr>
<tr>
<td>L2</td>
<td>Ankle, Sagital Plane</td>
</tr>
<tr>
<td>L3</td>
<td>Knee</td>
</tr>
<tr>
<td>L4</td>
<td>Hip, Transverse Plane</td>
</tr>
<tr>
<td>L5</td>
<td>Hip, Coronal Plane</td>
</tr>
<tr>
<td>L6</td>
<td>Hip, Sagital</td>
</tr>
<tr>
<td>L7</td>
<td>Hip, Sagital</td>
</tr>
<tr>
<td>L8</td>
<td>Hip, Coronal Plane</td>
</tr>
<tr>
<td>L9</td>
<td>Hip, Transverse Plane</td>
</tr>
<tr>
<td>L10</td>
<td>Knee</td>
</tr>
<tr>
<td>L11</td>
<td>Ankle, Sagital Plane</td>
</tr>
<tr>
<td>L12</td>
<td>Ankle, Coronal Plane</td>
</tr>
<tr>
<td>M2</td>
<td>Mass of Shank</td>
</tr>
<tr>
<td>M3</td>
<td>Mass of Thigh</td>
</tr>
<tr>
<td>M4</td>
<td>Mass of Upper Body</td>
</tr>
<tr>
<td>M11</td>
<td>Mass of Foot</td>
</tr>
<tr>
<td>Y2</td>
<td>Length of Shank</td>
</tr>
<tr>
<td>Y3</td>
<td>Length of Thigh</td>
</tr>
<tr>
<td>X6</td>
<td>Width of Hips</td>
</tr>
</tbody>
</table>
be approximated as a partial cone (see fig:B.3) and the torso by an elliptical cylinder (see fig:B.4).
With these measurements, an approximate model of each joint may be derived. The dynamic
equations for the model are derived from the algorithm laid out by Khalil et al.[39] The equations
are based on modeling of rigid links coupled together with the first link pinned to the floor. The
MATLAB™ code for generation of the models is given in Appendix A: Autogeneration of Models.

Table B.3: Table of Joint Measurements

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_i$</td>
<td>Major diameter of joint</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Minor diameter of joint</td>
</tr>
<tr>
<td>$L_i$</td>
<td>Length of Joint</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Proportion of joint length to center of mass</td>
</tr>
</tbody>
</table>
Figure B.2: Position of Center of Mass

Figure B.3: Joint Measurements

Figure B.4: Torso Measurements