Coherent Mitigation of Radio Frequency Interference in 10–100 MHz

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(ABSTRACT)

This dissertation describes methods of mitigating radio frequency interference (RFI) in the frequency range 10–100 MHz, developing and evaluating coherent methods with which RFI is subtracted from the afflicted data, nominally resulting in no distortion of the underlying signals. This approach is of interest in weak signal applications such as radio astronomy, where the signal of interest may have interference–to–noise ratio much less than one, and so can be easily distorted by other methods. Environmental noise in this band is strong and non–white, so a realistic noise model is developed, with which we characterize the performance of signal parameter estimation, a key component of the proposed algorithms. Two classes of methods are considered: “generic” parameter estimation/subtraction (PE/S) and a modulation–specific form known as demodulation–remodulation (“demod–remod”) PE/S. It is demonstrated for RFI in the form of narrowband FM and Broadcast FM that generic PE/S has the problem of severely distorting underlying signals of interest and demod–remod PE/S is less prone to this problem. Demod–remod PE/S is also applied and evaluated for RFI in the form of Digital TV signals. In both
cases, we compare the performance of the demod–remod PE/S with that of a traditional adaptive canceling method employing a reference antenna, and propose a hybrid method to further improve performance. A new metric for “toxicity” is defined and employed to determine the degree to which RFI mitigation damages the underlying signal of interest.
Dedication

This dissertation is dedicated to my wife, Jieun, my parents, and my parents in law, for all their love and support.
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Chapter 1

Introduction

Wide bandwidth is important to various applications such as radio frequency surveillance, radio astronomy, and ultra wideband (UWB) communication systems. However, bandwidth expansion of the receiver is limited by the presence of radio frequency interference (RFI). Therefore, to tackle the RFI issue, mitigation techniques are of interest in the applications requiring a wideband receiver. In this dissertation, we describe a method of mitigating RFI in wideband receivers operating in the frequency range 10–100 MHz, focusing on the development and evaluation of a coherent strategy in which RFI is subtracted from the afflicted data, nominally resulting in no distortion of the weak signals of interest.
This chapter is organized into four sections: Section 1.1 ("Wideband Direct Sampling Receiver") describes the assumed receiver architecture: the direct sampling receiver. Section 1.2 ("Applications of Wideband Low Frequency Receivers and Associated RFI Sources") describes applications where such receivers are likely to be used, focusing on an on-going project in radio astronomy. Also, sources of RFI in the 10–100 MHz range are described. Section 1.3 ("Challenges of RFI Mitigation for Radio Astronomy") reviews the existing RFI mitigation methods and addresses their limitations with respect to radio astronomy. In Section 1.4 ("Primary Application: LWA"), we describe the Long Wavelength Array (LWA) as a primary application of this work. Section 1.5 ("Contributions") summarizes the research contributions of this dissertation and Section 1.6 ("Organization of This Dissertation") describes the organization of the remainder of this dissertation.

1.1 Wideband Direct Sampling Receiver

Depending on instantaneous bandwidth, we can categorize receiver systems as either narrowband or wideband systems. A commonly-used criterion for this distinction is fractional bandwidth defined as $\frac{f_H - f_L}{f_H + f_L}$, where $f_H$ and $f_L$ are high and low frequency edges of the passband, respectively. If the fractional bandwidth is $\leq 20\%$, the receiver is normally considered to be narrowband. RFI is a relatively minor concern for narrowband receivers because most RFI can be filtered out at the
analog-RF stage in the receiver chain. Contrary to narrowband systems, wideband systems are quite vulnerable to RFI because relatively more RFI exists in the same passband and causes problems for the receivers even if it does not overlap the desired signals. This precludes the possibility of analog filtering of all RFI. Thus, in such applications, RFI mitigation is most effectively done in the digital domain. The direct sampling architecture shown in Figure 1.1 is desirable for wideband low frequency applications because the traditional architecture, i.e., heterodyne, including analog mixer, tends to generate excessive inband spurious frequency components [1][2].

1.2 Applications of Wideband Low Frequency Receivers and Associated RFI Sources

Current and future wideband low frequency system applications include radio astronomy, frequency-agile cognitive radio, UWB communication systems, earth...
sciences remote sensing, government surveillance, and military applications. In the case of emerging frequency-agile cognitive radio, its main objective is to improve spectral efficiency by sensing and using idle spectrum [3]. Even though this technology is still essentially narrowband, wideband receivers are desirable because the large swaths of spectrum must be searched to find out the unused channels [4]. UWB systems are inherently wideband and its allocated frequency for communication in U.S. is currently above 3.1 GHz [5], but there are some other UWB applications such as ground penetrating and airborne radars which may operate at frequencies below 100 MHz. In earth sciences remote sensing, frequency modulation-continuous wave (FM-CW) radar is used for applications such as ocean wave height sensing [6]. Other radar applications operating in this band require wideband bandwidth for high resolution [7][8]. In government surveillance, it is desired to extend bandwidth so as to more readily detect new emitters and more easily monitor multiple emitters simultaneously.

Although the work described in this dissertation is applicable to all of these applications, our main focus is radio astronomy. Traditionally, radio astronomy in the 10–100 MHz range has been able to get by with narrow protected frequency bands, e.g. 37.5–38.25 MHz and 73–74.6 MHz, or sometimes outside these bands when RFI is acceptably weak [9]. But new science drives observations outside the protected bands and to larger bandwidths. For example, a topic which is currently of intense
interest to the astronomical community is the possibility of studying the Epoch of Reionization (EoR) using the redshifted 21 cm emission of hydrogen (e.g., [10]), and is a prime motivator for a number of new radio telescopes currently under construction including the Low Frequency Array (LOFAR) [11], the Murchison Widefield Array (MWA) [12][13], and the Square Kilometer Array (SKA) [14][15]. This effort requires access to large contiguous segments of quiet spectrum ranging from 75 MHz to 250 MHz. For a variety of reasons beyond EoR studies, the new instruments LOFAR and the Long Wavelength Array (LWA) [16] intend to operate at frequencies as low as 10 MHz.

Anthropogenic RFI poses severe challenges to radio astronomy [17]. In particular, the spectrum below 100 MHz is heavily used by television, FM, and international broadcast communications, amateur radio, and mobile two-way radio. All of these signals can propagate enormous distances and are therefore sources of RFI to radio astronomy even for radio telescopes operating at sites very distant from population centers. Broadcast FM and Digital TV (DTV) are sources of RFI of particular concern to radio astronomers. Broadcast FM signals are allocated at frequencies from 88 MHz to 108 MHz in the U.S. They have bandwidths of about 200 kHz and are very complex, including a pilot signal and subcarriers for stereo, radio broadcast data system (RBDS), and subsidiary communications authorization (SCA). DTV is an emerging threat to radio astronomy in the 54–88 MHz range, and is due to
replace conventional analog TV by February 2009. (This deadline applies only to “full power” stations. The deadline for “low power”, “Class A”, and “translator” stations has not yet been set.) While in analog TV there exists usable spectrum between the video and audio carriers of weak TV stations, DTV signals are spectrally flat, leaving no such gap. Thus, applications such as radio astronomy that rely on this “white space” may be in trouble once the replacement is complete. The complex and persistent nature of FM and DTV signals limit the extent to which a strategy of avoidance can be successful, and motivate technical solutions in which RFI-afflicted data might instead be cleaned.

1.3 Challenges of RFI Mitigation for Radio Astronomy

Although extensive literature exists on the problem of mitigation of RFI, the vast majority of past work is oriented toward commercial and military applications (e.g., [18] and references therein), as opposed to radio science applications. The resulting emphasis is on the reduction of very strong RFI – i.e., interference-to-noise ratios (INR) orders of magnitude greater than 1. RFI whose INR is less than or equal to 1 is typically no longer limiting to communication performance. In radio science applications, in contrast, input INR ≤ 1 can be devastating; e.g.,
in spectroscopy involving long integrations. Thus, we seek algorithms which are
effective in suppressing weak RFI to INR ≪ 1. Additionally, we seek algorithms
which do this without distorting the underlying astronomy, which essentially equates
to not distorting the underlying noise.

The emphasis in this work is accomplishing RFI mitigation through a strategy
of RFI waveform estimation followed by coherent subtraction. Notable previous at-
ttempts to mitigate RFI in this manner includes the work of Barnbaum and Bradley
(1998) [19], which investigated the feasibility of using a time-domain adaptive can-
celer in this application. This approach requires an auxiliary signal which has nom-
inally a high-INR copy of the RFI and negligible SNR for the astronomical signal
of interest, and must therefore be obtained from a separate “reference” antenna.
Thus limitations of this approach are that (1) some a priori information about the
location of the signal is required so as to properly point the reference antenna(s),
and (2) the technique is limited to suppression which is related to the INR achieved
in the reference channel. In fact, this class of mitigation techniques tends to produce
output INR of about 1 regardless of input INR. This tends to limit the usefulness of
affected algorithms since RFI which appears weak over short time–frames can easily
ruin observations made over longer time periods. In the Bradley and Barnbaum
approach, the only alternative is to increase the gain of the reference antenna.
An interesting alternative approach can be found in the synthetic aperture radar literature. In Miller, McCorkle, and Potter (1997) [20], the RFI over a wide bandwidth is modeled as a set of sinusoids of unknown frequency, magnitude, and phase. The proposed algorithm then consists of dynamically estimating these parameters, synthesizing new (noise-free) versions of the signals using the estimated parameters, and then coherently subtracting these from the original data. This technique turns out to be highly effective, especially when the sample rate is much greater than the Nyquist criterion for any given RFI signal, since in this case narrowband RFI signals are well-modeled as unmodulated carriers, even over large numbers of samples. Furthermore, there is no need for a separate reference antenna. To accommodate frequency-modulated signals having bandwidth too great to model effectively as unmodulated sinusoids, this approach adds an additional parameter—the first derivative of frequency with respect to time—which in effect extends the signal model to include “linear chirp” signals. The disadvantage of this approach is that the algorithm has only a limited ability to distinguish between weak RFI and noise, and thus tends to suppress both. This can be problematic for the detection of weak signals underlying the RFI. Ironically, the performance of this approach is also constrained by the limitations of adaptive canceling; in fact severely so since, in the basic algorithm, there is no reference antenna to increase the input INR. Detailed discussion of this issue can be found in [21], and one contribution of this dissertation is to address this topic in greater detail. Ellingson and Hampson (2003) [22]
describes an application of the sinusoidal estimation and subtraction approach to L-band astronomy afflicted by ground–based aviation radar, where the limitations of both limited suppression and detection sensitivity (i.e., “You can not suppress what you can not detect.”) are observed.

Although one might consider some combination of the Barnbaum and Bradley “adaptive canceler approach” and the Miller, McCorkle, and Potter “sinusoidal estimation/subtraction approach” to overcome this difficulty; that is, using a reference antenna to increase the input INR and thereby improving the estimation of sinusoidal parameters. This is considered in Section 5.5.2. However this is not an ideal solution as we wish to avoid the implementation problems associated with a reference antenna.

Ellingson, Bunton, and Bell (2001) [23] addressed this difficulty by employing a priori information about the modulation of the RFI signal to improve the “effective” INR of the received RFI. They demonstrate more than 20 dB of suppression of a GLONASS (a direct–sequence spread spectrum satellite) signal received at an INR of −20 dB; i.e., suppression much greater than that possible by adaptive canceling alone. In this case, the fact that the modulated bandwidth is orders of magnitude greater than the message (pre-modulation) signal – i.e., large “processing gain” – is exploited to convert the problem from that of estimating the parameters of the
rapidly-changing modulated RF waveform with low INR to that of estimating the parameters of the slowly-varying message waveform with high INR. In terms of implementation, this turns out to be a simple matter of demodulating the signal to retrieve the information signal, and then remodulating the signal to obtain a noise-free version of the original signal. In this dissertation, this concept is referred to as “demod–remod” and is applied to several other modulations.

In this work, we attempt to extend this “modulation-savvy” approach to improve the performance of RFI mitigation, extending the multi–carrier parametric estimation and subtraction strategy of Miller, McCorkle, and Potter. For example, we exploit the significant (but relatively modest) processing gain and “constant modulus” (magnitude) properties of broadcast FM signals to achieve the benefits of this approach with reduced distortion of the underlying signals of interest. DTV signals have much in common with GLONASS signals, in the sense that they are digitally modulated (“finite alphabet”) and therefore, the general strategy of RFI waveform estimation through demodulation and remodulation can be adopted.

1.4 Primary Application: LWA

The Long Wavelength Array (LWA) is a new radio telescope, now in design process. The LWA will consist of many phased array “stations” distributed over the
state of New Mexico. As currently conceived, each station will consist of 256 broadband active dipole antennas. The output of each antenna will capture the entire spectrum from 10 to 88 MHz, using a direct sampling receiver of the type described in Section 1.1. RFI mitigation techniques described in this dissertation will be directly applicable to LWA.

In this section, we describe a demonstration of prototype LWA hardware in order to more clearly show the nature of the desired and RFI signals that LWA must contend with. The measurement site is located near Blacksburg, Virginia. This site is located a few miles from the Virginia Tech main campus. A prototype LWA receiver chain is used with a thin-blade active dipole antenna. A block diagram of the system is shown in Figure 1.2. The active antenna is described in the work of Ellingson, Simonetti, and Patterson [24]. The active antenna provides 24 dB gain with 1 dB compression point $-3 \text{ dBm}$ and 250 K noise temperature. This antenna is designed for a different system with narrower bandwidth (29–47 MHz), but provides a usable level of sensitivity for the LWA frequency range of interest. The antenna is connected to an analog receiver via 150-ft RG-58 coaxial cable. The analog receiver has 20-80 MHz passband, 54 dB gain, 6 dB noise figure, and $-16 \text{ dBm}$ input third–order intercept point (IIP3). A detailed description of the analog receiver is in [25]. The output of the analog receiver is fed into a custom digitizer [26]. This digitizer uses the 12–bit Analog Devices AD9230 analog–to–digital converter chip, sampling
Figure 1.2: Block diagram of the prototype LWA antenna/receiver demonstration.

at 200 Million samples per second (MSPS). The digitized data is captured by an Analog Devices HSC-ADC-EVALC digital capture board [27] connected to a laptop PC via USB. Additional details of the experimental setup are given in [28].

The measured power spectral density (PSD) is shown in Figure 1.3. Many HF (3–30 MHz), television (54–88 MHz), and FM broadcast signals (88–108 MHz) are visible. Especially notable is a DTV signal at about 63 MHz, and there are few NTSC (analog TV) carriers. However it is also seen that much of the spectrum is apparently empty. A goal of this work is to eliminate or suppress the existing RFI.
Figure 1.3: Results of the LWA antenna/receiver demonstration. PSD averaged over 30 ms.

1.5 Contributions

The purpose of this work is to develop a method of mitigating RFI in wideband receivers for applications such as radio astronomy operating at frequencies below 100 MHz, using a coherent strategy exploiting a priori information such as modulation type. The contributions of this research include the following items.

1. Extended the PE/S strategy of [20]–[23] to narrowband FM and broadcast FM, employing a demod-remod strategy as opposed to assuming that these signals can be modeled as simple tones or chirps (Sections 5.3.2 and 5.3.3).
Demonstrated performance, identified pitfalls, and pros and cons are assessed with respect to the adaptive canceler strategy in [19], which requires a reference antenna (Sections 5.5.1).

2. Used demod–remod as way to enhance the INR of reference channel, further improving performance of adaptive canceling (Sections 5.5.2).

3. Developed and demonstrated a demod-remod PE/S algorithm for digital TV (ATSC) (Section 6.2 and 6.3), and again assessed pros and cons with respect to the adaptive canceler strategy (Section 6.4).

4. Described the special requirements of wideband radio frequency sensing applications (in particular radio astronomy) pertaining to protection of the underlying noise when canceling RFI, and developed a new metric for “toxicity” which evaluates this (Section 3.5.2).

5. Developed a method for generating coherent time-domain noise waveforms with the same non-white power spectral density observed in natural and man-made RFI environments (Section 2.3). Also developed a “reference interference scenario”, based on actual wideband measurements, which can be used as a signal model for future studies of RFI mitigation in the band of interest (Section 2.4 and Appendix D).

6. Used the new noise model to characterize the performance of signal parameter
estimation (in particular, frequency estimation) in the presence of realistic non-white (in particular, Galactic) noise spectra (Section 5.1.2).

7. Proposed a single consistent framework for detection of interfering signals of all kinds in the frequency range 10-100 MHz (Section 3.2).

1.6 Organization of This Dissertation

This dissertation is organized as follows. Chapter 2 (“Radio Frequency Environment”) summarizes the characteristics of radio environment below 100 MHz: RFI sources, propagation effects, and external noise. In Chapter 3 (“Theory of Interference Mitigation”), we summarize existing RFI mitigation methods, develop a detailed problem statement, and introduce the method of parametric estimation and subtraction (PE/S) as a solution to the problem. Performance metrics for RFI mitigation are defined. Chapter 4 (“Detection in the HF/Low–VHF Environment”) considers the problem of detection of RFI in a very wideband receiver bandpass covering large partitions of this frequency range. Chapter 5 (“Parametric Estimation and Subtraction”) provides several applications of PE/S, including examples of mitigation of narrowband FM and broadcast FM. The mitigation of emerging DTV broadcast signal is considered in Chapter 6 (“Mitigation of ATSC”). Conclusions and recommendations for future work are presented in Chapter 7.
Chapter 2

Radio Frequency Environment

In this chapter, the radio environment at frequencies in the range 10–100 MHz is described. Section 2.1 ("Sources of Interference") presents the possible sources of interference and mathematical models for transmissions are described. In Section 2.2 ("Channel Model"), channel propagation models are presented, and Section 2.3 ("Noise Model") introduces the radio noise environment. Section 2.4 ("Reference Interference Scenario") defines a reference interference scenario based on real measured data from the LWA project.
2.1 Sources of Interference

In this section, we describe the various signals existing in the 10–100 MHz range, which are for the most part legal and legitimate, but which are nevertheless RFI to radio astronomy and other applications using these frequencies.

2.1.1 Frequency Allocation and Services

Spectrum in the U.S. is allocated by federal law, following the provisions of international treaties established through meetings of the International Telecommunications Union (ITU). The process is described in useful detail in [9]. To summarize, spectrum is typically allocated to “services” (classes of users) on a “primary” basis or “secondary” basis. The difference between a primary allocation and a secondary allocation is essentially that the users of a secondary allocation must accept interference from the users of a primary allocation, and conversely must not interfere with the users of the primary service. Within this framework, national governments create and enforce additional regulations, typically to specify additional details and further elaborate on permitted uses of the spectrum. In the U.S., Federal use of spectrum is managed by National Telecommunications and Information Administration (NTIA), whereas non–federal (i.e., commercial, amateur, and passive scientific) use of spectrum is managed by Federal Communications Commission (FCC). FCC regulations concerning use of the spectrum are codified in Title 47 of the U.S. Code of
Federal Regulations (CFR) [29].

The frequencies in the range 10–100 MHz are often regarded as consisting of HF (3–30 MHz) and VHF (30–300 MHz). Frequency allocations are summarized in Table A.1 in Appendix A, which also has details for frequency assignment to services. From the perspective of the FCC, radio astronomy is a service (specifically, the “Radio Astronomy Service” or RAS) as is broadcasting, and so on.

The various modulations used in this frequency range are described next.

2.1.2 Analog Amplitude Modulation

Several different forms of amplitude modulation are used in 10–100 MHz. These include:

**Double Sideband (DSB) AM**  This modulation scheme is commonly used for shortwave broadcast, although it is being gradually replaced by Digital Radio Mondiale (DRM). An audio signal of 2–3 kHz bandwidth, \( m(t) \), is modulated as follows [30, Chapter 5]:

\[
\begin{align*}
    s_{RF}(t) &= \Re \{ s(t)e^{j\omega_c t} \} = A_c[1 - \mu m(t)] \cos \omega_c t \\
    s(t) &\triangleq A_c[1 - \mu m(t)]
\end{align*}
\]

(2.1)

where \( \mu \) is a modulation index, which must be \( \leq 1 \), i.e., a typical value is in 0.85–
0.95; $A_c$ and $\omega_c$ are the amplitude and frequency of the carrier, respectively; and $\Re$ means “take the real part”. $s_{RF}(t)$ is the transmitted signal, whereas $s(t)$ is said to be the “baseband representation” of $s_{RF}(t)$. Appendix B addresses models for the voice signal $m(t)$. Note that the passband spectrum is a shifted version of the baseband spectrum, therefore, the bandwidth of the AM signal is at least twice that of the audio signal; i.e., generally $\sim 10$ kHz. Channel spacing for sound broadcasting is addressed in [31]. Figure 2.1 exemplifies DSB-AM with a 3 kHz baseband voice signal.

![Figure 2.1: DSB–AM signal: $A_c = 1$, $\mu = 0.8$, and $m(t)$ is Model 1 (see Appendix B).](image)

Single Sideband (SSB) AM. A variant of DSB–AM is a DSB modulation of which is the same as DSB–AM except that the carrier is suppressed. Its modulation
is modeled by [30, Chapter 5]

\[ s_{RF}(t) = \mathcal{R}\{s(t)e^{j\omega_c t}\} = A_c m(t) \cos \omega_c t \] (2.2)

\[ s(t) \triangleq A_c m(t) \]

We refer to this as DSB suppressed carrier (DSB–SC). In a real–valued signal, the upper and lower parts of the spectrum have the same information. Therefore, it is possible to transmit only one sideband of a DSB–SC signal without any loss of information, but with reduced bandwidth and power. This technique is single sideband (SSB), and depending on which sideband is transmitted, this modulation may be referred to as either upper–sideband (USB) or lower–sideband (LSB). Mathematically, an SSB signal is the baseband analytic form of the information signal mixed to the carrier frequency, and is given by [30, Chapter 5]

\[ s_{RF}(t) = \mathcal{R}\{A_c m(t) * [1 \pm jh_Q(t)]e^{j\omega_c t}\} \] (2.3)

\[ s(t) \triangleq A_c m(t) * [1 \pm jh_Q(t)] \]

where \( h_Q(t) \) is the impulse response of Hilbert transform, and the ‘−’ sign is for USB and the ‘+’ is for LSB. A USB signal is shown in Figure 2.2. Additional details on the use of SSB at HF are given in [32].
2.1.3 Frequency Modulation

An FM signal is given by [30, Chapter 5]

\[ s_{RF}(t) = A_c \cos \left[ \omega_c t + D_f \int_{-\infty}^{t} m(\tau)d\tau \right] \]  \hspace{1cm} (2.4)

where \( D_f \) is the frequency deviation constant. The frequency modulation index \( \beta_f \) is a useful parameter to express the transmission bandwidth of FM signal. This is defined as

\[
\beta_f \triangleq \frac{\Delta F}{B}
\]

\[
\Delta F \triangleq \frac{D_f V_p}{2\pi}
\]

where \( V_p = \max |m(t)| \), and \( B \) is the bandwidth\(^1\) of an information signal. Then,

\(^1\)The FM modulation index is defined only for the case of single-tone modulation. But it is
the transmission bandwidth having 98% of total power is given by \textit{Carson's rule}: 

\[ B_T = 2(\beta_f + 1)B \quad (2.6) \]

FM can be broadly categorized as being either “wideband” FM (WBFM) or “narrowband” FM (NBFM).

\subsection*{2.1.3.1 Narrowband FM}

Narrowband FM is defined as FM whose modulation index \( \beta_f \) is less than 1. Another definition used by the FCC is a FM signal whose frequency deviation is less than \(+/-15\) kHz [33, Section 4.7]. For instance, two-way FM mobile radio has 1 as the modulation index and 5 kHz as the frequency deviation. Standard bandwidths for NBFM signals are 25 kHz, 12.5 kHz, and (now rare but likely to become more common) 6.25 kHz. NBFM signals in the U.S. generally follow TIA-603 [34]. This document includes a set of standards for Land Mobile FM or PM communication equipment, measurement, and performance. An example of two-way FM mobile radio is shown in Figure 2.3.

often used for other waveforms, where \( B \) is chosen to be the highest frequency or the dominant frequency in the waveform.
Figure 2.3: Narrowband 12.5 kHz FM signal: $A_c = 1$, $\beta_f = 1$, $\triangle F = 5$ kHz, $B = 5$ kHz, and $m(t)$ is Model 3 (see Appendix B).

### 2.1.3.2 Broadcast FM

Broadcast FM is classified as “wideband” FM due to its modulation index much larger than 1, and FCC definition is a FM signal whose frequency deviation is greater than +/-15 kHz [33, Section 4.7]. Broadcast FM is actually multiple signals combined through frequency division multiplexing (FDM) prior to modulation: mono audio, stereo audio, pilot, Radio Broadcast Data System (RBDS), and Subsidiary Communications Authority (SCA). They all are illustrated in Figure 2.4. The mono audio is located in the range 30 Hz to 15 kHz. The stereo audio is amplitude-modulated onto a 38 kHz suppressed carrier, which results in a DSB–SC signal in the range 23 to 53 kHz. A pilot tone is at 19 kHz, at exactly one–half of the 38 kHz sub–carrier frequency. RBDS provides digital text using a 57 kHz sub–carrier. This
Figure 2.4: Generation of a broadcast FM signal.

runs at 1187.5 bits per second, and uses shaped biphase symbols given by [35, Chapter 1]

\[
\{\delta(t) - \delta(t - t_d/2)\} * F^{-1}\{H_T(f)\} : \text{logic 1}
\]

\[
\{-\delta(t) + \delta(t - t_d/2)\} * F^{-1}\{H_T(f)\} : \text{logic 0}
\]  

(2.7)
where $\delta(t)$ is the Dirac impulse function and $H_T(f)$ is

$$
H_T(f) = \begin{cases} 
\cos \frac{\pi f t_d}{4} & \text{if } 0 \leq f \leq 2/t_d \\
0 & \text{if } f > 2/t_d 
\end{cases}
$$

(2.8)

and $t_d$ is symbol duration, $(1187.5)^{-1}$ s = 842 µs. The spectrum and waveforms of RBDS are shown in Figure 2.5. SCA Subcarrier channels may appear at 67 kHz and 92 kHz from the main carrier, although 67 kHz is the most used. SCA is a second analog audio signal. Figure 2.6 shows an example of a Broadcast FM signal.

2.1.4 Digital Radio Mondiale (DRM)

Digital Radio Mondiale (DRM) is a emerging HF band digital audio broadcasting technology [36]. DRM is also the name of world wide initiative [37], and the European Telecommunications Standards Institute (ETSI) released the technical
Figure 2.6: Broadcast FM: $\beta_f = 5$, $\Delta F = 75$ kHz, $B = 15$ kHz, and mono audio follows a model of Case 2 (see Appendix B).

specification for the DRM system in September 2001 [38]. Its modulation is orthogonal frequency-division multiplexing (OFDM), and it can deliver sound quality comparable to FM broadcast despite long–distance propagation at frequencies below 30 MHz.

2.1.5 NTSC

NTSC is the analog standard for TV in the U.S. It consists of a vestigial sideband (VSB) video signal and an FM aural signal, as shown in Figure 2.7. The composite baseband video signal in Figure 2.7 is a combination of a raw video signal and a digital waveform which is for synchronizing. The synchronizing is done with the period of 63.5 $\mu$s. VSB signal can be obtained by filtering DSB–AM (2.1) with
Figure 2.7: The overall configuration of NTSC transmitter system.

A VSB filter whose frequency response is given by [39, pages 272–274]

\[
H_{vsb}(\omega - \omega_c) = H_{ssb}(\omega - \omega_c) - H_{\beta}(\omega - \omega_c) \quad \omega > 0 , \text{ where} \quad (2.9)
\]

\[
H_{\beta}(\omega) = \begin{cases} 
  -H_{\beta}(-\omega) & |\omega| \leq \beta \\
  0 & |\omega| > \beta 
\end{cases}
\]

Figure 2.8 illustrates \( H_{vsb}(\omega) \). Then, the output of VSB filter is given by [40, pages 36–38]

\[
s_{RF}(t) = h_{vsb}(t) * s_{DSB-AM}(t) \quad (2.10)
\]

\[
= A_c \cos \omega_c t - A_c \mu \{ [h_I(t) * m(t)] \cos \omega_c t + [h_Q(t) * m(t)] \sin \omega_c t \}
\]

where \( h_{vsb}(t) = F^{-1}\{H_{vsb}(\omega)\} \), \( F^{-1}\{\bullet\} \) denotes the inverse Fourier transform, \( s_{DSB-AM}(t) \) is the RF DSB–AM signal, \( h_I(t) = h_{vsb}(t) \cos \omega_c t \), and \( h_Q(t) = h_{vsb}(t) \sin \omega_c t \). Thus,
VSB is similar to SSB in the sense that it has quadrature carrier form. An example of NTSC is shown in Figure 2.9. Note that most of the power in NTSC signals is near the video carrier and the aural carrier, so that NTSC signal can often be effectively modeled as the sum of the two carriers.

2.1.6 ATSC

The U.S. standard for DTV is named after the defining organization, the Advanced Television Systems Committee (ATSC) [41]. It uses 8-level digital VSB (8VSB) modulation. The symbol rate is 10.76 MHz. The details of the baseband data format are given in Appendix C. The bandpass modulated signal can be explained in terms of a baseband filter method of modulation illustrated in Figure 2.10 [42]. In
(a) Time domain: Composite video signal

(b) Frequency Domain: Composite video signal

Figure 2.9: NTSC: $\beta_f = 1.67$, $\Delta F = 25$ kHz, $B = 15$ kHz, and aural signal is Model 2 (see Appendix B).

This method, an I–channel signal $I_{sq}(t)$ at the input to a quadrature upconverter has an even symmetric frequency response, and a Q–channel signal $Q_{sq}(t)$ at the input to the quadrature upconverter has an odd symmetric frequency response. The output is an upconverted VSB spectrum signal. Then, the bandpass 8VSB signal is modeled as

$$s_{RF}(t) = I_{sq}(t) \cos \omega t + Q_{sq}(t) \sin \omega t,$$

where

$$I_{sq}(t) = p_{sq}(t) \ast \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_s)$$

and

$$Q_{sq}(t) = H\{p_{sq}(t)\} \ast \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_s)$$

Here, $p_{sq}(t)$ is a squared root raised cosine (SRRC) filter with 11.6% roll–off factor.
and first null bandwidth of 6 MHz, \( H\{\bullet\} \) denotes the Hilbert transform, and \( T_s \) is the symbol period. \( a_k \) are real-valued data symbols from the set \( \{-7, -5, -3, -1, +1, +3, +5, +7\} \). An example of 8VSB modulation is shown in Figure 2.11.

A pilot is inserted by adding a constant offset 1.25 to the baseband data sequence. The pilot is intended to assist in carrier acquisition and recovery.

### 2.2 Channel Model

In this section, we briefly describe the nature of propagation channels in the frequency range 10–100 MHz. For the purposes of this section “propagation channel” is defined as the transfer function between a transmitter and a receiver. The transfer
(a) Time domain: $I_{sq}$ without a pilot, and $Q_{sq}$ (b) Frequency Domain: $I_{sq}$ without a pilot, and VSB–filtered signals

Figure 2.11: ATSC signal.

function is typically defined in terms of an impulse response. The parameters which are used to characterize the impulse response of a propagation channel include path loss, delay spread, doppler spread, and coherence time.

The 10–100 MHz frequency range of interest spans multiple “bands,” as they are defined by common convention. The “high frequency” (HF) band is usually taken to be 3 MHz through 30 MHz, and the “very high frequency” (VHF) band is usually taken to be 30 MHz through 300 MHz. In many applications the term “VHF Low” is used to describe frequencies from 25 MHz to 50 MHz, which has significance primarily for regulatory as opposed to technical reasons. Significant differences exist in the nature HF and VHF propagation channels, so they are considered separately. Findings of this section are summarized in Table 2.1.
<table>
<thead>
<tr>
<th></th>
<th>HF</th>
<th>VHF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frequency Range</strong></td>
<td>3–30 MHz</td>
<td>30–300 MHz</td>
</tr>
<tr>
<td><strong>Primary Mechanism</strong></td>
<td>Sky wave refraction</td>
<td>LOS &amp; terrain scattering</td>
</tr>
<tr>
<td><strong>Typical Range</strong></td>
<td>Regional to intercontinental</td>
<td>~ to radio horizon (see “caveats”)</td>
</tr>
<tr>
<td><strong>Doppler</strong></td>
<td>&lt; 1 Hz typical</td>
<td>&lt; 50 Hz</td>
</tr>
<tr>
<td></td>
<td>up to 30 Hz</td>
<td>due to TX motion</td>
</tr>
<tr>
<td><strong>Source Freq. Error</strong></td>
<td>Can be comparable</td>
<td>Up to 20 ppm (up to 1 kHz at 50 MHz) per TIA–603</td>
</tr>
<tr>
<td></td>
<td>due to Doppler</td>
<td></td>
</tr>
<tr>
<td><strong>Delay Spread</strong></td>
<td>≪ 1 ms typical</td>
<td>&lt; 5 ms typical</td>
</tr>
<tr>
<td></td>
<td>up to 7 ms</td>
<td></td>
</tr>
<tr>
<td><strong>Coherence Bandwidth</strong></td>
<td>≫ 1 kHz typical</td>
<td>&gt; 200 kHz typical</td>
</tr>
<tr>
<td></td>
<td>Due to ionosphere</td>
<td>Due to terrain reflection</td>
</tr>
<tr>
<td><strong>Coherence Time</strong></td>
<td>~ 10 min typical</td>
<td>≥ 10 ms; much longer</td>
</tr>
<tr>
<td></td>
<td>Due to ionosphere</td>
<td>for broadcast</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Due to transmitter motion</td>
</tr>
<tr>
<td><strong>Caveats</strong></td>
<td>Disturbed ionosphere,</td>
<td>Sporadic E,</td>
</tr>
<tr>
<td></td>
<td>Multiple hop possible</td>
<td>Meteor scatter</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of HF- and VHF-band propagation.
2.2.1 HF Propagation

At frequencies below 3 MHz (\(\lambda > 100 \text{ m}\)), the Earth acts as a lossy dielectric and the dominant propagation mechanism is “ground wave” propagation in which the radiated wave is literally bound to the surface of the Earth [43]. Ground wave propagation generally has quite limited range as the lossy ground dissipates power as the wave propagates. As frequency increases, radiated waves are eventually able to decouple from the Earth, leading to the emergence of a “sky wave,” which is essentially free-space propagation. The midpoint of this transition from ground wave propagation to sky wave propagation occurs around 10 MHz (\(\lambda \sim 30 \text{ m}\)). Above that frequency, the Earth becomes sufficiently conductive that the ground wave mechanism is essentially “shorted out” and cannot efficiently propagate.

The efficacy of sky wave propagation is highly dependent on the ionosphere. The ionosphere is a layer of free electrons which exists above the Earth’s atmosphere [44]. The presence of free electrons has a refractive and dispersive effect on propagating sky waves. The electron density is time-varying and inhomogeneous over spatial scales ranging from wavelengths to continental dimensions. In order for a sky wave to return to earth, the electron density must be sufficiently large for refraction to redirect the sky wave toward Earth. The effectiveness of the ionosphere in reflecting a sky wave decreases with increasing frequency, resulting in a “maximum useable frequency” (MUF) above which the ionosphere no longer efficiently reflects
sky waves. The MUF varies with latitude, time of day, season, and phase within the 11-year solar cycle. Over a daily cycle, MUF varies roughly from a few MHz to tens of MHz.

A single reflection from the ionosphere is usually sufficient to allow propagation over continental distances or further. In the frequency range above a few MHz and below the MUF, both the ionosphere and the surface of the Earth are efficient reflectors, and thus the possibility of multiple reflections ("multiple hop") between the surface of the Earth and the ionosphere are possible. This allows HF band signals to sometimes propagate with very low loss over intercontinental distances. Thus, spectral occupancy in the HF band, from the perspective of a receiver on the ground, appears to vary according to a daily cycle as transmission from more or fewer stations are able to propagate over the required distance.

The time-varying and inhomogeneous electron density of the ionosphere imparts both Doppler (frequency) spread and multipath (delay) spread onto the sky wave. The associated impulse response is well-described by a wide sense stationary uncorrelated scattering (WSSUS) model [45, 46]:

\[ h(\tau; t) = \sum_{n=1}^{N} \alpha_n c_n(\tau; t) \delta(\tau - \tau_n) \] (2.12)

where \( \tau \) is the delay parameter, and \( t \) is used simply to index the current state of the time-varying impulse response function. In this equation, \( N \) is the number of resolvable discrete paths (typically a small number or just 1), \( \alpha_n \) is the attenuation
associated with path \( n \), \( \tau_n \) is the differential delay associated with path \( n \), \( \delta(\tau) \) is the Dirac delta function, and \( c_n(\tau; t) \) is a complex-valued quantity which describes both the unresolved portion of the delay spread as well as the doppler spread of path \( n \). The received signal is then given by the convolution of \( h(\tau; t) \) with the transmitted signal.

In general, \( c_n(\tau; t) \) is quite complex; e.g., see [45, 46, 47]. However, HF ionospheric propagation channels can usually be treated as stationary over 10’s of kHz and 10’s of minutes. Exceptions are times around sunset and sunrise, and during periods of extreme ionospheric disturbance. Also, for the purposes of modeling propagation of narrow bandwidth communication signals, it is usually reasonable to simplify \( c_n(\tau; t) \) to represent only Doppler shift, as the part of the delay spread modeled by this coefficient cannot be resolved over the limited bandwidth of most communications signals. The simplified model is then:

\[
h(\tau; t) = \sum_{n=1}^{N} \alpha_n e^{j\omega_{d,n}\tau_n} \delta(\tau - \tau_n)
\]

(2.13)

where \( \omega_{d,n} \) is the Doppler shift associated with path \( n \).

In terms of this model, the HF sky wave channel typically exhibits Doppler spreads less than 1 Hz, increasing to as much as 30 Hz during times of severe disturbance. Note that the Doppler spread can easily be as large as or larger than the expected frequency error associated with the transmitter: For example, 0.1 ppm frequency offset (representing a mediocre frequency standard at the transmitter) at 30 MHz is
a 3 Hz error. Doppler can arise independently due to motion of the transmitter; e.g., transmission from ground vehicles, ships, or aircraft. Generally, only aircraft speeds are large enough to generate Doppler shifts sufficient to dominate over ionospheric Doppler or source frequency error.

Delay spreads for the HF sky wave channel are typically $\ll 1$ ms, increasing to as much as 7 ms during times of severe disturbance. The expected delay spread does not typically have much impact on received signals because the associated coherence bandwidth (the bandwidth over which the channel can be assumed to be approximately constant, and roughly equal to the reciprocal of the delay spread) is typically $\gg 1$ kHz, whereas most HF-band modulations occupy only a few kHz. However at extreme values associated with highly disturbed ionospheric conditions (e.g., 7 ms), the associated coherence bandwidth shrinks to just a few hundred Hz. Under these conditions, the channel can therefore become “frequency-selective” even from the perspective of typical HF communications bandwidths. Usually, however, the channel can be assumed to be “flat”; i.e., constant over the bandwidth of the signal.

### 2.2.2 VHF Propagation

The efficiency of the ionosphere as a reflector of radio waves falls off sharply with increasing frequency. Above 30 MHz, efficient ionospheric reflection is a relatively
rare occurrence. As a result, the propagation of signals at VHF tends to be limited by the curvature of the earth, with antenna height being a significant factor. A common expression for this “horizon” distance assuming that one antenna is located at ground level is

\[ R = (4.12 \, \text{km}) \sqrt{\frac{h_a}{1 \, \text{m}}} \]  

where \( h_a \) is height of the other antenna above ground [48]. For example, a broadcast antenna mounted at a height of 100 m can cover a radius of about 41 km through line-of-sight (LOS) propagation, assuming no terrain blockage. If the LOS path within the circle defined by the radio horizon is blocked by terrain, then propagation is obviously impeded. In this case, a situation-specific propagation path loss prediction technique such as Longley-Rice (also known as the ”Irregular Terrain Model” (ITM)) [49]–[52] is typically employed. (The Okumura-Hata method [53] is another popular technique but is not believed to be valid below 150 MHz.) Longley-Rice path loss predictions are nominally valid in the range 20 MHz to 20 GHz, but are known to be vulnerable to a number of problems of both a theoretic and practical nature (e.g., [54]). Various efforts to refine and standardize methodologies are considered in [55] (30–1000 MHz) and [56] (100 MHz–800 MHz). It should also be noted that terrain scattering can allow communications beyond the radio horizon, although typically with attenuation that is much greater than free space.

Whereas the ionosphere is usually not a factor in VHF propagation, terrain features
such as mountain ranges are sufficiently large compared to a wavelength to become efficient reflectors. Since terrain features are utterly stationary, the associated multipath channels tend also to be highly stationary and free of doppler. However, VHF frequencies are commonly used for mobile communications (where, in contrast to HF frequencies, compact resonant-mode antennas are possible). Thus, considerable doppler is often observed due to motion of the transmitter. For example, the maximum Doppler shift expected from a vehicle moving at 100 mph at 50 MHz is about 25 Hz. Source frequency errors are probably dominated by the frequency stability of the transmitter. For example, the relevant NBFM specification TIA–603 [34] permits carrier frequency errors up to 20 ppm (i.e., 1 kHz at 50 MHz).

If significant multipath scattering exists at the transmitter’s location (regardless of whether it resolvable in time or not), then an interference pattern (commonly known as “Rayleigh fading”) is created which is spatially periodic approximately with a period of $\sim \lambda/2$ [48]. The associated coherence time can be taken to be roughly $1/10$ of the time it takes for the transmitter to traverse this distance. So for the example of a vehicle moving at 100 mph and transmitting at 50 MHz the coherence time is lower bounded at $\sim 20$ ms.

Delay spread in the VHF channel depends on the difference in propagation time between the most direct path and paths which are specularly-reflected from terrain features. To have significant strength, the longer path must typically lie entirely
within the radio horizon, which then places an upper bound on the maximum possible delay spread. For example, consider the $h_a = 100$ m example above, where the other end of the link is located at the radio horizon. A rough guess at the path length associated with the longest detectable multipath is $\sqrt{2}R$. The delay spread is then determined by the associated differential propagation delay $(\sqrt{2}R - R)/c \approx 57 \mu s$. In practice, maximum delay spreads of only about 1/10 of this; i.e., $\sim 5 \mu s$, are encountered in practice (e.g., [57]). Given that multipath at VHF and above appears under normal circumstances to be determined primarily by reflection from terrain features, it could be anticipated that the delay spread is frequency independent. This is consistent with the findings of experimental studies [48, 58]. The associated coherence bandwidth is lower-bounded to $\sim 1/(5 \mu s) = 200$ kHz and thus VHF-band communications, which have bandwidths of 200 kHz or less, typically do not experience significant frequency-selective fading.

Occasionally, ionospheric conditions become disturbed in a way that allows HF-type ionospheric propagation to prevail even at VHF frequencies. For example, “Sporadic E” ($E_s$) conditions can occur in which the ionosphere temporarily becomes an efficient reflector over a region of the Earth for minutes to hours at a time [44]. When this happens, a portion of the VHF band behaves very similarly to HF below MUF. The MUF for $E_s$ conditions is sometimes as high as 70 MHz, and occasionally much higher, but only very rarely extending into the FM broadcast band. Because
Eₙ conditions are localized, multiple reflections are typically not possible. Eₙ is most common over North America in daylight hours from April through July, and events range in length from a few minutes to a few hours.

Yet another phenomenon which can cause VHF to take on HF-like ionospheric propagation behavior is *meteor scatter* [59]. Meteors entering the Earth’s atmosphere leave a trail of ionization which can be sufficiently dense to become an efficient reflector of sky waves at frequencies throughout the VHF band. These ionization trails are very localized and typically last only for seconds; on the other hand there is a steady stream of meteors falling to earth throughout the day. From the perspective of the receiver, propagation via meteor scatter is perceived as only an intermittent “ping” during which a transmitter which would not normally be detectable is briefly observed. Meteor scatter is sufficient to allow communications at VHF frequencies over continental distances.

### 2.3 Noise Model

This section reviews characteristics of radio noise, focusing on the frequency range of interest, 10–100 MHz. A widely-used summary of the characteristics of radio noise is available in Recommendation ITU-R P.372-8 [60]. Components of external radio noise include:
1. Radiation from celestial radio sources, dominated by the ubiquitous Galactic synchrotron radiation background [61, 62].

2. Atmospheric noise from natural atmospheric processes, primarily lightning discharges in thunderstorms. Generally this is negligible unless storms are nearby, except at frequencies $\ll 10$ MHz.

3. Man–made noise from unintended radiation of electrical machinery, electrical and electronic equipment, power transmission lines, internal combustion engines, and other sources. The contribution of man–made noise to total noise usually depends on population density and land use.

The contributions of these components have been measured and characterized in considerable detail [60]. The aggregate strength of external noise may be characterized in terms of an “environmental noise factor” defined as

$$F_a = \frac{P_n}{kT_0B} + 1 \quad (2.15)$$

where $P_n$ is available noise power from the antenna, $k$ is Boltzmann’s constant ($= 1.38 \times 10^{-23}$ J/K), $T_0$ is the reference temperature taken as 290 K, and $B$ is the noise equivalent bandwidth of the receiving system. Alternatively, the environmental noise can be described in terms of a mean noise temperature $T$, following a power law $a f^{-b}$ where $f$ is the frequency, and a variance with respect to location $\sigma^2$. The total noise temperature is the linear sum of the Galactic noise and the applicable category
Table 2.2: Parameters for noise temperature $T = af^{-b}$ [K], $f$ in Hz [Courtesy S. M. Hasan, Virginia Tech].

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Quiet Rural</th>
<th>Rural</th>
<th>Residential</th>
<th>Business A/B</th>
<th>Galactic</th>
</tr>
</thead>
<tbody>
<tr>
<td>3–30</td>
<td>a</td>
<td>3.81 × 10$^{25}$</td>
<td>2.53 × 10$^{26}$</td>
<td>8.54 × 10$^{26}$</td>
<td>2.30 × 10$^{27}$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>2.86</td>
<td>2.77</td>
<td>2.77</td>
<td>2.77</td>
</tr>
<tr>
<td>30–100</td>
<td>a</td>
<td>-</td>
<td>2.53 × 10$^{26}$</td>
<td>8.54 × 10$^{26}$</td>
<td>2.30 × 10$^{27}$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-</td>
<td>2.77</td>
<td>2.77</td>
<td>2.77</td>
</tr>
<tr>
<td>σ</td>
<td></td>
<td>5.3 dB</td>
<td>5.3 dB</td>
<td>4.5 dB</td>
<td>6.6 dB</td>
</tr>
</tbody>
</table>

of man-made noise. The values of $a$ and $b$ have been summarized in Table 2.2, derived from data provided in [60]. Furthermore, [60] describes the variation in noise power as a function of location in terms of “decile variations” $D_u$ and $D_l$; i.e. the values exceeded 10 percent and 90 percent of the time. Assuming Gaussian statistics (consistent with the observation that $D_u$ and $D_l$ are found in [60] to be symmetric about the mean), it is straightforward to derive $\sigma$ from these values; these are also reported in Table 2.2. $S_{ext} = kT$ is the noise power spectral density due to the external noise. Then, (2.15) can be rewritten as

$$F_a = \frac{S_{ext}}{kT_0} + 1$$

(2.16)

Typical environmental noise figures are shown in Figure 2.12.
In the work reported in this dissertation, it is necessary to have a realistic time domain simulation of noise. One method is to begin with spectrally white Gaussian noise \( z_{wg}(t) \) whose PSD is 1 W/Hz and filter it according to an impulse response \( h_{rn}(t) \) as follows:

\[
z_{rn}(t) = h_{rn}(t) \ast z_{wg}(t)
\]

where \( \ast \) denotes convolution and \( h_{rn}(t) \) is related to the inverse Fourier transform of the noise power spectral density \( S(\omega) \) as follows:

\[
h_{rn}(t) = cF^{-1}\{\sqrt{S(\omega)}\}, \text{ where}
\]

\[
S(\omega) = kT_0F_a(\omega)
\]

Note that \( h_{rn}(t) \) is a real-valued impulse response due to the fact that \( S(\omega) \) is also
Figure 2.13: The (averaged) PSD of Galactic radio noise simulated using the method
described in the text: “mean” is plotted 5 dB lower than its value for clarity.

real valued and has Hermitian symmetry. \( c \) in (2.18) is a scale factor defined by

\[
c = \sqrt{\frac{\int_{-\infty}^{\infty} S(\omega) d\omega}{\frac{1}{T} \int_{T} h_{rn}^2(t) dt}}
\]  
(2.19)

This scale factor guarantees the power of \( z_{rn}(t) \) is same with that of \( S(\omega) \). Figure 2.13 shows this simulated noise PSD, compared to Galactic noise using parameters from Table 2.2.

An issue which becomes important is the extent to which the noise spectrum appears
to be colored. To address this, we define the following quantity:

\[
\frac{S_1}{S_2} = \left( \frac{f_1}{f_2} \right)^{-b} = \left( \frac{f - B/2}{f + B/2} \right)^{-b}
\]  
(2.20)

\( S_1 \equiv kaf_1^{-b} \) and \( S_2 \equiv kaf_2^{-b} \)
Figure 2.14: Degree to which Galactic noise ($a = 1.07 \times 10^{23}$ and $b = 2.52$) is colored with respect to observed bandwidth (6 MHz, 200 kHz, and 20 kHz).

where $B$ is the observed bandwidth. Figure 2.14 shows plots of (2.20) with three different bandwidths $B$: 6 MHz, 200 kHz, and 20 kHz, which represents the bandwidths of TV, broadcast FM, and narrowband FM signals, respectively. This implies that TV and broadcast FM are less affected by the colored Galactic noise due to their allocated frequencies: The maximum $\frac{S_1}{S_2}$ is $\sim 1$ dB for TV and $\sim 0.5$ dB for broadcast FM. In contrast, we see that 20 kHz bandwidths are never significantly colored, and that 200 kHz bandwidths are potentially significantly colored only below about 50 MHz. The actual impact of colored noise will be addressed further in Chapter 5.
2.4 Reference Interference Scenario

This section describes a “reference interference scenario (RIS)” based on an analysis of actual measurements. The purpose of this is to (1) provide a convenient method to simulate a realistic RFI scenario, and (2) support future work on the evaluation of algorithms addressed in this dissertation. The measurement was conducted about 60 miles west of Socorro, N.M. (34.1° N, 106.9° W), near the center of the Very Large Array (VLA) radio telescope, which will be the location of the first LWA station. The most appropriate noise model for this site is “Galactic” due to its very remote location. The data were collected from 17:00 local time Nov. 28, 2006 though 17:00 the following day. Details about the measurement are in [63]. Tables in Appendix D summarize the RIS which reflects the data from the measurement. A time domain signal is then generated using the models described in this chapter, assuming Galactic noise. Figure 2.15 shows the resulting power spectral density.
Figure 2.15: The PSD for the Reference Interference Scenario (integration over 1.5 ms). For comparison, the noise PSD alone is also plotted, but 5 dB lower for clarity.
Chapter 3

Theory of Interference Mitigation

The process for interference mitigation consists of the processes of detection and mitigation. These two topics are discussed in this chapter, providing the theoretical background for subsequent chapters. Section 3.1 ("Isolation of Interference") addresses a preliminary issue pertaining to the spectral separability of interference sources. Section 3.2 ("Detection of Interference") describes detection generally, and Section 3.3 ("Detection of TV Signals") discusses the special case of detection of TV signals, which have relatively large bandwidth. Section 3.4 ("Suppression of Interference") describes methods for the suppression of interference once it is detected. Section 3.5 ("Metrics of Interference Mitigation") defines metrics for evaluation of mitigation performance, including special considerations for radio astronomy.
3.1 Isolation of Interference

Before discussing the detection and mitigation of signals, we must first address the important issue of spectral separability of signals. This is an issue particularly with wideband receivers, which are prone to have multiple signals simultaneously in the passband. When the mitigation processing is performed in the discrete–time domain, the frequency resolution depends on the sampling rate. If the sampling rate is sufficiently low (or can be reduced to a sufficiently low rate) each of the interferers can be isolated via a bandpass FIR filter of acceptable length, and the estimation procedure for each of the interferers boils down to that for a single interferer, and thus mitigation is relatively simple. Furthermore, in this case, it may be possible to approximate noise as spectrally white over the new passband even though the noise is colored in the original passband, as shown in Figure 2.15. This possibility is discussed in Section 5.1.2. If FIR bandpass filters of sufficient length are not available, or if the sample rate is not sufficiently low, then this spectral isolation of interferers is not possible. In this case, the mitigation processing has to deal with the possibility of multiple simultaneous signals. This leads to procedures in which multiple signals must be considered jointly, and we may have to consider colored noise.
A special case is that of TV (NTSC and ATSC) signals. Due to their large fractional bandwidth, signals corresponding to separate channels are always spectrally separable, whereas signals using the same channel are never spectrally separable. We assume the latter case is rare since frequencies are allocated specifically to prevent this, so if two signals are detectable in a channel, one must be very weak relative to the other.

### 3.2 Detection of Interference

This section considers the detection problem, i.e., how to determine whether an interferer is present. For simplicity, we consider here only signals which can be modeled as tones, which is our case might apply to all RFI other than TV signals (These are treated in a later section). Detection is an important issue because we may not wish to use an RFI suppression algorithm to mitigate an interferer which does not exist, since (1) we may unnecessarily degrade the desired signal by processing it and (2) we may consume hardware or software resources unnecessarily. For this discussion, let us begin with a “multi–tone” input signal, i.e., input consisting of a finite number of sinusoidal signals:

\[
\begin{align*}
x(t) &= \sum_{l=1}^{L} s_l(t) + s_a(t) + z(t), \text{ where} \\
s_l(t) &= a_l e^{j(\omega_l t + \theta_l)} ,
\end{align*}
\]
\( L \) is the number of tones, and \( a_l, \omega_l, \) and \( \theta_l \) are the magnitude, frequency, and phase of the \( l \)th tone, respectively. It should be noted here that \( \omega_l \) is considered to be the “nominal” frequency in the sense that we neglect small errors in \( \omega_l \) such as those due to Doppler or oscillator offsets. In effect, \( \omega_l \) is really selecting a channel (i.e., range of frequencies) although the channel width/spacing depends on frequency band and allocation, and is not explicitly considered here. Fine adjustments in \( \omega_l \) to account for small frequency displacements within a channel are considered later. \( s_a(t) \) is the astronomical signal of interest and \( z(t) \) is noise, possibly colored. \( s_a(t) \) is assumed to be much weaker than \( z(t) \) and so is not a consideration in the RFI detection problem.

Let the set of all possible frequencies at which the \( L \) tones might exist be:

\[
\{\omega_p\}^P_{p=1} = \{\omega_1, \omega_2, \cdots, \omega_P\}. \tag{3.2}
\]

Note that \( L \leq P \) and \( \{\omega_l\}^L_{l=1} \subset \{\omega_p\}^P_{p=1} \). The number of tones \( L \) and the set \( \{\omega_p\}^P_{p=1} \) may not be known \textit{a priori}. Thus, we consider four possible scenarios: (1) Known \( L \) and \( \{\omega_p\}^P_{p=1} \); (2) Known \( L \) but unknown \( \{\omega_p\}^P_{p=1} \); (3) Unknown \( L \) but known \( \{\omega_p\}^P_{p=1} \), and (4) Unknown \( L \) and unknown \( \{\omega_p\}^P_{p=1} \). The most appropriate detection process is potentially different for each scenario, as shown in Figure 3.1.

In Case (1), we have the problem of \( L \) tones present with unknown \( a_l \) and \( \theta_l \), but \( \omega_l \) is constrained to be in the known set \( \{\omega_p\}^P_{p=1} \). This is essentially the Miller,
Figure 3.1: Different strategies for model order and center frequency determination in each of four cases assuming the $L$–tone signal model of (3.1).
Potter, and McCorkle (1997) [20] scenario, and is optimally solved as a joint linear least squares estimation problem. However we potentially have the problem that the noise may be colored, which is not addressed in [20]. This solution is discussed in Section 4.2.1. If the spacings between the known \( \{ \omega_p \}_{p=1}^P \) are sufficiently large so that adjacent tones can be spectrally isolated (e.g., using digital bandpass filter), then the problem is potentially simplified by first using a filter bank to separate the channels \( \{ \omega_p \}_{p=1}^P \), and then treating the output of each filter bank channel as a problem of detecting and estimating a single tone, which is addressed in Section 4.1.1. Recent work from the cognitive radio area has revealed the value of multi–taper spectral estimation as a particularly effective means for detecting and isolating narrowband signals [64, 65].

In Case (2), we have the problem of \( L \) tones present with unknown \( a_l \) and \( \theta_l \) (as in Case 1), but no constraints on possible \( \omega_l \). In this case, the optimal solution is a generalized version of the search described for Case 1, except that this now requires a joint search for the \( L \) unknown \( \omega_p \). In general, this problem can be solved only by a “brute force” search over the range of possible frequencies simultaneously with the tone parameters. This is enormously computationally intensive and will not be considered further here. Alternatively, we can search for frequencies first. Since \( L \) is known \textit{a priori}, a number of excellent techniques exist for estimating frequencies including Nonlinear Least Squares (NLS) and ESPRIT [66]. NLS is optimal but
requires a computationally intensive search. ESPRIT approaches NLS performance and does not require an intensive search. In practice, therefore, ESPRIT is usually preferable [66]. It is important to note that if the noise is sufficiently colored, then the performance of NLS and ESPRIT is degraded. In general, NLS is known to be relatively robust to this, whereas ESPRIT is known to be vulnerable. This is addressed further in Section 4.2.1.3 and 5.1.2.3.

Case (3) and (4) are similar in that in each case the number of tones is not known a priori. However Case (3), in which the set \( \{\omega_p\}_{p=1}^P \) is known a priori, is far simpler in the sense that there is a finite number of possible frequencies to consider. Essentially, Case (3) is the same as Case (2) with the complication that we now must also search over the set of possible frequency combinations; i.e., \( L = 1 \), one \( \omega_p \) at a time, then \( L = 2 \), \( \omega_p \)'s taken 2 at a time, then \( L = 3 \), \( \omega_p \)'s taken 3 at a time, and so on. Thus the number of “Case 2” type joint blind NLS searches required is \( P + \binom{P}{2} + \binom{P}{3} + \cdots + \binom{P}{P} \); i.e., prohibitively large unless \( P \) is very small.

Alternatively, we can treat Cases (3) and (4) as problems of model order detection, in which no a priori information about possible frequencies is assumed. An optimal solution to this problem is described by Wax and Ziskind (1989) [67] for the white noise case and Wax (1992) [68] for the colored noise case. Both techniques require a joint search over frequencies and model order. A simpler suboptimal approach is using a model order estimator such as Minimum Description Length (MDL) [69]
which yields only an estimate of $L$, from which we can proceed as in Case (2).

When the actual received signal does not conform to the model of Equation (3.1) (i.e., is not a countable number of tones), then the detection problem is considerably more complicated and there are no comparably simple algorithms. In this case, we must assume spectral separability of the signals. Chapter 4 discusses this in further detail.

### 3.3 Detection of TV Signals

TV signals have sufficiently large fractional bandwidth that they can not reasonably be modeled as tones unless the sample rate is extraordinarily high and the observation time is very short. In the case of NTSC, we can bypass this problem by treating a single signal as separate audio and video carriers, which can possibly be detected jointly for improved performance. In the case of ATSC, the optimal detection approach is via matched filtering with symbol-synchronous sampling of the output decision metric; i.e., essentially the first stage of an ATSC demodulator. This is described in detail in Chapter 6. Additional work on this topic is reported in [71, 72].
3.4 Suppression of Interference

Two methods of interference suppression applicable to our problem can be categorized as follows: (1) adaptive canceling and (2) parametric estimation and subtraction (PE/S). This section briefly introduces the methods and discusses their limitations.

**Adaptive canceling:** In this approach, the basic idea is to identify and subtract the interference from the received signal [70, pages 246–252]. This is illustrated in Figure 3.2. The estimate of the interference is computed by an adaptive filter which compares the input to a reference signal. The reference signal is nominally an exact copy of the interfering signal. Since an exact copy is typically not available, a more common approach is to separately obtain or generate a model for the interferer which is likely to be highly correlated with it, but uncorrelated with the desired signal components. In the Barnbaum and Bradley (1998) [19] implementation of this scheme, the reference signal is provided by a highly–directional antenna which points at the source of the interference. This approach has the drawback that interference rejection ratio (IRR) is bounded by the interference noise ratio (INR), as mentioned in Section 1.3. It has the additional limitation that it is not guaranteed to preserve the weak astronomical signal; in fact any astronomy which is correlated with the reference signal tends to be cancelled. Another suboptimal version of the
Figure 3.2: An adaptive (notch or whitening) filter. The delay block is to accommodate delay in the adjustable filter.

Adaptive canceler involves taking the output of the canceler as the reference signal, which results in a “whitening” filter. Although simpler, this approach is potentially damaging to weak astronomical signal since the correlation between reference signal and astronomical signal might be very high.

**Parametric estimation and subtraction (PE/S):** If the interference has a well-defined signal model, a noise-free copy of the interference can be synthesized using this model combined with estimates of the model parameters obtained by analysis of the input signal. The interference can then be suppressed by subtracting this copy from the afflicted signal. The conceptual block diagram is shown in Figure 3.3. Compared to the adaptive canceler method, this method ideally leads to 1) no distortion of the desired signal since nominally an exact copy of the inter-
ference is being subtracted and 2) an increase in performance by virtue of the use of *a priori* information about the signal. The simplest scenario suitable for PE/S is the multi–tone model of (3.1), in the special case where $L = 1$. For simplicity, let us assume the signal exists (i.e., neglect the detection issue). The optimal estimates of the parameters, $a_l$, $\theta_l$, and $\omega_l$, are given by [73, Chapter 7]

\[
\hat{\omega}_l = \arg \max_{\omega_l} | < x(t)e^{-j\omega_l t} > |^2
\]

\[
\hat{a}_l = | < x(t)e^{-j\hat{\omega}_l t} > |
\]

\[
\hat{\theta}_l = \angle < x(t)e^{-j\hat{\omega}_l t} >
\]

where the angle brackets denote time averaging. In the $L > 1$ case, however, the procedure becomes dramatically more computationally intensive, especially if done jointly (optimally). A popular suboptimal approach is to decouple the frequency search from the search for the other parameters; in fact frequency information will
typically already be available from the detection process, as noted in Section 3.2. As in the detection process, the situation becomes more complicated when Equation (3.1) is not a reasonable model; i.e., the interference signals are not well-modeled as simple tones. Miller, Potter, and McCorkle (1997) [20] deal with this by defining a second class of sinusoids which are frequency modulated in a simple way: specifically, as a linear (chirp) modulation which can be described through the introduction of one additional parameter – the rate of change of frequency with respect to time. This modification accommodates a broader class of signal. However, one would not expect their approach to work well for signals which are not well-modeled as linear chirp waveforms, as we shall demonstrate in Chapter 5.

PE/S is improved by improving the fidelity of the signal model. A very simple example is Ellingson and Hampson (2003) [22], who use a PE/S approach to mitigate interference from a ground-based aviation radar, consisting of short tone pulses beginning and ending with distinct transient envelopes. In their approach, they develop a model of the pulse from measurements. The signal model then consists of the model pulse plus parameters for magnitude, phase, and delay. The estimation of the parameters is via a correlation receiver derived from the model pulse.

Another example of PE/S with an even more sophisticated signal model is Ellingson, Bunton, and Bell (2001) [23], who demonstrate the mitigation of the direct se-
quence spread spectrum (DSSS) emission from Russian Global Navigation Satellite System GLONASS. In this case, the signal model parameters are complex gain (i.e., magnitude and phase), frequency (accounting for Doppler), and code phase (delay). The estimation stage in this case is essentially the same as a GLONASS receiver, which outputs an estimate of the low data rate (50 b/s) message signal. However, the enormous processing gain associated with the DSSS demodulator allows the parameters to be estimated accurately even at very low INR. At the same time, the act of regenerating the signal from parameters is very effective in excluding astronomical signal from entering the synthesized canceling signal, and so precludes the possibility of corrupting the astronomical signal. This is a very attractive feature for radio science and surveillance applications generally. In this dissertation, we seek to extend the strategy of partial demodulation to exploit processing gain available in wideband signals below 100 MHz.

3.5 Metrics of Interference Mitigation

In the study which follows, it is desirable to have simple metrics to characterize performance. This section presents two metrics of mitigation suitable for radio astronomy: Interference Rejection Ratio (IRR) and quality metric ($Q$). IRR measures the extent to which interference is mitigated and $Q$ measures the extent to which the desired signal $s_a(t)$ in Equation (3.1) is (unintentionally) suppressed due to the
mitigation.

3.5.1 Interference Rejection Ratio (IRR)

The interference rejection ratio is defined as the total interference power present at the input of the interference mitigation process, relative to the total interference power present at the output. Given an input signal

\[ x(t) = s(t) + s_a(t) + z(t) \]  \hspace{1cm} (3.4)

where \( s(t) \) is interference, \( s_a(t) \) is the astronomical signal of interest, and \( z(t) \) is additive white Gaussian noise, and given an output signal

\[ y(t) = \tilde{s}(t) + \tilde{s}_a(t) + \tilde{z}(t) \]  \hspace{1cm} (3.5)

where \( \tilde{s}(t) \), \( \tilde{s}_a(t) \), and \( \tilde{z}(t) \) are components of interference, the astronomical signal, and AWGN, respectively after the mitigation process, then, IRR is given by

\[ \text{IRR} \triangleq \frac{P_s}{P_{\tilde{s}}} \] (Unity gain processor)  \hspace{1cm} (3.6)

where \( P_s = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |s(t)|^2 dt \) and \( P_{\tilde{s}} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |\tilde{s}(t)|^2 dt \). Note that this definition assumes a unity gain processor. If the gain of the processor is possibly different from unity, it is more convenient to use the general definition:

\[ \frac{\text{IRR}}{\text{INR}_{\text{in}}/\text{INR}_{\text{out}}} = \frac{P_s/P_z}{P_{\tilde{s}}/P_{\tilde{z}}} = \frac{P_s}{P_{\tilde{s}}} \frac{P_z}{P_{\tilde{z}}} \] \hspace{1cm} (General case)  \hspace{1cm} (3.7)
where \( P_z \) and \( \tilde{P}_z \) are defined similarly to \( P_s \) and \( \tilde{P}_s \), and we can see that the factor \( P_z/P_{\tilde{z}} \) serves as an estimate of the gain of the processor.

### 3.5.2 Quality Metric: \( Q \)

The quality metric \( Q \) accounts for the *toxicity* of the processing; in particular, the possibility that the processing suppresses the desired astronomy \( (s_a(t) \) in Equation (3.1)) as well as the interference. We define this metric as follows:

\[
Q \triangleq \left| \frac{< s_a(t), \tilde{s}_a(t) >}{< s_a(t), s_a(t) >} \right| \quad \text{(Unity gain processor)} \tag{3.8}
\]

where \(< f(t), g(t) >\) is the correlation operation defined as

\[
< f(t), g(t) > \triangleq \int_{-\infty}^{\infty} f(t)g^*(t)dt \tag{3.9}
\]

We see that \( Q = 1 \) represents nominal performance, whereas \( 0 \leq Q < 1 \) represents suppression of the desired signal (toxicity). Evaluation of toxicity obviously requires controlled conditions and is not possible in field conditions. Thus, to use this metric, \( s_a(t) \) is a given and must be specified as part of the \( Q \)-metric evaluation. \( \tilde{s}_a(t) \) can be determined using the simple experiment shown in Figure 3.4. It should be noted that \( \tilde{s}_a(t) \) obtained in this way is not exact, as the estimation errors \( \varepsilon_1(t) \) and \( \varepsilon_2(t) \) are not guaranteed to be exactly same. However, given that we expect the power in \( s_a(t) \ll P_s + P_{\tilde{z}} \), we expect the difference to be negligibly small.
Figure 3.4: The experiment configuration for measuring $\tilde{s}_a(t)$, which is part of calculation for the toxicity of interference mitigation processing.
Chapter 4

Detection in the HF/Low–VHF Environment

In Section 3.2 (“Detection of Interference”), the detection problem has been classified according to four cases based on the extent of available \textit{a priori} knowledge on the number and the frequency of interferers. This chapter now addresses the details of those procedures, taking into account the relevant properties of the HF/Low–VHF radio frequency environment as described in Chapter 2. The discussion is divided into two parts, according to the “spectral separability” criteria explained in Section 3.1.
Section 4.1 (“Spectrally–Separable Interference Case”) addresses the detection when interferences in received signal are spectrally separable, and Section 4.2 (“Spectrally–Inseparable Case”) deals with the case that spectral separation is not possible.

4.1 Spectrally–Separable Interference Case

In this section, we consider the case that the channelization of the receiver is possible such that interferers can be detected and processed one at a time. Noise is assumed to be additive white Gaussian (AWGN). This is a reasonable assumption since the change in noise PSD over the bandwidth of the widest signal channel (broadcast TV, 6 MHz) is about 1 dB or less as shown in Figure 2.14. There is the possibility of more than one interferer in a channel due to reuse of frequencies; however, we will neglect this situation for the moment. As pointed out in Section 3.2, the detection of tones is straightforward problem whereas the detection of modulated carriers is potentially complex. Thus, in this section, we consider 3 cases separately: (1) tone detected using tone model, (2) modulated carrier detected using tone model (“carrier as tone”), and (3) modulated carrier detected using modulated carrier model (“carrier as carrier”).
4.1.1 Tone Detected using Tone

This section assumes a receiver has the sum of tone interferences.

4.1.1.1 Single Tone Detector

In this situation, we can exploit the strategy of “the detection of one tone at one channel”. Then, from the standpoint of detector, the input signal in one channel is given by

\[
x(t) = s(t) + z(t), \text{ where}
\]

\[
x(t) = Ae^{j(\omega t + \theta)},
\]

\(s(t)\) is baseband model, \(s_{RF}(t) = \Re\{s(t)e^{j\omega t}\}\), and \(z(t) \sim \mathcal{N}(0, \sigma_w^2)\); i.e., WGN since we assume noise is spectrally flat in the passband over which detection is attempted. The problem is that the signal may be present \((A \neq 0)\) or absent \((A = 0)\) and we wish to know which is true. \(A, \theta, \) and \(\sigma_w\) are generally unknown, whereas \(\omega\) may or may not be known but can be constrained to a narrow range \(\{\omega_{\min}, \omega_{\max}\}\). Since there is no constraint on \(A\), the optimal detector will take the form of a threshold test in which the threshold is user–selected according to a trade–off between detection sensitivity (ability to sense small \(A\)) and probability of false alarm [74, Chapter 7]. The relevant detection metric is

\[
r(x; \hat{\omega}, T) \triangleq \left| \frac{1}{T} \int_{T} x(t)e^{-j\hat{\omega}t} dt \right|^2
\]
and the detection algorithm can be expressed as

\[
\begin{aligned}
\max_{\hat{\omega}} r(x; \hat{\omega}, T) > \gamma & : \text{assume signal present (}H_1\text{)} \\
\max_{\hat{\omega}} r(x; \hat{\omega}, T) < \gamma & : \text{assume no signal (}H_0\text{)}
\end{aligned}
\] (4.3)

The (frequency) search is obviously not required if the true value of \(\omega\) is known \textit{a priori}. \(T\) is selected to be as large as possible so as to maximize the INR of the detection metric, but shorter than the expected duration of the signal and expected coherence time of channel. Based on the considerations of channel coherence time in Section 2.2, a conservatively small value for \(T\) is 10 ms. \(\gamma\) implements a user–selectable trade–off between maximizing the conditional probability of detection:

\[
P_{D} \triangleq \Pr \left[ \max_{\hat{\omega}} r(x; \hat{\omega}, T) > \gamma \mid H_1 \right]
\] (4.4)

and the conditional probability of false alarm:

\[
P_{FA} \triangleq \Pr \left[ \max_{\hat{\omega}} r(x; \hat{\omega}, T) > \gamma \mid H_0 \right]
\] (4.5)

For the hypothesis \(H_0\), \(r(x; \hat{\omega}, T)\) in (4.2) is said to be distributed as \(\chi^2_2\), a central chi–square random variable with 2 degrees of freedom, and for the hypothesis \(H_1\), to be distributed as \(\chi^2_2(\lambda)\), a noncentral chi–square random variable with 2 degrees of freedom and a noncentrality parameter \(\lambda\) [74, Chapter 2]. Then, with the predefined threshold \(\gamma\) and \(r(x; \hat{\omega}, T)\) in (4.2), \(P_{FA}\) and \(P_D\) are described by [74, Chapter 7]

\[
P_{FA} = Q_{\chi^2_2} \left( \frac{2\gamma}{\sigma_w^2} \right) = e^{-\frac{\gamma}{\sigma_w^2}}
\] (4.6)

\[
P_D = Q_{\chi^2_2(\lambda)} \left( \frac{2\gamma}{\sigma_w^2} \right)
\]
where $\lambda$ is given by $E_s/\sigma_w^2$, where $E_s$ is the energy of tone $s(t)$ over $T$, $\sigma_w^2$ is the power of (white) noise, and $Q_{\chi^2_2}(x)$ and $Q_{\chi'^2_2}(\lambda)(x)$ are the right tail probabilities of $\chi^2_2$ and $\chi'^2_2(\lambda)$, respectively. The analytic plot of $P_D$ is shown in Figure 4.1, with different $P_{FA}$ associated with different threshold $\gamma$. The threshold is denoted assuming $\sigma_w^2 = 1$. Note that an informed decision on selecting $\gamma$ requires that $\sigma_w$ be known. However, recall from Chapter 2 that it is desirable and feasible for the receiver to be external noise dominated, and therefore $\sigma_w$ is simply a measurement of this external noise. If the receiver is known to be Galactic noise limited, then this value is known \textit{a priori}. Thus, we shall assume $\sigma_w$ is known.
Because the optimal (coherent) detector described above can in some cases be computationally expensive to implement, it is worth considering two other, suboptimal (incoherent) detectors. First is the “energy detector”, for which the detection metric is simply

\[ r(x; T) \triangleq \frac{1}{T} \int_T |x(t)|^2 \, dt. \]  

(4.7)

For the given threshold \( \gamma \), \( P_{FA} \) and \( P_D \) are given by [74, Chapter 5]

\[
\begin{align*}
P_{FA} &= Q_{\chi^2_N} \left( \frac{\gamma \sigma_w^2}{\sigma_w^2} \right) \\
P_D &= Q_{\chi^2_N(\lambda)} \left( \frac{\gamma}{P_s + \sigma_w^2} \right)
\end{align*}
\]

(4.8)

where \( P_s \) is the power of the signal \( s(t) \) and \( N \) is the number of samples in \( T \) assuming Nyquist sampling of \( x(t) \). Figure 4.2 shows \( P_D \) for optimal and energy detectors, through simulation. The result shows the performance of energy detector is much less than that of optimal detector. The energy detector becomes severely impaired as the bandwidth increases, since noise power increases with bandwidth whereas tone power does not. This can be addressed by using filter–bank channelization techniques such as the fast Fourier transform (FFT) to reduce the bandwidth prior to energy detection. However in this case, the distinction between single–tone and multi–tone (non–spectrally isolated) detection is lost, and so further consideration of incoherent techniques is deferred to Section 4.1.4.
Figure 4.2: The performance of optimal (coherent) tone detection and energy detection, compared.

4.1.2 Carrier Detected as Tone

We now consider the case in which we attempt detection using a tone model, in the case in which the interferer is perhaps not well-modeled as tone. Then, \( s(t) \) in (4.1) becomes

\[
s(t) = A(t)e^{j(\omega t + \theta(t))}
\]

(4.9)

In (4.9), \( A(t) \) and \( \theta(t) \) are time-varying, depending on the type of modulation described in Section 2.1. The resulting model mismatch will lead to the degradation of detection performance. An analysis of the degraded performance is left as future work.
4.1.3 Carrier Detected as Carrier

Despite the attractive simplicity of the tone detector, there are some cases in which this is unacceptable. This is the case if improved or optimal performance is required, or if the apparent bandwidth of the carrier (with respect to sample rate) is too large to allow the signal to be reasonably modeled as a tone. The alternative is to employ detectors which exploit the known characteristics (features) of a signal. Additional work is reported in [75]. Detectors in this category include the optimal matched filter detector, and the suboptimal “cyclostationary” detectors.

In the optimal matched filter, the received signal $x(t)$ is passed through a filter which inverts the propagation channel $H(\omega)$ (equalization) and matches the spectrum $S(\omega)$ of the known waveform $s(t)$, as shown in Figure 4.3. The length of the filter depends on the channel and source characteristics, whereas $T$ is selected based on channel coherence time. Detection is accomplished by subjecting the output to a threshold test, as discussed in Section 4.1.1.1.
This approach has the obvious complication that the channel $H(\omega)$ must be known, and invertible. Details for dealing with this issue are well known [76] and will not be considered further here. It is worthwhile noting, however, that the matched filter detector is closely related to the demodulator of the optimal communication receiver, and all the relevant theory applies to this problem. Specifically, if $T$ is taken to be the symbol period, and $s(t)$ is taken to be a symbol from modulation signal set, then Figure 4.3 is the optimal detector for that symbol. It is also worth noting that in many cases, the channel is well modeled as a single complex constant, which is irrelevant to the detection process and can be ignored.

The cyclostationary detector extracts the cyclic features hiding in a signal which is well modeled as a cyclostationary random process. These detectors are summarized in Table 4.1.

Advanced digital modulations such as DRM (Section 2.1.4) and ATSC (Section 2.1.6) have features which make them easier to detect. For example, the PN 63 and PN 511 preambles of ATSC (see Appendix C), which are used for equalizer training, provide detection metrics as side information.

### 4.1.4 Filter Bank Approach

In some cases, none of the detection approaches described in Sections 4.1.1–4.1.3 are convenient or possible to employ. In this case, a filter bank approach should
<table>
<thead>
<tr>
<th>Method</th>
<th>Applicable to</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matched Filter</td>
<td>Any modulation (optimal)</td>
<td>[74]</td>
</tr>
<tr>
<td>Demod.+Threshold test</td>
<td>Any modulation for which demodulation can be defined</td>
<td>[76]</td>
</tr>
<tr>
<td>Cyclostationary detector</td>
<td>Any modulation (suboptimal)</td>
<td>[77, 78, 79]</td>
</tr>
</tbody>
</table>

be considered. In this approach, some or all of the passband is decomposed into \( N \) contiguous channels, where the bandwidth of each channel is approximately equal to the bandwidth of the largest signal to be detected. The computational burden of the filter bank is minimized using the FFT, albeit with some compromise in channel–to–channel isolation due to spectral leakage. A greatly improved filter bank–type method is the multitaper method of spectral detection [64, 65]. The filter bank outputs can be detected coherently or incoherently, as described earlier. It should be noted that this approach is applicable to both spectrally–separable and spectrally–inseparable cases, as the latter case can be converted to the former using a filter with a sufficiently large number of channels.
4.2 Spectrally–Inseparable Interference Case

In Section 4.1, we considered the case in which interferers were spectrally separable, and then could be dealt with one at a time. In this case, we also assumed the noise was sufficiently close to white that it could be assumed to be white. We here consider the case in which signals are too closely spaced with respect to the sample rate that they can not be separated and detected separately. In this case, we may also need to consider the impact of colored noise, since we may be considering bandwidths larger than that of any one signal.

4.2.1 Tone Detected as Tone

In contrast to Section 4.1.1, we must now employ a multi–tone model including colored noise:

\[ x(t) = \sum_{l=1}^{L} s_l(t) + z_c(t), \text{ where} \]

\[ s_l(t) = A_l e^{j(\omega_l t + \theta_l)} \]

The definitions of parameters in (4.10) are same with those in (4.1) with the change that \( z_c(n) \) represents noise which is colored in the manner described in Section 2.3.

The procedure for selecting a detection algorithm was outlined in Section 3.2 (see Figure 3.1). In this section, we shall only outline the remaining details in the
relevant algorithms. When the number of frequencies $L$ is known (Cases 1 and 2), we have: joint linear least squares (LS) (known frequency grid), nonlinear least squares (NLS) (unknown frequency grid), or ESPRIT. When the number of frequencies $L$ is unknown (Cases 3 and 4), we have either joint LS (known frequency grid) or MDL (unknown frequency grid). In subsequent sections, we explain each and show how each is modified to deal with non–white noise.

4.2.1.1 Multi–tone Linear Least Squares

Multi–tone Linear Least Squares (LS) can be used for both white and colored noise, because of its insensitivity to noise model [66, Chapter 4]. The method is applicable where $\{\omega_p\}_{p=1}^{P}$ is available, Cases (1) and (3) in Figure 3.1. The strategy is shown in Figure 4.4. Multi–tone LS is conveniently described using discrete–time

![Figure 4.4: Linear Least Squares for multi–tone detection.](image)
notation. The data model is

\[ x(t_k) = \sum_{p=1}^{P} \alpha_p e^{j\omega_p t_k} + z_c(t_k), \quad k = 1, 2, \cdots, N \]  

(4.11)

where \( \alpha_l = A_l e^{j\theta_l} \) and \( N \) is the number of samples to be considered. Given sample period \( T_S \), \( N \) is nominally \( \sim \tau'_c/T_S \) where \( \tau'_c \) is the lesser of the channel coherence time (\( \sim 10 \) ms as described in Section 2.2) and a small fraction of the signal inverse bandwidth, if the signal is modulated. Then, the matrix form of (4.11) is given by

\[ x = H a + z_c, \quad \text{where} \]  

(4.12)

\[ x = [x(t_1) \ x(t_2) \ \cdots \ x(t_N)]^T, \]

\[ a = [\alpha_1 \ \alpha_2 \ \cdots \ \alpha_P]^T, \]

\[ z_c = [z_c(t_1) \ z_c(t_2) \ \cdots \ z_c(t_N)]^T, \]

\[ H = \begin{bmatrix}
e^{j\omega_1 t_1} & \cdots & e^{j\omega_P t_1} \\
e^{j\omega_1 t_2} & \cdots & e^{j\omega_P t_2} \\
\vdots & \vdots & \vdots \\
e^{j\omega_1 t_N} & \cdots & e^{j\omega_P t_N}
\end{bmatrix} \]

The linear LS estimate of \( a \) for the given (4.12) is thus:

\[ \hat{a} = (H^H H)^{-1} H^H x \]  

(4.13)

Then, based on (4.13), the detection metric of \( \omega_p \) is

\[ r(x; \omega_p, N) = |\hat{\alpha}_p|^2 \]  

(4.14)

where \( \hat{\alpha}_p \) is simply the \( p^{th} \) element of \( \hat{a} \). In Case (1), the detection procedure is to choose the \( L \) largest elements in the obtained set of \( \{ r(x; \omega_p, N) \}_{p=1}^{P} \). In Case (3),

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the procedure is to subject each output to a threshold test (per Equation (4.3)), resulting in both $L$ and $\{\omega_i\}_{i=1}^L$.

### 4.2.1.2 Multi–tone Nonlinear Least Squares

Multi–tone Nonlinear Least Squares (NLS) is applied to the spectrally–inseparable case where only $L$ is available; i.e., Case (2) in Figure 3.1. In contrast to Section 4.2.1.1, $\{\omega_p\}_{p=1}^P$ is not known \textit{a priori}. Therefore, NLS requires a search over frequencies:

$$\{\hat{\omega}_i\}_{i=1}^L = \arg \max_{\{\omega_i\}_{i=1}^L} \{x^H H (H^H H)^{-1} H^H x\}$$  \hspace{1cm} (4.15)

The $L$–dimensional search in (4.15) is likely to lead to an unacceptable computational burden. Thus, this approach is not attractive in many cases, even though the variance of the estimate, asymptotically, approaches the theoretical best (Cramer-Rao Lower) bound. The recommended alternative to NLS for obtaining the frequency estimates for use in (4.13) and (4.14) is ESPRIT, which will be presented in the next section.

### 4.2.1.3 ESPRIT

Estimation of Signal Parameters by Rotational Invariance Techniques (ESPRIT) is one of a class of ”subspace methods” to estimate frequencies of sums of tones. ESPRIT exploits the underlying rotational invariance of the signal space spanned
by two temporally–displaced data vectors [66, 80]. The covariance matrix associated with (4.12) is given by

\[ \mathbf{R} \triangleq E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{H}\mathbf{A}\mathbf{H}^H + E\{\mathbf{z}_e\mathbf{z}_e^H\}, \]

where

\[ \Lambda \triangleq E\{\mathbf{a}\mathbf{a}^H\} \]

It should be noted that ESPRIT assumes white noise. Using ESPRIT in our case constitutes a model violation, since we know the noise is colored to some degree. This will be addressed in more detail in Section 5.1.2.3. For now, we assume \( \mathbf{z}_e \) is white noise with a variance \( \sigma^2 \), even though it is not. Therefore, (4.16) becomes

\[ \mathbf{R} = \mathbf{H}\mathbf{A}\mathbf{H}^H + \sigma^2 \mathbf{I} \] (4.17)

The steps in ESPRIT are as follows: (1) Obtain \( \hat{\mathbf{S}} \), the matrix of eigenvectors of \( \hat{\mathbf{R}} \),

(2) Obtain \( \hat{\mathbf{S}}_1 \) and \( \hat{\mathbf{S}}_2 \), which are given by

\[ \hat{\mathbf{S}}_1 = [\mathbf{I}_{N-1} \mathbf{0}] \hat{\mathbf{S}} \quad \text{and} \]

\[ \hat{\mathbf{S}}_2 = [\mathbf{0} \mathbf{I}_{N-1}] \hat{\mathbf{S}}. \]

where \( \mathbf{I}_{N-1} \) is the identity matrix whose dimension is \( (N-1) \times (N-1) \) and \( \mathbf{0} \) is the \( (N-1) \times 1 \) zero matrix.

(3) Calculate \( \hat{\mathbf{\Psi}} = (\hat{\mathbf{S}}_1^H \hat{\mathbf{S}}_1)^{-1} \hat{\mathbf{S}}_1^H \hat{\mathbf{S}}_2 \), and (4) The frequencies \( \{\omega_l\}_{l=1}^L \) are estimated as

\[ -\arg \hat{\nu}_l \]

where \( \{\nu_l\}_{l=1}^L \) are the eigenvalues of the matrix \( \hat{\mathbf{\Psi}} \). It should be noted that in Step (3) \( \mathbf{\Psi} \) can be solved either a least squares sense or total least squares sense (TLS) by using \( \hat{\mathbf{S}}_1 \hat{\mathbf{\Psi}} = \hat{\mathbf{S}}_2 \).
4.2.1.4 Minimum Description Length

In the case that $L$ is not available and we do not have reliable information about the frequency grid (Case 4), we may consider information theoretic criteria (ITC) approach to select $L$. In ITC, two approaches have been commonly used: the Akaike information criterion (AIC) proposed by Akaike [81] and Minimum Description Length (MDL) by Rissanen [82] and Schwartz [83]. Wax and Kailath bring them into the signal processing area and show that MDL yields a consistent estimate, while the AIC yields an inconsistent estimate [69]. In this work, MDL is chosen because of its consistent estimate.

MDL uses a given set of $M$ observations: $\{x(t_m)\}_{m=1}^{M}$, where

$$x(t_m) = [x(t_m) \ x(t_{m+1}) \ \cdots \ x(t_{m+N-1})]^T.$$  

The estimate of the covariance matrix is obtained by

$$\hat{R}_x = \frac{1}{M} \sum_{m=1}^{M} x(t_m)x^H(t_m) \quad (4.18)$$

Assuming $k$ signals are present, the eigenvalues of $\hat{R}_x$ are denoted by $\{\hat{\lambda}_i\}_{i=0}^{k-1}$ and they are the maximum likelihood estimates of eigenvalues of the true covariance matrix [84]. The MDL criteria for deciding $k$ consists of computing:

$$MDL(k) = \log \left( \prod_{i=k+1}^{N} \hat{\lambda}_{i-\hat{\lambda}^{M-k}}^{1} \right)^{(N-k)M} + \frac{1}{2} k(2N-k) \log M \quad (4.19)$$

for the range of possible $k$ (=1, 2, \cdots). Then, the model order $L$ is decided as $k$
which results in the minimum value of (4.19). It should be noted that this approach assumes noise is AWGN. Therefore, if noise is somehow colored, MDL requires preprocessing such as a whitening filter. An alternative approach is suggested by Wax and briefly introduced in the next section.

4.2.1.5 Joint Nonlinear Detection with MDL–based Subspace Mapping

Wax [68] proposes a detection method with only two assumptions: that the noise is zero–mean Gaussian, and a signal is also Gaussian and is independent of the noise. This model order detection utilizes MDL for signal mapping to signal and noise spaces, and, assuming that \( k \) is the supposed number of signals, and \( \omega^{(k)} \) is a vector having \( k \) frequency elements, it can be summarized as finding MDL of the sum of three terms: 1) the description length of noise subspace components in a given \( \omega^{(k)} \), 2) the description length of signal subspace components in a given \( \omega^{(k)} \) and noise space, and 3) the description length of \( \omega^{(k)} \). The explicit expression of this can be found in [68] and note that this approach detects the number of signal and estimates the frequencies, jointly.
Chapter 5

Parametric Estimation and Subtraction

This chapter investigates interference cancellation through parametric estimation and subtraction (PE/S) which is briefly introduced in Section 3.3 (“Suppression of Interference”). It is assumed that detection (Chapter 4) is already done correctly. To address the importance of the signal model and its effect on the interference cancellation, we first examine the tone–based Linear Least Squares method of Miller, Potter, and McCorkle (1997) [20] to show its limitations in terms of the quality factor defined in Section 3.5.2. We then show how improved performance is possible using the PE/S approach, i.e., using explicitly the known signal model in greater detail. The approach is demonstrated for NBFM and Broadcast FM.
This chapter is organized as follows: Section 5.1 ("Tone Mitigated as Tone") addresses the case that the interference is the sum of tones (sinusoids) and the signal model used is a tone. In Section 5.2 ("Carrier Mitigated as Tone"), the interference is the sum of modulated tones, while the signal model is still sinusoidal. Section 5.3 ("Carrier Mitigated as Carrier") describes the fully implemented PE/S approach for common modulations. Section 5.4 ("Channel Compensation for Broadcast FM Demod–Remod") expands demod–remod PE/S to compensate the multipath effect in Broadcast FM, and Section 5.5 ("Comparison to Adaptive Cancelling, and a Hybrid Technique") evaluates adaptive cancelling (Barnbaum and Bradley approach) and proposes a hybrid technique which combines the demod–remod PE/S and adaptive cancelling.

5.1 Tone Mitigated as Tone

This section considers the suppression of tone interference when the signals are (correctly) assumed to be tones. In Section 5.1.1, we consider a single tone interferer whose frequency is known \textit{a priori} and show the corresponding PE/S mitigation structure and performance. Section 5.1.2 considers the case of a single tone interferer whose precise frequency is unknown. We describe the frequency estimation problem and propose a two-stage frequency search solution. The Cramer Rao Lower Bound (CRLB) is evaluated to determine limits of estimation error, and the interference
rejection ratio (IRR) and quality metric $Q$, defined in Section 3.5.2, are evaluated as
the metrics of mitigation performance. Section 5.1.3 extends the single tone model
to multiple tones. Additional work for frequency estimation is reported in [85, 86].

5.1.1 Single Tone with Known Frequency

This section assumes that “the estimation of one tone at a time” is possible, i.e.,
the spectrally-separable case, and the frequency of the tone is known. The model
for input to the estimator is given by (4.1). With known frequency, the goal of the
single tone estimator is simply to estimate $\alpha = A e^{j\theta}$. The optimal estimate of $\alpha$ is
obtained by

$$\hat{\alpha} = \frac{1}{T} \int_0^T x(t) e^{-j\omega t} dt$$

which is seen to be an intermediate step in computing the detection metric of Equa-
tion (4.2). $T$ is chosen as large as possible, up to the coherence time or, if this

Figure 5.1: Single tone PE/S with a known frequency.

Additional work for frequency estimation is reported in [85, 86].
method is being used with a digital modulation, a small fraction of the symbol time. Then, the interferer is synthesized and subtracted as shown in Figure 5.1, where the output is denoted by \( \tilde{x}(t) \), and \( \varepsilon(t) \) represents the estimation error which is defined as

\[
\varepsilon(t) = (\alpha - \hat{\alpha}) e^{j\omega t} \tag{5.2}
\]

and the goal is to make \( |\varepsilon(t)| \) as small as possible.

The unknown \( \alpha \) consists of two parameters: magnitude \( A \) and phase \( \theta \). The Cramer Rao Lower Bound (CRLB) gives us a lower bound on the variance of any unbiased estimator, and in this case can therefore be used to determine the accuracy with which we can estimate \( A \) and \( \theta \). The detailed derivation of the CRLB can be found in [73, Chapter 3]. Let us assume \( \alpha \) is estimated using \( N \) samples of \( x(t) \), and that these samples are taken at a rate of no greater than the Nyquist rate for the associated noise bandwidth, so that the associated samples of \( z(t) \) can be assumed to be independent. With the assumption that \( N \) is “large”, the CRLB of the unknown parameters is given by

\[
\begin{align*}
\text{var}(\hat{A}) & \geq \frac{2\sigma_r^2}{N}, \quad \text{and} \\
\text{var}(\hat{\phi}) & \geq \frac{2\sigma_r^2}{NA^2} \tag{5.3}
\end{align*}
\]

where \( \sigma_r^2 = \frac{1}{T} \int_0^T |\Re\{z(t)\}|^2 dt \), the power for real part of \( z(t) \), and \( T \) is in this case \( N \) times the sample period. To demonstrate, the performance of single tone parameter
estimation is compared with the CRLB through Monte Carlo simulation. The result is shown in Figure 5.2. We see that the variance of the estimate almost meets the CRLB when \( N > 40 \), even at low INR.

The performance of the single tone PE/S procedure can be described in terms of IRR defined in Section 3.5.1. In this case the power of \( \Re\{s(t)\} \), \( P_s \), is \( A^2/2 \). The power in \( s(t) \) after PE/S, \( P_s \), is the power in \( \varepsilon(t) \) in (5.2). Assuming the phase estimation errors are uniformly distributed with zero mean, this in turn is simply \( \frac{1}{2} \text{var}(\hat{A}) \); i.e., one half the estimation variance in the magnitude of \( \alpha \), given in (5.3).

Thus, we have:

\[
\text{IRR} \triangleq \frac{P_s}{P_s} = \frac{A^2/2}{\frac{1}{2} \text{var}(A)} \approx \frac{A^2/2}{\sigma_r^2/N} = N \frac{A^2/2}{\sigma_r^2}
\]

and since \( A^2/2\sigma_r^2 = \text{INR}_{in} \), \( \text{IRR} \approx N \times \text{INR}_{in} \), assuming we achieve the CRLB. This is proof of the principle identified in Section 1.3, that IRR is approximately proportional to \( \text{INR}_{in} \). Again, it should be emphasized that it is assumed that the noise (\( z(t) \)) samples are independent, which will be the case if the sample rate is at the Nyquist rate for the noise bandwidth or slower. Figure 5.3 shows a comparison of theoretical and simulation results, which confirms the analytical result of (5.5).

We see that IRR improves linearly with \( \text{INR}_{in} \). We also see that IRR improves linearly with the number of samples, but only for \( N \gg 1 \).
Figure 5.2: Single tone estimation performance, known frequency.
Figure 5.3: Single tone PE/S performance, known frequency. Mean over 2000 trials.
5.1.2 Single Tone with Unknown Frequency

This section removes the restriction that frequency is known \textit{a priori} and addresses our attention to the estimation of the frequency.

5.1.2.1 Frequency Estimation Methods

The optimal estimator of the parameters, $A$, $\theta$, and $\omega$, can be expressed in terms of a simple modification to the known frequency estimator from (5.1) in Section 5.1.1. Now, the procedure is

$$
\hat{\omega} = \arg \max_\omega \left| \frac{1}{T} \int_0^T x(t)e^{-j\omega t} \right|
$$

(5.6)

$$
\hat{\alpha} = \frac{1}{T} \int_0^T x(t)e^{-j\hat{\omega}t} dt
$$

and then $\hat{A} = |\hat{\alpha}|$ and $\hat{\theta} = \angle \hat{\alpha}$ as before. The CRLB in this case is [73, Chapter 3]

$$
\text{var}(\hat{\omega}) \geq \frac{12}{\eta N(N^2 - 1)}
$$

(5.7)

$$
\text{var}(\hat{A}) \geq \frac{2\sigma_r^2}{N}
$$

(5.8)

$$
\text{var}(\hat{\phi}) \geq \frac{2(2N - 1)}{\eta N(N + 1)}
$$

(5.9)

where $\eta$ is INR$_{\text{in}} = A^2/2\sigma_r^2$. It is interesting to note that the magnitude lower bound is independent of the frequency estimate (compared to (5.3)), whereas the phase lower bound is increased.
Because the frequency search in (5.6) is computationally intensive, there are two
categories of spectral estimation technique to consider: non-parametric methods and
parametric methods [66]. Non-parametric methods rely on the evaluation of power
spectral density (PSD). The frequency is estimated as the location of the peak of
the PSD. These methods are typically based on periodogram theory and tend to
be limited in resolution. However, periodogram methods can be used to initialize
the optimal algorithm implied by Equation (5.6), and this is how they will be used
here. Parametric methods assume the signal of interest conforms to the paramet-
ric signal model of $s(t)$ in (4.1), where $\omega$ appears explicitly as a parameter. If the
assumed model closely approximates the observed signal, parametric methods can
provide more accurate frequency estimation than non-parametric techniques; other-
wise, these methods may yield biased estimation due to model error. Of parametric
methods, ESPRIT described in Section 4.2.1.3 is generally considered to be supe-
rior [66]. However ESPRIT is also known to be vulnerable to error when noise is
non–white. Thus, in the single tone case, we have two possibilities: (1) The optimal
procedure of (5.6), initialized using a periodogram approach, which is computa-
tionally expensive but robust to non–white noise; and (2) ESPRIT, which has relatively
low computationally burden but is more vulnerable to non–white noise. We outline
the procedures for (1) and (2) in Section 5.1.2.2 and 5.1.2.3, respectively.
5.1.2.2 Frequency Search Algorithm

To implement (5.6), we define

$$P_{xx}(\omega) \triangleq \left| \frac{1}{T} \int_{0}^{T} x(t)e^{-j\omega t} \right|^2$$  (5.10)

such that the estimator becomes

$$\hat{\omega} = \arg \max_{\omega} P_{xx}(\omega)$$  (5.11)

In the discrete time domain, we use $N$ samples of $x(t)$ and $P_{xx}(\omega)$ becomes

$$P_{xx}(\omega) = \left| \frac{1}{N} \sum_{n=1}^{N} x(t_n)e^{-j\omega t_n} \right|^2$$  (5.12)

which is recognized to be the Discrete Time Fourier Transform (DTFT). A practical consideration is that the Fast Fourier Transform (FFT) is usually used in practice, yielding a frequency estimate which is quantized to the grid of FFT frequency bins. Rather than selecting $N$ very large to yield the desired frequency resolution, a two-stage frequency search method is adopted for frequency estimation of single tone [87, 88, 89]. This method consists of the following steps: 1) Coarse frequency search with FFT, 2) Fine frequency search based on the result of coarse frequency search. The details for each of these steps are as follows.

**Stage 1: (Coarse search):** The implementation of this is simply done with FFT whose length ($L_{FFT}$) should be $\frac{F_S}{\Delta f}$ where $F_S$ is sampling frequency and $\Delta f$ is approximately equal to the bandwidth of the actual (modulated) signal. If $N \gg$
Then, we average squared magnitudes of FFTs for as long as possible in order to reduce the variance associated with noise. Then, $\omega_L$ and $\omega_H$ for the fine search are chosen as the edges of the (FFT) bin having the largest value.

**Stage 2 (Fine search):** The fine frequency search is done using an iterative and convergent binary search. This approach is considered a suitable method for hardware implementation because a binary search relies only on multiply and accumulate (MAC) operations, and its performance is comparable to the CRLB with enough iterations. With the frequency range of $[\omega_L, \omega_H]$ obtained from the coarse search, the fine (binary) search is performed to find the peak of the periodogram, (5.12), through the algorithm detailed in Table 5.1.

The matter remains of how to determine $f_{\text{resol}}$, the desired resolution for binary search. This can be obtained as follows: If we for the moment assume perfect estimation of $\hat{A}$ and $\hat{\omega}$, and assume these to be 1 and 0, respectively, we have

$$\text{IRR} = \frac{|e^{j\theta}|^2}{|e^{j\theta} - e^{j\hat{\theta}}|^2} = |1 - e^{j\Delta\theta}|^{-2}$$

(5.13)

where $\Delta\theta = |\theta - \hat{\theta}|$. Over time $T$, the associated frequency error is $f_{\text{resol}} = \Delta\theta / 2\pi T$. Thus, $f_{\text{resol}}$ is determined by minimum acceptable IRR.

To demonstrate the performance of fine frequency search in Galactic and white noise, the frequency of a single tone is estimated in simulation. The noise bandwidth (NBW) is 1 MHz, 100 samples at 2.048 MSPS are used, and 10,000 trials
Table 5.1: The binary fine frequency search algorithm

Given: $\omega_L$ and $\omega_H$ from Stage 1, and

$f_{resol}$ (desired resolution).

1) Compute $P_{xx}(\omega_L)$ and $P_{xx}(\omega_H)$.

2) if $P_{xx}(\omega_L) > P_{xx}(\omega_H)$, then

$$\omega_H \leftarrow (\omega_L + \omega_H)/2$$

else

$$\omega_L \leftarrow (\omega_L + \omega_H)/2$$

3) if $\omega_H - \omega_L > 2\pi f_{resol}$, then go to 1)

4) if $P_{xx}(\omega_L) \geq P_{xx}(\omega_H)$ then $\hat{\omega} = \omega_L$, else $\hat{\omega} = \omega_H$.

are performed. In this experiment, we assume that detection and coarse search are already done and the FFT bin size ($\Delta f$) is 8 kHz with $L_{FFT} = 256$. For $f_{resol}$, we suppose that we would like to achieve IRR up to 80 dB and therefore, $f_{resol}$ is set as 0.325 Hz by Equation (5.13). It should be noted, however, that we know from the CRLB that this level of IRR can not actually be achieved, and is used here only to ensure a conservative number of search iterations. The results are shown in Figure 5.4. Figure 5.4(a) presents bias at $\text{INR}_{in} = -10, 0, \text{ and } 10 \text{ dB}$. This shows that bias of fine search has little dependence on noise type but has some dependence on $\text{INR}_{in}$; specifically smaller bias with higher $\text{INR}_{in}$. RMS error shown in
Figure 5.4(b) demonstrates that it is close to CRLB and that the noise type has negligible impact on RMS error. Note the bias is relatively large because only about 49 µs (100 samples at 2.049 MSPS) are used. The behaviors of the bias and RMS error, according to INR$_{in}$, are shown in Figures 5.4(c) and 5.4(d). Figure 5.4(c) shows some dependence of bias on INR$_{in}$, but as can be seen, the dependence is weak. From these results, we can conclude that fine search can be used in Galactic (colored) noise without significant loss of performance. Before going further, it is worthwhile noting that in 5.4(d) the RMS error is lower than the CRLB for INR$_{in}$ = $\sim$ −10 dB. This occurs due to the quantization of the coarse result due to 8 kHz FFT bin size: at INR$_{in}$ = $\sim$ −10 dB, the CRLB is 3.571 kHz and the obtained RMS error of frequency estimation with 8 kHz FFT bin size is 2.31 kHz in worst case$^1$.

5.1.2.3 ESPRIT

ESPRIT is already described in Section 4.2.1.3. This section addresses the primary concern about ESPRIT, that is, its performance in the non–white radio noise environment below 100 MHz, described in Section 2.3. To do this, the performance of single tone ESPRIT was evaluated in simulation with a noise bandwidth (NBW) of 1 MHz. This NBW is wide enough to have several narrowband interferences

$^1$Worst case means the error of frequency estimation has uniform distribution from -4 kHz to 4 kHz.
Figure 5.4: Comparison of fine search performance for white and Galactic noise (Mean over 10,000 trials. NBW=1 MHz. $N=100$).
Figure 5.4: Comparison of fine search performance for white and Galactic noise
(Mean over 10,000 trials. NBW=1 MHz. N =100) (con’d).
Figure 5.4: Comparison of fine search performance for white and Galactic noise
(Mean over 10,000 trials. NBW=1 MHz. $N=100$) (con’d).
Figure 5.4: Comparison of fine search performance for white and Galactic noise (Mean over 10,000 trials. NBW=1 MHz. $N = 100$) (con’d).
which are well-modeled as tones. For one frequency estimate, 100 samples with 2.048 MSPS are used, and 10,000 trials are performed. Figure 5.5(a) and 5.5(b) show bias and RMS error as function of frequency at $\text{INR}_{in} = -10, 0, \text{and} 10 \text{ dB}$. Figure 5.5(a) demonstrates the bias performance of ESPRIT, that is, its dependence on $\text{INR}_{in}$ and noise type (colored or white). This experiment shows that ESPRIT is somehow affected by Galactic noise and that increasing INR decreases the effect of Galactic noise on frequency estimation of ESPRIT. This is consistent with the previously-known result that bias decreases with increasing INR for ESPRIT [90]. The largest bias differences are $\sim 40 \text{ kHz}$, $\sim 0.2 \text{ kHz}$, and $\sim 0.02 \text{ kHz}$ for $\text{INR}_{in} = -10, 0, \text{and} 10 \text{ dB}$, respectively. Figure 5.5(b) demonstrates that ESPRIT has almost same RMS error in both white and Galactic noise. But note that the RMS error is always larger than the CRLB, and are also much larger than biases in Figure 5.5(a). Figure 5.5(c) and 5.5(d) shows the bias and RMS error as function of $\text{INR}_{in}$.

From this study, we can conclude that Galactic (colored) noise mainly acts to increase bias of ESPRIT but its effect is minor, compared to the RMS error, which is approximately independent of noise color and frequency. The more significant problem with ESPRIT in this application is its degraded performance compared to the CRLB and fine frequency estimation $\text{INR}_{in}$ below 0 dB, as can be seen by comparing Figures 5.5(d) and 5.4(d). For this reason, the fine frequency search estimate
is preferable if the computational burden can be accommodated.

Figure 5.5: Comparison of ESPRIT performance for white and Galactic noise (Mean over 10,000 trials. NBW=1 MHz.).

5.1.2.4 IRR Performance

The performance of single tone PE/S when the frequency is unknown can be analyzed in a manner very similar to that for the known frequency case. Now we have

$$
\varepsilon(t) = \alpha e^{j\omega t} - \hat{\alpha} e^{j\hat{\omega}t}
$$

and the power of \( \varepsilon(t) \), \( P_s \), can become quite large compared to the single tone with
Figure 5.5: Comparison of ESPRIT performance for white and Galactic noise (Mean over 10,000 trials. NBW=1 MHz.) (con’d).
Figure 5.5: Comparison of ESPRIT performance for white and Galactic noise (Mean over 10,000 trials. NBW=1 MHz.) (con’d).
Figure 5.5: Comparison of ESPRIT performance for white and Galactic noise (Mean over 10,000 trials. NBW=1 MHz.) (con’d).
known frequency. Figure 5.6 shows the results with white noise, analogous to those in Figure 5.3. The single tone parameters are obtained by Equation (5.6). The sample rate is 512 MHz, and for frequency estimation, the fine search algorithm is used with the coarse search ($L_{FFT} = 1024$ and FFT bin size=0.5 MHz). The frequency resolution in the fine search is set as 27 Hz based on the assumption that the minimum acceptable IRR, Equation (5.13), is 80 dB with 300 samples (the maximum number of samples in this simulation), being conservative as explained in Section 5.1.2.2.

It can be observed that IRR is degraded for the unknown frequency case, compared to the known frequency case, and that the degradation is greatest at high INR, and negligible at low INR. To demonstrate the impact of “fine” frequency estimation method and noise type, we simulate IRR considering binary search and ESPRIT, with white and Galactic noise. NBW is chosen as 1 MHz. To investigate “coloring” of Galactic noise, the single tone is placed at an RF frequency of 20 MHz where the degree of Galactic noise “coloring” is significant. The results are shown in Figure 5.7, where it can be recognized that in this range of INR, binary search is always better, and Galactic noise little degrades IRR performance.
Figure 5.6: Single tone PE/S performance (white noise). Mean over 2000 trials.
Figure 5.7: Single tone PE/S performance with two stage and ESPRIT for frequency estimation (Mean over 200 trials and NBW=1 MHz).
5.1.3 Multiple Tones

This section discusses multiple tones, which is modeled by (4.11). When frequencies are known, linear least squares solution is used to estimate magnitude and phase, leading to optimal solution, as already discussed in Section 4.2.1.1. If frequencies are not available, nonlinear least squares estimation is possible, which is described in Section 4.2.1.2. As in the single tone case, the high computational effort of the \( L \)- (the number of tones) dimensional search motivates two alternatives: periodogram “peak counting” and ESPRIT. These techniques generalize directly from the single to multiple tone case, and the pro and cons are assumed to be analogous. Additional work for the estimation of multiple–tones can be found in [91, 92, 93].

5.2 Carrier Mitigated as Tone

In this section, we consider the performance of PE/S assuming a single tone applied to realistic modulated signals in realistic noise. With simulation, we investigate three modulations, USB, NBFM, and broadcast FM, with varying the integration time \( T \) in Equation (5.6). As mentioned earlier, \( T \) should be chosen as large as possible, up to the minimum expected coherence time, \( \sim 10 \) ms. Since it can be expected that the optimal \( T \) is related to the modulation bandwidth, the simulation uses \( T = \frac{1}{100B}, \frac{1}{10B}, \) and \( \frac{1}{B} \), where \( B \) is the signal bandwidth. The assumed values
of $B$ are 3, 12.5, and 200 kHz for USB, NBFM, and broadcast FM, respectively. NBW=40 kHz is used for USB and NBFM, and 400 kHz for broadcast FM. The modulation parameters and voice model for USB and NBFM are the same as those described in the captions of Figures 2.2 and 2.3, respectively. For broadcast FM, the captions of Figure 2.6 gives the modulation parameters, voice model, and FDM subcarrier choices used in this simulation.

The simulation result is shown in Figure 5.8. With $T = \frac{1}{B}$, we observe that IRR for every modulated signal becomes saturated after a certain INR$_{in}$: $\sim -5$ dB for USB, $\sim 5$ dB for NBFM, and $\sim 10$ dB for Broadcast FM. Therefore, it can be concluded that $\frac{1}{B}$ is too large for $T$, causing severe model mismatch error at high INR. The apparently good performance at low INR is also in fact bad: in this case the RFI is partially correlated with the noise, such that the noise is partially cancelled as well, which is obviously not desired. All cases with $T = \frac{1}{B}$ give an non–saturating IRR for INR$_{in} \leq 10$ dB. In case of $T = \frac{1}{100B}$, IRR is better than that in case of $T = \frac{1}{10B}$ for some INR region, but only slightly so. Because we wish small $T$ to deal with time varying channel, $\frac{1}{10B}$ is the best choice as a rule of thumb.

To investigate the effect of model mismatch on “fine” frequency estimation in both white and Galactic noise, we simulate binary search and ESPRIT for USB, NBFM, and broadcast FM. To consider the effect of Galactic noise, it is assumed that a signal is located at 11 MHz. $T$ is $\frac{1}{10B}$, NBW is 400 kHz, and RMS error is
evaluated. The sample rate is 1 MHz. For binary search, we assume coarse search is done with $L_{FFT} = 128$, which leads to the FFT bin size of 7.8 kHz, and the frequency resolution is set as 0.5 Hz, assuming that a minimum acceptable IRR, Equation (5.13), is 80 dB with $T = 33.33 \mu s$ matched to the narrowest modulation, USB. The results are shown in Figure 5.9. The frequency estimation performance for the two noise types are almost same since the NBW of 400 kHz is much smaller than 1 MHz. In the results, RMS error of frequency estimation for tone can be the reference for measuring RMS error caused by model mismatch. At $\text{INR}_{in} = 30 \text{ dB}$ in white noise, the RMS error of frequency estimation for tone is $\sim 100 \text{ Hz}$ for ESPRIT and binary search, as shown in Figure 5.9(a). At same conditions, the RMS error for USB is $\sim 1 \text{ kHz}$ in Figure 5.9(a), the RMS error for NBFM is $\sim 3 \text{ kHz}$ in Figure 5.9(b), and the RMS error for Broadcast FM is $\sim 5 \text{ kHz}$ with binary search and $\sim 20 \text{ kHz}$ with ESPRIT in Figure 5.9(b). Compared to the RMS error for tone, $\sim 100 \text{ Hz}$, we see that modulated signals experience degradation of frequency estimation caused by model mismatch, i.e., all RMS errors are larger than $\sim 100 \text{ Hz}$. Especially, broadcast FM has the most degradation and this motivates the introduction of a “chirp” parameter for broadcast FM, which is described in Section 5.3.3.1. For Galactic noise case (Figure 5.9(c) and 5.9(d)), we have the same conclusion, implying that the coloring effect of Galactic noise is negligible with the NBW of 400 kHz.

\footnote{To evaluate the effect of model mismatch, high $\text{INR}_{in}$ is selected.}
Figure 5.8: IRR performance for carrier mitigated as tone (Mean over 50,000 trials).
In this section, we account for the fact that the received signal is a modulated carrier and exploit this to obtain the copy of interference. When the modulation can be described in terms of slowly-varying parameters as in the tone case, this is a form of PE/S. However, an alternative approach is to partially demodulate the signal, which removes noise, and then remodulate. We call this “demodulation–remodulation (demod–remod) method”. Demod–remod exploits the inherent processing gain available in most modulations. Additional work is reported in [94].

Figure 5.8: IRR performance for carrier mitigated as tone (Mean over 50,000 trials) (con’d).

5.3 Carrier Mitigated as Carrier

(c) Broadcast FM (NBW=400 kHz).
Figure 5.9: Frequency estimation with modulated carriers, white noise ($T = 1/10B$, Mean over 50,000 trials, and NBW=400 kHz.)
Figure 5.9: Frequency estimation with modulated carriers, Galactic noise ($T = 1/10B$, Mean over 50,000 trials, and NBW=400 kHz.)
As in previous sections, this section models the received signal as

\[ x(t) = \sum_{n=1}^{N} s_n(t) + s_a(t) + z(t) \]  \hspace{1cm} (5.15)

However we generalize the interference signal model as follows:

\[ s_n(t) = a_n(t)e^{j(\omega_n(t)t + \theta_n(t))} \]  \hspace{1cm} (5.16)

The meanings and behaviors of the parameters in (5.16) depend on the modulation of the RFI signal. Correspondingly, the optimal method of estimation is different for the various modulations. However, identification of the modulation type at low INR is a difficult additional problem, so we first consider a generic estimation procedure which is implied by (5.16). It is shown in Figure 5.10 and is applicable to all modulations which are sufficiently narrowband to be considered as well–modeled as tone over some period. For simplicity, we assume the signal of interest is first isolated using a bandpass filter (BPF) centered at \( \omega_n \). Then, the parameters, \( a_n, \omega_n, \) and \( \theta_n \), are estimated by assuming that the signal conforms to the single tone model (4.1) over the period of \( T = T_2 \) since the last update of the parameters. The parameters \( a_n, \omega_n, \) and \( \theta_n \) are determined using the procedure of Section 5.1.2, since the instantaneous frequency is unknown and varying due to the modulation. The bandpass filter should be wide enough to include the entire bandwidth of the signal, plus any error in the initial coarse frequency estimate. The update rate \( T_2 \) for sinusoidal parameter estimation is a trade–off between sensitivity (favoring
relatively long $T_2$) and desire to accurately track parameter variations resulting from modulation (favoring relatively short $T_2$). In practice, $T_2$ should normally be approximately one–tenth of the inverse bandwidth of the signal, as demonstrated in Section 5.2.

![Generic estimation procedure diagram](image)

**Figure 5.10: Detail of generic estimation procedure.**

### 5.3.1 Narrowband Amplitude Modulation

For amplitude-modulated signals including SSB, in (5.16), $a_n(t)$ is the audio “message” signal and its form is described in Section 2.1.2; $\omega_n(t)$ is the carrier frequency which varies only slightly and slowly due to a combination of the limited frequency stability of transmitting equipment and Doppler effects; and $\theta_n(t)$ is an arbitrary phase offset which also varies very slowly, primarily in response to propagation conditions. For SSB, the bandwidth is about 5 kHz and thus $T_2$ should be about 20 $\mu$s which corresponds to one–tenth of signal bandwidth; and since this is $\ll$ channel
coherence time, other effects such as frequency stability and Doppler need not be separately accounted for. To avoid jitter resulting from endpoint effects in the estimation of parameters, it is recommended to apply a Bartlett (triangular) window over the time-domain samples to be processed to obtain any given update.

PE/S does not work well for AM band modulations, and will not be considered further here. The fundamental problem is that PE/S can not distinguish between signals and noise for the associated time–bandwidth product $T_2 \times B = (20 \ \mu s)(5$ kHz)$=0.1$. As a result PE/S suppresses noise as well as interference, resulting in a spectral “hole”. An example of this effect is shown for NBFM in Figure 5.14. In general, we will need some way to increase the effective time–bandwidth product for PE/S to be effective. This is not possible for AM–band modulations, but is possible with FM–band modulations as will be shown in the next section.

5.3.2 Narrowband FM Modulations

5.3.2.1 Generic Procedure Parameters

For NBFM signals, in (5.16), the message is represented by $\theta_n(t)$, and $a_n(t)$ varies only slightly and slowly, relative to the inverse message bandwidth, due primarily to multipath fading as discussed in Section 2.2. Based on signal characteristics, the parameters for the generic approach in Figure 5.10 are as follows: The bandwidth of bandpass filter should be $\sim 15$ kHz, and $T_2$ should be 6.7 $\mu$s. Results will be
demonstrated by example in Section 5.3.2.3.

5.3.2.2 Demod–Remod for Narrowband FM

Because $a_n(t)$ is nominally constant, FM signals are said to have “constant modulus”, a property that can be exploited here. As noted in Section 2.1.3.1, standard bandwidths for NBFM signals are 25 kHz, 12.5 kHz, and (now rare but likely to become more common) 6.25 kHz. The relatively large occupied bandwidth compared to the message bandwidth means NBFM signals contain a considerable degree of redundant information; this is essentially the “processing gain”. This processing gain is exploited in NBFM systems to achieve improved audio quality; however here we take advantage of it in much the same manner it is exploited in [23]; that is, to enhance discrimination between signal and noise. Figure 5.11 shows the improved procedure which is applicable to NBFM signals. In this case, $x_n(t)$ is demodulated by computing the time-derivative of $\theta_n(t)$, which yields the message signal. This is then low-pass filtered to suppress extraneous noise. The processing gain is obtained by the fact that the message signal is mostly preserved, whereas any noise outside the low-pass bandwidth is suppressed. The message signal is then remodulated by time-domain integration and addition of initial phase effect to obtain the improved estimate of $\theta_n$. Concerning LPF design, the passband must include the entire message spectrum, and passband ripple must be minimized to prevent error.
in the remodulated signal. Note that the principal advantage in this procedure is that the new sinusoidal phase estimates now respond primarily to the signal, as opposed to the combination of signal plus noise. Thus, this procedure reduces the extent to which the algorithm distorts \( s_n(t) + z(t) \), as will be demonstrated in the next section.

A second modification for the NBFM-specific procedure of Figure 5.11 is that the constant modulus property of NBFM is exploited to improve the estimation of \( a_n \). This is achieved by averaging the estimates over a period \( T_3 \) which is as long as possible, but much shorter than the channel coherence time (typically \( \leq 10\text{ms} \) as described in Section 2.2).

Figure 5.11: Detail of improved estimation procedure for NBFM signals.
5.3.2.3 Experiment

To assess the effectiveness of the proposed algorithms, we applied them to a real-world example of NBFM signals. The data consist of four separate weather radio stations operated by the U.S. National Oceanic and Atmospheric Administration located in Arizona (Southwest USA). This includes a station at 162.400 MHz located at Flagstaff, a station at 162.425 MHz located at Payson (Mt. Ord), a station at 162.500 MHz located at Globe (Signal Peak), and a station at 162.550 MHz located at Phoenix. Although these signals are not in the primary frequency range of interest for this work (10–100 MHz), the data are in all other respects applicable to the present problem, except for the degree to which the noise might be non-white noise. The data provided to the algorithm is in complex baseband form sampled at 256 kSPS, and the total length of the dataset is ~1 seconds long. The average spectrum is shown in Figure 5.12. It can be seen that the modulation is of the 12.5 kHz variety, and that the INRs are ~8.7 dB, ~10 dB, ~19 dB, and ~35 dB within each signal’s passband. Note that the spectrum of each signal includes a strong narrow peak which seems to be independent of the expected modulation. The explanation for this feature can be seen in Figure 5.13, which shows the dataset in the form of a spectrogram. The narrow peak corresponds to periods, visible in the spectrogram, over which the carrier is effectively unmodulated. These unmodulated periods occur

\[3\text{This data is provided courtesy of Michael Gray (http://www.kd7lmo.net)}\]
because these stations continue to transmit even when there is no voice activity.

![Figure 5.12: Power spectral density (PSD) of the real-world NBFM dataset averaged over 0.967 s.](image)

The algorithm parameters used to process this data are as follows: The carrier frequency is $\sim 162$ MHz. Therefore, assuming the RFI transmitter is moving with the speed of 30 m/s, $T_C$ becomes 26 ms; thus a reasonable value for $T_3$ is 3.9 ms, i.e., much shorter than $T_C$ and much longer than noise correlation distance induced by the BPF. First, we consider the results using the generic processing method of Figure 5.10 with $T_2 = 6.7 \mu s$. The result is shown in Figure 5.14. Note that the generic processing algorithm results in the deep suppression of all four NBFM signals. However, noise which occupies the same bandwidth as the RFI signals are
Figure 5.13: Spectrogram (power spectral density vs. time and frequency) for the real-world NBFM dataset. Time–frequency resolution is 3.2 ms × 321.5 Hz.

also suppressed. This is due to the generic algorithm’s inability to discriminate RFI from noise, as noted in Section 5.3.1.

Next, we consider the results using the NBFM-specific processing method of Figure 5.11 with $T_2 = 6.7 \mu$s and $T_3 = 3.9$ ms. The bandwidth of the LPF following the differentiator is set at 4 kHz, which includes the entire message spectrum. $\omega_n$ is estimated just once, i.e., $T_4 \sim 1$ s. The result is shown in Figure 5.15. Note in this case the noise spectrum is preserved, but the RFI is not as deeply suppressed.

Finally, we consider the potential toxicity of the RFI processing to weak astronomical signals, $s_a(t)$. To do this, we created simulated spectral lines in the form of
Figure 5.14: PSD before (top) and after (bottom) application of the generic mitigation algorithm (Figure 5.10).

sinusoids added to the data, with two cases of frequency offset, 1 and 5 kHz, above the center frequency of each of the 4 RFI signals, and with magnitude such that it has SNR of $\sim 6$ dB with respect to the resolution bandwidth of the spectrum shown in Figure 5.12. The spectrum of these signals by themselves is shown in Figure 5.16 for 1 kHz offset. Figures 5.17–5.19 show the results for generic and NBFM-specific processing. In Figure 5.17, it can be seen that the generic processing method suppresses all signals, including the artificial spectral lines. Thus, the generic algorithm is not suitable for spectroscopy. In Figures 5.18 and 5.19, the suppression results with NBFM–specific processing are seen to depend on the frequency offsets. In case of 1 kHz offset, the simulated spectral lines are suppressed by $\sim 50\%$. In case of
Figure 5.15: PSD before (top) and after (bottom) application of the NBFM–specific mitigation algorithm (Figure 5.11).

5 kHz offset, they are preserved. The reason the lines with 5 kHz offset are preserved whereas the lines with 1 kHz offset are not is explained by the bandwidth of the LPF in the “demod–remod” path in Figure 5.11, which is 4 kHz in this example. Thus, the choice of the bandwidth of this LPF is a trade–off between a large value (e.g., 5 kHz), which provides the most effective suppression, and a small value (e.g., 3 kHz), which protects largest fraction of bandwidth against the toxic effect of the mitigation algorithm.
Figure 5.16: PSD of 4 sinusoids located at 1 kHz above the center frequencies of the RFI signals. The dash line is the mean of noise power spectral density.
Figure 5.17: PSD after application of the generic mitigation algorithm (Figure 5.10) with 4 additional sinusoid signals at 1 kHz offset.
Figure 5.18: PSD after application of the NBFM–specific mitigation algorithm (Figure 5.11) with 4 additional sinusoid signals at 1 kHz offset. Markers indicate the proper frequency and level of the additional signals.
Figure 5.19: PSD after application of the NBFM–specific mitigation algorithm (Figure 5.11) with 4 additional sinusoid signals at 5 kHz offset. Markers indicate the proper frequency and level of the additional signals.
5.3.3 Broadcast FM

The complex structure of the broadcast FM signal was documented in Section 2.1.3.2. The bandwidth for broadcast FM signals is $\sim 200$ kHz and quite large compared to the combined bandwidth of the information signal. As a result, broadcast FM signals contain a considerable degree of redundant information; i.e., large “processing gain”. Because the ratio of information bandwidth to occupied bandwidth is smaller for broadcast FM than for NBFM, we can expect better performance from a demod–remod technique for broadcast FM than narrowband FM. Like narrowband FM, broadcast FM also has the constant modulus property.

5.3.3.1 Generic Procedure Parameters

Based on the results of Section 5.3.2, the use of the generic algorithm of Figure 5.10 is not advisable here. However, we might consider the use of a similar procedure in which we introduce an additional “chirp” parameter, as in [20]. The revised generic procedure is shown in Figure 5.20. Only difference from generic procedure in Figure 5.10 is that $\omega_n$ is linearly time–varying with chirp rate $c$. $\omega$, $c$, and $\alpha$ are given by

$$\begin{align*}
\{\hat{\omega}, \hat{c}\} &= \arg \max_{\omega, c} \left| \frac{1}{T} \int_0^T x(t) e^{-j(\omega + \hat{c} t)t} dt \right| \\
\hat{\alpha} &= \frac{1}{T} \int_0^T x(t) e^{-j(\hat{\omega} + \hat{c} t)t} dt
\end{align*}
$$

\[(5.17)\]
Note that this is a search like Equation (5.6), except now in two dimensions (over $c$ and $\omega$, where $\omega$ is initialized by fine frequency search and $c$ is initialized to zero).

![Diagram of generic chirp estimation procedure](image)

Figure 5.20: Detail of generic chirp estimation procedure.

From the specification of the signal, the bandwidth of bandpass filter should be chosen as 200 kHz, which include entire bandwidth of the signal and any error in coarse frequency search, and the update rate $T_2$ for chirp parameter estimation should be 0.5 $\mu$s. Results will be demonstrated by example in Section 5.3.3.3.

### 5.3.3.2 Demod–Remod for Broadcast FM

Figure 5.21 shows an improved procedure based on “demod–remod” principle which is applicable to broadcast FM signals. In this case, $x_n(t)$ is demodulated by computing the time derivative of $\theta(t)$, which yields the message signal consisting of the various subcarriers. Each subcarrier is processed independently. Referring to Figure 2.4, the subcarrier processing is as follows: Mono audio, stereo audio, and
SCA are simply filtered using bandpass filters of order 2048 having 3 dB bandwidths of 0–15 kHz, 22.5–53.5 kHz, and 59.5–76 kHz, respectively. The pilot signal is processed to obtain magnitude and phase using known–frequency single tone estimation per (5.1), and then a “clean” tone is synthesized with these parameters. The digital RBDS subcarrier is demodulated (i.e., binary data recovered) and then remodulated to obtain clean copy. In this procedure, the processing gain is obtained by the fact that each of sub–carriers is preserved, whereas any noise outside the audio signal bandwidth is suppressed. Finally, the message signal is then remodulated by time–domain integration and addition of phase offset to obtain the improved estimate of $\theta_n$. Note that the principal advantage in these procedures is that the new sinusoidal phase estimates now respond primarily to the signal alone, as opposed to the combination of signal plus noise. Thus, these procedures reduce the extent to which the algorithm distorts $s_a(t) + z(t)$, as will be demonstrated in the next section.

A second modification for the broadcast FM–specific procedure of Figure 5.21 is that the constant modulus property of the broadcast FM is exploited to improve the estimation of $a_n$ in an enhanced way, as we shall now explain. The procedure is derived as follows: In Figure 5.21, the complex envelope of $x_n(t)$, at output of complex envelope extraction, is given by

$$x_n(t)e^{-j\omega_nt} = a_ne^{j\theta_n(t)} + z(t)e^{-j\omega_nt}$$

$$= a_n e^{j\theta_n(t)} + b(t)e^{j\theta_z(t)}$$

(5.18)
where $b(t)$ and $\theta_z(t)$ are magnitude and phase, respectively, of $z(t)e^{-j\omega_n t}$. Note that $\theta_z(t)$ follows zero-mean uniform distribution if $z(t)$ is Gaussian and the power of $s_n(t)$ is much smaller than $z(t)$. After obtaining the improved estimate of $\theta_n(t)$, the magnitude–normalized complex envelope is synthesized as

$$u(t) \equiv e^{j \theta_{n,\text{imp}}(t)}$$  \hspace{1cm} (5.19)$$

where $\theta_{n,\text{imp}}(t)$ denotes the improved estimate of $\theta_n(t)$ from the procedure described above. Then, conjugating (5.19) and multiplying the result with (5.18) leads to

$$\tilde{A}(t) \equiv a_n e^{j \epsilon} + b(t)e^{j(\theta_z - \theta_{n,\text{imp}})}$$  \hspace{1cm} (5.20)$$

where $\epsilon(t) \equiv \theta_n(t) - \theta_{n,\text{imp}}(t)$. When $\epsilon \ll 1$, (5.20) is approximated by

$$\tilde{A}(t) = a_n + j a_n(t) \epsilon + b(t)e^{j(\theta_z - \theta_{n,\text{imp}})}$$  \hspace{1cm} (5.21)$$

Figure 5.21: Detail of improved estimation procedure for broadcast FM signals.
By taking the real part of (5.21) and averaging it, we can exclude the part from phase estimation error, suppress the noise effect, and obtain $a_n(t)$:

$$E\{\Re[\tilde{A}(t)]\} \approx E\{a_n + b(t)\cos(\theta_z - \theta_{n,imp})\} = a_n.$$ \hspace{1cm} (5.22)

This is implemented as shown in Figure 5.21.

Now, we must decide the period $T_3$. In this case, the effect of multipath on $a_n$ must be considered since Broadcast FM, in contrast to NBFM, is likely to experience frequency selective fading. If the channel has no multipath, the averaging should be done over a period $T_3$ which is as long as possible, but much shorter than the channel coherence time as the narrowband FM case. For Broadcast FM, the carrier frequencies are $\sim 100$ MHz. Therefore, assuming the RFI transmitter is moving with the speed of 67 mi/hr, $T_C$ becomes 43 ms; thus a reasonable value for $T_3$ is 3.9 ms, i.e, much shorter than $T_C$ and much longer than noise correlation time induced by the BPF. Note also that this is conservative with respect to the “10 ms” rule of thumb from Table 2.1.

However, if the channel has significant multipath, $a_n$ is amplitude–modulated by the multipath. This issue is addressed by Corrington [95] and Walton, et al. [96]. Walton, et al. gives the following expressions for envelope and instantaneous frequency (FM demodulated signal) of FM, assuming a two–ray channel and a sinusoidally–
modulated FM:

\[ a_n(t) \approx a_n \sqrt{1 + \alpha^2 + 2\alpha \cos \phi - (2\alpha \beta \omega_m \tau \sin \phi \cos \omega_m t} \quad (5.23) \]

\[ \omega(t) = \omega_c + \chi [\cos(\omega_m t - \xi)] \quad (5.24) \]

where \( \alpha \) is a ratio of gains for the two–ray channel, \( \tau \) is the path delay difference, \( \omega_m \) and \( \omega_c \) are the frequency of the sinusoidal–modulating signal and the frequency of the unmodulated FM signal, respectively; \( \phi = \omega_c \tau \), and \( \chi \) and \( \xi \) are functions of the signal and channel parameters. \( \beta \) is the modulation index. It should be noted that the approximation in Equation (5.23) is valid with the restrictions of \( \tau \ll 2\pi/\omega_m \) and \( \beta \leq 3 \). Equation (5.23) shows that \( a_n(t) \) is partially amplitude–modulated by multipath interference and this suggests that the period \( T_3 \) should be determined by considering the bandwidth of the modulating signal. This issue is already addressed in Section 5.2 and thereby, as rule of thumb, \( 1/10B \) is an acceptable choice, where \( B \) is the bandwidth of the modulating signal. Then, the matter remain is how to decide \( B \) for broadcast FM. As shown in Figure 2.4(a), the modulating signal consists of FDM subcarriers. The power of mono audio signal is much greater than those of the others, and the power of mono audio signal is concentrated in low frequency range (This will be demonstrated with real–world data in Figure 5.26.). Here, we choose \( B = 5 \) kHz which has 80% power of the modulating signal. Then, \( T_3 \) becomes 20 \( \mu \)s. Note that this is much shorter than 3.9 ms, which one would have selected in the absence of frequency–selective fading.
5.3.3.3 Experiment

To demonstrate the effectiveness of the proposed algorithm, we applied it to a simulated broadcast FM signal and a real–world broadcast FM signal. The simulated signal is considered first. The simulated signal was generated as follows: The mono and stereo audio signals were generated using the Case 2 audio model in Appendix B, the SCA audio using the Case 3 audio model in Appendix B, and the RBDS signal is generated from a pseudorandom bit sequence. Doppler effects are neglected both because they are small compared to the modulation bandwidth and because Doppler has negligible effect on FM demodulation. The composite signal spectrum was shown already, in Figure 2.6. Furthermore, we assume white noise (as opposed to colored noise), since the change in noise power spectrum density across 200 kHz at 88 MHz (lowest carrier frequency for broadcast FM) is seen from Figure 2.14 to be just $\sim 0.03$ dB, and AWGN channel, i.e., no multipath, is assumed. Finally, to facilitate comparison to the real–world example to follow, the INR is set to 18 dB. First, we consider the results using the generic (chirp) processing method of Figure 5.20 with $T_2 = 5.9 \, \mu s$. The result is shown in Figure 5.22. Note that the generic processing algorithm results in deep suppression. However, noise which occupies the same bandwidth as the RFI signal is also suppressed. This is due to the generic algorithm’s inability to discriminate RFI from noise, as we have seen previously.

Note that the sample period $T_s$ is larger than the value of $T_2 = 0.5 \, \mu s$ used in Section 5.3.3.1. Our choice of $T_2 = 5.9 \, \mu s$ comes from the limitation of sample rate, 512 kHz.
Figure 5.22: PSD before (top) and after (bottom) application of the generic (chirp) mitigation algorithm (Figure 5.20) to the simulated broadcast FM signal. INR$_{in}$ = 18 dB, 5 s averaging.
Next, we consider the results using the broadcast FM-specific processing method of Figure 5.21 with $T_3 = 3.9 \, \mu s$ due to the assumption of AWGN channel. $\omega_n$ is estimated just once, i.e., $T_4 \sim 1 \, s$. The result is shown in Figure 5.23. Note in this case, we do not see the “noise eating” problem of the generic chirp algorithm. However, the noise spectrum is not perfectly white either. To investigate further, we use the quality metric procedure of Section 3.5.2 to consider the toxicity to weak astronomical signals, $s_a(t)$, for this algorithm. Recall that the ideal case (no toxicity to $s_a(t)$) corresponds to $Q = 1$. Figure 5.24 shows $Q$ for the generic (chirp) and the Broadcast FM algorithms. As expected, the generic (chirp) algorithm is quite

![Figure 5.23: PSD before (top) and after (bottom) application of the broadcast FM–specific mitigation algorithm (Figure 5.21) to the simulated FM signal. INR$_{in} = 18$ dB, 10 s averaging.](image)
Figure 5.24: Assessment of toxicity of RFI processing for simulated broadcast FM using the quality metric $Q$, assuming a spectral line having signal-to-noise ratio 6 dB with respect to the resolution bandwidth of the spectrum shown in Figure 5.23.
toxic to spectral lines; thus, the generic algorithm is not suitable for spectroscopy.
The broadcast FM–specific algorithm is seen to be dramatically better, although still with significant toxicity. This outcome might be useful for detection of spectral lines (albeit with decreased sensitivity), but probably not for spectral line level estimation.

The real–world data is broadcasted by station KMPX located in Phoenix Arizona (Southwest USA)

\(^5\). Its over–the–air frequency is 96.9 MHz. The data provided to the algorithm is in complex baseband form sampled at 512 kSPS, and the total record to be evaluated is \(\sim 1\) seconds long. The average spectrum is shown in Figure 5.25.

It can be seen that the modulation is of the 200 kHz variety, and that its INR is approximately 18 dB within \(\sim 300\) kHz–wide occupied bandwidth. The PSD of the demodulated signal is shown in Figure 5.26. Mono, pilot, stereo, RBDS, and SCA1 sub–carriers can be seen. Figures 5.27–5.29 show the results of the generic (chirp) and the Broadcast FM–specific demod–remod methods with the real–world data. The generic algorithm results are analogous with those with the simulated FM signal; In case of the Broadcast FM–specific method, we consider both \(T_3 = 3.9\) ms and 20 \(\mu\)s, since we have no \textit{a priori} knowledge about the channel. For \(T_3 = 3.9\) ms, some performance degradation can be seen with respect to the simulated data results of Figure 5.23, and for \(T_3 = 20\) \(\mu\)s, the performance is almost

\(^5\)Data provided courtesy of Michael Gray (http://www.kd7lmo.net)
Figure 5.25: Power spectral density (PSD) of real world test dataset averaged over 0.98 s.

same with that in Figure 5.23. This implies that real-world data has frequency-selective fading arising from multipath scattering, as described in Section 2.2, and the selection of $T_3 = 20 \mu s$ is a better choice taking into account the resulting amplitude modulation of signal. It should be noted that the broadcast FM-specific demod-remod method with $T_3 = 20 \mu s$ only considers the multipath effect on the magnitude of broadcast FM signal. Therefore, performance improvement is probably possible if the procedure for improved $\theta_n(t)$ in Figure 5.21 also reflects the multipath effect. We will deal with this problem in the next section.
Figure 5.26: Power spectral density (PSD) of information in real world test dataset, averaged over 0.98 s.

5.4 Channel Compensation for Broadcast FM Demod–Remod

The FM demod–remod procedure described in the previous section accounts for multipath only in that $T_3$ is chosen based on the bandwidth of information delivered, as explained in Section 5.3.3.2. Therefore, limited performance is possible in the real–world broadcast FM demod–remod result for $T_3 = 20 \mu s$ case, shown in Figure 5.28, due to multipath, which is not part of the PE/S model used in Section 5.3. There are two ways multipath could cause problems: (1) Multipath could be degrading estimation performance. If so, an improvement may be possible
Figure 5.27: PSD before (top) and after (bottom) application of the generic (chirp) mitigation algorithm (Figure 5.20) to a real-world broadcast FM. $\text{INR}_m = 18 \text{ dB}$, 0.98 averaging.
Figure 5.28: PSD before (top) and after (bottom) application of the broadcast FM–specific mitigation algorithm (Figure 5.21) to a real–world broadcast FM. INR_{in} =18 dB, 0.98 averaging.
Figure 5.29: Assessment of toxicity of RFI processing for real-world broadcast FM using the quality metric $Q$, assuming a spectral line having signal-to-noise ratio 6 dB with respect to the resolution bandwidth of the spectrum shown in Figure 5.28.
through equalization before the demod–remod procedure; and (2) It is possible that
the synthesized RFI, which is free of multipath, is not properly cancelling with the
actual RFI, which may have multipath. Therefore, improvement may be possible
by processing the demod–remod output through a filter which is equivalent to the
channel which the received signal experiences.

Figure 5.30: The expanded FM demod–remod algorithm, now including equalizer
and channel compensation.

This section explores an expanded version of the FM broadcast mitigation algo-
rithm that includes both equalization and channel compensation. The overall block
diagram for the expanded version is illustrated in Figure 5.30, and the details are
explained in the following sections.
5.4.1 Equalization

The purpose of an equalizer is a deconvolution procedure against channel impulse response. In Section 2.2, we noted that we do not expect significant frequency selective fading for Broadcast FM signals. However, it could be that even slight frequency selectivity may be sufficient to degrade mitigation performance.

Among linear equalizers, the Minimum Mean Square Error (MMSE) equalizer is used here because the zero–forcing equalizer amplifies noise appearing near the spectral nulls of channel impulse response [76, Section 10.2]. MMSE requires a “reference” signal which is highly correlated with the observed signal and uses a FIR filter to minimize the mean square error between the reference signal and the filtered observed signal by adjusting the coefficient of filter. For broadcast FM, the proposed configuration of the MMSE equalizer is shown in Figure 5.31. The reference signal is obtained directly from the received signal by demod–remod.

![Diagram](image_url)

Figure 5.31: The configuration of equalizer for Broadcast FM.
Let us assume that the observed signal $x(t_n)$ and equalizer output $x_{eq}(t_n)$ are defined by

\[
x(t_n) \triangleq [x(t_0) \ x(t_1) \ \cdots \ x(t_{M-1})]^T \tag{5.25}
\]

\[
x_{eq}(t_n) \triangleq \sum_{k=1}^{M} c_k^* x_{n-k+1} \tag{5.26}
\]

where $c \triangleq [c_1 \ c_2 \ \cdots \ c_M]^T$. Then, by applying the orthogonality property [73, Chapter 12], we have

\[
E \left\{ x(t_n) \left[ \hat{s}^*(t_n) - x^H(t_n)c_{eq} \right] \right\} = 0 \tag{5.27}
\]

This directly results in a matrix form of the normal equation, which is expressed as

\[
Rc_{eq} = d, \quad \text{where} \tag{5.28}
\]

\[
R \triangleq E \left\{ x(t_n)x^H(t_n) \right\}
\]

\[
d \triangleq E \left\{ x(t_n)\hat{s}^*(t_n) \right\}
\]

which can be solved for the equalizer coefficients $c_{eq}^*$. The integration time for $R$ and $d$ is as long as possible, but should be restricted by the channel coherence time. In contrast to mobile communication in VHF, broadcast FM can be assumed to have a negligible Doppler shift, therefore the coherence time for Broadcast FM is much longer than 10 ms, described in Section 2.2. With the assumption of the fixed broadcast station and receiver, it is expected that the coherence time no longer restricts the integration time. Therefore, we choose 439.45 ms as the integration time for $R$ and $d$. 

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5.4.2 Channel Compensation

Channel compensation considers the channel effect by processing the demod–remod output through a filter equivalent to the channel. Let us assume that the transmitted signal $s(t_n)$ and the received signal $x(t_n)$ are defined by

$$s(t_n) \triangleq [s(t_0) \ s(t_1) \ \cdots \ s(t_{M-1})]^T \tag{5.29}$$

$$x(t_n) \triangleq c_c^H s(t_n) + z(t_n) \tag{5.30}$$

where $c_c \triangleq [c_{c1} \ c_{c2} \ \cdots \ c_{cM}]^T$ is the channel impulse response, and $z(t_n)$ is noise. Then, the reference correlation vector is obtained by

$$d = E \{s(t_n)x^*(t_n)\} \tag{5.31}$$

$$= E \{s(t_n)s^H(t_n)\} c_c + E \{s(t_n)z^*(t_n)\}$$

$$= R c_c$$

where $R = E \{s(t_n)s^H(t_n)\}$, and the third equality in Equation (5.31) is justified by the fact that $s(t_n)$ and $z(t_n)$ are independent. For broadcast FM, we exploit $\hat{s}(t_n)$ instead of $s(t_n)$, which is the output of the demod–remod processing block, whose input is $x_{eq}(t_n)$. The configuration is illustrated in Figure 5.32. Then, Equation (5.31)
can be rewritten as

$$R_c c = d, \text{ where}$$

$$R \triangleq E \{\hat{s}(t_n)\hat{s}(t_n)\}$$

$$d \triangleq E \{\hat{s}(t_n)x^*(t_n)\}$$

Then, we can obtain the channel filter, $c_c^*$ by solving Equation (5.32).

Figure 5.32: The configuration of channel compensation for Broadcast FM.

5.4.3 Algorithm Design and Performance

In this section, we determine a specific design for a demod–remod algorithm for Broadcast FM that possibly includes both equalization and channel compensation. For this, we first consider 6 cases based on combinations of equalization and channel compensation; and two channel conditions: the impulse channel and a simple two–ray channel. The number of taps for filters is chosen initially as 101, with sample rate 1.024 MSPS, and the time difference between rays in the two–ray channel is 5 µs. 101
(\sim 98.6 \mu s) for the number of taps is probably excessive, and the optimal filter length will be investigated later. 20 \mu s is selected for \( T_3 \) to accommodate the possibility of multipath–induced amplitude modulation, as discussed in Section 5.3.2.2. The parameters and models for generating the simulated broadcast FM are same as those in Section 5.3.3.3, and the noise is assumed to be zero–mean white Gaussian. The 6 cases are:

1. No equalizer and no channel compensation (NEQ/NCC) with impulse channel.

2. No equalizer and no channel compensation (NEQ/NCC) with two–ray channel.

3. No equalizer, but channel compensation using a perfectly–known reference signal (NEQ/CCi) with two–ray channel. This is to evaluate the best possible performance of channel compensation.

4. Equalizer and channel compensation using a perfectly–known reference signal (EQ/CCi) with two–ray channel.

5. No equalizer, but channel compensation (NEQ/CC), with two–ray channel.

6. Equalizer and channel compensation (EQ/CC) with two–ray channel.

For Cases 5 and 6, the reference signals are synthesized by the method described in Sections 5.4.1 and 5.4.2, and for Cases 3 and 4, it is assumed that the reference signals are known. Figure 5.33 shows the IRR performance. In low INR_{in}
Figure 5.33: IRR performance for 6 cases (combinations of equalization and channel compensation, and channel types). FIR filters with 101 taps. Mean over 20 trials and in each of trials, 0.98 s data is evaluated.
range (Figure 5.33(a)), most of cases except Cases 3 and 4 have almost same performance. This can be explained by that demod–remod performs inadequately in this low INR$_{in}$ and, thereby, the effect of EQ and CC is negligible. Note that IRR in Cases 3 and 4 (EQ/CCi and NEQ/CCi) outperforms the others, due to utilization of the noise–free reference signals. In high INR$_{in}$ range (Figure 5.33(b)), the IRR of Case 5 (NEQ/CC) is close to low performance bound of Case 2. Overall, Case 6 (EQ/CC) has a better IRR performance than NEQ/CC and NEQ/NCC, although the IRR improvement is not much. In other words, both equalization and channel compensation are useful, but only for INR$_{in} \gtrapprox 0$ dB and providing less than 1 dB of improvement.

As mentioned earlier, 101 might be an excessive filter length. This would be undesirable as it would increase computational cost. Therefore, we investigate IRR performance according to the length of filter: $M = 11, 25, 51, \text{ and } 101$. Figure 5.34 shows the result indicating that $M = 25$ is optimal of these choices in the sense that it is the minimum length that appears to provide all the benefits of EQ/CC.

The PSDs before/after mitigation with EQ/CC are in Figure 5.35–5.39. At 18 dB, the impulse channel case in Figure 5.35 exhibits some “noise eating”, while the two–ray channel in Figure 5.36 has a bit of RFI remaining after suppression. Figure 5.37 shows the result for applying EQ/CC to the real–world data used in Section 5.3.3.3. The result is almost same with that in Figure 5.28, which is to be expected given
Figure 5.34: IRR performance for EQ/CC according to the length of filter (two-ray channel). Mean over 20 trials and in each of trials, 0.98 s data is evaluated.
the results shown in Figure 5.33.

![Graph showing PSD comparison before and after application of the expanded broadcast FM-specific mitigation algorithm.](image)

Figure 5.35: PSD before (top) and after (bottom) application of the expanded broadcast FM–specific mitigation algorithm (Figure 5.30: EQ/CC with $M = 25$). Simulated broadcast FM signal. Impulse channel. INR$_{in}$ =18 dB. 0.98 s averaging.

Figure 5.38 and 5.39 are PSDs before/after suppression at INR$_{in}$ =-10 dB. These show that FM demod–remod causes some noise eating after suppression, although it is not as severe as generic procedure in Figure 5.27. This noise eating property also distorts the underlying astronomical signal. To evaluate this, the Q metric for toxicity is evaluated for three cases: EQ/CC, NEQ/CC, and EQ/NCC. The results at INR$_{in}$=10, 0, and −10 dB are shown in Figure 5.40, 5.41, and 5.42, respectively. In most cases, Q has a small value near the carrier frequency, ranging from 0.3 to 1. It is observed that there is no dramatic difference in the relative performance of
Figure 5.36: PSD before (top) and after (bottom) application of the broadcast FM–specific mitigation algorithm (Figure 5.30: EQ/CC with $M = 25$). Simulated broadcast FM signal. Two–ray channel, $\text{INR}_{in} = 18$ dB, 0.98 s averaging.
Figure 5.37: PSD before (top) and after (bottom) application of the broadcast FM–specific mitigation algorithm (Figure 5.30: EQ/CC with $M = 25$). Real–world broadcast FM. $\text{INR}_{\text{in}} = 18 \text{ dB}, 0.98 \text{ s averaging.}$
EQ/NCC, EQ/CC, and NEQ/CC in terms of toxicity except perhaps that EQ/NCC is slightly less toxic closer to the modulation band edge.

Figure 5.38: PSD before (top) and after (bottom) application of the broadcast FM–specific mitigation algorithm (Figure 5.30: EQ/CC with $M = 25$). Simulated broadcast FM signal. Impulse channel, INR$_{in} = -10$ dB, 0.98 s averaging.

5.5 Comparison to Adaptive Canceling and a Hybrid Technique

As described in Barnbaum and Bradley (1998) [19], adaptive canceling is also a possibility for suppression of broadcast FM, using a separate high–gain reference antenna that is pointed at the source of the interference. In this section, we show the
Figure 5.39: PSD before (top) and after (bottom) application of the broadcast FM–specific mitigation algorithm (Figure 5.30: EQ/CC with $M = 25$). Simulated broadcast FM signal. Two–ray channel, INR$_{in} = -10$ dB, 0.98 s averaging.
Figure 5.40: Q metric for the toxicity of the FM broadcast demod–remod (Figure 5.30), INR$_{in}$=10 dB.

...performance of the Barnbaum and Bradley compared to the expanded demod–remod algorithm developed in Section 5.4 and propose a hybrid technique that combines both methods for the possibility of improved overall performance.

### 5.5.1 Adaptive Canceler Algorithm and Performance

Figure 5.43 illustrates the Barnbaum and Bradley approach. The system has two antennas, primary and reference. The primary antenna receives the astronomical signal $s_a(t)$ via the main beam, and RFI $s_p(t)$ via the sidelobes. The reference antenna is pointed at interference so that its input is well–approximated as being RFI...
Figure 5.41: Q metric for the toxicity of the FM broadcast demod–remod (Figure 5.30), INR$_{in}=0$ dB.

$s_r(t)$ only. Then, it can be seen that $s_p(t)$ and $s_r(t)$ are strongly correlated, therefore the adaptive filter algorithm (MMSE) estimates this correlation and generates $y(t)$, the estimate of $s_p(t)$, at the output of filter. The mean square error between signal the primary antenna and $y(t)$ can be expressed as

$$E\{\varepsilon^2(t)\} = E\{s_p^2(t)\} + E\{[s_p(t) - y(t)]^2\}$$  \hspace{1cm} (5.33)

Because $E\{s_p^2(t)\}$ is constant, the minimization of Equation (5.33) leads to the minimization of $E\{[s_p(t) - y(t)]^2\}$. Thereby, the canceling $s_p(t)$ is done by subtracting $y(t)$ from a signal from the primary antenna. Note that the system noise in the primary and the reference antennas are nominally independent.
Figure 5.42: Q metric for the toxicity of the FM broadcast demod–remod (Figure 5.30), INR$_{in}$ = −10 dB.

To evaluate the performance of adaptive canceling, IRR, PSDs, and toxicity are investigated by simulations with same conditions as in Section 5.4.3. The length of the adaptive filter $M$ is chosen as 25 at 1.024 MSPS. The signal from the reference antenna is assumed to have 10 dB greater INR and 10 dB less SNR for astronomical signal compared to the primary antenna. The impulse and the two–ray channels are considered for the primary signal, and for both cases, it is assumed that the reference signal experiences only the impulse channel. The IRR performance is shown in Figure 5.44. The performance in the two–ray channel case is almost same with that of impulse channel. Thus, this algorithm is highly effective under the assumptions made here.
Figure 5.43: Adaptive canceling using a separate reference antenna.

Figure 5.45–5.48 present PSDs before and after suppression. In most cases, the RFI is effectively suppressed without noticeably affecting background noise. But it can be observed that multipath may degrade the suppression performance by comparing Figures 5.45 and 5.46. Figure 5.47 and 5.48 show that adaptive canceling works effectively even when the INR in the primary channel is very low, presumably due to the relatively higher INR in the reference channel.

The Q metric of toxicity is shown in Figure 5.49. This result implies that adaptive canceling with the reference antenna is almost free from the distortion of astronomical signal caused by mitigation procedure, compared to FM demod–remod shown in Figure 5.40–5.42.
The above results show that the Barnbaum and Bradley adaptive cancelling method is overall superior to the demod–remod PE/S approach described in Section 5.3.3.2. However, it should be emphasized that the adaptive canceling approach requires (as we have assumed here) a high–gain reference antenna, whereas the demod-remod PE/S approach does not. It should also be noted that we have assumed relatively favorable conditions for adaptive canceling; i.e., no multipath in the reference channel.
Figure 5.45: PSD before (top) and after (bottom) application of the adaptive filter algorithm (Figure 5.43). Simulated broadcast FM signal. Impulse channel. INR_{in} for the primary antenna is 18 dB and 0.98 s averaging.
Figure 5.46: PSD before (top) and after (bottom) application of the adaptive filter algorithm (Figure 5.43). Simulated broadcast FM signal. Two-ray channel. INR$_{in}$ for the primary antenna is 18 dB and 0.98 s averaging.
Figure 5.47: PSD before (top) and after (bottom) application of the adaptive filter algorithm (Figure 5.43). Simulated broadcast FM signal. Impulse channel. INR_in for the primary antenna is $-10$ dB and 0.98 s averaging.
Figure 5.48: PSD before (top) and after (bottom) application of the adaptive filter algorithm (Figure 5.43). Simulated broadcast FM signal. Two-ray channel. INR_{in} for the primary antenna is \(-10\) dB and 0.98 s averaging.
Figure 5.49: Q metric for the toxicity of adaptive canceling, assuming the spectral line of astronomical signal having signal–to–noise ratio 6 dB at the primary antenna and signal–to–noise ratio –4 dB at the reference antenna, with respect to 1 kHz resolution frequency (Mean over 20 trials).
5.5.2 Hybrid Technique

This section proposes a hybrid technique which uses adaptive canceling but introduces a demod–remod stage to further improve the reference channel INR, as shown in Figure 5.50. In the figure, \( z_p(t) \) and \( z_r(t) \) are noise at the primary antenna and the reference antenna, respectively.

![Diagram](image)

Figure 5.50: The configuration of channel compensation for a broadcast FM.

With the same simulation conditions used in the previous section, the performance of this hybrid technique is evaluated. The IRR performance is shown in Figure 5.51. In the INR\(_{in}\) range of \(-5\) to \(+10\) dB, hybrid technique always has improved performance. This improvement is obtained by INR enhancement at the output of FM
demod–remod block. Figure 5.52 shows the INR enhancement, INR$_{in}$ versus INR$_{out}$.

The performance degradation for INR$_{in} < -5$ dB in Figure 5.51 can be attributed to increased estimation error in the demod–remod technique at low INR$_{in}$. Similarly the apparent “saturation” of the performance at high INR$_{in}$ represents the fact that the input already has such high INR$_{in}$ that the demod–remod technique can only degrade it. In the region between, the estimation error injected by demod–remod is less than the noise power at the input, and thus the overall result is better in this region.

Figure 5.51: IRR performance of adaptive filter described in Figure 5.50.

PSDs before and after suppression with the hybrid technique are shown in Figure 5.53–5.54. INR$_{in}$ is 4 dB, corresponding to a point at which the performance
Figure 5.52: Reference channel INR enhancement by demod–remod (INR_{in} is the input INR and INR_{out} is the output INR for demod–remod block). INR_{in} for reference channel is 10 dB higher than that for primary channel.
of the hybrid technique is about 1.62 dB better than the original algorithm. In the figures, it can be observed that multipath degrades the quality of PSD after suppression (PSD in Figure 5.53 is relatively flat compared to that in Figure 5.54). For the comparison with original adaptive canceling, Figure 5.55 and 5.56 shows PSDs before and after suppression with original adaptive canceling and INR$_{\text{in}} = 4$ dB. By comparing them (Figure 5.53 and 5.55 for the impulse channel, and Figure 5.54 and 5.56 for the two-ray channel), we see that the hybrid technique has slightly better performance consistent with Figure 5.51.

![Figure 5.53: PSD before (top) and after (bottom) application of the hybrid technique (Figure 5.50). Simulated broadcast FM signal. Impulse channel. INR$_{\text{in}}$ for primary antenna is 4 dB. 0.98 s averaging.](image)

Now, we want to compare the Q metric for hybrid technique and original adaptive
Figure 5.54: PSD before (top) and after (bottom) application of the hybrid technique (Figure 5.50). Simulated broadcast FM signal. Two-ray channel. INR\textsubscript{in} for primary antenna is 4 dB. 0.98 s averaging.
Figure 5.55: PSD before (top) and after (bottom) application of the original adaptive canceling (Figure 5.43). Simulated broadcast FM signal. Impulse channel. INR$_{in}$ for primary antenna is 4 dB. 0.98 s averaging.
Figure 5.56: PSD before (top) and after (bottom) application of the original adaptive canceling (Figure 5.43). Simulated broadcast FM signal. Impulse channel. $\text{INR}_{\text{in}}$ for primary antenna is 4 dB. 0.98 s averaging.
canceling. Figure 5.57 presents the difference of the Q metric for hybrid technique and original adaptive canceling. Near the center frequency, the difference is minor, but it is interesting that the difference increases as frequency offset increases to the band–edge frequency (∼90 kHz). In the hybrid technique, the astronomical signal in reference channel is suppressed by demod–remod, therefore the output of demod–remod is hardly correlated with astronomical signals at the primary channel. But the original adaptive canceling algorithm does not have an explicit function to suppress the astronomical signal in the reference channel. Therefore, the hybrid technique is slightly less toxic, especially at low INR.

![Graph showing Q metric difference for toxicity (hybrid technique and original adaptive canceling).](image)

**Figure 5.57**: Q metric difference for the toxicity (hybrid technique and original adaptive canceling), assuming the spectral line astronomical signal having signal–to–noise ratio 6 dB at the primary antenna and signal–to–noise ratio −4 dB at the reference antenna, with respect to 1 kHz resolution frequency (Mean over 20 trials).
Chapter 6

Mitigation of ATSC

In this chapter, we develop a coherent mitigation technique for ATSC. ATSC was described in Section 2.1.6 (“ATSC”) and Appendix C. The organization of this chapter is as follows: Section 6.1 (“Demodulation of ATSC”) describes the implementation for the demodulation of 8VSB, which is used in the PE/S procedure. Section 6.2 (“Demod-Remod PE/S for ATSC”) presents ATSC remodulation and derives the theoretic IRR performance. Section 6.3 (“Effectiveness for Spectroscopy”) assesses the performance of the algorithm. Section 6.4 (“IRR Evaluation of Hybrid Technique for ATSC”) considers a hybrid technique analogous to that developed in Section 5.5.2, based on adaptive canceling, for ATSC.
6.1 Demodulation of ATSC

To implement demod–remod, we require only the part of the 8VSB receiver up to symbol decisions. The overall procedure of 8VSB digital demodulation is in Figure 6.1. The first step is carrier synchronization, which is done with the help of the pilot tone. For this, we use Equation (5.6) in Section 5.1.2. The estimate of the pilot is used for coherent downconversion to baseband. This is followed by I-channel squared–root raise cosine (SRRC) filtering, and removal of DC. Then, with the transmitted 8VSB signal \( s_{RF}(t) \) in (2.11), the I-channel baseband 8VSB signal is expressed as

\[
x_{bs}(t) = \sum_{l=1}^{L} \Re\{s_{RF}(t - \tau_l)e^{-j\omega t_1 e^{j\phi_l}}\} + z(t)
\]

where \( I(t) = a_k \sum_{k=-\infty}^{\infty} p_{rc}(t - kT_s) \) and

\[
\tilde{Q}(t) = a_k \sum_{k=-\infty}^{\infty} p_{ct}(t - kT_s)
\]

where \( L, \alpha_l, \tau_l, \) and \( \phi_l \) are the number of multipaths, the real–valued path gain of \( l \)th path, the delay of \( l \)th path, and the phase offset of \( l \)th path, respectively. \( z(t) \) is noise. \( I(t) \) is the sequence consisting of raised–cosine (RC) pulses \( p_{rc}(t) \) defined as \( p_{sq}(t) \ast p_{sq}(t) \) and \( \tilde{Q}(t) \) is the sequence consisting of a pulses \( p_{ct}(t) \) defined by \( p_{sq}(t) \ast H\{p_{sq}(t)\} \), which is the cross–talk component when the phase offset \( \phi_l \) is
Symbol timing is obtained by the correlation of the known PN511-sequence \( s_{511}(t) \) with \( x_{bs}(t) \), seeking the symbol timing offset

\[
\hat{T}_{off} = \arg \max_\tau \{ R_{x_{bs} s_{511}}(\tau) \} \tag{6.2}
\]

where \( R_{x_{bs} s_{511}}(\tau) \) is the correlation given by

\[
R_{x_{bs} s_{511}}(\tau) = \langle x_{bs}(t), s_{511}(t - \tau) \rangle \tag{6.3}
\]

In practice, \( \hat{T}_{off} \) is used as an initial estimate for symbol timing, and a symbol timing tracker (described below) is used to continuously update.

To compensate the multipath effect, we use the PN511–sequence again. Here we use a fractionally–spaced equalizer (FSE). The tap delay for the FSE is set to \( T_S/20 \) where \( T_S \) is the 8VSB symbol duration. To estimate the equalizer coefficients, the
MMSE criterion is used, which minimizes mean square error between the received PN511 waveform and the known PN511 waveform. Then the equalizer coefficients $c_{eq}$ are obtained by solving the normal equation:

$$\mathbf{R}c_{eq} = \mathbf{d},$$ \hspace{1cm} (6.4)

$$\mathbf{R} = E\{x_{bs}(t_n)x_{bs}^H(t_n)\} \text{ and } \mathbf{d} = E\{x_{bs}(t_n)s_{511}^*(t_n)\}.$$ 

Note that $x_{bs}(t_n) = [x_{bs}(t_n) \ x_{bs}(t_{n-1}) \ \cdots \ x_{bs}(t_{n-M_e+1})]^T$ and it’s the PN511 part in the received signal from the DC removal block. $M_e$ is the order of equalizer, and in this implementation, $M_e$ is chosen as 4325 (20 $\mu$s at the sample rate of 215.24 MHz). This $M_e$ is selected based on the measured delay spread for VHF channel of ATSC in [57], whose maximum is 14.88 $\mu$s.

Symbol rate sampling is achieved using a variance–based symbol timing tracker. Assuming there is no cross–talk from the Q–component and noise $z(t)$ is negligible, the input to the tracker is given by

$$x_{eq}(t) = \sum_{k=-\infty}^{\infty} a_k p_{rc}(t - kT_s - \tau_a)$$ \hspace{1cm} (6.5)

where $T_s$ is the symbol duration (92.9 ns) and $\tau_a$ is the unknown time delay which is to be estimated by the tracker. Note that $p_{rc}(t)$ in (6.5) is assumed to be maximum at $t = kT_s + \tau_a$. Assuming initial symbol timing is done with Equation (6.2), the sampling instant can be expressed as $t_s = iT_s + \delta$, where $i$ is an integer and $\delta$ is the
timing offset. Then, we have the sampled version of Equation (6.5):

\[ x_{eq}(t_s) = x_{eq}(i; \delta) = a_ip_{rc}(\delta - \tau_a) + \sum_{k=-\infty \atop k \neq i}^{k=\infty} a_k p_{rc}\{(i - k)T_s + \delta - \tau_a\} \]  

(6.6)

Note that the last term in Equation (6.6) is ISI caused by symbol timing error. Assuming correct symbol decisions, i.e., that the eye is initially open at \( t_s \), we can obtain the error \( \varepsilon(i; \delta) \) due to the time difference of \( \delta \) and \( \tau_a \), as follows:

\[ \varepsilon(i; \delta) \equiv x_{eq}(i; \delta) - a_i \]

(6.7)

\[ = [p_{rc}(\delta - \tau_a) - 1]a_i + \sum_{k=-\infty \atop k \neq i}^{k=\infty} a_k p_{rc}\{(i - k)T_s + \delta - \tau_a\} \]

where \( a_i \) is the decision value of the \( i \)th symbol. With (6.7), we can estimate \( \tau_a \) through a search for \( \delta \), as follows:

\[ \hat{\tau}_a = \arg \min_{\delta} \left[ \text{var}\{\varepsilon(i; \delta)\} \right] \]

(6.8)

\[ \text{var}\{\varepsilon(i; \delta)\} = [p_{rc}(\delta - \tau_a) - 1]^2 \text{var}\{a_i\} + \sum_{k=-\infty \atop k \neq i}^{k=\infty} \text{var}\{a_k\} p_{rc}^2\{(i - k)T_s + \delta - \tau_a\} \]

Equation (6.8) is justified by two facts: (1) \( p_{rc}(0) = 1 \) and (2) no ISI components at \( \delta = \tau_a \). This symbol timing tracker is implemented as shown in Figure 6.2.

The symbol timing tracker has \( 2M - 1 \) samplers, each of which samples at times

\[ t_{s,m} = iT_s + (\delta_0 + m\delta_r) \]

where \( m = -(M - 1), \cdots, 0, \cdots, (M - 1) \), \( \delta_0 \) is the initial timing error from symbol timing using PN511, and \( \delta_r = T_s/2(M - 1) \). With the assumption that \( \hat{T}_{off} \) from (6.2) is reliable, the initial timing error \( \delta_0 \) is small, and
Figure 6.2: The variance–based symbol timing tracker.
then some of the $2M - 1$ samplers will have the correct (hard) decision symbols.

The symbol timing tracker in Figure 6.2 seeks the path which satisfies (6.8) within the $\delta$ search range of $\delta_0 - \frac{T_s}{2}$ to $\delta_0 + \frac{T_s}{2}$.

For the implementation of Figure 6.2, we have to decide $M$ and the evaluation time $T_u$ of the variance of error (6.7) for each of $2M - 1$ paths. For the given $T_u$, the variance of error is estimated by

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{I_u} \sum_{i=1}^{I_u} \varepsilon^2(i; \delta)$$

(6.9)

where $I_u = T_u/T_s$. With the symbol set of 8VSB, $\{ -7, -5, -3, -1, +1, +3, +5, +7 \}$, the maximum possible $\sigma_\varepsilon^2$, the variance of $\varepsilon(i; \delta)$, must be $< 1$, since we assume the eye is open. Figure 6.3 shows the variance of $\hat{\sigma}_\varepsilon^2$ according to $I_u$, assuming $\sigma_\varepsilon^2 = 1$.

In this figure, $\hat{\sigma}_\varepsilon^2$ is reliable at $I_u > 100$. With this result, we choose $I_u = 150$, corresponding to $T_u = 14 \mu s$. The updating time for choosing a path having the smallest variance is chosen as $T_u = 14 \mu s$ too. This choice is reasonable since $T_u$ is much smaller than the coherence time 10 ms described in Section 2.2. To decide $M$, we consider $\delta_r = T_s/(2(M - 1))$ for the given sample rate. In the implementation, we use $20 \times$ oversampling, so that the minimum allowable $\delta_r$ is $T_s/20$. With this minimum $\delta_r$, $M$ is 11. But in the using $M = 11$, there is the possibility that the samplers at $-(M - 1)$ and $(M - 1)$ are sampling the previous or the next symbol. To avoid this, $M = 9$ is used in the implementation, resulting in 17 sampler paths.

As can be expected, the performance of ATSC mitigation is related to symbol er-
error rate (SER). Assuming perfect carrier synchronization and AWGN, the SER for ATSC (8VSB) is the same as 8PAM [76, Section 5.2.6] shifted by 3 dB, since the power in the ATSC signal is divided into I– and Q– components. Thus, the analytic SER for 8VSB is given by

\[ p_e = \frac{2(M_t - 1)}{M_t} Q \left( \sqrt{\frac{3E_S}{(M_t^2 - 1)N_0}} \right) \]  

(6.10)

where \( M_t \) is the number of symbols (8) and \( E_S \) is the average symbol energy. Figure 6.4 compares SER determined from Monte Carlo simulation and Equation (6.10). We will derive the relation of SER to IRR in Section 6.2.
Figure 6.4: Symbol error rate of 8VSB in AWGN (6400 symbols are evaluated in simulation, assuming perfect carrier synchronization and symbol timing).

6.2 Demod-Remod PE/S for ATSC

The structure of our proposed demod–remod ATSC mitigation algorithm is shown in Figure 6.5. First, the signal is demodulated as described in Section 6.1, yielding $\hat{a}_k$’s and modulation parameters, i.e., symbol timing $t_{sym}$, estimates of channel $c_{ch}$, carrier frequency $\hat{\omega}$, and carrier offset $\hat{\phi}$. Before discussing the remodulation (synthesis), it is worth noting the updating times for modulation parameters. The updating time $T_u$ for the symbol timing $t_{sym}$ is 14 $\mu$s. This is much less than the coherence time and enough to obtain reliable variance of error (6.9), as shown in Figure 6.3. For carrier parameters $\hat{\omega}$ and $\hat{\phi}$, the update times are 29 $\mu$s and 14 $\mu$s.
respectively. The update time for $c_{ch}$ is set to 24.2 ms since the PN511 sequence is used for this, which is periodically received every 24.2 ms.

The synthesis of a noise free copy of the received signal uses the baseband filter method in Figure 2.10. A channel compensation filter is applied to account for channel effects, which is important due to large (frequency selective) bandwidth. Details are as follows:

(1) The baseband data sequence is generated as follows

$$a(t) = \sum_{i=-\infty}^{\infty} \hat{a}_k \delta(t - t_{sym}) \text{, where}$$

$$t_{sym} = iT_s + \hat{\tau}_a$$

where $\hat{\tau}_a$ comes from Equation (6.8).

(2) To do I– and Q– pulse shapings, $a(t)$ is filtered by $p_{sq}(t)$ and $H\{p_{sq}(t)\}$, respectively. $p_{sq}(t)$ is SRRC defined in Section 2.1.6.

(3) To add the channel effect, the outputs of the I– and Q–pulse shaping block are filtered using the coefficients $c_{ch}$. This channel impulse response (CIR) is estimated by comparing the known PN511 sequence to the PN511 sequence component of the received signal. With the MMSE criterion, $c_{ch}$ estimation can be accomplished by solving the following equation

$$\mathbf{R}c_{ch} = \mathbf{d} \text{, where}$$

$$\mathbf{R} = E\{s_{511}(t_n)s_{511}^H(t_n)\} \text{ and } \mathbf{d} = E\{s_{511}(t_n)x_{bs}^*(t_n)\}$$

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where $s_{511}(t_n) = [s_{511}(t_n) \ s_{511}(t_{n-1}) \ \cdots \ s_{511}(t_{n-M_c+1})]^T$ and $x_{bs}(t)$ is the received PN511 waveform sequence. $M_c$ is the order of filter. For this implementation, $M_c$ is 4325 with a sample rate 215.24 MHz, same as the order of equalizer $M_e$.

(4) VSB modulation is done by the method in Figure 2.10. Note that for $\hat{\omega}$ and $\hat{\phi}$, Equation (5.6) is used.

We now attempt to predict the performance of the proposed method. The synthesized 8VSB waveform is given by

$$\hat{s}(t) = \sum_{k=-\infty}^{\infty} \hat{a}_k \{p_{sq}(t - kT_s) - jp_{sqh}(t - kT_s)\} e^{j(\hat{\omega}t + \hat{\phi})}$$  \hspace{1cm} (6.13)$$

where $p_{sqh}(t) = H\{p_{sq}(t)\}$. To determine the best possible performance of this technique, AWGN channel condition and perfect carrier synchronization, (i.e., $\hat{\omega} = \omega$
and \( \hat{\phi} = \phi \) are assumed. Remaining after coherent subtraction is

\[
\epsilon_s(t) = s(t) - \hat{s}(t) = \sum_{k=-\infty}^{\infty} \epsilon_k \{p_{sq}(t - kT_s) - j p_{sqh}(t - kT_s)\} e^{j(\omega t + \phi)} \tag{6.14}
\]

where \( \epsilon_k = a_k - \hat{a}_k \). Therefore, the power of \( \epsilon_s(t) \) is given by

\[
P_{\tilde{s}} = \lim_{T \to \infty} \frac{1}{T} \int_T^T \epsilon_z(t) \epsilon_z^*(t) dt \tag{6.15}
\]

\[
= \lim_{T \to \infty} \frac{1}{T} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \epsilon_k \epsilon_l \times \\
\left[ \int_0^T p_{sq}(t - kT_s)p_{sq}(t - lT_s)dt + \int_0^T p_{sqh}(t - kT_s)p_{sqh}(t - lT_s)dt \right]
\]

\[
= \lim_{T \to \infty} \frac{1}{T} \sum_{k=-\infty}^{\infty} \epsilon_k^2 \left[ \int_0^T p_{sq}^2(t - kT_s)dt + \int_0^T p_{sqh}^2(t - kT_s)dt \right]
\]

\[
= \lim_{T \to \infty} \frac{1}{T} \sum_{k=-\infty}^{\infty} \epsilon_k^2 (E_{p_{sq}} + E_{p_{sqh}}), \text{ where}
\]

\[
E_{p_{sq}} \equiv \int_0^T p_{sq}^2(t - kT_s)dt \text{ and } E_{p_{sqh}} \equiv \int_0^T p_{sqh}^2(t - kT_s)dt
\]

Note that \( E_{p_{sq}} \) and \( E_{p_{sqh}} \) are the energies of \( p_{sq}(t) \) and \( p_{sqh}(t) \), respectively. In (6.15), the second equality comes from the fact that \( P_{\tilde{s}} \) is the positive real value, implying the imaginary part is zero, and the third equality is justified by noting that \( \int_0^T p_{sq}(t - kT_s)p_{sq}(t - lT_s)dt \) and \( \int_0^T p_{sqh}(t - kT_s)p_{sqh}(t - lT_s)dt \) are zero for \( k \neq l \).

Note that \( E_{p_{sq}} = E_{p_{sqh}} \) for 8VSB so that (6.15) can be rewritten as

\[
P_{\tilde{s}} = 2E_{p_{sq}} \sigma_{\epsilon_k}^2, \text{ where}
\]

\[
\sigma_{\epsilon_k}^2 \equiv \lim_{T \to \infty} \frac{1}{T} \sum_{k=-\infty}^{\infty} \epsilon_k^2
\]

At moderate to high INR, we might assume most of the errors contributing to \( \sigma_{\epsilon_k}^2 \) are limited to adjacent symbol errors; i.e., errors corresponding to the minimum
Euclidean distance \( d_{\text{min}} \) in the 8VSB constellation. In this case, \( \sigma_k^2 \simeq d_{\text{min}}^2 p_e \), where \( p_e \) is the SER. Therefore, (6.16) can be rewritten as

\[
P_s \approx 2E_{pq} d_{\text{min}}^2 p_e.
\]

(6.17)

Then, IRR becomes

\[
\text{IRR} = \frac{P_s}{P_s} = \frac{P_s}{2E_{pq} d_{\text{min}}^2 p_e}.
\]

(6.18)

Equation (6.18) shows that IRR is inversely proportional to \( p_e \) as long as Equation (6.17) holds true, i.e., for moderate to high INR. Figure 6.6 compares the prediction of (6.18) to simulation in a AWGN scenario. In the simulation, IRR is computed in two ways: one is to simulate IRR directly, and the other is to compute IRR using (6.18) with a simulated SER \( p_e \) which is function of INR as shown in Figure 6.4.

It can be seen that (6.18) is very accurate for SER \( \leq 0.3 \) or INR \( \geq 12 \) dB. We also see that under the assumed conditions that the proposed mitigation algorithm gives IRR \( \approx \text{INR}_{\text{in}} \) for INR \( \leq 12 \) dB, and much better performance for INR \( \geq 12 \) dB. The reason for the improved performance for INR \( \geq 12 \) dB is that the “finite alphabet” property of digital modulation increases the “effective” INR.

### 6.3 Effectiveness for Spectroscopy

We now consider the performance of the technique in terms of spectroscopy and toxicity. Figure 6.7 shows the averaged power spectral density with and without
Figure 6.6: Interference rejection ratio (IRR) of 8VSB in AWGN mitigation, assuming an AWGN channel, for INR\textsubscript{in} equal to 21.7 dB, 12.6 dB, and 6.6 dB. Although the IRR results are consistent with those predicted in Figure 6.6, we see that the “noise eating” phenomenon first noted in Chapter 5 is also apparent. However, the effect is dependent on INR\textsubscript{in}. The noise–eating effect of the procedure is caused by fact that symbol decisions are mainly determined by noise, as opposed to signal, at low INR\textsubscript{in}; In effect, the 8VSB demodulator behaves as a quantizer of the noise. Therefore, the synthesized signal becomes correlated with the noise, and tends to cancel it. At high INR, the noise has relatively little effect on symbol decisions, and so the demodulated signal is effectively decorrelated from the noise. It appears this can only be improved by improving the performance of the demodulator. For example, including Viterbi decoding can be expected to decrease the correlation between input noise and symbol decisions.
(a) High INR$_{in}$: 21.7 dB (the resulting INR$_{out}$ = $-6.3$ dB)
(b) Moderate INR$_{in}$: 12.6 dB (the resulting INR$_{out}$ = 0.5 dB)
(c) Low INR$_{in}$: 6.6 dB (the resulting INR$_{out}$ = $-0.7$ dB)

Figure 6.7: Averaged power spectral density (PSD) of simulated ATSC in AWGN, averaging 0.6 ms.

Figure 6.8 shows the $Q$ metric according to INR$_{in}$. This shows that the toxicity of demod–remod PE/S for ATSC is severe at moderate and low INR$_{in}$, implying the correlation of the synthesized signal and the desired astronomical signal $s_a(t)$
Figure 6.8: $Q$ metric for the toxicity of the demod–remod PE/S for ATSC, assuming a spectral line of $s_a(t)$ has SNR 6 dB with respect to the resolution bandwidth of the spectrum in Figure 6.7.

through the same mechanism described in the previous paragraph.

We now demonstrate demod–remod PE/S for ATSC with real–world data. The real world data are captured as part of a study by the IEEE 802.22 (WRAN) working group\(^1\). The sample rate of data is 21.524 MSPS and the center frequency is 5.38 MHz, i.e., it is IF–sampled. The actual center frequencies are 599 MHz and 677 MHz, corresponding to Channels 35 and 48, respectively. INR\(_{in}\) is $\sim$25 dB. Two channel cases were examined: no multipath and severe multipath. Figure 6.9(a) shows the result for the data having little multipath effect, and the result for data having

\(^1\)http://grouper.ieee.org/groups/802/22/Meeting_documents/2006_May/
significant multipath effect is shown in Figure 6.9(b). Note that the real-world
ATSC data in Figure 6.9(a) has strong two narrowband signals, unrelated to the
ATSC signal, at the upper and lower sides of ATSC spectrum. As can be observed
in Figure 6.9, there is some signal left after suppression. In Figure 6.9, it should
be noted that the suppression performance is not great even at high INR\textsubscript{in}. This
is probably due to cross-talk from Q-components caused by non-zero carrier phase
offset.

From Figure 6.6–6.9, it can be realized that the application of demod–remod PE/S
for ATSC must be restricted to the case with little multipath channel and high
INR\textsubscript{in}. But in that case, demod–remod PE/S gives quite desirable characteristics:
\( Q \) very close to 1 and flat PSD after suppression. These are vital requirements in
spectroscopy.

### 6.4 IRR Evaluation of Hybrid Technique for ATSC

As described in Section 5.5, adaptive canceling with a reference antenna exhibited
very good performance, which could be further improved using demod–remod in
the reference path. This section applies adaptive canceling to the ATSC signal and
evaluates IRR by simulation, and then considers the Hybrid technique to investigate
the possibility of IRR improvement as in Broadcast FM case. As in Section 5.5, it
is assumed that the RFI in the reference antenna is 10 dB higher than that in the
primary antenna. Two channel conditions are considered for the primary antenna: impulse channel and two-ray channel, and it is assumed that the reference antenna has very high gain (directional pattern), and thus experiences only an impulse channel. The time difference of the two paths in the two-ray channel is 5 µs, and their gains are 1 and 0.9. The length of the adaptive filter is set as 4325 with sample rate 215.24 MHz, corresponding to 20 µs, which is enough to span both paths of the two-ray channel. The IRR performance of adaptive canceling is shown in Figure 6.10. As expected, IRR is proportional to INR$_{in}$ and roughly the same for both impulse and two-ray channels.

For the hybrid technique, two conditions of demod–remod PE/S are simulated: perfect carrier synchronization, and carrier synchronization recovery according to Equation (5.6) with fine frequency search as addressed in Section 5.1.2.2. The former will give the best possible IRR of the hybrid technique and the latter will show the IRR limited by the performance of the carrier recovery method. The IRR performance of hybrid technique is shown in Figure 6.11. The hybrid technique with perfect carrier synchronization starts to outperform adaptive canceling alone above INR$_{in}$ ≥10 dB (INR$_{in} = 20$ dB at reference antenna). This can be expected from Figure 6.4 showing that SER is relatively poor below about 20 dB.

When imperfect carrier recovery is used, IRR becomes saturated above INR$_{in} = 10$ dB. This is probably due to increased Q–channel cross–talk, caused by carrier phase
error. As a result, increase of INR_in does not improve IRR. With this result, it can be thought that in practice the performance of hybrid technique depends on that of carrier recovery, and the improvement of carrier recovery should be a focus of future work.
(a) \( \text{INR} \): 25 dB (Ch 35 and no multipath). Signals at \(-0.5\) MHz and 6 MHz are unrelated to the ATSC signal.

(b) \( \text{INR} \): 25 dB (Ch 48 and multipath).

Figure 6.9: Averaged power spectral density (PSD) of real-world ATSC in AWGN, averaging 7 ms. Top and bottom are before and after mitigation, respectively.
Figure 6.10: IRR performance of adaptive filter described in Figure 5.43 (INR$_{in}$ is for a primary antenna). Mean over 20 trials.
Figure 6.11: IRR performance of adaptive filter with demod–remod PE/S on reference channel (INR$_{in}$ is for a primary antenna). Mean over 20 trials.
Chapter 7

Conclusions

In this dissertation, we describe methods for mitigating RFI in wideband receivers operating in the frequency range 10–100 MHz focusing on the development and evaluation of a coherent strategy in which RFI is subtracted from the afflicted data, nominally resulting in no distortion of the weak signals of interest.

7.1 Findings

The important findings in this dissertation are summarized, as following:

1. In Section 5.1.2, ESPRIT was compared to a fine-frequency search using a nonlinear least squares-based cost function in order to determine which gives better accuracy frequency estimation in the presence of Galactic (colored)
noise. The latter was found to be significantly better for low INR for noise bandwidth of 1 MHz, and was also found to be relatively insensitive to the coloring of the noise. As shown in Figure 5.7, IRR of 13–17 dB (the tone is located at 20 MHz where the degree of Galactic noise coloring is significant and impulse channel conditions are assumed) can be expected for “tone as tone” coherent mitigation when $\text{INR}_{in} = 0$ dB, and IRR varies in proportion to $\text{INR}_{in}$ for $\text{INR}_{in}$ in the range $-20$ dB to $+10$ dB.

2. In Section 5.3, it was shown that analog modulations including AM and NBFM can be deeply suppressed using the generic PE/S method of Figure 5.10, but that the result is quite damaging to underlying weak signal of interest. This toxicity problem can be partially overcome for NBFM and Broadcast FM by using a demod-remod approach to PE/S for NBFM (Figure 5.11), which exploits the bandwidth expansion inherent in FM as processing gain to improve INR and suppress noise in the estimation stage. For NBFM, input INRs in the range 9 to 35 dB can be suppressed to $\sim 3$ dB (Figure 5.15), with only mild toxicity (Figure 5.19).

3. A demod-remod PE/S algorithm for broadcast FM was developed (Figure 5.21) and was shown to provide IRR roughly equal to INR for $\text{INR}_{in}$ in the range $-20$ dB to $+20$ dB (Figure 5.33) with some toxicity ($Q \sim 0.6$) at the center frequency improving to $Q \sim 0.9$ 100 kHz offset from center. The basic perfor-
formance is roughly the same for simulated and real-world data, and regardless of multipath, although in the multipath case the resulting PSD shows some residual artifacts (Figure 5.28). These artifacts were shown to be attributable to amplitude modulation induced by multipath fading, and can be significantly reduced simply by reducing the integration time \( T_3 \) for magnitude estimation, which allows tracking of the amplitude modulation.

4. Since broadcast FM has sufficient bandwidth to be potentially frequency-selective, we considered in Section 5.4 additional equalization (to improve parameter estimation) and channel compensation (to improve coherent subtraction) stages with the goal of improving IRR. Improvement was obtained, but was less than 1 dB.

5. In Section 5.5 we compared the demod-remod PE/S performance to the Baum and Bradley [19] adaptive canceler approach, which requires a separate high-gain (10 dB assumed) reference antenna. As expected, the adaptive canceler approach is much better (both in terms of IRR and toxicity), owing to the improved INR\(_{in}\) provided by the reference antenna. We then considered the question of whether the performance could be further improved by using demod-remod to further enhance the reference channel INR. We found that this technique improves performance on the order of 1 dB over the range INR\(_{in}\) = \(-5\) dB to +10 dB, and so has only marginal value under the conditions
6. A demod-remod PE/S canceler was developed for ATSC in Chapter 6. The IRR performance was quantified both theoretically and by Monte Carlo simulation, assuming AWGN channel conditions and perfect carrier synchronization, and is shown in Figure 6.6. This work indicates that IRR of $\sim 9$ dB is possible for $\text{INR}_{in} = 0$ dB, and slowly increases with increasing $\text{INR}_{in}$ for $\text{INR}_{in} < \sim 15$ dB. The results were verified with simulated ATSC signals (Figure 6.7) but shown to be extremely toxic for astronomy for $\text{INR}_{in}$ below 15 dB. For $\text{INR}_{in} = 24$ dB, the results, including toxicity, were extremely good. For real world data (Figure 6.9), the observed IRR was somewhat less ($\sim 17$ dB for $\text{INR}_{in} = 25$ dB) and was attributed to carrier phase estimating accuracy.

In summary, we found ATSC demod/remod PE/S to be very effective at high $\text{INR}_{in}$, but dangerous (especially in terms of toxicity to astronomy) at lower $\text{INR}_{in}$.

7. Finally in Section 6.4, we again compared the demod-remod PE/S performance to the adaptive canceler approach (requiring a reference antenna) for ATSC, and once again found that the adaptive canceler approach is much better. We also again considered the question of whether the performance could be further improved by using demod-remod to further enhance the reference channel $\text{INR}_{in}$. This time we found that significant improvement was possible,
but only for high $\text{INR}_{\text{sn}}$ and assuming perfect carrier phase synchronization (Figure 6.11).

It should be pointed out that while we have not explicitly quantified the computational complexity of the proposed algorithms, we emphasize that it is well within the capabilities of existing conventional off-the-shelf real-time digital signal processing. In fact, this processing is similar or identical to processing performed by the communications equipment traditionally associated with these signals; e.g., detection, demodulation, and so on. Because these are typically commercial/commodity products, there has been considerable previous effort invested in identifying minimum cost/complexity implementations; see for example [97].

7.2 Future Work

The recommended future investigations are as follows:

1. An aspect of noise which is known to be important at these frequencies—impulsive noise [98, 99]—should be considered.

2. ATSC demod-remod might be improved by improving carrier synchronization. Also, one could consider implementing more of the receiver e.g., Viterbi decoding as opposed to just relying on hard symbol decisions one at a time. As noted in Section 6.3, this might help mitigating the toxicity problem seen at
low INR.

3. Superimposed signals were not considered. It is possible, for example, that weak ATSC signals from distant transmit stations might be found at levels damaging to radio astronomy in the same channels that strong (local) ATSC signals are found.

4. Input INR might be improved by using a “multisite” architecture. Specifically, if there are multiple receiving sites, then a site which is being interfered with by a distant transmitter might utilize a signal relayed to it from a site which is closer to the transmitter, and thereby providing higher INR. Thus, detection and PE/S canceling might both be improved. LWA is planned to consist of 53 sites scattered throughout the state of New Mexico, so this concept is especially attractive for LWA and other systems distributed over large geographical areas.

5. For Broadcast FM, constant modulus algorithm (CMA) and whitening/prediction filtering can improve IRR performance through improving the compensation of channel effect.

6. Methods for real-time implementation of the algorithms proposed here which are efficient in terms of both computational cost and power.
Appendix A

US Frequency Allocations:

10-100 MHz

Table A.1: Summary of FCC Frequency Allocations (P denotes Primary User and S denotes Secondary User) [29].

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<th>Freq. [MHz]</th>
<th>Service</th>
<th>Modulation</th>
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<tbody>
<tr>
<td>10.005-10.100</td>
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<td>See Table A.2</td>
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<td>10.150-11.175</td>
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<td>13.260-13.360</td>
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Table A.1: Summary of FCC Frequency Allocations (con’d)

| Freq. [MHz] | Service                      | Modulation                                      
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<td>18.168-18.780</td>
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<td>Freq. [MHz]</td>
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Table A.1: Summary of FCC Frequency Allocations (con’d)

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Table A.1: Summary of FCC Frequency Allocations (con’d)

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<td>P: AERONAUTICAL RADIONAVIGATION</td>
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</tr>
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<td>88.000-88.200</td>
<td>FM BROADCASTING: Channel 201</td>
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</tr>
<tr>
<td>88.200-88.400</td>
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<td></td>
</tr>
<tr>
<td>88.400-88.600</td>
<td>FM BROADCASTING: Channel 203</td>
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</tr>
<tr>
<td>88.600-88.800</td>
<td>FM BROADCASTING: Channel 204</td>
<td></td>
</tr>
<tr>
<td>88.800-89.000</td>
<td>FM BROADCASTING: Channel 205</td>
<td></td>
</tr>
<tr>
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<td>FM BROADCASTING: Channel 206</td>
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</tr>
<tr>
<td>89.200-89.400</td>
<td>FM BROADCASTING: Channel 207</td>
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</tr>
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<td>89.400-89.600</td>
<td>FM BROADCASTING: Channel 208</td>
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</tr>
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<td>FM BROADCASTING: Channel 212</td>
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<td>FM BROADCASTING: Channel 215</td>
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</tr>
<tr>
<td>92.600-92.800</td>
<td>FM BROADCASTING: Channel 224</td>
<td></td>
</tr>
<tr>
<td>92.800-93.000</td>
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WBFM (see Section 2.1.3).
Table A.1: Summary of FCC Frequency Allocations (con’d)

<table>
<thead>
<tr>
<th>Freq. [MHz]</th>
<th>Service</th>
<th>Modulation</th>
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</tr>
<tr>
<td>93.600-93.800</td>
<td>FM BROADCASTING: Channel 229</td>
<td></td>
</tr>
<tr>
<td>93.800-94.000</td>
<td>FM BROADCASTING: Channel 230</td>
<td></td>
</tr>
<tr>
<td>94.000-94.200</td>
<td>FM BROADCASTING: Channel 231</td>
<td></td>
</tr>
<tr>
<td>94.200-94.400</td>
<td>FM BROADCASTING: Channel 232</td>
<td></td>
</tr>
<tr>
<td>94.400-94.600</td>
<td>FM BROADCASTING: Channel 233</td>
<td></td>
</tr>
<tr>
<td>94.600-94.800</td>
<td>FM BROADCASTING: Channel 234</td>
<td></td>
</tr>
<tr>
<td>94.800-95.000</td>
<td>FM BROADCASTING: Channel 235</td>
<td></td>
</tr>
<tr>
<td>95.000-95.200</td>
<td>FM BROADCASTING: Channel 236</td>
<td></td>
</tr>
<tr>
<td>95.200-95.400</td>
<td>FM BROADCASTING: Channel 237</td>
<td></td>
</tr>
<tr>
<td>95.400-95.600</td>
<td>FM BROADCASTING: Channel 238</td>
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<td>95.600-95.800</td>
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<tr>
<td>96.800-97.000</td>
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</tr>
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<td>97.200-97.400</td>
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<td>97.400-97.600</td>
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<td>97.600-97.800</td>
<td>FM BROADCASTING: Channel 249</td>
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</tr>
<tr>
<td>97.800-98.000</td>
<td>FM BROADCASTING: Channel 250</td>
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</tr>
<tr>
<td>98.000-98.200</td>
<td>FM BROADCASTING: Channel 251</td>
<td></td>
</tr>
<tr>
<td>98.200-98.400</td>
<td>FM BROADCASTING: Channel 252</td>
<td></td>
</tr>
<tr>
<td>98.400-98.600</td>
<td>FM BROADCASTING: Channel 253</td>
<td></td>
</tr>
<tr>
<td>98.600-98.800</td>
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<tr>
<td>98.800-99.000</td>
<td>FM BROADCASTING: Channel 255</td>
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<tr>
<td>99.000-99.200</td>
<td>FM BROADCASTING: Channel 256</td>
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</tr>
<tr>
<td>99.200-99.400</td>
<td>FM BROADCASTING: Channel 257</td>
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</tr>
<tr>
<td>99.400-99.600</td>
<td>FM BROADCASTING: Channel 258</td>
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</tr>
<tr>
<td>99.600-99.800</td>
<td>FM BROADCASTING: Channel 259</td>
<td></td>
</tr>
<tr>
<td>99.800-100.000</td>
<td>FM BROADCASTING: Channel 260</td>
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</tr>
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</table>

WBFM (see Section 2.1.3)
Table A.2: Modulation types for services (see Table A.1). See Section 2.1 for acronyms.

<table>
<thead>
<tr>
<th>Services</th>
<th>Modulation types</th>
</tr>
</thead>
<tbody>
<tr>
<td>AERONAUTICAL MOBILE</td>
<td>DSB, FM, PM, SSB, Position/Phase modulation, Tone, Pulse, and others</td>
</tr>
<tr>
<td>MARITIME MOBILE</td>
<td>DSB, FM, PM, SSB</td>
</tr>
<tr>
<td>FIXED SERVICE</td>
<td>CW, AM, FM, SSB, ISB*, RTTY**, and Data (FSK or PSK)</td>
</tr>
<tr>
<td>LAND MOBILE</td>
<td>FM with ±4 or ±5 kHz deviation</td>
</tr>
<tr>
<td>BROADCASTING (HF)</td>
<td>DSB (10 kHz), SSB (5 kHz), and DRM</td>
</tr>
<tr>
<td>AMATEUR</td>
<td>CW, SSB, AM, RTTY**, and PSK.</td>
</tr>
</tbody>
</table>

* ISB (Independent sideband): A method of double-sideband transmission in which the information carried by each sideband is different.
**RTTY: Radioteletype (FSK)
Appendix B

Speech/Audio Signal Model

In speech/audio production, the (speech/audio) signal can be modeled as the output of a linear time–varying filter with an excitation [100, Chapter 9]. In the case of speech, the filter represents the vocal tract, and Quatieri and McAulay [101] propose the use of a sum of sinusoids to describe the excitation. In this model, the filter output, speech/audio signal $m(t)$, is expressed as

$$m(t) = \sum_{l=1}^{L} A_l(t) \cos \phi_l(t)$$

(B.1)

where $L$ is the number of sinusoidal components, and $A_l(t)$ and $\phi_l(t)$ are the magnitude and the phase of the $l$th sinusoidal component, respectively. Note that the magnitude and the phase are time–varying.
To use (B.1), we need to estimate \(A_l(t)\) and \(\phi_l(t)\). This estimation is facilitated by dividing the time into contiguous windows whose duration is \(T\), which is set as 16 ms in our case. Within an interval \(T\), we can assume: (1) \(A_l(t)\) is constant, and (2) \(\phi_l(t)\) is expressed as \(\omega_l t + \phi_l\) where \(\omega_l\) and \(\phi_l\) are constant over the observation time interval. Then, (B.1) in the \(n\)th interval is

\[
m(t) = \sum_{l=1}^{L} A_l \cos(\omega_l t + \phi_l) , \quad t_n - \frac{T}{2} < t < t_n + \frac{T}{2}
\]

(B.2)

where \(t_n\) is the center of the \(n\)th interval. The procedure for estimating \(A_l(t)\) and \(\phi_l(t)\) is 1) Take a short time Fourier Transform (STFT) on the interval, 2) Choose the peaks in the magnitude of the STFT (the total number of the picked peaks becomes \(L\) in (B.2)), and 3) Calculate \(\omega_l, \phi_l,\) and \(A_l\) for the chosen peaks using the procedures of Section 5.1.2.

We have used this method to develop three models for use in this dissertation. The parameters for these methods are extracted from actual signals. We use the parameterized version of these signals rather than the signals themselves to facilitate mathematical analysis of the modulated signal, should this ever be necessary or desired. In the following models, \(L\) is chosen as \(2^{12}\).

**Model 1:** \(m(t)\) is obtained from a real voice signal whose bandwidth is 3 kHz and time duration is 0.95 s. The voice signal comes from a weather radio station operated by the U.S. National Oceanic and Atmospheric Administration (Same data as in Section 5.3.2.3.).
Model 2: $m(t)$ is obtained from a real music signal whose bandwidth is 15 kHz and time duration is 0.97 s. (The music is broadcasted by station KMPX, which is used in Section 5.3.3.3.).

Model 3: $m(t)$ is obtained by applying a 5 kHz low-pass filter to Model 2.

The PSD of the modeled signals for Models 1 and 2 are shown in Figure B.1.

Figure B.1: Averaged PSD of audio models, Top: source data, Bottom: derived model (shifted down 40 dB for clarity). In each case, 0.9 s averaging.
Appendix C

ATSC Data Format

ATSC modulation was described in Section 2.1.6. In this appendix, we provide details of the baseband data format, portions of which are used in the ATSC canceling algorithm described in Chapter 6.

ATSC signals have a frame structure as shown in Figure C.1. The frame has two data fields, each consisting of 313 data segments. The first data segment of each data field is a field sync followed by 312 data segments. Figure C.2 illustrates the structures of a field sync and the other data segments. Each data segment consists of 832 symbols. The first 4 symbols, called segment sync, are (+5, -5, -5, +5). This is used for segment synchronization. The remaining 828 symbols are generally trellis, Reed-Solomon (RS) encoded, and interleaved symbols. The field sync is a special pseudo-random noise (PN) binary sequence employed for data field synchronization.
and channel estimation. The field sync is composed of segment sync, a PN511, three PN63s, and VSB mode. The PN511 is used in long equalizers for mitigating channel distortion over a long time length and the PN63 is for a short equalizer.

**PN 511 generator.** The PN511 is defined by $X^9 + X^7 + X^6 + X^4 + X^3 + X + 1$ with a pre-load value of “010000000”. The generator of PN511 is shown in Figure C.3.

**PN 63 generator.** The PN63 is defined by $X^6 + X + 1$ with a pre-load value of “100111”. The generator of PN63 is shown in Figure C.4.

The overall ATSC transmit system is illustrated in Figure C.5. The input data is a packet of 188 bytes consisting of interspersed video, audio, and ancillary data. It
Figure C.2: The structure of ATSC segments.

Figure C.3: A PN511 generator in a Field Sync.
is randomized first to whiten the data spectrum and then processed for forward error correction (FEC) with RS coding. The RS coding adds 20 parity bytes to the end of each packet, primarily as an aid in burst noise correction. After RS coding, the data bytes are interleaved and mapped to symbols through 2/3-rate trellis coding.
Appendix D

Reference Interference Scenario

Section 2.4 describes a reference interference scenario (RIS) derived from measurements and which can be used in future work as realistic model for RFI over the band 10–100 MHz. This RIS is reported in detail here. This should be used with Galactic noise model which is described in Figure 2.13. The RIS itself is in Table D.1, and Table D.2 shows information on signal models.

Table D.1: Reference Interference Scenario

<table>
<thead>
<tr>
<th>Center freq.[MHz]</th>
<th>Modulation</th>
<th>BW [kHz]</th>
<th>Power[dBm]/carrier</th>
<th>Duty Cycle [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.050</td>
<td>NBFM</td>
<td>20</td>
<td>−115.0</td>
<td>5</td>
</tr>
<tr>
<td>13.100</td>
<td>USB</td>
<td>3</td>
<td>−111.6</td>
<td>5</td>
</tr>
<tr>
<td>13.200</td>
<td>USB</td>
<td>3</td>
<td>−100.1</td>
<td>5</td>
</tr>
<tr>
<td>13.260</td>
<td>NBFM</td>
<td>20</td>
<td>−85.9</td>
<td>10</td>
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<tr>
<td>13.310</td>
<td>DSB-AM</td>
<td>6</td>
<td>−88.5</td>
<td>10</td>
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<tr>
<td>13.360</td>
<td>DSB-AM</td>
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<td>−109.8</td>
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<td>13.460</td>
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<td>−115.3</td>
<td>50</td>
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Table D.1: Reference Interference Scenario (con’d)

<table>
<thead>
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<th>Center freq.[MHz]</th>
<th>Modulation</th>
<th>BW [kHz]</th>
<th>Power[dBm]/carrier</th>
<th>Duty Cycle [%]</th>
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<td>100</td>
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<td>–102.0</td>
<td>100</td>
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<td>–108.9</td>
<td>100</td>
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<td>13.770</td>
<td>USB</td>
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<td>–83.5</td>
<td>100</td>
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<td>6000</td>
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<td>100</td>
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<td>NTSC (video-carrier)</td>
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<td>NTSC (video-carrier)</td>
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<td>Center freq.[MHz]</td>
<td>Modulation</td>
<td>BW [kHz]</td>
<td>Power [dBm]/carrier</td>
<td>Duty Cycle [%]</td>
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<td>NTSC</td>
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<td>–92.7</td>
<td>100</td>
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<tr>
<td>81.760</td>
<td>NTSC</td>
<td>6000</td>
<td>–92.9</td>
<td>100</td>
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<td>87.740</td>
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<td>6000</td>
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<td>100</td>
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<td>broadcast FM (WBFM)</td>
<td>200</td>
<td>–89.4</td>
<td>100</td>
</tr>
<tr>
<td>88.300</td>
<td>broadcast FM (WBFM)</td>
<td>200</td>
<td>–93.4</td>
<td>100</td>
</tr>
<tr>
<td>88.500</td>
<td>broadcast FM (WBFM)</td>
<td>200</td>
<td>–88.9</td>
<td>100</td>
</tr>
<tr>
<td>88.700</td>
<td>broadcast FM (WBFM)</td>
<td>200</td>
<td>–89.6</td>
<td>100</td>
</tr>
<tr>
<td>88.900</td>
<td>broadcast FM (WBFM)</td>
<td>200</td>
<td>–79.7</td>
<td>100</td>
</tr>
<tr>
<td>89.100</td>
<td>broadcast FM (WBFM)</td>
<td>200</td>
<td>–79.0</td>
<td>100</td>
</tr>
<tr>
<td>89.300</td>
<td>broadcast FM (WBFM)</td>
<td>200</td>
<td>–91.1</td>
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Table D.2: Modulation model information for RIS

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