Modeling, Analysis, and Experiments of Inter Fiber Yarn Compaction Effects in Braided Composite Actuators

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ABSTRACT

The braided composite actuator is a pressure-driven muscle-like actuator capable of large displacements as well as large blocking forces. It consists of an elastomeric tube reinforced by a sleeve braided by high performance fibers. In addition to the actuation properties, this actuator can also exhibit a large change in stiffness through simple valve control when the working fluid has a high bulk modulus. Several analytical models have been previously developed that capture the geometrical and material nonlinearities, the compliance of the inner liner, and entrapped air in the fluid. The inter fiber yarn compaction in the fiber layer, which is shown to reduce the effective closed-valve stiffness, is studied. A new analytical model for uniformly deformed actuators is developed to capture the compaction effect. This model considers the inter fiber yarn compaction effect and the fiber extensibility as well as the material and geometric nonlinearities. Analysis and experimental results demonstrate that the new compaction model can improve the prediction of the response behavior of the actuator. The compaction model is improved by considering the yarn bending stiffness. The governing equations are derived and the solution algorithm is presented.
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Chapter 1 Introduction

The braided composite actuator is a high mechanical advantage actuator capable of generating large forces and large strains when pressurized. It consists of an elastomeric tube covered by a braided sleeve having high performance fibers, such as aramid or carbon fibers. The braided composite actuator, also known as the McKibben actuator [1-3] or pneumatic artificial muscle (PAM) [4, 5] is structurally and functionally similar to the flexible matrix composite (FMC) actuator described by Philen and Shan et al. [6, 7]. Common amongst these actuators is the use of an inner liner to prevent leaks at high pressures. The primary structural difference between these actuators is that fibers are impregnated in a soft elastomer for the FMC actuator while the fibers remain dry for the braided composite actuator. Both the braided composite actuator and the FMC actuator are able to contract or elongate with internal pressure, depending on the fiber orientations, as shown in Figure 1-1. According to Philen et al. [6], FMC actuators were able to achieve more than 20% axial free strain and a blocking stress 20 times greater than the input pressure in the laboratory. These interesting actuation properties can be carried over to form adaptive skins with multi-directional actuations by integrating multiple braided composite actuators or FMC actuators into a continuous structural system as illustrated in Figure 1-2.

When the braided composite actuator or the FMC actuator is combined with an internal fluid having a high bulk modulus (e.g. water, oil), one can obtain significant changes in the effective stiffness through simple valve control [8]. The actuator can be very flexible with an open valve and very stiff when the valve is closed since the high bulk modulus fluid resists any volume change from reorientation of the fiber reinforcement. In fact, a stiffness change greater than $10^3$ can potentially be achieved with this technology [8] and a stiffness change greater than 52 has been measured in the laboratory [9].

Significant research work has been conducted on the braided composite actuator and the FMC actuator. A full coverage of the research work in this area will be included in
the following literature review. Both of the braided composite actuator and the FMC actuator are named “pressurized artificial muscle” in the following context in this chapter.

Figure 1-1: FMC actuator structure converting internal pressure to axial motion [6].

Figure 1-2: Fabrication of various multi-cellular-braided-composite-actuators adaptive structures performing different maneuvers.
1.1 Review: pressurized artificial muscle

1.1.1 Brief introduction of pressurized artificial muscles

Pressurized artificial muscles have been studied mostly for prosthetic/orthotic and robotic applications. Different names can be found such as “McKibben actuator” “rubbertuator”, “pneumatic artificial muscle”, “braided artificial muscle”, “fluidic muscle”, etc. The pressurized artificial muscle converts internal pressure to linear force, where the internal pressure can be achieved by either air or liquid. The pressurized artificial muscle consists of an elastomeric tube, double stiff helical fibers as constrains or reinforcements, and two end fittings. The stiff fibers transfer the elastomer radial expansion to the axial contraction. The pressurized artificial muscle has many great advantages. The most notable one is its high power/weight ratio. The power/weight ratio of pneumatic McKibben artificial muscles can be 500W/Kg - 2KW/Kg [10], which surpasses the ratio that is in the order of 100W/Kg for electric motors [11]. This characteristic suggests the potential of pressurized artificial muscles to replace heavier electro-mechanical devices. The high power/volume ratio which is 1.1W/cm³ [12] also allows the pressurized artificial muscle to be integrated into a system more feasible. In addition, it can be manufactured at a low cost because of its simple structure, and its compliance makes itself friendly to operators. These advantages have been attracting more and more interest in pressurized artificial muscles from researchers.

1.1.2 History of pressurized artificial muscles

The McKibben artificial muscle was first invented by American physician Joseph L. McKibben in the 1950s to assist patients with paralyzed hands. However, similar structures had been shown in the earlier patents. Pierce [13] proposed a mining cartridge which consisted of a rubber tube and an expansible braided cover. Haven [14] invented a tension device for converting fluid pressure into a linear tension force. This device obtained a pulling force of 1500 pounds with an internal pressure of 400psi. Morin [15] developed an elastic diaphragm, of which the elastic cylinder was covered by substantially stiff threads. Later Yarlott [16] and Kukolj [17] developed similar actuators,
which were “Fluid Actuator” and “Axially Contractable Actuator”, respectively. Gaylord [18] proposed the first equation describing the force-pressure relationship in his patent “Fluid Actuator Motor System and Stroking Device”.

After the artificial muscle was first used by physician McKibben in orthotics, more applications using the McKibben muscle appeared in this area. Nickel et al. [19] used the pneumatic McKibben muscle on finger-driven flexor-hinge splints to provide the prehension force. Engen [20] used the pneumatic McKibben muscle in upper extremity orthotic systems. The McKibben muscle was eventually abandoned in the 1960s in this area because of the disadvantage of using a cumbersome gas tank to provide the gas, replaced by the smaller electric motor devices [21].

Not until the 1980s the McKibben muscle was revived by the Bridgestone Corporation, which commercialized the muscle and renamed it “rubbertuator”. Two types of robots with the rubbertuator for industrial use were marketed. They were the horizontal multi-joint robot called RASC and the suspended multi-joint robot called SOFT ARM [22]. The pneumatically operated SOFT ARM was used for painting or coating, in the environment where there is a danger of fire or explosion with electric circuits.

Although Bridgestone discontinued their development in 1990s, there were some other groups continuing the work on the pressurized artificial muscle. The Shadow Robot Company developed the famous “Shadow Dexterous Hand” by integrating 40 air muscles. The Festo Corporation has developed the “Fluidic Muscle DMSP/MAS” [23]. The double helix aramid fiber reinforcement is embedded in the chloroprene rubber, which is resistive to chemicals and heat while keeping a high physical strength. With a service life between 100000 and 10 million cycles [24], the “Fluidic Muscle DMSP/MAS” can be widely used in industry.

More details about the McKibben artificial muscle history can be found in other papers [2, 4].
1.1.3 Manufacturing of pressurized artificial muscles

The McKibben artificial muscles are made of a hyper-elastic bladder covered by a braided fiber sleeve. The bladder material can be latex rubber, silicone rubber and other synthetic rubbers. The fibers can be metal, nylon, carbon fiber, glass fiber and other synthetic high strength fibers. The advantage is that the actuator can be easily manufactured since the bladder tubings and woven fiber sleeves are usually commercially available. The disadvantage, however, is that high friction exists in the crossover region of fiber strands when the actuator operates. The friction is believed to be one of the factors leading to the hysteresis phenomenon [2, 12]. The friction can also lead to the fatigue failure of the fiber sleeve [25].

Daerden et al. [26] developed a pleated pneumatic artificial muscle to eliminate inter-fiber friction and hysteresis. The flexible but stiff membrane, which is made of a Kevlar® 49 quasi-unidirectional fabric lined with a polypropylene film, is folded together along the long axis at rest. Later they [27] improved the manufacturing process and obtained the more reliable second generation pleated pneumatic artificial muscle. The pleated artificial muscle can achieve larger maximum contraction and blocking force than the conventional McKibben muscle. It is a variation of the McKibben muscle, with a fiber angle of zero to the longitudinal axis. This type of McKibben muscles were also developed by Bertetto et al. [28], Nakamura and Shinohara [29] and Saga et al. [30].

The Festo Corporation’s fluid muscle is manufactured by embedding two aramid fiber layers into the chloroprene rubber. Fibers are kept a distance apart by the rubber so that there is no friction between the fibers [23]. The bladder and the fiber sleeve are integrated into a single unit. Similar structure was also shown by Philen et al. [6], who made the FMC actuator by filament winding using a carbon fiber-polyurethane matrix system. Fibers in the FMC actuator can be winded at any one angle or combination of angles, which cannot be easily realized with the traditional McKibben muscle.
The end fittings are the critical parts of the pressurized artificial muscle and can be the determining factor in achieving the maximum actuation force and life. Many of the commonly used fitting designs can be found in these papers and patents [24, 25, 31-33]. The basic idea of these designs is to clamp the bladder and the sleeve to a plug by a metal tube. The plug is usually grooved or barbed to increase the friction and help prevent failure. The metal tube can be compressed by hose bands or a swaging process.

1.1.4 Modeling of pressurized artificial muscles

1.1.4.1 Static model

According to the literature, the two main approaches for modeling the response of the pressurized artificial muscles are the virtual work and the continuum mechanics methods.

The virtual work principle was applied by Chou [1, 12], Caldwell [34, 35] and Tondu [2]. The idea is that the input energy by the internal pressure ($P$) is converted to the work done by the muscle tension ($F$): Equation Chapter (Next) Section 1

$$ P \cdot \delta V = F \cdot \delta L. $$

(1-1)

With the assumption of inextensible fibers and uniform radial expansion, the geometric relationship between the volume ($V$) and the muscle length ($L$) can be easily obtained.

The static model through the force balance method was found in Schulte’s work [31]. It also assumed inextensible fibers and uniform radial expansion. Schulte got the same equation as the virtual work method shown in Equation(1-2), where $\theta$ is the braiding angle and $D_0$ is the diameter of the muscle at the braiding angle of 90°,

$$ F = \frac{P \pi D_0^2}{4} \left(3 \cos^2 \theta - 1 \right). $$

(1-2)
Although both methods generated the same equation predicting the McKibben muscle behavior, errors exist between the model and experiments. Researchers have made a lot of efforts in improving the models by considering other factors. Since the artificial muscle is clamped at both ends, the shape of the actuator is not cylindrical with a constant radius when pressurized. To capture the end effect, Tondu and Lopez [2] added an empirical coefficient greater than 1 to the axial strain, which needs to be experimentally determined. Tsagarakis and Caldwell [35] divided the artificial muscle into three parts: cylindrical muscle body and two curved end-parts, and the total contraction force was the summation of contraction forces from the three parts. They obtained the contraction forces from the end-parts through integration. They claimed their model was 30-50% more accurate than unmodified models. Doumit et al. [36] modeled the end part as a frustum cone. Kothera et al. [3] corrected the actuator length based on the physical observation of the approximate tip shape. The bladder’s thickness and elasticity were also considered. Chou and Hannaford [12] modified the fluid volume by subtracting the bladder’s volume from the total volume of the actuator. It was found that the bladder’s thickness can be ignored if the thickness/diameter ratio is below 1:10. Klute and Hannaford [37] later modified the model by including the strain energy storage in the elastic bladder made of natural latex rubber. It was assumed that the bladder was thin and made of the Mooney-Rivlin material. This improved the accuracy in the model. Chou and Hannaford [1, 12] as well as Tsagarakis and Caldwell [35] also proposed an empirical way to adjust the drive pressure, by subtracting the pressure to overcome the bladder elasticity. For the existing friction between the fibers, there is currently no accurate model that can capture this effect because of its complexity. Chou and Hannaford found the friction was the frequency insensitive Coulomb friction and they suggested add a friction value of 2.5N in the model for correction. Tondu and Lopez [2] proposed that the friction was a product of a friction coefficient, fiber contact areas and pressure. The friction coefficient, however, was determined by experiments. Davis and Caldwell [38] later modified the friction model by adding a constant scalar to more precisely predict the contact areas for circular-cross-section fibers. Similar work to predict the contact area more accurately was also done by Doumit et al. [36]. Fiber elongation is considered.
Davis et al. [4] calculated the extension of the fiber strand, and the new model showed better results than the original one. Kothera et al. [39] incorporated the elastic energy stored in both the bladder and fiber strands at the beginning. That is, the work done by pressure is converted to the work done by the muscle tension, plus the elastic energy storage in the bladder and fiber strands.

The continuum mechanics approach to model the McKibben actuator has been proposed by Liu and Rahn [40]. The McKibben actuator was modeled as a thin elastic membrane with continuously distributed inextensible fiber reinforcement undergoing finite deformation. This approach is based on the work of Adkins and Rivlin [41] and Kydoniefs [42]. Adkins and Rivlin investigated the deformations of thin membranes reinforced by inextensible cords. Kydoniefs obtained an exact solution for initially cylindrical membranes subjected to internal pressures. The membranes are assumed to be isotropic, incompressible material possessing a strain energy function $W(I_1, I_2)$, and reinforced by two families of flexible and inextensible cords. Matsikoudi-Iliopoulou [43] developed a solution for the axisymmetric deformation of a pressurized cylindrical membrane reinforced by one family of perfectly flexible and inextensible cords. In this continuum mechanics approach to model the McKibben actuator, the tangent of the meridian of the deformed tube is nowhere parallel to the tube axis except in the middle because of symmetry, which is quite different from other aforementioned approaches. This approach would obtain the pressurized thin-wall artificial muscle shape exactly, including the shape near the end. The drawback of this approach is that it is difficult to solve the differential equations, which is a boundary value problem (BVP), even with the numerical approach.

Another continuum mechanics approach has been developed by Shan [7], based on the important work by Luo and Chou as well as Luo and Taban [44, 45]. Luo and Chou [44] developed two dimensional constitutive equations for finite deformation of composite lamina which are reinforced by unidirectional extensible fibers. Luo and Taban [45] later modified these equations in terms of Lagrangian stress resultants. Shan et al. [7] developed the finite axisymmetric deformation model by combining Luo and
Taban’s constitutive law and the membrane deformation theory by Green and Adkins [46]. Fiber is allowed to be extensible in this approach. This approach can also get the exact solution for pressurized McKibben actuators with thin walls. Same as Liu and Rahn’s approach, it is complex to solve the differential equations. Besides, the fiber reinforced membrane’s material property—the constitutive law—has to be determined by experiments.

Compared to the continuum mechanics approaches which were able to capture end effects, previous straightforward modeling approaches using virtual work methods were simple in describing the McKibben muscle shape as a uniform circular cylinder. If the geometry description in Liu and Rahn’s [40] work is used in the virtual work methods, the exact solution to the actuator can also be derived.

The finite element method has been applied to calculate the McKibben muscle behavior. Bertetto et al. [28] discretized the bladder by a mesh of incompressible hyper-elastic elements with eight nodes. The braided cords were modeled using linear bar elements. Ramasamy et al. [47] used a preliminary result for linear-static analysis. Zhang et al. [48] used geometrically nonlinear anisotropic membrane elements to simulate the nonlinear structural behavior of pneumatic muscle actuators. For the muscle with fibers parallel to the longitudinal axis, Daerden obtained the exact analytical solution using the continuum mechanics approach [11]. However, the muscle membrane is assumed to be linear elastic.

These above models either neglected the effect of the bladder or assumed that the bladder is thin enough that the stress in the thickness direction could be ignored. As a result, the fluid pressure is fully transmitted to the fiber layer. Currently, a model that captures the real pressure between the bladder and the fiber layer does not exist. However, Shan et al. [9] has developed a linear elastic model that can capture the effect of thick bladders for small deformations using 3D constitutive equations. This model is to predict the axial stiffness of a sealed flexible matrix composite (FMC) containing a high bulk
modulus fluid. This paper revealed another important application for these actuators, which is tuning the effective axial stiffness using simple valve control.

1.1.4.2 Dynamic model

The dynamic model of the pressurized artificial muscle is extremely difficult to develop for its nonlinear and large deformations with internal damping. The dynamic property of the muscle varies with different conditions, which are internal pressure, axial load, and boundary conditions. Philen [49] developed a simplified dynamic model of the fluid-filled flexible matrix composite (FMC) combined with active valve control for force tracking. A two degree of freedom linear model was developed for a thin cylindrical shell undergoing axial contraction and uniform radial expansion. The coupled FMC/fluid governing equations were derived for the FMC tube, working fluid, servovalve, and feedback control system.

Tondu and Lopez [2] included kinetic frictions in the static model to get a dynamic force output of the McKibben muscle. Colbrunn et al. [50] and Kang et al. [51] also proposed empirical expressions of the kinetic inter-fiber friction to correct the dynamic force output. The parameters in their formula were determined by experiments.

In most applications, the structural dynamics of the McKibben actuator is ignored. It is believed that the McKibben actuator has a much swifter response than the internal fluid pressure. Therefore people are more interested in predicting the internal fluid pressure dynamics. Chou and Hannaford [1] proposed an enlightening electrical circuit which was analogous to the pneumatic circuit. The pressure difference between the gas source and the actuator is analogous to the voltage difference between two nodes. The gas flow is analogous to the current. The gas viscosity caused by the tubing and the connections was modeled as a linear resistor, and the accumulator was modeled as a linear capacitor. The capacitance is shown in Equation (1-3), where $V$ is the volume, $R$ is the gas constant and $T$ is the temperature. The gas inertia effect was further modeled as an inductance. The analogous electrical circuit was proved to be able to predict the pressure dynamics very
well. Based on this analogy, Davis et al. [4] obtained the cut-off frequency of the McKibben muscle system shown in Equation (1-4), and successfully increased the cut-off frequency of the actuator by reducing the capacitance, i.e. the volume of the actuator. The volume was reduced by filling the actuator’s internal space with granular, solid and liquid fillers. The bandwidth was increased up to 400%. Colbrunn et al. [50] and Kang et al. [51] both assumed the ideal gas and employed a mass flow rate expression proposed by McCloy and Martin [52] to predict the pressure dynamics. The mass flow rate through a valve is shown in Equation (1-5), where $A_p$ is the air passage area of the solenoid valve, $P_{up}$ is the upstream pressure, $C_q$ is the flow rate coefficient, $C_m$ is the flow rate parameter of the solenoid valve and $T_{am}$ is the upstream air temperature.

$$C = \frac{V}{RT} \quad (1-3)$$

$$f_c = \frac{1}{2\pi(R_f + R_s)C} \quad (1-4)$$

$$\dot{m} = A_p P_{up} C_q C_m / \sqrt{T_{am}} \quad (1-5)$$

Reynolds et al. [53] developed a three-element model as an analogy to the pneumatic McKibben muscle. The model consists of a contractile element, a spring element, and a damping element in parallel. The parameters in the model were obtained through precise and accurate experiments. A frequency modeling method of rubbertuators was proposed by Thongchai et al. [54]. The frequency response of the McKibben actuator system was measured experimentally, and the transfer function of the system was identified from the frequency response information. A flat differential model as a system identification method was employed by Sanchez [55].

In summary, almost every model on the dynamic performance of a McKibben actuator system depends on experiments. Since it is difficult to develop accurate dynamic models — which includes fluid dynamics, fluid/structure interactions, damping and large
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deformations — these experimentally based approaches provide simple but effective models for control.

1.1.5 Fatigue study on pressurized artificial muscles

While the McKibben artificial muscle has the high power/weight ratio and low cost, its durability is very important for wide acceptance. Although the structure is simple, improper construction processes may lead to easy and early failures. Common failures observed include sleeve pullout, bladder bursting through the mesh hole, pinhole leaks in the bladder [33, 56], and bladder ruptures [57]. Klute and Hannaford [57] launched the first study on the fatigue characteristics of the McKibben artificial muscle. Based on the crack propagation model, they proposed a method to estimate the fatigue limit of the McKibben actuator’s bladder. Fatigue tests were carried out. The results showed that smaller magnitude of actuator contraction led to a higher fatigue limit, and that the latex rubber had a much higher fatigue limit than the synthetic silicone rubber. Kingsley and Quinn [56] reported various construction materials and methods and their impacts on fatigue life. A spandex sheath was added between the bladder and the mesh, to prevent the fibers of the mesh from pinching the bladder during operation; the bladder was pre-stretched, allowing the mesh to wholly reform between cycles, which prevented the bladder from being pinched by the mesh fibers; tighter woven mesh would extend actuator life effectively. Festo’s fluidic muscle seems to work well. One important reason is that the fibers are embedded in the rubber and they do not cross each other. Therefore fiber damage is reduced as a result of reduced friction. Festo has successfully extended the service life of DMSP/MAS to 0.1-10 million cycles [24].

While the number of maximum cycles for Klute and Kingsley’s actuators were less than 18000 cycles, Woods et al. [25] increased the actuator life significantly which was over 120 million cycles by a new construction method. A stepped end fitting was designed with refined circumferential grooves. The bladder (latex rubber), the braided sleeve (PET plastic) and the end fittings were bonded together by an epoxy adhesive. A thin-wall metal tube was swaged around them to provide smooth and tight clamping.
Actuators with nominal diameters of 1.25 inch and initial braiding angles of 34.5° were tested for ultimate tensile force. The mean failure force was 1659 lbs with a standard deviation of 26 lbs. Fatigue tests were carried out at 30 Hz with a dynamic stroke of 3.7%, 5.3% and 5.9%. Signs of minor braid filament failure due to wear were observed after 120 million cycles as shown in Figure 1-3. During tests, surface temperatures of the actuators were about 130 °F, 30-40 °F higher than ambient temperature. The performances of the actuators barely changed after millions of cycles and the realization of long operating life is encouraging and indicates vast potential applications.

Figure 1-3: Minor braid filament damage after 120 million cycles ([25]).

1.1.6 Hysteresis

Almost all researchers found the hysteresis of the actuator’s tension-contraction curve from experiments. They explained that the phenomenon is a result of internal frictions, e.g. frictions between the bladder and fibers and frictions between fibers themselves [1, 2, 22, 38]. Chou and Hannaford [1] indicated that the hysteresis is velocity independent. Minh et al. [58] and Yeh et al. [59] both constructed a hysteresis model by applying a Maxwell-slip model to improve the accuracy of the existing McKibben actuator model. Van Damme et al. [60] proposed a Preisach-based hysteresis model, which is said to be the most satisfactory mathematical model of hysteresis available. The parameters in these hysteresis models were identified through experiments.
1.1.7 Driving methods of pressurized artificial muscles

1.1.7.1 Fluidic driving

Most pressurized artificial muscles are driven by air, for which the pressurized artificial muscle sometimes is called the pneumatic artificial muscle (PAM). Pneumatic artificial muscles are light-weight compared to the hydraulic driven muscles. It is also more compliant due to the compressibility of the air, which is good for soft touch and safe interaction. The compliance however impedes the precision with position control. Therefore sophisticated nonlinear control strategies are required.

The conventional McKibben muscles are driven by air pressure ranging 0-7 bar. The typical operating pressure of hydraulic systems can be more than 100 bar [61]. Researchers employed liquid fluids (oil, water, etc) to drive the McKibben artificial muscle to generate much higher force. Mori et al. [33] applied a hydraulic pressure of 4MPa to a McKibben artificial muscle (40mm in diameter, 700mm in length and 25% in maximum contraction ratio) and generated a blocking force of 28 kN. Focchi et al. [61] found that the McKibben artificial muscle’s operation bandwidth could be increased by the use of water instead of air: “Closed loop positioning experiments showed that a water powered muscle is more reactive to load variation and therefore positioning accuracy is improved.” But their dynamics positioning experiments showed worse control performance, for which they explained was due to the more under-damped characteristics of water. They addressed that it costs less energy to use water than to use air for operating the actuator since the mass flow is significantly less with water. However, the problems with the use of water as the working fluid may be corrosion, algae, filtering and, of course, the weight increase.

1.1.7.2 Chemical driving

A pH-activated McKibben artificial muscle has been developed by Tondu et al. [62, 63]. The actuator was filled with an ion-exchange resin. The resin has a ball-like microscopic structure favorable to flow circulation through the inner chamber. The ion
exchange with the NaOH solution (chemical reaction) causes the swelling of the resin up to 100% in volume. The reacted resin can exchange ions again with the HCl solution (chemical reaction) which de-swells itself. The innovative driven method converts chemical energy to mechanical energy, which is closer to the biological muscle. A maximum force of 100N was measured for a muscle with an initial cross section of 2 cm². However, it took 41 minutes to reach that state. They replaced the ion resin with a specific hydrogel to reduce the time. With the same ion-exchange principle, the hydrogel showed a time response of about 8 minutes to the same force of about 100N. Because NaOH and HCl solutions are too extreme to the environment, Tondu et al. [64] later used weak-base-weak-acid buffers for ion exchange. The pH range was limited to 4.5-8.3 in contrast to 0-14 for the NaOH and HCl solutions.

A “chemomuscle” actuation system has been developed by Shen and Christ [65]. Hydrogen peroxide, as a liquid fuel (namely monopropellant), was used to generate hot gas to drive the actuator. This type of liquid fuel is decomposed with a catalyst, producing gas as well as a large amount of heat. This monopropellant-powered muscle actuation system is compacted and it does not need an external energy source. Therefore, with this type of system, the pressurized artificial muscles are more likely to be employed in mobile robotics and prosthetics/orthosis.

1.1.7.3 Electrical driving

Yokota et al. [66] recently used an electro-conjugate fluid (ECF) as a pressure source for a small size McKibben artificial muscle. The ECF is a type of dielectric functional fluid. When a high DC voltage is applied, a powerful jet flow is generated between two electrodes which are a few hundred micrometers away in the ECF as shown in Figure 1-4. An ECF jet generator of a needle-ring electrode pair was used in this paper, in which the needle went through the hole of the ring. The needle diameter was 0.13mm and the electrode gap was 0.2mm. A pressure of 13.2KPa was measured from the ECF jet generator with an applied voltage of 8 KV. They indicated that the ECF jet becomes more powerful as the electrode pair becomes more compact. Therefore the ECF jet is suitable
for micro-fluid power systems, such as a micro McKibben actuator. No bulky fluid power sources are needed. As claimed in this paper, current is as low as several microamperes in spite of high voltage, which results in power consumption in the scale of milliwatts.

![ECF jet generator with a needle-ring electrode pair](image)

**Figure 1-4:** ECF jet generator with a needle-ring electrode pair [66].

### 1.1.8 Control methodologies

Since the pressurized artificial muscle is highly nonlinear and there are challenges with developing a precise analytical model, it is difficult to control. Many researchers have developed various control approaches to solve this problem. The sliding model control (SMC) has been applied widely [63, 67-74]. The fuzzy control approaches [54, 75-78] and the neural network approaches [79-84] can also be found. Other approaches include adaptive control [85, 86], adaptive pole-placement control [34, 87], gain scheduling control [88, 89] and variable structure controls [90-92]. More detail categories can be found in Shen’s work [68].

The McKibben actuator mathematical models used in the control approaches include: the phenomenological modeling [53] and polynomial approximation and frequency modeling (system identification). Experiments are required to determine the parameters in these models. Minh et al. [58] and Yeh et al. [59] both constructed a hysteresis model.
by applying a Maxwell-slip model to improve the accuracy of the existing McKibben actuator model and allowed for simpler controllers to be designed. Minh et al. used a simple PI controller and Yeh et al. devised an inverse control method combined with LTR feedback control. Both results showed good tracking performance.

While most developed controllers require independent pressure control for each actuator, Shen [68] developed a dynamic model including the flow dynamics, pressure dynamics, force dynamics and load dynamics. The control model only took the valve command as the input.

1.1.9 Applications

1.1.9.1 Prosthetics/orthotics

Due to the high power/weight ratio and the compliance, the pressurized artificial muscles are widely used in prosthetics/orthotics. In addition to the aforementioned orthotics applications in the 1950s and the 1960s, more applications in this area can be found. Bharadwaj et al. [93] developed a rehabilitation device made of a spring and a McKibben muscle in parallel to realize bi-directional actuating. The University of Michigan [5, 94] developed a powered ankle-foot orthosis (AFO) that used McKibben artificial muscles to generate active plantar flexor torque. They later extended the AFO to a KAFO (knee-ankle-foot orthosis) [95], as shown in Figure 1-5. Costa and Caldwell [96] developed a lower limb exoskeleton system, which were wearable and powered by the McKibben artificial muscles, for force augmentation and active assistive walking training. Ding et al. [97] developed a power-assisting device that could enable “pinpointed” motion support, rehabilitation and training by explicitly specifying target muscles. Sasaki et al. [98] developed a wearable master-slave lower limb training device constructed with the McKibben actuators. Zhang et al. [99] developed a wearable elbow exoskeleton with curved McKibben actuators, which weakened the coupling relationship between the output torque and contracting stroke of the actuators. Iwaki et al. [100] also developed a wearable hand assistive system employing the McKibben actuators.
1.1.9.2 Robotics

The pressurized actuators have been commonly used in robotic developments. Caldwell et al. [101] developed a humanoid and bipedal robot. The University of Washington at Seattle developed a muscle-like device [102, 103]. Two McKibben actuators were paralleled with a hydraulic damper and were placed in series with two-springs. The device had the general performance of a bio-muscle: higher activation pressure generated higher output forces, faster contractions resulted in lower output forces and longer muscle produced higher output forces. Kawashima et al. [104] developed a pneumatic humanoid robot arm as construction machinery, which was controlled remotely to ensure the safety of workers during excavation. The HANDAIFrontier Research Center in Osaka University [105, 106] has developed a biped robotic walker with McKibben actuators. Tondu et al. [107] developed a 7-degrees-of-freedom robot arm actuated by antagonistic McKibben artificial muscle pairs. Vrije University Brussel developed a biped walking robot named “Lucy” actuated with pleated pneumatic artificial muscles [108, 109]. Festo developed a humanoid muscle robot with their fluidic muscles [110] as shown in Figure 1-6. Shibata et al. [111] developed an underwater
robotic manipulator with arms driven by McKibben actuators, taking advantage of the simple mechanism and water resistance of the actuators.

Figure 1-6: Festo [110] humanoid muscle robot.

1.1.9.3 Other actuation applications

Recently Wereley’s group in the University of Maryland [112] has used the McKibben actuators in aerospace applications. The McKibben actuators have been applied in a morphing cell for a wing section [113] and in a trailing edge flap system [114, 115]. More recently, a morphing spoiler concept was designed and fabricated using flexible matrix composite actuators and results demonstrate that the morphing spoiler can achieve more than 12 cm of tip displacement under expected aerodynamic loading [116]. Other applications of McKibben actuators include: a manipulator rig for nuclear waste retrieval operations [117], used in parachute systems for soft landing and steering control [48], a haptic device for a virtual environment system [118], a linear axis driving device [69], posture control of parallel manipulator [85] and used as muscles in an artificial fish [119].
Figure 1-7: (a) Designed final spoiler prototype, (b) assembled spoiler before casting without honeycomb core, (c) assembled spoiler with honeycomb core, (d) shape of active spoiler after actuation.

1.1.9.4 Variable stiffness and vibration control

In addition to the conventional actuations, the pressurized artificial muscle has been used as variable stiffness elements in systems and structures. Researchers working on pneumatic artificial muscles noticed the characteristic of variable stiffness, which depends on the pressure and the length. Sardellitti et al. [120] controlled the torque and the stiffness of a joint — which was actuated by antagonistic pneumatic muscles — by controlling the pressure. However, the capability of variable stiffness of pressurized artificial muscle was not fully recognized until Philen et al. [8, 9] developed a variable stiffness adaptive structure based upon fluidic flexible matrix composites (F²MC). Combined with a high bulk modulus fluid (e.g. water, oil), F²MC can be very flexible with an open valve since the fluid is unconstrained, and it can be very stiff when the valve is closed since the high bulk modulus fluid resists any volume change from reorientation of the fiber reinforcement. The stiffness change theoretically can be up to
several orders of magnitude and a stiffness ratio of 56 has been measured in the laboratory [9]. One advantage of the F^2MC technology is that the material properties, such as the composite lamina properties, the fluid, as well as the tube geometry, can be tailored for different applications. Shown in Figure 1-8 is the possible design space of the F^2MC tube where each open circle represents a feasible design obtained through parameter studies. Because of the great tailorability, one can change to get different types of stiffness. For example one can change different winding angles, elastomers or fluid to get different stiffness ratios. Each small circle represents the different design points one can do with the composite actuators when filled with fluid. The F^2MC tubes can also be incorporated in different matrices to achieve variable stiffness structures [9, 121]. Philen [49] recently demonstrated through analysis and experiment that active valve control can track a desired force-displacement curve using a high-speed servo-valve and a combined observer/regulator feedback control system.

![Figure 1-8: Design space of F^2MC tubes and other variable modulus materials (open circles indicate feasible designs) [9].](image)

Due to the variable stiffness of F^2MC, it can be used for semi-active vibration absorbers/isolators. The vibration energy can be stored in the high-stiffness state as the potential energy of high bulk modulus fluid, and it can be dissipated in the low-stiffness state when the fluid is free to flow. Lotfi-Gaskarimakalle et al. [122] investigated the use
of a Zero Vibration state switch technique for the F$^2$MC system and showed that the optimal ZV controller outperforms an optimal Skyhook controller. Lotfi-Gaskarimakalle et al. [123] showed that the primary mass forced vibration can be suppressed at the tuned frequency using a pressurized air accumulator coupled with a single F$^2$MC tube. Scarborough et al. [124] demonstrated that the isolation frequency of an F$^2$MC vibration isolator can be tuned through adjustment of the air pressure in the system. Philen [125] developed an analytical model of a isolation mount based on F$^2$MC with a proportional valve. Results showed that the F$^2$MC based isolation mount is effective for reducing the force transmitted to the foundation. Li and Wang [126] investigated a dual-F$^2$MC cellular structure as a vibration absorber, whose absorption frequency can be tailored by designing the fiber angle combination or flow port dimension.

1.2 Problem statement and research objectives

As mentioned in Section 1.1.4.1, Shan et al. [7] developed the finite axisymmetric deformation model for the FMC actuator by combining Luo and Taban’s constitutive law and the membrane deformation theory by Green and Adkins [46]. Later structure/fluid analytical models with infinitesimal deformations were developed by Shan et al. [9] and Philen et al. [8, 127] to predict the axial stiffness variation between open- and closed-valve states. These models include the compliance of the inner liner and entrapped air in the fluid. In Philen et al.’s work [127], the FMC composite tube elements were manufactured with significantly smaller diameters than previously reported. This resulted in a reduced volume fraction of the internal working fluid with respect to the thicker inner liner and FMC laminate, which as a result, reduced the maximum closed-valve stiffness and increased the sensitivity to the through-the-thickness wall compliance of the FMC laminate. While there was relatively good agreement between analysis and experiment, the results did suggest that additional nonlinearities were not being captured in the analysis model.

As noted above, the wall compliance of the FMC laminate in the radial direction can reduce the effective closed-valve stiffness of the FMC actuator. The same problem also
exists within the braided composite actuator, of which the wall compliance is mainly due to the flattening of undulating yarns, as illustrated in Figure 1-9. The wall compliance is referred to ‘compaction’ throughout the rest of this dissertation. It has been observed in the experiments that the fiber yarns experience compaction at intersection areas when the braided composite actuator is subjected to an axial tensile force or internal pressure. Currently, to the authors’ knowledge, no model exists that considers the compaction effect on the performance of braided composite actuators or FMC actuators, especially for achieving large changes in effective stiffness between open- and closed-valve states. The compaction effect can have a significant influence on the stiffness variation.

Figure 1-9: Braided composite actuator sample and illustration of inter fiber yarn compaction.

The inter yarn compaction effect in the sleeve layer can reduce the performance of the braided composite actuator and the maximum closed-valve stiffness. Understanding this effect and capturing the behavior requires new analysis tools for prediction. Therefore the objectives of this research are to (a) develop a new analytical model that
includes compaction effects between overlapping fiber yarns, fiber extensibility, and the material and geometric nonlinearities, (b) perform analysis for a braided composite actuator, and (c) validate the analysis results in the laboratory through experiments.

1.3 Outline of dissertation

This dissertation consists of seven chapters, which are organized as follows.

The first chapter reports the research works relevant to the braided composite actuators muscles. The research motivation and objectives are stated.

The second chapter introduces the preliminary knowledge of braided composite actuator modeling, upon which the compaction model is developed.

The third chapter develops the compaction model for braided composite actuators reinforced with braided sleeves, assuming zero fiber yarn bending stiffness.

The fourth chapter describes the experimental validation for the compaction model and the fifth chapter analyzed the compaction effect to the performance of braided composite actuators by using the proposed compaction model.

The sixth chapter investigated the yarn bending effect to the compaction model and improved the compaction model by including the yarn bending energy.

Conclusions and future work are in the seventh chapter.
Chapter 2 Modeling braided composite actuators without compaction

The modeling of braided composite actuators involves material nonlinearity (finite strains) and geometry nonlinearity (finite strains and fiber reorientation).

As mentioned in section 1.1.4.1, the two main approaches for modeling the response of the braided composite actuator are the continuum mechanics approach and the virtual work approach.

2.1 Continuum mechanics approach

2.1.1 Rubber reinforced by unbounded fibers

Liu and Rahn [40] used the continuum mechanics approach to model the braided composite actuator based upon the theory of Kydoniefs [42] and Matsikoudi-Iliopoulou [43].

Assumptions were made to facilitate the modeling, as listed below:

a) Fibers are perfectly flexible, inextensible and helicoidal;
b) No two fibers of the same family are in brought into contact as a result of deformation;
c) Intersecting fibers of the two families do not slide relative to each other at their point of intersection.
d) The lengths of the intercepts on any one fiber of the one family by two adjacent fibers of the other family are independent of the position on the surface and these lengths are small compared with the radii of the curvature at any point of the deformed or undeformed membrane.
e) The rubbery inner liner is incompressible and very thin.
The geometric description is shown in Figure 2-1.

Figure 2-1: Braided composite actuator model and coordinate system definition.

The undeformed configuration ($C_0$) is denoted by the polar coordinates ($\rho, \Psi, \eta$) and the deformed configuration ($C$) is denoted by the polar coordinates ($r, \psi, z$). The deformed meridian element is denoted by $d\xi$. Since the structure is axisymmetric, we have

$$r = r(\xi),$$

(2-1)

$$z = z(\xi),$$

(2-2)

and

$$\psi = \Psi.$$  

(2-3)

Define
\[ \lambda_1 = \frac{d\xi}{d\eta}, \quad (2-4) \]

\[ \lambda_2 = \frac{r}{R}, \quad (2-5) \]

and

\[ \sin \sigma = \frac{dr}{d\xi}. \quad (2-6) \]

For inextensible fibers, we have

\[ ds = \frac{d\xi}{\cos \theta} = \frac{rd\psi}{\sin \theta} = ds_0 = \frac{d\eta}{\cos \theta_0} = \frac{Rd\Psi}{\sin \theta_0}, \quad (2-7) \]

where \( ds_0 \) and \( ds \) are the elements of fiber length in the undeformed and deformed state, respectively. Therefore we have

\[ \sin \theta = \lambda_2 \sin \theta_0 \]
\[ \cos \theta = \lambda_1 \cos \theta_0. \quad (2-8) \]

The stress for the braided composite actuator is the summation of the fiber and rubber stresses. The stress resultants (force per unit length) from the rubber in the meridian and the circumferential directions are \( n'_1 \) and \( n'_2 \) respectively, as shown in Equation (2-9),

\[ n'_1 = \frac{2h_0}{\lambda_2} \frac{\partial W}{\partial \lambda_1}, \]
\[ n'_2 = \frac{2h_0}{\lambda_1} \frac{\partial W}{\partial \lambda_2}. \quad (2-9) \]
where $W$ is the strain energy density of the rubber and $h_0$ is the undeformed thickness of the rubber. The stress resultants from the fibers in the meridian and the circumferential directions are $n_1^\prime\prime$ and $n_2^\prime\prime$ respectively, as shown in Equation (2-10),

$$
\begin{align*}
    n_1^\prime\prime &= 2\tau \cos^2 \theta \frac{\lambda_2}{\lambda_1} \\
    n_2^\prime\prime &= 2\tau \sin^2 \theta \frac{\lambda_2}{\lambda_1}
\end{align*}
$$

(2-10)

where $\tau$ is the tension from the fibers per unit length in the transverse direction. The stress resultants for the actuator are

$$
\begin{align*}
    n_1 &= n_1' + n_1^\prime\prime \\
    n_2 &= n_2' + n_2^\prime\prime
\end{align*}
$$

(2-11)

The equilibrium equations for a membrane undergoing large deformation are

$$
\begin{align*}
    \frac{d (\lambda_2 n_1)}{d \lambda_2} &= n_2 \\
    \frac{d (\lambda_2 n_1 \cos \sigma)}{d \lambda_2} &= P \cdot r
\end{align*}
$$

(2-12)

With the boundary conditions shown in Equation (2-13), Equation (2-12) can be solved.

$$
\begin{align*}
    \lambda_2(L) &= 1 \\
    \sigma(0) &= 0 \\
    n_1(0) &= \frac{P\lambda_2(0)R}{2\pi} + \frac{F}{2\pi\lambda_2(0)R}
\end{align*}
$$

(2-13)
2.1.2 **Rubber reinforced by embedded fibers**

In the theory introduced in section 2.1.1, since the fibers are unbounded with the rubber, the material properties of the rubber remain unchanged. Furthermore, the fiber rigidity significantly exceeds that of the rubber material. Therefore it is appropriate to assume the fiber is inextensible and apply the constitutive behavior of the rubber. However, the flexible matrix composites (FMCs) have a large fiber volume fraction in the rubbery matrix. The presence of fibers causes the transverse and shear rigidities to significantly exceed those of the unreinforced rubbery material. To account for these transverse reinforcement effects, it is appropriate to apply the theory of orthotropic composite membranes.

Luo and Chou [128] developed a two-dimensional constitutive law for finite deformation of composite lamina with unidirectional extensible fibers. They considered an in-plane deformation of a composite lamina element, shown in Figure 2-2. $X_1$-$X_2$ is the coordinate system in the undeformed plane. Point $(X_1, X_2)$ moves to point $(x_1, x_2)$ due to deformation. $x_1$ and $x_2$ can be expressed by $X_1$ and $X_2$ from geometry in Equation (2-14).

![Figure 2-2: Deformation of a composite element.](image)
\[ x_1 = \lambda_1 X_1 + \lambda_2 X_2 \tan \gamma \]
\[ x_2 = \lambda_1 X_1 \tan \varphi + \lambda_2 X_2 . \]  

(2-14)

The deformation gradient \( g_{ij} \) is determined from Equation (2-14) as

\[
g = [g_{ij}] = \begin{bmatrix} \frac{\partial x_i}{\partial X_j} \\ \frac{\partial x_j}{\partial X_i} \end{bmatrix} = \begin{bmatrix} \lambda_1 & \lambda_2 \tan \gamma \\ \lambda_1 \tan \varphi & \lambda_2 \end{bmatrix} . \]  

(2-15)

The Lagrangian strain tensor \( E_{ij} \) of the laminate is determined as

\[
E = [E_{ij}] = \begin{bmatrix} \frac{1}{2} (g_{ki} g_{kj} - \delta_{ij}) \end{bmatrix} . \]  

(2-16)

where \( \delta_{ij} \) is the Kronecker Delta.

A new coordinate system \( \bar{X}_1 - \bar{X}_2 \) is defined, which coincides with the initial principal material coordinate. Axis \( \bar{X}_1 \) is parallel to the original fiber longitudinal direction and \( \bar{X}_2 \) is in the transverse direction. A transformation matrix \( \mathbf{a} \) (Equation (2-17)) is used to express the Lagrangian strains in the principal material coordinates \( \bar{X}_1 - \bar{X}_2 \).

\[
\mathbf{a} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \cos \theta_0 & -\sin \theta_0 \\ \sin \theta_0 & \cos \theta_0 \end{bmatrix} \]  

(2-17)

The Lagrangian strains with respect to \( \bar{X}_1 - \bar{X}_2 \) is

\[
\bar{E}_{11} = E_{11} \cos^2 \theta_0 + 2E_{12} \sin \theta_0 \cos \theta_0 + E_{22} \sin^2 \theta_0 \\
\bar{E}_{22} = E_{11} \sin^2 \theta_0 - 2E_{12} \sin \theta_0 \cos \theta_0 + E_{22} \cos^2 \theta_0 \\
\bar{E}_{12} = \bar{E}_{21} = (E_{22} - E_{11}) \sin \theta_0 \cos \theta_0 + E_{12} (\cos^2 \theta_0 - \sin^2 \theta_0) . \]  

(2-18)
Luo and Chou also proposed a strain energy density function $W\left(\overline{E}_{11}, \overline{E}_{22}, \overline{E}_{12}^2\right)$ for the lamina, which is in the fourth-order polynomial form. The coefficients in the function are estimated from experiments. With the strain energy density function, one can obtain the constitutive equation of a composite lamina subjected to in-plane finite deformations with respect to the $X_1 - X_2$ coordinates. The Lagrangian stress tensor is obtained as

$$\Pi_{ji} = \frac{\partial W}{\partial g_{ij}}. \quad (2-19)$$

Assume the laminate consists of “$n$” layers of lamina with each layer having an identical thickness $t$, as shown in Figure 2-3. The Lagrangian stress resultants $(N_{ij})$ can be expressed as:

$$N_{ij} = \int_{-h/2}^{h/2} \Pi_{ij} dz = \sum_{k=1}^{N} \Pi_{ij}^{(k)} t, \quad (2-20)$$

where $\Pi_{ij}^{(k)}$ is the Lagangian stresses of the $k$th layer.

A symmetric composite laminate with fiber orientation sequence of $+\theta/ -\theta/ +\theta/ -\theta$ under biaxial deformation is shown in Figure 2-4. If the inter-laminar shear deformation is ignored, the shear deformation of the laminate is eliminated, i.e. $\gamma=\varphi=0$. In this figure, $n_{ij}$ are the Cauchy stress resultants and they can be determined by

$$n_{ij} = (\text{det}[g])^{-1} g_{ik} N_{kj}. \quad (2-21)$$

Shan et al. [7] developed the finite axisymmetric deformation model of the FMC composite tube by combining the introduced Luo and Chou’s constitutive law and the membrane deformation theory by Green and Adkins [46]. The governing equations are the same as in Equation (2-12) and the boundary conditions are the same as in Equation (2-13).
Figure 2-3: A composite laminate consists of $N$ layers of lamina.

Figure 2-4: Biaxial deformation of a symmetric $\pm \theta_0$ composite laminate.
2.2 Virtual work approach

The energy approach has been adopted by many researchers to model the McKibben actuators as mentioned in section 1.1.4.1. The geometry descriptions in their models were not as precise as that in Figure 2-1, i.e. with assumptions to simplify the actuator’s shape description. This section is to combine the virtual work approach and the exact shape description to derive the exact governing equations.

The concept of the virtual energy approach for the braided composite actuator or the flexible matrix composite actuator can be presented as

$$\delta E_{\text{strain}} = \delta W_P + \delta W_F , \quad (2-22)$$

where $W_P$ is the work done by the internal pressure $P$ and $W_F$ is the work done by the axial force $F$. $E_{\text{strain}}$ is the strain energy stored in the rubbery liner for the braided composite actuator or the strain energy stored in the composite laminate for the flexible matrix composite actuator. The frictions between the fibers in the fiber layer and the frictions between the fibers and the rubber are neglected. The strain energy in the fiber layer is also neglected here.

For the braided composite actuator, if the rubber is assumed to be the incompressible Mooney-Rivlin material, of which the strain energy density can be expressed as

$$W = C_1 (I_1 - 3) + C_2 (I_2 - 3)$$

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \lambda_2^{-2} ,$$

$$I_2 = \lambda_1^{-2} + \lambda_2^{-2} + \lambda_1^2 \lambda_2^2$$

where $C_1$ and $C_2$ are material constants which can be determined by experiments.
By using the same geometry description as in Figure 2-1, the virtual strain energy and the virtual works in Equation (2-22) can be written as

\[
\delta E_{\text{strain}} = \delta \int_0^L \left[ C_1 (I_1 - 3) + C_2 (I_2 - 3) \right] 2\pi r \frac{h_0}{\lambda_1 \lambda_2} d\xi ,
\]

\[
= \delta \int_0^L \left[ C_1 (I_1 - 3) + C_2 (I_2 - 3) \right] 2\pi Rh_0 d\eta ,
\]

\[
\delta W_p = P \delta V
\]

\[
= P \delta \int_0^L \pi r^2 \cos \sigma d\xi
\]

\[
= P \delta \int_0^L \pi r^2 \sqrt{1 - \left( \frac{dr}{d\xi} \right)^2} d\xi ,
\]

\[
= P \delta \int_0^L \pi \lambda_2^2 R^2 \sqrt{1 - \left( \frac{Rd \lambda_2}{\lambda_4 d \eta} \right)^2} \lambda_4 d\eta
\]

and

\[
\delta W_r = F \delta L
\]

\[
= F \delta \left( L_0 - \int_0^L \cos \sigma d\xi \right) .
\]

\[
= F \delta \left( L_0 - \int_0^{t_0} \sqrt{1 - \left( \frac{Rd \lambda_2}{\lambda_4 d \eta} \right)^2} \lambda_4 d\eta \right)
\]

Define the terms in Equation (2-24), Equation (2-25) and Equation (2-26) as follows

\[
(\lambda_2' = d\lambda_2 / d\eta):
\]

\[
\mathcal{F}(\lambda_2, \lambda_2') = -P \pi \lambda_2^2 R^2 \sqrt{1 - \left( \frac{Rd \lambda_2}{\lambda_4 d \eta} \right)^2} \lambda_4 .
\]
\[ \mathcal{S} (\lambda_2, \lambda'_2) = F \sqrt{1 - \left( \frac{Rd \lambda_2}{\lambda_2 d\eta} \right)^2} \lambda_1, \quad (2-28) \]

and

\[ \mathcal{S} (\lambda_2) = \left[ C_1 (I_1 - 3) + C_2 (I_2 - 3) \right] 2 \pi R h. \quad (2-29) \]

By substituting equations from Equation (2-27) to Equation (2-29) into Equation (2-22), we can obtain

\[ \int_{0}^{t_0} \delta \mathcal{S} (\lambda_2, \lambda'_2) d\eta + \int_{0}^{t_0} \delta \mathcal{S} (\lambda_2, \lambda'_2) d\eta + \int_{0}^{t_0} \delta \mathcal{S} (\lambda_2) d\eta = 0. \quad (2-30) \]

From Equation (2-30), we can obtain

\[ \int_{0}^{t_0} \left( \frac{\partial}{\partial \lambda_2} \delta \lambda_2 + \frac{\partial}{\partial \lambda'_2} \delta \lambda'_2 \right) d\eta + \int_{0}^{t_0} \left( \frac{\partial}{\partial \lambda_2} \delta \lambda_2 + \frac{\partial}{\partial \lambda'_2} \delta \lambda'_2 \right) d\eta + \int_{0}^{t_0} \frac{d}{d\lambda_2} \delta \lambda_2 d\eta = 0. \quad (2-31) \]

Integrating by parts, we get

\[ \int_{0}^{t_0} \left( \frac{\partial}{\partial \lambda_2} \delta \lambda_2 + \frac{\partial}{\partial \lambda'_2} \delta \lambda'_2 \right) d\eta + \left. \frac{\partial}{\partial \lambda'_2} \delta \lambda'_2 \right|_{0}^{t_0} - \int_{0}^{t_0} \left( \frac{d}{d\eta} \left( \frac{\partial}{\partial \lambda'_2} \right) \delta \lambda'_2 \right) d\eta = 0. \quad (2-32) \]

Since \( \delta \lambda_2 \bigg|_{t_0} = 0 \) and \( \lambda'_2 \bigg|_{0} = 0 \) according to the boundary conditions, the equilibrium equation is

35
\[
\frac{\partial \mathcal{F}}{\partial \lambda_2} + \frac{\partial \mathcal{G}}{\partial \lambda_2} + \frac{\partial \mathcal{H}}{\partial \lambda_2} - \frac{d}{d\eta} \left( \frac{\partial \mathcal{F}}{\partial \lambda_2'} \right) - \frac{d}{d\eta} \left( \frac{\partial \mathcal{G}}{\partial \lambda_2'} \right) = 0.
\]

(2-33)

By solving this differential equation with the boundary conditions \((\lambda_2(L_0) = 1, \lambda_2'(0) = 0)\), we can obtain the deformed shape of the actuators. It is a second order differential equation. The second derivative of \(\lambda_2\)—\(d^2\lambda_2 / d\eta^2\)—can be explicitly expressed by \(\eta, \lambda_2\) and \(\lambda_2'\). Therefore the Runge–Kutta method can be applied to solve the equation numerically. However, we need to try different values for \(\lambda_2(0)\) to find the correct solution that satisfies the boundary condition \(\lambda_2(L_0) = 1\).
Chapter 3 Compaction modeling for uniformly deformed braided composite actuators

Nomenclature

\( a \) fiber yarn dimension, see Figure 3-5
\( b \) fiber yarn dimension, see Figure 3-5
\( d \) fiber yarn width
\( n \) number of carriers that fiber sleeve is braided by
\( p \) pressure applied on the sleeve in the compaction law
\( A_{\text{int}} \) overlapping area of two intersected yarns
\( B \) bulk modulus of fluid
\( C_1, C_2 \) material constants for the Mooney-Rivlin material
\( F_A \) horizontal force component between overlapping yarns
\( F_N \) vertical force component between overlapping yarns
\( F_T \) tension in a single yarn
\( L_0 \) undeformed actuator length
\( N_1 \) axial Cauchy stress resultant
\( N_2 \) circumferential Cauchy stress resultant
\( P \) internal pressure of actuator
\( P_0 \) initial internal pressure in closed-valve actuator
\( \bar{P} \) increased pressure in closed-valve actuator
\( T \) tension applied on the actuator ends
\( R_0 \) undeformed actuator radius
\( R_{0\text{T}} \) actuator radius for the transitional state
\( V \) cavity volume of closed-valve actuator
\( V_0 \) initial cavity volume of closed-valve actuator
\( V_A \) air volume of closed-valve actuator
\( V_F \) fluid volume of closed-valve actuator
\( V_A^0 \) initial air volume of closed-valve actuator
\( V_F^0 \) initial fluid volume of closed-valve actuator
The compaction model for woven fabrics has been studied early by the textile community to investigate tensile properties of textiles. Mechanical deformation of woven fabrics was first investigated theoretically by Peirce [129]. He introduced certain quantities to facilitate general comparison between fabrics on the basis of their geometrical form and deduced the relations between spacings and crimps of warp and weft. Many researchers modified the model by relaxing the assumptions, such as inducing yarn bending rigidity and various deformable cross sections. Grosberg and Kedia [130] analyzed the initial load-extension modulus of a cloth. The yarns are assumed to be thin and inextensible beams. Leaf and Kandil [131] analyzed the initial load-extension behavior of plain woven fabrics using the Castigliano’s theorem. They used a saw-tooth model in which yarns are modeled as straight lines and compressive forces between yarns are assumed to be point forces. Kawabata et al. [132] developed a biaxial load-deformation model for plain weaves, also using the saw-tooth geometry. Hearle and Shanahan [133, 134] proposed an energy method to determine the plain
woven fabrics states based on Peirce’s geometry. Huang [135, 136] analyzed the finite biaxial extension of plain woven fabrics with the yarns treated as curved rods. He found that the extension of the fabric with low tensile stress level is governed by both the extension and the bending of yarns and the extension of the fabric with high tensile stress level is dominated by the yarn extension. Other relevant work includes Ghosh et al. [137], Anandjiwala et al. [138] and Hearle et al. [139]. A detailed literature review in the textile area can be found in Dastoor’s paper [140]. Recent years the compaction behavior of fabrics has been studied in the composites industry. The compaction of fabric preform can help to get a desired high fiber volume fraction after the manufacturing process, which results in performance improvement of the composites. In contrast to works in the textile area, the composite community concentrates more on the compaction subjected to out-of-plane pressures. Gutowski [141] proposed a simple elastic deformation model for transverse compression of aligned bundles. Simacek and Karbhari [142, 143] developed a constitutive equation for fiber yarn and then modeled the fabric compaction based on the beam theory. Chen and Chou [144] treated the fiber yarn as a transversely isotropic beam with deformable cross section shape. In the analysis, linear beam theory was applied and the in-plane strains of the fabric during compaction were ignored. Chen et al. [145] developed a micromechanical model including both micro compaction of yarn cross section and macro deformation of yarn undulation flattening. They also applied the linear beam theory and therefore ignored the fabric extension. Experimental studies for fabric compaction has been performed by Matsudaira and Qin [146], Hu and Newton [147], De Jong et al. [148] and Chen et al. [149].

3.2 Fabric structure

The elementary pattern of the braided sleeves is the twill 2x2 shown in Figure 3-1 and the schematic of the element is shown in Figure 3-2. Different from the previous works on fabrics compaction, the sleeve of the braided composite actuator is not planar.
3.3 Technical approach

According to the previous research works, the compaction was investigated with fabrics subjected to the loadings of either in-plane tensions or out-of-plane pressures. In this work, we consider both types of loadings for the fabric and the approach in this paper is based upon the Kydoniefs’ deformation theory [42] on membrane reinforced by two families of fibers and the saw-tooth geometry description [131, 132] on woven fabrics. In the current model, the braided composite actuator is assumed to be long enough that it undergoes uniform radial expansion when deformed. The inner rubber tube is assumed to
be thin and to be Mooney-Rivlin material. The fiber yarns are assumed to be perfectly flexible, i.e. zero bending stiffness.

3.4 Compaction modeling for braided composite actuators

3.4.1 Equilibrium analysis for fiber yarn

According to the saw-tooth model, the force between two intersecting yarns is assumed to be a point force. Since fiber yarns are not embedded in the rubbery matrix, we assume that friction forces are ignored at the contact areas. This provides for uniform tension in one yarn. And it is assumed that the two families of yarns are pinned at intersection points. Force balance at the intersection point M is shown in Figure 3-3. $F_T$ is the tension in the longitudinal direction of yarn. $F_C$ is the force from yarn F to yarn D caused by the yarn tension. $F_A$ and $F_N$ are the two components of $F_C$. $\phi$ is the bending angle of yarn D. Thus, equilibrium equations at point M can be written as

$$
F_T \cos \phi + F_A = F_T
$$

$$
F_N = F_T \sin \phi + P \cdot A_{\text{int}},
$$

where $P$ is the internal pressure of the braided composite actuator and $A_{\text{int}}$ is the overlapping area by two intersected yarns.
Figure 3-3: Simplified yarn intersection model.

The out-of-plane component $F_N$ is the compressive force for fiber compaction. The compressive forces are simplified to point forces and we assume that the compressive forces are distributed on a small parallelogram area, as shown in Figure 3-4. As the fibers rotate, the area of the parallelogram changes, which is included in this model. In the figure, $d$ is the width of the fiber yarn and $\theta$ is the braiding angle by the fiber and the generator of the cylinder.

Figure 3-4: Compressive forces distributed on the intersection area.
Shown in Figure 3-5, the solid lines and the dashed lines are the yarns before and after compaction, respectively. The thickness of the sleeve is reduced by $\Delta$. If the fiber yarns are inextensible, the length of the element edge is increased. Because the element dimension is much smaller than the actuator’s radius, the element can be treated as flat. Therefore the length of the element edge is increased by approximately $2b(\cos\phi_1 - \cos\phi_0)$, where $\phi_0$ and $\phi_1$ are the yarn bending angles before and after compaction respectively.

![Figure 3-5: Yarn compression schematic for braided sleeve element.](image)

3.4.2 Compaction law of sleeves

The elongation of the element edge depends not only on yarn tension and initial bending angle but also on the compressive property of the woven sleeve, which is governed by a compaction law. Several compaction experiments have been carried out and several compaction laws have been proposed [138, 146, 148, 150, 151]. In this work, we compressed a section of the sleeve of the braided composite actuator, as shown in Figure 3-6. Because the sleeve is almost 100% covered by the fiber yarns, we can determine the relationship between the sleeve thickness reduction $\Delta$ and the pressure $p$ applied on the sleeve by measuring the compressive forces, displacements, and the compressed area. We name this relationship the compaction law of the sleeve and it can be expressed as $p = f(\Delta)$. There is no yarn tension in the compaction law test. Here we are assuming that influence of yarn tension to the compaction law is negligible. Using the experimentally determined compaction law, we can get the equation below, in which the left side of the equation is the pressure applied on the yarn intersection area.
\[ \frac{F_N}{A_m} = f(\Delta). \] (3-2)

Figure 3-6: Compaction law test by compressing a piece of sleeve sheet.

3.4.3 Governing equations for braided composite actuators with compaction

The sleeve considered is braided with two families of fiber yarns at an angle \( \theta_0 \), as shown in Figure 3-7. The fiber yarns will be reoriented with pressure inside the tube or with tension at the ends of the tube.

Figure 3-7: The braided composite actuator schematic.

3.4.3.1 Compaction model with inextensible fibers

In Kydoniefs’ model [42], the fiber yarn crimp, which is the percentage excess of length of the yarn axis over the fabric length, was not considered. If there were no compaction between intersected yarns, the Kydoniefs’ model could be applied directly to solve the braided composite actuator tube response to the external loadings, by
employing the helix (the dotted line in Figure 3-5) as the fiber length. With compaction, 
the helix of the tube is elongated.

The compaction is coupled with radial expansion which is followed by fiber rotation 
when the tube is deformed. When the tube is pressurized, the radius is expanded and the 
sleeve layer is compacted. The compaction effect further increases the tube radius. Due to 
the changed radius, the equilibrium state of the braided composite actuator will be shifted, 
which will affect the compaction as a feedback. Eventually, the braided composite 
actuator will achieve an equilibrium state governed by the compaction law and the 
Kydoniefs’ deformation theory. To avoid large amounts of iterative computations, we 
propose a different path leading to the final equilibrium state. The braided composite 
actuator starts to deform with the sleeve layer compacted shown as state (2) — which is 
called ‘the transitional state’ — in Figure 3-8. The tube at the transitional state has the 
same helix length as the deformed tube (3) — which is the tube at the final equilibrium 
state — while keeping the same braiding angle $\theta$ as the undeformed tube (1). The 
deformation from state (2) to state (3) is governed by the Kydoniefs’ theory only. This is 
a simple way to decouple the compaction and the tube contraction. The matrix strain 
energy in state (2) is ignored since the tube radius and length increase due to the 
compaction with respect to state (1) is very small.
In Figure 3-8, the braided composite actuator length and radius both increase due to the compaction from state (1) to state (2). The increased radius can be computed by the following equation:

\[
R_{0T} = \left( \frac{a + b \cos \phi_1}{a + b \cos \phi_0} \right) R_0, \tag{3-3}
\]

where \( R_{0T} \) is the radius of the tube at the transitional state. If the braided composite actuator is braided by \( n \) carriers, the equilibrium equation at both ends of the tube can be written as

\[
n \cdot F \cdot \cos \theta_1 = P \cdot \pi \left( \lambda_2 R_{0T} \right)^2 + T, \tag{3-4}
\]

where \( \lambda_2 \) is the radius strain (ratio of the radius at the deformed state to the transitional state). By Equation (3-1), Equation (3-2) and Equation (3-4), the compaction law can be written as:
\[
\frac{P \cdot \pi (\lambda_2 R_{ut})^2 + T}{d^2 n \cos \theta_1} \sin \phi_1 \sin 2\theta_1 + P = f \left[ b(\sin \phi_0 - \sin \phi_1) \right].
\] (3-5)

From the transitional state to the deformed state, we can solve for the radial and axial strains as well as the fiber rotation using the equilibrium equations presented in Equation (2-12). Since the braided composite actuator is assumed to be long enough that it undergoes uniform radial expansion when deformed, Equation (2-12) is simplified to the following:

\[
N_1(\lambda_1, \lambda_2) = \frac{P\lambda_2 R_{ut}}{2} + \frac{T}{2\pi \lambda_2 R_{ut}}.
\]
\[
N_2(\lambda_1, \lambda_2) = P\lambda_2 R_{ut}.
\]
\[
\sin \theta_1 = \lambda_2 \sin \theta_0
\]
\[
\cos \theta_1 = \lambda_1 \cos \theta_0
\] (3-6)

Notice that we can solve for all of the unknowns (\(\lambda_1, \lambda_2, \phi_1, R_{ut}\) and \(\theta_1\)) by Equation (3-3), Equation (3-5) and Equation (3-6). The nominal strain is defined as the length ratio or radius ratio of the deformed state (3) to the undeformed state (1). The nominal strains are expressed as

\[
\lambda_{tn} = \lambda_1 \left( \frac{a + b \cos \phi_1}{a + b \cos \phi_0} \right),
\]
\[
\lambda_{2n} = \lambda_2 \left( \frac{a + b \cos \phi_1}{a + b \cos \phi_0} \right).
\] (3-7)

### 3.4.3.2 Compaction model with extensible fibers

When braided composite actuator is subjected to large internal pressure, the tension in the fiber yarn can be so large that the extensibility of the fiber cannot be ignored. The fiber extensibility is also included in the transitional state. Suppose the fiber yarn is
extended by a ratio of \((1 + \varepsilon_Y)\), which results in a modification of Equation (3-3) as the following

\[
R_{0T} = \left( \frac{a + b \cos \phi}{a + b \cos \phi_0} \right) (1 + \varepsilon_Y) R_0 .
\]  

(3-8)

Through experiment, an expression for the fiber yarn tensile property can be obtained:

\[
\varepsilon_Y = g \left( F_T \right) .
\]  

(3-9)

All of the unknowns \((\lambda_1, \lambda_2, \phi_1, R_{0T}, \theta_i \text{ and } \varepsilon_Y)\) can be solved by Equation (3-8), Equation (3-9), Equation (3-5) and Equation (3-6). The nominal strains are

\[
\lambda_{1n} = \lambda_1 \left( \frac{a + b \cos \phi}{a + b \cos \phi_0} \right) (1 + \varepsilon_Y) .
\]

\[
\lambda_{2n} = \lambda_2 \left( \frac{a + b \cos \phi}{a + b \cos \phi_0} \right) (1 + \varepsilon_Y) .
\]  

(3-10)

3.4.4 Pressure increase in the closed-valve actuator under tension

When the braided composite actuator is filled with a high bulk modulus fluid in the closed-valve scenario, it becomes significantly stiff in the axial direction, i.e. large effective stiffness value. With the application of an axial force on the tube, the braided composite actuator cavity volume tends to decrease from fiber reorientation. This results in fluid compression and causes the fluid pressure to increase. The bulk modulus of the fluid \((B)\) can be defined as:

\[
B = -V_F \frac{dP}{dV_F} ,
\]  

(3-11)
where $V_F$ is the volume and $P$ is the pressure. The pressure of the compressed fluid is

\[ \bar{P} = P_0 + B \ln \frac{\lambda_{2nb}^2}{\lambda_{2n}^2 \lambda_{n}^n}. \tag{3-12} \]

Similar to Shan et al.’s model [9], the entrapped air effect can be also included in the compaction model. The initial volumes of the entrapped air and the fluid are defined as $V_A^0$ and $V_F^0$ respectively. By assuming constant temperature, we have Equation (3-13) for the entrapped air where $V_A$ is the compressed air volume under the pressure of $\bar{P}$,

\[ P_0 \cdot V_A^0 = \bar{P} \cdot V_A. \tag{3-13} \]

Equation (3-12) is modified as shown in Equation (3-14), where $V_F$ is the compressed fluid volume under the pressure, $V_0$ is the initial volume ($V_0 = V_A^0 + V_F^0$), $V$ is the compressed volume ($V = V_A + V_F$) and $\alpha_A^0$ is the initial air volume ratio ($\alpha_A^0 = V_A^0 / V_0$),

\[ \bar{P} = P_0 + B \ln \frac{V_F^0}{V_F} = P_0 + B \ln \left[ \frac{V_0 (1 - \alpha_A^0)}{V - P_0 \alpha_A^0 V_0 / \bar{P}} \right]. \tag{3-14} \]

The ratio of $V/V_0$ can be obtained by Equation (3-15) as

\[ \frac{V}{V_0} = \frac{\lambda_{2n}^2}{\lambda_{2nb}^2}. \tag{3-15} \]
Chapter 4 Compaction experiments for uniformly deformed actuators

Nomenclature

\( p \) pressure applied on the sleeve in the compaction law
\( C_1, C_2 \) material constants for the Mooney-Rivlin material
\( F_T \) tension in a single yarn
\( P \) internal pressure of the actuator
\( P_0 \) initial internal pressure of the actuator
\( T \) tension applied on the actuator ends
\( R_0 \) undeformed actuator radius
\( \alpha_0^0 \) initial air volume ratio of closed-valve actuator
\( \alpha, \beta, \gamma \) experimentally determined coefficients in the compaction law
\( \varepsilon_Y \) yarn strain
\( \lambda \) axial deformation ratio of elastomer tube
\( \theta_0 \) braiding angle of undeformed actuator
\( \sigma \) Cauchy axial stress of elastomer tube
\( \Pi \) Lagrangian axial stress of elastomer tube
\( \Delta \) reduced thickness of the fabric

4.1 Determination of compaction law, elastomer material constants, and fiber yarn tensile property

4.1.1 Compaction law

As mentioned in section 3.4.2, the compressive property of the woven sleeve is required to determine the final strain of the braided composite actuator. The testing frame ADMET MTESTQuattro™ material testing system was used for the experiments, which provides programmable force and displacement control. It is capable of performing all types of tests including tension, compression, creep, etc. It has a minimum testing speed of 0.005mm/min with a position control resolution of 0.095μm. The load cell is the
INTERFACE SM-1000 force transducer with a load capacity of 1000lbf. The sampling rate can be up to 1kHz.

The force and the displacement were recorded in the experiment while compressing a section of braided sleeve sheet. Considering the fact that the base on which the sheet was compressed is not rigidly stiff, we also compressed the base without the sheet and recorded the force and displacement. The force-displacement relationship of the sleeve sheet was obtained by subtracting the data without a sheet from the case with a sheet. Three types of sleeve sheets using aramid fibers, carbon fibers and glass fibers were experimentally tested. For each type, sleeves with different fiber angles were compressed. For each angle, the sleeves were compressed multiple times. Results for the aramid sleeves and the fitted curves for three types of fiber sleeves are shown in Figure 4-1. Since there is no apparent deviation for different braiding angles, a compaction law which does not depend on the braid angle is assumed. After noticing that the compaction law curve starts at the origin and approaches infinity at some displacement value, the following function is proposed to fit the curves:

\[
p = \alpha \coth (\beta - \gamma \cdot \Delta) - \alpha \coth \beta .
\]  

(4-1)

Coefficients for aramid sleeves, carbon fiber sleeves and glass fiber sleeves are listed in Table 4-1. The compressed area is 1.61×10^{-4} \text{ m}^2.
Figure 4-1: Compaction laws: (a) Experimental and fitted results for the aramid fiber sleeve; (b) Fitted results for three types of fiber sleeves.
Table 4-1: Compaction law coefficients for three types of sleeves.

<table>
<thead>
<tr>
<th></th>
<th>( \alpha ) (MPa)</th>
<th>( \beta )</th>
<th>( \gamma ) (m(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aramid sleeve</td>
<td>1.91</td>
<td>4.426</td>
<td>23110</td>
</tr>
<tr>
<td>Carbon fiber sleeve</td>
<td>0.73</td>
<td>3.423</td>
<td>15940</td>
</tr>
<tr>
<td>Glass fiber sleeve</td>
<td>1.08×10(^{-3})</td>
<td>8.846×10(^{-3})</td>
<td>70</td>
</tr>
</tbody>
</table>

4.1.2 Material constants for the elastomer

The inner elastomer is assumed to be a Mooney-Rivlin material. Experiments were carried out to obtain the two constants \( C_1 \) and \( C_2 \). The ADMET MTESTQuattro\(^{TM}\) material testing system was employed for the experiments. A thin-wall elastomeric tube was loaded with axial tension and the axial strains and forces were recorded as the axial loading was increased. Since it is assumed that there are no stresses in the circumferential direction and thickness direction, the Cauchy axial stress can be expressed as

\[
\sigma = 2C_1 \left( \lambda^2 - \frac{1}{\lambda} \right) - 2C_2 \left( \frac{1}{\lambda^2} - \frac{1}{\lambda} \right).
\]

(4-2)

Thus the Lagrangian axial stress is

\[
\Pi = \left( 2C_1 + \frac{2C_2}{\lambda} \right) \left( \lambda - \frac{1}{\lambda^2} \right).
\]

(4-3)

The test result and the fitted curve are shown in Figure 4-2. The two constants are obtained as \( C_1 = 136854 \text{Pa} \) and \( C_2 = 152225 \text{Pa} \).
4.1.3 Fiber yarn tensile property

The fiber yarn tensile property defined in Equation (3-9) is obtained by applying tension to fiber yarns in the braided sleeves while recording the yarn extension. Tests were performed for different aramid fiber yarns, carbon fiber yarns and glass fiber yarns in the sleeves. The four test results and fitted curve for the aramid fiber yarns are shown in Figure 4-3(a). The discretized small steps observed in the experimental results are due to the displacement encoder. The lines appear linear, and the slope for the aramid yarn is about 6.64×10^{-5} N^{-1}, i.e.

\[ \varepsilon_Y = 6.64 \times 10^{-5} \cdot F_T. \]  

(4-4)

Similar linear strain-tension curves were obtained for carbon fiber yarns and glass fiber yarns shown in Figure 4-3(b) and the slopes are 3.85×10^{-5} N^{-1} and 1.15×10^{-4} N^{-1} respectively.
Figure 4-3: Fiber yarn tensile properties: (a) Experimental and fitted results for the aramid fiber yarn; (b) Fitted results for three types of fiber yarns.
4.2 Experiments of braided composite actuators

4.2.1 Compaction model validation: radial expansion measurement at actuator’s blocking state

According to the compaction model, the radius of braided composite actuator will expand when the actuator is pressurized with both ends clamped, i.e. the blocking state. However, this will not happen in Kydoniefs’ deformation theory since the fibers are assumed inextensible and yarn compaction is not included in the theory. In order to validate the compaction model, experiments measuring the radial expansion of the actuators at the blocking state were carried out. Hydraulic swage fittings were attached at both ends of the actuator and then were clamped in the ADMET MTESTQuattro™ testing frame. A high pressure ball-valve was mounted at each end, and a pressure transducer was mounted between the valves to measure the internal pressure. A fiber-optic displacement sensor measuring the actuator’s radial expansion was mounted perpendicular to the actuator axis pointing to the axis as shown in Figure 4-4. A metalized polyester film was attached to the surface of the actuator to increase the reflection. The fiber-optic displacement sensor in the experiment was the PHILTEC™ Model D20, which has a tip diameter of 0.81mm. It has a total operating range of 0~1.27mm and a DC output range of 0~5.0V after calibration. The far side sensitivity is 8 mV/μm and the near side sensitivity is 80mV/μm.

In the experiments, the internal pressure was increased gradually by using a valve connected to a 1.38MPa (200psi) air pressure tank. Data from the pressure transducer, the displacement encoder and the fiber-optic displacement sensor were recorded simultaneously at the rate of 10 Hz by a LabVIEW™ program driving a data acquisition board. The displacement encoder in the ADMET MTESTQuattro™ testing frame measured the actuator’s axial contraction. Even though the actuator was clamped at both ends, there was still some contraction for the actuator since the testing frame is not rigidly stiff. The radial expansion caused by this axial contraction was included in the analysis results. In the experiments, we began to record data with the internal pressure greater than 69KPa (10psi) to ensure the axial tension was large enough to keep the actuator’s axis
from shifting/rotating due to slack. The data from the fiber optic displacement sensor was used to determine the expanded diameter.

Figure 4-4: Experiment setup for radial expansion measurement at blocking state.

The experimental results for the actuators made of aramid fibers, carbon fibers and glass fibers with the braiding angle of 21° are shown in Figure 4-5, Figure 4-6 and Figure 4-7. In addition, the actuator made of aramid fibers with an angle of 29° was also tested, shown in Figure 4-8. Differences between the experimental results and the results for the model without compaction (dotted lines) indicate that radial expansions at the blocking state do exist. Apart from being caused by axial contraction, radial expansion is due to fiber extension and/or the inter fiber yarn compaction. The dash-dotted lines in figures from Figure 4-5 to Figure 4-7 are the results from the non-compaction model considering the fiber extensibility, where it is observed that fiber extension does contribute to the radial expansion. From the analysis results, it is clear that most of the radial expansion comes from the inter fiber yarn compaction at low pressures (< 10psi). More accurate predictions of the experiment are obtained from the compaction model, which are shown
as the dashed lines in the figures. The non-smooth behavior of the experimental results during pressurization is primarily a result of the sensitivity of the orientation of the fiber optic sensor with respect to the surface during radial expansion. While these variations in the results are undesirable, the trends in the experimental results do confirm the effects of compaction on the radial expansion in the blocked state.

Figure 4-5: Experimental vs. computational results of aramid fiber actuators’ radial expansions at blocking state, $R_0=4.6\text{mm}$, $\theta_0=21^\circ$. 
Figure 4-6: Experimental vs. computational results of carbon fiber actuators’ radial expansions at blocking state, $R_0=4.6\text{mm}$, $\theta_0=21^\circ$.

Figure 4-7: Experimental vs. computational results of glass fiber actuators’ radial expansions at blocking state, $R_0=4.6\text{mm}$, $\theta_0=21^\circ$. 
4.2.2 Tensile test experiments for actuators in the closed-valve scenario

Tensile test experiments were carried out for water-filled actuators with the valve closed. In this scenario, the effective stiffness of the actuators is maximized. The experiment setup is the same as in Figure 4-4 except that there is no fiber-optic displacement sensor measuring radial displacement. In the experiment, the actuator was initially pressurized using a pump with both ends fixed. After pressurization, both valves were closed and tests began. Axial forces, axial displacements, and internal pressures were recorded for each tensile test. The pressure transducer used in the experiment was the ASHCROFT® Model K1 transducer. The range is 0~2000psi with a linear DC output of 1~5V.

Experimental results for actuators made of three types of fibers with low initial pressures are shown in the following figures as discrete circles. Computational results with various conditions are also shown in the figures. In the analysis, we assumed a series of entrapped air volume ratios ($\alpha^0 = 0, 0.02, 0.04, 0.06, 0.08$) at atmospheric pressure since it is difficult to directly measure the air volume ratio in the water. The bulk
modulus of the working fluid used in the analysis is 2.2GPa and fiber extensibility is included in computations. As seen in the figures, the results from the analysis model that includes both the compaction and the entrapped air effects have better agreement with the experimental results. The model without compaction and without entrapped air predicts the largest closed-valve stiffness but it has the most error when compared to the experimental results. Both compaction and entrapped air reduce the closed-valve stiffness at the initial phase of loading, which is easily observed in the initial part of the loading curves. From Figure 4-9 to Figure 4-14, it can be inferred that the initial entrapped air volume ratio is around 0.06 since the analysis results match the experimental results best at this ratio. As the internal pressure and axial tension increase, the effects of compaction and entrapped air on reducing the effective stiffness decrease. Under large loadings, the slopes of curves are almost identical for both models, which is primarily governed by the fiber extensibility.

Experimental results for actuators made of three types of fibers with both low and high initial pressures are shown in Figure 4-15. The compaction and the entrapped air effects can be reduced with high initial pressure. As the loadings increases, the curve slope depends on the fiber extensibility only, no matter how different there initial pressures are. Carbon fiber yarns have the largest stiffness and glass fiber yarns have the smallest stiffness. Therefore actuators with carbon fibers have the largest closed-valve stiffness and actuators with glass fibers have the smallest closed-valve stiffness under larger loadings.
Figure 4-9: Tension-strain curve for closed-valve aramid fiber actuators with starting pressure at 1.26 psi: (a) with compaction; (b) without compaction.
Figure 4-10: Internal pressure-strain curve for closed-valve aramid fiber actuators with starting pressure at 1.26 psi: (a) with compaction; (b) without compaction.
Figure 4-11: Tension-strain curve for closed-valve carbon fiber actuators with starting pressure at 0psi: (a) with compaction; (b) without compaction.
Figure 4-12: Internal pressure-strain curve for closed-valve carbon fiber actuators with starting pressure at 0 psi: (a) with compaction; (b) without compaction.
Figure 4-13: Tension-strain curve for closed-valve glass fiber actuators with starting pressure at 0 psi: (a) with compaction; (b) without compaction.
Figure 4-14: Internal pressure-strain curve for closed-valve glass fiber actuators with starting pressure at 0psi: (a) with compaction; (b) without compaction.
Figure 4-15: Experimental results for actuators made of glass fibers, carbon fibers and aramid fibers.
Chapter 5 Analysis and discussion for uniformly deformed actuators

As we can see from the experimental results in Chapter 4, the compaction effect does exist and significantly decrease the stiffness of braided composite actuators filled with high bulk modulus fluid. And the computational results from the proposed compaction model agree well with the experimental results. The agreement shows that the proposed model is able to better predict the behavior of braided composite actuators than the original model without compaction. Not only is the stiffness of filled actuators affected by the compaction, other performances such as the contractile force as well as the blocking force could also be influenced. Thus, it is better to use the compaction model to predict other performances of the braided composite actuators. Some details in the compaction model, such as the fiber yarn bending angle change, fiber sleeve thickness decrease, fiber angle reorientation, fiber yarn extension, etc, will also be studied in this chapter, in order to better understand effects of compaction on the response of the actuators.

5.1 Compaction effect on the actuation performance

As an actuator with the fiber angle $\theta$ less than 54.7°, the braided composite actuator can perform contractile actuation when it is subjected to internal pressure. The contractile force depends on the fiber angle, the internal pressure and the contracted length. With the maximum contracted length, the contractile force is zero and with zero contracted length, the contractile force reaches the maximum which is named the blocking force. Researchers have applied both virtual energy method [1, 4, 37] and continuum mechanics method [7, 40] to derive the relationship between contractile force output and contracted length. Both methods lead to the same expression. Without considering the inter fiber yarn compaction effect, the calculated contractile force outputs are always higher than the forces output measured from experiments [4, 7, 37]. They modified the model either by including the fiber extensibility [4] or by including the inter fiber frictions [2]. However,
the inter fiber yarn compaction has not yet been considered. This may have some effect on the contractile force output decrease.

Two types of aramid braided composite actuators with the same initial radius of 4.6mm are studied in this section. One type of the actuator has an initial braiding angle of 21° and the other one has the initial braiding angle of 34°. An internal pressure of 0.6MPa is applied to both types of actuators. In addition, an internal pressure of 1.2MPa is applied to the actuator with the braiding angle of 21°. Therefore three cases are studied.

In each case, the axial tension is gradually increased from zero to the blocking force. The calculated force outputs and corresponding contractile strains are shown in Figure 5-1.

![Figure 5-1: Braided composite actuator contractile force vs. contractile axial strain.](image)

From Figure 5-1, we can see for various initial fiber angles and internal pressures, the predicted contractile forces by the compaction model are lower than the model without compaction with the same contractile strain. Inter fiber yarn compaction as well as other contributing factors, such as friction, can all reduce the performance of the actuator, and therefore including the compaction effects can improve the original model.
and our understanding of the actuator. Nevertheless, the original model can be improved by including the compaction effect. From the figure, we can also state that the actuator does not contract as much as predicted by the original model without compaction, under the same external loadings.

During the calculation, the maximum strain of a single fiber yarn is less than 0.1 percent and it is much less than the contractile strain whose maximum is over 30 percent. Therefore the fiber extensibility is ignored in this section, i.e. where the actuator’s axial strain is large.

Due to the compaction, the radial strains have been increased as shown in Figure 5-2. Figure 5-3 is a part of Figure 5-2 which has been zoomed in. For the same initial braiding angle $\theta_0$, the radial strain only depends on the axial strain if there were no compaction. However, due to the existence of compaction, larger radial strain is expected with the same axial strain. In addition, larger loading causes more radial strain as shown in Figure 5-3, because the fiber sleeve is compressed more in thickness and the fiber yarns are stretched more.

![Figure 5-2: Braided composite actuator radial strain vs. contractile axial strain.](image)

$R_0 = 4.6 \text{ mm, } \phi_0 = 10.4^\circ$

- Inextensible fiber without compaction
- Inextensible fiber with compaction

$\theta_0 = 21^\circ$, $P = 1.2 \text{ MPa}$

$\theta_0 = 34^\circ$, $P = 0.6 \text{ MPa}$
Similarly the predicted braiding angle is also shifted in the compaction model as shown in Figure 5-4. If there were no compaction, the deformed braiding angle depends only on the axial strain. But it is now increased at the same axial strain and it is increased more with larger internal pressure as shown in Figure 5-5.

Figure 5-3: Braided composite actuator radial strain vs. contractile axial strain (zoomed in).

Figure 5-4: Reoriented braiding angle vs. contractile axial strain.
Shown in Figure 5-6 and Figure 5-7 are the deformed fiber yarn bending angle ($\phi_1$ in Figure 3-5) and the fiber sleeve thickness decrease $\Delta$. They are dependent on each other through the geometric relationship $\Delta = b(\sin\phi_0 - \sin\phi_1)$ as shown in Figure 3-5. The initial bending angle $\phi_0$ is $10.4^\circ$ and the initial fabric sleeve thickness is about 0.6mm. The bending angle and the fabric sleeve thickness do not change much as the axial tension increases. The most of the changes occur at the beginning of the loading as the internal pressure increases from 0 to 0.6MPa or 1.2MPa. This is also indicated by the experimental results in section 4.2.1.

The two components ($F_T \cdot \sin\phi$ and $P \cdot A_{\text{int}}$, see Equation (3-1)) of the compressive force on the yarn intersection, which come from the yarn tension and internal pressure respectively, are shown in Figure 5-8 and Figure 5-9. These two components are comparable in terms of magnitude. The non-monotonicity of the internal pressure component $P \cdot A_{\text{int}}$ is because of the non-monotonic change of the yarn contact area. The contact area is

Figure 5-5: Reoriented braiding angle vs. contractile axial strain (zoomed in).
As the axial tension increases, the braiding angle $\theta$ decreases from around $52^\circ$ to $22^\circ$ which causes the non-monotonicity of the contact area.

Figure 5-6: Fiber yarn bending angle vs. contractile axial strain.
Figure 5-7: Fiber sleeve thickness decrease vs. contractile axial strain.

Figure 5-8: Compressive force on one yarn intersection from fiber yarn tension vs. contractile axial strain.
At the blocking state (zero contraction) of the braided composite actuator, the radius is larger than the undeformed radius as shown in Figure 5-11. Because of the existence of compaction, the undulated fiber yarn is flattened, which produces extra strains in both axial and circumferential directions. Since the axial strain is fixed to be zero, all of the extra strains go to the radial direction, along with fiber reorientation. Therefore the fiber angle is increased as well as shown in Figure 5-12. And the blocking force (force to keep zero axial strain) is decreased as shown in Figure 5-1 and the following Figure 5-10. The blocking force can be reduced by up to several hundred newtons at small $\theta_0$’s.

However, we know that the blocking force depends not only on the initial braiding angle but also on the actuator’s radius. A larger braiding angle reduces the blocking force while a larger radius increases the blocking force. The compaction effect causes not only larger braiding angle but also larger radius. So it is possible that the blocking force becomes larger with compaction effect.
Figure 5-10: Blocking force vs. initial braiding angles.

Figure 5-11: Radial strain vs. braiding angle at blocking state.
Figure 5-12: Reoriented braiding angle vs. initial braiding angle at blocking state.

5.2 Compaction effect and fiber extensibility to the variable stiffness performance of actuators

As mentioned in section 1.1.9.4, the braided composite actuators can be used as variable stiffness elements in systems and structures. Combined with a high bulk modulus fluid, braided composite actuators can be very flexible with an open valve since the fluid is unconstrained, and it can be very stiff when the valve is closed. The axial tension will cause the fiber rotation, which will reduce the cavity volume of the actuator. The fluid will be compressed. Because of the high bulk modulus, the fluid pressure will be increased dramatically, which will resist further volume compression. Therefore further rotation of the fiber is resisted. The pressure increase is calculated by Equation (3-14). With the proposed compaction model, analysis studies are performed to understand the effects of fiber extensibility and compaction on the closed-valve braided composite actuator response. The aramid braided composite actuator with an initial radius of 4.6 mm and an initial braiding angle of 21° is studied.

Shown in Figure 5-13 and Figure 5-14 are the comparisons of the tension-strain curves and pressure-strain curves for the closed-valve braided composite actuator. The
curves are the results from the models: inextensible fiber without compaction, extensible fiber without compaction, inextensible fiber ($\varepsilon_Y = 0$ in Equation (3-9)) with compaction and extensible fiber without compaction respectively. Based on the calculation results, the fiber yarn can be extended by as much as 0.26% with a tension of 3300N and an internal pressure of 4.1MPa. The braided composite actuator diameter is 13% larger than the undeformed actuator due to fiber extension as well as compaction. The fiber yarn strain is comparable to the axial strain. We can see there is apparent deviation between these two models and fiber extensibility does make a difference, especially in high pressure and high tension scenarios. The fiber extensibility can greatly lower the stiffness. If we notice the slope of the curves, we can see the compaction plays a dominant role at small strains. This is because most of the compaction occurs at the beginning of the loadings. As the loadings increase, the slope of the curve primarily depends on the fiber extensibility. For example, the extensible fiber models with and without compaction have the same slope at large axial strains. Thus a stiffer fiber is preferred for achieving the maximum stiffness in the closed-valve braided composite actuator.

\[
R_0 = 4.6 \text{ mm}, \theta_0 = 21^\circ, \phi_0 = 10.4^\circ, P_0 = 0.00\text{MPa}
\]
Figure 5-13: Closed-valve curve comparisons with starting pressure at 0.00MPa: (a) Tension-strain; (b) Internal pressure-strain.

(a)

(b)
5.3 Compaction law sensitivity analysis

The compaction law used in the calculation has been averaged from the experiments, as shown in Figure 4-1(a). To investigate how the deviation from the averaged compaction law affects the results, a sensitivity analysis of the compaction law is performed. In addition to the averaged compaction law, two other compaction laws are used for calculation. One of the additional compaction laws is from the left bound of the dots in Figure 4-1(a) and the other one is from the right bound of the dots in Figure 4-1(a), as shown in Figure 5-15. The compaction law curves are fitted by Equation(4-1). The coefficients of the three compaction law curves including the averaged one are listed in Table 5-1.
Figure 5-15: Deviated compaction laws: (a) on the left bound; (b) on the right bound.
Table 5-1: Compaction law coefficients for compaction law curves on the left bound, in the middle (averaged) and on the right bound.

<table>
<thead>
<tr>
<th></th>
<th>( \alpha ) (MPa)</th>
<th>( \beta )</th>
<th>( \gamma ) (m(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>2.81</td>
<td>4.997</td>
<td>28280</td>
</tr>
<tr>
<td>Middle (Averaged)</td>
<td>1.91</td>
<td>4.426</td>
<td>23110</td>
</tr>
<tr>
<td>Right</td>
<td>2.15</td>
<td>4.447</td>
<td>20030</td>
</tr>
</tbody>
</table>

These three compaction laws are used to reproduce the radial expansion at the blocking state as in Figure 4-5. The closed-valve tension-strain curve and the pressure-strain curve in Figure 4-9(a) and Figure 4-10(a) are also reproduced.

The radial expansions at the blocking state due to the three compaction laws are shown in Figure 5-16. The calculated tension-strain curve and the pressure-strain curve are shown in Figure 5-17 and Figure 5-18 respectively. From these figures very small deviations are found due to these three different compaction laws. Therefore using the averaged compaction law is acceptable. The curve fit errors from the experimental results will not induce significant analysis errors.

![Figure 5-16: Calculated radial expansions at the blocking state using three compaction laws.](image-url)
Figure 5-17: Calculated closed-valve tension-strain curve using three compaction laws.

Figure 5-18: Calculated closed-valve pressure-strain curve using three compaction laws.
Chapter 6 Compaction modeling for uniformly deformed braided composite actuators with fiber yarn bending stiffness included

Nomenclature

\(a\) amplitude of the sinusoidal fiber yarn

\(f(x)\) function of the reference curve in the Cartesian system, see Figure 6-4

\(h_0\) thickness of the inner rubber tube

\(l\) length of reference line in the compaction law test, see Figure 6-10

\(l_r\) length of reference curve in the deformed actuator, see Figure 6-4

\(l_{0}\) length of reference curve in the undeformed actuator

\(n\) number of the cell in the fabric of the actuator

\(n_{\text{test}}\) number of the cell in the fabric in the compaction law test

\(p\) pressure applied on the fabric in the compaction law

\(q\) weight per unit length of the fiber yarn

\(u\) axial displacement of the fiber yarn centerline

\(w\) deflection of the fiber yarn or the Euler-Bernoulli beam

\(w_0\) deflection of the fiber yarn in the undeformed fabric

\(x\) horizontal coordinate in the Cartesian system, see Figure 6-4

\(x_{1}\) horizontal coordinate of the yarn end in the Cartesian system, see Figure 6-4

\(A\) cross section area of the fiber yarn or the Euler-Bernoulli beam

\(A_{\text{test}}\) compressed fabric area in the compaction law test

\(C_1, C_2\) material constants for the Mooney-Rivlin material

\(E\) elastic modulus of the fiber yarn or the Euler-Bernoulli beam in the axial direction

\(E_{\text{fabric}}\) strain energy of the fabric in the actuator

\(E_{\text{rubber}}\) strain energy of the inner rubber tube in the actuator

\(E_{\text{yarnaxial}}\) total yarn axial strain energy in the fabric

\(E_{\text{yarnbending}}\) total yarn bending energy in the fabric

\(E_{\text{yarnthickness}}\) total strain energy in the yarn thickness direction in the fabric

\(F\) axial tension applied on the actuator

\(I\) moment of inertia of the cross section of the fiber yarn or the Euler-Bernoulli beam
In the previous compaction modeling in Chapter 3, the fiber yarns between intersections are assumed to be straight lines. However, the yarns are actually smooth curves in the longitudinal directions as shown in Figure 1-9. Some researchers have taken very good pictures for fabrics woven by fiber yarns consisting of thousands of filaments, such as Barbero et al. [152], King et al. [153] and Hivet and Boisse [154]. Based upon the photomicrographs of fabrics, the fiber yarn shapes are usually assumed to be the sinusoidal functions. Fiber yarns are usually modeled as beams to investigate the fabrics compaction behavior. Simacek and Karbhari [142, 143] developed a constitutive equation for fiber yarn and then modeled the fabric compaction based on the beam theory. Chen and Chou [144] treated the fiber yarn as a transversely isotropic beam with deformable cross section shape. In the analyses, the linear beam theory was applied and the in-plane strains of the fabric during compaction were ignored. Chen et al. [145] developed a
micromechanical model including both micro compaction of yarn cross section and macro deformation of yarn undulation flattening. They also applied the linear beam theory.

Since the yarn shapes in compaction are smooth curves instead of straight lines between intersections, it may be more accurate to model the yarns as curved beams and include the yarn bending stiffness. Therefore the objective of this chapter is to develop a new compaction model by including the yarn bending strain energy and to investigate the contribution of the model with the inclusion of the bending energy of the yarn.

6.1 Technical Approach

For the braided composite actuator, the compaction is the thickness reduction of fabric layer. But the consequence of the compaction is more than just thickness reduction of fabric layer. The thickness reduction causes the in-plane-bi-directional strains of the fabric, as shown in Figure 3-8. The in-plane strains were not included in research works in the composite industry [144, 145, 149]. The yarns were assumed to be linear beams and therefore the displacements in the axial direction were ignored. But the in-plain strains are critical for the braided composite actuator analysis, especially for the closed-valve scenario. Therefore in the current research the yarn must be considered as geometrically nonlinear beams, i.e. large deflection beams.

It is difficult to obtain the forces between two intersected fiber yarns with the nonlinear beam assumption. The energy method provides a more convenient approach to model the compaction behavior without solving the inter fiber yarn forces. The governing equation is written as

\[
\delta E_{\text{fabric}} + \delta E_{\text{rubber}} = \delta W_f + \delta W_p .
\]  

(5-2)

Compared to Equation (2-22), the additional term is the strain energy in the fabrics. The fabrics strain energy consists of three parts: yarn bending strain energy, yarn strain energy in the thickness direction and yarn strain energy in the longitudinal direction.
Several assumptions are made to simplify the modeling:

(a) Fiber yarns are assumed to be Euler-Bernoulli beams with large deflections. The inter-filaments effects are ignored.
(b) Fiber yarns are assumed to be in sinusoidal shapes with respect to their projections in the center surface of the braided sleeve.
(c) Fiber yarns have zero bending energy when the actuator is undeformed.
(d) Fiber yarn bending stiffness remains constant, even when its cross section is compressed.
(e) Intersected fiber yarns do not slide relative to each other at the intersection points.
(f) Energy loss due to frictions is ignored.
(g) The twisting of yarn is ignored.

6.2 Compaction modeling with yarn bending energy using energy method

6.2.1 Geometry of fiber yarns in braided composite actuators

One fiber yarn in a braided composite actuator undulates sinusoidally in a “curvy surface”, which is normal to the cylinder wall. Therefore the fiber yarn is twisted and bent in the 3D space. A cell of fabric of the braided composite actuator is illustrated in Figure 6-1(a). Because the dimensions of the cell are much smaller than the cylinder radius in the real structure, we can assume that the piece of yarn in the cell is bent in a plane normal to the cylinder wall and ignore the space twisting. To better observe how the yarn bends, the actuator is cut by a plane parallel to the “curvy surface” as shown in Figure 6-1(b) and the schematic is shown in Figure 6-2. The section view of the cell on the cylinder wall is shown in Figure 6-1(c). The fiber yarn cross sections are assumed to be lenticular, as suggested by many researchers.

The cross section in Figure 6-1(c) can be simplified to Figure 6-3. Undulating yarns lie on the top of an ellipse, which is the the cross section of the inner tube. The cutting plane can be adjusted so that the yarn center (point E in Figure 6-3) lies on the minor axis
of the ellipse. The dashed curve in Figure 6-3 is the centerline of the fabric. It is the reference curve around which the yarn undulates and it is parallel to the ellipse curve.

Figure 6-1: (a) CAD model of the braided composite actuator; (b) CAD model with a cutting plane; (c) section view of intersecting fiber yarns.
6.2.2 Fiber yarn shape equation

We assume the sinusoidal shape for the yarn. Different from yarns in plane fabrics as commonly assumed by other researchers [144, 145, 149], the sinusoidal function is described in a curvilinear coordinate system $\eta-\xi$, as shown in Figure 6-4. The fiber yarn with a thickness is simplified to a line which is the centerline of the fiber yarn. The yarn centerline undulates sinusoidally around the reference curve — the fabric centerline.
(dashed curve). It is appropriate to assume the yarn at its both ends is parallel to the reference curve. If the middle point of the fabric centerline ‘O’ is chosen as the origin of the $\eta$-$\xi$ system, the yarn centerline can be expressed as

$$\eta = -a \cos \left(2\pi \frac{\xi}{l_r}\right), -\frac{l_r}{2} \leq \xi \leq \frac{l_r}{2},$$

(5-3)

where $l_r$ is the length of the reference curve and $a$ is the amplitude of the sinusoidal curve which is also one half of the yarn thickness or a quarter of the fabric thickness.

Figure 6-4: Simplified yarn in the parameterized coordinate systems.

It is necessary to express the yarn centerline shape in the Cartesian coordinate system to derive its bending energy. The reference curve can be expressed in the Cartesian coordinate system ($x_0y$) as

$$f(x) = \sqrt{(R + 2a)^2 - x^2 \sin^2 \theta}, -x_1 \leq x \leq x_1,$$

(5-4)
where $R$ is the radius of the inner rubber tube of the actuator or the semi-minor axis of the ellipse in Figure 6-3, and $\theta$ is the braiding angle in the braided composite actuator or the cutting plane angle in Figure 6-2. The length of the reference curve is

$$l_r = \int_{-\xi_1}^{\xi_1} \sqrt{1 + \left[ f'(x) \right]^2} \, dx . \quad (5-5)$$

For an arbitrary point $P_0$ on the reference curve with a coordinate of $(\xi_0, 0)$, it can be expressed in the Cartesian coordinate systems as $(x, f(x))$, where

$$\int_{0}^{x} \sqrt{1 + \left[ f'(x) \right]^2} \, dx = \xi . \quad (5-6)$$

Since the points on the yarn centerline are one-to-one corresponding with the points on the reference curve, the counterpart of $P_0$ on the yarn centerline $P_1$ can be expressed in the Cartesian coordinate system as

$$P_1: \left[ x + \frac{f'(x)}{\sqrt{1 + \left[ f'(x) \right]^2}} a \cos \left( \frac{2\pi \xi}{l_r} \right), \right]$$

$$f(x) - \frac{1}{\sqrt{1 + \left[ f'(x) \right]^2}} a \cos \left( \frac{2\pi \xi}{l_r} \right) \right] , \quad (5-7)$$

where $x \in [-x_i, x_i]$. From Equation (5-4) to Equation (5-7), we know the yarn shape is determined by $R$, $\theta$ and $a$.

6.2.3 Fiber yarn strain energy

Due to the symmetry and repetitiveness of fiber yarns, we can obtain the total strain energy in a cell by analyzing one yarn in the cell since there are 8 yarns in one cell, i.e. the cell strain energy is 8 times as much as that in one yarn.
Once we have obtained the yarn shape expressed by the variables $R$, $\theta$ and $a$, the strain energy of the yarn can be derived. The yarn strain energy consists of three parts: the longitudinal tensile strain energy, the bending strain energy and the compressive strain energy in the thickness direction.

The yarn thickness is $2a$ and the strain energy in the thickness direction can be expressed as a function of $a$, noted as $U_t(a)$ which is the strain energy per yarn in the cell.

For a large deflection Euler-Bernoulli beam with infinitesimal strains, the total strain energy is

\[
U_{E-B} = \int_0^{\text{Length}} \left[ \frac{1}{2} EA \left( \frac{du}{dX} + \frac{1}{2} \left( \frac{dw}{dX} \right)^2 \right)^2 + \frac{1}{2} EI \left( \frac{d^2w}{dX^2} \right)^2 \right] dX ,
\]

(5-8)

where $EA$ is the axial stiffness of the beam, $EI$ is the bending stiffness, $u$ is the centerline axial displacement and $w$ is the centerline deflection. An infinitesimal piece of the Euler-Bernoulli beam before deformation ($dX$) and after deformation ($(1+\varepsilon)dX$) is shown in Figure 6-5.
Figure 6-5: Schematic of beam with vertical deflection and horizontal displacement.

From Figure 6-5, we get

\[
(1 + \varepsilon)^2 dX^2 = (dX + du)^2 + dw^2 ,
\]

(5-9)

where \(\varepsilon\) is the infinitesimal axial strain which is assumed to be uniform along the beam. By ignoring high order small terms, Equation (5-9) can be simplified to

\[
\varepsilon = \frac{du}{dX} + \frac{1}{2} \left( \frac{dw}{dX} \right)^2 .
\]

(5-10)

Therefore Equation (5-8) can be written as

\[
U_{E-B} = \int_0^{\text{Length}} \left[ \frac{1}{2} EAe^2 + \frac{1}{2} EI \left( \frac{d^2 w}{dX^2} \right)^2 \right] dX ,
\]

(5-11)
which is actually a combination of the axial strain energy and the bending strain energy.

For the fiber yarns in the braided composite actuator, they are assumed to have zero strain energy when there are no loadings applied to the actuator. The following equation is proposed to approximate the strain energy:

$$ U_{\text{Yarn}} = \int_0^{\text{YarnLength}} \left[ \frac{1}{2} E A \varepsilon_Y^2 + \frac{1}{2} EI \left( \frac{d^2 w}{dX^2} - \frac{d^2 w_0}{dX^2} \right)^2 \right] dX + U_t(a) , \quad (5-12) $$

where $\varepsilon_Y$ is the axial strain in the yarn the same as in Chapter 3, $w_0$ is the deflection of the undeformed yarn and $U_t(a)$ is the strain energy in the thickness direction. Recall that we assume the bending stiffness of the yarn does not change with the thickness change, so the bending strain energy and the thickness strain energy can be obtained independently.

The next step is to derive the expression for $d^2 w / dX^2$. One yarn in the braided composite actuator is shown in Figure 6-6. It is parallel to the reference curve at both ends. We assume the fiber yarn is a cantilever beam with one end clamped, e.g. left end clamped. A local coordinate system $XO'Y$ is created with the origin $O'$ at the yarn’s left end.
Figure 6-6: Fiber yarn as a cantilever beam in its local XO’Y coordinate system.

According to Equation (5-7), the left end of the yarn, i.e. O’, can be expressed in the global Cartesian coordinate system \(x0y\) as:

\[
O’: \left[ -x_i - \frac{af’(-x_i)}{\sqrt{1+[f’(-x_i)]^2}}, f(-x_i) + \frac{a}{\sqrt{1+[f’(-x_i)]^2}} \right]. \tag{5-13}
\]

Since the \(X\) axis in the local \(XO’Y\) coordinate system has the same slope as yarn left end, it can be expressed in the \(x0y\) system as:

\[
y = f’(-x_i)x + f(-x_i) + \frac{a}{\sqrt{1+[f’(-x_i)]^2}} - f’(-x_i) \left[ -x_i - \frac{af’(-x_i)}{\sqrt{1+[f’(-x_i)]^2}} \right]. \tag{5-14}
\]

For an arbitrary point \(P_1\) with a local coordinate of \((X, 0)\) on the undeformed fiber yarn centerline (i.e. straight yarn), its global coordinate in the \(x0y\) system on the
deformed yarn is presented in Equation (5-7). Since the functions of the $X$ axis is known, the distances of $P_1$ to the $X$ axis can be obtained as follows:

$$w(x) = \frac{f'(-x_1) \left[ x + \frac{f'(x) a}{\sqrt{1 + [f'(x)]^2}} \cos \left( \frac{2\pi \xi}{l_r} \right) \right] - f(x) + \frac{a}{\sqrt{1 + [f'(-x_1)]^2}} \cos \left( \frac{2\pi \xi}{l_r} \right)}{\sqrt{1 + [f'(-x_1)]^2}} \right), (5-15)$$

$$f(-x_1) + \frac{a}{\sqrt{1 + [f'(-x_1)]^2}} = f'(-x_1) \left[ -x_1 - \frac{af''(-x_1)}{\sqrt{1 + [f'(-x_1)]^2}} \right]$$

where $w$ actually is the deflection of $P_1$ in the local coordinate system $XO’Y$. $\xi$ can be obtained by solving the equation

$$(1 + \varepsilon_X) X = \int_{-l_r/2}^{\xi} \sqrt{1 + \frac{4\pi^2 a^2}{l_r^2} \sin^2 \left( \frac{2\pi \xi}{l_r} \right)} \, d\xi , \quad (5-16)$$

and $x$ can be obtained by solving Equation (5-6).

By defining

$$\Gamma(\xi) = \int \sqrt{1 + \frac{4\pi^2 a^2}{l_r^2} \sin^2 \left( \frac{2\pi \xi}{l_r} \right)} \, d\xi$$

$$H(x) = \int \sqrt{1 + [f'(x)]^2} \, dx$$

Equation (5-16) and Equation (5-6) can be written as follows:

$$(1 + \varepsilon_X) X = \Gamma(\xi) - \Gamma(-l_r/2)$$

$$\xi = H(x) - H(0) \quad . \quad (5-18)$$

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The first derivative of $w$ with respect to $X$ can be written as
\[
\frac{dw}{dX} = \frac{dw}{dx} \frac{d\xi}{dX},
\]
where
\[
\frac{dx}{d\xi} = \left( \frac{d\xi}{dx} \right)^{-1} = \left[ \frac{dH(x)}{dx} \right]^{-1} = \frac{1}{\sqrt{1 + \left[ f'(x) \right]^2}}.
\]
\[
\frac{d\xi}{dX} = \left( \frac{dX}{d\xi} \right)^{-1} = \left[ \frac{1}{1 + \varepsilon_Y} \frac{d\Gamma(\xi)}{d\xi} \right]^{-1} = \frac{1 + \varepsilon_Y}{\sqrt{1 + \frac{4\pi^2 a^2}{l_r^2} \sin^2 \left( \frac{2\pi \xi}{l_r} \right)}}.
\]

So the second derivative of $w$ with respect to $X$ can be written as
\[
\frac{d^2w}{dX^2} = \frac{d}{dx} \frac{dw}{dx} \frac{d\xi}{dX} \frac{dx}{d\xi}.
\]

The yarn strain energy can be obtained by Equation (5-12), which is determined by $\varepsilon_Y, R, \theta$ and $a$.

6.2.4 Constrains among fiber yarn shape parameters

For the braided composite actuator without considering compaction effect and fiber extensibility, $R$ and $\theta$ are not independent to each other. They have a relationship which is expressed in Equation (5-22), where $R_0$ and $\theta_0$ are the radius and fiber angle before deformation
\[
\sin \theta = \frac{R}{R_0} \sin \theta_0.
\]
For the braided composite actuator without considering compaction effect but including the fiber extensibility, only two variables are independent among $R$, $\theta$ and $\varepsilon_Y$, as shown in Equation (5-23)

\[(1+\varepsilon_Y)\sin\theta = \frac{R}{R_0}\sin\theta_0 .\]  

(5-23)

Similarly, for the braided composite actuator including the compaction effect and the fiber extensibility, only three variables are independent among $R$, $\theta$, $a$ and $\varepsilon_Y$.

We assume the sinusoidal undulation amplitude is $a_0$ and the length of the reference curve is $l_{r0}$ before deformation. We have the following equation based on the geometry:

\[\frac{l_t}{l_{r0}} = \frac{2\pi(R+2a)/\sin\theta}{2\pi(R_0+2a_0)/\sin\theta_0} .\]  

(5-24)

Since the yarn is extended by a ratio of $(1+\varepsilon_Y)$ after deformation, we have the following equation:

\[(1+\varepsilon_Y)\int_{-l_{r0}/2}^{l_{r0}/2} \sqrt{1+\frac{4\pi^2a_0^2}{l_{r0}^2}\sin^2\left(\frac{2\pi \frac{\varepsilon}{l_{r0}}}{r}ight)} \, \varepsilon \, d\varepsilon = \int_{-l_t/2}^{l_t/2} \sqrt{1+\frac{4\pi^2a^2}{l_t^2}\sin^2\left(\frac{2\pi \frac{\varepsilon}{l_t}}{l_t}ight)} \, d\varepsilon ,\]  

(5-25)

where the right side is the deformed yarn length in one cell. This equation can be simplified to:

\[4(1+\varepsilon_Y)\int_{0}^{\pi/2} \sqrt{\frac{l_{r0}^2}{4\pi^2}+a_0^2 \sin^2\theta} \, d\theta = 4\int_{0}^{\pi/2} \sqrt{\frac{l_t^2}{4\pi^2}+a^2 \sin^2\theta} \, d\theta .\]  

(5-26)

$a$ and $l_t$ can be determined by $R$, $\theta$ and $\varepsilon_Y$ with Equation (5-24) and Equation (5-26). Therefore the yarn strain energy is expressed with three independent variables $R$, $\theta$ and $\varepsilon_Y$. 

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Since the actuator is subjected to internal pressure, the fabric is compressed in the thickness direction. The amplitude of fiber yarn undulation becomes smaller. Moreover, the reference line must be extended but it cannot exceed the fiber yarn length. Therefore we have the following rules:

\[ 0 < a < a_0, \quad (5-27) \]

and

\[ l_{r0} < l_r < (1 + \varepsilon_Y) \int_{l_{r0}/2}^{l_{r0}/2} \sqrt{1 + 4\pi^2 a_0^2 \sin^2 \left(2\pi \frac{x}{l_{r0}}\right)} \, d\xi. \quad (5-28) \]

From Equation (5-24) and inequality (5-28), we have the following inequality:

\[ 1 < \frac{l_r}{l_{r0}} = \frac{(R + 2a)(\sin \theta_0)}{(R_0 + 2a_0)\sin \theta} < \frac{(1 + \varepsilon_Y) \int_{l_{r0}/2}^{l_{r0}/2} \sqrt{1 + 4\pi^2 a_0^2 \sin^2 \left(2\pi \frac{x}{l_{r0}}\right)} \, d\xi}{l_{r0}}. \quad (5-29) \]

This inequality can be rewritten as

\[ a > \frac{1}{2} \left[ \frac{(R_0 + 2a_0)\sin \theta}{\sin \theta_0} - R \right] \]

\[ a < \frac{1}{2} \left[ \frac{(1 + \varepsilon_Y) \int_{l_{r0}/2}^{l_{r0}/2} \sqrt{1 + 4\pi^2 a_0^2 \sin^2 \left(2\pi \frac{x}{l_{r0}}\right)} \, d\xi}{l_{r0}} \frac{(R_0 + 2a_0)\sin \theta}{\sin \theta_0} - R \right]. \quad (5-30) \]

To satisfy inequality (5-27), we have the following requirements for \( R, \theta \) and \( \varepsilon_Y \):
\[ \frac{1}{2} \left[ \frac{(R_0 + 2a_0) \sin \theta}{\sin \theta_0} - R \right] > 0 \]

\[ \frac{1}{2} \left[ (1 + \varepsilon_Y) \int_{-l_{10}/2}^{l_{10}/2} \left[ 1 + \frac{4\pi^2 a_0^2}{l_{10}^2} \sin^2 \left( \frac{2\pi \xi}{l_{10}} \right) \right] d\xi \left( \frac{R_0 + 2a_0}{\sin \theta_0} \right) - R \right] < a_0 \quad . \quad (5-31) \]

The inequality (5-31) can be rearranged into the following form:

\[ (1 + \varepsilon_Y) \int_{-l_{10}/2}^{l_{10}/2} \left[ 1 + \frac{4\pi^2 a_0^2}{l_{10}^2} \sin^2 \left( \frac{2\pi \xi}{l_{10}} \right) \right] d\xi \left( \frac{R_0 + 2a_0}{\sin \theta_0} \right) - R - 2a_0 < R < \frac{R_0 + 2a_0}{\sin \theta_0} \sin \theta \quad . \quad (5-32) \]

This inequality indicates that even though \( R, \theta \) and \( \varepsilon_Y \) are independent, they still have physical constrains which is useful in solving the governing equations.

### 6.2.5 Governing equations for braided composite actuator

The governing equations can be derived from Equation (5-2). There are three independent variables \( R, \theta \) and \( \varepsilon_Y \), so there should be three governing equations.

The virtual work done by the axial tensile force is

\[ \delta W_F = F \cdot \delta L = F \cdot \delta \left[ \frac{2\pi R}{\tan \theta} \left( 1 + \varepsilon_Y \right) \right] \quad . \quad (5-33) \]

\[ = F \cdot \frac{2\pi}{\tan \theta} \left( 1 + \varepsilon_Y \right) \delta R - F \cdot \frac{2\pi R}{\sin^2 \theta} \left( 1 + \varepsilon_Y \right) \delta \theta + F \cdot \frac{2\pi R}{\tan \theta} \delta \varepsilon_Y \]

The virtual work done by the internal pressure is
\[
\delta W_{\text{pressure}} = P \cdot \delta V
\]
\[
= P \cdot \delta \left[ \frac{\pi R^2}{\tan \theta} \cdot \frac{2\pi R}{\tan \theta} \left(1 + \varepsilon_\gamma \right) \right]
\]
\[
= P \cdot \delta \left[ \frac{2\pi^2 R^3}{\tan \theta} \right] \left(1 + \varepsilon_\gamma \right) \]. \quad (5-34)
\]
\[
= P \frac{6\pi^2 R^2}{\tan \theta} (1 + \varepsilon_\gamma) \delta R - P \frac{2\pi^2 R^3}{\sin^2 \theta} (1 + \varepsilon_\gamma) \delta \theta + P \frac{2\pi^2 R^3}{\tan \theta} \delta \varepsilon_\gamma
\]

The virtual strain energy in the rubber liner is
\[
\delta E_{\text{rubber}} = \delta \left[ C_1 \left( I_1 - 3 \right) + C_2 \left( I_2 - 3 \right) \right] 2\pi R_0 L_0 h_0
\]
\[
= 2\pi R_0 L_0 h_0 \left( \Lambda_1 \delta R + \Lambda_2 \delta \theta + \Lambda_3 \delta \varepsilon_\gamma \right) \]. \quad (5-35)
\]

where
\[
I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \lambda_2^{-2}
\]
\[
I_2 = \lambda_1^{-2} + \lambda_2^{-2} + \lambda_1^2 \lambda_2^2
\]
\[
\lambda_2 = \frac{R}{R_0}, \quad \Lambda_2
\]
\[
\lambda_1 = \frac{2\pi R}{2\pi R_0} \left(1 + \varepsilon_\gamma \right) = \frac{R \tan \theta_0}{R_0 \tan \theta} \left(1 + \varepsilon_\gamma \right)
\]
\[
\lambda_3 = \frac{1}{\lambda_1 \lambda_2}
\]

and
\[ \Lambda_1 = C_1 \left[ \frac{2R \tan^2 \theta_0 (1 + \varepsilon_Y)^2 + 2R}{R_0^2 \tan^2 \theta} - \frac{4R_0^4 \tan^2 \theta}{R^5 \tan^2 \theta_0 (1 + \varepsilon_Y)^2} \right] \\
+ C_2 \left[ \frac{-2R_0^2 \tan^2 \theta}{R^2 \tan^2 \theta_0 (1 + \varepsilon_Y)^2} - \frac{2R_0^2}{R^3} + \frac{4R_3^2 \tan^2 \theta_0 (1 + \varepsilon_Y)^2}{R_0^4 \tan^2 \theta} \right] \]

\[ \Lambda_2 = C_1 \left[ \frac{2R^2 \tan^2 \theta_0 \cos \theta}{R_0^2 \sin^3 \theta} (1 + \varepsilon_Y)^2 + \frac{2R_0^4 \sin \theta}{R^4 \tan^2 \theta_0 (1 + \varepsilon_Y)^2 \cos \theta} \right] \\
+ C_2 \left[ \frac{2R_0^2 \sin \theta}{R^2 \tan^2 \theta_0 (1 + \varepsilon_Y)^2} - \frac{2R^4 \tan^2 \theta_0 \cos \theta (1 + \varepsilon_Y)^2}{R_0^4 \sin^3 \theta} \right] \]

\[ \Lambda_3 = C_1 \left[ \frac{2R^2 \tan^2 \theta_0 (1 + \varepsilon_Y)}{R_0^2 \tan^2 \theta} - \frac{2R_0^4 \tan^2 \theta}{R^4 \tan^2 \theta_0 (1 + \varepsilon_Y)^3} \right] \\
+ C_2 \left[ -\frac{2R_0^2 \tan^2 \theta}{R^2 \tan^2 \theta_0 (1 + \varepsilon_Y)^3} + \frac{2R^4 \tan^2 \theta_0 (1 + \varepsilon_Y)}{R_0^4 \tan^2 \theta} \right] \].

(5-37)

The virtual strain energy in the fabric is

\[ \delta E_{\text{fabric}} = 8n \cdot \delta U_{\text{Yarn}} \]

\[ = 8n \delta \left\{ \int_0^{\text{YarnLength}} \left[ \frac{1}{2} EA \varepsilon_Y^2 + \frac{1}{2} EI \left( \frac{d^2w}{dx^2} - \frac{d^2w_0}{dx^2} \right)^2 \right] \, dX + U_i(a) \right\}, \quad (5-38) \]

where \( n \) is the number of the cell in the fabric.

By defining

\[ G(R, \theta, \varepsilon_Y) = \frac{d^2w}{dx^2} - \frac{d^2w_0}{dx^2}, \]

(5-39)

Equation (5-38) can be converted to
\[
\delta E_{\text{fabric}} = 8n \cdot \left[ \int_0^{\text{Yarnlength}} \left( EA \varepsilon_Y \delta \varepsilon_Y + EIG \delta G \right) dX + \delta U_i(a) \right]
\]
\[
= 8n \cdot \delta R \left[ \int_0^{\text{Yarnlength}} EIG \frac{\partial G}{\partial R} dX + \frac{\partial U_i(a)}{\partial R} \right] 
+ 8n \cdot \delta \theta \left[ \int_0^{\text{Yarnlength}} EIG \frac{\partial G}{\partial \theta} dX + \frac{\partial U_i(a)}{\partial \theta} \right] 
+ 8n \cdot \delta \varepsilon_Y \left[ \int_0^{\text{Yarnlength}} \left( EA \varepsilon_Y + EIG \frac{\partial G}{\partial \varepsilon_Y} \right) dX + \frac{\partial U_i(a)}{\partial \varepsilon_Y} \right] 
\]

The three governing equations are:

\[
8n \left[ \int_0^{\text{Yarnlength}} EIG \frac{\partial G}{\partial R} dX + \frac{dU_i(a)}{da} \frac{\partial a}{\partial R} \right] + 2\pi R \varepsilon h_0 \Lambda_1 
- F \frac{2\pi}{\tan \theta} \left( 1 + \varepsilon_Y \right) - P \frac{6\pi^2 R^2}{\tan \theta} \left( 1 + \varepsilon_Y \right) = 0 
\]

\[
8n \left[ \int_0^{\text{Yarnlength}} EIG \frac{\partial G}{\partial \theta} dX + \frac{dU_i(a)}{da} \frac{\partial a}{\partial \theta} \right] + 2\pi R \varepsilon h_0 \Lambda_2 
+ F \frac{2\pi R}{\sin^2 \theta} \left( 1 + \varepsilon_Y \right) + P \frac{2\pi^2 R^3}{\sin^2 \theta} \left( 1 + \varepsilon_Y \right) = 0 
\]

and

\[
8n \left[ \int_0^{\text{Yarnlength}} \left( EA \varepsilon_Y + EIG \frac{\partial G}{\partial \varepsilon_Y} \right) dX + \frac{dU_i(a)}{da} \frac{\partial a}{\partial \varepsilon_Y} \right] 
+ 2\pi R \varepsilon h_0 \Lambda_3 - P \frac{2\pi^2 R^3}{\tan \theta} - F \frac{2\pi R}{\tan \theta} = 0 
\]

6.2.6 Procedures to solve the governing equations

The procedures to solve the governing equations are listed below.

(a) Pick a trial solution \((R, \theta, \varepsilon_Y)\). The solution must satisfy the inequality (5-32).
(b) Solve for \( a \) and \( l_r \) with Equation (5-24) and Equation (5-26).

(c) Obtain the function of the reference curve \( f(x) \) with Equation (5-4).

(d) Determine the boundary of the reference curve in the Cartesian coordinate system \([-x_1, x_1]\) with Equation (5-5).

(e) Calculate the yarn length before deformation with \( a_0 \) and \( l_{r0} \) based upon the equation below:

\[
\text{YarnLength} = \int_{-\alpha_0/2}^{\alpha_0/2} \sqrt{1 + \frac{4\pi^2 a_0^2}{l_{r0}^2} \sin^2 \left( \frac{2\pi \xi}{l_{r0}} \right)} \, d\xi .
\tag{5-44}
\]

(f) Discretize the undeformed yarn length to get a series of points with the coordinates of \( X \), determine the corresponding points on the reference curve (\( \xi \)) with Equation (5-16) and further determine the corresponding \( x \) coordinates with Equation (5-6).

(g) Calculate \( dw/dx \) with Equation (5-15), \( dx/d\xi \) and \( d\xi/dX \) with Equation (5-20).

(h) Calculate \( dw/dX \) and \( dw^2/d^2X \) with Equation (5-19) and Equation (5-21) respectively. Then \( G(R, \theta, \varepsilon_Y) \) in Equation (5-39) is determined.

(i) Repeat (b) to (h) with perturbed trial solutions to obtain \( \partial G/\partial R \), \( \partial G/\partial \theta \), \( \partial G/\partial \varepsilon_Y \), \( \partial a/\partial R \), \( \partial a/\partial \theta \) and \( \partial a/\partial \varepsilon_Y \). For example, use perturbed trial solutions \((R+\varepsilon R, \theta, \varepsilon_Y)\) and \((R-\varepsilon R, \theta, \varepsilon_Y)\) to obtain \( \partial G/\partial R \) and \( \partial a/\partial R \), where

\[
\begin{align*}
\frac{\partial G}{\partial R} &= \frac{G(R+\varepsilon R, \theta, \varepsilon_Y) - G(R-\varepsilon R, \theta, \varepsilon_Y)}{2\varepsilon R}, \\
\frac{\partial a}{\partial R} &= \frac{a(R+\varepsilon R, \theta, \varepsilon_Y) - a(R-\varepsilon R, \theta, \varepsilon_Y)}{2\varepsilon R},
\end{align*}
\tag{5-45}
\]

and \( \varepsilon \ll 1 \).

(j) Plug \( R, \theta, \varepsilon_Y, G(R, \theta, \varepsilon_Y), \partial G/\partial R, \partial G/\partial \theta, \partial G/\partial \varepsilon_Y, \partial a/\partial R, \partial a/\partial \theta \) and \( \partial a/\partial \varepsilon_Y \) into the governing equations (Equations (5-41)-(5-43)) to calculate the residues. Whenever all the residues are zero, the trial solution is the exact solution.
6.3 Determination of fiber yarn bending stiffness and $U_i(a)$ through experiments

6.3.1 Fiber yarn bending stiffness

A simple test was performed to obtain the yarn bending stiffness. We clamped a piece of fiber yarn at one end and let the other end be free. The aramid fiber, carbon fiber and glass fiber yarns with various lengths are tested. Pictures were taken to record the yarn length and the yarn deflection. The deflection was read from the pictures. The testing pictures are shown in Figure 6-7.
Figure 6-7: Fiber yarn bending stiffness measurement: (a) before deflection; (b) after deflection.

For an Euler-Bernoulli beam with large deflection and small strains as shown in Figure 6-8, the deflection caused by its weight is

\[
w(x) = \frac{1}{EI} \left( \frac{1}{24} q x^4 - \frac{1}{6} q L_b x^3 + \frac{1}{4} q L_b^2 x^2 \right)
\]

(5-46)

where \( EI \) is the bending stiffness, \( L_b \) is the beam length and \( q \) is the fiber yarn weight per unit length. The bending stiffness can be obtained by

\[
EI = \frac{q L_b^4}{8w(L_b)},
\]

(5-47)

where \( w(L_b) \) is the tip deflection of the beam.
The measured deflections and the corresponding lengths are listed in Table 6-1 for three types of fiber yarns. The weights per unit length for each fiber are listed in Table 6-2. The mass for the fibers were measured by the METTLER TOLEDO™ XS205 DualRange Analytical Balance, which has a resolution of 0.01 mg.

Table 6-1: Measured fiber yarn lengths and deflections (Units: grid, 1 grid = 5.06mm).

<table>
<thead>
<tr>
<th>Sample #</th>
<th>Aramid fiber</th>
<th>Carbon fiber</th>
<th>Glass fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Length</td>
<td>Deflection</td>
<td>Length</td>
</tr>
<tr>
<td>1</td>
<td>3.8</td>
<td>12.5</td>
<td>8.6</td>
</tr>
<tr>
<td>2</td>
<td>7.1</td>
<td>16.1</td>
<td>2.9</td>
</tr>
<tr>
<td>3</td>
<td>2.1</td>
<td>12</td>
<td>1.1</td>
</tr>
<tr>
<td>4</td>
<td>10.7</td>
<td>19</td>
<td>7.1</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>20</td>
<td>3.9</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>12.2</td>
<td>5.1</td>
</tr>
<tr>
<td>7</td>
<td>1.7</td>
<td>10.8</td>
<td>8.9</td>
</tr>
<tr>
<td>8</td>
<td>6.6</td>
<td>17.1</td>
<td>6.1</td>
</tr>
<tr>
<td>9</td>
<td>2.6</td>
<td>10.4</td>
<td>3.5</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>14.5</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td>16.1</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>7.8</td>
<td>16.2</td>
<td>1.5</td>
</tr>
<tr>
<td>13</td>
<td>16.6</td>
<td>27</td>
<td>14.2</td>
</tr>
</tbody>
</table>
Table 6-2: Fiber weight per unit length.

<table>
<thead>
<tr>
<th></th>
<th>Aramid fiber</th>
<th>Carbon fiber</th>
<th>Glass fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td>q (N/m)</td>
<td>0.0097</td>
<td>0.0079</td>
<td>0.0109</td>
</tr>
</tbody>
</table>

The calculated bending stiffness of the three types of fiber yarns are shown in Figure 6-9 together with their mean values. The glass fiber has a larger bending stiffness but has a larger deviation. The average bending stiffness for these three types of fiber yarns are listed in Table 6-3. The tested yarns were from the braided sleeves. Creep behavior has occurred to some yarns, which might have caused the large deviations in the bending stiffness.

Table 6-3: Estimated bending stiffness.

<table>
<thead>
<tr>
<th></th>
<th>Aramid fiber</th>
<th>Carbon fiber</th>
<th>Glass fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td>EI (N·m²)</td>
<td>1.7044×10⁶</td>
<td>1.6818×10⁶</td>
<td>5.0615×10⁶</td>
</tr>
</tbody>
</table>
6.3.2 Determination of the $U_t(a)$

The strain energy density in the thickness direction of the yarn was obtained from the compaction law test in Chapter 4. Because the fabric energy is the combination of the yarn bending energy and the strain energy in the thickness direction, $U_t(a)$ is obtained by subtracting the yarn bending energy from the total strain energy.

The yarn in the compaction law test has a simpler shape function than it is in the actuator since the reference curve is a straight line. We still assume it is in the sinusoidal shape with an amplitude of $a$, as shown in Figure 6-10.

Figure 6-9: Measured bending stiffness for three types of fiber yarns and their averages

Figure 6-10: Sinusoidal fiber yarn shape in the compaction law test.
The yarn deflection in Figure 6-10 can be expressed as

\[ w(x) = -a \cos \left(2\pi \frac{x}{l}\right). \]  

(5-48)

Without the loss of generality, the yarn is assumed to be a cantilever beam of which the left end is clamped. For any point \((x, w(x))\) in the current configuration, its corresponding point in the initial configuration is \((X, 0)\), where \(X\) can be expressed as:

\[ X = \int_{-l/2}^{x} \sqrt{1 + \frac{4\pi^2 a^2}{l^2} \sin^2 \left(\frac{2\pi x}{l}\right)} \, dx. \]  

(5-49)

The bending energy in the yarn can be derived as:

\[ U_{\text{bending}}(a) = \int_0^{\text{YarnLength}} \frac{1}{2} EI \left(\frac{d^2 w}{dX^2} - \frac{d^2 w_0}{dX^2}\right)^2 dX, \]  

(5-50)

where \(w_0\) is the yarn deflection of the fabric before the compaction test.

The first derivative of \(w\) with respect to \(X\) is

\[ \frac{dw}{dX} = \frac{dw}{dx} \frac{dx}{dX} = \frac{2\pi a}{l} \sin \left(\frac{2\pi x}{l}\right) \frac{1}{\sqrt{1 + \frac{4\pi^2 a^2}{l^2} \sin^2 \left(\frac{2\pi x}{l}\right)}}. \]  

(5-51)

Therefore,
\[
\frac{d^2w}{dX^2} = \frac{d}{dx} \left( \frac{dw}{dX} \right) dx
= \frac{4\pi^2a}{l^2} \cos \left( \frac{2\pi x}{l} \right) \frac{1}{1 + \frac{4\pi^2a^2}{l^2} \sin^2 \left( \frac{2\pi x}{l} \right)}
- \frac{16\pi^4a^3}{l^4} \sin^2 \left( \frac{2\pi x}{l} \right) \cos \left( \frac{2\pi x}{l} \right) \frac{1}{1 + \frac{4\pi^2a^2}{l^2} \sin^2 \left( \frac{2\pi x}{l} \right) \left[ \frac{2\pi x}{l} \right]^2}.
\] (5-52)

In the compaction law test, the thickness decrease of the fabric was caused by the yarn flattening and the yarn thickness decrease. The input energy from the testing frame is

\[
W_{test}(a) = \int p \cdot A_{test} \, d\Delta, \quad (5-53)
\]

where

\[
\Delta = 4(a_0 - a), \quad (5-54)
\]

\( A_{test} \) is the compressed area and \( a_0 \) is the amplitude before the test.

The input energy is equal to the strain energy change in the fabric. The strain energy in the yarn thickness direction can be obtained as

\[
U_s(a) = \frac{W_{test}(a)}{8n_{test}} - U_{bending}(a) \quad (5-55)
\]

where \( n_{test} \) is the number of the cell in the tested fabric.

For the aramid fabric, the average intersection count per mm\(^2\) for the fabric is 0.49, based on which the \( n_{test} \) can be calculated as
\[ n_{\text{test}} = \frac{0.49 A_{\text{test}}}{16}, \]

where \( A_{\text{test}} \) is the compressed area in mm\(^2\) and the calculated \( n_{\text{test}} \) is rounded to integer. \( a_0 \) is 0.15mm, which is a quarter of the measured fabric thickness (0.6mm) before the compaction law test and the measured \( l_0 = 5.3 \)mm. With these parameters and the estimated bending stiffness of yarn in Table 6-3, we are able to obtain the bending strain energy change in the fiber yarn as well as the strain energy change in the yarn thickness direction as shown in Figure 6-11.

![Figure 6-11](image)

Figure 6-11: The input energy from the testing frame and the aramid yarn strain energy components in the compaction law test.

According to Equation (5-55), we can obtain the thickness-directional strain energy in one yarn as a function of the amplitude \( a \), as shown in Figure 6-12. We assume the yarns on the cylindrical surface of the braided composite actuator have the same strain-energy function in the thickness direction.
Figure 6-12: The strain energy for one yarn in the thickness direction as a function of $a$.

6.4 Fiber yarn bending energy estimation in deformed braided composite actuators

The procedures to solve the governing equations are listed in section 6.2.6. There are many methods to solve a set of nonlinear equations, such as the newton method, quasi-newton methods, the conjugate gradient method, etc. But these numerical methods all require a good initial guess of the solution, i.e. the trial solution falls in the domain where the trial solution can converge to the exact solution. It is very difficult to find a good trial solution, especially with three independent variables. The governing equations for the braided composite actuator are very sensitive to the variables even for those without the compaction effects. This indicates a very small converging domain in the solutions. In addition, it is time consuming to run through the solution steps. Considering that solving the governing equations require many iterations to converge, it will take significantly more time to solve the governing equations for compaction model with yarn bending effect than the model without the yarn bending included.

However, it is important to investigate the value in solving the complicated governing equations and the relative impact on the solutions. In other words, it may be
too expensive to include the yarn bending stiffness while there is little improvement in prediction. One way is to assume a deformed shape and compare the yarn bending energy with other strain energies in the fabric, e.g. strain energy in the thickness direction and in the axial direction. Three cases are studied in this section. The sample in the calculation is a braided composite actuator made of aramid fiber with $R_0 = 4.6\text{mm}$, $\theta_0 = 21^\circ$ and a total length of 0.3m. Additional parameters are $a_0 = 1.54E-4$ m and $l_{0\theta} = 5.3E-3$ m. The deformed shapes ($R$, $\theta$, $\varepsilon_Y$) are estimated from the compaction model in Chapter 3. The amplitude of the sinusoidal yarn $a$ is estimated with the compressed fabric thickness from the compaction model in Chapter 3. Then the bending energy, the axial strain energy and the strain energy in the thickness direction in the fabric can be estimated by corresponding terms in Equation (5-12). The results are listed in Table 6-4.

Table 6-4: Parameters in braided composite actuator with different loadings (All units are in SI).

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>0.6E6</td>
<td>0.6E6</td>
<td>1.2E6</td>
</tr>
<tr>
<td>$F$</td>
<td>0</td>
<td>482</td>
<td>0</td>
</tr>
<tr>
<td>$E_{\text{fabric}}$</td>
<td>$E_{\text{yarnbending}}$</td>
<td>9.8E-3</td>
<td>9.3E-6</td>
</tr>
<tr>
<td></td>
<td>$E_{\text{yarnthickness}}$</td>
<td>5.5E-2</td>
<td>7.7E-2</td>
</tr>
<tr>
<td></td>
<td>$E_{\text{yarnaxial}}$</td>
<td>1.8E-2</td>
<td>6.1E-2</td>
</tr>
<tr>
<td>$E_{\text{rubber}}$</td>
<td>9.6</td>
<td>5.6</td>
<td>9.9</td>
</tr>
<tr>
<td>$W_P$</td>
<td>27</td>
<td>3.0</td>
<td>58</td>
</tr>
<tr>
<td>$a$</td>
<td>1.17E-4</td>
<td>1.15E-4</td>
<td>1.15E-4</td>
</tr>
<tr>
<td>$R$</td>
<td>10E-3</td>
<td>4.8E-3</td>
<td>10E-3</td>
</tr>
<tr>
<td>$\theta$</td>
<td>52$^\circ$</td>
<td>22$^\circ$</td>
<td>53$^\circ$</td>
</tr>
<tr>
<td>$\varepsilon_Y$</td>
<td>3.8E-4</td>
<td>6.7E-4</td>
<td>8.1E-4</td>
</tr>
</tbody>
</table>

The actuator in Case I and Case III is free of axial force. The input energy by the internal pressure $W_P$ is much larger than the strain energy in the rubber $E_{\text{rubber}}$ and the strain energy in the fiber layer $E_{\text{fabric}}$. Please note here that even though the axial force is 0 it does not mean there is no energy input from the axial force. To achieve the final state as listed in Case I and Case III from the undeformed actuator quasi-statically, a gradually
decreasing axial force applied on the actuator is required. The work done by the axial force is negative. This is why the input energy is much larger than the strain energy stored in the actuator. Case II is very close to the blocking state, which is why the input energy from the internal pressure is close to the strain energy. They are not exactly the same because the force is a slightly larger than the blocking force in addition to numerical errors.

The fiber yarn bending energy in Case II is almost 4 orders of magnitude smaller than the strain energy in the thickness direction or in the axial direction. It is significantly smaller than Case I and Case III, although the external loadings in Case II is larger than Case I. The fiber yarn bending energies in Case I and Case III are about the same magnitude. According to section 6.2.3, the yarn shape depends not only on the sinusoidal amplitude $a$ but also on the radius $R$ and the braiding angle $\theta$ after deformation. $R$ and $\theta$ determine the curvature of the reference curve as shown in Figure 6-6. $R$ and $\theta$ in Case I and Case III are very close in magnitude, which leads to the close bending energies. Case II and Case III have the same magnitude of $a$ but there is a large difference in the bending energy. Therefore it can be inferred that most of the yarn bending energy comes from the curvature change of the reference curve (global bending) and the contribution from the amplitude change (local bending) can be ignored.

Significant curvature change of the reference curve only occurs with the large-stroke actuation of the actuator. As discussed in section 5.1, the actuation performance will not be influenced by the inter fiber yarn compaction as much as the closed-valve performance. Recall that the motivation of the compaction modeling is to improve the closed-valve stiffness prediction. In the closed-valve scenario, there is little change for the reference curve so the yarn bending energy is small enough to be ignored as Case II shows. Therefore it is reasonable that we ignored the yarn bending stiffness as we did in Chapter 3.
In summary, the compaction model may be more precise by including the bending energy in the yarn. But it is not worth doing so considering the limited improvement on the performance prediction versus the expensive computation.

6.5 Summary

In this chapter, a compaction model including the fiber yarn bending effect was developed by applying the virtual work method.

The fiber yarn bending energy is caused by both fiber rotation and the inter-fiber-yarn compaction. With the fiber rotation and the change of the actuator radius, the global curvature of the fiber yarn changes which induces part of the bending energy. The inter-fiber-yarn compaction reduces the amplitude of the undulating fiber yarn, which induces the other part of the bending energy. The model includes both parts of the bending energy, which results in a set of complicated governing equations to solve. Fortunately we are able to show that the yarn bending energy in the closed-valve scenario is significantly smaller in magnitude compared to the other strain energies in the fiber yarn.

We were also able to show that the global bending energy dominates the local bending energy when the actuator is in the large-stroke actuation scenario. Actually the global bending of the fiber yarn can be included in the actuator’s model without considering the compaction effect. Almost all of the previously developed actuator’s models ignored the yarn bending effect since the fibers were assumed to be flexible.

The author believes it is possible to include either the global bending energy or the local bending energy in the actuator’s model depending on the application. The local bending energy is not significant in the large-stroke actuation scenario. By including the global bending energy, one can improve the previously developed ‘non-compaction’ model. In the closed-valve scenario, there is little global bending so it can be neglected in the model. The governing equations may be simplified and thus may be easier to solve.
Chapter 7 Conclusions and Future work

7.1 Conclusions

The braided composite actuator is an elastomeric tube reinforced by braided fiber sleeves. It is also known as the McKibben actuator or the pneumatic artificial muscle. Another similar type of actuator is the flexible matrix composite actuator, which functions similarly but the fibers are embedded in the elastomer during fabrication. With internal pressure, the actuator can perform behaviors, such as contracting and extension, as a result of the orientation of the stiff reinforcing fibers. Its high power to weight or power to volume ratio makes itself appealing to many researchers. A comprehensive literature review is provided in Chapter 1. It covers almost all aspects of the actuator, i.e. modeling, hysteresis study, driving methods, manufacture, fatigue study and control methodologies.

Another important property of the braided composite actuator is to the ability to exhibit variable stiffness when using a high bulk modulus working fluid. It has low stiffness when the valve of the actuator is open and it becomes very stiff when the valve is closed after it is filled with the working fluid. Based on the previously developed models for the actuator, the effective stiffness can be predicted. However no model has included the fiber yarn compaction effect. The compaction effect is negligible for the application of large-stroke actuation since the strain induced by the compaction is much smaller than the strain during the actuation. But the strains are comparable in the variable-stiffness application. The compaction effect can significantly lower the predicted closed-valve stiffness. Improving the prediction of the closed-valve stiffness of the braided composite actuator by including the inter fiber yarn compaction effect is the motivation of this research work.

In this dissertation, the actuator model without compaction is introduced in Chapter 2. There are two main approaches: the continuum mechanics approach and the virtual work approach. The two models using the continuum mechanics approach are elaborated. The first model assumes inextensible fibers surrounding an elastomeric tube whereas the
second model presented considers extensible fibers embedded in the elastomer. This model has been developed for predicting the response of the flexible matrix composite (FMC) actuators. The virtual work approach has been adopted by many researchers in the McKibben actuator field, but they made assumptions on the deformed shape of the actuator instead of using the exact geometry description as it is used in the continuum mechanics approach. The model using the virtual work approach combined with the exact geometry description is derived.

The detailed compaction modeling is described in Chapter 3. The new model not only includes the compaction effect, but also includes the fiber extensibility and the entrapped air effects.

Experiments are described in Chapter 4. The material properties which are required in the model are determined experimentally. Two types of experiments are performed to validate the compaction model. By measuring the radial expansion of the blocked actuator, we confirmed the existence of compaction. The good agreement between the computational results and experimental results validates the compaction model. The tensile tests of the closed-valve actuator verify the improved stiffness prediction with the new compaction model.

The compaction model is used to analyze the performance of the braided composite actuator. The influence of the compaction is small on the actuation response of the actuator. The analysis for the closed-valve actuator shows that both the compaction and the fiber extensibility contribute to lowering the closed-valve stiffness. The difference between them is that the compaction dominates the contribution at the beginning of the loadings but the fiber extensibility is more important under the large loadings. Most of the compaction finishes with small loadings.

In the developed compaction model, the fiber yarns are assumed to be perfectly flexible. It may improve the model if the assumption is relaxed. Therefore the compaction model including the yarn bending stiffness is developed using the virtual
work approach. Calculated results from this model indicate that the bending energy is significantly less than the compressive energy and the axial strain energy in the yarn. The most of the yarn bending energy is attributed to the global bending (curvature change of the reference curve). For the closed-valve actuator, the global bending can be ignored since there is little radial expansion and fiber angle rotation. Therefore it is appropriate to assume that the yarn has zero bending stiffness to predict the actuator’s closed-valve stiffness.

7.2 Future work

In section 4.2.1, the radial expansion of the clamped actuator was measured by the fiber optic displacement sensor to prove the existence of compaction. While the results did capture the trend, the curves were not smooth. This can be attributed to: the angle between the sensor tip and the reflective surface, non-uniform expansion of the actuator surface and shifting axis of the actuator during the pressurization. To improve the experimental results, a sensor that can directly measure the actuator’s diameter is required. Actually a flexible electro-conductive rubber sensor has been developed and was used to estimate the diameter change of the McKibben actuator [155]. To better understand the compaction behaviors between fiber yarns in the braided composite actuator, more details need to be captured such as the yarn shape change, fabric thickness change, braiding angle change, etc. More advanced measuring equipment are required.

Both models in Chapter 3 and Chapter 6 ignore the frictions between fiber yarns. The frictions are believed to be the cause of the hysteresis phenomenon of the McKibben actuator. Researchers Davis and Caldwell [38] considered the inter yarn compression in the McKibben actuator for deriving a friction. Therefore incorporating the inter fiber yarn frictions into the model may be a direction to improve the closed-valve stiffness prediction.

The compaction model with yarn bending stiffness is able to include the bending energy of the fiber yarn. The most of the yarn bending energy is attributed to the global
bending (curvature change of the reference curve). It can also be incorporated in the previously developed non-compaction model to make an improvement, i.e. the models developed in Chapter 2.

There is not much global bending in the closed-valve scenario. By ignoring the global bending while keeping the local bending (amplitude change of the sinusoidal fiber yarn), the governing equations in section 6.2.5 may be simplified.
Bibliography


