Sound from Rough Wall Boundary Layers

William Nathan Alexander

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William J. Devenport
Stewart Glegg
Roger L. Simpson
Joseph A. Schetz

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ABSTRACT

Turbulent flow over a rough surface produces sound that radiates outside the near wall region. This noise source is often at a lower level than the noise created by edges and bluff body flows, but for applications with large surface area to perimeter ratios at low Mach number, this noise source can have considerable levels. In the first part of this dissertation, a detailed study is made of the ability of the Glegg & Devenport (2009) scattering theory to predict roughness noise. To this end, comparisons are made with measurements from cuboidal and hemispherical roughness with roughness Reynolds numbers, $hu_r/v$, ranging from 24 to 197 and roughness height to boundary layer thickness ratios of 5 to 18. Their theory is shown to work very accurately to predict the noise from surfaces with large roughness Reynolds numbers, but for cases with highly inhomogeneous wall pressure fields, differences grow between estimation and measurement. For these surfaces, the absolute levels were underpredicted but the spectral shape of the measurement was correctly determined indicating that the relationship of the radiated noise with the wavenumber wall pressure spectrum and roughness geometry appears to remain relatively unchanged. In the second part of this dissertation, delay and sum beamforming and least-squares analyses were used to examine roughness noise recorded by a 36-sensor linear microphone array. These methods were employed to estimate the variation of source strengths through short fetches of large hemispherical and cuboidal element roughness. The analyses show that the lead rows of the fetches produced the greatest streamwise and spanwise noise radiation. The least-squares analysis confirmed the presence of streamwise and spanwise aligned dipoles emanating from each roughness element as suggested by the LES of Yang & Wang (2011). The least-squares calculated source strengths show that the streamwise aligned dipole is always stronger than that of the spanwise dipole, but the relative magnitude of the difference varies with frequency.
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Nomenclature

Roman

\( c_\infty \) Speed of sound

\( C_f \) Skin friction coefficient

\( f \) Frequency

\( h \) Roughness height

\( k_o \) Acoustic wavenumber

\( \text{Re}_\theta \) Momentum thickness Reynolds number

\( U_c \) Convection velocity

\( U_o \) Nozzle exit velocity

\( U_m \) Maximum local velocity

\( U_e \) Edge velocity (equivalent to \( U_m \))

\( u_t \) Friction velocity

\( x \) Observer position

\( y \) Source position

Greek

\( \Gamma \) Wavenumber filter function defined by the roughness geometry

\( \delta \) Boundary layer thickness

\( \delta (\cdot) \) Uncertainty

\( \delta^* \) Displacement thickness

\( \theta \) Momentum thickness

\( \kappa \) Wavenumber

\( \nu \) Kinematic viscosity

\( \Phi_{pp}(\mathbf{x}, \omega) \) Far field pressure spectrum

\( \Phi_{pp}(\mathbf{k}, \omega) \) Wavenumber wall pressure spectrum

\( \Phi_{pp}(\omega) \) Near field pressure spectrum

\( \rho \) Density

\( \tau_w \) Wall shear stress

\( \omega \) Angular frequency
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Chapter 1 Introduction

1.1 Motivation

Roughness noise is a low-level noise source that is often overshadowed by the noise created by turbulent flow over edges or by jets. However, for applications at low Mach number and high Reynolds number with small perimeter to surface area ratios, roughness noise can become a significant contributor to the total noise produced. This is a typical scenario for marine vehicles. As other noise sources have been studied and diminished, interest in roughness noise has been increased, but little knowledge has been ascertained about the generation mechanism until very recently. Research on the subject has been hampered by inadequate facilities as measurements of roughness noise eluded researchers even in laboratory settings because of contaminating background noise.

The development of the anechoic wall-jet facility at Virginia Tech has led to significant advancements in roughness noise research. The design, construction, and iterations of this facility are detailed in Grissom (2007). The final product was a wall-jet that allowed for far field measurement of radiating roughness noise without having to immerse microphones in regions of flow. This facility has extremely low background noise operating levels that allow for straightforward measurement of roughness noise from even hydrodynamically smooth surfaces.

Recent theoretical progress and experimental studies in this wall-jet tunnel, such as those presented in Glegg & Devenport (2009), Devenport et al. (2010), and Devenport et al. (2011) have shown that the roughness noise may be a predictable function of the surface geometry and the wavenumber wall pressure spectrum. This study details the analysis of noise from discrete element cuboidal and hemispherical roughness in a turbulent wall-jet flow. Measurements of the radiated far field and surface pressure spectrum inside these surfaces are used to compare with theoretical predictions of the roughness noise using the theory of Glegg & Devenport (2009) assuming a homogeneous wall pressure field. The noise from short fetches of large, discrete cuboidal and hemispherical roughness are also analyzed using a linear microphone array to determine the source strength variation through fetches of roughness and to analyze the directivity of individual sources.

1.2 Previous Roughness Noise Research

Lighthill (1952) first made a theoretical link between turbulent fluid motion and the acoustic field it generates. Lighthill’s acoustic analogy is a rearrangement of the Navier-Stokes equation into a form resembling the wave equation as shown in Equation 1-1.

\[
\frac{\partial^2 (\rho')}{\partial t^2} - c_\infty^2 \frac{\partial (\rho')}{\partial x_i^2} = \frac{\partial T_{ij}}{\partial x_i \partial x_j}
\]

Eq. 1-1

where \( T_{ij} = \rho u_i u_j + p_{ij} - \rho' c_\infty^2 \delta_{ij} \)

\( T_{ij} \) is the Lighthill stress tensor, \( \rho u_i u_j \) is the Reynolds stress, and \( c_\infty \) is the speed of sound.
This formulation requires no assumptions or approximations and gives the exact solution to the radiated sound outside of the flow where the acoustic propagation would be equal to $c_\infty$. In this analysis, Lighthill found that the turbulent noise produced from a free field flow radiates as a quadrupole scaling on the eighth power of velocity.

Curle (1955) reformulated Lighthill’s solution using a Greens function for a bounded region. This resulted in a dipole source field generated on the surface of the boundary. These dipoles vary with velocity to the sixth power and are therefore more efficient producers of noise than quadrupole sources at low Mach number. Curle’s extended version of Lighthill is shown in Equation 1-2. This is an exact formulation, like Lighthill’s equation, except that it assumes a solid, impenetrable boundary.

$$\rho'(x, t)c_\infty^2 = \frac{\partial}{\partial x_i} \int_S \left[ p_{ij} + \rho u_i u_j \right]_{t=\tau^*} \frac{n_j dS(y)}{4\pi|x-y|} + \frac{\partial^2}{\partial x_i \partial x_j} \int_V \left[ T_{ij}(y, \tau) \right]_{t=\tau^*} \frac{dv(y)}{4\pi|x-y|}$$

where $\tau^*$ is the retarded time that corrects for the time delay for the acoustic wave to propagate from source to observer, $x$ is the observer position, and $y$ is the source location. Powell (1960) extended Curle’s analysis for a rigid, planar, infinite boundary case and found that using an image method the dipole source field on the surface is cancelled leaving only the quadrupole term.

Skyurdzyk & Haddle (1960) present some of the earliest experimental work on roughness noise. They studied the noise from a rotating cylinder with different surface roughness applied submerged in the Garfield Thomas Water Tunnel at Pennsylvania State University. Skyurdzyk & Haddle (1960) suggest that flow noise is directly related to the surface drag, and therefore, hydrodynamically smooth roughness contained within the laminar sublayer will not produce increased far field noise. They argue that the noise is created by the increased turbulence generated by the roughness. Therefore, the onset of roughness noise generation can be determined by a critical velocity below which roughness noise will not be produced. They studied four different rough surfaces including 180grit and 60grit sandpaper roughness coating the outside of the spinning cylinder and measured the radiated noise using hydrophones mounted on the inside of the tank. They found that the boundary layer noise was enhanced by the presence of roughness confirming the existence of roughness noise and that the total noise levels increased with velocity at a slope steeper than that produced by the smooth wall flow. Although, for the different rough walls, they observed that the slope of the velocity dependence changed and did not indicate a specific source mechanism typical of a dipole or quadrupole source. They also found a dependency on the density of the roughness.

Chanaud (1969) measured the radiated far field noise of a spinning disk with circumferential rings of roughness placed around the center of the disk on the top and bottom and with roughness placed on the ¼” wide edge of the disk. His experiment was conducted in the Herrick Laboratories’ anechoic room. The roughness had a maximum height of 0.032” and measurements were made at disk tip speeds of 158ft/s to 315ft/s. The disk had a diameter of 24”. Chanaud concluded that the directivity of the roughness noise source was indeed a dipole, but his measurements were contaminated with edge noise due to flow over the periphery of the disk. There was a clear increase of noise due to the surface roughness above 3150Hz. He also concludes through time-delay correlation measurements that the increase in noise is produced at the location of the roughness elements.
Cole (1980) presents results from one of the first roughness noise experiments conducted in a conventional turbulent boundary layer flow. His research was performed at the David W. Taylor Naval Ship Research and Development Center’s Anechoic Flow Facility using a flat plate covered in 80 and 40 grit sandpaper roughness. The far field noise of the roughness was measured using a parabolic-reflector microphone system. Both dipole and quadrupole scalings were examined to normalize the data, but the results were inconclusive both collapsing the data within a few decibels. Cole suggests that the source may be an admixture of the two types, and therefore, both scaling laws apply to some degree. He also notes that the presented data may be influenced by the background noise of the tunnel contaminating the far field noise recorded by the reflector.

The first conclusive characterization of the roughness noise source came from Hersh (1983). He measured the sound radiated from roughened pipe flow with exit speeds up to 120m/s using 36 to 180 grit sandpaper surfaces. He found the roughness noise scaled as a dipole source to the sixth power of velocity. Care was taken to ensure that the measured noise was not created by an enhanced lip dipole source from the pipe exit due to the presence of the roughness. Examination of the lip dipole showed its power levels well below that of the jet noise and roughness noise. Hersh also observed that the roughness noise was dependent on the size of the roughness. Larger roughness produced greater noise, but also shifted the maximum noise levels to lower frequencies. Hersh attempted to scale the data using two methods: first using the friction velocity and the equivalent sand roughness height and second with the friction velocity and momentum thickness. A better collapse was produced using the equivalent sand roughness height as the length scale.

Howe (1984) developed a scattering theory of roughness noise generation which details the process of converting the near field pressure fluctuations to radiating acoustic waves. He states that the assumption of surface dipoles cancelling each other as described by Powell (1960) is invalid with the introduction of surface irregularities. Therefore, roughness noise will behave as a distribution of dipole sources over the rough surface. Howe theoretically models a rough surface as an arrangement of hemispherical bosses on a planar wall and compares his far field results with the noise recorded by Hersh (1983). Absolute comparisons were not possible because of unknowns in Hersh’s experiment, but the data show similar trends. Howe agreed with the sixth power of velocity scaling indicating a dipole source and the spectral shapes of his far field spectra were also similar to that of Hersh.

The analysis of Howe (1984) assumed inviscid flow over the rough surface and was therefore only applicable for large roughness Reynolds numbers, \( hu_{e} / v \). Therefore, in 1986, Howe continued his analysis of the rough wall flow and introduced viscous effects into the analysis specifically including the no-slip condition on the wall. This analysis particularly regarded roughness completely contained within the viscous sublayer having \( hu_{e} / v < 5 \). It was found that the inclusion of viscous effects raised the predicted far field by up to 2-3dB. Howe (1988) uses Chase’s (1987) wavenumber wall pressure spectrum to make roughness noise predictions, but empirically modifies his formulation to include the effects of the interstitial flows on the rough wall. He adds the effects produced by diffraction of the enhanced Reynolds stresses due to the roughness with Chase’s spectrum for a smooth wall. Again, Howe uses Hersh’s data for spectral shape comparison and to adjust the constants in his empirical formula but cannot evaluate the accuracy of the absolute predicted values. The shape of the produced spectrum was very similar to experimental data. Overall, the greatest contribution of Howe’s work was to relate the radiated far field noise to the wavenumber wall pressure spectrum and the rough surfaces characteristics including roughness density and size.
The scattering mechanism was theoretically investigated by Howe, but cannot be applied to very large roughness as it does not account for self-generated drag induced dipoles from local vortex shedding. More recently, Glegg et al. (2007) investigated the scattering and drag dipole mechanisms for roughness of varying size to determine their relative strengths. For roughness that does not extend beyond the log layer, they find that the far field is dominated by the scattering of the wall pressure fluctuations and effects of vortex shedding can be ignored. Also, the two source mechanisms are suggested to scale differently. Glegg et al. (2007) suggest scaling the scattered pressure fluctuations using a frequency scaling based on the roughness lengthscale. The suggested frequency scaling for vortex shedding is based on the roughness height. Therefore, the two source mechanisms should be easily separated, and for most roughness noise cases, the dominant source will be the scattering of turbulent pressure fluctuations.

Citing the findings of Howe (1988) and noting the importance of the wavenumber wall pressure spectrum on the radiated acoustics, Farabee & Geib (1991) experimentally analyzed the wavenumber wall pressure spectrum of rough wall flows. Their study was conducted in the Anechoic Flow Facility at David Taylor Research Center. They applied three types of surface roughness to the wall of the tunnel, two in the fully rough regime and one that was transitionally rough. The roughness fetches were 2m long in the streamwise direction and a six sensor wall pressure microphone array was positioned immediately downstream of the rough surface. Rough wall boundary layers recover slowly back to smooth wall conditions according to the study of Antonia & Luxton (1972). Therefore, although this array was placed in a smooth wall condition, effects of the wall roughness extended over the microphone array. Their measurements show that wall roughness increases the intensity of the pressure fluctuations on the convective ridge of the wavenumber wall pressure spectrum and tend to shift it to lower frequencies. This indicates that the roughness creates larger turbulence Reynolds stresses and slows the convected eddies traveling over the surface. They also recorded increases in the sonic component of the wavenumber wall pressure spectrum and conclude that the increased levels in this region are due to scattering of the convective region by the rough surface. They note that their findings in the subconvective and supersonic regions may be corrupted by convective pressures and facility background noise, respectively, but that their findings in the convective and supersonic regions qualitatively agree with Howe (1988).

Liu & Dowling (2007) investigated the diffraction theory of Howe (1988) using several smooth wall wavenumber pressure spectra models that were adapted to the higher friction velocities and boundary layer thicknesses of rough wall flows. They compared predictions of Hersh’s (1983) experiment using these modified models with Howe’s (1988) estimation and found that the Howe model fit the data best, but that all of the models produced similar results. This validated Howe’s theory of a scattering mechanism. Liu & Dowling (2007) applied Howe’s theory to open-jet wind tunnel measurements of roughness noise from hemispherical roughness. Again, they found that the wavenumber pressure spectrum model used in the theoretical predictions made an insignificant difference and that the theoretical predictions were better at low frequency than higher frequencies. They attribute this to the assumption in the scattering theory that the boundary layer thickness to acoustic wavelength ratio is much less than 1. The estimations begin to deviate from the measurements at the same frequencies this assumption becomes invalid.

Anderson et al. (2007) completed wall pressure spectrum measurements using methods similar to the measurements of Farabee & Geib (1991), but also measured the radiated far field noise using a 63-sensor far field microphone array. Measurements of the wavenumber wall pressure spectrum agree with the results of Farabee & Geib (1991) showing increases in the convective and sonic regions with increasing roughness height. The far field microphone array produced measurements of the variation in
source strength along the length of the examined rough surfaces. They found that the front of the roughness fetch produced the greatest noise with the strength diminishing in the streamwise direction. They applied several recommended scalings to the recorded far field spectra from the rough surfaces. These included Cole’s (1980) inner and outer variable dipole and quadrupole suggested normalizations. None of the suggested scalings worked to collapse the spectral data completely and neither the dipole nor quadrupole velocity scaling was confirmed. Their data appear to suggest a combination of the two source types.

Liu et al. (2008) conducted another roughness noise experiment using phased microphone array technology. They studied the noise from two rough surfaces made of discrete hemispherical elements having different roughness heights and densities using both a high and low frequency microphone array. They analyzed their beamformed data using standard techniques that assume monopole sources. They then compared their measured source maps to a beamformed source map of a theoretical field of dipole sources with a source strength distribution as calculated using the method of Liu & Dowling (2007). The simulated and measured roughness noise produced similar beamformed solutions confirming the dipole nature of the source and the approximate source strength solution of Liu & Dowling (2007). The highest frequencies of their measurement were overestimated by 3dB and the streamwise decline in source strengths was underpredicted.

Coinciding with the theoretical and experimental work on roughness noise, Yang & Wang (2009, 2010, 2011) have been developing computational predictions of roughness noise from cuboidal, hemispherical, and cylindrical fetches of discrete elements using high resolution LES. The flow field is solved using the LES, and the acoustics are calculated with the Curle (1955)-Powell (1960) integral solution to Lighthill’s (1952) acoustic analogy. In Yang & Wang (2009), specific attention is brought to the noise produced by an individual hemispherical element and the results of interaction between two hemispherical elements. The studied roughness elements have roughness Reynolds numbers, \( \frac{h u_r}{\nu} \), equal to 95. They found that local vortex shedding is not a significant contributor to the total noise produced by the shedding element but the trailing element’s noise is enhanced by the vortices impinging on its surface. They also describe the presence of streamwise and spanwise aligned dipoles emanating from each roughness element.

Yang & Wang (2010, 2011) concern the noise from 40 element fetches of roughness arranged in 4 spanwise by 10 streamwise grid patterns with \( \frac{h u_r}{\nu} = 168 \). They compare the differences in sound produced by hemispherical, cuboidal, and cylindrical roughness and find the cuboidal roughness produces the greatest noise. The source strength variation throughout each fetch also varies for the differing roughness geometries. The lead row is the weakest producer of noise for both the cylindrical and hemispherical surfaces while it is the strongest for the cuboidal. One of the advantages of a computational simulation is that quantitative results are available for all locations in the domain. Therefore, they were able to calculate local RMS surface pressure and surface pressure spectra around and on the surface of roughness elements and correlate the results with the streamwise and spanwise dipole fluctuations. They found that the roughness noise sources are confined within the very near region of the roughness elements so that there is little coherence of the noise produced by adjacent elements. The majority of sound produced by the individual elements was created by the impinging vortices and the edges on the lead side of the cuboidal and cylindrical elements. Very little sound was produced by the downstream portions of the roughness elements.
1.3 Recent studies at Virginia Tech

An anechoic wall-jet facility was constructed at Virginia Tech in 2005 specifically for the study of roughness noise. The design of this facility is discussed in detail by Grissom (2007) along with presentations of the noise from stochastic and wavy wall surfaces. The aerodynamic characteristics of smooth and rough wall flows in the facility were examined in Smith (2008). This wall-jet tunnel was designed to negotiate around the problems encountered by previous studies of roughness noise, mainly the contaminating background noise. Grissom (2007) recorded noise from hydrodynamically smooth surfaces confirming the existence of a scattering mechanism as suggested by Howe (1988) for small roughness. Also, he noticed that the shape of the radiated far field noise varied between measurements from the wavy wall surfaces and the stochastic surfaces. Therefore, he concluded that inner or outer variable scalings would be unable to collapse the data from these differing roughness geometries. Grissom also found that the wavy wall surface exhibited specific directionality. It was tested with ribs aligned at several angles to the flow direction thereby rotating the wavenumber vector of the surface. The surface generated the most far field noise with the ribs aligned perpendicular to the flow producing a signal-to-noise ratio of approximately 25dB. When the ribs were aligned parallel to the flow direction very little far field noise was recorded. None of the previously proposed simple scaling models (Cole, 1980, Howe, 1988, Farabee & Geib, 1991, Glegg et al., 2007) would account for such a directional effect because they do not contain a physical explanation of the roughness noise generation mechanism. Grissom’s (2007) study suggests that future studies need to focus on capturing the interaction and behavior of the wavenumber wall pressure spectrum with specific roughness geometries.

The theory proposed in Glegg & Devenport (2009), which was the result of experimental and theoretical collaboration between researchers at Virginia Tech and Florida Atlantic University, is the first to provide a clear relationship between non-specific roughness geometry, the wavenumber wall pressure spectrum and the radiated far field noise. Therefore, unlike the theory of Howe (1988), which was derived for a distribution of hemispherical surface roughness, their theory could be applied to any surface geometry to estimate the far field noise. Their “Unified Theory of Roughness Noise” is shown in Equation 1-3.

\[
\rho'(x, \omega) c_\infty^2 \approx \frac{-ik_\omega e^{ik_\omega|x|}}{2\pi|x|} \int p_s(\kappa_1, \kappa_3, \omega) \left\{ \frac{x_1 \xi^{(1)}}{|x|} + \frac{x_2 \xi^{(3)}}{|x|} - i k_\omega h \xi^{(2)} \right\} (2\pi)^2 d\kappa_1 d\kappa_3
\]

where \(k_\omega\) is the acoustic wavenumber, \(p_s\) is the wavenumber surface pressure spectrum, \(h\) is the roughness height, \(\xi^{(1)}\) and \(\xi^{(3)}\) are the wavenumber transforms of the surface slope in the longitudinal and lateral directions, and \(\xi^{(2)}\) is the wavenumber transform of the surface. They show if a distribution of hemispherical bosses is assumed, that the result yields the same solution as presented in Howe (1998). They also develop their theory for two other example surface geometries: a wavy wall and a piecewise continuous surface. Their analysis of the wavy wall surface shows that a sinusoidal surface will radiate the near field pressure spectrum corresponding to the wavenumber vector of this surface thus providing an indirect measure of the wavenumber wall pressure spectrum. This explains the directional phenomenon observed in Grissom’s (2007) measurements of a wavy wall surface. The convective ridge of the theoretical wavenumber wall pressure spectrum peaks at spanwise wavenumbers of zero. Therefore, the wavy wall surface will produce a maximum sound when the ribs are aligned perpendicular to the flow creating a wavenumber vector without a spanwise component.
The noise from wavy wall surfaces was investigated further in Devenport et al. (2010). In this study, they used Glegg & Devenport’s (2009) theory to make measurements of the wavenumber wall pressure spectrum of the wall-jet flow and compared their results to the model of Chase (1980, 1987). By rotating the ribbed surface, which alters the wavenumber vector of the surface, they were able to make measurements corresponding to an arc passing through the wavenumber wall pressure spectrum as shown in Figure 1-1. Their measurements showed that the convection velocity of the wall-jet (~0.41 \( U_e \)) was considerably slower than that for conventional turbulent boundary layers (~0.60 \( U_e \)) and that the peak of the convective ridge may also be broader.

![Diagram of measurement of wavenumber wall pressure spectrum on an arc of wavenumber vectors](image)

**Figure 1-1. Diagram of measurement of wavenumber wall pressure spectrum on an arc of wavenumber vectors**

The theoretical piecewise surface discussed in Glegg & Devenport (2009) was analyzed as a representation of stochastic surfaces that have vertical sides. The results show that this configuration is wavenumber white assuming \( \omega h/U_c < 1 \) so that the specific geometry of the surface is not needed. The far field sound can be approximated with only general information about the observer location and wall pressure spectrum. The resulting predicted far field spectrum when normalized on the wall pressure spectrum should produce a slope of \( \omega^2 \). The theory of Glegg & Devenport (2009) for surfaces with vertical sides shows that the far field noise would be related to a homogeneous wall pressure spectrum as shown in Equation 1-4.

\[
\Phi_{pp}(x, \omega) \approx C \left( \frac{x_1}{|x|} \right)^2 \frac{(k_o h)^2 \Sigma \Phi_{pp}(\omega)}{|x|^2}
\]

Eq. 1-4

where \( C = \frac{a^2}{4\pi^2} \)

\( \Phi_{pp}(x, \omega) \) is the radiated far field, \( \Sigma \) is the roughness area, and \( \Phi_{pp}(\omega) \) is the single point wall pressure spectrum. Therefore, if the far field noise is normalized on the single point wall pressure spectrum, the resulting curve should only be a function of \( (k_o h)^2 \). This relationship was partially confirmed in Alexander (2009) and Devenport et al. (2011). These studies show the measured noise from stochastic
roughness ranging from hydrodynamically smooth to fully rough conditions using 20grit to 180grit sandpaper. The results as reported in Alexander (2009) are shown in Figure 1-2. The studies find the \((k_o h)^2\) relation as expected at lower frequencies, below 5kHz. Above this, the data fan out deviating from the \(\omega^2\) slope in order of roughness height. The smallest roughness retains the \(\omega^2\) slope to higher frequencies than the larger roughness sizes. They term this frequency where the expected result appears to differ from the observed behavior the “break frequency”. They attribute this “break frequency” to a dependency on the roughness grain sizes because the relationship shown in Equation 1-4 is only valid for homogeneous wall pressure fields. Therefore, the hypothesized relation is void when the turbulence scales become smaller than the individual roughness grains.

![Figure 1-2. Normalization of noise from stochastic roughness presented in Alexander (2009)](image)

The roughness noise problem is speculated to change when the roughness elements are large compared to the boundary layer thickness. The wall pressure field over such a surface would be highly inhomogeneous. The noise from rough surfaces with large roughness height to boundary layer thickness ratios was studied in Rasnick (2010). He calculated the per-element average non-dimensional drag dipole spectra for turbulent wall-jet flow over multi-element gravel, cuboidal, and hemispherical roughness fetches using the theory of Glegg et al. (2007). These elements were as large as 55% of the boundary layer thickness. His experiment involved only a single microphone placed in the far field to record the radiated noise, and therefore, he was unable to separate the contributions from individual elements.

1.4 The Wavenumber Wall Pressure Spectrum

The theoretical descriptions of roughness noise as given by Howe (1988) and Glegg & Devenport (2009) relate the radiated far field sound to the wavenumber wall pressure spectrum. Therefore, it is important to understand the development and form of a few of the current and most used models. The wavenumber wall pressure spectrum decomposes the wall pressure frequency spectrum into the contributing lengthscales at each frequency so that the intensity of pressure fluctuations from various size
flow structures can be observed. One of the first models of this spectrum was presented by Corcos in 1964 using curve-fits to measured spatial correlations in turbulent boundary layers. The Corcos model as given by Howe (1998) is shown in Equation 1-5.

\[
\frac{\Phi_{pp}(\kappa, \omega)}{\Phi_{pp}(\omega)} = \frac{l_1}{\pi \left[ 1 + l_1^2 (\kappa_1 - \omega/U_c)^2 \right] \pi \left[ 1 + l_3^2 \kappa_3^2 \right]}
\]

Eq. 1-5

Where \( l_1 \approx 9 U_c/\omega \) and \( l_3 \approx 1.4 U_c/\omega \)

\( \Phi_{pp}(\kappa, \omega) \) is the wavenumber wall pressure spectrum, \( \Phi_{pp}(\omega) \) is the wall pressure frequency spectrum, \( \kappa_1 \) and \( \kappa_3 \) are the longitudinal and lateral wavenumbers, and \( U_c \) is the convection velocity. This model provides good estimations of the convective ridge, the most intense region of turbulent pressure scales, but overpredicts wavenumbers below this peak in the subconvective region.

There were a few attempts to improve the modeled spectrum of Corcos. Efimtsov (1982), using the same empirical method as Corcos (1964), incorporated the effects of boundary layer thickness on the spatial correlations. His result was similar to the model shown in Equation 1-5, but replaces the definitions of the correlation lengths \( l_1 \) and \( l_3 \) with Equation 1-6 as reported in Graham (1997).

\[
l_1 = \delta \left[ \left( \frac{a_1 (\omega \delta)}{u_\tau U_c} \right)^2 + \frac{a_2^2}{(\omega \delta u_\tau)^2 + (a_2/a_3)^2} \right]^{-1/2}
\]

Eq. 1-6

\[
l_3 = \delta \left[ \left( \frac{a_4 (\omega \delta)}{u_\tau U_c} \right)^2 + \frac{a_5^2}{(\omega \delta u_\tau)^2 + (a_5/a_6)^2} \right]^{-1/2}
\]

\[
l_3 = \delta \left[ \left( \frac{a_4 (\omega \delta)}{u_\tau U_c} \right)^2 + a_7^2 \right]^{-1/2}
\]

for \( M_\infty < 0.75 \)

for \( M_\infty > 0.9 \)

where \( \delta \) is the boundary layer thickness, \( u_\tau \) is the friction velocity, and \( a_7 \) through \( a_7 \) are constants with values 0.1, 72.8, 1.54, 0.77, 548, 13.5, and 5.66, respectively. This model was an improvement, but still overpredicted the low wavenumber region.

Smol’yakov & Tkachenko (1991) give a model of the wavenumber wall pressure spectrum empirically fitting measured spatial correlations similar to Corcos (1964) and Efimtsov (1982). In Smol’yakov & Tkachenko, rather than accounting for the longitudinal and lateral correlation lengths separately, as done in the previous models, they calculated the Fourier transform using the combined root sum of squares correlation length. Their suggested model is given in Equation 1-7. This model has a narrower convective peak than previous versions and also diminishes the levels at low wavenumbers while retaining the supposed wavenumber white shape observed in experiment in this region.
\[
\frac{\Phi_{pp}(\kappa, \omega)}{\Phi_{pp}(\omega)} = 0.02467A(\omega)h_{ST}(\omega)[F(\kappa, \omega) - \Delta F(\kappa, \omega)]
\]

\[
A(\omega) = 0.124 \left(\frac{u_c}{\omega}\right)^2 \left(1 - \frac{u_c}{4\omega \delta_x} + \left(\frac{u_c}{4\omega \delta_x}\right)^2 \right)^{1/2}
\]

\[
h_{ST}(\omega) = \left[1 - \frac{m_1 A}{6.515\omega}\right]^{-1}
\]

\[
m_1 = \frac{1 + A^2}{1.025 + A^2} \quad G = 1 + A^2 - 1.005m_1
\]

\[
F(\kappa, \omega) = \left[A^2 + \left(1 - \frac{\kappa_1 U_c}{\omega}\right)^2 + \left(\frac{\kappa_3 U_c}{6.45\omega}\right)^2\right]^{-3/2}
\]

\[
\Delta F(\kappa, \omega) = 0.995 \left[1 + A^2 + \frac{1.005}{m_1}\left\{\left(m_1 - \frac{\kappa_1 U_c}{\omega}\right)^2 + \left(\frac{\kappa_3 U_c}{\omega}\right)^2 - m_1^2\right\}\right]^{-3/2}
\]

Ffowcs Williams (1982) derived a model from Lighthill's acoustic analogy with constants left to be determined from experiment. A simplified version of their formulation was given by Hwang & Geib (1984) assuming incompressible flow, but this model is divergent in high wavenumber. Its high wavenumber region does not satisfy the requirement that the integral of the normalized wavenumber wall pressure spectrum equal one as given in Equation 1-8. Therefore, this model is not considered further but is worthy of note since it was a theoretical attempt to model the wavenumber pressure spectrum and not simply an empirical fit of experimental data.

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\Phi_{pp}(\kappa, \omega)}{\Phi_{pp}(\omega)} d\kappa_3 d\kappa_1 = 1 \quad \text{Eq. 1-8}
\]

Chase's (1980, 1987) models, similar to Ffowcs Williams, are built on theoretical and experimental analysis and therefore include a number of adjustable constants to fit observations. His recommended model, assuming incompressibility, is given in Equation 1-9 as reported in Chase (1987). This is an update of his 1980 version to better model the subconvective wavenumber region. His estimated spectrum is lower in this region than the greatly overpredicted values of Corcos (1964), but Chase (1987) does not produce a wavenumber white shape. The convective regions of both the 1980 and 1987 models are nearly the same. Chase (1987) attempts to address the compressibility issues relating to the sonic and supersonic wavenumbers, but unknown parameters in his model, as encountered by the evaluation of Ffowcs Williams (1982), limit reliable application of his incompressible model in the acoustic and subconvective domains.

\[
\frac{\Phi_{pp}(\kappa, \omega)}{\Phi_{pp}(\omega)} = \frac{\rho^2 u_t^3}{\Phi_{pp}(\omega)[k_+^2 + (b \delta)^2]^{5/2}} \left[C_T|\kappa|^2 \left[k_+^2 + (b \delta)^2\right]^{-1} + C_M k_+^2 \right]
\]

Where \( h_c = 3 \quad C_T h_c = 0.014 \quad C_M h_c = 0.466 \quad b = 0.75 \quad \text{Eq. 1-9}\)

And \( k_+^2 = \left(\frac{\omega u_c - \kappa_1}{h_c^2 u_t^2 + k^2}\right)^2 \)
The single point spectrum of Chase was given in two parts in 1980 and 1987. The resultant formulation is given as:

\[
\Phi_{pp}(\omega) = \Phi_T(\omega) + \Phi_M(\omega)
\]

Where \(\Phi_T(\omega) = \pi C_T h_c \rho^2 u_T^4 \omega^{-1} (1 + \alpha^{-2})\) and \(\Phi_M(\omega) = r_M a_+ \rho^2 u_T^4 \omega^{-3} (1 + \mu^2 \alpha^2)\)

\[
\alpha = [1 + (U_c/\omega b \delta)^2]^{-1/2} \\
r_M = 1 - C_T h_c / (C_T h_c + C_M h_c)
\]

\[
a_+ = (2\pi h_c/3)(C_T + C_M) \\
\mu = h_c u_T / U_c
\]

Eq. 1-10

The previously discussed models, excluding Ffowcs Williams (1982), are compared in Figure 1-3. The models of Corcos (1964) and Efimtsov (1982) are similarly shaped with the values of Efimtsov falling slightly below that of Corcos. Smol'yakov & Tkachenko (1991) and Chase (1987) have much narrower peaks at these conditions with much lower subconvective and viscous regions. The wavenumber white subconvective regions, as observed in experiment, are apparent in the Corcos, Efimtsov, and the Smol’yakov & Tkachenko spectra where as the Chase spectrum continues to decline with decreasing wavenumber. There is little conclusive evidence as to which specific model of the wavenumber wall pressure spectrum is the best as reported in a review of each model by Graham (1997). Therefore, several models should be examined in reference to a particular issue. Both the Chase and Corcos spectra are analyzed further in this report through the theory of Glegg & Devenport (2009) that relates the wavenumber wall pressure spectrum to roughness noise.

Figure 1-3. Comparison of wavenumber wall pressure spectrum models with inputs \(U_c=30\text{m/s}\) and \(\omega=10000\ \text{rad/m}\)
1.5 Microphone Array Data Processing

Phased microphone arrays have been used in previous roughness noise studies (Anderson et al., 2007, Liu et al., 2008) to determine source directivity and strength distributions. This study also details microphone array measurements of roughness noise, and the data are analyzed using beamforming and least-squares analysis techniques. Therefore, a brief background and discussion of processing methods for phased microphone arrays is needed to understand current applications and practices.

An early study of phased microphone arrays was conducted by Billingsley & Kinns (1976) to locate noise sources on jet engines. They used a linear array of fourteen ¼” Bruel & Kjaer microphones. By weighting the components of the measured cross-spectral matrix to account for various sensor to source distances and summing the results, they were able to focus their array on different locations over a defined region. This type of analysis is termed delay and sum beamforming. A similar form of their analysis which does not assume uncorrelated sources is given by Tester & Glegg (2008) shown in Equation 1-11.

\[ B = G_{inv} C G_{inv}^H = W Q W^H \]  
Eq. 1-11

where \( B \) is the beamformed solution of windowed source strengths, \( G_{inv} \) is the inverse monopole steering matrix, \( C \) is the cross-spectral matrix, \( W \) is the windowing function, and \( Q \) is the cross-spectral density of the source strength. The Billingsley & Kinns (1976) analysis assumes monopole sources which have arbitrary directionality. They found that the resolution of the array in accurately pinpointing the source locations was affected by its aperture, which is the angle drawn from the array’s extremities to the source, and the frequency. The resolution becomes finer with increasing microphone spacing and frequency.

This beamforming method works well but the steering vectors generally assume monopole sources. This is typically the case encountered in aeroacoustic applications. Many noise sources are dipoles which have a specific directionality. Jordan et al. (2002) investigated the effects of monopole beamforming techniques on measurements of dipole sources. They found that measurement of a dipole source using a linear array perpendicular to its axis produced a void in the beamformed map at the location of the source when using the conventional monopole technique. This is due to the lobes of the dipole, which are 180° out of phase, cancelling each other when the steering vector is focused at its origin. They correct for this by observing the focal location where the phase of the microphones passes through 180°. This is the location of the dipole source. The phase of the microphone signals on either side of the dipole source can then be adjusted accordingly so that the summation is constructive instead of the out of phase signals cancelling each other. Jordan et al. (2002) find the beamforming method incorporating the modified cross-spectral matrix correctly identifies the source location and suppresses other sound sources. The drawback of their experiment is that it involved only a single dipole with known orientation. The problem becomes much more complicated if multiple dipoles are included with various orientations.

Beamforming methods often work well to identify source locations, but determining source strengths can be difficult, especially if multiple sources contained within a single lobe of a source map need to be independently analyzed. If the source location and type is known, an accurate source strength
solution can be determined using inverse methods like that presented in Nelson & Yoon (2000). Nelson & Yoon (2000) use the inverse of a transformation matrix altering the source strength to the measured noise, to solve for the source strength using a least-squares method. The first step to their method is modeling
the noise source into a discrete distribution of sources so that the frequency response function of the
source measurement can be determined as shown in Equation 1-12.

$$\bar{P} = Gq$$  \hspace{1cm} Eq. 1-12

where $\bar{P}$ is a complex vector of the measured acoustic response, $q$ is a vector of the models source
strengths, and $G$ is the transformation matrix dependent on the source model. If this representation is
extended to solve for the cross-spectral density of source strengths, $Q$, using the measured cross-spectral
matrix of acoustic pressures, $G_{pp}$, Equation 1-12 can be rearranged into Equation 1-13.

$$Q = G^+G_{pp}G^{+H}$$  \hspace{1cm} Eq. 1-13

where $H$ is the Hermitian transpose and the plus sign indicates a pseudo-inverse. The pseudo-inverse has
to be used when the source to sensor ratio is not one creating a matrix $G$ with a differing number of rows
and columns. In this case, the exact inverse cannot be determined. The solution to this matrix equation is
then solved using a least-squares analysis to minimize the error between the determined matrix of source
strengths and the measured cross-spectra.

Nelson & Yoon (2000) find that the accuracy of their solution using this least-squares method is
dependent upon the condition number, $\sigma$, of the matrix $G$ defined by:

$$\sigma(G) = \|G\| \|G^+\|$$  \hspace{1cm} Eq. 1-14

A large condition number increases the sensitivity of the solution to perturbations in the measured cross-
spectral matrix, $G_{pp}$, and the transformation matrix, $G$. They find that there are several geometrical factors
that influence the condition number. The condition number is negatively impacted by low source spacing
to wavelength ratios, increasing distance between source and sensor locations, differing source and sensor
geometrical configurations, and asymmetry of the source and sensor positions.

Yoon & Nelson (2000) discuss two different methods of increasing the accuracy of the least-
squares solutions affected by ill-conditioned transformation matrices: Tikhonov regularization and
singular value discarding. Tikhonov regularization is used to improve the conditioning of the calculation
by weighting various parts of the matrix $G$ to reduce its norm. Singular value discarding involves the
removal of the smallest values in the matrix $G$ that are responsible for the poor conditioning of the
inverted matrix. These are the values corresponding to the ratio of largest to smallest singular values that
determine the condition number. They compare their least-squares source strength solutions to
experimental measurements of a randomly vibrating plate and vary the condition number of the
measurements by changing the assumed source model and microphone configuration. They find their
method works well when the condition number is made sufficiently small either by experimental set-up or
the discussed matrix conditioning techniques.
1.6 Objectives

A reliable method to predict roughness noise is needed. The extent to which the scattering theory of Glegg & Devenport (2009) applies, especially for surfaces with large roughness Reynolds numbers or roughness height to boundary layer thickness ratios, has not been examined. Previous studies show that their theory works well to collapse the noise produced by stochastic roughness up to a “break frequency”, but the origin of this “break frequency” is not well understood. Also, for surfaces with very large roughness elements which have local vortex shedding, the relationship of the radiated noise with the wavenumber wall pressure spectrum may change. These surfaces produce inhomogeneous wall pressure fields. Therefore, the radiated noise may vary locally over the surface. This study attempts to address these issues by focusing on the noise generated by fetches of discrete cuboidal and hemispherical roughness. The noise from these surfaces is analyzed using far field and surface pressure measurements and is compared to results from current roughness noise theory. A linear microphone array was designed to measure the generated roughness noise produced by the fetches and a novel least-squares source strength analysis method was developed to decompose the contributions from individual sources to the radiated far field. The explicit objectives of this study are:

- To analyze the relationship of roughness noise with boundary layer, wall pressure, and rough wall characteristics by measuring the far field noise and wall pressure fluctuations in fetches of discrete cuboidal and hemispherical roughness elements.

- To assess the theory of Glegg & Devenport (2009) by estimating the produced far field noise for fetches of discrete cuboidal and hemispherical surface roughness and comparing with experimental measurements.

- To determine the effect of multiple discrete roughness elements and their relative locations on the produced far field noise from each source location.

- To support or refute the discrete element roughness noise model composed of a pair of uncorrelated dipole sources aligned streamwise and spanwise to the flow direction.

- To provide the information needed to validate computational predictions of roughness noise.
Chapter 2 Experiment Description, Instrumentation, and Analysis Techniques

2.1 Anechoic Wall-Jet Facility

All data were taken in the Virginia Tech Anechoic Wall-Jet Facility shown in Figure 2-1. This tunnel was purpose-built in 2005 to study roughness noise. A wall-jet style tunnel was chosen because far field microphones could be easily placed outside of the flow to record radiated roughness noise. The tunnels aerodynamic and acoustic characteristics have been analyzed in several papers including Grissom (2007), Smith (2008), Alexander (2009), and Devenport et al. (2011).

Figure 2-1. Schematic of Virginia Tech Anechoic Wall-Jet Facility (dimensions in mm)
The tunnel structure is made of MDF reinforced with 2” steel box beam. A Cincinnati Fan model HP-8D20 centrifugal blower powers the tunnel and is separated from the settling chamber by a flexible rubber hose. A SSA-8 discharge silencer is attached to the blower outlet to dampen acoustic vibrations through the hose. The settling chamber has several acoustic baffles that block direct noise radiation from the hose inlet through to the nozzle. The initial two thirds of the settling chamber are acoustically treated leading up to the nozzle section. The acoustic baffles are lined with fiberglass blankets to absorb acoustic energy.

A detailed view of the nozzle section can be seen in Figure 2-2. The bottom half of the nozzle is stationary and was designed using the relation in Fang et al. (2001) shown in Equation 2-1.

\[ y = (h_1 - h_2) \left[ 1 - \frac{1}{X_m^2} \left( \frac{x}{L} \right)^3 \right] + h_2 \quad x < X_m \]

\[ y = \frac{(h_1 - h_2)}{(1 - X_m)^2} \left( 1 - \frac{x}{L} \right)^3 + h_2 \quad x > X_m \]

Eq. 2-1

where \( h_1 \) is the nozzle exit height, \( h_2 \) is the initial height from the reference plane, \( X_m \) is the matched point, \( x \) is the distance from the nozzle, and \( L \) is the nozzle length. The values used for this contraction were \( h_1 = 0.681 \text{m} \), \( h_2 = 0 \), \( X_m = 0.254 \text{m} \), and \( L = 0.610 \text{m} \). The top half of the nozzle can be adjusted to vary the outlet opening height and therefore has no ramp leading to the nozzle exit. The nozzle height was kept at a constant 12.7mm for all reported data and was set by lowering the upper half of the nozzle down onto a set of gauge blocks. The top half of the nozzle was milled from two pieces of PVC joined at the center line of the plate. Its shape is a fusion of a quarter ellipse having a 3:1 ratio inside of the settling chamber spliced with a 38.1mm radius circular profile exit. The nozzle is 1206mm wide in the spanwise direction.

![Figure 2-2. Detailed view of nozzle and shelf of acoustic enclosure (dimensions in mm)](image)

Flow is accelerated through the nozzle exit and is exhausted over a flat aluminum plate 3058mm long and 1600mm wide enclosed in a removable acoustically treated chamber. As the flow travels along the plate the edge velocity reduces. All edges are kept far away from any regions of significant flow to
avoid the production of edge noise. The flow travels off of the end of the plate which is rounded to promote a Coanda effect where it diffuses out through the bottom of the acoustic enclosure.

The anechoic chamber has 457mm wedges on the fore and aft walls. The sidewalls and ceiling are covered in 89mm egg crate foam with a total thickness of 203mm. The bottom of the chamber is open and is approximately 70mm off of the floor allowing the flow to exhaust into the lab atmosphere. The acoustic enclosure has a shelf over the nozzle exit to block direct noise radiation from the nozzle to microphones placed above it. The shelf is made of MDF and is covered in the egg crate foam and 25.4mm thick flat foam. The wall-jet’s maximum velocity occurs well below the bottom of the shelf which is 330mm above the plate.

Measurements presented in Grissom (2007) show that the operating background noise of the wall-jet is dominated by turbulence noise that scales with the eighth power of velocity. Further analysis of the background levels of the wall-jet are included in Section 2.5.1.3. Measurements in Grissom (2007) and Alexander (2009) show the tunnel is sufficiently quiet to measure roughness noise even from hydrodynamically smooth surfaces. A study of the acoustic response of the anechoic chamber in Alexander (2009) concluded that the chamber did not significantly influence far field measurements of a source emanating from the flat plate.

A schematic of a wall jet profile is shown in Figure 2-3. $U_m$ is the profile’s maximum velocity, $y_{1/2}$ is the height above the boundary layer where the velocity is half of its maximum value, $\delta$ is the boundary layer thickness, and $\delta_{90}$ is the height inside the boundary layer at which the velocity is 90% of its maximum value. Aerodynamic characteristics of the Virginia Tech Anechoic Wall-Jet can be approximated by relations similar to those found in Wygnanski et al. (1992) and Narasimha et al. (1973) shown in Equation 2-2.

$\frac{U_m}{U_o} = 4.97Re_j^{n+1}Re_x^n$

$\frac{y_{1/2}}{b} = 0.0335Re_j^{m-2}Re_x^m$  \hspace{1cm} Eq. 2-2

$\frac{\delta^*}{b} = 0.0156Re_j^{p-2}Re_x^p$

where $U_o$ is the nozzle exit velocity, $Re_j$ is the Reynolds number based on nozzle exit height, $Re_x$ is the Reynolds number based on the streamwise distance from the nozzle, $\delta^*$ is the boundary layer displacement height, and $b$ is the nozzle exit height. These relations have been customized for this tunnel by curve fitting data from measured profiles at various locations downstream of the nozzle exit to determine the values of $n$, $m$, and $p$ which are -0.512, 1.0451, and 0.888, respectively. These relations have been updated from Smith (2008) after taking aerodynamic measurements with the acoustic enclosure in place over the flat plate. The aerodynamic measurements in Smith (2008) were recorded with the acoustic enclosure removed. The updated results are reported in Devenport et al. (2011). The wall-jet profile is self-similar and therefore the boundary layer thickness and momentum thickness can be determined from the simple approximations given in Equation 2-3.
Spanwise measurements in Smith (2008) show that the center 810mm of the flow remains two-dimensional as far as 1867mm downstream of the nozzle exit. All rough surfaces were placed well within the limits of this two-dimensional region. Rough surfaces were typically placed 1257mm downstream of the nozzle exit where \( U_m \) varies from 7-22m/s and \( \delta \) varies from 21-16mm for nozzle exit velocities of 20-60m/s.

\[ \delta \approx 15.4\delta^* \]
\[ \theta \approx 0.74\delta^* \]

Figure 2-3. Wall-jet profile and definition of parameters

The coordinate system used to describe microphone and roughness locations is shown in Figure 2-4. The origin is located at the spanwise center of the nozzle exit plane. The y-axis is perpendicular from the plane of the plate, the x-axis is measured in the streamwise direction, and the z-axis completes the right-hand rule.

Figure 2-4. Coordinate system

2.2 Far Field Microphone Instrumentation

The following is a description of the single point far field measurement instrumentation. Discussion of the linear microphone array is in Section 2.4. Bruel & Kjaer’s \( \frac{1}{2}" \) free-field microphones type 4190 were used to record the far field noise produced by the rough surfaces. These microphones were used because of their high sensitivity and low noise floors and have a flat frequency response up to
20kHz. The microphones were used with a B&K Nexus 2690 A0S4 signal conditioning amplifier. The signals were then high-pass filtered at 250Hz using a Krohn-Hite model 3364 filter to improve the digitized resolution of the frequency range where roughness noise was perceptible. An Agilent E1432 16-bit digitizer was used with Agilent Vee Pro data acquisition software. Data were recorded at 51200Hz with a digital anti-aliasing filter. Frequency spectra are the result of 1000 averages of 2048 samples each. Microphone calibrations were completed frequently using a Bruel & Kjaer type 4228 pistonphone.

Microphones were placed in various positions around the acoustic chamber using microphone holders with small cross sectional areas that were attached to acoustically treated scaffolding described in detail in Alexander (2009) and Rasnick (2010). A typical arrangement is shown in Figure 2-5. The scaffolding is made from 80/20 beams and is securely attached to the legs that support the flat plate of the wall-jet. The scaffolding stands vertically on either side of the plate and then has two horizontal bars that extend over the plate. Microphones were attached to the 80/20 bar and therefore could be easily positioned anywhere in the chamber free from any vibrations that may be present in the structure of the acoustic enclosure. Since many microphone locations were used during the experiment, the exact location of the microphones will be presented with the corresponding data.

![Figure 2-5. Typical far-field microphone arrangement](image)

2.3 Wall Pressure Microphone Instrumentation

Sennheiser type KE-4-211-2 electret microphones were used to record wall pressure fluctuations. These microphones have a 1mm diameter sensing area and have a flat frequency response out to 10kHz within 1dBm. Their nominal factory sensitivity is 10mV/Pa. The microphones were used with 5V DC power supplies and 2.5 gain amplifiers designed in-house and described in Mish (2003). The microphone pinhole size was reduced to 1/4mm using 5.1mm diameter circular cut-outs of 0.26mm thick brass shim stock with 1/4mm holes drilled through the centers. These pieces were then super glued to the tops of the Sennheiser microphones. A modified Sennheiser microphone is shown in Figure 2-6. The
modified pinhole allowed for higher frequency measurements without contamination due to spatial averaging over the original larger diameter sensing area.

Figure 2-6. Sennheiser KE-4-211-2 microphone with a brass 1/4mm pinhole

The Sennheiser microphones were calibrated in the anechoic chamber of the wall-jet tunnel. The calibration set-up is shown in Figure 2-7. The flat plate was covered with 25.4mm thick melamine foam that dampened acoustic reflections from the large flat surface. A University Sound model ID60C8 speaker was positioned on the shelf of the chamber and was driven with a Carver power amplifier model TFM-6CB which was provided a white noise signal by an Agilent VXI data acquisition system. Microphones were placed on a short speaker stand with a small cross-sectional area approximately 2m across the chamber on the end of the plate. A 1/8” Bruei & Kjaer type 4138 microphone was first used to record the white noise signal for reference. The Bruel & Kjaer microphone was calibrated itself before each measurement using a B&K type 4228 pistonphone. The 1/8” microphone has a flat frequency response up to 140kHz and was used with a Nexus 2690-A-OS2 signal conditioning amplifier. The modified Sennheiser microphone was then placed in the same position as the reference microphone and the white noise recorded again. Data for both microphones was recorded at 51200Hz by an Agilent E1432 16-bit digitizer and are the result of 1000 averages of 2048 samples. The calibration is the result of dividing the measured voltage response of the Sennheiser by the measured pressure values recorded by the calibrated 1/8” B&K microphone. To reduce uncertainty, a frequency averaging scheme like that employed in Smith (2008) and Alexander (2009) was used to smooth the Sennheiser calibration. The calibration below 800Hz was assumed to be the average value between 500Hz and 800Hz. From 800-2000Hz, 1/24th octave bands were averaged centered on each frequency. Above 2kHz, the calibration was averaged on 1/12th octave bands.
Figure 2-7. Wall pressure microphone calibration arrangement

Figure 2-8 shows the effect of the smaller pinhole size on the response of the Sennheiser microphone. The smaller pinhole reduces the sensitivity of the microphone at higher frequencies but is a necessary adjustment for the wall pressure measurements since the pressure fluctuations travel across the face of the microphone and not perpendicular to it. There is also a resonant frequency range 2-5.5kHz where the microphone actually becomes more sensitive to pressure fluctuations just before the steep polynomial decline in sensitivity.

Figure 2-8. Sennheiser calibration with a 1mm and 1/4mm pinhole
Figure 2-9 shows a diagram of the wall pressure microphones installed in the plate of the wall-jet. During measurements the microphones were placed through nylon bushings that increased the outer diameter of the microphone in order to securely fit into 9.5mm holes drilled through the flat plate of the wall-jet. The microphones were pushed up through the bottom of the plate with the face of the nylon bushing flush with the wall. The microphones were adjusted so that the brass microphone face was either flush with the wall or, if roughness was present, with the substrate of the rough surface.

![Diagram of Wall Pressure microphone installation](image)

**Figure 2-9. Diagram of Wall Pressure microphone installation**

Wall pressure measurements were recorded using an Agilent Vee Pro data acquisition system along with a 16-bit E1432 digitizer. Data were collected at 51200Hz and spectra are the average of 1000 records of 2048 samples each.

### 2.4 Microphone Array Instrumentation

Figure 2-10 and Figure 2-11 show a 36-sensor linear microphone array designed specifically for this study. The same high sensitivity ½” B&amp;K 4190 microphones used for single point far field measurements were used for the microphone array. The microphones were placed as close together as possible with a spacing 15.1mm center to center. This kept the aperture of the array small so that the array could be placed in various regions of influence of the produced roughness noise. In particular was the desire to place the array in regions dominated by the spanwise or streamwise aligned dipoles produced by the roughness. The linear array allowed for measurements on axes that would entirely isolate either the spanwise or streamwise dipoles.
Figure 2-10. Diagram of linear microphone array
The array structure was assembled from a single Delrin block with aluminum pins inset into the face on either side. The microphones fit through precisely drilled holes in the Delrin and were held snug with Nylon set screws. The microphone faces were flush with the face of the Delrin. The array was suspended by the end pins using two aluminum mounts that attached to the 80/20 scaffolding in the tunnel. The array would then hang 220mm below the scaffolding. The receiving angle of the array could be adjusted to point directly at the noise source on the plate.

Figure 2-12 shows several arrangements of the microphone array in the anechoic chamber. The array was positioned in four separate locations which will be referred to as Array Positions 1-4 as labeled. The first location was on the edge of the plate outside of the flow. The array was resting on the aluminum plate with the middle of the array centered on the leading edge of where the roughness elements were placed, \(x=1257\)mm, so that the position of individual sensors only varied in the \(x\)-dimension. This location should be dominated by noise from the theoretical spanwise dipole of a roughness source. The second location was above the shelf of the anechoic chamber suspended from the 80/20 scaffolding. This position evenly distributed the microphones on either side of the roughness element so that the microphone positions varied in the \(z\)-dimension only. This array position would record noise dominated by the streamwise dipole. The last two locations were also suspended from the 80/20 frame directly above the spanwise center of the plate at \(x=1257\)mm with the axis of the array aligned spanwise and then streamwise. Measurements of the sensor locations at each array position were recorded using a FaroArm Fusion coordinate measuring machine. The locations of the array center for the four positions are given in Table 2-1.
Figure 2-12. Microphone Array Positions

<table>
<thead>
<tr>
<th></th>
<th>$x$, mm</th>
<th>$y$, mm</th>
<th>$z$, mm</th>
<th>$\Theta$</th>
<th>$\varphi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Array Position 1</td>
<td>1258</td>
<td>25.4</td>
<td>-715</td>
<td>0°</td>
<td>90°</td>
</tr>
<tr>
<td>Array Position 2</td>
<td>915</td>
<td>585</td>
<td>0</td>
<td>-56°</td>
<td>0°</td>
</tr>
<tr>
<td>Array Position 3</td>
<td>1366</td>
<td>626</td>
<td>0</td>
<td>-90°</td>
<td>-90°</td>
</tr>
<tr>
<td>Array Position 4</td>
<td>1257</td>
<td>597</td>
<td>-1</td>
<td>-90°</td>
<td>0°</td>
</tr>
</tbody>
</table>

Table 2-1. Locations of array center for the four measured positions

Data were collected from the array using the Bruel & Kjaer Pulse 14 Time Recorder platform along with six B&K 3050-A LXI six-channel modules with 24-bit A/D conversion. The modules were tied together with a NetGear ethernet switch. Data were sampled over 40s at 65536Hz. The final spectra are the average Fourier transform of 2560 to 5119 records of 1024 samples each. Before analysis, these narrow band spectra were frequency averaged over 1/10th octave bands to reduce uncertainty and the recorded tunnel background noise was subtracted. Data from the microphone array were calibrated using single microphone measurements of roughness noise recorded from the same central point of the microphone array in Array Position 2. The procedure was used to eliminate a large scalloping effect due to the presence of the Delrin block surrounding the microphones. The calibration procedure involved multiplying the array measured cross-spectral matrix by the ratio of the single microphone autospectrum and array measured autospectrum of the roughness noise.
2.5 Array Data Processing Methods

Data from the array were analyzed using two main methods. A conventional beamforming method which assumes monopole sources was used to visualize the data while a least-squares method was used to separate and estimate the source strengths of the individual dipole noise sources.

2.5.1 Beamforming

2.5.1.1 Beamforming Algorithm

A conventional delay and sum beamforming method was used to visualize the data recorded at each microphone array position separately. Delay and sum beamforming uses the cross-spectral matrix of the recorded array data and the known locations of the microphone sensors. The phase and magnitude of the measurements are adjusted to account for the change in distance from each microphone to a focal point. The cross-spectral matrix is then summed. This procedure is then repeated for many locations over a defined focus area and the summations can be plotted producing a source map. Peaks occur in the source map when the cross-spectral matrix adds constructively indicating the location of a source. The summation and phase adjustment were completed using the matrix calculation from Tester & Glegg (2008) shown in Equation 2-4.

\[
\mathbf{\tilde{b}} = \text{diag}(\mathbf{G}_{inv}^{H} \mathbf{C} \mathbf{G}_{inv})
\]

Eq. 2-4

where \( \mathbf{C} \) is the background-subtracted cross-spectral matrix and \( \mathbf{G}_{inv} \) is the inverse monopole transformation matrix or steering matrix which applies a magnitude and phase adjustment to the individual elements of the cross-spectral matrix for multiple focal points. The superscript \( H \) denotes the Hermitian transpose. The resultant \( \mathbf{b} \) vector is the phase delayed and summed solution for all focal points.

The Green’s function for the acoustic propagation of a monopole source at \( \mathbf{y}_j \) to a sensor at \( \mathbf{x}_m \) is shown in Equation 2-5.

\[
G_{j,m} = \frac{e^{ikr_{j,m}}}{4\pi r_{j,m}} \quad r_{j,m} = |\mathbf{x}_m - \mathbf{y}_j|
\]

Eq. 2-5

where \( k \) is the acoustic wavenumber and \( r_{j,m} \) is the absolute distance between the microphone and source location. Therefore, the elements of \( \mathbf{G}_{inv} \) are defined as in Equation 2-6.

\[
G_{inv\,j,m} = r_{j,m} e^{-ikr_{j,m}} / r_c \sqrt{M}
\]

Eq. 2-6

where \( r_c \) is a reference distance and \( M \) is the number of microphones. A reference distance of 1m was used for all presented source maps.

2.5.1.2 Qualities of the Linear Array’s Source Maps

Figure 2-13 shows a simulated measurement of a 10368Hz noise source located 1m away from the microphone array. The sensors of the linear microphone array only vary in the \( z \)-direction. Three point source images are shown in Figure 2-14 produced by steering the array over the highlighted areas. Plane A is contained in the \( x-z \) plane, Plane B is in the \( y-z \) plane, and Plane C is in the \( x-y \) plane. These source maps demonstrate the inability of the linear array to accurately locate sources outside of its orientation.
axis. As the focal point moves along the array’s axis there is a clear peak at the source location, but when the focal point moves closer and further away or perpendicular to the array’s axis, the point source’s location becomes uncertain. Further, to determine the source strength, the source maps must be integrated over a selected area of which the choice is often unclear. Therefore, sweeps through the two dimensional source maps along the array’s axis, as shown in Figure 2-15, will often be used to compare results from several roughness types or flow conditions and only qualitative observations and comparisons will be made from the results.

Figure 2-13. Point source simulation
Figure 2-14. Source maps in $\text{Pa}^2/\text{Hz}$ for 10368Hz point source located at (0, 0, 0)

Figure 2-15. Sweep through source map of Plane A at $x=0$
Although the small aperture of the array allows it to be easily placed in a narrow region of noise dominated by a single lobe of the directionality field, it limits the array’s ability to distinguish between multiple sources especially at lower frequencies where the main lobe peak of a beamformed source becomes wider. Figure 2-16 displays simulated source maps at two different frequencies, 3008Hz and 10368Hz, generated by focusing on Plane A of Figure 2-13 again this time with two sources centered around (0,0,0) separated by 0.1m on the z-axis. At the lower frequency the two sources appear as one and at the higher frequency dual peaks are visible. The array’s application in this study required measurements of deterministic surfaces with elements as close as 5.5-16.5mm. Therefore, individual sources produced by roughness elements will not be distinguishable from one another within the measured frequency range, but general trends can be observed over the entire roughness fetch areas.

Figure 2-16. Source maps of two sources located at (0.05, 0) and (-0.05, 0) at a) 3008Hz and b) 10368Hz

2.5.1.3 Background Subtraction Method

Background noise measurements of the tunnel were made at each array position and subtracted from the recorded roughness noise data. The background and roughness noise were both averaged over 1/10th octave bands before subtraction to reduce uncertainty. The background noise as recorded from each array position at 10368Hz with a nozzle exit velocity of 60m/s is shown in Figure 2-17. These source maps are focused at the location rough surfaces were placed on the plate. Array Position 1 shows the background noise predominantly coming from the direction of the nozzle exit. This is the only microphone array position that has a direct line of sight to the nozzle. Array Position 2 which is shielded by the shelf of the wall jet has the lowest recorded background noise levels. The background noise from this position is dominated by turbulence noise and shows the spanwise variation of the noise. Due to the positioning of the array’s sensors, the array does not have sufficient resolution to conclude the location of the peak source strength in the x-direction. There is an asymmetry in this source map showing the negative z-direction to be slightly louder. The same result is viewed in the spanwise variation from Array Position 4. A similar asymmetry was recorded in the aerodynamic properties by Morton (2011) using a quadwire probe and is due to the slight spanwise variation of the wall-jet flow. The effect of this flow variation on the recorded noise is discussed in Chapter 5 along with the presentation of the beamformed
data. Array Position 3 shows the background noise coming from the direction of the nozzle exit at this position. The peak of the noise occurs around $x=1220$ mm, but the general trend of the source map is an increase in the direction of the nozzle. This peak may be an effect of the shelf, which was shown to increase the growth of the mixing layer by 25% in Devenport et al. (2011), and the noise from the nozzle exit.

Figure 2-17. Background noise as recorded from each array position at 10368Hz and $U_o=60$m/s

Figure 2-18 shows a measurement of a single 3mm cubic element with and without the background noise subtracted as recorded from Array Position 4 at 10368Hz. This displays the clear improvement of the measurement and isolation of the noise produced by the roughness element alone. This also shows the need for the microphone array to isolate the noise produced by the roughness element from that of the background of the tunnel. The background noise is significantly stronger than that produced by the cube as recorded from this array position, but after subtraction the source map clearly identifies the roughness noise source. Using the background-subtracted microphone array data, the source
strengths of individual roughness elements can be determined. This source strength analysis will be discussed in Section 2.5.2.

Figure 2-18. Measurement of single cubic element from Array Position 4 a) without b) and with the background noise subtracted

2.5.1.4 Acoustic Source Convection Correction

Preliminary array measurements of roughness noise recorded in the wall jet tunnel at Array Position 1 showed peak source locations downstream of the physical location of the roughness. To study this phenomenon, array measurements were made of white noise radiating from four locations on the flat plate which bound the region where rough surfaces were placed. The white noise was created using an Agilent VXI data acquisition system which provided a white noise signal to a Koss SparkPlug SP3 earbud speaker. This speaker was positioned from the bottom of the plate through each one of four 9.5mm diameter holes separately so that the noise was projected away from the flat surface and recorded by the microphone array in the far field. Data were recorded at all four array positions and nozzle exit velocities ranging from 20-60m/s. Figure 2-19 shows a sweep through the beamformed source map with the array at Array Position 1 measuring a source at \( x = 1259 \text{mm}, z = 40 \text{mm} \) for nozzle exit velocities 20-60m/s. The source appears to convect as far as 30.1mm downstream at the highest velocity. To accurately determine the convection distances for each experimental condition, the measured cross-spectral phase was compared with the ideal phase distribution for a source at the known location. The RMS error between the measured and ideal phase were adjusted to a minimum by manipulating the ideal phase via numerically shifting the \( x \)-location of the simulated ideal source. This method worked well for Array Position 1. Convection estimates could not be determined for Array Positions 2 and 4 because the array was oriented spanwise. Therefore, the array’s \( x \)-dimension resolution was poor and the error never produced a clear minimum. For Array Position 3, the acoustic propagation vector from source to array occurred in only a short region of significant flow velocity. Therefore, the estimated source convection distances are not as large as the Position 1 results. The tabulated convection results for Array Position 1 and 3 are given in Table 2-2 and Table 2-3, respectively.
Figure 2-19. Sweep through 10368Hz source map at $z=37.5$mm showing convection effect.

Table 2. Source convection results for Array Position 1

<table>
<thead>
<tr>
<th>Source Position, mm</th>
<th>$U_o$, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>60</td>
</tr>
<tr>
<td>$x=1259$ $z=-43$</td>
<td>25</td>
</tr>
<tr>
<td>$x=1259$ $z=40$</td>
<td>30.1</td>
</tr>
<tr>
<td>$x=1358$ $z=40$</td>
<td>27.1</td>
</tr>
<tr>
<td>$x=1357$ $z=-42$</td>
<td>23.5</td>
</tr>
</tbody>
</table>

Table 2-3. Source convection results for Array Position 3

<table>
<thead>
<tr>
<th>Source Position, mm</th>
<th>$U_o$, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>60</td>
</tr>
<tr>
<td>$x=1259$ $z=-43$</td>
<td>14.1</td>
</tr>
<tr>
<td>$x=1259$ $z=40$</td>
<td>15.1</td>
</tr>
<tr>
<td>$x=1358$ $z=40$</td>
<td>11.6</td>
</tr>
<tr>
<td>$x=1357$ $z=-42$</td>
<td>12.8</td>
</tr>
</tbody>
</table>
These convection results were consistent with simple estimates due to the local velocity shown in Equation 2-7.

\[ d = \frac{r}{c_\infty} U_m \]  

Eq. 2-7

where \( r \) is the distance the source vector is immersed in significant flow velocity, \( c_\infty \) is the speed of sound, and \( U_m \) is the local maximum velocity. Referring to the aerodynamic results of Smith (2008) and Devenport et al. (2010), at Array Position 1 operating conditions for the 60m/s case were approximately \( r=440\text{mm} \) and \( U_m=22\text{m/s} \), yielding a convection distance of 28mm, within a few millimeters of the observed distances.

To correct for the convection effect, the source maps produced from measurements at Array Position 1 and Array Position 3 were skewed using the measured convection distances calculated from the white noise calibrations. Convective distances were determined for every point in the source maps by interpolating and extrapolating the results over the entire focus area. The calculated convective distances were subtracted from the \( x \)-locations of the source map to realign the figures as if no flow was present. Since there was no observed source convection for Array Positions 2 and 4 these maps were not altered. An original and modified source map are shown in Figure 2-20 for the measurement of roughness noise from a single 3mm cubic element located at \( x=1257, z=0 \) with a nozzle exit velocity of 60m/s. The original map displays the source downstream of the physical location while the modified picture correctly identifies the source location in the \( x \) direction.

![Figure 2-20. Source maps from Array Position 1 for a single 3mm cube located at \( x=1257, z=0 \) (black dot) at a nozzle exit velocity of 60m/s a) unaltered b) and corrected (Pa²/Hz).](image)

2.5.2 Least-Squares Source Strength Analysis

2.5.2.1 Least-Squares Source Strength Calculation

Figure 2-16 shows that the linear microphone array’s resolution limits its potential to distinguish the strength of multiple sources spaced closely together. This was the motivation to investigate a least-squares approach of source strength estimation. A novel least-squares method was developed to separate and estimate the source strengths of individual elements in multi-element fetches of discrete roughness.
Derivation of the least-squares method begins by writing the recorded pressures at a single sensor as the summation of the response due to individual sources in Equation 2-8.

\[ P_m = q_1 G_{m,1} + q_2 G_{m,2} + \cdots q_j G_{m,j} \quad \text{Eq. 2-8} \]

where \( P_m \) is the acoustic pressure recorded at microphone \( m \), \( q_j \) is the source strength of source \( j \), and \( G_{m,j} \) is the Green’s function for the acoustic propagation of source \( j \) to sensor \( m \).

For this analysis, each roughness element was assumed to radiate uncorrelated spanwise and streamwise aligned dipoles with half-space Green’s functions defined by Equation 2-9.

\[ G_{m,j} = \frac{i \kappa \cos \theta_{m,j} \text{e}^{-ikr_{m,j}(1+i/kr_{m,j})}}{2\pi r_{m,j}} \quad \cos \theta_{m,j} = \hat{x} \cdot (x_m - y_j)/r_{m,j} \quad \text{Eq. 2-9} \]

where \( \hat{x} \) is the dipole directionality unit vector. The cross-spectrum between two microphones, \( m \) and \( n \), can be calculated using Equation 2-8 multiplying \( P_m \) by the conjugate of \( P_n \). As the signals from the uncorrelated sources are averaged, the cross source terms disappear leaving only a linear combination of the source components as in Equation 2-10.

\[ \overline{P_m P_n^*} = q_1^2 G_{m,1}^* G_{n,1}^* + q_2^2 G_{m,2}^* G_{n,2}^* + \cdots q_j^2 G_{m,j}^* G_{n,j}^* \quad \text{Eq. 2-10} \]

This equation can be rewritten for all terms in the cross-spectral matrix as:

\[ \overline{G_{pp}} = \mathbf{G}_{GG} \overline{Q} \quad \text{Eq. 2-11} \]

where \( \overline{Q} \) is a vector of mean squared source strengths, \( \mathbf{G}_{GG} \) is a transform matrix of cross-spectral Green’s functions, and \( \overline{G_{pp}} \) is the measured cross-spectral matrix put into vector form. The source strengths can then be calculated using a least-squares algorithm with a non-negative solution constraint.

### 2.5.2.2 Comparison with Nelson & Yoon (2000)

This method differs from previous least-squares methods in that it assumes the sources are uncorrelated. Nelson & Yoon (2000) developed a least-squares method for solving for the cross-spectral source strength matrix as shown in Equation 2-12.

\[ Q = \mathbf{G}^+ \mathbf{G}_{pp} \mathbf{G}^{+H} \quad \text{Eq. 2-12} \]

where \( Q \) is a matrix of source strengths, \( \mathbf{G} \) is a transfer function matrix with dimensions \( mj \times m \), and \( \mathbf{G}_{pp} \) is the cross-spectral matrix. This method leads to solutions in the off-diagonal terms of the matrix \( Q \) which are known to be zero for uncorrelated sources.

A simple comparison between methods displays the weaknesses of a more general least-squares approach such as Nelson & Yoon (2000) to correctly determine the acoustic source strengths given conditions consistent for measurements of roughness noise sources. A simulated measurement is shown in Figure 2-21. Five sensors spaced 10mm apart are located 1m away from five white noise monopole sources in a similar arrangement in free-space. This simulation’s dimensions are approximate to the roughness noise measurements that were taken in the anechoic wall-jet. The sources produce an
uncorrelated white noise that is measured by each sensor. Using the proposed method which assumes uncorrelated sources, the exact source strength solution is calculated for all sources and frequencies.

Figure 2-21. Simulated measurement of acoustic sources

Figure 2-22 shows the total percent RMS error in the source strength matrix calculation using Nelson & Yoon’s (2000) formulation. The error increases towards lower frequencies giving values that are 100% off at 7kHz and steadily decreases with a slope of $f^{-8}$. If the simulation measurement and calculation is completed again for an increased source spacing of 100mm, the produced errors are over a million times smaller. This displays the dependence of the source spacing to wavelength ratio. These findings agree with Nelson & Yoon who also state some other geometric conditions that can adversely affect the solution. They conclude that their method works best with a geometrically similar arrangement of sensors and sources and with low sensor to source spacing. Unfortunately, these stipulations make use of their least-squares method impractical for this study.
The uncorrelated source method preferred in this study also enables the data taken from multiple array positions to be used in the same calculation. The transform matrix and vector of cross-spectral terms can be appended to include any number of independent equations because this least-squares method does not require cross-spectral terms between all of its components. The Nelson & Yoon (2000) method requires a full cross-spectral matrix of all input data. These data are unknown between measurements at different array positions restricting the calculation to only one array position at a time. Therefore, Nelson & Yoon’s method was not considered during the analysis of the roughness noise data.

### 2.6 Rough Surfaces

Fetches of deterministic roughness were studied to analyze the produced far field noise dependence on surface geometry including element size, shape, and configuration. Fetches of 3mm hemispherical and cubic elements were used as well as fetches of 1mm hemispheres. All rough surfaces were placed starting at approximately 1257mm downstream of the nozzle exit where incoming aerodynamic conditions were as shown in Table 2-4 giving roughness height to boundary layer ratios of 5-18%.

<table>
<thead>
<tr>
<th>$U_o$, m/s</th>
<th>$\delta$, mm</th>
<th>$\delta^*$, mm</th>
<th>$\Theta$, mm</th>
<th>$U_m$, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>20.9</td>
<td>1.36</td>
<td>1.01</td>
<td>7.5</td>
</tr>
<tr>
<td>30</td>
<td>19.1</td>
<td>1.24</td>
<td>0.92</td>
<td>11.1</td>
</tr>
<tr>
<td>40</td>
<td>17.9</td>
<td>1.16</td>
<td>0.86</td>
<td>14.8</td>
</tr>
<tr>
<td>50</td>
<td>17.1</td>
<td>1.11</td>
<td>0.82</td>
<td>18.4</td>
</tr>
<tr>
<td>60</td>
<td>16.4</td>
<td>1.06</td>
<td>0.79</td>
<td>21.9</td>
</tr>
</tbody>
</table>

Table 2-4. Aerodynamic characteristics at lead of roughness fetch
The rough surfaces were made of molded epoxy with a combination backing including a sheet of Kevlar for strength topped with a sheet of sketch paper that sealed the porous surface. The two-layer backing made the substrate, and therefore the outer perimeter step of the roughness sheet, 0.30mm thick. Far field noise and wall pressure measurements made with a molded smooth surface showed no increased pressure fluctuations due to the step perimeter. This is an improvement from the earlier study of Alexander (2009) whose noise and wall pressure results may have been influenced by the larger 1.2-1.6mm step perimeter around the tested deterministic roughness. Figure 2-23 shows the 3mm cubic and hemispherical elements molded on the Kevlar/paper backing.

![Figure 2-23. Molded 3mm hemispherical and cubic elements](image)

The perimeters of the rough surfaces were taped to the plate using 0.08mm thick packing tape. This kept the flow from lifting the surface and filled the sharp corner of the step created by the boundary. Double sided tape was used on the underside of the 610x305mm fetches to keep the surfaces lying flat. For the wall pressure measurements, the area surrounding each microphone location was fixed using double sided tape to prevent the surface from fluttering. Many different combinations of fetch size and element size and shape were studied using various analysis techniques. Each will be described below along with the type of analysis conducted.

### 2.6.1 Single Element to 42 Element Fetches

Microphone array data were taken for fetches of 3mm cubic and hemispherical elements. The fetch sizes ranged from a single element to 42 element sheets arranged in six spanwise rows and seven streamwise columns. The microphone array measured cubic surfaces are shown in Figure 2-24. Similar measurements were made from hemispherical elements arranged in the 6x7, 6x5, 6x2, 6x1, and 1x1 configurations. The elements in these fetches were spaced 16.5mm center-to-center in a grid pattern. Each measurement was completed with the desired rough surface attached to the plate. The front center of the roughness fetches were placed in the same starting location of the plate \((x=1257\text{mm}, z=0)\) for all measurements.
Wall pressure measurements and single point far field measurements were made separately inside the 42 element fetches by punching holes in the surface. Figure 2-25 shows a diagram of these measurement locations and their relative position to the roughness elements. Wall pressures were recorded at fourteen positions in the cubic element fetch and nine positions in the hemispherical element fetch. For these measurements, the surface was moved relative to a single microphone which was fixed in the wall at $x=1358\text{mm}$, $z=-43\text{mm}$. Therefore, the front spanwise center of the fetch’s position varied from $x=1255$-$1366\text{mm}$ and $z=-92.5$-$6.5\text{mm}$. All holes at positions not being measured were covered with packing tape so as to not interfere with the measurement in progress. Table 2-5 and 2-6 give detailed descriptions of each wall pressure microphone location.

Figure 2-25. Wall pressure measurement diagram
<table>
<thead>
<tr>
<th>Location Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
</tr>
<tr>
<td>C2</td>
</tr>
<tr>
<td>C3</td>
</tr>
<tr>
<td>C4</td>
</tr>
<tr>
<td>C5</td>
</tr>
<tr>
<td>C6</td>
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<td>C12</td>
</tr>
<tr>
<td>C13</td>
</tr>
<tr>
<td>C14</td>
</tr>
</tbody>
</table>

Table 2-5. Description of wall pressure measurement locations inside the 3mm cubic element fetch

<table>
<thead>
<tr>
<th>Location Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
</tr>
<tr>
<td>H2</td>
</tr>
<tr>
<td>H3</td>
</tr>
<tr>
<td>H4</td>
</tr>
<tr>
<td>H5</td>
</tr>
<tr>
<td>H6</td>
</tr>
<tr>
<td>H7</td>
</tr>
<tr>
<td>H8</td>
</tr>
<tr>
<td>H9</td>
</tr>
</tbody>
</table>

Table 2-6. Description of wall pressure measurement locations inside the 3mm hemispherical element fetch
2.6.2 610x305mm Fetches

Far field and wall pressure measurements were made for three different 610x305mm fetches of hemispherical elements. The first fetch was made of 3mm hemispheres spaced 16.5mm apart. There were a total of 703 elements with 19 spanwise rows and 37 streamwise columns. The second fetch was a similarly scaled fetch of 1mm hemispheres spaced 5.5mm apart increasing the number of rows and columns to 54 and 109, respectively. The third fetch was composed of 6209 randomly located 1mm hemispherical elements with a maximum spacing between two elements of 9.4mm. The element positions were exactly known but were distributed over the extent of the roughness fetch using the pseudo-random Mersenne Twister algorithm to generate values for the $x$ and $z$ locations. The locations of all the elements are given in Appendix A. The shorter dimension for all three fetches was aligned in the streamwise direction. Like the 42 element fetches, all wall pressure measurements were made from a single downstream location ($x=1403$mm, $z=0$) while the surface was moved in relation. The wall pressure was measured at only one location relative to the roughness elements in the random surface. This location, shown in Figure 2-26, was 163.5mm from the leading edge of the fetch and 25mm off of the spanwise center of the fetch in the $-z$ direction.

![Figure 2-26. 1mm random hemispherical element array and detailed view of wall pressure microphone](image)

The wall pressure was measured at four locations in the grid patterned 1mm hemisphere surface each with different relative positions to the nearest roughness elements. All four measurements were made between 143-148.5mm downstream of the leading edge and within the middle 15 spanwise columns of the roughness. Eight relative locations were measured in the 3mm hemisphere surface between the ninth and tenth spanwise rows 132-148.5mm downstream of the lead row of roughness. The wall pressure measurement locations for both grid patterned surfaces are shown in Figure 2-27. The locations are described in Table 2-7 and 2-8. Some of the wall pressure locations were less than a microphone radius away from the base of the nearest elements in the 3mm hemispherical element fetch. This was done by removing the conflicting element then installing the microphone flush with the substrate. The element was then reattached to the surface partially covering the brass face of the microphone. Figure 2-28 shows a detailed view of this microphone arrangement. Elements on the 1mm surface were not replaced if they conflicted with the microphone position. The elements were simply removed and the measurement taken without them. The missing elements are correctly omitted in Figure 2-27b.
Figure 2-27. Diagram of wall pressure measurement locations relative to the roughness elements of the a) 3mm hemispherical surface b) and the 1mm hemispherical surface

<table>
<thead>
<tr>
<th>Location Description</th>
<th>Measurement Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>LH1 2.8mm upstream and 2.8mm to the left of the element center</td>
<td>LH1</td>
</tr>
<tr>
<td>LH2 2.4mm downstream and 3.2mm to the left of the element center</td>
<td>LH2</td>
</tr>
<tr>
<td>LH3 4.1mm upstream of element center</td>
<td>LH3, LH4</td>
</tr>
<tr>
<td>LH4 Centered between elements</td>
<td>LH5, LH6, LH7, LH8</td>
</tr>
<tr>
<td>LH5 4.3mm downstream of element center</td>
<td></td>
</tr>
<tr>
<td>LH6 Centered between two elements</td>
<td></td>
</tr>
<tr>
<td>LH7 3.1mm downstream and 2.7mm to the right of the element center</td>
<td></td>
</tr>
<tr>
<td>LH8 2.9mm upstream and 2.7mm to the right of the element center</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-7. Description of wall pressure measurement locations inside the 610x305mm 3mm hemispherical element fetch
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SH1</td>
<td>Microphone in location of roughness element</td>
</tr>
<tr>
<td>SH2</td>
<td>1mm upstream and 1mm to the left of the removed element center</td>
</tr>
<tr>
<td>SH3</td>
<td>Centered between elements</td>
</tr>
<tr>
<td>SH4</td>
<td>1mm downstream and 1mm to the right of the removed element center</td>
</tr>
</tbody>
</table>

Table 2-8. Description of wall pressure measurement locations inside the 610x305mm 3mm hemispherical element fetch

Figure 2-28. Detailed view of the wall pressure microphone configuration around a 3mm hemispherical element
Chapter 3 Roughness Wavenumber Analysis

Roughness noise is dependent upon the specific roughness geometry as observed in both Grissom (2007) and Alexander (2009). Therefore, the noise from roughness with different shapes will be unlikely to collapse using a simple inner or outer variable scaling even if the rough surfaces have equal roughness heights. Glegg & Devenport (2009) developed a relationship between the roughness geometry and the radiated far field noise. This relationship is dependent on the wavenumber content of the surface and its slope. The following section derives analytical solutions to the wavenumber content of the surface and slope for the hemispherical and cuboidal roughness used in this study. These solutions are used to make preliminary far field noise estimations. The noise from a 40 grit fetch of roughness is predicted and compared to the results presented in Alexander (2009).

3.1 Discussion of the “Unified Theory of Roughness Noise” from Glegg & Devenport (2009)

The generation of roughness noise as described in Glegg & Devenport (2009) is the result of interaction between the wavenumber surface pressure spectrum and the surface wavenumber content. The simplified form of their theory is shown in Equation 3-1 assuming a homogeneous wall pressure field.

\[
\Phi_{pp}(x, \omega) \approx \frac{4\pi^2(k_o h)^2\Sigma \Phi_{pp}(\omega)}{|x|^2} \int \Psi_{pp}(\kappa_x, \kappa_z, \omega) \Gamma(\kappa_x, \kappa_z, k_o) d\kappa_x d\kappa_z \quad \text{Eq. 3-1}
\]

where \(\Psi_{pp}\) is the wavenumber wall pressure spectrum normalized on the single point pressure spectrum and \(\Gamma\) is a non-dimensional wavenumber filter function defined by the rough surface characteristics as defined in Equation 3-2.

\[
\Gamma(\kappa_x, \kappa_z, k_o) = \frac{1}{\Sigma h^2} \mathbb{E} \left[ \left| \left( \frac{x\xi^{(x)}}{|x|} + \frac{z\xi^{(z)}}{|x|} \right) - i k_o h \xi \right|^2 \right] \quad \text{Eq. 3-2}
\]

\(\xi^{(x)}\) and \(\xi^{(z)}\) are the Fourier transforms of the surface gradients in the \(x\) and \(z\) directions, \(\xi\) is the Fourier transform of the surface shape, \(h\) is the roughness height, \(k_o\) is the acoustic wavenumber, \(x\) and \(z\) are the observer location along the \(x\) and \(z\) axes, \(x\) is the observer’s location vector, and \(\Sigma\) is the roughness area. Equation 3-2 is different from the equation given in Glegg & Devenport (2009). There is an error in their paper decreasing the magnitude of the filter function by a factor of 7. Several specific surface types are discussed in Glegg & Devenport (2009) and their wavenumber content analyzed including stochastic, deterministic, and wavy-wall surfaces. A thorough analysis of the noise produced by a wavy-wall surface is given in Devenport et al. (2010) showing that the far field noise produced can be used to probe the wavenumber surface pressure spectrum at regions currently immeasurable due to limits imposed by the size of existing sensors. Similarly, their results show the possibility of predicting the far field noise using measurements of a rough surface and the overriding wall pressure spectrum. The theoretical wavenumber solutions for a discontinuous stochastic surface as well as a surface composed of randomly located and sized hemispheres are given in Glegg & Devenport (2009). Sources on both surfaces are assumed
acoustically compact so that \( k_a h \) is very small and therefore the wavenumber spectrum of the surface is negligible compared to the wavenumber spectrum of the surface slope. This surface slope dictates the shape of the theoretical collapse of the far field data when normalized by the measured single point wall pressure spectrum. Glegg & Devenport (2009) predict that a surface composed of randomly distributed hemispheres will produce an \( \omega^4 \) collapse which is different from the theoretical and observed \( \omega^2 \) collapse of noise from stochastic roughness shown in Alexander (2009).

A wavenumber analysis of three of the rough surfaces used in this study is detailed below giving some insight into the inherit differences of the examined surfaces. Also, estimations of produced far field noise will be compared to the expected results from Glegg & Devenport’s (2009) theory.

### 3.2 Surface Wavenumber Spectrum of 42 Element Fetch of 3mm Cubes

A single cuboid of dimensions \( a, b, \) and \( c \), as shown in Figure 3-1, protruding from a flat surface can be defined by a series of two-dimensional Heaviside functions along the \( x \) and \( z \) axes as given in Equation 3-3.

![Figure 3-1. Cubic element](image)

\[
f(x, z) = c \cdot H \left( \frac{a}{2} - z \right) H \left( z - \frac{a}{2} \right) H \left( \frac{b}{2} - x \right) H \left( x - \frac{b}{2} \right)
\]

Eq. 3-3

The Fourier transform will be defined as in Equation 3-4 shown with its inverse pair.

\[
F(\kappa_x, \kappa_z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, z) e^{i(\kappa_x x + \kappa_z z)} \, dx \, dz
\]

Eq. 3-4

\[
f(x, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\kappa_x, \kappa_z) e^{-i(\kappa_x x + \kappa_z z)} \, d\kappa_x \, d\kappa_z
\]

The equation for a three-dimensional box can be inserted into the transform and the limits of integration can be adjusted to the region where \( f(x, z) > 0 \).

\[
F_{\text{cuboid}}(\kappa_x, \kappa_z) = \frac{c}{(2\pi)^2} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} e^{i(\kappa_x x + \kappa_z z)} \, dx \, dz
\]

Eq. 3-5
This can be integrated to yield the analytic Fourier transform solution for a single cuboid in Equation 3-6.

\[
F_{\text{cuboid}}(\kappa_x, \kappa_z) = \frac{c}{\kappa_x \kappa_z \pi^2} \sin \left( \frac{b}{2} \right) \sin \left( \frac{a}{2} \right)
\]

This solution in wavenumber space is shown in Figure 3-2 for a cube with \( a = b = c = 3\) mm. The flat vertical faces of the cube aligned on the \( x \) and \( z \) axes produce a highly directional wavenumber pattern indicating sound may be produced more efficiently in directions normal to the cubes faces.

![Figure 3-2. Wavenumber transform of a single 3mm cubic element](image)

The Fourier transform of a fetch of cubes with dimensions \( a, b, \) and \( c \) can be calculated by a linear addition of Equation 3-6 offsetting each cube to a different \((x_j, z_j)\) location using Equation 3-7.

\[
F_j(\kappa_x, \kappa_z) = F(\kappa_x, \kappa_z) e^{i(\kappa_x x_j + \kappa_z z_j)}
\]

This can be done to model a 42 element fetch of cubes spaced 16.5mm apart in a grid pattern simulating one of the fetches used experimentally in this study. The resulting Fourier transform is shown in Figure 3-3. The same directionality is observed, but the spectrum appears noisier because its smallest scale is now determined by the full size of the fetch rather than just a single element.
Figure 3-3. Wavenumber transform of a 42 element fetch of 3mm cubes spaced 16.5mm apart

Though the Fourier transform of these elements seems to indicate a specific directional efficiency, through the assumption of compact sources, it was determined that the wavenumber spectrum of the surface was negligible compared to the wavenumber spectrum of the surface slope. It can be shown that the Fourier transform of the surface can be related to the Fourier transform of the slope in the $x$-direction as in Equation 3-8.

$$
\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial f(x,z)}{\partial x} e^{i(\kappa_x x + \kappa_z z)} \, dx \, dz = -i\kappa_x F(\kappa_x, \kappa_z)
$$  \hspace{1cm} \text{Eq. 3-8}

The equation is similar in $z$ and can be generalized for the slope in any direction with unit vector $\vec{e}_s$.

$$
\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \vec{e}_s \cdot \nabla f(x, z) = \vec{e}_s \cdot [\kappa \vec{k} F(\kappa_x, \kappa_z)]
$$  \hspace{1cm} \text{Eq. 3-9}

Rearranging Equation 3-2 using the result of Equation 3-9 leaves a solution for the wavenumber filter function in terms of the surface wavenumber spectrum. The only assumption made is that the sources are compact so that the surface wavenumber term of Equation 3-2 vanishes.

$$
\Gamma(\kappa_x, \kappa_z) = \frac{1}{\Sigma h^2} \text{Ex}[|\vec{e}_s \cdot [\kappa \vec{k} F(\kappa_x, \kappa_z)]|^2]
$$  \hspace{1cm} \text{Eq. 3-10}

Figure 3-4 shows a 2D plot of the Fourier transform of the surface slope in the $x$-direction for the 42 element fetch of 3mm cubes.
Figure 3-4. Wavenumber transform of slope of 42 element fetch of 3mm cubes

The result seems to be wavenumber white in the $\kappa_x$ dimension, but the wavenumbers which will be most effective at scattering sound will be the wavenumbers that align with the convective ridge of the wavenumber wall pressure spectrum. This study will later show measured far field noise in a frequency range 250Hz-20kHz. This frequency range corresponds with a convective ridge that peaks between $\kappa_x \approx 200-14000$ rad/m for $\kappa_z = 0$ according to the wall pressure wavenumber spectrum models of Chase (1987) and Corcos (1964).

Figure 3-5 is a closer look at these much lower wavenumbers. The peak values are still seemingly flat in the $\kappa_x$ direction but there is a grid pattern present in the wavenumber space due to the grid configuration and shape of the cubic elements.

Figure 3-5. Low wavenumber region of wavenumber transform of surface slope of 42 3mm cubic element fetch
3.3 Surface Wavenumber Spectrum of 42 Element Fetch of 3mm Hemispheres

The fetch of hemispherical elements can be analyzed in the same way as the cubes using a Hankel transform. A Hankel transform of order zero can be used to express a two-dimensional Fourier transform of a radially symmetric function. Switching to polar coordinates, the Fourier transform of Equation 3-4 can be written as

\[
F(u, v) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(r) e^{2\pi i (ux + vz)} \, dx \, dz \quad \text{Eq. 3-11}
\]

where

\[
\begin{align*}
    r^2 &= x^2 + z^2 & x &= r \cos(\theta) \\
    q^2 &= u^2 + v^2 & z &= r \sin(\theta)
\end{align*}
\]

\text{Eq. 3-12}

The Fourier transform can be reformulated using the definition of a Hankel transform into Equation 3-13.

\[
F(q) = \frac{1}{2\pi} \int_{0}^{\infty} f(r) J_0(-2\pi qr) r \, dr
\]

\text{Eq. 3-13}

where \(J_0(-2\pi qr)\) is a Bessel function of order zero. For a hemisphere of radius \(R\), the function \(f(r)\) can be defined as

\[
f(r) = \begin{cases} 
    \sqrt{R^2 - r^2} & \text{for } r < R \\
    0 & \text{for } r \geq R
\end{cases}
\]

\text{Eq. 3-14}

Integration of Equation 3-13 then yields the solution

\[
F_{hemi}(q) = \frac{1}{2\pi} \left( \frac{\sin(2\pi q R) - 2\pi q R \cos(2\pi q R)}{(2\pi q R)^3} \right)
\]

\text{Eq. 3-15}

Substituting

\[
\kappa_x = 2\pi u \quad \kappa_z = 2\pi v \quad \kappa = 2\pi q \quad \kappa^2 = \kappa_x^2 + \kappa_z^2
\]

\text{Eq. 3-16}

The final solution in wavenumber space is

\[
F_{hemi}(\kappa) = \frac{\sin(\kappa R) - \kappa R \cos(\kappa R)}{(2\pi \kappa)^3}
\]

\text{Eq. 3-17}

Figure 3-6 shows this solution for a single 3mm hemispherical element. Unlike Figure 3-2, there is no strong directionality present, which is expected since a hemisphere is radially symmetric. The solution produces a spike around the origin and then decays as \(\kappa^2\) in all directions.
Figure 3-6. Wavenumber transform of a single 3mm hemispherical element

Using Equation 3-7 to simulate the 42 element fetch with a grid spacing of 16.5mm yields the solution shown in Figure 3-7. The addition results in a grid pattern present in the wavenumber in both the $\kappa_x$ and $\kappa_z$ directions. A similar pattern is present in Figure 3-3 generated using the 42 element fetch of cubic elements in a grid configuration.

Figure 3-7. Fourier transform of 42 element fetch of 3mm hemispherical elements spaced 16.5mm apart

Using Equation 3-8, the wavenumber of the surface slope is generated for an observer in the $x$-direction as in Equation 3-18.

$$F_{hemi}(k) = \kappa_x \frac{\sin(\kappa R) - k R \cos(\kappa R)}{2\pi R^3}$$  \hspace{1cm} \text{Eq. 3-18}$$

$$20 \log (F(\kappa_x, \kappa_z))$$
This solution approaches zero in the limit as $\kappa$ approaches zero but produces a singularity at the origin. The computation of the wavenumber field was corrected to satisfy the condition that $\Gamma(\kappa = 0)$ must be nil for a simple shape protruding from a flat surface. Figure 3-8 compares the wavenumber slope in the $\kappa_x$ direction for both the large wavenumber and small wavenumber fields. The wavenumber of the surface slope in the $\kappa_x$ direction behaves similarly.

At large wavenumbers, the solution appears relatively flat. The low wavenumber region shows that the solution is again radially symmetric except for the steep decline around $\kappa_x = 0$. The hills and valleys of this slope wavenumber space will interact differently than the cubic element fetch with the wavenumber wall pressure spectrum scattering different portions of the wavenumber space for various frequencies.

![Figure 3-8](image)

**Figure 3-8.** Wavenumber transform of the surface slope of the 42 element fetch of 3mm hemispheres at a) high b) and low wavenumber regions.
3.4 Surface Wavenumber Spectrum of 6209 Element Fetch of 1mm Randomly Distributed Hemispheres

The procedure for finding the wavenumber transform of the surface and the surface slope of the 610x305mm fetch of randomly distributed 1mm hemispheres directly follows that for the smaller fetch of 3mm hemispherical elements. Using the exact locations of all 6209 elements, the resulting Fourier transform of the surface is shown in Figure 3-9. An incorrect assumption of this procedure occurs when elements overlap. Linearly adding the Fourier transform of two overlapping hemispheres of exactly the same size would mathematically represent the Fourier transform of a single hemisphere of double the original height. It is assumed for this calculation that the total effect of this occurrence is negligible. On the entire fetch of 6209 elements only 560 overlap at all and no two elements have the exact same position. Just as the result of the single hemispherical element, the Fourier transform of this surface is radially symmetric in wavenumber space.

![Wavenumber transform of fetch of 6209 randomly located 1mm hemispheres](image)

Figure 3-9. Wavenumber transform of fetch of 6209 randomly located 1mm hemispheres

The Fourier transform of the surface slope for large and small wavenumbers is shown in Figure 3-10. Again at large wavenumbers, the spectrum appears relatively flat, but at low wavenumbers a pattern emerges. This result looks similar to that in Figure 3-8 for the 3mm hemispheres except that the underlying grid pattern in the spectrum is removed. This is due to the random relative positioning of the elements. Also, the radially symmetric ridges are further spaced in the random surface as compared to the 42 element fetch of larger hemispheres. In Figure 3-8b, there are four concentric rings surrounding the origin producing hills and valleys in wavenumber space compared to only one over the same wavenumber range in Figure 3-10b.
Figure 3-10. Wavenumber transform of surface slope for random fetch of 1mm hemispheres at a) high b) and low wavenumbers

3.5 Sound Estimates from Deterministic Roughness

The wavenumber decomposition of the surface slope for each fetch of deterministic roughness can be used to make absolute estimates of the sound produced by these surfaces using the theory of Glegg & Devenport (2009). By dividing Equation 3-1 by the single point wall pressure spectrum, any assumption of the form of this spectrum can be removed. If the observer is located a unit distance away in the negative x-direction and assuming compact sources, Equation 3-1 can be simplified to

$$\frac{\Phi_{pp}(x, \omega)}{\Phi_{pp}(\omega)} \approx 4\pi^2k_o^2 \int \Psi_{pp}(k_1, k_3, \omega)\Gamma(k_1, k_3, k_o)dk_1dk_3$$

Eq. 3-19

Where $$\Gamma(k_1, k_3, k_o) = |\zeta(x)|^2$$
The Corcos and Chase wavenumber wall pressure spectrum are both used to estimate the normalized wavenumber wall pressure spectrum, \( \Psi_{pp} \), for comparison. The Corcos spectrum as given by Howe (1998) is shown in Equation 3-20.

\[
\frac{\Phi_{pp}(\kappa, \omega)}{\Phi_{pp}(\omega)} = \frac{l_1}{\pi \left[ 1 + l_1^2 (\kappa_1 - \omega / U_c)^2 \right]} \frac{l_3}{\pi \left[ 1 + l_3^2 \kappa_3^2 \right]}
\]

Eq. 3-20

Where \( l_1 \approx 9 \frac{U_c}{\omega} \) and \( l_3 \approx 1.4 \frac{U_c}{\omega} \)

The analytical form of the integrated single point spectrum is

\[
\Phi_{pp}(\omega) = \frac{(\rho u_t^2)^2 \left( \frac{\omega \delta^+}{U_e} \right)^2}{\left( \frac{U_e}{\delta^+} \right) \left[ \frac{\omega \delta^+}{U_e} \right]^2 + \alpha_p^2}^{3/2}
\]

Eq. 3-21

Where \( \alpha_p = 0.12 \)

The Chase spectrum, which is much more complicated than the Corcos spectrum, provides a better model of the wavenumber wall pressure spectrum at low wavenumbers. This model spectrum as given by Chase (1987) is

\[
\Phi_{pp}(\kappa, \omega) = \frac{\rho^2 u_t^3}{[k_+^2 + (b \delta)^{-2}]^{5/2}} \left\{ C_T |\kappa| \left[ \frac{k_+^2 + (b \delta)^{-2}}{|\kappa|^2 + (b \delta)^{-2}} \right] + C_M \kappa_+^2 \right\}
\]

Eq. 3-22

Where \( h_c = 3 \quad C_T h_c = 0.014 \quad C_M h_c = 0.466 \quad b = 0.75 \)

And

\[
k_+^2 = \frac{\omega \delta^+}{U_e} \kappa_+^2 + \kappa^2
\]

Chase’s single point spectrum is given as the summation of two parts defined separately in Chase (1980) and Chase (1987).

\[
\Phi_{pp}(\omega) = \Phi_T(\omega) + \Phi_M(\omega)
\]

Where \( \Phi_T(\omega) = \pi C_T h_c \rho^2 u_t^4 \omega^{-1} \alpha^{-1} (1 + \alpha^{-2}) \) and

\[
\Phi_M(\omega) = r_m a_+ \rho^2 u_t^4 \omega^{-1} \alpha^{-3} (1 + \mu^2 \alpha^2)
\]

Eq. 3-23

\[
\alpha = [1 + (U_c / \omega b \delta)^2]^{-1/2} \quad r_m = 1 - C_T h_c / (C_T h_c + C_M h_c)
\]

For both wall pressure spectrum models, the friction velocity of the wall jet flow was estimated using Bradshaw & Gee’s (1960) skin friction relationship for a wall jet, \( C_T = 0.0315 \Re_\delta^{-0.182} \). The convection velocity was taken from the analysis of Devenport et al. (2010). They found, through analysis
of far field noise from rib roughness, that the convection velocity was approximately 41% of the edge velocity for a 60m/s nozzle flow. This is lower than values from conventional boundary layers, which have convection velocities around 60%, but the difference was attributed to interaction with the wall-jet’s large slower moving mixing layer. All further input values and relationships used in this analysis to estimate the Corcos and Chase spectra for a nozzle exit velocity of 60m/s are given in Equation 3-24.

\[
\begin{align*}
\rho &= 1.102 \text{ kg/m}^3 \\
u_r &= U_e \sqrt{C_f/2} \\
C_f &= 5.137 \times 10^{-3} \\
U_c &= 0.41 U_e
\end{align*}
\]

Eq. 3-24

\[\delta = 16.4 \text{mm} \quad U_e = 22 \text{m/s}\]

A 3D plot of the Chase wall pressure wavenumber spectrum is shown in Figure 3-11. Isocontours are highlighted to show the shape of the convective ridge. Comparison of two slices through the wavenumber spectra models of Corcos and Chase as a function of frequency are shown in Figure 3-12 at \(\kappa_x = 2000\) rad/m and \(\kappa_z = 0\). The Chase spectrum has a broader convective ridge and decays more rapidly at higher and lower frequencies, but both spectra produce a convective peak at the same approximate location.

![Figure 3-11. Convective ridge in wavenumber wall pressure spectrum](image-url)
The wavenumber pressure spectrum models can now be combined with the wavenumber of the surface slope and integrated to produce the estimated far field noise. Although the Fourier transforms of the surface slopes were computed analytically, the integration in Equation 3-19 was done numerically over a range of \( \kappa_x = [0 \text{ to } 20000] \text{ rad m}^{-1} \) and \( \kappa_z = [-10000 \text{ to } 10000] \text{ rad m}^{-1} \) with a resolution of 10 rad m\(^{-1}\). The resolution of this integration was examined to make sure the solution was converged. Also, the limits of the integration were studied and were found to be sufficient so that any extension did not significantly affect the results. Figure 3-13 to Figure 3-15 show the predicted noise from each fetch of deterministic roughness under the described condition for an observer located 1m upstream from the lead of the roughness fetch. Each figure has the predicted noise calculated using the Corcos and Chase wall pressure spectra as well as lines with slopes of \( \omega^2 \) and \( \omega^4 \) for comparison. None of the deterministic surfaces produce a clearly defined region where the normalization produces a \( \omega^2 \) curve. Each surface produces a collapse that resembles an \( \omega^4 \) region at low frequencies and then transitions to shallower slopes. For the cubic element fetch, the result approaches a straight line with an \( \omega \) slope. Both normalizations from the hemispherical fetches resolve to a flat spectrum at higher frequencies above approximately 1kHz for the 3mm hemispheres and 5kHz for the 1mm hemispheres. The predicted normalization from the randomly positioned element fetch is much smoother, especially at lower frequencies, than the results from the grid patterned fetches. This is because the grid patterned fetches produced slope wavenumber solutions with frequent hills and valleys due to the patterned spacing of elements. The integrated spectrum is reflecting the result of the convective ridge passing through this wavenumber pattern as the frequency is increased. At higher frequencies, the associated convective ridge is less narrow smoothing the hills and valleys in the slope wavenumber that are radiated to the far field.

An interesting note is that the estimates using the Corcos and Chase spectra produce very similar results. The Corcos spectrum is more sensitive to changes in the slope wavenumber spectrum because of its narrow convective ridge which produces lumps in the predicted noise normalization. The Chase spectrum with its broad convective ridge tends to smooth this effect.
Figure 3-13. Normalized noise estimate from 42 element fetch of 3mm cubic elements

Figure 3-14. Normalized noise estimate from 42 element fetch of 3mm hemispherical elements
The assumption of compact sources can be tested by comparing the estimate in Figure 3-14 for the fetch of 42 3mm hemispherical elements with a similar estimate using the full form of Equation 3-2. This calculation does not ignore the $i k_o h \xi$ term in the wavenumber filter function where $\xi$ is the wavenumber of the surface shape normalized on the roughness height. This comparison is shown in Figure 3-16 using the Chase model of the wavenumber wall pressure spectrum. The two spectra are almost identical over the considered frequency range indicating that the assumption of compact sources is valid. The 3mm hemispherical elements are the largest used in this study. Therefore, the assumption of compact sources can be extrapolated to the other studied surfaces.
3.6 Comparison with Data of Alexander (2009)

Figure 3-13 to 3-15 show only a very narrow $\omega^2$ region unlike the observed normalizations of stochastic surfaces in Alexander (2009). These stochastic surfaces differ from the deterministic surfaces considered here in the element size and randomness of the elements' shape. The 40 grit surface in Alexander (2009) contained elements that were nominally 0.425mm, less than half as large as the smallest considered deterministic surface in this analysis. To study the effect of extremely small element size, the same method as given in Equation 3-19 was used to estimate the normalized noise from a single cubic and hemispherical element with characteristic dimensions of 0.425mm. The results are shown in Figure 3-17 for an observer at $x'=(-1m, 0, 0)$. The predicted noise from both elements follows a slope slightly less than $\omega^4$. The form of this normalization appears to be almost independent of element shape. This can be explained in the limit as the roughness element size goes to zero. When considering elements which are very small compared to the wavelengths of disturbances on the convective ridge, the surface slope of any elementary shape will appear as two Dirac delta functions of opposite signs separated by a distance $2\alpha$ regardless of the actual element shape. This is essentially a low resolution view of the increasing slope on one side of the element and the decreasing slope on the other side. The low wavenumber region of the Fourier transform of the surface slope will appear similar between the two shapes. The resulting spectral shape is then only a function of the wavenumber wall pressure spectrum.

![Figure 3-17](image_url)

Figure 3-17. Estimated noise from a single 0.425mm hemispherical and cubic element at a nozzle exit velocity of 60m/s

To model the effect of the various grain sizes on a single fetch and their scattered positions, the entire 40 grit surface can be crudely approximated as a combination of randomly sized hemispheres and cuboids scattered over an area 610x305mm to match the fetch size studied in Alexander (2009). The grain heights were assigned randomly between values of 0.325 and 0.525mm. The total number of elements used to simulate the surface was calculated by counting the grains in two 5x5mm white light profilometry scans of 40 grit sandpaper to find the grain density. The computed grain density was 1.36 grains per mm$^2$, resulting in 253026 elements on the simulated fetch. An even number of cubes and hemispheres were
used. The resulting noise prediction is shown in Figure 3-18 compared with the data of Alexander (2009). The prediction has been computed for an observer at $x=(-0.2283, 0.473, 0)$m, which is the far field microphone location used in Alexander (2009).

The prediction is closer to the $\omega^4$ curve especially at lower frequencies. This could be a result of inadequately modeling the surface as discrete cubes and hemispheres. A 5x5mm scan of the 40 grit surface is shown in Figure 3-19. There doesn’t appear to be a truly flat substrate at any point in the scan. Therefore, the slope is never consistently zero for any distance on the surface. Also, the opposing Dirac delta functions may not accurately represent the slopes of each grain on the stochastic surface. The elements themselves do not have a simply approximated slope over their surface. The slope changes from sharply positive to negative not just at the front and back of each grain but over their entire surface area. Though, for the frequency range considered here, the low wavenumber region of the surface slope will dominate, 200 rad/m to 13000 rad/m, and the smallest details on the surface should be negligible. Details such as element spacing could be more important contributors in this region. The stochastic surfaces may be best described by a random distribution of slopes in wavenumber space which would be wavenumber white and produce the observed $\omega^2$ normalization.

The difference between the estimated and measured normalizations is not just the spectral shape but also the 5-9dB difference in magnitude. Although this analysis is only an approximation of the noise and does not collapse on the same curve, it may provide some insight into the “break frequency” described in Alexander (2009) where the normalization ceases to collapse the data. The estimated and measured normalization begin to “break” from their respective curves at approximately the same frequency around 5kHz. This indicates the “break frequency” could be dependent on the convective ridge passing through the wavenumber space of the surface slope. Since the error between the estimated and measured curves in Figure 3-18 could be due to the modeling of the stochastic roughness, a more accurate measure of the capabilities of the theory proposed in Glegg & Devenport (2009) may be derived from a

![Figure 3-18. Noise prediction for a 610x305mm fetch of 40grit sandpaper at a nozzle exit velocity of 60m/s compared to the results reported in Alexander (2009) at a range of nozzle exit velocities](image)
comparison between the estimates and measurements of the deterministic surfaces. These results are analyzed in the following chapter.

Figure 3-19. 5x5mm scan of the 40 grit sandpaper surface
Chapter 4 Estimations of Far Field Noise

Wall pressure and far field measurements were recorded from cuboidal and hemispherical roughness with various configurations and roughness heights. Unlike the previously studied stochastic surfaces in Grissom (2007), Alexander (2009), and Devenport et al. (2011), the exact analytical Fourier transforms of these surfaces can be calculated so that the accuracy of Glegg & Devenport’s (2009) roughness noise theory can be examined. This section details the measurement of the wall pressure field in the deterministic roughness along with the radiated far field noise. Estimations are made of the wall pressure normalized far field noise and the absolute far field noise from the deterministic roughness. Also, the form of the wavenumber wall pressure spectrum is studied as it relates to the estimation of roughness noise.

4.1 Measurements from 42 Element 3mm Cubic Roughness

4.1.1 Wall Pressure

The smooth wall pressure spectrum of the wall-jet behaves as a conventional turbulent boundary layer as according to Blake (1986). The low frequency region varies as $\omega^{-1}$ and the high frequency approaches $\omega^{-5}$. The wall pressure spectra measured in the wall-jet differ from conventional turbulent boundary layers most notably in the low frequency region. Figure 4-1 shows a comparison between the undisturbed wall pressure spectrum measured in the wall-jet at $x=1358\text{mm}$ and $U_o=60\text{m/s}$ with the unperturbed wall pressure spectrum from the computational study of Yang & Wang (2011) and the experimental studies of Rusche (2011) and Farabee & Cassarella (1986). These spectra are normalized on outer boundary layer variables to collapse the low frequency regions. Due to the wall-jet’s large slow moving mixing layer, the low frequency region of the wall pressure spectrum rises above the wall pressure spectrum continuing the $\omega^{-1}$ slope to the extent of the measured frequencies. The data of Rusche (2011) and Farabee & Cassarella (1986) collapse well at lower frequencies at a slope shallower than $\omega^{-1}$ common due to the low Reynolds numbers achievable in laboratory experiments. The Reynolds numbers of the compared wall pressure spectra are shown in Table 4-1. The shift of the transition region to higher frequency for the data of Rusche (2011) is due to the comparably higher Reynolds number which extends the length of the $\omega^{-1}$ region due to the convected turbulence in the logarithmic portion of the boundary layer. The LES data of Yang & Wang (2011) have a similar magnitude in the low frequency region but drop off at a much lower non-dimensional frequency than any of the experimental measurements. This is most likely due to insufficient resolution of the LES study at these high non-dimensional frequencies. Although the low frequency regions differ, the shape of the wall-jet’s spectrum at high frequencies is similar to that of Rusche (2011) and Farabee & Cassarella (1986).

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<td>$U_o=60\text{m/s}$</td>
<td>4079</td>
<td>7281</td>
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Table 4-1. Previous studies and their corresponding momentum thickness Reynolds numbers
The wall pressure was recorded at 14 locations in and around the fetch of 42 cubic elements as shown in Figure 4-2. The same notation as in Figure 4-2 will be used to identify the results at each measurement location. The wall pressure measurement location was held fixed at \( x = 1358 \text{mm} \) and \( z = -43 \text{mm} \) as the rough surface wave moved in relation. The streamwise variation through the fetch can be analyzed by focusing on positions C1-C5. Figure 4-3a shows the wall pressure recorded at these positions for \( U_o = 60 \text{m/s} \) compared to the smooth wall undisturbed wall pressure fluctuations. At position C1, which is 8.25mm upstream of the fetch, the wall pressure is slightly elevated from the smooth wall at all frequencies. The three internal measurement locations, C2, C3, and C4, all produce similar results below 5kHz which are significantly elevated from the upstream position. The scale of the wall pressure fluctuations on the convective ridge at 5kHz are approximately 1.8mm, roughly half the size of the roughness elements. Above 5kHz, the pressure fluctuations which begin 4dB above the smooth wall fluctuations at C2 tend back to the smooth wall spectrum with increasing streamwise distance. The wall pressure at Position C5, 8.25mm behind the roughness fetch, does not continue this trend. Behind the roughness fetch the wall pressure fluctuations increase at all frequencies and are above the smooth wall spectrum by up to 6dB. This suggests that the boundary layer is being displaced by the fetch of cubic roughness elements and that the presence of the downstream elements affects the upstream pressure fluctuations. The flow reattaches downstream of the roughness fetch creating a region of increased wall pressure fluctuations, especially in the frequency region 1kHz to 10kHz.

Positions C3 and C6 to C9 were recorded to analyze the spanwise change of wall pressure fluctuations over the fetch. These measurement locations have the same relative spacing among the nearest elements but are displaced spanwise between element rows. Positions C6 and C9 are located outside of the fetch, therefore, these positions only have elements in one spanwise direction. Figure 4-3b
shows the recorded wall pressure at these various locations for $U_o=60$m/s. There is very little change between positions. The two outside locations, C6 and C9, have slightly weaker low frequency fluctuations, approximately 1.5dB. At higher frequencies the opposite is true, and these outside positions have a slightly stronger response.

Figure 4-2. Wall pressure measurement locations inside 42 element fetch of cuboidal roughness

Figure 4-3. Variation of wall pressure in fetch of 3mm cubic elements at $U_o=60$m/s in the a) streamwise direction b) and spanwise direction
Since the relative wall pressure spectra recorded at similar interior locations across the span of the roughness fetch produce consistent results, positions C3 and C10 to C14 can be combined to compare the wall pressure around a single representative central element. Figure 4-4 compares the wall pressure spectra recorded at these locations with the smooth wall spectrum at $U_o=60\text{m/s}$. The wall pressure over this large element surface is clearly not homogeneous. The spectra not only differ in magnitude but in shape as well. Devenport et al. (2010) estimated the convection velocity of the smooth wall flow to be approximately 9m/s for $U_o=60\text{m/s}$. Therefore, frequencies below 7kHz correspond to flow structures convected with a characteristic dimension greater than 1.3mm. Below this frequency, the wall pressure fluctuations at every position were increased relative to the smooth plate. Above 10kHz, equivalent to flow structures smaller than 0.9mm, only the C13 and C14 positions, which are the closest measured downstream positions, recorded significantly elevated pressure fluctuations. The C11 position directly in front of the element showed a decrease in the high frequency pressure fluctuations. All others resembled the smooth wall data in this region with levels increased by approximately 2dB. Comparing positions C11 and C13, which are 1.7 element heights directly upstream and downstream of an element center, shows the wall pressure fluctuations at the upstream position are significantly lower than a similar distance downstream of the element center over all observed frequencies. These recorded wall pressure spectra are consistent with a horseshoe vortex wrapped around the base of the elements along with a region of separation behind the elements as seen in the LES results of Yang & Wang (2010). An upstream recirculation region produces an increase in low frequency pressure fluctuations and diminishing high frequency fluctuations as shown in wall pressure spectra of Farabee & Cassarella (1986) for the flow in front of a forward facing step. This is exactly the phenomenon as observed at position C11 upstream of the nearest roughness element. A similar effect is shown in the downstream separation region of a backward facing step in Farabee & Cassarella (1986). Position C13, downstream of the roughness element, has significantly increased low frequency fluctuations, but the high frequency fluctuations are also slightly greater than the smooth wall spectrum. This may be due to the relatively shorter separation region behind the roughness elements as compared to the step flows studied in Farabee & Cassarella (1986). Above 3kHz, pressure fluctuations recorded at the C14 position were greater than the recorded spectra at any other location. This position is downstream of the leading edges of the cubic roughness element from which the trailing legs of the horseshoe vortex would extend. Fluctuations above 3kHz correspond with structures equal to and smaller than the size of the roughness element, 3mm.
Figure 4.4. Wall pressure at various positions relative to nearest 3mm cubic element in 42 element fetch at $U_w=60m/s$

Figure 4-5 shows a direct comparison of the measured wall pressure spectra with the LES results of Yang & Wang (2011) at similarly scaled locations around cuboidal roughness. The cuboidal roughness in the LES study had a roughness height of 0.17", and elements were spaced 1". The incoming boundary layer thickness was 1.304" with an edge velocity of 45ft/s and the roughness Reynolds number was 168. These spectra are normalized on outer variables to collapse the low frequency region. The difference between measured spectra in the low frequency region is similar to the difference shown for the unperturbed wall pressure spectra shown in Figure 4-1 due to the mixing layer of the wall-jet. Like the unperturbed wall pressure spectra, the LES spectra roll-off at a much lower non-dimensional frequency than the measured wall-jet data. Again, this is most likely due to the large grid-scale of the LES unable to fully resolve the high frequency region. In the lower frequency region, the LES results produce more separation between the various measurement locations. This may indicate that the mixing layer of the wall-jet influences the roughness generated wall pressure fluctuations in this region while the local roughness has more of a dominating effect on the low frequency pressure fluctuations in a conventional turbulent boundary layer.

Comparison with the wall pressure spectra of Rusche (2011) shows the difference in experimentally measured wall pressure under a conventional turbulent boundary layer using the same configuration of cuboidal roughness used in this study with roughness heights of 3mm and a grid spacing of 16.5mm. The boundary layer in Rusche was 60mm thick and the edge velocity was 26.5m/s. The spectra in Figure 4-6 show the streamwise variation through the fetch of roughness. The low frequency regions of the recorded spectra show some differences between the two flows. The wall pressure fluctuations at position C2, one row into the roughness, do not rise significantly above the levels recorded in front of the fetch, C1, for the conventional turbulent boundary layer. Also, the data of Rusche (2011) do not show the same increased pressure fluctuations behind the fetch indicating a separated boundary.
layer. This is most likely due to the larger boundary layer thickness in Rusche’s experiment, 60mm, compared to the wall-jet’s 16.4mm thickness at $U_o=60m/s$.

![Figure 4-5. Comparison of measured normalized wall pressure spectra from a fetch of 3mm cuboidal roughness compared to similar cuboidal roughness relative locations from the computational study of Yang & Wang (2011)](image)

![Figure 4-6. Normalized wall pressure spectrum through a fetch of 3mm cuboidal roughness compared to the streamwise variation presented in Rusche (2011)](image)
To apply Glegg & Devenport’s (2009) theory of roughness noise as given in Equation 3-19, a representative single point wall pressure spectrum is needed, particularly, a spectrum that is representative of the portion of turbulent pressure fluctuations that are scattered into radiated noise. Since this large element fetch has a highly inhomogeneous wall pressure field, no single point wall pressure spectrum could perfectly represent the portion of wall pressure fluctuations scattered to the far field. Instead, an approximation is assumed using the average of the wall pressure spectra measured at each of the presented locations. The average wall pressure was calculated using the recorded pressures at positions in Figure 4-4 along with the upstream and downstream fetch locations, C1 and C5, for nozzle exit velocities between 20 and 60m/s. These averaged spectra are shown in Figure 4-7. The recorded pressures at positions C2, C4, and C6 to C9 were ignored so that the central location between elements was not biased over other positions in the average.

Another approach would be to use the wall pressures recorded at the C11 position to represent the scattered turbulent fluctuations. According to Yang & Wang (2009), the pressure fluctuations on the front of the roughness element are the most efficient scatterer of sound in the streamwise direction. These pressure fluctuations recorded at various nozzle exit velocities are also shown in Figure 4-7 compared to the averaged spectra.

![Figure 4-7. Wall pressure spectra for cubic element fetch at various nozzle exit velocities recorded at position C11 (dot) and averaged over the surface (solid) compared to the smooth wall (dash)](image)

4.1.2 Far Field Noise

Far field noise was recorded during each separate wall pressure measurement. The far field microphone was located upstream of the roughness at $x=1029$mm, $y=469$mm and $z=0$mm. The position of the roughness fetch was shifted for each wall pressure measurement so that the lead row of roughness began at $x=1255$mm to 1366mm and the spanwise center varied from $z=-92.5$mm to 6.5mm. Even though the fetch location was shifted for each near field measurement, the recorded far field noise was not significantly altered. The measured noise from the cubic element fetch is shown in Figure 4-8a compared
to the background levels of the tunnel. At 20m/s there is very little significant sound produced by the 3mm cubic roughness fetch. As the nozzle velocity is increased, the signal-to-noise ratio increases to a maximum of 18dB at \( U_o = 60 \text{m/s} \). Figure 4-8b shows the background-subtracted noise. To reduce uncertainty of the presented data, only roughness noise with a signal-to-noise ratio over 1dB is plotted. The uncertainty for the subtracted noise is 3.5dB, 1.5dB, and 1.1dB for regions with signal-to-noise ratios of 1dB, 5dB, and 10dB, respectively. As the nozzle velocity is increased from 20-60m/s, the maximum recorded far field produced by the rough surface increases by 26dB.

![Figure 4-8. Far field sound from cubic element fetch at various nozzle exit velocities compared to the background noise a) raw b) and background subtracted](image)

### 4.1.3 Radiated Noise Normalization and Estimate Comparison

Figure 4-9 and Figure 4-10 show the far field noise of Figure 4-8b normalized on the average wall pressure spectra and the wall pressure measured just in front of the roughness element, respectively. The averaged wall pressure spectra collapse the radiated noise on a slope of \( \omega^{1.25} \). The single-point wall pressure spectra recorded in front of the element produces an \( \omega^2 \) collapse over the majority of the analyzed frequency range. As the frequency increases above 4kHz, the collapse begins to break down and the data fan out in order of velocity using both methods. These measured normalizations have uncertainties of 3.64dB, 1.8dB, and 1.5dB for regions where the measured far field has a signal-to-noise ratio of 1dB, 5dB, and 10dB, respectively.

Noise estimates computed similarly as before in Chapter 3 are shown on each figure for comparison assuming acoustically compact sources. These estimates use Chase’s (1980, 1987) wavenumber wall pressure spectrum and the local conditions for nozzle exit velocities of 40 and 60m/s for a smooth wall at \( x = 1257 \text{mm} \). This includes the convection velocities as determined by Devenport et al. (2010) of 44% and 41% for \( U_o = 40 \) and 60m/s, respectively. The estimations have been calculated for the far field measurement location using observer distances in Equation 3-1 measured relative to the spanwise center of the plate at \( x = 1257 \text{mm} \).
The estimated normalized spectral shapes more closely resemble the measured normalization in Figure 4-9 using the averaged wall pressure spectra, although, the absolute results are approximately 5-9dB below the measured normalization. The error could be due to the assumption of a homogeneous wall pressure field inherent in Equation 3-19 which is obviously not the case as shown in Figure 4-4. The absolute levels of the normalized curves will be highly dependent on the measurement location of the single point wall pressure spectrum chosen to collapse the data. The 40m/s and 60m/s estimations do not produce a collapse as tight as the measured spectra, but only differ by approximately 2dB above 6kHz. At frequencies below 6kHz, the normalizations have large humps in them created by the convective ridge of the wall pressure wavenumber spectrum as it amplifies various regions of the surface slope wavenumber spectrum at different frequencies. The two estimated spectra only collapse within 5dB in this region. At higher frequencies, the convective ridge is much broader amplifying larger regions of the wavenumber space of the surface slope and the estimated normalized result becomes more linear.

The assumption of a homogeneous wall pressure spectrum depends on the scale of the pressure fluctuations scattering to the far field. The dominate portion of these scattered fluctuations are those on the convective ridge of the wavenumber wall pressure spectrum. Therefore, fluctuations at 500Hz and 20kHz are dominated by structures with sizes of 18mm and 0.45mm, respectively. The cubic roughness has dimensions of 3mm. Eddies of a similar size would pass at a frequency of 3kHz. Consequently, the assumption of a homogeneous wall pressure spectrum may be more valid at frequencies below 3kHz where the fluctuations are larger than the roughness elements. There isn’t a clear difference between the higher and lower frequency regions showing that the estimated values are closer to the measured normalizations at lower frequencies in Figure 4-9 which uses the average wall pressure spectra, but Figure 4-10, using the single point measurement, shows a clear deviation from the estimation at approximately 3kHz. The lower frequency side of the measured data in both figures has a higher uncertainty, near 3.64dB, slightly less than the difference in the spectra showing that these estimations are quite reasonable below 3kHz. Dividing the spectra in this way according to structure size implies that the low frequency fluctuations encountered by the element would be dominated by the large eddies in the turbulent boundary layer and that the high frequency would be attributed to local effects of the roughness. The shapes of the measured spectra are correctly estimated using the average wall pressure spectra. This suggests differences in the long eddy wavenumber regions and short eddy wavenumber regions associated with the average local flow must still behave in a form consistent with the Chase wavenumber wall pressure spectrum having a convection velocity that is approximately 41% of the edge velocity.
Figure 4-9. Measured far field normalization using average wall pressure compared to estimation

Figure 4-10. Measured far field normalization using single point wall pressure compared to estimation
4.2 Measurements from 42 Element 3mm Hemispherical Roughness

4.2.1 Wall Pressure

A similar analysis of the wall pressure variation throughout a 42 element fetch of 3mm hemispheres was conducted. The wall pressure measurement locations are given in Figure 4-11. Figure 4-12 shows the streamwise and spanwise variation through the roughness fetch at similar locations as measured in the cubic element fetch. Figure 4-12a shows, as the streamwise position increases, the wall pressure fluctuations increase as well until midway through the fetch at position H3. Beyond this streamwise point there is little variation in the measured spectra. Downstream of the fetch at position H5, the pressure fluctuations remain the same as the upstream values. An increase at all frequencies, as observed in the cubic element fetch, is not measured. This indicates that a similar boundary layer displacement effect may not be present for the hemispherical fetch. This agrees qualitatively with the findings in Yang & Wang (2011) that observes the difference between fetches of hemispherical and cuboidal roughness using LES. The LES shows that the cuboidal roughness creates a larger disturbance in the flow having a bigger separation region downstream of the elements.

There is little spanwise variation of the wall pressure as measured through the center of the roughness fetch shown in Figure 4-12b. Similar to the cubic roughness, position H8, which is bound by roughness elements in only one spanwise direction, has a slightly suppressed lower frequency region and increased higher frequency region. The measurements at all frequencies are 2 to 5dB above the smooth wall pressure fluctuations.

![Figure 4-11. Wall pressure measurement locations inside 42 element fetch of 3mm hemispheres](image)
The uniformity of the spanwise measured wall pressure enables a comparison of the varying element relative positions as if they were recorded in relation to similar elements. Figure 4-13 shows a comparison between a measurement at a central position of the roughness fetch, H3, and a position located in-line streamwise between two elements, H9. The wall pressure fluctuations are greater for all measured frequencies in the wake of an element at H9 by up to 9dB. This is consistent with the findings of Yang & Wang (2010) which show that the RMS pressure fluctuations are greater in the wake of the elements than between streamwise columns. Again, this figure highlights the great inhomogeneity of the wall pressure fluctuations in large element surfaces as compared to the previously studied small boundary layer to roughness height ratio stochastic roughness in Alexander (2009) and Devenport et al. (2011).

Figure 4-14 shows a comparison of the measured data with the streamwise variation through a hemispherical roughness fetch with equivalent dimensions as recorded by Rusche (2011). Rusche shows a dip in the high frequency pressure fluctuations downstream of the lead row of roughness at a non-dimensional frequency of 10 that is not observed in the wall-jet data. The low frequency region of Rusche’s spectra do not sequentially increase with streamwise distance into the roughness fetch, but the large fluctuations in his presented spectra, especially for the H2 position, suggest that the trend differences with the wall-jet may be due to uncertainty associated with the measurements given in Rusche (2011).
Figure 4-13. Wall pressure at various positions relative to nearest 3mm hemispherical element in 42 element fetch at $U_o=60\text{m/s}$

Figure 4-14. Comparison of the normalized wall pressure spectrum from the fetch of 3mm hemispherical roughness with the study of Rusche (2011)
The average wall pressure over the hemispherical element fetch was calculated using the spectra recorded at positions H1, H3, H5, and H9 for each nozzle exit velocity. These average spectra are compared to the measured smooth wall undisturbed pressure fluctuations in Figure 4-15. The smooth and rough wall spectra have a similar shape but are increased by approximately 4-5dB for all frequencies and velocities. These calculated average wall pressures will be used to normalize the recorded far field spectra from the hemispherical roughness.

Figure 4-15. Average wall pressure spectra for hemispherical element fetch at various nozzle exit velocities averaged over the surface (solid) compared to the smooth wall (dash)

4.2.2 Far Field Noise

The hemispherical element fetch produced very little measurable far field noise discernable from the tunnel’s background noise. Figure 4-16 shows the raw and background subtracted noise from the hemispherical element fetch as measured from the same far field position as the cuboidal roughness at \( x=1029\text{mm}, y=469\text{mm} \) and \( z=0\text{mm} \). Again, the location of the lead spanwise center of the fetch varied over \( x=1255\text{mm} \) to \( 1366\text{mm} \) and \( z=-92.5\text{mm} \) to \(-10\text{mm} \) and made a negligible difference in the recorded far field. The signal-to-noise ratio was very low for this surface, so that Figure 4-16b uses a lower limit of 0.5dB to exclude noise from the presented data instead of the 1dB limit that was used for the far field results from the cuboidal roughness. This lower tolerance is responsible for the noisier appearance of Figure 4-16b as compared to Figure 4-8b. This increases the maximum uncertainty level of the presented spectra in Figure 4-16b to 5.3dB for frequencies where the signal-to-noise ratio is 0.5dB.
4.2.3 Radiated Noise Normalization and Estimate Comparison

Using the averaged measured wall pressure, the far field/near field normalization is computed in Figure 4-17 for the 42 element fetch of 3mm hemispherical roughness. The normalization is flat unlike the cuboidal roughness results. These normalized results are similar to those found in Alexander (2009) from a 3mm hemispherical surface. In this previous study, the reason for the flat spectral shape and lack of collapse was attributed to an inaccurate representation of the inhomogeneous wall pressure field of the surface. Again, shown on Figure 4-17, Glegg & Devenport’s (2009) roughness noise formulation was implemented to provide estimates of the normalization using the Chase single point wall pressure spectra for nozzle exit velocities of 40m/s and 60m/s. Like the cuboidal roughness fetch, the estimated results correctly predict the shape of the spectra but do not accurately predict the magnitude. The estimated and normalized spectra only agree within 11dB at lower frequencies, but are more accurately predicted at higher frequencies above 7kHz. These measured spectra have a higher uncertainty than the cubic fetch data presented in Figure 4-9 due to the extended lower limit of the signal-to-noise ratio of the presented data, 0.5dB. For the spectra in Figure 4-17 corresponding to regions where the signal-to-noise ratio of the far field data is approximately 0.5dB, the uncertainty is as large as 5.5dB. This resolves some of the difference between measurement and estimation. Unfortunately, there is not a sufficiently large frequency range of measurable roughness noise to make any further conclusions on the quality of the prediction, specifically regarding the relation of the 40m/s and 60m/s normalizations, but clearly the functional relationship of the roughness geometry and radiated far field noise is being captured by the theory of Glegg & Devenport (2009) to predict this dramatic shift in expected normalized shape between the cuboidal and hemispherical roughness. Combined with the results of Figure 4-9 and Figure 4-10 for the cuboidal roughness, these estimations show that predictions of roughness noise for surfaces with even large \( h/\delta \) can produce reasonable results within approximately 5dB for regions where the turbulent structures of the boundary layer are significantly larger than the roughness elements.
4.2.4 Comparison of 42 Element Surfaces of Hemispherical and Cuboidal Roughness

Figure 4-18 shows a comparison between similar element relative wall pressure measurement locations, between the final two spanwise rows of the roughness centered between elements, in the cuboidal and hemispherical roughness fetches. The low frequency fluctuations up to 5kHz are approximately equivalent. Above 5kHz, the pressure fluctuations produced by the hemispherical surface rise above the cubic element spectra by 5dB. This could be due to the proximity of the wall pressure measurement location to the roughness elements. Although the roughness heights and grid spacings are equivalent between the two surfaces, the base of the hemispherical elements is twice that of the cuboidal elements. Therefore, the space between streamwise columns of hemispherical roughness is 3mm narrower than the cuboidal roughness. This difference in relative distances may influence the results above 5kHz, especially since the size of the eddies in this frequency range are smaller than the roughness elements.

The far field sound from the cuboidal roughness is greater than that produced by that hemispherical roughness for the entire frequency range where noise was discernable above the background as shown in Figure 4-19. The difference increases with frequency by up to 18dB at 20kHz. At lower frequencies, below 1kHz, the noise from the hemispherical elements may surpass that of the cuboidal roughness, but no definitive conclusion can be made about the shape of the spectra in this region. Referring back to the measured wall pressure spectra in Figure 4-18, it can be concluded that the far field sound of a discrete large element rough surface cannot be inferred from the surrounding wall pressure alone. The hemispherical roughness produces more intense pressure fluctuations in the same frequency range that the cuboidal roughness has an 18dB advantage in the far field.
Figure 4-18. Comparison of wall pressure in cubic and hemispherical element fetch at $U_o=60\text{m/s}$

Figure 4-19. Background subtracted far field noise from cubic and hemispherical roughness fetch at $U_o=60\text{m/s}$
4.3 Measurements from 610mmx305mm Fetch of 3mm Hemispherical Roughness

4.3.1 Wall Pressure

A closer analysis of the variation in wall pressure around 3mm hemispherical elements was conducted using a 610x305mm fetch with 703 elements arranged with the same grid spacing as the 42 element fetch, 16.5mm. The wall pressure measurement locations are shown in Figure 2-27a. These element relative measurement locations are at a greater distance into the rough surface as compared to the similar measurements in the 42 element fetches, roughly 140mm as compared to 50mm from the leading edge. The wall pressure results from these positions at $U_o=60$m/s are shown in Figure 4-20. Locations LH7 and LH8 are omitted from this figure because these results were equivalent to those recorded at locations LH2 and LH1, respectively, due to the symmetry of the flow. Like the 42 element fetch, the local elements dominate the surrounding wall pressure field producing widely varying spectral shapes and magnitudes over the roughness area. The spectra vary less at lower frequencies especially below 1kHz where the structure scale on the convective ridge have length scales of approximately 9mm, much larger than the size of the roughness. These larger eddies are less impacted by individual roughness elements, and therefore, the measured wall pressure spectra in this region should be more uniform. The magnitudes differ by up to 12dB at 20kHz depending on the measurement location. The spectra at positions LH3 and LH5 may indicate a separation region just upstream and downstream of the roughness elements which produce greater low frequency fluctuations with high frequencies similar to or below the smooth wall spectrum. Unlike the pressure spectra recorded from the cubic roughness, the upstream position produces stronger pressure fluctuations for all frequencies than at a similar distance downstream of an element.

Figure 4-21 displays a wall pressure comparison between similar roughness element relative measurement positions for the 42 element and 703 element fetches. Although very little difference was observed in the streamwise development of the wall pressure spectrum beyond the halfway point of the roughness in Figure 4-12a, the comparison in Figure 4-21 shows some variation between the recorded spectra at the H3 and LH4 measurement positions which only differ in streamwise distance from the leading edge. The difference between the two spectra is no greater than 3dB. The difference between the H9 and LH6 positions is greater. These positions are recorded in-line streamwise between two roughness elements. The spectra are similar at lower frequencies but differ by 4dB above 5kHz. These results indicate that the flow is still developing over the 42 element fetch only at a much slower rate than observed over the initial four rows of roughness as measured in the 42 element fetch.
Figure 4-20. Wall pressure variation around elements in the 610x305mm element fetch of 3mm hemispherical roughness at $U_o=60\text{m/s}$

Figure 4-21. Wall pressure spectra comparison of 42 element and 703 element fetch of 3mm hemispheres at $U_o=60\text{m/s}$
4.3.2 Far Field Noise

The far field noise, as recorded from $x=1029\,\text{mm}$, $y=469\,\text{mm}$ and $z=0\,\text{mm}$, produced by the larger fetch of 3mm hemispheres is shown in Figure 4-22. The spanwise center of the lead row of roughness was located at $x=1255\,\text{mm}$ to $1271\,\text{mm}$ and $z=-66\,\text{mm}$ to $66\,\text{mm}$. The increase in number of roughness elements increases the level of radiated noise, therefore, only data with a signal-to-noise ratio above 1dB is shown in Figure 4-22b. This is the same far field measurement position as used to record the far field data from the 42 element fetches. At nozzle exit velocities of above $40\,\text{m/s}$ there is a broad spike in the data at $13\,\text{kHz}$. The magnitude of the lump increases with velocity, but the frequency does not vary. This is a similar phenomenon as observed in Alexander (2009). Alexander (2009) recorded a pair of these spectral peaks at $9\,\text{kHz}$ and $13\,\text{kHz}$. An explanation of these peaks could not be given and were thought to be an error in the measurement, but the consistency of the occurrence indicates that it is a function of the surface. The surfaces used in each study contained exactly the same number of 3mm hemispherical elements with the same element spacing. The only difference between the surfaces was the manufacturing process. The surfaces used in this study were made from epoxy molded onto a Kevlar backing producing a substrate 0.30mm thick. The surfaces used in Alexander (2009) were created by molding a thicker rubber material and therefore had a 1.6mm thick substrate.

![Figure 4-22. Far field sound from 3mm hemispherical element fetch at various nozzle exit velocities compared to the background noise a) raw b) and background subtracted](image)

The same spectral peak at $13\,\text{kHz}$ is not seen in the recorded far field noise from the 42 element fetch of 3mm hemispherical roughness, but the cuboidal roughness produces a very slight lump in the far field spectra at a similar frequency as shown in Figure 4-8b. Figure 4-23 shows a comparison between the 42 element fetch and 703 element fetch of 3mm hemispherical roughness at a nozzle exit velocity of $60\,\text{m/s}$. The magnitude of the far field noise is increased with the addition of more elements by approximately a factor of four while the number of elements increased by a factor of nearly seventeen. Therefore, unlike the far field noise recorded by Rasnick (2010) from large rock and cubic roughness,
these data from 3mm hemispherical roughness would not scale similarly on number of elements even within 3-4dB. As observed in the previous wall pressure measurements of Figure 4-21, the flow is still developing in the streamwise direction through the 42 element fetch. Therefore, the trailing elements in the larger 610x305mm fetch of roughness may produce different noise spectra than that of the leading elements as the flow develops through the roughness in the streamwise direction. Rasnick (2010) collapsed the noise from 610x305mm fetches of roughness with heights double the size of the 3mm hemispheres. Use of these larger elements may explain the unvarying per element source strength calculated in his study for multiple fetch sizes. The larger elements may have a more even source strength distribution because their noise is dominated by self-generated unsteady forces and not a scattering mechanism. Rasnick examined only smaller fetches of 3mm roughness elements and therefore may not have measured this change in streamwise source strength due to the developing flow field.

![Figure 4-23. Far field comparison of noise from 42 element and 703 element fetch of 3mm hemispheres at $U_o=60$ m/s](image)

### 4.3.3 Radiated Noise Normalization and Estimate Comparison

The normalizations of the measured far field spectra on the average wall pressures recorded at positions LH1 through LH6 are shown in Figure 4-24 for multiple nozzle exit velocities. Again, the estimated spectra using Glegg & Devenport (2009) are shown alongside the measured data. For this larger fetch the estimated normalization more accurately predicts the observed data below 3kHz, where turbulent structures are larger than the roughness height, but estimations differ from measurement by up to 9dB at higher frequencies. Still, these differences are reasonable considering the inhomogeneity of the wall pressure fluctuations and that even approximate estimations of roughness noise have been unattainable until now. The levels of the average measured wall pressure spectra changed similarly to the levels of the measured far field in relation to the results from the 42 element fetch. Therefore, the measured normalized values of the two rough surfaces remained closely the same magnitude. The
estimated spectral levels increased because the magnitude of the wavenumber filter function, Γ, was increased due to the addition of more roughness elements. Although, the agreement between measurement and estimation seems to have increased for low frequencies, this is a poor indicator of the accuracy of this method. Calculating the measured far field normalizations for these large discrete element surfaces that produce highly inhomogeneous wall pressure fields entails choosing the proper single point wall pressure spectrum to collapse the data. This choice greatly impacts the level of the normalization. If the average used to normalize Figure 4-24 included the wall pressure at the front of the fetch, which has a magnitude comparable to the smooth wall spectrum, the disagreement between estimation and measurement would change. Therefore, the absolute level of the measured normalization is highly dependent on the chosen normalization spectrum.

The estimation fails to reproduce the peak created at 13kHz. The measured normalization produces the tightest collapse in this region, 6-20kHz, disregarding the peak magnitude at 13kHz. This spike may be an effect of the roughness geometry manipulating the flow field. This type of effect would not be reproduced by the simplified scattering theory of Equation 3-1, especially when applying a homogeneous smooth wall wavenumber spectrum to estimate the normalized results. Since the spike frequency does not depend on velocity, it must be related to a characteristic dimension of the surface. The characteristic acoustic lengthscale of a 13kHz signal is approximately 26mm. This is almost twice the element spacing of the rough surface but does not have a direct relation with any physical size of the surface. Also, this frequency gives a Strouhal number based on element height of 1.8 for \( U_o = 60 \text{m/s} \) which is exceedingly higher than the expected value for bluff body shedding, roughly 0.2. Aerodynamic measurements by Morton (2011) over the 42 element fetches show that the \( \overline{v^2} \) and \( \overline{u^2} \) turbulence levels, corresponding to the \( y \) and \( x \) directions, respectively, increase with streamwise distance through the fetch, but the \( \overline{w^2} \) values, corresponding to the \( z \) direction, decrease. This suggests the flow is being channeled between rows. Although the 13kHz spike was not seen for the 42 element roughness, the increased streamwise length of the larger fetch may allow additional development of this effect creating this resonant peak.
Figure 4.24. Measured far field normalization using average wall pressure from 610x305mm fetch of 3mm hemispherical roughness compared to estimation

4.4 Measurements from 610mmx305mm Fetch of Ordered 1mm Hemispherical Roughness

4.4.1 Wall Pressure

The inhomogeneous wall pressure fluctuations generated by the 3mm roughness made the application of Glegg & Devenport’s (2009) theory given in Equation 3-1 contradictory to the main assumption in its derivation that the wall pressure field was uniform over the rough surface. To improve the comparison between the experimental conditions and the assumptions inherent in the applied theory, the roughness size was reduced to create a more homogeneous wall pressure field. Fetches of 1mm hemispherical roughness were chosen as this element size provided a deterministic roughness on the scale of the largest stochastic roughness studied in Alexander (2009), which was a 20 grit sandpaper surface. The sizes of the elements (1mm) and their spacing (5.5mm) were scaled similarly, but the number of elements was increased to 5886 keeping the planar area of the roughness fetch the same. Four wall pressure measurement locations were measured inside of this fetch and are diagramed in Figure 2-27b. Figure 4-25 shows the wall pressure spectra recorded at these four locations for $U_o=60$ m/s. At this condition, $h^+=66$ and therefore, this flow is still in the fully rough regime, but $h/\delta$ is reduced from 18% for the 3mm case to 6%. The wall pressure spectra are not completely homogeneous, but show less variation than from the larger hemispherical roughness. Although, measurements were not completed at all the same locally scaled distances around individual elements as in Figure 4-20 for the 3mm hemispherical roughness. For the measured locations, the wall pressure spectra above 3kHz begin to differ by approximately 3dB. Eddies 1mm in size are associated with frequencies of 9kHz. The observed
effect of the roughness location extends down to 3kHz which corresponds to structures three times as large as the roughness height. At lower frequencies, the spectra appear to be uniform as expected.

The only direct comparison between similarly scaled element relative locations for the 3mm and 1mm 610x305mm hemispherical element fetches can be made between the LH4 and SH3 positions, which were both centered among a group of 4 elements. This comparison is shown in Figure 4-26. The wall pressure spectrum in the 1mm hemispherical roughness does provide less of a disturbance from the smooth wall at frequencies below 3kHz. Above this, both rough surfaces produce similar spectra. It was assumed that the smaller disturbances generated by the 1mm roughness translated to smaller disturbances on the wall pressure field as a whole. In this way, the wall pressure field could more justifiably be approximated as homogeneous over the considered frequency range. This assumption includes the wall pressure field over the surface of the roughness elements since this would be the part of the wall pressure field associated with the unsteady drag radiated to the far field.

Figure 4-25. Wall pressure variation in a grid patterned fetch of 1mm hemispherical roughness at $U_o=60m/s$
4.4.2 Far Field Noise

The far field noise was measured in an upstream position, the same as used for the previously presented far field measurements, and is shown in Figure 4-27. The lump at 13kHz, as observed from the 3mm hemispherical roughness in Figure 4-22b, is not seen for this surface, but if the phenomenon is a function of the roughness geometry or manipulation of the flow around the large roughness elements, it should not be expected for this smaller element roughness. If the broad spike in Figure 4-22b were to have a similar dimensional scaling of its wavelength as compared to the roughness size and spacing for this surface of 1mm hemispheres, the frequency of the phenomenon would be beyond the limits of this figure at 39kHz. There was very little signal-to-noise ratio at a nozzle exit velocity of 20m/s, but the noise shows a clear rise from the background at all other velocities. The maximum far field noise and peak frequency increase with velocity, similarly to the noise from the previously studied surfaces, but in this case, the shape of the spectra also change. A lump begins to appear at higher velocities between 2kHz and 3kHz altering the shape of spectra. As is all roughness noise, this lump is either generated by the unsteady loading due to vortex shedding or a scattering mechanism due to the imposed long wave pressure fluctuations of the overriding boundary layer flow. If the simplified theory of Glegg & Devenport (2009) correctly predicts this result than the scattering mechanism is confirmed.
4.4.3 Radiated Noise Normalization and Estimation Comparison

The subtracted radiated far field noise in Figure 4-27b was normalized on the average wall pressure spectra computed from the four measurement positions for each nozzle exit velocity. The smaller roughness produces an $\omega^2$ region that then deviates from this slope at approximately 2kHz. Above 2kHz, the data fan out in order of velocity. The estimations using Glegg & Devenport (2009) are very accurate over the measured frequency range within 5dB of the measured normalization at all frequencies and show a similar spectral behavior. Although the lump in the 60m/s spectra is not exactly predicted, the estimated 60m/s spectrum comes very close to recreating this behavior. The estimated spectrum produces a wavy normalized spectral shape that resembles the measured spectrum. A similar wavy pattern is estimated for the 40m/s spectrum but is shifted to lower frequencies. This may explain the presence of the suspect lump in the 60m/s spectrum that is absent for the 40m/s case. The lump may be beyond the extent of the measured data. The estimated and measured spectra are very similar and indicate that the sound production mechanism for roughness that is a small fraction of the boundary layer thickness is correctly captured by Glegg & Devenport’s (2009) scattering theory. Differences between the estimated and measured spectra may be attributed to inaccurate modeling of the wavenumber wall pressure spectrum. Like the analysis in Devenport et al. (2010) using a wavy wall roughness, these rough wall results may provide an opportunity to probe the wavenumber wall pressure spectrum’s form in the low wavenumber region.
4.5 Measurements from 610mmx305mm Randomly Distributed 1mm Hemispherical Roughness

4.5.1 Wall Pressure

The wavenumber spectra of the ordered arrays are discrete at multiples of the element spacing wavenumbers. A randomly distributed roughness of 1mm hemispheres was tested to examine the difference between the continuous wavenumber spectrum of a random surface and these ordered surfaces. This is also a more accurate representative case of roughness encountered in engineering applications which are likely to be random in nature. The element locations were chosen randomly but their locations were known so that the exact analytical Fourier transform could still be determined as presented in Section 3.4.

The wall pressure was only recorded in one central location of the random 1mm hemispherical element fetch due to limits imposed by the size of the surface pressure microphone. This location was central to the roughness fetch and is shown in Figure 2-24. The position is 163.5mm from the leading edge of the roughness. The wall pressure fluctuations recorded in this location resemble the smooth wall spectral shapes but are elevated by 4-7dB for all velocities and frequencies.
Figure 4-29. Wall pressure spectra for 1mm hemispherical element fetch at various nozzle exit velocities (solid) compared to the smooth wall (dash)

Figure 4-30 is a comparison of the wall pressure measured inside of the fetch of randomly located 1mm hemispherical roughness to that of the averaged wall pressure spectrum from the ordered 1mm roughness. The fluctuations at this single position are slightly elevated from this averaged spectrum but the two spectra have approximately the same shape.

Figure 4-30. Comparison of average wall pressure measured in ordered fetch of 1mm hemispherical elements and the wall pressure measured inside of the random 1mm element fetch at $U_o=60m/s$
4.5.2 Far Field Noise

Far field noise from the random fetch of 1mm hemispherical roughness recorded upstream of the roughness at the same location as presented for the other surfaces, \(x=1029\text{mm}, y=469\text{mm}\) and \(z=0\text{mm}\), is displayed in Figure 4-31. The background-subtracted spectra have a different spectral shape than that of the ordered fetch of 1mm hemispherical roughness.

![Figure 4-31. Far field sound from randomly distributed 1mm hemispherical element fetch at various nozzle exit velocities compared to the background noise a) raw b) and background subtracted](image)

Figure 4-32 shows a comparison of the background noise from the two 1mm element fetches at \(U_o = 40\text{m/s}\) and \(60\text{m/s}\). The random fetch has slightly more elements 6209 as opposed to 5886 in the ordered fetch, but the spectra are not just scaled versions of each other due to this difference in number of elements. The disagreement between the far field spectra grows with frequency. The ordered fetch has a steeper roll-off with frequency so that the far field from the random fetch is 4dB greater than that of the ordered fetch at 20kHz for the \(U_o = 60\text{m/s}\) spectrum. Also, the lump in the ordered \(U_o = 60\text{m/s}\) spectrum between 2kHz and 3kHz is not present in the random fetch spectrum. Therefore, this demonstrates that the element size is not the only important factor that determines the level of the radiated noise, but that the roughness element configuration is a formative property in predicting the radiated levels.
4.5.3 Radiated Noise Normalization and Estimation Comparison

The far field noise of Figure 4-31 can be normalized on the measured wall pressure spectra at all velocities producing the collapse shown in Figure 4-33. This normalization works poorly to collapse the data onto a single curve and fans out in order of velocity above 2kHz as observed for the ordered fetch of 1mm roughness, but this clear \( \omega^2 \) normalization and break frequency resemble the break frequency discussed in Alexander (2009) with respect to stochastic roughness. The estimated spectra shown on this figure predict the spectral values remarkably well. The spectral shape is correctly estimated with the absolute predicted the levels within 2dB below 2kHz. Above this, the difference between estimation and measurement grows up to 5dB, but as stated earlier, the difference may be attributed to inaccurate modeling of the wavenumber wall pressure spectrum. The estimated spectra appear to roll-off at a lower frequency. Although, the estimated normalizations show that the break frequency is a predictable function of the wavenumber wall pressure spectrum and the roughness geometry. The relative difference in break frequency is correctly predicted for the 40m/s and 60m/s spectra as well as the shape of the break. The estimations fan out in order of velocity with the higher velocity rising above and holding on to the \( \omega^2 \) shape slightly longer than that of the slower case.

Figure 4-34 illustrates the reason for the break frequency. This figure shows a slice through the wall pressure wavenumber spectrum at 3000Hz and 500Hz for the local conditions at the leading edge of the roughness fetch with \( U_o = 60m/s \). The peak produced in the map corresponds to the convective ridge. At these frequencies, the convective ridge peaks at a wavenumber of \( \kappa_x=2100 \) rad/m and \( \kappa_x=350 \), respectively. Figure 4-34 also shows the filter function, \( \Gamma \), from Equation 3-2 derived from the Fourier transform of the surface slope for the random fetch of 1mm hemispheres. The voids in \( \Gamma \) are a function of the diameter of the roughness so that the 3mm hemispheres would have three times as many voids over
the same wavenumber space. The filter function is an exact Fourier transform calculated by displacing the Fourier transform of a single hemisphere’s surface slope as calculated in Chapter 3 using the known location of each roughness element. Highlighted on this figure are two locations corresponding to the peak of the convective ridge of the wall pressure wavenumber spectrum for two different frequencies, 500Hz and 3000Hz. Increasing from 500Hz, the convective ridge peaks at lower wavenumbers and corresponds with an increasing region in the surface slope wavenumber spectrum. At 3kHz, the convective ridge aligns with a maximum in the surface slope wavenumber spectrum. As the frequency is increased further, the convective ridge will coincide with a sharp decline in the surface slope wavenumber spectrum associated with increasing $\kappa$. The shape of the estimated normalization is highly dependent on the interaction between the integrated product of the $\Gamma$ function and the shape of the convective ridge at each frequency. Referring back to Figure 4-33, the estimated spectrum for the $U_o = 60$m/s case breaks from the linear slope at approximately 3kHz precisely where the plot of the $\Gamma$ function indicates it should. To more accurately match the measured 60m/s normalized spectrum, this break needs to be shifted to a higher frequency. This could be accomplished by changing the characteristics of the model wavenumber wall pressure spectrum so that the integrated value of the normalized wavenumber pressure spectrum, $\Psi$, and wavenumber filter function of the surface, $\Gamma$, from Equation 3-2 peak at a higher frequency.

Figure 4-33. Measured far field normalization compared to estimation
Although the magnitudes of the estimations differ from measurement for the larger deterministic roughness, the accuracy of the predicted spectral shapes (and magnitudes for the $h=1$mm surfaces) for all of the different rough surfaces and velocities indicates that the produced far field noise, even from these transitionally to fully rough surfaces, is highly dependent on the wavenumber spectrum of the surface slope. Also, the form of the unsteady drag on the roughness elements associated with the self-generated and incoming wall pressure field is characterized well by the Chase (1980, 1987) wavenumber wall pressure spectrum. The full unreduced form of Glegg & Devenport (2009), Equation 1-3, which does not assume a homogenous wall pressure spectrum, cannot be used experimentally because it would involve integrating the wavenumber pressure spectrum over the entire rough surface. This is not currently feasible due to technological limits on sensor size. Using the simplified version, given in Equation 3-1, the error in magnitude between measurement and estimations appears to rely on the accuracy of the assumption of a homogeneous wall pressure field. Although the roughness elements do produce self-generated turbulence, the wavenumbers important in generating the observed far field noise are in the low wavenumber range 2000 to 14000 rad/m associated with disturbances $\frac{1}{2}$ to 3 times as large as the 1mm roughness. Therefore,
the majority of the observed noise spectra from these 1mm surfaces are due to the incoming pressure field from large upstream disturbances, relative to the size of the roughness elements, being scattered by the rough wall geometry.

The single point wall pressure spectrum produced by the integration of the Chase’s wavenumber spectrum is compared to the measured smooth wall undisturbed wall pressure fluctuations and the averaged wall pressure spectrum for the 42 element cubic roughness fetch case in Figure 4-35. The Chase model does not incorporate viscous terms and therefore does not produce the $\omega^{-5}$ roll-off at high frequencies associated with the dissipation region. The shape of the lower frequency region is captured, but the estimated results fall below the measured smooth wall pressure spectra for both the 40m/s and 60m/s cases. The estimated and measured smooth wall spectra differ by approximately 4-5dB over this range except at the lowest frequencies of the 60m/s spectra where the difference grows to 9dB. The Chase spectra perform much worse representing the averaged measured rough wall spectra. Differences are as great as 12dB. The slopes are approximately the same at the lowest frequencies, but again the estimates do not represent the shape of the dissipation region. Even though Chase’s formulation does a poor job of estimating the single point rough wall pressure spectra, the model wavenumber wall pressure spectrum still accurately reproduces the shape of the far field normalizations for all roughness geometries and sizes. These estimations are accurate because the single point wall pressure spectrum is normalized out of the equation and therefore Chase’s single point model does not directly factor into the result.

Referring to Equation 3-1, the $\Psi$ function, which contains the wavenumber wall pressure term, is normalized on the single point wall pressure spectrum. This means that the integrated value of the wavenumber wall pressure spectrum does not factor into the normalization of the far field because the $\Psi$ function will always have an integrated value of unity at each frequency. The $\Psi$ function’s only operation is essentially that of a shape function that amplifies different regions of the surface slope wavenumber spectrum that creates the resulting spectral form of the normalization. Minor adjustment of the
wavenumber wall pressure spectrum cannot increase or decrease the levels of the estimated far field normalization significantly without a filter function, \( \Gamma \), that varies dramatically over wavenumber space aligning perfectly with the convective ridge. Simple manipulations of Chase’s spectrum were examined to study the effects on the normalized outcome.

Chase (1980, 1987) gives suggested values for the constants in Equation 3-22. If the constant \( \mu \) from Equation 3-23 is assumed to be the same as used in Chase (1987), 0.176, instead of being determined from the constant value of \( h_c=3 \), the value of \( h_c \) changes correspondingly. The constant value of \( h_c \) in Chase’s formulation is described as the velocity dispersion coefficient determined from space-time correlations. Using the relationship in Equation 4-1 from Chase (1987), \( h_c \) falls to 1.4 for the wall jet at \( U_o=60\text{m/s} \) assuming a convection velocity of 41%.

\[
h_c = \frac{\mu(U_c)}{u_r}
\]

Equation 4-1

The change in \( h_c \) manipulates the levels of the subconvective and convective regions of the wall pressure wavenumber spectrum through changes in the constants \( C_T \) and \( C_M \), respectively. The adjustment of these constants changes the cross-section of the wavenumber wall pressure spectrum as shown in Figure 4-36. The narrowing of the convective ridge with a decrease in \( h_c \) manipulates the shape of the normalization through the integral in Equation 3-1.

![Figure 4-36. Comparison of Chase wavenumber wall pressure spectrum with various constants at \( \kappa_x=2000 \) and \( \kappa_z=0 \)](image)

Figure 4-37 shows the estimated normalization for the 42 element fetch of cubes for \( U_o=60\text{m/s} \) from Figure 4-9 compared to the estimated normalization using the manipulated constants for the Chase wall pressure wavenumber spectrum given in Equation 4-2.

\[
h_c = 1.4 \quad C_T h_c = 0.014 \quad C_M h_c = 0.466 \quad b = 0.75
\]

Equation 4-2
The result using Chase’s suggested value of \( h_c=3 \) fits the spectral shape better than the modified version. The narrower convective ridge peak of the modified Chase spectrum produces a lumpier normalization amplifying all of the small details in the \( \Gamma \) function of the 42 element fetch. Even though the shape of the normalized results is altered, the mean magnitude of the estimated curve is unchanged.

The \( b \) value in the Chase model can also be modified. This value controls the source layer thickness as a fraction of the boundary layer, but analysis showed that increasing or decreasing this value from 0.5 to 1 had an insignificant effect on the estimated outcome.

Figure 4-37. Comparison of normalization shape with adjusted Chase spectrum values

The estimation in Figure 4-33 for the random distribution of 1mm hemispheres deviates from the linear slope of \( \omega^2 \) at a lower frequency than the measured normalization. The cause of the “break frequency” was explained using Figure 4-34 which shows that the normalization will deviate at the frequency that aligns the convective ridge with the maximum location in the \( \Gamma \) function. Therefore, to improve the estimation, the convection velocity can be increased which decreases the slope of the convective ridge in the wavenumber wall pressure spectrum. With a higher convection velocity, eddies on the convective ridge at any given frequency are larger and therefore have a lower wavenumber. This affects the estimation by moving the “break frequency” to a higher value. Figure 4-39 shows the measured normalization for the random fetch of 1mm hemispheres at \( U_o=60\text{m/s} \) compared to the original calculated estimation using a convection velocity that is 41% of the edge velocity and a higher value of 52%.

Moving the convection velocity to a higher percentage of the edge velocity reduces the disagreement between measurement and estimation, but this is an arbitrary adjustment based on the desire to fit the measured and estimated values. The convective velocity of 0.41\( U_m \) was calculated in Devenport et al. (2010) using the noise radiated from a hydrodynamically smooth surface. The presence of the 1mm roughness should lower this convective velocity, not increase it. Also, the roughness used in this
measurement has a surface slope wavenumber spectrum that is not single valued as did the rib roughness used in Devenport et al. (2010). Therefore, the estimated values are affected by larger regions of the wavenumber wall pressure spectrum. Differences between the estimated and measured normalizations could be due to the assumed form of the wall pressure wavenumber spectrum.

To test the effect of the shape of the wall pressure wavenumber spectrum, the estimated normalization was calculated using a wavenumber wall pressure spectrum with a rectangular cross section. The cross-section was centered on the peak of the Chase spectrum, assuming the convective velocity of 0.41\(U_m\), for each frequency so that the slopes of the two spectra were the same. The rectangle was 2000\(\kappa_x\) x 6000 \(\kappa_z\) rad/m in wavenumber space. The contained area had an integrated value of unity and all points outside this area had values of zero. Figure 4-38 shows an illustration of a cut through the Chase wavenumber wall pressure spectrum at a frequency of 3kHz with the outline of the rectangular wall pressure spectrum at this frequency.

![Figure 4-38. Rectangular wavenumber wall pressure spectrum cross-section compared to the Chase wavenumber wall pressure spectrum at 3kHz](image)

The normalized curve produced by this crude wavenumber wall pressure spectrum is also shown in Figure 4-39. Above 1kHz, the rectangular spectrum produces values similar to that of the Chase spectrum. This suggests that the exact shape of the wavenumber wall pressure spectrum is not that important as long as the location of the convective ridge is approximately accurate. Crude forms of the wavenumber wall pressure spectrum can produce reasonable results at least over the frequency range observed in this experiment. Below 1kHz, the rectangular cross-section produces a normalized spectrum that follows a shallower slope of approximately \(\omega^2\). This slope may actually fit the lower frequency region of the measured normalization better than that produce by the Chase spectrum. The data from other velocities in Figure 4-33 clearly show the measured normalization at a shallower low frequency slope than the estimated spectra. The “break frequency” using this rectangular cross-section also changes shifting to a slightly higher frequency. This indicates that the earlier adjustment of the convective velocity might not be justified to settle the disagreement between measurement and estimation, but that the difference may be due to the shape of the assumed wavenumber wall pressure spectrum.
4.7 Total Sound Far Field Estimates

Glegg & Devenport’s (2009) theory can more usefully be applied to estimate the radiated far field noise from a rough surface and not just the normalized spectral shape. The previous comparisons of measured and estimated far field/near field normalizations were convenient because they excluded assumptions of the estimated single point wall pressure spectra. However, comparison with recorded normalized values required the measurement of the rough wall pressure spectra. In the following total far field sound estimates, a single point wall pressure spectrum is assumed in the estimated far field spectra instead. These estimations are calculated using the Chase wavenumber wall pressure spectrum and the analytical form of Chase’s (1980, 1987) single point wall pressure spectrum given in Equation 3-23 with appropriate inputs for the $U_c = 40\text{m/s}$ and $60\text{m/s}$ cases. The compared results for the five discussed deterministic rough surfaces are shown in Figure 4-40 to Figure 4-44 compared to the measured background subtracted roughness noise data.
Figure 4-40. Far field noise estimation using the Chase single point wall pressure spectrum compared to measured sound spectra from 42 element fetch of 3mm cubic roughness.

Figure 4-41. Far field noise estimation using the Chase single point wall pressure spectrum compared to measured sound spectra from 42 element fetch of 3mm hemispherical roughness.
Figure 4-42. Far field noise estimation using the Chase single point wall pressure spectrum compared to measured sound spectra from 703 element fetch of 3mm hemispherical roughness.

Figure 4-43. Far field noise estimation using the Chase single point wall pressure spectrum compared to measured sound spectra from fetch of ordered 1mm hemispherical roughness.
For all far field estimates, the measured spectral slope of the high frequency region above 2kHz is underpredicted due to the exclusion of viscous effects in the Chase single point wall pressure spectrum as shown in Figure 4-35. This is most apparent in the results for the 703 element fetch of 3mm hemispherical roughness in Figure 4-42. The high frequency slope of the estimation follows an $\omega^{-1}$ curve while the measured far field produces a slope nearly $\omega^{-5}$. The steep roll-off observed in the far field noise is controlled by the viscous effects in the near field damping the pressure fluctuations on the surface of the elements.

The smooth wall pressure spectrum model of Goody (2004) is a modified form of the Chase model but empirically accounts for the dissipative $\omega^{-5}$ region. His formulation is shown in Equation 4-3.

$$
\Phi_{pp}(\omega) = \frac{3 \left( \frac{\omega \delta}{U_e} \right)^2 \tau_0 \delta}{U_e \left[ \left( \frac{\omega \delta}{U_e} \right)^{0.75} + 0.5 \right]^{3.7} + \left[ 1.1 R_T^{-0.57} \left( \frac{\omega \delta}{U_e} \right) \right]^7}
$$

Eq. 4-3

where

$$
R_T = \left( \frac{\delta}{U_e} \right) / \left( \frac{\nu}{U_e} \right)
$$

Comparison of the Goody (2004) model with the wall-jet’s measured smooth wall spectra and the average rough wall pressure fluctuations measured in the 42 element fetch of cuboidal roughness is shown in Figure 4-45.
The Goody (2004) does a much better job than the Chase spectrum reproducing the shape of the measured wall pressure fluctuations. The low frequency region does not have the increased slope of the wall-jet near $\omega_z^{-1}$, but the high frequency region is much better modeled. The Goody model is a smooth wall pressure spectra model but overpredicts the smooth wall spectral levels of the wall-jet in the dissipative region by up to 5dB. Coincidentally, the difference in the smooth wall spectra is similar to the increased pressure fluctuations due to the addition of the roughness. Therefore, total far field estimations using this single point wall pressure model along with the Chase wavenumber wall pressure spectrum should produce better estimates of the measured far field spectra especially in the higher frequency region. The estimated far field results for all five of the studied deterministic surfaces are shown in Figure 4-46 to Figure 4-50. The results of the estimations are quite accurate over the frequencies associated with dissipation region in the single point wall pressure spectra. The accuracy of the predictions is greatest for the 1mm roughness.
Figure 4-46. Far field noise estimation using the Chase single point wall pressure spectrum compared to measured sound spectra from 42 element fetch of 3mm cubic roughness.

Figure 4-47. Far field noise estimation using the Chase single point wall pressure spectrum compared to measured sound spectra from 42 element fetch of 3mm hemispherical roughness.
Figure 4-48. Far field noise estimation using the Chase single point wall pressure spectrum compared to measured sound spectra from 703 element fetch of 3mm hemispherical roughness.

Figure 4-49. Far field noise estimation using the Chase single point wall pressure spectrum compared to measured sound spectra from fetch of ordered 1mm hemispherical roughness.
The total far field estimations using the Chase (1987) and Goody (2004) single point wall pressure spectra qualitatively predict the spectral peak shift to greater frequencies and the increase in far field noise produced by higher velocities, but the estimated spectral magnitudes differ with the far field measurements. The magnitude of the far field estimation is dependent on the wall pressure spectrum used in the estimation. Figure 4-35 shows the Chase single point spectrum well below the average recorded rough wall pressure spectra from the 42 element cubic roughness case. The relatively close resemblance of the Goody (2004) spectrum to the average recorded rough wall pressure spectra at high frequencies for the 1mm surfaces resulted in the exceptional accuracy of the estimations in this region. Use of the elevated rough wall pressure spectra in the measurements would bring the estimated far field closer to agreement with the measured values. The estimated normalizations in the previous sections for the 3mm roughness show that use of the recorded average rough wall pressure spectra will still unsuccessfully recreate the measured data, but the measured rough wall pressure spectra from the 1mm surfaces should accurately recreate the measured far field data.

Figure 4-51 shows a prediction of the far field noise from the fetch of randomly located 1mm hemispheres using the recorded rough wall pressure spectra for the 40m/s and 60m/s nozzle exit velocities compared to the measured background-subtracted far field noise. The estimated results are within 5dB of the measured values. The largest observed difference is due to the dip created by the model convective ridge passing through the first void in the Φ function shown in Figure 4-34 for each velocity. The accuracy of this prediction demonstrates that the far field noise produced by roughness that is a small fraction of the boundary layer thickness can be determined if the surface geometry and rough wall pressure spectrum are known.
4.8 Estimates of Average Pressure Fluctuations on Roughness Elements

Instead of trying to measure the wall pressure spectrum over the entire rough surface to calculate the true wavenumber wall pressure spectrum distribution, the average single point surface pressure spectrum that would exactly predict the radiated far field, assuming Chase’s wavenumber wall pressure spectrum, can be calculated using the measured far field data as shown in Equation 4-4.

$$\Phi_{pp}(\omega) \approx \frac{\Phi_{pp}(\kappa_{x}, \omega)|x|^2}{4\pi^2(k_{o}h)^2 \sum \Psi_{pp}(\kappa_{x}, \kappa_{z}, \omega) \Gamma(\kappa_{x}, \kappa_{z}, k_{o}) d\kappa_{x} d\kappa_{z}}$$  \hspace{1cm} Eq.4-4

Figure 4-52 shows the estimated near field spectra needed to collapse the measured and estimated far field for the 703 element fetch of 3mm hemispheres. This estimation is compared to the measured wall pressures inside the rough surface. This estimated spectrum represents the single point surface pressure fluctuations responsible for the measured far field. A portion of these pressure fluctuations are responsible for the unsteady drag on the elements, but the surface pressure spectra also contain contributions from lengthscales unassociated with the unsteady drag. The frequency range of calculated values is limited to regions where background-subtracted data was available in the calculation. The estimated spectrum does not differ in magnitude greatly from the measured wall pressure spectra over the majority of the calculated frequency range. Ignoring the lump at 13kHz, the spectral shape is similar to some of the interior measurements positions, particularly LH3 and LH5 located just ahead and behind roughness elements, respectively.
The estimated single point surface pressure fluctuations needed to resolve the difference between the predicted and measured spectra can be calculated for all five studied deterministic surfaces in a similar way. The results are shown in Figure 4-53 for a nozzle exit velocity of 60m/s. The majority of the spectra have a steep slope near or exceeding $\omega^5$ above 6kHz. The cubic element fetch produces has the shallowest slope in this frequency range. The shapes of the curves seem to follow a slope closer to $\omega^{-1}$ below 2-6kHz. The 1mm surfaces deviate from the steeper slope at approximately 6kHz while the larger surfaces tend to become shallower at lower frequencies. The cubic element spectrum is also stronger than estimated results for the hemispherical surfaces. This is consistent with the observation that the cubic roughness produces greater far field noise. The 42 element hemispherical roughness spectrum shows that this smaller fetch is a more efficient producer of noise per element than the larger 703 element fetch. This could be due to the fact that the ratio of leading to trailing elements is decreased by a factor of 3 in the larger fetch. The increase in number of trailing elements, which produce less noise, decreases the per element efficiency of the fetch and therefore the strength of the estimated average single point wall pressure spectrum.
Figure 4-53. Estimated average pressure fluctuation on surface of roughness elements for all five studied deterministic surfaces at $U_o = 60\text{m/s}$
Chapter 5 Source Map Analysis

The directivity and variation of noise through a fetch of roughness fetch was analyzed using the linear microphone array discussed in Section 2.4. A delay and sum beamforming technique was used to solve for source strengths and distributions throughout fetches of \( h = 3 \text{mm} \) discrete element surface roughness. As observed in Figure 2-14, the configuration of the linear microphone array limits the use of the array in directions out of the axis of its sensors. Therefore, the array was positioned in several locations in the anechoic chamber to record the variation of source strengths along individual axes. These microphone array positions are shown in Figure 2-12. The following sections detail the source strength analysis for the single to 42 element fetches of cuboidal and hemispherical roughness using a delay and sum beamforming technique.

5.1 Cuboidal Roughness – Single Element to 42 Element Fetch

Figure 5-1 shows a diagram of a microphone array measurement of roughness noise from Array Position 1 whose exact sensor location is detailed in Section 2.4. The microphone array was positioned in the spanwise direction of the roughness with sensor locations only varying along the \( x \)-axis. The focal area was approximately 300mm square centered around \( x = 1257 \text{mm}, z = 0 \). This area is large enough to contain all of the roughness elements in the single to 42 element fetches. The lead row of all roughness began at \( x = 1257 \text{mm} \).

![Figure 5-1. Diagram of Array Position 1 measurement and beamformed source map](image)

The sensors of the linear array make a receiving angle of \( \pm 34^\circ \) relative to the axis of a spanwise dipole emanating from \( x = 1257 \text{mm}, z = 0 \), the central lead row location of the roughness noise measurements. The power of a dipole source varies as the cosine squared of the receiving angle.
Therefore, the calculated source maps will be dominated by measurement of the spanwise dipole strength but are not an exclusive measurement of this source. The microphones on either end of the microphone array will be more influenced by the streamwise dipole than the central sensors. Figure 5-2 shows two cosine squared functions representing equal strength dipoles aligned in the streamwise and spanwise directions. The thick black lines mark the maximum receiving angles drawn from source to sensor of the microphone array. Even at these largest receiving angles, the spanwise dipole efficiency exceeds the streamwise efficiency by over a factor of two.

![Figure 5-2. Directivity of theoretical streamwise and spanwise dipoles compared to receiving angles encountered by linear array sensors](image)

Figure 5-3 shows the source map produced by measurement of the radiated far field noise at this spanwise position from a single 3mm cubic element located at \(x=1257\text{mm}, z=0\) for \(U_o = 60\text{m/s}\) at three frequencies, 6336Hz, 10368Hz, and 13696Hz. Although the source maps produce no source peak in the z-direction because of the sensor alignment, the produced maps clearly show the peak source strength at the location of the roughness element in the x-direction. The resolution of the calculated source location increases with frequency. Comparing the width of the main lobe on each map, defined as the width at half the peak value through \(z=0\), the size decreases by a factor of 2 from approximately 60mm to 30mm. The increase in resolution with frequency is a typical characteristic of this beamforming technique. Comparing magnitudes of the measured source maps, the strength of the spanwise source appears strongest at the lower frequency and decreases with increasing frequency.
Figure 5-3. Source maps from a single 3mm cubic element at $x=1257$, $z=0$ using Array Position 1 with $U_o = 60$ m/s for a) 6336Hz, b) 10368Hz, c) and 13696Hz in N$^2$/m$^4$Hz

A similar analysis of the streamwise dipole can be completed using the measured cross-spectrum with the microphone array at Array Position 2. The sensors at this position vary along the $z$-axis and the array is positioned upstream of the roughness in a position shielded from the jet noise produced by the nozzle. Figure 5-4 shows a diagram of the roughness noise measurements using Array Position 2. The source map focal area is the same size as that used in Array Position 1 and is centered about the same point, $x=1257$mm, $z=0$. This upstream microphone location is located 585mm above the flat plate of the wall jet and is angled down 56° from horizontal, so that points at the trailing edge of the focal area are 250mm further from the center of the microphone array than the leading edge. The sensors of the microphone array are centered around $z=0$ and have receiving angles that vary ±14° to the theoretical streamwise dipole axis of the single roughness element. Therefore, as shown similarly in Figure 5-2 for the spanwise dipole, the noise recorded at this position will be dominated by the streamwise radiating source.
Figure 5-5 shows the produced source maps for the single 3mm cubic element at \(x=1257\text{mm}, z=0\) with \(U_o = 60\text{m/s}\) calculated using Array Position 2 for three different frequencies. The same narrowing effect of the main lobe is observed as the frequency is increased changing by a factor of two. The source strength decreases as well with increasing frequency. Like the source maps of the spanwise dipole shown in Figure 5-3, the peak source strength occurs at the location of the single roughness element. These source maps are given to show that the frequency variation only affects the resolution of the maps but that the main lobe even at the highest recorded frequencies is significantly wider than the width of the single element. The remaining analysis will be focused mainly at 10368Hz which is a central frequency of the measurable roughness noise.

Array Positions 3 and 4, which place the array nearly vertical over the single roughness element with sensors aligned in the streamwise and spanwise direction, respectively, produce source maps that again locate the correct position of the roughness element. Figure 5-6 shows the results as recorded from Array Positions 3 and 4 for the single cubic element at 10368Hz. These array measurement locations both produce a symmetrical stripe through the center of each map. If the sources coming from the single roughness element were a combination of only streamwise and spanwise dipoles sources, the resulting directivity pattern would have a void perpendicular to the plane of the wall above the roughness. This void is at a point perpendicular to both dipole axes. None of the sensors of the linear array are at exactly this point for Array Positions 3 or 4. In Array Position 3, the sensors are in a geometrical position that straddles this void point with all of the array’s sensors in a position perpendicular to the origin of the spanwise dipole. Therefore, for the single element case, this is a measurement of the streamwise dipole solely. Array Position 4 is straddling the void point with sensors perpendicular to the streamwise dipole and is therefore a measurement of the spanwise source. Source convection effects due to the local flow velocity, discussed in Section 2.5.1.4, push the effective source location slightly downstream. As a result, the streamwise dipole will have some influence on the measurement at Array Position 4, but the receiving angle subtended from source to array for the streamwise source’s axis is \(\sim 87^\circ\) at the highest local velocity. Consequently, the streamwise source should be negligible compared to the spanwise source at this array position.
Figure 5-5. Source maps from a single 3mm cubic element at $x=1257, z=0$ using Array Position 2 with $U_o = 60\text{m/s}$ for a) $6336\text{Hz}$, b) $10368\text{Hz}$, c) and $13696\text{Hz}$ in $\text{N}^2\text{m}^4\text{Hz}$
To compare multiple source maps, sweeps through the maps at the roughness element location are taken in the direction of the array’s sensor axis. This allows the shape and magnitudes of the maps from each array position to more easily be examined. Figure 5-7 and Figure 5-8 show sweeps through the source maps at $z=0$ and $x=1257\text{mm}$ for Array Positions 1 and 2, respectively, calculated at various nozzle exit velocities. The results from Array Position 1, displaying the spanwise dipole, show that the source map peaks at the location of the roughness element for all velocities, and the magnitude of the peaks increases with velocity. At $U_o=30\text{m/s}$, the roughness noise is barely discernable above the background creating the large lumps in the normalized source maps away from the location of the roughness especially in the direction of decreasing $x$ which is the towards the nozzle. The source maps recorded at Array Position 2, showing the streamwise dipole, also correctly identify the location of the roughness element at all measured velocities. For this array position, even at 30m/s, the background noise from the tunnel is well below the recorded roughness noise. This is due to the shielding of the array from the nozzle exit by the acoustically treated shelf as shown in Figure 5-4. There is no direct line of sight from nozzle exit to sensors as there is for the spanwise measurement position, Array Position 1. Both figures show that the shape of the measured source maps remain the same at all examined velocities. This single element analysis also demonstrates that the applied beamforming method works well isolating the roughness noise from the background and that the technique performs as expected identifying the source locations and the variation in source strengths.
If the number of elements is increased to a single spanwise row of six elements, the observed shape of the source maps is unaltered as measured from Array Position 1, but the source location becomes broader as observed from Array Position 2. This is due to the roughness elements being aligned along the same axis of the microphone array in Array Position 2 and out of axis in Array Position 1. These maps are shown for 10368Hz in Figure 5-9 along with the results as recorded from the two remaining microphone array locations. The source maps show that the array cannot distinguish between individual roughness elements which are all contained within one main source lobe. The resolution is not fine enough even at the highest observable frequency, 20kHz, to distinguish between individual roughness elements.

Figure 5-10 compares sweeps through the six element maps with the results from the single cubic element. For comparison, the single cubic element source map was manipulated by increasing its magnitude by a factor of six. Also, in Figure 5-10b and d, the measured single cubic element map was
displaced in the spanwise direction corresponding to each element location and summed to simulate a measurement of the 6x1 arrangement.

Figure 5-10a shows that the beamformed shape of the single cube and 6x1 arrangement are similar as measured from Array Position 1. The 6x1 configuration increased the peak level of the source map by approximately a factor of 6.7 from the single element result, slightly beyond the simple addition of six of the single element’s sweep. The same enhancement in spanwise dipole strength is found in the measurement from Array Position 4. From this vantage point, the linear microphone records the spanwise variation of the spanwise dipole. Again, the results show that the spanwise noise of the 6x1 fetch is greater than that found from the addition of six individual sources. This indicates that the spanwise noise generated per element may be enhanced by the presence of spanwise adjacent elements, but it must be noted that these results do not account for the change in distance from the source-to-observer for the elements in the 6x1 configuration. The distributed single element map used for comparison is simply offset to simulate the 6x1 configuration of sources with no account for changes in the measured signals as would be recorded at each of the array sensors.

Alternatively, the streamwise dipole strengths of the 6x1 fetch, as measured from Array Positions 2 and 3, are approximately recreated by the addition of the single element results. There is no enhancement of the per element noise in the streamwise direction due to the addition of the spanwise elements. Array Position 2 shows the spanwise variation of the streamwise dipole is approximately constant. The source map is almost indistinguishable from the distributed single element results except for the slight asymmetry of the source map.

Both spanwise sweeps, in Figure 5-10b and d, show this asymmetric effect with spanwise and streamwise source strengths favoring the negative z-direction. This is believed to be due to variation of the in-flow conditions and is not a function of the surface geometry. This small variation in the flow in the spanwise direction was measured by Morton (2011) using a hotwire probe.
Figure 5-9. Beamformed source maps of a single spanwise row of six cubic elements at 10368Hz using a) Array Position 1, b) Array Position 2, c) Array Position 3, d) and Array Position 4 with $U_o = 60\text{m/s}$ in $\text{N}^2/\text{m}^4\text{Hz}$.
Figure 5-10 shows the variation in beamformed source maps with the addition of spanwise adjacent roughness elements. The effect of downstream elements on the streamwise and spanwise dipoles was studied by incrementally increasing the number of trailing spanwise rows up from the 6x1 configuration to a 42 element fetch of cubic elements in a six spanwise by seven streamwise arrangement. Figure 5-11 shows a comparison of the source maps produced by measurements of all the multi-element cubic roughness fetches as recorded from Array Position 1 at $U_o = 60$ m/s. These sweeps are through the spanwise center of the roughness fetch and show the streamwise variation of the spanwise dipole strength. Figure 5-11a shows that trailing rows of roughness increase the peak of the source map from the original 6x1 case, but a consistent increase with the total number of elements is not observed. The source map sweeps for fetches 6x2 to 6x7 have peaks that vary by 17% in a seemingly random order.

Figure 5-11b focuses on the shift in source map shape with the addition of trailing rows by normalizing the source maps on the peak values. The dashed lines indicate the streamwise location of the 1st and 2nd spanwise rows into the roughness. The addition of the trailing rows of roughness shifts the
peak of the source maps downstream. The 6x2 curve shows that the addition of a single trailing row of six spanwise elements produces a peak between the two rows. When a third row is added, the source map appears to peak at the location of the second row. The peak remains at this location with additional trailing rows, but the backside of the source map begins to widen further creating a broad base to the main lobe.

Figure 5-11. Sweeps through source maps at z=0 for the multi-element cubic fetches at $U_o = 60$ m/s for Array Position 1 (dashed lines indicate the streamwise location of the 1st and 2nd spanwise rows)

Figure 5-12 shows a comparison of the source maps produced by the multi-element cubic fetches as recorded from Array Position 2. The compared sweeps are through the streamwise location of the lead row at $x=1257$ mm and display the spanwise variation in the streamwise dipole. The shape of the source maps remains approximately constant with the small asymmetric effect becoming more pronounced with the increase in elements. In general, the source map magnitude increases with the addition of elements, but the magnitudes of the sweeps through the 6x3, 6x4, and 6x5 maps are relatively constant. The map strength then increases with the addition of 6th and 7th spanwise rows. The peak of all the maps is well contained within the boundaries of the roughness fetches shown as the two vertical dashed lines. For comparison, the single row source map was displaced streamwise accordingly to recreate the 6x7 configuration assuming each row produces a streamwise dipole strength equal to that of the 6x1 cubic fetch. The magnitude of the measured source map from the 6x7 fetch is significantly weaker than the summed result calculated assuming this equal strength. Therefore, this shows the per row strength distribution decreases with the inclusion of trailing rows in the roughness fetch. This indicates these trailing rows may have weaker streamwise dipole strengths than the lead row which lower the per row average strength of the radiated noise. To confirm this observation, the radiated noise needs to be analyzed from a vantage point that isolates the streamwise variation of the streamwise dipole.
Figure 5-12. Sweeps through source maps at \( x = 1257 \)mm for the multi-element cubic fetches at \( U_o = 60 \)m/s for Array Position 2 (dashed lines indicate the spanwise boundaries of the roughness fetch)

Figure 5-13 displays the results from Array Position 3 which is in a favorable position that isolates the streamwise variation of the streamwise dipole source. Like the spanwise dipole, the peak strengths occur near the front of the fetch, but again the peak magnitudes do not continuously increase with the number of elements. The peak levels rise and remain approximately constant beyond the addition the third row except for the 6x5 arrangement which has a magnitude similar to the 6x2 fetch. The shape of the source maps, indicating the relative source strength distribution through each roughness fetch, also resemble the spanwise dipole results in Figure 5-11. The peak of the map shifts downstream but remains within the lead two rows. The base becomes wider with increasing number of downstream elements creating a larger trailing side of the source map. A constant strength source distribution would produce a symmetrical source map different from the measured values. This suggests that the spanwise and streamwise dipole strengths throughout the fetch are not constant and favor the leading rows.

Figure 5-13. Sweeps through source maps at \( z = 0 \) for the multi-element cubic fetches at \( U_o = 60 \)m/s for Array Position 3
The fourth microphone array vantage point used in this study detects the spanwise variation of the spanwise dipole. These results for the multi-element cubic fetches are shown in Figure 5-14. The spanwise dipole behaves similarly to the streamwise dipole with an approximately even source distribution. Again, the slight asymmetric effect becomes more dominant as the number of elements increases. Like Figure 5-12, results from the 6x1 source map are distributed and summed to simulate a 6x7 fetch with spanwise source strengths equal to the 6x1 fetch for comparison. These results greatly overpredict the measured source map of the 6x7 fetch nearly doubling the observed peak value. This should be expected given the results of Figure 5-11 which show the trailing rows producing weaker spanwise dipole strengths. The streamwise source distributions of the spanwise and streamwise dipoles will be analyzed further in Section 5.4.

![Figure 5-14. Sweeps through source maps at x=1257mm for the multi-element cubic fetches at U_o = 60m/s for Array Position 4](image)

**Figure 5-14.** Sweeps through source maps at x=1257mm for the multi-element cubic fetches at $U_o = 60 \text{m/s}$ for Array Position 4

### 5.2 Hemispherical Roughness – Single Element to 42 Element Fetch

The far field noise from the hemispherical roughness was far below that produced by the cuboidal roughness as shown in Section 4.2.4. Beamformed maps of the noise from a single hemispherical element did not produce clearly defined peak over the focal area due to the extremely low signal-to-noise ratio of the measurement. Therefore, only source maps produced by the multi-element fetches are presented for the hemispherical roughness. Figure 5-15 and Figure 5-16 show sweeps through the multi-element source maps as recorded from Array Position 1 and Array Position 3. The source maps from both array locations are similar to the results from the cubic element fetch and display the streamwise variation of the spanwise and streamwise dipole strengths, respectively. The addition of trailing rows tends to shift the apparent peak source strength downstream and broaden the source area of both the spanwise and streamwise dipoles. Due to the low level sound produced by the hemispherical roughness, the background noise makes a greater appearance in the calculated source maps, especially from the direction of the nozzle in the negative x-direction. The source map shape of the streamwise dipole in Figure 5-16 appears to change with the size of the roughness fetch. The 6x5 fetch produces a broader peak than the 6x7 fetch
indicating a more even distribution of source strengths. No results are given for the 6x1 fetch in Figure 5-16 because the roughness noise was indistinguishable above the background levels.

Figure 5-15. Sweeps through source maps at z=0mm for the multi-element hemispherical fetches at $U_o = 60$ m/s for Array Position 1

Figure 5-16. Sweeps through source maps at z=0 for the multi-element hemispherical fetches at $U_o = 60$ m/s for Array Position 3

Figure 5-17 displays the results from Array Position 2 measuring the spanwise variation of the streamwise dipole. Similar to the cuboidal roughness, an approximately constant source distribution is observed across the span of the roughness. The same slight asymmetric noise production is measured most notably for the larger fetches. It is also observed that the per element streamwise noise production is diminished with the addition of trailing rows compared to the summed solution of the distributed 6x1 source strengths. Plots of the spanwise variation of the spanwise dipole could not be produced because there was not sufficient signal-to-noise ratio as measured from Array Position 4.
Figure 5-17. Sweeps through source maps at $x=1257\text{mm}$ for the multi-element hemispherical fetches at $U_o = 60\text{m/s}$ for Array Position 2

5.3 Comparison of Cuboidal and Hemispherical Roughness

Although the source maps vary in magnitude greatly, the hemispherical and cuboidal roughness produce similarly shaped source maps. Figure 5-18 shows the comparison of the source maps as recorded by Array Positions 1. The similarity of the maps indicates that the spanwise dipoles vary similarly in the streamwise direction for both roughness fetches.

Figure 5-18. Comparison of 42 element source maps from cuboidal and hemispherical roughness as recorded from Array Position 1

The results from Array Positions 2 and 3 are used to observe the spanwise and streamwise variation of the streamwise dipole, respectively. These results are shown in Figure 5-19. The streamwise dipole distribution in the streamwise direction shown in Figure 5-19a varies between the two roughness
geometries. The hemispherical roughness has a broader peak that is shifted slightly further downstream than the result of the cubic roughness. This indicates that the streamwise source strength distribution in the hemispherical roughness may decrease at a slower rate than the cuboidal roughness even though their spanwise dipole strengths, shown in Figure 5-18, were almost identical. Although the streamwise distribution may be different for both surfaces, the spanwise variation of the streamwise dipoles is approximately the same as shown in Figure 5-19b.

A similar comparison from Array Position 4 of the spanwise variation of the spanwise dipole could not be completed because the hemispherical roughness did not produce a sufficient signal-to-noise ratio.

![Figure 5-19. Array Position 3 and 2 comparison of noise from 42 element fetch of cubes and hemispheres at $U_o=60m/s$](image)

**5.4 Deconvolution of Source Maps**

The source maps in Figure 5-18 and 5-19 show specific source strength distributions that vary slightly between the cuboidal and hemispherical roughness, but determining quantitative source strengths from the beamformed maps can be a difficult task. Conventional methods of determining source strengths from beamformed maps require the integration of the source map strength over a user defined area which is often a challenging choice especially in cases where the noise produced by individual sources cannot be isolated in the source map. The choice of integration area is an even greater problem when using a linear microphone array because of the lack of spatial resolution in the out of sensor axis direction. Although the source strength may be difficult to determine, the source strength variation can be easily deduced from the measured source maps. The shape of the measured maps can be recreated by computing source maps of distributed dipoles with similarly varying strengths to the actual case. By examining various source strength distributions and comparing them to the measured maps, the strength distribution through a fetch of roughness can be determined.

The simplest case would be that of a single roughness element. In this case, there is not a distribution of sources but only a single dipole source located at the roughness element location. Figure 5-20 shows the source map comparison between the measurement of a single cubic roughness element and the point source image generated by a dipole source at the same location. The maps have been
normalized on their peak strengths to compare shapes. The main lobes have a very similar shape, but the measured map is slightly wider and has elevated side lobes due to the background noise not present in the point source map.

![Graph showing comparison of measured single cube element at Uo=60m/s spanwise dipole with dipole point source](image)

**Figure 5-20. Comparison of measured single cubic element at Uo=60m/s spanwise dipole with dipole point source**

A similar but more complex analysis can be done for the full 6x7 fetch of cubes in which a source strength distribution must be assumed. Figure 5-21 shows a few model source distributions by row for the streamwise variation of the streamwise and spanwise dipoles. Four different source distributions are studied for the strength of radiated noise in the streamwise and spanwise directions: a constant strength distribution, a linearly decreasing distribution, a dominant second row case, and the RMS force distribution as calculated by the LES of Yang & Wang (2011) for the corresponding streamwise or spanwise dipole. The RMS values reported in Yang & Wang (2011) may not be representative of the fluctuating forces at the frequencies where roughness noise was recorded in this study, and therefore, the comparison between generated source maps should not be considered a validation of the LES results. The RMS distribution was used solely as another distribution model shape. The absolute levels of the distributions are unimportant because the compared source maps will be normalized on their peak values the result being a contrast of the distribution alone.
Figure 5-21. Model source strength distributions for streamwise variation of streamwise and spanwise dipoles in cubic roughness

Figure 5-22 shows the measured source map of the spanwise dipole sources from a 6x7 fetch of cubic roughness compared to the source maps produced by the model source distributions at three different frequencies. The estimated source maps assume a single source strength at the spanwise center of each row. Comparison with source maps having six spanwise distributed model source distributions showed negligible differences with the source map shape produced by assuming only a single spanwise central distribution. The peak of the source map for this fetch of elements peaks downstream of the lead row, but the linearly decreasing model clearly fits the measured sweeps the best at all frequencies. This shift in peak location is due to spatial broadening in the beamformed source map due to the low resolution of the source image. The relative difference between the linear source distribution and the measured source strengths differ by frequency. The strengths of the trailing rows are slightly underpredicted with the linear model at the lowest frequency, but as the frequency increases, the relative strength of the trailing rows diminishes. The linear model rises above the measured values on the backside of the main lobe peak at 13696Hz. The measured source distribution tends to level off towards the trailing rows similar to that of the Yang & Wang’s (2011) RMS results but to a much less degree creating the hump on the backside visible at the two higher frequencies. The maximums in the normalized source maps produced by the second row dominant, the RMS distribution of Yang & Wang (2011), and the constant strength source distributions are all downstream of the maximum calculated in the measured map suggesting that the spanwise dipole is strongest for the lead row of cubic roughness.

Analyzing the streamwise dipole strength in a similar manner, Figure 5-23 shows that again the linear distribution of sources strengths decreasing by 1dB per row fits the measured maps best. The 6336Hz differs the most peaking upstream of all the model source distributions. The measured source map’s relation to the model maps changes with frequency indicating that the distribution of streamwise dipole strengths, like the spanwise dipoles, varies with frequency. For all analyzed frequencies, the results indicate that the streamwise and spanwise dipoles are strongest for the lead row of the cubic roughness fetch and that the strength of the trailing rows decreases at an approximately linear rate of 1dB per row.
Figure 5-22. Comparisons of measured source map of 6x7 cubic fetch at z=0 with source maps of model source distributions for the spanwise dipole at $U_o=60\text{m/s}$

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A similar analysis of the streamwise variation of source strengths through a fetch of hemispherical roughness is completed using the source distributions in Figure 5-24. The second row dominant distribution and linearly decreasing distributions are considered similar to the cubic roughness analysis. The constant strength distribution is not considered further because, as shown in Figure 5-22 and 5-23, it provides a poor fit to the measured source maps. The Yang & Wang (2011) RMS unsteady force distributions are the result of an LES study for a 4x10 fetch of hemispherical roughness. The RMS distributions of the hemispherical roughness differ from that of the cuboidal roughness. The lead row of the hemispherical roughness produced the weakest spanwise and streamwise RMS dipole strengths over the entire rough surface. The source strength of the second row was strongest and then the calculated strengths quickly level off. Again, these distributions are used as shape functions only and should not be considered a validation of the LES study.
Figure 5-24. Model source strength distributions for streamwise variation of streamwise and spanwise dipoles in hemispherical roughness

The spanwise dipole results are shown in Figure 5-25 for three frequencies. The best fit of the measured source map is again the linearly decaying source distribution model. This is expected since the comparison in Figure 5-18a shows that the cuboidal and hemispherical source maps are very similar in the spanwise direction. The other source distribution models are too heavily weighted on the downstream elements. Like the cuboidal roughness fetch, the source distribution tends to favor the front of the roughness fetch with increasing frequency. The trailing rows become relatively weaker as the frequency is increased. This can be related back to measurements of the wall pressure spectra in both the cuboidal and hemispherical roughness in Figure 4-3a and 4-12a, respectively. These figures show that the recorded high frequency pressure fluctuations decrease in the streamwise direction. At 6336Hz, the spectra at all streamwise positions inside the roughness fetches vary only slightly in the streamwise direction, but at 13696Hz, there is a more dramatic reduction in pressure fluctuations between the front and rear of the roughness fetches.

Figure 5-26 shows the results of the streamwise dipole variation. The results of the hemispherical fetch in this case are different from that of the cuboidal roughness as shown in Figure 5-19. Comparing the model source distributions, none of the examples fit the measured source map precisely. The source strength appears to peak at the lead row like the linearly decreasing source distribution, but the shape of the measured distribution varies greatly for the three different frequencies. At the lowest frequency, 6336Hz, the peak occurs very near the lead row indicating that the trailing rows are significantly weaker with strengths decreasing much greater than 1dB per row as does the linearly decreasing distribution. At 10368Hz, the source seems evenly distributed around the lead rows of the fetch. The source distribution may be a cross between the results of the linearly decreasing model and the second row dominant model. The lead row appears to be the strongest producer of noise at 13696Hz but noise from the trailing rows generates an unusually shaped source map downstream that is not recreated by any of the model source distributions. These lumps on the trailing side of the source map, same as in Figure 5-25, may be an effect of background noise in the measurement and not due to the actual source distribution.
Figure 5-25. Comparisons of measured source map at $U_o=60\text{m/s}$ of 6x7 hemispherical roughness fetch at $z=0$ with source maps of model source distributions for the spanwise dipole.
Figure 5-26. Comparisons of measured source map at $U_r=60$m/s of 6x7 hemispherical roughness fetch at $z=0$ with source maps of model source distributions for the streamwise dipole

The shapes of the source maps have been approximately determined with this source distribution deconvolution method, but the absolute levels of the discrete source strengths have yet to be evaluated. To do this, a method needs to be developed that minimizes the error between the estimated and measured source maps. Using the measured source map results from Figure 5-3 and Figure 5-5 for a single cubic roughness element, assuming each figure is the result of either the spanwise or streamwise dipole alone, respectively, the strengths of the theoretical dipoles that would recreate the calculated source maps can be determined by a simple least-squares analysis. Figure 5-27 shows the source maps produced by the measurement of the single cube at Array Positions 1 and 2 compared to two curves produced by source maps of a dipole point source image with a source strength determined by a least-squares fit of the measured cross-spectral matrix assuming a dipole emanating from the location of the roughness element. The result for each source map was determined individually. The estimated and measured source maps agree very well showing the viability of the least-squares method to reproduce the measured source maps.
The calculated strength of the streamwise aligned dipole was $2.07 \times 10^{-13} \text{ N}^2/\text{Hz}$ and the strength of the spanwise aligned dipole was $3.82 \times 10^{-14} \text{ N}^2/\text{Hz}$.

This method seems to work well for the single cubic element, but as the number of sources is increased so does the error. When solving for seven independent source strengths the least-squares solution assigns source strengths of zero which is an artificial result, but the accuracy of this process can be improved by combining the array data from all four measurement positions. This process creates a highly overdetermined system of equations that solves for the separate streamwise and spanwise dipole strengths simultaneously. Chapter 6 details this least-squares analysis of the streamwise and spanwise dipole strengths.
Chapter 6 Least-Squares Analysis

Individual sources in the beamformed source maps of the noise from the cuboidal and hemispherical roughness fetches could not distinguished even at the highest measured frequencies. The desire to estimate individual source strengths inside multi-element fetches of discrete roughness led to the development of the least-squares analysis detailed in Section 2.5.2. This method is used to solve for the streamwise and spanwise source strengths using the data from all four array position simultaneously. The individual elements of the cross-spectral matrices were solved for assuming all of the roughness noise sources were uncorrelated. The following section details the least-squares calculation of source strengths from individual roughness elements in fetches of 3mm cuboidal and hemispherical roughness of various sizes. The roughness configurations range from single elements up to 42 elements.

6.1 Single Element Results

6.1.1 Spanwise and Streamwise Dipole Strengths

The measurements of noise from single cuboidal and hemispherical elements were the simplest cases studied since only two sources were assumed in the analysis: a spanwise and streamwise aligned dipole located at the position of the roughness element. The source strength analyses of the noise from single cubic and hemispherical elements reveal the large difference in source strengths of the two element shapes. Figure 6-1 shows the estimated spanwise and streamwise dipole strengths for both elements at a nozzle exit velocity of 60m/s. The estimated cubic element source strengths are over an order of magnitude stronger than that of the hemisphere. This agrees with the single point analysis of the far field noise in Figure 4-19 showing the large difference in radiated spectra from fetches of the two surface geometries. Comparison of the cubic element’s spanwise and streamwise dipole show that the streamwise dipole is strongest for all frequencies and decreases at approximately the rate of $f^{-3.5}$. Although the conventional delay and sum beamforming method was unable to produce clear results at all of the microphone array positions for the single hemispherical element even at the highest studied velocity, the least-squares technique did converge over a limited frequency range. The results for the hemispherical element in Figure 6-1 are not as certain due to the very low signal-to-noise ratio and thus limited the frequency range over which a non-negative least-squares solution could be obtained. Further extensive analysis of the uncertainty of these estimations will be given in Chapter 7. A definitive conclusion cannot be made about the relative strengths of the hemisphere’s spanwise and streamwise dipoles from this narrow frequency range of estimated data.
Figure 6-1. Streamwise (solid) and spanwise (dashed) dipole strengths of single cubic and hemispherical element at $U_0=60\text{m/s}$

The source strengths as determined by this least-squares method are in fact the unsteady drag and side forces on the element. Using the theory of Glegg & Devenport (2009), the single point pressure spectra needed to resolve the difference between the measured and estimated far field normalizations for the fetches of cuboidal and hemispherical roughness were calculated in Section 4.8. These wall pressure spectra results are given in Figure 4.53. They were calculated by dividing the measured far field recorded at an upstream location by the integrated value of the Chase wavenumber wall pressure spectrum and the surface filter function for each frequency. This analysis is shown in Equation 4.4. In the scattering model, the single point surface pressure spectrum measured in a rough surface may not accurately represent the unsteady drag on a surface’s roughness elements because there is no account for the lengthscale of the boundary layer pressure fluctuations with respect to the roughness geometry which scatters only a portion of the fluctuations. Although, if the pressure fluctuations are the local result of unsteady forces on the element resulting from its own eddy shedding, then a more accurate representation might be expected. To examine this issue, the estimated single point wall pressure spectrum for the cuboidal roughness was converted to a crude approximation of the per element average unsteady drag by multiplying this spectrum by the cubes’ frontal area, 9mm$^2$. The result of this calculation is shown in Figure 6-2 along with the least-square solution of the streamwise and spanwise dipole strength calculated for a single cubic element. The average estimatede unsteady pressure from the fetch of 42 elements overpredicts the average unsteady force from the single cubic element over much of the frequency range, but there is close agreement with the estimated streamwise dipole strength between 3-9kHz. Outside of this range, the disparity rises up to about 5dB. This surprisingly good agreement between the unsteady forces in the
streamwise direction and the surface pressure spectrum responsible for the noise projected in the streamwise direction suggests that eddy shedding from the cubic elements may play a significant role both in the generation of sound and of the pressure fluctuations. If the convection velocity of the shed vortices is similar to that of the unperturbed boundary layer, 9kHz correlates to structures \( \frac{1}{3} \) the size of the roughness element while 3kHz corresponds to eddies equal to the size of the element.

![Figure 6-2. Estimated streamwise and spanwise source strengths of a single cuboid compared to the calculated average unsteady pressure on the roughness elements in a 42 element fetch of cuboids.](image)

A similar analysis can be done with the hemispherical roughness using the unsteady pressure spectra calculated in Figure 4-53 for the 42 element hemispherical roughness fetch. Again, the estimated single point wall pressure spectrum can be crudely converted to the unsteady drag spectrum by multiplying the wall pressure by the single hemisphere’s forward projected area, \( 9\pi/2\text{mm}^2 \). Figure 6-3 shows the least-squares estimated results for the single hemisphere’s dipole strengths along with the unsteady drag force calculated using the forward projected area and estimated single point wall pressure spectrum. The great difference between the estimated spectral levels reveals that, unlike the cuboidal roughness, the far field produced by the hemispherical roughness may be created by a scattering effect and not vortex shedding over this compared frequency range. This analysis shows that the noise produced by the unsteady drag cannot be determined from the projected area and single point wall pressure spectrum alone. The scale of the pressure fluctuations must be taken into account through the form of the wavenumber wall pressure spectrum and Fourier transform of the surface slope to determine the portion of the wall pressure fluctuations that are scattered to the far field.
6.1.2 Single Element Directivity

The source strengths determined for the cubic elements two dipole sources shown in Figure 6.1 have dissimilar spectral shapes. The spectra at 3kHz are separated by almost an order of magnitude. At 11kHz, the spectral values are nearly equal. This indicates that the directivity pattern of the noise from this single element varies with frequency. Figure 6-4 shows the implied radial directivity of noise in the plane of the wall at a distance of 0.65m produced by a single cubic element at $U_o=60$ m/s for two frequencies, 3872Hz and 10368Hz. The given decibel levels are not absolute, but are shown on similar scales to show the relative difference in directivity. The streamwise axis is along the 0° and 180° directions and the spanwise axis is along 90° and 270°. At 3872Hz, the streamwise dipole is much stronger than the spanwise so that the radiation pattern is 7dB stronger in than streamwise direction. At 10368Hz, the dipoles are approximately equal and the directivity pattern is nearly constant in all directions varying by only 1dB.
Figure 6-4. Directivity of noise radiation from single cubic element at a) 3872Hz b) and 10368Hz on a decibel scale

6.1.3 Effect of Flow Speed on the Dipole Strength of a Single Cubic Element

The source strength spectra from the single cube were determined at multiple nozzle exit velocities and are shown in Figure 6-5. No estimation could be made for the 20m/s nozzle exit velocity because the signal-to-noise ratio was too low. At all velocities, the determined spanwise strength was less than the streamwise dipole strength. The strengths at each frequency increased by approximately three orders of magnitude with a doubling of the local velocity from 11-22m/s which corresponds to the nozzle exit velocity increase of 30-60m/s. Like the 60m/s case, the other nozzle exit velocities also produce spanwise and streamwise dipole spectra that vary in relation to each other.
The estimated source strengths and frequencies can be normalized using the velocity of the unperturbed boundary layer evaluated at a height equivalent to the top of the roughness element, which is approximately 80% of the local edge velocity, and the roughness height. With this normalization, the three orders of magnitude spread shown in Figure 6.5 is reduced to a single order of magnitude as shown in Figure 6.6. Ignoring the $U_o=30\text{m/s}$ spectra, the collapse of the three remaining curves reduces further within about 5dB over the majority of the non-dimensional frequency range.

The relative change in the strength of the spanwise and streamwise dipoles with velocity is analyzed by integrating the results of Figure 6-6 over the normalized frequency range $0.625<fh/U<2.5$. The ratio of this integration is shown in Figure 6.7 displaying an increase in the relative strength of the streamwise to spanwise dipole as the velocity is increased. The strength of the streamwise dipole grows more rapidly nearly doubling in relation to the spanwise dipole over this non-dimensional frequency range. The slope of this change can be approximated as linear with a slope of 0.12 per local velocity. The shape of this curve may be leveling off at higher velocities not following a continually linear slope, but more data is needed at different local velocities to make any further conclusion.

**Figure 6-5.** Streamwise and spanwise dipole strengths of single cubic element at various nozzle exit velocities

![Graph showing the strength of streamwise and spanwise dipoles at various velocities](image)
Figure 6-6. Normalized source strengths for the single cubic element at multiple nozzle exit velocities: 60m/s (square), 50m/s (triangle), 40m/s (circle), 30m/s (diamond)

Figure 6-7. Integrated source strength ratio of dipoles produced by single cube at various maximum local velocities
6.1.4 Comparison with Rasnick (2010)

The least-squares method converges to produce source strength spectra even at frequencies where
the roughness noise is indistinguishable in the measured autospectra. Rasnick (2010) estimated the source
strength of an individual 3mm cubic element in an upstream position but was unable to estimate the
source strength of a single hemisphere because he used only a single microphone to record the far field
noise. The noise from the single hemisphere could not be separated from the background levels. A
microphone array records information that exists in the cross-spectral data as well as the autospectra.
These data were not available for Rasnick’s calculation. To validate the use of the least-squares method,
Rasnick’s (2010) results for a single cubic element can be compared to the source strength spectra
calculated from the least-squares method. Figure 6-8 shows this comparison for $U_o=60m/s$. Rasnick’s data
was corrected to account for the normalization using the velocity at the roughness height which was 80%
of the edge velocity, and the half-space Green’s function. The source strength for a single source was
declared in Rasnick (2010) as given in Equation 6-7.

$$C_D^2W_{ff} = \frac{(4\pi c_o)^2 \Phi(x,\omega)|x|^2}{\left(\frac{1}{2} \rho U^2 A\right)^2 \omega_r \left(\frac{\omega}{\omega_r}\right)^2 \cos^2(\theta)}$$  

Eq. 6-7

where $c_o$ is the speed of sound, $\Phi(x,\omega)$ is the recorded far field noise at vector position $x$ from the
roughness element, $A$ is the projected frontal area, $\omega_r = \frac{U}{h}$, and $\theta$ is the receiving angle of the sensor to
the roughness element. $C_D^2W_{ff}$ represents the non-dimensional source strength and is the steady drag
coefficient, $C_D$, multiplied by the source spectral shape function $W_{ff}$. The source strength definition used
in this study is given in Equation 6-8.

$$Q = \frac{4\pi^2 |x|^2 \Phi(x,f)}{(\frac{\omega}{c_o})^2 \cos^2(\theta)}$$  

Eq. 6-8

Converting $\Phi$ to per angular frequency and normalizing this definition on $\rho^2 U^3 h^5$, Equation 6-8 can be
rewritten as:

$$\frac{Q}{\rho^2 U^3 h^5} = \frac{(4\pi c_o)^2 \Phi(x,\omega)|x|^2}{\left(\frac{1}{2} \rho U^2 A\right)^2 \frac{h}{U} \omega^2 \cos^2(\theta)} \left(\frac{2\pi}{16}\right)$$  

Eq. 6-9

Therefore, the relationship between the normalized data presented here and the strength spectra
presented in Rasnick (2010) is a factor of $2\pi/16$. The spectral data for the streamwise dipole and Rasnick’s
data agree well as shown in Figure 6-8. Since Rasnick’s data was measured in an upstream position, the
measured source strength was that of the streamwise dipole. This independent agreement corroborates the
least-squares estimated results and verifies the use of the calculation method. The advantage of the least-
squares method is apparent in Figure 6-8. The least-squares estimated results extend to a much lower
frequency than calculated by Rasnick (2010). Also, the strengths of multiple sources are solved for
simultaneously. Rasnick’s method can only determine the strength of one source and therefore the
streamwise and spanwise sources are combined in his calculation. To separate sources using the method
of Rasnick (2010), far field microphones need to be placed in positions that isolate individual sources in
the directivity field. Of course, this microphone placement method would only work for individual roughness elements so that there are voids in the directivity pattern of the streamwise and spanwise dipole radiation. For multi-element fetches there is not a far field position that would eliminate the contribution from either the streamwise or spanwise dipoles completely. Also, Rasnick’s single microphone method would be unable to separate the different streamwise or spanwise dipole strengths from multiple roughness elements.

Figure 6-8. Single cubic element least-squares strength estimations compared to estimated strength from Rasnick (2010)

6.1.5 Comparison with LES

Yang & Wang (2011) completed a computational study of a wall flow involving fetches of cuboidal and hemispherical roughness arranged in a 4x10 pattern, 4 elements in the spanwise direction and 10 elements in the streamwise direction, in a conventional turbulent boundary layer. Their roughness height to boundary layer thickness ratio was 0.124 which is comparable to the ratio of 0.183 for the \( U_o = 60\text{m/s} \) case presented in this study. Also, their momentum thickness Reynolds number was 3065 where as for the \( U_o = 60\text{m/s} \) case the \( \text{Re}_\theta = 1021 \). Their roughness elements had a radius of 4.3mm and were spaced 25.4mm. The presented spectra in Yang & Wang (2011) are the average of the streamwise and spanwise source strengths across each spanwise row. The averaging was performed to reduce the uncertainty of the results. They found that the lead row of cuboidal roughness was the strongest producer of RMS sound in the streamwise direction and nearly the strongest in the spanwise direction following only the second row. For the hemispherical roughness, the RMS source strength of the lead row was the
weakest producer of noise in both the streamwise and spanwise direction. The least-squares estimated data only covers a small frequency range of the LES analysis near the highest calculated frequencies where LES analyses are typically most unreliable. Comparison between the results may corroborate both methods in this frequency range.

Figure 6-9 compares the per element streamwise and spanwise source strength results from the lead row of the cuboidal and hemispherical roughness compared to the least-squares estimated results for a single cuboidal and hemispherical element over the full presented LES range of data and a narrower range focusing on the region of comparison. Their calculated spectra in Yang & Wang (2011) was double sided and per angular frequency so their spectra have been adjusted by a factor of 4π. The velocity at the top of the roughness height was 70% of the edge velocity in their computation. Only the LES results from the lead row results are presented so that effects of upstream elements are not present in the comparison with the least-squares result for the single elements.

The least-squares estimated results are greater than the LES results at almost all non-dimensional frequencies. Disagreement between the LES estimated spectra and the least-squares calculated spectra grows with non-dimensional frequency up to approximately 15dB. At lower non-dimensional frequencies below \( \frac{fh}{U} < 1 \), the estimated spectra agree much better within a factor of 5 or 8dB. The streamwise and spanwise dipoles calculated from the experimental cuboidal roughness split at a non-dimensional frequency of 0.8 for the \( U_o = 50 \text{ m/s} \) and 60m/s cases. This split resembles the split in the LES results that occurs at a lower non-dimensional frequency of 0.5. These differences are large but may be attributed to differences in experimental conditions. The LES was conducted for a conventional turbulent boundary layer without the mixing layer present in the wall-jet. This mixing layer may be a significant influencing factor on the pressure fluctuations that are radiated to the far field. Also, the growing difference with frequency may be influenced by limits due to the resolution of the LES study. Although the absolute values differ, the difference in spectral magnitude between the hemispherical and cuboidal curves is the same for both the LES and experiment.
Figure 6-9. Single element source strength results compared to Yang & Wang (2011) LES results.
6.2 Multi-Element Cubic Fetches

6.2.1 Source Strengths of Individual Elements in Fetches

The least-squares method of source strength analysis was developed to decompose the source strength distribution further than was capable through examination of the beamformed source maps presented in Chapter 5. To show the effect of the addition of multiple elements on the least-squares analysis, the number of elements in the roughness fetch can be increased from an individual roughness element to a single spanwise row of six 3mm cubic elements. The least-squares results from this analysis are shown in Figure 6-10. The source maps in Chapter 5 show that the streamwise and spanwise dipoles at 10368Hz remain relatively constant across the span of the roughness fetch. This frequency corresponds to a non-dimensional frequency of 1.77. At this frequency in Figure 6-10, there is very little spread among the streamwise and spanwise dipole strengths agreeing with the source map results. However, at lower frequencies the results fan out more. Also, the least-squares method fails to calculate source strengths for all roughness elements at all frequencies. Source strengths of zero are assigned to some of the roughness elements occurring more often at lower frequencies. This is probably due to the acoustic wavelength growing with decreasing frequency creating a condition that makes the individual roughness elements indistinguishable from one another. This causes the source strength at one element to be identified as coming from another location. Therefore, to reduce uncertainty, the spanwise average of the source strengths in the single row of roughness elements was calculated to produce single spanwise and streamwise dipole strength spectra that represent the per-element average produced by each roughness element in a spanwise row.

Figure 6-10. Spanwise and streamwise source strengths of individual elements in a 6x1 fetch at $U_o=60m/s$, elements located at (x, z) mm: square (1257, -0.0413); triangle (1257, -0.0248); circle (1257, -0.0083); diamond (1257, 0.0083); pentagram (1257, 0.0248); upside-down triangle (1257, 0.0413)
The per-element averaged streamwise and spanwise dipole strengths for the 6x1 cubic element fetch at \( U_o=60 \text{m/s} \) are shown in Figure 6-11 compared to the results from a single cubic element. This average is calculated by summing the results including the values indicating zero strength and dividing by the number of spanwise elements, 6. The resulting spectra are much smoother than the curves produced by the strength spectra calculated for each roughness element. Like the results from the single cubic element, the streamwise dipole is stronger at all observed frequencies than the spanwise radiating dipole. The source maps in Chapter 5 indicated that the addition of spanwise adjacent elements may increase the spanwise dipole radiation per element, but these least-squares results confirm that the spanwise noise from the 6x1 fetch is approximately the same as that from the single element. The streamwise results from both the 6x1 and single element roughness are also approximately equal. This method of spanwise averaging the dipole source strengths will be used in the presentation of data from all of the multi-element fetches since it is reasonable to assume that there is little variation across the span of the roughness as shown in the beamformed source maps of Chapter 5.

![Figure 6-11. Spanwise averaged spanwise and streamwise dipole strengths of a 6x1 fetch of cubic roughness compared to single element source strengths at \( U_o=60 \text{m/s} \)](image-url)
6.2.2 Variation of the Lead Row’s Source Strength with the Addition of Downstream Elements

The effect of adding spanwise adjacent elements was studied in Figure 6-11 by comparing the results of the single element’s dipole strengths to the spanwise average strengths in a 6x1 configuration of roughness. The effect of adding trailing elements can also be examined by comparing the spanwise averaged source strengths of the initial spanwise row of cuboidal roughness for each of the roughness configurations from 6x1 to 6x7. Figure 6-12 and Figure 6-13 show the spanwise per-element averaged streamwise and spanwise dipole strengths, respectively, for the initial spanwise row of cuboidal roughness in each of the examined configurations. These results are for the condition \( U_o = 60 \) m/s.

Figure 6-12 shows that the calculated streamwise dipole strength of the lead row increases slightly with the addition of trailing elements. However, compared to the single cube results the single row of roughness (labeled 6x1) also shows a slight increase in the streamwise dipole above a non-dimensional frequency of 1.77. The addition of the spanwise elements from the single cube configuration causes the spectral levels to increase by a factor of 1.28. The small increase in strength spectra due to the trailing rows may be an artificial amplification caused by the large wavelength to source separation ratio so that source strengths of trailing rows or the background noise are included in the strength of individual elements. Even at 20kHz, the highest considered frequency, the acoustic wavelength is approximately 17mm while the roughness elements are only spaced 16.5mm. As the frequency is decreased, increasing the acoustic wavelength, the calculated strength of the lead row’s streamwise dipole, shown in Figure 6-12, increases further with the addition of almost every trailing row. The result at high frequencies may be real, but below a non-dimensional frequency of 1.33, the frequency at which the data begins to separate by number of elements, the results may be artificial. It is unlikely that the addition of the 6th or 7th row of roughness influences the source strengths of the lead row. The uncertainty of this calculation over the presented frequency range will be analyzed further in Chapter 7.

The results for the average spanwise dipole strength of the lead row in Figure 6-13 show a similar behavior with frequency. The data begin to fan out at lower frequency in order of number of elements suggesting that the addition of the 7th spanwise row of roughness affects the spanwise dipole strength of the lead row. Again, this is most likely an artificial effect due to the long acoustic wavelengths at these frequencies. At higher non-dimensional frequencies, there is very little difference between the estimated spanwise source strengths for the varying size fetches ranging from a single element to a full 42 element fetch. This indicates that the addition of multiple elements has no effect on the spanwise dipole strength of the lead row at these frequencies.
Figure 6-12. Streamwise dipole strength of the lead row for all of the examined cuboidal roughness fetches at $U_o=60\text{m/s}$

Figure 6-13. Spanwise dipole strength of the lead row for all of the examined cuboidal roughness fetches at $U_o=60\text{m/s}$
6.2.3 Streamwise Source Strength Distribution through a 42 Element Fetch of 3mm Cuboidal Roughness

The source strength variation through the 6x7 fetch of cuboidal roughness is shown in Figure 6-14 compared to the source strengths determined from the single cubic element presented in Figure 6-1 for a nozzle exit velocity of 60m/s. The results are spanwise averaged so that 14 total strength spectra are shown representing the streamwise and spanwise per-element averaged dipole strengths for each of the 7 spanwise rows. In general, the relative estimated strength of the streamwise dipole for each row is stronger than that of the corresponding spanwise source. The estimated streamwise strengths for the trailing rows in Figure 6-14 mostly fall below the curve created by the lead row so that the lead row is the strongest producer of noise in the spanwise and streamwise direction. As shown in Figure 6-12 and Figure 6-13, both the streamwise and spanwise dipole are slightly stronger for the lead row of the 42 element fetch as compared to the single cubic element with the difference increasing towards lower non-dimensional frequencies, but this is within the uncertainty of the estimation as will be shown in Chapter 7.

Figure 6-14. Source distribution through 42 element cubic roughness fetch at $U_o=60$ m/s: solid lines (1st Row), square (2nd Row), diamond (3rd Row), triangle (4th Row), upside-down triangle (5th Row), right-pointing triangle (6th Row), pentagon (7th Row) compared to single cubic element strengths in grey (solid) streamwise dipole (dashed) spanwise dipole

$N^2/Hz/\rho^2U^3h^5$ vs $f_h/U$
6.2.4 Comparison with LES

Direct comparison of the estimated strengths in Figure 6-14 can be made with the LES study of Yang & Wang (2011). This comparison is shown in Figure 6-15. The LES results are the same as presented in comparison with the estimated single cubic element source strengths in Figure 6-9. The LES data are the per-element averaged streamwise and spanwise dipole strengths for the lead row of cuboidal roughness arranged in a 4x10 grid pattern. The least-squares estimated streamwise dipole strength is larger than that determined from LES at all non-dimensional frequencies. Estimation of the spanwise dipole is actually closer to the LES results, but the slopes of the estimated spectra differ slightly so that comparison of the streamwise and spanwise dipoles agree better at lower non-dimensional frequencies. The spanwise results agree within a factor of two over the entire non-dimensional frequency range. The disagreement between the streamwise dipole results increases from a factor of 2 to 17 with increasing non-dimensional frequency.

Figure 6-15. 42 cubic element roughness fetch streamwise and spanwise dipole strengths at $U_o=60$m/s compared to lead row results from LES of Yang & Wang (2011): symbols same as Figure 6-14
6.2.5 Confirmation of Streamwise and Spanwise Dipole

This least-squares analysis is dependent upon the assumed presence of a streamwise and spanwise dipole located at each roughness element location. To test this presumption, the analysis for the 42 element cubic fetch at $U_0=60\text{ m/s}$ was completed assuming only a streamwise dipole, only a spanwise dipole, and a complete representation including both. A diagram of the three source radiation models is shown in Figure 6-16 as applied to single cubic roughness elements. For the two models that assumed only a streamwise or spanwise radiating dipole, the least-squares method was used to calculate only 42 total sources over the roughness fetch, one for each roughness element. The full representation included the simultaneous computation of 84 independent sources.

![Diagram of assumed roughness noise models](image)

Figure 6-16. Diagram of assumed roughness noise models a) streamwise-only dipole b) spanwise-only dipole c) and the full representation including both dipole sources

The least-squares analysis was conducted three times using the three different source models. The results were compared to the corresponding single point far field measurement in Figure 4-8 for $U_0=60\text{ m/s}$. To calculate the estimated single point far field noise using the least-squares determined source strengths and models the far field noise was attained using a similar transformation matrix of Greens functions originally used to determine the source strengths as defined in Equations 2-9, 2-10, and 2-11. The estimated source strengths are combined into vector form and are multiplied by a vector of Greens function transformations that apply the assumed radiation pattern to transfer the dipole strengths to the far field noise produced at the considered far field location. This process is shown in Equation 6-10.

$$ G_j = \frac{ik\cos\theta_j e^{i(kr_j)}}{2\pi r_j} \left(1 + \frac{i}{kr_j}\right) \cos\theta_j = \hat{x} \cdot (x - y_j)/r_j $$

$$ PP^T = \sum_{j} q_j^2 G_j G_j^* + \sum_{j} q_j^2 G_j G_j^* + \cdots q_j^2 G_j G_j^* $$

\[ \text{Eq. 6-10} \]

$$ PP^T = \bar{G} \bar{G}^* \bar{Q} $$

where $\theta_j$ is the receiving angle from the far field location to the axis of source j’s dipole, $r_j$ is the distance from observer position to the source, $x$ is the position vector of the observer, $y_j$ is the position vector of
the source, \( \overline{PP^T} \) is the measured autospectrum, \( q_j \) is the strength of source \( j \). \( \overline{G}_{GG} \) is a vector of Greens function transformations with dimensions of one by number of sources, and \( \overline{Q} \) is a vector of source strengths with dimensions of number of sources by one.

The comparison of the results using the three dipole radiation models is shown in Figure 6-17. Using only a spanwise dipole, the analysis produced estimated far field results that were far below the measured single point values since the compared position was in the streamwise direction. The individual calculated sources using the spanwise-only dipole model radiate very inefficiently in the streamwise direction producing low level far field results at the single point measured far field position which was at \( x=1029\text{mm} \), \( y=469\text{mm} \) and \( z=0\text{mm} \). Therefore, these results are not displayed in Figure 6-17. When assuming only a streamwise dipole, the measured far field was overpredicted. This is due to the streamwise dipole having to compensate for the large volume of sound measured in the spanwise direction by the microphone array. The least-squares calculation for this case does not have an assumed spanwise dipole source. So the only way for the calculation to resolve the large spanwise radiated noise is to increase the strength of the roughness noise model’s streamwise dipoles. The complete representation produced the best results agreeing with the measured single point far field roughness noise spectrum within 1dB. This corroborates the assumption of both spanwise and streamwise aligned dipoles that emanate from each roughness element to produce the radiated far field noise.

![Figure 6-17. Comparison of the far field noise determined by extrapolation of the least-squares estimated results assuming both streamwise and spanwise dipole sources and only a streamwise dipole source with the measured single-point far field spectrum](image-url)
6.3 Multi-Element Hemispherical Fetches

Multi-element fetches of hemispherical roughness were also analyzed using the least-squares method. Figure 6-18 displays the spanwise-averaged source strength results for the 42 element fetch of 3mm hemispheres at $U_o=60$m/s. The single hemispherical element results from Figure 6-1 are shown for comparison. The strengths calculated from the multi-element fetch are similar to the strengths calculated from the single roughness element. Again, the estimates show that the lead row of roughness is the largest producer of sound in the streamwise and spanwise directions, but the low signal-to-noise ratio produced by the hemispherical roughness increases the uncertainty of this measurement. Therefore, no definitive conclusion can be made about the relative strengths of the trailing rows. Also, like the results from the cuboidal roughness, the calculated streamwise dipole strengths for the hemispherical roughness are stronger than the corresponding spanwise dipole strengths for each spanwise row.

These data for the 42 element fetch of hemispheres are compared to the source strength data calculated from the LES of Yang & Wang (2011) for a 40 element fetch of hemispheres. This comparison is shown in Figure 6-19. The data differ by up to 30dB disagreeing more than the comparison of the cuboidal roughness studies shown in Figure 6-15. Like the cuboidal roughness comparison, the source strength estimations of the hemispherical roughness agree better at lower non-dimensional frequencies.
The least-squares estimated spectra have a shallower slope than the LES spectra so that the difference grows with non-dimensional frequency. The presented data is clearly in the upper frequency limits of reliability for the LES analysis. The LES estimated strengths show significant noise in the estimations at the highest presented frequencies indicating that these results may be limited by the resolution of the computational study. This may explain the difference in spectral slope over the calculated frequency range.

Figure 6-19. Comparison of source strengths from a hemispherical roughness fetch determined by the least-squares method (symbols same as Figure 6-14) and Yang & Wang's (2011) computational study (streamwise dipole: black-solid, spanwise dipole: black-dashed)
Chapter 7 Uncertainty of Least-Squares Analysis

The least-squares method provides estimates of the streamwise and spanwise dipole source strengths for single and multi-element roughness fetches, but the signal-to-noise ratio and number of sources in the analyses appear to have an effect on the quality of the solutions. The following section analyzes the uncertainty of the least-squares source strength estimations and the sensitivity of the solution to uncertainty in the microphone array sensors, the signal-to-noise ratio, the error in source location, and the number of sources.

7.1 Jitter Analysis

A jitter analysis was conducted for several of the least-squares source strength estimations to determine the sensitivity of the calculation to the uncertainty of the measured cross-spectra of the linear microphone array sensors. The uncertainty of the cross-spectral matrix was determined by repeat measurements during the experiment and is due to uncertainty inherent in the instrumentation. These uncertainties were found to be 1dB amplitude of the cross-spectral matrix and a 10° phase variation, at 10368Hz. Therefore, each cross-spectral matrix pair in the least-squares analysis was jittered by these amounts separately and the results summed to determine the total uncertainty as shown in Equation 7-1.

\[ \delta \left( q^2(f) \right) = \sqrt{\frac{\partial q^2(f)}{\partial A_{i,j}} \delta(A_{i,j}) + \frac{\partial q^2(f)}{\partial P_{i,j}(f)} \delta(P_{i,j}(f))} + \cdots \]  

Eq. 7-1

where \( \delta \left( q^2(f) \right) \) is the uncertainty in the determined mean squared source strength at a given frequency, \( \frac{\partial q^2(f)}{\partial A_{i,j}} \) is the sensitivity of the calculated source strength to the amplitude of the index \((i, j)\) and its conjugate pair \((j, i)\), \( \delta(A_{i,j}) \), of the measured cross-spectral matrix. \( \frac{\partial q^2(f)}{\partial P_{i,j}(f)} \) is the sensitivity of the calculated source strength to the phase uncertainty at a given frequency of the index \((i, j)\) and its conjugate pair \((j, i)\), \( \delta(P_{i,j}(f)) \), in the measured cross-spectral matrix.

The results of the jitter analyses for the source strength spectra of the single roughness elements shown in Figure 6-1 are given in Figure 7-1. The results for the single cubic element are only mildly affected by the uncertainty of the measured cross-spectral matrix. The uncertainty bounds are wider on the high and low frequency extremes of the estimation. The estimated values for the single hemispherical element are much more sensitive. The results for the two dipole sources have uncertainties that are as much as a factor of 2 different from the estimated value. This shows that the sensitivity of the analysis is dependent upon the signal-to-noise ratio of the measurement itself.

A similar jitter analysis was conducted for the results of the streamwise and spanwise dipole strengths of the lead row of the 42 element fetch of cuboidal roughness. The results for the spanwise-averaged dipole strengths are shown in Figure 7-2. The addition of more roughness elements increases the sensitivity of the least-squares analysis results to the uncertainty of the cross-spectral matrix. The uncertainty is greatest over the middle of the calculated frequency range and extends up to a factor of 2 for the streamwise dipole and a factor of 1.5 for the spanwise dipole. Overall, the uncertainty determined
by the jitter analysis seems reasonable over the entire calculated frequency range for all of the studied surfaces, but this analysis does not calculate or give any indication as to the accuracy of the estimated source strengths. It only portrays the sensitive of the analysis to the uncertainty of the sensors used. The jitter analysis itself is dependent on the number of assumed sources and the signal-to-noise ratio of the measurement. Therefore, an analysis of these effects on the accuracy of the estimation is also studied.

Figure 7-1. Single element jitter analysis for the cuboidal and hemispherical roughness (streamwise dipoles: solid, spanwise dipoles: dashed)
7.2 Uncertainty Due to Noise in Measurement

To calculate the uncertainty of the estimated source strength values due to noise in the measurement, a simulated measurement was conducted of a pair of dipoles with origins at $x=1257\text{mm}$, $z=0$ having constant source strengths of $2.244\times10^{-5} \text{N}^2/\text{Hz}$, $1\text{dB}$ with reference $4\times10^{-10}\text{Pa}^2$, aligned in the streamwise and spanwise directions. The cross-spectral matrices were calculated for this pair of sources over the entire least-squares estimated frequency range as would be recorded using the linear microphone array at the four positions used in this study. Varying levels of noise were added to the cross-spectral matrix of the signal. The cross-spectral matrix of the noise varied by $3\text{dB}$ and from $0$ to $2\pi$ in phase. The level of the signal-to-noise ratio for each sensor was different due to the dipole source directivity, varying sensor to source distances, and $3\text{dB}$ variation of the noise, but average signal-to-noise ratios were computed for each array position and are used to denote each noise level. Five different noise levels were examined which were a constant strength below the measured signal at each array location over all frequencies. An illustration of the studied signal-to-noise ratios is shown in Figure 7-3. The far field autospectrum of the signal, and therefore noise level spectra, have a slope of $\omega^2$ because of the dipole efficiency factor, $k_o^2 = (\omega/c_\infty)^2$.
The least-squares analysis was conducted for this pair of dipoles with the varying levels of noise added to the spectra. These results are shown in Figure 7-4. For the case with the largest signal-to-noise ratio, the exact source strength is very accurately computed at all frequencies. As the signal-to-noise ratio is diminished from a factor of 4 to 1/10, the computed source strengths become progressively more inaccurate. With a 1/10 signal-to-noise ratio, the error is as great as a factor of 2 which is still a reasonable estimation method for these very low signal-to-noise ratio sources.

The measured noise from the single cubic roughness element had a signal-to-noise ratio at $U_o=60\text{m/s}$ as high as a factor of 0.6 as measured from microphone array Position 2. This indicates that the estimations of the source strengths at this condition are only slightly influenced by the background noise. The signal-to-noise ratio of the far field noise recorded by the single hemispherical element is extremely small. If the relative difference of noise recorded from the 42 element fetches is an indication of the single element differences, the SNR of the single cube is 63 times stronger than that of the single hemisphere.

**Figure 7-3. Example signal-to-noise ratio at far field position**

![Graph showing signal-to-noise ratio vs frequency](image)
7.3 Sensitivity to Error in Source Position

The source locations for the experimental analysis of the measured roughness noise were assumed to be located at the central position of each of the roughness elements. The physical source locations were adjusted in the analysis to correct for an acoustic wave convection effect due to the local flow velocity. This resolved the difference between the observed and physical source location. The source maps in Chapter 5 indicate that this correction method works well to visually identify the source locations. The same corrections were applied in the least-squares analysis, and the sensitivity of these numerical solutions is an important aspect of this method to consider.

A simulation of a single dipole pair with a signal-to-noise ratio of ¼ was used to analyze the effect of source location error on the determined source strengths. This is the same case examined in Figure 7-4. Figure 7-5 shows the least-squares analysis of the streamwise and spanwise dipole strengths with independent 6mm source location errors in the x and z directions compared to the nominal solution and the exact solution without any noise in the measurement. Interestingly, the solution is minimally affected with source location errors equal to twice the roughness height over this frequency range. Error in the z-location has very little effect on the computed spanwise dipole strength. Error in the x-direction has an impact on both the estimated streamwise and spanwise dipole strengths. In both cases, the error remains within the maximum error produced by the analysis of the ¼ signal-to-noise ratio using the exact
known source location. The deviation from this nominal case grows with frequency showing that the error due to source location increases with the ratio of the location error to the acoustic wavelength.

Combining the results of the jitter analysis, signal-to-noise ratio analysis, and the source location error, the total estimated uncertainty of the single cubic element source strength estimations appears to be within 1dB, a factor of 1.5, at the highest calculated frequencies and within 3dB, a factor of 2, at the lowest frequencies. The source strengths determined from the single hemispherical element have a higher uncertainty. This limited frequency range of results should only be trusted within 5dB, or a factor of 3.

![Graph](image)

**Figure 7-5. Single source least squares results with 1/3 SNR with varying position error in the x and z directions**

### 7.4 Effect of Multiple Source Locations

Using three of the signal-to-noise ratios illustrated in Figure 7-3, the addition of multiple source locations was studied. The first analysis involves two source locations with varying source spacings. At each source location the same roughness noise model was assumed having a spanwise and streamwise aligned dipole source with constant, equal source strengths of $2.244 \times 10^{-5}$ N$^2$/Hz. Therefore, this analysis included four total sources. The location of one dipole source pair was held fixed at (0, 0) and the other was offset in the positive $z$-direction by 8.25mm, 16.5mm, and 24.75mm. The level of the signal-to-noise ratios listed in Figure 7-3 is approximately doubled by the addition of a second uncorrelated source. Figure 7-6 shows the result of the least-squares analysis for all four independent source strengths. To stay consistent with the labeling in Figure 7-3, the legend of this figure lists the signal-to-noise ratio as applicable to the signal from one source. The true average signal-to-noise ratio for the sensors at each
array position are approximately double these values. The level of noise was not increased because the increased signal-to-noise ratio is a more accurate representation of the experimental condition when the number of sources is increased as the background levels remain approximately constant.

As the source spacing increases from 8.25mm to 24.75mm, the error reduces as well. Also, the instances of zero value solutions assigned to source strengths diminishes with increased source spacing. An interesting observation from all three source spacings is that the value of the two streamwise dipoles are approximately inversely related as well as the values of the spanwise dipoles for the two source locations. Therefore, as the estimated value of one is underpredicted, the value of the other is overpredicted to compensate for the measured signal strength. As sources are assigned strengths of zero, the corresponding source from the other location is approximately double the exact source strength. This confirms the value of the spanwise-averaging method used to present the average dipole strengths in Chapter 6 from the multi-element fetches. Averaging does reduce the uncertainty of the solution by diminishing the effect of estimated source strength fluctuations by reassignment and distribution of the true strengths between roughness elements.

The grid-spacing of the studied roughness fetches was 16.5mm. At this spacing there is considerable error with decreasing signal-to-noise ratio for individually calculated source strengths, but the average values are very near the actual source strengths. To study this averaging effect to reduce uncertainty, a 6x7 arrangement of sources will be simulated recreating the measurement of the 42 element cuboidal and hemispherical fetches.
Figure 7-6. Estimated source strengths of at two source positions separated by $z=8.25\text{mm}$, b) 16.5mm, c) and 24.75mm with various background noise levels: source $(0,0)$ streamwise dipole-solid, spanwise dipole-dashed; source $(0, z)$ streamwise dipole-dash/dot, spanwise dipole-dot
7.5 Uncertainty of the 42 Element Roughness Fetch Strength Analysis

The 42 element cuboidal roughness fetch had an 18dB signal-to-noise ratio at the highest measured frequency, 20kHz. This means the signal was a factor of 63 greater than the background noise. Using the cross-spectral matrix of noise that created the average signal-to-noise ratio of 2 as calculated for the single source location, corresponding to the cyan curve in Figure 7-3, the addition of 42 streamwise and spanwise aligned dipole sources in the 16.5mm grid arrangement with source strengths of $2.244 \times 10^{-5}$ N$^2$/Hz results in an average signal-to-noise ratio of 75 at each array position. The signal-to-noise ratio is not increased by a factor of 42 from 2 to 84 because the directivity pattern of the dipole sources reduces the average signal recorded at each array position. Therefore, this simulation is an experimentally similar estimate of the uncertainty of the 42 element cuboidal fetch results. A least-squares analysis of the spanwise-averaged streamwise and spanwise dipole strengths was conducted under these simulated conditions to determine their deviation from the known values.

Figure 7-7 shows the estimated spanwise-averaged streamwise dipole strengths calculated for each row of the source fetch. These results are accurate within a factor of 1.2 diminishing with increased frequency. All rows have similar levels of estimated uncertainty. The least-squares analysis does not appear to favor any source location positions. This indicates that the streamwise dipole strengths estimated in Figure 6-12 and 6-14 are accurate within a factor of 2.5 over the entire frequency range and are even more accurate at higher frequencies above 9kHz within 10%. The results from the lead row of the cuboidal roughness fetch may indeed have this low level of uncertainty, but the trailing rows appear to have more uncertainty possibly due to their lower signal-to-noise ratios. At some frequencies, an entire spanwise row has an average estimated source strength value of zero and the strength reallocation effect shown in Figure 7-6 works to transfer source strengths between the spanwise rows. The total fetch-averaged streamwise and spanwise dipole strengths should be a more accurate representation of the streamwise and spanwise dipole radiation but would not provide an understanding of the source strength distribution. Therefore, the uncertainty of the spanwise-averaging analysis needs to be quantified with a justifiable approach. The uncertainty of the estimated source strengths for the trailing rows cannot be calculated as factor of the estimated value because zero values are assigned. The only way to analyze the results is by observing all of the source strengths and making a judgement on the level of accuracy at each frequency solution. A lower cut-off is needed beyond which estimated source strengths are ignored. The predicted strength of the lead row of the cuboidal roughness fetch is a repeatable measurement with varying size roughness fetches and is therefore, most likely an accurate representation of the source strength. The majority of estimated source strengths for the trailing rows are contained within two orders of magnitude of these lead row dipole strengths. Below this, there are no repeatably estimated spectral values and the estimated strengths may be carefully ignored by assuming that the source strengths through this short fetch of large discrete roughness do not vary by over a factor of 100. In Figure 6-14, if these values are ignored, the results of these trailing rows appear to have an uncertainty that is within 5dB.

The spanwise-averaged spanwise dipole results are shown in Figure 7-8. These results are far more uncertain at frequencies below 9kHz corresponding to an acoustic wavelength of 38mm which is slightly over double the source separation. The arrangement of the four microphone array positions appears to bias measurement of the streamwise dipole producing much greater uncertainty of the spanwise dipole source. Above 9kHz, the average spanwise dipole results are extremely accurate and deviate negligibly from the true value. A noticeable change in the source strength spectra as observed in Figure 7-8 below 9kHz is not seen in Figure 6-13 and 6-14. This may be due to differences in the
simulated experiment with the roughness noise measurement. The consistent estimated spectral shape of the spanwise noise from the cuboidal roughness indicate that the uncertainty may be better than the simulation suggests. Like the streamwise dipole, the trailing rows of the spanwise dipole appear to have an uncertainty within a factor 5, ignoring results more than two orders of magnitude less than the lead row’s estimations, while the lead row’s results, estimated as the strongest, are less uncertain within a factor of 2.5. Above 9kHz, the lead row’s results are uncertain within 10%. For both the streamwise and spanwise dipoles, the uncertainty analysis indicates that the results for all rows are more accurate at higher frequencies.

This simulation was representative of the evaluation of the noise from the cuboidal roughness, but the hemispherical roughness produced a much lower total signal-to-noise ratio. The 6x7 array of hemispherical roughness was barely distinguishable in the measured autospectra with only a ¼ total SNR. Therefore, the estimated streamwise and spanwise dipole strengths from this fetch are far more uncertain. If the estimated values that fall more than two orders of magnitude below the lead row’s calculated source strengths are ignored, the estimations shown in Figure 6-19 for the 42 element hemispherical roughness fetch are most likely accurate within 10dB, or an order of magnitude.

Figure 7-7. Spanwise averaged streamwise dipole strengths
Figure 7-8. Spanwise averaged spanwise dipole strengths
Chapter 8 Conclusions

Far field noise and wall pressure spectra have been recorded from turbulent wall-jet flow over discrete hemispherical and cuboidal roughness. Also, the scattering theory of Glegg & Devenport (2009) has been investigated and used to make predictions of roughness noise for stochastic and deterministic rough surfaces at conditions with boundary layer to roughness height ratios ranging 0.06 to 0.18 and roughness Reynolds numbers 46 to 198. These predictions were also used to explore the form and accuracy needed of the model wavenumber wall pressure spectrum to make absolute predictions of the far field noise. A linear microphone array was used to record the far field noise from fetches of discrete cuboidal and hemispherical roughness. These array data were examined using standard delay and sum beamforming techniques as well as a novel least-squares source strength analysis to determine the strength and variation of source strengths through fetches of discrete roughness.

This analysis showed that the scattering theory of Glegg & Devenport (2009) can be used to accurately predict the radiated roughness noise even from surfaces with large roughness Reynolds numbers and moderately large roughness height to boundary layer thickness ratios. These predictions only entail knowledge of the surface geometry and form of the wall pressure spectrum, specifically the convection velocity. These estimations are improved with the addition of measurements of the rough wall pressure frequency spectrum, but the accuracy of this prediction is only mildly affected by the form of the wavenumber wall pressure spectrum. Therefore, further study of the wavenumber wall pressure spectrum will have little effect on the predicted results. The “break frequency” as described in Alexander (2009) and Devenport et al. (2011) was found to be a predictable function of the surface geometry that is related to the size of the roughness elements.

The least-squares method of source strength analysis was shown to be a viable method to decompose source strengths through a fetch of discrete roughness noise sources. Using this technique, the existence of streamwise and spanwise aligned dipoles located at each roughness element was confirmed. Also, the estimated relative strengths of the dipole sources were found to vary with frequency.

The following list details the remaining conclusions of this study:

Measurements of Roughness Noise and Wall Pressure Spectra

- Noise spectra produced by different roughness geometries varies in form and so cannot be collapsed on one another with simple inner or outer boundary layer variable scaling.
- For rough surfaces with large element height to boundary layer thickness ratios, the produced wall pressure spectra are highly inhomogeneous. The measurements show increased pressure fluctuations in the wakes of elements and a wall pressure field around elements consistent with a horseshoe vortex wrapped around the base of elements with separation zones just upstream and downstream of individual elements.
- Normalizations of the far field noise using the average measured wall pressure through fetches of discrete roughness produce similar collapses as observed in Alexander (2009) and Devenport et al. (2011) for stochastic and deterministic surfaces. The normalized spectral shapes differ with the roughness geometry. The normalized spectra produced by the fetch of randomly distributed 1mm hemispherical roughness resembles the $\omega^2$ collapse and “break frequency” observed from
the stochastic roughness in Alexander (2009). After the “break frequency” the data fan out in order of velocity.

**Roughness Noise Predictions**

- Predictions of the far field noise normalized on the single point wall pressure spectrum accurately reproduce the spectral shapes of the measured normalized curves for all of the surfaces. Absolute predictions for the smaller roughness heights are closest to the measurements, with the normalized spectrum becoming somewhat underpredicted for the larger roughness sizes. For the smaller 1mm roughness, the normalized spectra are accurately predicted within 3dB before the “break” and 6dB after.
- The origin of the “break frequency”, as observed in Alexander (2009) and Devenport *et al.* (2011), was shown to be a predictable function of the roughness geometry. The “break frequency” occurs at the frequency where the peak of the convective ridge lines up with a major lobe in the wavenumber filter function.
- Predictions of the absolute far field noise show that the radiated roughness noise for roughness that is a small fraction of the boundary layer thickness can be very accurately determined if the surface geometry and rough wall pressure spectrum are known.

**Microphone Array Beamforming Analysis**

- Source strengths were observed to vary only slightly across the span of the roughness, but the lead rows of the roughness produced stronger source strengths in both the streamwise and spanwise directions.

**Least-Squares Source Strength Analysis**

- A single hemispherical element produced weaker far field noise, and therefore, the drag and side force spectra are over an order of magnitude lower than that of the cube. For multi-element fetches, the lead row of roughness was found to be the strongest producer of sound in the streamwise and spanwise direction with weaker estimated source strengths from all of the following rows.
- The relative magnitude of the estimated streamwise and spanwise drag spectra for a single cubic element changed with frequency indicating that the directivity varies by frequency.
- The LES calculated strengths of Yang & Wang (2011) for the lead row of cuboidal and hemispherical roughness and the source strengths calculated by the least-squares method are similar at lower non-dimensional frequencies. The disagreement between the computational and experimental studies grows with non-dimensional frequency. The streamwise dipole source strength calculated by the least-squares method is greater than the LES estimated strength at all non-dimensional frequencies.
## Appendix A: Random Roughness Locations

The following is a list of the roughness element locations for the roughness fetch of 6209 randomly located 1mm hemispheres. These positions are given in millimeters relative to a corner of the roughness fetch so that all values are positive. The x-values (left column) are in the streamwise direction and z-values (right column) are the spanwise orientation.

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