Implementation of Iterative Reconstruction of Images from Multiple Bases
Representations

by

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(ABSTRACT)

Usually, image compression techniques that use only one transform exhibit some poor properties. For instance, the Discrete Cosine Transform (DCT) cannot efficiently represent high frequency components, resulting in blurred images. The Multiple Bases Representation (MBR) compression technique, which uses two or more transforms, is found to be superior to the single transform techniques in terms of representation efficiency. However, some bits in the MBR representation are needed to track the basis information. The MBR image quality is deteriorated by discontinuities at block boundaries, as is the standard DCT transform.

In this thesis, test images are distorted by MBR compression using a Recursive Residual Projection algorithm. This algorithm is a sub-optimal method to find the best basis vector subset for representing images based on multiple orthogonal bases. The MBR distorted images are reconstructed by the iterative method of Projection onto Convex Sets (POCS). Many constraints that form convex sets are reviewed and examined.
Due to the high distortion at the block boundaries, some constraints are introduced particularly to reduce artifacts at the boundaries. Some constraints add energy to the reconstructed images while others remove energy. Thus, the initial vectors play a key role in the performance of the POCS method for better MBR reconstruction. This thesis also determines the most appropriate initial vector for each constraint.

Finally, the composite projections associated with the sign, minimum decreasing and norm-of-slope constraints are used to improve the reconstruction of the MBR distorted images and the effect of ordering of the projections is investigated.
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# Table of Contents

1. Introduction .................................................................................................................. 1  
   1.1 Background ........................................................................................................... 1  
   1.2 Overview of the Hybrid Compression Method ..................................................... 2  
   1.3 Overview of Iterative Image Reconstruction ....................................................... 3  
   1.4 Thesis Preview .................................................................................................... 4  

2. Theoretical Review ....................................................................................................... 6  
   2.1 Representation of Signals: Introduction ............................................................. 6  
   2.2 Representation of Signals: Multiple Bases Representation (MBR) .................... 9  
   2.3 Recursive Residual Projection (RRP) ............................................................... 11  
   2.4 RRP Outcomes and Statistics .......................................................................... 14  

3. Iterative Image Reconstruction .................................................................................... 17  
   3.1 Mathematics Review ......................................................................................... 17  
   3.2 Projection Onto Convex Sets (POCS) ............................................................... 20  
   3.3 MBR Constraint .............................................................................................. 22  
   3.4 Local Constraint Definitions and Projection Operators .................................. 29  

4. Blocking Artifact ....................................................................................................... 39  
   4.1 Boundary Characteristics ............................................................................... 39  
   4.2 Slope Constraint .............................................................................................. 41  
   4.3 Norm-of-Slope Constraint ............................................................................. 42  
   4.4 Estimate of the Scalar E ............................................................................... 48
5. Implementation Description ................................................................. 50
   5.1 Implementation Tool ....................................................................... 50
   5.2 Implementation Preview ............................................................... 51
   5.3 Quantization Method ...................................................................... 51
   5.4 Error Signal Determination ............................................................. 52
   5.5 Image Quality Measurement ......................................................... 52
   5.6 Initial Vector .................................................................................. 54
   5.7 Types of Test Images ...................................................................... 60
   5.8 Number of Iterations ...................................................................... 60

6. Performance Improvements Due to Constraints ............................... 61
   6.1 Additional Information Determination .......................................... 61
   6.2 Effects of Initial Vectors on Performance Due to Individual Constraints .... 63
   6.3 Performance Improvements Due to the Boundary Constraints ......... 79
   6.4 Implementation on Other Images ................................................... 88
   6.5 Sequence of Projection Operators .................................................. 93

7. Conclusion ......................................................................................... 101

References ............................................................................................ 103
Vita ........................................................................................................ 105
List of Figures

Figure 2.1 The RRP implementation ................................................................. 13
Figure 3.1 The MBR constraint ..................................................................... 25
Figure 4.1 Pixel intensity at the boundaries ................................................ 40
Figure 4.2 Choices of vectors for $C_{NS}$ .................................................... 45
Figure 6.1 PSNR of the images reconstructed by the sign constraint .......... 65
Figure 6.2 64×64 Subimage for subjective evaluation ................................. 69
Figure 6.3 Performance of the sign constraint at various added noise levels .................................................. 71
Figure 6.4 PSNR of the images reconstructed by the minimum increasing constraint ............................................. 73
Figure 6.5 PSNR of the images reconstructed by the spike constraint ......... 76
Figure 6.6 Comparison of the reconstructed images ..................................... 78
Figure 6.7 PSNR of the images reconstructed by the slope constraint ........ 80
Figure 6.8 Performance due to the norm-of-slope constraint with actual norms .............................................................. 83
Figure 6.9 Exact and estimated boundary norm values .................................. 85
Figure 6.10 Performance due to the norm-of-slope constraint with estimated norms ................................................................. 86
Figure 6.11 Comparison of the images reconstructed by the boundary constraints .................................................. 87
Figure 6.12 Reconstructed landscape images ................................................ 91
Figure 6.13 Reconstructed X-ray images ....................................................... 92
Figure 6.14 PSNR of different projection sequences used on the landscape image ....... 96
Figure 6.15 Reconstructed portrait images by using composite contraints .............. 98
Figure 6.16 Reconstructed landscape images by using composite contraints .......... 99
Figure 6.15 Reconstructed X-ray images by using composite contraints .......... 100
List of Tables

Table 6.1 Improvement of overall PSNR (dB) from the MBR observed signal .......... 90
Table 6.2 Improvement of boundary PSNR (dB) from the MBR observed signal ...... 90
Table 6.3 Overall PSNR of different projection sequences .................................. 94
Table 6.4 Boundary PSNR of different projection sequences. ............................... 94