Development of Data Analysis Algorithms for Interpretation of
Ground Penetrating Radar Data

Samer Lahouar

Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in

Electrical Engineering

Dr. Imad L. Al-Qadi, Co-Chair
Dr. Gary S. Brown, Co-Chair
Dr. Richard W. Conners
Dr. William A. Davis
Dr. David A. de Wolf

October 3, 2003
Blacksburg, Virginia

Keywords: Ground Penetrating Radar, Nondestructive Testing, Layer Thickness, Dielectric Constant, Layer Detection, Flexible Pavements

Copyright 2003 by Samer Lahouar
Development of Data Analysis Algorithms for Interpretation of Ground Penetrating Radar Data

Samer Lahouar

Abstract

According to a 1999 Federal Highway Administration statistic, the U.S. has around 8.2 million lane-miles of roadways that need to be maintained and rehabilitated periodically. Therefore, in order to reduce rehabilitation costs, pavement engineers need to optimize the rehabilitation procedure, which is achieved by accurately knowing the existing pavement layer thicknesses and localization of subsurface defects. Currently, the majority of departments of transportation (DOTs) rely on coring as a means to estimate pavement thicknesses, instead of using other nondestructive techniques, such as Ground Penetrating Radar (GPR). The use of GPR as a nondestructive pavement assessment tool is limited mainly due to the difficulty of GPR data interpretation, which requires experienced operators. Therefore, GPR results are usually subjective and inaccurate. Moreover, GPR data interpretation is very time-consuming because of the huge amount of data collected during a survey and the lack of reliable GPR data-interpretation software. This research effort attempts to overcome these problems by developing new GPR data analysis techniques that allow thickness estimation and subsurface defect detection from GPR data without operator intervention. The data analysis techniques are based on an accurate modeling of the propagation of the GPR electromagnetic waves through the pavement dielectric materials while traveling from the GPR transmitter to the receiver. Image-processing techniques are also applied to detect layer boundaries and subsurface defects. The developed data analysis techniques were validated utilizing data collected from an experimental pavement system: the Virginia Smart Road. The layer thickness error achieved by the developed system was around 3%. The conditions needed to achieve reliable and accurate results from GPR testing were also established.
Acknowledgments

I would like to express my sincere gratitude to my advisor Dr. Imad L. AL-Qadi for his guidance, encouragements, and support during this research. Special thanks are also given to my co-advisor Dr. Gary S. Brown for his contribution in enhancing the quality of this research. My thanks go also to Dr. Richard W. Conners, Dr. William A. Davis, and Dr. David A. de Wolf for accepting to be in my committee to evaluate and improve this work.

My highest gratitude goes to my parents Chaabane and Fethia, my brothers Souhail and Saher, my sisters Souhir and Afek, and all the other members of my family, for their love and encouragement. Without your love and support I would not have accomplished this work.

I would like to thank my friend Amara Loulizi for his assistance and advice during this research. Special thanks go also to my friends and colleagues in the Roadway Infrastructure Group: Alex Appea, Mostafa Elseifi, William (Billy) Hobbs, Edgar de Leon, Samer Katicha, and the rest of the team.

Finally, I would like to express my thanks and appreciation to all my friends back home in Tunisia and all the new friends that I made during my five-year stay in Blacksburg.

Samer Lahouar

October, 2003
# Table of Contents

Abstract ........................................................................................................................................... ii

Acknowledgments .......................................................................................................................... iii

Chapter 1 Introduction .................................................................................................................... 1
  1.1 Background .............................................................................................................................. 1
  1.2 Problem Statement .................................................................................................................. 3
  1.3 Research Objectives ................................................................................................................ 3
  1.4 Scope ....................................................................................................................................... 4

Chapter 2 Current State of Knowledge ........................................................................................... 5
  2.1 Pavements ............................................................................................................................... 5
    2.1.1 Flexible pavements .......................................................................................................... 5
    2.1.2 Rigid pavements .............................................................................................................. 8
    2.1.3 Composite pavements .................................................................................................... 10
  2.2 Ground Penetrating Radar (GPR) Systems .......................................................................... 10
    2.2.1 Ground penetrating radar types ..................................................................................... 11
      2.2.1.1 Frequency modulated GPR .................................................................................... 11
      2.2.1.2 Synthetic pulse GPR ............................................................................................ 12
      2.2.1.3 Pulsed (or impulse) GPR ..................................................................................... 12
    2.2.2 Ground penetrating radar antennas .............................................................................. 14
  2.3 Electromagnetic Theory Pertinent to GPR Systems ............................................................. 14
    2.3.1 Electromagnetic propagation .......................................................................................... 15
    2.3.2 Electromagnetic scattering ............................................................................................. 18
      2.3.2.1 Scattering from a planar interface .......................................................................... 18
      2.3.2.2 Scattering from multiple planar layers .................................................................... 21
  2.4 GPR Applications to Pavements ......................................................................................... 23
    2.4.1 Measurement of pavement layer thicknesses ................................................................. 24
    2.4.2 Detection of subsurface distresses ................................................................................... 28
    2.4.3 Estimation of concrete properties .................................................................................... 29
    2.4.4 GPR imaging techniques ............................................................................................... 31
2.5 Potential Benefits of Using GPR for Pavement Assessment ................................. 32

Chapter 3 Research Approach ......................................................................................... 34
3.1 Virginia Smart Road ............................................................................................... 34
3.2 GPR System Description ....................................................................................... 37
   3.2.1 Control unit ...................................................................................................... 37
   3.2.2 Antennas .......................................................................................................... 38
   3.2.3 Data acquisition control .................................................................................. 41
   3.2.4 Performance tests ......................................................................................... 42
3.3 Data Collection ....................................................................................................... 45
3.4 Research Tasks ....................................................................................................... 47

Chapter 4 Ground Penetrating Radar Data Analysis Techniques ........................................ 49
4.1 Introduction ............................................................................................................ 49
4.2 Preprocessing ......................................................................................................... 51
   4.2.1 Coupling pulse removal .................................................................................. 51
      4.2.1.1 Use of the coupling pulse in GPR data interpretation ................................. 53
      4.2.1.2 Coupling pulse removal procedure ......................................................... 54
      4.2.1.3 Performance evaluation of coupling pulse removal ............................... 57
   4.2.2 Noise filtering ................................................................................................. 59
      4.2.2.1 Noise sources ......................................................................................... 59
      4.2.2.2 Filter design ......................................................................................... 61
      4.2.2.3 Forward-backward filtering ................................................................. 69
   4.2.3 Depth resolution enhancement ...................................................................... 71
      4.2.3.1 Importance of high depth resolution for GPR data analysis .................... 71
      4.2.3.2 Ideal depth resolution enhancement ...................................................... 79
      4.2.3.3 Inverse filtering .................................................................................. 80
      4.2.3.4 Predictive deconvolution ................................................................. 91
      4.2.3.5 Pulse spiking ..................................................................................... 99
      4.2.3.6 Pulse shaping ............................................................................... 109
      4.2.3.7 Homomorphic deconvolution ...................................................... 120
      4.2.3.8 Iterative decomposition of reflected signals ........................................ 130
      4.2.3.9 Comparison between the different depth resolution enhancement techniques 132
List of Tables

Table 3-1: Pavement Sections at the Virginia Smart Road ............................................................. 36
Table 3-2: SIR-10B Specifications ................................................................................................. 39
Table 3-3: Performance Tests Results .......................................................................................... 46
Table 4-1: Minimum Filter Orders for Different Types of Lowpass Filters for a Given Specification .......................................................................................................................... 67
Table 4-2: Flexible Sections Used for Detector Performance Evaluation .................................. 152
Table 4-3: Summary of Dielectric Constant Variations for the Different Mixes ....................... 172
Table 5-1: Flexible Sections Used for GPR Data Analysis ........................................................ 194
Table 5-2: Comparison between Core Thicknesses and GPR Thicknesses Obtained by the Overall and CMP Analysis Techniques .............................................................................. 201
Table 5-3: Comparison between Core Thicknesses and GPR Thicknesses Obtained by the Individual and Individual + Loss Analysis Techniques ..................................................... 203
Table 5-4: Comparison between Individual Layer Thicknesses Measured on Cores and Estimated by GPR .......................................................................................................................... 205
List of Figures

Figure 2-1: Pavement Types (a) Conventional Flexible Pavement, (b) Full-Depth Asphalt Pavement (c) Rigid Pavement ................................................................................................ 6
Figure 2-2: Four Different Types of Concrete Pavements: (a) JPCP, (b) JRCP, (c) CRCP, (d) PCP ......................................................................................................................................... 9
Figure 2-3: FM-CW Time-Frequency Diagram for two Targets .................................................. 11
Figure 2-4: Block Diagram of an Impulse GPR System ............................................................... 13
Figure 2-5: Oblique Reflection and Transmission from a Flat Interface: (a) TE, (b) TM ............ 18
Figure 2-6: Reflection and Transmission from Multiple Planar Layers at Normal Incidence .... 21
Figure 2-7: Reflection from a Two-Layer System and Corresponding Time Domain Reflected Signal .................................................................................................................................... 25
Figure 3-1: Copper plate under 21-B material in section B .......................................................... 37
Figure 3-2: Block Diagram of the SIR-10B GPR System ............................................................ 38
Figure 3-3: GPR Survey Van with the Air-Coupled and Ground-Coupled Antennas .................. 40
Figure 3-4: Incident GPR Pulse Shape ......................................................................................... 41
Figure 3-5: Distance Measuring Sensor ........................................................................................ 42
Figure 3-6: Antenna Setup for TTI Performance Tests ............................................................. 43
Figure 3-7: Typical Reflected Signal from a Copper Plate ............................................................ 44
Figure 4-1: Flowchart of the Proposed GPR Data Analysis System ............................................ 50
Figure 4-2: EM Propagation at Transmission and Reception for: (a) Bistatic and (b) Monostatic GPR Systems .......................................................................................................................... 52
Figure 4-3: Major Reflections in a Scan Collected by an Air-Coupled, Bistatic GPR System .... 52
Figure 4-4: Major Reflections in a Scan Collected by a Ground-Coupled, monostatic GPR System ................................................................................................................................... 53
Figure 4-5: Air-Coupled GPR Response as a Function of Antenna Height ............................... 54
Figure 4-6: Coupling Removal Procedure: (a) Raw Signals, (b) Incorrect Coupling Pulse Removal (c) Cross-correlation Function, (d) Aligned Signals .................................................. 56
Figure 4-7: Reflected Signal with Coupling Pulse Removed ....................................................... 57
Figure 4-26: Average Estimated $SNR_{red}$, $SER$, and $PCR$ vs. Inverse Filter Length ........................................ 87
Figure 4-27: Magnitude Spectra for Different Filter Lengths: (a), (c), and (e): Reflected Signal and Inverse Filter; (b), (d), and (f): Deconvolved Signal ................................................................. 89
Figure 4-28: Deconvolved GPR Signal Obtained by an Inverse Filter of Optimal Length $N_{opt} = 60$ ........................................................................................................................................... 90
Figure 4-29: Deconvolved GPR Data Obtained by an Inverse Filter of Optimal Length $N_{opt} = 60$, Showing Copper Plate Reflections at the: (1) WS/BM-25.0, (2) BM-25.0/OGDL, (3) OGDL/21A, (4) 21A/21B, and (5) 21B/Subgrade Interfaces ................................................................. 91
Figure 4-30: Prediction Error Filter: (a) Direct Implementation, (b) Alternative Implementation ........................................................................................................................................... 93
Figure 4-31: Average Estimated (a) $SNR_{red}$ (dB), (b) $SER$ (%), and (c) $PCR$ (%) vs. Prediction Filter Length and Prediction Distance ........................................................................................................................................... 95
Figure 4-32: Optimal Prediction Filter Performance Parameters vs. Prediction Filter Length .... 97
Figure 4-33: Deconvolved GPR Signal Obtained by a Prediction Error Filter, $N_{opt} = 60$ and $\alpha_{opt} = 1$ ............................................................................................................................................. 98
Figure 4-34: Deconvolved GPR Data Obtained by a Prediction Error Filter of $N_{opt} = 60$ and $\alpha_{opt} = 1$, Showing Copper Plate Reflections at the: (1) WS/BM-25.0, (2) BM-25.0/OGDL, (3) OGDL/21A, (4) 21A/21B, and (5) 21B/Subgrade Interfaces ........................................................................................................................................... 98
Figure 4-35: Spiking Filter Input and Actual Output for a Filter Length $N = 90$ and Lag Values $l = 0, 45, \text{ and } 90$ ........................................................................................................................ 101
Figure 4-36: Spiking Filter Performance Parameter $P$ (in %) vs. Filter Length $N$ and Lag $l$ .... 102
Figure 4-37: Maximum Spiking Filter Performance Parameter $P$ (in %) and Corresponding Optimal Lag vs. Filter Length.................................................................................................... 103
Figure 4-38: Average Estimated (a) $SNR_{red}$ (dB), (b) $SER$ (%), and (c) $PCR$ (%) vs. Spiking Filter Length and Desired Output Lag ........................................................................................................................................... 105
Figure 4-39: Optimal Spiking Filter Performance Parameters vs. Filter Length........................ 107
Figure 4-40: GPR Signal after Processing by Spiking Filter of Length $N_{opt} = 95$ and Output Lag $l_{opt} = 60$ ............................................................................................................................................. 108
Figure 4-41: GPR Signal after Processing by Spiking Filter of Length $N_{opt} = 95$ and Output Lag $l_{opt} = 60$, Showing Copper Plate Reflections at the: (1) WS/BM-25.0, (2) BM-25.0/OGDL, (3) OGDL/21A, (4) 21A/21B, and (5) 21B/Subgrade Interfaces ................................................. 108
Figure 4-58: Comparison between the Performance of the Threshold and Matched Filter Detectors: (a) 3, (b) 4 Thick Layers, (c) 3, (d) 4, and (e) 5 Thin Layers ........................................ 154
Figure 4-59: Comparison between the Performance of the Threshold and Matched Filter Detectors for 4 Mostly Thin Layers ........................................................................................................ 157
Figure 4-60: Performance Parameters of Detecting GPR Deconvolved Pulses for Various Layer Categories: (a) Square Error Ratio, (b) Number of Detected Layers ........................................ 160
Figure 4-61: Typical Reflected GPR Signal from a Wearing Surface Backed by a Copper Plate ............................................................................................................................................. 165
Figure 4-62: Magnitude Spectra of the Incident and Reflected GPR Signals ......................... 166
Figure 4-63: Multiple Reflections Model .................................................................................... 167
Figure 4-64: Comparison between Measured and Theoretical Reflection Coefficient Variations vs. Frequency: (a) Real Part, (b) Imaginary Part .......................................................................................... 169
Figure 4-65: Dielectric Constant Variations for the Different Mixes: (a) SM-9.5A, (b) SM-9.5A with High Lab Compaction, (c) SM-9.5D, (d) SM-9.5E, (e) SM-12.5D, (f) SMA-12.5.... 171
Figure 4-66: Measured and Modeled GPR Reflected Signals: (a) SM-9.5D, (b) SM-9.5E ....... 173
Figure 4-67: Second Layer Dielectric Constant Variations versus First Layer Loss ................. 176
Figure 4-68: Common Midpoint Geometry Using Ground-Coupled Monostatic and Bistatic Antennas ............................................................................................................................................. 178
Figure 4-69: Modified Common Midpoint Geometry Using a Ground-Coupled Monostatic System and an Air-Coupled Bistatic System .................................................................................... 178
Figure 4-70: Dielectric Constant Variations versus Time Delay Difference ......................... 182
Figure 4-71: Raw Reflected Pulses: (a) Normalized Time-Delays, (b) Normalized Amplitudes, Tracked Reflections: (c) Normalized Time-Delays, (d) Normalized Amplitudes............ 190
Figure 5-1: Typical HMA Core Extracted from Section H at the Virginia Smart Road .......... 200
Figure 5-2: Variation of Absolute GPR Error versus Wearing Surface Thickness ................. 202
Figure B−1: Weiner Filter Design .............................................................................................. 224
Figure C−1: Schematic of a Layered Pavement System ............................................................. 231
Figure D−1: Thickness of the Overall HMA Layer: (a) Section E, (b) Section A, (c) Section F, (d) Section G, (e) Section H, and (f) Section K ................................................................. 235
Figure D−2: Thicknesses of the Individual Lossless HMA Layers: (a) Section E, (b) Section A, (c) Section F, (d) Section G, (e) Section H, and (f) Section K ........................................... 236
Figure D–3: Thicknesses of the Individual Lossy HMA Layers: (a) Section E, (b) Section A, (c) Section F, (d) Section G, (e) Section H, and (f) Section K ................................................. 237

Figure D–4: Thickness of the Overall HMA Layer, CMP Method: (a) Section E, (b) Section A, (c) Section F, (d) Section G, (e) Section H, and (f) Section K..................................................... 238
Chapter 1

Introduction

1.1 Background

RADAR, an acronym for RAdio Detection And Ranging, is a device invented during the first decades of the 20th century to remotely detect and range objects (such as airplanes and ships) using electromagnetic (EM) waves. The detection principle behind radar goes back to the first experiments conducted by Hertz at the end of the 19th century. Hertz tested the EM theories of Maxwell, and showed that EM waves are reflected by conductors and dielectrics. Those findings were not utilized until the 1900s when a German engineer patented a device for ship and obstacle detection using EM waves. However, due to its low range of detection (around one mile), the device was not very successful as an operational sensor.

A few years prior to World War II (WWII), the first continuous wave (CW) radar systems were tested in different countries. These radar systems operated mainly in the HF (high frequency: 3 to 30MHz) and VHF (very high frequency: 30 to 300MHz) frequency bands and achieved detection ranges of up to 50 miles. The CW radars used the Doppler-frequency shift introduced by moving targets as a detection basis without any information about target range or position. During WWII, radar systems were systematically used as a tool to improve military defense systems, by providing early detection of hostile aircrafts and ships. During that period, pulse radars were introduced to provide range estimation based on the measurement of the time delay (that can be converted to distance, knowing the constant speed of EM waves through air) between the transmitted pulse and the returned echoes from the target. Since then, radar systems continued to be developed and improved both in hardware (radar transmitter, receiver, antennas, etc.) and, later, in software, after computers were introduced as tools for radar data analysis and interpretation. Presently, radars are largely used in different civil applications such as air traffic
control, aircraft and ship navigation, weather forecasting, law enforcement and military applications, such as surveillance, navigation, and control and guidance of weapons.

Ground penetrating radar (GPR) is a special kind of radar designed specifically to look into the ground by penetrating the surface. Experiments with GPR systems were first performed in 1929 in Austria to measure the depth of a glacier. After that, GPR technology was largely ignored until the late 1950s, when a US Air Force aircraft crashed in Greenland because its radar altimeter failed to respond to the weak reflections from the snow surface, causing it to overestimate the airplane’s altitude during landing. This incident prompted many experiments to evaluate the capability of radar to “see” through the surface of an opaque dielectric medium, other than snow, in order to visualize and estimate the shape, size, and depth of buried inhomogeneities. The introduction of commercial GPR systems to the market in the 1970s by Geophysical Survey Systems Inc. (GSSI) led to the proliferation of GPR systems and their widespread application in different domains.

Currently GPR is used in many areas as a nondestructive investigation tool. For example, it is used in geophysics to estimate the structure of the earth sediments and to find the depth of bedrock, water tables, etc. Ground penetrating radar is also used in archeology to locate buried archeological structures before digging to prevent their accidental damage. Moreover, GPR is used as a safety tool to locate landmines and as an investigation tool for law enforcement to locate buried bodies. Finally, GPR is used in civil engineering testing to evaluate the performance of civil structures, such as buildings, bridges, pavements, tunnels, etc.

In spite of the development of GPR technology, however, its use for evaluating pavements and bridges remains minimal. In fact, only a few (around 11) Departments of Transportation (DOT) currently own a GPR system. Yet, these DOTs do not systematically use GPR in their Pavement Management System (PMS) decision-making [1]. The limited use of GPR for pavement evaluation is mainly due to the lack of reliable automated procedures for data analysis and the difficulty of manually interpreting the large amount of GPR data. Additionally, manual analysis of GPR data generally requires an experienced operator and, therefore, usually leads to subjective, inaccurate results and extensive processing times.
The goal of this research is to analyze GPR data collected from pavements to provide accurate and reliable information about the subsurface layers. The developed techniques are expected to help DOTs to integrate GPR systems easily into their pavement assessment practices, therefore reducing the cost of pavement rehabilitation by providing reliable information for decision makers.

1.2 Problem Statement

Unlike other imaging techniques, such as X-ray or Computed Tomography (CT) scans, a GPR system does not provide an “image” of the subsurface. Consequently, extracting useful and accurate information about the surveyed structure from GPR scans is a relatively complex process.

For pavement systems, accurately estimating the thickness of a pavement layer or the depth to a pavement defect from GPR response is directly linked to the dielectric properties of all the overlaying layers. On the other hand, the dielectric properties are usually unknown and vary from one location to another, as they depend on many parameters. Current GPR data analysis techniques either assume a dielectric constant for the entire surveyed pavement section or estimate it based on a finite set of cores taken from the road. Therefore, current GPR data analysis techniques are neither accurate nor reliable. The large amount of data collected during a GPR survey further degrades the performance of the GPR nondestructive investigation technique because data is usually analyzed manually over limited segments of the total surveyed area.

1.3 Research Objectives

To overcome the aforementioned problems, this research effort proposes to develop GPR data analysis algorithms specifically for flexible pavement thickness measurement and internal flaws detection. These algorithms will allow the following:

- Identification of the different layers composing a pavement system
- Accurate estimation of the dielectric constant of each layer
- Measurement of the thickness of each layer above the deepest detectable interface
- Detection of pavement defects
• Presentation of the processed GPR data in a format easily understandable by inexperienced GPR users
• Performance evaluation of the developed algorithms

1.4 Scope

This document is divided into six chapters and four appendices. The first chapter provides an introduction about the research and its objectives. The second chapter presents the current state of knowledge about the use of GPR for pavements. The third chapter discusses the research approach adopted to achieve the set objectives. The fourth chapter thoroughly discusses the different steps and algorithms developed for GPR data interpretation. The fifth chapter presents the validation and performance assessment of the developed GPR data analysis system. The performance evaluation is accomplished based on field GPR data collected from an experimental pavement system. Lastly, the sixth chapter presents the various findings, conclusions, and recommendations of the study.

The detailed GPR analysis algorithms developed in this research are grouped in Appendix A. Appendices B and C, respectively, discuss Wiener filtering and least-squares fitting as a support material for chapter four. Finally, Appendix D presents the GPR results obtained for the validation of the developed analysis system.
Ground penetrating radar systems are based on sending an EM wave through the surface of the ground and then analyzing the scattered signals collected by the receiver. The propagation of EM waves through a medium depends largely on the medium’s dielectric properties and structure. Therefore, to better study the propagation of GPR signals through pavements, it is necessary to start by examining the common types of pavement systems in use today and the information that can be reported by GPR measurements of those structures.

2.1 Pavements

Pavements are planar-layered media with different materials composing each layer. Based on their main components, pavements are divided into three categories: flexible (hot-mix asphalt) pavements, rigid (concrete) pavements, and composite pavements.

2.1.1 Flexible pavements

Flexible pavements are layered systems composed of different layers that are placed in such a way that layer strength is greater at the top, where the stresses caused by traffic loading are high. This approach allows cheaper local materials to be used in pavement construction. As depicted in Figure 2-1a, flexible pavements can be composed of the following [2]:

- **Surface course** (or **wearing surface**) is the top layer of a flexible pavement. It is constructed by a dense graded hot-mix asphalt (HMA) material. This mix provides a balance in aggregate size, where a high resistance to traffic-load requires large aggregate and a smooth and skid-resistant riding surface requires strong, small
aggregate. The wearing surface thickness is usually between 25mm and 50mm (1in to 2in).

- **Binder course** (or asphalt base course) is an HMA layer that is composed of larger aggregates and less asphalt binder content than the surface course. Because of its low asphalt binder content, the base course resistance to fatigue cracking is less than the surface mix. The lower resistance to cracking is justified by the lower stresses applied to the base course layer since it is deeper in the pavement. To ensure bonding between the two HMA layers, tack coat (asphalt emulsion) is usually sprayed over the bottom layer before placing the top layer. The asphalt base course thickness usually varies between 50mm and 250mm (2in to 10in).

- **Base course** is a layer composed of crushed stone that can be either untreated (loose) or stabilized (by adding small quantities of cement or asphalt). The base course is usually 100mm to 300mm thick (4in to 12in).

![Figure 2-1: Pavement Types (a) Conventional Flexible Pavement, (b) Full-Depth Asphalt Pavement, (c) Rigid Pavement](image_url)
• **Subbase course** is similar to the base course; however, it is typically composed of lower quality (weaker) aggregates for economic reasons. The subbase course is usually 100mm to 300mm thick (4in to 12in).

• **Subgrade** is the bottom layer supporting all the aforementioned layers. It can be either the original in-situ soil or a placed layer of selected material. In both cases, the subgrade should have a high density attained by good compaction.

It should be noted that a typical pavement is composed of any combination of two or more layers in the order specified above (top to bottom). A flexible pavement should have, at least, an HMA layer and an aggregate (base course) layer. The layer thicknesses used in the design depend primarily on the importance of the road, the volume of traffic it is expected to carry, the properties of the construction materials used, and the local environmental conditions.

Some flexible pavements, known as full-depth asphalt pavements, have a unique 50mm to 100mm (2in to 4in) wearing surface layer and a 50mm to 510mm (2in to 20in) asphalt base course as shown in Figure 2-1b. This type of pavements is more suitable for heavy loads; furthermore, it provides better resistance to environmental problems, such as moisture and frost that usually weaken the base and subbase granular layers. Such design is not economical and, therefore, is rarely used. However, perpetual pavements are currently gaining momentum in the United States.

Flexible pavement distresses are mainly due to repeated traffic loading and to varying environmental conditions, such as temperature, rain, and frost. Distresses could appear directly on the surface of the pavement (wearing surface) in the form of cracks or potholes, in which case they are visible and, therefore, are easily detectable by visual inspection. Distresses could also start underneath the surface, propagating over time until they reach the surface. These internal defects are typically caused by extensive loading, weak subgrade, and/or entrapped moisture. Moisture accumulates in the subgrade or the aggregate base layers, causing a decrease in the material resistance to permanent deformation caused by heavy traffic loads. Moisture can also be trapped within the HMA layers, thus producing rupture in the bond between asphalt and aggregates, which can result in raveling (disintegration of the surface), bleeding (asphalt binder movement to the surface), and a weak layer.
2.1.2 Rigid pavements

Rigid pavements are constructed of 150mm to 300mm thick (6in to 12in) Portland cement concrete (PCC) slabs as illustrated in Figure 2-1c. The slabs can be placed either directly on the prepared subgrade surface or on a 100mm to 300mm thick (4in to 12in) granular base layer. Four types of PCC pavements can be identified based on the reinforcement configuration [2]:

- **Jointed Plain Concrete Pavements (JPCP)**, shown in Figure 2-2a, are non-reinforced concrete slabs separated by transversal contraction joints to allow for concrete expansion caused by environmental conditions. The joint spacing used in these pavements is typically between 5m and 10m (15ft to 30ft).

- **Jointed Reinforced Concrete Pavements (JRCP)**, shown in Figure 2-2b, are concrete slabs reinforced by a wire mesh or deformed bars. The reinforcement introduced in this case does not improve the structural capacity of the pavement, but rather protects the concrete against concrete shrinkage and temperature variation cracking. Thus, it increases the joint spacing, which can vary in this case from 10m to 30m (30ft to 100ft).

- **Continuous Reinforced Concrete Pavements (CRCP)**, shown in Figure 2-2c, are reinforced by steel reinforcing bars (rebars) in the longitudinal and transversal directions. Use of reinforcement in this case eliminates the contraction joints, which are the main weakness in concrete pavements; moreover, it reduces the thickness of the slab.

- **Prestressed Concrete Pavements (PCP)**, shown in Figure 2-2d, are built in such a way that a compressive stress is naturally imposed on the slabs in the absence of any traffic load. Consequently, tensile stresses applied by vehicles on the concrete slab will be greatly reduced, which in turn reduces pavement damage. This is based on the fact that concrete is strong in compression but weak in tension. Concrete is typically prestressed using post-tensioned steel wire strands.
Rigid pavement distresses are essentially due to repeated traffic loading and to varying environmental conditions. These distresses may be divided into two categories: distresses caused by the base (or subgrade) layer failure and distresses originating in the concrete slab itself. The first category is caused by weakening of the base (or subgrade) layer either due to moisture accumulation or material loss produced by soil and particle pumping at the slab joints or the existing cracks. The second category can be due to the corrosion of the steel rebar in reinforced concrete, joint faulting, freezing and thawing, alkali-silica reaction in concrete, and/or other chemical attacks.
2.1.3 Composite pavements

Composite pavements are composed of concrete slabs overlaid by HMA, thus providing the simultaneous strength of concrete as a base layer and the smoothness of HMA. Due to the high cost of such pavements, they are rarely constructed as new pavements; however, they usually result from the rehabilitation of old concrete pavements by adding an HMA overlay at an appropriate thickness. Flexible pavements may also be overlaid with concrete, which is known as whitetopping.

Distresses in composite pavements are similar to those found in flexible pavements. However, a characteristic crack type, the joint reflection cracking, is usually found in HMA layers overlaying jointed concrete slabs. This type of cracking starts at the concrete joints and propagates over time towards the surface.

2.2 Ground Penetrating Radar (GPR) Systems

Unlike traditional radar systems, GPR systems are mainly used to detect and measure the depth of inhomogeneities (either defects or layers) in a dielectric medium. Detection could be achieved by comparing the power of the scattered EM waves produced by the contrast in the dielectric properties between medium and inhomogeneity to a prefixed threshold above the receiver noise level. Depth estimation, however, is more complicated because it requires precise measurement of the time delay between the transmitted signal and the reflected signal. The time delay can then be converted to depth by multiplying it by the speed of EM waves through the studied medium. Original radar systems, working with a CW, did not have this feature because with a CW, it is difficult to set a time marker representing the start of wave transmission and another one representing the reception of the reflections. This problem was overcome in radars and in GPR systems by introduction of modulation of the CW. Depending on the modulation technique used, three types of GPR systems can be identified: the frequency modulated GPR, the synthetic pulse radar, and the amplitude modulated (or impulse) GPR [3] and [4].
2.2.1 Ground penetrating radar types

2.2.1.1 Frequency modulated GPR

Frequency modulation of a continuous wave (FM-CW) could be used as a time marker to precisely locate the transmission and reflection events in time. As shown in Figure 2-3, the simplest implementation of this technique is to linearly change the frequency of the CW over time between two limits in a known periodic manner using a voltage controlled oscillator (VCO). The frequency difference, $f_d$, between the transmitted signal and the reflected signal is proportional to the two-way travel time, $t_d$, to the inhomogeneity (or target). The frequency difference is determined by mixing the reflected signal with the transmitted signal via a coherent mixer.

The center frequency and bandwidth of the transmitted signal from a FM-CW GPR should be suitably selected according to the investigated medium to minimize attenuation and distortion of the reflected signal. Additionally, it is more suitable to use FM-CW GPR systems to detect single shallow inhomogeneities, where the need for a short pulse (or equivalently large bandwidth) to resolve the inhomogeneity is difficult to achieve using the other types of GPR. Multiple inhomogeneity detection, however, is more difficult to accomplish because the difference signal will contain multiple frequency components that need a narrow bandpass filter bank to be resolved.
2.2.1.2 Synthetic pulse GPR

Synthetic pulse GPR, also called stepped frequency GPR, is similar to the FM-CW GPR where the carrier frequency is modulated. In this design, however, the carrier frequency is varied discretely, rather than continuously, between two frequency limits in $N$ steps. The amplitude and phase of the reflected signals are then precisely measured and recorded for each frequency step, yielding the Fourier Transform (FT) of the reflected signal. A simple inverse fast Fourier Transform (IFFT) can then be applied to the recorded samples to reconstruct the time domain signal from which the detection and range information are extracted. Alternative methods for detection and range estimation can also be applied directly in the frequency domain [5].

2.2.1.3 Pulsed (or impulse) GPR

Impulse GPR is the most common GPR system used currently because its data is easier to interpret. An impulse GPR system can be regarded as an amplitude-modulated radar, where the transmitted signal is reduced to a very short pulse (in the order of one nanosecond or less) with a wide spectrum (a few GHz). Because of their wide spectrum, impulse GPR systems are referred to as ultra-wide bandwidth (UWB) radars. According to the Federal Communications Commission (FCC), a device is considered UWB if it has a fractional bandwidth greater than 0.25 or if its bandwidth occupies 1.5GHz or more of spectrum when its center frequency is greater than 6GHz. The fractional bandwidth is defined as $2(f_h - f_l)/(f_h + f_l)$, where $f_h$ and $f_l$ are, respectively, the upper and lower frequencies of the -10dB relative to the maximum emission point, and $(f_h + f_l)/2$ is commonly known as the center frequency.

The principle of an impulse GPR system is based on sending a short EM pulse through the antenna to the ground and then recording the reflected pulses from the surface and the internal inhomogeneities. The two-way travel time to the targets can then be measured in the time domain between the reflected pulses.

As depicted in the block diagram in Figure 2-4, impulse GPR systems function as follows. A trigger pulse is first generated in the radar control unit. This trigger pulse is then sent to a transceiver where it is modulated and amplified to become a bipolar transmit pulse with a much higher amplitude and bandwidth. The pulse is sent through the transmitting antenna to the ground. After a short time (in the order of few nanoseconds), the reflected signal is collected by
the receiving antenna and is then transmitted to the receiver circuitry. It should be noted that the pulse generator produces a large number of pulses at a fixed pulse repetition frequency (PRF). Due to the relatively short range of a GPR system (around 1m [3ft] for pavement assessment), the PRF can be in the order of a few hundred kHz without any ambiguities in range [6]. The resulting reflected signals, which are assumed to be similar because they are collected over the same location, are integrated together in the receiver to produce a single scan with a higher signal to noise ratio (SNR). Because of the wide bandwidth of the received signals, the receiver does not require a superheterodyne architecture like traditional receivers. However, it is usually composed of a low-noise RF amplifier, a wideband bandpass filter to limit the frequency content of the signal, a high-speed sampler, and a high-speed digital-to-analog converter (ADC). Since ADCs cannot work at high sampling frequencies (in the GHz range to meet the Nyquist sampling frequency limit), the entire reflected signal is digitized over a set of identical reflected signals resulting from different trigger pulses, with different samples acquired from each signal. The digitized signal is then transferred to a digital signal processor (DSP), where it is amplified and filtered according to the user-selected parameters. The collected data is finally displayed for immediate interpretation and is stored on magnetic media for later processing.

Due to the pulsed nature of the impulse GPR, it can transmit high peak power EM pulses to ensure an appropriate depth of penetration, with an overall low average power. The transmitted low average power makes impulse GPR systems safer to use than other CW systems.

![Figure 2-4: Block Diagram of an Impulse GPR System](image-url)
2.2.2 Ground penetrating radar antennas

Like other systems working with EM waves, a GPR system needs an antenna to transmit and receive EM energy. Depending on the number of antennas used, a GPR system can be monostatic (a single antenna is used for transmit/receive), bistatic (one antenna is used for transmission and another antenna is used for reception), or multistatic (a single antenna or multiple antennas are used as transmitters and multiple antennas are used as receivers). In the case of a monostatic GPR, the transceiver should include a duplexer to protect the receiver from the high power signals during transmission and to direct the low power received signals to the receiver during reception.

Depending on the way antennas are deployed, GPR systems are classified as air-coupled (or launched), or ground-coupled systems. In air-coupled systems, the antennas (usually horn antennas) are typically 150 to 500mm above the surface. These systems produce a clean radar signal and allow for highway speed surveys. However, the drawback of these systems is the low depth of penetration into the pavement structure since part of the EM energy, sent by the antenna, is reflected back by the pavement surface. In contrast, a ground-coupled system’s antenna is in full contact with the ground, which gives a higher depth of penetration (at the same frequency) but limits the speed of the survey. Ground-coupled antennas are usually in the form of bowtie dipoles.

For easier data interpretation, the GPR antennas are designed to radiate a wave that can be approximated in the far field ($r > 2D^2/\lambda$) by a normal-incidence transverse electromagnetic (TEM) plane wave. This applies also to the case of a bistatic configuration, where the incidence angle is small but different from zero. Moreover, the incident wave is generally linearly polarized. The half power (3dB) beamwidth varies typically between 20° and 90°.

2.3 Electromagnetic Theory Pertinent to GPR Systems

In order to correctly interpret GPR data, it is essential to examine how the EM waves transmitted by the GPR antenna interact with a pavement system. There are two mechanisms involved during the travel of EM waves between the transmitting and receiving antennas: first
propagation of the waves through the homogeneous probed medium, second scattering of the EM waves from the encountered inhomogeneities.

2.3.1 Electromagnetic propagation

Electromagnetic wave propagation through a homogeneous medium is governed by Maxwell’s equations and the constitutive relations. These equations relate the electric field and the magnetic field at every point to the sources that create them, and to the electrical properties of the medium. For a source free medium represented by its permittivity, \( \varepsilon \), conductivity, \( \sigma \), and permeability, \( \mu \), the reduction of Maxwell’s coupled equations to a single field equation, yields the wave equation. For time-harmonic EM fields with angular frequency \( \omega \) (assuming time variations of the form \( e^{j\omega t} \)), the wave equation for the complex electric field \( E(\mathbf{r}) \) at a spatial point defined by vector \( \mathbf{r} \), is given by equation (2-1):

\[
\nabla^2 E(\mathbf{r}) + \omega^2 \varepsilon \mu (1 - j \frac{\sigma}{\omega \varepsilon}) E(\mathbf{r}) = 0 \tag{2-1}
\]

If the electric field is assumed parallel to the \( x \)-axis and it depends only on the \( z \) coordinate, the second order differential equation (2-1) can be reduced to:

\[
\frac{d^2}{dz^2} E_x(z) + \omega^2 \varepsilon \mu (1 - j \frac{\sigma}{\omega \varepsilon}) E_x(z) = 0 \tag{2-2}
\]

A solution to (2-2) is given as follows:

\[
E_x(z) = E^+_0 e^{-j\beta z} + E^-_0 e^{j\beta z} \tag{2-3}
\]

where:

\[
k = \omega \sqrt{\varepsilon \mu (1 - j \frac{\sigma}{\omega \varepsilon})} = \beta - j\alpha , \quad \alpha, \beta \geq 0 \tag{2-4}
\]

\( k \) is the wavenumber, \( \alpha \) is the attenuation constant (Np/m) and \( \beta \) is the phase constant (rad/m). The first term in (2-3) represents a wave traveling in the \( +z \) direction, and the second term
represents a wave traveling in the \(-z\) direction. A time varying representation of the wave traveling in the \(+z\) direction is given as follows:

\[
E_x(z,t) = \text{Re}\{E_0 e^{-j(\beta - j\alpha)z} e^{j\omega t}\} = E_0 e^{-\alpha z} \cos(\omega t - \beta z), \text{ with } z > 0
\] (2-5)

This represents a uniform plane wave since the amplitude and phase of the field at all points in the \(xy\) plane at the coordinate \(z\) is the same. This wave is exponentially attenuated by a factor \(\alpha\) as it propagates in the \(+z\) direction. The phase velocity, \(v\), which is, in this case, equal to the energy (or group) velocity, can be obtained by setting the phase of the wave (as a function of time) to a constant and then taking its time derivative:

\[
\frac{d}{dt} (\omega t - \beta z) = 0 = \omega - \beta \frac{dz}{dt} \Rightarrow v = \frac{dz}{dt} = \frac{\omega}{\beta}
\] (2-6)

Expressions for \(\alpha\) and \(\beta\) as a function of frequency and the constitutive parameters can be derived by squaring equation (2-4), equating its real and imaginary parts, and then solving for \(\alpha\) and \(\beta\) simultaneously:

\[
\begin{align*}
\alpha &= \omega \sqrt{\mu \epsilon} \left[ 0.5 \left( \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} - 1 \right) \right] \quad (2-7) \\
\beta &= \omega \sqrt{\mu \epsilon} \left[ 0.5 \left( \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} + 1 \right) \right] \quad (2-8)
\end{align*}
\]

For a good dielectric material at a high frequency, such as a pavement material probed by GPR at 1GHz nominal frequency, the quantity \((\sigma/\omega \epsilon)^2\) is usually small compared to 1. Moreover, due to the non-magnetic nature of the pavement materials, the permeability \(\mu\) can be approximated by the permeability of free space \(\mu_0\). Therefore, approximate expressions for \(\alpha\), \(\beta\) and \(v\) can be derived according to the following equations:

\[
\alpha = \frac{\sigma}{2} \frac{\eta_0}{\sqrt{\epsilon_r}}
\] (2-9)
\[
\beta = \frac{\alpha}{c} \sqrt{\varepsilon_r} \quad (2-10)
\]

\[
v = \frac{c}{\sqrt{\varepsilon_r}} \quad (2-11)
\]

where:

- \( \eta_0 \): wave impedance of free space, \( \eta_0 = \frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} \approx 120\pi \ \Omega \)
- \( \varepsilon_r \): relative permittivity or dielectric constant of the medium
- \( c \): speed of light in free space, \( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \times 10^8 \text{m/s} \)

It should be noted that the dielectric constant, \( \varepsilon_r \), is usually complex, with the real part representing energy storage in the media and the imaginary part representing loss due to dielectric effects. To account for that loss, the imaginary part of \( \varepsilon_r \) should be incorporated into the attenuation factor \( \alpha \). However, because conduction loss is usually much higher than dielectric effects loss, the dielectric constant can be considered as a real number, provided that conduction loss is accounted for, as specified by equation (2-9). Moreover, for many materials, the dielectric constant of a medium depends on the frequency of the EM waves. Consequently, signal components of different frequencies will travel with different speeds, which results in the distortion of the signal (dispersive media). Nevertheless, the dielectric constant of pavement materials does not vary significantly in the GPR bandwidth (For dried concrete \( \varepsilon_r \) varies between 5 and 4.8 and \( \sigma \) varies between 0.003S/m and 0.009S/m, for 9% saturated concrete, \( \varepsilon_r \) varies between 8.5 and 7.5 and \( \sigma \) varies between 0.06S/m and 0.14S/m ([7] and [8]) in the frequency band 0.5GHz to 1.5GHz. Other authors reported higher values depending on the mixes used [9]). Hence, \( \varepsilon_r \) can be considered as a constant (versus frequency) [10] to facilitate data interpretation.
2.3.2 Electromagnetic scattering

Electromagnetic scattering occurs when an incident wave encounters a discontinuity in the electromagnetic properties of the traversed medium. In the case of GPR surveys on pavements, the discontinuity can be either the interface between two layers in the pavement system or an irregularly shaped defect within a layer. Due to the discontinuity, the wave is reflected, refracted, or diffracted depending on the geometry of the discontinuity, the properties of the materials, and the wavelength of the incident signal. Wave scattering is generally tightly related to the problem geometry; thus, the solutions are problem specific. Since pavements are planarly layered media, only scattering from planar interfaces will be presented.

2.3.2.1 Scattering from a planar interface

As depicted in Figure 2-5, scattering from a planar interface yields a reflected signal and a transmitted signal. The reflection and transmission coefficients can be found using the boundary conditions at the interface (continuity of the tangential components of the electric and magnetic fields across the interface). For an oblique incident wave, two solutions can be found for the reflection and transmission coefficients, depending on the polarization of the incident field.

![Figure 2-5: Oblique Reflection and Transmission from a Flat Interface: (a) TE, (b) TM](image)

Polarization of the incident field is defined with respect to the plane of incidence, which is the plane formed by the normal to the interface and the direction of propagation of the incident
wave. A transverse electric (TE or perpendicular, Figure 2-5a) wave represents a wave with the electric field perpendicular to the plane of incidence. A transverse magnetic (TM or parallel, Figure 2-5b) wave represents a wave with the electric field parallel to the plane of incidence (the magnetic field is therefore perpendicular to this plane). By satisfying the boundary conditions at the interface between medium 1 (characterized by \( \varepsilon_1 \) and \( \mu_1 \)) and medium 2 (characterized by \( \varepsilon_2 \), and \( \mu_2 \)), the reflection and transmission coefficients can be found as follows [11]:

For TE polarization:

\[
\gamma_\perp = \frac{-\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t} \quad (2-12)
\]

\[
\tau_\perp = \frac{2\eta_2 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t} \quad (2-13)
\]

For TM polarization:

\[
\gamma_\parallel = \frac{-\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t} \quad (2-14)
\]

\[
\tau_\parallel = \frac{2\eta_2 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t} \quad (2-15)
\]

where \( \gamma \) and \( \tau \) represent the reflection and transmission coefficients, respectively, \( \eta_1 \) and \( \eta_2 \) are the impedances of mediums 1 and 2 given by \( \eta_{1,2} = \sqrt{\frac{\mu_{1,2}}{\varepsilon_{1,2}}} \), and \( \theta_i \) and \( \theta_t \) are the angles of incidence and transmission, respectively. The angles \( \theta_i \) and \( \theta_t \) are related by Snell’s law of refraction:

\[
k_i \sin \theta_i = k_2 \sin \theta_t \quad (2-16)
\]

It can be shown that the surface of a pavement system could be considered smooth when compared to the wavelength of the incident signal using Rayleigh’s criterion for a plane wave. Rayleigh’s criterion states that a surface is considered smooth if the condition given by the following equation is satisfied [12]:

\[
\frac{\lambda}{\lambda_s} = \frac{\lambda_{k,1}}{\lambda_{k,2}} \quad (2-17)
\]
\[ \sigma < \frac{\lambda}{8 \cos \theta_i} \] (2-17)

where \( \sigma \) is the standard deviation of the ground irregularity and \( \lambda \) is the wavelength of the incident signal. For a normal incidence signal with a wavelength \( \lambda = 0.2m \) (for \( f_{\text{max}} = 1.5\text{GHz} \)), the ratio \( \lambda / (8 \cos \theta) = 25\text{mm} \). The standard deviation, \( \sigma \), of the pavement surface irregularity could be estimated from pavement texture data measured longitudinally by a laser profiler. A typical maximum value of \( \sigma \) for a paved road system is around 1mm. Rayleigh’s criterion is thus satisfied, and the pavement surface could be considered smooth at the frequencies of interest.

Due to the relatively smooth pavement surface, the linearly polarized GPR incident signal will not depolarize on reflection [11]. Consequently, for the TE case, the reflected and transmitted complex electric fields would be the following, respectively: \( \mathbf{E'} = \gamma_\perp \mathbf{E'}^i \) and \( \mathbf{E'} = \tau_\perp \mathbf{E'}^i \) where \( \mathbf{E'}^i \) is the incident electric field. For the TM case, the reflected and transmitted complex magnetic fields would be the following respectively: \( \mathbf{H'} = \gamma_\parallel \mathbf{H'}^i \) and \( \mathbf{H'} = \frac{\eta_1}{\eta_2} \tau_\parallel \mathbf{H'}^i \) where \( \mathbf{H'}^i \) is the incident magnetic field.

At normal incidence (\( \theta_i = \theta_t = 0^\circ \)), the reflection and transmission coefficients will reduce to the following:

\[
\begin{align*}
\gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} \quad (2-18) \\
\tau &= \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\sqrt{\varepsilon_{r1}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} \quad (2-19)
\end{align*}
\]

where \( \varepsilon_{r1} \) and \( \varepsilon_{r2} \) are the dielectric constants of mediums 1 and 2, respectively. The right-hand side of both equations is obtained for a good dielectric, non-magnetic medium. For the normal incidence case, the reflected and transmitted fields are related to the incident fields according to the following equations:

\[
\mathbf{E'} = \gamma \mathbf{E'}^i \text{ and } \mathbf{H'} = -\gamma \mathbf{H'}^i \quad (2-20)
\]
\[ \mathbf{E}' = \tau \mathbf{E}' \quad \text{and} \quad \mathbf{H}' = \frac{\eta_1}{\eta_2} \tau \mathbf{H}' \quad (2-21) \]

### 2.3.2.2 Scattering from multiple planar layers

For GPR data interpretation purposes, a pavement system could be modeled as a set of \( N+1 \) homogeneous planar layers, as shown in Figure 2-6. Each layer \( i \) is assumed to have a finite thickness \( d_i \) (except for the \( N^{th} \) layer, which is considered semi-infinite), a dielectric constant \( \varepsilon_{r,i} \), a conductivity \( \sigma_i \), and a magnetic permeability equal to that of free space because of the non-magnetic nature of pavement materials. The top layer, layer 0, could either be composed of air, for an air-coupled system, or the pavement surface layer, for a ground-coupled system. At normal incidence, the reflection and transmission coefficients at each layer interface \( i \) are obtained from equations (2-18) and (2-19) as follows:

\[ \gamma_i = \sqrt{\frac{\varepsilon_{r,i} - \sqrt{\varepsilon_{r,i+1}}}{\varepsilon_{r,i} + \sqrt{\varepsilon_{r,i+1}}} \quad (2-22) \]

\[ \tau_i = \frac{2 \sqrt{\varepsilon_{r,i}}}{\sqrt{\varepsilon_{r,i} + \sqrt{\varepsilon_{r,(i+1)}}}} = 1 + \gamma_i \quad (2-23) \]

**Figure 2-6: Reflection and Transmission from Multiple Planar Layers at Normal Incidence**
Because of the relatively small contrast between the dielectric constants of the pavement layers, the reflection coefficients at the layer interfaces $\gamma_i$ could be considered small in absolute value (i.e. $|\gamma_i| << 1$). Therefore, multiple reflections between the interfaces could be neglected, especially since they would be attenuated because of material loss in each layer. Consequently, it could be assumed that the reflected signal at the receiver results only from the principal reflections at each of the layer interfaces.

Under these assumptions and for a normal incidence signal, the input reflection coefficient $\Gamma_{in}(\omega)$ referenced to the transmitting/receiving antenna could be approximated as follows:

$$
\Gamma_{in}(\omega) = \gamma_0 e^{-2 j k_0 d_0} + \gamma_1 (1 - \gamma_0^2) e^{-2 j (k_0 d_0 + k_1 d_1)} + \cdots + \gamma_{N-1} (1 - \gamma_0^2) \cdots (1 - \gamma_{N-2}^2) e^{-2 j (k_0 d_0 + k_1 d_1 + \cdots + k_{N-1} d_{N-1})} \tag{2-24}
$$

where $k_i$ is the wavenumber of the $i^{th}$ layer and $\gamma_i$ is the reflection coefficient at the interface between layers $i$ and $i+1$ as given by equation (2-22). The terms $(1-\gamma_i^2)$ in (2-24) are due to the double transmission of the wave across each interface: first downward with a transmission coefficient $\tau_i = 1 + \gamma_i$ and then upward with a transmission coefficient $\tau_i = 1 - \gamma_i$. The exponential coefficients $e^{-2 j k_i d_i}$ represent the wave propagation through each layer $i$. Equation (2-24) can be rewritten in a more compact form, as follows:

$$
\Gamma_{in}(\omega) = \sum_{n=0}^{N-1} \gamma_n \left[ \prod_{i=0}^{n-1} (1 - \gamma_i^2) \right] e^{-2 j \sum_{i=0}^{n} k_i d_i} \tag{2-25}
$$

Using the assumption $|\gamma_i| << 1$, equations (2-24) and (2-25) can be further simplified by neglecting the quantities $\gamma_i^2$ with respect to 1, which results in the following equations, respectively:

$$
\Gamma_{in}(\omega) = \gamma_0 e^{-2 j k_0 d_0} + \gamma_1 e^{-2 j (k_0 d_0 + k_1 d_1)} + \cdots + \gamma_{N-1} e^{-2 j (k_0 d_0 + k_1 d_1 + \cdots + k_{N-1} d_{N-1})} \tag{2-26}
$$

$$
\Gamma_{in}(\omega) = \sum_{n=0}^{N-1} \gamma_n e^{-2 j \sum_{i=0}^{n} k_i d_i} \tag{2-27}
$$
Replacing the wavenumbers \( k_i \) and the reflection coefficients \( \gamma_i \) with their respective expressions given by equations (2-4), (2-9), (2-10), and (2-22) yields the input reflection coefficient as a function of the pavement parameters:

\[
\Gamma_{in}(\omega) = \frac{\sqrt{E_{r,0}} - \sqrt{E_{r,1}}}{\sqrt{E_{r,0}} + \sqrt{E_{r,1}}} e^{-2j\frac{\omega}{c} \sqrt{E_{r,0}d_0}} e^{-\sqrt{E_{r,0}}} + \cdots +
\]

\[
\frac{\sqrt{E_{r,N-1}} - \sqrt{E_{r,N}}}{\sqrt{E_{r,N-1}} + \sqrt{E_{r,N}}} e^{-2j\frac{\omega}{c} \sqrt{E_{r,0}d_0} + \cdots + \sqrt{E_{r,N-1}d_{N-1}}} - \eta_0 \left( \frac{\sigma_{d_0}}{\sqrt{E_{r,0}}} + \cdots + \frac{\sigma_{d_{N-1}}}{\sqrt{E_{r,N-1}}} \right)
\]

(2-28)

This equation can be presented in a more compact form as follows:

\[
\Gamma_{in}(\omega) = \sum_{n=0}^{N-1} \frac{\sqrt{E_{r,n}} - \sqrt{E_{r,n+1}}}{\sqrt{E_{r,n}} + \sqrt{E_{r,n+1}}} e^{-2j\frac{\omega}{c} \sum_{i=n}^{n+1} \sqrt{E_{r,i}d_i}} - \eta_0 \sum_{i=0}^{N-1} \frac{\sigma_{d_i}}{\sqrt{E_{r,i}}}
\]

(2-29)

### 2.4 GPR Applications to Pavements

The use of GPR for pavement evaluation has been developed considerably during the past three decades. This development is essentially due to the evolution of the GPR technology and the pavement engineers’ need for fast and reliable tools to investigate and assess pavement systems. In fact, traditional investigation techniques, such as coring, are destructive to the pavement, and are very time consuming. Moreover, they require diverting the traffic from the lane under evaluation and are usually not very representative of the entire pavement since they are performed over a small set of points. These techniques can be contrasted to GPR, which is nondestructive, can be operated at highway speeds, and can provide information about the pavement under assessment at a much higher spatial sampling frequency, varying from a few feet to a few inches, depending on the GPR system used and the speed of the survey. According to the literature, a GPR system could be used for pavement assessment as either a quality control tool to check newly constructed pavements or as a road structure evaluation tool to evaluate the structure of existing in service pavements [13]. These two main functions are divided into the following applications.
2.4.1 Measurement of pavement layer thicknesses

The main application of GPR for pavement evaluation is the measurement of the thicknesses of the layers composing a pavement system. For new pavements, measuring the thickness of the layers assures that the constructed pavement conforms to the design, which ensures a defect-free pavement over its service life. For existing pavements, accurate measurement of layer thicknesses is very useful to pavement engineers for optimization of pavement maintenance or rehabilitation. The thickness is also needed as input to other nondestructive techniques such as Falling Weight Deflectometer (FWD). It should be noted that layer thicknesses of old pavements are usually unavailable due to undocumented repairs or differences between design and construction.

For an impulse GPR system, measuring the two-way travel time $\Delta t$ between the two interfaces of a homogeneous layer and knowing the EM velocity $v$ in (or dielectric constant $\varepsilon_r$ of) the traversed medium yields the thickness $d$ of the layer, as given by equation (2-30):

$$d = \frac{v\Delta t}{2} = \frac{c\Delta t}{2\sqrt{\varepsilon_r}}$$  \hspace{1cm} (2-30)

As depicted in Figure 2-7, the two-way travel time is the time that the wave takes to go from one interface to the other and back. It corresponds to the time difference between two consecutive reflected pulses in the GPR collected signal. To ensure a non-overlap between the reflected pulses from the interfaces of a homogeneous layer with a dielectric constant $\varepsilon_r$, its thickness should be greater than a minimum thickness known as the depth (or range) resolution. For a given layer, the depth resolution $\Delta d$ is given by [6]:

$$\Delta d = \frac{cT}{2\sqrt{\varepsilon_r}}$$  \hspace{1cm} (2-31)

where $T$ is the transmitted pulse width.
The main difficulty in interpreting GPR data when attempting to measure the thickness of a layer is illustrated by equation (2-30). Specifically, if it is assumed that the two-way travel time $\Delta t$ can be accurately measured from the GPR signal, the dielectric constant of the material remains unknown. In fact, the dielectric constant of a pavement layer is greatly affected by moisture content and mixture proportions of the layer’s different components. Therefore, use of a predetermined value for $\varepsilon_r$ leads to inaccurate thickness estimations.

Maser et al. [14] and Wimsatt et al. [15] developed a data analysis system that can automatically estimate the thickness of the HMA layer and that of the base layer in a flexible pavement system from air-coupled GPR data. The data analysis system assumes that the pavement is composed of three homogeneous layers, namely: HMA layer, base layer, and subgrade layer. With this system, the layer thicknesses are determined from the two-way travel time estimated from the reflected time domain signal, as shown above. The respective dielectric constants are estimated from the amplitudes of the reflected pulses and the amplitude of the incident pulse according to the following equations:

$$\varepsilon_a = \left[ \frac{A_{inc} + A_a}{A_{inc} - A_a} \right]^2$$  \hspace{1cm} (2-32)
\[ \varepsilon_b = \varepsilon_a \left[ \frac{F - R}{F + R} \right]^2 \]  

(2-33)

where:

- \( \varepsilon_a \) is the dielectric constant of the HMA layer,
- \( \varepsilon_b \) is the dielectric constant of the base layer,
- \( A_{\text{inc}} \) is the maximum amplitude in absolute value of the incident wave. It is obtained by collecting data over a copper plate placed on the surface of the pavement and then taking the negative of the resulting wave.
- \( A_a \) is the maximum amplitude in absolute value reflected from the HMA surface.
- The factor \( F \) is defined as: \( F = \frac{4\sqrt{\varepsilon_a}}{1 - \varepsilon_a} \)
- \( R \) is the ratio of the maximum reflected amplitude from the top of the base layer to the maximum reflected amplitude from the top of the HMA layer.

Equations (2-32) and (2-33) are found from the general input reflection coefficient, given in equation (2-29), by neglecting the conductivity of the layers and assuming that the dielectric constants are independent of frequency. While the latter assumption could be considered valid for the dielectric constants and GPR bandwidths used, the former assumption is not. In fact, neglecting any attenuation in the HMA layer, for example, will result in underestimating the value of \( \varepsilon_b \); consequently, the thickness of the base layer would be overestimated.

The GPR data analysis system developed by Maser et al. and Wimsatt et al. assumes that the HMA and base layers are homogeneous throughout their entire thicknesses. This assumption is usually not valid since a pavement layer typically contains inhomogeneities, such as localized defects and moisture accumulation in addition to thin overlays or differences in properties of successive lifts in the original pavement. As a result, this GPR data analysis system provides relatively accurate results for new pavements; however, the results degrade considerably for existing pavements.

To account for the thin layers that might be present in a pavement system, Kurtz et al. [16] incorporated multiple layers in their pavement model. Their data analysis system iteratively
detects all the reflected pulses from the time domain signal and computes the corresponding
dielectric constants using the reflection amplitudes, without taking into account material loss.
The layer thicknesses are then measured based on the two-way travel times and the
corresponding EM speed in each layer. In this case, layer detection is performed either by
matched filter or pattern recognition based on the reflected pulse slope changes, which are
assumed similar to those of the incident pulse. It was found that the matched filter technique
outperforms the slope change technique because it is more immune to noise. The major
drawback of the technique, however, is that it underestimates the dielectric constants of the
subsurface layers because of neglecting material loss and, therefore, overestimates the layer
thicknesses.

Spagnolini [17] used EM inversion in the time domain to estimate the dielectric constant
profile of multilayered dielectric media, such as pavement systems. The technique divides the
medium into \( N \) thin homogeneous layers with thicknesses less than the GPR resolution. Then it
tries to synthesize the reflected signal based on a theoretical time domain model that uses
predetermined values for material loss. A Newton-Gauss iterative approach is subsequently used
to find the best dielectric constant profile that minimizes the mean square error between the
synthesized reflected signal and the measured signal. To resolve the non-uniqueness problem of
the inversion procedure, the dielectric constant profile is further modeled using cubic B-splines
either in the depth direction (1D inversion) or in both depth and longitudinal directions (2D
inversion) to account for the lateral continuity of the layers. This inversion procedure gives good
results if the material loss values used are adequate. However, it is computationally intensive.

Gentili et al. [18] used the same EM inversion technique but computed the reflected signal
based on a frequency domain model instead of the time domain model. The performance of this
system was comparable to Spagnolini’s system.

Rampa and Spagnolini [19] and [20] developed a system that uses a likelihood test to
detect and track layers in a multilayered pavement system. The a-priori probability density
function (PDF) for detecting a reflected pulse is determined using a Markov model that computes
the PDF for a given scan from the a-posteriori PDF of the previous scan. The a-posteriori PDF
for current scan is then computed using Bayes theorem. This approach is extended to the multi-
layer case by iterating through the layers and removing the echo of each detected layer from the total reflected signal. The process is repeated until all the “known” layers are extracted or the residual reflected signal drops below a fixed threshold.

Lau et al. [21] used a frequency domain model to synthesize the reflected signal from a planarly layered medium. The model parameters are the layer thicknesses and the corresponding dielectric properties. Starting from an initial guess of layer thicknesses and dielectric properties, the theoretical time domain reflected signal is computed (by computing each spectral component and then using an inverse Fourier transform) and then compared to the collected reflected signal in the mean square error sense. Subsequently, the thicknesses and dielectric properties are iteratively changed until a minimum mean square error is achieved. Because of the evaluation of an inverse Fourier transform for each set of model parameters, this technique is computationally intensive. Moreover, due to the non-uniqueness of the solutions, the search algorithm might converge to a local minimum of the mean square error, thus leading to incorrect results.

2.4.2 Detection of subsurface distresses

As a road structure investigation tool, GPR is also used to locate initial subsurface defects in old rigid or flexible pavements before they propagate and reach the surface, in which case they become costly to repair. Subsurface defect detection is also imperative before placement of new overlays to ensure the effectiveness of the rehabilitation procedure.

In their automated pavement thickness measurement system presented above, Maser et al. [14] described a technique for estimating the base layer moisture content. The base layer moisture content is computed from the base dielectric constant, already calculated from the amplitude of the reflected signal, and the complex refractive index model mixture law. The parameters of this model are the base layer dielectric constant, the base layer moisture content, the aggregate dielectric constant, the dry density of the mixture, and the density of aggregates. With the exception of the base layer moisture content, which is the model unknown, and the base layer dielectric constant, all the other parameters are either assumed or determined from a core and are then assumed constant along the surveyed area. The accuracy of the results obtained by this technique is closely related to the precision of the estimates of the dielectric constant and dry density of the base material and their uniformity along the tested section.
Rmeili et al. [22] used GPR to locate stripped areas in HMA layers. Stripping is typically caused by moisture trapped in the HMA layers, which causes a rupture of the bond between aggregates, thus resulting in a low density layer. In this technique, stripping identification is based on detecting an intermediate negative reflection between the positive reflections from the surface and the bottom of HMA layer. This is based on the fact that because the stripped layer is less dense than a normal HMA layer, it has a lower dielectric constant. Therefore, the reflection coefficient at that interface would be negative. The major drawback of this technique that might lead to false alarms is the possibility that thin lifts with different properties may be present within the HMA layer, which might produce the same kind of reflection. Consequently, knowing the exact structure of the surveyed pavement is imperative for this technique to give accurate results.

Saarenketo et al. [23] used GPR to estimate the air void content of newly placed and compacted HMA in order to estimate its density, which should be within a specified range to ensure a defect-free pavement over its service life. It was found that for given aggregate and binder types, there exists an exponential relation between the HMA layer dielectric constant and its air void content.

Saarenketo [24] used GPR to detect mix segregation in the HMA layer. Segregation can be defined as the change in asphalt-binder-coated aggregate gradation due to improper handling of the mix during construction and mostly ill distribution of the aggregate mix, which results in periodic low-density areas in the HMA layer that can create long-term problems. The technique is based on estimating the HMA dielectric constant over the surveyed section and monitoring its variations. The dielectric constant over segregated areas would be lower than the rest of the pavement.

2.4.3 Estimation of concrete properties

The typical concrete properties that could be estimated from a GPR survey include the following: porosity (ratio of volume of voids to total volume of concrete), saturation (degree of saturation of air voids with water), chloride content, and reinforcing bar location, size, and cover depth. Studies of these properties could be used to estimate the potential for initiation of distresses in a concrete slab or its current degree of deterioration.
Halabe et al. [25] and [26] tested a GPR inversion technique that could estimate the porosity, degree of saturation, pore water salinity, and top reinforcing bar cover depth of reinforced concrete structures. The technique uses an empiric model for the reinforced concrete structure in the frequency domain to synthesize the reflected waveform based on the incident signal and an initial guess of the parameters stated above. A correction factor is then applied to the parameters based on the root mean-square error (RMSE) computed between synthesized and collected GPR signals. The process is repeated until the RMSE falls below a fixed threshold. The results show that the inversion technique is dependent on the initial guess. However, it was found that the total water content (i.e., porosity multiplied by saturation) and the depth of the rebar are almost independent of the initial guess.

Molyneaux et al. [27] used artificial neural networks (ANN) to detect rebar presence in a concrete structure and to estimate its diameter and cover depth. The network used is composed of three fully connected layers. The ANN used the Fourier transform of the received signal over the center of the rebar as input and a vector of seven elements as output. The first element of the output vector denotes rebar presence (1 present and 0 not present), whereas the other 6 elements represent 6 possible discrete depth bins, with 1 representing the correct depth bin used. The network was trained with a set of scans collected over a simulation tank containing a water-oil emulsion (which was shown to have same electromagnetic properties as concrete) and steel rebar at different depths. Another set of scans collected over the same simulation tank was used for testing the ANN. The tests showed a combined failure rate of 42% for mistakenly detecting rebar where no rebar exists or for assigning the rebar to the wrong depth bin. In addition, use of ANN with the described configuration failed to estimate the size of the rebar.

Heiler et al. [28] investigated an approach to automatically interpret GPR data collected over bridge decks using ANN. The authors divided GPR data interpretation into three categories: (1) Data quality control, (2) Locating events (or reflections), and (3) Identifying events. The authors define a “normal” scan over a bridge deck as a scan that contains exactly three reflections (A, C and D) corresponding to the air/HMA, HMA/concrete and concrete/rebar interfaces, respectively. An anomaly in a scan could include the presence of an extra reflection between A and C, the presence of noise, or the absence of a reflection. For task (1) ANN were used with the reflected signal as input and two nodes representing the presence or absence of an
anomaly as output. In this case, ANN achieved a maximum error rate of 21%. Tasks (2) and (3) were combined using an ANN with the reflected signal as input (124 input nodes) and three nodes representing the location of each event of interest ($A$, $C$ or $D$) as output. In this case, the ANN gave an average error between the calculated peak and the actual peak between 1.5 and 2.5 data points.

Millard et al. [29] described the use of a simulation tank to simulate concrete slabs with different configurations. The tank is filled with an oil-water emulsion, which perfectly simulates the electromagnetic properties of concrete. The tank is used to obtain GPR scans corresponding to different concrete configurations obtained by varying rebar size, spacing, and cover depth in addition to the incorporation of different types of voids with different shapes and sizes. The simulation results indicated that it is difficult to quantitatively determine rebar size; however, a qualitative comparison was possible. It was also shown that there is a limit to resolving individual bars as a function of bar spacing and cover depth: when cover depth increases, the bars become more difficult to resolve. Furthermore, it was found that at a given cover depth, the bars could cause a Faraday cage effect, thus masking deeper features. This phenomenon appears when the bars are closer to each other or when they have large diameters. A GPR scan with the antennas parallel to the bars (i.e., the electric field perpendicular to the bars) can reduce this effect.

2.4.4 GPR imaging techniques

Different attempts to obtain an “image” of subsurface inhomogeneities from GPR data were tried by different authors. The problem, in this case, is to find the subsurface object distribution function, which is proportional to the dielectric property distribution in the surveyed area. In fact, when displayed in 2D or 3D, the object distribution function could be considered as a good image of the subsurface objects.

Johansson and Mast [30] presented a 3D GPR imaging technique based on focusing the time domain reflected signals using Synthetic Aperture Radar (SAR) data. This technique assumes that imaging is done in a homogeneous medium with known parameters. To get the 3D subsurface image, GPR data is collected over a grid of points covering the surveyed area. The object distribution function is then obtained by averaging all the reflected signals at the point of
The averaging procedure should take into account the time delay between the antenna position where the reflected signal was collected and the point of interest in the distribution function.

Devaney [31] used diffraction tomography techniques to create images of weak reflective inhomogeneous formations in a homogeneous background using GPR. The technique is based on linearizing the wave equation’s solution, which has an integral form in this case. This equation expresses the scattered field at any point in space as a function of the inhomogeneity object distribution function and the field within the inhomogeneity. The linearization uses either the Born or the Rytov approximation for weak scatterers. These approximations assume that the scattered field is small compared to the incident field, which requires the magnitude of the object distribution and the total volume of the object to be small. This second condition could be relaxed with the Rytov approximation. After linearization, the object distribution function would be linearly related to the scattered field when the Born approximation is applied, and to the complex phase of the scattered field when the Rytov approximation is applied. With these two approximations, the object distribution function is solved by a simple matrix inversion.

Johansson and Mast [32] used a GPR imaging technique based on diffraction tomography. The algorithm used in this technique tries to find the spatial distribution of a buried object from the reflected waves by using a plane-to-plane backward propagation technique that is based on a spatial linear filter. The results found for each frequency component of the reflected signal are then superimposed to give the global solution. It should be noted that, in this case, GPR data is acquired within a rectangular area (2D) rather than along a longitudinal line.

2.5 Potential Benefits of Using GPR for Pavement Assessment

Accurate layer thickness estimation and distress localization are important issues for pavement engineers. For new pavements, layer thickness measurement is essential to ensure that the constructed layers meet the design specifications as part of the quality control and quality assurance procedure. For example, the Virginia Department of Transportation (VDOT) allows a maximum thickness error of ±25mm (±1.0in) for aggregate layers and ±15mm (±0.6in) for HMA layers [33]; the contractor is penalized if he/she does not meet the pavement thickness requirements. For old pavements, layer thickness measurement and subsurface defect
localization are important to make appropriate economical rehabilitation decisions. Layer thickness estimation is also essential as input for pavement management systems.

Traditionally, layer thickness estimation and distress localization are based on direct measurements and observations of a finite number of cores taken from the road. This technique has two major drawbacks: first, it introduces perturbation to the pavement structure that might cause weakening of the entire area surrounding the core locations; second, it is time-consuming and requires that the surveyed lane be closed to traffic, which is not an economical practice—especially on highways.

According to the Bureau of Transportation Statistics [34], U.S. governmental agencies spend more than 90 billion dollars annually to rehabilitate and maintain the public roadway infrastructure. If the efficiency of the decision-making process could be enhanced by introducing reliable and easy-to-use nondestructive evaluation techniques, the reduction in maintenance costs would be considerable.

Ground penetrating radar is considered as one of the prospective nondestructive tools that will provide such enhancement. In fact, GPR is nondestructive, can be operated at highway speeds, and can provide the required information (layer thickness and flaw locations) about the pavement being assessed at a high spatial sampling frequency that could be as low as a few centimeters.
Chapter 3

Research Approach

In order to achieve the objectives of this research, GPR data was collected from a known experimental pavement site, the Virginia Smart Road, with a commercially available impulse GPR system. Collected data was then analyzed, and the results were compared to the ground truth to develop the best combination of data analysis techniques that will allow for accurate GPR data interpretation. A description of the Virginia Smart Road, the GPR system that was used, and the main research tasks followed are presented in the following sections.

3.1 Virginia Smart Road

The Virginia Smart Road in Southwest Virginia is a unique, state-of-the-art, full-scale research facility for pavement research and evaluation of Intelligent Transportation Systems (ITS) [35]. When completed, the Virginia Smart Road will be a 9.6km (6mile) connector highway between Blacksburg and I-81 in Southwest Virginia, with the first 3.2km (2miles), which has already been constructed, designated as a controlled test facility. This connection will serve an important role in the I-81/I-73 transportation corridor. After completion of the project, provisions will be made to route traffic around controlled test zones on the Virginia Smart Road to allow for ongoing testing.

The Virginia Smart Road test facility has 12 flexible pavement sections and a continuously reinforced concrete pavement section. The flexible pavement sections were heavily instrumented (using around 500 sensors) during construction to allow pavement performance monitoring under real vehicular loading and environmental conditions. The sensor array includes different types of strain gages (to measure longitudinal and transversal strain), pressure cells (to measure vertical stress), thermocouples, time-domain reflectometry (TDR) probes (to measure layer moisture content), and resistivity probes (to measure frost depth) [36]. These
sensors provide continuous data for the different parameters of interest underneath each pavement layer.

The design of the Virginia Smart Road pavement sections is summarized in Table 3-1. Each section is composed of different layers that are each in accordance with the Virginia Department of Transportation (VDOT) specifications. The different layers include the following:

- **Wearing surface:** Seven types of wearing surface were used (SM-9.5A, SM-9.5A with high laboratory compaction, SM-9.5D, SM-9.5E, SM-12.5D, SMA-12.5, and open-graded friction course (OGFC)). The main difference between these mixes is the aggregate size and the binder type. All mixes, with the exception of the OGFC, were constructed at a thickness of 38mm.
- **Intermediate hot-mix-asphalt base layer,** designated BM-25.0, was placed at different thicknesses ranging from 100 to 244mm.
- **Three sections have the SM9.5A surface mix placed under the BM-25.0** to examine the benefits of such a design in reducing fatigue cracking.
- **Open Graded Drainage Layer (OGDL):** out of the 12 sections, three sections were built without OGDL. Seven sections were treated with asphalt cement, and two were treated with Portland cement. The thickness of this layer was kept at 75mm throughout the road.
- **Cement Stabilized base,** termed as 21-A layer, was used in ten sections at a thickness of 150mm. This layer was stabilized by adding 3.5% (by weight) cement to the aggregate.
- **Subbase aggregate layer,** designated as 21-B, was placed over the subgrade at different thicknesses ranging from 75 to 150mm.
- **The concrete slab was 250mm thick throughout its length.**
Table 3-1: Pavement Sections at the Virginia Smart Road

<table>
<thead>
<tr>
<th>Section</th>
<th>Length (m)</th>
<th>Wearing Surface (38mm)</th>
<th>BASE BM-25.0 (mm)</th>
<th>BASE SM-9.5A (mm)</th>
<th>OGD</th>
<th>21-A Aggr. Cem. Stab. (mm)</th>
<th>21-B Aggr. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>371</td>
<td>SM-12.5D/CP</td>
<td>150/CP</td>
<td>0</td>
<td>75</td>
<td>150/CP</td>
<td>175/CP</td>
</tr>
<tr>
<td>B</td>
<td>90</td>
<td>SM-9.5D/CP</td>
<td>150</td>
<td>0</td>
<td>75</td>
<td>150/CT/CP</td>
<td>175/CT/CP</td>
</tr>
<tr>
<td>C</td>
<td>87</td>
<td>SM-9.5E/CP</td>
<td>150/CP</td>
<td>0</td>
<td>75</td>
<td>150/CT</td>
<td>175/CT</td>
</tr>
<tr>
<td>D</td>
<td>117</td>
<td>SM-9.5A/CP</td>
<td>150/CP</td>
<td>0</td>
<td>75</td>
<td>150/CT</td>
<td>175/CT</td>
</tr>
<tr>
<td>E</td>
<td>76</td>
<td>SM-9.5D</td>
<td>225/CP</td>
<td>0</td>
<td>0</td>
<td>150/CT</td>
<td>75/CT/CP</td>
</tr>
<tr>
<td>F</td>
<td>94</td>
<td>SM-9.5D</td>
<td>150/CP</td>
<td>0</td>
<td>0</td>
<td>150/CT</td>
<td>150/CT</td>
</tr>
<tr>
<td>G</td>
<td>90</td>
<td>SM-9.5D</td>
<td>100/CP</td>
<td>50/CP</td>
<td>0</td>
<td>150/CT/CP</td>
<td>150/CT/CP</td>
</tr>
<tr>
<td>H</td>
<td>90</td>
<td>SM-9.5D</td>
<td>100/CP</td>
<td>50/CP</td>
<td>75</td>
<td>150/CT</td>
<td>75</td>
</tr>
<tr>
<td>I</td>
<td>98</td>
<td>SM-9.5A*/CP</td>
<td>100/CP/GM</td>
<td>50</td>
<td>75</td>
<td>150/CT</td>
<td>75</td>
</tr>
<tr>
<td>J</td>
<td>92</td>
<td>SM-9.5D</td>
<td>225</td>
<td>0</td>
<td>75/SR/CP</td>
<td>0/GT</td>
<td>150/CT</td>
</tr>
<tr>
<td>K</td>
<td>86</td>
<td>OGFC^</td>
<td>244/SR/CP</td>
<td>0</td>
<td>75/C</td>
<td>0/GT</td>
<td>150</td>
</tr>
<tr>
<td>L</td>
<td>104</td>
<td>SMA-12.5/Cp</td>
<td>150/GM</td>
<td>0</td>
<td>75/C</td>
<td>150</td>
<td>75</td>
</tr>
</tbody>
</table>

CP: Copper plate * High lab compaction
SR: Stress Relief Geosynthetic GT: Woven Geotextile/Separator
^ 19-mm-thick OGFC over 19-mm SM-9.5D GM: Galvanized Metal Mesh

The Virginia Smart Road offers a good opportunity to explore the feasibility of using GPR to accurately measure layer thicknesses and to monitor pavement performance over time [37]. To achieve this objective, 31 copper plates were embedded in the flexible pavement during construction at the bottom of different layers, and four plates were placed underneath the concrete slab in the rigid pavement. Copper, with a conductivity of $5.7 \times 10^7$ S/m is considered to be a very good conductor and, therefore, was chosen to serve as a perfect EM reflecting material. Copper sheets measuring 0.91m $\times$ 1.22m (3ft $\times$ 4ft) were placed at the center of the instrumented lane at several locations in all tested sections, as indicated in Table 3-1. All the copper plates of the same section were placed at a distance of 5m from each other to avoid masking of the bottom plates by the top ones. Figure 3-1 shows a copper plate placed on top of the subgrade layer in section B. The benefit of copper plates is to indicate where the interface between each two layers occurs. In fact, because some of the pavement materials do not have significant differences in their dielectric properties, a very small amount of energy is reflected back from their interface. This reflected energy is further attenuated by the layers and, thus, it is obscured
by the GPR receiver noise. In this case, accurate determination of the interface becomes difficult.

After completion of the Virginia Smart Road construction, two methods were used to ensure that the as-built layer thicknesses conform to the design. The first method was based on the copper plates. In fact, during the construction of the Virginia Smart Road, the positions of the copper plates were surveyed in the x, y and z directions prior to being placed. The layer thicknesses near the copper plates were then determined based on the z coordinates of the plates. The second method was a direct measurement of the layer thicknesses of cores taken from the road (around 120) for other studies. It should be noted that the cores provide only the thickness of the HMA layers because the aggregate layers cannot be extracted without destroying the core. These two methods showed that the wearing surface thickness error varied between -8% and 40%, whereas the BM-25 thickness error varied between -10% and 26%, depending on the core location. The core data is also used in the current research to check the GPR results and estimate their accuracy.

![Copper plate under 21-B material in section B](image)

Figure 3-1: Copper plate under 21-B material in section B

### 3.2 GPR System Description

#### 3.2.1 Control unit

The GPR system used in this research is a SIR-10B (acronym for Subsurface Interface Radar) system manufactured by Geophysical Survey Systems, Inc. (GSSI). The SIR-10B is an
impulse GPR system that has two Input/Output channels, allowing two sets of antennas to be connected simultaneously, as depicted in Figure 3-2. Internally, the two channels are multiplexed to a single receiver. Therefore, GPR signals are transmitted and received successively from the antennas, rather than simultaneously. This design minimizes interference problems in case both antenna systems are used at the same time. The EM pulse shape and frequency content are imposed by the transceiver and the transmitting antennas used. The transceiver is separate from the control unit and is sometimes incorporated into the antenna. Consequently, the same SIR-10B system can work with a variety of antennas for different types of investigations (geophysical, archeological, civil, etc.).

![Figure 3-2: Block Diagram of the SIR-10B GPR System](image)

### 3.2.2 Antennas

Two types of antennas were used simultaneously in this research: an air-coupled antenna and a ground-coupled antenna. The air-coupled system is composed of a pair of separate horn antennas (one serves as a transmitter and the other as a receiver) with a frequency bandwidth of 1GHz, which corresponds to a pulse width of nearly 1ns. The antenna has a 3dB beamwidth of approximately 60°. The ground-coupled system is comprised of a single bowtie antenna (operating as transmitter and receiver) that delivers a signal with a 900MHz bandwidth, corresponding approximately to a pulse width of 1.1ns. The 3dB beamwidth of this antenna is...
around 90°. The main specifications of the SIR-10B GPR system and the two antennas are presented in Table 3-2.

As depicted in Figure 3-3, the two antenna systems were mounted behind the survey van, with the control unit set inside of it. This configuration allows data to be collected by both systems at the same time. To prevent reflections from the ground-coupled antenna while collecting data with the air-coupled antenna, the ground-coupled antenna was placed away from the footprint of the air-coupled antenna.

**Table 3-2: SIR-10B Specifications**

<table>
<thead>
<tr>
<th>Radar Parameters:</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antennas</td>
<td>Can handle up to 2 antenna inputs simultaneously</td>
</tr>
<tr>
<td>Sampling frequency rate</td>
<td>up to 100GHz</td>
</tr>
<tr>
<td>Analog Quantization</td>
<td>8 or 16 bits, selectable</td>
</tr>
<tr>
<td>Analog to Digital Sampling rate</td>
<td>128, 256, 512, 1024, 2048, 4096 or 8192 samples/scan, selectable</td>
</tr>
<tr>
<td>Scan Rate</td>
<td>2 to 220 scans/second, selectable</td>
</tr>
<tr>
<td>Programmable time window</td>
<td>2-10,000nanoseconds full scale, selectable</td>
</tr>
<tr>
<td>Programmable sampling window increments</td>
<td>10picoseconds</td>
</tr>
<tr>
<td>Dynamic range of time variable gain</td>
<td>150dB</td>
</tr>
<tr>
<td>Dynamic range of input AΔ</td>
<td>25 bits</td>
</tr>
<tr>
<td>Dynamic range of input averaging filter</td>
<td>48 bits</td>
</tr>
<tr>
<td>Dynamic range of DSP</td>
<td>24 bits</td>
</tr>
<tr>
<td>Clock frequency of DSP</td>
<td>38.5MHz</td>
</tr>
<tr>
<td>Programmable stacking range</td>
<td>2-32768</td>
</tr>
</tbody>
</table>

**Antennas:**

| Ground-Coupled                           | Model 3101                                         |
| Frequency Bandwidth                       | 900MHz                                             |
| Pulse width                               | 1.1ns                                              |
| Air-Coupled                               | Model 4208, TEM horn antenna                       |
| Frequency Bandwidth                       | 1GHz                                               |
| Pulse width                               | 1ns                                                |
| Pulse Repetition Frequency (PRF)          | up to 400KHz                                       |
Chapter 3: Research Approach

It is worth noting that GPR antennas play an important role in selecting the transmitted pulse shape [38]. In fact, antennas can be regarded as highpass filters that only radiate signals with frequency contents higher than a minimum cutoff frequency fixed by the antenna size. On the other hand, the ultra-wide bandwidth GPR pulses have most of their energy concentrated in the lower frequencies, thus outside the antenna spectrum. This frequency mismatch between antenna and radiated signals could cause considerable EM energy loss and distortion of the transmitted signals. Therefore, the GPR pulse shape should be chosen in such a way that its spectrum falls approximately within that of the transmitting antenna.

The EM energy lost because of frequency mismatch can be quantified by the spectral efficiency, which is defined as the ratio of the portion of the transmitted energy that falls within the antenna frequency band to the total pulse energy [38]. Hence, a spectral efficiency near zero indicates a high frequency mismatch between the transmitted signal and the antenna, whereas a spectral efficiency near one indicates a low mismatch. For any pulse shape, the spectral efficiency increases uniformly with the pulse bandwidth since more energy would fall within the antenna band as the pulse’s bandwidth increases. Moreover, it was reported [38] that, in general, bipolar pulses have higher spectral efficiency than their unipolar counterparts since they have less low frequency content. In particular, it could be shown that a bipolar bell-shaped pulse (Mexican hat) has the highest spectral efficiency (greater than 90% for low bandwidths and
converges rapidly to approximately 98% for higher bandwidths) compared to simpler bipolar pulses, such as rectangular or triangular shapes, which have spectral efficiencies around 85% even for large bandwidths [38]. For this reason, a Mexican hat pulse (Figure 3-4) shape is used as a transmitted pulse for the considered GPR system.

![Incident GPR Pulse Shape](image)

**Figure 3-4: Incident GPR Pulse Shape**

### 3.2.3 Data acquisition control

To precisely locate the collected GPR data spatially, a distance-measuring instrument (DMI) connected to the survey vehicle wheel is used, as depicted in Figure 3-5. The DMI output is used to control the trigger pulses generated by the pulse generator, as illustrated in Figure 3-2. In this case, data is collected as a function of distance (i.e., \( n \) scans per meter) rather than as a function of time (i.e., \( n \) scans per second).
3.2.4 Performance tests

The Texas Transportation Institute (TTI) [39] developed a set of performance tests to ensure that an air-coupled GPR system could be reliably used for bridge or pavement assessment. Even though these tests are not standard, they provide a good indication about the performance of a GPR system. Texas Transportation Institute performance tests are carried out using the air-coupled GPR system, with the antenna deployed at its nominal height (45cm or 18in for the antennas used in this research), as summarized below:

- A noise-to-signal ratio test is performed by collecting a single scan over a copper plate large enough (0.91m × 1.22m) to cover the foot print of the antenna, as shown in Figure 3-6. The noise-to-signal ratio, $NSR$, is defined as follows:

$$NSR = \frac{A_N}{A_{MP}}$$ (3-1)

where $A_{MP}$ is the maximum amplitude reflected from the copper plate and $A_N$ is the maximum amplitude occurring after the copper plate reflection, as depicted in Figure 3-7.

- A signal stability test is performed by collecting 100 scans over the copper plate as shown in Figure 3-6. Signal stability, $SS$, is defined as follows:
where $A_{\text{max}}$ is the maximum amplitude reflected from the copper plate over the 100 scans, $A_{\text{min}}$ is the minimum amplitude reflected from the copper plate over the 100 scans, and $A_{\text{avg}}$ is the average amplitude reflected from the copper plate over the 100 scans.

- A long term signal stability test is carried out by collecting GPR data over a copper plate for a period of two hours at a rate of 1 scan every two minutes. The long term signal stability, $LTSS$, is defined as follows:

$$LTSS = \frac{A_{\text{max}} - A_{20}}{A_{20}}$$  \hspace{1cm} (3-3)

where $A_{20}$ is the amplitude reflected from the copper plate after 20min and $A_{\text{max}}$ is the maximum amplitude reflected from the copper plate between 20min and two 2hrs.

Figure 3-6: Antenna Setup for TTI Performance Tests
Chapter 3: Research Approach

Figure 3-7: Typical Reflected Signal from a Copper Plate

- An end reflection test is performed by collecting a single scan over the copper plate as shown in Figure 3-6. The end reflection ratio, $ERR$, is defined as follows:

$$ERR = \frac{A_{end}}{A_{MP}}$$  \hspace{1cm} (3-4)

where $A_{end}$ is the maximum amplitude of the coupling signal (or end reflection) between the transmitting and receiving antennas and $A_{MP}$ is the maximum amplitude reflected from the copper plate, as depicted in Figure 3-7.

- A time calibration factor test is performed by collecting 3 scans over a copper plate at 3 different heights, namely: 0.260m, 0.406m, and 0.552m. The time calibration factor, $TCF$, is defined as follows:

$$TCF = \frac{C_1 - C_2}{(C_1 + C_2)/2}$$  \hspace{1cm} (3-5)

where $C_1$ is the EM speed evaluated from the reflection at height 1 (i.e., height 1 divided by time of travel 1) and $C_2$ is the EM speed between heights 2 and 3 (i.e., difference of heights 3 and 2 divided by difference of times of travel 3 and 2).
• A symmetry of metal plate reflection test is performed by collecting a single scan over the copper plate, as depicted in Figure 3-6. The symmetry of the reflection is evaluated by measuring the time between the minimum negative peak following the copper plate reflection and the first subsequent zero crossing point.

• A concrete penetration test is performed by collecting a single scan over a 150mm (6in) thick concrete slab with a copper plate placed underneath it. The concrete penetration, \( CP \), is defined as follows:

\[
CP = \frac{A_{MP}}{A_{surf}}
\]  

(3-6)

where \( A_{MP} \) is the maximum reflection amplitude from the slab/copper plate interface and \( A_{surf} \) is the maximum reflection amplitude from the slab surface.

The performance of the SIR-10B system was evaluated using all of TTI tests described above. The test results are summarized in Table 3-3. According to the results, the SIR-10B system meets TTI specifications except for the end reflection test, in which it was 2% higher than the limit. However, this failure to meet the specification should not affect the accuracy of GPR results since the end reflection is usually removed from the collected data in the preprocessing phase. Furthermore, the end reflection amplitude can be controlled by the distance separating the transmitting and receiving antennas. In fact, if this distance is increased from its nominal value of 50mm to a distance of 150mm, the end reflection decreases to 15%.

3.3 Data Collection

GPR surveys have been conducted approximately every two months over the experimental section of the Virginia Smart Road since its completion in late 1999. During each survey, data were collected by the ground-coupled and air-coupled antenna systems using the survey van depicted in Figure 3-3. The frequency of the surveys allowed for data collection over different environmental conditions of the pavement (i.e., various temperatures and moisture content). However, all the surveys were performed over a dry pavement surface and at least two days after rain to obtain the maximum depth of penetration possible.
### Table 3-3: Performance Tests Results

<table>
<thead>
<tr>
<th>Test</th>
<th>Result</th>
<th>TTI Spec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise-to-signal ratio</td>
<td>4.7%</td>
<td>≤ 5%</td>
</tr>
<tr>
<td>Signal stability</td>
<td>1.0%</td>
<td>≤ 1%</td>
</tr>
<tr>
<td>Long term signal stability</td>
<td>1.1%</td>
<td>≤ 3%</td>
</tr>
<tr>
<td>End reflection</td>
<td>27%</td>
<td>&lt; 25%</td>
</tr>
<tr>
<td>Time calibration factor</td>
<td>2 %</td>
<td>&lt; 2%</td>
</tr>
<tr>
<td>Symmetry of metal plate</td>
<td>0.59 ns</td>
<td>≤ 0.75 ns</td>
</tr>
<tr>
<td>Concrete penetration test</td>
<td>30.3%</td>
<td>≥ 25%</td>
</tr>
</tbody>
</table>

During the surveys, data were collected continuously over the lane containing the copper plates, which also contains all the sensors used for pavement monitoring. For the sections with embedded reinforcing steel meshes (i.e., sections I and L), data were also collected over the non-instrumented lane to avoid the high reflections from the meshes. In fact, because the mesh is made out of steel, its reflections usually mask the deeper features.

In addition to the continuous (or dynamic) surveys, two static calibration measurements were conducted at the end of each survey. In each of these measurements, at least 100 scans were collected while the survey vehicle was stopped. During the data analysis phase, these scans were averaged to yield one scan with a higher signal-to-noise ratio. These measurements were taken as follows:

- The first measurement was carried out with a copper plate placed on the pavement surface directly underneath the air-coupled antenna. As will be shown in the following chapter, because copper is a perfect EM reflector, this data was used as an approximation to the transmitted pulse.
- The second measurement was conducted with the air-coupled antenna pointing towards sky. This data was used as an estimate for the coupling pulse shape, as will be shown in the next chapter.

To be able to compare GPR data collected during different periods, the surveys were always performed in the same driving direction, starting from a fixed point located at the
beginning of section A. Data collection was controlled longitudinally by the distance measuring instrument (DMI) with a spatial sampling rate of either 5 or 10 scans per meter. Because of the use of the DMI, the van speed did not affect the regularity of the scan positions. However, a low speed was maintained (around 16km/h or 10mph) to cope with the GPR’s data acquisition speed. To help locate special features on the road (such as the beginning of a section or a bridge) in the GPR collected data, manual marks where inserted by means of a push-button connected to the system. Laterally, the surveys were conducted on the center of the lane where the locations of the copper plates are clearly marked on the surface of the pavement.

3.4 Research Tasks

In order to achieve the objectives of this research, the following research tasks were adopted:

1. Collect GPR data using a commercially available GPR system from the Virginia Smart Road, which is an experimental pavement site with a known and well-documented structure.
2. Divide the analysis system into smaller independent processing stages that are only related by their inputs/outputs, based on the collected GPR data. The processing stages are as follows:
   a. Preprocessing of raw GPR data in order to enhance it. This includes investigation of noise filtering and deconvolution techniques. The deconvolution is used to separate the incident GPR signal from the pavement reflectivity function to improve the depth resolution, which is linked to the pulse width of the transmitted signal.
   b. Examining the layer interface detection in the time domain. The output of this stage is the layer interface reflection time-delays and amplitudes for each collected scan. The detection techniques that will be investigated include mainly correlation detectors, such as the matched filter detector.
   c. Estimating the dielectric properties of the different pavement layers using three methods, as follows:
i. Estimation of the dielectric constant variations of HMA over the GPR bandwidth.

ii. Estimation of the dielectric constant based on the reflection amplitudes.

iii. Estimation of the dielectric constant based on reflection time-delays using a modified common midpoint technique.

d. Separating layer interfaces and subsurface defects using a classification algorithm based on the layer interfaces and dielectric constants found from consecutive scans. The classifier will use the natural longitudinal extensions of the layers to extract localized subsurface defects. The output of this stage includes the estimated layer thicknesses and subsurface distress locations.

3. Evaluate the performance of each processing stage individually using field data collected from the Virginia Smart Road. For each stage, choose the best/optimum analysis technique based on performance.

4. Evaluate the performance (primarily accuracy) of the overall GPR data analysis system using field data collected from the Virginia Smart Road.
4.1 Introduction

Automating GPR data interpretation could be simplified by dividing the data analysis system into smaller subsystems. These subsystems are only interconnected by their inputs and outputs. This configuration allows the performance of each subsystem to be evaluated as if it were a standalone module. Moreover, it allows for the evaluation of any combination of consecutive subsystems. Another advantage of this configuration is the possibility of distributing the data processing over multiple processors to speed up the analysis.

The flowchart of the proposed data analysis system is depicted in Figure 4-1. The system includes the following components:

- **Preprocessing**: raw GPR data should be first preprocessed in order to enhance its quality. Preprocessing mainly includes the following: removal of the coupling (or end reflection) pulse, noise filtering, and pulse compression in order to improve depth resolution. The output of this subsystem is an enhanced version of the GPR data.

- **Layer interface detection**: the function of this subsystem is to accurately detect all layer interface reflections and to estimate their locations within the reflected signal. The output of this subsystem is the number of detected layers in the pavement system along with their relative positions in time delay units (nanosecond or samples).

- **Dielectric properties estimation**: in this stage, the dielectric properties of each layer identified by the previous subsystem are estimated from the reflected GPR signal. The outputs of this module are the layer thicknesses and their respective dielectric properties.
• *Layer/Distress separation:* based on the layer thicknesses and dielectric properties found by the previous stage, this subsystem would be used to separate pavement layers and pavement defects.

![Flowchart of the Proposed GPR Data Analysis System](image)

*Figure 4-1: Flowchart of the Proposed GPR Data Analysis System*

Detailed analyses of these different processing subsystems and comparisons between different techniques that could be used at each stage will be investigated in the following sections.
Unless otherwise specified, all GPR signals involved in the following analysis are assumed to be discrete and causal (i.e. equal to zero for negative time values). The variable $t$, where $t$ is an integer number, will be used to represent the sample numbers.

### 4.2 Preprocessing

In its raw format, GPR data is usually contaminated by noise and clutter. Noise is predominantly caused by interference from other radio-wave-emitting devices (such as CB radios, cell phones, etc) that are present in the vicinity of the GPR system during data collection. Clutter, on the other hand, represents any unwanted reflections present in the GPR signal that render GPR data interpretation more difficult. For pavement systems, GPR clutter could be classified into two categories: clutter inherent to the GPR system and clutter due to the surveyed pavement structure. Clutter inherent to the GPR system corresponds to the coupling (or end reflection) pulse, which is caused by EM energy passing directly from transmitting to receiving antenna. It should be noted that this reflection is always present in the GPR data. Ground penetrating radar clutter caused by the surveyed pavement structure corresponds to the multiple reflections in the GPR signal that are usually attributable to EM energy trapped between two highly reflective layer interfaces. In this case, EM energy goes back and forth between the two reflectors and, thus, produces an unwanted echo on the GPR trace. The effect of this type of clutter, which depends on the surveyed structure, could be minimized by suitably selecting the length of the GPR scan.

In order to reduce the effects of these unwanted signals, the raw GPR data should be passed through a cascade of suitably designed digital filters. These filters constitute the preprocessing stage of the GPR data analysis system. The preprocessing stage would also be used to enhance the depth resolution of the GPR system by compressing the reflected pulses. This procedure would allow for the estimation of the thickness of thin pavement layers (such as the wearing surface layer), which are thinner than the minimum detectable thickness.

#### 4.2.1 Coupling pulse removal

As was mentioned in Chapter 2, the transmitting and receiving antennas of a bistatic GPR system are deployed next to each other (typically 50mm [2in] apart). Because of this
configuration (shown in Figure 4-2a), a portion of the transmitted EM energy is passed directly from transmitter to receiver without undergoing any reflections. For an impulse GPR system, this energy corresponds in the time domain reflected signal to a pulse at the beginning of the scan. This pulse is commonly referred to in literature as a *coupling* pulse [16]. Figure 4-3 depicts the coupling pulse along with the major reflections identified in a scan collected by an air-coupled bistatic GPR system. For a monostatic GPR system (shown in Figure 4-2b), antenna impedance mismatch causes a reflection at the end of the antenna. This spurious reflection is recorded by the receiver as a pulse located at the beginning of the time domain signal. It is commonly referred to as *end reflection* pulse [16]. Figure 4-4 shows the end reflection pulse and the other important subsurface reflections in a scan collected by a monostatic ground-coupled GPR system. It should be noted that, in this case, the end reflection pulse coincides with the surface reflection, as illustrated in the figure.

![Figure 4-2: EM Propagation at Transmission and Reception for: (a) Bistatic and (b) Monostatic GPR Systems](image)

![Figure 4-3: Major Reflections in a Scan Collected by an Air-Coupled, Bistatic GPR System](image)
4.2.1.1 Use of the coupling pulse in GPR data interpretation

When analyzing air-coupled GPR data, the coupling pulse might be used as a time marker to indicate the position of the antenna in the reflected time domain signal. The time difference (indicated by “Air” in Figure 4-3) between the surface reflection and the coupling pulse corresponds to twice the height of the antenna. Therefore, using the measured time difference and knowing the EM speed in air (assumed equal to the speed of light in free space, $c$), the antenna height could be estimated. The benefit of knowing the antenna height is that it could be used in the subsequent data analysis stages to correct for signal amplitude as a function of the propagation distance (i.e., to correct for geometric spreading loss). It should be noted that even though the antenna is securely fixed to the survey van, its height might change during the survey due to vehicle and antenna bounce when traveling at highway speeds.

In practice, to be able to estimate the antenna height from the reflected GPR signal, the antenna should be deployed at a minimum elevation that guarantees a non-overlap between the coupling pulse and the surface reflection. Figure 4-5 illustrates how the antenna height affects the GPR response. When the antenna is 0.10m (4in) high, the strong surface reflection entirely overlaps the coupling pulse. However, when the antenna height increases, the two pulses become more distinguishable until the height of 0.475m (18in, the height recommended by antenna manufacturer) where the coupling pulse and the surface reflection become completely separated.
As illustrated in Figure 4-4, in ground-coupled GPR data, the coupling pulse coincides with the surface reflection. Therefore, it is used in data interpretation to measure the two-way travel time between the interfaces of the top layer.

4.2.1.2 Coupling pulse removal procedure

As was mentioned above, the coupling (or end reflection) pulse is necessary to interpret ground-coupled GPR data; therefore, it should not be removed. On the other hand, for air-coupled GPR data, the coupling pulse should sometimes be removed in order to correctly apply the subsequent data processing stages, as will be shown in the following sections. In particular, the coupling pulse should be removed if the consequent processing stages involve theoretical modeling of the reflected signal. In fact, because the coupling pulse occurs in the near field of the antenna, it is difficult to model theoretically.

The reflected signal measured by an air-coupled GPR system can be modeled by the following equation:

\[ y_r(t) = y_e(t) + x(t) \gamma(t) + n(t) \]  

(4-1)

where:

- \( y_r(t) \): reflected GPR signal,
• $y_c(t)$: coupling pulse,
• $x(t)$: transmitted GPR signal,
• $\gamma(t)$: reflectivity function of the pavement system,
• $n(t)$: random additive noise,
• $*$ is the convolution operator.

According to equation (4-1), if the coupling pulse shape $y_c(t)$ were known and assuming that the effect of the random noise could be neglected ($n(t) \approx 0$), the signal reflected by the pavement system (i.e. $x(t)*\gamma(t)$) would be recovered by subtracting $y_c(t)$ from the reflected GPR signal $y_r(t)$. Equation (4-1) also indicates that if the pavement reflectivity function $\gamma(t)$ is zero, the GPR reflected signal $y_r(t)$ would be equal to $y_c(t)$. A technique to measure $y_c(t)$ would then be to point the GPR antennas towards the sky [16], which guarantees that $\gamma(t) = 0$. The reflected signal (equal to $y_c(t)$ in this case) will be then subtracted from the all the collected GPR data over the pavement to yield the GPR data without coupling pulse.

Figure 4-6a shows a normal GPR scan collected over a pavement system (signal $y_r(t)$) along with the signal recorded with the antennas pointing towards the sky (signal $y_c(t)$). It is noted that due to the GPR system’s data acquisition procedure, the two signals might be out of phase (e.g., Figure 4-6a shows that $y_c(t)$ is leading $y_r(t)$). Therefore, the simple subtraction of $y_c(t)$ from $y_r(t)$ will not remove the coupling pulse as expected. This is depicted in Figure 4-6b, where the amplitude of the coupling pulse increased after the subtraction operation. To measure the time shift between the two signals, their cross-correlation sequence is evaluated. The cross-correlation would be maximized at the lag corresponding to the time shift between the two signals.

A biased estimate of the cross-correlation sequence between two real $N$-sequences $x_n$ and $y_n$ is given by the following equation [40]:

$$
r_{xy}(l) = \begin{cases} 
\frac{1}{N} \sum_{n=0}^{N-l-1} x_n y_{n-l} & l \geq 0 \\
\frac{1}{N} \sum_{n=0}^{N+l+1} y_n x_{n+l} & l < 0 
\end{cases}
$$

(4-2)
Figure 4-6c depicts the normalized cross-correlation sequence $r_{y_r y_c}(l)$ between the signals shown in Figure 4-6a. The cross-correlation has a maximum at lag $l_m = +10$; therefore, in this case, $y_c(t)$ is leading $y_r(t)$ by 10 samples. Shifting signal $y_r(t)$ by 10 samples produces a perfect alignment between the two signals, as illustrated in Figure 4-6d.

Figure 4-7 shows the reflected signal with the coupling pulse removed. This signal is the result of subtracting signal $y_c(t)$ from the time-shifted signal $y_r(t)$. It is noted in this figure that after the coupling pulse is removed, there still exists a residual coupling pulse with relatively lower amplitude than the original. The residue is mainly due to the following:
• Amplitude fluctuations between the signals \( y_c(t) \) and \( y_r(t) \) due mostly to ADC quantization errors,

• Imperfect alignment between the signals due to synchronization errors when sampling \( y_c(t) \) and \( y_r(t) \). After aligning the waveforms as mentioned above, this error could be up to one sampling period (for example, for a 20ns window and 512samples/scan the maximum misalignment error is around 40ps).

4.2.1.3 **Performance evaluation of coupling pulse removal**

To evaluate the performance of the aforementioned coupling pulse removal procedure, the end reflection test, presented in Chapter 3 for testing the performance of the GPR system according to TTI specifications [39], is used. The end reflection ratio is given by equation (3-4) and is repeated here:

\[
ERR = \frac{A_{end}}{A_{MP}}
\]  

(4-3)

where \( A_{end} \) is the maximum amplitude of the coupling pulse and \( A_{MP} \) is the maximum amplitude reflected from a copper plate placed underneath the antennas (see Figure 3-7).

The performance of the coupling pulse removal procedure could be evaluated by comparing the end reflection ratios before and after coupling pulse removal for a large set of

![Figure 4-7: Reflected Signal with Coupling Pulse Removed](image-url)
data. The end reflection ratios were computed for GPR data collected during different surveys conducted between August 1999 and December 2002, as specified in 3.3. The main differences among the surveys were the weather conditions and, sometimes, the GPR configuration parameters (acquisition window length and amplifier gain). These differences allow the coupling pulse removal procedure to be evaluated under various conditions.

Figure 4-8 depicts the difference between the end reflection ratio measured before and after coupling pulse removal for different survey dates. According to the figure, for all the surveys, the coupling pulse removal procedure allowed a significant decrease in the \( ERR \). Specifically, before coupling removal, the \( ERR \) has an average of 31%, with a minimum of 27% and a maximum of 34%. On the other hand, after coupling pulse removal, the \( ERR \) average decreased to 4%, with a minimum of 1% and maximum of 6%. On average the decrease in \( ERR \) for all surveys is around 88%, with a maximum of 97% and a minimum of 77%.

According to TTI specifications [39], the end reflection ratio should be less than 25%. It is noted that this condition is not met for the raw GPR data; however, it is largely satisfied after applying the coupling pulse removal procedure for all the surveys considered.

![Figure 4-8: Comparison between ERR before and after Coupling Pulse Removal for Different Surveys](image-url)
4.2.2 Noise filtering

Because of the ultra-wide bandwidth of impulse GPR receivers, GPR signals are susceptible to noise corruption. The main sources of noise that might affect GPR reflected signals when performing GPR surveys along highways are cell-phone towers, cell-phones, CB radios, and any other EM devices emitting in the GPR bandwidth [41]. It should be noted that even though GPR antennas are mainly directed towards the ground, they still interfere with EM devices in their vicinity. For GPR data analysis, the major effect of noise is that when its level increases significantly within the GPR reflected signal (signal-to-noise ratio (SNR) decreases), the probability of detecting reflected pulses decreases, as shown in 4.3.

4.2.2.1 Noise sources

During the previous two years, a considerable amount of GPR data was collected from different locations in Virginia. This data will be used to show the different sources of noise that might corrupt GPR data.

Figure 4-9a and Figure 4-9b show a Linescan view of raw GPR data collected over Interstate I-81 using the air-coupled antenna and ground-coupled antenna, respectively. A Linescan view represents a set of along-track scans stacked together vertically. The amplitude of each scan is quantized and coded into a solid color. Therefore, the x-axis in this type of figure represents the scan number (proportional to the surveyed distance) and the y-axis represents the reflection time that can be converted to depth, if the dielectric properties of each layer are known. The amplitude-to-color transformation function used to obtain Figure 4-9a is given at the left side of the figure. In order to enhance low reflections, a nonlinear transformation function was used, with black representing reflections near zero, bright colors representing positive reflections, and dark colors representing negative reflections.

Although data shown in these figures were collected simultaneously, Figure 4-9a appears to be more noisy than Figure 4-9b. Specifically, Figure 4-9a shows a noisy band between scans 50 and 100 and scattered noise at the bottom, which are not present in Figure 4-9b. Hence, it could be concluded that noise affecting the air-coupled antenna is exterior to the system. This noise does not affect the ground-coupled antenna because it is in full contact with the ground, which acts as a low pass filter that attenuates noise. Furthermore, it was noticed during the
survey that the noisy band in Figure 4-9a resulted from a CB radio used by the traffic-control crew that was helping to divert traffic from the GPR survey vehicle. The presence of many cell-phone towers was also noticed in the area, which confirms the sources of noise observed in Figure 4-9a. On the other hand, Figure 4-10 depicts raw GPR data collected by the air-coupled antenna (using the same GPR settings as for the I-81 data) over section A of the Virginia Smart Road. This figure seems to be noise free when compared to the data presented in Figure 4-9a. This is because the Virginia Smart Road is relatively far from any sources of noise, and it is closed to traffic. Figure 4-10 further proves the origins of EM noise affecting GPR signals.

Figure 4-9: Raw GPR Data Collected over I-81 North, Milepost 139, (a) Air-Coupled Data, (b) Ground-Coupled Data
A common method for reducing the effects of random noise in signals is through the use of filters. Ground penetrating radar systems usually have digital filters implemented by a DSP processor (see Figure 2-4) to filter the collected data in real time. These filters are commonly set on high bandwidths to prevent the attenuation of useful data (for the 1 GHz air-coupled antenna, a boxcar lowpass filter with a cutoff frequency of 8.5GHz is usually used). To further decrease noise effects, narrower digital filters could be used during the data processing phase.

### 4.2.2.2 Filter design

It is well known that the first step in designing a filter entails knowing the frequency content of the signal that will be passed through the filter. The filter is then designed in a way that will pass the frequency content of the desired signal and block the other frequencies. Therefore, a study of the GPR reflected signal is necessary in order to design the adequate filter.

After coupling pulse removal, the GPR reflected signal could be modeled by the following equation, found by removing the coupling pulse from equation (4-1):

\[ y_r(t) = x(t) \star r(t) + n(t) \]  \hspace{1cm} (4-4)
where \( y_r(t) \) is the reflected GPR signal, \( x(t) \) is the transmitted GPR signal, \( \gamma(t) \) is the reflectivity function of the pavement system, and \( n(t) \) is random additive noise. Taking the Fourier transform of equation (4-4), gives its frequency domain equivalent as follows:

\[
Y_r(\omega) = X(\omega)\Gamma(\omega) + N(\omega) \tag{4-5}
\]

where \( Y_r(\omega), X(\omega), \Gamma(\omega), \) and \( N(\omega) \) are, respectively, the Fourier transforms of \( y_r(t), x(t), \gamma(t), \) and \( n(t) \). According to equation (4-5), the factor dominating the spectrum of the reflected signal is the spectrum of the transmitted signal \( X(\omega) \). In fact, \( N(\omega) \) is the unwanted signal to be removed, and the input reflection coefficient \( \Gamma(\omega) \) is a modulating function that will change the shape of \( X(\omega) \) to yield \( Y_r(\omega) \) without adding any spectral components (as shown in 2.3.2.2). Therefore, knowing the frequency content of the transmitted signal is sufficient to design an adequate filter for any reflected signal.

As was mentioned earlier, the GPR transmitted signal could be recovered by placing a sufficiently large (practically \( 0.91 \times 1.22 \text{m} [3 \times 4\text{ft}] \)) copper plate (\( \sigma = 5.7 \times 10^7 \text{ S/m}, \) perfect EM reflector) underneath the antenna. The reflected signal, in this case, would be the reverse of the GPR transmitted signal. Figure 4-11 shows the GPR transmitted pulse, as reflected by the copper plate, after coupling pulse removal and then multiplication by -1 (the reflection coefficient of copper).

Computing the Fourier transform of the transmitted GPR pulse via a fast Fourier transform (FFT) gives its magnitude spectrum, as depicted in Figure 4-12. This figure shows that the -10dB limits of the transmitted energy are located between fractional frequencies 0.0147 and 0.0557 (the fractional frequency is defined here as \( f/f_s \), where \( f \) is the considered frequency and \( f_s \) is the sampling frequency). The signal’s frequency content appears, however, to extend beyond these limits. To ensure that no valuable information is lost during the filtering operation, the frequency content of the transmitted signal will be considered to be between 0 and 0.08 (approximately -30dB below the maximum point). Consequently, the necessary filter should be a lowpass filter with a fractional cutoff frequency of 0.08. This chosen fractional frequency corresponds to a frequency of about 2GHz.
Based on their impulse response, digital filters are divided into two categories: finite impulse response (FIR) filters and infinite impulse response (IIR) filters [42]. FIR filters have an all-zero polynomial transfer function; thus, they are always stable. This structure of the transfer function makes FIR filters act only on the input signal (current sample plus a finite number of past samples) to produce their output. Therefore, FIR filters can be considered in the time domain as weighted moving-average filters. This property makes the impulse response of an
FIR filter finite (i.e., goes to zero after a certain time delay). For a length \( N \) FIR causal filter, if the impulse response is symmetric about the point \((N-1)/2\), the filter would have exactly a linear phase response [42], which produces a constant phase delay in the output. IIR filters, on the other hand, have transfer functions with both zeros and poles; hence, they can be unstable (if poles were chosen outside the unit circle in the \( z \) plane). This structure of the transfer function is usually implemented with a feedback of the output (i.e., an IIR filter can act on samples of the input signal as well as past values of the output signal). Consequently, IIR filters have a theoretically infinite impulse response. For IIR filters, the combination of a pole near the pass-band edge and a zero near the stop-band edge gives them very short transition regions between pass-band and stop-band [42]. This characteristic makes IIR filters more efficient (i.e., lower order) in achieving particular specifications than are FIR filters. For this reason, IIR filters were chosen for noise reduction for GPR data. The main drawback of IIR filters, however, is their nonlinear phase response in the pass-band, which might cause some distortion in the output signal. As will be shown later, this problem could be corrected when filtering data off-line (i.e., the entire input signal is available) by using a forward-backward filtering technique.

Digital filter design techniques are well developed in the literature [42]-[45]. The design procedure is usually based on finding the best transfer function or impulse response function (corresponding to a FIR or IIR filter) that approximates the desired frequency response. Since analog filter design is a well developed field, a very important set of digital filter design methods is based on mapping analog filters into digital filters, using a common transformation, the bilinear transformation, which maps the analog \( s \)-plane into the \( z \)-plane [43]. This design technique consists of designing the filter in the analog domain and then converting it to the digital domain using the bilinear transformation given by the following equation [43]:

\[
s = \frac{2}{T} \left( 1 - z^{-1} \right) \left( 1 + z^{-1} \right)
\]

(4-6)

where \( s \) is the Laplace complex variable and \( T \) is the sampling period. The bilinear transformation given by equation (4-6) maps the \( j\Omega \) axis in the \( s \)-plane (\( \Omega \) is the analog frequency) into the unit circle in the \( z \)-plane. It also maps the left-half plane and the right-half plane of the \( s \)-plane into, respectively, the inside and outside of the unit circle in the \( z \)-plane [43].
Consequently, the stability of the analog filter is preserved during the mapping to the digital filter.

Four standard frequency response approximation functions are commonly used in analog filter design. These functions usually represent low-pass filter prototypes that could be transformed to any other configuration using special frequency transformations [42]. The four analog filter functions are as follows [43]:

- **Butterworth** filters are all-pole filters characterized by a monotonic frequency response in both the pass-band and stop-band, as shown in Figure 4-13a. These filters have a relatively wide transition band between pass-band and stop-band that could be reduced by increasing the order of the filter.

- **Type I Chebyshev** filters are all-pole filters with a monotonic frequency response in the stop-band and an equiripple in the pass-band as depicted in Figure 4-13b. Their transfer function is based on the $N^{th}$ order Chebyshev polynomial. These filters have a narrower transition band between pass-band and stop-band than do Butterworth filters with the same order. The specification of the pass-band ripple and the transition bandwidth are used during the design process to determine the minimal filter order that should be used to satisfy the required specifications. A decrease in the transition bandwidth and/or the pass-band ripple amplitude causes the filter order to increase.

- **Type II Chebyshev** filters have both zeros and poles in the transfer function. They are symmetric to Type I Chebyshev filters in the sense that their frequency response is monotonic in the pass-band and exhibits equiripple in the stop-band, as shown in Figure 4-13c. Type II Chebyshev filters have also narrower transition band than Butterworth filters with the same order. The minimal order to satisfy the required specifications is also found based on the width of the transition band and the stop-band ripple.

- **Elliptic** (or Cauer) filters have both zeros and poles in their transfer function, which is based on the $N^{th}$ order Jacobian elliptic function. Their frequency response is characterized by equiripple in both the pass-band and stop-band, as illustrated in Figure 4-13d. The pass-band ripple, the stop-band ripple, and the transition bandwidth are used during the design process to find the minimal filter order to satisfy the required specifications.
specifications. The main advantage of elliptic filters when compared to the other three filter types is that they would satisfy the required specifications with a minimum order; consequently they are the most efficient filters. In particular, elliptic filters have the minimum transition band than the other types with the same order, as shown in Figure 4-13. On the other hand, the major drawback of elliptic filters is that they exhibit a more nonlinear phase response in the pass-band than do Butterworth or Chebyshev filters, especially at the edge.

![Figure 4-13: Frequency Response of Standard Analog Lowpass Filters: (a) Butterworth, (b) Type I Chebyshev, (c) Type II Chebyshev, (d) Elliptic](image-url)
Table 4-1 shows the minimum filter order and transition bandwidth (fractional frequency) of a set of lowpass filters (Butterworth, Chebyshev I, Chebyshev II, and elliptic) found to meet the following specifications: 0.01dB maximum ripple in the pass-band, stop-band 30dB below the maximum pass-band value, pass-band edge at 0.08 fractional frequency, and stop-band edge at 0.10 fractional frequency. The transition bandwidth between pass-band and stop-band is computed from the actual filter frequency response between the point at which the pass-band ripple exceeds the specified 0.01dB and the point at which the stop-band ripple drops below -30dB.

Table 4-1: Minimum Filter Orders for Different Types of Lowpass Filters for a Given Specification

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>FIR</th>
<th>Butterworth</th>
<th>Chebyshev I</th>
<th>Chebyshev II</th>
<th>Elliptic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. Order</td>
<td>126</td>
<td>28</td>
<td>11</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>Transition Bandwidth</td>
<td>0.019</td>
<td>0.020</td>
<td>0.016</td>
<td>0.016</td>
<td>0.015</td>
</tr>
</tbody>
</table>

According to Table 4-1, the elliptic filter is the most efficient filter based on its order (almost half the order of a Chebyshev filter, one-fifth the order of a Butterworth filter, and one-twentieth the order of a FIR filter) and its transition bandwidth. For this reason, elliptic filters were chosen to filter GPR data in the preprocessing stage. The problem of the elliptic filter nonlinear phase in the pass-band could be solved by forward-backward filtering, as will be shown in the following sections.

The frequency response of an elliptic lowpass filter that could be used for GPR data noise reduction is presented in Figure 4-14. This filter was designed with the following specifications: 0.08 fractional cutoff frequency, 0.01dB ripple in the pass-band, and a stop-band 40dB below the maximum pass-band value. The minimum elliptic filter order needed to meet these specifications was found to be 7. Based on the amplitude spectrum presented in Figure 4-14a, the designed filter highly approximates an ideal lowpass filter. However, as shown in Figure 4-14b, its phase response is nonlinear in the pass-band, especially at the band edge. This nonlinearity results in a frequency dependent group delay (defined by: \(-d\theta/d\omega\)), which would cause undesirable distortions in the output signal. Figure 4-15 shows a comparison between a raw GPR scan and its filtered version obtained by direct filtering (i.e., by convolution or Fourier
Transform) using the above elliptic filter. Even though the filtered version shows a significant decrease in the noise level, it also undergoes some distortions because of the nonlinear phase of the filter. To avoid this problem, a forward-backward filtering procedure that results in a zero-phase delay could be used.

Figure 4-14: Frequency Response of Designed Lowpass, Order 7 Elliptic Filter: (a) Amplitude Spectrum, (b) Phase Spectrum
4.2.2.3 Forward-backward filtering

Forward-backward filtering is mainly used for the implementation of noncausal filters and zero-phase filtering using IIR filters [46]. The forward-backward filtering procedure is summarized by the block diagram depicted in Figure 4-16, where $X(z)$ and $H(z)$ are the $z$-transforms of the input sequence $x_n$ and the filter impulse response $h_n$, respectively. As indicated by its name, forward-backward filtering consists of two stages. In the first stage, the forward stage, the filter $H(z)$ is applied to the input sequence $x_n$. In the second stage, the backward stage, the filter $H(z)$ is applied to the time-reversed output sequence of the forward stage. Finally, the output of the backward stage is in turn time reversed to yield the output of the overall system. Based on this filtering scheme, forward-backward filtering can only be used when the entire input signal is available (i.e. off-line processing).

![Forward-Backward Filtering Block Diagram](image)
Using the time reversal property of the $z$-transform, which states that if $x_n$ has a $z$-transform $X(z)$ then $x_{-n}$ has a $z$-transform $X(1/z)$, the $z$-transform of the output of the overall system represented by the block diagram of Figure 4-16 would be as follows:

$$Y(z) = X(z)H(1/z)H(z)$$  \hspace{1cm} (4-7)

The Fourier transform of the overall system output is obtained by evaluating equation (4-7) over the unit circle (i.e. $z = e^{j\omega}$) as follows:

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{-j\omega})H(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})|H(e^{j\omega})|^2$$  \hspace{1cm} (4-8)

Consequently, the system presented in Figure 4-16 is equivalent to a filter with frequency response given by $|H(e^{j\omega})|^2$, which has exactly zero phase. The output signal will then have the same phase as the input signal, but its frequency content will be limited by $|H(e^{j\omega})|^2$. This result is illustrated in Figure 4-17, which depicts a comparison between a raw GPR scan and its filtered version obtained by forward-backward filtering. It is evident in this figure that the filtered signal has the same shape (no distortion) and phase (no phase delay) as the raw data; moreover, it has much less noise.

![Figure 4-17: Comparison between Original Signal and Elliptic Filter Output Obtained by Forward-Backward Filtering](image)
Passing the raw GPR data collected by the air-coupled antenna from I-81 North (depicted in Figure 4-9a) through the designed lowpass elliptic filter results in the data presented in Figure 4-18. A comparison between both figures shows the effectiveness of the filter in removing the external noise while keeping the integrity of the GPR reflected signal. In fact, Figure 4-18 shows that the noisy band between scans 50 and 100 was eliminated along with the scattered noise in the bottom half of the figure. At the same time, the deep, low amplitude reflections in the middle of the figure were preserved.

Figure 4-18: Filtered Air-Coupled GPR Data Collected from I-81 North, Milepost 139

4.2.3 Depth resolution enhancement

4.2.3.1 Importance of high depth resolution for GPR data analysis

According to equation (2-31), the depth resolution of an impulse GPR system depends on the scanned medium (by its dielectric constant) and the GPR system used (by the transmitted pulse width). Figure 4-19 depicts the depth resolution, computed according to equation (2-31), for different pulse widths and different pavement construction materials. The pulse widths used in the computation ($T=0.6$, $1.0$, and $1.1$ ns) are commonly utilized for GPR surveys over pavements. They correspond to GPR signal bandwidths of $1.5$, $1.0$, and $0.9$ GHz, respectively. Figure 4-19 shows that an impulse GPR system achieves better depth resolution with narrow pulses than with wide pulses. However, because narrow pulses have higher frequency content than wide pulses, they undergo higher attenuation while propagating through the pavement.
layers due to material loss. In other words, narrow pulses have shallower depths of penetration than do wide pulses, assuming that the transmitted EM energy is the same in both cases. Hence, for a GPR system with fixed transmitted EM energy, there is always a trade-off between depth of penetration and depth resolution.

If the thickness of a pavement layer is smaller than or equal to the depth resolution of the GPR system used to scan the pavement structure, the reflected pulses from the top and bottom interfaces of the considered layer will overlap in the time-domain signal. In this case, distinction between the individual interface reflections in this case would depend mainly on the following:

- The difference between the layer thickness and the depth resolution of the GPR system used,
- The contrast between the reflection amplitudes at both layer interfaces.

This result is illustrated in Figure 4-20a, which depicts a GPR scan collected by the air-coupled antenna over section A at the Virginia Smart Road. As shown in this figure, there are minor distortions at the trailing edge of the surface reflection and the leading edge of the HMA/base reflection. These distortions are not due to the incident pulse, which is depicted in

![Figure 4-19: GPR Depth Resolution for Different Pulse widths and Different Pavement Materials](image-url)
the top right corner of Figure 4-20a. Moreover, since there are no similar distortions in the deeper base/subgrade reflection located between 12 and 14ns, it could be concluded that the distortions in the shallower reflections are not due to dispersion in the pavement layers.

![Diagram](image)

*Figure 4-20: GPR Scans Showing Effect of Thin Layers on GPR Reflected Signal: (a) Raw GPR Scan, (b) Decomposition of GPR Scan into Individual Reflections*
As described in 3.1, the HMA layers in section A include a 38mm wearing surface (WS), a 150mm HMA base layer (BM-25.0), and a 75mm OGDL layer. Even though these three layers have similar components (asphalt binder, aggregates, and air voids), according to the dielectric mixture theories of heterogeneous materials [47], they would have different dielectric constants since the mix proportions are different from one layer to the other:

- The WS layer has the highest asphalt content and the highest density (less air voids),
- The BM-25.0 layer is composed of larger aggregates and has less asphalt binder than the wearing surface,
- The OGDL layer is composed of large uniform aggregates (more air voids) and has the lowest asphalt content.

Because of the dielectric constant contrast between the three HMA layers in section A, GPR reflections are expected at their individual interfaces. Figure 4-20b shows a decomposition of the raw reflected signal of Figure 4-20a into the individual reflections at all the layer interfaces. The two curves plotted in Figure 4-20b were obtained by changing the dielectric constants of all the layers, knowing their respective thicknesses from the pavement design, until a minimum root mean square error (RMSE) between the curve of Figure 4-20a and the sum of the two curves of Figure 4-20b was achieved. Since the thicknesses of the WS and OGDL layers fall within the depth resolution of a 1ns impulse GPR system (see Figure 4-19), the reflections at these layer interfaces are overlapped. For the wearing surface layer, the high-amplitude surface reflection overlaps and masks the subsequent reflection at the WS/BM-25.0 interface. For the OGDL layer, the high-amplitude reflection from the OGDL/Base interface masks the preceding reflection at the BM-25.0/OGDL interface. Additionally, it is noticed in Figure 4-20a that the WS/BM-25.0 interface reflection is more noticeable than the BM-25.0/OGDL interface reflection. As illustrated in Figure 4-20b, this is due to the fact that the amplitude of the reflection at the WS/BM-25.0 interface is higher than that at the BM-25.0/OGDL interface, which undergoes more attenuation while propagating deeper into the pavement system. For both layer interfaces, however, it is difficult to accurately locate the exact positions of the reflections in the raw GPR signal shown in Figure 4-20a. In fact, a comparison between Figure 4-20a and Figure 4-20b shows a difference between the probable and actual locations of the reflections.
The effect of poor GPR depth resolution is more obvious in Figure 4-21, which depicts raw air-coupled GPR data collected from section A at the Virginia Smart Road. For a clearer view of the plotted GPR data, negative reflections were masked. This figure shows the reflections from normal layer interfaces as well as reflections from the copper plates located between the different layers (i.e., WS/BM-25.0, BM-25.0/OGDL, OGDL/21A, 21A/21B, and 21B/Subgrade). According to this figure, the only layer interface reflection that corresponds to a copper plate reflection is the one at the OGDL/21A interface (reflection 3 in the figure). Because of the low dielectric constant contrast between the aggregate materials that comprise the base layers, the reflections from the 21A/21B and 21B/subgrade interfaces are only visible when there is a copper plate at the interface (reflections 4 and 5 in the figure). For the HMA layers, the reflections from the copper plates at the WS/BM-25.0 and BM-25.0/OGDL interfaces are seen in Figure 4-21 (reflections 1 and 2), respectively, above and below the continuous reflections visible within the HMA layer. Since these continuous reflections are not aligned with the copper plate reflections, it could be concluded that they are spurious and, therefore, do not correspond to real layer interfaces. The spurious reflections are attributed to the distortions of the reflections at the pavement surface and the OGDL/21A interface. These distortions are due to the poor GPR resolution compared to the thin WS and OGDL layers.

Figure 4-21: Air-Coupled GPR Data over Section A Showing Copper Plate Reflections at the: (1) WS/BM-25.0, (2) BM-25.0/OGDL, (3) OGDL/21A, (4) 21A/21B, and (5) 21B/Subgrade Interfaces
If some of the overlapped reflected pulses in the time domain GPR signal are missed during data analysis, it could lead to incorrect reports regarding the number of layers in the pavement system as well as incorrect estimates of the layer thicknesses. The first type of thickness error that results from considering multiple layers with different dielectric properties as a single homogeneous layer is caused by using an incorrect dielectric constant (usually that of the top layer) in the computation of the thickness of the other layers. According to equation (2-30), it could be shown, that for an error $\Delta \varepsilon_r$ in the dielectric constant of a homogeneous layer with an actual dielectric constant $\varepsilon_r$, the estimated layer thickness would be the following:

$$d + \Delta d = \frac{c\Delta t}{2\sqrt{\varepsilon_r + \Delta \varepsilon_r}}$$  \hspace{1cm} (4-9)$$

Therefore, the thickness error $\Delta d$ due to $\Delta \varepsilon_r$ would be as follows:

$$\Delta d = \frac{c\Delta t}{2\sqrt{\varepsilon_r + \Delta \varepsilon_r}} - \frac{c\Delta t}{2\sqrt{\varepsilon_r}} = \frac{c\Delta t}{2\sqrt{\varepsilon_r}} \left[1 + \left(\frac{\Delta \varepsilon_r}{\varepsilon_r}\right)^{-1/2}\right] - 1$$  \hspace{1cm} (4-10)$$

From which the relative thickness error $\Delta d/d$ could be found as follows:

$$\frac{\Delta d}{d} = \left[1 + \left(\frac{\Delta \varepsilon_r}{\varepsilon_r}\right)^{-1/2}\right] - 1$$  \hspace{1cm} (4-11)$$

For small values of $\Delta \varepsilon_r/\varepsilon_r$ ($\Delta \varepsilon_r/\varepsilon_r << 1$), the quantity $(1+\Delta \varepsilon_r/\varepsilon_r)^{-1/2}$ could be approximated by a first order Taylor series expansion about 0: $(1-\Delta \varepsilon_r/2\varepsilon_r)$. Hence, the relative layer thickness error $\Delta d/d$ would be approximated by the following:

$$\frac{\Delta d}{d} \approx -\frac{\Delta \varepsilon_r}{2\varepsilon_r}$$  \hspace{1cm} (4-12)$$

The minus sign in this equation indicates that the variations of the thickness error are the inverse of the variations of the dielectric constant error (i.e., when the dielectric constant increases, the thickness decreases and vice versa). It is clear from equation (4-12) that for small values of $\varepsilon_r$ (for example for HMA where $\varepsilon_r$ varies from 3 to 6), a small error in the estimation of
the dielectric constant gives a large error in the layer thickness. This result is illustrated in Figure 4-22, which depicts the relative thickness error versus the dielectric constant error for different values of $\varepsilon_r$. It should be noted in Figure 4-22 that when the dielectric constant error becomes large in absolute value, the relative thickness error deviates from what is given by equation (4-12). This is because Figure 4-22 is plotted for the exact expression of the relative error given in equation (4-11), whereas equation (4-12) is truncated to two terms of the Taylor series.

![Figure 4-22: Relative Layer Thickness Error versus Error in Dielectric Constant Estimation for Different Values of $\varepsilon_r$](image)

The second type of thickness error that results from considering multiple layers with different dielectric properties as a single homogeneous layer is due to the estimation of each layer’s dielectric constant from the reflected time domain GPR signal. In fact, when the dielectric constants of the HMA and base layers are estimated using equations (2-32) and (2-33), respectively, an error in the reflection amplitude would affect the computed dielectric constant values. Figure 4-23 shows the relative dielectric constant error of the pavement top layer as a function of the surface reflection amplitude error for different values of the dielectric constant.
According to this figure, an error of ±1000 in the amplitude produces a relative dielectric constant error of approximately ±20%. This amplitude error is comparable to the one caused by pulse overlapping as shown through a comparison of the amplitude of the overlapped surface reflection of Figure 4-20a (around 9000) to the amplitude of the actual surface reflection of Figure 4-20b (around 10000). The same result is noticed for the overlapped HMA/base interface reflection of Figure 4-20a (amplitude around 3500) and the actual HMA/base interface reflection of Figure 4-20b (amplitude around 3000). In both cases, the amplitude errors are due to the summation of signals of opposite amplitudes within the overlapped pulses. According to equation (4-12), a dielectric constant error of ±20% would produce an approximate relative thickness error of ±10%.

![Figure 4-23: Relative Dielectric Constant Error versus Reflection Amplitude Error for Different Values of εr](image)

In summary, the previous discussion shows the importance of GPR depth resolution for a correct and reliable analysis of GPR data for pavement thickness evaluation. Since the depth resolution of an impulse GPR system for any given pavement material is limited by the transmitted pulse width, which in turn is fixed by hardware, it is imperative to enhance the depth resolution of the collected signal by software during the data analysis phase.
4.2.3.2 Ideal depth resolution enhancement

As given by equation (4-4), the GPR reflected signal can be represented in the time domain as a convolution of the GPR transmitted signal \( x(t) \) and the reflectivity function of the pavement system \( \gamma(t) \). The reflectivity function \( \gamma(t) \) is obtained by taking the inverse Fourier transform of the input reflection function \( \Gamma_{in}(\omega) \), given by equation (2-29), as follows:

\[
\gamma(t) = F^{-1}\{\Gamma_{in}(\omega)\} = \sum_{n=0}^{N-1} \frac{\sqrt{\varepsilon_{r,n} - \sqrt{\varepsilon_{r,n+1}}} e^{-\eta_d \sum_{i=0}^{N-1} \sqrt{\varepsilon_{r,i}}} \delta(t - \frac{2}{n} \sum_{c=0}^{N-1} \varepsilon_{r,c} d_i)}{\sqrt{\varepsilon_{r,n} + \sqrt{\varepsilon_{r,n+1}}}}
\]

(4-13)

where \( F^{-1}\{\} \) is the inverse Fourier transform and \( \delta(t) \) is the Dirac delta function. Therefore, under the assumptions presented in 2.3.2.2, the reflectivity function of the pavement could be ideally represented by a train of time-delayed spikes that correspond to the interface locations. Accordingly, enhancement of the GPR depth resolution could be accomplished by extracting the reflectivity function from the reflected signal. This result could be achieved by a deconvolution operation.

Ideally, if \( q(t) \) represents the deconvolution filter that when applied to the reflected signal \( y(t) \) would give the reflectivity function \( \gamma(t) \):

\[
y(t) * q(t) = y(t) * x(t) * q(t) = \gamma(t)
\]

(4-14)

then \( q(t) \) should satisfy the following equation:

\[
x(t) * q(t) = \delta(t)
\]

(4-15)

In other words, \( q(t) \) should be the inverse of signal \( x(t) \).

Knowing the incident signal \( x(t) \), equation (4-15) could be solved for the deconvolution filter \( q(t) \) either directly in the time domain or in the frequency domain using a Fourier transform. However, since the deconvolution operation involves measured discrete signals that might be corrupted by noise, this direct solution is usually unstable \([48]\). Hence, alternative deconvolution methods should be investigated.
Most of the deconvolution techniques that will be presented in the following sections have been used to enhance the depth resolution of seismic data collected during geophysical surveys. These techniques will be tried on GPR signals because of their similarity to seismic signals (seismograms). In fact, like GPR data, seismograms are composed of signals resulting from the reflection of an incident wave at the different subsurface sedimentary layer interfaces. However, instead of EM waves, the incident signals used during geophysical surveys are usually produced by dynamite explosions or by air-gun shots at the surface, depending on the type of the survey. The reflected signals are then recorded with one or more seismic sensors, such as geophones, placed at the surface. Because of the long wavelength of the incident signals used in geophysical surveys and the random nature of the structure of the earth, seismograms typically suffer from a poor depth resolution in addition to corruption by multiple reflections (or reverberations) caused by energy entrapment between strong reflecting interfaces. For this reason, different deconvolution techniques have been developed to reduce the effects of these inherent anomalies in seismograms.

4.2.3.3 Inverse filtering

As indicated by equation (4-15), a suitable filter that could be used to deconvolve GPR data is the inverse of the incident signal \( x(t) \). The inverse filter could be designed in the form of a Wiener filter [43], which is designed, in general, to minimize the difference between a desired output and the actual filter output in the least-squares sense. The detailed derivation of Wiener filters is presented in Appendix B. Using the notations of Figure B−1 for Wiener filter design, the desired output signal, in this case, would be \( d(t) = \delta(t) \). Hence, the cross-correlation function between the desired signal \( d(t) \) and the input signal \( x(t) \), assumed causal, would be given according to equation (B−6), Appendix B:

\[
r_{dx}(i) = \sum_{t} \delta(t)x(t-i) = \begin{cases} x(0) & \text{for } i = 0 \\ 0 & \text{for } i = 1, \ldots, N-1 \end{cases}
\]

(4-16)

Substituting \( r_{dx}(i) \) into equation (B−9), Appendix B, gives the matrix relation of the Wiener inverse filter of length \( N \) as follows:
The only unknown in equation (4-17) is the autocorrelation function $r_{xx}(l)$ of the incident signal $x(t)$. If the incident signal $x(t)$ is known, its autocorrelation function would be easily computed using the biased autocorrelation estimate of equation (B-11), Appendix B. The designed inverse filter would be, in this case, a spiking filter (i.e., a filter that transforms a known signal into a spike). Detailed analysis of spiking filters is presented in 4.2.3.5.

Even when the incident signal $x(t)$ is unknown, an estimate of its autocorrelation function could be obtained. From equation (B-11), it is noted that the autocorrelation function of the reflected signal $y(t)$ could be put in a convolution form as follows:

$$\gamma = [(4-17)$$]

Substituting $y(l)$ by its expression presented in equation (4-4) (assuming a noise free signal) yields:

$$r_{yy}(l) = y(l) * y(-l)$$  \hspace{1cm} (4-18)

Finally, using the associative and commutative properties of convolution, the autocorrelation function of the reflected signal $y(l)$ would be given by:

$$r_{yy}(l) = [\gamma(l) * y(-l)] = r_{xx}(l) * \gamma(l)$$  \hspace{1cm} (4-19)

Thus, the last equation shows that the autocorrelation function of the reflected signal $y(t)$ is the convolution of the autocorrelation function of the incident signal $x(t)$ and the reflectivity function $\gamma(t)$. Assuming that the reflectivity function $\gamma(t)$ is composed of small amplitude uncorrelated samples (fact observed in the reflectivity function given by equation (4-13)); its autocorrelation function would be approximated by a spike at lag zero as follows:
\[ r_{yy}(l) = E_{\gamma} \delta(l) \]  

where \( E_{\gamma} \) is the energy in \( \gamma(t) \). Substituting equation (4-21) in (4-20) gives the following expression of the autocorrelation function of the reflected signal:

\[ r_{yy}(l) = E_{\gamma} r_{xx}(l) \]  

(4-22)

Hence, it is shown that except for the proportionality factor \( E_{\gamma} \), the autocorrelation function of the incident signal could be approximated by that of the reflected signal, which can then be used in the design of the Wiener inverse filter according to equation (4-17). It should be noted that the factors \( x(0) \) in (4-17) and \( E_{\gamma} \) in (4-22) could be factored out to yield a scaled version of the matrix equation of the designed inverse Wiener filter of length \( N \):

\[
\begin{bmatrix}
    r_{yy}(0) & r_{yy}(1) & r_{yy}(2) & \cdots & r_{yy}(N-1) \\
    r_{yy}(1) & r_{yy}(0) & r_{yy}(1) & \cdots & r_{yy}(N-2) \\
    r_{yy}(2) & r_{yy}(0) & r_{yy}(1) & \cdots & r_{yy}(N-3) \\
    \vdots & \ddots & \ddots & \ddots & \vdots \\
    r_{yy}(N-1) & \cdots & \cdots & r_{yy}(0)
\end{bmatrix}
\begin{bmatrix}
    a_0 \\
    a_1 \\
    a_2 \\
    \vdots \\
    a_{N-1}
\end{bmatrix}
= \begin{bmatrix}
    1 \\
    0 \\
    0 \\
    \vdots \\
    0
\end{bmatrix}
\]  

(4-23)

The outputs of the filters of equation (4-17) and that of (4-23) would have the same shape but different amplitudes. Thus, this difference would not affect the deconvolution results.

Figure 4-24 shows the output of an inverse filter of length 50 having as input the GPR scan presented in Figure 4-20a. Comparison between the two figures shows that, after filtering, the reflected pulses were compressed in time by at least 50% (original pulse-width is around 2ns, whereas the deconvolved pulse-width is around 1ns). As depicted in Figure 4-24, this compression allowed the distinction between the surface reflection and the wearing surface/BM-25.0 reflection, originally overlapped in the raw scan. The BM-25.0/OGDL reflection, however, is still difficult to localize accurately. The other two reflections that were originally visible in the raw GPR scan (OGDL/base and base/subgrade reflections) are still detectable in the processed scan. Moreover, Figure 4-24 illustrates that the noise level in the filtered signal increased with respect to the signal level. In other words, inverse filtering caused a reduction in the SNR of the processed GPR scan. This reduction might affect the performance of the subsequent pulse.
detection stage by increasing the number of false alarms (i.e., noise peaks mistakenly detected as reflected pulses).

The only design parameter that could be used to improve the performance of the inverse filter is its length $N$. Figure 4-25 shows the output of inverse filters having as input the signal presented in Figure 4-20a and the respective lengths of 2, 50, and 150.

A comparison between the three signals shown in Figure 4-25 illustrates the effect of filter length on the performance of inverse filters when utilized for deconvolution of GPR signals. According to this figure, the filter length mainly controls the pulse-width of the deconvolved signal as well as its SNR. Increasing the length of the inverse filter has two antagonist effects on its performance: as the filter length increases, the SNR decreases (fact observed, for example, for the part of the signal between 10 and 13ns, which is composed of noise only) and the widths of the deconvolved pulses also decrease. The SNR decrease is primarily caused by the addition of a processing noise resulting from the filtering operation.

In order to find the optimal inverse filter length that could be used effectively for GPR data deconvolution, measurable criteria about the deconvolved signal should be minimized versus the filter length. For the minimization purposes, the following easy-to-estimate criteria were chosen:
Figure 4-25: Deconvolved GPR Signals Obtained by Inverse Filters of Length $N = 2$, 50, and 150

- Signal-to-noise ratio reduction (in dB) between original signal and deconvolved signal, defined by the following:

$$SNR_{red} = SNR_{orig} - SNR_{dec}$$  \hspace{1cm} (4-24)

where $SNR_{orig}$ and $SNR_{dec}$ are the signal to noise ratios (in dB) of the original signal and that of the deconvolved signal, respectively. From this definition, a higher filter performance is equivalent to a lower SNR ratio reduction.

- Pulse compression ratio, defined as follows:

$$PCR = \frac{N_{orig} - N_{dec}}{N_{orig}}$$  \hspace{1cm} (4-25)

where $N_{orig}$ is the width of a reflected pulse in the original GPR signal and $N_{dec}$ is the width of the same pulse after deconvolution. According to this definition, a larger pulse compression ratio leads to a higher filter performance.

- Square error ratio, defined by the following:
The square error ratio is defined as:

$$SER = \frac{\sum_{t=0}^{M-1} (y(t) - y_f(t))^2}{\sum_{t=0}^{M-1} y(t)^2}$$  \hspace{1cm} (4-26)$$

where \(y(t)\) is the measured GPR signal, \(M\) is its length, and \(y_f(t)\) is a synthesized signal obtained by fitting the detected pulses in the deconvolved signal to a theoretical time domain GPR reflection model. Ground penetrating radar signal fitting to a theoretical reflection model is discussed in Appendix C. The square error ratio is introduced to ensure that the detected pulses in the deconvolved signal correspond to actual reflections in the measured GPR signal. According to this definition, the smaller the square error ratio (or equivalently \(y(t) - y_f(t)\) tends towards zero), the higher the similarity between the measured GPR signal and the synthesized signal. The high similarity between the signals translates, in turn, to a more accurate localization of the reflected pulses.

Joint minimization of the \(SNR_{red}\), the \(SER\), and maximization of the \(PCR\) are difficult to achieve theoretically since the designed inverse filter does not have a closed form. Consequently, numerical minimization/maximization on experimental GPR data is necessary to find the optimal inverse filter length that ensures effective deconvolution of GPR data.

Practically, to evaluate the \(SNR_{red}\), estimates of the SNR of the reflected pulse (\(SNR_{orig}\)) and that of the deconvolved pulse (\(SNR_{dec}\)), used in equation (4-24), could be computed according to the following:

$$SNR = 10\log \frac{E_p}{E_n}$$  \hspace{1cm} (4-27)$$

where \(E_p\) is the average power contained in the reflected (or deconvolved) pulse \(y_p(t)\) of length \(N_p\). The average power \(E_p\) is estimated as follows:

$$E_p = \frac{\sum_{t=0}^{N_p-1} y_p(t)^2}{N_p}$$  \hspace{1cm} (4-28)$$
The factor $E_n$ is the noise power estimated from a portion of the reflected (or deconvolved) signal that is assumed to be composed of noise only (i.e., does not contain any reflected pulses). Practically, the trailing portion of the reflected (or deconvolved) signal (between 16 and 20ns in this case) is usually composed of noise only; therefore, the noise power $E_n$ could be estimated over that portion. As for the signal power, the noise power could be estimated according to the following:

$$E_n = \frac{\sum_{t=0}^{N_n-1} n(t)^2}{N_n}$$

(4-29)

where $n(t)$ is the noise signal extracted from either the reflected signal or the deconvolved signal (depending on the evaluated signal to noise ratio $SNR_{orig}$ or $SNR_{dec}$) and $N_n$ is its length.

Experimental minimization/maximization of the criteria presented above were conducted on a large set of GPR scans collected over different periods of time from the different flexible sections of the Virginia Smart Road. For this analysis, inverse filters of varying lengths were applied to each scan of the considered GPR data, and the $SNR_{red}$, $SER$, and $PCR$ were computed based on the deconvolved signal after detection of all the reflected pulses. A detailed analysis of pulse detection in the GPR signal is presented in 4.3. For each filter length, the $SNR_{red}$, $SER$, and $PCR$ results found for all the processed scans were averaged to yield single estimates of the measured quantities for each filter length. Figure 4-26 depicts the average estimated $SNR_{red}$, $SER$, and $PCR$ found for inverse filters of length $N$, varying from 1 to 200. In this figure, the $SNR_{red}$ and $PCR$ were evaluated for the surface reflection because it is the most accurate to localize. On the other hand, the $SER$ was evaluated using all the detected pulses.

According to Figure 4-26, the optimal inverse filter length $N_{opt}$ that ensures the simultaneous minimization of $SNR_{red}$, $SER$, and maximization of $PCR$ is in the interval 30 to 60 points, with a best performance achieved at $N_{opt}$ equal 60. Beyond the 60-points limit, the $SNR_{red}$ increases steadily from 4 to 12dB and the $SER$ increases from 4 to 12%. The $PCR$, on the other hand, decreases from 55% to 40%. For filter lengths less than 30, the $SNR_{red}$ and $SER$ fluctuate slightly around 5dB and 5%, respectively, but the $PCR$ decreases largely, especially for filter
lengths less than 10. It should be noted that the 60-points limit corresponds approximately to the length of the incident pulse.

Figure 4-26: Average Estimated $SNR_{red}$, SER, and PCR vs. Inverse Filter Length

As observed in Figure 4-26, the variation of the performance of the inverse filter (used for GPR data deconvolution) versus its length could be explained by the principal of operation of this type of filters. As indicated by equation (4-23), the developed inverse filter essentially transforms the reflected GPR signal into a spike at lag zero, based on its autocorrelation function, which theoretically approximates that of the incident signal (see equation (4-22)). Since for small filter lengths the inverse filter would be designed based on a fraction of the autocorrelation function of the reflected signal, the filter would not correctly transform the reflected pulses into the desired spike shape. This would lead to minor compression of the original reflected pulses (small $PCR$) and would not affect the noise level (small $SNR_{red}$). As the filter length $N$ increases, the inverse filter would be designed using larger lags of the autocorrelation function of the reflected signal. This would better approximate the autocorrelation function of the incident signal. Hence, the pulse compression ratio $PCR$ would increase. However, because the spiking operation would add high frequency components to the inverse filter output, the noise level of the deconvolved signal would also increase (larger $SNR_{red}$). When the filter length is chosen equal to the length of the incident signal, the inverse filter would be designed based on an autocorrelation function that is approximately identical to that of the incident signal. Therefore,
the reflected pulses in the raw GPR signal would be ideally converted into spikes at zero lags (relative to the original positions of the reflected pulses). Beyond this length, the autocorrelation function of the reflected signal would have extra components that do not exist in the autocorrelation function of the incident signal. This would lead to an inverse filter with a lower pulse compression ratio. The noise level would further increase, in this case, because of the addition of high frequency components caused by the spiking operation. It should be noted that the $SER$ variations versus the inverse filter length are expected to follow the same trend as the $SNR_{red}$, as shown in Figure 4-26. In fact, the $SER$ was introduced to ensure that the detected pulses in the deconvolved signal correspond to actual reflections in the measured GPR signal. Since, as is presented in 4.3, a decrease in the SNR of the processed signal (or equivalently an increase in the $SNR_{red}$ of the deconvolved signal in this case) would degrade the performance of the pulse detector, the $SER$ would increase.

Frequency analysis of the inverse filtering operation could also be used to explain the performance of the inverse filter used for deconvolution of GPR data. Figure 4-27 depicts the magnitude spectra of the GPR reflected signal, the corresponding inverse filter, and the deconvolved signal for different filter lengths ($N = 2, 60, \text{ and } 200$). For a small filter length ($N = 2$), a comparison between the magnitude spectra of the original reflected signal and that of the corresponding inverse filter (Figure 4-27a) shows that even though the inverse filter is rich in high frequency components, it fails to approximate the inverse of the reflected signal. Consequently, the output of such a filter (Figure 4-27b) would have almost the same shape as its input. As shown in Figure 4-27c, if the inverse filter length is equal to the incident signal length ($N = 60$), the shape of the magnitude spectrum of the inverse filter would be approximately identical to the inverse of the magnitude spectrum of the reflected signal, except for some oscillations that are due to the reflectivity function of the pavement $\gamma(t)$. In fact, a comparison between Figure 4-27c and Figure 4-12 shows that the magnitude spectrum of the inverse filter, in this case, is identical to the inverse of the magnitude spectrum of the incident signal. Therefore, as presented in Figure 4-27d, application of this filter to the reflected signal would remove the effect of the incident signal and leave only the reflectivity function. Finally, for a large filter length ($N = 200$), the magnitude spectrum of the inverse filter would be the same as the inverse of the magnitude spectrum of the reflected signal, as shown in Figure 4-27e. The inverse filter
would then severely flatten the spectrum of the reflected signal, as shown in Figure 4-27f. Hence, the performance of the filter would degrade.

![Magnitude Spectra for Different Filter Lengths](image)

Figure 4-27: Magnitude Spectra for Different Filter Lengths: (a), (c), and (e): Reflected Signal and Inverse Filter; (b), (d), and (f): Deconvolved Signal
Figure 4-28 presents the deconvolved GPR signal, corresponding to the signal depicted in Figure 4-20a. The deconvolution operation was conducted using an inverse filter of a length $N_{opt}$ equals 60. The layer interface assignments were based on the actual interface positions of Figure 4-20b. As seen in Figure 4-28, all reflected pulses, originally masked in the raw data, became visible after deconvolution. The amplitudes of the deconvolved reflections are all above noise level except for the BM-25.0/OGDL reflection, which originally had low amplitude before deconvolution. Therefore, this reflection would likely be missed in the pulse detection stage.

![Deconvolved GPR Signal](image)

**Figure 4-28: Deconvolved GPR Signal Obtained by an Inverse Filter of Optimal Length $N_{opt} = 60$**

Figure 4-29 shows the deconvolved version of the raw air-coupled GPR data collected over section A at the Virginia Smart Road, originally depicted in Figure 4-21. The deconvolution operation was achieved by an inverse filter of length $N = N_{opt} = 60$ points. This figure shows a continuous reflection at the level of the reflection from the copper plate underneath the WS (reflection 1 in the figure). Therefore, it could be concluded that this continuous reflection corresponds to the WS/BM-25.0 interface originally masked in Figure 4-21. A second continuous reflection is detected in Figure 4-29 slightly above the reflection from the copper plate underneath the BM-25.0 layer (reflection 2 in the figure). This reflection represents a better approximation to the BM-25.0/OGDL interface than does the original reflection shown in
the raw data of Figure 4-21. Thus, Figure 4-29 proves the effectiveness of inverse filtering for the deconvolution of GPR data.

Figure 4-29: Deconvolved GPR Data Obtained by an Inverse Filter of Optimal Length $N_{opt} = 60$, Showing Copper Plate Reflections at the: (1) WS/BM-25.0, (2) BM-25.0/OGDL, (3) OGDL/21A, (4) 21A/21B, and (5) 21B/Subgrade Interfaces

In summary, this analysis showed the effectiveness of inverse filtering for GPR data deconvolution. The inverse filter length should be chosen, in this case, anywhere between the half-width to the entire-width of the incident signal, which is approximately equal to the width of a reflected pulse. Within this range of suitable inverse filter lengths, high values would produce better results, and low values would allow faster processing.

4.2.3.4 Predictive deconvolution

Predictive deconvolution is commonly used to deconvolve seismograms collected during geophysical surveys [49]-[51]. This technique was found to be especially useful for the elimination of multiple reflections typically present in seismograms. The major advantage of predictive deconvolution over other deconvolution techniques is that it does not require the knowledge of the incident signal $x(t)$. Instead, as for inverse filtering, predictive deconvolution uses the measured reflected signal $y(t)$ to estimate the reflectivity function $\gamma(t)$. 
As inferred by its name, predictive deconvolution is based on a type of filters known as prediction filters [52]. A prediction filter is a filter that uses the values of the input signal up to time sample $t$ to predict its future value at time sample $t+\alpha$, where $\alpha$ is a positive integer known as the prediction distance. Typically, a prediction filter is designed in the form of a Wiener filter. A detailed derivation of general Wiener filters is presented in Appendix B.

To design a prediction filter for the reflected signal $y(t)$, the desired output signal (see Figure B−1) should be taken as the input signal advanced by $\alpha$ samples. That is, using the notations of Appendix B and replacing $x(t)$ by $y(t)$, the desired signal would be $d(t) = y(t+\alpha)$. Thus, the cross-correlation between desired output and input signal would be according to equation (B−6), Appendix B:

$$r_{dy}(i) = \sum_t d(t)y(t-i) = \sum_t y(t+\alpha)y(t-i) = \sum_t y(t)y(t-(i+\alpha)) = r_{yy}(i+\alpha) \quad (4-30)$$

The matrix relation for a prediction filter (for signal $y(t)$) of length-$N$ and prediction-distance $\alpha$ is then obtained by substituting the result of equation (4-30) into equation (B−9), Appendix B, and replacing $x(t)$ by $y(t)$ as follows:

$$\begin{bmatrix}
  r_{yy}(0) & r_{yy}(1) & r_{yy}(2) & \cdots & r_{yy}(N-1) \\
  r_{yy}(1) & r_{yy}(0) & r_{yy}(2) & \cdots & r_{yy}(N-2) \\
  r_{yy}(2) & r_{yy}(0) & \ddots & \ddots & \vdots \\
  \vdots & \ddots & \ddots & r_{yy}(N-3) \\
  r_{yy}(N-1) & \cdots & \cdots & \cdots & r_{yy}(0)
\end{bmatrix}
\begin{bmatrix}
  a_0 \\
  a_1 \\
  a_2 \\
  \vdots \\
  a_{N-1}
\end{bmatrix}
= 
\begin{bmatrix}
  r_{yy}(\alpha) \\
  r_{yy}(\alpha+1) \\
  r_{yy}(\alpha+2) \\
  \vdots \\
  r_{yy}(\alpha+N-1)
\end{bmatrix} \quad (4-31)$$

Based on equation (4-31), the prediction filter coefficients $\{a_k\}$ could be efficiently solved by using the recursive Levinson-Durbin algorithm [43]. Practically, the autocorrelation function $r_{yy}(l)$ would be estimated directly from the collected GPR data using the biased estimate given by equation (B−11), Appendix B.

For a prediction filter $a_k$ of length $N$ and prediction distance $\alpha$, the difference between the actual signal value and the predicted value is known as the prediction error. As depicted in Figure 4-30a, the prediction error corresponding to prediction filter $a_k$ is given as follows:

$$e(t) = y(t) - \hat{y}(t) \quad (4-32)$$
Substituting \( \hat{y}(t) \) by its expression as output of the prediction filter leads to the following expression of the prediction error:

\[
e(t) = y(t) - y(t - \alpha) * a_t = y(t) - \sum_{r=0}^{N-1} a_r y(t - \alpha - \tau)
\]  

(4-33)

Or equivalently:

\[
e(t) = \sum_{r=0}^{N-1} f_r y(t - \tau), \text{ with } f_r = 1, 0, \ldots, 0, -a_0, -a_1, \ldots, -a_{N-1}
\]  

(4-34)

Hence, as depicted in Figure 4-30b, it is shown that the filter \( f_k \) could be applied directly to the signal \( y(t) \) to produce the prediction error at its output. The filter \( f_k \) is called the prediction error filter corresponding to prediction filter \( a_k \).

![Figure 4-30: Prediction Error Filter: (a) Direct Implementation, (b) Alternative Implementation](image)

It could be shown [50] that, except for a scale factor, a prediction error filter \( f_k \) of length \( N \) and prediction distance unity (i.e. \( \alpha = 1 \)) is equivalent to an inverse filter that transforms a pulse of an unknown shape into a spike at zero lag. More generally, it could be shown [50] that a prediction error filter of length \( N \) and a prediction distance \( \alpha \) greater than one could be used to transform an unknown pulse of arbitrary length \( \alpha+N \) into another pulse of length \( \alpha \). This transformation would, thus, accomplish the required deconvolution operation. In another way, if
the reflected GPR signal \( y(t) \) is assumed to be composed of a predictable part (the incident signal \( x(t) \)) and an unpredictable part (the reflectivity function \( \gamma(t) \)), then the prediction error filter would ideally remove the predictable part. Hence, the output of the prediction error filter would be the reflectivity function \( \gamma(t) \) [49].

The main advantage of predictive deconvolution over inverse filtering is that it provides a second parameter (the prediction distance \( \alpha \)) that could be used along with the filter length to control the performance of the filter. It should be noted here that the length of the prediction error filter is equal to \( N+\alpha \), where \( N \) is the length of the corresponding prediction filter.

The same procedure used to determine the optimal length of the inverse filter could be used to find the prediction filter optimal length \( N_{opt} \) and prediction distance \( \alpha_{opt} \) that ensure the correct deconvolution of GPR data. In this case, the two parameters should be varied in order to minimize the \( SNR_{red} \), \( SER \), and to maximize the \( PCR \).

Experimental minimization/maximization of the filter performance parameters (\( SNR_{red} \), \( SER \), and \( PCR \)) were conducted on a large set of GPR scans collected over different periods of time from the different flexible sections of the Virginia Smart Road. Prediction filters of varying lengths and prediction distances were applied to each scan of the considered GPR data, and the \( SNR_{red} \), \( SER \), and \( PCR \) were computed based on the deconvolved signal. For each filter length and prediction distance, the \( SNR_{red} \), \( SER \), and \( PCR \) results found for all the processed scans were then averaged to yield single estimates of the measured quantities per filter length and prediction distance. For this analysis, the prediction filter length was varied between 1, and 150 points and the prediction distance was varied between 1 and 15 points.

Contour plots of the average estimated \( SNR_{red} \) (in dB), \( SER \) (in %), and \( PCR \) (in %) are presented in Figure 4-31. As for the inverse filter, the \( SNR_{red} \) and \( PCR \) were evaluated for the surface reflection, whereas the \( SER \) was evaluated based on all the pulses detected in the deconvolved GPR signal.
Figure 4-31: Average Estimated (a) $SNR_{red}$ (dB), (b) $SER$ (%), and (c) $PCR$ (%) vs. Prediction Filter Length and Prediction Distance
According to Figure 4-31a, the $SNR_{red}$ increases considerably with the prediction filter length $N$ and increases slightly with the prediction distance $\alpha$. This behavior is expected from the prediction error filter. In fact, for the filter length, the performance should be similar to that of the inverse filter. On the other hand, it is known that for a prediction error filter, the prediction error increases with an increase in the prediction distance [52]. Thus, the noise in the filter output would increase and so would the $SNR_{red}$. Similarly, the $SER$ depicted in Figure 4-31b is found to increase with both the filter length and the prediction distance. The $SER$ increase with the prediction distance is less severe than its increase with the filter length. Thus, as with the inverse filter, the $SER$ and the $SNR_{red}$ have similar variation trends versus the filter length and the prediction distance. Finally, the $PCR$ shown in Figure 4-31c is found to be jointly dependent on the filter length and the prediction distance. As expected from the theoretical analysis (i.e., the prediction error filter transforms a signal of length $N+\alpha$ into a signal of length $\alpha$), the $PCR$ decreases with the prediction distance and increases with the filter length until it reaches a peak after which it starts decreasing again. The peak of the $PCR$ is attained for filter lengths spread around the length of the incident signal (60 samples in this case).

Based on these results, it is found that the joint minimization of $SNR_{red}$, $SER$, and maximization of the $PCR$ is achieved for a prediction filter length $N_{opt}$ between the half-width and entire-width of the incident signal and a prediction distance $\alpha_{opt}$ between 1 and 7. To find the optimal filter parameters within this range, the variations of the maximum $PCR$, minimum $SNR_{red}$, and minimum $SER$ are plotted in Figure 4-32 versus the prediction filter length $N$. This figure also depicts the optimal prediction distance $\alpha$ resulting in the maximum $PCR$ and the corresponding $SNR_{red}$ and $SER$. According to Figure 4-32, the maximum $PCR$ is obtained for decreasing values of the prediction distance $\alpha$ versus the filter length $N$. Moreover, the best filter performance is achieved for a prediction filter length around 60 points and a prediction distance $\alpha$ equals 1. Below this length, the $PCR$ degrades slightly, but it remains above 50%, and the $SNR_{red}$ and $SER$, which, as shown in the figure, are approximately equal to the minimum achievable values, stay constant around 5dB and 5%, respectively. On the other hand, for the prediction distance $\alpha$, the best deconvolution results are achieved for small values of $\alpha$ (less than 3). It should be noted that choosing a small value of $\alpha$ would also guarantee a faster filtering operation since the length of the prediction error filter is the sum of the length of the corresponding prediction filter and the prediction distance (i.e., total filter length equals $N+\alpha$).
Figure 4-33 presents the deconvolved GPR signal obtained by a prediction error filter of length $N_{opt} = 60$ and prediction distance $\alpha = 1$ and corresponding to the raw signal depicted in Figure 4-20a. It is noted that the signal plotted in this figure is very similar to the output of an inverse filter of the same length (Figure 4-28), except for a scale factor. According to Figure 4-33, all reflected pulses, originally masked in the raw data, became visible after deconvolution. The amplitudes of the deconvolved reflections are all above noise level, with the exception of the BM-25.0/OGDL reflection, which originally had a low amplitude before deconvolution.

Figure 4-34 shows the deconvolved scans of the raw air-coupled GPR data collected over section A at the Virginia Smart Road (see Figure 4-21). The deconvolution operation was achieved by a prediction filter of length $N_{opt} = 60$ points and prediction distance $\alpha_{opt} = 1$. As for the inverse filter output of Figure 4-29, this figure shows the continuous reflections of the WS/BM-25.0 interface and the BM-25.0/OGDL interface originally masked in the raw data of Figure 4-21.

In summary, this analysis showed how predictive deconvolution allows for the enhancement of depth resolution of GPR data. The best performance was found for a prediction filter length between the half-length and the entire-length of the incident signal and a small
prediction distance (less than 3). Within this range, a longer prediction filter length allows for better performance, whereas a shorter filter length allows for faster processing while keeping the performance reasonable. For small prediction distances, it was found that the prediction error filter behaves approximately like an inverse filter.

Figure 4-33: Deconvolved GPR Signal Obtained by a Prediction Error Filter, \( N_{\text{opt}} = 60 \) and \( \alpha_{\text{opt}} = 1 \)

Figure 4-34: Deconvolved GPR Data Obtained by a Prediction Error Filter of \( N_{\text{opt}} = 60 \) and \( \alpha_{\text{opt}} = 1 \), Showing Copper Plate Reflections at the: (1) WS/BM-25.0, (2) BM-25.0/OGDL, (3) OGDL/21A, (4) 21A/21B, and (5) 21B/Subgrade Interfaces
4.2.3.5 Pulse spiking

Pulse spiking is a filtering procedure that uses the principles of Wiener filtering (see Appendix B) to design a filter that transforms a signal with a known shape into a delta function (or spike) [53]. Therefore, pulse spiking processing could be regarded as a type of pulse compression rather than deconvolution. A pulse spiking filter $a$ that would ideally shrink a signal of known shape $x(t)$ into a spike at lag 0 could be designed according to the matrix relation presented in equation (4-17).

For GPR data, if the spiking filter $a$ is designed based on the incident signal $x(t)$, in other words if the incident signal $x(t)$ satisfies:

$$x(t) * a_i = d(t) = \delta(t)$$

then ideally when the spiking filter is applied to the measured reflected signal $y(t)$, its output signal would be as follows:

$$y(t) * a_i = \gamma(t) * x(t) * a_i = \gamma(t) * \delta(t) = \gamma(t)$$

(4-36)

Because, as indicated by equation (4-13), the reflectivity function $\gamma(t)$ is assumed to be composed of a series of time delayed delta functions, the spiking filter would ideally accomplish the required depth resolution enhancement of the GPR data.

It could be shown [53] that the performance of spiking filters could be greatly enhanced by changing the lag of the desired output signal. In other words, instead of designing the filter to have its desired output $d(t)$ equal $\delta(t)$, it is designed to have its output equal $\delta(t-l)$, where $l$ is a positive integer. The choice of the lag $l$ depends mainly on the energy concentration in the incident signal [53] or, equivalently, on its phase spectrum characteristics (minimum-phase, maximum-phase, or mixed-phase [43]). When this kind of spiking filters is applied to the reflected signal, its ideal output would be a replica of the reflectivity function delayed by $l$ samples, as follows:

$$y(t) * a_i = \gamma(t) * x(t) * a_i = \gamma(t) * \delta(t-l) = \gamma(t-l)$$

(4-37)
When the spiking Wiener filter is designed such that the desired output signal \( d(t) \) equals \( \delta(t-l) \), the cross-correlation between input signal and desired output, given by equation (B−6), Appendix B, would be changed to the following:

\[
r_{dk}(i) = \sum_{t} \delta(t-l)x(t-i) = \begin{cases} x(l-i) & \text{for } i \leq l \\ 0 & \text{for } i = l + 1, \ldots, N-1 \end{cases}
\] (4-38)

Therefore, the designed spiking filter \( a_k \) of length \( N \) and lag \( l \) should satisfy, according to equation (B−9), Appendix B, the following matrix relation:

\[
\begin{bmatrix}
  r_{xx}(0) & r_{xx}(1) & r_{xx}(2) & \cdots & r_{xx}(N-1) \\
  r_{xx}(1) & r_{xx}(0) & r_{xx}(1) & \cdots & r_{xx}(N-2) \\
  r_{xx}(2) & r_{xx}(0) & r_{xx}(N-3) & \cdots & r_{xx}(N-1) \\
  \vdots & \ddots & \ddots & \ddots & \ddots \\
  r_{xx}(N-1) & \cdots & \cdots & r_{xx}(0) & r_{xx}(N-1)
\end{bmatrix}
\begin{bmatrix}
  a_0 \\
  a_1 \\
  a_2 \\
  \vdots \\
  a_{N-1}
\end{bmatrix}
= \begin{bmatrix}
  x(l) \\
  x(l-1) \\
  x(0) \\
  0 \\
  \vdots \\
  0
\end{bmatrix}
\] (4-39)

For this analysis, the shape of the signal \( x(t) \) used for the filter design is assumed known. Consequently, its autocorrelation function could be computed according to equation (B−11), Appendix B. Practically, the signal \( x(t) \) used for the spiking filter design is chosen as the GPR incident signal. For the case of an air-coupled GPR system, the incident signal could be collected using a copper plate on top of the pavement surface, as was mentioned in 4.2.2.2. For a ground-coupled GPR system, however, this pulse spiking technique cannot be applied since it is difficult to acquire the shape of the incident signal.

It should be noted that, theoretically, there are no restrictions on the values of the filter length \( N \) or the lag \( l \) for the design of the spiking filter (i.e., \( l \) could be greater, equal, or less than \( N \)). However, it could be shown that the choice of these two parameters severely affects the performance of the spiking filter [53], which could be measured during the design phase by the parameter \( P \) of equation (B−21), Appendix B. The effect of spike lagging in the desired output signal on the performance of the spiking filter is shown in Figure 4-35. This figure depicts the input signal used for designing a spiking filter of length \( N = 90 \), along with the filter output signals obtained for the same input signal for different values of the lag \( l \). The amplitudes of the
signals in this figure were scaled for clarity purposes. According to Figure 4-35, the performance of the spiking filter degrades when the lag $l$ is either too small (for $l = 0, P = 0.15$) or too high (for $l = 90, P = 0.15$). However, when $l$ is in the middle of the lag interval, the filter performance increases significantly, with a performance parameter $P$ tending to 1 (for $l = 45, P = 0.90$) and an output signal approximating the shape of a spike at the specified lag value $l$.

![Figure 4-35: Spiking Filter Input and Actual Output for a Filter Length $N = 90$ and Lag Values $l = 0, 45, and 90$](image)

In order to study the joint effect of filter length and desired output lag on the performance of the spiking filter, contour plots of the performance parameter $P$ (in %) are presented in Figure 4-36 for filter lengths $N$ varying between 2 and 130 and lag values $l$ varying between 2 and 120. In this figure, the performance parameter $P$ was evaluated for spiking filters designed for GPR incident signals collected over different periods of time. The results were then averaged to yield a single estimate of $P$ per filter length and lag value. Figure 4-36 illustrates that for a fixed value of $l$, as the filter length $N$ increases, the performance of the filter increases until it reaches a maximum value after which it stays almost constant. This result is valid for Wiener filters in general. In fact, increasing the filter length would allow a better match between the actual output signal and the desired output signal and, therefore, would minimize the mean square error ($MSE$). After a certain filter length, however, the effect of the added terms of the Weiner filter
would be insignificant since the filter parameters would tend toward 0 because the high-order elements of the cross-correlation function between the desired output and the input signal would also tend toward 0. Therefore, it could be concluded that the choice of the lag value \( l \) is very critical to attaining a high spiking filter performance (for example, for \( l = 10 \), \( P < 80\% \) for any value of \( N \) whereas for \( l = 30 \), \( P > 90\% \) for \( N > 50 \)).

![Figure 4-36: Spiking Filter Performance Parameter \( P \) (in %) vs. Filter Length \( N \) and Lag \( l \)](image)

On the other hand, Figure 4-36 shows that for a fixed value of the filter length \( N \), the performance of the filter increases versus the lag \( l \) until it reaches a maximum value where it stays constant. Then, after a certain upper limit of \( l \), the performance begins to degrade again. In particular, the spiking filter performance degrades severely (\( P < 20\% \)) when \( l \) becomes greater than \( N \). This performance behavior of the spiking filter could be explained by the right-hand side of the matrix relation of equation (4-39), which represents the cross-correlation between the input signal and the desired output signal. Based on this equation, for \( l \) less than \( N \) the cross-correlation vector would be: \([x(l), x(l-1), \ldots, x(0), 0, \ldots, 0]^T\). Therefore, for small values of \( l \), the cross-correlation vector would be composed mostly of zeros. This translates into a low correlation between the input signal and output signal, which in turn leads to a low filter performance. As \( l \) increases, the cross-correlation vector would include more significant terms of the input signal \( x(t) \). Hence, the spiking filter would perform better. When the lag \( l \) becomes
greater than $N$, the cross-correlation vector becomes: $[x(l), x(l-1), \ldots, x(l-N+1)]^T$. For values of $l$ larger than a certain minimum value, this vector would be composed of insignificant elements at the trailing portion of the input signal $x(t)$. Thus, the performance of the spiking filter would start decreasing again. In conclusion, in order to get the best spiking filter performance, the parameters $N$ and $l$ should be chosen such that the cross-correlation vector contains the most significant terms of the input signal $x(t)$.

In order to find the optimal values of the filter length $N$ and lag $l$ that ensure a high performance of the spiking filter, the maximum value of the parameter $P$ and the corresponding optimal lag $l$ for each filter length are extracted from the contour plots of Figure 4-36 and are depicted in Figure 4-37. According to this figure, it is confirmed that the performance parameter $P$ converges to 100% as the filter length increases. For filter lengths larger than the length of the incident signal (60 samples in this case), the rate of increase of $P$ diminishes and $P$ becomes almost constant. On the other hand, it is found that the optimal lag $l$ increases linearly with the filter length $N$ ($l = 0.62N$ with $R^2 = 0.97$). Hence, it could be concluded that for the type of GPR signals used in this study, an optimal spiking filter should have a length equal to the length of the incident signal and an optimal lag found by the linear relation specified in Figure 4-37 (for $N_{opt} = 60$, $l_{opt} = 38$). A longer filter would not significantly enhance the performance.

![Figure 4-37: Maximum Spiking Filter Performance Parameter $P$ (in %) and Corresponding Optimal Lag vs. Filter Length](image)
The aforementioned performance analysis of the spiking filter was conducted for the incident GPR signal, which was used firstly to design the filter and then secondly as the filter input for the performance analysis. To study the performance of the filter when a reflected GPR signal is used as input, the SNR reduction ($SNR_{red}$), square error ratio ($SER$), and pulse compression ratio ($PCR$) described in 4.2.3.3 could be used.

Experimental minimization/maximization of these three parameters were conducted on a large set of GPR scans collected over different periods of time from the different flexible pavement sections of the Virginia Smart Road. For this study, spiking filters of varying lengths $N$ and desired output lags $l$ were designed based on the incident GPR signals collected during the surveys. Only the combinations of $N$ and $l$ that gave a performance parameter $P$ greater than 95% (or equivalently $N$ greater than 60 and $l$ greater than 40, according to Figure 4-37) were used in the computations of the $SNR_{red}$, $SER$, and $PCR$. In fact, experimental results showed that for lower values of $P$, the signal at the output of the spiking filter becomes very noisy. The spiking filters were then applied to each scan of the corresponding GPR data, and the $SNR_{red}$, $SER$, and $PCR$ were computed based on the spiking filter output signal after detection of all the reflected pulses. For each filter length $N$ and output lag $l$, the $SNR_{red}$, $SER$, and $PCR$ results were found for all the processed scans and were then averaged to yield single estimates of the measured quantities per filter length and lag value.

Contour plots of the average estimated $SNR_{red}$ (in dB), $SER$ (in %), and $PCR$ (in %) are presented in Figure 4-38. As for the inverse filter, the $SNR_{red}$ and $PCR$ were evaluated for the surface reflection whereas the $SER$ was evaluated based on all the pulses detected in the output signal of the spiking filter. As shown in Figure 4-38, even though the performance parameter $P$ of the designed spiking filter is greater than 95%, its performance degrades for some combinations of $N$ and $l$, when applied to GPR reflected signals. According to Figure 4-38, the spiking filter performs better (i.e., low $SNR_{red}$ and $SER$ and high $PCR$) when the filter length $N$ and the desired lag $l$ are far from the edges of the region of the 95% performance depicted in Figure 4-36. This result suggests that in order to have a correct depth resolution enhancement using spiking filters, the performance parameter $P$ of the designed filter has to be a little larger than 95%. Further fine-tuning to obtain the optimum value of the $P$ parameter is presented below.
Figure 4-38: Average Estimated (a) $SNR_{red}$ (dB), (b) $SER$ (%), and (c) $PCR$ (%) vs. Spiking Filter Length and Desired Output Lag
Moreover, Figure 4-38a illustrates that the $SNR_{red}$ is relatively higher (minimum $SNR_{red}$ is greater than 10dB) than what was found with the inverse filter or the error prediction filter. The slightly large $SNR_{red}$ value found for the spiking filter proves its susceptibility to contamination by noise due to the introduction of high frequency components by the spiking operation. Actually, the output signal of the spiking filter usually has to be filtered in order to yield a usable signal. Experimental testing on GPR data showed that a filter such as the one described in 4.2.2 is adequate to filter the output of the spiking filter and to reduce the noise level.

In order to find the optimal spiking filter parameters $N$ and $l$ that ensure a correct enhancement of GPR data depth resolution, the variations of the maximum $PCR$, minimum $SNR_{red}$, and minimum $SER$ versus the length $N$ of the spiking filter are extracted from the results illustrated in Figure 4-38 and then depicted in Figure 4-39. These curves were obtained by minimizing the $SNR_{red}$ and $SER$ and maximizing the $PCR$ as a function of the lag $l$ for each filter length $N$. Figure 4-39 also depicts the optimal lag values $l_{opt}$, which result in the minimum $SNR_{red}$, along with the corresponding $PCR$ and $SER$ values. Since the $SNR_{red}$ was found to be the most vulnerable parameter to the changes of $N$ and $l$, it was chosen to determine the optimal lag values presented in the figure. Fitting these optimal lag values to a line shows that, in this case, the optimal lag has a linear dependence on the filter length $N$ ($l_{opt} = 0.60N$, with $R^2 = 0.86$).

Figure 4-39 shows that the $SER$ and $PCR$ corresponding to the optimal lag values found for the minimum $SNR_{red}$ are approximately equal to the minimum $SER$ and maximum $PCR$, respectively. Hence, choosing an optimal lag value based on the minimum $SNR_{red}$ will lead to optimal values of the other two parameters. Based on Figure 4-39, the $SNR_{red}$ and $SER$ are highest for small filter lengths (less than 80) and then become almost constant at 11dB and 5%, respectively, as the filter length increases. Similarly, the $PCR$ is lowest for small filter lengths (less than 80), and then it increases to approximately 55%, where it remains relatively constant. Consequently, the joint minimization of the $SNR_{red}$ and $SER$ and maximization of the $PCR$ would be achieved for an optimal filter length $N_{opt}$ greater than 80 and a corresponding desired optimal lag $l_{opt}$ equal to $0.60N_{opt}$. Increasing the length of the spiking filter would not largely enhance its performance. Figure 4-37 shows that a filter length of 80 would give a performance parameter $P$ of around 97%. Hence, it could be concluded that a suitable spiking filter for GPR data should have a minimum performance parameter $P$ around 97%.
Figure 4-40 presents the output of an optimal spiking filter of length $N_{opt}$ equals 95 and desired output lag $l_{opt}$ equals 60 and having as input the raw GPR signal depicted in Figure 4-20a. The layer interface assignments in this figure were based on the actual interface positions located in Figure 4-20b. As seen in Figure 4-40, all reflected pulses, originally masked in the raw data, became visible after the application of the filter. The amplitudes of these pulses are all above noise level except for the BM-25.0/OGDL reflection, which originally had low amplitude. Moreover, examination of the portions of the filtered signal where there were originally no reflected pulses (i.e., between 8 and 12ns and above 14ns) shows the extent of the noise increase caused by the spiking filter.

Figure 4-41 depicts GPR scans after processing by a spiking filter of length $N_{opt}$ equals 95 and output lag $l_{opt}$ equals 60. These scans correspond to the raw air-coupled GPR data collected over section A at the Virginia Smart Road and presented in Figure 4-21. As for the inverse filter and prediction error filter, this figure shows the continuous reflections of the WS/BM-25.0 interface and the BM-25.0/OGDL interface that were originally masked in the raw data of Figure 4-21. In fact, as could be seen in Figure 4-41, the WS/BM-25.0 and BM-25.0/OGDL interface reflections are at the same depth levels as the copper plate reflections (1) and (2).
In summary, this analysis showed the effectiveness of spiking filters to enhance the depth resolution of GPR data. The choice of the spiking filter length and the desired output lag largely
influences the performance of the filter. It was found that for a higher performance of the spiking filter, the desired output lag has to be greater or equal to the length of the incident signal but less than the length of the filter. To ensure the highest pulse compression ratio and the lowest noise level increase in the filtered signal, the spiking filter should be designed such that it has a performance of at least 97%.

The main advantage of the pulse spiking technique over the inverse filtering or predictive deconvolution is that the parameters of filter are computed only once based on the incident signal, then the filter is applied for all the reflected scans. Hence, this technique will result in faster processing. On the other hand, the major drawbacks of the spiking filter include the following: the incident signal has to be known in order to design the filter, and the output of the spiking filter would usually be corrupted by noise due to the high frequency components introduced by the spiking operation.

4.2.3.6 Pulse shaping

Pulse shaping processing is similar to pulse spiking processing except for the shape of the desired output pulse. In fact, for pulse spiking, the desired output pulse is a delta function delayed by a specified lag; in contrast, for pulse shaping the desired output pulse has a specified shape, a specified width (larger than the spike), and a specified lag. There are two main advantages to using pulse shaping instead of pulse spiking. First, because a relatively large pulse has a narrower spectrum than that of a spike, pulse shaping would perform better than pulse spiking especially because the incident GPR signal, based on which the filter design is accomplished, has also a limited spectrum [53]. Second, pulse shaping introduces another design parameter (the desired pulse width), which would be used to control the degree of depth resolution of the processed signal.

If a shaping filter \( a_t \) is designed based on the incident GPR signal \( x(t) \) to produce a desired output signal \( s(t) \), then, ideally, \( x(t) \) would be related to \( s(t) \) according to the following equation:

\[
x(t) * a_t = s(t)
\]  

(4-40)

Therefore, when the shaping filter \( a_t \) is applied to the measured reflected GPR signal \( y(t) \), its output signal would be as follows:
\[ y(t) * a_t = \gamma(t) * x(t) * a_t = \gamma(t) * s(t) \] (4-41)

Since the reflectivity function \( \gamma(t) \) is theoretically composed of a train of time-delayed spikes, as given by equation (4-13), the output of the shaping filter \( a_t \) would be according to equation (4-41), composed of a train of pulses with the shape of the signal \( s(t) \). Hence, the choice of the desired output pulse \( s(t) \) and its width would set the depth resolution of the processed signal.

As for a spiking filter, a shaping filter could be designed in the form of a Wiener filter. A detailed description of Wiener filters in general is presented in Appendix B. If the desired output of the shaping filter is designated \( s(t-l) \) where \( l \) is the desired output lag, then the cross-correlation between input signal \( x(t) \) and desired output signal \( s(t-l) \) would be determined according to equation (B−6), Appendix B, as follows:

\[ r_{x_d}(i) = \sum d(t)x(t-i) = \sum s(t-l)x(t-i) = \sum s(t)x(t-(i+l)) = r_{sx}(i+l) \] (4-42)

where \( r_{sx}(i) \) is the cross-correlation function between desired output signal \( s(t) \) and input signal \( x(t) \). Hence, the designed shaping filter \( a_k \) of length \( N \) and lag \( l \) would be designed according to the matrix relation derived from equation (B−9), Appendix B, as follows:

\[
\begin{bmatrix}
  r_{xx}(0) & r_{xx}(1) & r_{xx}(2) & \cdots & r_{xx}(N-1) \\
  r_{xx}(1) & r_{xx}(0) & r_{xx}(1) & \cdots & r_{xx}(N-2) \\
  r_{xx}(2) & r_{xx}(0) & r_{xx}(1) & \cdots & r_{xx}(N-3) \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  r_{xx}(N-1) & \cdots & \cdots & r_{xx}(0) & a_{N-1}
\end{bmatrix}
\begin{bmatrix}
  a_0 \\
  a_1 \\
  a_2 \\
  \vdots \\
  a_{N-1}
\end{bmatrix}
= 
\begin{bmatrix}
  r_{xx}(l) \\
  r_{xx}(l+1) \\
  r_{xx}(l+2) \\
  \vdots \\
  r_{xx}(l+N-1)
\end{bmatrix}
\] (4-43)

As for spiking filters, the shape of the signal \( x(t) \) used for the shaping filter design is assumed known. Consequently, its autocorrelation function \( r_{xx}(i) \) could be estimated according to equation (B−11), Appendix B. Likewise, the cross-correlation function \( r_{sx}(i) \) between the desired output \( s(t) \) and the input signal \( x(t) \) could be estimated using equation (B−11), Appendix B, by substituting \( x(t) \) by \( s(t) \). Practically, the signal \( x(t) \) used for the shaping filter design is chosen as the GPR incident signal. For the case of an air-coupled GPR system, the incident signal could be collected using a copper plate on top of the pavement surface, as was described.
in 4.2.2.2. On the other hand, for a ground-coupled GPR system, the pulse shaping technique is not applicable since it is difficult to acquire the shape of the incident signal using a ground-coupled GPR system.

The choice of the desired shape and width of the output pulse would generally influence the performance of the shaping filter [53]. Theoretically, any pulse shape could be used as desired output as long as it is shorter than the incident GPR signal. However, it would make more sense to choose an output pulse that would increase the detectability of the subsequent pulse detection stage while increasing the performance of the pulse shaping filter itself. For these reasons, output pulses with the shape of known window functions were chosen for this application. The chosen windows, pictured in Figure 4-42, are as follows: Gaussian, rectangular, triangular, Blackman, Hamming, Hanning, and Kaiser [43].

![Window Functions Used as Desired Output Pulses](image)

**Figure 4-42: Window Functions Used as Desired Output Pulses**

As could be observed in Figure 4-42, except for the rectangular and triangular windows, all the other window types have almost similar time domain shapes. The main difference between these windows, however, is their frequency content: specifically the width of the main lobe that
contains most of the signal’s energy and the degree of attenuation of the side lobes [43] and [52]. The frequency content difference between these windows could influence their performance in reshaping the incident GPR signal.

As for spiking filters, the performance of shaping filters could be evaluated, during the filter design phase, by the performance parameter $P$ of equation (B–21), Appendix B. In this case, in addition to the filter length $N$ and the desired output lag $l$, the performance of the shaping filter would also depend on the desired output shape and the desired output pulse width. In order to study the joint effect of all these factors on the performance of the shaping filter, it would be impractical to use contour plots of the parameter $P$ versus the filter length and the desired lag, as was done in Figure 4-36 for the spiking filter. In fact, the number of combinations of different desired pulse shapes and pulse widths would lead to a huge number of plots. Instead, it was decided to fix the latter two parameters (i.e., the desired pulse shape and the desired pulse width) and then to find the optimal values of the filter length and output lag that would give a maximum parameter $P$. This procedure was then repeated for different pulse shapes and pulse widths to yield the maximum performance parameter $P$, optimal filter length $N$, and optimal lag $l$ for each filter type versus the desired output pulse width. Practically, the filter length was varied between 5 and 130, the output lag between 5 and 120, and the desired output pulse width between 1 and 25. It should be noted that an output pulse width of 25 would ideally give a $PCR$ of around 58% if the incident signal is 60 samples long. For all the combinations of these design factors, the maximum performance parameter $P_{\text{max}}$, the corresponding optimal filter length $N_{\text{opt}}$, and optimal output lag $l_{\text{opt}}$ were evaluated for shaping filters designed based on GPR incident signals collected over different periods of time. The results were then averaged to yield single estimates of $P_{\text{max}}$, $N_{\text{opt}}$, and $l_{\text{opt}}$ per desired output shape and desired output pulse width.

The variations of the maximum performance parameter $P$ for each window type versus the desired output pulse width are presented in Figure 4-43a, and the corresponding optimal lag values are presented in Figure 4-43b. For all the studied cases, the optimal filter length was found to be the maximum value used in the computations (i.e., 130). This result conforms to the findings when spiking filter was applied. Both approaches suggested that as the length of the filter increases, its performance parameter $P$ increases and converges monotonically towards 1.
Figure 4-43: Performance of Various Shaping Filters vs. Desired Output Pulse Width: (a) Maximum Performance Parameter $P$ and (b) Optimal Desired Output Lag

Figure 4-43a illustrates the effects of varying the desired output shape and output pulse width on the performance of the shaping filter. Specifically, the figure shows that for all window
types, the performance of the shaping filter is inversely proportional to the desired output pulse width, with the Blackman window having the highest performance and the rectangular and Kaiser windows having the lowest performance. Hence, it could be deduced that the spiking filter (which could be seen as a shaping filter having a desired output pulse width of 1) performs better than any of the other shaping filters studied. This result contradicts the findings of Treitel and Robinson [53], who showed that the spiking filter, which is a particular case of the general class of shaping filters, usually achieves the lowest performance when compared to the other filter types.

Further investigation of shaping filters showed that they could indeed outperform spiking filters for special cases. Figure 4-44 depicts the maximum performance parameter $P$ obtained for different shaping filters versus the desired output pulse width. According to this figure, the performance of the considered shaping filters increases proportionally with the desired output pulse width until it reaches a peak value – after which it starts decreasing again. The peak value of the performance parameter and the corresponding desired pulse width depend on the type of window used as desired output pulse.

![Figure 4-44: Maximum Performance Parameter $P$ of Various Shaping Filters vs. Desired Output Pulse Width](image)

It should be noted that the performance results shown in Figure 4-43a and Figure 4-44 were computed from shaping filters designed based on the same incident GPR signals. The only difference between the two cases, however, was in the way the incident GPR signals were processed before designing the filters. In the design of the filters that gave the results in Figure 4-43a, the incident GPR signal was first cleaned from the coupling pulse and was then truncated to keep the incident pulse only. Next, the resulting signal was filtered to remove any additive noise. The same processing steps were followed for the design of the filters that gave the results in Figure 4-44, except that the truncation to the incident pulse was performed, in this case, after noise filtering rather than before it. Thus, the first processing (called processing type 1 in the following analysis) would result in a “smoother” transition between signal and no signal than would the second processing (called processing type 2). This minor difference in the processing steps of the incident signal could be considered as the origin of the large differences between the performance of the shaping filters as pictured in Figure 4-43a and Figure 4-44. In order to further investigate the effects of the processing differences of the incident signal on the performance of the shaping filters, a frequency analysis was conducted.

The magnitude spectra of the input (or equivalently the GPR incident signal), desired output, and actual output signals of different shaping filters are depicted in Figure 4-45, along with the corresponding magnitude response of the filter. Figure 4-45a shows these different magnitude spectra for a spiking filter designed after following the steps of processing type 1. Figure 4-45b shows the same results for a shaping filter with a width-ten Hanning window as desired output. Figure 4-45c and Figure 4-45d show the same results as Figure 4-45a and Figure 4-45b, respectively, except that, in this case, processing type 2 was used instead of processing type 1. The performance parameters found for the four cases of Figure 4-45 were 0.98, 0.86, 0.21, and 0.76, respectively.

As seen in Figure 4-45a, because a filtering operation was conducted after truncation of the incident signal, the magnitude spectrum of the input signal is smooth (i.e., does not have any oscillations due to the truncation) and so is the magnitude response of the corresponding spiking filter. In contrast, in Figure 4-45b, even though the shaping filter was designed based on the same input signal as the one used in the previous case, its magnitude response has considerable oscillations especially after frequency 0.22. This corresponds approximately to the upper
boundary of the main lobe of the spectrum of the desired output. Since the only difference between the two aforementioned cases is the shape of the desired output signal, the appearance of oscillations in the magnitude response of the shaping filter shown in Figure 4-45b are mainly due to the discontinuities and side lobes observed in the spectrum of the desired output. It should be noted that for the spiking filter of Figure 4-45a, the magnitude spectrum of the desired output is flat, at 0dB, and hence does not present any discontinuities or oscillations.

![Figure 4-45: Magnitude Spectra of Input Signal, Shaping Filter, Output Signal, and Desired Output Signal for: (a) Processing Type 1 and Output Width 1, (b) Processing Type 1 and Output Width 10, (c) Processing Type 2 and Output Width 1, (d) Processing Type 2 and Output Width 10](image-url)
For the shaping filter of Figure 4-45b, the presence of oscillations in the magnitude response of the filter would propagate to the spectrum of the filter’s output signal, thus causing a discrepancy between the desired output and the actual output, as observed in Figure 4-45b. For the spiking filter, however, because of the oscillation-free spectra, the discrepancy is less severe, as seen in Figure 4-45a. As a result, it is found that for the processing type 1, the spiking filter performs better ($P = 0.98$) than does the shaping filter ($P = 0.86$). The reason for the high performance of the spiking filter compared to that of the shaping filter could be explained by the fact that it would be easier for the filter to transform a “smooth” spectrum to a different “smooth” spectrum than for it to transform a “smooth” spectrum to an oscillatory spectrum. Investigation of the effects of processing type 2 on the performance of shaping filters confirms this conclusion.

As seen in Figure 4-45c, truncating the incident signal manifests in the magnitude spectrum as appearance of oscillations. In fact, because truncating a signal is equivalent to multiplying it by a rectangular window, the spectrum of the truncated version would be the convolution of the spectrum of the original signal with that of the rectangular window, which is composed of a main lobe followed by attenuated side lobes [43]. In the case illustrated in Figure 4-45c, the oscillations in the spectrum of the input signal propagate to the magnitude response of the spiking filter and from there to the spectrum of the output signal, thus causing discrepancies between the desired output and the actual output signals. The same phenomenon is observed for the case of the shaping filter shown in Figure 4-45d. A comparison between the performance results of the spiking filter and the shaping filter, when designed after following processing type 2 steps, shows that the latter performs much better ($P = 0.76$) than the former ($P = 0.21$). The differences in the performance could again be explained by the fact that it would be easier for the filter to transform a signal with oscillatory spectrum to a signal with an oscillatory spectrum than for it to transform a signal with an oscillatory spectrum to a signal with a “smooth” spectrum.

The results of the maximum performance parameter of the different shaping filters, pictured in Figure 4-43a, agree with the aforementioned explanation. In fact, according to Figure 4-43a, the Blackman window achieves the highest performance, while the rectangular window and the Kaiser window achieve the lowest performance, with all the other windows having a performance parameter in between. An examination of the different window functions shows
that the Blackman window has its first side lobe level at -58dB, the rectangular window has its first side lobe level at -13dB, and the other window types have their first side lobe level in between these limits [43]. Hence, the Blackman window has the “smoothest” spectrum and, therefore, has the highest performance, whereas the rectangular window has the most oscillatory spectrum and thus the lowest performance. All the other windows have their spectra in between as well as their performances.

Figure 4-46 presents the desired output and the actual output signals obtained by the spiking filter (graphs [a] and [c]) and the width-ten-Hanning-window shaping filter (graphs [b] and [d]). The input signal used for these filters was the GPR incident signal depicted in Figure 4-35 and processed according to the steps of processing type 1 (graphs [a] and [b]) or the steps of processing type 2 (graphs [c] and [d]). The high performance of the spiking filter when its input signal is processed according to type 1 processing is evident in Figure 4-46a, where the desired output and the actual output signals overlap each other. The low performance of the spiking filter when its input signal is processed according to the steps of processing type 2 is also clear in Figure 4-46c, where the output signal is composed of a wide pulse at the desired output lag in addition to a spurious spike around sample 200. The approximate agreement between the desired Hanning window shape and the actual output of the shaping filter could also be clearly seen in Figure 4-46b, where the input signal was processed according to type 1 processing, and in Figure 4-46d, where the input signal was processed following type 2 processing.

Finally, it should be noted that the differences in the processing steps of the input signal (type 1 versus type 2) have a higher impact on the performance of the spiking filter than on the performance of the shaping filter. In fact, for the spiking filter, the performance parameter decreased from $P = 0.98$ for processing type 1 to $P = 0.21$ for processing type 2. On the other hand, for the shaping filter, the performance parameter decreased from $P = 0.86$ for processing type 1 to $P = 0.76$ for processing type 2. The big jump in the performance parameter of the spiking filter is a good indication of its sensitivity to the frequency content variations in the input signal. A comparison between the four plots in Figure 4-46 clearly shows the effects of the type of filter used and the type of processing conducted on the performance of shaping filters.
In summary, this analysis investigated the possibility of using shaping filters to enhance the depth resolution of GPR data. It was found that if the input signal used to design the shaping filter is correctly processed prior to designing the filter, a shaping filter with a desired output signal of width 1 (i.e., a spiking filter) would outperform any other shaping filter of any desired output shape or width. On the other hand, if the input signal was not correctly processed or if it was contaminated by noise, a shaping filter would outperform the spiking filter. The desired

Figure 4-46: Desired Output and Actual Output Signals for Different Shaping Filters: (a) Processing Type 1 and Output Width 1, (b) Processing Type 1 and Output Width 10, (c) Processing Type 2 and Output Width 1, (d) Processing Type 2 and Output Width 10
output shape and desire pulse width that would give a maximum performance, in this case, would depend mainly on the input signal used.

Since it was found that adequate processing of the input signal would guarantee that the spiking filter would outperform any shaping filter, no further investigation was conducted regarding the performance of shaping filters used on actual GPR reflected signal (i.e., analysis of $SNR_{red}$, $SER$, and $PCR$ of the output signals of the filter). An analysis of the performance of spiking filters when used to enhance the depth resolution of GPR data was described in 4.2.3.5.

4.2.3.7 Homomorphic deconvolution

Homomorphic deconvolution is based on a nonlinear processing technique, first introduced in the sixties, known as cepstral analysis [44]. In general, cepstral analysis has the motivation of transforming signals related by convolution into signals related by an addition operation. Thus, in the new cepstral domain, separation between the processed signals, which became superposed, would be achieved by a simple linear filtering operation. The term “homomorphic” comes from the property of superposition that the transformed signals obey in the cepstral domain.

Homomorphic processing in general and homomorphic deconvolution in particular have been used for different applications, such as speech, image [44], and seismic data processing [54], [55]. For seismic data, homomorphic deconvolution has been mainly used to extract the incident wavelet, the reflectivity function of the earth structure, and the multiple trace (i.e., trace of multiple reflections) from the measured seismic signal. The procedure consists of transforming the convolution product relating the three signal components into a simple superposition in the cepstral domain through the use of cepstral processing. If the cepstra of the three signal components occupy different regions of the measured signal’s cepstrum, the undesirable signal parts could be removed by linear filtering. Then, when the filtered cepstrum is transformed back to the time domain, it would include only the desired signal component (either incident wavelet, reflectivity function, or multiple trace). Therefore, the success of the method would depend mainly on the degree of separability of the different signal components in the cepstral domain.

The block diagram shown in Figure 4-47 illustrates the different steps involved in homomorphic deconvolution. The process could be decomposed into three main stages:
• The cepstrum computation stage (top dashed block in Figure 4-47) is used to transform the processed signal from the time domain, where the signals are related by convolution, to the cepstral domain, where the signals are related by addition.

• The linear filtering (LF) stage is used to eliminate one of the signal components in the cepstral domain. If the signals occupy different regions in the spectrum, the filtering operation would be achieved by forcing the undesired parts of the cepstrum to zero.

• The inverse cepstrum computation stage (bottom dashed block in Figure 4-47) is used to transform the filtered cepstrum back to the time domain, thus obtaining the deconvolved signal.

![Figure 4-47: Block Diagram of Homomorphic Deconvolution](image)

As shown in the top dashed block of Figure 4-47, the computation of the cepstrum of a signal is accomplished through the cascade of the following three operations:

• Fourier transform (FT) used to convert the convolution of the time domain signals to a product of their spectra as shown in the figure.

• Complex logarithm used to convert the product of the spectra to a sum of the logarithms of the Fourier transforms of the different signal components, as follows:

$$\bar{F}(f) = \log[X(f)\bar{\Gamma}(f)] = \log[X(f)] + \log[\bar{\Gamma}(f)] = \bar{X}(f) + \bar{\Gamma}(f) \quad (4-44)$$
The logarithm used in this operation could be in any base; however, a natural logarithm (i.e., base $e$) is usually used. The complex logarithm of a complex number $X(f)$ could be defined as follows:

$$\log[X(f)] = \log[|X(f)|e^{j\theta(f)}] = \log[|X(f)|] + j\theta(f)$$  \hspace{1cm} (4-45)

where $\theta(f)$ is the unwrapped phase of $X(f)$. The unwrapped phase could be defined as the continuous version of the phase, which might be different from its principal value defined in the interval $(-\pi, \pi]$ [44].

- Inverse Fourier transform (IFT) used to obtain the complex cepstrum of the signal. It should be noted that because of the use of the IFT, the complex cepstrum domain is similar to the time domain. The time-like variable used in the cepstral domain is termed as the “quefrency” $q$.

After linear filtering, the complex cepstrum could be converted back to the time domain using the bottom dashed block of Figure 4-47, which represents the inverse cepstrum computation block. As shown in the figure, this procedure is accomplished through the cascade of three operations representing the inverse of the cepstral computation block operations:

- Fourier transform used to transform the complex cepstrum to the logarithm of the Fourier transform of the deconvolved signal.
- Exponential operation used to get the Fourier transform of the deconvolved signal
- Inverse Fourier transform used to obtain the deconvolved time domain signal.

Despite its name, the complex cepstrum of a real sequence is a real sequence [44]. The term “complex” is added here to indicate that the cepstrum is computed based on the complex logarithm. If the imaginary part in equation (4-45) is dropped, the cepstrum computed by the inverse Fourier transform would be termed as the real cepstrum [44]. Because the real cepstrum does not include any phase information, it is not invertible.
As shown in Figure 4-47, the complex cepstrum of the reflected GPR signal $y(t)$ can be expressed as the sum of the complex cepstra of the incident signal $x(t)$ and reflectivity function $\gamma(t)$ as follows:

$$\tilde{y}(q) = \tilde{x}(q) + \tilde{\gamma}(q)$$

(4-46)

Using the expression of the reflectivity function given by equation (C-3) and assuming that the first reflection time $t_0$ equals zero, the spectrum of the reflectivity function would be expressed as follows:

$$\Gamma(f) = A_0 \left[ 1 + \sum_{n=1}^{N-1} \frac{A_n}{A_0} \exp(-j2\pi f \sum_{i=1}^{n} t_i) \right]$$

(4-47)

Therefore, the logarithm of the spectrum of the reflectivity function would as follows:

$$\log[\Gamma(f)] = \log[A_0] + \log \left[ 1 + \sum_{n=1}^{N-1} \frac{A_n}{A_0} \exp(-j2\pi f \sum_{i=1}^{n} t_i) \right]$$

(4-48)

Using a Taylor series expansion about 0 of the logarithm function (i.e. $\log(1+a) \approx a - 1/2a^2 + 1/3a^3 + \ldots$ for $|a| < 1$) and assuming that the terms $A_n/A_0$ are small in absolute value (in other words, ignoring the terms with powers greater than 1 in the Taylor expansion), equation (4-48) could be simplified, as follows:

$$\log[\Gamma(f)] \approx \log[A_0] + \sum_{n=1}^{N-1} \frac{A_n}{A_0} \exp(-j2\pi f \sum_{i=1}^{n} t_i)$$

(4-49)

Hence, the complex cepstrum of the reflectivity function would be given by the following equation:

$$\tilde{\gamma}(q) = F^{-1}[\log[\Gamma(f)]] = \log[A_0] \delta(q) + \sum_{n=1}^{N-1} \frac{A_n}{A_0} \delta(q - \sum_{i=1}^{n} t_i)$$

(4-50)

where $F^{-1}\{}$ is the inverse Fourier transform. Substituting the complex cepstrum of the reflectivity function of equation (4-50) in equation (4-46) gives the expression of the complex cepstrum of the reflected GPR signal, as follows:
\[
\bar{y}(q) = F^{-1}\left\{ \log[X(f)] \right\} + \log[A_0] \delta(q) + \sum_{n=1}^{N-1} \frac{A_n}{A_0} \delta(q - \sum_{i=1}^{n} t_i)
\]

(4-51)

where \(X(f)\) is the spectrum of the incident signal \(x(t)\). According to this equation, the complex cepstrum of the reflected signal is composed of the complex cepstrum of the incident signal plus a train of time-delayed spikes, with time delays corresponding to the interface reflection times \(t_i\). Since the cepstrum of the incident signal usually occupies the low region of frequencies [55], the cepstrum of the reflected spikes would be extracted by high-pass filtering the complex cepstrum of the reflected signal. On the other hand, the complex cepstrum of the incident signal would be obtained by a low-pass filtering operation. The high-pass and low-pass filtering operations could be achieved by forcing the undesired parts of the cepstrum to zero. After filtering, the time domain signals are obtained by computing the inverse cepstrum, as shown in Figure 4-47.

The main problem associated with using the aforementioned homomorphic deconvolution technique is how to choose the width of the window to be used in the filtering procedure. This problem is illustrated in Figure 4-48, which depicts the recovered incident signals and reflectivity functions after homomorphic deconvolution of two GPR scans collected from section A at the Virginia Smart Road. For both scans, the filtering operation was achieved using the same window width (12 samples). As shown in Figure 4-48a, even though the utilized filtering windows were similar, the incident signals recovered from the processed scans are different, with the top signal representing a better approximation of the real incident GPR signal depicted in Figure 4-11. The same result is observed in the reflectivity functions shown in Figure 4-48b. In fact, the top graph shows that, with the exception of the BM-25.0/OGDL reflection, the originally masked reflections in the raw GPR reflected signal (Figure 4-20a) became visible after deconvolution. In contrast, the bottom graph shows that some of the originally visible reflections (like the OGD/ Base reflection) became masked by noise after deconvolution. The erroneous results obtained for the second scan are mainly due to the inadequacy of the width of the filtering window used in the processing. Indeed, in this case, the width of the window was chosen such that the recovered incident signal computed from the first scan would have an identical shape to the real incident signal. Hence, this analysis shows the importance of choosing an appropriate filter width to achieve an optimal deconvolution of the GPR signals.
A further investigation of the homomorphic deconvolution results of the scan corresponding to the bottom plots of Figure 4-48 showed that the incident signal and the reflectivity function could not be recovered correctly from this GPR scan for any width of the
filtering window. To study the causes of the failure of homomorphic deconvolution in this case, the complex cepstra of the incident and reflected GPR signals along with the complex cepstrum of a simulated reflectivity function are depicted in Figure 4-49. The reflected signal and reflectivity function whose cepstra are shown in this figure correspond to the deconvolved signals shown in the bottom of Figure 4-48. As illustrated in Figure 4-49, there is an overlap between the complex cepstrum of the incident signal and that of the reflectivity function, especially for low quefrency values. Because of this overlap, low-pass filtering (to recover the incident signal) or high-pass filtering (to recover the reflectivity function) would result in removing parts of the cepstrum necessary for correct inversion of the filtered cepstrum and, therefore, would lead to erroneous deconvolved signals. To resolve this problem, comb filtering of the reflected signal’s cepstrum would be a better alternative, in this case, than would the window filtering [55]. However, this solution is not very practical since, as seen in Figure 4-49, the processed (or reflected) cepstrum has many peaks that have ambiguous origins (i.e., due to incident signal or reflectivity function), which would render the determination of the correct comb filter function difficult to estimate.

![Figure 4-49: Complex Cepstra of Incident GPR Signal, Reflected GPR Signal, and Simulated Reflectivity Function](image)
According to the literature, the use of the power cepstrum for the estimation of the time of arrival of seismic waves is more appropriate than the use of the complex cepstrum [55]. The power cepstrum is defined as the power spectrum of the logarithm of the processed signal’s power spectrum [49]. In equation format, the power cepstrum of signal \( y(t) \) could be obtained as follows:

\[
C_y(q) = \left| F\left\{ \log\left| F\{y(t)\} \right|^2 \right\} \right|^2
\]  
(4-52)

If the signal \( y(t) \) is the result of the convolution of two signals \( x(t) \) and \( \gamma(t) \) then the power cepstrum would be expressed as follows:

\[
C_y(q) = \left| F\left\{ \log[\Phi_x(f)\Phi_\gamma(f)] \right\} \right|^2 = \left| F\left\{ \log[\Phi_x(f)] + \log[\Phi_\gamma(f)] \right\} \right|^2
\]  
(4-53)

where \( \Phi_x(f) \) and \( \Phi_\gamma(f) \) are the power spectra of \( x(t) \) and \( \gamma(t) \), respectively. The power spectrum of the reflectivity function could be determined from equation (4-47) as follows (assuming \( t_0=0 \)):

\[
\Phi_\gamma(f) = \| \Gamma(f) \|^2 = A_0^2 \left\{ 1 + \sum_{n=1}^{N-1} \frac{A_n}{A_0} \cos(2\pi f \sum_{i=1}^{n} t_i) \right\}^2 + \left\{ \sum_{n=1}^{N-1} \frac{A_n}{A_0} \sin(2\pi f \sum_{i=1}^{n} t_i) \right\}^2
\]  
(4-54)

Assuming that the coefficients \( A_n/A_0 \) are small in absolute value, the previous equation could be approximated by neglecting the terms with a power greater than 1:

\[
\Phi_\gamma(f) \approx A_0^2 \left\{ 1 + 2 \sum_{n=1}^{N-1} \frac{A_n}{A_0} \cos(2\pi f \sum_{i=1}^{n} t_i) \right\}
\]  
(4-55)

Substituting the above equation in equation (4-53) gives the following expression of the power cepstrum of \( y(t) \):

\[
C_y(q) = \left| F\left\{ \log[\Phi_x(f)] + \log\left[ A_0^2 \left\{ 1 + 2 \sum_{n=1}^{N-1} \frac{A_n}{A_0} \cos(2\pi f \sum_{i=1}^{n} t_i) \right\} \right] \right\} \right|^2
\]  
(4-56)

This equation could be further simplified using a first-order Taylor series expansion of the logarithm function and neglecting the high-order terms as follows:
Based on the above equation, the logarithm of the power spectrum of $y(t)$ (i.e., the term inside braces) is a periodic function of the frequency $f$. The frequencies of oscillations of this function are set by the sum of the reflection times $t_i$. Hence, the frequency $f$ and the reflection times $t_i$ used within the cosine function of equation (4-57) play, in this case, reverse roles. The power spectrum of the logarithm of the power spectrum (or power cepstrum) $C_y(q)$ would, consequently, be composed of spikes corresponding to the reflection times $t_i$ and, therefore, would represent a good approximation of the deconvolved reflectivity function. However, because the power cepstrum is based on estimating power spectra, the polarity of the estimated reflections would be lost. It is worth noting that in this case also the independent variable, or quefrency, $q$ of the power cepstrum has units of time.

Practically, it was found that the use of a nonparametric power spectral estimation technique, such as the periodogram or Welch’s method [52], would yield better estimates of the power cepstrum than would the direct application of the Fourier transform. Moreover, since the logarithm of the power spectrum of the reflected signal $y(t)$ includes the logarithm of the spectrum of the incident signal $\Phi_x(f)$ that is usually non-oscillatory versus the frequency $f$, the complex cepstrum would comprise a peak at the origin with a much higher amplitude than the rest of the reflections.

Figure 4-50 shows the power cepstrum of the raw GPR signal depicted in Figure 4-20a. The power spectra used to compute this power cepstrum were estimated using periodograms. Because of the high amplitude of the peak at the origin of the power cepstrum and in order to expose the required interface reflections, the plotted power cepstrum was scaled using a scaling factor that is linearly increasing with time. Figure 4-50 shows that the originally masked WS/BM-25.0 reflection in the raw data became visible after deconvolution, but the BM-25.0/OGDL reflection remained masked due to its low amplitude. Additionally, in contrast to the complex cepstrum results, it was found that all the interface reflections that were originally visible in the raw data remained visible after deconvolution using the power cepstrum technique. All these reflections are above noise level even after applying the scaling factor.
Figure 4-50: Power Cepstrum of a Typical GPR Signal

Figure 4-51 depicts the estimated power cepstrum of the raw air-coupled GPR scans collected over section A at the Virginia Smart Road, which was presented in Figure 4-21. As for the plot of Figure 4-50, the computed power cepstrum was linearly scaled to reveal the deep reflections, which have significantly lower amplitudes than the peak at the origin representing the surface reflection. Figure 4-51 shows the continuous reflection of the WS/BM-25.0 interface originally masked in the raw data of Figure 4-21. In fact, as could be seen in Figure 4-51, the continuous WS/BM-25.0 interface reflection is at an identical depth level as the reflection of the copper plate located at the same layer interface (reflection 1 in Figure 4-21). On the other hand, the BM-25.0/OGDL interface reflection falls within the noise level and, therefore, remains masked after deconvolution.

In summary, this analysis investigated the feasibility of using two homomorphic techniques, namely the complex cepstrum and the power cepstrum, to deconvolve GPR data and, thus, to enhance its depth resolution. It was found that because of the typical overlap between the cepstrum of the incident signal and that of the reflectivity function, the complex cepstrum procedure usually fails to correctly deconvolve GPR data. In contrast, the power cepstrum technique was found to give more accurate results, with the exception of when the original interface reflections have low amplitudes. Moreover, it was found that the power cepstrum
results should generally be scaled using an increasing scale factor in order to balance between
the high amplitude of the peak at the origin and the amplitude of the remaining peaks
Corresponding to the wanted interface reflections.

Figure 4-51: GPR Signal after Homomorphic Deconvolution, Showing Copper Plate Reflections at the: (1) WS/BM-25.0, (2) BM-25.0/OGDL, (3) OGDL/21A, (4) 21A/21B, and (5) 21B/Subgrade Interfaces

4.2.3.8 Iterative decomposition of reflected signals

In contrast to all of the depth resolution enhancement techniques discussed thus far, iterative decomposition of the GPR reflected signals is the only technique that was used for GPR data deconvolution [16]. This method is based on the fact that the GPR reflected time-domain signal could be modeled, as indicated by equation (C−6) in Appendix C, as the sum of scaled and time-delayed replicas of the incident signal. Hence, if the incident signal is known (as in the case when using an air-coupled GPR system, for example), thin layer reflections would be detected using the following iterative algorithm:

1. Detect the strongest reflected pulse in the GPR reflected signal,
2. Time-delay and scale the incident signal so that it coincides with the detected reflected pulse,
3. Subtract the scaled and time-delayed incident signal from the reflected signal,
4. Repeat at step 1 until the remaining signal falls below a fixed threshold.

Based on the aforementioned algorithm, when an interface reflection is masked by a stronger reflection at its vicinity, the weak reflection would become detectable after subtraction of the strong reflection. This result is illustrated in Figure 4-52, which shows that after removal of the surface reflection in the first iteration, the WS/BM-25.0 reflection became visible. Similarly, after removal of the OGDL/Base reflection in the second iteration, the BM-25.0/OGDL reflection became visible.

![Figure 4-52: Iterative Decomposition of a GPR Reflected Signal](image)

The main drawback of this technique is that it is very dependent on the interface reflection detector used to detect and locate the reflected pulses. In fact, it can be seen from the algorithm presented above that an error in estimating the position of a reflected pulse would lead to the
appearance of spurious reflections due to the subtraction of the scaled and time-delayed incident pulse from the reflected signal where no reflected pulses really exist. Spurious reflections might also appear because of the change of the reflected pulses’ shape due to dispersion while propagating in the pavement layers. Because of this shape change, the subtraction operation would lead to a result different from zero. As depicted in Figure 4-52 for the third iteration, even a small change in the shape of the reflected pulse caused spurious reflections to appear. These reflections might have amplitudes at the same level as the real reflections and, therefore, would be mistakenly selected as reflected pulses in the subsequent iterations.

4.2.3.9 Comparison between the different depth resolution enhancement techniques

The performance parameters introduced in 4.2.3.3 - signal to noise ratio reduction ($SNR_{red}$), square error ratio ($SER$), and pulse compression ratio ($PCR$) - could be used as criteria to find the best GPR depth enhancement technique among all those studied. This analysis was conducted on a large set of GPR data collected from the different sections of the Virginia Smart road during different periods of time. For each analyzed scan, the $SNR_{red}$, $SER$, and $PCR$ were computed after the studied deconvolution method was applied. The results were then averaged to yield a single value of the performance parameters for each deconvolution technique. It should be noted that the design parameters used for each technique were the aforementioned optimal values, which are summarized in the following:

- Inverse filter has an optimal filter length $N_{opt} = 60$,
- Predictive deconvolution using a filter of length $N_{opt} = 60$ and prediction distance $\alpha_{opt} = 1$,
- Pulse shaping filter length $N_{opt} = 95$ and output lag $l_{opt} = 60$.

The results found for the different deconvolution techniques are depicted in Figure 4-53. Because the iterative decomposition technique does not provide a deconvolved output signal similar to the output of all the other techniques, the $SNR_{red}$ and $PCR$ were not applicable for this case; however, the $SER$ was computed based on the detected pulse locations. As could be seen in Figure 4-53, the inverse filter, predictive deconvolution, and pulse spiking performed identically when the $SER$ and $PCR$ were compared. Nonetheless, the latter technique suffered from the highest SNR reduction compared to the other two (almost double). This is mainly due to the high frequency signals introduced by the spiking operation. The power cepstrum method
gives a slightly higher SER than the previous techniques; however, it achieves an SNR\textsubscript{red} of zero and a much higher PCR. Finally, the iterative decomposition technique is found to have the highest SER among all the studied techniques. Hence, it could be concluded that the power cepstrum is the best technique that could be used for GPR depth enhancement.

![Comparison between the Different Deconvolution Techniques](image)

**Figure 4-53: Comparison between the Different Deconvolution Techniques**

### 4.2.3.10 Summary

In summary, this analysis showed the importance of increasing the depth resolution of a GPR system during the data analysis phase in order to provide reliable thickness estimates. It was found that the power cepstrum technique is the most effective—in the sense that it would provide a high pulse compression while not severely affecting the performance of the subsequent data analysis stages. All the other deconvolution techniques, which were based on Wiener filtering, had lower performances in part because Wiener filters need infinite history for their design to get accurate correlation functions. It should be noted, finally, that noise effects were neglected for the derivation of the various filters used in this analysis. However, in practice, if the noise level is high, then the performance of all these deconvolution techniques would degrade.
4.3 Detection of Layer Interface Reflections

As was mentioned in the previous sections, GPR signals collected over pavements consist of time-delayed pulses resulting from the reflections of the incident GPR pulses at the different layer interfaces and/or distresses located within the pavement system. In contrast to ultrawideband radars, where the incident pulses get distorted when propagating from the transmitter to the receiver [38], the shapes of the GPR reflected pulses do not significantly differ from the original incident pulse shapes. This is due to two main points: firstly, the propagation paths are relatively short for GPR (sum of layer thicknesses usually does not exceed one meter), and secondly, the reflecting targets (layer interfaces) are large when compared to the probing signal wavelength. The only distortion that can occur in the GPR reflected pulses usually results from the overlap of adjacent reflections when the considered layer is thin compared to the incident pulse width, as was shown in 4.2.3. Thus, in the time domain, the reflected GPR signal \( y_r(t) \) is given by the following equation, which is derived from equation (C-6), Appendix C, by adding a noise component:

\[
y_r(t) = \sum_{i=0}^{N-1} A_i x(t - \sum_{j=0}^{i} t_j) + n(t) \tag{4-58}
\]

where \( x(t) \) is the incident GPR pulse, \( N \) is the total number of layers composing the pavement, \( A_i \) \((i = 0, 1, \ldots, N-1) \) are the relative amplitudes of the reflected pulses, \( n(t) \) is additive noise, and \( t_j \) \((j = 0, 1, \ldots, N-1) \) are the reflection time-delays or two-way travel times. The incident GPR signal \( x(t) \) could be assumed known, as was specified in 4.2.2.

In order to analyze GPR data, the individual reflected pulses should be detected and their exact time-delays within the reflected signal estimated. Detection of layer-interface reflections represents one of the most important stages of GPR data analysis in the sense that its outcome considerably affects the accuracy of the results of the overall GPR system. Erroneous detection of the reflected pulses within the GPR signal results in reporting an incorrect number of layers and, therefore, incorrect layer thicknesses. This type of error can be divided into two categories:
• Detecting a reflected pulse where no actual pulse is present. In radar terminology, this detection error is referred to as a type I error or false alarm [56]. For GPR data, this error usually occurs when the noise level becomes comparable to the level of the reflected pulses.

• Not detecting a reflected pulse where an actual pulse is present. In radar terminology, this detection error is referred to as a type II error or missed detection [56]. For GPR data, this error could be caused either by an overlap of the reflected pulses or by a low reflected pulse level in comparison to the noise level.

The performance of any signal detector is usually measured by the probabilities of detection and false alarm. A good detector should have a high probability of detection while keeping the probability of false alarm at a minimum. When the detected signal is composed of multiple pulses, another performance parameter should be accounted for. This parameter is the resolution with which the pulses are detected. The resolution of a detector represents the minimum time-delay between any two consecutive pulses that ensures their successful detection.

On the other hand, incorrectly estimating the time-delay of the reflected pulses within the GPR reflected signal leads to incorrect estimates of the layer thicknesses and/or distress depths. In fact, based on equation (2-30), which gives a layer thickness as a function of the time-delay, it could be seen that a relative error $\Delta t/t$ in the time-delay estimate would result in the same relative error $\Delta d/d$ in the layer thickness. The performance of any time-delay estimation technique could be measured by its accuracy (or inversely its bias), which quantifies the degree of conformity of the estimated time-delay value to its true value.

For GPR data analysis, detection of interface reflections could be viewed either as a detection problem or as an estimation problem. A comparison between these two processing schemes is presented in the following sections, and then the proposed interface reflection detection technique is detailed. The reflected GPR signal $y_r(t)$ is assumed to be filtered in this case and to have its coupling-pulse removed, as was detailed in section 4.2, but not to be deconvolved. Since it was previously shown that deconvolution reduces the SNR of the processed signal, detection of interface reflections from the deconvolved signals are studied separately in 4.3.4.
4.3.1 Detection of interface reflections

In order to detect all of the reflected pulses present within the GPR signal without prior knowledge about their number or locations, the time domain signal \( y_r(t) \) could be divided into a set of range cells – each with a width equal to the width of the assumed known incident pulse \( x(t) \). This technique is typically used for radar data processing and is known as range-gating [6]. After carrying out range-gating, each range cell is separately tested for the presence of a reflected pulse. Since the GPR reflected signal \( y_r(t) \) is composed of attenuated and time-delayed replicas of the known incident pulse \( x(t) \), signal detection within the range cells could be performed using any of the common detection techniques of a known signal in noise [56]. In this case, when a reflected pulse is successfully detected, its range cell location is used as an estimate of its time-delay. Hence, using this technique, the reflected pulses would be detected with low time-delay accuracy and resolution, which are equal to the incident pulse width.

To increase the accuracy and resolution of time-delay estimates of the detected pulses, the range cells, obtained by range-gating, could be selected in a way that the adjacent cells overlap each other. Using cell overlap, the highest possible time-delay estimation accuracy and resolution would be attained when any two successive range cells are delayed by one sample (or in other words, any two adjacent range cells have a length \([M_i-1]\)-sample overlap, where \(M_i\) is the width of the incident pulse \( x(t) \)). Although this overlap would guarantee high time-delay estimation accuracy and resolution, it would also cause the detection of the same reflected pulse within multiple adjacent range cells, thus increasing the number of false alarms. In order to solve this multiplicity problem, the range cell that has the highest probability of a pulse’s presence should be selected as the location of the detected reflected pulse. All the adjacent cells in which a reflected pulse is also detected should then be discarded. The discarded cells are usually located within a pulse width from the selected time-delay estimate of the detected pulse. Consequently, this technique would provide an accurate time-delay estimate of the reflected pulse, but it would not increase the time-delay resolution (i.e., if two reflected pulses are next to each other, the weakest among them would be discarded).

Previous studies regarding the problem of detecting and estimating the time-delays of multiple pulses in presence of noise have shown that the abovementioned technique could be implemented optimally by passing the reflected signal through a matched filter (MF) [57]
designed based on the known incident pulse $x(t)$ [56]. The output signal of the matched filter would then peak at times corresponding to the time-delays $t_i$. The amplitudes of these peaks can be considered as good estimates of the amplitudes $A_i$. Thus, the problem of time-delay estimation is transformed into a simpler problem of local maxima search on the output signal of the MF. It should be noted that for this search to be successful (i.e., detection of all the reflected pulses present within the reflected signal), the time-delay between any two successive pulses should be greater than the duration of the autocorrelation function of the incident signal $x(t)$ [57]. If this condition is not satisfied, the peaks in the MF output would be obscured by the stronger reflections in their vicinity. Hence, for the MF detection technique, the time-delay accuracy is equal to the sampling period; however, the time-delay resolution is equal to the width of the incident signal $x(t)$. This low resolution would lead to a high missed-detection rate – especially for pavements composed of relatively thin layers such as flexible pavements.

In summary, it was shown that simple detection of the reflected pulses within the GPR signal would not always succeed since the condition that the minimum time-delay separation between any two consecutive reflected pulses has to be greater than the incident pulse width, is rarely verified for field GPR data. Because this minimum time-delay separation constraint is derived from the fact that the detection problem is converted, in this case, into a search problem for local peaks, use of any detection technique other than the MF would be subject to the same constraint.

### 4.3.2 Estimation of interface reflection time-delays

With the aim of increasing the time-delay resolution of the detected pulses present within the GPR reflected signal, layer interface detection could be viewed as an estimation problem instead of a traditional detection problem. With the estimation approach, a given number of pulses are assumed to be present within the GPR reflected signal, and an optimal estimator is used to estimate their respective time-delays. It is known in estimation theory that one of the most optimal (i.e., unbiased with minimum variance) and realizable estimators is the maximum likelihood estimator (MLE) [40]. The MLE is defined as the estimator that maximizes the likelihood ratio, which is the ratio of the probability of presence of the processed signal to the probability of its absence (i.e., probability of noise only) [58]. Since there are no restrictions on
the estimated time-delays with an estimation approach except that they should be within the duration of the reflected GPR signal, this technique would provide high time-delay resolution and accuracy.

Several studies concerning the time-delay estimation of multiple pulses within a signal have shown that, in the case of white Gaussian noise, the MLE is equivalent to a nonlinear least-squares fitting (NLS) problem [59]. This property becomes invalid if the noise is colored, but the NLS would still give accurate time-delay estimates [60]. For time-delay estimation purposes, least-squares fitting could be performed either in the time domain or in the frequency domain. In the time domain, least-squares fitting is achieved by minimizing the error $I$ of equation (C–7), Appendix C, where the unknown parameters are the relative amplitudes $A_i$ and the time-delays $t_i$. Because the time-delays are unknown, the solution presented in Appendix C is invalid for this case, and the problem is highly nonlinear, with no trivial solution.

In the frequency domain, the reflected signal could be obtained by combining equations (4-5) and (2-29) and simplifying the result to obtain the following equation:

$$
Y_r(\omega) = X(\omega) \sum_{i=0}^{N-1} \alpha_i \exp\left(-j\omega t_i \sum_{k=0}^{i} \alpha_k \right) + N(\omega)
$$

(4-59)

where:

- $Y_r(\omega)$, $X(\omega)$, and $N(\omega)$ are the discrete Fourier transforms (DFT) of the reflected signal $y_r(t)$, incident signal $x(t)$, and noise signal $n(t)$, respectively. The angular frequency $\omega$ is in this case discrete.
- $N$ is the total number of layers,
- $\alpha_i$ are the reflection amplitudes that might be complex or real valued,
- $t_k$ are the time-delays to be estimated. It is worth noting that the time-delays $t_k$ represent the time differences between consecutive reflected pulses, as shown in Figure C–1, Appendix C. The time-delay $t_0$ of the first reflection (i.e. surface reflection) could be chosen zero.
If equation (4-59) is divided by $X(\omega)$, then the quantity $Y_r(\omega)/X(\omega)$ would have the form of a sum of complex exponentials with random amplitudes and frequencies (frequency and time should be swapped to put equation (4-59) in the classic form). In this form, the amplitudes and time-delays could be estimated using any of the known spectral estimation techniques, such as MUSIC, Prony’s method, or ESPRIT [52]. The major flaw with this procedure, however, is that when dividing by $X(\omega)$, the resulting noise term $N(\omega)/X(\omega)$ will become colored even if $N(\omega)$ were white [59]. Hence, the spectral estimation techniques would not work in this case unless $X(\omega)$ is assumed to be constant within the bandwidth of interest, as was done in [61]. This condition is usually not verified for real GPR data since $X(\omega)$ usually varies considerably within the GPR bandwidth, as was shown in 4.2.2.

In the frequency domain, the error term that should be minimized in the least-squares sense is given as follows:

$$I = \sum_{\alpha} \left| Y_r(\omega) - X(\omega) \sum_{i=0}^{N-1} \alpha_i \exp \left( -j \omega \sum_{k=0}^{i} t_k \right) \right|^2$$

(4-60)

where $Y_r(\omega)$ is the DFT of the measured GPR reflected signal. It is evident from equation (4-60) that minimizing the error $I$ versus $\alpha_i$ and $t_k$ is a highly nonlinear problem because of the presence of the unknown time-delays $t_k$ inside the exponential term. This minimization usually involves a search for the minimum of a function in an $N$-dimensional space of the possible time-delay values $t_k$ [57]. Because the number of layers $N$ is usually unknown, the problem is further complicated.

The different solutions proposed for this NLS problem were mainly focused on reducing the size of the search space. For example, Li et al [59] were able to reduce the $N$-dimensional search space into a set of $N$ one-dimensional search spaces over all the possible values of the time-delays. In their simulations, the variance of the developed estimator was found to approach the Cramer-Rao Lower Bound (CRLB) [40]. However, it should be noted that because of the large number of the possible time-delay values, the search space remains, in this case, relatively large and thus the technique is computationally intensive. When the number of layers $N$ is
unknown, the estimation procedure is usually iteratively repeated by increasing $N$ starting from 1 until a stopping condition (such as invariance of the error $I$ with increasing $N$) is reached.

In summary, using an estimator to determine the time-delays of the multiple reflected pulses present within the GPR signal would usually provide accurate and high-resolution time-delay estimates at the expense of high processing times. The processing time is further increased because the number of pulses (or layers) $N$ is generally unknown.

### 4.3.3 Proposed technique

Appendix C shows that if the reflection time-delays are known, then the reflection amplitudes are optimally found using least-squares fitting, which by definition guarantees a minimum error between the measured signal and the modeled signal. Thus, if a maximum number of possible reflected pulses and their corresponding time-delays are found using a given detector, then least-squares fitting can be used to select the optimum set of reflection time-delays (among all detected pulses) that ensures a minimum error between measured and modeled signals in the least-squares sense. Thus the modeled signal found can then be subtracted from the measured signal to yield a difference signal composed mainly of weak reflections originally masked by the stronger reflections in their surroundings. The detection process can then be repeated on the difference signal to find the time-delays of the weak reflections. These time-delays are then combined with the first set of time-delays to obtain the time-delays of all the reflected pulses. This stage can then be followed by another least-squares fitting operation to find the optimum set of reflection time-delays that yields a minimum square error between the measured signal and the modeled signal. The whole process could then be repeated until no new reflections are detected within the difference signal. The combined detection/estimation procedure can be summarized by the following algorithm:

1. Assign the measured reflected signal $y_r(t)$ to $d(t)$,
2. Use a detector to detect all possible reflected pulses within the signal $d(t)$,
3. Use the time-delays of the detected pulses to find the optimum synthesized reflected signal $y_{rs}(t)$ that approximates the signal $y_r(t)$ in the least-squares sense as shown in Appendix C,
4. Compute the difference signal $d(t) = y_r(t) - y_{rs}(t)$,
5. Repeat from step 2 until no more pulses are detected in the signal \( d(t) \),

6. Combine all reflection time-delays found by the previous steps and determine the optimum set that yields a minimum least-squares error between the measured reflected signal \( y_r(t) \) and the synthesized signal \( y_{rs}(t) \). This step is mainly introduced to minimize the false alarm rate.

7. Determine the reflection amplitudes \( A_i \) from the optimum least-squares fit found.

The proposed technique combines the advantage of high processing speeds for the detection procedures and high accuracy and high resolution for the estimation procedures. It should be noted that this algorithm is similar to the deconvolution algorithm presented in 4.2.3.8. The main dissimilarity, however, is that for the detection algorithm, the difference signal \( d(t) \) represents the minimum error between the measured signal and the synthesized signal. In contrast, the difference signal \( d(t) \) for the deconvolution algorithm is obtained by subtracting a scaled and time-delayed replica of the incident signal from the measured GPR signal \( y_r(t) \), for each detected pulse. The scaling operation is based on the maximum amplitude of the detected pulse. Hence, if the amplitude of the detected pulse was affected by the weaker reflections in its vicinity, spurious reflections would appear in the difference signal after the subtraction operation, as was shown in 4.2.3.8. This problem is minimized in the detection algorithm since the amplitudes of the synthesized pulses are obtained based on curve fitting in the least-squares sense.

According to the aforementioned algorithm, least-squares fitting is used to correct any detection errors made by the detector; therefore, the detection stage can be realized by different detector types, even if they do not have high performance. It should be noted, however, that the only errors that would be corrected by the least-squares stage are the false alarms. Thus, the detector should have a high probability of detection without severe constraints on the probability of false alarm. For this reason, different types of detectors can be used in the detection stage of the proposed technique.

4.3.3.1 Threshold detector

In detection theory, a detector is generally used in a decision scheme to decide which hypothesis among several alternatives is in effect at a particular time. This general problem
could be simplified in the case of binary detection where the decision is made between two hypotheses only. For GPR data analysis, the detection problem is binary by nature: the considered hypotheses are the presence or absence of a reflected pulse within a segment of the analyzed GPR signal. In this case, detection errors are mainly caused by noise corrupting the reflected signals, either by masking weak reflections (missed detection) or by creating false reflections (false alarm). The performance of a detector is measured by the probability of detection $P_d$ (or percentage of successful detections) and the probability of false alarms $P_f$ (or percentage of false detections). The probability of missed detections $P_m$ is related to the probability of detection $P_d$ by the following equation:

$$P_m = 1 - P_d$$  \hspace{1cm} (4-61)

Thus, a high probability of detection is equivalent to a low probability of missed detections and vice versa. A good detector should have a probability of detection near one and a probability of false alarm near zero. Since these limits are theoretical, a more practical criterion for defining the goodness of detectors used for radar (or GPR) data processing is the Neyman-Pearson criterion [56]. With this criterion, the probability of false alarm is chosen as large as tolerable, and the detector is designed to maximize the probability of detection (or, alternatively, to minimize the probability of missed detections).

The simplest detector that can be used to detect a signal embedded in noise is a threshold detector. A threshold detector typically compares the level of the analyzed signal to a fixed threshold; then it declares the signal present or absent depending on whether or not the analyzed signal’s level exceeds the specified threshold. This detection technique assumes that the minimum detectable reflections are above noise level, which can usually be used as a suitable value for the threshold. Yet, if the chosen threshold is very low, false alarms would occur whenever the noise increases unexpectedly.

For the case of GPR data, since the reflected pulses might have different polarities than the incident pulse (depending on the dielectric properties of the pavement materials), two thresholds should be used: a positive threshold to detect “positive” reflections and a negative threshold to detect “negative” reflections. Due to the shape of the GPR pulses, this approach would lead to the detection of at least three separate peaks (one positive peak surrounded by two negative
peaks or vice versa) corresponding to each reflected pulse, as illustrated in Figure 4-54. Although the obtained peaks can be grouped together to form the actual detected pulses, this procedure is cumbersome because a different number of peaks are detected for each pulse, as shown in Figure 4-54. For example, only one peak is detected for the relatively weak reflection around 10ns; however, for the leading stronger reflections, three peaks are detected. Hence, it would be difficult to decide, in this case, to which pulse a particular peak belongs, especially if the reflected pulses are close to each other.

![Figure 4-54: Typical Detections Obtained from a Threshold Detector](image)

The multiple-peaks problem could be eliminated if an envelope detector were used before the threshold detector. Envelop detectors are commonly used in communication systems that use Amplitude Modulation (AM). The role of an envelope detector is to extract the modulation envelope (containing the transmitted information) of the received signal while rejecting the high frequency carrier signal [63]. Even though GPR signals are composed of ultra-wide band pulses that do not have a carrier frequency, an envelope detector applied to this type of signals would still extract the envelope describing the variations of their amplitude. Mathematically, the real envelope $x_e(t)$ of any real signal $x(t)$ is defined as the magnitude of the corresponding analytic signal, as given by the following equation [63]:

$$x_e(t) = |x(t)|$$
where $x_a(t)$ and $\hat{x}(t)$ are the analytical signal and Hilbert transform [43] of the signal $x(t)$, respectively.

With an envelope detector, a pulse is declared detected if the amplitude of the analyzed signal’s envelope exceeds a fixed threshold selected above noise level. Since the envelope of a signal is always positive by definition, only a positive threshold value should be used. Figure 4-55 shows the output of an envelope detector for a typical GPR signal along with the reported detected pulses. As illustrated in the figure, only three pulses are detected instead of the seven pulses found when using a threshold detector only. An algorithm that can be used to find the local maxima (corresponding to the reflected pulses) of the envelope of the analyzed GPR signal is as follows:

1. Compute the envelope of the analyzed signal,
2. Find the maximum value of the envelope,
3. Compare the maximum value found to the selected threshold. If the maximum value is less than the threshold, then terminate the detection procedure; otherwise, take the time-delay of the maximum value as the time-delay of the detected pulse,
4. Set the envelope values around the obtained time-delay to zero for a length equal to the width of the incident pulse,
5. Repeat at step 2.

For radar data analysis, the threshold value used by the threshold detector is generally selected based on the Neyman-Pearson criterion. With this criterion, the probability of false alarm is chosen as large as tolerable by the application. The detector is then designed to maximize the probability of detection. If the noise embedded in the GPR signal is assumed to be white, Gaussian, and has a variance $\sigma^2$, then at the output of the envelope detector it will have a Rayleigh probability density function (PDF) [6] given by the following equation [63]:

$$p(u) = \frac{u}{\sigma^2} \exp\left(\frac{-u^2}{2\sigma^2}\right), \text{ with } u \geq 0$$ (4-63)
When noise level exceeds the selected threshold value, a false alarm error is produced. Therefore, the probability of false alarm is the probability that noise level exceeds the threshold. This probability can be found from the PDF equation (4-63) as follows:

\[ P_f = p(v_t < u < \infty) = \int_{v_t}^{\infty} \frac{u}{\sigma^2} \exp\left(-\frac{u^2}{2\sigma^2}\right) du = \exp\left(-\frac{v_t^2}{2\sigma^2}\right) \]  

(4-64)

where \( v_t \) is the selected threshold. From the previous equation, the detector threshold can be found as a function of the probability of false alarm according to the following equation:

\[ v_t = \sqrt{-2\sigma^2 \log P_f} \]  

(4-65)

Equation (4-65) shows the dependence of the threshold used in the detection procedure on noise power. Typically, when noise power increases, the threshold increases accordingly so that the probability of false alarm \( P_f \) stays within a tolerable range. Practically, the noise variance \( \sigma^2 \) could be estimated from the processed GPR reflected signal by assuming that a segment at the end of the signal is composed of noise only (like in the case of Figure 4-55 for the signal part.)
after 12ns, for example). This assumption is usually valid for GPR data collected over pavements since a “buffer zone” is usually added to the range of the deepest detectable layer to ensure that all interfaces would be detected if the layer thicknesses increase unexpectedly [64].

### 4.3.3.2 Matched filter detector

An optimal detector that can be used to decide between the presence and absence of a known signal \( x(t) \) embedded in white Gaussian noise of variance \( \sigma^2 \) can be derived from the likelihood ratio test (LRT). The likelihood ratio is defined as the ratio of the probability of the known signal’s (to be detected) presence to the probability of its absence. The LRT is given by the following equation [56].

\[
\Lambda(\mathbf{r}) = \frac{p_1(\mathbf{r})}{p_0(\mathbf{r})} > S_t
\]  

where \( \Lambda(\mathbf{r}) \) is the likelihood ratio, \( p_1(\mathbf{r}) \) and \( p_0(\mathbf{r}) \) are respectively the probabilities of presence and absence of the known signal \( x(t) \), \( \mathbf{r} \) is the vector representing the tested signal (the elements of \( \mathbf{r} \) are the different samples \( r(t) \) collected over time), and \( S_t \) is a threshold determined based on the Neyman-Pearson criterion. After computing the likelihood ratio \( \Lambda(\mathbf{r}) \) for the measured signal \( \mathbf{r} \), the signal \( x(t) \) is decided to be present in \( \mathbf{r} \) if \( \Lambda(\mathbf{r}) > S_t \); otherwise it is decided to be absent.

Using the Gaussian property of the noise, the probabilities \( p_0(\mathbf{r}) \) and \( p_1(\mathbf{r}) \) can be found, respectively, according to the next equations [56].

\[
p_0(\mathbf{r}) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left( -\frac{1}{2\sigma^2} \mathbf{r}^\top \mathbf{r} \right)
\]  

\[
p_1(\mathbf{r}) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left( -\frac{1}{2\sigma^2} (\mathbf{r} - \mathbf{x})^\top (\mathbf{r} - \mathbf{x}) \right)
\]

where \( N \) is the length of vector \( \mathbf{r} \), \( \mathbf{x} \) is the vector whose elements are the samples of \( x(t) \), and the superscript \( T \) is the transpose operation. Substituting \( p_0(\mathbf{r}) \) and \( p_1(\mathbf{r}) \) in equation (4-66), taking its logarithm (which is a monotonic increasing function that does not affect the LRT), simplifying the result, and embedding all the constants into \( S_t \) yields a simplified version of the
detector, commonly known as the sufficient statistics \( l(r) \), which is given by the following equation [56].

\[
l(r) = r^T x > S_t
\]  

(4-69)

Hence, the optimal detector, known as a correlator detector, found based on the LRT, correlates the measured signal \( r \) with the known signal \( x \) and then compares the result to a fixed threshold \( S_t \). The signal \( x \) is declared detected or not, within the signal \( r \), depending on whether the result of the correlation is higher or lower than the threshold. The threshold \( S_t \) is determined from the maximum tolerable probability of false alarm \( P_f \) based on the Neyman-Pearson criterion. The probability of false alarm \( P_f \) is given in this case by the following equation [56].

\[
P_f = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{S_t/(\sigma^2 E)^{1/2}} \exp \left( -\frac{u^2}{2} \right) du = \text{erfc} \left( \frac{S_t}{\sqrt{\sigma^2 E}} \right)
\]  

(4-70)

where \( E \) is the total energy of the known signal \( x(t) \) and \( \text{erfc}(u) \) is the complementary error function defined as \( \text{erfc}(u) = 0.5 - \text{erf}(u) \), with \( \text{erf}(u) \) is the error function defined in [65]. The threshold \( S_t \) can then be found as a function of the probability of false alarm \( P_f \) as follows.

\[
S_t = \text{erfc}^{-1}(P_f) \sqrt{\sigma^2 E}
\]  

(4-71)

where \( \text{erfc}^{-1}(u) \) is the inverse complementary error function. As for the threshold detector, the threshold found for the correlator detector increases with the noise power in order to keep the false alarm rate within tolerable limits.

If \( h(t) \) is the impulse response of a filter defined as the time reversal of the known signal \( x(t) \), then it could be shown that the correlation operation of equation (4-69) is equivalent to a filtering operation. The impulse response of the filter \( h(t) \) is given by the next equation.

\[
h(t) = x(T - t)
\]  

(4-72)

where \( T \) is the duration of signal \( x(t) \) introduced to make the filter \( h(t) \) causal and, therefore, realizable. Since the filter \( h(t) \) matches the signal \( x(t) \), it is known as a matched filter (MF).
Applying the matched filter $h(t)$ to the measured signal $r(t)$ produces the output $y_{MF}(t)$, as given by next equation.

$$y_{MF}(t) = r(t) * h(t) = \sum_{\tau} r(\tau) h(t - \tau) = \sum_{\tau} r(\tau) x(T - t + \tau)$$ \hspace{1cm} (4-73)

It is clear from this equation that the output of the matched filter $y_{MF}(t)$ is equal to the correlation result of equation (4-69) at time $T$. Hence, the optimal detector of equation (4-69) can be implemented in the form of a matched filter. Under this form, the detection decision (i.e., comparison to the threshold $S_i$) is made based on the matched filter output sampled at time $T$.

Finally, it can be shown that at time $T$ the matched filter output exhibits a maximum signal to noise ratio [56]. Hence, the output of the matched filter would have a maximum (or a minimum, depending on whether the polarity of the detected signal is reversed compared to the original signal or not) at time $T$. Using these properties of the MF, the following algorithm for detecting multiple reflected pulses within the GPR reflected signal $y_r(t)$ could be derived:

1. Compute the matched filter impulse response $h(t)$ as the time reversal of the incident GPR signal $x(t)$,
2. Filter the measured GPR signal $y_r(t)$ using the matched filter $h(t)$ to obtain the filtered signal $y_{MF}(t)$,
3. Find the maximum absolute value of the matched filter output $y_{MF}(t)$,
4. Compare the maximum value found to the selected threshold $S_e$. If the maximum value is less than the threshold, then terminate the detection procedure; otherwise, subtract the width $T$ of the known signal $x(t)$ from the time-delay corresponding to the maximum value of the MF output to find the time-delay of the detected pulse,
5. Set the detected pulse in $y_r(t)$ to zero,
6. Repeat at step 2.

As with the threshold detector, the noise power $\sigma^2$ used to compute the matched filter detector threshold could be estimated from the reflected signal $y_r(t)$ by assuming that the trailing part of the signal is composed of noise only. It should be noted that the assumption that noise embedded in the reflected GPR signal is zero mean, white Gaussian could be verified.
experimentally from the collected data. Figure 4-56 presents a comparison between the normalized autocorrelation functions (ACF) of the noise extracted from the GPR signal and randomly generated white Gaussian noise. The similarity between the two autocorrelation functions is clearly seen in the figure, which shows that the ACFs peak at zero lag and then converge to zero for all other lags. The ACF’s behavior proves the whiteness of the noise signal and, therefore, the independence between the different samples.

![Figure 4-56: Comparison between the Normalized ACF of White Gaussian Noise and Noise Embedded in the GPR Reflected Signal](image)

4.3.3.3 Performance comparison between the different detectors

Typically, the performance of a detector is evaluated using the receiver operating characteristic (ROC) [56], which gives the detector’s probability of detection versus its probability of false alarm for various values of the SNR. A good detector should have a probability of detection near one and a probability of false alarm near zero even for low values of the signal to noise ratio. In order to get the ROC, the probability of detection should be expressed as a function of the probability of false alarm and of the SNR.
The probability of detection of the threshold detector and that of the matched filter
detector, in case of additive white Gaussian noise, could be derived respectively from [6] and
[56]. The probabilities of detection of both detectors are presented in the next two equations.

\[
P_{d,TH} = 0.5 \left[ -2 \text{erf} \left( \sqrt{2} \left( -\ln P_f - \sqrt{\text{SNR}} \right) \right) + \frac{\exp \left[ -\left( -\ln P_f - \sqrt{\text{SNR}} \right)^2 \right]}{4 \sqrt{2 \pi \text{SNR}}} \right] \\
\left[ 1.25 - 0.25 \frac{\ln P_f}{\sqrt{\text{SNR}}} + \frac{1 + 2 \left( -\ln P_f - \sqrt{\text{SNR}} \right)^2}{16 \text{SNR}} \right]
\]

(4-74)

\[
P_{d,MF} = \text{erfc} \left( \text{erfc}^{-1} \left( P_f \right) - \sqrt{2 \text{SNR}} \right)
\]

(4-75)

where \( P_{d,TH} \) and \( P_{d,MF} \) are the probabilities of detection of the threshold and matched filter
detectors, respectively, \( P_f \) is the probability of false alarm, \( \text{SNR} \) is the signal-to-noise ratio, \( \text{erf}(u) \)
is the error function, and \( \text{erfc}(u) \) and \( \text{erfc}^{-1}(u) \) are the complementary and inverse complementary
error functions, respectively. The error functions have the same expressions as previously.

Figure 4-57 shows a comparison between the ROC of the matched filter and that of the
threshold detector for three values of the signal to noise ratio (0, 5, and 10dB). It is clear from
the figure that the matched filter outperforms the threshold detector for low SNR values.
However, as the SNR increases, the performance of both detectors becomes comparable.
The main reason that the matched filter detector has better performance for low SNR values is that
the filtering operation carried out before the detection decision is made increases the SNR of the
processed signal; hence, the probability of detection is increased and the probability of false
alarm is decreased. In contrast, for high SNR values of the processed signal, the SNR
enhancement introduced by the filtering operation is insignificant for the detection decision (i.e.,
the LRT result would be much higher or lower than the selected threshold). Therefore, the
performance of the matched filter detector and that of the threshold detector become similar for
high SNR values. Finally, it should be noted that in developing the ROC of the matched filter
detector, the detected signal was assumed to be a known signal embedded in noise.
Consequently, any deviation of the detected signal from the shape of the known signal would
result in performance deterioration. Since this assumption is not applicable for the threshold
detector, its performance is not affected by the shape of the detected signal and it depends solely on the SNR.

![Figure 4-57: Comparison between the ROCs of the Matched Filter Detector and Threshold Detector](image)

Practically, it is difficult to utilize the ROC to evaluate the performance of the aforementioned detectors when used to detect and estimate the time-delays of the reflected pulses in GPR signals. The difficulties arise from the following points:

- The correctness of the time-delays reported by the detector cannot be checked since the true time-delays are difficult to find (even visually from the collected scans), especially when overlap occurs between the reflected pulses. This leads to an inability to experimentally estimate the probability of detection.
- There is no control over the SNR of the reflected GPR signals. The only way to change the SNR of the processed signal is to add randomly generated noise to it. This operation would not give an accurate estimate of the detector performance since the processed data would be simulation data rather than real field data.

A more appropriate method for comparing the performance of the detectors when applied to field GPR data could be the use of the square error ratio (SER), defined previously by equation (4-26). The SER represents, in this case, the error between the measured GPR signal \( y(t) \) and the
signal synthesized based on the estimated reflection time-delays and amplitudes. Since this quantity represents an error measure, the lower its value, the higher the performance of the detector. Another parameter that could be considered as a good indicator of the detector’s performance is the number of detected layers $N$. In fact, if the number of layers composing the pavement is known a priori, then the number of detected layers would indicate the occurrence of false alarms if the reported number is greater than the known number and missed detections in the converse case. Nevertheless, for the number of detected layers to be a valid performance parameter, it should be used concurrently with the SER in order to ensure that the detected reflected pulses correspond to real reflections and not to false alarms.

To compare the performance of the threshold detector to that of the matched filter detector, a large set of GPR data collected over different periods of time from the different flexible sections at the Virginia Smart Road were used. For each collected GPR scan, both detectors were applied with various probabilities of false alarm ranging from $10^{-16}$ to 0.4, and the SER and the number of detected layers $N$ were recorded. The results were then averaged over sections with similar number of layers and layer thicknesses. The sections that were considered analogous for this analysis are summarized in Table 4-2. The number of layers presented in this table represents the number of layers with sufficient contrast in their dielectric constants to produce a detectable reflection in the GPR signal. These layers are mainly the HMA layers plus the top base layer (21A or 21B, depending on the section). Since layer thicknesses influence the degree of overlap between the successive reflected pulses and, thus, might affect the performance of the detector, sections with different layer thicknesses were considered dissimilar even if they had an equal number of layers.

Table 4-2: Flexible Sections Used for Detector Performance Evaluation

<table>
<thead>
<tr>
<th>Sections</th>
<th>Number of Detectable Layers</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>3 (WS, BM-25.0, Base)</td>
<td>Mostly thick layers except for WS</td>
</tr>
<tr>
<td>A, B, C, D, J, L</td>
<td>4 (WS, BM-25.0, OGDL, Base)</td>
<td>Mostly thick layers except for WS</td>
</tr>
<tr>
<td>F</td>
<td>3 (WS, BM-25.0, Base)</td>
<td>Relatively thin layers</td>
</tr>
<tr>
<td>G</td>
<td>4 (WS, BM-25.0, SM, Base)</td>
<td>Relatively thin layers</td>
</tr>
<tr>
<td>H</td>
<td>5 (WS, BM-25.0, SM, OGDL, Base)</td>
<td>Relatively thin layers</td>
</tr>
</tbody>
</table>
The variations of the average SER and number of detected layers \( N \) versus the probability of false alarm found with the threshold and the matched filter detectors are presented in Figure 4-58a through 4-58e for the five layer categories of Table 4-2, respectively. The x-axis in these figures represents the probability of false alarm in logarithmic scale. Based on Figure 4-58, it is found that for all the studied cases and with both detectors, the SER increases as the probability of false alarm decreases, whereas the number of detected layers \( N \) decreases and converges to the real number of layers. Hence, a minimum SER between the measured and synthesized GPR signals is equivalent, in all cases, to the detection of a number of layers greater than the real number (i.e., high false alarm rate). This result is similar to what is found in the ROC, where a high probability of detection produces a high probability of false alarm. Therefore, for a correct
Figure 4-58: Comparison between the Performance of the Threshold and Matched Filter Detectors: (a) 3, (b) 4 Thick Layers, (c) 3, (d) 4, and (e) 5 Thin Layers
evaluation of the performance of a detector in the case of GPR data, both the \textit{SER} and the number of layers $N$ should be used jointly.

For the cases of three and four relatively thick layers and the case of three relatively thin layers, the performance results depicted in Figure 4-58a, b, and c, respectively, show that in accordance with what was found in the ROC of Figure 4-57, the matched filter outperforms the threshold detector. In fact, it is seen that for the three cases, the \textit{SER} that results from the matched filter is lower than the one resulting from the threshold detector for all probabilities of false alarm. At the same time, the number of detected layers is higher than the real number of layers (i.e., three or four) for both detectors for high probabilities of false alarm, but it converges to the real number of layers for lower probabilities of false alarm. Thus, when the probability of false alarm $P_f$ is suitably selected, the reflections detected by the matched filter would be better approximations of the real reflections than those detected by the threshold detector. The relatively large difference in the \textit{SER} of the two detectors is a good indicator that some of the interface reflections detected by the threshold detector do not correspond to real reflections, even though the number of detected layers is almost equal for the low probabilities of false alarm. It should be noted that the large difference between the number of layers reported by the matched filter and those reported by the threshold detector for high probabilities of false alarm is mainly due to the way the detection thresholds are chosen for both detectors (equations (4-65) and (4-71)).

In contrast to what was presented by the ROC shown in Figure 4-57, the performance results illustrated in Figure 4-58d and Figure 4-58e show that the threshold detector outperforms the matched filter detector for the cases of four and five relatively thin layers. In fact, these figures show that the \textit{SER} found by the matched filter is higher than the \textit{SER} found by the threshold detector for all utilized probability of false alarm values. In particular, for the four layers case (Figure 4-58d), the \textit{SER} found when the number of layers detected by the matched filter is approximately equal to the correct number of layers (i.e., when $N = 4$ for $P_f \approx 10^{-6}$) is larger than the \textit{SER} found by the threshold detector for the same condition (i.e., when $N \approx 4$ for $P_f \approx 10^{-16}$). Moreover, for the matched filter it is noticed that the number of detected layers decreases below the real number of layers for the low false alarm probability values. This result is an indication of missed detections if the probability of false alarm is not chosen adequately.
At the same time, the number of layers detected by the threshold detector is higher than the real number of layers for high probabilities of false alarm, but it converges towards the true number of layers when the probability of false alarm decreases. The same variations of the number of detected layers are observed for the five thin layers (Figure 4-58c), except that, in this case, the number of layers reported by both detectors (maximum average of 4.25 layers for the MF and 4.30 layers for the threshold detector) is less than the real number of layers (i.e., five) even for high probabilities of false alarm. Hence, both detectors failed in this situation to detect all the layers composing the pavement, yet the threshold detector still shows a better performance than the matched filter for any probability of false alarm value.

The degradation in the performance of the matched filter detector compared to that of the threshold detector in finding the interface reflections from GPR data when the number of layers composing the pavement system increases and the layers become thin could be explained by the breakdown of the assumptions made in the derivation of the matched filter detector. In fact, for the performance of the matched filter to be similar to the theoretical performance, the pulses that compose the reflected GPR signals should not deviate very much from the incident pulse (i.e., the known pulse). This condition is usually more difficult to satisfy when the layers become thin, as was shown in section 4.2.3. In fact, overlap between the reflections from thin layer interfaces would cause distortion of the reflected pulses in the GPR signal. These distortions generally grow more severe as the layers become thinner and their number increases. Nevertheless, when only one thin layer is present, the performance of the matched filter is not degraded much and, indeed, it outperforms the threshold detector (as in the cases of Figure 4-58a and Figure 4-58b). On the other hand, since no restrictions were imposed on the threshold detector, its performance is not very much affected by the presence of thin layers.

To further confirm the degradation of the performance of the matched filter when the number of thin layers within a pavement system increases, GPR data collected over section K at the Virginia Smart Road is used. Section K is composed of two very thin layers (19mm each) as a wearing surface, a thick HMA base layer (244mm), and a relatively thin OGDL layer (75mm). In addition, section K incorporates a thin (5mm) polyvinyl chloride (PVC) geocomposite membrane within the HMA base layer that is used for stress relief. Even though the layers composing this section are thin compared to the incident pulse width and, therefore, would not
produce detectable reflections, they would distort considerably the reflected pulses from the other interfaces and would, thus, alter their shape and deviate it from the incident pulse shape. Figure 4-59 shows the performance results of the matched filter and the threshold detector when they are used with the GPR data collected over section K. Based on this figure, it is noticed that for this section the threshold detector outperforms the matched filter. In fact the $SER$ found for the matched filter, in this case, approximately equals 12% for all the selected probabilities of false alarm, whereas the $SER$ found for the threshold detector is around 4% only. Moreover, the number of layers detected by the matched filter is less than the real number of layers for all probabilities of false alarm, which indicates missed detections. In contrast, the number of layers detected by the threshold detector is slightly higher than the actual number of layers for high probabilities of false alarms, yet it converges to the real number for the low probability values. Thus, it could be concluded that utilizing the threshold detector for this situation would achieve much better results than the matched filter.

In summary, the aforementioned analysis showed that depending on the thicknesses and the number of the layers composing a pavement system, various detectors should be used to
locate the interface reflections. In particular, if most of the pavement layers are relatively thick, a matched filter detector would be the optimal detector to use. However, if most of the layers are relatively thin, a threshold detector should be used. For detection purposes, the pavement layers are considered thin or thick by comparison to the GPR range resolution given by equation (2-30). Finally, it should be noted that with both detectors, the probability of false alarm parameter should be adequately chosen in order to be able to detect the correct interface reflections and, thus, the correct number of layers. If the real number of layers is a priori known, the probability of false alarm should be chosen so that the number of layers reported by the detector equals the real number of layers. However, if the real number of layers is unknown, the probability of false alarm should be chosen as the lowest possible value that would yield a number of layers converging to an almost constant value (versus the probability of false alarm).

4.3.4 Detection of reflected pulses from deconvolved signals

In the previous section, it was shown that a matched filter or a threshold detector could be used in a specially developed detection algorithm to find multiple interface reflections in raw GPR signals and to estimate their respective time-delays, even in the case of overlap between the reflected pulses. In this paragraph, the reflected GPR signals will be assumed deconvolved, as presented in section 4.2.3, before applying the detector. The reason for this analysis is that any overlap between the successive reflections would be relatively eliminated before the detection stage is carried out. The deconvolution technique used for this analysis is the homomorphic power cepstrum deconvolution technique described in 4.2.3.7. The power cepstrum technique was found to have the highest performance for deconvolving GPR data than any of the other techniques presented in section 4.2.3.

Since the reflected pulses in the deconvolved GPR signal theoretically do not overlap each other, a classic detector, such as the matched filter, could be used for their detection. The detector should be applied iteratively to detect all reflections present within the signal. At the end of each iteration, the detected pulse should be removed (i.e., the processed signal forced to zero at the time-delay of the detected pulse) in order to detect the remaining pulses. The only problem in the matched filter detector design, for this case, is that the noise corrupting the deconvolved signal becomes colored (i.e., correlated noise samples) and non-Gaussian after
application of the power cepstrum, even if the original noise signal was white Gaussian. Consequently, the matched filter impulse response of equation (4-72) is not applicable. An alternative expression of the frequency response of the matched filter that accounts for the colored noise is given by the following equation [66].

\[ H(j\omega) = \frac{X(-j\omega)}{S_n(\omega)} \exp(-j\omega T) \]  

(4-76)

where \(X(j\omega)\) is the Fourier transform of the signal to be detected \(x(t)\), \(S_n(\omega)\) is the power spectrum of the noise, and \(T\) is the duration of the signal \(x(t)\). This form of the matched filter correlates the processed signal with a version of the known signal distorted by the noise spectrum. This is equivalent to pre-whitening the deconvolved signal prior to applying the matched filter. Practically, the signal to be detected \(x(t)\) is taken as the power cepstrum of the incident GPR signal. The noise power spectrum \(S_n(\omega)\) used to develop the matched filter is estimated from the processed signal by assuming that the trailing portion of the signal is composed of noise only, as was done when estimating the noise power in the case of the threshold and matched filter detectors.

The same parameters used to evaluate the performance of the threshold and matched filter detectors could be used to assess the performance of detecting the reflected pulses based on the deconvolved GPR signals. These parameters are the square error ratio \(SER\) and the number of detected layers \(N\). To estimate these two parameters in the case of detecting the reflected pulses from the deconvolved GPR signal, the same data set used to evaluate the performance of the threshold and matched filter detectors was processed by the cascade of the power cepstrum and the colored-noise matched filter detector. The average square error ratio \(SER\) and the average number of detected layers \(N\) found by this detection technique for the different layer categories of Table 4-2 and for various probability of false alarm values are depicted in Figure 4-60a and b, respectively. According to Figure 4-60a, the minimum \(SER\) found by this technique is approximately 3% for all probability of false alarm values. This \(SER\) value was found for the section composed of three mostly thick layers. It is seen that this value is higher than the \(SERs\) found for the matched filter and threshold detectors for all the layer categories, as presented in section 4.3.3.3. At the same time, Figure 4-60b shows that the number of layers found for each
layer category and for any probability of false alarm value is higher than the real number of layers composing the pavement, except for the five-thin-layer category where the number of detected layers is less than the actual number.

The SER and number of detected layers results illustrate that the detection after deconvolution technique is worse than the threshold or matched filter detectors. In fact, on the
one hand, the relatively high number of detected layers $N$ proves that this detection technique suffers from a high false alarm rate. On the other hand, the relatively high $SER$ indicates a high missed detection rate. Combining these two results proves that the analyzed detection technique suffers from a high detection error rate for all the studied layer categories and for all the probability of false alarm values. Accordingly, the threshold and matched filter detectors used in the detection algorithm presented in 4.3.3 would be better alternatives for detecting overlapped pulses than the detection based on the deconvolved GPR signals.

The main reasons that the matched filter and the threshold detectors outperform the detection from the deconvolved GPR signals could be summarized as follows:

- The deconvolved GPR signals were assumed to have non-overlapping pulses. This assumption usually breaks down if the pavement system is composed of thin layers, especially if the deconvolved pulses are not ideal spikes (delta functions). On the contrary, as was found based on field GPR data, a deconvolved pulse has a small width different from zero. The overlap between the deconvolved pulses can usually cause missed detections if two or more consecutive reflections are considered as a single reflection.

- Noise effects were neglected in the derivation of the power cepstrum deconvolution technique. Yet if noise increases unexpectedly, it would produce some high amplitude spurious pulses in the power cepstrum of the processed GPR signal. This is due to the additive nature of noise, which would cause the addition of periodic terms (mainly the pavement reflectivity function) to the power cepstrum expression of the reflected GPR signal, as could be seen in equation (4-53). The spurious pulses would then cause the false alarm rate to increase and, therefore, the number of detected layers to increase larger than the real number of layers.

- The noise power spectrum $S_n(\omega)$ estimate might not be very representative of the real spectrum because of the relatively small number of samples used for its determination.

In summary, it is shown that detection of interface reflections based on GPR signals deconvolved by power cepstrum suffers from high detection error rates because of the distortions introduced to the signal during the deconvolution process. These error rates are a little higher
than the errors resulting from the use of a threshold detector or a matched filter detector. Finally, it is worth noting that deconvolving the GPR signals with other deconvolution techniques would yield, at most, the same performance results as when the power cepstrum is utilized since the latter technique was shown to have the highest performance in terms of pulse compression ratio and signal-to-noise ratio reduction.

4.3.5 Summary

In this section, possible schemes for detecting the GPR pulses reflected from pavement layer interfaces were examined. It was shown that because of the high resolution and accuracy needed for detecting the reflected pulses within GPR signals, traditional techniques normally used for radar target detection are not applicable. Instead, an iterative detection algorithm using either a threshold or a matched filter detector was shown to successfully detect the reflected pulses – even in the case of overlap between the reflections resulting from the interfaces of thin layers. Particularly, the detector based on the matched filter was found to have the highest performance when the pavement system is composed of relatively thick layers. On the other hand, the threshold detector was shown to have the highest performance when the number of relatively thin layers within the pavement increases. These two detectors were also found to outperform a matched filter detector applied to a deconvolved version of the GPR signal that, theoretically, does not have any overlapped reflections. The performance of the various detectors was measured based on their ability to synthesize a reflected signal approximately equal to the measured signal, using exclusively the time-delays of the reflected pulses. Moreover, since the GPR data used to evaluate the performance was collected over pavements with a priori known number of layers, the number of detected layers was also used as a performance parameter for the various detectors.

4.4 Dielectric Properties Estimation

As was mentioned in section 2.1, pavement layers are typically composed of mixtures of various construction materials, such as asphalt binder, aggregate, air-voids, water, etc. Even though the layers are physically inhomogeneous, they are seen by GPR as homogeneous materials because of the small size of the layer constituents (usually less than 30mm) compared to the wavelength of the probing GPR signals (greater than 100mm for most pavement materials
Mixture theory [47] could be used in this case to define bulk dielectric properties that characterize the pavement layers at a macro level. According to all mixture theories, the bulk dielectric properties of an inhomogeneous material are typically a combination of the dielectric properties and volume proportions of its individual components. Hence, the dielectric properties of pavement layers would vary depending on the mixtures used. Furthermore, because it is difficult to have a uniform mixture throughout a pavement system, the dielectric properties of the layers would vary depending on the surveyed location within the pavement. The dielectric properties of pavement layers are also greatly affected by environmental conditions. The most important environmental factor is rain and the resulting moisture accumulated within the pavement layers. In fact, since water has a dielectric constant of approximately 81, whereas the other pavement materials have dielectric constants ranging between 3 and 12, water (or moisture) has a significant influence on the bulk dielectric properties of pavement layers. Consequently, it could be concluded that the dielectric properties of pavement layers are usually unknown and difficult to predict.

On the other hand, the dielectric constant of a layer is needed to determine its thickness (equation (2-30)) by estimating the speed of the electromagnetic wave within the material and by measuring its travel time between the two layer interfaces. Monitoring the variations of the dielectric properties along pavements could also be used as a tool to detect subsurface distresses, such as moisture accumulation, HMA stripping, or low layer density due to high air voids. Hence, estimating the dielectric properties of pavement layers is very important for correct GPR data interpretation.

The dielectric properties of pavement layers could be estimated either destructively by direct measurements on pavement cores or nondestructively based on the reflected GPR signals. Since the destructive technique is not very accurate and would introduce perturbations to the pavement structure, it is not treated here. In the following subsections, in-situ dielectric constant estimation from GPR data is discussed. In the first part, the dependence of the dielectric properties of HMA layers on frequency is introduced. Next, different techniques for estimating the dielectric properties from GPR data are presented.
4.4.1 Dielectric properties dependence on frequency

In all the preceding sections, the reflected GPR pulses were assumed to preserve their shape while propagating through the different pavement layers. In other words, the pavement layers were assumed to be non-dispersive and, therefore, their dielectric properties were considered to be independent of frequency. Yet these assumptions contradict the known fact that the dielectric properties of most materials vary considerably with frequency [11]. In the following sections, the effects of the variations of the dielectric properties of different HMA mixes used at Virginia Smart Road will be examined.

4.4.1.1 In-situ HMA dielectric properties estimation approach

The surface mixes investigated in this study were those used at the Virginia Smart Road, namely: SM-12.5D, SM-9.5D, SM-9.5E, SM-9.5A, SM-9.5A with high lab compaction, and SMA-12.5. The main differences between all these mixes include the binder type, binder content, and/or the nominal aggregate size. In order to estimate the dielectric properties of these mixes as placed in the field, GPR scans were performed over the different surface mixes where copper plates were previously placed during the pavement construction. Since copper is assumed to be a perfect EM energy reflector, all the EM energy transmitted by the GPR through the HMA surface mix would be reflected back to the GPR receiver, with the exception of a fraction of the energy that would be lost within the layer due to material loss (dielectric and conduction losses). Comparing the measured GPR reflected signal to the transmitted signal and using a theoretical model for the surveyed system (i.e., HMA layer plus copper plate) yields the dielectric properties of the HMA layer, which are the only unknowns in the theoretical model.

Theoretically, the GPR reflected signal is related to the GPR incident signal in the frequency domain through the overall reflection coefficient of the surveyed system, as given by the following equation:

$$\Gamma_{in}(f) = \frac{F\{y_r(t)\}}{F\{x(t)\}} = \frac{F_y(f)}{F_x(f)}$$  \hspace{1cm} (4-77)

where $\Gamma_{in}(f)$ is the overall reflection coefficient, $F\{\}$ is the Fourier transform, $f$ is frequency, and $y_r(t)$ and $x(t)$ are the reflected signal from the pavement surface and the incident signal.
respectively. The Fourier transform is computed from the time domain signal using a fast Fourier transform algorithm. Figure 4-11 depicts a typical time-domain incident GPR signal, whereas Figure 4-61 shows a typical GPR reflected signal collected over a wearing surface that is backed by a copper plate. It should be noted that because the wearing surface is thin (design calls for a thickness of 38mm) compared to the GPR depth resolution, the reflected signal is composed of an overlap between the reflection from the surface of the pavement and the reflection from the copper plate. Moreover, because the air/HMA surface interface and the copper plate are strong reflectors, EM energy would be trapped in between (i.e., EM energy would go back and forth between the interfaces until all energy is dissipated). This would result in a ringing phenomenon or multiple reflections, which, in turn, would overlap with the other two reflections. Figure 4-62 depicts typical magnitude spectra of the incident and the reflected GPR signals. According to this figure, the GPR bandwidth is found to be between approximately 300 and 2000MHz.

![Figure 4-61: Typical Reflected GPR Signal from a Wearing Surface Backed by a Copper Plate](image)
On the other hand, the overall reflection coefficient $\Gamma_{\text{in}}(f)$ of the considered system (i.e., HMA layer plus copper plate) can be determined theoretically using the multiple reflection model presented in Figure 4-63. It should be noted that the GPR transmitted wave is assumed, in this case, transverse electromagnetic (TEM) propagating normally to the pavement surface. As the electromagnetic wave reaches the road surface, part of the signal is reflected back with a reflection coefficient $\gamma$, and the rest is transmitted through the air/HMA interface with a transmission coefficient equal to $(1+\gamma)$. The reflection coefficient $\gamma$ is given by the following equation [11]:

$$
\gamma(f) = \frac{1 - \sqrt{\varepsilon_r^*(f)}}{1 + \sqrt{\varepsilon_r^*(f)}}
$$

where $\varepsilon_r^* = \varepsilon_r' - j\varepsilon_r''$ is the complex dielectric constant of the HMA layer with the real part $\varepsilon_r'$ representing energy storage and the imaginary part $\varepsilon_r''$ representing dielectric loss. The frequency $f$ is added to show the dependency of the considered parameters on frequency.

The wave then propagates through the HMA layer with a propagation factor $T$ until it reaches its bottom, just above the copper plate. The propagation factor $T$ is given for a layer of complex dielectric constant $\varepsilon_r^*$ and thickness $d$, by this equation [11].
where $\omega$ is the wave angular frequency, $c$ the speed of light in free space, and $j^2 = -1$. The right-hand side of the previous equation was obtained by substituting the square root of the dielectric constant $\varepsilon_r^*$ by a function of the reflection coefficient $\gamma(f)$ obtained from equation (4-78). As soon as the wave hits the copper plate, it reverses polarity (because the reflection coefficient of the copper plate is -1) and starts propagating back to the surface, where it is transmitted through the HMA/air interface with a transmission coefficient $(1 - \gamma)$. Hence, the second reflection coefficient $\gamma_1$ is given as follows:

$$
\gamma_1(f) = \left[1 - \gamma^2(f)\right]T^2(f)
$$

\hspace{5cm} (4-80)

At the HMA/air interface, the remaining part of the wave is reflected back into the HMA layer with a reflection coefficient $-\gamma$. The process repeats until all the transmitted GPR energy is dissipated within the HMA layer. Each time that part of the signal is transmitted through the HMA/air interface, the reflection coefficient $\gamma_i$ at the surface would be given by the following:

$$
\gamma_i = -\gamma^{i-1}(1 - \gamma^2)T^{2i}, \text{ with } i = 1, 2, \ldots, N
$$

\hspace{5cm} (4-81)
where \( i \) is the reflection number and \( N \) is the total number of multiple reflections. The frequency dependence of \( \gamma_i \), \( \chi \), and \( T \) was removed from the last equation for clarity purposes. Thus, the theoretical overall reflection coefficient at the surface of the HMA layer is the sum of all the reflection coefficients, which can be simplified as given by the following equation.

\[
\Gamma_{in}(f) = \gamma + \sum_{i=1}^{N} \gamma_i = \gamma - (1 - \gamma^2)T^2 \sum_{i=0}^{N-1} (\gamma T^2)^i
\]  

(4-82)

The summation term represents the sum of a geometric series with a base \( \gamma T^2 \). Therefore, equation (4-82) could be further simplified to yield the final expression of the overall reflection coefficient at the pavement surface:

\[
\Gamma_{in}(f) = \gamma - (1 - \gamma^2)T^2 \frac{1 - (\gamma T^2)^N}{1 - \gamma T^2}
\]  

(4-83)

Equating equations (4-77) and (4-83) leads to an equation where the reflection coefficient \( \gamma \) is the only unknown:

\[
g(\gamma) = \gamma - (1 - \gamma^2)T^2 \frac{1 - (\gamma T^2)^N}{1 - \gamma T^2} - \frac{F_s(f)}{F_s(f)} = 0
\]  

(4-84)

Equation (4-84) should be solved for \( \gamma \) for the different frequencies within the GPR bandwidth. The HMA complex dielectric constant could then be recovered from the reflection coefficient \( \gamma(f) \) using equation (4-78).

In order to solve equation (4-84), the number of multiple reflections \( N \) occurring within the HMA layer should be estimated. Because of the overlap between the reflected pulses, this task is difficult to achieve based on the time domain signal presented in Figure 4-61. An alternative technique for finding \( N \) could be based on the overall reflection coefficient \( \Gamma_{in}(f) \). This technique consists of comparing the variations of the measured reflection coefficient with those of the computed reflection coefficient, obtained from equation (4-83), for different values of \( N \). Since the comparison, in this case, is based on the overall curve of the reflection coefficient, the dielectric constant used in the computation could be assumed to be constant in the range of

Chapter 4: Ground Penetrating Radar Data Analysis Techniques

168
typical values of the dielectric constant of HMA (for example 4). Figure 4-64a and Figure 4-64b show the variations, versus frequency, of the real and imaginary parts of the overall reflection coefficient $\Gamma_{in}(f)$ measured over a HMA surface mix using equation (4-77). Computing the overall reflection coefficient values for different values of $N$ and for a dielectric constant of 4 showed that the best match between the measured and computed reflection coefficients occurs for $N$ equals 5, as depicted in Figure 4-64. This result was similar for all the six types of HMA layers used.

![Figure 4-64: Comparison between Measured and Theoretical Reflection Coefficient Variations vs. Frequency: (a) Real Part, (b) Imaginary Part](image)
Because of the periodicity introduced by the complex factor $T$, equation (4-84) typically has multiple solutions, depending on the number of multiple reflections $N$, the frequency $f$, the HMA dielectric constant $\varepsilon_r^*$, and the HMA thickness $d$. However, in this case it was found that within the GPR bandwidth and the maximum range of physical complex dielectric constant values, a unique solution exists for each frequency value.

For $N$ equals 5, equation (4-84) does not have a closed form solution; therefore, it has to be solved numerically. A Gauss-Newton algorithm [67] was found to converge rapidly to the required solution. Because the real and imaginary parts of the dielectric constant are continuous functions of frequency, the initial solution of the Gauss-Newton algorithm at any frequency value was chosen to be the exact solution found at the previous frequency value. The initial guess for the first frequency value was arbitrarily chosen to be 4, which is a typical dielectric constant value of HMA.

### 4.4.1.2 Dielectric constant estimation results

Applying the previous technique to GPR data collected from the different HMA mixes at the Virginia Smart Road yields their dielectric constant variations over the GPR bandwidth. The real and imaginary parts of the dielectric constant found are depicted in Figure 4-65a through Figure 4-65f for the SM-9.5A, SM-9.5A with high lab compaction, SM-9.5D, SM-9.5E, SM-12.5D, and SMA-12.5, respectively. A summary of the statistics for the dielectric constant variations (average, standard deviation, coefficient of variance (COV), and maximum) for the different mixes is presented in Table 4-3. According to the results of Figure 4-65 and Table 4-3, the variations of the real part of the dielectric constant within the GPR bandwidth are not large. In fact, the real part’s coefficient of variance is found to vary between a minimum of 3.6% for the SM-9.5D mix to a maximum of 12.7% for the SM-9.5E mix. The standard deviation for the imaginary part is comparable to its average value, which suggests a high coefficient of variance (high fluctuations around the average). However, since the imaginary part’s maximum value is approximately 0.7 for all the mixes, these fluctuations are not very significant. Table 4-3 also presents the loss tangent ($\tan\delta$) of the studied mixes. The loss tangent represents the ratio of the imaginary part of the dielectric constant to its real part. For the different mixes, the loss tangent is found to vary between 0.020 and 0.057, which suggests that the dielectric loss within the HMA layers is low, especially if the layers are not thick.
Figure 4-65: Dielectric Constant Variations for the Different Mixes: (a) SM-9.5A, (b) SM-9.5A with High Lab Compaction, (c) SM-9.5D, (d) SM-9.5E, (e) SM-12.5D, (f) SMA-12.5
Table 4-3: Summary of Dielectric Constant Variations for the Different Mixes

<table>
<thead>
<tr>
<th>Mix</th>
<th>Real Part</th>
<th>-Imaginary Part</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>SM-9.5A</td>
<td>4.9</td>
<td>0.38</td>
</tr>
<tr>
<td>SM-9.5A*</td>
<td>4.3</td>
<td>0.25</td>
</tr>
<tr>
<td>SM-9.5D</td>
<td>4.3</td>
<td>0.16</td>
</tr>
<tr>
<td>SM-9.5E</td>
<td>4.6</td>
<td>0.59</td>
</tr>
<tr>
<td>SM-12.5D</td>
<td>4.7</td>
<td>0.53</td>
</tr>
<tr>
<td>SMA-12.5</td>
<td>4.0</td>
<td>0.48</td>
</tr>
</tbody>
</table>

* high lab compaction.

In order to investigate the effects of using a constant dielectric constant instead of a frequency-dependent dielectric constant on the time-domain GPR signals, the averaged complex dielectric constants of Table 4-3 were used to generate theoretical GPR reflected signals based on the overall reflection coefficient $\Gamma_{in}(f)$ of equation (4-83). After computing the overall reflection coefficient $\Gamma_{in}(f)$ for all frequency values within the GPR bandwidth, the theoretical Fourier transform of the GPR reflected signal $F_r(f)$ was found by multiplying $\Gamma_{in}(f)$ by the measured Fourier transform of the incident signal $F_i(f)$. The theoretical time-domain reflected signal was then found from $F_r(f)$ via an inverse Fourier Transform.

Figure 4-66a and Figure 4-66b depict the measured and modeled reflected GPR signals for the SM-9.5D and SM-9.5E, respectively. These two mixes were chosen for the reflected GPR signal modeling because they represent the mixes with the lowest and highest dielectric constant coefficient of variance (as seen in Table 4-3), respectively. For the SM9.5D, the root mean square error (RMSE) between the measured and reflected signals was approximately 0.8%, whereas for the SM-9.5E, the RMSE was approximately 2.6%. Therefore, even for high dielectric constant fluctuations within the GPR bandwidth, it was found that the signal generated using a constant dielectric constant perfectly fits the measured signal. The dielectric constant variations’ low influence on the GPR signal could be attributed first to the relatively small fluctuations of the dielectric constant within the GPR bandwidth, and second to the small thickness of the layers where the GPR signals propagate.
In summary, the previous analysis showed that for various HMA mixes, the variations of the dielectric constant within the GPR bandwidth are insignificant. Therefore, use of an invariant complex dielectric constant, in this case, would not affect the accuracy of GPR results.

Figure 4-66: Measured and Modeled GPR Reflected Signals: (a) SM-9.5D, (b) SM-9.5E
4.4.2 Estimation of static dielectric constant from reflection amplitudes

To obtain the full advantage of the nondestructive nature of the GPR tool, the dielectric constant of the different pavement layers, needed for layer thickness estimation, should be determined directly from the collected GPR signals. Since the amplitudes of the reflected pulses depend on the contrast of the dielectric constants of the adjacent layers, they can be used to get good estimates of the needed dielectric constant values.

The relationship between the layer dielectric constants and the relative amplitudes of the reflected pulses $A_n$ are given by equation (C−4), Appendix C. The relative reflection amplitudes $A_n$ represent the ratio between the amplitudes of the reflected pulses and the amplitude of the incident signal. As describe in 4.3.3, the amplitudes $A_n$ are readily computed at the interface reflection detection stage via the least-squares fitting.

Substituting the layer thickness of equation (2-30) into the expression of the relative reflection amplitude $A_n$ given by equation (C−4), Appendix C, reduces $A_n$ to the following.

$$A_n = \frac{\sqrt{\varepsilon_{r,n}} - \sqrt{\varepsilon_{r,n+1}}}{\sqrt{\varepsilon_{r,n}} + \sqrt{\varepsilon_{r,n+1}}} \prod_{i=0}^{n-1} \left(1 - \gamma_i^2\right)^{-\eta_0 \frac{\sigma_i \varepsilon_i}{2 \varepsilon_{r,n}}}$$ (4-85)

where $\varepsilon_{r,n}$, $\sigma_n$, and $t_n$ are, respectively, the dielectric constant, conductivity, and two-way travel time at layer $n$ and $\gamma_n$ is the reflection coefficient at layer interface $n$. All these parameters are depicted in Figure C−1, Appendix C.

Based on equation (4-85), the relative reflection amplitude $A_n$ at a layer interface $n$ depends on the dielectric constants of both layers at that interface and the conductivities, dielectric constants, and two-way travel times of the ($n$-1) layers above it. For an air-coupled GPR system, the top layer (i.e., layer 0) is composed of air. Therefore, its dielectric constant $\varepsilon_{r,0}$ equals one and its conductivity $\sigma_0$ equals zero. Substituting these values in equation (4-85) yields the dielectric constant of the first pavement layer (i.e. layer 1):

$$\varepsilon_{r,1} = \left(\frac{1 - A_n}{1 + A_0}\right)^2$$ (4-86)
Similarly, the dielectric constant of the second pavement layer (i.e., layer 2) is determined by solving equation (4-85) for $\varepsilon_{r,2}$. The end result of this operation is given by the next equation.

\[
\varepsilon_{r,2} = \varepsilon_{r,1} \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 - \gamma_1 A_4 - A_2
\]

The dielectric constant of the second pavement layer depends on both the dielectric constant $\varepsilon_{r,1}$ and the conductivity $\sigma_1$ of the first layer. The dielectric constant $\varepsilon_{r,1}$ is given by equation (4-86), whereas the conductivity $\sigma_1$ is unknown and cannot be determined. Consequently, the correct value of $\varepsilon_{r,2}$ cannot be found based solely on the reflection amplitudes (more unknowns than equations). An approximate value of $\varepsilon_{r,2}$ can be estimated by assuming the value of $\sigma_1$.

The dielectric constant of the third pavement layer is also determined from equation (4-85) by letting $n$ equal 2 and solving for $\varepsilon_{r,3}$ as follows.

\[
\varepsilon_{r,3} = \varepsilon_{r,2} \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 - \gamma_1 A_4 + A_2
\]  

As for $\varepsilon_{r,2}$, the dielectric constant of the third layer $\varepsilon_{r,3}$ depends on the dielectric properties of the two layers above it. In particular, $\varepsilon_{r,3}$ depends on the unknown conductivities $\sigma_1$ and $\sigma_2$ and on the dielectric constant $\varepsilon_{r,2}$, which was roughly estimated. Hence, $\varepsilon_{r,3}$ would exhibit a higher error than $\varepsilon_{r,2}$. In general, the dielectric constant $\varepsilon_{r,n}$ of the $n^{th}$ layer could be deduced from equation (4-85) by inspection of equation (4-88). The end result for $\varepsilon_{r,n}$ is given in the following equation:

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 - \gamma_1 A_4 - A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]

\[
\left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2 \left( 1 - A_0^2 \right)^2
\]

\[
\varepsilon_{r,n} = \varepsilon_{r,n-1} + A_{n-1}
\]
According to the previous equation, the dielectric constant of the $n^{th}$ layer depends on the unknown conductivities and the roughly estimated dielectric constants of the $(n-1)$ layers above it. Consequently, it could be concluded that the error of estimating the dielectric constant of a layer based on the GPR reflected pulses amplitudes would increase with the depth of the layer.

With the exception of the case of high moisture accumulation within the pavement layers, pavement materials are generally considered as insulators or, at most, as semiconductors; therefore, their conductivity is typically low (as can be deduced from the imaginary part of the dielectric constant in Table 4-3). A good approximation for $\sigma_n$ would then be zero, which implies that the layers are lossless. This choice of $\sigma_n$ would simplify the dielectric constant estimation equations. However, it would also cause the dielectric constant of the deep layers to be underestimated since the reflection amplitudes, based on which the computations are carried out, would have been higher if the layers were truly lossless. As an example, the variation of the dielectric constant of the second pavement layer, computed based on equation (4-87), versus loss at the first layer is plotted in Figure 4-67. The loss part in this figure corresponds to the exponential term in the equation. The significance of assuming values for the conductivity of the layers on the accuracy of the GPR thickness estimation is presented in Chapter 5.

Figure 4-67: Second Layer Dielectric Constant Variations versus First Layer Loss
4.4.3 Common midpoint technique

The common midpoint (CMP) technique (also known as common-depth point, CDP) is often used in seismic tests as a stacking technique to improve the SNR of an ensemble of seismic observations generated by a set of sources and recorded by another set of receivers [68]. This technique is also useful in estimating the velocity of seismic waves, transmitted by one source and received by multiple receivers, after traveling through the earth layers. Similarly, the CMP technique can be used to estimate the velocity $v$ of EM waves in a material [69] and, therefore, to find its dielectric constant $\varepsilon_{r,1}$ according to equation (2-11).

Figure 4-68 illustrates a simple CMP configuration that can be used to estimate the average EM velocity $v$ within a single layer or within multiple layers with comparable dielectric constants, using simultaneously monostatic and bistatic antennas. As can be seen from this figure, the monostatic system is centered between the transmitter and the receiver of the bistatic system, which are separated by a predetermined distance $x$. It can be assumed that on average, the signals collected by both antenna systems are coming from the same point $P$ at the bottom interface of the layer. Therefore, using ray tracing, the following equations can be derived.

\[ vt_1 = 2d_1 \quad (4-90) \]

\[ vt_2 = 2 \sqrt{d_1^2 + \left(\frac{x}{2}\right)^2} = \frac{ct_2}{\sqrt{\varepsilon_{r,1}}} \quad (4-91) \]

where $d_1$ is the unknown thickness of the layer, $t_1$ and $t_2$ are the two-way travel times of the monostatic and bistatic systems, respectively, $x$ is the distance separating the transmitter and receiver of the bistatic system, and $c$ is the speed of light in free space.

Combining equations (4-90) and (4-91) by eliminating the unknown $d_1$ and then solving for $\varepsilon_{r,1}$ results in the following relation:

\[ \varepsilon_{r,1} = \frac{c^2}{x^2} \left( t_2^2 - t_1^2 \right) \quad (4-92) \]
It should be noted that because the CMP method $\varepsilon_{r,1}$ is determined from the reflections at the bottom of the layer, it represents an average value of the dielectric constant that accounts for any inhomogeneities within the layer. This result is more accurate than that found by equation (4-86), which is based on the top reflection only and, therefore, does not consider any dielectric constant changes within the layer.

![Common Midpoint Geometry Using Ground-Coupled Monostatic and Bistatic Antennas](image)

Figure 4-68: Common Midpoint Geometry Using Ground-Coupled Monostatic and Bistatic Antennas

The aforementioned CMP configuration assumes that both systems are ground-coupled. However, the bistatic GPR system used in this research is air-coupled. Thus, slight modifications in the geometry, as depicted in Figure 4-69, and the resulting equations are needed.

![Modified Common Midpoint Geometry Using a Ground-Coupled Monostatic System and an Air-Coupled Bistatic System](image)

Figure 4-69: Modified Common Midpoint Geometry Using a Ground-Coupled Monostatic System and an Air-Coupled Bistatic System.
In the modified CMP technique (Figure 4-69), the bistatic antennas are set at a height $d_0$ above ground. Because of the relatively large distance separating the transmitter and receiver of the bistatic system (known as baseline in bistatic radar terminology [70]), the wave incidence is no longer normal, but it becomes oblique. According to this figure, it is clear that estimating the dielectric constant of the first layer based on equation (4-92) holds when replacing $x$ by $x_1$, which is the distance between the incidence point and the reflection point of the air-coupled system on the pavement surface. In this case, the two-way travel time $t_2$ is measured between the two interfaces of the first layer. The distance $x_1$ is unknown because the dielectric constant $\varepsilon_{r,1}$, the incidence angle $\theta_i$, and the transmission angle $\theta_t$ are unknown. On the other hand, the two-way times of travel $t_1$ and $t_2$ are measured directly from the ground-coupled and air-coupled responses, respectively.

The angles $\theta_i$ and $\theta_t$ are related by Snell’s law of refraction given in equation (2-16) and simplified in the case of non-magnetic material as follows:

$$\sqrt{\varepsilon_{r,0}} \sin \theta_i = \sqrt{\varepsilon_{r,1}} \sin \theta_t$$

(4-93)

where $\varepsilon_{r,0}$ is the dielectric constant of air ($\varepsilon_{r,0} = 1$).

Using the geometry of Figure 4-69 and elementary trigonometric relations, the following equations relating the different unknowns can be deduced:

$$2h_0 \tan \theta_i + x_1 = x_0$$

(4-94)

$$\tan \theta_t = \frac{x_1}{2h_1} = \frac{x_1 \sqrt{\varepsilon_{r,1}}}{ct_1}$$

(4-95)

where $x_0$ is the distance between the air-coupled transmitter and receiver, $d_0$ is the antenna elevation with respect to ground, and $d_1$ is the first layer thickness. Combining equations (4-92) and (4-95) and solving for $\theta_t$ as a function of the two-way times of travel $t_1$ and $t_2$ yields the following:
Finally, combining equations (4-92), (4-93), and (4-94) gives the following relation, which has $\theta_i$ as the only unknown:

$$
2h_0 \tan \theta_i + c \frac{\sin \theta_i}{\sin \theta_r} \sqrt{t_2^2 - t_1^2} = x_0
$$

After determining the angles $\theta_t$ and $\theta_i$, all the other unknowns can be estimated using the previous equations. In summary, to estimate the dielectric constant of the top layer(s) of a pavement system using the modified CMP technique, the following algorithm should be used:

1. Measure the reflection times $t_1$ and $t_2$ corresponding to the two-way travel times within the layer, obtained by the ground-coupled and air-coupled systems, respectively,
2. Calculate the transmission angle $\theta_t$ using equation (4-96),
3. Solve equation (4-97) numerically to find the angle of incidence $\theta_i$,
4. Use Snell’s law of refraction, equation (4-93), to estimate the dielectric constant of the pavement layer $\varepsilon_{r,1}$.

It should be noted that to avoid interference between the two antenna-systems during the survey, the ground-coupled antenna should not be placed directly underneath the air-coupled antenna, as implied by Figure 4-69. Instead, it should be placed at a distance $d$ (at least 1m) in front of it (Figure 3-3). Thus, the scans collected by the air-coupled antenna at a specific point are delayed with respect to those collected by the ground-coupled antenna by a number of scans corresponding to the distance $d$.

Using the CMP technique to estimate the dielectric constants of pavement layers has many advantages when compared to the dielectric constant estimation technique based on the reflection amplitudes. The main advantages are as follows:

- The CMP technique measures the bulk dielectric constant of the layer based on the GPR reflections at its bottom. Therefore, this technique takes into account any
inhomogeneities present within the layer to estimate the dielectric constant. The CMP technique should be used when the reflection interfaces of the adjacent layers are overlapped and not separable. This situation usually occurs when GPR data is collected over in-service pavements with newly placed thin wearing surface overlays. The CMP technique should also be used when the dielectric constant of the considered layer varies gradually with depth due to moisture accumulation or other distresses.

- The CMP technique yields more accurate dielectric constant results than does the amplitude technique since the former is based on reflection time delays and the latter is based on reflection amplitudes. The reflection time delays are much more stable than the reflection amplitudes, which can fluctuate because of scattering at the pavement surface or overlap between the consecutive reflections.

- The conductivity or material loss of the pavement layers do not enter in the dielectric constant estimation based on the CMP technique.

On the other hand, the CMP technique has some shortcomings that should be considered during the data analysis phase. The main problems that would result in erroneous dielectric constant estimations when using the CMP method are listed below:

- The CMP method cannot be used to estimate the dielectric constant of thin layers or layers with high dielectric constants since the time delays \( t_1 \) and \( t_2 \) would have comparable values (become equal in the limit of the GPR sampling rate) and the aforementioned equations would result in nonphysical dielectric constant values. This problem can be counteracted by decreasing the height \( d_0 \) of the air-coupled antenna and increasing the baseline distance \( x_0 \) between the bistatic transmitter and receiver, thus increasing the difference between the time delays \( t_1 \) and \( t_2 \). However, there are physical limits to the maximum ranges of \( d_0 \) (should be greater than 300mm for the GPR system used otherwise the coupling-pulse would overlap the surface reflection) and \( x_0 \) (limited by the directivity of the antennas, which transmits most of the energy in the normal direction) that should be taken into account.

- The dielectric constant results found by the CMP technique are sensitive to the magnitude of the time difference \( (t_2-t_1) \). This is illustrated in Figure 4-70, which shows the dielectric
constant variations versus the time delay difference \((t_2-t_1)\). This figure shows that for low time delay differences, a small variation in the time delays leads to a big jump in the dielectric constant value. However, as the time difference increases, the dielectric constant value stabilizes. To reduce the effects of this problem, the time delays \(t_1\) and \(t_2\) should be estimated with high accuracy. This can be achieved by increasing the GPR data acquisition sampling rate.

![Dielectric Constant Variations versus Time Delay Difference](image)

**Figure 4-70: Dielectric Constant Variations versus Time Delay Difference.**

- It is difficult to measure accurately the time-delay \(t_1\) of the ground-coupled system because of the overlap of the end-reflection pulse with the surface reflection pulse, in this type of systems, which results in an ambiguous time-delay reference. This problem can be solved by using a monostatic air-coupled antenna instead of the ground-coupled antenna. Alternatively, calibration measurements can be conducted on some cores extracted from the surveyed road to obtain a time-delay offset to be added to the time-delay measured from the ground-coupled response. This offset would account for the uncertain time-delay reference.

- The CMP technique requires that twice the quantity of data taken during a normal GPR survey to be collected. Therefore, more storage space and analysis time are needed to use the CMP method. Moreover, since both the air-coupled and ground-coupled antennas are controlled and triggered by the same control unit, the scan rate is reduced to half...
(compared to the use of a single antenna) to cope with the ADC speed. This results in slowing the GPR survey speed.

- The CMP technique is cumbersome to apply for multiple-layer systems because it is difficult to solve for the incident and transmission angles at the different layer interfaces.

### 4.4.4 Summary

Various techniques to estimate pavement materials dielectric properties from GPR reflected signals were presented. Based on field data, it was found that the variations of HMA materials dielectric properties within the GPR bandwidth are insignificant. Therefore, use of an invariant complex dielectric constant, in this case, would not affect the accuracy of GPR results. Similar findings were reported in another study for Portland cement concrete slabs [8]. The dielectric constant of pavement materials could be estimated from GPR response based on either the reflection amplitudes measured by an air-coupled GPR system or the reflection time-delays measured simultaneously by an air-coupled system and a ground-coupled system. The time-delay method uses a modified common midpoint technique (CMP) for the analysis. Estimating the dielectric constant from reflection amplitudes is easier to implement, yet it requires an assumption of the material loss within the layers in order to yield accurate dielectric constant results. On the other hand, the time-delay technique is more complex to apply but yields more accurate dielectric constant estimates if applied correctly.

### 4.5 Layer Interface/Distress Separation

This processing stage is intended to divide the detected reflected pulses into two categories: reflections from layer interfaces and reflections from subsurface distresses. Hence, the degree of deterioration of the surveyed pavement could be assessed. The separation procedure is based on the properties of the two pavement features: the layer interfaces typically extend longitudinally along the surveyed pavement, whereas subsurface distresses, such as air-voids or accumulated moisture, are generally localized within small areas of the road. The layer interfaces have, theoretically, the same length as the surveyed pavement except for some cases where the layers are localized to a section of the pavement. This situation occurs when full-depth repairs are performed on segments of the surveyed pavement, in which case the layer interfaces of the repaired section would not necessarily correspond to the layer interfaces of the
rest of the pavement [71]. Yet the layer interfaces of the repaired sections would usually be longer than the localized distresses. The localized reflections can also correspond to false alarms resulting from the interface detection stage, but these reflections were usually found to be scattered and not present in more than a maximum of three or four consecutive GPR scans. Hence, false alarm detections can easily be distinguished from the localized distresses and can then be filtered out by removing any detected pulses that are not present over several consecutive scans.

The layer interface/distress separation stage could be carried out using the estimated layer interface depths and layer dielectric constants. In fact, for sound pavement layers, the layer interfaces and dielectric constants are continuous functions along the surveyed pavement. However, when distresses are present, abrupt changes in the interface depths and calculated dielectric constants occur. Using these two parameters, layer interface/distress separation is performed in two stages. The first stage consists of grouping the detected reflected pulses that have comparable depths and dielectric constants and considering them as coming from the same layer interface or distress. In the second stage, layer interfaces are separated from distresses based on their respective lengths, according to the following:

- Relatively large reflected pulse groups with lengths comparable to the surveyed pavement length are considered as layer interfaces;
- Small reflected pulse groups are considered as pavement distress;
- Groups composed of three or four reflected pulses are considered as false alarm detections and, thus, should be discarded.

The thresholds used in this classification scheme of reflected pulse groups are usually distinct from each other. For example, a one-km pavement probed with a GPR system at 10 scans per meter (i.e., a total of 10,000 scans), would have layer interface reflections spread over at least 9900 scans (990m); pavement distress reflections would be spread over 30 scans (3m); and false alarms would be spread over a maximum of four scans or less. Practically, the layer interface reflections are usually not distributed over the whole pavement length because some missed detections might occur during the interface detection stage.
The only problem with the aforementioned procedure is that if false alarm detections are present and are mistakenly used to compute the layer depths and dielectric constants, they would lead to non-continuous layer interfaces and dielectric constants even if the actual layer interfaces were continuous and no distresses were present. More appropriate parameters for grouping the reflected pulses in the first stage were found to be the interface reflection time-delays and the corresponding reflection amplitudes. In fact, the reflection time-delays and amplitudes are also continuous functions along the surveyed pavement when they correspond to a layer interface and exhibit abrupt changes when they correspond to subsurface distresses. Furthermore, reflection time-delays are not affected by false alarms.

One method for classifying the detected GPR pulses into groups with comparable time delays and amplitudes is the use of a clustering algorithm [72]. This technique was found to work only if the layer interfaces are sufficiently distant from each other and the layer thicknesses do not change much along the surveyed pavement. These two conditions are necessary to obtain separable clusters. This technique assumes that the pavement distresses are not at the same depth as the layer interfaces; otherwise, they would be grouped in the same cluster.

Another alternative for classifying the reflected pulses is to use a tracking algorithm. Typically, a tracking algorithm tries to follow the changing locations of a group of objects versus time using successive observations and then decides on the trajectory (or track) followed by each object. For the case of layer interfaces, the algorithm could be adapted to track the layer interface reflection time delays and amplitudes along the collected GPR scans or, equivalently, along the surveyed pavement. The various tracks formed by this process would correspond either to layer interfaces or to distresses. For the tracking algorithm to work, some conditions on the tracked parameters should be satisfied [73] as follows:

- The tracked quantities should remain relatively unchanged between the different observations. This condition is verified for layer interfaces since the reflection time-delays and reflection amplitudes corresponding to the same layer interface usually change slowly between the collected scans – especially if the GPR scan rate is high (can be up to three scans per meter or more at survey speeds of 90km/h, depending on the GPR system used). However, a sudden change of the reflection time-delay and amplitude would
occur in the case of a transition between a layer interface and a distress or vice-versa. In this case, the tracking algorithm should create new tracks that correspond to the new features found.

- The velocity of the tracked object should not change much between the different observations. This condition is already satisfied because GPR scans are usually triggered by a distance measuring instrument (DMI), which ensures data collection at fixed-user-selected intervals.
- The direction of motion of the tracked object should not change abruptly between the different observations. This condition also holds for reflection time delays and amplitudes since the layer interfaces are parallel to each other.

To get a correct classification of the reflected pulses, interface tracking should be performed in a 3D space, using the following parameters:

- The reflection time delays are used to ensure tracking of the layer interfaces,
- The reflection amplitudes are used to ensure tracking of the dielectric constants,
- The survey distance or scan number is the variable along which the other two parameters are tracked.

Since the reflection time-delays and amplitudes have different orders of magnitude, they need to be normalized in order to have the same importance for interface tracking. The normalization of the time-delays is performed based on the known maximum range of the GPR system. The normalization of the reflection amplitude is done based on the known word-length of the ADC (for example, with a signed 16-bit ADC, the maximum amplitude is 32768). The maximum range of the GPR system and the word-length of the ADC are selected during the GPR data acquisition phase.

As for object tracking, layer interface tracking is based on the continuity of the tracked variables, which are, in this case, the reflection time delays and reflection amplitudes. For a given track, these two variables should be continuous with respect to the collected GPR scans or surveyed distance. A track is defined, in the considered 3D space, as a set of points $P^k_i(x^k_i, t^k_i, a^k_i)$ corresponding to a continuous layer interface or a pavement distress. The point $P^k_i$ represents the
$k^{th}$ point (i.e., found in the $k^{th}$ scan) of the $i^{th}$ track; its coordinates are the scan number or longitudinal survey distance $x^k$, the reflection time-delay $t^k_i$, and the reflection amplitude $a^k_i$. It should be noted that because GPR data is collected at fixed intervals, all $k^{th}$ points of the different tracks have the same survey distance $x^k$ independent of the considered track $i$. To choose the track from among the already formed tracks to which a new point $P^{k+1}$ belongs, a nearest neighbor search is used. With this search, the distances between the point $P^{k+1}$ and the last points found for all tracks are computed. The point $P^{k+1}$ is then assigned to the track with the least distance. All distances considered here are Euclidean distances. To account for the appearance of new layer interfaces or distresses along the surveyed pavement, a maximum distance $d_{\text{max}}$ should be introduced. When the distances between the unassigned point $P^{k+1}$ and the last points of all known tracks are greater than $d_{\text{max}}$, a new track should be created. Since the tracked parameters were normalized before processing, a suitable value of the distance $d_{\text{max}}$ could be fixed heuristically for GPR data collected from most pavement systems. However, $d_{\text{max}}$ should be reduced if the surveyed pavement is composed of thin layers. To be independent of the user-selected intervals at which GPR data is collected, the distances used to perform the nearest neighbor search and the maximum distance $d_{\text{max}}$ should be computed in a 2D space rather than in the considered 3D space. The coordinates of the track points in the 2D space are the reflection time delays and corresponding amplitudes.

After grouping the different reflected pulses into tracks based on the distance that two consecutive points of the same track are allowed to have, the continuity of the formed tracks is further checked by a deviation function $\phi$. The deviation function is introduced to account for the smoothness of the tracks. For three consecutive points of the same track, the deviation function is defined as given by the following equation:

$$\phi(P^{k-2}_i, P^{k-1}_i, P^k_i) = 1 - \cos \theta$$

(4-98)

where $P^{k-2}_i$, $P^{k-1}_i$, and $P^k_i$ are the before previous, previous, and current points of the $i^{th}$ track, respectively, and $\theta$ is the angle formed between the lines defined by $P^{k-2}_i$ and $P^{k-1}_i$ on the one hand, and $P^{k-1}_i$ and $P^k_i$ on the other hand. According to the previous equation and considering that the GPR scans are taken in the forward direction only, the deviation function has its values in the $[0, 1]$ interval. The value 0 of the deviation function corresponds to no deviation of the
last point (the three points are aligned), whereas the value 1 corresponds to the highest deviation (both lines are orthogonal). For computational simplicity, the preceding equation could be put in a vector format, as follows:

\[
\phi(P_i^{k-2}, P_i^{k-1}, P_i^k) = 1 - \frac{P_1 \cdot P_2}{\|P_1\| \|P_2\|}
\] (4-99)

where \(P_1\) and \(P_2\) are the vectors formed by the points \(P_i^{k-2}\), \(P_i^{k-1}\), and \(P_i^k\) respectively, the “\(\cdot\)" operator is the dot product, and \(\| \|\) is the vector 2-norm. Unlike the nearest neighbor search, which was performed in a 2D space formed by the reflected time-delays and amplitudes, the deviation function has to be computed in the 3D space that also includes the survey distance or the scan numbers. After finding all possible tracks based on the nearest neighbor search, the deviation function is applied for the various track points in an exchange loop. Within this loop, two points constrained by the distance \(d_{\text{max}}\) but belonging to two distinct tracks would be swapped if it is found that they would yield lower deviation after the swapping operation is performed. Hence, after applying the deviation function, all tracks would have a minimum global deviation, which can be defined as the sum of deviations given by equation (4-99) for all points of all tracks.

In the previous analysis, the tracks found (corresponding to layer interfaces or distresses) were assumed to be composed of continuous and uninterruptible sets of points. Hence, each track was assumed to have a point in each of the scans spanning its length. In practice, this assumption is not always true because of the missed detections caused by the reflected pulse detector. Because of the presence of missed detections, the tracking algorithm would report multiple tracks for the same layer interface. These tracks would correspond to the contiguous segments of reflected pulses. To solve this problem, phantom points should be used. Phantom points are points that are not really detected by the detector but are added to fill the missing points of the tracks. Phantom points are added whenever no true points within the constraints of the maximum distance \(d_{\text{max}}\) are found. Since layer interfaces are usually parallel to each other, the added phantom points should have the same coordinates as the last known true point of the processed track except for the \(x\) coordinate, which depends on the scan number or the surveyed distance. All added phantom points should be marked so that they can be removed at the end of
the tracking procedure. With the introduction of phantom points, the different layer interfaces can be easily tracked even if several points are missing.

The detailed algorithm for layer interface tracking is presented in A.3, Appendix A. To test its performance, this tracking algorithm was applied to GPR data collected over a 40m stretch of section A at the Virginia Smart Road. Figure 4-71a shows the normalized reflection time-delays just after the interface detection stage, and Figure 4-71b depicts the corresponding normalized reflection amplitudes. Other than the long layer interfaces represented in these two figures by various patterns, numerous false alarm detections could be observed. The dissimilarity of patterns used to represent interfaces at the same depth is an indication that reflections corresponding to the same layer interface would be reported by the detector as belonging to different layer interfaces. On the other hand, use of the same patterns to represent layer interfaces at different depths is an indication that those reflections would be reported as belonging to the same interface. The differences of patterns representing the layer interfaces are caused by the interface detector processing, which is performed on a scan-by-scan basis. Therefore, the order and number of the detected reflections is not maintained across the scans.

Figure 4-71c shows the normalized reflection time-delays after processing by the tracking algorithm, and Figure 4-71d depicts the corresponding normalized amplitudes (the reflection time-delays and amplitudes corresponding to the same layer interface are represented in the figures by the same patterns). According to these two figures, all false alarm detections were removed by the tracking algorithm, and all interface reflections at the same depth are now represented by the same patterns. Hence, these reflections would be reported as corresponding to the same layer interfaces, which are in Figure 4-71c from bottom to top: the pavement surface, the WS/BM-25.0, the BM-25.0/OGDL, and the OGDL/base interfaces. The assumption made for the development of the layer interface/distress separation that layer interfaces have almost the same length as the surveyed section is therefore valid. In addition to the layer interfaces, Figure 4-71c shows localized reflections at survey distances of approximately 42, 47, and 52m. These reflections correspond to some of the copper plates embedded in section A. Since the Virginia Smart Road is relatively new and is closed to traffic, no subsurface defects have developed within the pavement yet. Consequently, no other localized reflections are seen in the figure. Finally, it is worth noting that even though the copper plate reflections at 47 and 52m are seen in
Figure 4-71c at the same depth, they were represented by different patterns after layer tracking. This indicates that they do not belong to the same interface. These two localized tracks were not grouped together because they have different reflection amplitudes, as seen in Figure 4-71d (first reflection is positive and the second is negative).

![Figure 4-71: Raw Reflected Pulses: (a) Normalized Time-Delays, (b) Normalized Amplitudes, Tracked Reflections: (c) Normalized Time-Delays, (d) Normalized Amplitudes](image)

Based on Figure 4-71c, it is evident that it is easy to separate layer interfaces from distresses (simulated here by the copper plates) after layer interface tracking, based uniquely on the track lengths. After the separation procedure, the dielectric constants of the various layers
are estimated, as shown in section 4.4. Then, using the reflection time-delays and the dielectric constants estimates, the layer thicknesses and the depths to distresses are computed based on equation (2-30).

In summary, the previous section showed how the reflected pulses detected from the GPR scans can be grouped together to form contiguous layer interfaces and localized subsurface distresses. The formed layer interfaces and distresses could then be separated based on their respective lengths. A tracking algorithm that follows the variations of the reflection time-delays and amplitudes across the GPR scans was found to work perfectly to accomplish this clustering task. Since the tracked parameters are normalized (based on their expected maximum values) before the tracking algorithm is applied, the algorithm can be used for a variety of pavement configurations without any changes in the algorithm thresholds. The tracking algorithm can also improve the performance of the preceding pulse detector stage by removing the scattered false alarm detections.

4.6 Summary

This chapter presented various techniques that can be used to automatically interpret GPR data collected over pavements. The data analysis procedure was divided into four consecutive processing stages: (1) preprocessing, (2) layer interface reflection detection, (3) dielectric properties estimation, and (4) layer interface/distress separation. In all of these processing stages, the physical and geometrical properties of pavement layers were exploited to simplify the analysis. The preprocessing stage consisted mainly of enhancing the quality of the GPR signals by filtering unwanted noise and removing any spurious reflections not resulting from the pavement layers. The preprocessing stage was also used to enhance the range resolution of the GPR system by deconvolving the collected scans. In the layer interface detection stage, all pulses present within the reflected GPR signal were detected, and their time-delays and amplitudes were estimated. The detection stage consisted of a cascade of a detector (threshold detector or matched filter) and a least-squares estimator. After detecting all possible pulses within the GPR signal, their respective time-delays are used to fit a theoretical reflection model to the measured reflected signal in the least squares sense. The synthesized signal is then subtracted from the original reflected signal to reveal any pulses originally overlapped by the
stronger reflections in their vicinities. The detection procedure is then repeated to find all possible reflected pulses using multiple iterations. It was found that this procedure yields better results than direct detection of the reflected pulses from the deconvolved GPR signal since deconvolution generally reduces the SNR of the processed signal, which degrades the performance of the detector. After finding all reflection time-delays and amplitudes, these two parameters are tracked along all GPR scans to form separate groups of layer interfaces and localized subsurface distresses, which are normally contained within small areas of the pavement system. Using the tracked reflections, the dielectric constants of the various layers are estimated based on the reflection time delays measured on GPR scans collected by ground-coupled and air-coupled antennas arranged in a common midpoint technique. The dielectric constants could alternatively be estimated based on the amplitudes of the reflected pulses collected by monostatic or bistatic air-coupled antennas. For accurate results, this method needs assuming values for the material loss of the various layers. Finally, using the interface reflection time-delays and the dielectric constant estimates, the layer thicknesses and the distress depths are computed.

The performances of all the processing stages presented in this chapter, except for the layer dielectric constant and layer thickness estimation, were evaluated using field GPR data collected from the Virginia Smart Road. The performance evaluation was carried out using special performance parameters adapted to the evaluated processing stage. The performance of the dielectric constant estimation technique was not evaluated since there are currently no known methods to measure accurately the in-situ dielectric constants of subsurface layers. The only way to evaluate the performance of this processing stage is to estimate the accuracy of the layer thicknesses reported by the last processing stage. This will be presented in next chapter.
Chapter 5

GPR Data Analysis Results and System Validation

In Chapter 4, the various stages needed for correct GPR data analysis were presented. The performance of each of the processing stages, with the exception of the dielectric constant estimation stage, was evaluated based on performance parameters computed mainly from the stage output and the measured GPR signal. The performance of the dielectric properties estimation stage could not be evaluated because, currently, there are no known techniques that can be used to measure accurately the in-situ dielectric properties of subsurface layers and to then compare their output to the dielectric properties found by GPR. The only method that can be used to evaluate the performance of the dielectric properties estimation stage is to evaluate the accuracy of the layer thickness reported by GPR. In fact, according to equation (2-30), the accuracy of estimating the layer thicknesses is closely related to the accuracy of estimating the dielectric constants. Furthermore, estimating the accuracy of reported layer thicknesses allows the overall GPR data analysis system to be assessed.

In this chapter, GPR data collected from the Virginia Smart Road will be analyzed using the aforementioned processing stages. The accuracy of the overall GPR data analysis system will be estimated based on a comparison of the thickness output obtained from the data analysis system and thicknesses measured directly on field cores taken from the pavement.

5.1 GPR Data Analysis Results

Ground penetrating radar data collected from the Virginia Smart Road was analyzed using the various data analysis stages presented in Chapter 4. For the analysis purposes, the Virginia Smart Road sections were divided into six categories based on the number and thicknesses of the layers composing each section. This division is identical to the one adopted for the performance evaluation of the pulse detectors studied in 4.3.3.3. It is used here because, on the one hand,
different detectors would be utilized for the different pavement system categories and, on the other hand, the overall data analysis system would perform differently based on the pavement system category. The various pavement categories used for GPR data analysis, the chosen section among each category, and the utilized pulse detector are presented in Table 5-1.

Table 5-1: Flexible Sections Used for GPR Data Analysis

<table>
<thead>
<tr>
<th>Section</th>
<th>Number of Detectable Layers</th>
<th>Detector</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>3 (WS, BM-25.0, Base)</td>
<td>Matched Filter</td>
<td>Mostly thick layers except for WS</td>
</tr>
<tr>
<td>A</td>
<td>4 (WS, BM-25.0, OGDL, Base)</td>
<td>Matched Filter</td>
<td>Mostly thick layers except for WS</td>
</tr>
<tr>
<td>F</td>
<td>3 (WS, BM-25.0, Base)</td>
<td>Matched Filter</td>
<td>Relatively thin layers</td>
</tr>
<tr>
<td>G</td>
<td>4 (WS, BM-25.0, SM, Base)</td>
<td>Threshold</td>
<td>Relatively thin layers</td>
</tr>
<tr>
<td>H</td>
<td>5 (WS, BM-25.0, SM, OGDL, Base)</td>
<td>Threshold</td>
<td>Relatively thin layers</td>
</tr>
<tr>
<td>K</td>
<td>5 (2WS, BM-25.0, OGDL, Base)</td>
<td>Threshold</td>
<td>Mostly thin layers + Geocomposite</td>
</tr>
</tbody>
</table>

Since the bottom of the base layers at the Virginia Smart Road are not detectable by GPR, their thicknesses could not be estimated. For this reason, the data analysis presented here is exclusive to the HMA layers, which are composed (depending on the section) of the wearing surface, BM-25.0, OGDL, and SM-9.5A layers. Based on the discussions in Chapter 4, four variants of GPR data analysis could be identified:

1. Estimation of the overall HMA layer thickness without separation between the thin layers of each pavement section,
2. Estimation of the individual thicknesses of the HMA layers assuming that the layers are lossless,
3. Estimation of the individual thicknesses of the HMA layers assuming that the layers are lossy. Material loss for each layer is estimated based on field cores,
4. Estimation of the overall HMA layer thickness using the common midpoint technique.

Because the layers’ dielectric constants are estimated from the reflection amplitudes, all these data analysis variants were applied to GPR data collected by the air-coupled antenna, with
the exception of the fourth variant, which uses both air-coupled and ground-coupled data to estimate the layers’ dielectric constants from the reflection time-delays.

### 5.1.1 Thickness of the overall HMA layer thickness

To estimate the thickness of the overall HMA layer, the various processing stages presented in Chapter 4 were applied to the air-coupled GPR data collected from the Virginia Smart Road. The data was processed in the following order: coupling pulse removal, noise filtering, layer interface detection (using a suitable detector depending on the pavement configuration, as presented in Table 5-1), layer interface/distress separation, dielectric constant estimation based on reflection amplitudes, and layer thickness computation from the time-delays and the layer dielectric constants. For each GPR scan, the pulse detector was used to detect the strong reflections resulting from the pavement surface and the HMA/base interface and to ignore all weak and masked reflections. Therefore, this processing resulted in finding the thickness of the overall HMA layer, which is composed of the wearing surface, BM-25.0, OGDL, and SM-9.5A in some sections.

The overall HMA layer thickness results found for the six sections in Table 5-1 are depicted, respectively, in Figures D−1a through D−1f, Appendix D. The x-axis in these figures represents the survey distance referenced to the beginning of section A. For comparison purposes, the overall HMA design thicknesses are presented in the same figures. It should be noted that the design thicknesses do not necessarily represent the as-built thicknesses of the layers, as was mentioned in 3.1; however, they could be considered as good approximations. According to the thickness results, GPR tends to overestimate the thickness of the overall HMA layer, especially for section K (Figure D−1f, Appendix D), which is composed of 5 relatively thin layers plus a geocomposite membrane. This apparent thickness overestimation is mainly due to the assumption that the different HMA layers are a single homogenous layer with its dielectric constant determined uniquely from the amplitude of the surface reflection.

### 5.1.2 Thicknesses of individual HMA layers: lossless layers

The same processing stages used to determine the thickness of the overall HMA layer were applied to find the thicknesses of each pavement section’s individual layers. The only difference
was the implementation of the pulse detector, which in this case was used iteratively, as described in 4.3.3, to detect all the strong and weak reflections resulting from all the HMA interfaces. The dielectric constant of each layer was estimated using the reflection amplitudes, as given by equation (4-89), where the layers were assumed to be lossless (i.e., the conductivities $\sigma_n$ were considered equal to zero). The individual thicknesses were then determined using the time-delays and dielectric constants relative to each layer. This analysis resulted in finding the thicknesses of the wearing surface, BM-25.0, OGDL, and SM-9.5A layers of each pavement section.

The thickness results of the individual HMA layers found for the six sections in Table 5-1 are depicted, respectively, in Figures D−2a through D−2f, Appendix D. For comparison purposes, the design thicknesses are also shown in the same figures. According to these results, the individual layer thicknesses are approximately equal to the design thicknesses for all the considered sections; therefore, the overall thicknesses are also equal. This improvement in the GPR performance is mainly the result of separating the layers of different dielectric properties and considering them as multiple layers instead of a single homogeneous layer. The only layers that could not be separated in this analysis were the SM-9.5A and OGDL layers in section H, as shown in Figure D−2e. However, as can be seen in the figure, missing the SM-9.5A/OGDL interface reflection did not affect the layer thickness results since the two layers have comparable dielectric constants. In contrast, the interface between the third and second BM-25.0 lifts (respectively BM-25.0 [3] and BM-25.0 [1+2] in Figure D−2f, Appendix D) of section K was detected, and the thicknesses of both parts of the layer were successfully determined. For this particular section, the interface between the different lifts of the BM-25.0 layer was detectable even though the same HMA material was used throughout the layer because of the Polyvinyl Chloride (PVC) geocomposite membrane installed at that interface. The geocomposite membrane (with a dielectric constant of approximately 6) caused a contrast in the dielectric constants of both layer lifts, which resulted in the reflection.

5.1.3 Thicknesses of individual HMA layers: lossy layers

This analysis is similar to the aforementioned procedure except that the layers, in this case, are considered lossy. The conductivities of the various pavement layers were computed
iteratively using the known thicknesses and dielectric constants measured directly on a calibration core and were then considered invariant along the pavement. The conductivity $\sigma_n$ of the $n^{th}$ layer could be found by transforming equation (4-89), as given by the next equation:

$$\sigma_n = \frac{\sqrt{\varepsilon_{r,n}}}{d_n} \left\{ \frac{1}{\eta_0} \log \left[ \frac{1-A_0^2}{\sqrt{\varepsilon_{r,n+1}} \left( \sum_{i=1}^{n-1} \gamma_i A_i - A_n \right) - \sqrt{\varepsilon_{r,n}} \left( \sum_{i=1}^{n-1} \gamma_i A_i + A_n \right)} \right] \right\} = \sum_{i=1}^{n-1} \sigma_i d_i$$

where $n = 1, \ldots, N$ (the total number of layers), $\varepsilon_{r,n}$ and $d_n$ are, respectively, the dielectric constant and thickness of the $n^{th}$ layer, $\gamma_i$ and $A_i$ are the reflection coefficient and relative reflection amplitude of the $i^{th}$ interface, and $\eta_0$ is the wave impedance of free space. An expression of the reflection coefficient $\gamma_i$ is given by equation (2-22). To find the layer conductivities from the previous equation, the dielectric constants $\varepsilon_{r,n}$ need to be known. Practically, the dielectric constants were estimated using equation (2-30) by substituting the layer thicknesses measured on the calibration core and the reflection time-delays estimated from the GPR response collected over the core location before coring then solving for the dielectric constants. For the four HMA layers (i.e., wearing surface, BM-25.0, OGDL, and SM-9.5A), an average conductivity of 0.01S/m (or equivalently 8dB/m) was found when applying this technique.

For the GPR data analysis, the suitable detector for each section was iteratively applied to detect all layer interface reflections from which the layer dielectric constants (accounting for loss) and thicknesses of the individual layers were estimated. The thickness results found for the six sections of Table 5-1 are depicted, respectively, in Figures D−3a through D−3f, Appendix D. For comparison purposes, the design thicknesses are also shown in these figures. When comparing these results to the layer thicknesses reported in Figures D−2, Appendix D, it is noticed that the GPR system performs similarly when the layers are considered lossy or lossless. This similarity in performance is due to the low values of the conductivities, which did not considerably affect the estimated layer dielectric constants and, therefore, did not have a great influence on the layer thicknesses.
5.1.4 Common midpoint technique

As discussed in 4.4.3 the dielectric constant of the top pavement layer can be estimated based on the time-delays measured from GPR data collected simultaneously by bistatic air-coupled antennas and a monostatic ground-coupled antenna. This technique was applied to GPR data collected from the Virginia Smart Road to find the thickness of the overall HMA layer, which was assumed in this case to be homogenous. It should be noted that even though the HMA layer is composed of multiple layers of different dielectric constants, this technique would yield more accurate results than would the technique adopted in 5.1.1. In fact, with the CMP method, the dielectric constant is determined based on the reflection at the bottom of the layer, whereas with the other technique, the dielectric constant is based on the reflection at the top of the layer. Consequently, the dielectric constant estimated with the CMP technique would represent an average (or bulk) dielectric constant that accounts for all inhomogeneities. Moreover, because the dielectric constants of the various HMA layers are comparable, refraction of the oblique-air-coupled incident signal at the different layer interfaces could be neglected. Hence, the CMP equations developed in 4.4.3 could be applied here to find the dielectric constant of the overall HMA layer without considerable errors.

After coupling pulse removal and noise filtering of the air-coupled data, a detector was used to find the time-delays of the surface and bottom of the layer reflections in the same way as was described in 5.1.1. A similar process, with the exception of the coupling pulse removal, was applied to the ground-coupled data. The inherent coupling pulse was not removed from the ground-coupled data because it represents, in this case, the sum of surface reflection and the coupling (or end reflection) pulse. Therefore, its removal would eliminate valuable information from the reflected GPR signal. The estimated time delays from the air-coupled and ground-coupled data were then substituted into the CMP equations to find the dielectric constant of the overall HMA layer and, hence, its thickness. Practically, because the coupling pulse and surface reflection are overlapped in the ground-coupled data, the exact time-delay of the latter reflection could not be determined precisely. To resolve this problem, the time-delay found by the detector on the ground-coupled data was offset to account for the overlap. The value of the offset was determined by trial and error using the known thickness measured on a calibration core and the time-delays estimated from the GPR data collected over the core location before the actual
coring. The offset was then applied to the time-delays estimated from the ground-coupled data for all the studied sections. For this particular case, an offset of 0.2ns was used.

The thickness of the overall HMA layers of the six sections in Table 5-1 are depicted, respectively, in Figures D−4a through D−4f, Appendix D. For comparison purposes, the design thicknesses are also shown in the same figures. Comparing these figures with the results presented in Figure D−1, Appendix D, shows that the thicknesses found with the CMP method are better approximations of the design thickness. Accordingly, it can be concluded that the CMP technique outperforms the classic technique when estimating the overall thickness of multiple inhomogeneous layers. Among all the studied sections, it is further noticed that the CMP technique has the lowest performance for section K. This degradation in the performance is mainly due to the break down in the assumption that the refraction effects are negligible across the different HMA interfaces. This is primarily caused by the PVC geocomposite membrane under the third lift of the BM-25.0, which has a higher dielectric constant than HMA.

5.2 Accuracy of GPR Layer Thicknesses

To assess the accuracy of the GPR tool, the layer thicknesses reported by GPR should be compared to the real thicknesses of the pavement layers. These thicknesses usually differ from the design thicknesses because the paving procedures used during pavement construction are inaccurate. Hence, GPR accuracy should not be evaluated based on a comparison of the GPR thicknesses to the design thicknesses. A more appropriate performance evaluation technique is to compare the GPR results to thicknesses measured directly on pavement cores. For this assessment method to be reliable, the GPR data used for the comparison should be collected prior to coring from the same locations where the cores would be taken from. In fact, since coring is destructive to the pavement, the presence of core pits, even filled with HMA material, would affect the GPR signals.

The accuracy of the developed GPR analysis system was evaluated based on the thicknesses measured on 19 cores taken from the different sections of the Virginia Smart Road. Figure 5-1 shows a typical core taken from Section H. Four different layers can be identified in the core: wearing surface between 0 and 1.5in, BM-25.0 between 1.5 and 5in, SM-9.5A between 5 and 8in, and OGDL between 8 and 11in. The different layers can be clearly separated in the
core based on aggregate size. All cores were extracted from the pavement after all necessary GPR data was collected. To facilitate finding the core locations on the GPR data, the cores were taken at a distance of 0.8m (2.6ft) from the centers of the copper plates embedded in the pavement, which were surveyed and marked on the pavement surface after the Virginia Smart Road construction was finished. Then, based on the copper plate signature, the GPR data corresponding to the core locations was identified by moving a number of scans corresponding to the offset distance. Locating core data in GPR files was facilitated by the use of the distance measuring instrument, which ensured data collection at regular intervals (practically 1 scan every 0.1m or 4in).

The GPR data collected at the core locations was analyzed using the four data analysis variants presented in 5.1. Table 5-2 presents a comparison between the total core thicknesses and the thicknesses measured by GPR using the two analysis techniques: overall HMA thickness estimation and the CMP technique. The distance column in Table 5-2 represents the absolute distance, starting from the beginning of section A, at which the cores were taken. According to these results, the average absolute error in estimating the HMA thickness using the overall analysis method is 12.0%, whereas the average error introduced by the CMP technique is only 3.9%. For the overall thickness estimation technique, the error varies between a minimum of
1.1% for section A to a maximum of 27.6% for section K. For the CMP technique, the absolute error varies between a minimum of 0.6% for section A to a maximum of 14.4% for section J.

Table 5-2: Comparison between Core Thicknesses and GPR Thicknesses Obtained by the Overall and CMP Analysis Techniques

<table>
<thead>
<tr>
<th>Core #</th>
<th>Dist (m)</th>
<th>HMA Thick (mm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Core</td>
<td>Overall</td>
</tr>
<tr>
<td>A1</td>
<td>45.0</td>
<td>282</td>
<td>313</td>
</tr>
<tr>
<td>A2</td>
<td>49.9</td>
<td>273</td>
<td>276</td>
</tr>
<tr>
<td>A3</td>
<td>54.8</td>
<td>266</td>
<td>259</td>
</tr>
<tr>
<td>A4</td>
<td>60.8</td>
<td>268</td>
<td>252</td>
</tr>
<tr>
<td>B2</td>
<td>148.5</td>
<td>283</td>
<td>322</td>
</tr>
<tr>
<td>D1</td>
<td>307.3</td>
<td>276</td>
<td>315</td>
</tr>
<tr>
<td>D2</td>
<td>312.6</td>
<td>265</td>
<td>301</td>
</tr>
<tr>
<td>E1</td>
<td>542.9</td>
<td>292</td>
<td>317</td>
</tr>
<tr>
<td>E2</td>
<td>547.9</td>
<td>285</td>
<td>302</td>
</tr>
<tr>
<td>F1</td>
<td>608.9</td>
<td>211</td>
<td>247</td>
</tr>
<tr>
<td>F2</td>
<td>613.9</td>
<td>210</td>
<td>247</td>
</tr>
<tr>
<td>F3</td>
<td>619.1</td>
<td>206</td>
<td>239</td>
</tr>
<tr>
<td>G1</td>
<td>723.3</td>
<td>195</td>
<td>209</td>
</tr>
<tr>
<td>G2</td>
<td>733.3</td>
<td>204</td>
<td>214</td>
</tr>
<tr>
<td>H2</td>
<td>798.1</td>
<td>286</td>
<td>302</td>
</tr>
<tr>
<td>J1</td>
<td>979.2</td>
<td>286</td>
<td>330</td>
</tr>
<tr>
<td>J2</td>
<td>986.4</td>
<td>355</td>
<td>405</td>
</tr>
<tr>
<td>K1</td>
<td>1091.2</td>
<td>297</td>
<td>376</td>
</tr>
<tr>
<td>K2</td>
<td>1096.2</td>
<td>297</td>
<td>379</td>
</tr>
</tbody>
</table>

Average Absolute Error (%) 12.0 3.9

According to the results of Table 5-2, it is evident that the CMP technique outperforms the overall thickness estimation technique. Again, the high errors introduced by the latter analysis method are mainly due to the erroneous estimates of the dielectric constant, which were computed solely from the amplitudes of the surface reflection. The error is reduced when the CMP method is used since the dielectric constant is determined from the reflections at the bottom of the HMA layer. A regression analysis of the absolute error between core thicknesses and GPR thicknesses estimated by the overall analysis technique shows that the error tends to be inversely proportional to the wearing surface thickness, as shown in Figure 5-2. This confirms that the dielectric constant estimates computed from the amplitude of the surface reflections are
influenced by the wearing surface thickness, which causes the superposition of the reflections resulting from both layer interfaces if the layer is relatively thin. The superposition of the reflected pulses typically causes the reduction of the surface reflection amplitude from its true value and therefore, according to equation (4-86), the decrease of the HMA dielectric constant. The errors resulting from the CMP technique could not be correlated to any of the pavement sections characteristics, such as the number of layers or their relative thicknesses. However, since the errors found for cores extracted from the same section have generally comparable values, it is apparent that the CMP error is due to the inadequacy of the calibration factor used, which was computed based on core A3 and was then kept invariant for all the other sections.

![Graph showing variation of absolute GPR error versus wearing surface thickness](image)

**Figure 5-2: Variation of Absolute GPR Error versus Wearing Surface Thickness**

Table 5-3 presents a comparison between the total thicknesses measured from cores and the thicknesses determined by GPR using the separation into individual layers technique for both the lossless and the lossy layers (loss equals 8dB/m for each layer) cases. The average absolute errors found between core thicknesses and GPR thicknesses estimated by both techniques are, respectively, 3.1% and 3.3%. For the lossless layers technique, the absolute error varies between 0.2% for section F and 9.4% for section K, whereas for the lossy layers case, the error varies, respectively, from 0.2% to 11.1% for the same sections. Consequently, assuming the pavement layers lossless when analyzing GPR data yields slightly more accurate results than when the layers are considered lossy. The degradation of the thickness estimation performance when
material loss is introduced could be attributed to the variations of material loss along the
pavement sections; whereas for the analysis, the loss was computed from core A2 and was then
kept invariant along all the pavement sections. Hence, it could be concluded that using the
wrong material loss values during the GPR analysis phase would decrease the accuracy of the
results. The GPR thickness errors introduced by incorrect material loss values would increase as
the layer thicknesses increase. Finally, it is worth noting that for the CMP method, no
correlation between the thickness errors and the characteristics of the pavement sections (i.e.
number of layers and their relative thicknesses) were found. However, according to Table 5-3,
the thickness error generally increases for both analysis techniques when the number of thin
layers within the pavement increases (such as for sections H and K).

Table 5-3: Comparison between Core Thicknesses and GPR Thicknesses Obtained by the Individual and
Individual + Loss Analysis Techniques

<table>
<thead>
<tr>
<th>Core #</th>
<th>Dist (m)</th>
<th>HMA Thick (mm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Core</td>
<td>Individual</td>
<td>Individual + Loss</td>
</tr>
<tr>
<td>A1</td>
<td>45.0</td>
<td>282</td>
<td>272</td>
</tr>
<tr>
<td>A2</td>
<td>49.9</td>
<td>273</td>
<td>276</td>
</tr>
<tr>
<td>A3</td>
<td>54.8</td>
<td>266</td>
<td>259</td>
</tr>
<tr>
<td>A4</td>
<td>60.8</td>
<td>268</td>
<td>252</td>
</tr>
<tr>
<td>B2</td>
<td>148.5</td>
<td>283</td>
<td>266</td>
</tr>
<tr>
<td>D1</td>
<td>307.3</td>
<td>276</td>
<td>271</td>
</tr>
<tr>
<td>D2</td>
<td>312.6</td>
<td>265</td>
<td>255</td>
</tr>
<tr>
<td>E1</td>
<td>542.9</td>
<td>292</td>
<td>298</td>
</tr>
<tr>
<td>E2</td>
<td>547.9</td>
<td>285</td>
<td>283</td>
</tr>
<tr>
<td>F1</td>
<td>608.9</td>
<td>211</td>
<td>211</td>
</tr>
<tr>
<td>F2</td>
<td>613.9</td>
<td>210</td>
<td>211</td>
</tr>
<tr>
<td>F3</td>
<td>619.1</td>
<td>206</td>
<td>204</td>
</tr>
<tr>
<td>G1</td>
<td>723.3</td>
<td>195</td>
<td>193</td>
</tr>
<tr>
<td>G2</td>
<td>733.3</td>
<td>204</td>
<td>202</td>
</tr>
<tr>
<td>H2</td>
<td>798.1</td>
<td>286</td>
<td>270</td>
</tr>
<tr>
<td>J1</td>
<td>979.2</td>
<td>286</td>
<td>292</td>
</tr>
<tr>
<td>J2</td>
<td>986.4</td>
<td>355</td>
<td>367</td>
</tr>
<tr>
<td>K1</td>
<td>1091.2</td>
<td>297</td>
<td>269</td>
</tr>
<tr>
<td>K2</td>
<td>1096.2</td>
<td>297</td>
<td>275</td>
</tr>
<tr>
<td>Average Absolute Error (%)</td>
<td>3.1</td>
<td>3.3</td>
<td></td>
</tr>
</tbody>
</table>
A comparison of the overall HMA thickness results obtained by the four GPR data analysis variants (Tables 5–2 and 5–3) shows that separation of the individual HMA layers (regardless of material loss consideration) yields the highest thickness accuracy of the GPR tool. It should be noted that the average error reported for this technique (i.e., 3.1%) is similar to the error observed when measuring the thickness on a pavement core from different spots (error of 2.9% reported in [74]). The errors encountered when measuring pavement cores are mainly due to fuzzy layer interfaces (as can be observed in Figure 5-1), large aggregates at the interface, or missing chunks from the bottom of the core.

Analyzing GPR data after separation of the pulses reflected from all the layer interfaces yields the individual layer thicknesses. Thus, GPR’s accuracy in determining multiple layer thicknesses could be evaluated by comparing the thicknesses estimated by GPR to the thicknesses of the individual layers measured on cores. A comparison between the thicknesses reported by GPR in the lossless layers case and the thicknesses measured directly on cores is presented in Table 5-4. The studied layers are the wearing surface, BM-25.0, SM-9.5A, and OGDL. From the results, it is found that the average absolute thickness error for the wearing surface is 26.1%, varying between 0.8% for core E1 and 46.1 for core K2. For the BM-25.0 layer, the thickness error is 7.1%, varying between 0.4% for section E2 and 30.3% for core G2. The SM-9.5A is only present in the cores taken from section G, and it has an average error of 15.0%, varying from 7.1% to 22.9%. Finally, the OGDL layer has an average thickness error of 8.0%, varying between 1.0% for core A1 and 16.5 for core B2.

Based on the average errors shown in Tables 5–3 and 5–4, GPR’s accuracy in estimating the thicknesses of the individual HMA layers seems to be worse than its accuracy when computing the overall thickness even if the same data analysis technique is used (individual errors of 26.1%, 7.1%, 15.0%, and 8.0% compared to an overall error of 3.1%). These errors appear to be large because of the small thicknesses of the considered layers. In fact, if the errors are converted to thicknesses instead of percentages, they become comparable in values (respectively, 10, 11, 8, and 6mm for the WS, BM-25.0, SM-9.5A, and OGDL layers compared to 8mm for the overall thickness). The slightly higher errors found for the WS and BM-25.0 layers are caused, in part, by the thickness measurement on cores, since the interface between these two layers is sometimes not well defined, as can be noticed in Figure 5-1. For the
individual layer thicknesses, no correlation was found between the GPR accuracy and the layer types, thicknesses, depths, or the pavement structure in general.

Table 5-4: Comparison between Individual Layer Thicknesses Measured on Cores and Estimated by GPR

<table>
<thead>
<tr>
<th>Core #</th>
<th>Core (mm)</th>
<th>GPR (mm)</th>
<th>Error (%)</th>
<th>Core (mm)</th>
<th>GPR (mm)</th>
<th>Error (%)</th>
<th>Core (mm)</th>
<th>GPR (mm)</th>
<th>Error (%)</th>
<th>Core (mm)</th>
<th>GPR (mm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>51</td>
<td>40</td>
<td>-21.1</td>
<td>168</td>
<td>169</td>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
<td>64</td>
<td>63</td>
<td>-1.0</td>
</tr>
<tr>
<td>A2</td>
<td>50</td>
<td>37</td>
<td>-25.5</td>
<td>164</td>
<td>183</td>
<td>11.8</td>
<td></td>
<td></td>
<td></td>
<td>60</td>
<td>56</td>
<td>-6.1</td>
</tr>
<tr>
<td>A3</td>
<td>54</td>
<td>35</td>
<td>-35.2</td>
<td>161</td>
<td>173</td>
<td>7.7</td>
<td></td>
<td></td>
<td></td>
<td>52</td>
<td>51</td>
<td>-1.3</td>
</tr>
<tr>
<td>A4</td>
<td>52</td>
<td>40</td>
<td>-23.6</td>
<td>155</td>
<td>166</td>
<td>6.9</td>
<td></td>
<td></td>
<td></td>
<td>60</td>
<td>56</td>
<td>-15.0</td>
</tr>
<tr>
<td>B2</td>
<td>43</td>
<td>54</td>
<td>24.6</td>
<td>169</td>
<td>153</td>
<td>-9.3</td>
<td></td>
<td></td>
<td></td>
<td>71</td>
<td>59</td>
<td>-16.5</td>
</tr>
<tr>
<td>D1</td>
<td>53</td>
<td>59</td>
<td>12.0</td>
<td>146</td>
<td>130</td>
<td>-11.2</td>
<td></td>
<td></td>
<td></td>
<td>77</td>
<td>82</td>
<td>7.0</td>
</tr>
<tr>
<td>D2</td>
<td>50</td>
<td>56</td>
<td>12.0</td>
<td>149</td>
<td>142</td>
<td>-4.5</td>
<td></td>
<td></td>
<td></td>
<td>67</td>
<td>57</td>
<td>-14.5</td>
</tr>
<tr>
<td>E1</td>
<td>41</td>
<td>40</td>
<td>-0.8</td>
<td>251</td>
<td>257</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>251</td>
<td>257</td>
</tr>
<tr>
<td>E2</td>
<td>39</td>
<td>36</td>
<td>-8.5</td>
<td>246</td>
<td>247</td>
<td>0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>246</td>
<td>247</td>
</tr>
<tr>
<td>F1</td>
<td>38</td>
<td>29</td>
<td>-24.3</td>
<td>173</td>
<td>182</td>
<td>5.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>173</td>
<td>182</td>
</tr>
<tr>
<td>F2</td>
<td>39</td>
<td>29</td>
<td>-25.6</td>
<td>171</td>
<td>182</td>
<td>6.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>171</td>
<td>182</td>
</tr>
<tr>
<td>F3</td>
<td>39</td>
<td>26</td>
<td>-33.9</td>
<td>166</td>
<td>178</td>
<td>7.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>166</td>
<td>178</td>
</tr>
<tr>
<td>G1</td>
<td>45</td>
<td>29</td>
<td>-36.0</td>
<td>91</td>
<td>119</td>
<td>30.3</td>
<td>58</td>
<td>45</td>
<td>-22.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G2</td>
<td>43</td>
<td>29</td>
<td>-32.6</td>
<td>96</td>
<td>103</td>
<td>7.7</td>
<td>65</td>
<td>70</td>
<td>7.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H2</td>
<td>41</td>
<td>29</td>
<td>-28.7</td>
<td>92</td>
<td>96</td>
<td>4.0</td>
<td>153</td>
<td>145</td>
<td>-5.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J1</td>
<td>44</td>
<td>30</td>
<td>-32.3</td>
<td>242</td>
<td>262</td>
<td>8.3</td>
<td>N/A</td>
<td>N/A</td>
<td>NA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J2</td>
<td>42</td>
<td>30</td>
<td>-28.6</td>
<td>238</td>
<td>258</td>
<td>8.6</td>
<td>75</td>
<td>79</td>
<td>5.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K1</td>
<td>55</td>
<td>30</td>
<td>-45.1</td>
<td>242</td>
<td>239</td>
<td>-1.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K2</td>
<td>56</td>
<td>30</td>
<td>-46.1</td>
<td>241</td>
<td>245</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Abs. Error</td>
<td>26.1</td>
<td>7.1</td>
<td>15.0</td>
<td>8.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.3 Summary

In this chapter, the developed GPR data analysis system was validated and its performance in estimating layer thicknesses was assessed. The performance of the data analysis system was evaluated based on field data collected from the Virginia Smart Road. The system validation and assessment were carried out for four variants of the data analysis: estimation of the overall HMA layer thickness, estimation of the thicknesses of the individual HMA layers assuming that the layers are lossless, estimation of the thicknesses of the individual HMA layers assuming that the layers are lossy, and estimation of the overall HMA layer thickness using the common midpoint technique. The thickness accuracy of the GPR data analysis techniques was estimated based on comparisons of the thicknesses reported by GPR and the thicknesses measured directly on pavement cores taken from the Virginia Smart Road.
It was found that estimating the HMA overall thickness without separation between the individual layers produces relatively high thickness errors, in the range of 12%. This error is reduced to approximately 4% when the CMP method is used. Separating the individual HMA layers before estimating the thickness further reduces the error of the overall thickness to around 3%. For this analysis variant, considering the pavement layers as composed of lossless materials resulted in slightly more accurate results than when incorrect material loss values were used (error of 3.1% for the lossless case, versus 3.3% for the lossy case). When the layers are separated, the errors on the individual layer thicknesses relative to their actual thicknesses were much higher than were the errors on the overall thickness found by the same analysis technique; however, the absolute errors were comparable.

The error observed for the overall HMA layer thickness, when computed without separation of the individual layers, was found to decrease with increasing wearing surface thickness. This suggests that this error is mainly caused by the superposition of the reflected pulses from the two interfaces of that layer, which leads to the decrease of the dielectric constant used to compute the overall thickness. For the other three data analysis variants, the thickness errors could not be correlated to any of the pavement characteristics, such as the number of layers, their thicknesses, depths, or compositions. However, the errors were found to increase slightly when the number of thin layers within the pavement increases.

Finally, it is worth noting that the CMP method developed in this research effort gives accurate thickness results only if some conditions are satisfied. In particular, for this technique to be successful, the GPR data should be acquired with a high sampling rate, the distance between the bistatic antennas should be as large as possible, and the height of the air-coupled antenna should be as low as possible. For this particular study, the monostatic antenna used with the CMP method was ground-coupled, which required some calibration on the field cores. Using an air-coupled monostatic antenna instead of the ground-coupled antenna might enhance the technique by eliminating the need for core calibration and by increasing the accuracy of the system.
Chapter 6

Findings and Conclusions

6.1 Summary

Accurate layer thickness estimation is an important issue for pavement engineers, for both newly constructed and old pavements. For new pavements, layer thickness measurement is essential to ensure that the placed layers meet the design specifications, as part of quality control and quality assurance. For old pavements, layer thickness measurement and subsurface defect localization are important to make appropriate economical rehabilitation decisions. This information is also essential for pavement management systems. Traditionally, layer thickness estimation is based on direct measurements of a finite number of cores taken from the road. This technique has two major drawbacks: first, it introduces perturbations to the pavement structure near the core locations; second, it is time-consuming and requires closing of the surveyed lane for traffic, which is not an economical practice, especially on highways. Ground Penetrating Radar (GPR) represents a prominent alternative to coring since it can be used for nondestructive pavement profiling and subsurface distress detection at highway speeds. However, GPR suffers from a major problem: the huge amount of collected data that is usually analyzed manually by experienced operators, which could yield to inaccurate and biased results.

The objective of this research was to develop a data analysis system that can estimate the pavement layer thicknesses and locate subsurface defects from GPR data without the need for destructive coring. The developed processing system is divided into five processing stages that work successively to provide the required output. The five processing stages are as follows:

1. Preprocessing: this stage is intended to enhance the quality of the GPR signal by removing any unwanted noise and spurious reflections. In particular, in this stage the coupling pulse inherent to the GPR signal is removed, and a filter is then applied to filter
out the noise corrupting the signal. The optimum filter found to process the GPR signal is an elliptic filter applied in a forward-backward procedure to eliminate any phase distortions. Afterwards, the GPR signal is deconvolved to enhance its depth resolution. Six different deconvolution methods were tried on the signal: inverse filtering, predictive deconvolution, pulse spiking, pulse shaping, homomorphic deconvolution, and iterative decomposition of the GPR signal. The homomorphic deconvolution was found to outperform all the other techniques.

2. Detection of layer interface reflections and estimation of their time-delays and amplitudes: The detection procedure was carried out iteratively. The algorithm starts by detecting all possible reflected pulses and estimating their time delays. The time delays are then used in a least-squares fitting procedure to yield a theoretical signal that approximates the measured GPR signal. Next, the synthesized signal is subtracted from the measured signal, resulting in a difference signal composed mainly of weak and originally masked reflected pulses. The whole procedure is then repeated on the difference signal until all reflections are detected. Within each iteration, the detection of the reflected pulses was performed either by a threshold detector or by a matched filter detector.

3. Separation of layer interfaces from distresses: this stage is carried out in two steps. First, based on the time delays and amplitudes of the reflected pulses, all reflections resulting from the same layer interface or distress are grouped together using a layer tracking algorithm. Second, the layer interfaces are separated from distresses based on their longitudinal extensions: layer interfaces usually have a length comparable to the surveyed distance whereas distresses are generally localized to small areas.

4. Dielectric constant estimation of the detected layers: this stage is carried out using the reflection amplitudes or alternatively using the reflection time-delays in a modified common midpoint (CMP) technique.

5. Estimation of layer thicknesses and distress depths using the reflection time-delays and layer dielectric constants.

The developed GPR data analysis system was validated using field data collected from an experimental pavement system with a known and well documented structure: the Virginia Smart
The accuracy of the GPR results obtained by four slightly different variants of the analysis system was evaluated by comparing the GPR results to direct thickness measurements on cores taken from the road. The four data analysis variants include the following: estimation of the overall HMA layer thickness, estimation of the thicknesses of the individual HMA layers assuming that the layers are lossless, estimation of the thicknesses of the individual HMA layers assuming that the layers are lossy, and estimation of the overall HMA layer thickness using the common midpoint technique.

6.2 Findings

Various findings concerning GPR data analysis were encountered during this research. These findings are summarized as follows:

- The different deconvolution procedures applied to GPR data were found to affect the performance of the subsequent detection stage because they decrease the signal-to-noise ratio (SNR) of the signal by introducing processing noise. A better alternative for enhancing the depth resolution of the GPR system is to detect the weak and masked reflections after subtracting from the measured signal a synthesized signal obtained by least-squares fitting of the strong reflections to a theoretical model. This procedure is repeated iteratively until all reflections are detected.
- The dielectric properties of HMA layers were found to be almost invariant within the GPR bandwidth. Thus, using an average dielectric constant does not reduce the performance of the GPR system.
- Even though material loss enters in the computation of the layer dielectric constant, the pavement layers could be assumed to be lossless without a significant effect on the GPR accuracy. On the other hand, if high and incorrect material loss values are used in the analysis, the layer thickness error would increase.
- Estimating the dielectric constant of the pavement materials based uniquely on the surface reflection and then assuming it invariant across the layers yields incorrect layer thickness estimates. The error with this technique usually increases as the thickness of the wearing surface decreases.
• A modified common midpoint technique (CMP) can be used to accurately estimate the bulk dielectric constant of multiple layers that have slightly different dielectric constants and then to determine their overall thickness. This technique could be applied successfully only if some conditions about the configuration of the antennas (such as air-coupled antenna height) and the GPR data acquisition (such as minimum sampling rate) are satisfied.

• The presence of a geocomposite membrane or relatively thick interlayer systems within the pavement layers usually decreases the performance of the layer interface detector and degrades the thickness accuracy of the overall GPR system.

### 6.3 Conclusions

A system for analysis and interpretation of GPR data collected over pavements was developed. Within this system, GPR data is analyzed on a scan by scan basis by passing through successive processing stages. The results obtained from the individual scans are then grouped together to form the different pavement layer interfaces and subsurface distresses based on which the layer thicknesses and distress depths are estimated. The performance of the developed system was evaluated using data collected from a pavement system with a known structure. Based on this research, the following conclusions could be drawn:

• Ground penetrating radar can be used reliably to nondestructively assess pavement systems and to estimate their layer thicknesses and distress locations.

• For accurate GPR thickness results in the case of a multiple-thin-layer pavement system (typical characteristic of flexible pavements), the reflections from all the layer interfaces should be separated or deconvolved. The dielectric constant and thickness of each layer should then be determined individually rather than as a single thick layer.

• The thickness error found for the overall HMA layer when the multiple layers composing HMA are separated is around 3%. In contrast, when the layers are not separated, the thickness error reaches 12%.

• The relative thickness error for the individual HMA layers increases with decreasing layer thickness; however, the absolute error is almost constant and is similar to the absolute error observed on the overall thickness.
• The accuracy of the developed GPR data analysis system is almost independent of the pavement structure, such as the number of layers, their thicknesses, compositions, or depth. However, the thickness error increases slightly when the number of thin layers within the pavement (such as overlays) increases.

6.4 Recommendations

As a continuation to this research, the following recommendations are proposed:

• In this study, the pavement layer material loss was considered to be invariant along the surveyed pavement. A thorough investigation of the significance of the variations of the pavement layers’ material loss on GPR results should be conducted.

• Subsurface distresses were assumed in this research to be localized and to have dielectric properties different from the dielectric properties of the regular layers. Study of the actual “shapes” and the GPR signature of field subsurface distresses would enhance the ability of GPR to detect them and identify their types (i.e., stripping, moisture accumulation, voids, etc.).

• The accuracy and reliability of the common midpoint technique when two air-coupled antennas are deployed (instead of an air-coupled and ground-coupled configuration as used in this research) should be examined.

• It is recommended to investigate the GPR accuracy when used over flexible pavements incorporating reinforcement. In fact, reinforcement, such as steel mesh, would produce multiple spurious reflections within the GPR response, which would mask the real reflections resulting from the deeper layers.

• Further research is needed to incorporate the GPR results into an Expert System that would assess the condition of the surveyed pavement, predict its remaining service life, and suggest adequate rehabilitation procedures in case of severe deterioration. The results of other nondestructive testing procedures, such as the Falling Weight Deflectometer, could also be incorporated into the Expert System for better decision making.
References


Appendix A

Detailed GPR Data Analysis Algorithms

Detailed algorithms that could be used for pavement thickness estimation from GPR data are presented in the following paragraphs. These algorithms could be implemented using Matlab® with the Signal Processing toolbox.

A.1 Preprocessing

A.1.1 Coupling pulse removal

For a detailed discussion of the coupling pulse removal procedure, refer to section 4.2.1. The coupling pulse removal algorithm is as follows:

1. Load coupling pulse signal \( y_c(t) \),
2. For each GPR scan \( y_r(t) \) do:
   a. Compute the cross-correlation function between \( y_r(t) \) and \( y_c(t) \),
   b. Find lag \( l_m \) corresponding to maximum cross-correlation,
   c. Time-shift signal \( y_c(t) \) by \( l_m \) samples,
   d. Subtract time-shifted signal \( y_c(t-l_m) \) from reflected signal \( y_r(t) \).

A.1.2 Noise filtering

For a detailed discussion of noise filtering of GPR data, refer to section 4.2.2. Forward-backward noise filtering could be implemented as follows:

1. For each GPR scan \( y_r(t) \) do:
   a. In the forward stage, filter \( y_r(t) \) to obtain the forward output \( y_{rf}(t) \),
   b. Time reverse the output of the first stage to obtain the signal \( y_{rf}(-t) \),
c. In the backward stage, filter \( y_r(-t) \) to get the backward output \( y_{rb}(-t) \),
d. Time reverse the output of the second stage to obtain the filtered output \( y_{r \text{filt}}(t) = y_{rb}(-t) \).

### A.1.3 Depth resolution enhancement

For a detailed discussion of depth resolution enhancement of GPR data, refer to section 4.2.3 and the subsequent paragraphs. The different depth resolution enhancement techniques could be implemented as follows:

#### A.1.3.1 Inverse filtering

1. Choose a filter length \( N \),
2. For each GPR scan \( y_r(t) \) do the following:
   a. Compute the autocorrelation function \( r_{yy}(l) \) of the reflected signal \( y_r(t) \) for \( l = 0, 1, \ldots, N-1 \),
   b. Construct the Toeplitz matrix \( R_{yy} \) corresponding to \( r_{yy}(l) \),
   c. Use the Levinson-Durbin algorithm to solve for the inverse filter coefficients \( \{ a_k \} \) based on equation (4-23),
   d. Filter the reflected signal \( y_r(t) \) using the filter \( \{ a_k \} \) to obtain the deconvolved GPR signal.

#### A.1.3.2 Predictive deconvolution

1. Choose a filter length \( N \) and a prediction distance \( \alpha \),
2. For each GPR scan \( y_r(t) \) do the following:
   a. Compute the autocorrelation function \( r_{yy}(l) \) of the reflected signal \( y_r(t) \) for \( l = 0, 1, \ldots, \alpha+N-1 \),
   b. Construct the Toeplitz matrix \( R_{yy} \) corresponding to \( r_{yy}(l) \),
   c. Use the Levinson-Durbin algorithm to solve for the prediction filter coefficients \( \{ a_k \} \) based on equation (4-31),
   d. Construct the prediction error filter coefficients \( \{ f_k \} \) as in equation (4-34),
   e. Filter the prediction signal \( \hat{y}_r(t) \) using the filter \( \{ f_k \} \) to obtain the deconvolved GPR signal.
A.1.3.3 Pulse spiking

1. Choose a filter length $N$ and a desired output lag $l$,
2. Compute the autocorrelation function $r_{xx}(l)$ of the GPR incident signal $x(t)$ for $l = 0, 1, \ldots, N-1$,
3. Construct the Toeplitz matrix $R_{xx}$ corresponding to $r_{xx}(l)$,
4. Use the Levinson-Durbin algorithm to solve for the spiking filter coefficients $\{a_k\}$ based on equation (4-39),
5. For each GPR scan $y_r(t)$ do:
   a. Remove the coupling pulse from the reflected signal $y_r(t)$ to obtain the “clean” reflected signal $y_{rc}(t)$,
   b. Filter the signal $y_{rc}(t)$ with the filter $\{a_k\}$ to obtain the deconvolved signals.

A.1.3.4 Homomorphic deconvolution

1. For each GPR scan $y_r(t)$ do the following:
   a. Remove the coupling pulse from the reflected signal $y_r(t)$ to obtain the “clean” reflected signal $y_{rc}(t)$,
   b. Compute the power spectrum $\Phi_y(f)$ of the signal $y_{rc}(t)$ using periodograms or any other nonparametric power spectral estimation technique,
   c. Compute the natural logarithm of the power spectrum $\Phi_y(f)$,
   d. Compute the power spectrum $C_y(q)$ of the logarithm of the power spectrum $\Phi_y(f)$ to obtain the deconvolved GPR signal.

A.2 Detection of Layer Interface Reflections

A.2.1 Threshold detector

For a detailed discussion of the detection algorithm based on the threshold detector, refer to sections 4.3.3 and 4.3.3.1. The detection algorithm is as follows:

1. Assign the measured reflected signal $y_r(t)$ to $d(t)$,
2. Use a threshold detector to detect all reflected pulses in the signal $d(t)$ according to the following:
a. Compute the envelope of the signal $d(t)$,
b. Find the maximum value of the envelope,
c. Compare the maximum value found to the selected threshold. If the maximum value is less than the threshold, then terminate the detection procedure and go to step 3; otherwise, take the time-delay of the maximum value as the time-delay of the detected pulse,
d. Set the envelope values around the obtained time-delay to zero for a length equal to the width of the incident pulse,
e. Repeat at step b.

3. Use the time-delays of the detected pulses to find the optimum synthesized reflected signal $y_{rs}(t)$ that approximates the signal $y_r(t)$ in the least-squares sense, as shown in Appendix C,

4. Compute the difference signal $d(t) = y_r(t) - y_{rs}(t)$,

5. Repeat at step 2 until no more pulses are detected in the signal $d(t)$,

6. Combine all reflection time-delays found by the previous steps and determine the optimum set that yields a minimum least-squares error between the measured reflected signal $y_r(t)$ and the synthesized signal $y_{rs}(t)$,

7. Determine the reflection amplitudes $A_i$ from the optimum least-squares fit found.

### A.2.2 Matched filter detector

For a detailed discussion of the detection algorithm based on the matched filter detector refer to sections 4.3.3 and 4.3.3.2. The detection algorithm is as follows:

1. Assign the measured reflected signal $y_r(t)$ to $d(t)$,
2. Use a matched filter detector to detect all reflected pulses in the signal $d(t)$ according to the following:
   a. Compute the matched filter impulse response $h(t)$ as the time reversal of the incident GPR signal $x(t)$,
   b. Filter the difference signal $d(t)$ using the matched filter $h(t)$ to obtain the filtered signal $d_{MF}(t)$,
   c. Find the maximum absolute value of the matched filter output $d_{MF}(t)$,
d. Compare the maximum value found to the selected threshold $S_r$. If the maximum value is less than the threshold, then terminate the detection procedure and go to step 3; otherwise, subtract the signal duration $T$ of the known signal $x(t)$ from the time-delay corresponding to the maximum value of the MF output to find the time-delay of the detected pulse,
e. Set the detected pulse in $d(t)$ to zero,
f. Repeat at step b.

3. Use the time-delays of the detected pulses to find the optimum synthesized reflected signal $y_{rs}(t)$ that approximates the signal $y_A(t)$ in the least-squares sense as shown in Appendix C,

4. Compute the difference signal $d(t) = y_A(t) - y_{rs}(t)$,

5. Repeat at step 2 until no more pulses are detected in the signal $d(t)$,

6. Combine all reflection time-delays found by the previous steps and determine the optimum set that yields a minimum least-squares error between the measured reflected signal $y_A(t)$ and the synthesized signal $y_{rs}(t)$,

7. Determine the reflection amplitudes $A_i$ from the optimum least-squares fit found.

### A.3 Layer Interface Tracking

For a detailed discussion of layer interface/distress separation refer to section 4.5. The layer interface tracking algorithm can be implemented as follows:

1. Form initial tracks based on reflection time-delays and amplitudes:
   a. Take the detected pulses corresponding to the first scan as the initial points of the tracks,
   b. Assign each detected pulse from the current GPR scan to the track that has the least distance between its last point and the detected pulse. The distance should be within the maximum distance $d_{max}$. If no tracks are found within $d_{max}$, then create a new track for the processed reflected pulse,
   c. Extend tracks that were not assigned new points from the last scan using phantom points having the same coordinates as the last known true points,
d. Repeat at step b until all GPR scans are processed,

2. Exchange track points until a minimum global deviation of the detected tracks is reached:
   a. Let $k$ be the track point number, which corresponds to the scan number. Start from scan number 3,
   b. Take all combinations without repetitions of two tracks among all detected tracks,
   c. For each pair of tracks $i$ and $j$ that have their $k^{th}$ point within the distance $d_{max}$ constraint, compute the quantity:
   \[
   G = \phi(P_i^{k-2}, P_i^{k-1}, P_i^k) + \phi(P_j^{k-2}, P_j^{k-1}, P_j^k) - \phi(P_i^{k-2}, P_i^{k-1}, P_i^k) - \phi(P_j^{k-2}, P_j^{k-1}, P_j^k)
   \]
   d. If G is positive then swap the points $P_i^k$ and $P_j^k$,
   e. Repeat at step b until all points of all detected tracks are processed.
Appendix B

Wiener Filtering

In the following paragraphs, a brief description of Wiener filters will be presented. A more detailed analysis could be found in any digital signal processing book such as [43] or [52].

Wiener filters were introduced by Norbert Wiener, who was the first to develop optimal filtering techniques in the least-squares sense. As shown in Figure B−1, Wiener filters are usually designed to approximate an input signal $x(t)$, that might be unknown, with a desired signal $d(t)$ such that the error signal $e(t)$ between $x(t)$ and $d(t)$ is minimum in the least-squares sense. In the following derivation, the discrete signals $x(t)$ and $d(t)$ are assumed causal (i.e. $x(t) = d(t) = 0$ for $t < 0$).

Assuming that the Wiener filter is implemented using an FIR filter with a length $N$ impulse response $a_k$, the filter output $y(t)$ would be, according to Figure B−1, as follows:

$$y(t) = x(t) * a_k = \sum_{k=0}^{N-1} a_k x(t - k)$$  \hspace{1cm} (B−1)

which results in an error signal $e(t)$ between $x(t)$ and $d(t)$ given by:
\[ e(t) = d(t) - y(t) = d(t) - \sum_{k=0}^{N-1} a_k x(t - k) \]  \hspace{1cm} (B-2) 

The sum of the squares of the error over the length of the signals is therefore:

\[ I = \sum_{t} e(t)^2 = \sum_{t} \left[ d(t) - \sum_{k=0}^{N-1} a_k x(t - k) \right]^2 \]  \hspace{1cm} (B-3) 

To achieve a minimum mean-square error (MMSE), the sum of squared errors given by equation (B-3) should be minimized with respect to all filter coefficients \( a_k \). Thus, the partial derivative of \( I \) with respect to any filter coefficient \( a_i \) should be equal to zero:

\[ \frac{\partial I}{\partial a_i} = -2 \sum_{t} \left[ d(t) - \sum_{k=0}^{N-1} a_k x(t - k) \right] x(t - i) = 0, \text{ for } i = 0, 1, \ldots, N-1 \]  \hspace{1cm} (B-4) 

Rearranging and simplifying equation (B-4) leads to:

\[ \sum_{t} d(t)x(t - i) = \sum_{k=0}^{N-1} a_k \sum_{t} x(t - k)x(t - i), \text{ for } i = 0, 1, \ldots, N-1 \]  \hspace{1cm} (B-5) 

The left-hand side of equation (B-5) represents the cross-correlation function between the desired signal \( d(t) \) and the input signal \( x(t) \):

\[ \sum_{t} d(t)x(t - i) = r_{dx}(i) \]  \hspace{1cm} (B-6) 

The inside summation of the right-hand side of equation (B-5) represents the autocorrelation function of the input signal \( x(t) \), therefore:

\[ \sum_{t} x(t - k)x(t - i) = \sum_{t} x(t)x[t - (i - k)] = r_{xx}(i - k) \]  \hspace{1cm} (B-7) 

Substituting equations (B-6) and (B-7) into equation (B-5) leads to a set of equations, known as the normal equations:

\[ \sum_{k=0}^{N-1} a_k r_{xx}(i - k) = r_{dx}(i), \text{ for } i = 0, 1, \ldots, N-1 \]  \hspace{1cm} (B-8)
For real-valued signals \( x(t) \), the autocorrelation is an even function of the lag (i.e., \( r_{xx}(i) = r_{xx}(-i) \)). Using this result, the set of equations (B–8) could be put in a matrix form as follows:

\[
\begin{bmatrix}
  r_{xx}(0) & r_{xx}(1) & r_{xx}(2) & \cdots & r_{xx}(N-1) \\
  r_{xx}(1) & r_{xx}(0) & r_{xx}(1) & \cdots & r_{xx}(N-2) \\
  r_{xx}(2) & r_{xx}(0) & r_{xx}(1) & \cdots & r_{xx}(N-3) \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  r_{xx}(N-1) & \cdots & \cdots & \cdots & r_{xx}(0)
\end{bmatrix}
\begin{bmatrix}
  a_0 \\
  a_1 \\
  a_2 \\
  \vdots \\
  a_{N-1}
\end{bmatrix}
= 
\begin{bmatrix}
  r_{dx}(0) \\
  r_{dx}(1) \\
  r_{dx}(2) \\
  \vdots \\
  r_{dx}(N-1)
\end{bmatrix}
\]  

(B–9)

Or equivalently:

\[
R_{xx} \mathbf{A} = R_{dx}
\]  

(B–10)

Where \( R_{xx} \) is the autocorrelation matrix, \( \mathbf{A} \) is the filter coefficients vector, and \( R_{dx} \) is the cross-correlation vector.

The optimal least-squares filter (Wiener filter) that satisfies the block diagram of Figure B–1 could then be found by solving equation (B–10). Theoretically, according to this equation, the solution requires only the knowledge of the autocorrelation and cross-correlation functions without the specific knowledge of the signals \( x(t) \) or \( d(t) \). In practice, however, the correlation functions are typically estimated from the signals themselves. A biased estimate of the autocorrelation function of a signal \( x(t) \) having a length \( M \) is computed according to [49] as follows:

\[
\hat{r}_{xx}(l) = \begin{cases} 
\frac{1}{M} \sum_{n=0}^{M-|l|-1} x(n)x(n-l) & |l| < M \\
0 & |l| \geq M
\end{cases}
\]  

(B–11)

It can be shown that the bias of the estimated autocorrelation function (i.e., difference between the actual autocorrelation function \( r_{xx}(l) \) and the expected value of its estimate \( \hat{r}_{xx}(l) \)) of equation (B–11) is given by [49]:

\[
B(l) = r_{xx}(l) - E\{\hat{r}_{xx}(l)\} = \frac{|l|}{M} r_{xx}(l)
\]  

(B–12)
where \( E\{\cdot\} \) is the expected value operator. Equation (B–12) shows that the bias could be reduced by increasing the number of samples \( M \); however, when the lag \( l \) approaches \( M \), the bias can become large, depending on the value of \( r_{xx}(l) \). For this case, it could be also shown that the variance of the autocorrelation estimate is essentially independent of the lag \( l \) [49].

Alternatively, an unbiased estimate of the autocorrelation function could be computed as follows [49]:

\[
\hat{r}_{xx}(l) = \begin{cases} 
\frac{1}{M - |l|} \sum_{n=0}^{M-|l|-1} x(n)x(n-l) & |l| < M \\
0 & |l| \geq M
\end{cases}
\]  

(B–13)

The bias of the estimated autocorrelation function of equation (B–13) is zero; however, its variance, which could be approximated by [49]:

\[
\text{var}(\hat{r}_{xx}(l)) \approx \frac{1}{(M - |l|)^2} \sum_{n=-M+1}^{M-1} r_{xx}^2(n) + r_{xx}(n+l)r_{xx}(n-l)
\]

(B–14)

would tend to infinity as the lag \( l \) approaches \( M \). For this reason, use of the biased estimate of the autocorrelation function given by equation (B–11) is practically more appropriate for Wiener filter design.

For large values of \( n \), the matrix relation of equation (B–10) can be solved efficiently using the Levinson-Durbin algorithm [43]. This algorithm uses the Toeplitz property of the autocorrelation matrix \( R_{xx} \) (symmetric and all elements on the diagonal or any sub-diagonal are identical) to recursively solve for \( A \) without going through the computation of the inverse of \( R_{xx} \). With the Levinson-Durbin algorithm, the number of computations is reduced from the usual \( N^3 \) to \( N^2 \).

The performance of the designed Weiner filter could be evaluated based on the sum of the squared errors \( I \) given by equation (B–3) [53], which represents a measure of the difference between the desired output signal \( d(t) \) and the actual filter output \( y(t) \). Equation (B–3) could be expanded as follows:

---

Appendix B: Wiener Filtering

227
\[
I = \sum_{t} \left[ d^2(t) - 2d(t) \sum_{k=0}^{N-1} a_k x(t-k) + \sum_{k=0}^{N-1} a_k \sum_{i=0}^{N-1} x(t) x(t-i) \right] \quad \text{(B-15)}
\]

Rearranging and separating the terms in equation (B-15) gives the following expression of \( I \):

\[
I = \sum_{t} d^2(t) - 2 \sum_{k=0}^{N-1} a_k \sum_{t} x(t-k) d(t) + \sum_{k=0}^{N-1} a_k \sum_{i=0}^{N-1} x(t-i) x(t-k) \quad \text{(B-16)}
\]

The inner summation in the second term of this expression of \( I \) could be identified according to equation (B-6) as the cross-correlation function \( r_{dx}(k) \). Similarly, the inner summation in the third term could be identified according to equation (B-7) as the autocorrelation function \( r_{xx}(i-k) \). Thus, the expression of \( I \) is simplified to:

\[
I = \sum_{t} d^2(t) - 2 \sum_{k=0}^{N-1} a_k r_{dx}(k) + \sum_{k=0}^{N-1} a_k r_{xx}(i-k) \quad \text{(B-17)}
\]

The inner summation in the third term of equation (B-17) is the same as the left-hand side of equation (B-8), which when replaced by the right-hand side (i.e., \( r_{dx}(i) \)) would minimize the sum of squared-errors \( I \). Therefore, the minimum of \( I \) is given by:

\[
I_{\min} = \sum_{t} d^2(t) - 2 \sum_{k=0}^{N-1} a_k r_{dx}(k) + \sum_{k=0}^{N-1} a_k r_{dx}(k) \quad \text{(B-18)}
\]

or

\[
I_{\min} = \sum_{t} d^2(t) - \sum_{k=0}^{N-1} a_k r_{dx}(k) \quad \text{(B-19)}
\]

Dividing by the factor \( \sum_{t} d^2(t) \), which represents the energy of the desired signal \( d(t) \), the normalized minimum squared error between the desired filter output and its actual output is obtained as follows:
\[ E = \frac{I_{\min}}{\sum_i d^2(t)} = 1 - \sum_{k=0}^{N-1} a_k r_{de}(k) \]  \hspace{1cm} (B-20) 

It should be noted that because \( E \) is a sum of squares, it cannot be negative. Therefore, the values of \( E \) are comprised between 0 (best performance) and 1 (worst performance). The one’s complement of \( E \) could be also used as a valid measure of the performance of the filter:

\[ P = 1 - E = 1 - \sum_{k=0}^{N-1} a_k r_{de}(k) \]  \hspace{1cm} (B-21) 

Based on equation (B-21), the performance parameter \( P \) is also comprised between 0 (worst performance) and 1 (best performance).
Appendix C

Least-Squares Fitting of GPR Data to a Theoretical Reflection Model

Least-squares fitting is typically used to fit a set of measured data points to a theoretical model. For this type of data fitting, the model parameters are computed in a way that minimizes the error between the measured values and the corresponding computed values in the least-squares sense [67].

Under the assumptions stated in section 2.3.2.2, the reflectivity function $\gamma(t)$ of a pavement system composed of $(N+1)$ layers could be retrieved by taking the inverse Fourier transform of the input reflection coefficient $\Gamma_{in}(\omega)$ given in equation (2-25):

$$ \gamma(t) = F^{-1}\{\Gamma_{in}(\omega)\} = \sum_{n=0}^{N-1} \frac{\sqrt{E_{r,n} - \sqrt{E_{r,n+1}}}}{\sqrt{E_{r,n}} + \sqrt{E_{r,n+1}}} \left[ \prod_{i=0}^{n-1} (1 - \gamma_i^2) \right] e^{-\eta \sum_{i=0}^{n} \frac{\sigma_i d_i}{\sqrt{\epsilon_i}}} \delta(t - \frac{2}{c} \sum_{i=0}^{n} \sqrt{\epsilon_i d_i}) \quad (C-1) $$

where, as depicted in Figure C−1, $d_i$, $\epsilon_{r,i}$, and $\sigma_i$ are, respectively, the thickness, dielectric constant, and conductivity of layer $i$ and $\gamma_i$ is the reflection coefficient at interface $i$. The thickness of layer $i$ could be expressed as a function of the dielectric constant $\epsilon_{r,i}$ and the two-way travel time $t_i$ as follows:

$$ d_i = \frac{ct_i}{2\sqrt{\epsilon_{r,i}}} \quad (C-2) $$

where, as shown in Figure C−1, the two-way travel time $t_i$ is the time that the wave takes to go from one interface of the layer to the other and back. Practically, $t_i$ corresponds to the time
difference between two consecutive reflected pulses in the GPR collected signal, as presented in Figure 2-7.

\[ \gamma(t) = \sum_{n=0}^{N-1} A_n \delta(t - \sum_{i=0}^{n} t_i) \]  \hspace{1cm} (C-3)

where \( A_n \) is the relative amplitude (with respect to the incident pulse amplitude) of the reflection at interface \( n \). The expression of \( A_n \) is given by the following:

\[ A_n = \frac{\sqrt{\varepsilon_{r,n} - \sqrt{\varepsilon_{r,n+1}}} \left( \prod_{i=0}^{n-1} \left( 1 - \gamma_i^2 \right) \right)^{-\eta_0} \sum_{i=0}^{n-1} \frac{\sigma d_i}{\sqrt{\varepsilon_{r,n}}} } {\sqrt{\varepsilon_{r,n} + \sqrt{\varepsilon_{r,n+1}}}} e^{-\eta_0} \]  \hspace{1cm} (C-4)

For a noise free environment, the time domain GPR reflected signal \( y(t) \) could be found as a function of the incident GPR signal \( x(t) \) and the reflectivity function \( \gamma(t) \) as follows:

\[ y(t) = x(t) \ast \gamma(t) \]  \hspace{1cm} (C-5)

which yields, when substituting \( \gamma(t) \) by its expression of equation (C-3), to the following model:
\[ y(t) = \sum_{n=0}^{N-1} A_n x(t - \sum_{i=0}^{n} t_i) \]  

(C-6)

If the number of layers \((N+1)\) is known and the two-way travel times \(t_i\) are assumed measurable from the GPR collected signal \(y_r(t)\), then the only model parameters that need to be computed are the reflection amplitudes \(A_n\).

The sum of squares of the error between the collected GPR signal \(y_r(t)\) and the signal computed from the model given by equation (C-6) is given as follows:

\[ I = \sum_{t} \left[ y_r(t) - \sum_{n=0}^{N-1} A_n x(t - \sum_{i=0}^{n} t_i) \right]^2 \]  

(C-7)

To achieve a minimum mean-square error between the measured and modeled GPR signals, the sum of squared errors given by equation (C-7) should be minimized with respect to all the model parameters \(A_n\). Thus, the partial derivatives of \(I\) with respect to the parameters \(A_n\) should be set to zero:

\[ \frac{\partial I}{\partial A_k} = -2 \sum_{t} \left[ y_r(t) - \sum_{n=0}^{N-1} A_n x(t - \sum_{i=0}^{n} t_i) \right] x(t - \sum_{i=0}^{k} t_i) = 0, \text{ for } k = 0, 1, \ldots, N-1 \]  

(C-8)

Rearranging and simplifying equation (C-8) leads to:

\[ \sum_{n=0}^{N-1} A_n \sum_{t} x(t - \sum_{i=0}^{n} t_i) x(t - \sum_{i=0}^{k} t_i) = \sum_{t} y_r(t) x(t - \sum_{i=0}^{k} t_i), \text{ for } k = 0, 1, \ldots, N-1 \]  

(C-9)

Equation (C-9) could be expanded in the form of a set of equations according to:

\[
\begin{cases}
A_0 \sum_{t} x^2 (t - t_0) + \cdots + A_{N-1} \sum_{t} x(t - t_0 - \cdots - t_{N-1}) x(t - t_0) = \sum_{t} y_r(t) x(t - t_0) \\
A_0 \sum_{t} x(t - t_0) x(t - t_0 - t_1) + \cdots + A_{N-1} \sum_{t} x(t - t_0 \cdots - t_{N-1}) x(t - t_0 - t_1) = \sum_{t} y_r(t) x(t - t_0 - t_1) \\
\vdots \\
A_0 \sum_{t} x(t - t_0) x(t - t_0 - \cdots - t_{N-1}) + \cdots + A_{N-1} \sum_{t} x^2 (t - t_0 - \cdots - t_{N-1}) = \sum_{t} y_r(t) x(t - t_0 - \cdots - t_{N-1})
\end{cases}
\]  

(C-10)
In a matrix format, system (C–10) is equivalent to the following:

\[
\mathbf{M} \mathbf{A} = \mathbf{Y} \tag{C–11}
\]

where:

\[
\mathbf{A} = \begin{bmatrix} A_0 & A_1 & \cdots & A_{N-1} \end{bmatrix}^T \tag{C–12}
\]

\[
\mathbf{Y} = \begin{bmatrix} \sum_y y_r(t)x(t-t_0) & \sum_y y_r(t)x(t-t_0-t_1) & \cdots & \sum_y y_r(t)x(t-t_0-\cdots-t_{N-1}) \end{bmatrix} \tag{C–13}
\]

\[
\mathbf{M} = \begin{bmatrix}
\sum_t x^2(t-t_0) & \sum_t x(t-t_0-t_1)x(t-t_0) & \cdots & \sum_t x(t-t_0-\cdots-t_{N-1})x(t-t_0) \\
\sum_t x(t-t_0)x(t-t_0-t_1) & \sum_t x^2(t-t_0-t_1) & \cdots & \sum_t x(t-t_0-\cdots-t_{N-1})x(t-t_0-t_1) \\
\vdots & \vdots & \ddots & \vdots \\
\sum_t x(t-t_0)x(t-t_0-\cdots-t_{N-1}) & \sum_t x(t-t_0-t_1)x(t-t_0-\cdots-t_{N-1}) & \cdots & \sum_t x^2(t-t_0-\cdots-t_{N-1})
\end{bmatrix} \tag{C–14}
\]

Thus the model parameters \( A_n \) that ensure a minimum mean-square error between the measured GPR signal \( y_r(t) \) and the theoretical signal \( y(t) \) could be determined according to:

\[
\mathbf{A} = \mathbf{M}^{-1} \mathbf{Y} \tag{C–15}
\]
Appendix D

HMA Thickness Results
Figure D-1: Thickness of the Overall HMA Layer: (a) Section E, (b) Section A, (c) Section F, (d) Section G, (e) Section H, and (f) Section K
Figure D–2: Thicknesses of the Individual Lossless HMA Layers: (a) Section E, (b) Section A, (c) Section F, (d) Section G, (e) Section H, and (f) Section K
Figure D–3: Thicknesses of the Individual Lossy HMA Layers: (a) Section E, (b) Section A, (c) Section F, (d) Section G, (e) Section H, and (f) Section K
Figure D-4: Thickness of the Overall HMA Layer, CMP Method: (a) Section E, (b) Section A, (c) Section F, (d) Section G, (e) Section H, and (f) Section K
Vita

Samer Lahouar was born on April 24, 1973 in Bizerte, Tunisia. He received his high school diplomat “Baccalauréat” with honors in 1991. After that, he joined the “Ecole Nationale d’Ingénieurs, Monastir” (National School of Engineers, Monastir, Tunisia) where he received the diploma “Diplôme d’Ingénieurs Principal” from the department of Electrical Engineering in 1997. This diploma is equivalent to a Master of Science degree. After graduation, Samer worked as a computer engineer at a furniture factory in his home town Hammam Sousse, Tunisia. In August 1998, Samer decided to go back to school by entering the Ph. D. program of the Bradley Department of Electrical and Computer Engineering at Virginia Tech, which earned him a Ph. D. degree in 2003 under the joint supervision of Dr. Imad Al-Qadi and Gary Brown. During the same period Samer worked as a Graduate Research Assistant with the Roadway Infrastructure Group (RIG) at the Virginia Tech Transportation Institute. Currently, Samer is working with the same research group as a Senior Research Associate.

During his work with RIG, Samer has co-authored several technical reports and over 15 conference and journal papers.