Chapter 2  Warpage of Large Curved Composite Panels

2.1  Chapter Outline

The objective of the first phase of this research is to quantify the effects of several common manufacturing imperfections on the deformation of large radii, elevated temperature cured polymer-matrix fiber-reinforced curved panels used primarily for aerospace structures. Considered will be ply misalignments, thermal gradients during cure, and differences in ply thickness. Modeling consists of geometrically linear and nonlinear, two-dimensional and three-dimensional finite-element analyses using ABAQUS™ [1]. Interest has centered on large curved composite panels because they may be used to construct aircraft fuselages or large launch vehicles. Figure 2.1 depicts the problem statement as a four-step manufacturing process. The first step in the process is the lay-up of pre-impregnated composite layers upon the tool. The tool has specific dimensions, in this case a specific radius of curvature. This step is accomplished in a room temperature environment. A significant imperfection at this stage may be a ply misalignment, where some or all layers of the composite are slightly misaligned with respect to their intended orientation. Inherent imperfections of the prepreg materials, such as variations in ply thickness and fiber waviness within the prepreg, also constitute manufacturing tolerances within this step. The second step is the elevated temperature curing of the composite in an autoclave. A key issue at this stage is the thermal gradients that are established within the composite due to the autoclave heating characteristics and panel geometry. Because of the temperature gradient, the degree of cure and the stress-free temperature may be different from point to point within the laminate. Another issue during this stage is the difference in ply thickness from one ply to the next due to resin bleeding. When a laminate is cured at elevated temperatures, excess resin can flow out of the composite, leading to a nonuniform distribution of fiber and resin. Details of the consolidation and curing process, such as the part and mandrel geometry, prepreg resin content, bleeder material location may result in significant resin flow, leading to ply thickness variations. Step three, cooling to room temperature, is when many of the effects of the imperfections become evident. The temperature decrease
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Chapter 2. Warpage of Large Curved Composite Panels

from the stress free curing temperature to room temperature, coupled with the free thermal deformation characteristics of the material, ply misalignment, and ply thickness differences, cause the shape and dimension of the curved panel to be different than those of the uncured panel and tool. These deformations are generally unwanted and are often referred to as manufacturing distortions, or warpage. The presence of a thermal gradient during cure can lead to a temperature change that is not the same at all locations within the panel, resulting in further warpage problems. Warpage can lead to difficulties when trying to fit a number of panels together to assemble a complete fuselage or other aerospace structures. It is important to realize that the magnitude of ply misalignment, the magnitude of ply thickness variation, and the magnitude and exact nature of any thermal gradient are all uncertain. Therefore it may be necessary to conduct a large number of analyses to bound the problem, such as is done with Monte Carlo methods.

To serve as a baseline case, Section 2.2 investigates the deformation characteristic of the so-called perfect panel. There are inherent thermally-induced deformations with a perfect panel. One source of deformation is the fact that the out-of-plane thermal expansion characteristic is different than the inplane thermal expansion characteristics, i.e., the thermal expansion characteristics of a fiber-reinforced composite material are inherently orthotropic. This means that a traditional two-dimen-
sional theory could fall short of accurately capturing the thermally-induced deformations of even a perfect composite panel. To determine this, the investigation is conducted using both two-dimensional finite-element and three-dimensional finite-element models. Section 2.3 investigates the effect of ply misalignment on the warpage of curved composite panels. Ply misalignment can lead to an unsymmetric laminate, which, as is well known, will deform when the temperature is changed. The next Section, 2.4, focuses on the effect of not having each ply have the same thickness. There could well be thickness variations within a ply, but they will not be considered here. Ply thickness variations also lead to a somewhat unsymmetric laminate. Section 2.5 investigates the effect of thermal gradients in the large composite panel as it cures within an autoclave. The last section of this chapter draws conclusions from the various factors and their influences.

2.1.1 Cylindrical Coordinate Systems

Upon initial investigation of some of the finite-element results, it was found that due to the manner for accounting for rigid body motions with a finite-element model, every point on the reference surface of a curved panel can experience anomalous displacements due to a rigid body translation of the deformed panel with respect to the undeformed panel. To explain, Fig. 2.2(a) shows the cross-sectional view of a cylinder, dark lines representing the curved panel, before and after a uniform temperature decrease is applied. For reference, the remainder of the cylinder from which the panel can be considered to be cut from is shown with dashed lines. Relative to the cylindrical coordinate system, it can be seen that, due to the temperature decrease, every point on the cylindrical panel simply experiences the same inward radial displacement, with no circumferential displacement. Figure 2.2(b) shows the cross-section view of the same problem, but with a point in the center of the panel being constrained from any motion. This type of condition is commonly used in finite-element analyses to constrain rigid body motion. When this condition is used, the displacement field is translated relative to the true cylindrical coordinate system of Fig. 2.2(a). This translation affects the computed radial ($w$) and circumferential ($v$) displacements of every point of the cylindrical panel. The finite-element displacement calculations need to be adjusted if the results are to be interpreted in the context of a cylindrical coordinate system, which is the easiest way to consider the problem. The correction to the finite-element results can be made by relying on the assumption that the radial displacement of a complete cylinder depends primarily on the circumferential coefficient of thermal expansion of the laminate, $\bar{\alpha}_s$. Specifically,
use is made of the fact that for an axisymmetric problem, the circumferential strain and radial displacement are related by

\[ \varepsilon_s = \frac{w}{R} \]  

(2.1)

For the thermal problem, the circumferential strain is given by

\[ \varepsilon_s = \alpha_s \Delta T \]  

(2.2)

Therefore, the radial displacement in Fig. 2.2(a), and the resulting translation in Fig. 2.2(b) are given by

\[ w = R \alpha_s \Delta T \]  

(2.3)

where the mean radius of the cylindrical panel is used for \( R \). This value of \( w \) can be used to adjust the finite-element results so they are interpreted in a true cylindrical coordinate system. In particular, the radial and circumferential displacements are shifted by

\[ v = v_{FE} + R \alpha_s \Delta T \sin \phi \]  

(2.4)

\[ w = w_{FE} + R \alpha_s \Delta T \cos \phi \]  

(2.5)

thereby translating the finite-element results to the cylindrical coordinate system.

![Fig. 2.2 (a) Cylindrical coordinate system, (b) Finite-element coordinate system](image-url)
With these issues in mind, the analysis coordinate system and model geometry used throughout this study are shown in Fig. 2.3. The coordinate $x$ is used to identify axial locations and the arc-length coordinate $s$ is used to identify circumferential locations. The origin of the coordinate system is located in the geometric center of the uncured panel. The dimensions of the uncured panel are $L_x$ by $L_s$ by $H$. The radius of curvature of the mean thickness location is denoted by $R$. Fiber angle within a layer will be measured from the $x$-axis and will be denoted by $\theta$. To discuss the influence of various imperfections on warpage, some metric, or measure, of warpage is necessary. For the present, four metrics, illustrated in Fig. 2.4, will be used to quantify warpage. These are the radial, axial, and circumferential displacement of one corner, labeled corner $a$ in Fig. 2.3, and the overall twist of the panel. The twist will be measured by using the radial displacements of corner $a$ and corner $b$ and the dimension $L_x$. It can be said that purely circumferential and axial manufacturing induced deformations of the panel are not as important as the radial displacement and twist deformation due to the ability to make oversized panels and then trim the edges before assembly. Therefore primary focus will be on the radial displacements and twist.
2.2 The Perfect Curved Panel

This section focuses on the manufacturing induced deformations of a perfect panel. The term ‘perfect panel’ refers to the lack of imperfections during the manufacturing process, leaving the spatially uniform temperature change due to an autoclave cure cycle and the inherent laminate material properties as the sole causes of the deformations. Initial interests centers on the valid application of two-dimensional analytical simplifications for our model. The implementation of two-dimensional analytical simplifications is crucial for problems such as this, where, as mentioned before, the presence of uncertainties may require a large number of analyses to bound the problem. However, a two-dimensional analysis is not capable of modeling through-thickness effects. One such effect is the orthotropy of the thermal expansion, namely the thermal expansion

Fig. 2.4 Displacement-based warpage metrics
in the thickness direction of the laminate being so much larger than the thermal expansion in the plane of the laminate. The orthotropy of the thermal expansion characteristics leads to circumferential stress distributions through the thickness of a complete cylinder. The stress distribution can lead to a net circumferential moment through the thickness of the cylinder. Considering a section, or panel, within a complete cylinder, these moments are illustrated in Fig. 2.5. When the section, or panel, is cut from the complete cylinder, the edges become traction free, leading to a release of the moment and subsequent deformations of the panel. These deformations lead to what is often referred to as spring-out or spring-in, depending on whether the radius of curvature increases or decreases. To investigate the difference in the deformations predicted by a three-dimensional analysis compared to a two-dimensional analysis, two- and three-dimensional finite-element models were developed. In addition, with a view towards perhaps having to do a large number of analyses in a Monte Carlo type simulation, a Rayleigh-Ritz approach to computing the manufacturing deformations for the geometrically linear problem is also investigated. Although a Rayleigh-Ritz approach is generally quite problem specific, if it is felt to be accurate enough for particular needs, it can be quite computationally efficient. The results for the Rayleigh-Ritz approach will be compared with the finite-element results.

![Fig. 2.5 Stress distribution due to orthotropic thermal expansion characteristics](image)

**2.2.1 Finite-Element Models**

The stress distribution through the thickness can be computed by a three-dimensional finite-element model using three-dimensional solid elements. However, due to the three-dimen-
sional aspect ratio limits of the three-dimensional solid elements, the ratio between the height, width, and length of the finite element must remain reasonable. Therefore, this particular model will be a disproportionately thick four-layer laminate. Figure 2.6 shows the panel geometry being considered for this portion of the study.

The panel stacking sequence considered is \([+\theta]_S\), where \(\theta\) ranges from 0º to 90º. The ratio of \(R/H\) is equal to 600, consistent with realistic thin panel dimensions. The three-dimensional model consists of 1568 20 noded C3D20 solid brick elements. The model is divided into four layers of elements, each layer representing the constitutive response of the corresponding ply orientation. The two-dimensional model consists of 392 S9R5 shell elements. This shell element allows for shear deformations through the thickness by approximations of the first order. In addition, geometrically nonlinear two-dimensional and three-dimensional models will be evaluated to investigate the importance of geometrical nonlinearities for this problem. The layer material properties utilized for these models are listed in Table 2.1. A spatially uniform negative 280º F temperature change, corresponding to the cooling of a laminate from a 350º F stress free curing temperature to 70º F room temperature, is applied to all models.
2.2.2 The 33-Term Rayleigh-Ritz Model

The Rayleigh-Ritz approach utilizes the minimization of the total potential energy to find approximate solutions. Initially, polynomial series approximations of the displacement field were developed by fitting polynomials to selected solutions generated by the finite-element model for general and extremes deformation shapes. The result was the following approximations for the three components of displacement:

\[
\begin{align*}
    u(x, s) &= b_{01} s + a_{20} s^2 + a_{30} s^3 + (a_{01} + a_{11} s + a_{21} s^2 + a_{31} s^3)x + (a_{02} + a_{12} s + a_{22} s^2 + a_{32} s^3)x^2 \\
    v(x, s) &= b_{10} s + b_{20} s^2 + b_{30} s^3 + (b_{01} + b_{11} s + b_{21} s^2 + b_{31} s^3)x + (b_{02} + b_{12} s + b_{22} s^2 + b_{32} s^3)x^2 \\
    w(x, s) &= c_{00} + c_{01} s + c_{20} s^2 + c_{30} s^3 + (c_{01} + c_{11} s + c_{21} s^2 + c_{31} s^3)x + (c_{02} + c_{12} s + c_{22} s^2 + c_{32} s^3)x^2
\end{align*}
\]

(2.6) (2.7) (2.8)
The 33 constants $a_{01}$, $b_{01}$, etc., are determined through the minimization of total potential energy. These constants make up the 33-term Rayleigh-Ritz model. The assumed displacement fields are substituted into the geometrically linear kinematic relationships of Sanders [2] to compute the reference surface strains

\begin{align*}
\epsilon^0_x &= \frac{\partial u^0}{\partial x}, & \epsilon^0_s &= \frac{\partial v^0}{\partial s} + \frac{w^0}{R}, & \gamma^0_{xs} &= \frac{\partial v^0}{\partial x} + \frac{\partial w^0}{\partial s} \quad (2.9)
\end{align*}

and curvatures

\begin{align*}
\kappa^0_x &= -\frac{\partial^2 w^0}{\partial x^2}, & \kappa^0_s &= \frac{1}{R} \frac{\partial v^0}{\partial s} - \frac{\partial^2 w^0}{\partial s^2}, & \kappa^0_{xs} &= \frac{1}{R} \frac{\partial v^0}{\partial x} - 2 \frac{\partial^2 w^0}{\partial x \partial s} \quad (2.10)
\end{align*}

It was discovered that the Donnell [3] kinematic relations could not accurately capture the deformations of a perfect composite panel. This was due to the approximation inherent to the Donnell kinematics which assume that the magnitude of inplane deformations are always negligible with respect to the out-of-plane deformations. This assumption is generally valid for cylinder problems. However this particular problem involves an unusually large panel undergoing a free thermal expansion that results in large magnitudes of inplane deformations. In the absence of imperfections, the magnitude of inplane deformations can be greater than the out-of-plane deformations, resulting in the need for more accurate kinematic relations, such as those proposed by Sanders. Invoking the plane-stress assumption by assuming all components of stress in the thickness direction are zero, the total potential energy of the curved panel may be written as

\begin{align*}
\Pi = \frac{1}{2} \int_{-l \epsilon}^{+l \epsilon} \int_{-l \epsilon}^{+l \epsilon} \int_{-H}^{H} \left( (\sigma_x - \sigma_x^T) \epsilon_x + (\sigma_s - \sigma_s^T) \epsilon_s + (\tau_{xs} - \tau_{xs}^T) \gamma_{xs} \right) dx \, ds \, dz \quad (2.11)
\end{align*}

By assuming that the Kirchhoff hypothesis applies, the strains may be expressed in terms of reference surface strains and curvatures by

\begin{align*}
\epsilon_x &= \epsilon_x^0 + z \kappa_x^0 \quad (2.12) \\
\epsilon_s &= \epsilon_s^0 + z \kappa_s^0 \\
\gamma_{xs} &= \gamma_{xs}^0 + z \kappa_{xs}^0
\end{align*}
The reference surface strains and curvatures were defined in equations 2.9 and 2.10. Due to the assumption of plane-stress, the stress-strain relations may be written as,

\[
\begin{align*}
\sigma_x &= \bar{Q}_{11}(\epsilon_x - \alpha_x \Delta T) + \bar{Q}_{12}(\epsilon_s - \alpha_s \Delta T) + \bar{Q}_{16}(\gamma_{xs} - \alpha_{xs} \Delta T) \\
\sigma_s &= \bar{Q}_{12}(\epsilon_x - \alpha_x \Delta T) + \bar{Q}_{22}(\epsilon_s - \alpha_s \Delta T) + \bar{Q}_{26}(\gamma_{xs} - \alpha_{xs} \Delta T) \\
\tau_{xs} &= \bar{Q}_{16}(\epsilon_x - \alpha_x \Delta T) + \bar{Q}_{26}(\epsilon_s - \alpha_s \Delta T) + \bar{Q}_{66}(\gamma_{xs} - \alpha_{xs} \Delta T)
\end{align*}
\] (2.13)

where \( \Delta T \) is the temperature change from the reference (curing) temperature. This equation may be rewritten as

\[
\begin{align*}
\sigma_x &= \bar{Q}_{11}\epsilon_x + \bar{Q}_{12}\epsilon_s + \bar{Q}_{16}\gamma_{xs} - \sigma_x^T \\
\sigma_s &= \bar{Q}_{12}\epsilon_x + \bar{Q}_{22}\epsilon_s + \bar{Q}_{26}\gamma_{xs} - \sigma_s^T \\
\tau_{xs} &= \bar{Q}_{16}\epsilon_x + \bar{Q}_{26}\epsilon_s + \bar{Q}_{66}\gamma_{xs} - \tau_{xs}^T
\end{align*}
\] (2.14)

where

\[
\begin{align*}
\sigma_x^T &= (\bar{Q}_{11}\alpha_x + \bar{Q}_{12}\alpha_s + \bar{Q}_{16}\alpha_{xs})\Delta T \\
\sigma_s^T &= (\bar{Q}_{12}\alpha_x + \bar{Q}_{22}\alpha_s + \bar{Q}_{26}\alpha_{xs})\Delta T \\
\tau_{xs}^T &= (\bar{Q}_{16}\alpha_x + \bar{Q}_{26}\alpha_s + \bar{Q}_{66}\alpha_{xs})\Delta T
\end{align*}
\] (2.15)

The \( \bar{Q} \)'s are reduced stiffnesses and the \( \alpha \)'s are coefficients of thermal expansion. Equations 2.14 and 2.15 can be used to rewrite the total potential energy as

\[
\Pi = \frac{1}{2} \int_{-\frac{L_x}{2}}^{+\frac{L_x}{2}} \int_{-\frac{L_y}{2}}^{+\frac{L_y}{2}} \int_{-\frac{H}{2}}^{+\frac{H}{2}} \left\{ (\sigma_x - \sigma_x^T)(\epsilon_x^o + z\kappa_x^o) + (\sigma_s - \sigma_s^T)(\epsilon_s^o + z\kappa_s^o) \right. \\
+ (\tau_{xs} - \tau_{xs}^T)(\gamma_{xs}^o + z\kappa_{xs}^o) \right\} dx \, ds \, dz \] (2.16)

Carrying out the integration through-the-thickness leads to

\[
\Pi = \frac{1}{2} \int_{-\frac{L_x}{2}}^{+\frac{L_x}{2}} \int_{-\frac{L_y}{2}}^{+\frac{L_y}{2}} \left\{ (N_x - \hat{N}_x^T \Delta T) \epsilon_x^o + (N_s - \hat{N}_s^T \Delta T) \epsilon_s^o + (N_{xs} - \hat{N}_{xs}^T \Delta T) \gamma_{xs}^o \right. \\
+ (M_x - \hat{M}_x^T \Delta T) \kappa_x^o + (M_s - \hat{M}_s^T \Delta T) \kappa_s^o + (M_{xs} - \hat{M}_{xs}^T \Delta T) \kappa_{xs}^o \right\} dx \, ds \] (2.17)

The force resultants, \( N_x, N_s, N_{xs} \), and moment resultants, \( M_x, M_s, M_{xs} \), are defined as
The thermal force and moment resultants are defined as

\[
\begin{align*}
N_x & \equiv \int_{-H/2}^{+H/2} \sigma_x dz = A_{11} \epsilon_x^0 + A_{12} \epsilon_s^0 + A_{16} \gamma_{xx}^0 + B_{11} \kappa_x^0 + B_{12} \kappa_s^0 + B_{16} \kappa_{xs}^0 - \tilde{N}_x^T \Delta T \\
N_s & \equiv \int_{-H/2}^{+H/2} \sigma_s dz = A_{12} \epsilon_x^0 + A_{22} \epsilon_s^0 + A_{26} \gamma_{xs}^0 + B_{12} \kappa_x^0 + B_{22} \kappa_s^0 + B_{26} \kappa_{xs}^0 - \tilde{N}_s^T \Delta T \\
N_{xs} & \equiv \int_{-H/2}^{+H/2} \tau_{xs} dz = A_{16} \epsilon_x^0 + A_{26} \epsilon_s^0 + A_{66} \gamma_{xx}^0 + B_{16} \kappa_x^0 + B_{26} \kappa_s^0 + B_{66} \kappa_{xs}^0 - \tilde{N}_{xs}^T \Delta T \\
M_x & \equiv \int_{-H/2}^{+H/2} z \sigma_x dz = B_{11} \epsilon_x^0 + B_{12} \epsilon_s^0 + B_{16} \gamma_{xx}^0 + D_{11} \kappa_x^0 + D_{12} \kappa_s^0 + D_{16} \kappa_{xs}^0 - \tilde{M}_x^T \Delta T \\
M_s & \equiv \int_{-H/2}^{+H/2} z \sigma_s dz = B_{12} \epsilon_x^0 + B_{22} \epsilon_s^0 + B_{26} \gamma_{xs}^0 + D_{12} \kappa_x^0 + D_{22} \kappa_s^0 + D_{26} \kappa_{xs}^0 - \tilde{M}_s^T \Delta T \\
M_{xs} & \equiv \int_{-H/2}^{+H/2} z \tau_{xs} dz = B_{16} \epsilon_x^0 + B_{26} \epsilon_s^0 + B_{66} \gamma_{xx}^0 + D_{16} \kappa_x^0 + D_{26} \kappa_s^0 + D_{66} \kappa_{xs}^0 - \tilde{M}_{xs}^T \Delta T 
\end{align*}
\]

Substitution of the displacement field approximations, equations 2.6-2.8, into the Sanders kinematic relations, equations 2.9 and 2.10, leads to polynomial approximations of the strains. The stress components defined in equations 2.14 and 2.15 can be used together with the strains to form the expression for the total potential energy, equation 2.17. The Rayleigh-Ritz method calls for the minimization of the total potential energy with respect to the 33 unknown coefficients, i.e.,

\[
\frac{\partial \Pi}{\partial a_{01}} = 0, \quad \frac{\partial \Pi}{\partial a_{02}} = 0, \quad \frac{\partial \Pi}{\partial a_{03}} = 0, \quad \frac{\partial \Pi}{\partial a_{10}} = 0, \quad \ldots, \quad \frac{\partial \Pi}{\partial \epsilon_{32}} = 0
\]

(2.20)
leading to a set of algebraic equations which can be solved for the unknown coefficients to arrive at approximate solutions. The symbolic manipulations and numerical computations were conducted using Mathematica® [4].

2.2.3 Results

The results are shown in Figs. 2.7-2.10, where the four warpage metrics for a perfect curved panel are shown. Each warpage metric is plotted as a function of ply orientation $\theta$. The displacements have been normalized by the laminate thickness. As a matter of interest, in addition to the four finite-element analyses and the Rayleigh-Ritz analyses, the predictions of classical lamination theory (CLT) are included. It is evident that there exists some discrepancies between the analyses. Figure 2.7 shows the normalized radial displacement of corner $a$. Radial displacements on the order of two laminate thicknesses are predicted. All six analyses agree fairly well over the whole range. However, the three-dimensional analyses deviate from the other four as $\theta$ is increased. This result is expected due to deformations induced by the increased through-thickness thermal expansion effect as $\theta$ approaches 90º, where the effect is the greatest. The CLT analysis is based on equation 2.5 using $\bar{\alpha}_s$ as predicted by CLT. It can be seen that CLT provides a good estimate of displacement.
Figure 2.8 shows the normalized circumferential displacement of the perfect curved panel. Examination of Fig. 2.8 shows that the circumferential warpage predicted by CLT, Rayleigh-Ritz, and the geometrically linear two-dimensional finite-element results agree exactly. Specifically, they all predict zero circumferential displacements. On the average, two-dimensional models all agree with each other. However, the three-dimensional models deviate significantly from the two-dimensional models as \( \theta \) approaches 90°, where the through-thickness thermal expansion effect is the greatest. Except for \( \theta = 90° \), it appears the three-dimensional nonlinear model predicts less circumferential displacement than the three-dimensional linear model. It is important to note the relatively small scale of the solutions presented in Fig. 2.8, approximately 10% of a laminate thickness.
Fig. 2.8 Normalized circumferential displacement of a perfect panel

Fig. 2.9 Normalized axial displacement of a perfect panel
Figure 2.9 shows that the normalized axial displacements predicted by the six models agree well over the whole range of $\theta$. However, Fig. 2.10 shows large differences in predictions of twist warpage. It is of particular interest that the CLT, Rayleigh-Ritz, and geometrically linear two-dimensional finite-element models predict no twist throughout the whole range of $\theta$, while the geometrically nonlinear two-dimensional theory and the three-dimensional theory predicts twist as a function of ply orientation. Furthermore, the geometrically nonlinear two-dimensional theory predicts twist deformations opposite to that predicted by all the other analyses for $\theta$ values greater than 45°. Again, the geometrically nonlinear three-dimensional model predicts less deformation than the geometrically linear three-dimensional model. It should be noted that a twist of 0.02° leads to a difference in normalized radial displacement of corner $a$ relative to corner $b$ of 0.45 in., not a large effect considering the overall size of the panel.

The results shown in Fig. 2.8 and 2.10 above show that three-dimensional effects due to through-thickness circumferential stress distribution causes deformations that are not captured by the two-dimensional theory. It is also noted that the difference in predictions are especially severe for circumferential displacement and twist. It is seen that geometrically nonlinear effects are important when predicting certain deformation responses.
2.3 Ply Misalignment

The manufacturing of composite panels requires the layup of individual layers to form the composite laminate. The panel wall may be a laminate made up of 16 or more layers, or it may be of sandwich construction with laminates making up the face sheets. In either case, the layers may be positioned by hand, by a tow placement device, or other processes. There is generally a tolerance associated with placement of fiber directions. Though it is not known for certain, and it may well change from location to location and from ply to ply, it can be assumed there may be a few degrees of misalignment in the fiber direction, referred to here in Fig. 2.11 as $\Delta \theta$. With misaligned fibers, the laminate construction becomes unsymmetric. The combination of the unsymmetric construction with the negative 280°F temperature change, corresponding to the cooling of a laminate from the stress free curing temperature, can result in significant deformations.

![Fig. 2.11 Ply misalignment](image)

2.3.1 Three-Dimensional Effects

In Section 2.2, it was seen that through-thickness orthotropy, a three-dimensional effect, can lead to deformations that are not captured by traditional two-dimensional theories. However, it is important to note that the results were compared in the total absence of manufacturing imperfections. This section investigates the importance of three-dimensional effects in the presence of ply misalignments, a typical manufacturing imperfection. Therefore the analysis will be performed using the same finite-element models and Rayleigh-Ritz models developed in Section 2.2,
except for a 1° ply misalignment in the first layer of the four-layer curved panel considered. Recall the panel layup is $[\pm \theta]_S$, where $\theta$ ranges from 0° to 90°, and the ratio $R/H$ is equal to 600, consistent with realistic panel dimensions. A spatially uniform negative 280° F temperature change, corresponding to the cooling of a laminate from the stress free curing temperature, is applied to both models. The characteristics of the four warpage metrics in the presence of the ply misalignment are shown in Figs. 2.12-2.15 as a function of $\theta$. Figure 2.12 shows the normalized radial displacement of corner $a$ of the four-layer laminate. The results show that the deformations induced by a misaligned ply, specifically the lack of laminate symmetry, are greater than the deformations induced by through-thickness orthotropy. This can be observed by comparing the solutions for the misaligned ply, the five curves with dotted lines, with the solutions for the perfect laminate, the solid lines. The agreement between solutions for the imperfect case suggests that deformations induced by three-dimensional effects are negligible with respect to those induced by ply misalignments. The same trend may be observed for the other three warpage metrics, Figs. 2.13-2.15, leading to the conclusion that the two-dimensional theory can accurately predict deformations due to a slight lack of symmetry of the laminate. However, the geometrically linear solutions and the geometrically nonlinear solutions do not agree over most of the range of $\theta$. In particular, the predicted radial and circumferential warpages at $\theta=30^\circ$ are significantly different. In general, geometric nonlinearities reduce the magnitude of the deformations relative to the linear predictions. However, the influence of the misaligned ply may be over emphasized in this model due to its four-layer construction. Therefore, the effects of geometric nonlinearities need to be investigated in the context of a realistic laminate, such as a 16-layer panel. That will be done in the following section.
Fig. 2.12 Normalized radial displacement of panel with a misaligned ply

Fig. 2.13 Normalized circumferential displacement of panel with a misaligned ply
It is of interest to note that the character of each of the warpage metrics with respect to ply orientation angle appears to be split into three distinct zones. The $\theta$ range from 0º to 30º can be considered a region where the solutions are upper bounds and are relatively constant. The $\theta$ range from 30º to 90º can be considered a region where the solutions are lower bounds and are relatively constant.
from 30° to 60° can be considered the transition region where the solutions change rapidly, crossing through zero. Finally, θ range from 60° to 90° can be considered a region where the solutions are lower bounds and are relatively constant. Although the implications of this response are not known explicitly, the results clearly map out regions of minimized warpage and regions of dimensional insensitivity to ply misalignments for this particular geometry and material system.

### 2.3.2 The 16-Layer Finite-Element Models

To study the role of imperfections in realistic laminates, 16-layer curved panels are studied. Due to the findings in Section 2.3.1, three-dimensional models need no longer be considered. It appears the influence of imperfections dominates any three-dimensional effects, so two-dimensional models can be used. Therefore, the models are no longer constrained by three-dimensional modeling issues that led to the use of unrealistically thick four-layer laminates. The geometry of the model will be unchanged except for the thickness and the radius, the laminate will be a thin 16-layer composite instead of the thick four-layer models used previously. The radius was changed to keep the ratio $R/H$ the same as the previous models. Figure 2.16 describes the model geometry. Three classes of laminates will be considered; quasi-isotropic, axially-stiff, and circumferentially-stiff. Table 2.2 describes the layer orientations of these laminates.

![Fig. 2.16 Model geometry](image-url)
In order to investigate the importance of geometric nonlinearities, two finite-element models, geometrically linear and geometrically nonlinear, will be used. The models are initially used to investigate the effects of a 5° ply misalignment in the first layer of a quasi-isotropic curved panel. As before, the deformations are caused by a spatially uniform temperature change of negative 280° F. Figure 2.17 shows the deformed panel predicted by the geometrically linear two-dimensional finite-element model. Figure 2.18 shows the deformed panel predicted by the geometrically-nonlinear two-dimensional finite-element model. The deformations induced by a 5° ply misalignment in the first layer of a 16-layer quasi-isotropic curved panel are small compared with the influence of a 1° ply misalignment in the first layer of a four-layer quasi-isotropic curved panel. This result is expected since the model assumes that the remaining 15 layers are perfectly aligned, effectively reducing the influence of the single misaligned ply. For the four-layer case, with one ply misaligned, 25% of the layers are misaligned. For the 16-layer case, with one ply misaligned, only 6% of the layers are misaligned. Comparing Figs. 2.17 and 2.18, it is seen that the straight edges of the panel appear to deform in a similar manner. However the geometrically nonlinear analysis predicts that the warpage is confined to the edges. This leads to a relatively large unwarped section, with only warped edges. The linear model predicts that the warpage is evenly distributed over the whole panel.
Figure 2.19 shows the normalized radial displacements predicted by the geometrically linear and geometrically nonlinear finite-element models. Note that the radial displacements predicted by the linear model are over three laminate thicknesses. It can be clearly seen that geometric nonlinearities are very important for this problem. The geometrically linear solution over-predicts the radial displacement. The geometrically nonlinear solution reinforces the conclusion that the effect of the misaligned ply is confined primarily to the edges, a deformation mode that is not captured by the linear analysis. Therefore, for the remainder of this section, only the results predicted by the geometrically nonlinear two-dimensional finite-element models will be considered.
To put the results for the curved panel into context, two finite-element models were developed to investigate the effects of geometric nonlinearities in flat panels similar to the curved panels being discussed. Flat panels will be discussed in more detail in the chapter to follow, but showing some results now is of value. Figures 2.20 and 2.21 shows the geometrically linear and nonlinear predicted shapes of an initially flat quasi-isotropic laminate with 5° of misalignment on the first layer. The figures show that the predicted magnitude of the deformations are very different for the two analyses. Both shapes are saddles, with equal and opposite curvatures, but the saddle predicted by the geometrically nonlinear analysis is much shallower. Therefore geometric nonlinearities are important for this problem of cooling a flat laminate with a misaligned ply. The difference between the flat and curved panels can be attributed to the curvature-induced stiffness of the curved panels. This curvature-induced stiffness significantly influences the structural response. Figure 2.22 shows the normalized out-of-plane displacements for the two analyses. The influential effects of geometric nonlinearities are clearly visible.

Fig. 2.19 Two-dimensional finite-element solutions
Fig. 2.20 Geometrically linear finite-element solution for a plate

Fig. 2.21 Geometrically nonlinear finite-element solution for a plate
2.3.3 Results

Results presented in the context of warpage metrics based on displacement at specific locations on the panel may not be the best metrics, e.g., locations $a$ and $b$ in Figs. 2.3 and 2.6. This is partly due to the fact that metric quantities are highly dependent on the specific location on a panel where the displacements are being computed. The warpage of a particular laminate could thus be misrepresented by the discussion of the character of the metrics that are localized to a certain position on the laminate. These displacement-based warpage metrics are also dimensional. This means that the warpage is a function of panel dimensions $L_x$ and $L_y$. In order to remedy these shortcomings, warpage metrics independent of location on the curved panel are preferred. Therefore, normalized average warpage metrics defined in terms of curvatures, as follows, will be used:
These quantities will be referred to as the normalized axial, circumferential, and twist warpage, respectively. As can be seen, the metrics are the root mean squared normalized curvatures for the panel. The normalizing factor is the curvature of the uncured panel, $1/R$. This method is advantageous due to the reduction of curvature data to a single non-dimensional number. This simplifies the comparison of overall warpage from one laminate to another, somewhat independent of the size of the panel. Physical insight may be gained by computing the out-of-plane displacement, $w$, from the definition of curvature and the normalized warpage metrics. For example, a normalized circumferential warpage of 0.1 translates to eight laminate thicknesses of out-of-plane displacement at the corner of the panel shown in Fig. 2.16.

Figure 2.23 shows the normalized axial warpage of a 5º ply misalignment in one layer of a 16-layer quasi-isotropic composite panel as predicted by the geometrically nonlinear two-dimensional finite-element model. The perfect solution contains no ply misalignments, while the other 16 solutions represent a positive 5º ply misalignment in each layer, applied one layer at a time, while assuming the remaining layers are perfectly oriented. Figure 2.24 and 2.25 show the normalized circumferential warpage and normalized twist warpage, respectively. It can be observed that the influence of a misaligned ply on the magnitude of the axial warpage is, in general, less than half of the influence on the circumferential warpage, and less than half the influence on the twist warpage. This is because of the inherent geometric stiffening of the curved panel in the axial direction. Considering the axial and circumferential curvatures, it is seen that warpage is influenced by the $\pm 45º$ layers being misaligned, and the farther the layer is from the midthickness location, the more the influence. On the other hand, twist warpage is influenced by misalignments of the 0º and 90º layers. It therefore appears that for twist warpage, a 5º misalignment of a layer
intended to be 0° or 90° has much more influence than a 5° misalignment of a layer intended to be +45° or -45°. This can be traced to the variation of $Q_{16}$ and $Q_{26}$ with respect to the ply orientation angle. Also, the farther the misaligned ply is from the midthickness location, the more the influence.

Fig. 2.23 Normalized axial warpage of a misaligned quasi-isotropic panel
Fig. 2.24 Normalized circumferential warpage of a misaligned quasi-isotropic panel.

Fig. 2.25 Normalized twist warpage of a misaligned quasi-isotropic panel.
Figures 2.26-2.28 depict the normalized axial, circumferential, and twist warpages, respectively, for an axially-stiff curved panel. For the axial and circumferential warpage, the results are similar to the quasi-isotropic panel, namely the ±45° plies having the most influence. Also, for these two warpage metrics the magnitude of warpage resulting from a ply misalignment is dependent upon the through-thickness location of the misaligned ply. It is also noted that the orientation of the remaining layers can alleviate or aggravate the effective warpage, depending on the layup. For example, for the axially-stiff panel the circumferential warpage is about a factor of three times that for a quasi-isotropic panel. This is because the axially-stiff panel has less stiffness in the circumferential direction than the quasi-isotropic panel. Unlike the quasi-isotropic panel, for the twist warpage, the 0° and 90° plies do not have as much influence as the ±45° plies. Also, twist of the axially-stiff panel is about a factor of four more than that of a quasi-isotropic panel.

![Normalized axial warpage of a misaligned axially-stiff panel](image-url)
Fig. 2.27 Normalized circumferential warpage of a misaligned axially-stiff panel

Fig. 2.28 Normalized twist warpage of a misaligned axially-stiff panel
Figures 2.29-2.31 depict the normalized axial, circumferential, and twist warpages respectively, for a circumferentially-stiff curved composite panel. Once again, it is observed that the axial and circumferential warpages are sensitive to ply misalignments in the $\pm 45^\circ$ layers, much like the quasi-isotropic and axially-stiff panels. Unlike the quasi-isotropic panel, for the twist warpage, the $0^\circ$ and $90^\circ$ plies do not have as much influence as the $\pm 45^\circ$ plies. The results again show that the magnitude of warpage resulting from a ply misalignment is highly dependent upon the through-thickness location of the misaligned ply.

![Normalized axial warpage of a misaligned circumferentially-stiff panel](image-url)
Fig. 2.30 Normalized circumferential warpage of a misaligned circumferentially-stiff panel

Fig. 2.31 Normalized twist warpage of a misaligned circumferentially-stiff panel
The warpage resulting from a ply misalignment has common features for all three laminates. In all cases, the magnitude of warpage depends on three key factors. The intended orientation of the misaligned ply, through-thickness position of the misaligned ply, and the lamination sequence. It can be seen that the circumferential warpage due to misaligned plys in the axially-stiff case is an order of magnitude larger than for the circumferentially-stiff laminate, and four times that of the quasi-isotropic laminate. This is due to the absence of fibers in the circumferential direction of the axially-stiff laminate. Overall, the quasi-isotropic laminate and the circumferentially-stiff laminate are least susceptible to warpage caused by a misaligned ply. The quasi-isotropic laminate has sufficient stiffness in the circumferential directions to resist warpage, while the circumferentially-stiff laminate has considerable stiffness circumferentially and relies on the axial stiffness due to the curved geometry to suppress axial warpage.

The next section addresses another common manufacturing problem, namely, ply thickness variations.

2.4 Ply Thickness Variations

This section investigates the residual stress induced deformations of large radii curved composite panels due to ply thickness variations. When a symmetric laminate is constructed, symmetry is preserved if the thicknesses of the plies are spatially uniform, and identical, and that remains to be the case throughout the curing process. However, prepreg layers can vary in thickness within a layer and from one layer to another either initially, or as a result of cure. Consequently, the effective mechanical properties of all the layers will not be the same due to the resulting variations in fiber volume fraction. This occurs because the amount of fibers within each layer can be assumed to be the same, while the amount of resin varies, resulting in the ply thickness variations. This combination leads to an unsymmetric laminate that can have an effect on the cured shape of the composite. While there can be variations in ply thickness from one location to the next within a ply, that will not be considered here. Rather, it will be assumed that each ply is of uniform thickness, but the thickness varies from ply to ply. This situation is shown in Fig. 2.32. To be studied are the influences of having one ply 10% thicker than the remaining plies. Table 2.3 lists the material properties of a 10% thicker ply. These values were computed by averaging the properties predicted by various micromechanical models, including the rule of mixtures and
finite-element results. These properties should be compared with the properties listed in Table 2.1, the case of nominal ply thickness.

Fig. 2.32 Ply thickness variations
2.4.1 Geometrically Nonlinear Effects

To investigate ply thickness effects, geometrically linear and geometrically nonlinear finite-element models were developed. Figure 2.33 shows the normalized radial displacement, predicted by the two models, for a 16-layer quasi-isotropic laminate for the case where layer 2, the inner -45º layer, is 10% thicker than the rest. This figure shows that geometric nonlinearities are important for the deformations induced by a thicker ply. It can be seen that the linear solutions over-predicts the radial displacements. Furthermore, the geometrically nonlinear results predict that the deformations induced by the thick ply are confined to the edges of the panel, while the linear solution predicts that the deformations are evenly distributed over the whole panel. This effect has been seen earlier in this study. This large discrepancy between the two predictions suggest that geometric nonlinearities are important for this class of problems. Therefore the remainder of

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the results in this section will be computed by the geometrically nonlinear two-dimensional finite-element models.

![Geometrically Linear
Geometrically Nonlinear](image)

Fig. 2.33 Quasi-isotropic panel with layer 2 thicker by 10%

### 2.4.2 Results

This section presents the warpage induced by a 10% thicker ply within a 16-layer curved composite panel. Once again, a quasi-isotropic, axially-stiff, and circumferentially-stiff curved panels will be considered. Figures 2.34-2.36 show the normalized axial, circumferential, and twist warpages induced by the presence of one thicker ply in a 16-layer quasi-isotropic curved composite panel. The first bar to the left presented on each of the following figures will be the warpage induced by the cooling of a perfect panel. The bars to the right will be the warpage induced by cooling a laminate containing one thicker layer. The layer number along the bottom corresponds to the position of the thicker layer. The results suggest that this structure is sensitive to the through-thickness location of the thicker ply. The ply orientation is a factor. The normalized axial and twist warpage metrics are generally about one-fourth of those induced by a 5º ply misalignment in the same quasi-isotropic laminate. The normalized circumferential warpage is approxi-
imately one-half of that induced by a ply misalignment. To be noted is the sensitivity of the circumferential and twist warpage to the outer ±45° layers being thicker. The bars representing that ply in the twist warpage are the highest of all.

Fig. 2.34 Normalized axial warpage of a thicker quasi-isotropic panel
Fig. 2.35 Normalized circumferential warpage of a thicker quasi-isotropic panel

Fig. 2.36 Normalized twist warpage of a thicker quasi-isotropic panel
Figures 2.37-2.39 depict the normalized axial, circumferential, and twist warpages, respectively, for an axially-stiff 16-layer curved composite panel. Like the quasi-isotropic case, the magnitude of axial and circumferential warpage resulting from a thicker ply is dependent upon the through-thickness location, and also like the quasi-isotropic case, dependent on the orientation of the thicker ply. The magnitudes of warpage induced by the thicker ply in the axially-stiff panel are approximately the same as those induced in the quasi-isotropic panel. It is important to note that the normalized axial warpage and the normalized twist warpage induced by a thicker ply are considerably smaller than those induced by a ply misalignment. The normalized circumferential warpage is approximately 25% of that induced by a ply misalignment. Unlike the quasi-isotropic case, the twist warpage of the axially-stiff case is more sensitive to the inner ±45° layers being thicker. Recall, from Fig. 2.36, the quasi-isotropic panel was most sensitive to the outer ±45° layers being thicker.

![Normalized axial warpage of a thicker axially-stiff panel](image-url)
Fig. 2.38 Normalized circumferential warpage of a thicker axially-stiff panel

Fig. 2.39 Normalized twist warpage of a thicker axially-stiff panel
Figures 2.40-2.42 depict the normalized axial, circumferential, and twist warpages, respectively, for a circumferentially-stiff 16-layer curved composite panel. Once again, it is observed that the magnitude of warpage is dependent upon the through-thickness location and orientation of the thicker ply. In fact, the axial warpage caused by the outer 90° layer (layer 14) being thicker is highest of all, providing more evidence that ply orientation and ply location can be important. The overall character of the response is very similar to the results of a thicker ply in the quasi-isotropic case. However, the normalized axial warpages induced by a thicker ply are about an order of magnitude less than those induced by a ply misalignment. The normalized circumferential warpage is approximately the same for both ply misalignments and ply thickness variations.

Fig. 2.40 Normalized axial warpage of a thicker circumferentially-stiff panel
Fig. 2.41 Normalized circumferential warpage of a thicker circumferentially-stiff panel

Fig. 2.42 Normalized twist warpage of a thicker circumferentially-stiff panel
The warpage resulting from a thicker ply has common features for all three laminates. In general, the effect of a thicker ply is about 25-50% as influential as the ply misalignment, though there are exceptions. In all cases, the through-thickness position of the thicker ply is an important variable. The orientation of the thicker ply is influential for the quasi-isotropic and circumferentially-stiff laminates. However, the orientation of the thicker ply is not as important for the axially-stiff laminate. Overall, and as could be expected based on simply physical arguments, a ply misalignment or a thicker ply near the center of the laminate is much less of a factor in producing warpage than if the ply is elsewhere through the thickness.

Attention is now turned to the issue of an unwanted temperature gradient during cure.

2.5 Thermal Gradients

This section investigates deformations of large radii curved composite panels due to temperature distributions induced by thermal gradients in the autoclave during cure. Referring to Fig. 2.43, of particular interest are thermal gradients in the $x$-$s$ plane.

![Fig. 2.43 Temperature distribution](image)

Large composite panels cured in an autoclave are vulnerable to these kinds of thermal gradients due to the realities of autoclave curing. Non-uniform spatial positioning of heating elements and the uneven proximity of the laminate to the heating elements can lead to thermal gradients. The resulting temperature distribution leads to uneven curing and cooling, and subsequent deformations of the panel. Six types of temperature distributions will be considered in this section. Each
of these cases will be composed of a 0.1° F/in. thermal gradient. The first temperature distribution to be considered, labeled distribution 1 in Fig. 2.44(a), is a linear distribution in the axial direction and is given by

\[ T(x) = 276 + 0.1x \]  \hspace{1cm} (2.22)

This distribution leads to a maximum temperature of 280° F at one end of the 80 in. panel and 272° F at the other end. The second temperature distribution to be considered, labeled distribution 2 in Fig. 2.44(b), is a linear distribution along the circumferential direction and is given by

\[ T(s) = 277 + 0.1s \]  \hspace{1cm} (2.23)

This leads to a maximum temperature of 280° F at one side of the 60 in. wide curved panel and 274° F at the other side. The third temperature distribution to be considered, labeled distribution 3 in Fig. 2.44(c), is a linear distribution along the diagonal direction and is given by

\[ T(x, s) = 273 + 0.1x + 0.1s \]  \hspace{1cm} (2.24)

This leads to a maximum temperature of 280° F at one corner and 266° F at the other corner. The fourth temperature distribution to be considered, labeled distribution 4 in Fig. 2.44(d), is a bi-linear distribution along the axial direction and is given by

\[ T(x)_1 = 276 + 0.1x \hspace{1cm} x \geq 0 \]
\[ T(x)_2 = 276 - 0.1x \hspace{1cm} x \leq 0 \]  \hspace{1cm} (2.25)

This leads to a maximum temperature of 280° F at both ends and a minimum temperature of 276° F in the middle, along the s-axis. The fifth temperature distribution to be considered, labeled distribution 5 in Fig. 2.44(e), is a bi-linear distribution along the circumferential direction and is given by

\[ T(s)_1 = 277 + 0.1s \hspace{1cm} s \geq 0 \]
\[ T(s)_2 = 277 - 0.1s \hspace{1cm} s \leq 0 \]  \hspace{1cm} (2.26)

This leads to a maximum temperature of 280° F at both sides and a minimum temperature of 277° F in the middle, along the x-axis. The final temperature distribution to be considered, labeled distribution 6 in Fig. 2.44(f), is a four quadrant bi-linear distribution and is given by,
This leads to a maximum temperature of 280º F at the four corners and a minimum temperature of 273º F at the center of the panel.

\[
\begin{align*}
T(x, s)_1 &= 273 + 0.1x + 0.1s & x \geq 0 & s \geq 0 \\
T(x, s)_2 &= 273 - 0.1x + 0.1s & x \leq 0 & s \geq 0 \\
T(x, s)_3 &= 273 + 0.1x - 0.1s & x \geq 0 & s \leq 0 \\
T(x, s)_4 &= 273 - 0.1x - 0.1s & x \leq 0 & s \leq 0
\end{align*}
\]

(2.27)
Fig. 2.44 Temperature distributions
2.5.1 Geometrically Nonlinear Effects

The analyses of deformations induced by thermal gradients will be conducted using two-dimensional models for the reasons stated earlier. However, the importance of geometric nonlinearities needs to be re-evaluated for this class of problems. To investigate these effects, geometrically linear and geometrically nonlinear finite-element models were developed. Figure 2.45 shows the normalized radial displacements of the two models for the case of a circumferential temperature distribution, distribution 2. This figure shows that geometric nonlinearities are important for the deformations induced by non-uniform cooling. The geometrically linear solution over-predicts the radial displacement by approximately 300%. Therefore, the results in this section will be computed by the geometrically nonlinear two-dimensional finite-element models. It is important to note that the magnitude of the radial displacements are small, being considerably less than a laminate thickness.

Fig. 2.45 Quasi-isotropic panel deformation due to distribution 2
2.5.2 Results

This section investigates the response of three 16-layer curved composite panels due to a spatially non-uniform cooling resulting from thermal gradients during cure. The three laminates considered will be axially-stiff, circumferentially-stiff, and quasi-isotropic. The normalized axial, circumferential, and twist warpages will be presented for six temperature distributions that were presented in Fig. 2.44.

Figures 2.46-2.48 depict the normalized axial, circumferential, and twist warpages respectively, for a quasi-isotropic curved panel. A quick glance shows that overall, compared to a ply misalignment or a thicker ply, the warpages induced by the various temperature distributions are very small. In some cases, the magnitudes of warpages induced by the distributions are equivalent to the deformations induced by spatially uniform cooling of a perfect panel. However, the twist warpage of the quasi-isotropic laminate seems to be aggravated by the distributions 5 and 6.

![Normalized Axial Warpage](image)

Fig. 2.46 Normalized axial warpage of a quasi-isotropic panel
Fig. 2.47 Normalized circumferential warpage of a quasi-isotropic panel

Fig. 2.48 Normalized twist warpage of a quasi-isotropic panel
Figures 2.49-2.51 depict the normalized axial, circumferential, and twist warpages respectively, for an axially-stiff curved composite panel. The results suggest that the normalized circumferential and twist warpages for this axially-stiff panel are insensitive to the particular temperature distributions. This can be seen by the fact that the height of the bars are the same for each of the six temperature distributions. The normalized axial warpage is slightly aggravated by distribution 4 and 5 when compared to the remaining four temperature distributions. The twist response suggests that all six of these temperature distributions lead to about 11 times more twist when compared to the perfect laminate, but the twist warpage is still quite small.

![Normalized Axial Warpage](image)

Fig. 2.49 Normalized axial warpage of an axially-stiff panel
Fig. 2.50 Normalized circumferential warpage of an axially-stiff panel

Fig. 2.51 Normalized twist warpage of an axially-stiff panel
Figures 2.52-2.54 show the normalized axial, circumferential, and twist warpages, respectively, for a circumferentially-stiff curved composite panel. The results suggest that this circumferentially-stiff panel is sensitive to temperature gradients, but insensitive to the particular distribution. For all three warpage metrics, all six temperature distributions produce the same effect. All three metrics show that the warpages induced by the thermal gradient in circumferentially-stiff curved composite panels are much higher than those induced by a spatially uniform cooling. Since this laminate exhibits the most difference in warpage between the perfect case and the six temperature distributions, it can be concluded that the circumferentially-stiff laminate is the most sensitive to thermal gradients of the three laminates being considered.
Fig. 2.53 Normalized circumferential warpage of a circumferentially-stiff panel

Fig. 2.54 Normalized twist warpage of a circumferentially-stiff panel
It was found that geometrically linear models over-predicted warpages induced by thermal gradients by as much as 300% when compared to the geometrically nonlinear results. The circumferentially-stiff panel is the most sensitive to thermal gradients. Figures 2.52-2.54 show that for this panel the warpage induced by a thermal gradient, irrespective of the temperature distribution, is much larger than those induced by perfect uniform cooling. The axially-stiff and quasi-isotropic laminates are much less sensitive to thermal gradients. This can be observed from Figs. 2.46-2.51 for the quasi-isotropic and axially-stiff panels, which show that the warpages induced by the six temperature distributions can be similar to those induced by a spatially uniform cooling. The result also shows that the magnitude of normalized warpage is often insensitive to the temperature distribution. It is primarily a function of the existence of a thermal gradient. In general, temperature distribution 4, 5, or 6 lead to the most warpage. However, the warpages induced by thermal gradients are negligible in comparison to those induced by ply misalignments and ply thickness variations.

2.6 Chapter Conclusion

Chapter two focused on the manufacturing induced deformations of curved composite panels. Four problems were analyzed in detail. The first problem was the deformations induced in a perfect panel due to orthotropic thermal expansion characteristics of the laminate. It was found that the orthotropic thermal expansion characteristics of the composite led to deformations that were not captured by the two-dimensional theories. The next section investigated the warpage induced by a misaligned ply. This investigation led to the conclusion that deformations induced by orthotropic thermal expansion characteristics were negligible when compared to deformations induced by manufacturing imperfections such as ply misalignments. Furthermore, it was felt that deformations of a particular laminate may be misrepresented by using displacement based warpage metrics that are highly dependent on the position on the laminate. Therefore, normalized average warpage metrics based on the curvatures were developed. This allowed for the reduction of warpage metrics to a single number, somewhat independent of panel geometry, allowing for easier comparisons from one laminate to another. Another important result in this section was that geometric nonlinearities were not negligible for this problem. The geometrically linear solution over-estimated the deformations. Furthermore, it predicted that the warpage was evenly distributed over the whole laminate. In contrast, the geometrically nonlinear solutions predicted that the
warpage was confined to the edges of the panel. It was found that the warpages resulting from a ply misalignment had common features for all three laminates. In all cases, the magnitude of warpage depended on three key factors: the intended orientation of the misaligned ply, through-thickness position of the misaligned ply, and the lamination sequence. The axial warpage was the least sensitive of the warpage metrics to ply misalignments. This can be explained by the inherent geometrically-induced stiffness of the panel due to the initial curvature. It can be seen that the circumferential warpage due to misaligned plies in the axially-stiff case was an order of magnitude larger than for the circumferentially-stiff laminate and four times that of the quasi-isotropic laminate. This is due to the absence of fibers in the circumferential direction of the axially-stiff laminate.

The next section investigated the warpages induced by a thick ply. Again, it was found that geometric nonlinearities were important. The geometrically linear solution over-estimated the deformations by predicting that the warpage was evenly distributed over the whole laminate. In contrast, the geometrically nonlinear solutions predicted that the warpage was confined to be near straight sides of the panel. In general, the effect of a thicker ply was about 25-50% as influential as the ply misalignment. In all cases, the through-thickness position of the thick ply was found to be an important variable. The orientation of the thick ply was influential for the quasi-isotropic and circumferentially-stiff laminates. However, the orientation of the thick ply was not as important for the axially-stiff laminate. The last section investigated the effects of nonuniform cooling due to thermal gradients during autoclave curing. Assuming a constant thermal gradient of 0.1º F per in., six temperature distributions were investigated. Again, it was found that geometric nonlinearities were important for this problem. However, the warpages induced by thermal gradients were found to be much smaller than those induced by misaligned plies or thick plies. The circumferentially-stiff panel was found to be the most sensitive to temperature gradients. Figures 2.52-2.54 show that the warpages induced in this panel by a thermal gradient, irrespective of the temperature distribution, were much larger than those induced by a spatially uniform cooling. The other two laminates were much less sensitive to thermal gradients.

In summary, then, manufacturing distortions of curved composite panels are most sensitive to ply misalignments. Ply thickness variations are the next most important imperfection. The effect of these imperfections on the warpage of a panel is dependent on the through-thickness location, and is dependent on the orientation of the ply containing the imperfection. It was also
found that the lamination sequence can either mitigate or aggravate the resulting deformations. The developed analytical tools and conclusions can be used to design laminates that are insensitive to manufacturing induced deformations. Furthermore, the conclusions have isolated critical manufacturing parameters, ply orientation tolerances, and ply thickness tolerances, which can lead to reduced warpage and increased dimensional stability of autoclave cured curved large composite panels.

The cases considered in the sections dealing with ply misalignments and thicker layers have one feature in common. The laminates in these sections are all unsymmetric laminates. In the jargon of classical lamination theory, they have non-zero components to some or all elements of their $B$ matrices. They also have thermally-induced bending moments due to the lack of symmetry. This, of course, is unintentional. It is the existence of thermally-induced moments that causes the unwanted deformations. In the cases considered here, the magnitude of the thermally-induced moments was small, since they were due to an imperfection. However, for laminates constructed to be unsymmetric intentionally, to take advantage of the bending-stretching coupling present in unsymmetric laminates, the thermally-induced moments can be large enough to cause very significant out-of-plane displacements. Displacements 20-50 times the laminate thickness can occur. With manufacturing induced deformations that large, the laminate may not be useful. However, it may be that novel curing processes, such as electron beam curing, can be used to reduce or eliminate these large deformations. To put into context how large the deformations are, and to develop tools to predict these large deformations, the next chapter deals with severely unsymmetric laminates that are assumed to be cured in an autoclave. The results in this next chapter put into context the results of the chapter that follows, a chapter that investigates perfectly flat unsymmetric laminates under the assumption that they can be manufactured.