Chapter 2

Weak Radiating Cell Concept

“Everyday’s all the same,
same old ways never change,
goin’ From the Cradle to the Grave.”
Leo Kottke / Ron Nagle

2.1 Introduction

The first step needed to understand the WRC concept, begins with the review of the simple sound source. The volume velocity for simple source is then defined, and used to obtain the pressure, intensity, and power for both monopole and dipole sources. The dipole source characterizes the basis for this noise control method. Next, the WRC concept is presented. Finally, a depicted design of a WRC is introduced with a discussion of volume velocity results from a model given by Ross and Burdisso [26].

2.2 Simple Sound Sources

A simple sound source is defined as an object that vibrates with arbitrary velocity distribution producing outward waves, at frequencies such that the wavelength is much greater than any of its dimensions. Although the simply sound source is more of a mathematical concept, some realistic systems, although quite complicated for detailed analysis, can be grossly simplified for noise-control analysis to resemble a simple source.
One example is a small speaker mounted on one side of a small rigid box where the radiated sound wavelengths are large compared to the box dimensions.

The volume velocity or source strength $Q$ is the rate of volume of fluid displaced by the source defined as follows

$$Q e^{j \omega t} = \int_s \tilde{u} \cdot \vec{n} dS$$

where $\tilde{u}$ is the instantaneous velocity of a point on the surface of the source, $\vec{n}$ is the unit outward normal to the surface element $dS$ and the integrals is taken over the entire surface, $S$, of the source. Bold face text indicates complex values.

The significance of source strength comes when the wavelength of the sound being radiated is much larger than the source dimension. In this case, the details of the surface motion are not important for it radiates exactly the same as a pulsating sphere with the same source strength. Thus, the pressure field of a simple source is the same as produced by a pulsating sphere at low frequency with the same source strength. That is

$$p(r,t) = j \rho_o c \frac{Q k}{4 \pi r} e^{j(\omega t - kr)}$$

where $\rho_o$ is the fluid density, $c$ is the fluid’s speed of sound, $Q$ is the magnitude of the source strength, $r$ is the distance from the source to the observation point, $k = \omega / c$ is the acoustic wavenumber, $\omega$ is the frequency in rad/sec, and $j = \sqrt{-1}$. It is also possible to define the simple source intensity (or monopole at low frequency) as

$$I_m(r) = \frac{Q^2 k^2 \rho_o c}{(4 \pi r)^2}.$$ 

Integrating the intensity over a sphere centered at the source, the radiated sound power is as follows
Another important concept relevant to the work here is the dipole source. A dipole is obtained by coupling two monopole sources of equal strengths, separated by some distance \(d\) smaller than acoustic wavelength, i.e. \(kd \ll 1\), vibrating out-of-phase at the same frequency. This configuration is shown in Figure 2.1.

The acoustic far-field pressure at a point in the sound field \((r, \theta)\) is obtained as follows [30]

\[
p(r, \theta, t) = \frac{k^2 \rho c Q d}{4\pi r} \cos \theta \left(1 + \frac{1}{jkr}\right) e^{j(\omega t - kr)}
\]  

(2.5)

where \(\theta\) is the angle between the observed pressure location and of the axis of the dipole, \(Q\) is the source strength magnitude of one monopole and \(\cos \theta\) defines the directivity factor, such that normal to the axis of the dipole the pressure is zero while along the axis of the dipole the pressure is maximum. The radial component of the intensity vector of the dipole source is given as

\[
I_d(r) = \frac{d^2 Q^2 k^4 \rho c}{(4\pi r)^2} \cos^2 \theta
\]  

(2.6)

Once again the sound power can be determined by integrating the intensity over the surface of a sphere of radius \(r\) giving

\[
\Pi_d = \rho c \frac{d^2 Q^2 k^4}{12\pi}
\]  

(2.7)

The importance’s of a dipole system is realized when the power of a monopole, \(\Pi_m\), and that of a dipole source, \(\Pi_d\), is compared. Using Eq 2.4 and Eq 2.7, their ratio is determined as follows
\[ \frac{\Pi_d}{\Pi_m} = \frac{(kd)^2}{3}. \]  

(2.8)

It can be seen that as \( kd \) (the measure of the sources’ separation distance to wavelength ratio) decreases the efficiency of the dipole as a radiator of sound diminishes. This is explained by the equal and opposite response of each source passing the surrounding fluid back and forth locally [28-30].
Figure 2.1: Geometry used in derivation of radiation characteristics of a dipole source
2.3 The Weak Radiating Cell Concept

The methodology of volume velocity noise control has been developed in many forms [19-25]. However, most research involving volume velocity noise control has included some form of active control. Involvement of active control in noise reduction schemes has shown promising results, yet at the expense of increased cost, complexity, weight, power requirement, and unreliability. Here the passive volume velocity control concept referred as the Weak Radiating Cell (WRC) is presented.

Prior to developing an analytical system model of the WRC, it is helpful to briefly discuss the volume velocity control principles of the WRC, which will be useful for the interpretation of the analytical results in the following sections. At low frequency where the acoustic wavelength is greater than the source dimension, the radiated acoustic power is directly related to the volume velocity of the source as in the case of the monopole source in Eq 2.4. Thus, by simply minimizing the source strength $Q$ would lead to noise reduction, i.e., volume velocity control approach. However, there exist two basic methods to minimize the volume velocity of a source: (i) suppression of the source surface velocity $\bar{u}$ and (ii) modification of the source velocity distribution, i.e., $Q$ is minimized without reducing $\bar{u}$ in Eq 2.1. A dipole source is an example of the second approach, e.g., the source strength is zero without suppressing the motion of the two monopole sources. Hence, partitioning the structure’s surface into small sectors each of which responding independently as a dipole with zero volume velocity suggests, an efficient method to reduce structurally radiated sound. To this end, the concept of a weak sound radiating cell has been developed to reduce the low frequency radiated noise from structures.

The weak radiating cell consists of two mechanically coupled surfaces such that, when placed on a vibrating structure, the response of the two surfaces are nearly out-of-phase
and of equal strength over a wide frequency range. This response of the two surfaces forms a local acoustic dipole on the vibrating surface acting to reduce the far-field pressure. Figures 2.2a-c provide a simply illustration of the concept. Figure 2.2a illustrates the creation of a sound field by a vibrating structure in the surrounding fluid. Consider a small area of the vibrating structure, $S_s$, with dimensions smaller than the acoustic wavelength, such that the radiated acoustic power can be determined by the volume velocity $Q_s = v_s S_s$, where $v_s$ is the normal average velocity over the small area, $S_s$. With structure vibrating at low frequency, the velocity of the area is nearly uniform thus radiating sound as a monopole source. Note that the total radiated power from the plate is due to the contribution of all the areas of the structure including their far-field coupling. Figure 2.2b is a conceptual drawing of a WRC covering surface, $S_s$. Recall the concept of the WRC is to transform the smaller radiating like monopole surface area, $S_s$, into radiating dipole source. Thus the motion of the nearly uniform surface velocity, $v_s$ of surface $S_s$ is converted into the two surface velocities $v_{c1}$ and $v_{c2}$ corresponding to the two surface areas $S_{c1}$ and $S_{c2}$, respectively. The design of the WRC is such that the motion of these two surfaces are out-of-phase and of a relative magnitude which leads to a minimization of the volume velocity, i.e. $v_{c1}S_{c1} + v_{c2}S_{c2} \approx 0$. Figure 2.2c depicts a structure’s surface cover with an array of WRCs, which in turn converts the structure’s response into an array of acoustic dipoles, thus minimizing far-field sound radiation. To this end, the WRC will behave as a dipole source for frequencies that the wavelength is much larger than the cell dimension. Hence, a wider frequency range of noise reduction is expected with smaller cell’s dimensions.
Figure 2.2: Attenuation of low frequency structurally radiated sound using the weak radiating cell concept: (a) untreated plate, (b) single cell, (c) plate with array of cells
2.4 Weak Radiating Cell Device

Shown in Figure 2.3 is a design of the weak radiating cell developed by Ross and Burdisso [26]. The make-up of the WRC contains three separate pieces: an outer element, a flexible medium, and an inner element. The outer rigid element is a cylinder of area $S_{c2}$, which is referred to as the cell’s frame. The cell’s frame physically adheres to the vibrating structure. Since the structure in Figure 2.3 behaves as a piston with normal velocity $v_s$ and the cell’s frame is assumed to be very stiff, the velocity of $S_{c2}$ is nearly the same as the vibrating structure, $v_{c2} \cong v_s$. The inner element, referred as the cell’s plate, forms the second radiating surface of the cell, $S_{c1}$. The cell’s plate is attached to the cell’s frame through a flexible medium, indicated in Figure 2.3 by the spring-damper systems, which creates a closed cavity. The mechanical response of the cell can be modeled as a single degree-of-freedom system with base excitation. Thus the dynamic analysis is straightforward [26, 27, and 31]. The theoretical complex volume velocities of the structure’s surface response, $v_s$, and two surface responses, $v_{c1}$ and $v_{c2}$, are computed with eq (2.1). The source strength magnitude and phase of the two surfaces, $S_{c1}$ and $S_{c2}$, forming the WRC are presented in Figures 2.4. Moreover, the magnitudes of the source strength of both the cell and piston structure are shown in Figure 2.5.
Figure 2.3: Mechanically coupled cell: Simplified Model
These responses are plotted as a function of frequency normalized by the natural frequency of the cell’s plate-shim-cavity system, \( f_n \). In Figure 2.5, the general response of the cell can be described for the following frequency regions: below, around, and above the resonance of the plate-shim-cavity system. It is seen in Figure 2.4 that well below resonance, \( f/f_n < 0.4 \), the phase and magnitude of the two surfaces are the same (i.e., the surface velocities are the nearly identical.) Thus, the volume velocity of the two surfaces combine to equal that produced by the original structure, Figure 2.5. As frequencies approach \( f/f_n \approx 1 \), the cell’s plate response dominates over the piston due to dynamic amplification, and the phase goes through an 180° variation increasing the volume velocity and in turn increasing the sound radiated by the WRC. This increase in volume velocity and thusly-increased noise is seen as an adverse effect of the WRC and it is later addressed. Figure 2.4 also shows that above resonance the motion of the two surfaces is out-of-phase and nearly equal in magnitude. Coupling both the out-of-phase with nearly equal magnitude motion results in the desired dipole effect minimizing the sound radiated from the original source for \( f/f_n > 1.2 \). From Figure 2.5, it is clear that there is a frequency where the volume velocity \( Q_c \) is a minimum and thus the sound reduction a maximum. Note that the volume velocity does not vanish at this frequency because the damping of the system does not result in a perfect out-of-phase motion of the two cells’ surfaces, thus the WRC behaves as a nearly perfect dipole. This frequency is referred as the cell’s dipole frequency, \( f_d \).
Figure 2.4: Source strength of the WRC’s surfaces $S_{c1}$ (---) and $S_{c2}$ (----).
Figure 2.5: Magnitude of source strength of piston (—) and WRC (—).