Chapter 6

Active control of fan noise on a UHB engine

The model developed in this work was used to investigate the potential that active noise control techniques have for reducing fan noise on an ultra high bypass (UHB) turbofan engine. Active noise control is expected to perform differently on a UHB turbofan engine than on a conventional turbofan engine since the short and wide inlet duct inherent to the UHB engine promotes radiation from evanescent modes as well as interference between inlet and outlet radiation. These characteristics were shown in chapter 5 to impact the performance of an active control system. For this investigation, pure active control techniques as well as hybrid control techniques were studied. The results obtained are presented in this chapter.

6.1 Model configuration

The ducted engine fan modeled for this study is described in Figure 6.1. A duct of radius 1.5 m with an inlet and outlet length each of 1.74 m was modeled. These dimensions are representative of those associated with an ultra high bypass turbofan engine prototype being developed by Pratt and Whitney. A BPF of 1000 Hz, and a
uniform flow Mach number of 0.25, which is representative of landing or take off conditions, were considered.

The circumferential order \( m \) of the modes generated by the interaction of \( N_V \) stator vanes with \( N_B \) fan blades is given by \( m = n_h N_B - j N_V \) where \( n_h \) is the harmonic number and \( j \) is an integer (Tyler and Sofrin 1962). From this equation it can be seen that for a given number of fan blades, the number of stator vanes can be adjusted such that the only modes that could be generated are below their cut on frequency. Many of the current turbofan engines are designed such that no modes are cut on. These engines are said to be cut-off. Such engine design, however, features a large number of stator vanes that is now believed to be responsible for an increase in broadband noise compared to engines having a lower number of stator vanes. Future engine designs therefore consider reducing broadband noise by reducing the number of stator vanes, allowing some of the fan modes to be cut on in return. These propagating modes are expected to be of fourth or fifth circumferential order. Therefore, for this study, the generation of the fourth order circumferential modes (i.e., \( m=4 \)) was simulated.

Since no information was available regarding the modal amplitude of the fan modes that were modeled, the fan model based on radial arrays of spinning point dipoles was not used to simulate the fan noise. Instead, the spinning line source model was used. The radial distribution chosen for the strength of the sources was of the form

\[
\bar{F}(r) = F_{\psi}(r) \hat{\psi} + F_Z(r) \hat{Z}
\]

\[
= (a r + b) \hat{\psi} + (c r + d) \hat{Z}, \quad 0 \leq \frac{r}{r_d} \leq 0.98 \tag{6.1}
\]

This choice for the modeling of the blade loading force was based on the results obtained in section 5.1.2, which indicated that a realistic simulation of fan noise radiation could be obtained with such modeling of the blade loading. In order to simulate the generation of fourth order circumferential modes, four spinning line sources of identical strength were used. These line sources were located in the \((z = 0)\) plane as described in Figure 6.1.
Hence, this configuration of the line sources models a fan with four blades which (referring to section 3.1) will only generate fourth order circumferential modes at the fundamental blade passage frequency.

![Figure 6.1: Schematic of the model configuration used for the noise control study on a UHB turbofan engine.](image)

A plot of the calculated pressure field in a plane containing the axis of the duct is presented in Figure 6.2. This pressure field was calculated using the theory of chapter 2 and section 3.2.1. The plot in Figure 6.2 is composed of 200 by 200 computation points and was calculated in approximately 10 minutes on a PC (Pentium 2, 300MHz processor, 32 MB of RAM). Unlike the cases presented previously in section 5.3, a continuous pressure distribution is observed along the duct radius line located at $z = 0$, and is caused
by the presence of the line source at that location. Also, due to the dipole nature of this line source (the line source is composed of an infinite number of axial point dipoles), the acoustic pressure generated in the upstream direction is seen to be out of phase with the pressure generated downstream. Thus, to the right of the radius line located at \( z = 0 \), the pressure is first negative then positive and so on, while to the left of this same radius line the pressure is first positive then negative and so on.

Seven fourth order circumferential modes (i.e., the \((4,0)\), \((4,1)\), \((4,2)\), \((4,3)\), \((4,4)\), \((4,5)\) and \((4,6)\) modes) were cut on and propagated through the inlet and outlet of the duct. As expected, the modes are seen to propagate and radiate in the upstream direction with a greater intensity than in the downstream direction due to the Doppler effect (see section 5.3.1). The cut-off ratios of the first and last modes that were cut on (i.e., the \((4,0)\) and \((4,6)\) modes) are 5.34 and 1.1, respectively.

![Figure 6.2: Pressure field generated by the ducted fan.](image)

M=0.25, BPF=1000 Hz.
The goal was to reduce the noise that radiates within a sector of the far field including both inlet and outlet radiation, since for high bypass turbofan engines the inlet and outlet fan noise dominates at both approach and takeoff (Groeneweg et al. 1991). Thus, the 40° to 155° sector (measured from the inlet opening and with respect to the axis of the duct) was chosen as the target sector for noise reduction. This sector, which includes radiation towards the side of the engine, is also believed to strongly affect the EPNL.

The EPNL, or Effective Perceived Noise level, is a single number measure of complex aircraft flyover noise which approximates laboratory annoyance responses (Pearsons and Bennett 1974). It is derived from PNL (Perceived Noise Level) which refers to a summation, over all 1/3-octave frequency bands, of the sound pressure levels at a given observer point, with the level in each band weighted by a factor which represents the degree of annoyance to noise observed at that particular frequency (Groeneweg et al. 1991). The EPNL refers to a time integration of PNL received by an observer as the noise source (i.e., the plane) passes by. The noise certification measurement locations (or reference points) are shown in Figure 6.3.

The inlet and outlet of the aircraft engine point towards the approach and takeoff reference microphones (see Figure 6.3) only at the beginning and at the end of the approach and takeoff phases, that is, when the aircraft is far away from these reference microphones. Therefore, the noise radiating in the direction of the engine axis is well attenuated when it reaches these microphones. Furthermore, the aircraft flies over the approach reference microphone with a positive flight path angle. Consequently, the noise that radiates towards the side of the engine predominantly impacts the noise level measured at that reference microphone. When the aircraft approaches and flies over the takeoff reference microphone, its flight path angle is such that the noise that radiates towards the side of the engine predominantly impacts the noise level measured at that reference microphone. Finally, the noise level that is measured along the sideline reference line is obviously dictated by the noise that, again, radiates towards the side of
the duct engine. Therefore, the fan noise radiating at high angles from the engine axis appears to be of primary concern.

6.2 Pure passive control

The amount of reduction in sound power level that could be achieved within the target sector (the 40° to 155° sector) using pure passive control was first determined. It was assumed that the duct would be lined over its entire length, except over a length of 0.01 m at the tip of the duct inlet and outlet. The attenuation in sound power level that
could be obtained in other sectors of the far field using various values of the liner impedance was also computed. Only impedance values representative of realistic liners were considered (Motsinger and Kraft 1991).

Figure 6.4: Attenuation in Sound Power Level achieved with purely passive control.

a) 0 to 40 deg. sector  

b) 40 to 70 deg. sector  

c) 70 to 125 deg. sector  

d) 40 to 155 deg. sector  

Figure 6.4: Attenuation in Sound Power Level achieved with purely passive control.
The reduction in sound power level that could be achieved in the far field within the
0° to 40°, 40° to 70°, 70° to 125° and 40° to 155° sectors is shown in Figure 6.4. The
specific resistance of the liner was varied from 0 to 5 with a step of 0.2 and the specific
reactance was varied from –3 to 3, also with a step of 0.2. From Figures 6.4(a), (b) and
(c) it is observed that up to 1.5 dB reduction in sound power level could be achieved
within the 0° to 40° sector, 4.6 dB for the 40° to 70° sector, and 14.4 dB for the 70° to
125° sector. Hence, the liners worked poorly in attenuating the noise radiated toward the
region close to the duct axis and worked best in reducing the noise radiated toward the
sideline of the duct. This was expected since the higher order modes, which propagate
and radiate at larger angles from the duct axis, have higher attenuation rates than the

From figure 6.4(d), it is observed that the passive control system was able to achieve
a maximum reduction in sound power level of 4.6 dB within the target sector. It is also
seen that there is a relatively large region of the impedance plane over which maximum
or near maximum noise reduction level is attainable.

Next, it was determined whether the levels of reduction that were achieved in the
target sector with the pure passive control system could be improved when using active
control techniques.

**6.3 Fuselage error sensors technique**

A schematic of the control system is presented in Figure 6.5. An axial array of error
sensors was placed along the aircraft fuselage, 6 m from the axis of the duct. This
distance was estimated from studying the schematics of large commercial airplanes with
wing mounted engines. The error sensors were placed within the 40° to 70° sector when
controlling the inlet fan noise radiation, or within the 125° to 155° sector when
controlling the outlet radiation. With this configuration of the error sensors, the (4,4), (4,5) and (4,6) modes were primarily targeted for attenuation by the control system since the direction of peak radiation of these two modes is contained within the sectors covered by the error sensors. One or two circumferential arrays of control sources were placed in the duct and used to generate the control field.

Figure 6.5: Schematic of the control system configuration for the fuselage error sensors technique.
6.3.1 Pure active noise control

The duct inner wall was first considered to be rigid. An array of three fuselage error sensors placed within the $40^\circ$ to $70^\circ$ sector and a single array of control sources were used. The reduction in sound power level that could be achieved in the target sector (i.e., in the $40^\circ$ to $155^\circ$ sector) for different axial locations of the control source array was computed. The axial location of the control source array was stepped along the duct length from 1.7 m downstream of the fan to 1.7 m upstream of the fan in 0.1 m increments. The results obtained are presented in Figures 6.6 and Table 6.1.

![Figure 6.6: Sound power level reduction for the 40 to 155 deg. sector. Pure active noise control. Three fuselage error sensors.](image)
Table 6.1: Sound power level reduction. Pure active noise control.
One control source array, three fuselage error sensors.

<table>
<thead>
<tr>
<th>sector (deg.)</th>
<th>0-40</th>
<th>40-70</th>
<th>70-125</th>
<th>125-155</th>
<th>155-180</th>
<th>40-155</th>
</tr>
</thead>
<tbody>
<tr>
<td>reduction (dB)</td>
<td>-0.43</td>
<td>3.36</td>
<td>3.2</td>
<td>0.2</td>
<td>-8.2</td>
<td>2.5</td>
</tr>
<tr>
<td>max.</td>
<td>0.3</td>
<td>3.75</td>
<td>0.9</td>
<td>-4.8</td>
<td>-7.9</td>
<td>0.74</td>
</tr>
</tbody>
</table>

The maximum reduction in sound power level that could be achieved in the target sector was 2.5 dB, and was achieved when the control source array was placed 0.2 m upstream of the fan. Table 6.1 indicates that the configuration of the control system that led to a reduction of 2.5 dB in the target sector, also led to a reduction of 3.36 dB, 3.2 dB and 0.2 dB in the 40° to 70°, 70° to 125° and 125° to 155° sectors, respectively. The maximum reduction in sound power level that could be achieved in the 40 to 70 sector (where the error sensors were located) was 3.75 dB and was achieved when the control source array was placed 0.4 m upstream of the fan. Table 6.1 indicates that this configuration of the control system only led to a reduction of 0.9 dB and 0.74 dB in the 70° to 125° and 40° to 155° sectors, respectively and also led to an increase of 4.8 dB in the 125° to 155° sector. Thus, the maximum level of reduction achieved in the target sector was not obtained with the control system configuration that led to a maximum level of reduction in the 40° to 70° sector. Instead, it was achieved for a configuration of the control system that avoided the creation of spillover in the region of the far field of the outlet, while still maintaining a good level of attenuation within the 40° to 70° sector.

The reduction in sound power level that could be achieved in the target sector for different axial locations of a single control source array was recomputed by placing an array of seven error sensors, instead of three, along the fuselage within the 40° to 70° sector. The results are presented in Figure 6.7 and Table 6.2.
It is observed that a reduction in sound power level of 2.9 dB could be obtained in the target sector when the control source array was located 0.2 m upstream of the fan. This is an improvement of 0.4 dB over the preceding case where an array of only 3 error sensors was used. Thus, the use of additional error sensors did not lead to a very significant improvement in the maximum level of reduction that could be achieved in the target sector. The maximum level of reduction for the target sector was obtained, as in the preceding case, for a configuration of the control system that led to a simultaneous reduction of the noise that radiates into the far field of the inlet and outlet. This optimum solution is seen to remain sensitive to the axial location of the control source array.

Figure 6.7: Sound power level reduction for the 40 to 155 deg. sector.
Pure active noise control. Seven fuselage error sensors.
Table 6.2: Sound power level reduction. Pure active noise control.

One control source array, seven fuselage error sensors.

<table>
<thead>
<tr>
<th>sector (deg.)</th>
<th>0-40</th>
<th>40-70</th>
<th>70-125</th>
<th>125-155</th>
<th>155-180</th>
<th>40-155</th>
</tr>
</thead>
<tbody>
<tr>
<td>reduction (dB)</td>
<td>0.3</td>
<td>2.6</td>
<td>7.4</td>
<td>4.2</td>
<td>-7.1</td>
<td>2.9</td>
</tr>
<tr>
<td>(max.)</td>
<td>-0.23</td>
<td>4.41</td>
<td>-0.6</td>
<td>-5.6</td>
<td>-9.59</td>
<td>0.37</td>
</tr>
</tbody>
</table>

A comparison of the reduction levels obtained within the 40° to 70° sector for different locations of the control source array, when using seven versus three error sensors, is presented in Figure 6.8.

Figure 6.8: Comparison of the sound power level reduction achieved with three and seven fuselage error sensors.
It can be seen that improved coverage by the error sensors of the 40° to 70° sector increased the number of control source array locations that led to a reduction in sound power level, and almost eliminated the occurrence of spillover in this 40° to 70° sector. This resulted both in an overall decrease in the levels of spillover that could occur within the target sector, and also in an increase in the number of configurations of the control system that assured a reduction in sound power level in the target sector.

Next, the results discussed above were compared with the ones obtained while using two control source arrays, instead of one, to generate the control field.

First, an array of three fuselage error sensors was placed within the 40° to 70° sector. The axial positions of the two control source arrays were stepped along the duct inlet and outlet in 10 cm increments in order to find the configuration of the control system that would lead to a maximum reduction in sound power level in the target sector. The results are presented in Figure 6.9 and Table 6.3. The maximum attenuation that could be obtained was 5.3 dB when the control source arrays were located, respectively, 0.1 m and 0.4 m upstream of the fan. This is an improvement of 1.8 dB over the single control source array case. This optimum configuration of the control system led simultaneously to a reduction of 5.2 dB within the 40° to 70° sector, and to a reduction of 4.8 dB within the 125° to 155° sector. It can be seen that the addition of a second control source array improved the ability of the control system to simultaneously control the modes that propagated through the inlet and outlet of the duct. It was also noted that this active noise control system configuration could lead to a reduction of up to 7.3 dB in sound power level for the 40° to 70° sector, which is an increase of 3.55 dB over the level of reduction that could be achieved with a single control source array. Thus, the addition of a second control source array to the control system increased the system's controllability over the higher order radial modes that radiated within the target sector, or more precisely, within the 40° to 70° sector. Although the controllability of the system over the propagating
modes was improved, the optimum solution remained very sensitive to the location of the control source arrays. A reduction of 4.3 dB could be obtained with less sensitivity.

Figure 6.9: Sound power level reduction for the 40 to 155 deg. sector. Pure active noise control. Two control source arrays, three fuselage sensors.

Table 6.3: Sound power level reduction achieved with the optimum configuration of the control system. Pure active noise control. Two control source arrays, three fuselage sensors.

<table>
<thead>
<tr>
<th>sector (deg.)</th>
<th>0-40</th>
<th>40-70</th>
<th>70-125</th>
<th>125-155</th>
<th>155-180</th>
<th>40-155</th>
</tr>
</thead>
<tbody>
<tr>
<td>reduction (dB)</td>
<td>0.44</td>
<td>5.2</td>
<td>7.5</td>
<td>4.8</td>
<td>6.1</td>
<td><strong>5.4</strong></td>
</tr>
</tbody>
</table>
Placing seven error sensors instead of three along the fuselage, within the 40° to 70° sector, did not significantly improve the maximum reduction in sound power level that could be achieved in the target sector. Figures 6.10(a), 6.10(b), 6.10(c), and 6.10(d) show the sound power level reduction achieved for the 40° to 70° sector and for the 125° to 155° sector using three or seven fuselage error sensors. It is observed that while reduction in sound power level could be achieved relatively easy within the sector containing the error sensors, spillover would dominate in the far field of the outlet. However, it is noted that, for both of these sectors, the placement of additional error sensors within the 40° to 70° sector increased the number of control source array locations that led to reductions in sound power level and reduced the levels of possible spillovers. These two effects are also observed for the 40° to 155° sector (cf. Figure 6.11 and Table 6.4). Thus, a larger number of control source array locations that led to noise reductions in the target sector could be obtained with seven error sensors than with three, and a maximum spillover of only 2.6 dB would occur with seven error sensors versus a maximum of 10 dB with the three error sensors case.

Therefore, using the pure active noise control fuselage error sensors technique, it was observed that:

(i) Using two control source arrays instead of one increased controllability over the propagating modes and could increase, by up to 3 dB, the reduction that could be achieved in the target sector.

(ii) Using seven instead of three error sensors did not significantly change the levels of reduction that could be achieved in the target sector, but it reduced the level of possible spillovers and it significantly increased the number of control source array locations that led to noise reductions in the target sector (i.e., control system less sensitive to positioning of control source arrays).
(iii) By using two control source arrays and seven sensors, the active noise control system could match, or even exceed, the performance of the optimum pure passive control case.

Figure 6.10: Sound power level reduction. Pure active noise control. Two control source arrays. Fuselage error sensors.

a) 40 to 70 deg. sector. 3 error sensors.

b) 40 to 70 deg. sector. 7 error sensors.

c) 125 to 155 deg. sector. 3 error sensors.

d) 125 to 155 deg. sector. 7 error sensors.
Figure 6.11: Sound power level reduction for the 40 to 155 deg. sector.
Pure active noise control. Two control source arrays, seven fuselage sensors.

Table 6.4: Sound power level reduction achieved with the optimum configuration of the control system. Pure active noise control.
Two control source arrays, seven fuselage sensors.

<table>
<thead>
<tr>
<th>sector (deg.)</th>
<th>0-40</th>
<th>40-70</th>
<th>70-125</th>
<th>125-155</th>
<th>155-180</th>
<th>40-155</th>
</tr>
</thead>
<tbody>
<tr>
<td>reduction (dB)</td>
<td>0.41</td>
<td>5.8</td>
<td>5.6</td>
<td>5.6</td>
<td>6.2</td>
<td>6.0</td>
</tr>
</tbody>
</table>
6.3.2 Hybrid control

Next, it was determined whether the above mentioned results could be improved by adding a liner to the active noise control system, that is, by using a hybrid control system instead of a pure active or pure passive control system. The hybrid control system combines the use of active (control sources) with passive (liners) noise control techniques. Two different hybrid control systems were studied. For both of these systems, the duct was considered to be lined over its entire length, except over a length of 0.01 m at the tip of the duct inlet and outlet, as in the pure passive control case described previously in section 6.2. In the active part of the control system it is assumed that the control sources are point sources, and as such, do not have surface area. On a real engine however, each of the control sources that are embedded in the liner occupies a finite portion of the lining material surface. If the absence of lining material at the location of the control sources were to be accounted for, control sources of finite area would need to be modeled instead of point sources. This could be done by replacing the Dirac delta functions (in $\psi$ and $Z$) in Eq. (4.1) with step functions. This would model control sources of square or rectangular area.

In the first case of hybrid control studied, one control source array and an axial array of three fuselage error sensors placed along the fuselage, within the $40^\circ$ to $70^\circ$ sector, were considered for the active part of the control system. The maximum reduction in sound power level that could be achieved in the target sector by this hybrid control system was then determined by simultaneously optimizing the impedance of the liner and the axial location of the control source array as previously. The optimum configuration of the control system led to a maximum reduction of 8.4 dB. It was achieved for a liner specific impedance of 1.7+2.8i, when the control source array was located 0.2 m upstream of the fan.
Figure 6.12: Sound power level reduction achieved within the 40 to 155 deg. sector.

Hybrid control. One control array. Three fuselage error sensors.

a) Liner impedance: 1.7 + 2.8 i

b) Control source array location: 0.2 m
The robustness of this optimum solution with respect to the location of the control source array was evaluated next. To do so, the attenuation in sound power level that could be achieved in the target sector by the hybrid control system was computed for various locations of the control source array, while the duct wall was lined with the optimum liner of specific impedance 1.7+2.8i. These results are presented in Figure 6.12(a). This figure shows that although at least 4 dB of reduction could be achieved for almost all possible axial locations of the control source array, the optimum solution remains sensitive to variation in the control source array location.

The robustness of this optimum solution with respect to the liner impedance was also studied. The reduction in sound power level that could be achieved in the target sector was computed for various values of the liner impedance, while the control source array was fixed at its optimum location (0.2 m upstream of the fan). The results are presented in Figure 6.12(b). From this figure it is seen that levels of reduction close to the maximum could be obtained for a large number of impedance values. Thus, the optimum solution is robust with respect to variation in liner impedance.

In the second case, two control source arrays and an axial array of seven fuselage error sensors placed within the 40° to 70° sector were considered for the active control part of the hybrid system. The dimensions of the liner remained unchanged. Again, the control source arrays locations and the liner impedance combination that would lead to the best reduction in the 40° to 155° sector were determined. The best attenuation that could be achieved was 10.2 dB with a liner specific impedance of 1.7+2.8i and with the control arrays placed 0.2 m and 1.6 m, respectively, upstream of the fan. Figures 6.13(a) and 6.13(b) show the robustness of the optimum solution with respect to the liner impedance and with respect to the control source arrays locations. Again, the optimum solution appears to be more sensitive to the location of the control source arrays than to the liner impedance. Nevertheless, reduction occurred for almost all control source array locations.
a) impedance of the liner: $1.7 + 2.8i$

b) location of the control arrays: 0.2 and 1.6 m

Figure 6.13: Sound power level reduction achieved within the 40 to 155 deg. sector. Hybrid control. Two control arrays. Three fuselage error sensors.
Thus, using this hybrid control technique, it was observed that:

(i) The hybrid system increased the reduction in sound power level by 6 dB in the target sector over the pure active or passive noise control systems.

(ii) The presence of the liner greatly increased the number of control source array locations for which reduction occurred in the target sector.

(iii) The optimum solution was very robust with respect to the liner impedance and less with respect to the control source array location.

6.3.3 Control of inlet and outlet radiation

Placing the fuselage error sensors in the far field of the inlet insured a good reduction in that region of the far field; however, it also led to considerable spillover in the far field of the outlet (particularly within the 125° to 180° sector), hindering the level of noise reduction in the target sector (40° to 155° sector) and the robustness of the optimum solution.

Therefore, in addition to placing an axial array of three error sensors along the fuselage within the 40° to 70° sector, an array of three error sensors was also placed along the fuselage within the 125° to 155° sector, as depicted in Figure 6.14.
Pure active as well as hybrid control systems were studied with this configuration of the error sensors using one or two arrays of control sources. The maximum attenuation in sound power level that could be achieved in the target sector with each of these control systems is presented in Table 6.5(a). With the pure active control system, the possible reduction was up to 2.03 dB when one control source array was used, and up to 4.3 dB when two control source arrays were used. With the hybrid system, a reduction up to 7.3 dB could be achieved using one control source array and up to 9.6 dB with two control source arrays. These levels are slightly lower than the ones that could be achieved when the error sensors were placed only within the 40° to 70° sector (cf. Table 6.5(b)).
Table 6.5: Maximum reduction in sound power level achieved within the 40 to 155 deg. sector.

a) with fuselage error sensors placed in the far field of the inlet and outlet.

<table>
<thead>
<tr>
<th></th>
<th>pure ANC</th>
<th>hybrid control</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 control array</td>
<td>2.03</td>
<td>7.3</td>
</tr>
<tr>
<td>2 control arrays</td>
<td>4.3</td>
<td>9.6</td>
</tr>
</tbody>
</table>

b) with fuselage error sensors placed in the far field of the inlet only.

<table>
<thead>
<tr>
<th></th>
<th>pure ANC</th>
<th>hybrid control</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 control array</td>
<td>2.5</td>
<td>8.4</td>
</tr>
<tr>
<td>2 control arrays</td>
<td>6.0</td>
<td>10.2</td>
</tr>
</tbody>
</table>

Figures 6.15(a), 6.15(b) and 6.15(c) show, for the sectors spanning 40° to 155°, 40° to 70° and 125° to 155°, the reduction in sound power level that was achieved by the pure active control system for various locations of the two control source arrays used.
a) for the 40 to 155 deg. sector.

b) for the 40 to 70 deg. sector.  
c) for the 125 to 155 deg. sector.

Figure 6.15: Sound power level reduction achieved with two control source arrays and fuselage error sensors placed in the far field of the inlet and outlet.
It can be seen that the control source arrays locations that led to reduction in the target sector correspond well to the ones that also led to reduction in the 40° to 70° sector, but did not correspond well to the ones that led to reduction in the 125° to 155° sector. This was also observed for the other configurations of the control system being studied. This seemed to indicate that reducing the sound power level within the 40° to 70° sector would achieve better results for the overall reduction in the target sector than reducing the sound power level in the 125° to 155° sector. By adding error sensors to the far field of the outlet, less reduction was achieved in the 40° to 70° sector since the control system was made to perform at both the inlet and outlet regions of the far field. This impacted negatively the maximum level of overall reduction for the target sector, although better reduction (or at least less spillover) was obtained in the 125° to 155° sector as a result of the outlet far field error sensors.

The addition of error sensors to the far field of the outlet did not increase the maximum level of reduction for the target sector, but it did improve the robustness of the optimum solution by increasing the number of control source array locations that led to a reduction in the target sector. Although this was verified for each of the systems being studied, only one will be discussed. For each of the two hybrid control systems under consideration, the impedance of the liner was fixed to its optimum value. That is, the value that led each of these systems to achieve their maximum level of reduction in the target sector was used. Figures 6.16(a) and 6.16(b) show the reduction in sound power level that could be achieved in the 125° to 155° and 40° to 155° sectors by using an hybrid control system with three error sensors placed in the far field of the inlet and outlet and two control source arrays. These results were compared with the ones that were achieved by the hybrid control system when three error sensors were placed in the far field of the inlet only (cf. Figures 6.16(c) and 6.16(d)). These reduction levels were computed as functions of the location of the control source arrays.
Figure 6.16: reduction in sound power level. Hybrid control.
Fuselage sensors. Two control source arrays.
Therefore, by adding fuselage error sensors to the far field of the outlet, it was observed that:

(i) It increased the number of locations of the control source arrays that led to reduction in sound power level in the target sector (i.e., more robust control system).

(ii) It reduced the level of possible spillover.

(iii) It decreased the peak value of sound power level reduction (this might not be the case for an annular duct) by an average of 1 dB or 15.5% which is non-negligible.

6.4 Wavenumber error sensor technique

As explained in section 4.2.3, the wavenumber sensors technique minimizes certain components of the wavenumber spectrum in an attempt to reduce the acoustic radiation toward specific directions in the far field. This technique is based on the fact that the axial wavenumbers and the angles of peak radiation of the duct acoustic modes are uniquely related (as discussed in section 4.2.3). Thus, the control system targets the modes according to their axial wavenumbers in order to reduce noise radiation in specific directions of the far field.

Figure 6.17 shows a schematic of the control system used. An axial array of pressure sensors was placed along the duct inlet (or outlet) inner wall. The time domain pressure was measured at the location of these sensors, and then used to compute an estimate
$T(k_z^*)$ of the wavenumber spectrum component (using Eq. (4.43.a)) corresponding to a chosen axial wavenumber $k_z^*$ (or to a chosen angle of peak far field radiation). This axial wavenumber will be referred to as the “error” wavenumber. The control field was generated by a single control source array, and was aimed at canceling the duct mode propagating with the error wavenumber. This was achieved by minimizing the

$\sum_n p(Z_n) e^{i K_z n \Delta Z}$

Figure 6.17: Schematic of the control system configuration for the wavenumber sensors technique.

In the ducted fan noise case considered in this study, seven fourth order circumferential modes (i.e., the (4,0), (4,1), (4,2), (4,3), (4,4), (4,5) and (4,6) modes) are cut on. As indicated in Figure 6.18, these modes propagate toward the inlet with axial wavenumbers (in increasing radial order): 24.0 m$^{-1}$, 23.3 m$^{-1}$, 22.3 m$^{-1}$, 21.0 m$^{-1}$, 19.2 m$^{-1}$.
$16.8 \; \text{m}^{-1}$ and $13.3 \; \text{m}^{-1}$, respectively, and propagate toward the outlet with axial wavenumbers (also in increasing radial order): $-14.3 \; \text{m}^{-1}$, $-13.6 \; \text{m}^{-1}$, $-12.6 \; \text{m}^{-1}$, $-11.3 \; \text{m}^{-1}$, $-9.5 \; \text{m}^{-1}$, $-7.1 \; \text{m}^{-1}$ and $-3.5 \; \text{m}^{-1}$, respectively. It is seen in the figure that the spacing $(k_{z}^{m,n} - k_{z}^{m,n+1})$ between successive axial wavenumbers increases with the radial order of the modes being considered. Thus, the spacing between consecutive axial wavenumbers varies from a minimum value of $0.7 \; \text{m}^{-1}$ between the $(4,0)$ and $(4,1)$ modes, to a maximum value of $3.5 \; \text{m}^{-1}$ between the $(4,5)$ and $(4,6)$ modes.

Figure 6.18: Axial wavenumbers and angles of peak far field radiation.

$m=4, \; \text{BPF}=1000 \; \text{Hz}, \; r_{d}=1.5 \; \text{m}$ and $M= \pm 0.25.$
The ability of the control system to distinguish the spectral responses of the different modes propagating in the duct is dictated by the axial wavenumber resolution which is defined as

$$\Delta k_z = \frac{2\pi}{(N_c - 1) \Delta z}. \quad (4.45)$$

The resolution $\Delta k_z$ is a key parameter in obtaining a reliable and accurate spectrum (i.e., accurate estimates of $T(k_z^*)$). The smaller $\Delta k_z$ is (i.e., the longer the record length to be analyzed is), the more accurate and detailed the spectrum estimate will be. A “good” resolution implies that each peak of the spectrum estimate appears distinctively. Unfortunately, in addition to the record length, the implicit windowing that occurs during the computation of the wavenumber transform also limits the quality of the resolution that can be achieved. This is explained next.

The wavenumber transform (see Eq. (4.43)) of a finite data record $p(z)$ of length $L$ corresponds, in fact, to the wavenumber transform of an unlimited record $v(z)$ multiplied by a boxcar window denoted by $box(z)$,

$$box(z) = \begin{cases} 1, & \text{for } 0 \leq z \leq L \\ 0, & \text{for } z < 0 \text{ and } z > L \end{cases} \quad (4.46)$$

Thus,

$$p(z) = box(z) \cdot v(z) \quad (4.47)$$

Now, the wavenumber transform of this product is the convolution of the wavenumber transforms:

$$T(k_z) = BOX(k_z) \ast V(k_z)$$

$$= \int_{-\infty}^{\infty} BOX(\alpha) \cdot V(k_z - \alpha) \, d\alpha \quad (4.48)$$

where $BOX(k_z)$ is the wavenumber transform of the boxcar window depicted in Figure 6.19 (one-sided spectrum). Note the finite width of the main lobe of this spectrum. Because of the finite value of $L$, what would have been a delta function for infinite $L$ has
become a sinc function, $\frac{\sin z}{z}$. Thus, each of the spectral peaks which should have been concentrated at a single point has been spread out over a much broader range of axial wavenumbers. Consequently, if the sensor array cannot resolve a single spectral line which corresponds to a single radiation direction, the result is error information which corresponds to radiation over a range of angles. The net effect will be an averaged reduction over a sector rather than a large reduction at a single angle.

Note also in Figure 6.19, that the spectrum of the boxcar window has noticeable side lobes. These side lobes distort the main lobe of a response at another axial wavenumber of the spectrum, thus affecting the estimate of the wavenumber transform at that axial wavenumber. This phenomenon is referred to as “leakage”. In order to reduce side-lobe leakage, a window with smaller side lobes would have to be used instead of the boxcar window.
window. Unfortunately, windows with smaller side lobes than the boxcar window always have a broader main lobe (Hannan 1960). In the case of the wavenumber sensor technique, the quality of resolution is substantially limited (as it will be shown subsequently) by the short length of the data record used by the control system to compute estimates of wavenumber spectrum components. Therefore, to avoid further reduction of the quality of resolution, using a window with a main lobe of small width (such as the boxcar window) was preferred over reducing side-lobe leakage.

The influence that the resolution $\Delta k_z$ has on the computation of accurate estimates of the wavenumber spectrum components is explained in the following example. The propagation of fourth order circumferential modes in a duct of radius 1.5 m, at a BPF of 1000 Hz and a flow Mach number of 0.25 was considered. The wavenumber spectrum resulting from the propagation of these duct modes was computed with two different resolutions $\Delta k_z$. These results are presented in figure 6.20.

The spectra displayed in figure 6.20(a) and 6.20(b) were obtained from data records of lengths $(N_e - 1)\Delta z = 9$ m and $(N_e - 1)\Delta z = 1.75$ m, respectively. Referring to Eq. (4.45), this corresponds to computing these spectra with a resolution $\Delta k_z$ equal to 0.69 $\text{m}^{-1}$ and 3.59 $\text{m}^{-1}$, respectively. Comparing figure 6.20(a) with figure 6.20(b), it is seen that the definition of the spectrum diminished significantly with $\Delta k_z$ equal to 3.59 $\text{m}^{-1}$ instead of 0.69 $\text{m}^{-1}$: in the spectrum displayed in Figure 6.20(a), the peaks corresponding to the (4,4), (4,5) and (4,6) modes appear distinctively, while in figure 6.20(b) all the spectral components are grouped under a single broad lobe (only the spectral peak corresponding to the (4,6) mode partially stands out).
In the spectrum computed with a resolution of $0.69 \, \text{m}^{-1}$ and displayed in figure 6.20(a), the estimate $T(k_z^{4.6})$, for example, is only distorted by the side lobes of adjacent spectral peaks. Therefore, the existence of the peak at $k_z^{4.6}$ (i.e., the value computed for
T(k_{z}^{4,6}) results, almost exclusively, from the propagation in the duct of the (4,6) mode. Hence, minimizing the corresponding wavenumber component of the spectrum will translate into canceling the (4,6) mode.

On the other hand, the estimate T(k_{z}^{4,6}) computed with a resolution of 3.63 m\(^{-1}\) (and displayed in figure 6.20(b)) results from the combination of the main lobe of the spectral response at k_{z}^{4,6} with the main lobes of the spectral responses at adjacent axial wavenumbers, and as such, does not result exclusively from the propagation of the (4,6) mode in the duct. Instead, it is a result of the propagation of a combination of modes (the ones whose spectral peaks overlap the spectral peak at k_{z}^{4,6}). Therefore, minimizing T(k_{z}^{4,6}) will result in targeting for control several modes instead of one.

Note that in the spectrum displayed in figure 6.20(a), the spectral peaks of the (4,0), (4,2) and (4,3) modes can not be distinguished from the spectral peaks of the (4,1) and (4,4) modes respectively centered at k_{z} = 23.3 m\(^{-1}\) and 19.2 m\(^{-1}\). Since the spacing between each modal axial wavenumber is greater than the resolution \(\Delta k_z\) used to compute the spectrum, the implicit windowing that occurred during the computation of the wavenumber transform is responsible for the fact that the spectral peaks of the (4,0), (4,2) and (4,3) modes (and hence radiation towards those angles) are not distinguishable. Thus, these spectral peaks are probably of small amplitude and are masked by the leakage from the main lobes of the larger adjacent peaks corresponding to the (4,1) and (4,4) modes.

From figure 6.20(a), it is seen that unlike the spectral peaks of the (4,0), (4,2) and (4,3) modes, the small spectral peak centered at k_{z} = 16.8 m\(^{-1}\) and corresponding to the (4,5) modes is distinct from the much larger adjacent spectral peaks corresponding to the (4,4) and (4,6) modes. The spacing between k_{z}^{4,5} and k_{z}^{4,4} is equal to 2.4 m\(^{-1}\) which is
about 3.5 times the resolution $\Delta k_z$. Hence, in order to distinguish all the peaks of the spectrum, the resolution would have to be such that $\Delta k_z = (k_z^{4.0} - k_z^{4.1})/3.5 = 0.2\ m^{-1}$ (the spacing between the spectral peaks corresponding to the (4,0) and (4,1) modes being the narrowest). Referring to Eq. (4.45), this level of resolution is obtained with a data record of length $L = 2\pi / \Delta k_z = 31.4\ m^{-1}$ (i.e., with a pressure sensors array 31.4 m long which is of course not feasible).

The wavenumber sensors technique was used to reduce the sound power level within the 40$^\circ$ to 155$^\circ$ sector of the far field as depicted in figure 6.1. This was accomplished by targeting for control radiation from either the inlet or outlet of the duct. Radiation from the duct inlet was controlled by placing the array of pressure sensors along the inlet wall in order to estimate, and then minimize, the components of the wavenumber spectrum corresponding to positive axial wavenumbers (wavenumbers of modes travelling toward the inlet opening). Similarly, radiation from the duct outlet was controlled by placing the array of pressure sensors along the outlet wall in order to estimate, and then minimize, the components of the wavenumber spectrum corresponding to negative axial wavenumbers (axial wavenumbers of modes propagating toward the outlet opening).

Because of the short lengths of the duct inlet and outlet, and hence because of the limited length of the pressure sensors arrays that could be realistically placed along the duct, the resolution used by the control system to calculate estimates of the wavenumber spectrum components was poor. For example, when using an array of pressure sensors 0.6 m long (as described in section 6.4) the resolution is $12.56\ m^{-1}$. The spectrum that is computed with this level of resolution is presented in Figure 6.21. It is clearly seen from this figure that minimizing the estimate $T(k_z^*)$ of the spectral component corresponding to a specific axial wavenumber $k_z^*$, actually results in minimization of a range of spectral components (or axial wavenumbers).
In this work, pressure sensors arrays that cover only up to 50% of the duct inlet or outlet length were considered in order to conform to the same “duct length to array length ratio” that was used in the experimental work done by Burdisso and Smith (Smith et al. 1999). Cases where longer pressure sensors arrays are used were not studied.

Figure 6.21: Wavenumber spectrum. $\Delta k_z = 12.56 \text{ m}^{-1}$, BPF=1000Hz, $M=0.25$, $r_d=1.5\text{m}$ and $m=4$. 


6.4.1 Control of the inlet radiation – pure active control

An axial array of four pressure sensors was used for this case. The sensors were placed along the duct inlet inner wall between 1.55 m and 1.7 m with a spacing of 0.05m. Referring to Eq. (6.42), this configuration of the pressure sensor array corresponds to a resolution of 41.8 m\(^{-1}\) for the computation of the components of the wavenumber spectrum. The reduction in sound power level that could be achieved in various sectors of the far field by targeting different error wavenumbers (i.e., by minimizing estimates of different components of the wavenumber spectrum), and for various locations of the control source array, was computed. The control source array was stepped throughout the inlet and outlet of the duct between \(-1.7\) m and 1.5 m, with an increment of 0.1 m, while the values of the error wavenumbers targeted were varied between 5 m\(^{-1}\) and 29 m\(^{-1}\), with an increment of 1 m\(^{-1}\). The results are presented in Figures 6.22(a), 6.22(b) and 6.22(c). It is observed that the reduction in sound power level that could be achieved in the different sectors of the far field shows no dependence with respect to which wavenumber component was being minimized. The levels of reduction were found to be sensitive only to the location of the control source array. Thus, with a resolution as poor as 41.8 m\(^{-1}\), it was impossible for the control system to associate an estimate \(T(k^*_n)\) with the propagation of a specific mode (or of a small group of modes). Instead, the control system was attempting to control all the propagating modes in the same time, hence failing to reduce noise radiation toward different regions of the far field when targeting different axial wavenumbers.

Increasing the number of pressure sensors from four to six, and changing the spacing between them to 0.1m improved the resolution to 12.56 m\(^{-1}\). The optimization process of the control system was then repeated. The results are presented in Figures 6.23(a), 6.23(b) and 6.23(c). From these figures, it is seen that the level of reduction achieved in each sector varies with respect to the control source array location, as well as with respect to the error wavenumber. Thus, by increasing the resolution, the control system was able
to better estimate, and therefore to better differentiate the components of the wavenumber spectrum. As a result, the control system could better target specific sectors of the far field. With a resolution of 41.8 m\(^{-1}\), the control system was able to achieve a reduction in sound power level of up to 1.5 dB, 4.4 dB, 4.0 dB and 1.16 dB, respectively, for the (0°-40°), (40°-70°), (70°-90°) and (40°-155°) sectors, as compared to 1.8 dB, 5.2 dB, 4.6 dB and 2.3 dB, for the respective sectors, with a resolution of 12.5 m\(^{-1}\).

Figure 6.22: Reduction in sound power level achieved with four inlet wavenumber sensors and one control source array. Resolution of 41.8 m\(^{-1}\). Pure active noise control.
Figure 6.23: Reduction in sound power level achieved with six inlet wavenumber sensors and one control source array. Resolution of 12.56 m$^{-1}$. Pure active noise control.
Placing the control source array at its optimum location, the attenuation in sound pressure level that could be achieved in the far field of the inlet for different error wavenumbers was then computed. These results are presented in Figure 6.24. They do not show a very good correlation between the error wavenumber that was being targeted and the direction of the far field where reduction was achieved.

*Figure 6.24: Reduction of the sound pressure level in the far field of the inlet; Pure active control; Six inlet wavenumber sensors; Optimum control source location.*
6.4.2 Control of the inlet radiation – hybrid control

Next, the effects of adding a liner to the active control system were studied. The duct was assumed to be lined over its entire length except over a length of 0.01 m at the tip of the inlet and outlet. This hybrid control system was optimized by looking for the error wavenumber, the control source array location, and the liner impedance that would lead to the maximum reduction in sound power level within the target sector. These three parameters were optimized simultaneously. The value of the error wavenumber was varied between 5 m\(^{-1}\) and 30 m\(^{-1}\) with an increment of 1 m\(^{-1}\), while the axial location of the control source array was stepped along the duct length from 1.65 m downstream of the fan to 1.15 m upstream of the fan in 0.1 m increments. The specific resistance and reactance of the liner were varied between 0.1 and 5 for the resistance and between -3 and 3 for the reactance, both with an increment of 0.2.

The best attenuation that could be achieved in the target sector was 6.3 dB. It is an improvement of 4 dB over the pure active control case. This reduction in sound power level was obtained for a liner of specific resistance 0.1 and specific reactance 0.7, for a target error wavenumber of 7 m\(^{-1}\), and for a location of the control source array of 1.55 m downstream of the fan.

Placing the control source array at its optimum location, the attenuation in sound pressure level that could be achieved in the far field of the inlet for different error wavenumbers was then computed. This result is presented in Figure 6.25, and does show a good correlation between the error wavenumber that is being targeted and the direction of the far field toward which reduction is achieved.
Thus, it can be seen that as the error wavenumber increases, the zones of the plot where reduction is achieved tilted toward the axis of the duct. In the rigid wall case, the wavenumbers of the modes that are propagating toward the inlet of the duct are (in increasing radial order) 24.0 m⁻¹, 23.3 m⁻¹, 22.3 m⁻¹, 21.0 m⁻¹, 19.2 m⁻¹, 16.8 and 13.3 m⁻¹, while in this lined wall case, these wavenumbers are 23.7 m⁻¹, 22.9 m⁻¹, 21.8 m⁻¹, 20.2 m⁻¹, 18.4 m⁻¹, 15.3 m⁻¹ and 10.5 m⁻¹. It is noted that the difference $\Delta k_z$ between consecutive wavenumbers is, in average, larger in the lined wall case than in the rigid
wall case. In the rigid wall case, $\Delta k_z$ varied from 0.7 m$^{-1}$ for the first two modes that were cut on, to 3.5 m$^{-1}$ for the last two modes that were cut on, while in the lined wall case, $\Delta k_z$ varied from 0.8 m$^{-1}$ for the first two modes that were cut on, to 4.8 m$^{-1}$ for the last two modes that were cut on. Therefore, the good correlation observed between the error wavenumber being minimized and the region of the far field where noise reduction is achieved could be due to the fact that the presence of the liner in the duct causes the wavenumber spectrum to “stretch”, thus allowing the control system to better differentiate (and hence better estimate) the different components of the wavenumber spectrum.

The robustness of this optimum solution with respect to the error wavenumber and to the location of the control source array was investigated. While the specific impedance of the duct inner wall was set to its optimum value of 0.1+0.7i, the attenuation in sound power level that could be achieved in the target sector for various locations of the control source array and various error wavenumbers was computed. These results are presented in Figure 6.26(a). They show that although the optimum solution remained sensitive to the control source array location, a larger number of control source array location that led to reduction in the target sector was obtained, as compared to the rigid wall case. Thus, the presence of the liner increased the robustness of the control system.

From Figure 6.26(a), it can also be observed that reduction in sound power level was achieved in the target sector for almost all possible locations of the control source array when the lower error wavenumbers were targeted. Therefore, the control system demonstrated the ability to identify the range of error wavenumbers which corresponded to the modes that radiate within the target sector, since targeting the lower error wavenumbers corresponds to targeting the higher order radial modes (which are the ones that radiate within the target sector).
a) liner impedance: $0.1 + 0.7 \, \text{i}$

b) Control source array location: $-1.55 \, \text{m}$. Error wavenumber: $7 \, \text{m}^{-1}$

Figure 6.26: Sound power level reduction achieved within the 40 to 155 deg. sector. Hybrid control. One control array. Six inlet wavenumber sensors.
The robustness of the optimum solution with respect to the liner impedance was then assessed. Fixing the error wavenumber and the axial location of the control source array to their optimum values (7 m\(^{-1}\) for the error wavenumber and -1.55 m for the axial location of the control source array), the reduction in sound power level that could be achieved with this hybrid system was computed for various values of the liner specific impedance. These results are presented in Figure 6.26(b) and indicate that the optimum solution is not very robust with respect to the liner impedance.

### 6.4.3 Control of the outlet radiation – hybrid control

Next, the level of reduction that could be achieved in the target sector when targeting the negative wavenumber components of the spectrum was investigated. Thus, the modes that were propagating through the outlet of the duct were targeted for control. An axial array of 6 wavenumber sensors was placed in the duct outlet. The sensors were placed along the duct inner wall between -1.7 m and -1.2 m with a spacing of 0.1 m. The duct was considered to be lined (since this technique demonstrated poor performances with a rigid duct), and this hybrid control system was optimized. The combination of liner impedance, control array location, and error wavenumber that would lead to a maximum reduction in the target sector was determined. A maximum reduction of 5.94 dB could be achieved for a liner impedance of 0.8+1.1i, an error wavenumber of -3 m\(^{-1}\), and a location of the control source array of 1.55 m upstream of the fan.

Placing the control source array at its optimum location, the attenuation in sound pressure level that could be achieved in the far field of the inlet for different error wavenumbers was then computed. The results are presented in Figure 6.27. They show, again, a good correlation between the error wavenumber that is being targeted and the direction of the far field where reduction is achieved.
The level of reduction of 5.94 dB that was achieved within the target sector when controlling the negative wavenumber components is 0.36 dB less than what was obtained when the inlet wavenumber components were targeted. This difference in performance between the two approaches is probably due to the fact that, as it was observed earlier, controlling the modes that radiate through the inlet of the duct has more impact on the overall reduction achieved within the target sector than when controlling the modes that radiate through the outlet of the duct. However, this difference in performance is relatively small.

Figure 6.27: Reduction of the sound pressure level in the far field of the outlet. Six inlet wavenumber sensors. Optimum control source location and liner impedance.
It was also observed that the value of optimum liner specific impedance is not the same for the cases where positive or negative error wavenumbers were minimized. This could be due to the fact that, even though the modes that radiate through the outlet of the duct do it so at the same angle with respect to the duct axis as the modes that radiate from the inlet of the duct, the wave fronts of the outlet and inlet modes do not hit the duct wall (or liner) with the same angles (see Figure 6.28). For example, the axial wavenumbers of the (4,0) modes propagating toward the inlet opening and toward the outlet opening are 24.0 m⁻¹ and −14.3 m⁻¹, respectively. From Eq. (4.39), the angle between the modal wavefront and the duct axis is 9° for the mode propagating toward the inlet and 49° for the mode propagating toward the outlet. Nevertheless, as indicated by Eq. (4.42) or as seen in Figure 6.16, the angle of peak radiation of both modes is 11.2°.

Figure 6.28: Angles of propagation and peak radiation of a (m,n) mode propagating toward the inlet and outlet.
Regarding the wavenumber sensors technique, it was observed that:

(i) The performance of the wavenumber sensors technique depends on its ability to accurately evaluate the components of the wavenumber spectrum corresponding to each of the propagating duct modes.

(ii) The longer the pressure sensors array is, the smaller the resolution $\Delta k_z$ is, and thus, the better the control system performs.

(iii) The larger the spacing between consecutive modal axial wavenumbers is, the better the control system performs.

(iv) The presence of the liner improved the correspondence between the error wavenumber being targeted and the far field region where noise reduction was achieved. It also increased the number of locations of the control source array that led to reduction in the target sector.

(v) The hybrid system improves by 4 dB the reduction that could be achieved in the target sector, compared to the pure active noise control system.

(vi) The optimum solution is sensitive to both the liner impedance and to the control source array location.

(vii) Similar levels of reduction were achieved when targeting positive or negative axial wavenumbers.

Therefore, in-duct pressure sensors could be use to control fan noise radiation. This technique works best when controlling the radiation toward the sideline of the engine, that is, when targeting higher order radial modes (since the axial wavenumbers of these
modes are more spaced out than the axial wavenumbers of modes of lower radial order). Since the performance of this technique depends greatly on the resolution $\Delta k_z$ and on the spacing between consecutive modal axial wavenumbers, the wavenumber sensors technique should be used with the longest possible array of pressure sensors and combined with passive control.

6.5 Summary:

A table summarizing the optimum levels of reduction that could be achieved for the target sector using the different control systems that were studied is presented in Figure 2.23.

A maximum reduction of 4.5 dB could be achieved by the pure passive control system. This was an improvement of about 2 dB over what could be achieved by the pure active control systems when a single array of control sources was used to generate the control field. However, the performance of the passive control system was matched or exceeded when two control source arrays were used by the active control systems.

The performances of the hybrid control systems were better by an average of 4 dB than the ones achieved by the pure active or pure passive control systems. The addition of a liner to the active control systems also improved the robustness of the optimum solutions.

Finally, the hybrid control system based on the fuselage error sensors technique performed better than the hybrid system that was based on the wavenumber sensors technique. The former was also found to have a more robust optimum solution and to be easier to optimize: with the wavenumber sensors technique three parameters have to be optimized simultaneously (control source array location, error wavenumber and liner
impedance), versus two parameters with the fuselage error sensors technique (control source arrays locations and liner impedance).

Table 6.6: Maximum reduction in sound power level (dB) achieved within the 40 to 155 deg. sector with each of the control systems studied.

<table>
<thead>
<tr>
<th>Control System</th>
<th>Fuselage sensors technique</th>
<th>Wavenumber technique</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>inlet radiation</td>
<td>inlet + outlet radiation</td>
</tr>
<tr>
<td>Pure ANC</td>
<td>3 sensors</td>
<td>7 sensors</td>
</tr>
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