6.0 MODEL APPLICATIONS

6.1 Steady-State Performance

In the steady-state model, the numerical model of the dome is exposed to nighttime conditions. Figure 6.1 shows the model results for the gradient of the temperature on the dome in still air, when it is hypothesized that \( h = 0 \, \text{W/m}^2\text{K} \).

![Temperature gradient on glass dome exposed to nighttime conditions in still air](image)

Figure 6.1 Predicted temperature gradient on glass dome exposed to nighttime conditions in still air

In this case, the dome temperature spread is about 14°C. This magnitude may be reduced if convective heat transfer is increased. The temperature distributions for several cases are shown in Figure 5.2. The temperature below the base temperature is shown as a function of zenith angle on the dome. The \( h \) values for modeling these cases are calculated for uniform laminar flow over the outer surface of the dome. This is a realistic approximation for the Eppley ventilator units, which supply a ring of air around the outer dome.
In addition to affecting the magnitude of the gradient, increased convection also affects the slope of the temperature gradient with respect to the zenith angle. As shown in Figure 6.2, the slope of the temperature gradient for the higher values of convection approaches zero much nearer to the base. The location of a representative temperature for these gradients is closer to the base, because the temperature is almost uniform along the upper portions of the dome. A temperature that will represent the entire dome gradient is found along the portion of the dome with a greater spread of temperatures. This effect is explored in detail in Section 6.3.

Figure 6.2 Comparison of gradients on the dome from base to vertex for varying convective conditions

6.2 Transient Performance

Figures 6.3 through 6.10 show the results of the finite element model compared with experimental data. The temperature deviations of the models are shown along with the comparative plots. The warming model used for comparison calculates convection with a convection heat transfer coefficient of $h = 2 \text{ W/m}^2\text{K}$. The cooling model uses a heat transfer coefficient of $h = 0.8 \text{ W/m}^2\text{K}$.
Figure 6.3 Transient time history of steel rim thermistor (T4) subjected to warming conditions (model and experiment)

Figure 6.4 Temperature difference between steel rim thermistor and warming model results
Figure 6.5 Transient time history of dome base thermistor (T3) subjected to warming conditions (model and experiment)

Figure 6.6 Temperature difference between dome base thermistor and warming model results
Figure 6.7 Transient time history of dome 45-deg thermistor (T2) subjected to warming conditions (model and experiment)

Figure 6.8 Temperature difference between dome 45-deg thermistor and warming model results
Figure 6.9 Transient time history of dome vertex thermistor (T1) subjected to warming conditions (model and experiment)

Figure 6.10 Temperature difference between dome vertex center thermistor and warming model results
Figure 6.11 Transient time history of steel rim thermistor (T4) subjected to cooling conditions (model and experiment)

Figure 6.12 Temperature difference between steel rim thermistor and cooling model results
Figure 6.13 Transient time history of dome base thermistor (T3) subjected to cooling conditions (model and experiment)

Figure 6.14 Temperature difference between dome base thermistor and cooling model results
Figure 6.15 Transient time history of dome 45-deg thermistor (T2) subjected to cooling conditions (model and experiment)

Figure 6.16 Temperature difference between dome 45-deg thermistor and cooling model results
Figure 6.17 Transient time history of dome vertex thermistor (T1) subjected to cooling conditions (model and experiment)

Figure 6.18 Temperature difference between dome vertex thermistor and cooling model results
The model predictions appear to reproduce general trends in experimental measurements as shown in the temperature-difference graphs. These graphs do reveal the shortcomings of the model, however. For instance, in many cases for the cooling results, one set of data is overestimated while the other is underestimated. This is due to the difficulty of describing the actual convective conditions. During the experiment, the ambient conditions were often changed by causes such as duty cycles in the refrigerator or air conditioning cycles in the room. Nevertheless, the accuracy of the model despite these difficulties shows that the results are not overly affected by estimating this parameter. This is especially important for using the model to represent the instrument in outdoor operation. There is a sudden rise in the data minus model graph near the beginning, which is resolved quickly. This may be due to the condensation warming effect described in Chapter 4, which is not accounted for in the model.

6.3 Locating a Representative Temperature

The inside surface of the inner dome of the instrument exchanges thermal radiation with the sensor surface. The model results are used to determine the effect this exchange may have on the incident irradiance of the sensor and, consequently, on the instrument measurement. The inner dome is assumed to have a similar temperature distribution as the gradients determined for the dome model. Several gradients are analyzed to determine an equivalent single temperature for each. If the dome were uniform at this representative temperature, the sensor would exchange the same amount of radiation with the dome as it does with the respective dome temperature gradient. The representative temperature may be used in the place of \( T_d \) in the analytical model described in Chapter 2 to correct instrument measurements. Recall that

\[
E = U_{\text{eng}} \left( \frac{1}{c \alpha_s \tau_d} + \frac{4 \varepsilon_s \sigma (1 - \alpha_s \rho_d)}{S \alpha_s \tau_d} T_b^3 \right) \frac{\varepsilon_s \sigma (1 - \alpha_s \rho_d)}{\alpha_s \tau_d} T_b^4 - \frac{\varepsilon_s \sigma}{\tau_d} T_b^4 \quad (2.7c)
\]

Dome gradients for analysis are selected from model results of the dome exposed to a variety of conditions. The model results were analyzed for the steady-state case exposed to nighttime conditions with winds of 1 m/s, 10 m/s and 40 m/s. The transient model results are selected for analysis from the transient time history at the time corresponding to one time constant. This time was selected in order to maintain consistency in the selection. The time constant is defined in this case as the time at which the average nodal temperature
\[ T_{\text{avg}} = \frac{\sum_{i=1}^{N} T_i}{N} \]  

has a value equal to

\[ T_{\text{avg}} = \left(1 - e^{-1}\right) \times \left(T_{\text{init}} - T_{ss}\right). \]  

The dome is divided into the 17 rings, similar to the finite element model mesh shown in Figure 3.1. Each spans 15 deg in the zenith direction, except the first and last, which spans 7.5 deg. Each ring has a constant temperature, the nodal temperature corresponding to the node located at the center of the middle rings and the edge of the two outer rings. An Excel spreadsheet is set up to calculate the amount of heat resulting from thermal irradiance from the ring on the dome to the sensor surface, utilizing Equations 6.3 through 6.7. Included in these equations is a shape factor, or view factor [Howell], F. This quantity is determined for the geometry from each ring \(i\) to the sensor surface \(s\), as shown in Figure 6.19.

Figure 6.19 Determination of radiation configuration factor for individual rings to the sensor surface

According to Howell,

\[ F_{i,s} = \frac{(\frac{z_i}{a})^2 \frac{(z_i}{a})}{\left(1 - (\frac{z}{a})^2\right)^2 + 4(\frac{z}{a})^2 \left(\frac{z}{a}\right)^2}. \]
This factor permits the heating equation for the irradiance from each ring to be written

\[ q_i'' = A_i F_{d-s} \varepsilon \sigma T_i^4, \quad (6.4) \]

where

- \( A_i \) = Interior surface area of ring \( i \),
- \( \varepsilon \) = emissivity of the dome surface, 0.95,

and \( \sigma \) = Stefan-Boltzmann constant = \( 5.67 \times 10^{-8} \) W/m\(^2\)K\(^4\).

The total irradiance at the sensor surface resulting from the thermal exchanges with the dome is a sum of the individual irradiances from these rings,

\[ Q_{net} = \sum_{i=1}^{17} q_i''. \quad (6.5) \]

The sensor would receive the same amount of radiation from a uniform dome at the representative temperature. This temperature is found by reversing the flux calculation. The shape factor, \( F_{d-s} \) is calculated in a similar manner from the entire dome to the sensor surface, yielding

\[ F_{d-s} = \frac{1}{4} \left\{ \left[ (1 - (\frac{a}{r})^2) + 4(\frac{a}{r})^2 \right]^2 - \left( 1 - (\frac{a}{r})^2 \right) \right\}. \quad (6.6) \]

Then,

\[ T_{rep} = \sqrt[4]{\frac{Q_{net}}{F_{d-s} \sigma \varepsilon A_d}}. \quad (6.7) \]

This representative temperature is located on the existing dome gradient, and this position is the ideal measurement location along this gradient.

The transient warming model reaches a single time constant after 20 min. The average nodal temperature at this time is 285.5 K. The temperature on the dome varies from 282.2 K at the base to 287.1 K at the vertex. The total heat incident to the sensor surface due to thermal radiation from the dome is 0.0359 W. This is the same amount of heat that would be incident if the entire dome were uniform at a temperature of 285.83 K. A temperature close to this value could be found at the node located 40 deg from the base. This node indicates a temperature of 285.8 K, which is a negligible deviation from the calculated value.
The transient cooling model reaches a single time constant after 26 min. The average nodal temperature at this time is 279.6 K. The temperature on the dome varies from 281.0 K at the base to 275.9 K at the vertex. The total heat incident to the sensor surface due to thermal radiation from the dome is 0.0318 W. The representative temperature for this gradient is 277.42 K. Again, the node located at 40 deg from the base most closely matches this temperature. The node indicates a temperature of 277.3 K, a negligible deviation from the calculated temperature.

A summary of the results of the analyses of these gradients, as well as the analysis of the steady-state gradients, is given in Table 6.1.

<table>
<thead>
<tr>
<th>Description of Gradient</th>
<th>Base Temp (K)</th>
<th>Center Temp (K)</th>
<th>Representative Temp (K)</th>
<th>Deviation of 28-deg node (%)</th>
<th>Deviation of 34-deg node (%)</th>
<th>Deviation of 40-deg node (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warming Transient</td>
<td>282.2</td>
<td>287.1</td>
<td>285.8</td>
<td>0.301</td>
<td>0.157</td>
<td>0.013</td>
</tr>
<tr>
<td>Cooling Transient</td>
<td>281.0</td>
<td>275.9</td>
<td>277.4</td>
<td>0.247</td>
<td>0.103</td>
<td>0.042</td>
</tr>
<tr>
<td>Steady-State Natural Convection</td>
<td>273.2</td>
<td>259.2</td>
<td>262.8</td>
<td>0.690</td>
<td>0.283</td>
<td>0.067</td>
</tr>
<tr>
<td>Steady-State 1 m/s Ventilation</td>
<td>273.2</td>
<td>265.0</td>
<td>266.8</td>
<td>0.319</td>
<td>0.098</td>
<td>0.078</td>
</tr>
<tr>
<td>Steady-State 10 m/s Ventilation</td>
<td>273.2</td>
<td>269.6</td>
<td>270.2</td>
<td>0.062</td>
<td>0.015</td>
<td>0.067</td>
</tr>
<tr>
<td>Steady-State 40 m/s Ventilation</td>
<td>273.2</td>
<td>271.2</td>
<td>271.4</td>
<td>0.001</td>
<td>0.031</td>
<td>0.053</td>
</tr>
</tbody>
</table>

Table 6.1 Comparison of gradients and representative temperature locations

A comparison of the results in Table 6.1 reveals that the location of the representative temperature changes with increased convection. For the lower values of convective heat transfer, in which the magnitude of the gradient is larger, the 40-deg node is the closest nodal temperature to the calculated temperature. However, for the gradient corresponding to the ventilation of 10 m/s, it appears that the location of the closest nodal temperature moves closer to
the base of the dome, to the 34-deg node. The location for the 40 m/s gradient moves even
closer to the base, to the 28-deg node. However, due to the fact that the magnitude of these
gradients are smaller than the other cases, the accuracy of the 40-deg node still yields a result
within 1.0 percent of the calculated temperature. Transient case gradients are shown with their
respective representative temperature locations in Figure 6.20.

![Figure 6.20 Individual dome gradients and ideal locations to measure representative temperature](image)

Figure 6.20 Individual dome gradients and ideal locations to measure representative temperature

The finite element model has shown to be useful in describing the thermal state of the
instrument dome. This information builds confidence for the method of estimating the thermal
fluxes using a single representative temperature. The temperature of the 40-deg node is a fair
representation of the dome gradient for a variety of cases.

The results described in this chapter will be summarized in Chapter 7. In addition,
suggestions will be given for the development of the finite element model.