Voltage Stability and Control in Autonomous Electric Power Systems with Variable Frequency

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Dissertation submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in

Electrical Engineering

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September 21, 2007
Blacksburg, Virginia

Keywords: voltage stability, large-signal stability, lyapunov methods, synchronous generator excitation, stand-alone power systems, multi-pulse transformer rectifier
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Abstract

This work focuses on the safe and stable operation of an autonomous power system interconnecting an AC source with various types of power electronic loads. The stability of these systems is a challenge due to the inherent nonlinearity of the circuits involved. Traditionally, the stability analysis in this type of power systems has been approached by means of small-signal methodology derived from the Nyquist stability criterion. The small-signal analysis combined with physical insight and the adoption of safety margins is sufficient, in many cases, to achieve a stable operation with an acceptable system performance. Nonetheless, in many cases, the margins adopted result in conservative measures and consequent system over designs.

This work studies the system stability under large-perturbations by means of three different tools, namely parameter space mapping, energy functions, and time domain simulations. The developed parameters space mapping determines the region of the state and parameter space where the system operation is locally stable. In this way stability margins in terms of physical parameters can be established. Moreover, the boundaries of the identified stability region represent bifurcations of the system where typical nonlinear behavior appears. The second approach, based on the Lyapunov direct method, attempts to determine the region of attraction of an equilibrium point, defined by an operation condition. For this a Lyapunov function based on linear matrix inequalities was constructed and tested on a simplified autonomous system model. In addition, the third approach simulates the system behavior on a computer using a detailed system model. The higher level of model detail allows identifying unstable behavior difficult to observe when simpler models are used.

Because the stability of the autonomous power system is strongly associated with the characteristics of the energy source, an improved voltage controller for the generator is also presented. The generator of an autonomous power system must provide a good performance
under a wide variety of regimes. Under these conditions a model based controller is a good solution because it naturally adapts to the changing requirements. To this extent a controller based on the model of a variable frequency synchronous generator has been developed and tested. The results obtained show a considerable improvement performance when compared to previous practices.
Acknowledgements

Many people have helped me during my PhD study time, to all of them I would like to express my gratitude.

Special thanks go to my advisors, Dr. Dushan Boroyevich and Dr. Fred Wang, for their support, guidance, patience, and encouragement during my study. I have certainly learned a lot from both of them. I am grateful to Dr. Boroyevich for giving me the opportunity to start my PhD at CPES, and having a positive attitude about almost everything. I have always admired how good he is at teaching, expressing ideas, and communicating in general. Dr. Wang has been extremely helpful during my years of PhD studies; I am very indebted to him. I have always appreciated that, in spite his busy schedule, he always found some time to talk with me and discuss the issues that research was bringing. We have also had very good discussions about the engineering profession. I value his practical orientation and admire his multitasking abilities.

Dr. Rolando Burgos has helped me in many aspects related to this dissertation. I am very thankful for all that help, the ideas he has given me, the papers we wrote, and his tireless efforts to reach the best possible results. Dr. Yilu Liu has always helped me a lot, from when I started my graduate studies at Virginia Tech until when I was close to graduation and interviewing for jobs. I also want to express my deep gratitude to Dr. Virgilio Centeno who has helped me in many aspects that go beyond the academic environment. His advice, support, and encouragement have been extremely valuable for me during all this years of graduate studies. I also want to thank Dr. Dan VanWyk and Dr. Werner Kohler for being members of my advisory committee during my studies.

The staff at CPES has been of great support; special thanks to Marianne Hawthorne, Theresa Shaw, Trish Rose, Linda Gallagher, Bob Martin, Dan Huff, Beth Tranter, and Jamie Evans.

Some colleagues have also being of great help through not only technically helping me, but also their friendship and encouragement. Special thanks to Francisco Canales who is a good

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1 This work has been supported by The Boeing Company, the Office of Naval Research, and Thales Avionics Electrical Systems. This work made use of ERC Shared Facilities supported by the National Science Foundation under Award Number EEC-9731677.
friend and helped me a lot in the times where I open my eyes to power electronics. Thanks also to Jerry Francis who was a great support when we did the generator controller tests in France. Other colleagues came to visit the CPES lab and enriched my graduate student life; many thanks to Pedro Rodriguez, Josep Pou, Luisa Coppola, Luca Solero, and Marcelo Cavalcante.

In some ways the burden of long working hours was ameliorated by having the companionship of some very good people. I apologize if I forget someone, but I cannot avoid mentioning Sara Ahmed, Luis Arnedo, Siriroj Sirisukprasert, Carlos Cuadros, David Lugo, Bin Huang, Molly Zhu, Peter Barbosa, Xiangfei Ma, Chong Han, Xin Zhu, Wei Shen, Yan Jiang, Daniel Ghizoni, Tim Thacker, Rixin Lai, Hongfang Wang, Hongang Shen, Di Zhang, Callaway Cass, Puqi Ning, Kisun Lee, Carl Tinsley, Qian Liu, Michele Lim, and Parag Kshirsagar.

The number one local supporters that I had during these years have been Valarie and Edwin Robinson; I owe them a lot. In Blacksburg I have also met some wise people that enriched my life; thanks to Charless Aull, Fr. Richard Mooney, and Fr. John Prinelli. Moreover, many other people outside the lab and the school room gave my invaluable support, thanks to all of them, especially to Guy, David⁹, Tatiana, Yovanna, Hugo, Fulvia, Oscar, Cecilia, Jose Manuel, Adrian, Patricio, Lucho, David, Peter, Sarah, Alejandra, Dione, Sandra, Christine², Anand, Greg, Marcos, Rosanna, Christian, Alexandra, and many others.

I could not have finished this enterprise without the loving support under all circumstances of my wonderful wife Cecilia. She was always been there to help me when I needed, share my emotions when things did or did not work, and give me much more than a little push when my forces were weakening. I am certainly blessed for being able to share my life with her.

To my parents Jorge and Montse I am not only indebted for being a constant source of blessings, but most of all for giving me the most important thing that someone can give, which is his/her unconditional love. From them I have inherited the passion for knowledge. My siblings Moia, Javier, Damian and their respective families are always a big source of joy.

Most of all I would like to thank God.
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Chapter 1  Introduction

I. Scope of this Work

This research is focused on stability assessment in autonomous electric power systems. There are many uses of electric power that require energy from a source not connected to a network. Among these autonomous systems a special group corresponds to vehicular power systems like ships, aircrafts or automobiles. The use of electricity in vehicles is experiencing an increasing demand due to initiatives like the all electric ship, the more electric aircraft, or electric hybrid vehicles [1]-[2]. Increased power demand and higher performance requirements on the user side requires new power systems that bring new issues that must be analyzed. Fig. 1-1 shows schematically a generic architecture of this type of power system. There can be several generators in the system, but in standard operation mode a single generator provides energy to a set of varied loads with many of them processing the energy through power electronic converters [3]. The generator location, close to the prime mover, is remote to the main power consumptions; the interconnection is done by means of one or more feeders. In addition, the excitation controller regulates the voltage at the selected point of regulation that is usually chosen at the AC bus.

The studied distribution is based on a hybrid AC/DC architecture. The AC distribution supplies energy to three-phase loads, like motor drives (MD), or single-phase loads. This AC distribution can be at different voltage levels by means of transformers. Regulated loads of relatively low power consumption are connected through single-phase active rectifiers (1phAR). The MDs are usually fed from a DC bus. This DC voltage is obtained by means of rectifying the AC voltage. Both passive and active rectifier types are used in autonomous power systems. The first type generally corresponds to multi-pulse transformer rectifiers (MPTR). The three-phase active rectifiers (3phAR), on the other side are pulse-width modulated (PWM) converters. In addition, a single rectifier can provide more than one load connected to its DC output.

In most cases one of the loads, usually an MD, consumes a large percentage of the total generated power. In the case of aircrafts this large load corresponds to air blowers for cabin pressurization [4], in a ship integrated power system to the propulsion motors [5], and in electric
hybrid terrestrial vehicles to the driving motor [6]. Moreover, in some vehicular applications, it is common to use a variable frequency constant voltage AC source because of its reduced maintenance costs [7]. The frequency variation range can be greater than 2:1.

Fig. 1-1 Basic architecture and main components of an autonomous electric power system

The increase of the power consumption levels and use of new power conversion technologies, like power electronics, creates new requirements that must be studied [9]. One of such important aspects is the integrity of the system, which is related to its stability. More complex load dynamics require sophisticated controllers increasing the possibility of achieving unstable or close to unstable conditions. In addition, the wide operation range, which in part is due to the variable frequency operation, and the associated change in the system dynamics increase the requirements over the voltage regulation. The present work focuses on the stability analysis of autonomous power systems with variable frequency. More specifically, the focus is on the large-signal stability of such systems. Because of the close relation between the system stability and the voltage regulating performance of the source, this work also develops an improved voltage controller for the variable frequency synchronous generator.

II. Stability of an Electric Power System

The problem of stability of a system is referred to its ability to reach and remain at the desired operation condition, which is mathematically an equilibrium state of the system. More
formally, the behavior of a nonlinear system can be described by the following differential equation:

\[ \dot{x} = f(x, p) \]  

(1-1)

In the previous equation \( x \) represents the vector of state variables (or states), which is generically \( n \)-dimensional. Moreover, \( p \) is the \( q \)-dimensional vector of parameters. The dynamical system represented by equation (1-1) is unforced in the sense that there is no explicit input driving the trajectory of the system. The states that satisfy the condition,

\[ 0 = f(x, p) \]  

(1-2)

are called stationary states. The null derivative means that there is no system motion; therefore, a stationary state is an invariant set in the sense that no system trajectory can emerge from it unless an external perturbation modifies the condition. The point of the state space identifying this condition is called equilibrium point and to characterize it both the state and parameter values are required \( (x_0, p_0) \). Such point is then defined as a state that satisfies the condition in (1-2). The study of the system stability is closely related to the behavior of the system relative to its equilibrium points.

The stability of a dynamical system is usually classified in static, small- and large-signal. The static, or steady-state, stability refers to the existence of the equilibrium point. It could happen that for a given set of parameters \( p \), equation (1-2) has no solution. In the case that the equilibrium point has been determined, if the trajectories stay in a neighborhood around the point it is called a stable equilibrium point (SEP); on the contrary if the trajectories diverge it is referred as unstable equilibrium point (UEP). The small-signal stability analysis refers to the system behavior in the proximity of an equilibrium point when there is no significant change in the set of equations or parameters governing the system. On the contrary, the large-signal analysis studies the transition between an operation condition and a different equilibrium state. In this way, large-signal stability is related to the event that creates the transition. The difference between small- and large-signal stability requires different analytical tools to approach each of them.

The simulation results shown in Fig. 1-3 illustrate the small and large signal stability phenomena. The variables plotted in Fig. 1-3 correspond to the circuit schematic in Fig. 1-2. The
load is of a controlled type with the power consumption being regulated at the desired power level. Therefore, a circuit voltage $v_{dc}$ variation produces a change in the current $i_{dc}$ in order to keep $P_{load}$ constant.

![Diagram of electric circuit with voltage dependent dynamic load](image)

**Fig. 1-2** Schematic of electric circuit with voltage dependent dynamic load

The circuit responses for different steps in the load power level are shown in Fig. 1-3. While in Fig. 1-3 (a) the unstable condition is achieved after a succession of small-step load increments, in Fig. 1-3 (b) the instability is created by a sudden load connection. The power level of the load that drives the system unstable is smaller in the sudden connection than in the step-wise case. It is also worth mentioning that there exists also a third possibility regarding the stability of a system. This is given by an equilibrium state that vanishes or does not exist, called static stability.

![Graphs illustrating small and large signal stability phenomena](image)

**Fig. 1-3** Small and large signal stability phenomena illustration (a) instability after successive small load increments. (b) instability created by a sudden load step

In the large-signal analysis two types of events must be considered. In the first type, the operating conditions change because of a parameter variation while the structure of the system is preserved. In the second type the operating conditions change due to a change of the system structure. In electrical circuits, an example of the first event type corresponds to a load variation...
while an example of the second event type corresponds to the connection or disconnection of circuit branches. This last case can be created by faults or by the action of protection schemes.

A. Mathematical Stability Definitions

In the autonomous power system under analysis the primary energy comes from the generator prime mover. Its mechanical speed and torque define the input power of the electric system, and also its input variables. The analysis in this work considers a prime mover with a power capacity many times larger than the electric generator power. Because of this characteristic, the speed can be considered independent from the torque, and the mechanical power at the generator automatically matches the electrical power demand. Moreover, the speed changes are slow compared to the electrical variable transients. Therefore, the speed can be considered as a quasi-stationary variable in the electrical analysis. Under these conditions it is possible to apply a coordinate transformation, like the Park transformation [10], to refer the AC variables to a rotating reference frame. This is a standard procedure in the dynamic analysis of three-phase electric systems [35],[62]. If the frequency of the rotating frame and the AC system are the same, the transformed sinusoidal variables become constant and the equilibrium trajectories, equilibrium points. In addition, if the speed is assumed constant during certain time frame under analysis, the power system model can be considered to be unforced.

The definitions given below are standard and can be found in the literature on nonlinear control systems, like [11]-[12]. The equilibrium point of interest is considered to be located at the coordinate origin; if this is not the case it can be easily translated by means of a coordinate translation. Moreover, if the system in (1-1) is considered time invariant the set of parameters $p$ are constant along a system trajectory and can be considered equation coefficients.

**Stability in the sense of Lyapunov**

For the system defined by (1-1) with equilibrium at the origin and fixed parameters $p$, such equilibrium point $x_0 = 0$ is stable if, for each $\epsilon > 0$, there exist $\delta = \delta(\epsilon, t_0)$ such that:

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| < \epsilon, \forall t \geq t_0 \geq 0$$

The situation referred in this definition is depicted in Fig. 1-4. The scheme in the right of the Figure gives a possible time domain evolution that does not necessarily converge to the equilibrium point, as stated in the definition.
Asymptotic stability

For the same system of the previous definition, the equilibrium point $x_0 = 0$ is asymptotically stable if it is stable and there exist $\eta(t_0)$, such that $\|x(t)\| < \eta(t_0) \Rightarrow x(t) \to 0$, when $t \to \infty$

![Fig. 1-4 Stability in the sense of Lyapunov](image)

Exponential stability

The system is exponentially stable if there exist $\delta > 0$, $\epsilon > 0$, and $\alpha > 0$, such that $\|x(t)\| < \delta \Rightarrow \|x(t)\| < \epsilon \|x(t_0)\| e^{-\alpha(t-t_0)}$, $t > t_0$

In practical applications, generally but certainly not always, the interest is to determine the asymptotic stability of the equilibrium point. The concept of exponential stability is also related because trajectories taking long times to converge or presenting large deviations after a relatively long period of time will probably violate some of the system operation constraints. In many cases these constraints can be approximated by an exponential characteristic, e.g. a protective device time-current characteristic.

B. Practical Stability Definition

From a technical point of view, the stability of an electric power system can be synthesized in the following definition taken from [13]:

“An equilibrium point of a power system is stable if, when the initial state is in the given starting set, the system motion converges to the equilibrium set, and operation constraints are satisfied for all relevant variables along the entire trajectory”
In addition to the stable operation at the required condition this definition points out the importance of keeping the integrity of the system during the trajectory originated by the system transition from the initial to the final state.

III. Literature Review

The stability analysis of electrical systems has been object of many research efforts. In power systems the stability classification in steady-state, small-, and large-signal type, receive particularly the names of static, dynamic, and transient respectively [14]. Most of the early works were focused on the synchronous generator and its controls because interconnection was rare in early power systems [15],[16]. The nature of those power systems required analyzing the two major aspects of synchronous generator stable behavior: the angle (or synchronization) stability and the voltage stability. Static and small-signal disturbance were the main objects of discussion. Transient stability gained relevance when power system became interconnected. Moreover, the establishment of the equal area criterion [17] increased the understanding of the phenomena. This criterion set the basis for the analysis of transient stability by means of balancing the energy in the generator. Although having suffered many reformulations and extensions, a now more sophisticated dynamic equal area criterion constitutes the basis of an energy based methodology for transient stability analysis of power systems [18],[19].

Because of the complexity of the stability phenomena, like in the synchronous generator case, static analysis is a first step to approach a system stability study that provides useful understanding of the system behavior. In power systems, the load flow Jacobian, which is a non-linear static model of the network, was first used to determine the steady-state stability of the system [20]. In this way the vanishing of equilibrium points was one of the first identified mechanisms producing voltage collapse in interconnected power systems [21]. In large non-linear systems static stability analysis is non-trivial and has been object of various researches [22]. A system that is originally operational can lose its stationary operation by a series of small events that take the system to a condition where it is not operable anymore. Stability indexes attempting to measure how far the system is from achieving such inoperable condition have also been proposed [23].
The next step increasing the depth of system stability study is based on small-signal analysis tools. The small-signal analysis uses linearized models around the equilibrium point of interest. This allows using a wide variety of analytical tools that can be applied, which generally will produce good results in their domain of application. In large power systems, small-signal based analysis is commonly used in voltage stability analysis. Using singular perturbations theory the load-flow Jacobian has been applied to determine some types of dynamic, or small-signal, stability cases [21]. Later the introduction of some of the dynamic behavior in the power system model allowed the development of point of collapse methodology to study the voltage stability based on the eigenvalues of the system state matrix [24]. These type of methodologies based on the system matrix eigenvalues applied to power networks are still object of research [25].

In the dynamic of the voltage the behavior of the load assumes a prime importance; therefore, voltage stability has been sometimes also called load stability [26]. The use of bifurcation analysis also added understanding of the voltage stability mechanisms [27]. Other approaches focused on the similarities and differences of the angle/voltage stability phenomena in an attempt to decouple them [28].

In power electronics DC systems, the small-signal stability analysis is usually done through the so-called impedance criteria that was derived based on the relationship between the source output impedance and the load input impedance. Based on the relationship between the source output impedance and load input impedance [29] proposed one of the first criterions used to evaluate system stability. Following the same idea of comparing the impedances several other criteria have been proposed. These analysis techniques have also been extended to three-phase AC systems, where through the Park transformation all AC variables are transformed into constant magnitudes in the rotating reference frame [30]. The application of those tools is still under development [31]. Like in the power system voltage stability the characteristics of the load play a major role in power electronics system small- and large-perturbation stability [32]. For this reason the constant power characteristic of regulated loads have been studied, and models of those loads proposed.

Small-signal analysis combined with physical insight and the adoption of safety margins is sufficient in many cases to achieve a stable operation with a good overall system performance. Nevertheless, due to the intrinsic nonlinearity of power systems, the small-signal stability
analysis does not ensure stability in the large-signal sense as pointed out in [33]. Therefore, in many cases, the margins adopted with the small-signal approach result in very conservative measures and consequent system over designs. Furthermore, increasing requirements in terms of system performance including higher power quality standards demand better and more complete stability studies.

The study of transient stability in a power interconnected system has been object of many research efforts. In the transient stability analysis, the generator rotor dynamics play a fundamental role. Large signal voltage stability is also intrinsically linked to the rotor motion. The system under study has a major difference with those mentioned studies given by the absence of mechanical dynamics. Neglecting the mechanical dynamics is based on a prime mover power capacity that is several times bigger than the synchronous generator rated power. In this way, the rotational speed is controlled independently of the electric circuit circumstances and in many cases can be assumed constant for the stability analysis. This type of behavior is typical in aircraft power systems where the turbine providing the propulsion is also the prime mover. Under these circumstances the stability is mainly determined by the electrical interactions between generator and loads and the associated controllers play a major role. In other systems where the large power difference between mechanical prime mover and generator is not verified it is still possible that the voltage and angle stability are highly decoupled and the voltage stability depends mostly on the electrical dynamics.

Traditionally, and still nowadays, system stability and especially large-signal stability is studied with intensive use of computer simulations. These simulations involve integration of the system dynamic equations (1-1) in time domain. This is in principle a non-efficient approach in terms of use of the computational resources; however, can produce results that are as good as the models implemented in the computer. Therefore, some literature has been devoted to the modeling and implementation of stability studies in computers [35],[36]. There are numerous works that report intensive use of time domain simulations as basis for the stability analysis [37]. Some of those works focus on vehicular power system applications [38]. The work presented in [39] summarizes different methodologies for stability studies in a ship power system that includes small- and large-signal including time domain simulations.
The nature of the transient stability phenomena made it attractive for the application of Lyapunov’s direct, or second, method. This method is attractive for large-signal stability analysis because it can identify a region around the stable equilibrium point where the stable behavior is guaranteed. The details of the direct method can be found in chapter 5 section I. Work using Lyapunov’s direct method for transient stability analysis started in the Soviet Union in the 1940’s [40]. By using simplified generator and system models this early work set up the basis of energy based criterion for transient stability limit. Later on, the Lyapunov direct method became more widely used and the efforts concentrated on two areas: the development of the Lyapunov function and the determination of the critical energy value. Important results related to the Lyapunov function construction are summarized and discussed in [42]. Achievements in the search of the critical energy came with the development of the potential energy boundary surface (PEBS) and the closest UEP methods [43],[44]. A good summary of the transient energy function (TEF) method is presented in [45]. Despite the mentioned developments, the TEF method presented some drawbacks mainly related to the simplicity of the models that could be handled. The synchronous machines were represented in a very simplified with basically neither excitation nor speed controller. In addition, the transmission network was not allowed to have losses. Improvements in the transmission network model came after some time with the development of the structure preserving model [46] and the Lyapunov functions based on it [47]. Other model characteristics, like the generator excitation controller, could only be included in the TEF method only in an approximate manner [45].

Because of the voltage and angle dynamic link in a typical power network and the availability of Lyapunov functions developed for studying the transient stability problem some early work based on energy functions for voltage stability analysis used similar functions with an additional simple load dynamical model. This model included the load reactive demand originally neglected in the TEF method [48]. Improvements of the approach included power electronics controllers [49]. However, as it is pointed out by the same authors, the energy functions used did not meet the Lyapunov criteria strictly speaking. Therefore, there was no guarantee on the result accuracy. The work concentrated on the analysis of the dynamic aspects of voltage stability showing the effects of simplified modeling usually assumed in the angle stability study and the importance of the algebraic constraints of the system model [48].
More recent work focused on overcoming the main drawbacks that the use of direct methods still has. One of these main issues is related to the very simplified models of the system components. Specifically this was related to voltage regulation controllers and loads. Both these elements play a key role in voltage stability analysis. Lyapunov functions that allow for better modeling representation of loads are presented in [50],[51]. The goal of these studies is to improve the model of the reactive part of the load, which has a considerable influence in voltage stability. The use of improved voltage excitation control models in direct methods is discussed in [52] where the problem is transformed to a Lur’e formulation and a Lyapunov function of that type can be used. Other approaches concentrate on a better network representation [53]. In addition to component modeling; another aspect to improve is the representation of limits in the energy function. The work in [54] presents a contribution in this last direction where the geometrical properties of the region of attraction boundary are studied in order obtain the critical energy estimate.

The work using Lyapunov’s direct method to study the large-signal stability analysis in power systems started including power electronic converters in AC/DC systems using Lyapunov functions derived from previously AC system analysis [49]. The problem that can appear following this approach in an autonomous power system is the considerable different voltage dynamic behavior. In that sense, approaches developed that accurately model the generator excitation control, like [52] seemed more appropriate. This approach constructs the Lyapunov function based on the solution of a linear matrix inequalities (LMI) problem. The advantage of the LMI approach is that it can be formulated as a convex optimization problem and then solved in a computer [55]. Other uses of LMI applied to classical power system stability models have been reported. In the work presented in [53] LMI is also used to construct a Lyapunov function of Lur’e type for an improved network model. In this last study the approach is also linked to the controlling UEP approach.

More specifically in power electronics system, the work in [56] proposes using a quadratic Lyapunov functions to study the large signal stability of polytopic models of the original system. Although innovative the method produces stability predictions that are quite conservative. In [57] several methods are tested to generate quadratic Lyapunov functions that are then used for the estimate the stability of a DC power electronic system. Among the methods tested, the one based on linear matrix inequalities produces the best estimates of the system stability in most
cases. The construction of a Lyapunov function based on LMI does not solve the problem of finding the critical energy, which is the value of the energy function that defines the boundary of the region where the system is stable. In order to calculate this value, the work in [58] proposes to use optimization algorithms based on genetic algorithms.

IV. Motivation and Objectives

New autonomous power systems with new requirements require stability studies to assess a safe operation with good performance. The new equipment used in those systems, with high use of power electronics and associated controls has changed the dynamic behavior of the system. The most important of those changes came from the load “constant power” type behavior. Therefore, the stability of autonomous power systems requires new, deeper studies to understand the behavior and help define the conditions for a safe operation. The novelty of the problem requires the development of new tools to complement existing ones in order to gain better understanding of the stability behavior. In order to achieve the goal, proper models of the system components and their controllers are required.

An important aspect of the stability analysis is related to ability of the system to withstand or recover from large perturbations, which can be created by sudden changes in the operation or faults. Therefore, this work primarily focuses on the large-signal aspect of the stability analysis. Large-signal analysis of power system has been attempted many times, but still time domain simulations continue to be the most used practice, especially when detailed models are required. The simulation approach is highly conditioned by the type of study and system. Therefore, it is necessary to develop tools that systematically approach the analysis of the autonomous power system large-signal stability.

Analyzing some of the small signal stability characteristics is a good and necessary step to gain understanding of the whole system stability behavior. Small-signal analysis allows studying the stability behavior in the proximity of the equilibrium points determining if they will be stable or unstable. Although the analysis is local it can be repeatedly applied at different operation conditions dictated by changes in the system parameters. In this way the impact of critical system parameters on the stability can be studied. There exist many techniques to analyze the small-signal stability of power systems; however, using those tools to asses the impact of the system
parameters is not straightforward. Therefore, a tool that systematically and quickly can evaluate that impact is developed in this work.

After the small-signal study has been completed it is possible to approach the large-signal stability analysis with more confidence. The objective of the large signal study is to determine if the stationary operation condition after the large perturbation will be achieved safely. This will occur if the initial conditions of the post–event trajectory lie inside a region where that trajectory convergence is guaranteed. This region is called the region of attraction of an equilibrium point. Therefore, the objective of the large signal study is to find the region of attraction around the equilibrium points of the autonomous power system. There are different ways to approach the region of attraction search. The most widely accepted are based on Lyapunov’s direct method of stability analysis. The main challenge is to overcome the conservativeness of the results usually obtained with the Lyapunov direct method. The availability to solve systems of LMIs with the help of optimization algorithms allows approaching the problem in a new way that also allows accounting for the CPL nonlinearities.

Because the stability of the system is challenged by the control characteristics of the loads, adjustments and improvements on the source side are required to keep a stable behavior. This work also focuses on the voltage control characteristics of the generator. With the objective of improving the performance of the voltage regulation compared to existing practices a generator excitation controller is developed and tested.

V. Dissertation Outline and Summary of Contributions

Chapter 2 of the thesis discusses the modeling aspects of the major system components. These components are the synchronous generator, the active and passive rectifiers, and the loads. The necessity of developing a simple, yet accurate model, of the multi-pulse transformer rectifier (MPTR) was detected during the study. This new model is presented in this chapter.

Chapter 3 analyzes the nonlinear behavior of the power system under study. Because large-signal stability is closely related to the system nonlinearities it is relevant to analyze the nonlinear behavior of the equipment components individually and when they interact in the system. In this chapter several cases are analyzed and new behavior is identified and explained.
The system small-signal stability boundaries are analyzed through a mapping methodology developed in chapter 4. This methodology allows quantifying the impact of the system parameters on the local stability of the equilibrium points. Several scenarios are analyzed using the developed methodology allowing extracting conclusions on the criticality of some parameters for stable behavior.

Chapters 5 and 6 are devoted to the large-signal analysis. Chapter 5 approaches the region of attraction estimation by means of the Lyapunov direct method. To construct the Lyapunov function, linear matrix inequalities are used. Based on a Lur’e type model of the system a Lyapunov function is proposed to estimate the region of attraction in a system with constant power loads. This function is based on a simplified system model. Chapter 6 studies the large-signal stability by means of time domain simulations. The simulation study allows identifying and quantifying some stability aspects in a more accurate way. Some aspects of the pure large-signal behavior that are very difficult to analyze with the Lyapunov approach are also discussed.

Finally, chapter 7 develops a new generator voltage controller for a synchronous generator with variable frequency. Good generator voltage regulation is required for a good system voltage stability behavior. The controller development is based on a more complete generator model that considers a three-stage machine with brushless excitation. The controller was experimentally tested showing considerably better performance when compared to previous practices for the same type of applications.
Chapter 2  Modeling of the Autonomous Power System

I. Introduction - Modeling Review

The autonomous power system combines different types of equipment. When those pieces of equipment are interconnected they start interacting with each other. To assess the behavior of the system these interactions must be studied. The objective of this chapter is to present the power system under study and provide a description of the models of the main components. Fig. 2-1 shows the circuit schematic of the system under analysis.

![Fig. 2-1 Schematic of the complete system under analysis](image)

The previous figure shows the one-line diagram of the autonomous power system under study where a synchronous machine feeds several AC and DC loads. Alternatively all loads can be transferred to a second generator, but who do not operate simultaneously. The generator voltage regulator, which is represented by the block \( V_{\text{reg}} \) in Fig. 2-1, controls the voltage at the AC bus. Therefore this bus is referred as point of regulation (POR). Several branches are connected to that bus, most of them feeding motor drives (MD). Two of the branches are connected through three-phase active rectifiers (3phAR) while there are two MDs connected
through a multi-pulse rectifier (MPTR). The branch connected to the multi-pulse rectifier concentrates the largest power consumption; therefore, it receives particular attention in the analysis and discussion. In addition, there is a branch connecting single-phase active rectifiers (1phAR) of relatively low power consumption at a lower voltage level.

In order to achieve good results in an efficient manner a proper balance between model degree of detail and fidelity must be achieved [59]-[61]. Therefore, it is good to review and analyze the characteristics that models must have. Because there are several models available for a particular piece of equipment it is necessary to know which one is appropriate for the analysis. Moreover, a new model for the multi-pulse rectifier; was developed and is presented in this chapter.

II. Individual Equipment Modeling

The object of this section is to review and discuss the models of the system components. For that the models available in the literature are reviewed. The discussion focuses on models appropriate for stability studies.

A. Power Source Modeling

The electric power source is typically a synchronous generator. There exists a wide variety of synchronous machine models of different complexity levels employed in simulation studies [61]-[64]. The most common representation uses a reference frame rotating at synchronous speed and aligned with the generator rotor, which is called the $d$-$q$-$0$ reference frame. The stator and rotor damper windings are represented through their equivalents in the $d$ and $q$ axis. More than one damper winding can be considered in each axis giving for better representation during transients [61]; however, in many cases one equivalent damper winding provides an acceptable level of accuracy [35]. In case of balanced operation, or symmetrical faults, the $d$-$q$-$0$ models can be simplified to the $d$-$q$ components only.

There are several ways to reduce the order of a synchronous generator model. Several of them have been discussed in [62]. In low damped machines for instance the damper winding effect can be neglected. The model of a synchronous generator without damper is discussed in detail in Chapter 3, section II-A. A more accurate way to reduce the order of the model is to
assume instantaneous flux change in the stator windings while keeping the damper effect in the model, as discussed in [61]. Neglecting the stator dynamic, as well as the dynamic of the electrical network connected to the generator, is a regular practice in power system stability studies [35]. In addition to the mentioned stator dynamic simplification, the sub-transient generator model decouples the damper and field winding assuming a large difference between their time constants. However, the need for fast voltage regulation may make this assumption inaccurate. In such cases an alternative subtransient model that does not make such assumption can produce good results as described in [62][31]. In some cases, the damper effect in the $d$-axis can be neglected with minimum changes in the results. Further simplification of the generator damper effect produces a first order model that mainly represents the dynamics of the excitation circuit. This simplification, generically called the transient generator model, is preferred when simplicity is prioritized over accuracy. Finally, the simplest synchronous generator model considers a constant back-emf in series with the synchronous impedance.

In the autonomous power system under study the mechanical dynamics can be neglected because the prime mover has a power capacity several times larger than the synchronous generator power, as discussed in chapter 1, section II. More relevant for our study is the excitation controller that regulates the RMS value of the AC voltage at the POR, which in $d$-$q$ coordinates is calculated as the quadratic mean of the components $v_d$ and $v_q$.

$$\bar{v}_{rms} = \sqrt{\frac{1}{2} (v_d^2 + v_q^2)}$$  \hspace{1cm} (2-1)

The voltage regulation action is achieved by an external feedback loop $C_v(s)$. The characteristics of $C_v(s)$ corresponds to a proportional-integral (PI) type transfer function. Additionally, the excitation scheme includes an exciter and an excitation control loop $C_{ex}(s)$ that controls the excitation current $i_{ex}$. A generic excitation control scheme is illustrated in Fig. 2-2. The action of the two control loops produces the excitation voltage, $v_{ex}$ that is fed to the exciter. The exciter is directly connected to the generator field winding as shown in Fig. 2-2. The excitation current, which closes the excitation current loop, is measured at this interconnection. The exciter current is limited to take only positive values. For simplified analyses, the exciter and its controls may be simplified or even neglected. That simplification is compatible with the generator model without damper windings or the transient model. The details of the generator and controller models are given in chapter 3 sections II-A and -C.
B. Voltage Source PWM Converter Modeling

A three-phase AC/DC converter can be divided for modeling purposes in its power stage and controller. The main components of the converter are schematically shown in Fig. 2-3. Moreover, in case of a dedicated converter, it may be convenient to include the load in the model. The power stage model includes the semiconductor bridge and the passive components.

The switching model of the bridge can be easily implemented in a circuit simulator; ideal switches and diodes provide an appropriate level of accuracy for stability analysis. However, when simulating the complete system numerical convergence may become an issue due to the large amount of components in the circuit. For those cases a switching function modeling of the bridge may help to improve the convergence and also speed up the simulations. Switching function implementation for a three-phase converter is described in [65]. The advantage of this type of model is the possibility of smoothing the switching action by means of a first order low-pass filter. The cut-off frequency of this filter depends on the switching frequency of the converter and the desired accuracy. The space vector modulator (SVM) can be included in most
circuit simulators as an own logic defined block, which is better for flexibility. In the average representation the voltage source converter (VSC) and modulator are integrated in one stage. In many cases a model representation in the $d$-$q$ reference frame is preferred. The average model of the VSC is presented in chapter 3, section III-A.

The controller is fully implemented in both switching and average models. In the case of the motor drive a vector control strategy is used to control the motor speed comprised of a $d$-$q$ axes current regulator featuring PI controllers with decoupling terms and anti-windup loops, and an outer speed-loop PI regulator with anti-windup and a pre-filter compensator for optimum transient response. This last loop sets the reference for the $q$-axis control-loop that generates the torque current component [66]. For further simplification or reduction of a dedicated converter it is convenient to integrate it with the load. The model of the controller without antiwindups and limiting functions is described together with the VSC average model in chapter 3, section III-A. In addition, the complete model of the controller including antiwindups schemes is described in chapter 3, section III-C.

C. Load Modeling

The permanent magnet synchronous motor (PMM) has numerous advantages over other motor types with increasing number of applications including autonomous power systems. Therefore, the motor connected to the VSC in the power system under study is a PMM. The model of this motor is described together with the VSC in chapter 3, section III-A. That model is of second order and does not include the rotor damping effect, although this effect could be easily added depending on the machine characteristics. The model of the PMM can be reduced to a first order model in a similar way than the synchronous generator model. In such case the dynamics of the reduced order model correspond to the direction of the permanent magnet flux. In case more accuracy is required, magnetic saturation can be included in the model [68].

The control characteristics of the VSC controller gives the VSC connected to the PMM a dynamic behavior characterized by a constant power taken from the electric circuit. Therefore, in such case the load is referred as constant power load (CPL). Behavioral models of constant power loads are used in stability studies to represent loads like the VSC-PMM set. In this study CPL models are used in chapter 5; those models are described in that chapter, section III-B.
D. Multi-pulse Transformer-Rectifier Model

Multi-pulse transformer rectifiers (MPTR) are composed of a transformers or autotransformers and one or more sets of six-pulse diode bridges. The number of bridges is given by the number of phases in the secondary side of the transformer, with a minimum of three phases corresponding to one diode bridge. In order to reduce the output voltage ripple the number of phases in the secondary side is increased by an appropriate interconnection of windings. Several different approaches have been proposed to model rectifier bridges and or rectifiers directly connected to transformers. A first classification divides these models in switching and average models. Switching models having different degree of detail are used for detailed cases. For our study we prefer to use the average modeling approach. These average models can be broadly divided in two groups. The first type corresponds to behavioral models or “black box” approach. Models of this type are described in [69]-[71]. Some of these approaches model only the rectifier [69] while others also include the transformer [70]. In any case, interphase interactions of the magnetic flux are neglected. The rectifier model obtained by a first order approximation of the waveforms is static and uses parameters to relate input-output magnitudes identified at detailed models. The range of model validity is limited, but can be extended using parameter look up tables. An advantage of these models is its relative simplicity.

On the other side, a detailed model is obtained by a closer analysis of the physical phenomena, or “white-box” approach. Models of rectifiers connected to transformers or synchronous generators have been proposed in [72]-[76]. These models can represent the rectifier behavior quite accurately over a wide range. On the other side, they have the drawbacks that are not self-sustained, requiring knowing the value of the external circuit output impedance to calculate the commutation angle. A significant disadvantage for their use in our study is the model complexity, which makes them difficult to manipulate and understand the role the MPTR plays in the stability of the system. Therefore, the necessity of a developing a simpler model that balances the characteristics of the two approaches was detected. This model is presented later in this chapter, in section IV.

A simplified behavioral model of the MPTR combined with the synchronous generator and its voltage regulator was developed for this study and is presented in chapter 5, section III-A.
This model aims to provide a good representation of the dynamic electrical behavior of the generator-MPTR set at the DC circuit connection.

III. Characteristics of Models for Stability Studies

Models for stability studies require appropriate degrees of detail in structure and time scale. The interaction nature of the stability phenomena requires that the models have a similar range of validity in the time-frequency scale. Additionally, some of the requirements can be summarized as:

- Accounts for the dynamic behavior in the desired range
- Model reduction or expansion is possible according to desired degree of detail
- It has a straightforward implementation in simulation
- Simplified math complexity if possible

There are various ways to approach the stability analysis of an electric power system as was explained in the previous chapter. In most cases the analysis requires to simulate the model operation in a computer. According to the type of study there is a model that matches more closely the necessities of the simulation. Analytical dynamical studies usually employ average model of the system components. When the analysis is based on time domain simulations higher degree of detail is required. Table 2-1 summarizes the main characteristics of the models to use according to the type of computer based analysis.

**Table 2-1 Summary of the system component models used at different types of simulation based stability analysis**

<table>
<thead>
<tr>
<th>Model type</th>
<th>Source</th>
<th>Distribution Network</th>
<th>AC/DC rectification</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-state</td>
<td>Constant back-emf behind the synchronous reactance</td>
<td>Lumped parameters R and L.</td>
<td>Static I/O magnitude based on energy conservation</td>
<td>Constant back emf behind the synchronous reactance</td>
</tr>
<tr>
<td>Average</td>
<td>Simplified Transient one-axis w/V loop control</td>
<td>Lumped parameters R and L.</td>
<td>Reduced order model</td>
<td>Voltage dependent CPL model</td>
</tr>
<tr>
<td></td>
<td>Full-order Subtransient, q-axis dynamic simplified w/ detailed control</td>
<td>Lumped parameters R and L.</td>
<td>Full-order model</td>
<td>d-q axis model of motors Detailed controls</td>
</tr>
<tr>
<td>Full-order</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switching</td>
<td>Full order including stator and damper dynamics</td>
<td>Lumped parameters R and L.</td>
<td>Ideal diodes or switching function for PWM converter</td>
<td>Switching functions for converters</td>
</tr>
<tr>
<td>Switching/faults</td>
<td>Full order with saturation High accuracy: stator reference frame</td>
<td>Lumped parameters R and L + mutual coupling</td>
<td>Switches with protection logic. Include magnetic saturation if present</td>
<td>Ideal switches/diodes for converters. Include load protections</td>
</tr>
</tbody>
</table>
IV. Autotransformer-Rectifier Modeling

The modeling analysis to be presented can be applied to rectifiers of different number of pulses; the development concentrates on an eighteen pulse rectifier like the one in Fig. 2-4 that has a three-phase primary and a nine-phase secondary side. The nine-phases are equally spaced in the line frequency period being the electrical angle between phases of forty degrees. The output can also be seen as three sets of three-phases equally spaced and each one connected to a diode bridge rectifier. Each of this six-pulse rectifiers conduct one third of the total output power.

![Eighteen pulse diode rectifier circuit schematic](image)

When modeling these circuits usually it is assumed that the leakage inductances have the same value in all nine secondary phases. In order for the circuit to achieve good performance it is desirable that this leakage inductance is of reduced value. Nonlinearities due to the iron core of the transformer are also neglected. The circuit is usually connected to an LC filter in the DC side, this filter can be included as part of the model. The basic input-output voltage relationships are given by:

\[
[V]_{dc} \begin{bmatrix} S \end{bmatrix}_{abc} = V_{dc} = R_{dc}i_{dc} + L_{dc} \frac{di_{dc}}{dt} + V_{load}
\]  

(2-2)
\[ [i]_{abc} = [S]_{abc} i_{dc} \]  

(2-3)

Where \([S]_{abc}\) represents the switching action of the multi-pulse rectifier, \(L_{dc}\) is the inductance connected at the output and \(R_{dc}\) its resistance; \(V_{\text{load}}\) is the voltage at the load. The model to be developed in this section also assumes small harmonic content in the input AC magnitudes and small ripple in the output DC voltage and current when compared to the fundamental frequency component. Under these assumptions all the power is carried on by the fundamental frequency component. If additionally the rectification process is assumed lossless the power at both AC and DC sides must be the same. The power at the 3-phase AC side is given by:

\[ P_{ac} = 3 V_l I_l \cos \varphi \]  

(2-4)

where \(V_l\) and \(I_l\) are the rms of the line-ground voltage and the line current and \(\varphi\) the angle between the two. Otherwise, the power in the DC side is,

\[ P_{dc} = v_{dc} i_{dc} \]  

(2-5)

Neglecting the magnetizing current component, the current input and output must be the same. On the other side, the voltage experiences a drop due to the commutation process. Therefore, from (2-4) and (2-5) the following relationship can be established

\[ v_{dc} = G_{dc} \sqrt{v_d^2 + v_q^2} \cos \varphi \]  

(2-6)

Conversely, the currents in the AC side can be calculated from the DC side magnitudes.

\[ i_d = G_{dc} i_{dc} \frac{v_d \cos \varphi + v_q \sin \varphi}{\sqrt{v_d^2 + v_q^2}} \]  

(2-7a)

\[ i_q = G_{dc} i_{dc} \frac{-v_d \sin \varphi + v_q \cos \varphi}{\sqrt{v_d^2 + v_q^2}} \]  

(2-7b)

The factor \(G_{dc}\) must consider the conversion factor of the abc→dq0 matrix transformation and the transformer ratio of the autotransformer. For the matrix transformation of the nine-phase circuit considered, the following relationship applies.

\[ V_{ph} = \sqrt{2} V_l = \sqrt{\frac{2}{3}} \sqrt{v_d^2 + v_q^2} \]  

(2-8)

The phasor diagram in Fig. 2-5 represents the fundamental components of the AC magnitudes and their \(d-q\) decomposition. Given the equations (2-6) and (2-7) it also can be seen as a representation of the average values of the DC magnitudes, affected by a scale factor.
Fig. 2-5 Phasor diagram for the AC side of the rectifier

Considering the equation (2-6) the voltage drop created during the commutation is given by

\[ v_{\text{drop}} = G_{dc} \sqrt{v_d^2 + v_q^2} (1 - \cos \phi) \]  \hspace{1cm} (2-9)

Additionally, the DC voltage equation (3-25) under lossless conditions reduces the equation (2-10):

\[ v_d = V_p \sin \left( \frac{\pi}{9} \right) \left( \frac{18}{\pi} \right) \frac{9}{\pi} \omega L_c i_{dc} \]  \hspace{1cm} (2-10)

In this equation the first term on the right represents the ideal DC voltage in case the commutation effect is negligible. Moreover, the second term corresponds to the voltage drop produced by the commutation.

\[ v_{\text{drop}} = \frac{9}{\pi} \omega L_c i_{dc} \]  \hspace{1cm} (2-11)

From equations (2-9) and (2-11), and considering the peak component relationship in (2-8), the following expression provides the angle between the voltage and current fundamental components.

\[ \phi = \arccos \left( 1 - \frac{\omega L_c i_{dc}}{G_{dc} \sqrt{3/2} V_{pm} \frac{\pi}{9}} \right) \]  \hspace{1cm} (2-12)

The equations (2-2), (2-6), (2-7), and (2-12) provide the complete description of the simplified multi-pulse rectifier model. These equations can be represented in the circuit schematic of Fig. 2-6.
A. Discussion on the Coefficients and Angles

In the calculation of \( V_{dc} \), (2-6) assumed that the output voltage corresponds to a rectifier of infinite number of pulses. In such case the output voltage corresponds to \( 2V_{pk} \). However, a look at equation (2-10) shows that in the 18-pulse rectifier, the ideal output voltage is:

\[
v_x = V_{pk} \frac{2 \sin \left( \frac{\pi}{9} \right)}{9}
\]

(2-13)

The error introduced by the approximation is small, about 1.2%. In fact, the true ratio can be accounted for in the calculation of \( G_{dc} \). However, the influence on the output magnitudes is not significant and can be neglected considering the other assumptions in the model.

Equation (2-12) provides the phase shift angle between the fundamental components of the voltage and current in the DC circuit. This shift is introduced by the leakage reactance of the autotransformer and the inductance in the primary circuit, which are generically considered in \( L_e \). This inductance is also responsible for the commutation time interval given by the commutation angle \( \mu \). For a nine-pulse rectifier that commutation angle is given by, [74]:

\[
\mu = \arccos \left( 1 - \frac{\omega L_e i_{dc}}{V_{pk} \sin \left( \frac{\pi}{9} \right)} \right)
\]

(2-14)

A look at equations (2-12) and (2-14) show the close relation between these two angles. Fig. 2-7 shows the evolution of these two angles as function of the DC current for a generic eighteen-pulse rectifier.
B. Small-Signal Analysis

In most cases the small-signal behavior of a non-linear circuit can be analyzed using a linearization of the original model around the operation point. In particular, the input-output impedances (or admittances) are important to evaluate of the stability of a DC or AC system [30]. Therefore, the input admittance of both average models: full-order, and simplified was evaluated. In \(d-q\) reference the admittance has four components as indicated by (2-15).

\[
Y_{\omega} = \begin{bmatrix}
Y_{dd} & Y_{dq} \\
Y_{qd} & Y_{qq}
\end{bmatrix}
\] (2-15)

The admittance frequency responses of the simplified average model just presented and the full order model described in [74] are shown in Fig. 2-8. The reference frame is aligned with the \(d\)-axis of the \(d-q\) frame. In the Figure it is possible to see that the two models present a very close agreement in their admittance frequency response.

C. Large-Signal Analysis

This section evaluates the behavior of the multi-pulse rectifier average models beyond the description provided by the small-signal frequency response of the circuit. The evaluation is made through computer simulations.
Fig. 2-8 Input admittance of the multi-pulse rectifier full-order model under the same conditions

The transient behaviors of three different models: detailed switching, average full-order, and average simplified, was simulated in order to observe the evolution of the different circuit magnitudes. Fig. 2-9 shows the waveforms for $v_{dc}$ and $i_{dc}$ for a sudden load connection and disconnection.

The evolution of the AC current for the same transient than the previous Fig. 2-9 is shown in Fig. 2-10. In these two Figures it is possible to observe the closeness of the response of the three models. There is however slight differences mainly between the two average models and the switching models originated in the assumptions made during the modeling.
Fig. 2-9 Evolution of $v_{dc}$ and $i_{dc}$ during a transient originated by a load connection-disconnection.

Fig. 2-10 Evolution of the current components $i_d$, $i_q$ during a load connection-disconnection transient.
D. Large-signal stability behavior

An important aspect of the large-signal evaluation is related to the model ability to represent the large-signal stability behavior. The circuit in Fig. 2-11 was used for this large-signal evaluation. The DC side of the rectifier connects to a constant power load (CPL) type. In this load the current level is adjusted in order to maintain the power unchanged at any input voltage. For a voltage perturbation, the current does not follow the perturbation instantaneously. The response is of a first order type, with a time constant that corresponds to the bandwidth of the load controller.

![Circuit schematic of the circuit used for large-signal stability analysis](image)

Fig. 2-11 Circuit schematic of the circuit used for large-signal stability analysis

The circuit is also composed of a synchronous generator with excitation regulation. The DC filter is provided by $L$ and $C$; whereas $R_p$ represents a small resistive load connected in parallel to the CPL. The sudden connection of the controlled load was simulated. When the load power level is increased the circuit reaches a point where the load connection produces instability. It is important that a simulation study using the developed model is able to identify the load level producing instability. To compare model performances, the simulation using the circuit in Fig. 2-11 was done for the detailed switching model and the simplified average model. Fig. 2-12 shows the $v_{dc}$, $i_{dc}$ evolution for a stable case. Increasing the load in a 10kW steps, which corresponds to less than 10% of the total load, produces the unstable behavior of Fig. 2-13. As indicated by the simulations, both models agreed in identifying the stability limit inside a band of about 8% of the total power.
**E. Multi-pulse Rectifier in Stationary Operation**

Here the multi-pulse rectifier is analyzed from the point of view of the characteristics of the active and reactive power flow through it. Considering the rectifier connected to a Thevenin equivalent of the three phase AC source, like the circuit shown in Fig. 2-14 the voltage at the DC port of the rectifier is given by (2-16).

\[
v_{dc} = G_{dc} e_g - (z_l + z_g) \sqrt{3} i_{dc} - \frac{9}{\pi} \omega L_c i_{dc}
\]  

(2-16)

where \( G_{dc} \) is the transfer ratio related to the transformer ratio of the autotransformer; \( e_g \) and \( z_g \) are the equivalent parameters of the AC source; \( z_l \) is the impedance of the feeder; and \( L_c \) the
commutation inductance of the primary circuit. The dc current $i_{dc}$ can only be positive. This expression is rather simplified, but shows the main components of the AC circuit affecting the voltage at the DC side. From equation (2-16) it possible to conclude that whenever the parameters in the equation remain constant the output characteristics of the multi-pulse rectifier at the DC bus will remain linear. On the other side, the impact of the DC circuit on the AC side is given by the angle between the voltage and current phasors, namely $\phi$. This angle $\phi$ is created by the commutation phenomena and its value is related to the commutation inductance. For a load $i_{dc}$, connected at the DC side of the rectifier, the power factor seen at the three-phase AC terminals can be calculated from equations (2-7) giving as a result (2-17).

![Fig. 2-14 MD with limits at the current loop and duty cycle](image)

$$
\cos \phi = \frac{1}{1 + \frac{\pi}{9} \omega L_v \frac{i_{dc}}{v_{dc}}}
$$

Expression (2-17) shows that for any DC load, the power factor at the AC side will always be lagging. This has a beneficial effect on the stability of the synchronous generator, because it reduces the electrical angle of the generator, increasing the distance between the stable and unstable points.

**F. Model Evaluation**

In general, good agreement among the evaluated average models and the switching model was found in the model evaluation. The simplified average model has the advantage of providing good response with much less calculation effort than the full-order average model. From the analysis it is also possible to conclude that the dynamic inherent to the multi-pulse rectifier is of reduced impact when compared to the dynamics of the AC and DC circuits where it is connected. This is valid at least for the type of power systems of interest.
V. Summary

This chapter has described the model of the system to be studied in this work. The purpose of the description was to introduce the reader with the equipment and architecture of the system under analysis. The characteristics of the models needed for stability studies were also discussed. The analysis of all the models available in the literature showed the necessity of developing a simplified average model for the multi-pulse transformer rectifier. Therefore, a simplified average model of that rectifier was proposed. In addition, the model was evaluated in simulations and its response was compared to the ones of switching models and full-order average models.
Chapter 3  Nonlinear Dynamic Behavior of the Autonomous Power System

I. Introduction

The purpose of this chapter is to describe and analyze the nonlinear behavior at the power system under study. This nonlinear behavior can appear on a wide range of operation conditions. The analysis is approached in a general manner based on the mathematical formulation of the system components; the emphasis is in the nonlinear dynamical aspects of the models. A particular type of nonlinearity is the one representing hard limits; therefore, it is analyzed in detail at each of the models described.

Large-signal distinctive phenomena are created by the nonlinear model characteristics. In order to predict such type of behavior the nonlinearities must be accurately modeled. Some of the system nonlinear behavior is due to the constant power load (CPL) nature of the electrical consumption in the system. This has been studied for DC systems using simplified models [3]; however, the CPL mechanism is not the only nonlinearity in the power system affecting its stability as will be analyzed in the present chapter. It is well known that a nonlinear system has multiple equilibrium points defining its dynamic behavior and hence its stability. The nature of equilibrium points, i.e., stable or unstable, may be determined using the Lyapunov indirect or linearization method, which provides a basis for the small-signal stability analysis. The results obtained with this type of analysis are valid only locally.

The study of large-signal stability in AC power systems is usually referred to as transient stability. To study stability in an AC system the synchronous generator rotor dynamics is key, where the rotor electrical and mechanical angles receive special attention [83]. Other stability phenomena affecting power networks are classified as voltage stability, which is related to the constant power characteristic of the loads. These two phenomena, originally treated separately, show a coupled behavior in the presence of fast voltage regulation [80]. For large signal analysis of DC circuits, the work presented in [33] for instance analyzed the equilibrium points and their characteristics with respect to the system operational limits. In [24] the search for equilibrium points, their bifurcations, and eigenvalues were used for the analysis of an AC/DC system.
The analysis in this chapter focuses on the system in Fig. 3-1, which basically comprises a synchronous generator connected to motor drive that consumes a large portion of the generator power. The AC/DC rectifier can be either passive, like a multi-pulse rectifier, or an active front end. Some previous work has studied a system similar to the one in Fig. 3-1 for control design or stability analysis. The work in [81] focuses on the development of the control algorithm of the motor controller.

The analysis in this chapter uses the Lyapunov indirect method to approach to the nonlinear analysis because it allows determining the conditions where the system bifurcates. The behavior at those bifurcations is studied through computer simulations. Many of the bifurcation cases found are given by hard limit situations. Those limits exist in the controllers of the source and load. State-transition diagrams are presented when these limit situations are encountered.

![Fig. 3-1 Schematic of the simplified system focus of the analysis in this chapter](image)

Section two of this chapter is devoted to the analysis of the synchronous generator nonlinear behavior. Section three describes the motor controller while section four covers the interconnected system. The behavior of the interconnected system is summarized in section five.

II. Synchronous Generator Voltage Regulation

A. Generator and Voltage Regulator Model

Because in the system under study the power of the prime mover is many times bigger than the electric generator the mechanical transient can be neglected when studying the generator stability behavior. In this aspect, the large-signal stability analysis of the power system in this study is very different from the classical transient stability analysis of a power system. The analysis here focuses on the nonlinear characteristics of the regulated generator. The basic block
diagram of the generator excitation control was given in Fig. 2-2 in the previous chapter. The block diagram of the excitation control includes two nonlinearities. One of them is related to the control of the rms value of the voltage. In the d-q model of the machine the rms value is obtained by the mean root square of the voltage $d$ and $q$ components.

$$v_r = \sqrt{\frac{1}{2} (v_d^2 + v_q^2)}$$  \hspace{1cm} (3-1)

The second nonlinearity is given by the presence of the diode in the excitation circuit. This diode limits the excitation current to flow only in one direction. Another type of nonlinearity usually considered in the modeling of an electric machine is the rotational speed. However, because in our system this speed is an external variable, it can be considered as an independent parameter. Therefore, in the system under study it does not constitute a nonlinearity.

**B. Analysis of the Synchronous Generator Stationary Operation**

To analyze the nonlinear behavior of the regulated generator a simple circuit like the one in Fig. 3-2 can be used. A d-q model of the synchronous generator, formulated as in [61], is used in the analysis. To facilitate the analysis, the excitation controller is simplified in order to have only the voltage control loop. An additional simplification is made by neglecting the damper windings in the generator model. These simplifications do not change the conclusions of the analysis, but reduces the order of the system and the equations. The voltage loop controller, $C_v(s)$ is considered to have a characteristic of a lead-lag plus integrator as shown in equation (3-2).

$$C_v(s) = \frac{K_v s \omega_z^{-1} + 1}{s \omega_p^{-1} + 1}$$  \hspace{1cm} (3-2)

The controller parameters $K_v$, $\omega_z$, and $\omega_p$ are respectively the gain, and characteristic frequencies of the controller zero and pole. The input variable is the difference between the reference, $v_{ref}$, and actual voltage, $v_t$, which is called the error voltage $v_{er}$. The previous equation can also be expressed in state-space form.

$$\begin{bmatrix} \dot{v}_{d} \\ \dot{v}_{q} \end{bmatrix} = \begin{bmatrix} -\omega_p & K_v \omega_p \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_d \\ v_q \end{bmatrix} + \begin{bmatrix} K_v \omega_p \\ \omega_p \end{bmatrix} v_{er}$$  \hspace{1cm} (3-3)

Where $v_{er}$ is calculated from the d-q-0 components of the voltage at the point of regulation (POR), which in this case is the generator terminals.
The system model is obtained by combining the equations of all the system components; in this case: generator, controller, and load. The nonlinearities in this model are located at the generator excitation control. If the excitation is assumed constant, or its regulation is not included in the model, it is possible to combine the generator and load in a linear state-space model. Equation (3-6) combines the R-C load with the generator equations in a state-space formulation of the form:

\[
\dot{x} = Ax + Bu \\
y = Cx
\]  
(3-5)

Equation (3-6)(a) corresponds to the state equation itself and (b) to the output equation. In this last equation the voltage components \(v_d\) and \(v_q\) are the output magnitudes. The system model combines equation (3-6) with the voltage regulator state equation (3-7). Let us consider for now that the diode is not present at the excitation circuit. In this way, the excitation current is given the possibility of having both positive and negative values.

\[
\begin{bmatrix}
\dot{\lambda}_d \\
\dot{\lambda}_q \\
\dot{\lambda}_{\text{gl}} \\
v_{\text{ad}} \\
v_{\text{eq}}
\end{bmatrix} = 
\begin{bmatrix}
-(R_s + R_e) \frac{L_{\text{sl}}}{L_{\text{sl}}^{\text{eq}}} & \omega_r & (R_s + R_e) \frac{L_{\text{sl}}}{L_{\text{sl}}^{\text{eq}}} & 1 & 0 \\
-\omega_r & -(R_s + R_e) \frac{1}{L_q} & 0 & 0 & 1 \\
R'_{\text{ad}} \frac{L_{\text{sl}}}{L_{\text{sl}}^{\text{eq}}} & 0 & -R'_{\text{ad}} \frac{L_{\text{sl}}}{L_{\text{sl}}^{\text{eq}}} & 0 & 0 \\
-\frac{1}{C} \frac{L_{\text{sl}}}{L_{\text{sl}}^{\text{eq}}} & 0 & \frac{1}{C} \frac{L_{\text{sl}}}{L_{\text{sl}}^{\text{eq}}} & 0 & \omega_r \\
0 & -1/C & 0 & -\omega_r & 0
\end{bmatrix}
\begin{bmatrix}
\lambda_d \\
\lambda_q \\
\lambda_{\text{gl}} \\
v_{\text{ad}} \\
v_{\text{eq}}
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]  
(a)  
(3-6)
Chapter 3 - Nonlinear Dynamic Behavior of the Autonomous Power System

\[
\begin{bmatrix}
  v_d \\
v_q
\end{bmatrix} =
\begin{bmatrix}
-R_e \frac{L'_{fd}}{L_{djs}} & 0 & R_e \frac{L'_{md}}{L_{djs}} & 1 & 0 \\
0 & -\frac{R}{L_q} & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\lambda}_d \\
\dot{\lambda}_q \\
\dot{\lambda}'_{fd} \\
\dot{\lambda}'_{md} \\
v_{eq}
\end{bmatrix}
\]

(b)

\[
\begin{bmatrix}
  v'_{fd} \\
v'_{eq}
\end{bmatrix} =
\begin{bmatrix}
\frac{\omega_p}{N_{fd}} & K, \omega_p \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
v_{fd} \\
v_{eq}
\end{bmatrix}
+ \frac{K, \omega_p}{\alpha_p N_{fd}} \begin{bmatrix} 1 \end{bmatrix} v_{ref} \frac{1}{\sqrt{2}} \sqrt{v_{d}^2 + v_{q}^2}
\]

(3-7)

A first step in the analysis consists in determining the equilibrium points. In order to do that the system of nonlinear equations must be solved for the condition where the derivatives are equal to zero. Because of the quadratic nature of the voltage feedback control the system has two solution solutions that have the same absolute value but opposite sign. The equilibrium solutions are represented with capital letters and the two solutions \(X_1\) and \(X_2\) are generically given by (3-8).

\[
X_i^T = \begin{bmatrix}
\Lambda_{fd} & \Lambda_{q1} & \Lambda'_{fd1} & V_{eq1} & V'_{fd1} & V'_{eq1}
\end{bmatrix}
\]

\[X_2 = -X_1\]

(3-8)

The excitation voltage steady-state solutions are calculated as:

\[
V_{fd1} = \frac{N_{fd} R'_{fd}}{\omega_q \frac{L'_{md}}{L_{djs}}} \frac{\sqrt{2} V_i}{\text{sign} \left( \frac{B_r}{A_i} \right) \left[ 1 + \left( \frac{B_r}{A_i} \right) \right]^{\frac{1}{2}}} \left[ 1 + R_s W_{c2} - \omega_q L_q W_{c1} + \frac{B_r}{A_i} \left( R_s W_{c1} + \omega_q L_q W_{c2} \right) \right]
\]

\[
V_{fd2} = -V_{fd1}
\]

(3-9)

With:

\[
A_i = 1 + R_s W_{c2} - \omega_q L_q W_{c1}, \quad B_i = R_s W_{c1} + \omega_q L_q W_{c2}
\]

\[
W_{c1} = \frac{\alpha_p C_c}{\alpha_q C_c^2 R_i^2 + 1}, \quad W_{c2} = \frac{\alpha_p^2 C_c^2 R_i}{\alpha_q C_c^2 R_i^2 + 1}
\]

(3-10)

In order to show the impact of the load parameters on the generator operation the stationary values of the excitation voltage \(V_{fd1}\) and \(V_{fd2}\) are plotted as function of the load capacitance \(C_c\) for a fixed value of resistance \(R_c\). Fig. 3-3 shows the values of \(V_{fd1}\) and \(V_{fd2}\) as well as the value of the resulting load power factor. It is observed very clearly in the Figure how the excitation requirement is reduced as the power factor of the load becomes increasingly leading. The values of other magnitudes at the stationary condition can be calculated in the same way. The next Fig. 3-4, shows the value of the positive solution of the excitation field \(\Lambda_{fd1}\) for different \(C_c\) values.
This Figure also shows the resulting load active and reactive power provided by the generator. In addition, the Figure also shows the voltage relative angle, which is calculated as:

$$\delta = \arctan \left( \frac{v_q}{v_d} \right)$$  \hspace{1cm} (3-11)

**Fig. 3-3 Excitation voltage** $V_{fd}'$ **for the two possible solutions** $V_{fd1}'$ **and** $V_{fd2}'$ **of the unlimited system model**

**Fig. 3-4 Operation magnitudes for the positive stationary solution:** Generator active and reactive power, excitation field and generator rotor angle
C. Small-signal stability of the Synchronous Generator

The values of the state variables at stationary operation are also the coordinates of an equilibrium point in the state space. The possibility of solving the previously mentioned set of nonlinear equations indicated the existence of the equilibrium points. Once the equilibrium point has been determined it is possible to study the stability at a neighborhood of them. This can be done by means of the Lyapunov indirect method, which provides an evaluation of the local stability. The next paragraphs analyze the small-signal model of the generator with linear load. Because it has no impact on the small-signal stability this analysis neglects the presence of the diode in the excitation circuit. The impact of this hard limit in the current excitation will be studied later. The voltage error equation (3-4) equation can be linearized using the linear terms of its Taylor series expansion; this produces equation (3-12).

\[ \tilde{v}_{av} = \tilde{v}_{av} - \frac{V_d}{2V_i} \tilde{v}_{av} - \frac{V_q}{2V_i} \tilde{v}_{aq} \]  

(3-12)

Replacing this last equation in the controller equation (3-7) and combining it with the machine-load state equation (3-6) it is possible to obtain a linear state-space model for the complete system. This model is given in equation (3-13), where both the state and output equations are given.

\[
\begin{bmatrix}
\dot{\lambda}_q \\
\dot{\lambda}_d \\
\dot{\lambda}_d \\
\dot{\lambda}_q \\
\dot{v}_{av} \\
\vdots
\end{bmatrix} = \begin{bmatrix}
-(R_e + R_i) \frac{L_m}{L_{e_{av}}} & \omega & (R_e + R_i) \frac{L_{load}}{L_{e_{av}}} & 1 & 0 & 0 & 0 \\
-\omega & \frac{(R_e + R_i)}{L_e} & 0 & 0 & 1 & 0 & 0 \\
R_e \frac{L_{e_{av}}}{L_{e_{av}}} & 0 & -R_e \frac{L_{e_{av}}}{L_{e_{av}}} & 0 & 0 & 1 & 0 \\
\frac{1}{C_i} \frac{L_{e_{av}}}{L_{e_{av}}} & 0 & \frac{1}{C_i} \frac{L_{e_{av}}}{L_{e_{av}}} & 0 & \omega & 0 & 0 \\
0 & -\frac{1}{C_i} \frac{L_{e_{av}}}{L_{e_{av}}} & 0 & -\omega & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\lambda_q \\
\lambda_d \\
\lambda_d \\
\lambda_q \\
v_{av} \\
\vdots
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{bmatrix} \begin{bmatrix}
\frac{K}{\omega N_{av}} \\
\frac{\omega}{N_{av}} \\
\frac{V_{d}}{2V_i} \frac{L_{e_{av}}}{L_{e_{av}}} \\
\frac{V_{q}}{2V_i} \frac{L_{e_{av}}}{L_{e_{av}}} \\
\frac{V_{d}}{2V_i} \frac{L_{load}}{L_{e_{av}}} \\
\frac{V_{q}}{2V_i} \frac{L_{load}}{L_{e_{av}}}
\end{bmatrix}
\]

(3-13)
The eigenvalues of the state matrix of the linearized system (3-13) define the stability character of the equilibrium point under study. Moreover, the equilibrium point can move due to matrix coefficient variations linked to system parameters changing. This parameter change can be produced by a change in system itself; i.e. a generator with a different impedance is used; or an operation condition change; i.e. load consumption has increased. In any case (3-13) is a model valid in the proximity of the equilibrium point being considered. Using the Lyapunov indirect method over (3-13) makes possible to determine the stability of the equilibrium points for a quasi-stationary set of changing parameters.

To study the generator stability behavior it is useful to analyze the impact of the load parameters, \( R_c \) and \( C_c \), on the eigenvalues of the system matrix (3-13). For each pair \( R_c, C_c \) the local stability of the equilibrium point can be determined. Fig. 3-5 shows the result of described procedure. For a given load condition if the result of the Lyapunov indirect method indicates that the generator under such load is stable, the point is marked in blue in the \( R_c-C_c \) plane; otherwise, if it is unstable it is marked in red. For reference purposes the Figure also shows curve levels corresponding to various operation conditions. The area on the right of the curve marked as 250kVA, which is the nominal power of the generator, corresponds to powers lower than the nominal and in the normal range of operation. It is possible to observe in Fig. 3-8 that loads with a total power lower than the generator rated power, but power factor close to 0.85 leading produce an unstable operation.

The impact of other parameters can be studied in the same way. The generator model used to obtain equation (3-13) did not include the damper winding dynamics. Including the damper windings in the model only adds two state variables corresponding to the equivalent damper winding in the d and q coordinate axis. Using this more complete generator model the same type of parametric study can be done. Fig. 3-6 shows the same case of Fig. 3-5 but including the damper windings. By comparison between the two mentioned Figures it is possible to notice the
enlargement of the unstable zone due to the effect of the damper windings. Therefore, damper windings reduce the range of capacitive power factor that the generator is able to supply.

Fig. 3-5 Areas of stable and unstable generator operation for changing capacitive load parameters

Fig. 3-6 Areas of stable and unstable generator operation for generator with damper windings

The series connected capacitive load of Fig. 3-2 does not frequently appear in a practical application. More frequent is the situation where a capacitive type load is connected in parallel to a dissipative load, like a resistance. The schematic of this circuit is shown in Fig. 3-7. The same type of analysis previously described can be done for the parallel capacitive load configuration. The result of the load parameter impact on stability is shown in Fig. 3-7. In the mentioned
Figure, it is possible to observe that although the shape and location of the stable and unstable zones has changed when compared to its equivalent for series connection in Fig. 3-5 the relationship to the power factor curve levels and the rated generator power is very similar. Like in the series connection the condition of 0.85 leading power factor crosses the unstable zone indicating unstable behavior.

Fig. 3-7 Synchronous generator with linear parallel capacitive load

The information in Fig. 3-6 and Fig. 3-8 is useful for gaining insight on the stability of the synchronous generator, and can provide valuable information for design and operation, e.g. setting up limits for stable operation. However, the stability analysis based on models linearized around the equilibrium points provides only a local description. It does not tell how the generator will behave during a transient created by a sudden load connection for example. Neither can be used to predict how the generator will behave in the circumstance of an electric fault. The next subsection will approach the analysis under such transient conditions.

Fig. 3-8 Areas of stable and unstable generator operation for capacitive parallel load
D. Synchronous Generator Nonlinear Behavior

If a large-disturbance is applied, i.e. a load step, the generator must transition from an initial operating point to a final one. A first necessary condition for the system to be stable requires that both initial and final equilibrium points are locally stable. However, this does not guarantee that after the transition the system will stabilize at the desired condition because the trajectory may not converge to the intended equilibrium point even if this is locally stable. The next Fig. 3-9 is used to explain the different alternatives that appear when a transient modifies the generator operation conditions.

Fig. 3-9 shows a stable and an unstable equilibrium points (SEP and UEP respectively). The SEP corresponds to the desired operating condition. The UEP corresponds to the negative solution in (3-8). As mentioned earlier, the unstable solution appears due to the quadratic characteristic of the generator rms voltage control. The figure shows two trajectories converging and diverging from the SEP and UEP respectively (stable tr and unstb tr). Additionally, a trajectory Cc step is included. Trajectories stable tr and Cc step, both correspond to the same load condition. They initially start at the origin and reach unloaded rated voltage operation, marked by R in the Figure. On the contrary, after load the connection they depart from each other because while in the Cc step case the load is connected all at once, in stable tr the load capacitor is smoothly changed to reach the same final value as Cc step.

![Fig. 3-9 Trajectory of the unlimited synchronous generator](image-url)
The target equilibrium point in \textit{Cc step} is the same as \textit{stable tr}, but during the transient evolution the system falls into the attraction of the UEP and becomes unstable. Therefore, to have a complete picture of the system stability it is necessary to know not only the behavior around the equilibrium points, but also the region of the state space where those equilibrium points have influence, in other words, the characteristics and size of the region of attraction around the equilibrium points. This last is the object of discussion in a latter chapter of this dissertation.

\textit{E. Limited System Analysis}

The previous section has shown a simple example where the knowledge of the system behavior around equilibrium points does provide a complete characterization of its stability in the large-disturbance case. This fact is a well known nonlinear behavior and has been widely studied; however, the knowledge of the equilibrium points and their characteristics still provides useful information about the system. In a practical application, the synchronous generator excitation circuit is limited to only allow positive excitation currents. This limit is created by a diode connected in the excitation path shown previously in Fig. 2-2. Such constraint in the excitation current can be synthesized like shown in (3-14). If the voltage appearing on the diode is positive, the excitation current can be calculated from the machine fluxes like during regular operation. If the voltage on the diode reverses, the current is still related to the generator fluxes, but it is forced to be zero.

\[
\begin{align*}
    i'_{jd} &= \frac{L_d}{L_{dfe}} \left( \lambda'_{jd} - \frac{L_{sfe}}{L_j} \lambda_d \right) \quad \text{if} \quad v_{diode} < 0 \\
    i'_{jd} &= 0 \quad \text{if} \quad v_{diode} > 0
\end{align*}
\]  

(3-14)

In the second case of (3-14) the machine cannot operate with reverse excitation and the corresponding equilibrium point does not exist anymore. Therefore, if the system is applied the same load step described in Fig. 3-9, the trajectory is forced to converge to the only remaining equilibrium point. This new situation is shown in Fig. 3-10, where it is now possible to see that the previous unstable case has been stabilized. The two trajectories, \textit{unstb tr} and \textit{w-diode} are obtained under the same conditions. After the start-up (origin at point \textit{O} in Fig. 3-10) and stabilization of the voltage at its rated value (point \textit{R} in the Figure), the load is connected to the generator. Both mentioned trajectories evolve in a similar way until the excitation current would
change sign and become negative (point D). After that, \textcolor{red}{unstb tr} falls into the attraction of the UEP and diverges. On the contrary, the field current cannot become negative when the diode is present in the circuit. In such case, with the absence of field current, the trajectory \textcolor{red}{w-diode} starts traveling towards the origin because no excitation is applied. While on that evolution, conditions for positive excitation reappear at a different point. During its evolution the generator can experience more than one period of interruption of the excitation current, but finally the trajectory falls into the region of attraction of the SEP and converges to that equilibrium point.

Along the described trajectory the system transitions between two states based on the diode current. For diode currents greater than zero, the dynamic of the system is governed by the state given generically by equations (3-6) and (3-7), which is a seventh order nonlinear system. On the other side, when the diode current is negative the connection between the controller and generator is lost and the controller loses its control capability. In addition, the field current is zero meaning that the field flux follows the same evolution than the flux in the machine d-axis. In such case, the dynamic behavior of the generator is given by the fourth order model of equation (3-15).

![Graph showing comparison of trajectories with and without excitation current limitation](image)

\textbf{Fig. 3-10 Comparison of trajectories with (w-diode) and without excitation current limitation (unstb tr)}

The model of the generator-load subsystem with $i_{fd}$ equal to zero is linear; therefore, presents only one equilibrium point at the origin whose stability also depends on the circuit parameters. Because of the constraint in the excitation current, the only possibility for the global system to
converge to the equilibrium point at the origin would be in case the reference voltage was set to zero, which is not a practical situation.

\[
\begin{bmatrix}
\dot{\lambda}_d \\
\dot{\lambda}_q \\
\dot{v}_{sd} \\
\dot{v}_{sq}
\end{bmatrix}
= 
\begin{bmatrix}
-(R_q + R_e) \frac{L_d'}{2 L_{d_0}} & \omega_r & 1 & 0 \\
-\omega_r & -(R_q + R_e) \frac{1}{L_q} & 0 & 1 \\
-\frac{1}{C_e} \frac{L_d'}{L_{d_0}} & 0 & 0 & \omega_r \\
0 & -\frac{1}{C_e L_q} & -\omega_r & 0
\end{bmatrix}
\begin{bmatrix}
\lambda_d \\
\lambda_q \\
v_{sd} \\
v_{sq}
\end{bmatrix}
\tag{3-15}
\]

\[
\begin{bmatrix}
v_{sd} \\
v_{sq}
\end{bmatrix}
= 
\begin{bmatrix}
-R_e \frac{L_d'}{2 L_{d_0}} & 0 & 1 & 0 \\
0 & \frac{R_e}{L_q} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\lambda_d \\
\lambda_q \\
v_{sd} \\
v_{sq}
\end{bmatrix}
\]

The behavior under this condition can be analyzed with the help of a state transition diagram [84] like the one shown in Fig. 3-11. When the field current is greater than zero, the system operates with the dynamics given by the model on the right. When the field current is zero, the dynamics are given by the model on the left. The system will stay in this condition until the field current becomes positive again transitioning back to the right. The limited system can be seen as a subsystem of the original system because the limits imply cancellation of some of the state dynamics. The location of the equilibrium points in the state space change according to the subsystem they belong to. The trajectory of the global system will eventually converge to an equilibrium point if such point is compatible with the limits and the transitioning among subsystems stops.

The behavior of the transitioning system can greatly depart from each of the individual systems and requires specific analysis [85]. For the controlled generator, it was found that the described transition can produce three different types of behavior. The trajectories can converge to the equilibrium point as shown in Fig. 3-10, they can diverge from the equilibrium point or, under certain operating conditions the trajectories can become periodic. The three different cases are shown in Fig. 3-12. The first case in Fig. 3-12(a) involves several transitions of the state; after them the trajectories converge to the equilibrium of the unbounded system. Fig. 3-12(b) corresponds to the oscillatory case where the evolution does not converge to a point neither diverges. Finally, the case in Fig. 3-12(c) shows the diverging trajectory.
The existence of periodic orbits for a controlled generator connected to a power system has already been described in [82] for example. Fig. 3-13 shows the periodic orbit with more detail portrayed in the machine flux ($\lambda_d$, $\lambda_q$, $\lambda_{fd}'$) sub-space. The conditions are similar to the one just described for the w-diode case in Fig. 3-10, but the load parameters are different. In this case, the equilibrium point enclosed by the orbits is of the unstable type.

It is relevant to analyze the relationship of the operation conditions at the physical system, which in this case is the synchronous machine, to its stability condition. Fig. 3-14 shows the generator operation magnitudes of Fig. 3-4 together with the three possible stability behaviors appearing when the load capacitance is changed. Several features can be observed in Fig. 3-14; the most important the relationship of the negative excitation flux $\lambda_{fd}'$ to the unstable condition. The unstable, or limit cycles appear for very low, or negative values of $\lambda_{fd}'$. This condition is also related to rotor angles larger than ninety degrees.

Further analysis shows that the limit cycle condition appears for cases that without the limiting diode in the excitation circuit would have been unstable. This means that the diode has a
positive effect on the system by preventing the system from divergence and also limiting the values the magnitudes can take.

![Figure 3-13 Periodic orbits projected over the generator flux subspace](image)

**Fig. 3-13 Periodic orbits projected over the generator flux subspace**

![Figure 3-14 Relationship of generator operation conditions and its stability behavior](image)

**Fig. 3-14 Relationship of generator operation conditions and its stability behavior**

**F. Generator with Limited Exciter**

The previous sections analyzed the generator behavior under the assumption of infinite excitation capacity. The only limit in the excitation circuit was the unidirectionality of the excitation current. However, in real applications the excitation system can not feed the excitation winding with any current or voltage. Moreover, in most cases it is desired to restrict the maximum
excitation current in order to limit the voltage and current at the generator during a fault in the circuit. Therefore, regardless of its construction the exciter has limits in the maximum current or voltage that can be fed to the machine field. These limits require the implementation of an anti-windup scheme in the excitation controller. The scheme used in this study is shown in Fig. 3-15 where the voltage control loop is shown in detail. In case the voltage at the generator field \( v_{fd} \) differs from the controller output \( v_{ex} \) the anti-windup takes action by changing the integration rate at the controller integrator. The anti-windup scheme changes the dynamic response of the voltage controller. The state-space model of the voltage loop controller \( C_v(s) \) in Fig. 3-15 is given in equation (3-16). In case the exciter is unsaturated the dynamic response of the controller is the same as before and the state equations reduce to (3-3). The causes of the exciter saturation can be due to the excitation current or voltage exceeding the maximum capacity or the desired limits.

Fig. 3-15 Synchronous generator excitation regulator block diagram showing the anti-windup scheme associated to the exciter limitation

\[
\begin{bmatrix}
\dot{v}_{jd} \\
\dot{v}_{ref}
\end{bmatrix} =
\begin{bmatrix}
-\omega_p & \omega_p \\
-K_{mvr} & 0
\end{bmatrix}
\begin{bmatrix}
v_{jd} \\
v_{ref}
\end{bmatrix} +
\begin{bmatrix}
K_{e} \omega_p & 0 \\
0 & K_{mvr}
\end{bmatrix}
\begin{bmatrix}
v_e \\
v_w
\end{bmatrix}
\]  

(3-16)

The control block diagram in Fig. 3-15 shows that the antiwindup control activates also in case of the diode blocking a reverse excitation current. Now the case of reversing excitation has become a particular case of excitation saturation. Therefore, the state transition diagram of the generator can be modified to reflect this new situation shown in Fig. 3-16. The regular operation appears in the left. If the exciter saturates for any reason the dynamic is given by the exciter...
constrained system represented on the top right. The dynamic behavior of this system is given by the generator-load dynamics, and the controller equation (3-16). In case that the excitation current tries to reversers, the conditions expressed in (3-14) are still valid and the evolution of the generator is completely independent from the controller as expressed in (3-15). On the contrary, for other limited conditions the controller evolution changes due to the effect of the antiwindup. However, the stability condition does not change significantly because the unstable behavior is produced at the unlimited system.

\[
\begin{align*}
\dot{x} &= f_g(x, z, p) \\
0 &= g_g(x, z, p)
\end{align*}
\]

\[
\begin{align*}
0 &= v_{ex} = v_{jd} \\
0 &= v_{ex} \neq v_{jd}
\end{align*}
\]

\[
\begin{align*}
i_{fd} &< 0 \\
i_{fd} &= 0
\end{align*}
\]

**Fig. 3-16 State transition diagram for the generator with limited excitation capacity**

III. Motor Drive Load

**A. Motor Drive Average Model**

The motor drives (MD) in the system are used to drive PMM. The MD itself is composed of a voltage source inverter (VSI) and the controller. The PMM is used to drive a mechanical load coupled to its shaft. Because of the dedicated use of the converter it is convenient to analyze the MD, PMM, and mechanical load set together. Fig. 3-17 shows the schematic of the MD-PMM circuit.

The control strategy used to control the motor speed consist of a \(d-q\) axes current regulator featuring PI controllers with decoupling terms and anti-windup loops, and an outer speed-loop PI regulator with anti-windup and a pre-filter compensator for optimum transient response. This last loop sets the reference for the \(q\)-axis control-loop that generates the torque current component [66]. The purpose here is not to describe the controller in detail because it was already done in a
previous volume of this report. Rather the analysis concentrates on the nonlinear characteristics of the MD. In the nonlinear analysis it is also important to know the hard limits and saturations the system has. The block diagram in Fig. 3-18 shows the control scheme with limits and antiwindups. The antiwindup settings follow the guidelines established in [86]. The Figure shows a current limit, which is used to keep the q-axis current reference bounded during the transients. This is done primarily to thermally protect the equipment, but has also impact on the stability. In addition, there exists a limit produced by the maximum modulation capacity of the VSI, which in this work is considered to be one.

**Fig. 3-17 PWM VSC circuit schematic with sub-systems**

**Fig. 3-18 Block diagram of the MD control system including limits and anti-windups**
There exist several nonlinearities in the physical system under control. These nonlinearities appear at the VSI, the PMM and the mechanical load. The analysis that follows is based on the average model given in the previous chapter.

Equations (3-24) and (3-25) give the relationships at the input/output of the VSC average model. The PMM state equations in (3-17) are expressed as function of the back-emf and the duty cycles. The back-emf components $e_d$, $e_q$ are calculated according to equation (3-18). The advantage of having the formulation of the machine model as function of the back-emf components is related to the direct calculation of the machine angle. The angle equation is shown in (3-19). It is important to mention that there exist infinite possible solution angles as noted in the equation. The electromagnetic torque is given in equation (3-20).

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} & 0 \\ 0 & -\frac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \end{bmatrix} \begin{bmatrix} v_{d} \\ v_{q} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v_{dc}}{L_s} \end{bmatrix} \begin{bmatrix} d_d \\ d_q \end{bmatrix}$$ (3-17)

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} = \begin{bmatrix} 0 & \omega_r L_s \\ -\omega_r L_s & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$ (3-18)

$$\delta = \arctan \left( \frac{e_d}{e_q} \right) \pm n\pi \quad \text{with } n \text{ integer}$$ (3-19)

$$T_e = \frac{3}{2} p I_e \lambda_{PM}$$ (3-20)

The motor electromagnetic torque $T_e$ is related to the load torque $T_l$ and the rotor speed $\omega_r$ according to equation (3-21) where $J_m$ is the inertia of the rotating parts and $B_m$ the friction. The load torque is assumed to have a quadratic characteristic as shown in equation (3-23). This type of quadratic characteristic corresponds in practice to centrifugal type pumps or air blowers. This type of characteristic is, in fact, very beneficial for the PMM stability.

$$\dot{\omega}_r = \frac{1}{J_m} \frac{p}{2} (T_r - T_l) - \frac{B_m}{J_m} \omega_r$$ (3-21)

$$\delta = \omega_r - \omega_{ref}$$ (3-22)

$$T_l = k_m \omega^2$$ (3-23)

The average model of the voltage source converter (VSC) is described by the input-output electrical relationships of the average PWM VSC in the $d$-$q$ frame are given by,
\[
\begin{bmatrix}
    v_d \\
    v_q
\end{bmatrix}
= \begin{bmatrix}
    d_d \\
    d_q
\end{bmatrix} V_{dc}
\]  
(3-24)

\[
I_{dc} = \frac{3}{2} \begin{bmatrix}
    d_d \\
    d_q
\end{bmatrix}
\]  
(3-25)

where \(v_d, v_q\) are the output phase-voltage components, \(d_d, d_q\) are the duty cycles components which corresponds to the converter control input, \(V_{dc}\) is the DC voltage, \(I_{dc}\) is the DC-link current drawn by the converter, and \(i_d, i_q\) are the AC output current components.

The motion of the MD-PMM is governed by the MD controller. In the case that the control limits and anti-windups are inactive the reference of the \(q\)-axis current control is calculated from equations (3-26) and (3-27) where \(\omega_k\) is the state variable associated to the speed controller. The duty-cycle components are calculated by \(d\)- and \(q\)-axis current control loops. Equations (3-28) and (3-29) correspond to the \(d\) current control loop with \(i_{dk}\) being the state variable associated to the integrator. On the other side, equations (3-30) and (3-31) correspond to the \(d_q\) computation by the \(q\) current control loop and \(i_{qk}\) is the associated state variable.

\[
i_{qref} = \omega_k + K_{\omega} (\omega_{ref} - \omega_k)
\]  
(3-26)

\[
\dot{\omega}_k = K_{\omega} (\omega_{ref} - \omega_k)
\]  
(3-27)

\[
d_d = \frac{1}{V_{dc} \sqrt{3}} \left\{ i_{dc} + K_{i} (i_{ref} - i_d) - \omega_L i_q \right\}
\]  
(3-28)

\[
\dot{i}_{dk} = K_{i} (i_{ref} - i_d)
\]  
(3-29)

\[
d_q = \frac{1}{V_{dc} \sqrt{3}} \left\{ i_{qref} + K_{i} (i_{ref} - i_q) + \omega_L i_d + \lambda_{ref} \right\}
\]  
(3-30)

\[
\dot{i}_{qk} = K_{i} (i_{ref} - i_q)
\]  
(3-31)

B. Analysis of Stationary Operation

The previous equations describe mathematically the dynamic behavior of the motor drive when it is connected to an ideal voltage source \(v_{dc}\). From this set of equations it is possible to compute the equilibrium points of the system by making all derivatives equal to zero. Solving the resulting set of nonlinear equations it is found that the system has only one solution for all variables except the angle which has a solution every \(\pi\) radians. Further analysis of the small-signal stability around each equilibrium point shows that the ones at even multiples of \(\pi\) are
stable, while the ones at odd multiples of $\pi$ are unstable. This situation is pictured in Fig. 3-19; the SEP location corresponds to a difference of an integer number of turns. This characteristic of the PMM operation has been described in [87] where additional characteristics of the topology around the equilibrium points are provided. If during a transient the motor approaches one of the UEPs it will be re-directed to the closest SEP. High stresses can be created during such transient. Nevertheless, for the MD being analyzed this situation is unlikely to appear. The motor is able to deliver the rated torque while working at small rotor angles. This has been verified through computer simulations; Fig. 3-20 shows the angle during regular motor operation. When the motor is delivering its rated torque, the rotor angle is smaller than twenty degrees. The Figure also shows that the mechanical transient is very short and the maximum angle excursion during the transient is also small. The part (b) of the Figure shows the voltage and current evolutions during the mentioned transient. The rotor angle is related with the quality of the machine and its electrical parameters. The value of the angle during stationary operation is given by (3-32).

$$\delta = \arctan\left(\frac{L_s i_d}{L_s i_d - \lambda_{pm}}\right)$$

(3-32)

Fig. 3-19 MD equilibrium points in the $\delta$-$\omega_r$ plane; SEPs appear every $2\pi$ radians

The previous case analyzed the MD connected to an ideal DC voltage source. Different is the situation when the MD is connected through an impedance. For DC-fed motor controllers based on VSC it is usually required to use a DC filter at the DC bus connection. A simplified schematic of such connection is shown in Fig. 3-21, where $R_{fa}$ and $L_{fa}$ refer to the filter series resistance and inductance and $C_{fa}$ to the filter parallel capacitance. If the equilibrium point of the new system is sought the equation of the DC circuit must be added to the equation set. The filter equations for the DC current and voltage are given in (3-33) and (3-34).
Fig. 3-20 Evolution of the motor rotor angle for a load step

\[
\dot{i}_i = \frac{1}{L_{fa}} \left( v_s - v_{dc} - R_{fa} i_i \right) \quad (3-33)
\]

\[
\dot{v}_{dc} = \frac{1}{C_{fa}} (i_i - i_{dc}) \quad (3-34)
\]

Fig. 3-21 Motor controller connection to a non-ideal DC bus

When the MD is connected to the source through some impedance like in Fig. 3-21 the system has two possible stationary solutions for each rotor angle. This means that the number of equilibrium points has doubled compared to the connection to an ideal DC source. The values corresponding to both the stable and unstable equilibrium points of the MD of Fig. 3-21 for the parameters of the base case are shown in Table 3-1. Examining the characteristics of these solutions it is found that regardless of its stability condition one of the solutions is technically feasible while the other is not. The feasible solution corresponds to relative high voltage and low current operation while the unfeasible solution corresponds to low voltage and high current. In
practice the unfeasible solution saturates the current limit in the MD control system. It is also possible to observe from the values in the Table that the motor-load condition practically does not change between both solutions while the input filter and controller operation variables differ greatly.

The influence of the input filter parameters $L_{fa}$ and $C_{fa}$ can be analyzed by using the same mapping technique used for the generator. The result of such mapping is shown in Fig. 3-22. The Figure shows the result for the feasible solution. For certain values of the parameters the SEP turns to a UEP. Different is the situation at the unfeasible stationary solution. The corresponding equilibrium point is locally unstable along the whole range of mapped parameters.

| Table 3-1 State-variable values at the MD SEP and UEP |
|-------------------------|-------------------------|-------------------------|
| variable | Feasible (stable) | Unfeasible (unstable) |
| $i_d$ | 0.0 | 0.0 |
| $i_q$ | 262.04 | 262.04 |
| $\omega_r$ | 628.32 | 628.32 |
| $\delta$ | $0.2316 + n\pi$ | $0.2286 + n\pi$ |
| $i_{d\delta}$ | -64.605 | -671.018 |
| $i_{q\delta}$ | 293.61 | 2906.8 |
| $\omega_\delta$ | 262.04 | 262.04 |
| $v_{dc}$ | 460.21 | 79.788 |
| $i_L$ | 398.94 | 2301.1 |

The behavior of the MD can be verified at different points of the map by means of time domain simulations. The result of such simulation for a stable and an unstable case can be found Fig. 3-23 (a) and (b). The case (a) corresponding to the stable operation was obtained for values of the LC filter of $C_{fa} = 500 \mu F$, $L_{fa} = 250 \mu H$. On the other side, the case (b) corresponding to the unstable operation was obtained for values of the LC filter of $C_{fa} = 500 \mu F$, $L_{fa} = 400 \mu H$. In the stable case the MD is able to reach the desired operation while in the unstable case it oscillates around the intended operation. In this last case the MD operation runs into periodic
oscillations, or a limit cycle. Although the magnitude of these oscillations is not large and they may not mean a risk in the short term, this is an undesirable operation condition.

This subsection has analyzed the behavior of a simplified MD model that did not include the effect of the limits in the MD controller. However, those limits, which were presented in Fig. 3-18 greatly impact the behavior and the stability of the MD. The existence and magnitude of the limit cycle observed in Fig. 3-23 are given by the presence of the control limits. The next section focuses on the impact of the control limits on the MD behavior.

**Fig. 3-22** Influence of the input filter parameters $C_{fa}$, $L_{fa}$ on the local stability of the feasible solution of the MD

**Fig. 3-23** MD transient response for two different input filter values: (a) stable $C_{fa} = 500 \mu F$, $L_{fa} = 250 \mu H$ (b) unstable $C_{fa} = 500 \mu F$, $L_{fa} = 400 \mu H$
C. MD Limits and Nonlinear Behavior

To accurately analyze the nonlinear behavior of the MD it is required to add the limits and anti-windups in the model. When the limits are active the anti-windups loops take action changing the dynamic of the system. Therefore, the dynamic equations of the controller change; to account for that change equations (3-27), (3-29), and (3-31) are rewritten bellow.

\[
\dot{\omega}_d = K_\psi \left( \omega_{ref} - \omega_r \right) - K_{m} \left( \omega_k + K_p \left( \omega_{ref} - \omega_r \right) - i_{qref} \right) \tag{3-35}
\]

\[
\dot{i}_{dq} = K_\psi \left( i_{dref} - i_d \right) - K_{m} V_{dc} \left\{ \frac{1}{V_{dc}} \left[ i_{dref} + K_p \left( i_{qref} - i_d \right) - \omega_r i_q \right] - d_{dl} \right\} \tag{3-36}
\]

\[
\dot{i}_{qk} = K_\psi \left( i_{qref} - i_q \right) - K_{m} V_{dc} \left\{ \frac{1}{V_{dc}} \left[ i_{qref} + K_p \left( i_{qref} - i_q \right) + \omega_r i_d + \omega_r \lambda_{pm} \right] - d_{q} \right\} \tag{3-37}
\]

When the anti-windup schemes are active the values of \(i_{qref}\) and the d-q components of the duty-cycle, \(d_{dl}\) and \(d_{q}\) can be limited or not according to the operation condition. The unlimited values, or values before the limit are calculated by expressions (3-26),(3-28), and (3-30). The limited values are calculated according to the following expressions.

\[
i_{qref} = \min \left\{ \omega_k + K_p \left( \omega_{ref} - \omega_r \right), I_{lim} \right\} \tag{3-38}
\]

\[
d_{dq} = d_q \min \left\{ 1, \frac{1}{\sqrt{d_{d}^2 + d_{q}^2}} \right\} \tag{3-39}
\]

\[
d_{dl} = d_d \min \left\{ 1, \frac{1}{\sqrt{d_{d}^2 + d_{q}^2}} \right\} \tag{3-40}
\]

Like in the generator case the limit action cancels the existence of the unfeasible (or unstable) equilibrium point. Moreover, limits affect the MD operation. By restricting the maximum current on the motor armature the transient response is affected. Because of the two limits shown in Fig. 3-18 there are four possible operation situations with respect to the limits. These four possible situations corresponding to the combination of the limits are shown in Fig. 3-24. The first one on the top left corresponds to the unlimited case. The two on top right and bottom left represent the condition when only one of the limits is active at the same time. Finally, the case on the bottom right corresponds to the case when both limits are simultaneously active. The transition among the states is giving by the activation of the limit conditions also shown in the Figure.
Fig. 3-24 State transition diagram for the MD with limits at the current loop and duty cycle

The simulation results in Fig. 3-25 show the limit action during the MD operation. The case on the left, Fig. 3-25 (a) correspond to the same case in Fig. 3-23 (b); the Figure shows the activation of the duty cycle limit, which corresponds to the saturation of the modulator. This effect is created by an increased voltage drop in the inductance of the input filter $L_{fa}$. The case in Fig. 3-25 (b) corresponds to the simulation of the MD operation when $L_{fa}$ has been increased to 800\,$\mu$H. Because of this large value the voltage drop at the MD is considerably large requiring the controller of the MD to take larger currents to try to keep the desired operation condition. Those larger currents activate the current limit as shown in the Figure. As pointed out before the limit action keep the MD inside reasonable margins. However, the oscillatory conditions just described are operatively undesirable.

Fig. 3-25 Excitation of MD limits: (a) d limit only for $C_{fa}$=500\,$\mu$F, $L_{fa}$=400\,$\mu$H (b) d limit and I limit for $C_{fa}$=500\,$\mu$F, $L_{fa}$=800\,$\mu$H
IV. Nonlinear Behavior of the Interconnected System

The previous paragraphs analyzed the nonlinear characteristics of the components of the simplified autonomous power system shown in Fig. 3-1. This section will focus on analyzing the behavior of the generator connected through the MD by means of a multi-pulse rectifier.

A. Source-load Interaction and Nonlinear Behavior

In order to be able to change the operation conditions of the generator an additional linear passive load is connected to the system AC bus as shown in Fig. 3-26. The power and power factor of that load are changed for the study, but in all cases the MD is kept as the main load taking more than half of the total system power. When the generator stability was analyzed it was found that capacitive load types can drive the generator unstable; therefore, the additional AC load is set to have an adjustable power factor that can become leading under some circumstances. The load level and power factor of the AC load were changed during the study.

The bandwidth of the generator controller greatly affects the behavior of the generator-MD system. At higher bandwidths the system will transitions from stable operation to the collapse conditions in a small interval of load parameter change. On the contrary if the generator bandwidth is reduced a range of load level/power factor appears where the behavior corresponds to sustained oscillations. These oscillations are shown in Fig. 3-27. The DC voltage and current waveforms are shown in Fig. 3-27(a). In addition Fig. 3-27 (b) shows the activation of the MD current limit and duty cycle saturation during the same operation condition; both limits activate almost simultaneously.

The amplitude of the oscillations is limited by the action of the limits. In the simulation case shown in Fig. 3-27 the activated limits correspond to the MD while the generator excitation
limits were inactive. In our study it was not detected a case where the generator and MD limits act together being the main reason for this the unused generator capacity. Further bandwidth reduction will lead the system to a voltage collapse situation produced by the inability of the generator to follow the load variations that are relatively much faster [88].

![Figure 3-27](image)

**Fig. 3-27** Periodic behavior of the Generator-MD system: (a) current and voltages at the dc bus, (b) action of the MD antiwindup schemes; s-i represents activation of the current antiwindup while s-d the duty cycle one limit action

The periodic behavior of the system produce closed trajectories in the system state-space. Fig. 3-28 shows the projection of those trajectories over the generator $i_r$-$i_q$ current plane. The magnitude of the oscillation can produce undesired stresses on the equipment and may or may not lead to protection tripping. Therefore, it is important to evaluate the condition and possible issues it can bring over the safety of the power system.

The range where the system presents the behavior described above can be predicted from the equilibrium point map of the system in a similar way it was used for the load and the generator. For the generator-MD system one of the possible maps is shown in Fig. 3-29. The periodic oscillations just described correspond to a UEP in Fig. 3-29, which represents a locally unstable behavior. The unstable behavior originates at the generator. The reason of the small signal stability in this case is the combination of low regulation bandwidth with leading total power factor. However this power factor is in fact very close to one in this case. The level curves in Fig. 3-29 show that if the load power level is increased the unstable condition appears at a higher power factor.
Fig. 3-28 Orbits over the generator armature current plane

Fig. 3-29 mapping of generator active and reactive power level

The detected periodic oscillations appear at the region where the system bifurcates. This bifurcation is given by the boundary between the SEP and UEP region in Fig. 3-29.

B. Verification Using Full-order, Switching Model Simulations

The previous time domain simulations were done using average models of the system components. Therefore, it is necessary to verify the same behavior using more accurate models. Such verification can obtained from a detailed model of the same system in Fig. 3-26. In this
model a switched based power converter is used for the rectifier and the inverter at the motor controller. The controller of such inverter switching model includes the same algorithm than the controller in the average model.

The simulation results obtained with the detailed model are shown in Fig. 3-30. The Figure shows the evolution of the DC variables and the speed, as well as the duty cycle at the converter, which also saturates at some portion of the periodic oscillation.

![Fig. 3-30 Periodic behavior of the generator-MD system obtained from simulations done using detailed models of the system components](image)

V. Summary

This chapter analyzed the stability behavior of the main components of the autonomous power system under study. First the synchronous generator was analyzed finding that the nonlinear characteristic of the voltage regulator can introduce large-signal instabilities. To reduce the possibility of unstable behavior the generator excitation current is limited to only positive
values. This limitation cancels the undesired unstable equilibrium point improving the large signal stability. However, the generator can still go into periodic behavior that can create large stresses in the electric system.

The nonlinearities of the MD were also analyzed. The MD has several nonlinearities at the converter, motor, and mechanical load. The constraints (limits) in the operation are given by the current limit, which is a design choice, and the duty cycle, which is a physical limitation. The MD model, when connected to an ideal DC source through an L-C filter, presents two equilibrium points. One of the points corresponds to low-voltage, high-current operation and the other to high-voltage, low-current. The first operation condition is undesirable because of high-losses associated to the large current. Moreover, it is small-signal unstable for most of the parameter range. Operation at this point is prevented by the current limitation. Like in the generator case the limits introduced in the controller introduce periodic oscillation of the MD. The oscillations are also related to small-signal stability at the high-voltage, low-current operation point.

Finally the large-signal stability of the generator-MD system was analyzed. It was found that under certain conditions the system presents typical nonlinear behavior. The region in the system parameter space where this typical nonlinear behavior can appear corresponds to bifurcations of the system dynamic behavior where the equilibrium point has already become unstable from a small-signal point of view.

The conditions for periodic orbits to exist, and the characteristics of them, go beyond Lyapunov’s theory and the analytical tools derived from it. One way to approach the problem is by means of a state transition diagram like the ones shown in this chapter. This allows analyzing the sub-systems separately; using that analysis insight on the global system behavior can be obtained. However, there is not general way to predict the behavior of the global system and the sub-system analysis remains particular for the conditions analyzed.

The analysis of the stability characteristics under periodic operation requires appropriate mathematical tools. For simple systems, the Poincare-Bendixson criteria can be used to predict periodic behavior [11]. However, for higher order systems there are no established criteria for determining the existence of periodic behavior and its stability characteristics.
Chapter 4 Mapping the Parameter Space for Stable System Design

I. Introduction

Along the discussion in the previous chapter the eigenvalues of the state matrix corresponding to the linearized small-signal model were used to show ranges of parameters where the system operation was locally stable. The parametric information was obtained in the form of maps of parameter sub-spaces of the system like Fig. 3-5. Such type of information is relevant not only for analyzing the stability condition of a given system, but also for defining safe values of the system parameters during its design stage. The work in [92], for example, performs a thorough parameter search in order to assess the stability behavior of a boost converter. The usefulness of the mentioned parameter maps indicate the convenience of developing a tool that allows mapping the parameter space systematically and for more complex systems like the power system of Fig. 2-1. The mapping of such complex system is not straightforward due to the increased number of states. This chapter focuses on developing such systematic methodology to map the parameter space in terms of local stability.

To determine the local, or small-signal, stability of a system is relatively simple. It requires to obtain the linearized small-signal model and to calculate the eigenvalues of the state space matrix of such linear model. The Lyapunov indirect method can be used to determine the stability based on the eigenvalue real part. Because of the simplicity of this procedure it has been used in various systems and applications. The work in [93] calculates the eigenvalues of the linearized state-space matrix to determine areas of safe operation for a current source type MD. In [94] a similar procedure is used to evaluate the impact of magnetic saturation modeling in predicting the small-signal stability of a voltage sourced MD. More recently, similar methodology was used in an AC power electronics based distribution system [95]. All these studies were confined to a reduced set of components and a relatively small number of states. The work in [27] analyzes the stability and bifurcation behavior of an elementary power system with algebraic constraints.

In this chapter the work is based on the whole model of the autonomous power system of Fig. 2-1. Modeling considerations differentiate this system from power networks. In addition,
attention is given to the algebraic constraints in the model. The limits existing in the controllers of the loads and source are included in the models and their impact on the stability map is analyzed.

A. Lyapunov Indirect (linearization) Method

Provided the original system model represented by equation (1-1) is time invariant, and the function \( f(x, p) \) is continuously differentiable, it is possible to apply a Taylor series expansion to the equation (1-1). Applying that expansion around an equilibrium point \((x_o, p_o)\) produces:

\[
x_o + \delta x = f(x_o, p_o) + \left. \frac{\partial f(x, p)}{\partial x} \right|_{x_o, p_o} \delta x + \left. \frac{\partial f(x, p)}{\partial p} \right|_{x_o, p_o} \delta p + \text{h.o.t}
\]

Where the first term in the right corresponds to the stationary value at the equilibrium point, the second and third terms to the partial derivatives respect to the state variables and parameters respectively, and the last to the higher order terms (h.o.t.). Neglecting the h.o.t., changing the coordinate system in order to have the equilibrium state at the origin, and considering the set of parameters \( p \) invariant the dynamic of the system in is given by:

\[
\delta \dot{x} = \left. \frac{\partial f(x, p)}{\partial x} \right|_{x_o, p_o} \delta x
\]

Under the mentioned condition of \( p \) invariance the first factor on the right of (4-2) is constant and the previous equation can be related to the behavior of a linear time invariant system as shown in the next expression:

\[
\dot{x} = Ax
\]

where: \( A = \left. \frac{\partial f}{\partial x} \right|_{x_o} \)

where the matrix \( A \) is called the system Jacobian. Equation (4-3) approximates the behavior of the original system only in a close vicinity of the equilibrium point; therefore, the linear model given by the previous equation can be used to approximate the behavior of the original system. This approximation is valid only in the neighborhood of the equilibrium point.

Providing that the matrix \( A \) is nonsingular, the stability of the linear system (4-3) can be determined through the eigenvalues of \( A \). According to those eigenvalues the equilibrium points can be classified in stable equilibrium points (SEP) or unstable equilibrium points (UEP). This is
done according to the value of the real part of the matrix $A$ eigenvalues expressed in the following relationships:

$$\text{Re}\left(\lambda [A]_{x_i}\right) < 0 \quad \forall \lambda \quad \Rightarrow \text{SEP}$$

$$\text{Re}\left(\lambda [A]_{x_i}\right) > 0 \quad \text{for some } \lambda \quad \Rightarrow \text{UEP}$$

$$\text{Re}\left(\lambda [A]_{x_i}\right) = 0 \quad \text{for some } \lambda \text{ and no } \text{Re}\left(\lambda [A]_{x_i}\right) > 0 \quad \Rightarrow \text{no stability conclusion}$$

### B. Differential-Algebraic System Model

In order to better describe the operation of the electric power system under study it is required to add additional algebraic, usually nonlinear, equations in the model. These equations originate in the characteristics of the energy flow through the system. Associated to these algebraic equations there is a set of algebraic variables, here identified with $z$, that are required to fully define the system operation. Therefore, in a more complete way than (1-1) the generic model of the power system can be described by the set of equations in (4-5). The characteristics of this model, illustrated in an elementary power system, have been described in [96].

$$\dot{x} = f(x,z,p)$$

$$0 = g(x,z,p)$$

(4-5)

An equilibrium point now must satisfy the condition:

$$0 = f(x,z,p)$$

$$0 = g(x,z,p)$$

(4-6)

For the differential algebraic system, the small-signal linearized model is obtained from the first order terms of the series expansion of the two equations in (4-5). Therefore, the full Jacobian matrix of this linearized system is given by:

$$A = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial z} \end{bmatrix}_{x,z}$$

(4-7)

Note that each of the elements of matrix $A$ is by itself a matrix of the following dimensions:
\[ D_x f = \left[ \frac{\partial f}{\partial x} \right]_{x=x_0}, \quad D_z f = \left[ \frac{\partial f}{\partial z} \right]_{z=z_0}, \quad D_x g = \left[ \frac{\partial g}{\partial x} \right]_{x=x_0}, \quad D_z g = \left[ \frac{\partial g}{\partial z} \right]_{z=z_0} \] (4-8)

To evaluate the stability condition of the equilibrium points of a differential algebraic system all the components of the full Jacobian must be considered. An equilibrium point \((x_o, z_o, p_o)\) of the system (4-6) is stable if \(D_z g\) is nonsingular and the reduced Jacobian \(J\), which is given in equation (4-9), has all eigenvalues with negative real part.

\[ J = D_x f - D_x f \left( D_x g \right)^{-1} D_z g \] (4-9)

Note that in case the system dynamic does not depend on the algebraic variables, \(D_z f = 0\), the stability character of the equilibrium points is determined like in (4-4).

The feasibility region is useful concept in the stability analysis of a nonlinear system with algebraic constraints. It is identified as the set of all operation states that can be achieved quasistatically from a stable equilibrium point by mean of a continuous parameter variation. In this way, two equilibrium points belong to the same feasibility region if both are small-signal stable according to (4-9) and can be connected by a continuous variation of the parameters of the system. More formally this can be mathematically stated as follows (modified from [96]):

Given a stable equilibrium point \((x_o, z_o)\) for parameter values \(p_o\), the connected set of small-signal stable operating points, which contains \((x_o, z_o, p_o)\), is called the feasibility region of \((x_o, z_o, p_o)\). And the boundaries of this region where the stable condition is lost are the feasibility boundaries.

The mapping of the parameter space to be developed in this chapter is in fact the mapping of the system feasibility region.

II. Methodology

This section discusses the principles, tools and issues to face in order to develop the methodology to obtain the feasibility region map in order to determine the ranges of parameters where the system operation is stable.
A. Mapping Principle

To obtain the desired parameter space map means finding the equilibrium point local stability when the set of parameters $p$ in (4-5) change. The set of changing parameters can be associated to the load, the controllers, or any of the generator or motor physical parameters. A possibility to determine that stability would be to simulate the system operation at each of the desired $p$ values. However, implementing such possibility requires significant computation resources and would not be efficient. Therefore, a more efficient way to obtain the map is developed here.

The feasibility region is the set of equilibrium points that can be reached quasistatically from a stable equilibrium point by means of a smooth parameter variation. This means that the small-signal model obtained by linearizing the system around a stable equilibrium point can be used as base of the calculations. To obtain the small-signal model of power electronics circuits the average models of those circuits must be used. Once the system has been linearized, all tools developed for linear systems can be applied, like Hurwitz, Nyquist, Root-locus, etc. In our case we found it convenient to use the Lyapunov indirect method. This method expressed in (4-4) allows classifying the character of an equilibrium pointing in locally stable or unstable based on the eigenvalues of the state-space matrix.

Several tasks are required in order to map the feasibility region of a given system model generically expressed as (4-5). The first step requires determining the equilibrium point where the local stability will be analyzed. The possibility of simulating the system until reaching stationary operation has been discarded; in addition, it would not work in cases where the target point is unstable. An alternative is to estimate the values of the states at the desired operation using some state estimation technique. The mathematical details of such state estimation procedure are described in following paragraphs. Once the equilibrium point is found the small-signal linearized system can be obtained from its first order Taylor series expansion. The eigenvalues of the Jacobian matrix of the linearized system indicate the stability at the equilibrium point under study. The details of the developed mapping algorithm are explained in section III. The application to study the parameter effect on the stability of the power system under study is described in section IV. Section V evaluates the method and the results obtained, while section VI summarizes the main evaluation of the power system stability.
B. State estimation

To map the system feasibility region it is necessary to determine its equilibrium points. This means solving the set of nonlinear equations given by (4-6), which involves both differential and algebraic equations. For $n$ and $m$ number of differential and algebraic variables respectively, the total number of variables to solve for would be $n+m$. However, if it is possible to obtain explicit expressions for the $m$ algebraic variables from the $n$ algebraic equations the formers can be replaced in the differential equations. In such case it is possible to reduce the system to the set of $n$ differential equations represented by:

$$0 = f_g(x, p)$$

(4-10)

Where the sub-index $g$ indicates that the algebraic constraints have been eliminated from the system. This in fact is a non-trivial issue and actually has great impact on the convergence of the equilibrium point search algorithm.

To solve (4-10) a nonlinear equations solving technique like the Newton-Raphson method could be used. However, when the number of states increases finding the solution of a diverse set of nonlinear equations can be extremely difficult. Moreover, it could easily happen that the search does not converge to the intended solution. Therefore, an optimization technique is preferred. This type of technique will provide an answer even in case the exact solution could not be found. That solution will be the optimal in terms of an evaluation parameter usually related with the residual. Most commonly the optimality is evaluated with the least minimum squares of the residual. An optimization problem is generally formulated in the following manner:

$$\min_x j(x)$$

subject to: \hspace{1cm} $G_i(x) = 0 \quad i = 1, \ldots, h$ \hspace{1cm} (4-11)

In (4-11) $j(x)$ is called the objective function and $G_i(x)$ is the optimization constraints. In our case the function to minimize correspond to the state equations, $f_g$, making the derivatives of the state variables equal to zero. The constraints can for example be used to establish values for the system inputs, outputs, or can be even be used to force state variable values.

In this work a sequential quadratic programming (SQP) algorithm was used to search for the equilibrium points. SQP algorithms represent the state of the art in optimization methods [97].
The sequential quadratic programming algorithm method can be implemented in Matlab for the equilibrium point search through the instruction \texttt{trim} [98].

III. Automation of the Mapping Procedure

A computer based algorithm was implemented in order to map the parameter space of the electric power system in Fig. 2-1. The mapping procedure works over the average models of the system components. The code of the algorithm was implemented in Matlab language. The Matlab environment allows performing the operations required by the method over a model of the system implemented in Simulink. Therefore, a block diagram of the power system average model can be mapped using the algorithm developed in this chapter. The details of such algorithm are described in the next sections.

A. Algorithm Overview

Given a desired subspace of the parameter space that is desired to map, the mapping algorithm will determine the local stability of a set of equilibrium points in that subspace. Those equilibrium points will be classified in stable equilibrium points (SEP) or unstable equilibrium points (UEP). Given a set of parameters $p_o$, in order to do the classification the mapping algorithm must perform the following computational steps:

1. Estimate the equilibrium state $x_o$ corresponding to $0 = f_x(x_o, p_o)$

2. Obtain the linearized small-signal model around the equilibrium state $x_o$

   \[ \dot{x} = Ax \quad \text{where: } A = \frac{\partial f}{\partial x} \bigg|_{x_o} \]  

   (4-12)

3. Calculate the eigenvalues of the state-space matrix of the linearized model

4. Apply Lyapunov’s indirect method criteria expressed by:

   \[ \text{Re} \left( \lambda [A]_{\lambda} \right) < 0 \quad \forall \lambda \quad \Rightarrow \quad \text{SEP} \]

   \[ \text{Re} \left( \lambda [A]_{\lambda} \right) > 0 \quad \text{for some } \lambda \quad \Rightarrow \quad \text{UEP} \]
After the previous four steps have been covered the procedure can move to the next point to be mapped. The systematic application of this methodology to the system model provides the parameter map where conditions for SEPs or UEPs can be easily identified. The values of the state variables will also change as a consequence of the parameter change and those values at the equilibrium points can also be extracted from the procedure. A flowchart of the procedure described is presented in Fig. 4-1.

![Flowchart of the basic mapping procedure](image)

**Fig. 4-1 Flowchart of the basic mapping procedure**

Implementing each of the steps described in the methodology defined above requires solving mathematical problems. In a small (low-order) system the solution of such problems is relatively straightforward. On the contrary, dealing with a complex system with a large number of states
and large separation among time constants requires a higher computation effort. If the limits existing in the controllers of the source and loads are included in the system model the estimation and linearization tasks become even more complex. The situation created by those limits requires special treatment in order to obtain an accurate small-signal linearized model. Therefore, the steps one and two are analyzed with detail in the following two sections.

B. Estimation of the Equilibrium State

The difficulty of estimating the equilibrium states comes from the nonlinearity of the power system model. Nonlinear systems can have multiple equilibrium states and finding all of them becomes in many cases not practical. However, our interest is to find those states that are mathematically and physically feasible in terms of the power system operation. This provides a range of variables where the search must be conducted. Other difficulty comes from the fact that some of the equilibrium points to map are unstable; in such case the convergence of the state estimation algorithm may be challenged further.

To search for the equilibrium points of the system an optimization technique was used. The mathematical characteristics of such optimization, which uses a SQP algorithm, were described in the previous section. To improve the convergence of such algorithm it is desired that the algebraic equations in the original model in (4-5) be removed. There are several options to do that removal. The first one is to obtain explicit solutions of the algebraic equations that can be used to eliminate the algebraic variables. Nevertheless, in some cases this option is not available, the alternative then is to introduce auxiliary states that augment the system state and can be used to eliminate the algebraic variables. This is justified from singular perturbations point of view. Moreover, these auxiliary states can be related to parasitics or other characteristic physically existing in the system that can be then included in the model.

The SQP optimization algorithm used to find the equilibrium point bases its search procedure on an initial guess of such equilibrium point, which can be supplied externally. Therefore, it is important to provide the SQP algorithm with a good initial guess. This guess can be obtained from time domain simulation of the system provided the map starts at a stable equilibrium state, where the simulation converges. This is a good approach for the point where the map starts. If the parameter variation is smooth and the changes between successive mapping points are not
large, the previous equilibrium state can be used as initial guess for the next equilibrium point search. In this way the mapping space can be covered using small changes in the parameters being mapped. The described procedure is schematically shown in Fig. 4-2.

Since the algorithm used for equilibrium point search base its convergence criteria on the deviation norm it is useful to use per unit values of the system state variables. In case state variables are not used it is convenient to do some block algebra manipulations in order to make the stationary values of the state variables have the same order of magnitude.

**Fig. 4-2 Mapping procedure to facilitate the convergence of the equilibrium point search algorithm**

During the search of the equilibrium point it is relevant to detect if the variables that are subject to saturation (limited) have reached that situation and if their value is at the maximum or minimum limits. Detecting this situation is important when computing the linearized small-signal model as will be pointed out in the next section. It was detected during the implementation of the algorithm that due to oscillations of the state estimation algorithm a wrong result is obtained producing the consequence of a bad limit detection condition. Therefore, a special routine to detect the condition of variables subject to saturation and reject false limit situations was implemented. The flowchart of such routine is shown in Fig. 4-6. The saturation condition is detected by comparing to input and output of the limiting block. If the limit is hit for two consecutive points of the map the equilibrium point is considered as reaching a saturation situation. Otherwise, additional points in the neighborhood of the parameter space are checked to
further assess the situation of the limits. If this additional check finds more points where the variable under analysis reaches the same saturation condition, the limit will be marked as active and the mapping procedure will continue to the following steps. The reason behind this additional limit situation check was the detection of situations where the state estimation algorithm oscillates in and out of the limit situation for consecutive points of the map.

![Algorithm](image)

**Fig. 4-3 Algorithm used to reject false limit conditions**

**C. Linearized Small-Signal Model**

Once the equilibrium point is found the system model can be linearized around that point producing the small-signal model. In case a limited variable has reached the saturation condition it will change the small-signal dynamic of the system. As an example to explain this situation the speed control loop of the MD is considered; which is shown in Fig. 4-4. The output of the speed loop controller is the reference value for the q-axis current controller, \( i_{qref} \). If we consider that the value of \( i_{qref} \) is at a saturation level, small changes at the input will not affect the output. Small-perturbations do not propagate along a saturated path; therefore, the small-signal gain of a saturated limiting block is equal to zero. For small-signal calculations the saturated block can be replaced as block with zero gain, or a zero gain block can be added in series with the saturation block as shown in Fig. 4-4.

During the implementation of the mapping procedure it was found that in order to improve the convergence of the state estimation procedure the slope of the saturated part could be
adjusted. In Fig. 4-5 it is shown the saturation function with the slopes at the saturated and unsaturated portion being $\alpha$ and $\mu$, respectively. Increasing the value of $\mu$ usually improves the convergence of the state search. For the small-signal calculation the gain of the saturation path must be made equal to zero regardless of this change in the value of $\mu$. In practice it was found that this approach produced good estimates for the linearized small-signal model.

**Fig. 4-4 Saturation block and antiwindup scheme corresponding to the MD speed loop**

![Saturation block and antiwindup scheme](image)

**Fig. 4-5 Saturation function of a limited value: $\alpha$ is the slope at the unsaturated condition, usually equal to one, $\mu$ is the slope during saturation**

**D. Identification and Classification of Eigenvalues with Small Real Components**

Due to existence of fast dynamics eigenvalues with very high frequency and small real part can appear during the system small-signal linearization. Some of these eigenvalues can be created by resonances in parasitic circuits. In such case the energy associated to such high frequency phenomena is very small. Because of the system nonlinearity and the approximation of the equilibrium point search, sometimes these eigenvalues can cross the imaginary axis and
have a positive real part. However, while some of the system variables may present a sustained high-frequency oscillation the behavior of the system is still stable. It is important then to be able to detect these conditions and identify these types of eigenvalues separately from the ones that indicate a real stability problem. Fig. 4-6 shows the flowchart of the procedure used to identify those high frequency eigenvalues.

The algorithm in Fig. 4-6 computes the real part of the high frequency eigenvalue with the dominant stable one and stores them in maxP and minN respectively. If the value of maxP is smaller than a certain fraction \( k \) of minN then the eigenvalue is very close to the imaginary axis and the system operation is most probably stable. If the real part of the positive eigenvalue is larger than this threshold value then most probably it corresponds to an unstable situation. In any case the conclusion about the stability is not definitive and additional tests may be required. It is worth mentioning that a good system model able to produce good estimates of the equilibrium points does not present this type of eigenvalue situation very frequently.

![Fig. 4-6 Algorithm used to reject false limit conditions](image)

IV. Case Analysis

To study the impact of some of the system parameters of the power system under study several case studies were defined. Each of these case studies analyzes the impact of a particular parameter or group of parameters in the system stability. The list of those case studies is given in
Table 4-1. The following sections analyze the impact of those parameters in the system stable operation.

\textit{A. Case definition}

The cases presented in Table 4-1 represent a wide set of parameters covering the different equipment connected in the power system of Fig. 2-1. Some of the parameters change the system dynamic response and also the circuit stationary operation magnitudes. Example of these types of parameters can be series AC impedance. On the contrary, other parameters only change the dynamic behavior like the case of a controller bandwidth.

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
Case nr & Case description & Parameters to map \\
\hline
1 & MPTR capacitance, MD bandwidth & \(C_{fa}, T_e, fbw_{MD}\) \\
\hline
2 & MPTR inductance, MD bandwidth & \(L_{fa}, R_{fa}, T_e, fbw_{MD}\) \\
\hline
3 & Generator bandwidth, feeder length & Cable length, \(T_e, fbw_{g}\) \\
\hline
4 & Generator - MD controller interaction & \(fbw_{g}, fbw_{MD}, T_e\) \\
\hline
\end{tabular}
\end{center}

The MD load power level represented by the torque \(T_e\) is used as reference in some of the maps. The MD speed loop bandwidth is referenced by \(fbw_{MD}\) and the generator V loop bandwidth by \(fbw_{g}\). The following paragraphs describe in detail the mapping results corresponding to case one. The results corresponding to the other cases are also discussed while the information presented is reduced. Some of the cases are further analyzed in the following section.

\textit{B. Multi-pulse Transformer Rectifier Filter Parameter Analysis}

The purpose of case one is to study the impact of the capacitor of the multi-pulse rectifier filter on the system stability behavior. Together with case two, which studies the multi-pulse rectifier filter inductance, they determine the impact of that filter. The dimensioning of the filter is based not only on stability considerations but also on other requirements like the power quality in the system. Therefore, it is important to know the range where the filter basic dimensioning
(\(L_{fa}, R_{fa}, C_{fa}\)) does not impact on the system stability behavior. Several alternatives, or sub-cases, are considered; they are shown in Table 4-2.

The mapping corresponding to case 1-a is shown in Fig. 4-7. The filter capacitance was mapped from an original rated value of 500 µF to 25 µF. The current limit in the \(i_q\) current controller of both MDs connected to the DC bus was set at 1.3 times the nominal current. The first characteristic the \(C_{fa}-T_e\) map shows is the instability appearing at low capacitance values that also depends on the load level. The Figure also shows the limit action at 1.3 times the rated torque. At that point the speed regulator of the 100kW MD reaches the maximum allowed current and the input of the MD \(i_q\)-current loop becomes constant. Additional increments in the mechanical torque cause the modulator to also saturate. It is possible to observe in Fig. 4-7 that all points with load torque greater than 1.3 pu become stable meaning that when the current limit gets active it cancels the unstable oscillation appearing at low capacitance values. This denotes the dynamic interaction between the MD speed controller and the filter at the DC bus.

**Table 4-2 Parametric studies considered for case one**

<table>
<thead>
<tr>
<th>Case nr</th>
<th>Case description</th>
<th>Parameters to map</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-a</td>
<td>Filter capacitance vs. MD load level 400Hz</td>
<td>(C_{fa}, T_e)</td>
</tr>
<tr>
<td>1-b</td>
<td>Filter capacitance vs. MD load level 800Hz</td>
<td>(C_{fa}, T_e)</td>
</tr>
<tr>
<td>1-c</td>
<td>Filter capacitance vs. MD bandwidth 400Hz</td>
<td>(C_{fa}, fbw_{MD})</td>
</tr>
<tr>
<td>1-d</td>
<td>Filter capacitance vs. MD bandwidth 800Hz</td>
<td>(C_{fa}, fbw_{MD})</td>
</tr>
<tr>
<td>1-e</td>
<td>Filter capacitance vs. inductance 400Hz</td>
<td>(C_{fa}, L_{fa})</td>
</tr>
<tr>
<td>1-f</td>
<td>Filter capacitance vs. inductance 800Hz</td>
<td>(C_{fa}, L_{fa})</td>
</tr>
</tbody>
</table>

From the mapping algorithm it is also possible to obtain the characteristic frequency of the dominant eigenvalue (or dominant pair of complex conjugate eigenvalues). The dominant eigenvalues are the ones that are closest to the imaginary axis. In case the system is unstable and there exist eigenvalues in the right half plane those eigenvalues become dominant. Fig. 4-9(a) shows the frequency of the dominant eigenvalue. The \(x\) and \(y\) coordinates correspond to the map in Fig. 4-7 while the \(z\) coordinate is the mentioned frequency. It is possible to see that when the system becomes unstable the dominant frequency is in the range of 2000-3000 Hz. Fig. 4-9(b)
pictures the loci of the unstable eigenvalues in the complex plane when the parameters are changed.

The computation of the stationary operation variables during the mapping procedure allows obtaining additional data of the system operation. The pictures in Fig. 4-8 show some of those possible calculations. In Fig. 4-8(a) voltage at the MD input and the load torque Fig. 4-8(b) shows the 100kW MD speed as function of the load torque where it is possible to observe the loss of speed regulation due to the current limitation.

![Fig. 4-7 Mapping of the MPTR DC filter capacitor vs. MD mechanical load level](image1)

![Fig. 4-8 Eigenvalue analysis for 400 Hz operation: (a) characteristic frequency of the dominant eigenvalue. (b) unstable eigenvalue loci in the complex plane](image2)
The case 1-b map is shown in Fig. 4-10 where the unstable area has shrunk considerably. The eigenvalue analysis is presented in Fig. 4-11. The characteristic frequency of the dominant eigenvalue in Fig. 4-11(a) shows the existence of some high-frequency eigenvalues appearing when the capacitance has values close to the nominal. This eigenvalues do not lie in the right half plane and may indicate the presence of some resonance in the circuit. The eigenvalue loci in Fig. 4-11(b) show that the characteristic frequency of the unstable eigenvalue is in the same range as the 400 Hz case. This points the source of the instability to the DC side of the circuit because that characteristic frequency has not been affected by the change in the AC circuit operation.

---

**Fig. 4-9 Motor operation magnitudes at different load levels. (a) Voltage at MD input terminals. (b) Rotational speed**

**Fig. 4-10 MPTR filter capacitance vs. load mechanical torque at 800 Hz**
Fig. 4-11 Eigenvalue analysis for 800 Hz operation: (a) characteristic frequency of the dominant eigenvalue. (b) unstable eigenvalue loci in the complex plane

Fig. 4-12 present the mapping cases where the filter capacitance $C_{fa}$ is mapped against the speed bandwidth of the two MDs connected to the DC bus. The ratio between speed bandwidth and current loop bandwidth is kept constant during the speed bandwidth change. Fig. 4-12(a) corresponds to an AC operation frequency of 400 Hz while Fig. 4-12 (b) corresponds to an AC frequency of 800 Hz. The unstable region appears for relatively low DC capacitor values and high MD bandwidths. Like in the previous case the high frequency operation considerably improves the system stability. High frequency operation allows for a stable operation with smaller capacitances or higher bandwidth of the MD speed controller. The change of the motor bandwidth or DC bus capacitance do not imply a change in the stationary conditions; in this way no operational limit is activated during the mapping like in the previous case. The characteristic frequency of the dominant eigenvalues is very similar. This indicates a relation in the phenomena creating the instability in both cases.

The case where the filter capacitance and inductance are changed together is presented in Fig. 4-13(a) and (b) for the 400 Hz and 800 Hz respectively. During the computation of these maps the filter capacitance $R_{fa}$ was kept constant as well as all other system parameters. An inductance increment is non-beneficial for the system stability due to the increased voltage drops during load current changes. On the other side, a larger capacitance can mitigate the effect and improve the stability. Like in the previous two cases a higher frequency operation considerably enlarges the feasible operation region.
Fig. 4-12 MPTR filter capacitance vs. MD speed loop bandwidth at a system frequency of:  
(a) 400 Hz (b) 800 Hz

Fig. 4-13 MPTR filter capacitance vs. inductance at a system frequency of:  
(a) 400 Hz (b) 800 Hz

The large effect of the multi-pulse rectifier filter inductance observed in Fig. 4-13 motivates case study number two. This case study the impact of the filter inductance parameters, $L_{fa}$ and $R_{fa}$, on the stability by mapping these parameters against the load level at the MDs connected to the multi-pulse rectifier DC bus. The parametric studies for this case are specified in Table 4-3. The first two study the effect of the inductance and resistance separately while the last two consider a simultaneous change of inductance and resistance in order to maintain constant the ratio between them.
Table 4-3 Parametric studies considered for case two

<table>
<thead>
<tr>
<th>Case nr</th>
<th>Case description</th>
<th>Parameters to map</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-a</td>
<td>MPTR inductor filter L vs. MD load level 400Hz</td>
<td>$L_{fa}$, $T_e$</td>
</tr>
<tr>
<td>2-b</td>
<td>MPTR inductor filter L vs. MD load level 800Hz</td>
<td>$L_{fa}$, $T_e$</td>
</tr>
<tr>
<td>2-c</td>
<td>MPTR inductor filter R vs. MD load level 400Hz</td>
<td>$R_{fa}$, $T_e$</td>
</tr>
<tr>
<td>2-d</td>
<td>MPTR inductor filter R vs. MD load level 800Hz</td>
<td>$R_{fa}$, $T_e$</td>
</tr>
<tr>
<td>2-e</td>
<td>MPTR inductor filter L, R vs. MD load level 400Hz</td>
<td>$L_{fa}$, $R_{fa}$, $T_e$</td>
</tr>
<tr>
<td>2-f</td>
<td>MPTR inductor filter L, R vs. MD load level 800Hz</td>
<td>$L_{fa}$, $R_{fa}$, $T_e$</td>
</tr>
</tbody>
</table>

The mapping results for cases 2-a and -b are shown in Fig. 4-14(a) and (b) respectively. The detrimental effect of a larger inductance on the stability was expected from the previous case. The change of the load power makes the current limit at the 100 KW MD activate when the mechanical load reaches 1.3 pu of the rated power. The activation of the limit with the consequent cancellation of the speed loop control dynamics and the loss of the constant power load characteristics restore stable operation. The comparison of the 400 Hz and 800 Hz operation shows similarities with the previous cases; a higher frequency operation is beneficial for the system stability also in this case allowing for a larger filter inductance. The results in Fig. 4-14 (a) and (b) also show the significant impact of the AC operation frequency on the stability.

![Fig. 4-14 filter inductance vs. MD mechanical load level at a system frequency of: (a) 400 Hz (b) 800 Hz](image-url)
The increment of the filter resistance alone in the range considered does not create an unstable zone in the mapping. More interesting are the maps where both the inductance and resistance of the filter change together. The corresponding results can be seen in Fig. 4-15. The resistance increment changes the stationary operation of the system activating the limits if the series resistance is large enough. This limit activation also cancels the CPL behavior and restores stable conditions.

![Fig. 4-15 filter inductance and resistance vs. MD mechanical load level at a system frequency of: (a) 400 Hz (b) 800 Hz](image)

**C. Impact of AC Circuit Parameters**

The purpose of the case studies three and four in Table 4-1 is to analyze the impact of critical parameters of the AC portion of the power system. Case three concentrates on the parameters of the generator feeder while case four focuses on the total power connected to the AC system. The feeder introduces a series impedance in the link between load and generator. The characteristics of this impedance can impact the system operation and affect the stability behavior. The analysis considers the parametric studies summarized in Table 4-4. Along the study the per unit values of the feeder inductance and resistance, $L = 12 \, \mu H/m$ and $R = 1 m\Omega/m$, remain constant while the length is used as changing parameter. Moreover, remote point of regulation is assumed for the voltage reference of the generator excitation regulator, which is located at the system AC bus.

The mapping result corresponding to cases 3-a and -b are shown in Fig. 4-16. In that Figure it is easy to notice that the 400 Hz case is relatively more stable than the 800 Hz one. This constitutes a major difference when compared to the previous two cases. In the case in Fig. 4-16 the feeder series reactance is proportional to the AC system operation frequency; therefore, the
operation at 800 Hz faces considerably larger impedance than the 400 Hz operation. The total feeder length before the system becomes unstable is considerable large when compared to the base value of 30 m.

Table 4-4 Parametric studies considered for case three

<table>
<thead>
<tr>
<th>Case nr</th>
<th>Case description</th>
<th>Parameters to map</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-a</td>
<td>Feeder length vs. MD load level 400Hz</td>
<td>Length, $T_e$</td>
</tr>
<tr>
<td>3-b</td>
<td>Feeder length vs. MD load level 800Hz</td>
<td>Length, $T_e$</td>
</tr>
<tr>
<td>3-c</td>
<td>Feeder length vs. Generator AVR bandwidth 400Hz</td>
<td>Length, $f_{bw_g}$</td>
</tr>
<tr>
<td>3-d</td>
<td>Feeder length vs. Generator AVR bandwidth 800Hz</td>
<td>Length, $f_{bw_g}$</td>
</tr>
</tbody>
</table>

Fig. 4-16 Feeder length vs. MD mechanical load level at a system frequency of: (a) 400 Hz (b) 800 Hz

The maps in Fig. 4-17 correspond to the case where the feeder length is mapped against the bandwidth of the generator voltage control loop. This case again shows a better stability for the low frequency operation. For the 400 Hz the control loop bandwidth required to make the operation unstable is considerably large when compared to a base value of $f_{bw_g} = 300$ Hz. The case of 800 Hz shows that for long enough feeders the behavior becomes unstable at all bandwidths. In addition, Fig. 4-17(b) shows unstable behavior for low V loop bandwidths, corresponding to 100 Hz. Instability at very low generator frequency is expected for this type of system and will be further analyzed in another case.
Fig. 4-17 Feeder length vs. generator voltage loop bandwidth at system operation of:
(a) 400 Hz (b) 800 Hz

Case study four focuses on the generator voltage loop bandwidth impact on the system stability. The parametric studies related to this case are summarized in Table 4-5. The analysis studies the impact of the generator bandwidth against the MD load magnitude and speed control loop bandwidth. The maps corresponding to case 5-a and -b can be seen in Fig. 4-18(a) and (b) respectively. The 400 Hz shows an unstable area when the generator bandwidth is large and the MDs are operating above the rated conditions. On the other side, the 800 Hz case presents only a reduced unstable area at the generator low bandwidth condition; this low bandwidth value corresponds to 100 Hz.

Table 4-5 Parametric studies considered for case four

<table>
<thead>
<tr>
<th>Case nr</th>
<th>Case description</th>
<th>Parameters to map</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-a</td>
<td>Generator V bandwidth vs. MD load level 400Hz</td>
<td>fbw₉, Tₑ</td>
</tr>
<tr>
<td>4-b</td>
<td>Generator V bandwidth vs. MD load level 800Hz</td>
<td>fbw₉, Tₑ</td>
</tr>
<tr>
<td>4-c</td>
<td>Generator V bandwidth vs. MD bandwidth 400Hz</td>
<td>fbw₉, fbwMD</td>
</tr>
<tr>
<td>4-d</td>
<td>Generator V bandwidth vs. MD bandwidth 800Hz</td>
<td>fbw₉, fbwMD</td>
</tr>
</tbody>
</table>
The cases in Fig. 4-19 map the generator voltage loop bandwidth against the MD speed loop bandwidth. In this case, the generator control bandwidth is mapped on the lower side of the 300 Hz base value. The Figure shows that the stability of the interconnected system is very sensitive to low generator control bandwidths. The unstable behavior for low source bandwidths is rather independent of the load bandwidth as the almost vertical bound between the stable and unstable zone pictures. The comparison between the 400 Hz and 800 Hz indicates a larger requirement on the generator control bandwidth when the system is operating at higher frequencies.
V. Results Validation

This section discusses about validation of the results presented in the previous section. The validation concerns both to the methodology and the models used to obtain the results.

A. Validation with Time Domain Simulations

A fundamental part of the research work is to experimentally validate the results of the study. However, in some cases that validation is not possible because the required prototypes are not available. In such cases a possible way to verify the results is through computer simulations based on good models of the study system. These models must have been previously validated.

The maps showing the parameter effect on the system stability were obtained using a system model where all switching converters were modeled with average models. A first to verify consists in testing if the mapping algorithm worked appropriately identifying stable and unstable operating regions. This verification can be done with the same system average models used for the mapping. The second stage of verification is related to the validity of the models used in the study. For this verification the results obtained with the average models have to be verified with higher quality models; this requires a higher degree of detail. For example detailed circuits and switching models of power electronics converters can be used in this verification. In the paragraphs below some results show the verification with time domain simulation. In addition, some of the maps are verified by means of time domain simulation.

To verify the mapping algorithm time domain simulations with parameters corresponding to different points of the map can be used. If for example the parameter map of the multi-pulse rectifier filter capacitance vs. mechanical torque at 800 Hz of Fig. 4-10 is considered, some points at the unstable and stable region can be used for the verification. Fig. 4-20 shows the result for two points of the map obtained from the same model used for Fig. 4-10. The simulation in Fig. 4-20(a) corresponds to an operation with values of \( T_e = 1.2 \) pu and \( C_{fa} = 40 \) \( \mu \)F. On the other side the simulation in Fig. 4-20(b) corresponds to operation at \( T_e = 1.3 \) pu and \( C_{fa} = 40 \) \( \mu \)F. At this point the MD speed controller has reached a level where the current limit gets activated. Additional points could be also considered to show that the points in the map correctly identify the stability condition of the system.
More relevant is the verification with simulations done using detailed switching models of the system. Fig. 4-21 shows the result of such verification done for the mapping case of Fig. 4-7. The case in Fig. 4-21(a) corresponds to an unstable operation while the case in Fig. 4-21(b) corresponds to a stable condition.

A more complete and exhaustive verification of the results consists in doing the same type of parameter mapping but using detailed models of the system. In case switching models are used it is possible neither to estimate the stationary operation nor to use linearized small-signal models to determine the local stability behavior. Therefore, the map must be done by means of checking the stability at each of the desired operating points using time domain simulations. In this way it requires a considerably large amount of computational resources and simulation time. In
addition, the integration of the dynamic equations in time domain allows verifying the stability during transient conditions or large-signal stability. In order to do this verification the loads must be started in simulation in a similar way than in the actual circuit. In case the loads are connected smoothly the circuit will behave closely to the small-signal condition. Such type of time domain simulation can also be used to verify the mapping results obtained using the methodology explained here. Fig. 4-22 shows the comparison of the maps obtained with time domain simulation of the detailed switching system model. The points marked SEP and UEP were obtained simulating the detailed model while the black dotted lined corresponds to the stable and unstable points boundary obtained with the mapping algorithm discussed here.

![Fig. 4-22 Verification of the map with detailed switching models](image)

VI. Analysis of the Results

This section analyzes and discusses some of the results presented in the previous section providing more insight on the circuit behavior and the influence of the explored parameters on the stability.

A. Analysis of the Impact of the Multi-Pulse Rectifier Filter Parameters

The difference in the unstable areas at 400 Hz and 800 Hz motivate further investigation of the mechanism producing instability in the circuit. When the model of the MPTR was discussed
the commutation inductance concept was explained. The existence of this inductance on the primary side produces during the commutation process a DC voltage. The output voltage at the MPTR DC side neglecting parasitic losses can be calculated with (4-13) where the second term on the right side represents the voltage drop created by the commutation. This voltage drop is proportional to the DC current; therefore, it can be considered as created by a “commutation” resistance, $R_{com}$. For an 18-pulse rectifier an approximate calculation of such commutation resistance can be obtained from (4-14). The value of the commutation inductance $L_c$ is in fact difficult to determine, depending on the impedance seen from the AC side of the rectifier.

$$V_d = V_d \sin \left( \frac{\pi}{9} \right) \left( \frac{18}{\pi} \right) \frac{9}{\pi} \omega L_c I_{dc}$$

\[ (4-13) \]

$$R_{com} = \frac{9}{\pi} 2 \pi f_{dc} L_c = 18 f_{dc} L_c$$

\[ (4-14) \]

According to (4-14) the commutation resistance is proportional to the AC frequency of the three-phase circuit. Therefore, it doubles when the operation frequency changes 400 Hz to 800 Hz. The large resistance increases the damping in the circuit branch connecting the load to the source. A simplified circuit, like the one that was used in the resonance analysis, can be used to show the impact of the commutation resistance on the local stability. That circuit schematic corresponds to Fig. 4-23. The small-signal stability in that circuit can be analyzed through the impedance ratio in Fig. 4-23. To analyze the impedance at the motor terminals, $v_{dcm}$ in the circuit, the source impedance $Z_s$ must include the parameters of the source, including the commutation resistance and filter. For the circuit in Fig. 4-23 the damping is given in (4-16).

$$G_{stab} = \frac{1}{1 + Z_s / Z_{load}}$$

\[ (4-15) \]

**Fig. 4-23** Simplified circuit to analyze the impact of DC filter parameters and commutation resistance
\[ \xi = \frac{R_{\text{com}} + R_{fa}}{L_{fa}} \sqrt{L_{fa} C_{fa}} \frac{C_{fa}}{2} \]  

(4-16)

For the circuit under analysis typical values of the parameters involved highlight the impact of the commutation resistance. The MPTR filter resistance value \( R_{fa} = 50 \) mΩ. If the commutation inductance \( L_c = 4 \) µH the corresponding commutation resistance values are \( R_{\text{com}} = 28.8 \) mΩ at 400 Hz and \( 57.6 \) mΩ at 800 Hz. These values are comparable to the filter resistance and in the case of 800 Hz the commutation resistance actually doubles the damping in the circuit improving the local stability behavior.

However, the presence of \( R_{\text{com}} \) explains only part of the difference between the low and high AC frequency stability regions. The other part comes from the effect of the commutation angle in the AC side of the rectifier, the change of the AC circuit impedance, and the source impedance (generator) change with the frequency. To obtain an evaluation of these effects the \( Z_{\text{out}} \) and \( Z_{\text{in}} \) of the source and load at the MD input were computed. Fig. 4-24 shows a circuit schematic where those impedances were obtained. The impedances obtained are plotted in Fig. 4-25. The MD impedance does not change appreciably with the frequency and it is shown in light blue. On the contrary, the source impedance presents a significant difference. The impedance at low AC frequency (400 Hz) is shown in yellow, while the impedance at high frequency (800 Hz) is shown in purple. In the medium frequency range the higher frequency impedance is larger than the lower one. This is expected considering the inductive nature of the AC circuit. However, in the vicinity of the resonance observed at 3 kHz the lower frequency impedance increases more than the higher one. In this area some interactions creating instability problems appear.

![Simplified circuit to analyze the impact of DC filter parameters and commutation resistance](image)
Fig. 4-25 Input-output impedance comparison at the 100KW MD input at 400 Hz and 800 Hz

B. Analysis of the AC circuit parameters

The cases where the impact of the feeder impedance and generator bandwidth were studied presented a behavior that corresponded to what a priori can be expected. For example a long feeder can lead to unstable behavior due to the high-impedance. In addition, increasing the frequency operation made the situation worst because the AC reactance increases with the frequency. The behavior observed at low generator bandwidths is also expected because of the inability of the generator to follow faster load power variation requirements and the implied high source impedance.

The mapping procedure is based on the computation of the system eigenvalues at each operation point. The eigenvalues can also be used to obtain additional information like the characteristic dominant frequency. Another interesting aspect is given by the character of the eigenvalues. If they are complex conjugate they indicate the presence of oscillations. In a system where the variable excursions is bounded by saturation functions in the controllers this oscillations have, in most cases, limited amplitude. On the other side, real positive eigenvalues may indicate a more stressing situation.
To describe one of the cases where the eigenvalue analysis can provide additional information of the system behavior, the map of generator V loop bandwidth vs. MD speed loop bandwidth is repeated in Fig. 4-26(a) where additional lines mark the bounds where the system bifurcates. In the SEP-UEP bound a pair of complex conjugate eigenvalues cross the imaginary axis becoming unstable. The second bound (dotted line) corresponds to the eigenvalues becoming positive real. The eigenvalue loci in Fig. 4-26(b) describe the path of the unstable eigenvalue along the red portion of the map. These two bifurcations are related to the system behavior that is reflected in the simulation results of Fig. 4-27. The case in Fig. 4-27(a) corresponds to the complex conjugate eigenvalues, which corresponds to values of $\text{fbw}_g = 100$ Hz and $\text{fbw}_{MD} = 45$ Hz while the case in Fig. 4-27(b) to $\text{fbw}_g = 50$ Hz and $\text{fbw}_{MD} = 45$ Hz. In the first case (I) the system goes into a periodic behavior of limited oscillations. In the second case (II), the system variables collapse; the recovery from this situation may be harder to achieve.

Fig. 4-26 Eigenvalue analysis (a) map of generator V loop bandwidth vs. MD speed bandwidth indicating bounds where eigenvalues change character (b) eigenvalue loci in the complex plane

Fig. 4-27 Simulations in time domain showing: (a) the oscillatory behavior corresponding complex conjugate eigenvalues (b) system collapse for real positive eigenvalues
VII. Summary

This chapter developed a procedure to systematically map the parameter space of the power system classifying the equilibrium points according to their local stability point of view. The region of locally stable equilibrium points (SEP) is called the feasibility region of the system. The boundaries of this region represent bifurcations in the system dynamic behavior [27]. Therefore, the mapping algorithm detects the values of the system parameters where these bifurcations happen. The following paragraphs discuss the possible uses and characteristics of the developed methodology. They also summarize the main results obtained from the analysis of the autonomous power system under analysis.

A. Evaluation of the Methodology

The methodology was applied to the analysis of an autonomous power system. Two main aspects of system stability analysis can be obtained from the methodology. The first is the analysis of the system stability under defined conditions. Given an operating condition, stability margins can be obtained for the parameters that may lead the system to unstable behavior. These margins can be obtained from the change that the parameters are allowed to have before the system bifurcates. This possibility of quantifying stability margins in terms of parameter changes can also be advantageously used at the system design stage. At this stage, parameters that have been primarily designed based on criteria related to the individual equipment operation, or other system issue like power quality, can be verified and eventually modified from the stability point of view.

B. Summary of Stability Results

Based on the maps presented in this chapter the following main conclusions about the system stability behavior can be drawn from the study.

- MPTR DC filter capacitance and inductance requirements change with the operating frequency of the synchronous generator. The stability at low frequency operation is more critical. At high frequency operation, additional damping introduced by the MPTR commutation process and changes in the source impedance help to improve the stability behavior.
• The bandwidths of the motor drives affect the system stability in close relation to the DC filter parameters. Higher bandwidths require larger filter capacitances and smaller inductances. On the other side, the link between the bandwidth of MD loads and the generator bandwidth is weak.

• The impedance of the feeder connecting source and load can affect the stability behavior, though the operation margin at the base case is relatively large. High frequency operation limits the feeder length whose reactance increases proportionally to the operation frequency.

• The bandwidth of the generator is critical for stable operation. Low voltage control bandwidths produce instability that can produce sustained oscillations or even voltage collapse.

• The AC load power factor can affect the stable operation of the system because it affects the synchronous generator stable behavior. In this way having a lagging type power factor is relatively more stable.
Chapter 5  Stability Analysis Based on Energy Functions

I. Introduction

This chapter analyzes the large-signal stability of the autonomous power system using the energy function method. This method is based on the Lyapunov direct method. The application of the Lyapunov direct method requires using functions with specific characteristics called Lyapunov functions. When this method is applied on a physical system the properties of the Lyapunov functions resemble the energy of the system. For this reason the methodologies derived from the Lyapunov direct method are generically called energy function based methods. Previous chapters analyzed various aspects of the large signal behavior and stability of the autonomous power system. Most of that analysis was based on the Lyapunov indirect method, which provides a local stability description. In some cases the analysis was complemented with analysis of the system nonlinear behavior affecting the stability. The stability conclusions, therefore, were local or valid for particular cases.

For large-signal stability analysis it is required to know the characteristics of the operating area surrounding the stable equilibrium points, called the region of attraction of those points. In this sense the Lyapunov direct method provides more complete information about the stability. Lyapunov theory has been used to develop a variety of tools over the years for the analysis of large power systems, which in some cases included AC/DC interfaces. Therefore, there exists a large amount of work analyzing the stability of AC interconnected systems based on the Lyapunov direct method. This work originated the so-called transient energy function method [45].

The application of the Lyapunov direct method presents several challenges to overcome in order to achieve meaningful results. The first one is referred to its conservativeness. Because the Lyapunov theorem basis of the direct method states sufficient conditions for stability, the results it provides are in many cases conservative. This is true especially in case of large systems. Good understanding of the system behavior and instability mechanisms can help to reduce this conservativeness. The other challenge is related to the complexity of the models that can be handled. In power systems transient stability analysis the models used have been quite simple.
Some of the latest work on the transient energy function method for AC systems has been devoted to increase the complexity of the models that the method can handle [50],[51]. While improvements have been made on the load and network representation, the generator model still remains very simplified, not allowing for the representation of the excitation regulation mechanism. In some cases, like the system under study, that control action plays a fundamental role.

### A. Lyapunov Direct Method

The Lyapunov indirect method provides a local description of the stability behavior respect to a single equilibrium state. To fully understand the stability behavior of the complete system it is necessary to consider an extended area, beyond the limits of validity of the linear approximation, and to analyze the behavior during state change transitions. For that analysis, the region of attraction is a powerful concept. For the system defined in (1-1) with trajectories described by $\phi(x)$ the region of attraction (RA) is defined as the set of all points $x$ such that $\phi(x)$ is defined for all $t \geq 0$ and

$$\lim_{t \to \infty} f(x, p) = 0$$  \hspace{1cm} (5-1)

The region of attraction itself and the methodology to find it have been object of numerous studies, with particular development in transient stability analysis of power systems. Most of the approaches to estimate RA are based on the following Lyapunov theorem. This theorem is the cornerstone of the Lyapunov direct method [11].

**Theorem:** Given a dynamic system described by equation (1-1), with equilibrium point at the origin $x_e = 0$. Let $D \subset \mathbb{R}^n$ be a domain containing the equilibrium point, and $V(x)$ with $V: D \to \mathbb{R}$ a continuously differentiable function such that $V(0) = 0 \text{ and } V(x) > 0 \text{ in } D \setminus \{0\}$. If:

$$V(x) \leq 0 \text{ in } D \Rightarrow x_e = 0 \text{ stable}$$

$$V(x) < 0 \text{ in } D \Rightarrow x_e = 0 \text{ asymptotically stable}$$  \hspace{1cm} (5-2)

Because of these two requirements on the Lyapunov function $V(x)$, it allows identifying a region where all the trajectories inside it are guaranteed to converge to the equilibrium point; therefore, it is a fundamental tool in the search for regions of attraction. The Lyapunov theorem gives the condition of stability based on the function $V(x)$ and its derivative on a certain domain
$D$. The theorem conditions for stability imply that $V(x)$ will become null at the equilibrium state. It is important to mention that there exist a difference between $D$ and the region of attraction. While it can happen that a trajectory inside $D$ leaves it. On the contrary, the region of attraction is defined as the state space portion where all trajectories entering it cannot leave. In terms of the energy function this means that there exists certain value of $V(x)$ that marks the limit. Therefore, RA is given by:

$$R_A = \{ x \in R^n \mid V(x) \leq c \} \quad (5-3)$$

Additionally, the RA is contained in $D$. The value of $c$ defines the RA limit, in terms of the Lyapunov function can be defined as:

$$c = \max \{ V(x) = c \mid \dot{V}(x) < 0, \forall x \} \quad (5-4)$$

and is all the critical energy. Therefore, the region of attraction search based on the Lyapunov direct method comprises two main tasks. The first one is to construct a valid Lyapunov function. Then, using that function the critical energy value $c$ has to be determined.

**B. Region of Attraction Concept**

The formal definition of the region of attraction (RA) was already given in the previous section. A classical example on the region of attraction concept, which is also related to our work, is given by the transient stability study of a power system. The main goal of transient stability analysis is to determine if a system will be able to recover from a big disturbance, like a fault created by a short circuit in a transmission line. Fig. 5-1 is used in the brief explanation of the transient stability problem that follows.

Suppose a two dimensional electrical system is in a pre-fault state $x_{init}$ when a fault occurs. A trajectory starting at $x_{init}$ defines the faulted trajectory. After the fault is cleared the equilibrium state is given by $x_{pos}$. There is a region around this last point where every trajectory converges to $x_{pos}$. The faulted trajectory takes the system away from the equilibrium point. The goal of the transient stability direct methods is to identify when the faulted trajectory crosses the boundary of the region of attraction. If the fault is cleared before crossing that boundary the system is stable; otherwise, it is unstable. In other words, all system trajectories whose initial state belongs to the RA of an equilibrium point will converge to that equilibrium point. The time required for
the trajectory to reach the boundary is called critical clearing time or cct, $t_{\text{crit}}$. Many of the previously proposed methods usually produce conservative results of the RA. In such a case $t_{\text{crit}}$ is approximated by $t_{\text{c-est}}$.

![Region of attraction for a two-dimensional system](image)

**Fig. 5-1 Region of attraction for a two-dimensional system**

**C. Review of Approaches for Region of Attraction Search**

The Determination of the region of attraction for nonlinear systems other than power systems has been object of numerous researches. An old but still useful review on the region of attraction search topic is found in [101]. In the same reference the trajectory reversing method is proposed as an accurate but limited tool to find region of attraction estimates. A broad classification of region of attraction search techniques divides them in Lyapunov and non-Lyapunov based. Among the Lyapunov based techniques the ones based on the Lyapunov energy function and Lyapunov theorem extensions, like the La-Salle theorem, has been the most successful. To this category correspond most of the Lyapunov construction methods like Krasowskii, Ingwerson, the variable gradient method, Lur’e and Szego [103],[104]. The transient energy function (TEF) method for studying transient stability of electric power systems belong to this category. The use of matrix Lyapunov functions also belongs to this category.

The other approach still based on Lyapunov theory is due to Zubov. According to Zubov theorem the exact region of attraction can be found by solving a nonlinear differential equation in partial derivatives [101]. To find this solution is quite difficult; however some approaches
have been proposed for small order systems. Among the non-Lyapunov approaches the transient trajectory reversed presents simplicity and accuracy but is limited to low order systems. For system with more than three states the trajectory reversing method can become extremely difficult to apply [102]. Other approaches have proposed the use of some frequency response techniques like descriptive functions. However, use of those techniques has not become widely used.

More recently there has been some development on the application of quadratic Lyapunov functions region of attraction estimation [57]. These quadratic Lyapunov functions are based on the Lyapunov matrix equation.

\[ A^T P + PA = -Q \] (5-5)

In [56] quadratic Lyapunov functions are used to estimate the region of attraction of polytopic models, which correspond to a power electronic system. In [57] several methods are tested to generate the Lyapunov quadratic equations that are then used for the region of attraction estimation of a DC power electronic system. Among these methods the linear matrix inequality (LMI) [55] can produce good region of attraction estimates in some cases. The advantage of the LMI approach is that it can be formulated as a convex optimization problem and then solved in a computer. Other uses of LMI applied to classical power system stability models have been reported. In [52] LMI is used to construct a Lur’e type Lyapunov function to estimate the RA in a power system model with detailed voltage control. Lur’e type Lyapunov functions are the sum of a quadratic and non-quadratic terms. In the work presented in [53] LMI is also used to construct a Lyapunov function of Lur’e type for an improved network model. In this last one the approach is also linked to the controlling UEP approach.

II. Lyapunov Direct Method to Estimate the Region of Attraction

The use of the Lyapunov direct method to estimate the RA can be divided in two main steps. The first one corresponds to the construction of the Lyapunov function. The second one uses this function to determine the critical energy. The value of this energy defines the boundaries of the region of attraction of the equilibrium point under analysis [45].
A. Construction of the Lyapunov Function

The construction of a Lyapunov function classically follows two possible approaches. The first one is to select a physically related energy function. This approach is usually followed for controller design where the controlled system energy has to meet the Lyapunov theorem conditions to be stable. In [105],[106] this type of approach is followed to obtain a synchronous generator excitation controller whereas in [107]-[109] a Lyapunov function is proposed for design controllers for power electronic converters. Physically meaning energy functions has also been the most followed approach to obtain Lyapunov functions for transient stability analysis of electric AC power systems. Although Lyapunov functions for transient stability analysis were originally developed for the synchronization problem, recently several more complete Lyapunov functions have been proposed for the voltage stability problem [111]. Those functions also allow for various types of load models.

The derivation of the mentioned Lyapunov functions is usually based on the power flow model of the AC network and the generator swing equation. Singular perturbations are introduced in the load flow equations, which are static by nature, to produce a dynamic model where the Lyapunov function can be obtained by integration of the power terms. The state variables of an AC system model are generically given by:

\[
x = [\omega \delta V p q]
\]

Where \( \omega \) is the generator speed, \( V, \delta \) the bus voltage magnitude and angle, and \( p, q \) the dynamic load state variables. The Lyapunov function associated with such system model has the generic form:

\[
V(x) = V_k(\omega) + V_p(\delta,V) + V_l(\delta,V,p,q)
\]

In the previous equation \( V_k \) is the kinetic energy term, \( V_p \) the potential, and \( V_l \) the term corresponding to the load stored energy. These types of Lyapunov functions have the advantage of their close relation to the system physical energy. However, they are defined for a specific system model and allowing for modeling variations require modifications that may not only be difficult to implement, but also may not be even feasible.

The alternative approach is to construct the Lyapunov function from the complete set of mathematical model equations [53]. The resulting Lyapunov function in this case is usually
complex and has no direct link to physics. However, the function is guaranteed to meet the Lyapunov criteria, at least in some region within the equilibrium point, which later may produce an estimate of the RA. While the first mentioned approach produces a customized Lyapunov function that is physically sound, it must be reformulated in case the original model is changed; this second approach constructs a Lyapunov function that can be easily adapted to system model changes.

The construction of a Lyapunov function based on the system equations can be approached analytically or numerically. Since the Lyapunov theorem is based on the positive definiteness of $V(x)$ and negative definiteness of the derivative of $V(x)$ along the trajectories, methods to solve these inequalities have been proposed [55]. The Lyapunov theorem applied to linear systems like (5-8) states its stability if and only if a positive definite matrix $P$ can be found, such that $A^TP+PA=-Q$, with $Q$ also positive definite. Therefore, a way to find a matrix $P$ is obtained by solving the following set of linear matrix inequalities (LMI).

\[
\dot{x} = Ax \tag{5-8}
\]
\[
A^TP + PA < 0 \tag{5-9}
\]
\[
P > 0
\]

The equation set (1-1) gives a basis for using LMI to find a Lyapunov function. Quadratic type Lyapunov functions are attractive because of its natural positive definitiveness. The following steps, which follow [57], show how to use the (1-1) inequalities to obtain an LMI based quadratic Lyapunov function. To determine that function the matrix $P$ in (5-10) must be computed.

\[
V(x) = x^TPx \tag{5-10}
\]

The orbital derivative of system (5-8) when using a quadratic $V(x)$ is given by:

\[
\dot{V}(x) = 2x^TPf(x) \tag{5-11}
\]

This orbital derivative is negative definite at all points in a domain that includes the region of attraction $x_1,\ldots, x_r$. Therefore, for any one of these points the following inequality must also hold:

\[
diag \left( x_i^TPf(x_1),\ldots,x_i^TPf(x_r) \right) < 0 \quad \text{for } i = 1,\ldots,r \tag{5-12}
\]
Equations (5-9) and (5-12) form a set of inequalities that when solved together provide a quadratic Lyapunov function of the (5-10) form for system (5-8). In case the original system is nonlinear, a local linear approximation like (5-8) can be used. The issue of what points from the domain must be chosen to obtain a good estimate is a priori undefined. In principle, there are an infinite number of points to choose; however, the selection of those points will affect the resulting Lyapunov function and the related RA estimation. Therefore, the point selection can be optimized in order to produce the largest estimate of RA. In [57] an optimization based on genetic algorithms was proposed for this problem.

Quadratic Lyapunov functions are easy to construct; however because of the intrinsic locality they RA estimate is usually not good. Other possible LMI based construction formulations require to formulate the nonlinear system in its Lur’e type form. Electric power systems like many autonomous nonlinear systems can be represented as a feedback connection of a linear system and a nonlinear function located in the feedback path. This representation is actually a Lur’e type form, which is schematically shown in Fig. 5-2. The equations corresponding to that model are (5-13) and (5-14). In the actual model of the power system the nonlinearities correspond to the load, the generator voltage controller, and the multi-pulse rectifier. However, the most important nonlinearity is given by the constant power load characteristic.

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx \\
u &= -\Psi(y)
\end{align*}
\]

Fig. 5-2 Lur’e representation of a nonlinear system

For the Lyapunov direct method based analysis of a Lur’e type dynamical system the Lyapunov function in (5-15) can be used provided the nonlinearity meets a sector condition [12].
\[ V(x) = \frac{1}{2} x^T P x + \sum_{i=1}^{n} \int_{0}^{y_i} \psi_i(\sigma)^T d\sigma \]  
\hspace{10cm} (5-15) 

Where:

\[ \Psi(y) = \int_{0}^{y} \psi_i(\sigma)^T d\sigma \]  
\hspace{10cm} (5-16)

The quadratic term in (5-15) corresponds to the linear portion of the system in Fig. 5-2, while the integral term corresponds to the nonlinear feedback. In addition, the Lur’e formulation requires the nonlinearity meets a sector condition like (5-17). In order to achieve this condition it may be necessary to apply a loop transformation to the original model.

\[ \Psi(y)^T K^{-1} \Psi(y) \leq y^T \Psi(y) \]  
\hspace{10cm} (5-17)

Like in the quadratic Lyapunov function case, the matrix \( P \) and the coefficient \( \gamma \) can be found by solving a system of LMIs. Such set LMIs is obtained from the positive definiteness of \( P \) and negative definiteness of the Lyapunov function derivative as follows. From the derivative of the quadratic term in (5-15) and doing proper replacements it is possible to obtain:

\[ \dot{V}_1(x) = x^T (PA + AP) x - 2x^T PB \Psi(y) \]  
\hspace{10cm} (5-18)

And from the derivative of the integral term:

\[ \dot{V}_2(x) = \gamma x^T A^T \Psi(y) - \gamma \Psi(y)^T CB \Psi(y) \]  
\hspace{10cm} (5-19)

The sector condition (5-17) can also be expressed as:

\[ 0 \leq y^T \Psi(y) - \Psi(y)^T K^{-1} \Psi(y) \]  
\hspace{10cm} (5-20)

Therefore, the derivative of the Lyapunov function \( V(x) \) is less or equal than the sum of the three left hand sides terms of the previous three equations producing:

\[ \dot{V}(x) \leq x^T (PA + AP) x - 2x^T PB \Psi(y) + \gamma x^T A^T \Psi(y) - \gamma \Psi(y)^T CB \Psi(y) + y^T \Psi(y) - \Psi(y)^T K^{-1} \Psi(y) \]  
\hspace{10cm} (5-21)

The terms in this last equation can be grouped in the following matrix inequality:

\[ \dot{V}(x) \leq \left[ \begin{array}{c c} x^T & \Psi(y)^T \end{array} \right] \left[ \begin{array}{c c} A^T P + PA & x - 2x^T PB \\ -B^T P + \frac{\gamma}{2} CA + \frac{1}{2} C & -K^{-1} - \frac{\gamma}{2} (B^T C + CB) \end{array} \right] \left[ \begin{array}{c} x \\ \Psi(y) \end{array} \right] \]  
\hspace{10cm} (5-22)

This last inequality guarantees that the \( V(x) \) derivative will be negative definite if the following matrix inequality is true [52]:
Knowing the system model $A, B, C$ and a sector condition $K$ bounding the nonlinearity $\psi(y)$ this last inequality together with the positive definiteness of $P$ allow for the calculation of $P$ and $\gamma$. In this way the Lyapunov function in (5-15) is constructed.

To complete the Lur’e type Lyapunov function description some additional explanation of the sector condition must be given. The condition expressed in (5-17) represents a bound on the magnitude of the nonlinear function. In the one-dimensional case the interpretation is straightforward and a graphical representation is given in Fig. 5-3 where the light green area is the sector where the nonlinear function must be contained. The slope of the line limiting the sector corresponds to $K$ in the inequality (5-17).

\[
\begin{bmatrix}
A^T P + P A & x - 2x^T P B \\
-B^T P + \frac{\gamma}{2} CA + \frac{1}{2} C & -K^{-1} - \frac{\gamma}{2} (B^T C^T + C B)
\end{bmatrix} < 0
\]  

(5-23)

**Fig. 5-3 Nonlinear feedback function and sector condition for the on-dimensional case**

Bounded nonlinear functions can in many cases be transformed to a sector condition function by means of loop transformations [11]. Fig. 5-4 shows the block diagram of two possible loop transformations $K_1$ and $K$ applied to the Lur’e type system. The blocks, $K$ and $K_1$ correspond to constant gains related to the characteristic of the transformation applied.

The loop transformation consist in multiplying the system signal by appropriate gain factors in such a way that the system dynamic response is not changed, but the nonlinear function in the feedback path is transformed to a sector function.
B. Determination of the Critical Energy

The critical energy was defined as the value of the Lyapunov function $V(x)=c$ that defines the RA boundary. Every point where the Lyapunov function is $V(x)\leq c$ belongs to the RA; otherwise it is outside of the RA and the stable behavior of a trajectory starting at a point where $V(x)>c$ is not guaranteed. In transient stability analysis there exist two approaches to search for the critical energy. The controlling unstable equilibrium point method assigns the critical energy the value of the energy function at the UEP closest to the trajectory of the faulted system [110]. The other approach, the potential energy boundary surface (PEBS) identifies the critical energy as the value of the potential energy at the region of attraction limits [45]. In case the system has a reduced number of states, the controlling method can usually be identified as the closest UEP. The controlling UEP and PEBS approaches were derived based on the particular characteristics of the power system model under faulted conditions.

As a matter of fact finding the critical energy means finding the value $c$, defined in (5-4), that guarantees the derivative of $V(x)$ will be negative definite along all the trajectory. If the Lyapunov function is negative and can only decrease it has to end up in an invariant set that contains the equilibrium point. In practice it is easier to find the value of $c$ in the way expressed in (5-24). This means that if the derivative of $V(x)$ is positive at some point the stability of points with larger value of $V(x)$ may have negative derivative their stability is not guaranteed because the derivative sign can change.
\[ c = \min \left\{ V(x) = c \left[ \frac{\partial V(x)}{\partial x} > 0 \right] \right\} \] (5-24)

In the small dimensional case finding the value of \( c \) can be done by mapping the value of \( V(x) \) and its derivative throughout the state-space. On the contrary, for a large dimensional case (ten or more), mapping the whole space requires a vast amount of computational resources making it impractical. In such cases the problem represented by (5-23) can be formulated as an optimization problem. The work in [57] shows one of such possible optimization formulations. In the work developed here a state-space mapping is still possible and that approach will be followed.

III. System Modeling for the Lyapunov Direct Method

A. Rectified AC source Equivalent Model

Detailed synchronous generator models are available for analysis and simulation studies. Those models can accurately predict the behavior under transient and steady-state conditions. Reduced order models of the generator and controller are also frequently used. The models to be used here are relatively simple. The machine is represented by its back-emf and transient reactance. Additionally, the excitation regulator is added in the models. The necessity of including this regulator originates in the role the voltage regulator plays in the stability of the system under analysis. The model of the regulator can also be simplified. In general it suffices to include the outer voltage loop only, which can be represented by a first order controller. In this simple model the generator internal voltage \( e_g \) is given by:

\[
\dot{e}_g = \frac{1}{T_s} \left( V_{\text{ref}} - e_g + i_z \right)
\] (5-25)

The multi-pulse rectifier limits the active and reactive power flow through it. The average model presented in chapter two is simple, but quite accurate, and therefore selected for the analysis. Considering the rectifier connected to a Thevenin equivalent of the three phase AC source, the voltage at the DC port of the rectifier is given by (5-26).

\[
v_{dc} = G_{dc} e_g - \left( z_i + z_{\text{r}} \right) \sqrt{3} i_{dc} - \frac{9}{\pi} \omega L \ i_{dc}
\] (5-26)
where $G_{dc}$ is related to the transfer ratio of the autotransformer; $e_g$ and $z_g$ are the equivalent parameters of the AC source; $z_f$ is the impedance of the feeder; and $L_c$ the commutation inductance of the primary circuit. The dc current $i_{dc}$ can only be positive. This expression is rather simplified, but shows the main components of the AC circuit affecting the voltage at the DC side.

![Diagram of simplified equivalent source model](image)

**Fig. 5-5 Simplified equivalent source model**

![Graph of transient response for a short circuit at the DC bus](image)

**Fig. 5-6 Transient response for a short circuit at the DC bus: (a) full-order model (b) simplified model**

**B. Constant Power Load Equivalent Models**

A great simplification is obtained by representing a dynamic load like the MD as a voltage-dependent load with internal dynamics. This type of behavioral model has been widely used for the study of voltage stability in electrical power systems [78],[79]. The power demanded by this equivalent model has an exponentially voltage-dependent characteristic as shown in (5-27), where $P_o$ and $V_o$ are reference values, which correspond to the load operation, and $nps$ is a
coefficient related to the load characteristic. In the strictly constant power load (CPL) case nps is equal to zero. The dynamic can be formulated assuming either impedance or admittance as state variable. Equation (5-28) shows the admittance model in the s domain; equivalently, the current can be the variable of interest; in such case it can be calculated from (5-29).

To validate the CPL model it was compared in simulation against the MD full-order average model. Fig. 5-8 (a) and (b) show the response of both models for a voltage and mechanical torque step-change respectively, where the load was modeled as a strictly constant power load in both cases, i.e., the exponential coefficient nps was zero. The load torque step corresponded to a load power step in the CPL model. It is possible to see that in both cases the simple CPL model can follow the current change quite closely. However, it has the limitation of a first order model approximation, which for example cannot represent overshoots. To obtain the good accuracy shown in the Figure it is necessary to give the CPL models a time constant T₀ that relates to the converter speed loop bandwidth at the rated operation conditions, which can be different from the value chosen for the design.

\[
P(V) = P_0 \left( \frac{V}{V_0} \right)^{nps} \quad \text{(5-27)}
\]

\[
Y = \frac{P_0}{V^2} \frac{1}{s T_0 + 1} \quad \text{(5-28)}
\]

\[
I = \frac{P_0}{V} \frac{1}{s T_0 + 1} \quad \text{(5-29)}
\]

![Fig. 5-7 CPL response for a voltage step at the DC bus](image)
Better accuracy can be obtained by using a second order CPL model. In that case the dynamic of the equivalent CPL is given in equation (5-30). The characteristic frequency $\omega_h$, and damping ratio $\xi$, of the second order transfer function correspond to those of the motor controller speed loop gain at the rated operation conditions. Equation (5-31) gives an equivalent state-space formulation of the second order CPL model where the fictitious state variable $a_d$ has been added. Fig. 5-8 (a) and (b) show the responses of the CPL equivalent model of equation (5-30) or (5-31) and the motor controller full-order average model. The same two conditions of DC voltage and load torque step of the previous Fig. 5-7 were tested. The results are shown in Fig. 5-8 where it is possible to observe the close match between the currents taken by both models. In the voltage step case the match is excellent while in the torque step there is some difference in the transient responses. However, the second order CPL model improves considerable the response of the first order CPL model.

\[
I = \frac{P}{V} \frac{1}{s^2 + 2 \xi \omega_h s + \omega_h^2} \tag{5-30}
\]

\[
\dot{a}_d = \frac{P}{v_{dc}} \omega_h^2 - 2 \xi \omega_h a_d - \omega_h^2 i_{dc} \tag{5-31}
\]

Fig. 5-8 MD and CPL transient response for: (a) voltage step at the DC bus, (b) load torque at the MD or load power at the CPL
IV. Application of the Lyapunov Direct Method to Estimate RA in Circuits with CPL

A. CPL Connected to an Ideal Voltage Source

This section presents in detail an example using a Lyapunov function constructed through LMIs and the RA estimation for a circuit connecting the CPL to an ideal voltage source. The circuit under study is shown in Fig. 5-9. Although simple this example shows the nonlinear characteristics of the CPL and its effect on the large-signal stability. A first order equivalent model like equation (5-29) is assumed for the CPL. The state-equations corresponding to the circuit are given in (5-32),(5-33), and (5-34). The parameter ω₀, the inverse of T₀ in (5-29), represents the bandwidth of the CPL.

\[
i_{dc} = -\omega_0 i_{dc} + P_o \omega_0 \frac{1}{v_{dc}} \\
\dot{v}_{dc} = -\frac{1}{C} i_{dc} + \frac{1}{C} i_l \\
i_l = -\frac{1}{L} v_{dc} - \frac{R}{L} i_l + \frac{1}{L} v_i
\]

(5-32) (5-33) (5-34)

The previous set of state-space equations has two equilibrium points that are located at:

\[
V_{dc} = \frac{1}{2} \left( V_i \pm \sqrt{V_i^2 - 4 R P_c} \right), \quad I_{dc} = \frac{P_c}{V_{dc}}, \quad I_i = I_{dc}
\]

(5-35)

It is convenient to translate coordinate to one of the equilibrium points. The highest \( V_{dc} \) solution is chosen for this. In this way the translated state-space model in matrix from becomes:
After the coordinate translation the new location of the system equilibrium points is,

\[
(0,0,0) \quad \text{and} \quad \left( \frac{-R P_{e} \cdot V_{dc}^2}{R V_{dc}}, \frac{-R P_{e} \cdot V_{dc}^2}{V_{dc}}, \frac{-R P_{e} \cdot V_{dc}^2}{V_{dc}} \right) \quad (5-37)
\]

By inspection of equation (5-36)(a) it is possible to note the existence of a singularity point at \(v_{dc} = -V_{dc}\). In order to obtain the Lyapunov function it is convenient to relate the state-space equation to the Lur’e model. In this way the nonlinear feedback function is obtained as:

\[
\psi(C \cdot x) = \psi(v_{dc}) = \omega_{b} \cdot \frac{v_{dc}}{v_{dc} + V_{dc}}
\]

(5-38)

It is convenient now to consider the sector condition requirement on \(\psi(v_{dc})\). Expression (5-38) is not just a function but actually a family of functions parameterized by the value of \(V_{dc}\) which corresponds to the particular equilibrium point. In order to obtain better results it is recommended that the K slope of the sector condition is kept as low as possible. It is also required that the sector condition extends over a wide range of the \(\psi\) function; this range has to be larger than the RA. Therefore, it is recommended to apply a loop transformation to the nonlinear feedback function \(\psi\). The loop transformation to be used in this example corresponds to the blocks \(K_{1}\) in Fig. 5-4. The corresponding transformed feedback function is given in (5-39) and the effect of the loop transformation is shown in Fig. 5-10. The state-space equation is modified accordingly.

\[
\psi'(v_{dc}) = \frac{\omega_{b}}{10} \cdot \frac{v_{dc}}{v_{dc} + V_{dc}} - k_{i} \cdot \frac{V_{dc}}{10 P_{e}} \cdot v_{dc}
\]

(5-39)
Chapter 5 - Stability Analysis Based on Energy Functions

Fig. 5-10 Original and loop transformed sector condition for the nonlinear feedback function corresponding to a load level of 100 kW

It is now possible to integrate the modified feedback function $\psi_l$ of expression (5-38) in order to obtain the nonlinear term of the Lyapunov function. The integral to solve in (5-40) is one-dimensional and the corresponding result is given in (5-41).

$$V_{ad}(v_{dc}) = \frac{v_e}{10} \int_0^\sigma \left( \frac{\omega_s}{10} + \frac{V_{dc}}{10P_e} \right) d\sigma$$

(5-40)

$$V_{ad}(v_{dc}) = v_{dc} - V_{dc} \log \left( \frac{v_{dc} + V_{dc}}{v_{dc}} \right) - \frac{k_l}{2} \frac{V_{dc}}{10P_e} v_{dc}^2$$

(5-41)

To construct the LMIs that will calculate the values only the value of the sector condition $K$ that limits the transformed feedback function is needed. The solution of the inequality in (5-23) allows to calculate $P$ and $\gamma$ and fully determine $V(x)$.

$$V(x) = x^TPx + \gamma \left( v_{dc} - V_{dc} \log \left( \frac{v_{dc} + V_{dc}}{v_{dc}} \right) - \frac{k_l}{2} \frac{V_{dc}}{10P_e} v_{dc}^2 \right)$$

(5-42)

A possibility is to use the calculated Lyapunov function in (5-40) to estimate the RA of an operating respect to a power load step. In this case the load is operating at a given level and it is desired to determine if it can safely reach that power level from a previous lower or higher level. As an example, the RA for the load at a power level of 90 kW load operation is obtained below. The values of the circuit parameters are $C=300 \ \mu F$, $L=250 \ \mu H$, $R=0.2 \ \Omega$. Fig. 5-11 pictures the identified region of attraction in the $i_{dc}$, $v_{dc}$, $i_l$ space. This figure also shows a trajectory
corresponding to the load step connection. Since it is a stable trajectory, it must be contained inside the identified region of attraction. The evolution of the system variables $i_{dc}$, $v_{dc}$, and $i_l$ for the 90kW load connection is shown in Fig. 5-12(a). For the same transient Fig. 5-12(b) shows the evolution of the Lyapunov function. It is possible to appreciate in this last Figure the monotonically decreasing evolution of $V(x)$, which is indicative of a stable behavior. The Figure also shows the quadratic and the nonlinear term of $V(x)$. In addition the critical energy level was calculated for this example and is shown in Fig. 5-12(b). Because the case is stable, the energy is lower than the critical energy along the load connection transient. It is also possible to observe that there is still some margin between the maximum energy reached and the critical energy.

![RA boundary](image1)

**Fig. 5-11** RA boundary (magenta) in the state space and trajectory for 90 kW load connection (blue). The trajectory is totally contained inside the RA.

![Waveforms](image2)

**Fig. 5-12** 90kW CPL load connection: (a) transient waveforms of circuit $v_{dc}$, $i_{dc}$, and $i_l$ (b) Lyapunov function waveform during the transient connection.
In order to evaluate the conservativeness of the RA determination further time domain simulations were done. From these simulations it was possible to determine that the maximum load-step keeping the system stable was 25% higher than the Lyapunov prediction.

V. Region of Attraction Estimation for the Simplified Autonomous Power System

The purpose here is to use the stability assessment based on the RA estimation developed in the previous sections to evaluate the large-signal stability of the reduced autonomous power system composed of the generator with voltage regulation connected to the DC CPL through a multi-pulse rectifier. The system stability behavior is evaluated for two types of large-signal events. The first type corresponds to events that do not change the structure of the system. The equipment and their interconnections are preserved, but a considerable change in the operation is produced by a large change in the system parameters. Large load steps, for example, can be treated in this way. The second type of events does change the system structure implying the connection or disconnection of circuit branches. A fault and its clearance can be included in this category.

A. System Model Considerations

Based on the modeling considerations previously discussed the circuit to be studied in this section can be represented by the schematic in Fig. 5-13. The AC source is represented by its equivalent circuit at the DC side, while the load by its equivalent CPL second order model. The state-space equations corresponding to the simplified circuit are given in (5-43) to (5-47) while the corresponding circuit parameters are shown in Table 5-1.

![Fig. 5-13 Simplified equivalent circuit of the reduced aircraft power system composed of the regulated AC source, rectifier and CPL](image-url)

117
\[ \dot{e}_g = \frac{1}{T_e} \left\{ v_{ref} - e_g + i_i \frac{z_g}{G_{dc}} \right\} \]  
(5-43)

\[ i_i = -\frac{1}{L} v_{dc} - i_l \left\{ \frac{R}{L} + \sqrt{3} \frac{1}{L} (z_l + z_g) + \frac{9}{\pi} \frac{\omega L_c}{L} \right\} + \sqrt{3} \frac{1}{L} G_{dc} e_g \]  
(5-44)

\[ i_i = \frac{P_o \omega^2}{v_{dc}} - 2 \xi \omega \xi i_l - \omega^2 i_i \]  
(5-45)

\[ \dot{i}_d = i_{dc} \]  
(5-46)

\[ \dot{v}_{dc} = -\frac{1}{C} i_{dc} + \frac{1}{C} i_i \]  
(5-47)

### Table 5-1 Circuit parameters

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC source</td>
<td>e_g=100V, z_g=0.2, z_l=0.02, T_c=0.03</td>
</tr>
<tr>
<td>Rectifier</td>
<td>G_{dc}=1.5, L_c=2 mH</td>
</tr>
<tr>
<td>DC circuit</td>
<td>R=0.05 \Omega, L=250 \mu H, C=500 \mu F</td>
</tr>
<tr>
<td>CPL</td>
<td>P_o=100 KW, \omega_o=700, \xi=0.8</td>
</tr>
</tbody>
</table>

To construct the Lyapunov function it is convenient to obtain the Lur’e form of the previous model with the origin of coordinates at the equilibrium point of interest. The coordinates of such equilibrium point are represented with large caps (\(V_{dc}, I_b, I_d, I_{dc}, V_{bc}\)). The mentioned model in matrix form is given in (5-48).

\[
\begin{bmatrix}
    \dot{e}_g \\
    i_l \\
    \dot{i}_d \\
    \dot{i}_{dc} \\
    \dot{v}_{dc}
\end{bmatrix} = 
\begin{bmatrix}
    \frac{1}{T_c} & \frac{z_g}{G_{dc} T_c} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
    \sqrt{3} \frac{G_{dc}}{L} & \frac{R}{L} - \sqrt{3} \left( \frac{z_l + z_g}{L} \right) & \frac{9}{\pi} \frac{\omega L_c}{L} & 0 & 0 \\
    0 & 0 & 0 & 2 \xi \omega \xi & -\omega^2 \\
    0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & \frac{1}{C} \\
\end{bmatrix} 
\begin{bmatrix}
    e_g \\
    i_l \\
    \dot{i}_d \\
    \dot{i}_{dc} \\
    \dot{v}_{dc}
\end{bmatrix} + 
\begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
\end{bmatrix} 
\begin{bmatrix}
    \frac{P_o}{V_{dc}} \omega^2 \\
    \omega^2 \\
    V_{dc} + V_{bc} \\
\end{bmatrix} 
\]  
(5-48)

\[ v_{dc} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \]
Following the same procedure explained in the previous section the Lyapunov function $V(x)$ is constructed. The result is shown in the next expression (5-49). Like in the previous case the values of the matrix $P$ and the coefficient $\gamma$ are found by solving the LMI (5-23). The solution of the LMI set was obtained using the robust control toolbox of Matlab. This toolbox has the capability to perform the optimization procedure in order to find the values of the unknowns that meet the inequality constraints.

$$
V(x) = x^T P x + \gamma \left[ \omega_o^2 \left( v_{dc} - V_{dc} \log \frac{V_{dc} + V_{dc}}{v_{dc}} \right) - \frac{k_i}{2} \frac{V_{dc}}{10 P_o} v_{dc}^2 \right]
$$

(5-49)

The Lyapunov function (5-49) can now be employed to find the RA in the system of Fig. 5-13. The two cases of load step and faults were developed and are described in the next paragraphs.

B. Load Step Case Analysis

During load step transients the system structure is preserved; therefore the pre- and post-event only differ in their parameters, or the coefficients in the state-space equation. The Lyapunov function must be calculated for the post-event system. In the case studied the load is increased from an original very low value. The simulation results corresponding to load connection in the circuit of Fig. 5-13 are shown in Fig. 5-14. The part (a) of the Figure shows the evolution of the DC voltages and currents, while the part (b) shows the evolution of the Lyapunov function $V(x)$ and its quadratic and nonlinear terms. It is possible to observe the dominance of the quadratic term. In Fig. 5-14(b) the horizontal axis has been shifted in order to make the waveform start at the point where the load is connected. The value of the critical energy is also shown in the Figure.

To give more insight on the characteristics of the Lyapunov Fig. 5-15 shows the Lyapunov function level curves together with the system trajectory during the circuit start-up and load connection projected over the $i_{dc}-v_{dc}$ plane of coordinates. It is possible to observe that the trajectory has two differentiated portions. The first time corresponds to the system start up and ends at the approximate coordinates of (20, -100). The second portion corresponds to the load connection and because the system is stable it converge to the equilibrium point at the coordinate origin.
C. Fault Case Analysis

The large disturbance in the system can be originated by a fault. In this case the objective of the large signal analysis is to determine if the system will recover after the fault is cleared. The proposed Lyapunov function was tested for faults at the CPL terminals as shown in Fig. 5-16. In this example the fault is external to the load, as it happens in a parallel connected branch. Therefore, it can be removed separately with the post fault event corresponding to the fault branch disconnected. That system has the same configuration than the system described in previous section and also the same Lyapunov function.
Fig. 5-16 Circuit schematic for testing the system stability when faults at load terminals occur

The waveforms in Fig. 5-17 correspond to the simulation results of a fault case. The Figure shows the evolution of the Lyapunov function and its quadratic and nonlinear components. In the first case Fig. 5-17(a) the fault is light and does not drive the system out of the stability region. In the Figure it is possible to observe that the value of $V(x)$ is always lower than the critical energy value. Fig. 5-17(b) shows the transient for a heavier fault. In this case the fault is cleared after 10msec. Although the trajectory goes outside of the predicted region of attraction of the post-fault operating point, the system recovers after clearing the fault. This denotes certain degree of conservatism in the region of attraction estimation. Finally, Fig. 5-18 shows the Lyapunov function when the varying the two components $i_{dc}$ and $v_{dc}$ while the others are kept constant at zero value. The Figure shows the characteristic well-shaped conformation of $V(x)$. The evolution of $V(x)$ along the post-fault trajectory for the heavy fault case is also shown in the Figure.
VI. Method Evaluation

Slow voltage regulation is usually assumed in the transient stability analysis of a power system. However, for a stand-alone AC/DC system like the one studied in this work, the requirements of fast voltage regulation and power quality made that assumption invalid. The method presented in this paper allows using voltage regulation in the generator. On the other side the Lyapunov function construction is based on an optimization approach. The result of this optimization are the parameters of the Lyapunov function, which therefore cannot be linked to physical magnitudes as in the case of the Lyapunov functions used in transient stability analysis of power networks.

VII. Summary

This chapter presented a large-signal stability assessment of the simplified autonomous power system comprised of AC generator, multi-pulse transformer rectifier, and a dc-fed PWM motor drive. First it was shown that the characteristics of the AC/DC system allow approaching the stability analysis by means of reduced order models. Taking advantage of this fact, a Lyapunov energy function was proposed to determine the stability on the DC bus. Simulation
results were used to show the validity of the proposed approach illustrating how the method can be employed to determine the magnitude of the transient events that the system can withstand before becoming unstable. The conservativeness observed in the results obtained was within reasonable limits. Nevertheless, a thorough evaluation of the degree of conservativeness of the methodology developed here requires a deeper study.
Chapter 6 Analysis of Stability Behavior Using Time Domain Simulations

I. Introduction

This chapter studies the stability by analyzing the system response under various scenarios obtained through integration of the system dynamic equations in time domain. The scenarios considered in this chapter correspond to large-perturbations created by faults. The types of faults analyzed are short circuits at different points of the system. Because of the hybrid nature of the system those short circuits can be located at AC distribution (three- or single-phase) or DC distribution. The objective of the stability analysis under fault conditions is to determine if the system will recover stable operation after the fault is cleared. In terms of the Lyapunov stability discussed in the previous chapter this means that the post-fault trajectory converges to a stable post-fault equilibrium point. This requires that the post-fault equilibrium exists and is stable.

Stability analysis based on computer simulations has been the most traditional and widely used approach for transient stability analysis in large power systems. Numerous works cover the topics of modeling the system components and techniques for time domain integration of the models, which usually are sets of nonlinear ordinary differential equations. The work in [113] for example, discusses models of power equipment under fault conditions while [35] presents a comprehensive description of the stability analysis based on computer simulations. Although the models and computing capabilities have considerably change along the years, time domain simulation continues to be the most commonly used approach when the transient stability of a power system must be analyzed [36],[37].

Simulation based studies allow in principle to use any model of the system that can be integrated in time domain using a computer. However, those models must be carefully selected in order not only to optimize the computing resources, but also facilitate the data manipulation and result analysis [57],[60]. In addition, very detailed models could provide accurate results, but when introduced in a larger system they can compromise the convergence of the solution. Therefore, the degree of model detail must be evaluated according to the type of study [61]. The general characteristics of the models used in this work where described in chapter two; here we
will refer to the models used in every case study and the reasons for using them. In some cases it is not possible to model the whole power system with high degree of detail. The modeling approach must also consider the type and characteristics of the fault being analyzed. The path followed in this work models the equipment “electrically close” to the fault with higher accuracy while the other equipment is modeled with less detail.

The system response against faults is closely related to the protection schemes and settings. Therefore, protections for each of the system components were set up for the study. The description of those protection schemes are also provided in this chapter. The purpose of the study is not to define how to coordinate the protections in the system; rather to account for the effect of the protection on the system stability behavior.

A. Objectives of the Stability Analysis under Fault Conditions

The main objective of this chapter is to verify the system stable operation and to identify possible unstable behavior. In a system study there are two main aspects to consider. The first one is related to the integrity of the interconnections among the system components. The second one is related to the stable behavior of all the system components under the faulted operation and during the fault recovery. The simulation of the system under fault conditions requires computing the values of overcurrents and overvoltages created by the faults. In this way, the stresses (thermal, mechanical, etc) over the system components can be quantified and the protection settings can be verified and corrected in case of necessity. In addition, the coordination between protection schemes can be refined because in many cases the requisite of preserving the system may require to change the protection coordination.

The study assumes a basic protection scheme for each piece of equipment that was defined based on current practices for those types of equipment. Simulations were done with different protection settings in order to analyze the possible scenarios and also assess the impact of the protection on the system recovery after the fault clearance.

B. Considerations for Evaluation of the System Stability

The criteria to evaluate a system ability to recover stable operation after a fault clearance has been traditionally based on the critical clearing time (cct) [35]. The cct refers to the maximum
time a fault can be applied to a system and this will recover after that fault has been cleared. This concept is linked to the region of attraction concept explained in the previous chapter. A short cct implies that the fault under consideration makes the system unstable very easily while a large cct represents a fault that is less stressful for the system stable operation. Moreover, an infinite cct will mean that the fault does not represent a problem for the system stability. Fig. 6-1 can be used to clarify the cct concept; in Fig. 6-1(a) the fault is cleared before the cct while in Fig. 6-1(b) it is cleared after the cct making the system unstable.

**Fig. 6-1 Critical clearing time examples (a) fault clearance after cct, the system does not recover (b) fault clearance before cct, the system recovers**

In the analysis of system under fault conditions it is usually assumed a very small, ideally zero, fault resistance. This is based on a worst case scenario consideration. Contrary to high voltage system where faults can present a large resistance, small resistances are quite common in low voltage systems [14]. However, the simulation of such type of very low resistance faults may complicate the simulation convergence making in some cases necessary to give the fault resistance a value of several tens of milliohms.

II. System Protection

A basic protection scheme was set up for the system in order to have a reference for the simulated fault scenarios. The protection scheme set up is based on common practices for individual equipment protection. The MD protection includes AC overcurrent protection at the motor side, overvoltage and undervoltage protection at the DC bus feeding the converter and shoot-through protection at the inverter. The activation of any of these protections will cause the
inverter to turn-off. In addition, except for the case of undervoltage protection the circuit branch connecting the MD will also disconnect. The disconnection time depends on the characteristics of the circuit breaker (CB) employed. A typical CB opening time of 10msec was used in the study. A similar scheme is considered for the 3phAR protection except that there is no overcurrent protection at the DC output. Nevertheless, an AC overcurrent protection is provided at the CB connecting the 3phAR to the AC bus.

The feeders connecting the generator to the AC bus were provided with overcurrent protections at each of the three-phases. The activation of one phase will trip the three-phase breaker with a clearing time depending on the CB plus an additional time required for coordination with the protections at the individual branches. The 1phAR were provided with individual AC overcurrent protections at their point of connection. The protection scheme for the complete power system is shown in the one-line diagram of Fig. 6-2. Summary of the protection types and settings are given in Table 6-1.

![Fig. 6-2 System one-line diagram showing the protection schemes](image-url)
Table 6-1 Basic protection types and typical settings

<table>
<thead>
<tr>
<th>Individual equipment</th>
<th>Typical settings (pu)</th>
<th>T-off time (msec)</th>
<th>Disc. time (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1&gt;</td>
<td>shoot</td>
<td>V&gt;</td>
</tr>
<tr>
<td>Motor drive</td>
<td>1.5</td>
<td>2</td>
<td>1.3</td>
</tr>
<tr>
<td>3-phase active rectifier</td>
<td>1.5</td>
<td>2</td>
<td>1.3</td>
</tr>
<tr>
<td>Three-phase circuit breaker</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Single-phase circuit breaker</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

III. Fault Case Scenarios Definition

In order to study the system response against different types of faults a set of case studies was defined. The idea supporting the case study definition was to cover most of the equipment in the system. The fault parameters are the clearing time (ct), and fault resistance (R\textsubscript{fault}). In some cases the election of the fault resistance corresponds to the ability of the circuit simulator program to converge under the fault conditions, and especially during the fault connection and clearance. In Table 6-2 the cases 1, 2, BUS, DCt, and DCe correspond to generic system interconnections. On the other side cases 3, 4, 3distf, and 1phAR correspond to the connection of specific type of equipment.

Table 6-2 Fault scenarios basis of the analysis

<table>
<thead>
<tr>
<th>Case</th>
<th>Fault location</th>
<th>Fault parameters</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Feeder at generator terminals</td>
<td>ct, R\textsubscript{fault}</td>
<td>Fault – parallel feeder disconnect</td>
</tr>
<tr>
<td>2</td>
<td>Feeder at AC bus terminals</td>
<td>ct, R\textsubscript{fault}</td>
<td>Fault – parallel feeder disconnect</td>
</tr>
<tr>
<td>3</td>
<td>MPTR connection to AC bus</td>
<td>ct, R\textsubscript{fault}</td>
<td>Fault – MPTR branch disconnection</td>
</tr>
<tr>
<td>4</td>
<td>3phAR connection to AC bus</td>
<td>ct, R\textsubscript{fault}</td>
<td>Fault – 3phAR branch disconnection</td>
</tr>
<tr>
<td>BUS</td>
<td>AC bus generic output</td>
<td>ct, R\textsubscript{fault}</td>
<td>Fault – clearance</td>
</tr>
<tr>
<td>DCt</td>
<td>DC bus generic output</td>
<td>ct, R\textsubscript{fault}</td>
<td>Fault – clearance</td>
</tr>
<tr>
<td>DCe</td>
<td>DC link of 3phAR</td>
<td>ct, R\textsubscript{fault}</td>
<td>Fault – clearance</td>
</tr>
<tr>
<td>1phR1</td>
<td>3-ph AC distribution</td>
<td>ct, R\textsubscript{fault}</td>
<td>Fault – 3x1-phAR disconnection</td>
</tr>
<tr>
<td>1phR3</td>
<td>1-phAR connection to AC distribution</td>
<td>ct, R\textsubscript{fault}</td>
<td>Fault – 1-phAR disconnection</td>
</tr>
</tbody>
</table>
IV. Fault Case Studies Simulation Results

This section presents some of the results obtained from the simulation of the different cases in Table 6-2. The results shown here have the purpose to provide a basis for the analysis to be discussed in the next section. Additional results can be found in the Appendix. In order to optimize the computation resources the system model was adapted for each case study. The modeling guideline was to use detailed models of the equipment connected close to the fault while remote equipment was modeled with less detail. The actual model used in each case is given together with the results obtained from it. The schematic in Fig. 6-3 shows the magnitudes measured during the simulations. The waveforms to be presented in the following paragraphs would refer to the names given in Fig. 6-3.

![System model showing the magnitudes measured during the simulations](image)

**Fig. 6-3 System model showing the magnitudes measured during the simulations**

A. Fault at the Generator Feeders

The first two cases in Table 6-2 correspond to faults at the generator feeder and use the same model of the system. Because of the fault location, all the equipment connected to the AC bus was modeled with detail including switching models for rectifiers and nonlinear cores for magnetic circuits. The 1phAR are lumped in order to have one equivalent circuit connected at
each phase. The circuit schematic of the model is shown in Fig. 6-2. The color code of the equipment in the system is given as follows. Switching models of converters are represented in light blue, while average models in yellow. Magnetic cores modeling nonlinear magnetic characteristic are shown in purple while linear cores are in white.

![Circuit Schematic]

Fig. 6-4 Model to study faults at feeders. The transformer cores model magnetic saturation and the 3phAR converters models include switches. The loads, in yellow, are represented by average models.

The faults are simulated at the feeder terminals before the circuit breakers. It is assumed that the protection between the two feeders is coordinated in such a way that only the faulted feeder opens the fault. The first case corresponds to a fault at the generator terminals while the system was operating at 800 Hz. The corresponding waveforms are shown in Fig. 6-5 to Fig. 6-7. The generator waveforms are pictured in Fig. 6-5(a) where also the protection signals are shown. The protection at the faulted terminal detects the fault immediately, but it takes approximately 20 msec for the CB to open. The current limiter of the excitation controller limits the current at the generator terminals during the fault. Once the generator terminal is disconnected the overcurrent protection in the other feeder terminal activates tripping the CB 20 msec later. During the time between the two CB trips the fault is supplied from the AC bus terminal; during that time the feeder impedance also limits the value of the fault current.

Fig. 6-5(b) shows the waveforms at the 1phAR branch. The regulator of the 1phAR cannot keep the DC voltage during the first stage of the fault, but after the first CB trips the DC voltage
recovers increasing momentarily the current demand. After the fault complete clearance the AC bus experience an overvoltage due to the limited generator voltage regulator bandwidth. The overvoltage creates in the 1phAR branch, which is still under transient conditions, a large overcurrent that excites the overcurrent protection.

The waveforms in Fig. 6-6 correspond to the 3phAR branches of the circuit. Fig. 6-6(a) corresponds to the first 3phAR, which is based on a two-level converter topology, while Fig. 6-6(b) corresponds to the second 3phAR, which is based on a two-level converter topology. The voltage depression created by the fault turn-off the converters at both the rectifier and connected MDs, but the passive component of the converter remains connected. After the fault is cleared the overvoltage created at the AC bus produces also a large overvoltage at the DC bus which produces the trip of the AC circuit breakers. However, the bus will remain charged at the high voltage level. Finally, Fig. 6-7 shows the waveforms at the MDs connected to the multi-pulse rectifier; like the other MDs the undervoltage created by the fault turns-off the inverter. The simulation results corresponding to the low-speed (400 Hz) operation follow the same direction than the 800 Hz cases just presented. Additional results corresponding to faults at the generator feeder are included in the Appendix.
Fig. 6-6 Fault at generator terminals of feeder. V and I waveforms and trip signals at: (a) first 3phAR and MD (b) second 3phAR and MD

Fig. 6-7 Fault generator terminals of feeder: V and I waveforms and trip signals at MDs connected to the multi-pulse rectifier
B. Faults at AC Bus Generic Output

The simulation of the previous case showed the importance of using detailed models of all system loads. However, if switching models of all the converters present in the system of Fig. 6-2 are used the simulation may fail to converge. In such cases the system becomes too complex for the circuit simulator program that cannot handle it properly. Therefore, in that situation it is necessary to simplify the model of the system to be simulated. A simplification used in this study came from the elimination of one of the 3phAR branches. In order to keep the total power in the system constant the power of the other 3phAR, which is the two-level topology active rectifier, and its connected MD was double to reach a capacity of 50 KW. With this system simplification the schematic of the system model used in this part of the study is shown in Fig. 6-8.

The modified system model was used to analyze the effects of faults at various circuit locations. Instead of applying the fault at each AC bus connection it was decided to create a generic bus output where the faults can be applied. In this way the effect on all the branches can be analyzed in the same simulation. This is a common practice in power system protection studies [14]. The bus output and the fault location are shown in the Fig. 6-8; the resistance in the schematic corresponds to the fault resistance.

![Fig. 6-8 Model to study faults at a generic AC bus output](image-url)
Because the cases presented previously showed a high sensitivity of the system protections to disconnect non-faulted loads, in the simulations corresponding to this case the protection settings were changed according to the settings in Table 6-3.

**Table 6-3 Modified protection settings for the system stability fault analysis**

<table>
<thead>
<tr>
<th>Individual equipment</th>
<th>Typical settings (pu)</th>
<th>T-off time (msec)</th>
<th>Disc. time (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I &gt; Shoot V &gt; V &lt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Motor Controller</td>
<td>2 3 2 0.5 Inst</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Active Front End</td>
<td>2 3 2 0.5 Inst</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three-phase circuit breaker</td>
<td>2 - - - 10+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single-phase circuit breaker</td>
<td>2 - - - 10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The simulation results in Fig. 6-9, Fig. 6-10 and Fig. 6-11 correspond to a case with system operation at 400Hz and fault resistance of 10 mΩ. The clearing time was set at approximately 40 msec and the undervoltage protection was disabled. Fig. 6-9 shows the generator and 1phAR waveforms during the fault transient and clearance with minor differences compared to the previous case. In this case the fault is cleared at t = 0.202 sec.

In Fig. 6-10, which shows the waveforms corresponding to the 3phAR, MPTR, and MD, it is possible to observe a major difference compared to previous case. Fig. 6-10(a) shows that after the fault clearance the 3phAR does not recover normal operation. On the contrary, it gets disconnected by its overvoltage protection. When the fault is cleared the 3phAR attempts to recover its pre-fault operation; the recovery transient produces a large overvoltage that eventually excites the maximum voltage protection, this happen even with the protection setting considerably raised from its original value. In this situation the CB trips approximately 15 msec after the fault has been cleared. The trip causes the overvoltage to grow even further before its starts decreasing with the bus discharging. The difference between this case and the case with a clearing time of 30 msec (shown in Appendix) identifies the critical clearing time in the range between 30 msec and 40 msec. This time also depends on the severity of the fault (i.e. fault resistance) and overvoltage protection setting. The waveforms in Fig. 6-11 show the evolution of some of the 3phAR and MD control variables. The names of the control variables for the MD displayed in Fig. 6-11 correspond to the MD control schematic shown in Fig. 6-12.
Fig. 6-9 Fault at generic AC bus output. V and I waveforms and trip signals at:
(a) generator and (b) 1phAR loads

Fig. 6-10 Fault at generic AC bus output. V and I waveforms and trip signals at:
(a) 3phAR and connected MD (b) MTPR and MD loads
Fig. 6-11 Fault at generic AC bus output. V and I waveforms at:
(a) MD controller (b) 3phAR controller

The variables of the 3phAR shown in Fig. 6-13(b) correspond to the controller schematic in Fig. 6-13. The controller has two inner current loops and an outer voltage loop that regulates the DC voltage value.

Fig. 6-12 MD controller schematic showing the magnitudes measured during the simulations

V. Analysis of System Behavior under Faults

The purpose of this section is to analyze and discuss the results from the fault case study simulations. First the effects of faults at each of the individual circuit branches are discussed. Then some considerations are presented about the effects of the faults on the interconnected system.
Fig. 6-13 3phAR controller schematic showing the variables measured during the simulations

A. Synchronous Generator

The evolution of some of the synchronous generator main magnitudes during a fault at the system AC bus are shown Fig. 6-14. In that Figure, the waveform $v_{ar}$ corresponds to the voltage at the AC bus, which is the point of regulation. The waveform $i_{agen}$ is the current at the generator terminals. In the pre-fault condition the generator was operating at a point near its rated capacity. During the operation under the fault condition the current does not exceed 2.5 times its rated value. This is due to the limitation of the maximum value that the excitation current can achieve. In this way the excitation circuit limits the armature current during short circuits. The excitation current waveform is shown as $i_{exm}$ at the bottom of Fig. 6-14. In the same Figure it is also possible to observe the voltage angle, $delta$, which stays within the first quadrant during the transient created by the fault.

The evolution of the generator magnitudes during the different fault scenarios simulated in this study shows a similar response to the results in Fig. 6-14 for most of the cases. A case showing different behavior was presented in Fig. 6-5 where a considerable overvoltage appears at the AC bus after the fault clearance. This overvoltage can potentially excite some of the load protections and produce undesired tripping.
B. Motor Drives

The transient behavior of the MD during a fault located at its input terminal is explained with the help of the waveforms in Fig. 6-15. The figure shows the waveforms for the two MDs connected to the multi-pulse rectifier; no protection action is included in the simulation in this case. Because of the fault, the voltages at the connection points, $v_{dc}$ and $v_{dct}$, become very small. Because of the back-emf of the rotating motor the power flow in the MD reverses during the initial fault time. This is noticed by the large negative values of the DC currents, $i_{dc}$ and $i_{dct}$, observed in the Figure. The large DC current has a correlation at the AC motor side of the converter in $i_a$, which achieves a peak value around three times the rated current.

The reverse power flow causes the energy stored in the motor and mechanical load inertias to flow back to the electrical system and get dissipated in the fault. Therefore, the motor quickly reduces its speed. When the fault is finally cleared, the voltage at the DC link is restored allowing the VSC to restore the voltage at motor armature windings. The restoration transient causes some overcurrent in the motor armature as seen in Fig. 6-15. This overcurrent is limited to

Fig. 6-14 Evolution of generator magnitudes during a fault at the AC bus
about 1.5 times the rated current. Additionally, some overvoltage appears at the DC bus, but it is of reduced magnitude.

![Waveforms](image)

**Fig. 6-15** Waveforms corresponding to the MDs connected to the MPTR during a fault at the DC bus connecting the rectifier to the motor drives

### C. Three-phase Active Rectifier

The 3phAR presents some interesting behavior described with the help of Fig. 6-16. The case described in that Figure corresponds to a fault at the system AC bus. Fig. 6-16(a) on the left corresponds to a clearing time of 30 msec while Fig. 6-16(b) on the right to 40 msec. In the case of the shorter clearing time the 3phAR recovers normal operation after fault clearance. On the contrary, when the clearing time becomes 40 msec the normal operation is not restored after the fault clearance.

In the case in Fig. 6-16(b) the 3phAR controller attempts to restore the voltage at the DC output at the expense of a relatively large DC current, but fails to regulate it at the reference value. After some time the DC voltage becomes large enough to excite the overvoltage
protection that disconnects the 3phAR from the AC system. The lack of proper DC voltage control during the restoration transient creates the large voltage excursion. In the study it was found that a tighter current limit will improve the performance during the recovery allowing restoring the system operation for most cases.

![Diagram](image)

**Fig. 6-16 3phAR transient response for an AC bus fault: (a) clearing time equals 30 msec (b) clearing time equals 40 msec**

By comparison of the two cases shown in Fig. 6-16 it is possible to conclude that the 3phAR branch presents a critical clearing time with a value between 20 and 40 msec.

**D. Single-phase Active Rectifiers**

The response of the 1phAR circuits to an AC fault is shown in Fig. 6-17. Due to the large AC voltage drop, the controller of the 1phAR is unable to keep the DC voltage constant. When the fault clears, a large AC current excursion appears both at the primary and secondary side of the transformer connecting the rectifiers to the circuit (waveforms $i_{a1r1}$ and $i_{a1r}$ in Fig. 6-17). The overcurrent observed in the Figure reaches twice the circuit rated current. The DC overvoltage also shows an overshoot of 50% the reference DC value. This overshoots are related to the absence of antiwindup schemes in the 1phAR circuit controllers. The increased overcurrent on the AC circuit can also excite the overcurrent protections. In addition, they create additional losses and voltage drop that may affect other branches of the circuit.
Fig. 6-17 Transient response at the 1phAR circuit branch for a fault at the AC bus

E. Interconnected System

A fault appearing in a power system affects the operation of each individual component. It also affects the operation of the system as a set of interconnected equipment that must operate in a coordinated manner. Therefore, it is important in the fault study to analyze if the faults can affect that coordinated operation. In other words, the study must detect if undesired interactions among system components appear during the fault affecting any of the equipment. In such case the consequences of the fault are not isolated; they can cause additional stresses and related protection trips.

A case that leads to overvoltage at the AC bus during fault clearance is given by a short circuit located at the feeder generator terminals. This case is shown in Fig. 6-5 where the second CB trip leaves the system with overvoltage in the AC bus. After the first CB trip the generator takes a large excitation in an attempt to try to regulate the voltage at POR. The load seen from the generator terminals corresponds mostly to the impedance of the two feeders connected in
series, which is highly inductive. When the second CB opens the excitation conditions at the generator produce a large voltage at the POR. The magnitude the voltage can achieve is related to the maximum excitation current limit. The overvoltage created can cause some harmful effects on the equipment connected. In addition, it can activate the overvoltage protection at the individual equipment. The disconnection of the equipment during the recovery can favor further overvoltage increase creating a cascading disconnection.

VI. Summary

This chapter has analyzed the effect of faults on the system stability by means of time domain simulation. Various fault scenarios were considered in the analysis. For these studies proper models of the system were set. In order to facilitate the required computations with the available resources in a reasonable amount of time, the system models were settled according to the type of fault under study; this allowed for some model simplification.

Protection schemes were provided for the different system components. The settings of these protections greatly condition the system response during and after a fault. In order to visualize the fault effect on the system behavior, the protection settings were relaxed allowing for larger excursions of the circuit magnitudes during the transients. Under the condition of relaxed protection the system was able to recover after fault in most of the cases. Nevertheless, a safe operation requires proper protection settings in order to avoid equipment damage. In this sense overvoltage and overcurrent protections are necessary. It was observed in the simulations that large current or voltage excursions can occur due to a fault. In many cases this excursions are not compatible with the equipment integrity. The following paragraphs summarize the results of the study.

The faults studied through the simulations have been short circuits, which in most cases involved the three phases. Those faults create a large voltage depression that affects the equipment in the ways described in the previous sections. Some of those effects are expected from the fault characteristics, especially the effects occurring during the fault itself. This is the case of the regeneration observed at the MD for example. However, this effect is limited to a large current circulation between the MD and the fault point.
Some unstable behavior has been observed after the fault clearance in the 3phAR. This unstable behavior originates in a deficient operation of the antiwindup schemes creating large overvoltages and the disconnection of the 3phAR branch. However, as observed in the simulation results in Fig. 6-9 to Fig. 6-11 the overvoltage does not propagate to the AC bus or other equipment. The related large overcurrent creates an additional voltage drop, but under the testing conditions it does not prevent the AC voltage from recovery. This has also relation to the branch disconnection by the protection. In case the overcurrents further increase it could affect the voltage regulation and the whole system operation.

The 1phAR branch also experience large overcurrent during restoration, which is related to the fault duration. In the cases tested in this study, the overcurrent at the 1phAR branch did not affect the system operation because of the limited total power of the branch. In case the branch power becomes a large portion of the system the impact could become more significant. Both cases of the 3phAR and 1phAR are related to an improper operation of their controllers. Therefore, the undesired operation observed during the restoration can probably be minimized by an adequate control scheme. Nevertheless, these aspects must be considered when analyzing the system stability.

From the system point of view the fault at the generator can create large overvoltages at the AC bus. The overvoltage level may increase if additional equipment is disconnected after the fault clearance. If the higher voltage reaches a level that trips the overvoltage protection, a cascading effect may be produced disconnecting most of the loads in the system.
Chapter 7  Generator Excitation Controller

I. Introduction

The synchronous generator, energy source of the system, is the object of this chapter discussion. The system stability behavior is closely related to the generator stability. Moreover, different than in other type of power systems, in the autonomous power system under study the generator stability refers almost exclusively to its electrical stability. The electromechanical factors that are relevant in many stability studies do not take part in our system due to the several times larger prime mover power than the generator rated power. This difference can be more than one order of magnitude. The voltage dynamics of this type of machine is commanded by the voltage controller, which commands the excitation voltage applied to the machine field. Due to this fact, this chapter develops a voltage controller with a wide range of operating frequency. The proposed control algorithm is developed based on the synchronous generator model, which provides an improved and more uniform dynamics.

The proposed model base control uses a synchronous machine model formulated in $d$-$q$ coordinates. The calculation of the $d$-$q$ components of the circuit magnitudes makes necessary to know the position of the generator shaft during operation. In that way the $d$-$q$ components of the electrical magnitudes $v$ and $i$ can be calculated and used for control. Therefore, in addition to the algorithm formulation it is necessary to search for ways to detect the rotor position of the electric machines. This is done in the section II of this chapter. Section III presents the controller algorithm, while section IV shows the results obtained with the proposed algorithm. Finally, section V discusses the controller stability improvements with the proposed control method.

A closer look at the generator shows that it is actually a set of three coupled rotating machines. Fig. 7-1 shows the configuration of the system under study with the generator and generator control unit (GCU). The primary power for the generator excitation is taken from a permanent magnet generator (PMG) that feeds the exciter, which is a synchronous machine, through a controlled DC/DC converter. The GCU produces the signal that controls the excitation of the generator.
The rectifier can have different operation modes with correspondingly different characteristics. Continuous average rectifier models, like the one given by equation (1), provide an adequate description only when the diode bridge operates in rectifying mode, transferring power from AC (exciter armature) to DC (main generator field). In practice, the diode bridge can also operate in freewheeling mode. For instance, sudden load changes can induce a very large voltage in the field winding of the main generator, leading the diode rectifier to freewheeling. Fig. 7-2 shows a schematic of the main generator field and the diode bridge circuit, connected to the exciter. The voltage of the main generator field winding, which is connected to the rectifier, is given by equation (7-1)

\[
v_{fd} = r_{fd}i_{fd} + (L_{fd} + L_{md}) \frac{di_{fd}}{dt} - e_{dfd}
\]

where \(e_{dfd}\) is the emf induced in the field by the load change, given by:

\[
e_{dfd} = N_{fd} L_{md} \left( \frac{di_{fd}}{dt} - \frac{di'_{fd}}{dt} \right)
\]

In rectifying mode, \(v_{fd}\) is greater than zero. From the equation (7-2), it can be seen that a sudden change (increase) in \(i_{fd}\) can create an induced voltage large enough causing \(v_{fd}\) to try go negative. In such a case, the diodes are directly positively biased, applying no voltage to the main generator field, and any intended excitation change by the exciter will have no effect over the field of the main generator. The phenomenon can be seen clearly in the simulation shown in
Fig. 7-5, in which the induced voltage $e_{dld}$ increase causes the field winding voltage $v_{fd}$ trying to go negative and the diode-bridge to go into the freewheeling mode. The impact of the phenomenon on the output voltage is also shown.

![Diagram of Rectifier and Generator Field Equivalent Circuit](image)

**Fig. 7-2 Rectifier and generator field equivalent circuit**

The diode rectifier freewheeling mode limits the system controllability and no control can be executed in such a mode, making it difficult to implement a control fully based on the system model. However, it will be shown in subsequent sections that it is still possible to obtain the main controller characteristics from the models and achieve good performance, despite the rectifier freewheeling mode.

II. Evaluation of Rotor Position Detection Techniques

For using d-q model based control in synchronous generator excitation regulation it is required to know the position of the rotor shaft during operation. Because all three rotating
machines: PMG, exciter, and generator, are coupled on the same shaft it is possible to base the rotor position on any one of them. Due to its relative simplicity, the PMG is selected for the rotor position detection. Because of cost and reliability reasons it is desired that the position detection is done sensorless.

A. Review of Rotor Position Detection in PM Machines

Several methods have been proposed in the literature for the rotor position detection of permanent magnet motors. Traditionally those methods were grouped in two main categories according to their working principles: reluctance techniques [115], and state observers [116]. The first group principle is based on determining the response of the machine to an injected electric signal. In addition, it is generally applied where the detection is required to operate at standstill or low speeds. On the other side, the observer principle relies on the state determination based on the measurement of the available magnitudes and the machine model. For this purpose, a proper system model is required [117], [66]. Because the application under study that does not involve low speed operation, it is considered more suitable for the observer type approach. Many different observation techniques have been proposed and used for rotor position detection in permanent magnet motors [118]-[125]. After a wide review of those techniques, three of them were selected for a detailed evaluation in computer simulation. The simulation concluded in the selection of one of the algorithms for experimental testing. This last algorithm was then enhanced for its implementation in a DSP. The results obtained probe the appropriate operation of the implemented detection algorithm.

The observer based techniques can be classified from both the point of view of the observer itself and the filtering method. Observers can be of deterministic or stochastic type as well as the filtering techniques. In any case the challenge is to overcome errors that are mainly created by parameter uncertainty or measurement errors and noise. The machine model used in the observer can be formulated in the stationary reference frame a-b-c, like equation, in the $\alpha$-$\beta$-$\theta$ coordinates, or in the rotating reference frame d-q-o. The models in the different references are obtained from the following equation by application of standard matrix transformations.

$$V_{a,b,c} = R_{a,b,c} I_{a,b,c} + L_{a,b,c} \frac{d I_{a,b,c}}{dt} + \frac{d \phi_{a,b,c}}{dt}$$  \hspace{1cm} (7-3)
Among the three methods preliminary selected, one of them uses a stochastic observer while the other two are based on back-emf estimation using different deterministic machine modeling. The stochastic observer uses a sliding mode observer and an extended Kalman filter. The idea of this method is to use a sliding mode observer (SMO) over an expanded state of the PM machine for a robust current detection [118],[119]. The Kalman filter is used for filtering measurement noise and SMO oscillation of the signals [120]-[122]. The second method [125], calculates the back-emf from the flux linkage change. In order to improve the estimation and make the method less sensitive to the noise amplification due to the use of time step increments, the procedure averages the three-phase measurements.

The last approach is based on an observer built on the d-q-o model of the machine [123],[124]. The technique selected is described in [123], where the difference between the observed and measured voltages is used to iteratively correct the position estimation.

B. Evaluation of Position Detection in Simulation

The preliminary evaluation was done in simulation using a detailed model of the PMG, which included nonlinearities produced by non-uniform coil disposition and the rectifier load. The interest was to test the robustness of the algorithms for: machine modeling inaccuracies, parameter deviation due to miscalculation or changes in the operation conditions, non-linearity of the load circuit connected to the PMG, and noise in the voltage and current measurements. The method based on the SMO and EKF presents the larger error in ideal conditions, but it is the most robust against disturbances. The flux linkage method presents a very small angle estimation overall, but it is sensitive to the electromagnetic constant value. The method based on the observation in the d-q coordinates is quite sensitive to disturbances. The results are summarized in Table 7-1.

<table>
<thead>
<tr>
<th></th>
<th>SMO + EKF</th>
<th>Flux linkage</th>
<th>d-q observer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Typical angle error</strong></td>
<td>Large, about 1.5 deg</td>
<td>Very small, &lt; 0.1 deg</td>
<td>Very small, &lt; 0.1deg.</td>
</tr>
<tr>
<td><strong>Parameter deviation</strong></td>
<td>Low</td>
<td>$R_s, L_s, \text{low}$, $k_e, \text{high}$</td>
<td>Low</td>
</tr>
<tr>
<td><strong>Noise handling</strong></td>
<td>Good</td>
<td>Good</td>
<td>Fair</td>
</tr>
<tr>
<td><strong>Load nonlinearities</strong></td>
<td>Very small effect</td>
<td>$\theta$ small, $\omega$ large effect</td>
<td>Considerable effect</td>
</tr>
</tbody>
</table>
C. Experimental Evaluation in Case of Non-linear Load

For the application under study it was required to know the response in case of non-linear load. Therefore, a diode rectifier with capacitive filter, which in the final application will supply the power for excitation, was connected to the PMG. Fig. 7-4 shows the circuit used to test the algorithm response for this case.

![Circuit Diagram]

**Fig. 7-4 Schematic of the circuit used to test the position algorithm response in case of nonlinear load**

The voltage and current waveforms at the PMG terminals, used for detection, are shown in Fig. 7-5. Two line to line voltages were measured in the circuit of Fig. 7-4 from where the three phases to neutral magnitudes were calculated. Additionally, the three line currents were also measured in the circuit.

![Waveforms and Harmonic Spectra]

**Fig. 7-5 (a) Waveforms at the PMG with the rectifier load; line and phase voltages, and line current, (b) Harmonic spectra for the voltage and current used for the position detection**
The harmonic contents of the voltage and current magnitudes are shown in Fig. 7-5(a). In this case the total harmonic distortion (THD) in both voltage and current increased to about 12%. The response of the position detection scheme changes significantly under the harmonic regime described in Fig. 7-5. The angle error obtained is shown in Fig. 7-6 that also includes the case of resistive load for comparison purposes. The angle error increased to 0.08 radians (or 4.6 degrees) in the rectified load case. Although the considerable change in the response, the detected angle is still accurate enough for the application under analysis.

![Graph of Rotor position error obtained for rectified and resistive loads](image)

**Fig. 7-6 Rotor position error obtained for rectified and resistive loads**

III. Generator Voltage Controller

*A. Review of Works on Excitation Controllers*

There have been numerous previous work on model based synchronous machine controls for torque and/or speed regulations as in motor drive applications [127]-[129], where stator current, voltage, and excitation field are controlled together. Some motor control approaches take advantage of the full machine model [127]-[128] while some others use only a simplified or reduced-order model [129]. The models are usually used in auxiliary control loops for feed-forward actions, while a classical feedback loop is still used for the basic regulation. On the other hand, the model based control is less common and less straightforward in synchronous generator voltage regulation, where control is realized only with the excitation field. The predictive controller [130] is based on the machine model, although an additional profiler is applied to the controller output (in this case, field voltage or current command), which changes and usually
slows down the original machine dynamics in order to maintain stability. The approach was improved in the transfer function regulator method [131], which uses a model based auxiliary feed-forward control loop for load compensation, while the main excitation regulation is still performed by a conventional automatic voltage regulator (AVR).

The control approach adopted by this paper advances the concept introduced in the transfer function regulator method [131], by using model based controls for both the basic voltage regulation loop and the auxiliary control loop for load disturbance compensation. In addition, an inner feedback loop for exciter field current regulation is used for improved performance. The control concept utilized in this paper can fall under the category of the two-degree-of-freedom internal model control (IMC) as explained in [132]. IMC has been used in motor control for drive applications. A one-degree-of-freedom IMC is proposed in for current control [128], whereas a two-degrees-of-freedom IMC is used for motion control [133]. In another motion control application where the open-loop system is unstable with the full model, partial internal model control (PIMC) proposes using only the stable portion of the model [134].

B. Controller Principle

Fig. 7-7 illustrates the block diagram of the proposed controller for the main generator voltage regulation, which is based on three control actions. The first is an inner feedback loop that controls the excitation current of the exciter, the second is the outer feedback loop for direct output voltage regulation of the main generator, and the third is the disturbance rejection loop that compensates for the changes in the output voltage caused by the generator load (i.e. stator) current. This last control loop is a feed-forward loop. The excitation current command or reference for the inner-loop is generated by the sum of the regulation action $i_{e,reg}$ from the outer voltage regulator and the disturbance rejection $i_{e,change}$ from the feed-forward loop.

As mentioned in the previous subsection, the proposed control concept and design can also be analyzed in the context of internal model control (IMC) method. The IMC control strategy was first formalized in [136] for single-input-single-output (SISO) and in [136] for multi-input-multi-output (MIMO) systems. Although in our case no plant model is used explicitly to calculate the disturbance value, since the disturbance can be measured, the principle of the controller in Fig. 7-7 can be related to a two-degree of freedom IMC [132]. Note that the controlled exciter can be considered as part of the plant.
The postulate of IMC is to use the system model both to control the process and to reject the disturbances. In our case, both $C_v(s)$ and $C_d(s)$ (in Fig. 7-7) are obtained from the system model. For SISO systems, the design of the IMC controller is quite straightforward, requiring to invert the process transfer function with additional rules to avoid improper functions or right half plane poles. In the case of a MIMO system, the process is more complex [136]. In our case, there is only one control variable, i.e. exciter field current (or field voltage), but multiple output variables, considering the independent generator phase voltages or equivalent d and q voltage components. The following paragraphs provide more details on the proposed controller algorithm.

C. Exciter Field Current Regulator

The exciter field current loop ($i_{ex}$-loop) transfer function of $C_{ex}(s)$ is given in the next equation (7-4).

$$C_{ex}(s) = K_e \frac{s L'_{eqf}/r'_{eqf} + 1}{s/\omega_{pex} + 1}$$

(7-4)

$C_{ex}(s)$ is a lead-lag controller, whose zero is chosen to cancel the major pole of the equivalent exciter field winding. Although that pole is primarily a function of the field winding parameters, it is also related to the armature winding parameters and load depending on the operation conditions of the exciter. The impact from the armature side can be characterized through its effect in the equivalent inductance $L'_{eqf}$ of the field winding. An approximate calculation for $L'_{eqf}$ is given in (7-18). The parameters $\omega_{pex}$ and $K_e$ should be selected for desired bandwidth and
stability margin. Note that the bandwidth is practically limited by stability requirement, with one of limiting factors being the delay caused by measurement time-lag due to digital sampling. It is found that a good choice for $\omega_{p_{ex}}$ is to make it ten times larger than the regulator zero ($r_{fd}^e / L_{esf}$), provided that it is still considerably smaller than the sampling frequency. Once $\omega_{p_{ex}}$ is chosen, $K_e$ selection is straightforward given the regulator zero and pole, the desired margin, and the measurement time-lag or sampling frequency. With a properly designed and sufficiently large gain, the exciter field current tracks the command well.

D. Voltage Regulator

The outer feedback loop provides the basic generator voltage regulation. Because the additional disturbance rejection loop accounts for the load changes, the voltage regulator design is based on the no-load machine model. For the application under study, the rotation speed $\omega_r$ can be considered fixed at a given operating point. Therefore the machine model equations under such condition are linear and time-invariant, and can be arranged in matrix format of equation (7-5). Note that for simplicity, the saturation is neglected in the voltage regulator because of limited effect, though it could be compensated with a model if necessary.

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

(7-5)
in which, $x$, $y$, and $u$ are vectors for state variables, output variables, and control variables, respectively. The matrices $A$, $B$, and $C$ are linear-time-invariant coefficients determined from the machine model. For the main generator under no-load condition, the $x$, $y$, and $u$ vectors can be established as:

$$x^T = \begin{bmatrix} \lambda_d & \lambda_q & \lambda_{fd}' & \lambda_{qd}' \end{bmatrix}$$

$$y^T = \begin{bmatrix} v_d & v_q & v_{fd}' \end{bmatrix}$$

(7-6)

$$u = \begin{bmatrix} i_{fd}' \end{bmatrix}$$

Note that the control can be either the field voltage $v_{fd}'$ or current $i_{fd}'$. Since under no-load condition the rotor angle $\delta$ and $v_d$ are null, the $v_{q} - i_{fd}'$ transfer function can be used for designing the voltage controller. This transfer function can be expressed as:
\[
\frac{V_q(s)}{I_{jd}(s)} = \frac{1}{\omega_q} \frac{K_{iv}\cdot f(s)}{(s + r_{jq} / L_{jq}) \cdot \left(s + r_{iq} / (L_{iq} + L_{m}) \right) \cdot h_{fp}(s)}
\]  

(7-7)

As in the exciter case, the dominant pole of the main generator transfer function is determined by the field winding. However, the field winding pole has already been considered in the design of the exciter field current regulator. The remaining major pole is due to the q-axis damper winding.

The voltage regulator \( C_v(s) \) is chosen to be a lead-lag plus an integrator. The use of a lead-lag is dictated by the dominant pole and zero of the transfer function (7-7). The integrator is added to cancel any steady-state voltage error, and its introduction, as well as using only part of the machine model, necessitates a separate stability analysis [136]. However, this analysis can be done from the frequency response of the linearized small-signal model. The presence of the integrator makes it necessary to locate the controller zero at a lower frequency than the q-axis damper winding pole. The final placement depends on the desired stability margin, and a good choice is to shift the controller zero to at least half of the original frequency, which gives approximately 60° phase margin. The proposed regulator function is shown in (7-8), where \( c_{kq} \) accounts for the frequency shift for the zero. The stability analysis also shows that it is convenient to shift, \( \omega_p \), the pole of \( C_v(s) \) to higher frequency than the q-axis damper winding pole for better phase and gain margins. Note also in (7-6), to compensate for the machine characteristic changes with speed (frequency), the gain of \( C_v(s) \) is adapted in inverse proportion to the square of the rotating speed \( \omega_r \), whose detection is done together with the angle detection explained in previous section.

\[
C_v(s) = K_v \left( \frac{\omega_r}{\omega_{r_{max}}} \right)^2 \frac{1}{s} \frac{s \left(L_{iq} + L_{m} \right) c_{kq} / r_{iq} + 1}{s / \omega_p + 1}
\]  

(7-8)

**E. Load Disturbance Compensator**

In addition to the two feedback control loops for \( i_{ex} \) and \( v_{out} \), which are routinely used in conventional voltage regulators for synchronous generators in airplane systems, this work introduces a new load disturbance compensator loop in the controller for the variable frequency generator. The load compensator, which is represented by \( C_d(s) \) in Fig. 7-7, is termed \( i_d \)-loop since it changes the excitation current requirement based on the d-axis load current. On the other hand, the q-axis load current does not directly impact the excitation current. The operation
principle of $i_d$-loop is outlined below, while the details of the $i_d$-loop compensator derivation are explained in the load compensation transfer function sub-section.

First, we need to obtain the relationship between the controlling variable $i'_fd$ and the disturbances, i.e. the generator current components, $i_d$ and $i_q$. From the generator model given by equations (7-5) and (7-6), the input output relation can be generically given in frequency domain by:

$$I'_fd = G_{vd}V_d + G_{vq}V_q + G_{vfd}'V'_fd$$  \hspace{1cm} (7-9)

where $G_{vd}$, $G_{vq}$, $G_{vfd}$ are the respective transfer functions. The relationships between $V_d$, $V_q$ and $I_d$, $I_q$ are given by the machine equivalent output impedance. After some algebraic manipulation, it is found that the field current can be expressed as in (7-10):

$$I'_fd = H_d I_d + H_{sd}V'_fd$$  \hspace{1cm} (7-10)

The full expressions of $H_d$ and $H_{fd}$ in s-domain are given below. In our system $V'_fd$ is fed by the rectified exciter, which has a nonlinear $V$-$I$ relationship. This inconvenience can be overcome by linearization technique. At a given operating point, the excitation voltage $V'_fd$ can be represented by a Thevenin equivalent source with an equivalent open circuit source voltage $e'_fd$ and an equivalent source impedance $z'_fd$. Using the linear approximation in the field current equation (7-9) yields:

$$\Delta I'_fd = H_d \Delta I_d + H_{sd} \Delta E'_fd - H_{sa} Z'_sa \Delta I'_sd$$  \hspace{1cm} (7-11)

This equation can determine the change in the excitation current requirement as a function of the load change $\Delta I_d$ and the change of the rectified exciter source voltage $\Delta E'_fd$.

$$\Delta I'_fd = \frac{H_d}{1 + H_{sa} Z'_sa} \Delta I_d + \frac{H_{sd}}{1 + H_{sa} Z'_sa} \Delta E'_fd$$  \hspace{1cm} (7-12)

As can be seen clearly from (7-12), there can be two distinct forces to change the generator excitation field currents at a given operating point. There is a significant difference between $\Delta I_d$ and $\Delta E'_fd$. While the former is the disturbance itself, the latter is the result of the system controller action. As a matter of fact, the change in $E'_fd$ is governed by the action of the voltage regulator and exciter field current regulator. In addition, the time constant associated with $\Delta I_d$ (mainly a stator winding time constant) is generally much shorter than the time constant associated with $\Delta E'_fd$ (mainly a field winding time constant), which makes $\Delta I_d$ impact on $I'_fd$
faster. The same can be seen from the characteristics of the related transfer functions $H_d$ and $H_{fd}$, with the zeros of the $H_d$ at considerably lower frequencies than those of $H_{fd}$. It makes the early effect on $I'_{fd}$ by $\Delta I_d$ considerably more than by $\Delta E'_{fd}$. Based on these considerations, our approach is to make the $i_d$-loop compensate only the disturbance originated from the $\Delta I_d$ component in (7-12). The effect of $\Delta E'_{fd}$ is accounted for through voltage and exciter field current regulation loops. In a way, this approach is similar to the transfer function regulator method presented in [131] where the controller action is divided into a load compensation part, depending exclusively on the load current change, and an excitation compensation handled by a traditional AVR controller. However, our approach is different than [131] in how this load compensation is calculated. Considering only the impact of $\Delta I_d$, equation (7-12) can be simplified as:

$$\Delta I'_{fd} \approx \frac{H_d}{1 + H_{ld} Z'_{ld}} \Delta I_d$$ (7-13)

The transfer function of the disturbance compensator $C_d(s)$ is obtained accordingly from equation (7-13) to yield (7-14):

$$C_d(s) = H_d(s) \frac{1.5 L_m}{N_{mg} L'_{ld}}$$ (7-14)

with $H_d(s)$ given below in equation (7-21).

In summary, the block diagram in Fig. 7-7, together with regulators defined by equations (7-4), (7-8), and (7-14), completely describes the concept of the proposed model-based control scheme for variable frequency generator.

IV. Generator Transfer Functions for Control Design

This section describes the generator transfer functions and simplifications used to derived the controller transfer functions presented in the previous section.

A. Exciter Field Current and Main Generator Voltage Transfer Functions

The design of these control loops considers the main generator under no-load conditions, i.e. negligible armature current. For the exciter field current controller design, the exciter load must be considered. This load is the generator field winding circuit connected to the rotating rectifier.
The field $V$-$I$ relationship of the unloaded main generator is dominated by the field winding and can then approximated by its parameters. Additionally, the angle between the first harmonics of $V$ and $I$ at the rectifier is considered small. Under these assumptions, both $d$ and $q$ armature windings of the exciter are loaded by an equivalent inductance $Z_{eqv}$.

$$Z_{eqv} = \frac{k_i}{k_v} \left( r_{fd} + s L_{fd} \right) \quad (7-15)$$

The following equation gives a simplified $V$-$I$ excitation relationship based on a reduced order model of the exciter.

$$\frac{I'_{eq} (s)}{V'_eq (s)} = \left\{ s \left[ \frac{L'_{eqd} + L_{emd}}{L'_{eqd} + \frac{\omega_v^2 L_{emd} L_{eq}}{\omega_v^2 L_{cal} L_{eq} + (r_{eq} + Z_{eqv})^2}} \right] + r'_{fd} \right\}^{-1} \quad (7-16)$$

The transfer function (7-16) as a third order polynomial denominator, and a second order numerator. The denominator has one real and a pair of complex conjugate poles. All poles and zeros move when the operation point changes. Because the two zeros follow the movement of the complex conjugate poles, the excitation current regulator mainly needs to compensate the real pole. This real pole also varies in a smaller range, making it easier to compensate. In our study, it was found that a good placement for the zero of $C_{eq}(s)$ can be calculated by (7-17):

$$\omega_v \approx \frac{r'_{eqd}}{L'_{eqd} + L'_{eqd}} \quad (7-17)$$

with:

$$L'_{eq} = \frac{L_{emd}^2}{L_{cal} + \left( \frac{2 r_{eqd} k_i k_v}{\omega_v^2 L_{cal}} \right)^2} \quad (7-18)$$

**B. Load Compensation Transfer Functions**

The principle to obtain the load disturbance compensator $C_d(s)$ for a single synchronous machine was explained in the previous sub-section. However, the exciter and rectifier must also be included in $C_d(s)$. According to equation (7-13), the exciter field current compensation produced by the $i_{fd}$-loop is given in s-domain by:

$$\Delta I'_{fd} (s) = H_{i_{fd}} (s) \Delta I_d (s) \quad (7-19)$$
with:

\[ H_{d}(s) = \frac{H_{d}(s)}{1 + H_{d}(s)Z_{d}(s)} \]  

(7-20)

where \( H_{d}(s) \) and \( H_{td}(s) \) are obtained from the machine model. In our case they are equal to:

\[ H_{d}(s) = \frac{sL_{rd}(sL_{rd} + r'_{td})}{D(s)} \]  

(7-21)

\[ H_{td}(s) = \frac{s(L_{rd}' + L_{md}) + r'_{td}}{D(s)} \]  

(7-22)

With:

\[ D(s) = s^2 \left( (L'_{rd} + L_{rd})(L'_{rd} + L_{rd}) - L_{rd}^2 \right) + s \left( (L'_{rd} + L_{rd})r'_{td} + (L_{rd}' + L_{md})r'_{td} \right) + r'_{td}r'_{td} \]  

(7-23)

The calculation of \( Z'_{td}(s) \) is quite complex because it involves the output impedance of the exciter-rectifier \( Z_{out} \). For control design, it can be calculated from the average value model of the generator-rectifier, as done in [70]. Using the average modeling approach in calculating \( Z'_{td}(s) \), it is possible to observe that the denominator in (7-20) remains close to unity over a wide range of frequency. This is because

\[ Z_{td}(s) = Z_{out}(s) \frac{1.5}{N_{f}^2} \]  

(7-24)

where \( N_{f} \) is usually large. Fig. 7-8 shows \( H_{d}(s) \) and \( H_{d}(s) \) calculated from (7-20) for the machine A in Table 7-2. In fact \( H_{d}(s) \) varies within \( H_{d}(s) \) and \( H_{dim} \) (two dotted lines) corresponding to two extreme operating conditions. For most of the normal operation conditions, \( H_{d}(s) \) stays close to \( H_{d}(s) \). Therefore, the load compensation control was implemented using the approximation:

\[ H_{d}(s) = H_{d}(s) \]  

(7-25)

The compensation for the main generator field current due to a load change can be determined from (7-20) using (7-25). However, the controller acts through the exciter. The exciter field-armature current relation is:

\[ I'_{q}(sL_{qf} + r'_{qf}) = sL_{q}I_{q} + V'_{q} \]  

(7-26)
Fig. 7-8 Transfer function $H_d(s)$, and transfer function $H_{dM}(s)$ range defined by $H_{dM}(s)$ and $H_{dM}(s)$

In addition, the excitation current control loop gives the excitation current-voltage relation

$$V' = C_{ex}(s) \frac{1.5}{N_{ef}} \left(I' - I'_{ref}\right)$$  \hspace{1cm} (7-27)

where $C_{ex}(s)$ is given in (7-4) and $I'_{ref}$ is the control loop reference current. Replacing (7-27) in (7-26):

$$s I'_{ef} \frac{L'_{ef}}{L_{end}} \left(1 + \frac{K_e}{r_{ef} \left(s \tau_{pe} + 1\right)}\right) + I'_{ref} \frac{L'_{ef}}{L_{end}} - s I'_{ef} \frac{L'_{ef}}{L_{end}} \frac{K_e}{r_{ef} \left(s \tau_{pe} + 1\right)} = s I_{ef}$$  \hspace{1cm} (7-28)

Considering the effect of the exciter field current regulator, equation (7-28) can be approximated as:

$$s I'_{ef} \frac{L'_{ef}}{L_{end}} + I'_{ef} \frac{L'_{ef}}{L_{end}} = s I_{ef}$$  \hspace{1cm} (7-29)

Converting this equation to finite increments yields:

$$\Delta I'_{ef} = \Delta I_{ef} - \Delta I'_{ef} \frac{r'_{ef}}{L'_{ef}}$$  \hspace{1cm} (7-30)

And considering small $\Delta t$, it can be approximated by (7-31).

$$\Delta I_{ef}(t) = \frac{1.5}{N_{ef}} \frac{L_{end}}{L'_{ef}} \Delta I_{ef}(t)$$  \hspace{1cm} (7-31)

Assuming that the exciter armature current lies mainly on the d-axis, which is a good approximation for operating conditions requiring large excitation, then
\[ \Delta i_{id} \approx \Delta i_{is} / k_i \]  

(7-32)

The transfer function \( G_{id}(s) \) to be used in the load change compensation can then be obtained from (7-20) and (7-31), and affected by the corresponding stator-rotor referral ratios.

\[ G_{id}(s) = H_{id}(s) \frac{1.5^2 L_{cmd}}{N_{qf} N_{is} k_i L_{eqf}} \]  

(7-33)

The order of the numerator and denominator of the transfer function \( G_{id}(s) \) is the same, but one of the zeroes is at zero frequency as can be seen in (7-21). This may be undesirable and can be avoided with two alternative approaches: one approach is based on the fact that one pole of \( G_{id}(s) \) is at low frequency making it possible to cancel the pole with the zero at origin as was done in [135]; the other approach moves the zero from the origin and places it at a low frequency (at least ten times lower than the lowest pole). The two alternatives produced similar results when tested in simulation.

The proposed algorithm was intensively tested in simulation according to the wide range of operation conditions established in [137] and experiments.

V. Controller Evaluation

A. Simulation Verification

Computer simulations were used during the design process to validate the models and hypotheses, also to verify the behaviors of the proposed controller algorithm and compare its performance to previous approaches. The controller algorithm implemented in simulation followed the design rules explained in Section III. The zero of \( C_{ex}(s) \) was calculated according to equation (A-2); the pole of \( C_{ex}(s) \) was selected to be ten times larger than its zero. The zero of \( C_v(s) \) in (6) used a \( c_{kq} = 0.5 \) for 60° phase margin when combined with \( K_v \), and its pole was placed at 16 kHz or half the sampling frequency. The zero at origin of the \( i_d \)-loop compensator \( C_{id}(s) \) was canceled with the lowest frequency pole as explained in the Appendix. The controller formulation for three different generators was tested in simulation. The basic system information and their parameters are summarized in Table 7-2.

Simulation results in Fig. 7-9 compare the proposed control with a previous algorithm for machine B under impact load conditions. The previous controller uses two feedback control
loops: an exciter field current inner loop and a main generator output voltage outer loop of the PID type. The improvement with the new controller is clearly observed.

**Table 7-2 Machine and controller characteristics**

<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
<th>Machine C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power (kVA)</td>
<td>40</td>
<td>40</td>
<td>150</td>
</tr>
<tr>
<td>Voltage (V)</td>
<td>115</td>
<td>115</td>
<td>115</td>
</tr>
<tr>
<td>Speed (rpm)</td>
<td>9.6k-16k</td>
<td>9.6k-16k</td>
<td>11.1k-23.1k</td>
</tr>
<tr>
<td>$C_{ex}(s)$ Zero (rad/s)</td>
<td>86</td>
<td>110</td>
<td>25</td>
</tr>
<tr>
<td>$C_{e}(s)$ Zero (rad/s)</td>
<td>14</td>
<td>40</td>
<td>6</td>
</tr>
<tr>
<td>$H_{d}(s)$ Poles (rad/s)</td>
<td>3.6, 750</td>
<td>1.6, 452</td>
<td>2.3, 105</td>
</tr>
</tbody>
</table>

Certain critical conditions are deemed important and selected for testing. For example, high-speed capacitive load is relevant from the standpoint of stability, while inductive load at low-speed produces the slowest natural response. It is also important to evaluate the controller response in the case of nonlinear loads. The most relevant cases for controller performance test are summarized in Table 7-3. Fig. 7-10 shows simulation results for cases a, b, and c for machine A, comparing the control response with and without $i_{d}$-loop. The $i_{d}$-loop is especially helpful in the case of inductive loads where the excitation requirements are larger. On the other hand, since the parameters of the $i_{d}$-loop are not corrected for magnetic saturation it loses some effect at low speeds where saturation is rather significant. The case d is shown in the following subsection with experimental results.

**Table 7-3 Key cases for controller performance test**

<table>
<thead>
<tr>
<th>Case</th>
<th>Relevance</th>
</tr>
</thead>
<tbody>
<tr>
<td>a- Inductive load @ high-speed</td>
<td>Transient at disconnection</td>
</tr>
<tr>
<td>b- Inductive load @ low-speed</td>
<td>Regulation response speed</td>
</tr>
<tr>
<td>c- Capacitive load @ high-speed</td>
<td>Stability</td>
</tr>
<tr>
<td>d- Rectifier load</td>
<td>Transient and harmonics</td>
</tr>
</tbody>
</table>
Fig. 7-9 Comparison between the new and the previous controller with sudden connection and disconnection of a resistive load: 40 kW at 12000 rpm for Machine B

Fig. 7-10 Controller comparison with and without the $i_d$-loop for loads (a) inductive @ high-speed, (b) inductive @ low-speed, and (c) capacitive @ high-speed

B. Experimental Verification

The implemented DSP based GCU was experimentally tested for the voltage regulation of a 40 kVA generator. The results presented in this subsection were obtained with the digital GCU prototype shown in Fig. 7-11 and the synchronous generator system shown in Fig. 7-12, where the generator is coupled to its prime mover – an induction motor fed by a variable speed drive. The transient responses of the generator output voltage for an inductive load connection and disconnection are shown in Fig. 7-13 and Fig. 7-14 respectively. The case shown corresponds to an inductive load at about 30% of the rated capacity, which was the maximum power available in the laboratory tests. The color code in the Figure corresponds to: full controller (dark blue), vt-iex only (light blue), vt-iex and id loop (green), vt-iex and gain adapted speed (red). The results
clearly show the benefits of the added control features, i.e., load compensation \(i_d\) loop and adaptation of the voltage loop gain as function of generator speed. This case shows that the effect of the new control features is important even though the load level is relatively low and the speed is in the middle of the speed range.

Fig. 7-11 Picture of the digital GCU prototype showing the DSP board, the D/A circuits. The I/O board lies on a lower level

Fig. 7-12 : Picture of the 40 kVA generator used for the experiments and its primary motor

The response for rectified load (case d in Table 7-3) is shown in . No significant difference is noticed compared to linear load except for slight noise increase due to the rectifier.
The results in Fig. 7-16 and Fig. 7-17 show the controller performance under the test conditions specified in EN2282 Standard [137]. At full load and near full speed. The standard requires the connection and disconnection of a step load from 5% to 85% of the rated generator capacity for resistive, inductive (0.75 lagging power factor), and capacitive (0.95 leading power
factor) loads. The experimental results obtained from the lab tests confirm the performance of the controller obtained in the simulation and its appropriate behavior under all conditions.

Fig. 7-16 Voltage transient during connection of resistive, inductive, and capacitive loads at 15000 rpm and std. EN2282 limits

Fig. 7-17 Voltage transient during disconnection of resistive, inductive, and capacitive loads at 15000 rpm and std. EN2282 limits

VI. Analysis of the Proposed Controller Stability Characteristics

The simulation and experiments have demonstrated the advantages of the proposed model based control, especially with the model-based load disturbance compensation $i_d$-loop. In addition to further evaluations that can be done later on using the methods specifically discussed
in this work, this subsection explains the benefit of the $i_r$-loop analytically using linear system theory. The synchronous generator and its controller can be seen as a regulated voltage source. An important characteristic of a voltage source is given by its output impedance $Z_{\text{out}} = V_{\text{out}} / I_{\text{out}}$. When the source is modeled in the d-q reference frame, $Z_{\text{out}}$ is a two by two matrix with four components, which can be expressed as:

$$
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix} =
\begin{bmatrix}
Z_{dd} & Z_{dq} \\
Z_{qd} & Z_{qq}
\end{bmatrix}
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix}
$$

(7-34)

The general effect of the voltage regulation on $Z_{\text{out}}$ reduction, especially at low frequencies, is well known. Here, it is important to evaluate the change in $Z_{\text{out}}$ produced by the disturbance compensator $C_d(s)$. Because $i_d$ is used in the disturbance compensation, only $Z_{dd}$, $Z_{qd}$ are affected by $C_d(s)$ loop. The effect is illustrated in Fig. 7-18 and Fig. 7-19, where a reduction of the $Z_{\text{out}}$ components is observed in the mid-frequency range.

In the stability theory of linear systems, the stability margin is related to the ratio of the source output impedance over the load input impedance [30]. Generally speaking, a lower ratio indicates a more stable system. Therefore, the ratio reduction as a result of the change in the source $Z_{\text{out}}$ helps to improve the overall system stability. Clearly, this result agrees with observations from simulations and experiments.

![Fig. 7-18 Effect of control loops over the $Z_{dd}$ component](image-url)
VII. Summary

This chapter presented the development of a new controller algorithm for a synchronous generator with variable frequency for autonomous power system applications. Brushless excitation is a solution usually adopted for that type of applications. Because of the excitation configuration, the generator set that must be considered for a proper control design is composed of two synchronous machines and a diode rectifier.

The proposed three-loop model based control algorithm performs considerably better than other existing approaches as validated through simulation and experiments. The benefits of the new control include faster dynamics, better damping and stability margin, and more uniform response over the operating range. The improvement of the control mainly results from the model based $d$-axis load disturbance compensation loop and the adaptation of the outer voltage control loop gain with the speed. The analysis shows that the load disturbance compensation loop reduces the equivalent impedance of the generator and therefore improves the stability margin. In general, the new controller can account better for the wide operating frequency range and various load conditions. Since the controller parameters and characteristics are determined from the models, its tuning also becomes systematic and simplified.
Chapter 8  Conclusions and Future Work

This work has studied the stability of autonomous power systems with variable frequency most commonly used in vehicular applications. Although the main focus has been on the large-signal stability analysis, contributions to the small-signal analysis have also been made. In addition, an improved controller for synchronous generators of variable frequency and brushless excitation has been developed. The general accomplishments of the work are summarized in the following paragraphs.

I. Summary of Results

The study used three different ways to approach the large-signal stability analysis. These approaches are the parameter space mapping, the energy function determination of the RA, and the case simulation of fault scenarios. Each of the approaches allowed analyzing different aspects of the system nonlinear behavior that can produce instabilities. The main stability results are summarized as follows:

- The parameter mapping prediction of the stability region boundary is in close proximity to the large-signal stability boundary when the event creating the disturbance is a load step.
- During faults at the MD and the generator the larger stresses appear at the fault initial moments; the fault clearance usually restores the system to its normal operation.
- During faults at the 3phAR and 1phAR circuit branches the larger stresses appear after the fault clearance; these behaviors are related to the characteristics of active rectifier controllers.
- The overvoltages created during the recovery after a fault at the generator terminals can cause damage to the equipment and produce cascading disconnections by overvoltage protections.
- The protection settings impact the large-signal stability behavior and can produce undesired equipment disconnections.
II. Conclusions

To develop the stability analysis work the modeling of the autonomous power system was analyzed and a new simplified average model of the MPTR was proposed. The developed model was tested in simulations showing a good performance both in large- and small-signal response. The model has the advantage of compactness and relatively easy manipulation of the mathematical relationships. In this way it is appropriate for analytical studies.

The work has proposed a mapping methodology of the system parameters that allows identifying ranges of those parameters where the local stability is guaranteed. This is very useful information for the system designer because the ranges of parameter values required for a stable design can be known beforehand. The methodology can be applied to average models of systems with a relatively large number of states. In addition, the computation requirements are relatively low and the maps can be obtained in a reasonably short amount of computing time.

The large-signal stability analysis was approached by means of the Lyapunov direct method and time domain simulations. A Lyapunov function based on a Lur’e formulation of a simplified system model was constructed using LMI solution techniques to optimize the function coefficients. Based on this Lyapunov function, a new approach was developed to analyze the CPL effect on the autonomous power system.

A relatively complex model of the system was simulated in time domain in order to evaluate the impact of the faults on the system behavior. This allowed identifying behavior not easily identified when simplified reduced order models are used; this behavior is described in chapter 6. The type of undesirable behavior found can probably be corrected with proper control schemes. Nevertheless, the identification of those behaviors is an important aspect to consider when the stability of a particular component or system configuration is evaluated.

Finally, a new control algorithm for the excitation control of a variable frequency generator with brushless exciter was proposed in chapter seven. The developed controller adds the features of a feed forward loop compensating part of the distortion introduced by the load and adapts the voltage loop gain based on the operational speed. The controller transfer functions were derived from the machine characteristics simplifying the tune-up process and allowing for a more uniform response over the wide range of operation that the application requires. The model is based on a $d-q$ model of the machine requiring knowing the generator rotor position at all times.
Therefore, a sensorless position detection technique was evaluated and implemented. The proposed controller shows an overall performance improvement when compared to previous formulations.

III. Future Work

Some possible ideas for future work continuing the developments presented here are discussed in the following paragraphs.

The nonlinear behavior analysis presented in chapter three focused on the characteristics of the generator and MD. The configuration of generator-large MD is frequently found in autonomous power systems, especially in vehicular applications. Therefore, additional analysis of the behavior of that generator-MD system is required in order to understand the nonlinear phenomena and define stability limits that are not highly conservative. The analysis of the limit cycles could be approached with techniques appropriate for periodic orbit analysis where the conditions for stable or unstable orbits could be predicted.

The large-signal stability limit for load steps in the range of the rated power was found to be in close proximity with the small-signal stability limit predicted by the mapping methodology. Therefore, it would be good to determine if this condition holds in a generic way for all load step cases. More simulations can be done with this particular focus. In addition, the analysis in chapter three could be extended in order to determine the conditions where the small-signal is a good approximation to the load-step stability limit.

The mapping of the parameter space was presented as a tool that allows analyzing the stability behavior and determining stability boundaries. These boundaries can be used to establish criteria for design based on physical parameters. In this way, a possible direction of work consists in a comparison of different criteria in order to establish proper safety margins.

The Lyapunov function based analysis used a relatively simple model of the energy source. The complexity of the source model is a point that requires further development if the Lyapunov direct method analysis is pursued. The analysis of cases with multiple CPL type loads can be approached with the Lyapunov function construction methodology presented in this work.
Finally, most of the study was done considering a MPTR for the larger AC/DC power conversion. Additional studies can be done in order to assess the effect of other types of rectification, like active rectifiers of different topologies. In this way the main effect of each particular topology on the system stability would be evaluated and compared.
Appendix A

This appendix contains additional simulation data corresponding to fault cases that is not included in chapter 6.

A. Fault at Generator Feeders

The cases simulated corresponding to faults at the generator feeder faults are detailed in Table A-1.

Table A-1 Cases studies for faults at the generator feeder

<table>
<thead>
<tr>
<th>Case</th>
<th>System operation</th>
<th>Fault</th>
<th>ct (msec)</th>
<th>System parameters</th>
<th>Protection</th>
<th>Result Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>800 Hz</td>
<td>3-phase</td>
<td>20</td>
<td>Base</td>
<td>Basic settings</td>
<td>Prot. Coordination. Faulted feeder discon. Rectifiers do not discon. MD load turn-off by Vmin 1phAR disc. by Imax transient</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R_f= 50 mΩ</td>
<td></td>
<td></td>
<td>Basic settings</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>800 Hz</td>
<td>3-phase</td>
<td>20</td>
<td>Base</td>
<td>Basic settings</td>
<td>Prot. Coordination. Faulted feeder discon. Rectifiers do not discon. MD load turn-off by Vmin 1phAR disc. by Imax transient</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R_f= 50 mΩ</td>
<td></td>
<td></td>
<td>Basic settings</td>
<td></td>
</tr>
</tbody>
</table>

The results corresponding to case 2 are presented in Fig. A-1 to Fig. A-3. In this case the fault at the feeder is located at the AC bus terminals. Because of the adequate protection operation, only the faulted feeder gets disconnected. In this case both ends open at the same time because their protections see a current of similar value. Like in case 1, which was discussed in chapter 6, the excitation current limits the generator armature current during the fault. The 1phAR loads are disconnected by an overcurrent created during the post-fault transient shown in Fig. A-1 (b).

The responses of the 3phAR branches are shown in Fig. A-2. The voltage depression during the fault causes the 3phAR and MD inverters to turn-off while the passive part of the converters stay connected. Differently than case 1, the AC overvoltage after fault clearance is of reduced magnitude. Therefore, the voltage at the DC buses does not experience a large excursion and does not cause overvoltage protection trip. In this way the 3phAR branches remain connected to the circuit. The waveforms of the MDs connected to the MPTR are shown in Fig. A-3. The effect of the fault on those MDs causes undervoltage turn-off followed by overcurrent disconnection.
Fig. A-1 Fault case 2-a at generator feeder terminals at the AC bus. V and I waveforms and trip signals at: (a) generator and (b) 1phAR loads

Fig. A-2 Fault case 2-a at generator feeder terminals at the AC bus. V and I waveforms and trip signals at: (a) first 3phAR branch and (b) second 3phAR branch
Fig. A-3 Fault case 2-a at generator feeder terminals at the AC bus. V and I waveforms and trip signals at MDs connected to the MPTR

B. Fault at the AC Connection of the MTPR and 3phAR

Faults at some of the connections to the AC bus are applied in this section. The discrepancies observed in the modeling of the MD loads pointed out the necessity of using more accurate models of those converters, especially when they are connected to the branch under fault. The circuit model used to analyze the effect of the faults at the individual branches corresponds to Fig. A-4. One of the MD models in the MTPR supplied DC bus is now modeled with a detailed switch bridge instead of an average model. Faults were applied at the points indicated by 3 and 4 in the circuit schematic; the fault characteristics are listed in Table A-2.

Table A-2 Characteristics of the faults applied at the rectifiers’ connection to the AC bus

<table>
<thead>
<tr>
<th>Case</th>
<th>System operation</th>
<th>Fault</th>
<th>ct (msec)</th>
<th>System parameters</th>
<th>Protection</th>
<th>Result Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>400 Hz</td>
<td>3-phase R&lt;sub&gt;c&lt;/sub&gt; = 20mΩ</td>
<td>10</td>
<td>Base</td>
<td>Basic settings</td>
<td>MTPR branch disc. by I&lt;sub&gt;max&lt;/sub&gt; MD loads turn-off by V&lt;sub&gt;min&lt;/sub&gt; 3phAR do not disconnect 1phAR disc. by I&lt;sub&gt;max&lt;/sub&gt; transient</td>
</tr>
<tr>
<td>4</td>
<td>400 Hz</td>
<td>3-phase R&lt;sub&gt;c&lt;/sub&gt; = 20 mΩ</td>
<td>10</td>
<td>Base</td>
<td>Basic settings</td>
<td>3phAR branch disc. by I&lt;sub&gt;max&lt;/sub&gt; MD loads turn-off by V&lt;sub&gt;min&lt;/sub&gt; 1phAR disc. by I&lt;sub&gt;max&lt;/sub&gt; transient</td>
</tr>
</tbody>
</table>
Fig. A-4 Model to study faults at the MPTR and 3phAR branch connections to the AC bus

The waveform of the circuit magnitudes for the case three are shown in Fig. A-5 and Fig. A-6. The waveforms in Fig. A-5 (a) correspond to the generator voltage and current. The magnitude of the stresses at the generator is similar in case three and previous case two. The waveforms in Fig. A-5(b) correspond to the MDs connected to the multi-pulse transformer rectifier that is the branch that gets disconnected after the fault clearance.

Fig. A-6 shows the waveforms at the two 3phAR branches. In this branches the voltage depression created by the fault turn-off the converters of the motor drives. However, the rectifiers connected to the bus with an undervoltage protection set up at a lower level do not turn-off and remain operative. In this way they can be re-started immediately after the fault is cleared to continue with the operation. The 1phAR loads experience overcurrent disconnection produced during the post-fault transient. The situation created in the system by the fault of case four is similar to case three except for the 3phAR branch disconnection instead of the MPTR branch because of the fault location.
Fig. A-5 Fault case 3 at MPTR connection to the AC bus. V and I waveforms and trip signals at:
(a) generator and (b) MD loads connected to the MPTR

Fig. A-6 Fault case 3 at MPTR connection to the AC bus. V and I waveforms and trip signals at: (a) first 3phAR and (b) second 3phAR loads
C. Fault at AC bus generic output

Different cases were analyzed using the system model in Fig. 6-8. The protection scheme and settings were adjusted at the different cases. The main characteristic of each case are summarized in Table A-3. The waveforms shown below provide a description of those cases.

The results corresponding to case BUS-a are shown below in Fig. A-7 to Fig. A-9. The waveforms in Fig. A-7(a) correspond to the generator. The CB trip signal at the fault branch is also shown in the figure; the ct = 0.192 sec. The waveforms of the voltage and current at the 1phAR loads are shown in Fig. A-7(b). Because the overcurrent protection setting were raised, the load is not disconnected during the transient. The waveforms in Fig. A-8(a) and (b) correspond to the transient evolution of the magnitudes at the 3phAR and MPTR connected MDs. Because the undervoltage protections were disabled, the loads do not get disconnected during the operation under the faulted condition, neither disconnects the active rectifier. The AC current on the 3phAR branch increases because the attempts of the 3phAR controller to regulate the voltage. However, this current increment is limited by the control current limit. On the other side, the current on the DC bus at the MPTR becomes very small despite the CPL characteristic of the MDs. After the fault clearance all MD loads recover returning to the pre-fault condition.

<table>
<thead>
<tr>
<th>Case</th>
<th>System operation</th>
<th>Fault</th>
<th>ct (msec)</th>
<th>System parameters</th>
<th>Protection</th>
<th>Result Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUS-a</td>
<td>400 Hz</td>
<td>3-phase</td>
<td>30</td>
<td>iawup active</td>
<td>All raised</td>
<td>All loads recover</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R_f = 20 mΩ</td>
<td></td>
<td></td>
<td>All raised</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>BUS-b</td>
<td>400 Hz</td>
<td>3-phase</td>
<td>40</td>
<td>iawup active</td>
<td>All raised</td>
<td>1phAR reco., MPTR-MD reco. 3phAR disconnect by Vmax</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R_f = 10 mΩ</td>
<td></td>
<td></td>
<td>All raised</td>
<td>3phAR shows unstable behavior</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>None</td>
<td></td>
</tr>
</tbody>
</table>

The waveforms in Fig. A-9 (a) and (b) show the evolution of certain controller variables at the controllers of the 3phAR and MD. In Fig. A-9 (a) it is possible to observe the effect of the \( i_{qref} \) current limitation and quick recovery after fault clearance due to the action of appropriate antiwindup schemes. Fig. A-9 (b) shows the controller variables and duty cycles for the MD connected to the 3phAR branch. At the MD controller both \( i_{qref} \) and duty cycles reach the saturation level during the fault transient.
Fig. A-7 Fault case BUS-f at generic AC bus output. V and I waveforms and trip signals at:
(a) generator and (b) 1phAR load

Fig. A-8 Fault case BUS-f at generic AC bus output. V and I waveforms and trip signals at:
(a) 3phAR and connected MD (b) MPTR and MD loads
Fig. A-9 Fault case BUS-a at generic AC bus output. V and I waveforms at:
(a) MD controller (b) 3phAR controller

D. Faults at DC Bus Generic Output

The next set of cases analyses the system under faults at the DC bus connected to the MPTR feeding the two motor drives. The fault location is assumed to be at a generic bus connection similarly to the AC bus output case. The schematic of the system model used in this part of the study is shown in Fig. A-10 and the cases analyzed are summarized in Table A-4.

Fig. A-10 Model to study faults at the DC bus
Table A-4 Case studies for faults at the DC bus supplied from the MPTR

<table>
<thead>
<tr>
<th>Case</th>
<th>System operation</th>
<th>Fault</th>
<th>ct (msec)</th>
<th>System parameters</th>
<th>Protection</th>
<th>Result Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCt-a</td>
<td>400 Hz</td>
<td>3-phase</td>
<td>40</td>
<td>Iawup=1.6xIn</td>
<td>All raised</td>
<td>Everything recovers</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rf= 1mΩ</td>
<td></td>
<td></td>
<td>All raised</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Imax</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Vmax</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>vmin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCt-b</td>
<td>800 Hz</td>
<td>DC</td>
<td>5</td>
<td>Iawup=1.6xIn</td>
<td>All raised</td>
<td>1phAR recovers, MPTR-MD recover</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rf= 10mΩ</td>
<td></td>
<td>lexlim=75A</td>
<td>All raised</td>
<td>3phAR disconnect by Vmax</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3phAR shows unstable behavior</td>
</tr>
</tbody>
</table>

In this type of fault the larger stresses are applied to the multi-pulse rectifier with the loads experiencing the larger voltage depression being the ones connected “electrically close” to the fault location. The waveforms corresponding to the case DCt-a are presented below in Fig. A-11 to Fig. A-12. Fig. A-11 displays the evolution of the magnitudes at the generator and 1phAR loads while Fig. A-12 does the same for the 3phAR and MPTR and connected loads. The waveforms in those Figures show that all the system equipment recover after fault clearance. On the other side, some of those system components experience large stresses during the transients either under the faulted condition or after clearance. This is a point that requires further analysis when setting the protections.

![Waveforms](image_url)

**Fig. A-11** Fault case DCt-b at DC bus output. V and I waveforms and trip signals at: (a) generator and (b) 1phAR loads
Fig. A-12 Fault case DCt-a at DC bus output. V and I waveforms and trip signals at:
(a) 3phAR and MD load (b) MPTR and MD loads

The operation at 800Hz increases the risk of instability at the 3phAR that is affected by the voltage depression created by the fault. The simulation results for case DCt-b are shown in Fig. A-13 through Fig. A-14.

Fig. A-13 Fault case DCt-c at DC bus output. V and I waveforms and trip signals at:
(a) generator and (b) 1phAR loads
The case DCT-b illustrate the effect of the generator current limit on the voltage stability. In the waveforms in Fig. A-14 it is possible to observe that the 3phAR does not recover normal operation after the fault is cleared. Different is the case when the excitation current limit at the generator is increased to 100A; in such case the voltage depression during the fault is milder and the 3phAR recovers its normal operation after fault clearance.

![Graphs showing current and voltage waveforms](image)

**Fig. A-14 Fault case DCT-c at DC bus output. V and I waveforms at:**
(a) MPTR and connected MDs (b) 3phAR branch

### E. Fault at the 3phAR-MD DC link

The circuit model used in this case is shown in Fig. A-15. The fault is located in the DC link connecting the 3phAR to the MD. It was assumed in this case that the fault could be produced at some component connected to the DC link that can be independently removed. The case characteristics are summarized in Table A-5.

The waveforms corresponding to case DCE are shown in Fig. A-16 to Fig. A-18. A clear difference to the previous cases is the low impact of the fault in the non-faulted branches. This is evidenced in the voltage at the generator bus. A very light voltage sag appears with the fault, afterwards the generator excitation is able to recover its nominal voltage. The operation of the
motor drives connected to the MPTR continues with minimum perturbation. On the contrary, in the 3phAR branch the fault causes the CB at the three-phase side of the 3phAR to trip and shut down the rectifier and the MD. This is observed in Fig. A-18.

![Fig. A-15 Circuit schematic of the model to study faults at the 3phAR DC link](image)

**Table A-5 Case studies for faults located at DC link of the 3phAR**

<table>
<thead>
<tr>
<th>Case</th>
<th>System operation</th>
<th>Fault</th>
<th>t\text{_clear}</th>
<th>System parameters</th>
<th>Protection</th>
<th>Result Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCe-a</td>
<td>400 Hz</td>
<td>DC</td>
<td>40 msec</td>
<td>\text{lawup}=1.6\times I_n</td>
<td>All</td>
<td>3phAR trips by I_{\text{max}}&lt;br&gt;MD-MPTR recovery&lt;br&gt;1phAR recovery</td>
</tr>
</tbody>
</table>

![Fig. A-16 Fault case DCe at 3phAR DC link. V and I at: (a) generator and (b) 1phAR loads](image)
Fig. A-17 Fault case DCe at 3phAR DC link. (a) V and I waveforms at 3phAR and connected MD (b) 3phAR controller signals

Fig. A-18 Fault case DCe at 3phAR DC link. V and I waveforms and trip signals at MPTR and connected MD loads
F. Faults at the 1phAR load branch

The circuit schematic of the system model used to study the faults at the 1phAR load branches is shown in Fig. A-19. The main difference in the circuit of Fig. A-19 is the use of a larger amount of individual 1phAR units instead of the lumped 1phAR model used in previous cases. Nevertheless, the total power of the branch is maintained. The characteristics of the faults analyzed in this scenario are summarized in Table A-6.

![Fig. A-19 Circuit schematic of the model used to study faults at the 1phAR branch](image)

<table>
<thead>
<tr>
<th>Case</th>
<th>System operation</th>
<th>Fault</th>
<th>ct (msec)</th>
<th>System parameters</th>
<th>Protection</th>
<th>Result Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1phR3</td>
<td>400 Hz</td>
<td>3-phase add branch</td>
<td>30</td>
<td>Base</td>
<td>All</td>
<td>All</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R_f= 1mΩ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1phR1</td>
<td>400 Hz</td>
<td>Single-phase</td>
<td>10</td>
<td>Base</td>
<td>All</td>
<td>All</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R_f= 20mΩ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In both cases the fault is located at the remote 1phAR units. It is expected that the stresses created over the other system branches are reduced. Fig. A-20 and Fig. A-21 show the simulation results corresponding to case 1phR3. The magnitude of the voltage drop at the AC bus is reduced as observed in Fig. A-20. In that same Figure it is possible to observe that the excitation current does not reach saturation; thus it does not limiting the fault current value. The voltage and
currents at the faulted 1phAR are shown in Fig. A-20(b). The waveforms in Fig. A-21 correspond to the MPTR and 3phAR branches showing the fault-clearance transient.

Fig. A-20 Fault case 1phR3. V and I waveforms at: (a) generator and (b) 1phAR faulted branch

Fig. A-21 Fault case 1phR3. V and I waveforms at: (a) 3phAR and MD (b) MPTR and MD
The case of a single phase fault is illustrated in Fig. A-22 to Fig. A-24. The fault is located at the phase A of the most remote 1phAR. During the fault the generator experiences unbalance operation. This produces unbalance three-phase voltages and an oscillating excitation current. Nevertheless, the waveforms in Fig. A-23 show very little effect over the 3phAR and MPTR branches. Moreover, the results in Fig. A-24 show the waveforms at two 1phAR units connected at the same location but at different phases. The waveforms in Fig. A-24(a) correspond to the faulty phase, which is disconnected when the fault is cleared. Fig. A-24(b) on the contrary shows the waveforms at a healthy phase. In this phase the effect of the fault is reduced. On the primary side of the healthy phase it is possible to observe a current increase during the fault despite the fact that the fault is located at another phase. This is attributed to the current re-distribution at the transformer.

Fig. A-22 Fault case 1phR1. V and I waveforms during unbalanced operation at:
(a) generator and (b) generator each phase
Fig. A-23 Fault case 1phR1. V and I waveforms at: (a) 3phAR and MD (b) MPTR and MD

Fig. A-24 Fault case 1phR1. V and I waveforms at: (a) 1phAR connected to the faulty phase (b) 1phAR connected to a healthy phase
Appendix B

This appendix contains the Matlab code developed for the parameter space mapping presented in chapter four. The code presented here corresponds to case 4 of Table 4-1.

```matlab
% Eigenvalues of the linearized matrix
% The code refers to an average model of the system in Simulink
% The name of that model for this example is 'km5atru2MC_AfeViePfc_f'
clear lambda sumlam sizelam sumabla siglam rlamu;
clear x0 xp xps xpu xps xpl xplo ypl o wch;
for f=1:5,
    status=close(figure(f));
end

% #########################################################################
% Generator Controller parameters
vt=230;
wM=24000;
weM=2*pi*p1/60*wM;
Gvq=weM*Lmd*Nfds_gen/1.5;
Kvo=32;
wzo=100;
wp=10000;
% Generator Vloop bandwidth
gbw=300;
Kv=2*pi*gbw/Gvq;
wz=(Kv-Kvo)/2/Kvo*wzo+wzo;

%Factors for Park transform between generator&feeder and rest of circuit
dq=1.225;
qd=0.817;

% #########################################################################
% #########################################################################
%Mapping initialization values
%Read initial condition from simulation
x0=xFinal'; % Simulation must be run first to save initial operating point
% Use save to workspace check box in simulation parameters to
% assign final state variable name (xFinal)

%Initial value storage
st=1; % stable equilibrium point counter
us=1; % unstable equilibrium counter
cl=1; % critically stable counter
sts=1; % limit-hit system counter

% for plotting stable points (two maps at a time, 1 & 2)
x1s=[]; x2s=[]; y1s=[]; y2s=[];
% Parameters for mapping stored here (Xk-Yj)
wchs=[];wchr=[];
% Stores dominant eigenvalue char frequency
xps=zeros(length(x0),0);
% Stores stable states
```
% Stores parameters and stable states for case when limit is hit (same as % three above)
x1ss=[]; x2ss=[]; y1ss=[]; y2ss=[];
wchss=[];wchrss=[];
xpss=zeros(length(x0),0);

% for plotting unstable points (same as two cases above)
x1u=[]; x2u=[]; y1u=[]; y2u=[];
wchu=[];lamu=[];
xpu=zeros(length(x0),0);

% for plotting critically-stable points
x1cl=[]; x2cl=[]; y1cl=[]; y2cl=[];
wchl=[];
xpl=zeros(length(x0),0);

% Initial values for saturation block gains % (forces small-signal gain to zero during saturation)
gwMC1=1; gdMC1=1;
gwMC1b=1; gdMC1b=1;
gvAFE1=1; gdAFE1=1;
gwMC25=1; gdMC25=1;
gwMC253=1; gdMC253=1;
gvVIE=1; gnpVIE=1;
gvPFC=1;
hitpre=0;

% Changes from mapping to simulation mode
map=1;
sim=0;

%########################################################################%
% Circuit parameters
Cfa=500e-6; %DC filter cap initial value
Lfa=250e-6; %DC filter L initial value
Rfa=0.05;
Tel=1.0; %initial torque
Leng=30; %main feeder initial length
Lengp=10; %1phAR feeder length
Lenga=10; %3phAR 2-level feeder length
Lengv=10; %3phAR 3-level feeder length
freq=800; % Should agree with machine mechanical speed
%########################################################################%
% Definition of system parameters for mapping
% x1 main variable on x (m)
% x2 secondary variable on x (m)
% y1 main variable on y (n)
% y2 secondary variable on y (n)
% For this example mapping:
% x1 = Gen Vloop bandwidth
% x2 =
% y1 = MD wloop bandwidth
% y2 =
%########################################################################%
% Mapping parameters----------------------
% Initial values, these should equal the initial simulation minus the step
% BW simulation = 300 Hz = 320 - 20 = x1i - xp1s
x1i=320;
x2i=0;
y1i=32;
y2i=0;

% Parameter steps for the map
xp1s=-20; % Gen Vloop bandwidth
xp2s=0;
yp1s=-2; % MD wloop bandwidth
yp2s=0;

% Number of points in x & y axis
nxm=5;
nfi=5;

% Main loop starts -------------------------
m=1;
while m<=nxm,
    % x-axis variables to map
    x1(m)=x1i+m*xp1s;
x2(m)=x2i+m*xp2s;
n=1;
    while n<=nfi,
        % y-axis variables to map
        if m/2==floor(m/2);
            y1(n)=y1i+(nfi+1)*yp1s-yp1s*n;
y2(n)=y2i+(nfi+1)*yp2s-yp2s*n;
        else
            y1(n)=y1i+yp1s*n;
y2(n)=y2i+yp2s*n;
        end;
    end;

    % Operation point calculation (steady state)
    % Parameters changing in linearization
    % Generator Vloop bandwidth
    Kv=2*pi*x1(m)/Gvq;
wz=(Kv-Kvo)/2/Kvo*wzo+wzo;
    if Kv<Kvo
        wz=wzo;
    end

    %-----
    % x2(m)
    % MD speed loop bandwidth
    fs=y1(n);
    Hova=y1(n);
    % y2 (n);
    [x1(m), fs]

    % Automatic steady-state solution
    % Gain of sat blocks must be kept one for trimming
gwMC1=1; gdMC1=1;
gwMC1b=1; gdMC1b=1;
gvAFE1=1; gdAFE1=1;
gwMC25=1; gdMC25=1;
gwMC253=1; gdMC253=1;
gvVIE=1; gnpVIE=1;
gvPFC=1;
% Calculates equilibrium point:
[xlin,u,ylin,dx,options] = trim('km5atru2MC_AfeViePfc_f',x0);
x0=xlin;
xp(:,n)=x0;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Linearization Correction of saturation blocks
hits=0;
% Reads saturation outputs to detect if any limit is active
saty=ones(length(ylin),1)-abs(sign(ylin));
[saty, ylin]
% Check if the eq point has hit any limit
if sum(saty)<length(saty)
hits=1;
elseif hitpre==1
% If the limit has hit a perturbation is applied to verify that
% the simulation has hit the limit and it's not a numerical
% convergence problem (this is done 4 times)
while check < 5
    plpe=0; p2pe=0;
    if check==1
        plpe=-0.1;
    elseif check==2
        plpe=0.1;
    elseif check==3
        p2pe=-0.1;
    elseif check==4
        p2pe=0.1;
    end
    x1ch=x1(m)+xp1s*plpe;
    x2ch=x2(m)+xp2s*plpe;
    y1ch=y1(n)+yp1s*p2pe;
    y2ch=y2(n)+yp2s*p2pe;
% Send this parameters for eq point search
    Kv=2*pi*x1ch/Gvq;
    wz=(Kv-Kvo)/2/Kvo*wzo+wzo;
    if Kv<Kvo
        wz=wzo;
    end
    fs=y1ch;
    Hova=y1ch;
    [x1ch, fs]
% Find perturbed eq point
    gwMC1=saty(1); gdMC1=saty(2);
    gwMC1b=saty(3); gdMC1b=saty(4);
    gvAFE1=saty(5); gdAFE1=saty(6);
    gwMC25=saty(7); gdMC25=saty(8);
    gwMC253=saty(9); gdMC253=saty(10);
    gvVIE=saty(11); gnpVIE=saty(12);
    gvPFC=saty(13);
    [xlinc,u,ylinc,dx,options]=trim('km5atru2MC_AfeViePfc_f',x0);
% Linearization Correction of saturation blocks
    satyc=ones(length(ylinc),1)-abs(sign(ylinc))
    sumsat=sum(satyc);
    if sumsat < length(satyc)
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```
hits = 1;
check = 5;
end
check = check + 1;
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Linearization using matlab function
% Assigns limit gain for linearization procedure (zero if saturated)
gwMC1 = saty(1); gdMC1 = saty(2);
gwMC1b = saty(3); gdMC1b = saty(4);
gvAFE1 = saty(5); gdAFE1 = saty(6);
gwMC25 = saty(7); gdMC25 = saty(8);
gwMC253 = saty(9); gdMC253 = saty(10);
gvVIE = saty(11); gnpVIE = saty(12);
gvPFC = saty(13);

% Linearizes Simulink model 'km5atru2MC_AfeViePfc_f' with x(0)=xlin
[Am, B, C, D] = linmod('km5atru2MC_AfeViePfc_f', xlin);

% Eigenvalue calculation
lambda(:, n) = eig(Am);
siglam = sign(real(lambda(:, n)));
sumlam(n) = -sum(siglam);
sizelam(n) = length(siglam);

% Detects eigenvalue is close to Re~0
closeim = 0;
% if sumlam(n) < sizelam(n)       % some eig is =0 or <0
maxpos = 0;
minneg = 1e10;
minRn = 1e10;
maxRn = 0;
nrpl = 1;
for hn = 1:length(x0),
  % Real part is positive
  if siglam(hn) > 0
    % obtain max pos eig
    if abs(lambda(hn, n)) > maxpos
      maxpos = abs(lambda(hn, n));
    end
    if abs(real(lambda(hn, n))) > maxRn
      maxRn = abs(real(lambda(hn, n)));
      maxRm = abs(lambda(hn, n));
    end
    lam(nrpl) = lambda(hn, n);
    nrpl = nrpl + 1;
  end
  % Real part is negative
  elseif siglam(hn) < 0
    % obtain min(abs) neg eig
    if abs(lambda(hn, n)) < minneg
      minneg = abs(lambda(hn, n));
    end
    if abs(real(lambda(hn, n))) < minRn
      minRn = abs(real(lambda(hn, n)));
      minRm = abs(lambda(hn, n));
    end
end
```
end

if maxpos<3e-6*minneg
    closeim=1;
end

% Determine the stability of the equilibrium point
if size(lam(n))==sum(lam(n)) % it is a stable point
    index(n)=1;
    x1s(st)=x1(m);
    x2s(st)=x2(m);
    y1s(st)=y1(n);
    y2s(st)=y2(n);
    xps(:,st)=xp(:,n);
    % store freq of the dominant eigenvalue
    wchs(st)=minneg;
    wchrs(st)=minRm;
    st=st+1;

    % Stores point in limit-hit variables
    if hits==1
        x1ss(sts)=x1(m);
        x2ss(sts)=x2(m);
        y1ss(sts)=y1(n);
        y2ss(sts)=y2(n);
        xpss(:,sts)=xp(:,n);
        % store freq of the dominant eigenvalue
        wchss(sts)=minneg;
        wchrss(sts)=minRm;
        sts=sts+1;
    end

elseif closeim==1 % it has a zero (or close to zero) eigenvalue
    x1cl(cl)=x1(m);
    x2cl(cl)=x2(m);
    y1cl(cl)=y1(n);
    y2cl(cl)=y2(n);
    xpl(:,cl)=xp(:,n);
    % store freq of the dominant eigenvalue
    wchcl(cl)=minneg;
    cl=cl+1;
else % it is an unstable point
    index(n)=0;
    x1u(us)=x1(m);
    x2u(us)=x2(m);
    y1u(us)=y1(n);
    y2u(us)=y2(n);
    xpu(:,us)=xp(:,n);
    % calculate freq of the unstable eigenvalue
    wchu(us)=maxpos;
    if length(lamu)==0
        lamu(:,us)=lam;
        rlamu=size(lamu,1);
    end
    if length(lam)>rlamu
...
adlamu=zeros(length(lam)-rlamu,size(lamu,2));
lamu=[lamu;adlamu];
rlamu=size(lamu,1);
end
if length(lam)<rlamu
    adlam=zeros(length(lam)-rlamu,1);
    lam=[lam
    adlam];
end
lamu(:,us)=lam;
us=us+1;
end

% Mark hitting limit point for next eq. point
if hits==1
    hitpre=1;
else
    hitpre=0;
end

n=n+1;
end
m=m+1;
end

%########################################################################%
% s - stable equilibrium points
% u - unstable equilibrium points
% ss - stable eq pts limited
% cl - close to Imaginary axis
%########################################################################%
% Re-assignment of mapping parameters for output
% For this mapping ------------------------
% x1 = Gen Vloop bandwidth
% x2 =
% y1 = MD wloop bandwidth
% y2 =
%########################################################################%
% Output plots
figure(1);
plot(x1s,y1s,'+b',x1u,y1u,'sr',x1ss,y1ss,'oc',x1cl,y1cl,'dg','MarkerSize',3),
grid on;
figure(2);
xplo=[x1s x1u x1cl]
yplo=[y1s y1u y1cl]
wch=[wchrs wchu wchl];
XC=[min(xplo):range(xplo)/(nfi-1):max(xplo)];
YT=[min(yplo):range(yplo)/(nxm-1):max(yplo)];
[XP,YP]=meshgrid(XC,YT);
ZP=griddata(xplo,yplo,wch,XP,YP);
figure(3);
plot(real(lamu'),imag(lamu')), grid on;
% Calculation of the circuit operation parameters
Lengs=Leng;
Rfis=0.0012*Lengs;    Lfis=1e-6*Lengs;    Cfis=0.1e-9*Lengs;
vdds=Rfis./Lfis.*xps(56,:)-wr*xps(53,:);
vqds=Rfis./Lfis.*xps(53,:)+wr*xps(56,:);
vfs=sqrt(0.5*(vdds.^2+vqds.^2));
vos=sqrt(0.5*((xps(57,:)./Cfis).^2+(xps(58,:)./Cfis).^2));
vgs=sqrt(0.5*((xps(57,:)./Cfis+vdds).^2+(xps(58,:)./Cfis+vqds).^2));
angvs=atan((xps(57,:)./Cfis+vdds)./(xps(58,:)./Cfis+vqds));
angos=atan((xps(57,:)./Cfis)./(xps(58,:)./Cfis));
ids=xps(56,:)./Lfis;
igs=sqrt(ids.^2+iqs.^2);
angis=atan(ids./iqs);

if length(y1u)>0
    Lengu=Leng;
    Rfiu=0.0012*Lengu;    Lfiu=1e-6*Lengu;    Cfiu=0.1e-9*Lengu;
vddu=Rfiu./Lfiu.*xpu(56,:)-wr*xpu(53,:);
vqdu=Rfiu./Lfiu.*xpu(53,:)+wr*xpu(56,:);
vfu=sqrt(0.5*(vddu.^2+vqdu.^2));
vou=sqrt(0.5*((xpu(57,:)./Cfiu).^2+(xpu(58,:)./Cfiu).^2));
vgu=sqrt(0.5*((xpu(57,:)./Cfiu+vddu).^2+(xpu(58,:)./Cfiu+vqdu).^2));
angvu=atan((xpu(57,:)./Cfiu+vddu)./(xpu(58,:)./Cfiu+vqdu));
angou=atan((xpu(57,:)./Cfiu)./(xpu(58,:)./Cfiu));

end

else
    vggu=[];
vou=[];
fiu=[];
fiou=[];
end

if length(y1ss)>0
    Lengss=Leng;
    Rfiss=0.0012*Lengss;    Lfiss=1e-6*Lengss;    Cfiss=0.1e-9*Lengss;
vddss=Rfiss./Lfiss.*xpss(56,:)-wr*xpss(53,:);
vqdss=Rfiss./Lfiss.*xpss(53,:)+wr*xpss(56,:);
vfss=sqrt(0.5*(vddss.^2+vqdss.^2));
voss=sqrt(0.5*((xpss(57,:)./Cfiss).^2+(xpss(58,:)./Cfiss).^2));
vgs=sqrt(0.5*((xpss(57,:)./Cfiss+vddss).^2+(xpss(58,:)./Cfiss+vqdss).^2));
angvss=atan((xpss(57,:)./Cfiss+vddss)./(xpss(58,:)./Cfiss+vqdss));
angoss=atan((xpss(57,:)./Cfiss)./(xpss(58,:)./Cfiss));
idss=xpss(56,:)./Lfiss;
igss=xpss(53,:)./Lfiss;
igs=sqrt(idss.^2+iqss.^2);
angiss=atan(idss./iqss);

else
    vgss=[];
end
voss=[];
fiss=[];
fioss=[];
end

figure(4);
plot(vos,abs(cos(fios)),'+b',vou,abs(cos(fiou)),'sr',voss,abs(cos(fiss)),'oc','MarkerSize',3), grid on;
figure(5);
plot(xls,abs(cos(fios)),'+b',xlu,abs(cos(fiou)),'sr','MarkerSize',3), grid on;
figure(6);
plot(vgs,abs(cos(fis)),'+b',vgu,abs(cos(fiu)),'sr','MarkerSize',3), grid on;
REFERENCES


