Numerical Simulation of Injection and Mixing in Supersonic Flow

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Susan K. Cox-Stouffer

(ABSTRACT)

A numerical investigation of the performance of two candidate designs for injection into supersonic flow, including a comparison of two renormalized group theory (RNG) based K-ε turbulence models with a more conventional K-ε model. The chosen designs were an unswept ramp injector with four injection ports and a novel nine-hole injector array. The objectives of the investigation were to provide reliable computational solutions to the flowfields in question using both RNG and standard K-ε turbulence models and to compare the solutions to experiment, thereby to judge the relative performance of the turbulence models. A second objective of the investigation was to use the computed data to provide design insights for the nine-hole injector array.

This investigation made use of GASP™ version 2.2, a commercial computational fluid dynamics code that was augmented by the addition of one RNG-based K-ε turbulence model derived by Zhou, et. al. and one variant of Zhou’s model, which was derived by the author. Mesh sequencing studies were performed to measure solution quality, with the fine mesh for the injector array containing roughly one million grid nodes and the fine mesh for the ramp injector containing more than six million grid nodes. Results of these studies indicated that the injector-array solution was significantly under-resolved in the farfield, though the quality was better in the vicinity of the injector itself. The ramp-injector solution, while not perfectly grid-resolved, showed much better grid convergence in both the nearfield and farfield. Accordingly, comparison with
experiment was better for the ramp injector than for the injector array. For both injectors, the differences between solutions generated with RNG-based K-ε and standard K-ε turbulence models were negligibly small.

Despite inadequate grid resolution in the farfield, the computational investigation of the nine-hole injector array did yield several important design insights. Particularly, the significance to mixing and losses of the placement of the outer injectors of the second and third rows was determined.
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List of Symbols

A Any flow variable
\( \bar{A} \) Reynolds-averaged variable
\( \tilde{A} \) Fabre'-averaged variable
A' Fluctuating component of Reynolds-decomposed variable
A^+ Van Driest damping constant
a Sound speed
C_{R1} and C_{R2} Local interaction contribution coefficients in Reynolds stress equation in RNG turbulence model
C_{\tau_1}, C_{\tau_2}, and C_{\tau_3} Nonlocal interaction contribution coefficients in Reynolds stress equation in RNG turbulence model
C_{\epsilon_1} and C_{\epsilon_2} Modelling coefficients in dissipation equation of \( K-\epsilon \) turbulence model
C_\mu Modelling coefficient in eddy viscosity relation of \( K-\epsilon \) turbulence model
c_p Mixture specific heat at constant pressure
D_i Diffusion coefficient for species i
D_{ti} Turbulent diffusion coefficient for species i
d Diameter of each nozzle in the injector array
d_{eff} Effective diameter of injector array
e Specific internal energy
e_o Stagnation specific internal energy,
\( e_o = e + \frac{1}{2} (U^2 + V^2 + W^2) \)
F Inviscid flux vector for the x coordinate direction
F_{kleb} Klebanoff intermittency factor
\( \bar{F}_v \) Viscous flux vector for the x coordinate direction
\( \tilde{G} \) Inviscid flux vector for the y coordinate direction
\( \bar{G}_v \) Viscous flux vector for the y coordinate direction
$\overline{H}$ Inviscid flux vector for the $z$ coordinate direction

$\overline{H}_v$ Viscous flux vector for the $z$ coordinate direction

$h$ 

$k_i = \mu c_p / Pr$ Specific enthalpy 

$h_o$ Stagnation specific enthalpy,

\[ h_o = h + \frac{1}{2} (U^2 + V^2 + W^2) \]

$K$ Turbulence kinetic energy

$k$ Thermal conductivity ($k = \mu c_p / Pr$)

$k_t$ Turbulent thermal conductivity ($k_t = \mu c_p / Pr_t$)

$l_m$ Mixing length

$m_{f_i}$ Mass fraction of species $i$

$P$ Static pressure

$Pr_T$ Turbulent Prandtl number

$q$ Vector of conservative flow variables

$q$ Vector of primitive flow variables

$Q$ Heat-flux vector

$\overline{R}$ Residual

$Sc_l$ Schmidt number

$Sc_t$ Turbulent Schmidt number

$s$ Highest characteristic speed

$T$ Temperature

$T_o$ Time period of turbulent fluctuations used in Reynolds averaging

$t$ Time

$t.i.$ Turbulence intensity

$U_i$ Cartesian velocity vector in tensor notation

$u$ Flow speed

$V$ Magnitude of the velocity vector

$V_{\text{dif}}$ Velocity magnitude difference for use in wake function

$\hat{V}_{ij}$ Mass diffusion velocity of species $i$ in direction $j$

$\overline{V}_j$ Mass diffusion velocity vector of species $i$

$x_i$ Cartesian position vector in tensor notation

$y$ Normal distance above surface
\( y^+ \) Nondimensional law-of-the-wall coordinate

\( \alpha \) Transverse injection angle
\( \bar{\delta} \) Central difference
\( \Delta \) Forward difference
\( \nabla \) Backward difference
\( \delta_{ij} \) Kronecker delta
\( \epsilon \) Dissipation rate
\( \kappa \) Karman constant
\( \lambda \) Courant-Friedrichs-Levy (CFL) number
\( \mu \) Molecular viscosity
\( \mu_\tau \) Turbulent (eddy) viscosity
\( \mu_\tau^* \) Effective eddy viscosity used for heat and mass transfer in “mixing” RNG
\( \rho \) Mass density
\( \rho_i \) Mass density of species i
\( \sigma_\kappa \) Prandtl number for turbulence kinetic energy
\( \sigma_\epsilon \) Dissipation rate Prandtl number
\( \tau_{ij} \) Viscous shear stress
\( \tau_{\mu ij}^* \) Reynolds stress
\( \tau_{ij}^{**} \) RNG contribution to Reynolds stress
\( \omega \) Vorticity

Subscripts
i, j, or k Evaluated at plane i, j, or k
max Maximum or at maximum
w Wall value

Superscripts
n Evaluated at time-level n