Three Essays on Product Market and Capital Market Interaction

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ABSTRACT

The Industrial Organization literature investigates the product market decisions of a firm while the corporate finance literature explores the financing decisions of the firm. But the truth is both the financing decisions and the product market decisions are interdependent and should be modeled together to develop a better understanding of a firm’s decisions. This thesis takes a step in that direction.

The manager of a firm caters to the equity holders of the firm who are protected by limited liability. Ex-ante debt is issued and at the time of product market decision, debt is exogenous. The traditional product market capital market interaction literature has argued that debt financing leads to more aggressive product market strategies. If debt is treated as endogenous and/or the switching state of nature is endogenous, it can be shown that debt financing may lead to less aggressive product market strategies. Further, if external financing consists of both debt and equity financing, it is shown that a financially constrained firm shall produce less than what it would have produced if it was not financially constrained. Finally, managerial compensation is reported to be one of the reasons for product market aggressiveness of a firm in the context of product market capital market interaction.
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## Contents

1 Introduction 1

2 Limited Liability Effect with Endogenous Debt and Investment 7

2.1 Introduction .................................................. 7
2.2 The Existing Literature ...................................... 7
2.3 Brief Description of Oligopoly and Financial Constraint .... 9
2.3.1 A standard model ......................................... 9
2.3.2 Motivation of the paper .................................... 10
2.4 A Simple One Period Model of Duopoly and Financial Con-straint .......................................................... 11
2.4.1 The second stage of the game ............................ 12
2.4.2 The first stage of the game ............................... 13
2.5 Demand Uncertainty : Exogenous Debt and Switching State 14
2.5.1 Cournot Duopoly ........................................... 15
2.5.2 Stackelberg Equilibrium ................................ 16
2.5.3 Monopoly ...................................................... 16
2.5.4 No Debt ....................................................... 17
2.5.5 Comparison .................................................... 17
2.6 Demand Uncertainty : Endogenous Debt and Switching State 18
2.6.1 Cournot Duopoly ........................................... 18
2.6.2 Stackelberg Equilibrium ................................ 19
2.6.3 Monopoly Equilibrium ................................... 19
2.6.4 Proposition 3 .................................................. 20
2.7 Why Firms Produce Differently: An Insight ................ 21
2.7.1 No Debt ....................................................... 21
2.7.2 Exogenous Debt and Exogenous Switching State .... 21
2.7.3 Endogenous Debt and Endogenous Switching State ... 22
2.8 Demand Uncertainty: Endogenous Debt, Exogenous Switching State ................................................. 24
2.8.1 Cournot Duopoly ........................................... 24
2.8.2 Stackelberg Equilibrium .................................. 25
2.8.3 Monopoly Equilibrium .................................... 26
2.8.4 Proposition 4 .................................................. 26
2.9 Demand Uncertainty: Exogenous Debt, Endogenous Switching State .......................... 27
  2.9.1 Monopoly Equilibrium .................................... 27
  2.9.2 Cournot Duopoly ........................................... 28
  2.9.3 Proposition 5 .................................................. 29
2.10 Conclusion ......................................................... 29
2.11 Appendix .......................................................... 30
  2.11.1 Cournot Duopoly: Appendix A ............................. 30
  2.11.2 Stackelberg Equilibrium: Appendix B ......................... 31
  2.11.3 Appendix C: Monopoly Equilibrium ......................... 32
  2.11.4 Appendix D: Proof of Proposition 3 ......................... 33
  2.11.5 Appendix E: Cournot Duopoly ............................ 35
  2.11.6 Appendix F: Monopoly ....................................... 36
  2.11.7 Appendix G: Monopoly Firm .............................. 37
  2.11.8 Appendix H: Cournot Duopoly ............................ 41
  2.11.9 Appendix I: Proposition 5 ................................. 41

3 Does Endogenous Equity Affect Endogenous Investment under Limited Liability? 43
  3.1 Introduction ....................................................... 43
  3.2 The Existing Literature ......................................... 45
  3.3 The Model .......................................................... 47
    3.3.1 The Model Set-up ............................................ 48
    3.3.2 Proposition 1 .................................................. 52
    3.3.3 Proposition 2 .................................................. 57
  3.4 Conclusion ......................................................... 60
  3.5 Appendix .......................................................... 60
    3.5.1 Appendix A: Proof of Proposition 1 ......................... 60
    3.5.2 Appendix B ..................................................... 62

4 Executive Compensation, Financial Constraint and Product Market Strategies 68
  4.1 Introduction ....................................................... 68
  4.2 A Theoretical Model ............................................. 73
    4.2.1 Definition of Financial Constraint ......................... 73
    4.2.2 The Two Stage Game ......................................... 73
    4.2.3 The Three Stage Game ....................................... 79
  4.3 Hypothesis Development ......................................... 83
List of Figures

3.1 Relationship Between Equilibrium Output and Discounted Cost of Equity . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 56
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Descriptive Statistics of mean of sales growth based on two financial constraint criteria and inferences based on t test of the difference of the mean of sales growth</td>
<td>100</td>
</tr>
<tr>
<td>4.2</td>
<td>Fixed Effect Regression of Sales Growth on financial constraint. Dependent Variable is Sales growth with respect to industry.</td>
<td>101</td>
</tr>
<tr>
<td>4.3</td>
<td>Fixed Effect Regression of Sales Growth on Managerial Compensation. Instrumental Variable approach</td>
<td>102</td>
</tr>
<tr>
<td>4.4</td>
<td>Regression of Sales Growth on Managerial Compensation</td>
<td>103</td>
</tr>
<tr>
<td>4.5</td>
<td>Fixed Effect Regression of Managerial Compensation on Sales Growth</td>
<td>104</td>
</tr>
<tr>
<td>4.6</td>
<td>Fixed Effect Regression of Managerial Compensation on Financial Constraint. Financial Constraint is based on Dividend Payment</td>
<td>105</td>
</tr>
<tr>
<td>4.7</td>
<td>Fixed Effect Regression of Managerial Compensation on Financial Constraint. Financial Constraint is based on Long Run Credit rating of the firm</td>
<td>106</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Both the industrial organization literature and the corporate finance literature have developed in leaps and bounds shedding new light into product market decisions and financing decisions of a firm. The traditional industrial organization literature offers insights into the product market behavior of a firm. The corporate finance literature investigates external financing by a firm to finance investments. The literature of product market and capital markets interaction investigates how the output market decisions of a firm are affected by the financing decisions of the firm and vice versa, thereby linking the traditional industrial organization literature with the traditional corporate finance literature. But the interaction of these two major fields in Economics and Finance has not been explored rigorously. This is important because the product market decisions of a firm are often based on the financing of the firm and vice versa. Understanding just the product market decisions of the firm through the industrial organization literature is not sufficient. Similarly, understanding just the financing decisions of the firm is not sufficient either. One should develop a broader perspective of the decisions made by a firm, by simultaneously exploring both the product market decisions and financing decisions of the firm. Only then one can develop a deeper understanding of the decisions made by the firm. There are gaps in our knowledge of how the product market and financial market interact. This thesis is a humble attempt to bridge this gap.

Chapter 2 explains the basic concept of why the product markets should be interrelated with the financial markets. The seminal paper of Brander and Lewis in 1986 was one of the first to start this literature. They demonstrate how debt financing lead a firm to produce more aggressively in the output market. The manager of a firm caters to the equity holders of the firm. The equity holders are residual claimants of the firm. The firm issues debt to produce output. The revenue of the firm is uncertain and depends on the
state of the nature. When the state of nature is bad, the revenue of the firm is not enough to repay the debt issued by the firm. The firm does not have to pay back the entire debt if the revenue generated is lower than the debt issued. The firm and hence the equity holders are said to be protected by limited liability. The equity holders care only about the good states of nature. Once debt is issued, the equity holders want the firm to behave more aggressively in the output market. This entails the firm to maximize the revenue during the good states of nature with the cost of this aggressive output policy being the increased probability of not being able to pay back the debt. The equity holders are not worried about the prospect of not being able to pay back the debt as they are protected by limited liability. If the manager of the firm caters to the equity holders of the firm, the manager shall choose the more risky projects knowing fully well that the upper side of the risk shall benefit the equity holders while the downside risk is borne by the debt holders. This creates an agency problem between the debt holders and the equity holders. There is a transfer of wealth from the debt holders to the equity holders if the manager chooses more risky projects. Debt financing leads to more aggressive behavior by the firm in the product market. This is the traditional story of the industrial organization literature. Switching state of nature is that state of nature where revenue is just sufficient to repay the debt. In the traditional story of debt financing leading to more aggressive product market policies, the debt issued by the firm and the switching state of nature are treated as exogenous.

Chapter 2 explores four different cases. Both debt and switching state are exogenous, both debt and switching state are endogenous, debt is exogenous and switching state is endogenous and finally debt is endogenous and switching state is exogenous. The demand is assumed to be uncertain when the financing decisions and the production decisions are made. Here is the time line of events. It is assumed that the firm issues debt in the first stage. In the second stage, after the money has already been raised through debt, the firm makes the output decision. The output is produced before the demand is realized. The firm’s output decision is based on some estimated demand and not on the realized demand. In the third stage, the demand is realized and the revenue is generated. The debt holders are paid first, followed by the equity holders. In the typical industrial organization literature, debt is usually assumed to be exogenous to the production decision. At the first stage, debt is issued. In the second stage, the debt has already been issued and the firm decides on the output. The firm’s output market decision is exogenous to the amount of debt issued. This assumption may not be feasible as the debt contract often specifies the purpose of issuing debt. In chapter 1, debt is treated both as exogenous and endogenous. Endogenous debt re-
quires the additional assumption of debt being used to finance the cost of production. This assumption links the amount of debt issued to the amount of output produced thereby rendering the debt issued as endogenous. This linkage between debt and output was introduced in the literature by Povel and Raith in their well cited paper in 2004. Switching state of nature can also be endogenous. Recalling that switching state of nature is the state of nature at which revenue of the firm is exactly equal to the amount of debt issued, any rational manager can deduce that the switching state of nature depends on the output of the firm and hence is endogenous.

When both debt and switching state of nature are exogenous, the traditional result of debt financing leading to more aggressive production is arrived at. The intuition of the result is that the manager protects the interest of the equity holders. The equity holders are residual claimants of the firm’s earnings. The manager issues debt ex-ante and then decides on the output. After the debt has been issued, the manager produces aggressively in order to generate higher revenue in good states of nature knowing well that the downside risk is covered by limited liability protection. This is the typical agency problem argument between the equity holders and the debt holders. There is a transfer of wealth from the debt holders to the equity holders. When both debt and switching state of nature are endogenous, the above stated intuition breaks down. The debt issued is no longer ex-ante to the output produced. Both debt and output are simultaneously determined. The manager no longer chooses riskier projects after issuing debt thereby transferring wealth from the debt holders to the equity holders. There are benefits of acting aggressively which accrue to the equity holders. But there are also costs of acting aggressively which are borne by the equity holders. Acting aggressively implies producing more output and hence issuing more debt. So the range of the good states of nature is reduced. The costs to the equity holders are the revenue loss in the bad states of nature. More aggressive behavior increases the range of bad states of nature, thereby increasing the revenue loss due to bad states of nature and hence increasing the costs of acting aggressively. The manager weighs both the costs and benefits of acting aggressively before deciding whether to act aggressively. As a result, if the costs of acting more aggressively are more than the benefits, the firm may act less aggressively in the output market. When there is less uncertainty in demand, the benefits of acting aggressively are not large enough compared to the loss of revenue in the bad states of nature. This causes the firm to act less aggressively in the output market. But when the demand uncertainty is large, the benefits of acting aggressively in the output markets are large compared to the costs of acting aggressively and the firm acts more aggressively in the output market.
When either the switching state is endogenous and debt is exogenous or switching state is exogenous and debt is endogenous, the same result holds true. The argument is that there are both benefits and cost of acting aggressively. The costs are in form of loss of revenue in the bad states of nature. When both debt and switching state of nature are exogenous, both the range of bad states of nature and debt are exogenous. But when either debt and/or switching state of nature are endogenous, the range of bad states is endogenous. Hence the costs of acting aggressively are endogenous and increase with more aggressive output market strategies. The firm may act less aggressively in the output market. This result is in sharp contrast of the standard industrial organization result that debt financing lead to more aggressive output market strategies.

The traditional argument of debt financing leading to more aggressive output market strategies is based on the assumption that the equity holders are protected by limited liability and are residual claimants of firm value. They force the manager to maximize the revenue in the good states of nature while not caring about the bad states of nature. This assumption that the equity holders do not care about the bad states of nature is not realistic. When the firm is unable to meet its debt repayment requirement, the firm declares bankruptcy and is often liquidated. The assets of the firm are auctioned off to pay the debt holders with little probability of the equity holders receiving anything in case of liquidation. Further, the equity holders lose in terms of future revenue. This loss to the equity holders negates the assumption that the equity holders do not care of the bad states of nature. The assumption that equity holders do not care about the bad states of nature arises from the fact that debt financing is considered as the only source of external financing. In chapter 3, it is argued that one should consider both equity financing and debt financing as sources of external financing while investigating the interaction of financing decisions with product market decisions. The literature has focused on how debt financing affect output market decisions. The focus of chapter 3 is how both debt financing and equity financing affect output market decisions?

There are two firms, one financially constrained and the other financially unconstrained. External financing is used to finance production thereby linking the output produced with the amount of external financing. Both debt financing and equity financing are used as external financing. In case of bankruptcy, the assets of the firm are auctioned off and the proceeds are handed over to the debt holders. The equity holders are the residual recipients and it is assumed that they do not receive anything in case of liquidation. Further, the equity holders lose in terms of the future profits. These are the costs to the equity holders of acting aggressively in the output market. The
benefits of acting more aggressively in the output market are the transfer of wealth from the debt holders to the equity holders and increase in revenue during the good states of nature. But the costs involve the probability of liquidation of the firm and the costs that liquidation impose on the equity holders. The manager of the firm evaluates both the costs and benefits of aggressive product market strategies before deciding on which strategy to follow. It is shown that if a firm is both debt and equity financed, the firm is going to produce less than what it would have produced in case the firm was financially unconstrained and did not use external financing. Using comparative statistics, the relation between the cost of equity and the output is developed.

Chapter 3 investigates how the output of the firm gets affected if both debt and equity are used for external financing. Aggressive product market strategies of the firm may depend on some other sources also. In chapter 4, it is argued that the managerial compensation may be another reason why a financially constrained firm produces more aggressively in the output market.

A financially constrained firm by definition has higher cost of capital and hence higher cost of production. In order to compensate for the higher cost of capital, the financially constrained firm seeks to capture higher market share by aggressive product market strategies. Further, the financially constrained firm can also compensate for its higher production cost by inducing the manager to put in more effort. Managerial effort generates higher revenue thereby countering the higher production cost. The manager of the financially constrained firm shall put in more effort only when the compensation structure of the manager is more aligned with the value of the firm. The manager of a financially constrained firm has to be offered more incentives to increase the operating performance of the firm compared to a financially unconstrained firm. Connecting the threads, it can be argued that a financially constrained firm shall be more aggressive in the output market, it shall offer higher incentives to the manager to perform and finally more aggressiveness of the financially constrained firm can be explained by compensation structure of the manager.

In chapter 4, it is demonstrated both empirically and theoretically that a financially constrained firm shall be more aggressive in the output market compared to a financially unconstrained firm. It is also documented that managerial compensation is an explanatory variable in explaining industry adjusted sales growth after controlling for all other explanatory variables. Further, managerial compensation is higher for a financially constrained firm giving managers incentives to exert more effort compared to the financially unconstrained firm. Connecting these three observations, it is inferred that managerial compensation is an explanatory variable for product market ag-
gressiveness of a financially constrained firms.

The Industrial organization literature has developed models which investigate the product market behavior of the firms. The corporate finance literature explores how a firm generates external funding to support production. The recent capital market product market interaction literature connects these two important fields in Economics and Finance. One should not treat the production decisions and the financing decisions separately. To develop better understanding of the firm, both the financing and product market decisions should be explored simultaneously. Still there are questions to be answered in understanding how the product market and capital market work simultaneously.

This thesis investigates the important connection between the capital markets and the product markets. After the standard result of debt financing leading to more aggressive product market strategies is arrived at, further investigation reveals some interesting results. If debt is treated as endogenous and / or the switching state of nature is treated as endogenous, debt financing does not always lead to more aggressive product market strategies as evident in chapter 1. Instead of only debt financing, if both debt and equity financing are used as sources of external financing, the financially constrained firm produces less than what it would have produced if it were not financially constrained. Finally, managerial compensation is documented to be an explanatory variable for aggressive product market strategies.

Chapter 4 is the conclusion of the thesis.

One only hopes that this thesis shed some light into the interaction between two extremely important markets, the capital market and the product market and helps reveal some of the secrets of the black box called firm.
Chapter 2

Limited Liability Effect with Endogenous Debt and Investment

2.1 Introduction

The corporate finance literature has developed models of debt contracts where the threat of liquidation, i.e., higher costs of debt financing leads to underinvestment and less aggressive behavior in the output market. The industrial organization literature has shown that debt financing can lead to more aggressive behavior in the output market. The corporate finance literature deals with ex-post behavior of a firm in the sense that debt causes to act less aggressively in the output market because of higher probability of bankruptcy. The industrial organization literature, starting with the seminal paper of Brander and Lewis (1986) assumes that the firm is protected from losses by limited liability, but is a residual claimant to high earnings. This ‘risk shifting’ leads a firm to behave in a more aggressive manner. This paper’s objective is to further investigate the effects of the financial leverage on investment, in the context of the linkage between the product market and financial market and noting that debt and/or ‘switching state’ can be treated as endogenous.

2.2 The Existing Literature

The industrial organization literature focused on the firm’s strategies where the firm’s objective is to maximize profits. The corporate finance literature considered maximization of equity values. But in truth, the financial
market decisions and product market decisions are interlinked. Brander and Lewis (1986), in their seminal paper, studied the linkage between financial markets and product markets. They argued that limited liability may lead to a leveraged firm taking a more aggressive stand in the output market. Their 1988 follow-up paper included bankruptcy costs and came up with the same result. These two papers were followed by a series of papers. Maksimovic (1988) and Hendel (1996) argued that the firms become more aggressive while Glazer (1994) and Chevalier and Scharfstein (1996) argue that they become less aggressive. Either effect can happen in Showalter (1995). Fudenburg and Tirole (1986) have investigated the predatory action taken by competitors in the context of capital market imperfection and entry. Bolton and Scharstein (1990) discuss a two period model involving a financially constrained firm and a bank. The bank cannot verify the profits of the firm. A rival, who is not constrained financially, competes with the leveraged firm.

Maurer (1999) develops a model of product competition with debt as the optimal contract. He studies two firms, one of which is financially constrained. The managers of the two firms choose effort on innovation, with the outcome of the innovation being uncertain. Maurer's optimal financial contract is similar to Bolton and Scharfstein (1990). He shows the first best alternative is to exert effort unless the rival gains from predatory behavior.

Faure-Grimaud (2000) also derives debt as an optimal contract. He shows that debt causes firms to compete less aggressively because the positive liability effects on quantities is offset by the negative effect due to endogenous financial costs. But this paper also allows for renegotiation. The model assumes that the investors can withdraw their funds after the output decision is made without incurring any losses. This effectively means that the output choice is contractible, implying there is no moral hazard problem regarding output choice.

Povel and Raith (2004) investigate this moral hazard problem. They build a model with two firms. One of them is financially constrained. They include variable production cost, unlike most other models of industrial organization, and argue that costly production creates a feedback from the output market to the financial market. But their model argue that the future profit generated by the firm (this is the amount to be lost in case of liquidation) is constant. Further, the paper assumes that the manager of the firm acts in the interest of the equity holder and is afraid of liquidation. This assumption, though not unreasonable, is not always very realistic as the managers often are more concerned with their income rather than the firm's profits.

The empirical findings by Phillips (1995), Kovenock and Philips (1995) and Kovenock and Philips (1997) suggest that industrial concentration is an
important variable in the interaction between the financial market and the product market. An increase in the leverage of one firm decreases investments while the rivals increase investments. Philips argues that the close rivals can increase their market share while the leveraged firm lose out both in terms of investments and market shares. This finding is corroborated in Opler and Titman (1994). Chevalier (1995) investigates the consequence of leverage in the supermarket industry in the USA. She argues that debt weakens the competitive edge of the leveraged firm. Overall, the empirical literature has not supported the theoretical conclusions of Brander and Lewis.

2.3 Brief Description of Oligopoly and Financial Constraint

Let us begin by explaining the story of limited liability according to Brander and Lewis (1986,1988). The debt holders are residual claimants in case of bankruptcy. In case of no bankruptcy, the equity holders are residual claimants of the firm. Hence the equity holders of the firm are only concerned with the returns in the good state of nature. The manager of the firm caters to the equity holders of the firm. The manager pursues risky strategies which provide higher return for the good states of nature because the firm does not have to pay back the entire debt in case of bankruptcy. The firm is said to be protected by the limited liability effect of debt financing. The limited liability protection induces the levered firm to undertake more risky projects and behave more aggressively in the output market.

2.3.1 A standard model

Let us consider a duopoly market where the equity holders of each firm are protected by limited liability effects. The equity holders can ask for debt from outside investors if the equity capital is not sufficient to finance production. In stage 1, the firms choose a level of debt and equity, and the contract written cannot specify the output levels to be chosen in the next stage. In the next stage, the firms choose output levels given the debt and equity from the first stage. The manager of the firms are free to choose any level of output once the debt has been raised from the investors. Debt has to be repaid from the revenue of the firms before any dividend can be distributed to the equity holders. In the second stage, the firms play a Cournot game by choosing outputs. The outputs are chosen before the actual demands are generated. This often happens in practice that the firms choose output levels based on estimated demand, instead of actually observing demand and then
starting the production. There can be good states and bad states of nature. First, the debt holders are paid and then the equity holders are paid. If the revenue are not enough to repay the debt, the firms declare bankruptcy and the firms are liquidated. The firms’ assets are handed sold off and the debt holders are paid first followed by equity holders.

2.3.2 Motivation of the paper

Motivation of this paper is to look into the behavior of the firm if debt and/or the switching state of nature are endogenous. This paper investigates whether the standard Brander-Lewis results holds if debt and/or the switching state of nature are endogenous. If the firm has perfect foresight, it can perfectly forecast the switching state of nature, $\hat{z}$ which is defined by the following identity (in case of duopoly):

$$D = R(q_1, q_2; \hat{z})$$ (1)

It is the state of the nature at which the firm has earned just enough revenue to pay off its debt. Revenue of the firm depends on the state of the nature as the demand is uncertain. If the firm has perfect foresight, this switching state of nature, $\hat{z}$, can be treated as endogenous by the firm.

Further, this paper also investigates output market behavior when debt is either exogenous or endogenous. The firm issues debt to finance its production thereby linking the financing decision to the output decision. This assumption is introduced by Povel and Raith (2004). It is captured by

$$D = cq$$ (2)

where $c$ is the constant marginal cost of production.

Given both endogenous debt and endogenous switching state of nature, this paper investigates how the leveraged firm behaves in the output market. The manager caters to the equity holders and maximizes the equity value of the firm $R - D \text{Prob}(z > \hat{z})$, where $R$ is the expected operating profit, over the states of the nature $z > \hat{z}$. When the state of nature is bad, that is, $z < \hat{z}$, the firm does not earn enough revenue to pay back the debt $D$. If the firm does not pay back the debt $D$, the debt holders mop up the entire revenue $R(q, z)$ generated which is lower than the debt issued. The firm is said to be protected by limited liability as the debt issued need not be paid back when the state of nature is bad.

In section 4, we present a simple model of Cournot duopoly with one financially constrained firm and no uncertainty. In section 5, we present Monopoly, Cournot Duopoly and Stackleberg Duopoly model, where both
the firms issue on debt. Debt and ‘switching state of nature’ are exogenous to the firms. This is the standard Brander-Lewis framework. In section 6, we treat both the debt and switching state of nature as endogenous in the Cournot, Monopoly and Stackelberg framework. In section 7, we provide key insight as to why firms behave differently when the switching state and debt are endogenous. In section 8, the firms consider debt as endogenous but the firms do not have perfect foresight. So the ‘switching state’ of nature is exogenous and debt is endogenous. In section 9, firms treat debt as exogenous and the firms have perfect foresight. Hence, the ‘switching state of nature’ is endogenous and debt is exogenous. Finally we have the conclusion and the contributions of this paper.

2.4 A Simple One Period Model of Duopoly and Financial Constraint

Let the demand be deterministic and there be no uncertainty of any kind. We assume that there is no principal agent problem and the manager maximizes the profit of the firm.

The demand function is given as

\[ p = \theta - q = \theta - q_1 - q_2 \]

First let us derive the Cournot duopoly quantities of the two firms without imposing any type of financial constraints on any firm. Marginal costs of the firms are given by \( c \geq 0 \).

Firm 1 maximizes it’s profit,

\[ \max p q_1 - c q_1 = \max_{q_1} \theta q_1 - q_1^2 - q_1 q_2 - c q_1 \]

Solving, we get,

\[ q_1^* = \frac{\theta - c}{3}. \]

By symmetry, we get,

\[ q_2^* = \frac{\theta - c}{3}. \]

Hence, the Cournot Nash solution is

\[ q^* = q_1^* = q_2^* = \frac{\theta - c}{3}. \]

Now let us introduce financial constraints in firm 1. We assume that firm 1 is financially constrained and firm 2 is not financially constrained. To be
precise, firm 1 has to issue debt and equity in order to finance the production cost $cq_1$.

Initially firm 1 has an equity amount of $E^0_1$ from the old equity holders. $\max \{cq_1 - E^0_1, 0\}$ is needed by firm 1 to finance production of $q_1$. Firm 1 can issue equity $E^1_1$ and debt amount $D^1_1$ to meet the need. So we have

$$D^1_1 + E^1_1 = \max \{cq_1 - E^0_1, 0\} \quad (3)$$

In stage 2, the firms compete in quantity in a Cournot setup. Firm 1 does not treat the debt amount $D$ as exogenous while making output decision. In Brander and Lewis paper and the follow-up papers, the financially constrained firm treats $D$ as exogenous while deciding the output level in stage 2. But, in this model, following the Povel and Raith (2004), the firm treats the debt $D$ as endogenous at stage 2. The debt $D$ is used to finance the production of output thereby providing the essential link between the product market decisions and the financial market decisions.

As there is no uncertainty in the model, the debt is risk free and $r$ is the risk free rate of interest. $D^2_1$ be the amount to be paid back to the investor after one period.

### 2.4.1 The second stage of the game

First we consider the second stage of the game. In the second stage, the firms maximize profits by choosing output levels. Firm 1’s problem is

$$\max_{q_1} [\theta - q_1 - q_2] q_1 - D^2_1 \quad s.t. \quad \begin{cases} D^2_1 = D^1_1 (1 + r) \\ D^1_1 + E^1_1 = \max (cq_1 - E^0_1, 0) \end{cases} \quad (4)$$

where $D^i_j$ denotes the debt taken by firm $j$ in period $i$ and $E^i_j$ denotes the equity issued by the firm $j$ in period $i$.

We refrain from addressing the age old problem of conflict of interest between the old equity holders and the new equity holders. We assume that there are no old equity holders. There are only new equity holders. This means $E^0_1 = 0$ and hence,

$$D^1_1 + E^1_1 = cq_1 \quad (3a)$$

and

$$D^2_1 = D^1_1 (1 + r). \quad (3b)$$

Hence firm 1 maximizes its profits subject to constraints (3a) and (3b). Constraint (3b) links the amount borrowed at the beginning of the period $D^1_1$ with the amount returned back $D^2_1$. 

12
Firm 2 is assumed to have enough internal funds so as to finance its production. So firm 2’s problem is

$$\max_{q_2} (\theta - q_1 - q_2)q_1 - cq_2$$

Solving this Cournot duopoly game,

$$\tilde{q}_1 = \frac{\theta - c}{3} - \frac{2}{3}cr = q_1^* - \frac{2}{3}cr \quad \text{and} \quad \tilde{q}_2 = \frac{\theta - c}{3} + \frac{cr}{3} = q_2^* + \frac{cr}{3}$$

where $$q_1^* = q_2^* = \frac{\theta - c}{3}$$ denotes the Cournot Nash solution in case of no constraints.

**Proposition 1** In a Cournot duopoly with no demand uncertainty and with one firm financially constrained and the other unconstrained, the financially constrained firm behaves less aggressively after issuing debt. The financially constrained firm, firm 1, produces less than it would produce if it had no financial constraints. The opponent firm, firm 2 produces more than the Cournot Nash output level when both firms are unconstrained. It should be noted that this result is derived without any type of uncertainty in the model. There is no need for threat of liquidation as there is no uncertainty in the model and firm 1’s profit is known for sure. The link between the product market and the financial market is generated by the variable production cost, as has been introduced in the literature by Povel and Raith (2004). But unlike in their model where demand is uncertain, there is no uncertainty in this simple model. Leveraged firm acting less aggressively is a result which is also obtained in Povel and Raith (2004). But this simple model shows that one need not to have demand uncertainty in order to arrive at that result.

### 2.4.2 The first stage of the game

In the first stage, the firm maximize the equity value of the firm as the manager cater to the equity holders. At this stage, the debt and equity levels are chosen and the debt equity level is determined.

$$D_1^1 + E_1^1 = c\tilde{q}_1$$ \hspace{1cm} (3)

The question is what is the effective mix of debt and equity. The equity holders invest in the firm if the return they get is at least as large as the risk free rate of interest. This condition is captured by

$$\Pi_1 = (\theta - \tilde{q}_1 - \tilde{q}_2)\tilde{q}_1 - D_1^2 = \tilde{p}\tilde{q}_1 - D_1^1 (1 + r) \geq E_1^1 (1 + r)$$

13
which is equivalently written as

\[ \hat{p}q_1 \geq c\bar{q}_1 (1 + r). \]  

Equation (5) is the equity holder’s participation constraint.

In the first stage, the manager maximizes the equity value of the firm subject to constraints given by equation (1).

\[
\begin{align*}
\max_{D_1^1} & \quad \hat{p}q_1 - D_1^1 (1 + r) \\
\text{s.t.} & \quad E_1^1 + D_1^1 = c\bar{q}_1 \\
& \quad D_2^1 = D_1^1 (1 + r) \\
& \quad \hat{p}q_1 - D_2^1 \geq E_1^1 (1 + r)
\end{align*}
\]

(6)

Solving, in equilibrium, the optimal debt is zero and optimal equity is \(c\bar{q}_1\).

\[ D_1^1 = 0 \quad E_1^1 = c\bar{q}_1 \]

**Proposition 2** In a Cournot duopoly model with financial constraint and no uncertainty, the equilibrium optimal debt amount is zero.

In this model, debt and equity are perfect substitutes. There is no uncertainty in the model. But equity holders receive a return at least as large as the debt holders. Hence, none of the investors are willing to buy debt and every investors buy equity instead of debt.

In this simple model of Cournot duopoly with one financially constrained firm, we conclude that the financially constrained firm acts less aggressively. Optimal debt is zero. The model assumes endogenous debt by linking the product market decision to the financial market decision through variable production cost. The model does not have any kind of uncertainty. Hence in this model, there is no limited liability protection for the firm’s equity holders. The standard Brander and Lewis results are based on limited liability protection of the equity holders which requires uncertainty in revenue generation. Hence, in the following sections, we shall introduce demand uncertainty in the model and investigate how endogenising the debt \(D\) and/or ‘switching state of nature’ \(\hat{z}\) changes the standard Brander-Lewis results.

### 2.5 Demand Uncertainty: Exogenous Debt and Switching State

Both firms are financially constrained and issue debt.
Demand is given by
\begin{equation}
  P(q) = (\theta + z - q) .
\end{equation}

\(z\) is a random variable representing the state of the nature. Without loss of generality, we assume \(z\) follows an uniform distribution, following Chen (2005). This is done for numerical simplicity. Further assumptions on the support of density of the distribution are given by
\begin{align}
  z \in [\zeta, \overline{z}] \\
  \zeta = -\overline{z} \\
  f(z) = \frac{1}{\frac{1}{z} - \frac{1}{\overline{z}}}
\end{align}

2.5.1 Cournot Duopoly

Firms are protected by limited liability. Firm i’s equity holders shall earn a positive profit only when \(R_i > D_i\). This shall happen only when the state of nature \(z > \hat{z}\) where \(\hat{z}\) is defined by equation (1). The manager of the firm caters to the equity holders of the firm. So for any firm \(i\), \(i \in (1, 2)\), the manager of firm \(i\) solves
\begin{equation}
  \max_{q_i} V_i = \int_{\hat{z}}^{\overline{z}} (R_i - D_i) f(z) \, dz = \int_{\hat{z}}^{\overline{z}} (\theta + z - q_1 - q_2 - c) q_i f(z) \, dz - \int_{\hat{z}}^{\overline{z}} D_i f(z) \, dz 
\end{equation}

where \(\hat{z}\) is defined by
\begin{equation}
  D_i = R(q_1, q_2; \hat{z})
\end{equation}

and \(R_i\) is the operating profit of firm \(i\).

Maximizing (8a) with respect to \(q_i\), we get the first order condition as follows
\begin{equation}
  \frac{\partial V_i}{\partial q_i} = \int_{\hat{z}}^{\overline{z}} \frac{\partial R_i}{\partial q_i} \frac{1}{2\pi} \, dz = 2(\pi - \hat{z}) (\theta - 2q_1 - q_2 - c) + (\pi - \hat{z}) (\pi + \hat{z}) = 0 
\end{equation}

For Cournot duopoly, the first order condition is the same for both the firms. So the Cournot solution is given by the following
\begin{equation}
  q_i^* = \frac{\theta - c}{3} + \frac{\pi + \hat{z}}{6} .
\end{equation}
2.5.2 Stackelberg Equilibrium

For Stackelberg, we assume without loss of generality that firm 1 is the leader. The reaction function of firm 2 is given by

\[ q_2(q_1) = R_2(q_1) = \frac{2\theta - 2c + \bar{z} + \hat{z}}{4} - \frac{q_1}{2} = K - \frac{q_1}{2} \]

where \( K = \frac{2\theta - 2c + \bar{z} + \hat{z}}{4} \).

Firm 1's manager solves

\[
\max_{q_1} V_1 = \int_{\hat{z}}^{\bar{z}} (\theta + z - q_1 - q_2 - c) q_1 f(z) dz - \int_{\hat{z}}^{\bar{z}} D_1 f(z) dz
\]

Maximizing (10a) with respect to \( q_1 \), the first order condition is given by

\[
\frac{\partial V_1}{\partial q_1} = \int_{\hat{z}}^{\bar{z}} \frac{\partial R_1}{\partial q_1} \frac{1}{2\bar{z}} dz = 2(\theta - c - K - q_1) + (\bar{z} + \hat{z}) = 0
\]

Solving, we get

\[
\begin{align*}
q_1^* &= \frac{\theta - c + \bar{z} + \hat{z}}{2} \\
q_2^* &= \frac{\theta - c + \bar{z} + \hat{z}}{4}
\end{align*}
\]

2.5.3 Monopoly

The manager of the firm caters to the equity holders of the firm. So for the monopolist firm, the manager of the firm solves the following

\[
\max_q V = \int_{\hat{z}}^{\bar{z}} (\theta + z - q - c) q f(z) dz - \int_{\hat{z}}^{\bar{z}} D f(z) dz
\]

Maximizing (12a) with respect to \( q \), the first order condition is given by

\[
\frac{\partial V_i}{\partial q_i} = \int_{\hat{z}}^{\bar{z}} \frac{\partial R_i}{\partial q} \frac{1}{2\bar{z}} dz = 2(\theta - c - 2q) + (\bar{z} + \hat{z}) = 0.
\]

Solving, we get

\[
q_m^* = \frac{\theta - c + \bar{z} + \hat{z}}{2}
\]

In all these three cases, both the firms are leveraged. In all of these cases, the managers maximize the residual value of the firms. Now in order to investigate if the leveraged firms are producing more output, we need to compute the output of the firms when the firms are not leveraged.
2.5.4 No Debt

None of the firms are financially constrained and hence none of them issue debt. So for any firm \( i, i \in (1, 2) \), the manager of firm \( i \) solves the following

\[
\max_{q_i} V_i = \int_{-\bar{z}}^{\bar{z}} (R_i) f(z) dz = \int_{-\bar{z}}^{\bar{z}} (\theta + z - q_1 - q_2 - c) q_i f(z) dz
\]

\[
= (\theta - q_1 - q_2 - c) q_i
\]

where \( R_i \) is the operating profit of firm \( i \).

Firm \( i \)'s first order condition is given by

\[
\theta - c - 2q_1 - q_2 = 0.
\]

Solving, the Cournot solution is

\[
q_i^{c*} = \frac{\theta - c}{3}, i = 1, 2.
\]  \hspace{1cm} (14a)

Similarly, the Stackelberg solution, with firm 1 as the leader, is given by

\[
\left\{
\begin{array}{l}
q_1^{s*} = \frac{\theta - c}{2} \\
q_2^{s*} = \frac{\theta - c}{4}
\end{array}
\right.
\]  \hspace{1cm} (14b)

Similarly, the Monopoly solution is given by

\[
q_m^* = \frac{\theta - c}{2}.
\]  \hspace{1cm} (14c)

2.5.5 Comparison

Comparing equation (9) with equation (14a), we find out that

\[
q_i^{c*} = \frac{\theta - c}{3} + \frac{\bar{z} + \hat{\bar{z}}}{6} > q_i^{c*} = \frac{\theta - c}{3} \Leftrightarrow \frac{\bar{z} + \hat{\bar{z}}}{6} > 0.
\]

We note that \( z \) has an uniform distribution (from equation (8)). Hence, even if \( \hat{\bar{z}} \) is negative, \( \bar{z} \) is greater than \( \hat{\bar{z}} \), and \( \bar{z} \) is always positive. So the sum \( \frac{\bar{z} + \hat{\bar{z}}}{6} \) is always positive. So we get,

\[
\frac{\bar{z} + \hat{\bar{z}}}{6} > 0.
\]  \hspace{1cm} (15)

Hence,

\[
q_{debt}^{c*} > q_{noDebt}^{c*}.
\]
Comparing equation (11) with equation (14b) and noting equation (15), we can see that
\[ q_{1\text{debt}}^* > q_{1\text{noDebt}}^*, \]
\[ q_{2\text{debt}}^* > q_{2\text{noDebt}}^*. \]

Comparing equation (13) with equation (14c), and noting equation (21), we can see that
\[ q_{m\text{debt}}^* > q_{m\text{noDebt}}^*. \]

Combining all of these, we get
\[
\begin{aligned}
q_{c\text{debt}}^* > q_{c\text{noDebt}}^*, \\
q_{s\text{debt}}^* > q_{s\text{noDebt}}^*, \\
q_{m\text{debt}}^* > q_{m\text{noDebt}}^*.
\end{aligned}
\]

So we see that for all types of market form, Cournot duopoly, Monopoly and Stackelberg, debt financing leads to higher output. These are the standard Brander-Lewis results — which we replicate under the assumption that debt and switching state are exogenous.

### 2.6 Demand Uncertainty: Endogenous Debt and Switching State

We assume that the firms are issuing debt to finance production thereby interlinking debt with output as expressed by equation (2). Further, we assume that the firms perfectly foresee the switching state of nature \( \hat{z} \) which is defined by equation (1).

Combining equations (1) and (2), we get,
\[
D = R(q_1, q_2; \hat{z}) = cq_i = (\theta + \hat{z} - q)q_i \iff \hat{z} = c + q - \theta.
\]

A firm with perfect foresight should know that \( \hat{z} \) is a function of \( q \) and hence treat \( \hat{z} \) as endogenous.

#### 2.6.1 Cournot Duopoly

So for any firm \( i \in \{1, 2\} \), the manager of firm \( i \) maximizes the equity value of the firm:
\[
\max_{q_i} V_i = \int_{\hat{z}} R_i - D_i f(z) dz = \int_{\hat{z}} (\theta + z - q_1 - q_2 - c)q_i f(z) dz - \int_{\hat{z}} D_i f(z) dz
\]
where $R_i$ is the profit of firm $i$ and debt $D$ is given by equation (2).

Firm $i$ treats switching state $\hat{z}$ as endogenous. $\hat{z}$ is given by

$$\hat{z} = c + q - \theta. \quad (16)$$

The Cournot Duopoly solution is given by

$$q_1^* = q_2^* = q_c^* = \frac{\theta - c + \bar{z}}{4} \quad (19)$$

Proof: See Appendix A.

### 2.6.2 Stackelberg Equilibrium

We assume, without loss of generality, that firm 1 is the leader and the firm 2 is the follower. The reaction function of firm 2 with respect to firm 1 is given by

$$q_2 = R_2(q_1) = \frac{\theta - c + \bar{z} - q_1}{3}. \quad (20a)$$

Firm 1, the leader, operates on the reaction function of the follower. Both firm 1 and firm 2 have perfect foresight. For both, $\hat{z}$ and debt $D$ are endogenous variables. The leader’s optimizing problem is given by

$$\max_{q_1} V_1 = \int_{\hat{z}}^{\bar{z}} (R - D) f(z) dz = \int_{\hat{z}}^{\bar{z}} (\theta + z - q - c) q f(z) dz - \int_{\hat{z}}^{\bar{z}} D f(z) dz \quad (23a)$$

The Stackelberg equilibrium is given by

$$q_1^{s*} = \frac{\theta - c + \bar{z}}{3}, \quad q_2^{s*} = \frac{2(\theta - c + \bar{z})}{9}. \quad (22)$$

Proof: See Appendix B

### 2.6.3 Monopoly Equilibrium

A monopolist with perfect foresight solves the following problem:

$$\max_q V = \int_{\hat{z}}^{\bar{z}} (R - D) f(z) dz = \int_{\hat{z}}^{\bar{z}} (\theta + z - q - c) q f(z) dz - \int_{\hat{z}}^{\bar{z}} D f(z) dz \quad (23a)$$

where $R$ is the profit of the monopoly firm and debt $D$ is given by

$$D = cq. \quad (2)$$

The Monopoly solution is given by

$$q_m^* = \frac{\theta - c + \bar{z}}{3}. \quad (25)$$

Proof: See Appendix C
2.6.4 Proposition 3

Suppose the firms have perfect foresight and treat the switching state and debt as endogenous. Then for all three market forms, Cournot, Stackleberg and Monopoly, the leveraged firm produce less (more) than the unleveraged firm when there is less (more) uncertainty in the market. Hence, debt financing may lead to less (more) aggressive behavior in the product market depending on the level of demand risk.

Proof: See Appendix D

This Proposition is one of the main results of this paper. The Proposition stands in sharp contrast to the standard Brander and Lewis result which says debt financing leads to more aggressive behavior in the product market. The explanation for their result is as follows: The firm’s equity holders are protected by the limited liability effect, but are residual claimants to higher earnings. The manager of the firm caters to the equity holders and picks more risky projects knowing that the equity holders shall enjoy the upside risk while being protected from the downside risk by limited liability.

This type of ‘risk shifting’ behavior of the firm may change if the firm knows that it can influence the switching state of nature. In that case, the strategy of the manager of the firm is not only to maximize the profit in the good state, but also to change output in such a manner that the bad state occurrence is minimized. When the firm knows that it can affect the switching state of nature, the strategy of a perfectly rational firm is not simply ‘risk shifting’ but rather to maximize good state profit by choosing the quantity and the switching state. This change in firm strategy may induce the firm to produce less after issuing debt. The benefits of acting more aggressively are the profits in the good states of nature. The costs of acting aggressively are the revenue losses for the equity holders when the states of nature are bad. The manager compares the costs and benefits of acting aggressively and decides on the output strategies. When there is less uncertainty in demand, the benefits of acting aggressively are not large enough compared to the loss of revenue in the bad states of nature. This causes the firm to act less aggressively in the output market. But when the demand uncertainty is large, the benefits of acting aggressively in the output market are huge compared to the costs of acting aggressively. The manager of the firm acts more aggressively to capture this large upside risk knowing that the downside risk is fully covered by limited liability protection.

This result is in contrast to the standard Industrial Organization literature, which says that debt financing leads to aggressive behavior in the product market. Hence we shall try to understand what drives our result.
2.7 Why Firms Produce Differently: An Insight

In this section, we provide further insight who and why our results are different from the standard results of the literature.

2.7.1 No Debt

In case of no debt, the firm solves the following problem:

$$\max_{q_i} V_i = \int_{-\bar{z}}^{\bar{z}} (R_i) f(z) dz = \int_{-\bar{z}}^{\bar{z}} (\theta + z - q_1 - q_2 - c) q_i f(z) dz$$

$$= (\theta - q_1 - q_2 - c) q_i.$$  

Firm i’s first order condition is given by

$$\theta - c - 2q_1 - q_2 = 0.$$  

The reaction function of firm 1 is given by

$$q_1(q_2) = R_1(q_2) = \frac{\theta - c}{2} - \frac{q_2}{2}. \quad (R1)$$

The Cournot solution is

$$q_i^* = \frac{\theta - c}{3}, i = 1, 2. \quad (14a)$$

2.7.2 Exogenous Debt and Exogenous Switching State

In the Brander and Lewis case, where both debt and ‘switching state’ are exogenous, the manager of firm i solves

$$\max_{q_i} V_i = \int_{\hat{z}}^{\overline{z}} (R_i - D_i) f(z) dz = \int_{\hat{z}}^{\overline{z}} (\theta + z - q_1 - q_2 - c) q_i f(z) dz - \int_{\hat{z}}^{\overline{z}} D_i f(z) dz$$

where $\hat{z}$ is defined by

$$D_i = R(q_1, q_2; \hat{z}) \quad (1)$$

and $R_i$ is the operating profit of firm i.

Maximizing (8a) with respect to $q_i$, we get the first order condition as follows:
\[
\frac{\partial V}{\partial q_i} = \int_{\hat{z}}^{z} \frac{\partial R_i}{\partial q_i} \frac{1}{2\pi} dz = 2(\pi - \hat{z})(\theta - 2q_1 - q_2 - c) + (\pi - \hat{z})(\pi + \hat{z}) = 0. \quad (8b)
\]

The reaction function of firm 1 is given by

\[
q_1(q_2) = R_1(q_2) = \frac{\theta - c}{2} + \frac{z + \hat{z}}{2} - \frac{q_2}{2}. \quad (R2)
\]

Comparing the reaction function of firm 1 in case of no debt, \(R1\), and the reaction function of firm 1 in case of exogenous debt and ‘switching state’, \(R2\) has one extra term \((\hat{z} + \hat{z})/2\). As a result, the reaction function of firm 1 (and similarly for firm 2) moves outwards. Hence, both firm 1 and firm 2 shall produce more output in the Brander and Lewis case.

### 2.7.3 Endogenous Debt and Endogenous Switching State

Now let us consider the case where both debt and ‘switching state’ are endogenous. So for any firm \(i \in \{1, 2\}\), the manager of firm \(i\) solves

\[
\max_{q_i} V_i = \int_{\hat{z}}^{z} (R_i - D_i) f(z) dz = \int_{\hat{z}}^{z} [\theta + z - q_1 - q_2] q_i f(z) dz - \int_{\hat{z}}^{z} D_i f(z) dz. \quad (17a)
\]

where \(R_i\) is the revenue of firm \(i\) and \(D_i = cq_i\). Replacing \(\hat{z}\) by

\[
c + q - \theta, \quad (16)
\]

we get

\[
\max_{q_i} V_i = \frac{z - \hat{z}}{2\pi} \left[ (\theta - c - q) q_i + \frac{z + \hat{z}}{2} q_i \right] = \frac{z - \hat{z}}{2\pi} \left[ (\theta - c - q) q_i + \frac{z + c + q - \theta}{2} \right].
\]

Here if \(\hat{z}\) is exogenous, then again the reaction functions of both the firms shall shift outwards and hence both the firms shall produce more output. But in case the ‘switching state’ is endogenous, it is no longer clear if the profit function is increasing or decreasing due to the term \(\hat{z}\). In this case, the reaction function of the firm 1 (and similarly for firm 2) is given by

\[
q_1(q_2) = R_1(q_2) = \frac{\theta - c}{3} + \frac{z}{3} - \frac{q_2}{3}. \quad (R3)
\]

Comparing \(R3\) with the reaction function of no debt case \(R1\), the reaction function under endogenous debt and ‘switching cost’ \(R3\) shifts outwards if

\[
\frac{\theta - c}{3} + \frac{\pi}{3} > \frac{\theta - c}{2} \Leftrightarrow \pi > \frac{\theta - c}{2}. \quad (E)
\]
The reaction function (R3) is flatter compared to (R1). If the above condition (E) holds, the reaction function (R3) also moves outwards compared to (R1). If the reaction function (R3) moves outwards compared to (R1), the firms shall definitely produce more in case of endogenous debt and endogenous ‘switching state’. But if the reaction functions move inwards, then the firms may also produce more, provided the reaction functions (R3) become sufficiently flatter compared to (R1). From the proof of Proposition 3,

\[ q_{c}^{\text{debt}} > q_{c}^{\text{noDebt}} \iff z > \frac{\theta - c}{3}. \]

Let me consider the following condition:

\[ \frac{\theta - c}{3} < \frac{\theta - c}{2} < z. \]

Both firms treat debt and ‘switching state’ as endogenous. Here both condition (E) and condition (F) are satisfied. This is the scenario when the reaction functions of the firms shift outwards leading to increase in quantities. Further, as the reaction functions of the firms become flatter, the firms produce more. Overall, these two effects reinforce each other and the firms produce more.

Now let me consider the case

\[ \frac{\theta - c}{3} < z < \frac{\theta - c}{2}. \]

In this case, the reaction functions of the firms (R3) shift inwards leading to a decrease in quantities produced. But as the reaction functions become flatter, the quantities produced increase. The increase in quantities due to flatter reaction functions offsets the decrease in quantities due to inward movements of the reaction functions. Hence, the net effect is that the output produced increases.

Next consider

\[ z < \frac{\theta - c}{3} < \frac{\theta - c}{2}. \]

In this case, the reaction functions shift inward so much that there is huge decrease in output. The flatter reaction functions increase output. But the decrease in outputs due to inward shifts of reaction functions more than offsets the increase in output due to flatter reaction functions. Overall, the quantities produced decrease.

\( z \) captures the demand uncertainty. The benefits of acting more aggressively are the profits of the good states of nature. The costs of acting aggressively are the revenue losses for the equity holders when the states of nature
are bad. The manager compares the costs and benefits of acting aggressively and decides on the output strategies. When there is less uncertainty in demand, i.e., \( \bar{z} \) is lower, the benefits of acting aggressively are not large enough compared to the loss of revenue in the bad states of nature. This causes the firm to act less aggressively in the output market. But when the demand uncertainty is large, i.e., \( \bar{z} \) is higher, the benefits of acting aggressively are large compared to the costs of acting aggressively. The manager of the firm acts more aggressively to capture this large upside risk knowing that the downside risk is fully covered by limited liability protection.

From the proof of proposition 3,

\[
q_{c}^{\text{debt}} > q_{c}^{\text{noDebt}} \iff \bar{z} > \frac{\theta - c}{3}.
\]

(F)

The larger is the value of \( \bar{z} \), the greater is the chance that condition (F) holds. Hence, at higher levels of risk, debt financing leads to more aggressive behavior in the output market. At lower levels of risk, debt financing leads to less aggressive behavior in the output market. The latter happens when the following holds:

\[
q_{c}^{\text{debt}} < q_{c}^{\text{noDebt}} \iff \bar{z} < \frac{\theta - c}{3}.
\]

(F1)

2.8 Demand Uncertainty: Endogenous Debt, Exogenous Switching State

In this section, we assume again that the firm issues debt to finance production thereby making debt endogenous. But the firm does not have perfect foresight and does not know that it can affect the switching state of nature by choosing output level. The firm does not know equation (16). So for the firm, \( \hat{z} \) is a parameter and not an endogenous variable.

2.8.1 Cournot Duopoly

So for any firm \( i \in \{1, 2\} \), the manager of firm \( i \) maximizes the residual value of the firm given by

\[
\max_{q_i} V_i = \int_{\bar{z}}^{\tau} (R_i - D_i) f(z) dz = \int_{\bar{z}}^{\tau} (\theta + z - q_1 - q_2 - c) q_i f(z) dz - \int_{\bar{z}}^{\tau} D_i f(z) dz
\]

(26a)

where \( R_i \) is the profit of firm \( i \) and debt \( D \) is given by equation (2).

The firm endogenizes \( D \) but \( \hat{z} \) is treated as a parameter. Maximizing (26a) with respect to \( q_i \), we get the first order condition as follows:
\[
\frac{\partial V_i}{\partial q_i} = \int_{\hat{\tau}}^{\tau} \frac{\partial R_i}{\partial q_i} \frac{1}{2\pi} dz = 2(\tau - \hat{\tau})(\theta - 2q_1 - q_2 - c) + (\tau - \hat{\tau})(\tau + \hat{\tau}) = 0. \quad (26b)
\]

The Cournot Duopoly solution is given by

\[
q_1^* = q_2^* = q_c^* = \frac{\theta - c + \tau}{4}. \quad (27c)
\]

Proof: See Appendix E

2.8.2 Stackelberg Equilibrium

We assume, without loss of generality, that firm 1 is the leader and that
firm 2 is the follower. The reaction function of firm 2 with respect to firm 1 is given by

\[
q_2 = R_2(q_1) = \frac{\theta - c + \tau - q_1}{3}.
\]

Firm 1, the leader, operates on the reaction function of the follower. None of the firms have perfect foresight and hence neither one knows equation (16). Both treat \( \hat{\tau} \) as a parameter and both treat debt \( D \) as endogenous.

Firm 1’s equity holder, the leader, solves

\[
\max_{q_1} V_1 = \int_{\hat{\tau}}^{\tau} (\theta + z - q_1 - q_2)q_1 f(z)dz - \int_{\hat{\tau}}^{\tau} D f(z)dz
\]

\[
= \int_{\hat{\tau}}^{\tau} (\theta + z - q_1 - R_2(q_1))q_1 f(z)dz - \int_{\hat{\tau}}^{\tau} c q_1 f(z)dz. \quad (28a)
\]

The first order condition is given by

\[
\frac{\partial V_i}{\partial q_i} = \int_{\hat{\tau}}^{\tau} \frac{\partial R_i}{\partial q_i} \frac{1}{2\pi} dz = \frac{\tau - \hat{\tau}}{2\pi} \left[ \frac{2(\theta - c)}{3} - \frac{4q_1}{3} + \frac{\tau}{6} + \frac{\hat{\tau}}{2} \right] = 0. \quad (28b)
\]

Replacing \( \hat{\tau} \) by

\[
\hat{\tau} = q_1 + q_2 - (\theta - c) \quad (16)
\]

in equation (28b) and solving yields as Stackelberg equilibrium

\[
\begin{cases}
q_1^* = \frac{\theta - c - \tau}{\theta - c - \tau} \\
q_2^* = \frac{2(\theta - c - \tau)}{9}
\end{cases} \quad (29)
\]
2.8.3 Monopoly Equilibrium

The equity holder of a monopoly firm solves
\[
\max_q V = \int_{\hat{z}}^{\bar{z}} (R - D) f(z) dz - \int_{\hat{z}}^{\bar{z}} D f(z) dz = \int_{\hat{z}}^{\bar{z}} (\theta + z - q - c) q f(z) dz - \int_{\hat{z}}^{\bar{z}} D f(z) dz \quad (29a)
\]
where \( R \) is the profit of the firm and debt \( D \) is given by equation (2).

The monopoly firm treats \( \hat{z} \) as a parameter and \( D \) as endogenous.

The first order condition is as follows:
\[
\frac{\partial V}{\partial q} = \int_{\hat{z}}^{\bar{z}} \frac{\partial R}{\partial q} \frac{1}{2} z dz = 2(\bar{z} - \hat{z})(\theta - 2q - c) + (\bar{z} - \hat{z})(\theta + c) = 0. \quad (29b)
\]

The Monopoly solution is
\[
q^*_m = \frac{\theta - c + \bar{z}}{3}. \quad (30c)
\]

Proof: See Appendix F

2.8.4 Proposition 4

Suppose the firms do not have perfect foresight and treat the switching state as endogenous whereas they treat debt as endogenous. For all three market forms, Cournot, Stackleberg and Monopoly, the leveraged firm produces less (more) than the unleveraged firm when there is less (more) uncertainty in the market. Hence, debt financing may lead to less (more) aggressive behavior in the product market depending on the level of demand risk.

Proof: See Appendix D. The intuition for this proposition is similar to the intuition for Proposition 3.

We have considered three cases so far. In the first case, debt is exogenous and the ‘switching state’ is exogenous to the firm. We are able to replicate the Brander and Lewis results for the different market forms. We show that debt financing lead to more aggressive product market behavior. In the second case, the firm considered both the debt and the switching state as endogenous. In this case, we have shown that debt financing can lead to less aggressive behavior in the output market for certain ranges of the parameters. This result is in direct contradiction to the standard Brander and Lewis results. The next step is to find out what is driving the Brander and
Lewis results. So in the third case, the firm considered debt as endogenous, but the firm is not perfectly rational. Hence the firm does not know that it can change the switching state of nature by changing its output. Hence, the switching state of nature is exogenous. In this case, we also arrived at results contradictory to the Brander and Lewis findings. The scenario is very similar to the scenario when both debt and switching state are endogenous. When debt is endogenous and the ‘switching state’ as exogenous to the firm, the results are contradictory to Brander and Lewis results. The case left to be considered is when debt is exogenous, but the ‘switching state’ is endogenous. This case is investigated in the next section.

2.9 Demand Uncertainty : Exogenous Debt, Endogenous Switching State

In this section, we assume that the firm considers debt as exogenous. But the firm has perfect foresight in the sense that it knows that it can endogenize the switching state of nature.

2.9.1 Monopoly Equilibrium

The monopoly firm takes D as exogenous. But the switching state is endogenous as given below:

\[
D = (\theta + \widehat{z} - q)q - cq \leftrightarrow \widehat{z} = \frac{D}{q} + q - (\theta - c) \tag{31a}
\]

The monopoly firm knows equation (31a), and hence, \( \widehat{z} \) depends on its output decision and can be treated as endogenous. The residual value for the monopoly firm is given by

\[
V = \int_{\widehat{z}}^{\overline{z}} (\theta + z - q)q f(z)dz - \int_{\widehat{z}}^{\overline{z}} D f(z)dz
\]

\[
= \frac{[(\theta - c + z - q)q - D]}{4\overline{z}} \left[\theta - c + z - \frac{D}{q} - q\right]. \tag{31b}
\]

Equation (31b) can be written as

\[
V = \frac{\overline{z} - \widehat{z}}{4\overline{z}} \left[\theta - c + z - q)q - D\right]. \tag{31c}
\]

27
The monopoly firm maximizes the following function

$$V = \int_{\hat{z}}^{\hat{z}} (\theta + z - q) f(z) dz - \int_{\hat{z}}^{\hat{z}} D f(z) dz \quad s.t. \quad \hat{z} \in [-\bar{z}, \bar{z}], \quad \hat{z} = \frac{D}{q} + q - (\theta - c).$$  

(31c)

Solving, we get the monopoly output as

$$q_m^* = q_3^* = \frac{\theta - c + \bar{z}}{6} + \frac{\sqrt{(\theta - c + \bar{z})^2 + 12D}}{6}.$$  

(32c)

Proof: See Appendix G.

### 2.9.2 Cournot Duopoly

For any firm $i \in \{1, 2\}$, the manager of firm $i$ takes $D$ as exogenous. The switching state is known to the firms and the switching state is given as follows:

$$D = (\theta + \hat{z} - q_i - q_{-i})q_i - cq_i \iff \hat{z} = \frac{D}{q_i} + q_i + q_{-i} - (\theta - c).$$  

(35a)

The firms know equation (35a).

The manager of firm $i$ maximizes the firm’s equity value:

$$\max_{q_i} V = \int_{\hat{z}}^{\hat{z}} (\theta + z - q_i - q_{-i}) f(z) dz - \int_{\hat{z}}^{\hat{z}} D f(z) dz \quad s.t. \quad \hat{z} \in [-\bar{z}, \bar{z}], \quad \hat{z} = \frac{D}{q_i} + q_i + q_{-i} - (\theta - c).$$  

(36a)

The reaction function of firm $i$ is given by

$$q_i(q_{-i}) = R_i(q_{-i}) = \frac{\theta - c - q_{-i} + \bar{z}}{6} + \frac{\sqrt{(\theta - c - q_{-i} + \bar{z})^2 + 12D}}{6}.$$  

(36b)

Proof: Proof is similar to the proof given in Appendix G. Only difference is that $\theta - c$ is replaced by $\theta - c - q_{-i}$.

Given the reaction functions of firms 1 and 2, the Cournot equilibrium is given by

$$q_e^* = \frac{\theta - c + \bar{z}}{8} + \frac{\sqrt{(\theta - c + \bar{z})^2 + 16D}}{8}.$$  

(37a)

Proof : See Appendix H
2.9.3 Proposition 5

Suppose the firms have perfect foresight and treat the switching state as endogenous while they treat debt as exogenous. For the different market forms, the leveraged firm produce less (more) than the unleveraged firm when there is less (more) uncertainty in the market. Hence, debt financing may lead to less (more) aggressive behavior in the product market depending on the level of demand risk.

Proof: See Appendix I

2.10 Conclusion

We have considered four cases. In the first case, both debt and ‘switching state’ are exogenous to the firm. In this scenario, we could replicate the Brander and Lewis results for three market forms, Cournot, Stackelberg and Monopoly. In the second case, both debt and ‘switching state’ are endogenous to the firm. In this case, we find that debt financing may lead to less (more) aggressive behavior in the output market depending on lower(higher) amount of risk in the output market. This result is in direct contradiction to the standard Industrial Organization findings that a leveraged firm behaves more aggressively in the output market. In order to find out what is driving this contradictory result, we consider two more cases. In the third case, the firm treats debt as endogenous and ‘switching state’ as exogenous. Still we arrive at the same contradictory results. In the fourth and final case, the firm treats debt as exogenous and ‘switching state’ as endogenous. Here again, we find out that debt financing may lead to less (more) aggressive behavior in the output market depending on the levels of risk in the output market. Hence, either endogenizing debt or endogenizing ‘switching state’ or endogenizing both leads to the conclusion that leveraged firm may behave less aggressively in the output market, contradicting the standard Industrial Organization results. The paper shows that standard Brander and Lewis results may not hold if we either endogenize debt or ‘switching state’ or both. The relation between the debt financing and output behavior is non-monotonic. At lower levels of risk, debt financing leads to less aggressive behavior leading to a fall in output and profit. At higher levels of risk, debt financing leads to more aggressive behavior leading to a rise in the output and profit. This paper also introduces the concept of endogenizing the ‘switching state’. If a firm is perfectly rational, and hence has perfect foresight, the firm knows how the ‘switching state’ depends on its output. Hence, the firm can
treat the ‘switching state’ as endogenous. We also consider different market forms and find that the market forms prove not crucial for our results. What is important is whether debt and/or ‘switching state’ are endogenous. We arrive at the same results for all the different market forms considered.

In the present paper, the firm is debt financed if there is any external financing at all. Future extensions could consider both debt and equity financing. It would also be interesting to see how sensitive the results are to the introduction of bankruptcy costs.

2.11 Appendix

2.11.1 Cournot Duopoly: Appendix A

So for any firm $i \in \{1, 2\}$, the equity holder of firm $i$, with perfect foresight solves the following problem:

\[
\max_{q_i} V_i = \int_{\hat{z}}^{\bar{z}} (R_i - D_i) f(z) dz = \int_{\hat{z}}^{\bar{z}} (\theta + z - q_1 - q_2) q_i f(z) dz - \int_{\hat{z}}^{\bar{z}} D_i f(z) dz
\]

where $R_i$ is the revenue of firm $i$ and $D_i = cq_i$. Replacing $\hat{z}$ by $c + q - \theta$, we get

\[
\max_{q_i} V_i = \int_{\hat{z}}^{\bar{z}} (\theta - c - q + \hat{z} + c) q_i f(z) dz - \int_{\hat{z}}^{\bar{z}} D_i f(z) dz
\]

After simplification, firm 1 faces the following problem:

\[
\max_{q_i} V_i = \frac{\bar{z} - \hat{z}}{2\bar{z}} [(\theta - c - q) q_i + \frac{\bar{z} + \hat{z}}{2} q_i] = \frac{\bar{z} - \hat{z}}{2\bar{z}} [(\theta - c - q) q_i + \frac{\bar{z} + c + q - \theta}{2} q_i]
\]

\[
= \frac{\bar{z} - \hat{z}}{4\bar{z}} [(\theta - c - q + \bar{z}) q_i] = \frac{\bar{z} - \hat{z}}{4\bar{z}} (\bar{z} - \hat{z}) q_i = \frac{\theta - c - q + \bar{z}}{4\bar{z}} (\theta - c - q + \bar{z}) q_i.
\]

Alternatively, we can write the optimizing problem as

\[
\max_{\hat{z}} V_i = \frac{\theta - c - q + \bar{z}}{4\bar{z}} (\theta - c - q + \bar{z}) q_i \quad s.t. \left\{ \begin{array}{l} \hat{z} \in [-\bar{z}, \bar{z}], \\ \hat{z} = c + q - \theta. \end{array} \right. \quad (17c)
\]

Alternatively, we can write the optimizing problem as

\[
\max_{\hat{z}} V_i = \frac{\bar{z} - \hat{z}}{4\bar{z}} (\bar{z} - \hat{z}) (\bar{z} + \theta - c - q_{-i}) \quad s.t. \left\{ \begin{array}{l} \hat{z} \in [-\bar{z}, \bar{z}], \\ \hat{z} = c + q_i + q_{-i} - \theta. \end{array} \right. \quad (17d)
\]
The first order condition is given by either
\[ \hat{z} = \bar{z} \quad \iff \quad q_i^* = \bar{z} + \theta - c - q_{-i} \]  
(18a)

or
\[ \hat{z} = \frac{\bar{z}}{3} - \frac{2(\theta - c)}{3} + \frac{2}{3}q_{-i} \quad \iff \quad q_i = \frac{\bar{z}}{3} + \frac{\theta - c}{3} - \frac{q_{-i}}{3}. \]  
(18b)

Checking the SOC, we find that \( V_i \) is maximized at
\[ \hat{z} = \frac{\bar{z}}{3} - \frac{2(\theta - c)}{3} + \frac{2}{3}q_{-i} \quad \iff \quad q_i = \frac{\bar{z}}{3} + \frac{\theta - c}{3} - \frac{q_{-i}}{3}. \]  
(18b)

\( V_i \) is minimized at
\[ \hat{z} = \bar{z} \quad \iff \quad q_i^* = \bar{z} + \theta - c - q_{-i}. \]  
(18a)

Considering only equation (18b), the Cournot Duopoly solution is given by
\[ q_1^{c*} = q_2^{c*} = q_c^* = \frac{\theta - c + \bar{z}}{4}. \]  
(19)

### 2.11.2 Stackelberg Equilibrium: Appendix B

Firm 1, the leader, operates on the reaction function of the follower and solves
\[
\max_{q_1} V_1 = \frac{\theta - c - q_1 - q_2 + \bar{z}}{4\bar{z}}(\theta - c - q_1 + q_2 + \bar{z})q_1 \quad \text{s.t.} \quad \begin{cases} 
\hat{z} & \in [-\bar{z}, \bar{z}], \\
\hat{z} = c + q_1 - q_2 - \theta, \\
q_2 & = \frac{\theta - c + \bar{z} - q_1}{3}.
\end{cases} \]  
(20a)

Simplifying, we get
\[
\max_{q_1} V_1 = \frac{\theta - c - q_1 + \bar{z}}{9\bar{z}}(\theta - c - q_1 + \bar{z})q_1 \quad \text{s.t.} \quad \begin{cases} 
\hat{z} & \in [-\bar{z}, \bar{z}], \\
\hat{z} = \frac{2q_1}{3} + \frac{\bar{z}}{3} - \frac{2(\theta - c)}{3}.
\end{cases} \]  
(20b)

Alternatively, we get
\[
\max_{\hat{z}} V_1 = \frac{\bar{z} - \hat{z}}{4\bar{z}}(\bar{z} - \hat{z})[(\frac{3\bar{z}}{2}) + \theta - c - \frac{\bar{z}}{2}] \quad \text{s.t.} \quad \begin{cases} 
\hat{z} & \in [-\bar{z}, \bar{z}], 
\end{cases} \]  
(20c)

The First Order Condition is given by either
\[ \hat{z} = \bar{z} \quad q_1 = \theta - c + \bar{z} \]  
(21a)

or
\[ \hat{z} = \frac{5\bar{z}}{9} - \frac{4(\theta - c)}{9} \quad q_1 = \theta - c + \frac{\bar{z}}{3}. \]  
(21b)
The function $V_1$ shall be maximized at

$$\hat{z} = \frac{5\pi}{9} - \frac{4(\theta - c)}{9} \quad q_1 = \frac{\theta - c + \bar{z}}{3}$$

(21b)

and minimized at

$$\hat{z} = \bar{z} \quad q_1 = \theta - c + \bar{z}.$$  

(21a)

Corresponding to equation (21b), the follower’s output is given by

$$q_2^{*} = \frac{2(\theta - c + \bar{z})}{9}.$$ 

Hence, the Stackelberg equilibrium is given by

$$q_1^{*} = \frac{\theta - c + \bar{z}}{3} \quad q_2^{*} = \frac{2(\theta - c + \bar{z})}{9}.$$ 

(22)

2.11.3 Appendix C: Monopoly Equilibrium

For any monopoly firm, the equity holder of firm , with perfect foresight, solves

$$\max_q V = \int_{\hat{z}}^{\bar{z}} (R - D) f(z)dz = \int_{\hat{z}}^{\bar{z}} (\theta + z - q - c)q f(z)dz - \int_{\hat{z}}^{\bar{z}} D f(z)dz$$

(23a)

where $R_i$ is the revenue of the monopoly firm. Replacing $\hat{z}$ by $c + q - \theta$,

$$c + q - \theta,$$

we get

$$\max_q V = \frac{\bar{z} - \hat{z}}{2\pi} [(\theta - c - q)q + \frac{\bar{z} + \hat{z}}{2}q] = \frac{\bar{z} - \hat{z}}{4\pi} [\theta - c - q + \bar{z}]q$$

$$= \frac{\bar{z} - \hat{z}}{4\pi} (\bar{z} - \hat{z})q = \frac{\theta - c - q + \bar{z}}{4\pi} (\theta - c - q + \bar{z})q$$

(23b)

Thus the monopoly firm faces the following simplified problem:

$$\max_q V = \frac{\theta - c - q + \bar{z}}{4\pi} (\theta - c - q + \bar{z})q \quad s.t. \begin{cases} \hat{z} \in [-\pi, \bar{z}], \\ \hat{z} = c + q - \theta. \end{cases}$$

(23c)

Alternatively, we can write the optimizing problem as

$$\max_q V = \frac{\bar{z} - \hat{z}}{4\pi} (\bar{z} - \hat{z})(\hat{z} + \theta - c) \quad s.t. \begin{cases} \hat{z} \in [-\pi, \bar{z}], \\ \hat{z} = c + q - \theta. \end{cases}$$

(23d)
The first order condition is given by either
\[ \hat{z} = \bar{z} \quad \iff \quad q^* = \bar{z} + \theta - c \] (24a)
or
\[ \hat{z} = \frac{\bar{z}}{3} - \frac{2(\theta - c)}{3} \quad \iff \quad q = \frac{\bar{z}}{3} + \frac{\theta - c}{3}. \] (24b)

Checking the SOC, we find that \( V_i \) is maximized at
\[ \hat{z} = \frac{\bar{z}}{3} - \frac{2(\theta - c)}{3} \quad \iff \quad q = \frac{\bar{z}}{3} + \frac{\theta - c}{3}. \] (24b)

\( V_i \) is minimized at
\[ \hat{z} = \bar{z} \quad \iff \quad q_i^* = \bar{z} + \theta - c. \] (24a)

Considering only equation (24b), the **Cournot Duopoly** solution is given as
\[ q^*_m = \frac{\theta - c + \bar{z}}{3}. \] (25)

### 2.11.4 Appendix D: Proof of Proposition 3

**Cournot Duopoly**

\[ q^*_{\text{noDebt}} = \frac{\theta - c}{3}, i = 1, 2. \] (14a)

\[ q^*_{\text{debt}} = q^*_2 = q^*_c = \frac{\theta - c + \bar{z}}{4}. \] (19)

So comparing equation (14a) with equation (19), we get
\[ q^*_{\text{debt}} > q^*_{\text{noDebt}} \iff \bar{z} > \frac{\theta - c}{3}. \] (F)

So if the above condition is violated, the firm with no debt produces more than the leveraged firm. So there are range of parameters where the unleveraged cournot duopoly firms produce more than the leveraged Cournot duopoly firms.
Stackelberg Equilibrium

\[
\begin{aligned}
q_{\text{noDebt}}^1 &= \frac{\theta - c}{2}, \\
q_{\text{noDebt}}^2 &= \frac{\theta - c}{4}.
\end{aligned}
\]  

(14b)

\[
q_{\text{debt}}^1 = \frac{\theta - c + \varpi}{3}, \\ q_{\text{debt}}^2 = \frac{2(\theta - c + \varpi)}{9}.
\]  

(22)

So comparing equations (14b) and (22), we get

\[
q_{\text{debt}}^1 > q_{\text{noDebt}}^1 \iff \varpi > \frac{\theta - c}{2}.
\]

So if this condition is violated, the Stackelberg leader, without debt, shall produce more than a leveraged Stackelberg leader.

\[
q_{\text{debt}}^2 > q_{\text{noDebt}}^2 \iff \varpi > \frac{\theta - c}{8}.
\]

If this condition is violated, the Stackelberg follower, without debt, shall produce more than the leveraged Stackelberg follower. So we see that there are parameter ranges where the unleveraged Stackelberg firms produce more than the leveraged Stackelberg firms.

Monopoly

\[
q_{\text{debt}}^m = \frac{\theta - c + \varpi}{3};
\]  

(25)

\[
q_{\text{noDebt}}^m = \frac{\theta - c}{2}.
\]  

(14c)

Comparing equation (14c) and equation (25), we get

\[
q_{\text{debt}}^m > q_{\text{noDebt}}^m \iff \varpi > \frac{\theta - c}{2}.
\]

So if this condition is violated, the unleveraged monopoly firm produces more than the leveraged monopoly firm. Hence there are ranges of the parameters for which the unleveraged monopoly firm produces more than the leveraged one.

So for all three market structures, monopoly, Cournot duopoly and Stackelberg duopoly, there are ranges of the parameters for which the unleveraged firm(s) shall produce more than the leveraged firm(s). QED.
2.11.5 Appendix E : Cournot Duopoly

For any firm $i \in \{1, 2\}$, the equity holder of firm $i$, with perfect foresight, faces the following problem:

$$\max_{q_i} V_i = \int_{\hat{z}} \left( R_i - D_i \right) f(z) dz = \int_{\hat{z}} \left( \theta + z - q_1 - q_2 - c \right) q_i f(z) dz - \int_{\hat{z}} D_i f(z) dz. \quad (26a)$$

where $R_i$ is the profit of firm $i$ and debt $D$ is given by

$$D = cq. \quad (2)$$

Firm $i$ does not know that it can affect the switching state $\hat{z}$ by changing its output. $\hat{z}$ is given by

$$\hat{z} = c + q - \theta. \quad (16)$$

Equation (16) is unknown to the firm. The firm does not have perfect foresight and, hence, $\hat{z}$ is a parameter to the firm. So the firm endogenizes $D$ but not $\hat{z}$.

Maximizing (26a) with respect to $q_i$, we get the first order condition as follows:

$$\frac{\partial V_i}{\partial q_i} = \int_{\hat{z}} \frac{\partial R_i}{\partial q_i} 1 2\hat{z} = 2(\hat{z} - \hat{z})(\theta - 2q_1 - q_2 - c) + (\hat{z} - \hat{z})(\hat{z} + \hat{z}) = 0. \quad (26b)$$

Equation (16) is not known to the firm, though equation (16) holds at the equilibrium. Replacing $\hat{z}$ by $c + q - \theta$ (16) at the equilibrium, we get

$$[\hat{z} + \theta - c - q_2 - q_1][\theta - c + \hat{z} - 3q_1 - q_2] = 0. \quad (26c)$$

So the two optimal values of $q_1$ are given by either

$$q_1^* = \hat{z} + \theta - c - q_2 \quad (27a)$$

or

$$q_1^* = \frac{\theta - c + \hat{z}}{3} - \frac{q_2}{3}. \quad (27b)$$

Second order conditions yield that at

$$q_1^* = \hat{z} + \theta - c - q_2, \quad (27a)$$

35
V is minimized. At
\[ q_1^* = \frac{\theta - c + z}{3} - \frac{q_2}{3}, \]  
(27b)
the function V is maximized.

The Cournot solution is given by
\[ q_1^* = q_2^* = q_c^* = \frac{\theta - c + z}{4}. \]  
(27c)

### 2.11.6 Appendix F: Monopoly

For any monopoly firm the equity holder of the firm solves the following problem:

\[
\max_q V = \int_{\hat{z}}^{\bar{z}} (R-D)f(z)dz = \int_{\hat{z}}^{\bar{z}} (\theta + z - q - c)qf(z)dz - \int_{\hat{z}}^{\bar{z}} Df(z)dz \tag{29a}
\]

where \( R \) is the profit of the firm and debt \( D \) is given by
\[
D = cq. \tag{2}
\]

The monopoly firm does not know that it can affect the switching state \( \hat{z} \) by changing its output. \( \hat{z} \) is given by
\[
\hat{z} = c + q - \theta. \tag{16}
\]

Equation (16) is not known to the firm. The firm does not have perfect foresight and, hence, \( \hat{z} \) is a parameter to the firm. So the firm endogenizes \( D \) but not \( \hat{z} \).

Maximizing (29a) with respect to \( q_i \), we get the first order condition as follows:

\[
\frac{\partial V}{\partial q} = \int_{\hat{z}}^{\bar{z}} \frac{\partial R}{\partial q} \frac{1}{2z} = 2(z - \hat{z})(\theta - 2q - c) + (z - \hat{z})(\theta - \hat{z}) = 0. \tag{29b}
\]

Equation 16 is not known to the firm. But equation 16 holds at the equilibrium. Replacing \( \hat{z} \) by
\[
c + q - \theta \tag{16}
\]
at the equilibrium, we get
\[
[\bar{z} + \theta - c - q][\theta - c + z - 3q] = 0. \tag{29c}
\]
So the two optimal values of \( q \) are given by either
\[
q_m^* = \bar{z} + \theta - c \quad (30a)
\]
or
\[
q_m^* = \frac{\theta - c + \bar{z}}{3} \quad (30b)
\]
Second order conditions are checked to find that at
\[
q_m^* = \bar{z} + \theta - c, \quad (30a)
\]
\( V \) is minimized. At
\[
q_m^* = \frac{\theta - c + \bar{z}}{3}, \quad (30b)
\]
the function \( V \) is maximized. The Monopoly solution is given by
\[
q_m^* = \frac{\theta - c + \bar{z}}{3}. \quad (30c)
\]

2.11.7 Appendix G: Monopoly Firm

The monopoly firm maximizes the following function
\[
V = \int_{\xi}^{\bar{z}} (\theta + z - q)qf(z)dz - \int_{\xi}^{\bar{z}} Df(z)dz \quad s.t. \quad \hat{z} \in [-\bar{z}, \bar{z}], \quad \hat{z} = \left[ \frac{\theta}{q} + q - (\theta - c) \right]. \quad (31c)
\]
The profit function for the monopoly firm is given by
\[
V = \int_{\xi}^{\bar{z}} (\theta + z - q)f(z)dz - \int_{\xi}^{\bar{z}} Df(z)dz = \left[ (\theta - c + \bar{z} - q)q - D \right] \left[ \theta - c + \bar{z} - \frac{D}{q} - q \right]. \quad (31b)
\]
Noting equation (31b), \( V \) can be written as
\[
V = \bar{z} - \bar{z}[\left( \theta - c + \bar{z} - q \right)q - D]. \quad (31c)
\]
Unconstrained maximization of (31b) gives us the following FOC:
\[
\frac{\theta - c + \bar{z} - \frac{D}{q} - q}{4\bar{z}} [\theta - c + \bar{z} - 2q] + \frac{\left( \theta - c + \bar{z} - q \right)q - D}{4\bar{z}} \left[ \frac{D}{q^2} - 1 \right] = 0. \quad (32a)
\]
Corresponding to the FOC, the four values of \( q \) are given by
\[
q_i^* = \frac{\theta - c + \bar{z}}{2} + \frac{\sqrt{(-\theta + c - \bar{z})^2 - 4D}}{2}; \quad (33a)
\]
\[ q^*_2 = \frac{\theta - c + \bar{z}}{2} - \frac{\sqrt{(-(\theta - c) - \bar{z})^2 - 4D}}{2}; \quad (33\text{b}) \]

\[ q^*_3 = \frac{\theta - c + \bar{z}}{6} + \frac{\sqrt{(\theta - c + \bar{z})^2 + 12D}}{6}; \quad (33\text{c}) \]

\[ q^*_4 = \frac{\theta - c + \bar{z}}{6} - \frac{\sqrt{(\theta - c + \bar{z})^2 + 12D}}{6}. \quad (33\text{d}) \]

Looking closely, we find that \( q^*_4 \) is negative and hence is ruled out. Let
\[
(\theta - c + \bar{z}) = \sqrt{KD}. \quad (34\text{a})
\]

For \( q^*_1 \) and \( q^*_3 \) to be real, we need

\[ (-\theta - c - \bar{z})^2 - 4D > 0. \]

Then either
\[
\sqrt{KD} > 2\sqrt{D} \Leftrightarrow (\theta - c) + \bar{z} > 2\sqrt{D} \tag{A}
\]

or
\[
\sqrt{KD} < -2\sqrt{D} \Leftrightarrow (\theta - c) + \bar{z} < -2\sqrt{D}. \tag{B}
\]

B is not possible as \( \theta - c > 0 \) and \( \bar{z} > 0 \). Hence A is the only possibility. From A, we get
\[
K > 4. \quad (34\text{b})
\]

Given the FOC (32a) and inequality (34a), the SOC amounts to
\[
\frac{3q - 2\sqrt{KD} + \frac{D^2}{q}}{2\pi}. \quad (32\text{b})
\]

Now we know
\[
\hat{z} = \left[ \frac{D}{q} + q - (\theta - c) \right]. \quad (31\text{a})
\]

Correspondingly, the values of \( q \) are
\[
q_1 = \frac{\theta - c + \hat{z}}{2} + \frac{\sqrt{(-(\theta - c) - \hat{z})^2 - 4D}}{2};
\]
\[
q_2 = \frac{\theta - c + \hat{z}}{2} - \frac{\sqrt{(-(\theta - c) - \hat{z})^2 - 4D}}{2}.
\]

So when \( \hat{z} = \bar{z} \), the values of \( q \) are the values we get from the FOC, equation (33a) and equation (33b):
\[
q^*_1 = \frac{\theta - c + \bar{z}}{2} + \frac{\sqrt{(-(\theta - c) - \bar{z})^2 - 4D}}{2}; \quad (33\text{a})
\]

38
\[ q_2^* = \frac{\theta - c + \overline{z}}{2} - \frac{\sqrt{(-\theta - c - \overline{z})^2 - 4D}}{2} \] (33b)

But we know that the profit function is given by

\[ V = \frac{\overline{z} - \hat{z}}{4\overline{z}} \left[(\theta - c + \overline{z} - q)q - D\right]. \] (31c)

So when \( q \) attains the values \( q_1^* \) and \( q_2^* \), the profit function \( V \) becomes 0:

\[ V_{q_1^*} = 0, \quad V_{q_2^*} = 0, \] (35a)

\[ \hat{z} \in [-\overline{z}, \overline{z}]. \]

At \( \overline{z} \), the corresponding values of \( q \) are

\[ q_1^* = \frac{\theta - c + \overline{z}}{2} + \frac{\sqrt{(-\theta - c - \overline{z})^2 - 4D}}{2} \] (33a)

\[ q_2^* = \frac{\theta - c + \overline{z}}{2} - \frac{\sqrt{(-\theta - c - \overline{z})^2 - 4D}}{2} \] (33b)

Corresponding to \(-\overline{z}\), the values of \( q \) are

\[ q_{1-\overline{z}} = \frac{\theta - c - \overline{z}}{2} + \frac{\sqrt{(-\theta - c + \overline{z})^2 - 4D}}{2}; \] (33e)

\[ q_{2-\overline{z}} = \frac{\theta - c - \overline{z}}{2} - \frac{\sqrt{(-\theta - c + \overline{z})^2 - 4D}}{2}. \] (33f)

It can be shown that

\[ q_1^* > q_{1-\overline{z}}, \]

\[ q_2^* < q_{2-\overline{z}}. \]

Hence, it can be inferred that

\[ q_1^* > q_{1-\overline{z}} > q_{2-\overline{z}} > q_2^*. \]

Hence we can conclude that

\[ \hat{z} \in [-\overline{z}, \overline{z}] \iff q \in [q_2^*, q_1^*]. \]

So the profit maximization problem for the monopoly firm can be rewritten as

\[ \max_q V = \int_{\hat{z}}^{\overline{z}} (\theta + z - q)f(z)dz - \int_{\hat{z}}^{\overline{z}} Df(z)dz \quad s.t. \quad q \in [q_2^*, q_1^*]. \] (31d)
which can be rewritten as

$$\max_q V = \frac{[(\theta - c + \bar{z} - q)q - D]}{4\pi} [(\theta - c + \bar{z} - \frac{D}{q} - q)] \quad \text{s.t.} \quad q \in [q_1^*, q_2^*]. \quad (31e)$$

When $q = q_1^*$ or $q = q_2^*$, the profit function $V$ becomes 0:

$$V_{q_1^*} = 0, V_{q_2^*} = 0. \quad (35a)$$

Recall that unconstrained maximization gives us four candidate values of $q$ from the FOC:

$$q_1^* = \frac{\theta - c + \bar{z}}{2} + \frac{\sqrt{(-\theta - c) - \bar{z}^2 - 4D}}{2}; \quad (33a)$$

$$q_2^* = \frac{\theta - c + \bar{z}}{2} - \frac{\sqrt{(-\theta - c) - \bar{z}^2 - 4D}}{2}; \quad (33b)$$

$$q_3^* = \frac{\theta - c + \bar{z}}{6} + \frac{\sqrt{(\theta - c + \bar{z})^2 + 12D}}{6}; \quad (33c)$$

$$q_4^* = \frac{\theta - c + \bar{z}}{6} - \frac{\sqrt{(\theta - c + \bar{z})^2 + 12D}}{6}. \quad (33d)$$

$q_4^*$ is negative and hence ruled out. $q_1^*$ and $q_2^*$ yield $V=0$. The only possible value of $q$ to be verified is $q_3^*$. It can be shown that

$$q_1^* > q_3^* > q_2^*.$$

SOC is given by

$$\frac{3q - 2\sqrt{KD} + \frac{D^2}{q_1^*}}{2\pi}. \quad (32b)$$

Now we know that

$$(\theta - c + \bar{z}) = \sqrt{KD}. \quad (34a)$$

Hence $q_3^*$ can be written as

$$q_3^* = \frac{\sqrt{KD} + \sqrt{(\sqrt{KD})^2 + 12D}}{2\pi} \leftrightarrow \frac{D^2}{q_3^*} = \frac{6^4}{[\sqrt{K} + \sqrt{K + 12}]^4}.$$

The second order derivative can be written as

$$\frac{\sqrt{D}}{2\pi} [\frac{\sqrt{K} + \sqrt{K + 12}}{6}] 3 + \frac{6^4}{[\sqrt{K} + \sqrt{K + 12}]^4 - 2\sqrt{K}]. \quad (32c)$$
Expression (33c) is equal to 0 when $K = 4$ or $K = 12$. When $K > 4$, expression (33c) is less than 0:

$$K > 4 \Rightarrow \frac{\sqrt{D}}{2\pi} \left[ \frac{\sqrt{K} + \sqrt{K + 12}}{6} \right] \left[ 3 + \frac{6^4}{[\sqrt{K} + \sqrt{K + 12}]^4} \right] - 2\sqrt{K} < 0.$$ 

But we know that $K > 4$ from (34b). Hence the second order derivative at $q^*_3$ is negative. So the function $V$ attains the maximum at $q = q^*_3$, when the range of $q$ is given by $q \in [q^*_2, q^*_1]$.

### 2.11.8 Appendix H: Cournot Duopoly

The reaction function of firm $i$ is given by

$$q_i(q_{-i}) = R_i(q_{-i}) = \frac{\theta - c}{6} - q_{-i} + \frac{\sqrt{(\theta - c - q_{-i} + \overline{z})^2 + 12D}}{6}. \quad (36b)$$

At Cournot equilibrium,

$$q^*_1 = q^*_2 = q^*_c.$$

Putting this into equation (36b), we get

$$q^*_c = \frac{\theta - c - q^*_c + \overline{z}}{6} + \frac{\sqrt{(\theta - c - q^*_c + \overline{z})^2 + 12D}}{6}.$$

Solving for $q^*_c$, we get two values:

$$q^*_c = \frac{\theta - c + \overline{z}}{8} + \frac{\sqrt{(\theta - c + \overline{z})^2 + 16D}}{8}, \quad (37a)$$

$$q^*_c = \frac{\theta - c + \overline{z}}{8} - \frac{\sqrt{(\theta - c + \overline{z})^2 + 16D}}{8}. \quad (37b)$$

Ignoring the negative value of $q^*_c$, the Cournot equilibrium is given by

$$q^*_c = \frac{\theta - c + \overline{z}}{8} + \frac{\sqrt{(\theta - c + \overline{z})^2 + 16D}}{8}. \quad (37a)$$

### 2.11.9 Appendix I: Proposition 5

**Cournot Duopoly**

The Cournot equilibrium, in case of debt financing, is given by

$$q^*_c = \frac{\theta - c + \overline{z}}{8} + \frac{\sqrt{(\theta - c + \overline{z})^2 + 16D}}{8} \quad (37a)$$
\[ q^*_{\text{noDebt}} = \frac{\theta - c}{3}, \quad i = 1, 2. \]  

(14a)

So comparing equation 37a with equation 14a, we get,

\[ q^*_{\text{debt}} > q^*_{\text{noDebt}} \iff 9D > (\theta - c)(\theta - c - 3) \]  

(38a)

So if the above condition is violated, the firm with no debt produces more than the leveraged firm. So there are range of parameters where the unleveraged cournot duopoly firms produce more than the leveraged Cournot duopoly firms.

**Monopoly**

\[ q^*_{\text{debt}} = q_3^* = \frac{\theta - c + \bar{z}}{6} + \frac{\sqrt{(\theta - c + \bar{z})^2 + 12D}}{6} \]  

(33c)

\[ q^*_{\text{noDebt}} = \frac{\theta - c}{2} \]  

(14c)

Comparing equation 14c and equation 33c, we get,

\[ q^*_{\text{debt}} > q^*_{\text{noDebt}} \iff 4D > (\theta - c)(\theta - c - 2\bar{z}) \]  

(38b)

So if this condition is violated, the unleveraged monopoly firm produces more than the leveraged monopoly firm. Hence there are range of the parameters for which the unleveraged monopoly firm produces more than the leveraged one.

So for all the cases, monopoly and Cournot duopoly, there are ranges of the parameters for which the unleveraged firm(s) shall produce more than the leveraged firm(s). QED.
Chapter 3

Does Endogenous Equity Affect Endogenous Investment under Limited Liability?

3.1 Introduction

The equity market is an important source of external financing for firms. Though the literature has debated how debt financing may lead to more aggressive or less aggressive behavior in the product market, little is known about how equity financing may affect firm behavior. We investigate how a financially constrained firm behaves in the output market if it issues both debt and equity to finance output.

The corporate finance literature has developed models of debt contracts where the threat of liquidation, i.e., higher costs of debt financing leads to underinvestment and less aggressive behavior in the output market. In contrast, the industrial organization literature has shown that debt financing can lead to more aggressive behavior in the output market. The corporate finance literature deals with ex-ante behavior of a firm in the sense that debt causes a firm to decrease its cost of production and probability of bankruptcy by acting less aggressively in the output market, i.e., undertaking less risky strategies in the output market. The industrial organization literature, starting with the seminal paper of Brander and Lewis (1986) assumes that the firm’s equity holders are protected from losses by limited liability, but are a residual claimant to high earnings. The firm caters to the equity holders and undertakes risky projects. The equity holders reap the benefit of upside risk while being protected from the downside risk by limited liability. This ‘risk shifting’ leads a firm to behave in a more aggressive manner. All the subse-
sequent papers debated if the firm acts more aggressively or less aggressively following debt financing.

None of the papers in the literature look into the effect equity can have on firm output. One reason for ignoring the effect of equity on output strategies is because modeling equity involves solving the age old conflict of interest between the old shareholder and the new shareholder. Solving this problem is in itself quite challenging and can divert a paper’s focus from the relation between equity financing and firm output. Another reason why equity is ignored when investigating how firm output is related to external financing is that equity formed a small fraction of the total external financing by the firm as pointed out by Campello (2005). But Fama and French (2005) have shown that equity does constitute a significant portion of the firm’s external financing. Previous authors had considered only IPO and SEO as the form of equity financing. But Fama and French argue that there are in total seven different ways by which the firm can raise equity, not only IPO and SEO but other methods like employee stock options. Given the importance of equity in external financing, we introduce both debt and equity as external financing and investigate the effect of equity financing on the firm output. We use the framework of Povel and Raith (2004), who introduce variable production cost in the standard Brander and Lewis framework and make debt endogenous. In their framework, the firm issues debt to finance output. We introduce both endogenous debt and equity. In our framework, the firm issues both debt and equity to finance output. Our goal is to find out how equity affects the output behavior of the firm. We refrain from addressing the issue of conflict of interest between the old shareholder and the new shareholder. The previous literature investigates how debt financing can affect firm output. In this paper, we focus on the effect of equity financing on firm behavior in the output market.

We show that if the financially constrained firm issues both debt and equity to finance output, the output produced is always lower than the output that would have been produced if the firm was not financially constrained. Further, we investigate how the financially constrained firm’s output responds to the discounted cost of equity and derive a U-shaped firm output in response to the firm’s discounted cost of equity. In our model, we assume a three-stage game. In the first stage, the firm decides on the amount of debt and equity. In the second stage, the firm decides on the amount of output to be produced. In the third stage, the state of the nature is realized and the equity holders and debt holders are paid off. We assume that the return on equity is exogenous to the firm at the first and second stage. The firm makes output market decisions based on the assumption of an exogenous return to the equity holders. This exogenous return to equity can be
the long run equity return in that industry. But as the long run return to equity cannot be negative, we look only at the segment of the firm output where the return to equity is positive. We show that when there is no risk of default, the equity holder is paid the risk free rate. When the rate of equity return increases, the probability of default also increases and the firm initially reduces the output. When the default probability rises, the rate of return to equity also rises as the equity holder now bears the risk that the firm may go bankrupt. In case the firm goes bankrupt, the firm is liquidated and the firm’s assets are sold off. A portion of these assets are used to pay off the debt holders and the equity holders do not get anything. So in case the firm is liquidated, the equity holders lose their current investment and also lose the future profit the firm could have made. Hence bankruptcy is costly to the equity holders as well. As a result, as the default probability increases, the rate of return to the equity holders also increases to compensate for the increase in risk. As the return to equity increases, the output of the firm decreases and then subsequently increases. The manager of the financially constrained firm weighs the costs and benefits of increasing output. The costs of aggressive output market behavior results from a high probability of bankruptcy and liquidation in which case the equity holders lose their investments and future firm profits. The benefits of aggressive behavior arise from higher profits in case of the good state of nature. When the costs outweigh the benefits, the firm acts less aggressively in the output market which is the case when the return to equity is low. When the return to equity is high, the default probability is high, there is greater risk and correspondingly higher profits from good states of nature. The benefits outweigh the costs and the firm acts more aggressively in the output markets.

3.2 The Existing Literature

Traditionally the industrial organization literature focused on the firm’s strategies where the firm’s objective is to maximize profits. The corporate finance literature considered financial decisions of firms in terms of maximization of equity values. But in truth, the financial market decisions and product market decisions are interlinked. Brander and Lewis (1986), in their seminal paper, studied the linkage between financial markets and product markets. They argued that limited liability may lead to a leveraged firm taking a more aggressive stand. Their 1988 follow-up paper included the bankruptcy costs and arrived at the same conclusion. These two papers were followed by a series of others. Maksimovic (1988) and Hendel (1996) argued that the firms become more aggressive while Glazer (1994) and Chevalier
and Scharfstein (1996) argue that they become less aggressive. Either effect can happen in Showalter (1995). Poitevein (1989) and Fudenburg and Tirole (1986) have investigated the predatory action taken by competitors in the context of capital market imperfection and entry. Bolton and Scharfstein (1990) discuss a two period model involving a financially constrained firm and a bank. The bank cannot verify the profits of the firm. A rival, who is not constrained financially, competes with the leveraged firm.

Maurer (1999) develops a model of product competition with debt as the optimal contract. He studies two firms, one of which is financially constrained. The managers of the two firms choose effort on innovation, with the outcome of the innovation being uncertain. Maurer’s optimal financial contract is similar to Bolton and Scharfstein’s (1990). He shows the first best alternative is to exert effort unless the rival gains from predatory behavior.

Faure-Grimaud (2000) also derives debt as an optimal contract. He shows that debt causes firms to compete less aggressively because the positive liability effects on quantities is offset by the negative effect due to endogenous financial costs. But this paper also allows for renegotiation. The model assumes that the investors can withdraw their funds after the output decision is made without incurring any losses. This effectively means that the output choice is contractible, implying there is no moral hazard problem regarding output choice.

Povel and Raith (2004) investigate this moral hazard problem. They build a model with two firms. One of them is financially constrained. They include variable production cost, unlike most other models of industrial organization, and argue that costly production creates a feedback from the output market to the financial market. But they argue that the future profit generated by the firm (the amount to be lost in case of liquidation) is constant. Further, the paper assumes that the manager of the firm acts in the interest of the equity holder and is afraid of liquidation. This assumption, though not unreasonable, is not always very realistic as managers often are more concerned with their income rather than the firm’s profits. Our paper is closely related to their paper. In their model, they have endogenous debt and internal fund. They derive a U shaped firm output with respect to the firm’s internal funds. We use their model set-up, introduce both endogenous debt and equity and assess how cost of equity may affect the firm output. We do not have internal funds in our model, instead we have endogenous debt and endogenous equity. We derive a U shaped firm output with respect to discounted cost of equity $R_e$.

The empirical findings by Phillips (1995), Kovenock and Philips (1995, 1997) suggest that industrial concentration is an important variable in the interaction between financial markets and product markets. An increase
in the leverage of one firm decreases investments while the rivals increase investments. Philips argues that the close rivals can increase their market share while the leveraged firm loses out both in terms of investments and market shares. This finding is corroborated in Opler and Titman (1994). Chevalier (1995) investigates the consequence of leverage in the supermarket industry in the USA. She argues that debt weakens the competitive edge of the leveraged firm. Overall, the empirical literature has not supported the theoretical conclusions of Brander and Lewis. The empirical literature has provided evidence that debt financing leads to less aggressive firm behavior in the output market.

3.3 The Model

Let us consider a duopoly market where one firm is financially constrained and the other firm is not. We assume that firm $i$ is financially constrained and raises funds from external sources to finance its production cost. In stage 1, firm $i$ chooses a mix of debt and equity. In the next stage, stage 2, the firms $i$ and $j$ choose their outputs and engage in a Cournot duopoly game. The manager of firm $i$ uses the funds raised from the external market to finance production of output $q_i \geq 0$. There is no fixed cost of production. The variable cost of production is given by $c q_i$ and financed by debt and equity. So the amount of external financing determines in effect how much output firm $i$ is going to produce. Thus, the amount of output and the external financing are linked together. Povel and Raith (2004) introduced this idea of production being financed by external funding, thereby linking the two. Further, we assume that there is no agency problem between the manager and the equity holders. The manager truthfully maximizes the equity value of the firm. The outputs are chosen in the second stage before the actual demands are generated. This often happens in practice: the firms choose output levels based on estimated demand. The actual market demand depends on the state of nature. There can be good states and bad states of nature. The state of nature is random and ex ante unknown to the firm. In the third stage, after the firm has produced its output, the state of nature is realized and the actual demand is generated. If the state of nature and hence the demand is good (bad), the firm makes a profit (loss). A portion of the firm’s revenue is used to pay back the debt. If the state of nature is bad and the revenue is not enough to repay the debt, the firm declares bankruptcy and is liquidated and its assets are sold off. A portion of the revenue generated by the sale of the firm’s assets are paid to the debt holders and the rest is lost as deadweight loss. Here the assumption is that the revenue generated from
the sale of firm’s assets are not enough to pay off the entire debt and hence the equity holders are not paid anything in case of liquidation.

3.3.1 The Model Set-up

There are two risk neutral firms. One firm, firm i, is financially constrained and the other firm, firm j, is not. In stage 1, firm i issues a mix of debt and equity. In stage 2, the firms i and j compete in the output market as Cournot duopolists. The revenue of firm i is denoted by \( R^i(q_1, q_2, z) \). The state of the nature is represented by the random variable \( z \), which is distributed with density \( f(z) > 0 \) over a range \([\underline{z}, \overline{z}]\). The revenue of firm i has the standard properties:

\[
\begin{align*}
R_i^i &> 0 \quad R_{ii}^i < 0; \\
R_{ij}^i &< 0 \quad R_{ij}^i < 0; \\
R^i &\text{ is strictly concave and has a unique maximum in } q_i \text{ for given } q_j \text{ and } z; \\
R_{iz}^i &> 0 \quad R_{iz}^i > 0; \\
R_{ij}^i R_{jj}^j &> R_{ij} R_{ij}^j; \\
R^i(q, z) & = 0.
\end{align*}
\]

These assumptions are standard in the literature, like in Brander and Lewis (1986) and Povel and Raith (2004).

Firm i issues debt of size \( D_{im} \) in stage 1. \( D_{im} \) is the market value of debt or the amount of debt the firm raises in the market. Using this debt and the equity, the firm produces output \( q_i \). The state of the nature is realized and the firm pays back the debt holder the amount \( D_i \), the face value of debt, if the revenue generated is at least equal to \( D_i \).

Firm i can perfectly forecast the switching state of nature, \( \hat{z} \) which is defined by the following identity (in case of duopoly):

\[
\hat{z} = R(q_i, q_j, \hat{z}). \tag{1}
\]

\( \hat{z} \) is the state of the nature at which firm i has earned just enough revenue to pay off its debt obligations. If the firm has perfect foresight, this switching state of nature, \( \hat{z} \), can be treated as endogenous by the firm.

We note that both \( D_i \), the face value of debt and \( \hat{z} \) are endogenous and determined within the model. Both debt and equity issued by the firm are endogenous. The firm uses the external finance market to finance its variable cost of production. This means

\[
D_{im} + E = cq_i \tag{2}
\]

48
where $c$ is the constant marginal cost of production and $E$ is the amount of equity raised by the firm. We call this the cost of production constraint.

Later we discuss how the market value of debt $D^m_i$ and the face value of debt $D_i$ are related. Equation (4) captures the relation between them. The firm issues market value of debt $D^m_i$ and pays back the face value of debt $D_i$.

Firm $i$ maximizes $R_i - D_i$, where $R_i$ is the expected revenue, over all states of nature. When the state of nature is bad, i.e., $z < \hat{z}$, the firm does not earn enough revenue to pay back the face value of debt $D_i$. If the firm does not pay back the face value of debt $D_i$, the debt holder mops up the entire revenue $R(q, z)$, where $z < \hat{z}$. So in the bad states of nature, $z < \hat{z}$, the firm does not have to pay back the debt issued. Firm $i$ is said to be protected by limited liability as the debt issued need not be paid back by the equity holders when the state of the nature is bad.

The manager of firm $i$ maximizes the residual value of the firm after paying the debt holders. The residual value of the firm is given by

$$V_e = \int_{\hat{z}}^{\mathbb{Z}} (R^i(q_i, q_j, z) - D^i) f(z) dz.$$  \hfill (3)

This residual value is also the value of the firm to the equity holders. We refrain from the problem of conflict of interest between the old and the new shareholders. We assume that there are only new shareholders because the goal of this paper is not to address the conflict of interest between the old and new equity holders. Our goal is to investigate if there is a link between the equity financing and firm output. Firm $i$ initially did not have any shareholders. But when the firm issues equity, the equity holders enter the firm as the new equity holders. The equity holders invest $E$ and the return to the equity holders is given by $r_e$. In the short run, the firm promises to pay at least $r_e$ amount of return to the equity holders, that is

$$V_e \geq (1 + r_e)E.$$  

But in the long run, the firm shall pay a return of $r_e$, which is the same as given by others in the same or related industry. In the short run, firm $i$ may pay more return on equity than the return on equity prevalent in the industry in order to attract equity holders. But in the long run, the firm pays exactly the same return on equity as the other firms in the industry. The firm considers the return to equity $r_e$ as exogenous while making capital structure and output market decisions. The equity holders participation constraint is given by

$$V_e = (1 + r_e)E.$$
Firm $i$ issues debt with market value $D_i^m$ at a rate of return of $r_d$ and pays back the face value of debt $D_i$. $r_d$ is the prevailing rate of return in the bond market. The debt holders get back the face value of debt $D_i$ when the state of nature is good, $z > \hat{z}$. When the state of nature is bad, $z < \hat{z}$, the debt holders get the entire revenue of the firm, $R_i(q, z)$. Following Brander and Lewis (1988), we introduce bankruptcy cost in the model. Debt holders have to bear the cost of bankruptcy. The expected bankruptcy cost is given as

$$\gamma D_i(q, \hat{z}) \int_{\hat{z}}^{\hat{z}} f(z) dz$$

where $\gamma \in (0, 1)$.

When the firm declares bankruptcy, the firm has to pay the fraction $\gamma$ of the face value of debt $D_i(q, \hat{z})$ as bankruptcy cost. Whenever the firm declares bankruptcy, the debt holders have to bear this cost as the revenue payment to the debt holders shall be reduced by the cost of bankruptcy. We also assume when the firm declares bankruptcy, the firm is liquidated. We make this assumption to exempt us from the derivation of an optimal debt contract which is not the goal of this paper. We assume that due to liquidation, the firm’s assets are evaluated and sold off to a third party. We also assume that the total liquidation value of the firm is $L$ and this liquidation value is at least as large as the $\gamma$ fraction of the revenue generated by the firm in that bad state of nature, i.e. we assume $L > \gamma \int_{\hat{z}}^{\hat{z}} R_i(q, z) f(z) dz$. A fraction of the liquidation value $L$, $\gamma \int_{\hat{z}}^{\hat{z}} R_i(q, z) f(z) dz$ is paid to the debt holders as liquidation payment. The rest $L - \gamma \int_{\hat{z}}^{\hat{z}} R_i(q, z) f(z) dz$ is a deadweight loss. The equity holders are not paid anything in case of bankruptcy and subsequent liquidation. This is a standard assumption in the optimal debt contract framework. We justify this type of bankruptcy cost and liquidation value because it may often happen that the state of nature is so bad that the revenue generated $\int_{\hat{z}}^{\hat{z}} R_i(q, z) f(z) dz$ is extremely small and may be lower than the cost of bankruptcy $\gamma D_i(q, \hat{z}) \int_{\hat{z}}^{\hat{z}} f(z) dz$. In this case, the debt holders do not get back any portion of the debt and end up paying the bankruptcy cost from their own pockets. Hence, the debt holders get a negative payoff in case of bankruptcy. In order to avoid this kind of scenario, we assume the above type of liquidation payment to the debt holders. In the standard optimal contract literature, this type of payment to the debt holders due to liquidation of the firm is common. In reality, liquidation payments are made to the debt holders first and then to the equity holders. We are assuming here that the liquidation amount is so low that only the debt holders are
paid $\gamma \int_{\hat{z}}^{z} R^i(q, z)f(z)dz$ and the rest of the amount is lost as dead weight loss leaving the equity holders with nothing.

The debt holders invest the amount $D^m_i$ in the firm $i$. The debt holders are risk neutral. As a result, firm $i$ sets the face value of debt $D_i$ such that the expected amount to be paid back to the debt holder is $(1 + r_d)D^m_i$. The expected value of debt $D^m_i$ to the debt holders is given by

$$V_d = \int_{\hat{z}}^{z} R^i(q, z)f(z)dz + D^i(q, \hat{z}) \int_{\hat{z}}^{z} f(z)dz - \gamma D^i(q, \hat{z}) \int_{\hat{z}}^{z} f(z)dz - \gamma \int_{\hat{z}}^{z} R^i(q, z)f(z)dz.$$  

The first term is the amount of revenue the debt holders get when the state of the nature is bad $z < \hat{z}$, i.e., in case of bankruptcy. The second term is the face value of debt that holders get back when the state of the nature is good, $z > \hat{z}$. The third term is the bankruptcy cost which reduces the amount the debt holders get back in case of bankruptcy. The fourth term is the liquidation amount the debt holders get back when the firm is liquidated. All these four terms sum up as the expected value of debt to the creditors.

The manager of firm $i$ maximizes the residual value of the firm subject to the equity holders’ participation constraint, the debt holders’ participation constraint and the cost of production constraint. The maximization problem for the manager of firm $i$ is given by

$$\max_{q_i} V^i_e = \int_{\hat{z}}^{z} (R^i(q_i, q_j, z) - D^i) f(z)dz \quad \text{s.t.} \quad \begin{cases} V^i_e = (1 + r_e)E, \\ V^i_d = (1 + r_d)D^m_i, \\ D^m_i + E = cq_i, \\ D_i = R^i(q, q_j; \hat{z}). \end{cases}$$  

Firm $i$ is financially constrained. The firm $j$ is not financially constrained. Hence, the manager of firm $j$ maximizes
\[
\max_{q_i} V^j_q = \int_{\hat{z}}^{\bar{z}} (R^j_i(q_i, q_j, z) - c q_j) d z. \quad (6)
\]

The solution to the system (5) and (6) gives us the equilibrium output of firm i and firm j. If we assume that both the firms are financially unconstrained, we shall get the Cournot output by solving the following two equations.

\[
\int_{\hat{z}}^{\bar{z}} R^i_i(q_i, q_j, z) = c; \\
\int_{\hat{z}}^{\bar{z}} R^j_j(q_i, q_j, z) = c.
\]

Let us denote the unconstrained Cournot output by \((q^*_c, q^*_c)\).

In our model, in the first stage, the firm decides how to raise external funding to finance production. In the second stage, the firm decides how much output to produce. In the third stage, the state of nature is realized and the equity holders and debt holders are paid off. So the return to equity and debt are assumed to be exogenous when deciding how much to produce and how to finance the production.

### 3.3.2 Proposition 1

When a financially constrained firm finances it’s production through both debt and equity, it produces less than the unconstrained Cournot output \(q^*_c\) and the financially unconstrained firm produces more than the unconstrained Cournot output \(q^*_c\).

Proof: See Appendix A.

This proposition shows that the financially constrained firm is going to produce less than the unconstrained Cournot output under every possible states of nature. When the switching state of nature is extreme, that is, \(\hat{z} = \bar{z}\) or \(\hat{z} = \bar{z}\), the financially constrained firm produces the same amount of output, i.e. \(q^*_1\) is the same when \(\hat{z} = \bar{z}\) or \(\hat{z} = \bar{z}\). This output of the financially constrained firm \(q^*_1\) is less than the unconstrained Cournot output \(q^*_c\) while the financially unconstrained firm produces more than the unconstrained Cournot output \(q^*_c\). This result is in sharp contrast with the standard Brander Lewis result which states that the financially constrained firm shall produce more than unconstrained Cournot output. However, our result is in
line with the Povel and Raith (2004) result. The firm is financing the variable production cost with the money raised in the external capital market thereby linking the external financing to the output produced and rendering both debt and equity endogenous variables. Unlike Povel and Raith, who assume internal funds and debt to finance production, our model has both endogenous equity and debt to finance production. Our assumption is that if the firm goes bankrupt, the firm is liquidated. The equity holders of the firm are worse off because they lose their own investment in the firm and also all the future profits the firm could have made. If the firm issues more debt and equity and hence produce more output, the firm is increasing it’s risk of getting bankrupt and thereby becoming liquidated. But if the firm is liquidated, the equity holders of the firm are badly affected as they are not paid any of the revenue generated and further they loose any future profit. Brander and Lewis (1986) argued that the equity holders shall encourage the firm to pursue higher debt financing, undertake more risk and produce aggressively in the output market because the firm’s equity holders are protected by limited liability. By issuing more debt, the firm engages in more aggressive output market strategies. The firm can increase it’s profit when the state of the nature is good and does not care about the bad states of nature. The firm tries to leverage on the upside risk knowing that the downside risk is covered through the limited liability protection. Hence the firm raises huge amount of debt and floods the market with it’s output with the hope that the state of the nature shall turn out to be good and there shall be enough demand to absorb the firm’s output. The equity holders are better off with a huge amount of residual profit in case the state of nature is good and are protected by limited liability if the state of nature turns out to be bad. This is the argument in the literature showing how debt financing lead to more aggressive output market strategies.

But this argument is not very realistic. What the literature ignores is the interaction of equity financing and product market. The debt holders perfectly understand that the equity holders are better off as they are protected by limited liability. Hence the debt holders shall sign a debt contract only when their interest is sufficiently protected in case the firm goes bankrupt. In that scenario, the firm is liquidated and the debt holders are paid off a portion of the liquidation value. If the firm engages in aggressive debt financing and hence aggressive product market strategies, the firm runs into the risk of being bankrupt and liquidated. The equity holders are worse off in case of liquidation as they are the residual claimants and may receive nothing in case of bankruptcy. Further, they shall also lose any future profit opportunities. If we ignore liquidation costs to equity holders, then we can argue that the equity holders shall prefer aggressive debt financing and aggressive product
market strategies. But if we consider the costs to equity holders in case of bankruptcy and liquidation, as we considered in this paper, we show that the equity holders no longer want to produce more aggressively. As a result, the manager acts less aggressively in the output market. The maximum output of the financially constrained firm is less than the unconstrained Cournot output.

In the Brander and Lewis framework, it is assumed that the firm commits itself to some level of output before the demand is realized and the production is financed through the generation of subsequent revenue. Hence they do not assume that the debt is used to finance production. They assume that the debt issued by the firm is exogenous and after the debt is raised, the firm can do whatever it chooses to do with that debt. Hence, after the debt is raised, the firm undertakes riskier product market strategies as the equity holders care only about the good states of nature and are insulated from the downside risk by limited liability. There is a shift of wealth from debt holders to equity holders. Povel and Raith interlink debt issued with output produced by assuming the debt is used to finance output. In our model, we assume that the debt and equity issued are used to finance output thereby linking the aggregate external financing with output produced as well. This assumption is plausible as the debt contract often specifies the reason for which the debt is being raised. Further, the equity holders have the right to know how their money is being spent. Hence, the assumption that first the debt is issued and then the manager is free to choose whatever he wants to do with the issued debt is not very realistic. Due to that assumption in the Brander and Lewis framework, debt financing leads to aggressive output behavior. In our model, the assumption that both debt and equity are used to finance output leads to the result that the firm is less aggressive in the output market.

We control for the agency problem that may exist between the debt holders and the equity holders. Let us assume that bankruptcy is costless and the firm is not liquidated in case of bankruptcy. The firm issues debt for financing it’s project. The equity holders are protected by limited liability and the manager cater to the equity holders. It may happen that there are multiple projects available to the manager and the manager picks the most risky project. The equity holders get the benefit of risky project if the good state occurs and if the bad state occurs, the debt holders take the hit. So there is a transfer of wealth from the debt holders to the equity holders when the manager chooses a risky project. This is a typical agency problem between the equity holder and the debt holder. This kind of agency problem between the debt holder and the equity holder is avoided by making specific arrangements for the bad states of nature in the debt contract. In
our model, the debt contract is designed such that in case of bankruptcy, the firm is liquidated and the debt holders shall get a part of the liquidation amount \( \gamma \int_{\hat{z}}^{z} R(q, z) f(z) dz \). The equity holders shall not receive any amount of money in case of liquidation of the firm. Further, the equity holders shall lose any future profits of the firm. Hence, if the manager chooses a risky project, the debt holders are worse off but the equity holders are also worse off if the firm cannot pay back it’s debt. By assuming that the firm shall have to pay a liquidation fee to the debt holders and the firm shall be liquidated when it declares bankruptcy, we attempt to control for the agency problem that may arise between the debt holders and the equity holders.

So we conclude that the financially constrained firm shall always produce less than the unconstrained Cournot output. The firm produces less aggressively due to debt and equity financing because the costs of producing more aggressively and getting bankrupt is more than the gains of producing aggressively.

Our next goal is to find out what happens to the firm’s output when the returns to equity changes. In order to answer that question, we have to use comparative statics to see how the equilibrium output changes when the return to equity changes. In our model, in the first stage, the firm decides how to raise external financing to finance production. In the second stage, the firm decides how much to produce. In the third stage, the state of nature is realized and the equity holders and debt holders are paid off. So the return to equity and debt are assumed to be exogenous when deciding how much to produce and how to finance the production. \( r_e \) is the return on equity and \( r_d \) is the return on debt.

In Appendix B, we derive a U-shaped relation between the equilibrium firm output of financially constrained firm and the discounted cost of equity as depicted by figure 1. \( r_e \) is the return on equity and \( r_d \) is the return on debt. We define \( R_d = \frac{1}{1+r_d} \) as the discounted cost of debt and \( R_e = \frac{1}{1+r_e} \) as the discounted cost of equity. When \( \hat{z} = z \), \( r^\hat{z}_e = r_d > 0 \). When \( \hat{z} = \overline{z} \), \( r^\hat{z}_e = -\frac{1+r_d}{r_d} - 1 < 0 \) We recall the definition of the switching state.

\[
D_i = R(q_i, q_j, \hat{z})
\]

When the switching state is \( \hat{z} = z \), i.e., point A in figure 1, there is no state of nature which is below \( \hat{z} = z \). The firm issues debt given by \( D = R(q_i, q_j, \hat{z}) \). \( R(q_i, q_j, \hat{z}) \) is the minimum revenue the firm shall generate. Hence when \( \hat{z} = \overline{z} \), the default probability is zero. As a result, the firm pays the equity holders a rate of return equal to the prevailing bond rate of return, \( r^\hat{z}_e = r_d > 0 \). If \( \hat{z} \) rises above \( z \), the probability of default starts rising. If the firm defaults, the firm is liquidated and the firm’s assets are sold off to pay
Figure 1

Relationship between Equilibrium Output and Discounted Cost of Equity

Figure 3.1: Relationship Between Equilibrium Output and Discounted Cost of Equity
the debt holders. In order to compensate the equity holders for the increased risk, the equity holders are paid higher returns on equity. So as the \( \hat{z} \) starts increasing above \( z \), the probability of default is rising and the equity holders are paid more than \( r_d \).

When \( \hat{z} = z \), the probability of default is one. The return to the equity holders is negative. The revenue generated by the firm is guaranteed to be lower than the debt issued. The equity holders lose in terms of bankruptcy and liquidation cost and future profits. But we do not consider this case as a feasible case. We recall that \( r_e \) is the long run rate of return on equity. We postulate that the long run rate of return on equity cannot be negative. We recall that \( r_e \) is exogenous when the manager of the firm decides on output and debt and equity to be issued. The manager is catering to the equity holders. The firm shall be better off by not producing anything rather than assuming negative return for the equity holder. Hence in figure 1, we consider only the segment AOC for our investigation. At point A, the rate of return on equity \( r_e = r_d \). At the other end point C, the rate of return on equity \( r_e \) is infinity. As one moves away from point A, the probability of default increases till it reaches one at point C.

3.3.3 Proposition 2

The output of the firm is initially decreasing and then increasing with respect to the rate of return on equity.

As the return to equity increases, the output of the firm decreases and then subsequently increases. The manager of the financially constrained firm weighs the costs and benefit of increasing output. The costs of aggressive output market behavior results from a high probability of bankruptcy and liquidation in which case the equity holders lose their investments and future firm profits. The benefits of aggressive behavior arise from higher profits in case of the good state of nature. When the costs outweigh the benefits, the firm acts less aggressively in the output market which is the case when the return to equity is low. When the return to equity is high, the default probability is high, there is greater riskiness and correspondingly higher profits from good states of nature. The benefits outweighs the costs and the firm acts more aggressively in the output markets.

At point A, when \( \hat{z} = z \), the default probability is zero, \( r_e = r_d \) and the firm output is upward sloping with respect to \( R_e \). This is the point where the firm is producing the maximum output. When one moves away from A, the exogenous return on equity \( r_e \) increases above \( r_d \). We recall that \( r_e \) is the return on equity which is exogenous at the time when the output and financing decisions are made. Due to increase in the risk of default as one
moves away from point A, the firm starts acting cautiously in the output market and starts reducing the output. This continues till point O. When one moves from point O toward C, the firm start increasing the output as the rate of return on equity increases. After point O, the rate of return on equity becomes so high that in order to maintain that high level of return on equity, the firm has to increase output to order to increase profit. The expected marginal revenue is greater than the marginal cost of issuing debt \( E(R_i(q, z)) - R_i(q, \hat{z}) > 0 \). The firm’s output is below the first best level, or the unconstrained Cournot level of output because of which the firm can increase it’s profit by raising output. So when the rate of return is sufficiently high, the firm has to increase output to maintain that higher level of return to equity even if that means increasing the probability of default. Stated otherwise, as we move from point O toward point C, the risk of default increases and correspondingly the rate of return on equity increases.

When one moves from point O toward A, the probability of default keeps on falling and approaches zero. As the fear of default and subsequent bankruptcy and liquidation cost decreases, the firm starts behaving more and more aggressively by increasing output. The equity holders get lesser amount return on equity as the risk of default decreases. At point A, there is no risk of default and the equity holders get exactly the return on bond. The expected marginal revenue is greater than the marginal cost of taking debt \( E(R_i(q, z)) - R_i(q, \hat{z}) > 0 \). The firm’s output is below the first best level, or the unconstrained Cournot level of output because of which the firm can increase it’s profit by raising output. So as the discounted cost of equity is rising, i.e., as the return on equity is falling, i.e., one moves from point O toward A, the firm acts more aggressively in order to raise output given the fact that \( E(R_i(q, z)) - R_i(q, \hat{z}) > 0 \). The firm keeps on raising output till point A where \( \hat{z} = \tilde{z} \). At this point, there is no default probability and the rate of return on equity reaches it’s lowest value, i.e., \( r_e = r_d \). Now the equity holders receive only the rate of return prevailing in the bond market. The intuition is that as there is no chance that the firm is going to default, the manager is not going to compensate the equity holders for the risk of buying the shares of the firm as there is no risk that the equity holders end up losing the current investments and the future profits.

One may argue that the equity holders bear no risk and are protected by the limited liability effect in case of bankruptcy. But if the firm goes bankrupt, the firm is liquidated and the equity holders are not paid anything and further they loose the opportunity of sharing the firm’s future profit. The literature often ignores this cost of bankruptcy to the equity holders by arguing that the equity holders are protected by the limited liability effect. But we argue that the equity holders also stand to lose in case bankruptcy
occurs and the firm is liquidated. So we argue in this paper that in case of
bankruptcy, both the equity holders and debt holders bear the loss.

As we already pointed out, we ignore the possibility that the rate of return
on equity and the discounted cost of equity becoming negative. We recall
that the \( r_e \) is the long run rate of return. Hence we assume that \( r_e \) cannot go
beyond zero. When the rate of return on equity falls below zero, the firm’s
output curve is downward sloping. We ignore this downward portion of the
firm output, BC in figure 1. When the rate of return on equity goes below
zero, the value of the firm also becomes negative. The manager of the firm
maximizes the firm value. The manager shall maximize

\[
V_e = \int_z^\infty \left( R_i(q_i, q_j, z) - D^i \right) f(z) dz.
\]  

(3)

If we assume for the time being that the rate of return on equity is
negative, in that case the manager maximizes \( V_e \) subject to the constraints
given by equation 5. Given the rate of return on equity is negative, the firm’s
maximized value \( V_e \) is negative. But in this case, the manager always have
the option of not producing anything. We have assumed there is no fixed
cost of production or there is no sunk cost of production. So when the rate of
return on equity is negative, the firm shall be better off by not producing any
output. As a result, we ignore the downward portion of the firm output, BC
in figure 1.

We use the setup of Povel and Raith (2004). Povel and Raith use endoge-
nous debt and internal fund and derive U shaped firm output with respect to
firm’s internal funds. In this model, we have endogenous debt and endoge-
nous equity and derive U shaped firm output with respect to discounted cost
of equity \( R_e \).

We discuss how our model is different from the standard Modigliani-
Miller framework. The Modigliani-Miller assumptions are not true for our
model. The first basic difference is that in our model, we treat investment
as endogenous. In our model, the firm’s investment is equal to the variable
production cost \( cq_i \). In the Modigliani-Miller framework, the firm have fixed
investments and the firm’s problem is what should be the optimal debt equity
mix to finance exogenous fixed investment. Further, the Modigliani-Miller
framework assumes that the capital market is perfect. The firm can raise
debt and equity and they are perfect substitutes. In our model, the debt and
equity are not perfect substitutes. We also have bankruptcy and liquidation
cost in our model which are also not present in the standard Modigliani and
Miller framework. Overall, the framework of this paper is different from the
Modigliani-Miller framework.
3.4 Conclusion

In this paper, we investigate the output market behavior of a financially constrained firm when the firm issues both debt and equity to finance production. We show that the financially constrained firm produces less than what it shall produce in case it was not financially constrained. Further, we show that as the rate of return on equity increases, the firm output initially decreases and subsequently increases.

3.5 Appendix

3.5.1 Appendix A: Proof of Proposition 1

The maximization problem for the manager of firm i is

$$\max_{q_i, z} V^i_e = \int_{\hat{z}}^{\bar{z}} (R^i(q_i, q_j, z)f(z)dz - D^i)f(z)dz \quad \text{s.t.} \begin{cases} V^i_e = (1 + r_e)E, \\ V^i_d = (1 + r_d)D, \\ D + E = cq_i, \\ D = R^i(q_i, q_j; \hat{z}). \end{cases}$$

(5)

Forming Lagrange, we get

$$L = \max_{q_i, z} V^i_e + \lambda (\frac{V^i_e}{1 + r_e} + \frac{V^i_d}{1 + r_d} - cq_i).$$

Differentiating L with respect to $q_i$, we get,

$$\frac{\partial L}{\partial q_i} = (1 + \lambda R_e) \int_{\hat{z}}^{\bar{z}} R^i(q_i, q_j, z)f(z)dz - (1 + \lambda R_e)R^i(q_i, q_j, \hat{z})$$

$$+ \lambda [R_d(1 + \gamma) \int_{\hat{z}}^{\bar{z}} R^i(q_i, q_j, z)f(z)dz + R_dR^i(q, \hat{z})$$

$$- R_d(1 + \gamma) \int_{\hat{z}}^{\bar{z}} R^i(q_i, q_j, \hat{z})f(z)dz - c]$$

$$= 0 \quad \text{(A1)}$$

where

$$R_e = \frac{1}{1 + r_e}, \quad R_d = \frac{1}{1 + r_d}$$

60
Differentiating with respect to $\hat{z}$, we get,

$$\frac{\partial L}{\partial \hat{z}} = (1 + \lambda R_e) - \lambda (R_d - (1 + \gamma)R_d \int_{\hat{z}}^{\hat{z}} f(z)dz) = 0 \quad (A2)$$

Hence,

$$\lambda = \frac{1}{R_d - Re - (1 + \gamma)R_d \int_{\hat{z}}^{\hat{z}} f(z)dz}$$

Putting the value of $\lambda$ in equation A1, we get,

$$a(q, \hat{z}; R_e) = (1 + \frac{R_e}{R_d - Re - (1 + \gamma)R_d \int_{\hat{z}}^{\hat{z}} f(z)dz}) \int_{\hat{z}}^{\hat{z}} (R_i^i(q_i, q_j, z) - R_i^i(q_i, \hat{z}))f(z)dz + \frac{1}{R_d - Re - (1 + \gamma)R_d \int_{\hat{z}}^{\hat{z}} f(z)dz}[R_d(1 + \gamma) \int_{\hat{z}}^{\hat{z}} R_i^i(q_i, q_j, z)f(z)dz - c] \quad (A3)$$

When we differentiate $L$ with respect to $\lambda$, we get

$$R_d(1 + \gamma) \int_{\hat{z}}^{\hat{z}} R_i^i(q_i, q_j, z)f(z)dz + R_d R_i^i(q, \hat{z}) - R_d(1 + \gamma) \int_{\hat{z}}^{\hat{z}} R_i^i(q_i, q_j, \hat{z})f(z)dz - c$$

$$= 0 \quad (A4)$$

$A3$ and $A4$ are the equilibrium conditions for firm $i$, the financially constrained firm. The financially unconstrained firm, firm $j$, maximizes revenue minus cost. The equilibrium condition for firm $j$ is given by

$$\int_{\hat{z}}^{\hat{z}} R_j^j(q, z)f(z)dz = c \quad (A5)$$

The equations $A3, A4$ and $A5$ solve the equilibrium output $(q_i, q_j, z)$. When $\hat{z} = \hat{z}$ or $\hat{z} = \hat{z}$, equation 3 reduces to
\[ \int_{\hat{z}}^{z} R^i(q, z)f(z)dz = \frac{c}{R_d} \]

Solving this equation with equation A5, and comparing the outputs with the unconstrained Cournot output, we get,

\[ q_1^* < q_1^c \quad q_2^* > q_2^c \]

QED

3.5.2 Appendix B

We derive U shaped firm output with respect to discounted cost of equity \( R_e \).

When \( \hat{z} = \bar{z} \), the probability of default is zero. We know that when \( \hat{z} = \bar{z} \), then

\[ R_e = \frac{cq_i}{E(R^i(q, z))} > 0 \]

\[ E(R^i(q, z)) = \frac{c}{R_d} \]

Hence \( E(R^i(q, z)) = \frac{cq_i}{R_d} \) Applying that here,

\[ R_e = R_d \]

Similarly, when \( \hat{z} = z \), \( R_e \) is

\[ R_e = -\gamma R_d \]

We recall that

\[ R_e = \frac{1}{1 + r_e} \]

\[ R_d = \frac{1}{1 + r_d} \]

When \( \hat{z} = \bar{z} \), we have \( r^e = r_d > 0 \).

When \( \hat{z} = z \), we have \( r^e = -\frac{1 + r_d}{\gamma} - 1 < 0 \)

Now we have to find \( \frac{dq}{dr_e} \).

\(^1\text{Povel and Raith (2004) derive a similar U shaped firm output with respect to firm's internal funds}\)
These are the three equilibrium conditions, the endogeneous variables being $q_i, q_j, z$, which have to satisfy the system of equations (A3), (A4) and (A5). Now we have to do comparative statics. Differentiating these equations with respect to $R_e$ and using the Cramer’s rule, we get,
Now from the equations (A3), (A4) and (A5), we get,

\[
a^\hat{z} = -(1 + \gamma) (E(R^i(q, z)) - R^i(q, \hat{z})) f(\hat{z})
\]

\[
b^\hat{z} = R^i(q, \hat{z})(-R_e + R_d(1 + \gamma) \int_{\hat{z}}^{\bar{z}} f(z)dz)
\]

\[
b_i = R_d(1 + \gamma) \int_{\hat{z}}^{\bar{z}} R^i_q(q, z) f(z)dz + R_d R^i_i(q, \hat{z}) - R_d(1 + \gamma) R^i_i \int_{\hat{z}}^{\bar{z}} f(z)dz
\]

\[
+ R_e \int_{\hat{z}}^{\bar{z}} (R^i_q(q, z) - R^i_i(q, \hat{z})) f(z)dz - c
\]

\[
= R_e \int_{\hat{z}}^{\bar{z}} (R^i_q(q, z) - R^i_i(q, \hat{z})) f(z)dz - c + R_d R^i_i(q, \hat{z})
\]

\[
b_j = R_d(1 + \gamma) \int_{\hat{z}}^{\bar{z}} R^j_q(q, z) f(z)dz + R_d R^j_j(q, \hat{z})
\]

\[
-R_d(1 + \gamma) R^j_j(q, \hat{z}) \int_{\hat{z}}^{\bar{z}} f(z)dz + R_e \left( \int_{\hat{z}}^{\bar{z}} R^j_j(q, z) f(z)dz - R^j_j(q, \hat{z}) \right)
\]

\[
\frac{dq_i}{dR_e} = \frac{H_1}{H}
\]

\[
= - \frac{[E(R^i(q, \hat{z})) - R^i(q, \hat{z})] E(R^j_j)(1 + \gamma)[E(R^i_i(q, z)) - R^i_i(q, z)] f(\hat{z})}{-b^\hat{z}(a_i f_j - a_j f_i) + a^\hat{z}(b_i f_j - b_j f_i)}
\]

\[
= - \frac{[E(R^i(q, \hat{z})) - R^i(q, \hat{z})] E(R^j_j)(1 + \gamma)[E(R^i_i(q, z)) - R^i_i(q, z)] f(\hat{z})}{J}
\]

where

\[
A = [E(R^i(q, \hat{z})) - R^i(q, \hat{z})] E(R^j_j)(1 + \gamma)[E(R^i_i(q, z)) - R^i_i(q, z)] f(\hat{z})
\]

\[
J = -b^\hat{z}(a_i f_j - a_j f_i) + a^\hat{z}(b_i f_j - b_j f_i)
\]

\[
a_i f_j - a_j f_i > 0 \text{ due to the assumption } R^i_i R^j_j > R^i_i R^j_j
\]
\[ J = [(R_e - R_d + R_d(1 + \gamma) \int_{\hat{z}}^{\hat{z}} f(z) dz)] [R_{\hat{z}}^i [a_i f_j - a_j f_i)] \\
- (1 + \gamma) f(\hat{z}) E(R_{ij}^j) [E(R_i^i(q, z)] - R_{ij}^i(q, \hat{z})]^2] \\
+ (1 + \gamma) f(\hat{z}) [E(R_i^i(q, z)] - R_{ij}^i(q, \hat{z})] E(R_{ij}^j)[E(R_j^j(q, z)] - R_{ij}^j(q, \hat{z})] \\
+ (1 + \gamma) f(\hat{z}) [E(R_i^i(q, z)] - R_{ij}^i(q, \hat{z})] E(R_{ij}^j)[(1 + \gamma) \int_{\hat{z}}^{\hat{z}} R_{ij}^i(q, z) f(z) dz \\
+ R_{ij}^i(q, \hat{z}) \int_{\hat{z}}^{\hat{z}} f(z) dz - \gamma R_{ij}^i(q, \hat{z}) \int_{\hat{z}}^{\hat{z}} f(z) dz] \\
= B + C + D \quad (3.1) \]

where B, C and D are the first, second and the third term of the expression for J. Hence, the value of \( \frac{dq_i}{dR_e} \) is

\[
\frac{dq_i}{dR_e} = \frac{A}{B + C + D}
\]

Now we look at the second order condition when \( \frac{dq}{dR_e} = 0 \).

\[
\frac{d^2 q_i}{d^2 R_e} = \frac{H_1}{H} \\
= \left| \begin{array}{ccc}
-a_{\hat{z}} R_{\hat{z}} & a_j & a_{\hat{z}} \\
-b_{\hat{z}} R_{\hat{z}} R_e & b_j & b_{\hat{z}} \\
0 & f_j & 0 \\
\end{array} \right| \\
= \frac{E(R_{ij}^j(q, z)) a_{\hat{z}} R_{\hat{z}} R_e b_{\hat{z}}}{|H|} \\
= -\frac{E(R_{ij}^j) a_{\hat{z}} (R_{\hat{z}})^2}{a_i f_j - a_j f_i}
\]

We get this expression because when \( \frac{dq}{dR_e} = 0, a_{\hat{z}} = 0 \).

\[ a_{\hat{z}} = (1 + \gamma) f(\hat{z}) R_{ij}^i(q, \hat{z}) > 0 \]

Hence

\[ \frac{d^2 q_i}{d^2 R_e} > 0 \]
So we have proved that when
\[ \frac{dq_i}{dR_e} = 0 \quad \frac{d^2 q_i}{d^2 R_e} > 0. \]

Now we also know that when \( \hat{z} = z \) or \( \hat{z} = \bar{z} \), the firm produces the same output. Now we show that when \( \hat{z} = z \) and when \( \hat{z} = \bar{z} \),

\[ \frac{dq_i}{dR_e} > 0 \]

and when
\[ \frac{dq_i}{dR_e} < 0 \]

When \( \hat{z} = \bar{z} \),
\[
\frac{dq_i}{dR_e} = -\left[ \frac{E(R^i(q, z)) - R^i(q, \bar{z})}{(1 + \gamma) f(\bar{z}) [E(R^i(q, z)) - R^i(q, \bar{z})]} \right]_j R^d E(R^j(q, \bar{z})) > 0
\]

When \( \frac{dq_i}{dR_e} = \bar{z} \),
\[
\frac{dq_i}{dR_e} = -\left[ \frac{E(R^i(q, z)) - R^i(q, \bar{z})}{(1 + \gamma) f(\bar{z}) [E(R^i(q, z)) - R^i(q, \bar{z})]} \right]_j R^d E(R^j(q, \bar{z})) < 0
\]

When \( \hat{z} = \bar{z}, \ R_e = R_d \)
\[
\left[ \frac{dq_i}{dR_e} \right]_{\hat{z}} = -\left[ \frac{E(R^i(q, z)) - R^i(q, \bar{z})}{R_d E(R^j(q, \bar{z})) E(R^j)} \right] < 0
\]

This proves that there is an unique minimum. From appendix (B), we know that when \( z = \bar{z} \), we can solve for \( R_e \) from equation A4.

\[ R^\bar{z}_e = R_d > 0 \]

When \( z = \bar{z}, \ R_e \) is
\[ R^\bar{z}_e = -\gamma R_d < 0 \]

So the range of \( R_e \) is
\[ R_e \in [-\gamma R_d, R_d] \]
We show that when \( z = \bar{z} \), that is, when \( R_e = R_d \),

\[
\left[ \frac{dq_i}{dR_c} \right]_{z=\bar{z}} > 0
\]

Further, when \( z = \bar{z} \), that is, when \( R_e = -\gamma R_d \), then

\[
\left[ \frac{dq_i}{dR_c} \right]_{z=\bar{z}} < 0
\]

The output produced when \( \hat{z} = z \) is equal to the output produced when \( \hat{z} = \bar{z} \).

We prove that \( q \) is U shaped with respect to \( R_c \).

QED
Chapter 4

Executive Compensation, Financial Constraint and Product Market Strategies

4.1 Introduction

We show in this paper that the differences in managerial compensation may be one of the reasons for differences in the product market strategies between more financially constrained and less financially constrained firms. The literature on product market and financial market interaction has been typically concentrated on how debt financing hurts or boosts firm performance. Some authors like Brander and Lewis (1986), Maksimovic (1988) and Hendel (1996) argue that debt financing lead to more aggressive behavior by the firm in the output market. Ex-post, debt financing lead to ‘risk shifting’ because the firm’s equity holders are residual claimants and are protected by limited liability. The firm managers cater to the equity holders. They engage in riskier projects and act aggressively in the output markets. There is another group of researchers like Glazer (1994) and Chevalier and Scharfstein (1996) who believe that the firms behave less aggressively due to debt financing. They argue that ex-ante, debt financing increases the probability of bankruptcy. If the firm faces high liquidation costs and high bankruptcy costs, then the firm shall behave less aggressively in the output market avoiding risky output market strategies. So the typical question one asks is whether debt financing leads to more or less aggressive output market behavior.

This paper explores if managerial compensation can help explain difference in output market strategies between more financially constrained firms.
and less financially constrained firms. We investigate three related questions to answer this. The first question is whether product market strategies vary between more financially constrained and less financially constrained firms. If the answer to the first question is yes, we move on to the second question: Does managerial compensation affect output market strategies? If the answer to the second question is also yes, we ask the third and last question. Does managerial compensation vary between more financially constrained and less financially constrained firms? If the answer to all the three questions is yes, we can argue that managerial compensation varies among more financially constrained and less financially constrained firms. Managerial compensation affects product market strategies. Hence, differences in managerial compensation may be one of the reasons for differences in the product market strategies between more financially constrained and less financially constrained firms.

Degree of financial constraint is defined as the degree of difficulty in raising external capital. If a firm is financially unconstrained, the cost of internal capital and external capital should be the same. In real life, the firms face higher cost of external capital which can be attributed to agency costs of equity and debt. The higher is the divergence between the cost of internal capital and external capital, the more financially constrained is the firm. Hence, all firms are constrained, only the degree of financial constraint varies. Hence we use the terms more financially constrained firms and less financially constrained firms.

Suppose there is a duopoly set-up with one firm more financially constrained (firm B) than the other (Firm A). Both the firms raise capital from the external market and invest that capital in output market strategies. Both the firms may engage in risky and aggressive output market strategies like more spending in advertisement, more research and development and more sales growth. Aggressive output market strategies may lead to better operating performance for the firms, increasing the stock price of the firms. These are the benefits of aggressive output market behavior. But the aggressive output market strategies are often riskier than the less aggressive output market strategies. The costs of acting aggressively in the output market include the probability of declaring bankruptcy and liquidation cost. A firm weighs the costs of acting aggressively with the benefits of aggressive output market behavior and decides whether to be more aggressive or less aggressive in the output market. We assume if a firm has higher cost of capital, the firm has higher marginal cost of production and higher total cost of production. By assumption, firm B is financially constrained, has higher cost of capital and higher marginal cost of production compared to firm A. Output and profit of firm B decrease due to higher marginal cost. But if the degree
of product differentiation is higher, then firm B can increase its profit by acting more aggressively in the output market and increase output in order to counter its higher cost of production. There are two opposing effects of higher financial constraint on output strategy of firm B. Higher cost of capital lead to higher cost of production and less aggressive output market strategy. The first effect decreases the amount of output produced. The second effect is due to product differentiation and the second effect increases output. If the products of the two firms are sufficiently different, a firm can increase revenue by increasing its output. When the degree of product differentiation is sufficiently high, the higher cost firm can act more aggressively and produce more in order to compensate for higher cost of production. Which effect dominates depends on the extent to which firm B’s product is different from firm A’s product. We show theoretically that when the degree of product differentiation is sufficiently high, the more financially constrained firm acts more aggressively in the output markets. If the degree of product substitution is sufficiently high, firm B can act more aggressively by increasing output as the consumers cannot switch to firm A’s product. Further, we document empirically that financially constrained firms are more aggressive in the output markets. This answers our first question. The output market strategies are different for financially constrained and unconstrained firms because their costs of capital and hence cost of production are different and their products are not perfect substitutes.

With this result in hand, we move to our second question. Does managerial compensation affect product market decisions? We derive optimal contracts for the managers of the two firms in a duopoly. The optimal contract of the manager is linked to the value of the firm which, in turn, depends on the profit of the firm. If the profit of the firm is higher, the manager gets higher compensation. This compensation structure provides incentive to the manager to act more aggressively in the output market in order to secure higher profit. Managerial compensation should play a role in the output market strategies. For example, the more is the short term payoff of the manager (for example bonus), more the manager should be induced to boost current earnings and sales growth. Similarly, equity based compensation of the manager, like the percentage of shares held by the manager should also affect the aggressiveness of the firm. Aggressiveness in the output market improves operating performance of the firm leading to an increase in the current stock price. If the CEO holds more stocks of the firm, it shall be in her interest to boost current earnings and act aggressively because by doing so; she can improve the current stock price. So CEO stock holding, CEO’s short term payoff can be reasons why she acts more aggressively in the product market. Following Aggarwal and Samwick (1999), we decom-
pose CEO compensation into three components, the flow compensation, the change in the value of the stock holding and the change in the value of stock options. We provide evidence that the product market aggressiveness can be explained by managerial compensation. We document empirically that total compensation, which comprises salary, bonus, other annual short term compensation, explains industry adjusted sales growth after controlling for all other known variables which affect sales growth. Industry adjusted sales growth is sales growth of the firm minus the industry mean sales growth. Industry adjusted sales growth is the proxy for aggressiveness in the product market. Further, short run bonus, defined as ratio of bonus to total current compensation, explains industry adjusted sales growth after controlling for all other known variables which affect sales growth. We also show that stock holding of the CEO, change in the value of stock options and change in the value of stock holding are explanatory variables for industry adjusted sales growth regression.

We link the first and the second question by asking the third question: Is there a significant difference in the managerial compensation between the more financially constrained and less financially constrained firm? We find evidence that the shares owned by the CEO, the change in the value of stock option of the CEO, the total compensation of the CEO and the bonus as a percentage in total current compensation are higher for more financially constrained firms. Further, these are the variables which explain industry adjusted sales growth of a firm, which is our proxy for product market aggressiveness of the firm. This suggests that at least a part of the difference in the product market aggressiveness between the more financially constrained firms and less financially constrained firms can be attributed to the above components of managerial compensation.

There are some limitations of this paper. A theoretical model is created from which three hypothesis are developed. The three hypotheses are tested empirically. It should be noted that the theoretical model is not tested per se. This is the first limitation of the paper. Most of the assumptions made in the theoretical model are too simple to be realistic. For example, the assumption of constant marginal cost of production and linear demand function are made for obtaining closed form solutions even though they may not be true in reality. Nevertheless, these assumptions are often common in theoretical models. The theoretical model provides some theoretical justification for the empirical results we document in section 5. The empirical results can be theoretically conceivable from the theoretical model. The theoretical model also serves to provide economic intuition to the hypothesis developed and the empirical results provided. Another limitation of this paper is the use of industry adjusted sales growth of a firm as a measure of aggressiveness.
of the firm. Ideally one should use production level data to measure aggressive product market behavior of the firm. The theoretical model uses $q$, the output produced as a measure of aggressiveness. But the lack of reliable plant level production data forces us to use industry adjusted sales growth of a firm as a measure of aggressiveness of the firm. This measure of aggressiveness of the firm have been used in several recent published papers like Campello (2005).

This paper contributes to three different strands of the literature. First it documents how the financial constrained firms are different in terms of output market behavior. It contributes to the growing field of literature of how and why financially constrained firms are different. We look at the difference in the firm behavior in terms of output market. Industry adjusted sales growth is considered as a good proxy for the aggressiveness of the firm in the output market. We find that the financially constrained firms are more aggressive in terms of industry adjusted sales growth. A second strand of literature that this paper deals with is the literature of managerial compensation. We provide an alternative explanation of product market aggressiveness based on managerial compensation. This paper is the first paper to our knowledge to test if the output market strategies can be explained by the executive compensation structure. This paper shows empirically that short term pay-off, managerial stock holding, change in the value of stock options, total compensation, change in the value of stock holdings can explain the output market behavior like industry adjusted sales growth. The third strand of literature deals with the difference in managerial compensation between the constrained and unconstrained firms. In this paper, we show that the financially constrained firms have a different executive compensation package which leads them to behave differently. The more financially constrained firms’ managers have higher percentage of bonus in total current compensation, have higher stock ownership by the manager, have higher change stock options and have higher total compensation. To my knowledge there is only one other paper which deals with the executive compensation of the financially constrained firms. The paper by Wang (2005) shows that the CEO pay performance sensitivities are higher for the financially constrained firms where performance is measured in terms of stock returns. She also shows that the total compensation packages of the CEOs of the constrained firms are higher than the unconstrained firms. We decompose the total CEO component into three components, the flow component, the change in value of stock holding and the change in the value of stock options. We develop a variable called short run bonus which is the ratio of bonus to total current compensation (given by tcc in ExecuComp database). We show in this paper that the short run bonuses of the CEOs are more for the more constrained firms.
Moreover, the equity based compensation of the CEO is higher for the more constrained firms. More stock holding, higher ratio of bonus in total current compensation, more total compensation for more financially constrained firm may be reasons for aggressive behavior of the more financially constrained firms in the output market.

4.2 A Theoretical Model

In this section, we build a theoretical model linking the financial constraints with managerial compensation and output markets. Consider a duopoly with firm 2 being more financially constrained than firm 1.

4.2.1 Definition of Financial Constraint

If a firm is financially unconstrained, the cost of internal capital and the cost of external capital should be the same. Any wedge between the cost of internal capital and the cost of external capital is a measure of the degree of financial constraint. The cost of capital of firm 1 is \( r \) whereas the cost of capital of firm 2 is \( r+d \), where \( d \) is the extra cost of capital the more financially constrained firm, firm 2 faces. So \( d \) can be regarded as the degree of financial constraint. The higher is the degree of financial constraint, the higher is the value of parameter \( d \). In this model, we shall investigate how short term pay and firm output responds to changes in the degree of financial constraint, i.e., changes with respect to the value of parameter \( d \).

4.2.2 The Two Stage Game

In this section, the manager of a firm \( i \) treats the wage contract as exogenous. The wage contract is given by

\[
w_i = \alpha_i + \beta_i V_i^{1/3}, \quad i = 1, 2.
\]

(1)

\( \alpha_i \) and \( \beta_i \) are exogenous to the manager’s decision making process. \( V_i \) is the equity value of the firm.

The Set-Up

There are two firms who engage in a Cournot duopoly game to maximize their values. The manager of each firm chooses her effort and firm output.

In the first stage, the manager chooses her effort. Effort is unobservable to the equity holders and debt holders. Only the manager knows how much
effort she puts in. The equity holders design a contract to ensure that the
interest of the manager is aligned with that of equity holders. This is done
in order to tackle the agency problem between the manager and the equity
holders. The managerial wage is composed of two parts. The first compo-
nent is $\alpha_i$ which is the fixed component of managerial wage. The second
component is the variable compensation of the manager. The variable wage
of the manager depends on the value of the firm, $\beta_i V_i^{1/3}$. This compensation
structure is reasonable because in real world, manager’s compensation has
a fixed component which consists of the salary and a variable component,
which consists of bonus, perks and options granted. This variable component
depends on the performance of the firm. If the value of the firm increases,
the variable component of the compensation increases.

The manager decides on two things: How much effort to put and how
much output to produce. In the first stage, the manager maximizes her
utility by choosing her effort. The managerial utility is given by

$$U_i = w_i - \frac{e_i^2}{2}, i = 1, 2. \quad (2)$$

As manager’s variable wage depends on the value of the firm, the manager
has the incentive to maximize the value of the firm by putting in more effort.
But putting in more effort is a disutility for the manager which is captured
by the second term of equation 2.

The inverse demand function the firm faces is assumed to be

$$p_i = \theta + e_i + z_i - q_i - \lambda q_j \quad (3)$$

where $\lambda$ is the degree of product differentiation. $z$ is a random variable
representing the state of nature. Without loss of generality, we assume that $z$
has a uniform distribution, following Chen (2005). This is done for numerical
simplicity. Further assumptions on the support of density of the distribution
are given by the following

$$\begin{cases} z \in [z, \bar{z}] \\ \tilde{z} = -\bar{z} \\ f(z) = \frac{1}{\bar{z}-z} = \frac{1}{\bar{z}}. \end{cases} \quad (4)$$

If the manager puts more effort, the price increases leading to an increase
in revenue. If the state of the nature improves, the firm can charge a higher
price and hence increase revenue. The equity holders, the debt holders and
the rest of the world cannot distinguish between $e_i$ and $z_i$. Ideally, the
manager should cater to the equity holders of the firm. If there is a fall in
demand, the outside world cannot know if the manager did not put in enough

74
effort leading to a fall in demand or if the state of nature was bad leading to a decrease in demand. This creates an opportunity for the manager to act in her own interest which is the reason for the existence of a principle agent problem between the manager and the equity holders of the firm. The compensation structure of the manager is aligned with the equity value of the firm in order to mitigate the agency problem between the equity holders and the manager. The equity value of the firm is the value to the equity holders which is the value of the residual claims of the firm. The compensation structure of the manager consists of a variable component which depends on the value to the equity holders.

In the second stage, the manager of the firm chooses output to maximize the equity value of the firm. The firm raises money from the debt holder to fund its project. The debt amount is used to produce output $q$.

$$D = cq$$

where $c$ is the constant marginal cost of production, $q$ is the output produced and $D$ is the amount of debt. We endogenise the amount of debt by linking it to the amount of output produced.

The firm can forecast perfectly the switching state of nature, $\hat{z}$ which is defined by the following identity (in case of duopoly):

$$(1 + r)D = R(q_i, q_j, \hat{z}).$$

Switching state of nature, $\hat{z}$ is defined as that state of nature at which the revenue of the firm is just enough to payoff the debt and interest on debt. $r$ is the rate of interest on debt. For firm 1:

$$(1 + r)D = R_1(q_i, q_j, \hat{z}),$$

$$(1 + r)cq_1 = [\theta + e_1 + \hat{z}_1 - q_1 - \lambda q_2]q_1,$$

$$\hat{z}_1 = -[\theta + e_1 - (1 + r)c - q_1 - \lambda q_2]. \quad (4a)$$

For firm 2,

$$(1 + r + d)D = R_2(q_i, q_j, \hat{z}),$$

$$(1 + r + d)cq_2 = [\theta + e_2 + \hat{z}_2 - q_2 - \lambda q_1]q_2,$$

$$\hat{z}_2 = -[\theta + e_2 - (1 + r + d)c - q_2 - \lambda q_1]. \quad (4b)$$

Revenue of the firm depends on the state of nature as the demand is uncertain. If the firm has perfect foresight, this switching state of nature, $\hat{z}$, can be treated as endogenous by the firm.
This game is solved by backward induction. In the second stage, the manager of a firm engages in Cournot duopoly game with the other firm to maximize the value of her firm and obtain the optimal value of q in terms of managerial effort. In the first stage, the managers of each firm choose their efforts simultaneously to maximize their utilities. Optimal efforts are obtained in terms of wage contract parameters $\alpha_i, \beta_i$.

The Second Stage

Problem of the manager in stage 2 is to maximize equity value of the firm.

For firm 1;

$$\max_{q_1} V_1 = \int_{\hat{z}_1}^{\bar{z}} \left[ \theta + e_1 + \hat{z}_1 - q_1 - \lambda q_2 \right] q_1 - (1 + r) c q_1 \frac{1}{2z} dz. \quad (5a)$$

For firm 2;

$$\max_{q_2} V_2 = \int_{\hat{z}_2}^{\bar{z}} \left[ \theta + e_2 + \hat{z}_2 - q_2 - \lambda q_1 \right] q_2 - (1 + r + d) c q_2 \frac{1}{2z} dz. \quad (5b)$$

It can be shown that the maximized value of the firms are

$$V_1^* = \frac{(q_1^*)^3}{z} \quad (6a)$$

$$V_2^* = \frac{(q_2^*)^3}{z} \quad (6b)$$

where

$$q_1^* = \frac{\bar{z} + \theta + e_1 - (1 + r)c - \frac{\lambda}{3} [\bar{z} + \theta + e_2 - (1 + r + d)c]}{(3 - \frac{\lambda^2}{3})} \quad (7a)$$

and

$$q_2^* = \frac{\bar{z} + \theta + e_2 - (1 + r + d)c - \frac{\lambda}{3} [\bar{z} + \theta + e_1 - (1 + r)c]}{(3 - \frac{\lambda^2}{3})} \quad (7b)$$

The First Stage

In stage 1, the manager of each firm chooses her effort simultaneously with the manager of the other firm to maximize her own utility:

$$\max_{e_i} U_i = w_i - \frac{e_i^2}{2}, i = 1, 2,$$
\[
\max_{e_i} U_i = \alpha_i + \beta_i (V_i^*)^{\frac{1}{3}} - \frac{e_i^2}{2}, i = 1, 2, \tag{8}
\]

where \( (V_i^*)^{\frac{1}{3}} \) is given by equation 6. Solving for \( e_i \), we get,

\[
e_i^* = \frac{\beta_i}{(\frac{1}{3}) (3 - \frac{\lambda_2^2}{3})}, i = 1, 2. \tag{8a}
\]

We assume

\[
3 - \frac{\lambda_2^2}{3} > 0. \tag{A1}
\]

Justification for this assumption is that in reality, the pay performance sensitivity \( \beta_i \) is always positive. It does not make sense to have \( \beta_i \) as negative. If \( 3 - \frac{\lambda_2^2}{3} < 0 \), then \( \beta < 0 \) for effort to be positive.

Individual rationality constraint suggests that the utility of an individual manager must be greater than or equal to the reservation utility prevailing in the market. So

\[
U_i \geq \overline{U}.
\]

We assume that the labor market for managers is perfectly competitive which implies that a manager receives only the reservation utility. As a result,

\[
\overline{U} = \alpha_i + \beta_i V_i^{1/3} - \frac{e_i^2}{2}, i = 1, 2.
\]

Putting value of \( \alpha_i \) in the wage equation, we get,

\[
w_i = \overline{U} + \frac{e_i^2}{2}, i = 1, 2.
\]

Putting the value of \( e_i \) from equation (8a), the compensation contract of the manager is given by

\[
w_i = \overline{U} + \frac{\beta_i^2}{2(\frac{1}{3})(3 - \frac{\lambda_2^2}{3})^2}, i = 1, 2. \tag{9}
\]

The equilibrium outputs are given by

\[
q_1^* = \frac{\overline{Z} + \theta + \frac{\beta_1}{(\frac{1}{3})(3 - \frac{\lambda_2^2}{3})} - (1 + r)c - \frac{\lambda_1}{3}[\overline{Z} + \theta + \frac{\beta_2}{(\frac{1}{3})(3 - \frac{\lambda_2^2}{3})} - (1 + r + d)c]}{(3 - \frac{\lambda_2^2}{3})}\tag{10a}
\]
and
\[ q_2^* = \frac{\bar{z} + \theta + \frac{\beta_2}{(3-\lambda_2^2)} - (1 + r + d)c - \frac{\lambda_2}{3}[\bar{z} + \theta + \frac{\beta_1}{(3-\lambda_2^2)^2} - (1 + r)c]}{(3 - \lambda_2^2)}. \]

(10b)

\( \beta_i, i = 1, 2, \) are exogenous. Equilibrium outputs \( q_1^* \) and \( q_2^* \) depend on the values of the parameters \( \beta_i, i = 1, 2. \) \( \beta_i, i = 1, 2, \) can be interpreted as the percentage of short term variable compensation in total current compensation. \( \beta_i, i = 1, 2, \) can also be interpreted as the pay performance sensitivity of firm \( i. \)

**Proposition 1**

The more is the percentage of short term variable compensation in total current compensation of the manager of a firm, the more aggressive is the firm in the output market compared to its rival.

In Appendix A1, we show
\[ \frac{dq_2^*}{d\beta_2} - \frac{dq_1^*}{d\beta_2} > 0 \]

**Intuition:** The compensation contract of the manager of firm 2 is given by equation (1), \( w_i = \alpha_i + \beta_i V_i^{1/3}. \) Equity value of a firm depends on its output as given by equation (6). An increase in the value of the parameter \( \beta_2 \) implies more incentive to the manager of firm 2 to increase the value of firm 2 and hence increase firm 2 outputs. As this set-up is a Cournot duopoly, the output of rival firm, firm 1, decreases when firm 2 increases its output.

This illustrates why the compensation contract should play an active role in the output market strategies. The variable portion of managerial compensation is linked with the value of the firm, which crucially depends on the output of the firm. So an increase in the pay performance sensitivity, captured by the parameter \( \beta, \) (which can also be interpreted as the percentage of variable pay in total salary) provides incentive to the manager to act more aggressively in the output markets.

Up to now, we have assumed that the compensation contract is exogenously given and the manager cannot affect the compensation structure. Specifically, we have assumed that the manager’s decisions cannot affect the values of \( \alpha \) and \( \beta. \) This is too simplistic an assumption. The manager decides what should be the output which, in turn, determines the equity value of the firm. The equity holder maximizes the net equity value of the firm by choosing values of \( \alpha \) and \( \beta. \) Her maximization problem is given by
If the manager is rational, she can figure out that her output decision determines what should be the values of $\alpha$ and $\beta$ and hence the compensation contract is not exogenously given. Rather, we should have a three stage game which is described and solved below.

4.2.3 The Three Stage Game

The Set-Up

There are two firms who engage in a Cournot duopoly game to maximize their values. The manager of each firm chooses her effort and firm output. The equity holders choose optimal contracts for the manager. Let us explain the setup of the model in details.

There are three stages of this duopoly game. In the first stage, the equity holders choose the optimal contract $\alpha_i, \beta_i$ in order to maximize the net value of their respective firms.

$$\max_{\alpha_i, \beta_i} V_{net}^i = V_i - w_i$$

In the second stage, the manager of each firm chooses her effort to maximize her utility. The managerial utility is given by equation (2). Effort is unobservable to the equity holders and debt holders. The compensation contract is same as above and is given by equation (1).

In the third stage, the manager of each firm chooses output to maximize the equity value of her firm. The equity value of the firm is exactly the same as the two stage game.

This game is solved by backward induction. In the third stage, the manager of a firm engage in Cournot duopoly with the other firm to maximize the value of her firm and obtain the optimal value of $q$ in terms of managerial effort. In the second stage, the manager of the firm chooses her effort simultaneously with the manager of the other firm in Cournot game, to maximize her utility. Optimal efforts are obtained in terms of contract parameters $\alpha_i, \beta_i$. In the first stage, knowing the amount of effort of the manager in terms of the contract parameters, the equity holders of each firm choose the optimal managerial contracts to maximize the net value of the firms.

The Third Stage

This is same as stage 2 of the two stage game. Problem of the manager in stage 3 is to maximize the equity value of the firm. The maximization
The maximized value of the firms are

\[ V_1^* = \frac{(q_1^*)^3}{\xi} \]  \hspace{1cm} (6a)

\[ V_2^* = \frac{(q_2^*)^3}{\xi} \]  \hspace{1cm} (6b)

where

\[ q_1^* = \frac{\xi + \theta + \epsilon_1 - (1 + r)e - \frac{\lambda}{3}[\xi + \theta + \epsilon_2 - (1 + r + d)e]c}{(3 - \frac{\lambda^2}{3})} \]  \hspace{1cm} (7a)

and

\[ q_2^* = \frac{\xi + \theta + \epsilon_2 - (1 + r + d)e - \frac{\lambda}{3}[\xi + \theta + \epsilon_1 - (1 + r)e]c}{(3 - \frac{\lambda^2}{3})} \]  \hspace{1cm} (7b)

The Second Stage

This is same as the stage 1 of the two stage game. In stage 2, the manager of a firm chooses effort simultaneously with the manager of the other firm in order to maximize her own utility. The maximization problem is given by equation (8) and the maximized efforts are given by equation (8a). The compensation contract is given by equation (9).

The First Stage

Given the wage contract in terms of the contract parameters \( \alpha \) and \( \beta \), the equity holders of the company maximize the net value of the firm by choosing the optimal compensation contract. The equity holders choose the values of \( \alpha_i, \beta_i \) in order to maximize the net value of their respective firms.

\[ \max_{\alpha_i, \beta_i} V_{i}^{\text{net}} = V_i - w_i. \]

For firm 1, the optimization problem of the equity holders is

\[
\max_{\alpha_1, \beta_1} V_{1}^{\text{net}} = \frac{[\xi + \theta + \frac{\beta_1}{(\xi)^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})} - (1 + r)c - \frac{\lambda}{3}[\xi + \theta + \frac{\beta_2}{(\xi)^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})} - (1 + r + d)c]]^3}{\xi(3 - \frac{\lambda^2}{3})^3} - \frac{\beta_1^2}{2(\xi)^{\frac{5}{3}}(3 - \frac{\lambda^2}{3})^2} - U. \]  \hspace{1cm} (11a)
For firm 2, the optimization problem of the equity holders is

\[
\max_{\alpha_2, \beta_2} V_{net}^2 = \frac{[\tau + \theta + \frac{\beta_2}{(\tau)^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})}] - (1 + r + d)c - \frac{\lambda}{3}[\tau + \theta + \frac{\beta_1}{(\tau)^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})}] - (1 + r)c]^{\frac{3}{2}}}{\tau (3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}
- \frac{\beta_2^2}{2(\tau)^{\frac{1}{2}}(3 - \frac{\lambda^2}{3})^2} - U.
\]  

(11b)

The first order conditions with respect to \( \beta_1 \) are

\[
3[\tau + \theta + \frac{\beta_1}{(\tau)^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})}] - (1 + r)c - \frac{\lambda}{3}[\tau + \theta + \frac{\beta_2}{(\tau)^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})}] - (1 + r + d)c]^2
= (3 - \frac{\lambda^2}{3})^2 \frac{\beta_1}{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}.
\]

(12a)

The first order conditions with respect to \( \beta_2 \) are

\[
3[\tau + \theta + \frac{\beta_2}{(\tau)^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})}] - (1 + r + d)c - \frac{\lambda}{3}[\tau + \theta + \frac{\beta_1}{(\tau)^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})}] - (1 + r)c]^2
= (3 - \frac{\lambda^2}{3})^2 \frac{\beta_2}{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}.
\]

(12b)

The equilibrium values of \( \beta_1 \) and \( \beta_2 \) satisfy equations (12a) and (12b) simultaneously. Our goal is to find out how an increase in the cost of capital of financially constrained firm, firm 2, affect the equilibrium values of \( \beta_1 \) and \( \beta_2 \). As defined before, financially constrained firm has a higher cost of capital. Firm 2 in our model is financially constrained as it has higher cost of capital compared to firm 1, where the difference in cost of capital is given by \( d \). So \( d \) can be regarded as the parameter capturing the degree of financial constraint. We do not attempt to solve for \( \beta_1 \) and \( \beta_2 \) but find the values of \( \frac{d \beta_1}{dd} \) and \( \frac{d \beta_2}{dd} \). Differentiating equations (10a) and (10b) with respect to \( d \) and applying the first order conditions (10a) and (10b), we get,

\[
[1 - \frac{(3 - \frac{\lambda^2}{3})^2 \frac{\beta_1^*}{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}}{2 \sqrt{3} \beta_1^*^{\frac{1}{2}}} \frac{d \beta_1^*}{dd}] - \frac{\lambda}{3} \frac{d \beta_2^*}{dd} = -\frac{\lambda}{3} \frac{c \beta_1^*}{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}
\]

(13a)

\[
-\frac{\lambda}{3} \frac{d \beta_1^*}{dd} + [1 - \frac{(3 - \frac{\lambda^2}{3})^2 \frac{\beta_2^*}{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}}{2 \sqrt{3} \beta_2^*^{\frac{1}{2}}} \frac{d \beta_2^*}{dd}] = c \beta_2^* (3 - \frac{\lambda^2}{3}).
\]

(13b)
From (11a) and (11b), we solve for $\frac{d\beta_1^*}{dd}$ and $\frac{d\beta_2^*}{dd}$ which are given by

\begin{align}
\frac{d\beta_1^*}{dd} &= \frac{\lambda c \pi^3 (3 - \frac{\lambda^2}{\pi})^3}{2\sqrt{3}\beta_1^{1/2}D} \\
\frac{d\beta_2^*}{dd} &= \frac{c \pi^3 (3 - \frac{\lambda^2}{\pi})[1 - \frac{(3 - \frac{\lambda^2}{\pi})^2 \pi^4}{2\sqrt{3}\beta_1^{1/2}}] - \frac{\lambda^2}{9}}{D}
\end{align}

(14a)

(14b)

where

$$D = \left[1 - \frac{(3 - \frac{\lambda^2}{\pi})^2 \pi^4}{2\sqrt{3}\beta_1^{1/2}} \right]\left[1 - \frac{(3 - \frac{\lambda^2}{\pi})^2 \pi^4}{2\sqrt{3}\beta_2^{1/2}} \right] - \frac{\lambda^2}{9}$$

**Proposition 2**

When the degree of product differentiation is sufficiently high, the Pay Performance sensitivity of a financially constrained firm increases with the degree of financial constraint. Further, the difference between the Pay Performance sensitivity of a more financially constrained firm and less financially constrained firm increases with the degree of financial constraint.

**Proof:** This theorem is equivalent to showing that

$$\frac{d\beta_2^*}{dd} > 0$$

$$\frac{d\beta_2^*}{dd} - \frac{d\beta_1^*}{dd} > 0$$

for sufficiently high values of $\lambda$. Proof is in Appendix A2. We show in Appendix A2 that sufficient condition for $\frac{d\beta_2^*}{dd} > 0$ and $\frac{d\beta_2^*}{dd} - \frac{d\beta_1^*}{dd} > 0$ to hold is $\frac{3}{2} < \lambda$.

Intuition behind this theorem is that a financially constrained firm is one with a higher cost of capital, leading to higher marginal cost of production. Higher marginal cost of production reduces firm output which in turn decreases firm value. The compensation structure of a financially constrained firm has to be designed in such a manner so as to induce the manager to put more effort in order to offset higher cost of production. Financially constrained firm has to provide higher incentive for the manager to increase the value of the firm in order to compensate for the higher cost of capital. As the degree of financial constraint increases, the pay performance sensitivity should increase in order to induce the manager to put in more effort.
and counter the effect of higher cost of capital. Hence, the pay performance sensitivity increases due to increase in the degree of financial constraint.

**Proposition 3**

*Financially constrained firms are more aggressive in the product market in concentrated industries.*

Proof: See Appendix A3.

When \( \frac{3}{2} < \lambda \),

\[
\begin{align*}
\frac{dq_2}{dd} &> 0 \\
\frac{dq_1}{dd} &< 0 \\
\frac{dq_2}{dd} - \frac{dq_1}{dd} &> 0
\end{align*}
\]

For the financially constrained firm, one unit increase in the degree of financial constraint has two opposing effects. The first effect is that the marginal cost of production increases by \( c \), thereby reducing the output produced, \( q_2 \). The second effect is that the pay performance sensitivity \( \beta_2 \) also increases in order to induce the manager to put in more effort thereby increasing output \( q_2 \). The first effect reduces output \( q_2 \) whereas the second effect increases \( q_2 \). If the degree of product substitution is sufficiently high, a firm can act more aggressively by increasing output as the consumers cannot switch to rival’s product. The second effect of increase in output dominates the first effect of reduction in output.

**4.3 Hypothesis Development**

In this section, we develop three hypotheses corresponding to the three propositions. It should be noted that we are not testing the model per se. The theoretical model above provides some theoretical justification of my empirical results which follows below. We argue that the empirical results documented below are at least theoretically conceivable from the theoretical model above.
4.3.1 Aggressiveness In the Product Markets

Hypothesis 1

*Financially constrained firms are more aggressive in the output markets compared to financially unconstrained firms.*

This hypothesis follows directly from Proposition 3. This hypothesis answers the first question we pose in this paper. When faced with an investment opportunity, a firm raises capital from the external market. If the firm is more financially constrained, the cost of raising external capital is higher. The financially constrained firms should behave more aggressively in the product market in order to make up for their higher cost of capital. This is the traditional argument as to why financially constrained firms should behave more aggressively.

We move to our second question. What are the different reasons for aggressiveness in the product market? Our hypothesis is that managerial compensation is one of the causes for difference in the product market behavior.

4.3.2 Product Market Strategies and Managerial Compensation

In this section, we examine how various components of managerial compensation affect product market strategies.

Hypothesis 2

*Aggressiveness of a firm in the output market depends positively on managerial compensation.*

This hypothesis follows directly from Proposition 1. There can be endogeneity problem while testing this hypothesis. In the methodology section, we specify how we tackle this problem. This hypothesis answers our second question. Does managerial compensation affect output market strategies?

We examine how the different components of managerial compensation affect product market aggressiveness.

Hypothesis 2a

*The more is the percentage of short term bonus in the total current compensation; the more aggressive are the product market strategies.*

The manager’s compensation structure can play an important role in the decision making process of the manager. If the percentage of short term
bonus is more in the total current compensation, the manager shall try to boost the short term performance by undertaking aggressive strategies in the output market. Hence, the managerial short term payments lead to aggressive output market strategies thereby shifting wealth from debt holders to equity holders.

Hypothesis 2b

Aggressiveness in the output market is positively dependent on the number of shares owned by the manager.

The downside risk of the firm is covered by limited liability and the upward gains go to the equity holders. Undertaking risky strategies essentially means wealth transfer from the debt holders to equity holders. The manager also own shares in the firms. Hence, the more is the ownership of the manager, the more aggressively manager acts in the output markets.

4.3.3 Managerial Compensation And Financial Constraints

In this section, we answer the third question of the paper. Do those components of managerial compensation, which explain product market strategies, differ between the financially constrained and constrained firms?

Hypothesis 3a

Managerial compensation of more financially constrained firms is higher than that of less financially constrained firms.

Hypothesis 3b

Short term compensation for managers is higher for more financially constrained firms compared to less financially constrained firms.

This hypothesis follows from Proposition 2. This question answers the third question we raise in the introduction. Does managerial compensation vary between more financially constrained and less financially constrained firms? If we find that the managerial compensation are different among more financially constrained firms and less financially constrained firms, then we can attribute the difference in the product market strategies among the more financially constrained and less financially constrained firms to the difference in managerial compensation. Intuitively, a financially constrained firm has higher cost of production. In order to compensate for the higher cost of
production, the manager has to provide more effort. But the manager shall put in more effort only when the compensation structure of the manager rewards the manager for better performance. Wang (2005) shows that the pay performance sensitivity is higher for financially constrained firms. She argues that by choosing to work for a more financially constrained firm, the manager is taking more human capital risk and has to be compensated adequately. We show in the results section that the total compensation, the change in the value of stock options, the change in the value of the stock holdings, the bonus as a percentage of total compensation and the shares owned by the manager are higher for more financially constrained firms compared to less financially constrained firms. We argue that these differences in compensation among the two classes of firms may result in differences in the product market strategies across more financially constrained firms and less financially constrained firms.

4.4 Data and Methodology

In this section, we describe the data we use. The data sample includes all US firms listed on NYSE, AMEX or NASDAQ that are present in CRSP and COMPUSTAT for the period 1993 to 2007. The firm characteristics data is from COMPUSTAT. The executive compensation data is from ExecuComp. We use CRSP database to calculate firm’s return. The risk of the firm is calculated as the preceding sixty month variance of return. We define industry by the NAICS industry code. We exclude financial companies (SIC 6000-6999) and utility companies (SIC 4900 to 4999) to avoid the possible effects of regulations prevalent in these type of industries. We also exclude any firm with assets less than 10 million dollars.

4.4.1 Data Definition

See Appendix A4.

4.4.2 Criteria For Financial Constraint

We shall use two measures of financial constraint as has been used by the literature.

Payout Ratio

Firms have been classified based on Payout ratio in the seminal work of Fazzard, Hubbard and Peterson (1998) and subsequently by many oth-
ers. The intuition is that the firms who pay dividends are not financially constrained whereas the cash constrained firms are less likely to pay out dividends. The payout ratio is defined as the total dividends paid by the firm normalized by operating income (Compustat data items data 19 plus data21 divided by data178). Firms are classified in terms of the payout ratios and the top 30% of the firms are classified as financially unconstrained and the bottom 30% of the firms are considered financially constrained.

**S&P Long Term Credit Rating**

Data 280 of COMPUSTAT gives us the historical long term domestic issuer credit rating. Firms with credit rating of BBB- or better (data280 less than or equal to 12) are termed as unconstrained and all other firms are termed as constrained (data280 greater than or equal to 13). Whited (1992), Gilchrist and Himmelberg (1995) and Malmendier and Tate (2005) use this criteria to classify firms as constrained or unconstrained.

**4.4.3 Methodology**

For testing Hypothesis 1, we split the sample of firms into financially constrained and financially unconstrained based on the two above specified criteria. We conduct a $t$ test to test the difference in the means of sales growth and industry adjusted sales growth across financially unconstrained and financially constrained firms. As we have more than 2000 observations, $t$ test should serve as a good test for testing the difference in the means of the two groups. The $t$ test results reported here assumes inequality of variance between the two samples. $T$ test based on the assumption of equality of variance also provides similar results. $T$ statistic is the mean sales growth of financially unconstrained minus the mean sales growth of financially constrained firm divided by standard error and adjusted for degrees of freedom.

Descriptive statistics is not a comprehensive way for testing any hypothesis as there are no controls for relevant explanatory variables. One needs a regression framework for reliable inference. Regression is required to be run to test if the industry adjusted sales growth is higher for financially constrained firms. Following Opler and Titman(1994), Campello(2003), Campello and Fluck(2004), our baseline regression of industry adjusted sales growth is given by
For testing Hypothesis 1, we use a dummy variable for financial constraint and control for all the other explanatory variables given by the baseline regression. The following regression is run.

\[
SalesGrowth_{i,t} = c + \beta_0 SalesGrowth_{i,t-1} + \beta_1 Size_{i,t} \\
+ \beta_2 Profitability_{i,t} + \beta_3 Profitability_{i,t-1} + \beta_4 Investment_{i,t} \\
+ \beta_5 Investment_{i,t-1} + \beta_6 Leverage_{i,t} + \beta_7 Leverage_{i,t-1} \\
+ \beta_8 fcDummy + \epsilon_{i,t} .
\] (R1)

Two measures of financial constraint, one based on dividend payment and the other based on long run credit rating of the firm are used. If industry adjusted sales growth is higher for financially constrained firms, i.e., if hypothesis 1 is true, the coefficient \(\beta_8\) on the financial constraint dummy should be positive and significant.

To test for hypothesis 2, we use base line sales growth regression as defined by equation R1 and add components of managerial compensation into the regression.

\[
SalesGrowth_{i,t} = c + \beta_0 SalesGrowth_{i,t-1} + \beta_1 Size_{i,t} \\
+ \beta_2 Profitability_{i,t} + \beta_3 Profitability_{i,t-1} + \beta_4 Investment_{i,t} \\
+ \beta_5 Investment_{i,t-1} + \beta_6 Leverage_{i,t} + \beta_7 Leverage_{i,t-1} \\
+ \beta_8 ManagerialCompensation + \epsilon_{i,t} 
\] (R2)

The different managerial compensations tested are short run bonus, stocks owned by the CEO, total compensation, flow compensation, the change in the value of stock holding and the change in the value of stock options. If hypothesis 2 is correct, the coefficient \(\beta_8\) should be positive and significant.

One can argue that there can be endogeneity problem. The CEO compensation may be dependent on sales growth. To control for endogeneity, two common methods are used. First is the instrumental variable approach where the instrument for CEO compensation is lag of CEO compensation. In the
second approach, the industry adjusted sales growth in year t is regressed on previous year’s CEO compensation. It is unlikely that CEO compensation last year shall depend on industry adjusted sales growth this year. It can be reliably inferred that when industry sales growth in year t is regressed on CEO compensation in year t-1, there is no endogeneity problem in the model. To be on the safe side, CEO compensation is regressed on industry adjusted sales growth. It is shown that industry adjusted sales growth does not affect CEO compensation. This should clear any doubt about CEO compensation being endogenous in the regression of industry adjusted sales growth on CEO compensation.

Following Aggarwal and Samwick (1999), CEO compensation is composed of three components: flow compensation, the change in the value of stock holding and the change in the value of stock options. CEO compensation regression is famously called the pay performance sensitivity regression. The baseline regression, from Aggrawal and Samwick (1999) is

\[
CEO_{Compensation_{i,t}} = c + \beta_1 Ret_{i,t} + \beta_2 Ret_{i,t} \ast Tenure \\
+ \beta_3 \ast variance + \beta_4 \ast size + \epsilon_{i,t}
\]  
(R4)

Ret is the total dollar return to the share holder. Variance is the variance of the preceding 5 year stock return of the firm. Variance captures the risk of the stock. Tenure is proxy for CEO’s ability. Size is defined as log of assets, data6. Size captures the size effect, which is common in CEO compensation regression. Adding industry adjusted sales growth in the baseline regression, we have

\[
CEO_{Compensation_{i,t}} = c + \beta_1 Ret_{i,t} + \beta_2 Ret_{i,t} \ast Tenure \\
+ \beta_3 \ast variance + \beta_4 \ast size \\
+ \beta_5 \ast industryadjustedsalesgrowth + \epsilon_{i,t}
\]  
(R5)

Our goal is to check if the coefficient \( \beta_5 \) is statistically significant. If \( \beta_5 \) is not significant statistically, we do not have an endogeneity problem.

For testing Hypothesis 3, different components of managerial compensation are regressed on the financial constraint dummy. The three components of CEO compensation: flow compensation, the change in the value of stock holding and the change in the value of stock options as well as total compensation, short run bonus and stock holding of the manager are regressed
on the financial constraint dummy after controlling for all other explanatory variables given by the baseline regression R4.

\[
CEOCompensation_{i,t} = c + \beta_1 Ret_{i,t} + \beta_2 Ret_{i,t} \times Tenure \\
+ \beta_3 \times variance + \beta_4 \times size \\
+ \beta_5 \times fcDummy + \epsilon_{i,t} 
\] (R6)

fcDummy is a dummy variable for financial constraint. If hypothesis 3 is true, the coefficient on financial constraint dummy should be positive and significant.

The dataset is an unbalanced panel data of observations. Firm fixed effect and time effect are incorporated. When there is firm effect, running OLS regression shall bias the standard errors of estimates downwards. As a result, the t statistics shall be biased upwards and one shall falsely infer that a particular variable is significant when in fact it is not. Fixed effect panel data model is used to control for the firm fixed effect. The standard errors of the variable estimates are correctly estimated. We also use time dummies to control for time effect. To control for heteroskedasticity, we use heteroskedastic adjusted standard errors. To control for auto correlation, our dependent variable is not in levels. Sales growth is used as dependent variable instead of sales. Further, lag sales growth is included as an explanatory variable to control for any remaining autocorrelation.

4.5 Results

We test our first hypothesis by dividing the firms into financially constrained and unconstrained firms and estimating the means of industry adjusted sales growth. We use the standard t test to check if there is difference in the means of these variables between the two groups of firms.

Table 1 documents the means of sales growth and industry adjusted sales growth of the two groups of firms, grouped based on two criteria of financial constraint. Sales growth industry adjusted is the sales growth of the firm minus the median sales growth of the firms in the industry, where industry is defined by the three digit NAICS code. Every entry has three values. The first value is the value for the financially unconstrained firms and the second value is that of the financially constrained firm. We conduct a t test to test the difference in the means of sales growth across financially unconstrained
and financially constrained firms. The t test results reported here assumes inequality of variance between the two samples. T test based on the assumption of equality of variance also gives the similar results. The third value is the t statistic value of that t test. t statistic is the mean sales growth of financially unconstrained firm minus the mean sales growth of financially constrained firm divided by standard error and adjusted for degrees of freedom. For example, the first entry says that mean sales growth for financially unconstrained firms is 5.956 and mean sales growth for financially constrained firms is 21.269 where dividend payment is the financial constraint measure. The t value of the t test that tests the difference between the mean sales growth of unconstrained and constrained is -17.18 and it is significant at 1 percent significance level. We find that industry adjusted sales growth is higher for the financially constrained firms thus supporting hypothesis 1 that the financially constrained firms are more aggressive in the output markets.

Descriptive statistics is not very convincing as compared to regression set-up. We use base line regression R1 and use dummy variable to test for hypothesis 1 by running regression given by R2. The results are documented in table 2.

| Table 2 |

The coefficient on the dummy variable is positive and statistically significant thereby providing empirical evidence supporting hypothesis 1. So we can infer that the financially constrained firms are more aggressive in product markets. This conclusion supports hypothesis 1 and answers the first question we ask in the paper.

Having provided empirical evidence that the financially constrained firms are aggressive in the product market, we move to our second question. What are the different reasons for aggressiveness in the product market? To test hypothesis 2, we use baseline sales growth regression and add components of managerial compensation into the regression. We run regression R3 and report the results in table 3.

| Table 3 |

Table 3 reports the regression of industry adjusted sales growth on managerial compensation. For tackling endogeneity the instrumental variable approach is used. The instruments for CEO compensation are lag values of CEO compensation. The dependent variable is industry adjusted sales growth. The first column of table 3 reports the baseline regression, given by equation R1. In column 5, we include total compensation of the CEO. The coefficient of total compensation is positive and significant supporting
hypothesis 2. We break up the total CEO compensation into three components, flow compensation, change in the value of stock holding and the change in the value of stock options. Flow compensation coefficient is almost zero and statistically insignificant. Change in the value of stock holding and the change in the value of stock options both have positive and statistically significant coefficient. Coefficient of change in the value of stock holding and the change in the value of stock options support hypothesis 2.

Hypothesis 2b suggests that the more is the short run bonus of the CEO in the total current compensation of the CEO, the more aggressive is the firm in the output market. Short run bonus by definition depends on the current performance of the firm. So if the bonus as a percentage of total current compensation is larger, then the CEO has more incentive to improve current performance of the firm and act more aggressively in the output market. In column 2, we find that coefficient of short run bonus is positive and significant providing evidence for hypothesis 2a. Stocks owned by the CEO can also positively determine the industry adjusted sales growth (hypothesis 2b). The intuition is that if the CEO improves the sales growth by acting aggressively, the market shall notice the improved product market performance of the firm and reward the firm. The investors shall buy that firm’s stock because the product market fundamentals of the stock are improving because of aggressiveness of the CEO in the output market. As a result, the stock price increases and the CEO is better off as she holds the company stock. Column 4 provides evidence for hypothesis 2b. Coefficient of lag stock owned by CEO is positive and significant. Table 3 supports hypothesis 2, 2a and 2b.

For tackling endogeneity we can also use lag values of managerial compensation in the regression. The dependent variable is industry adjusted sales growth.

Table 4

The first column of table 4 reports the baseline regression, given by equation R1. In column 5, we include total compensation of the last year. The coefficient of lag total compensation is positive and significant supporting hypothesis 2. We break up the total CEO compensation into three components, flow compensation, change in the value of stock holding and the change in the value of stock options. Flow compensation has a negative and statistically significant coefficient whereas change in the value of stock holding and the change in the value of stock options both have positive and statistically significant coefficients. Coefficient of change in the value of stock holding and the change in the value of stock options support hypothesis 2. Similarly, the
coefficients for lag short run bonus, short run bonus and lag stocks owned by the CEO are positive and statistically significant documenting empirical evidence supporting hypothesis 2.2a and 2b.

To ensure there is no problem of endogeneity, CEO compensation is regressed on industry adjusted sales growth as per regression equation R5. We report the results in table 5.

Table 5

We use all the six explanatory variables of previous regression R3 as dependent variables, namely total compensation, flow compensation, change in the value of stock holding, the change in the value of stock options, short run bonus and shares owned by CEO. Industry adjusted sales growth is the explanatory variable. We can see that industry adjusted sales growth coefficient is almost zero and statistically insignificant. So it can be reliably concluded that endogeneity is not an issue for regression R3 of table 3 and 4.

We have answered our second question. Some components of managerial compensation explain industry adjusted sales growth. These are total compensation, change in the value of stock holding, change in value of stock options, total current compensation, bonus as a percentage of total current compensation, number of shares owned by the CEO.

We move to the third question of the paper. Do the financially constrained firms differ from the constrained firms with respect to these above mentioned variables which affect industry adjusted sales growth? We test hypothesis 3a and 3b by running regression R6. We use all the six explanatory variables of previous regression R3 as dependent variables, namely total compensation, flow compensation, change in the value of stock holding, the change in the value of stock options, short run bonus and shares owned by CEO. Financial constraint dummy is the explanatory variable. We report the results in table 6 and table 7.

Table 6

Table 7

The coefficient of financial constraint dummy is positive and significant using both the measures of financial constraints providing empirical evidence for hypothesis 3a and 3b.

We have documented empirical evidence that financially constrained firms have higher industry adjusted sales growth. This answers the first question we post and supports hypothesis 1. We also report that total compensation,
the change in the value of stock holding, the change in the value of stock options, short run bonus and shares owned by CEO can explain industry adjusted sales growth. This provides positive answer for our second question and proves hypothesis 2,2a and 2b. Further, we show that total compensation, flow compensation, change in the value of stock holding, the change in the value of stock options, short run bonus and shares owned by CEO are higher for financially constrained firms. This is the reply to our third and final question and confirms hypothesis 3. Hence, one of the explanations why financially constrained firms behave more aggressively in the output market is that the financially constrained firms have higher managerial compensation.

4.6 Conclusion

It is demonstrated both empirically and theoretically that a financially constrained firm shall be more aggressive in the output market compared to a financially unconstrained firm. It is also documented that managerial compensation is an explanatory variable in explaining industry adjusted sales growth after controlling for all other explanatory variables. Further, managerial compensation is higher for a financially constrained firm giving managers incentives to exert more effort compared to financially unconstrained firm. Connecting these three observations, it is inferred that managerial compensation is an explanatory variable for product market aggressiveness of a financially constrained firms.

4.7 Appendix

4.7.1 Appendix A1

Proof of Proposition 1

\[ q_1^* = \frac{\bar{z} + \theta + \frac{\beta_1}{(\bar{z})^2 (3 - \frac{\lambda^2}{3})} - (1 + r)c - \frac{\lambda}{3} \left[ \bar{z} + \theta + \frac{\beta_2}{(\bar{z})^2 (3 - \frac{\lambda^2}{3})} - (1 + r + d)c \right]}{(3 - \frac{\lambda^2}{3})} \]  

(10a)

and

\[ q_2^* = \frac{\bar{z} + \theta + \frac{\beta_2}{(\bar{z})^2 (3 - \frac{\lambda^2}{3})} - (1 + r + d)c - \frac{\lambda}{3} \left[ \bar{z} + \theta + \frac{\beta_1}{(\bar{z})^2 (3 - \frac{\lambda^2}{3})} - (1 + r)c \right]}{(3 - \frac{\lambda^2}{3})} \]  

(10b)
\[
\frac{dq_1^*}{d\beta_1} = \frac{1}{\pi^2(3 - \frac{\lambda^2}{3})^2} > 0 \\
\frac{dq_2^*}{d\beta_1} = -\frac{\lambda}{3} \frac{1}{\pi^2(3 - \frac{\lambda^2}{3})^2} \\
\frac{dq_1^*}{d\beta_2} = -\frac{\lambda}{3} \frac{1}{\pi^2(3 - \frac{\lambda^2}{3})^2} \\
\frac{dq_2^*}{d\beta_2} = \frac{1}{\pi^2(3 - \frac{\lambda^2}{3})^2} > 0 \\
\frac{dq_2^*}{d\beta_2} - \frac{dq_1^*}{d\beta_2} = \frac{1}{\pi^2(3 - \frac{\lambda^2}{3})^2} (1 + \frac{\lambda}{3}) > 0 \\
\]
for all values of \(\lambda\) as long as assumption A1 holds \((3 - \frac{\lambda^2}{3}) > 0\)

### 4.7.2 Appendix A2

\[
\frac{d\beta_1^*}{dd} = \frac{\lambda c \pi^{1/2} (3 - \frac{\lambda^2}{3})^3}{2\sqrt{3}\beta_2^{3/2}} D \\
\frac{d\beta_2^*}{dd} = \frac{c \pi^{1/2} (3 - \frac{\lambda^2}{3}) [1 - \frac{(3 - \frac{\lambda^2}{3})^2 \pi^{3/2}}{2\sqrt{3}\beta_1^{3/2}}] - \frac{\lambda^2}{9}}{D} \\
\]
where
\[
D = [1 - \frac{(3 - \frac{\lambda^2}{3})^{2}\pi^{3/2}}{2\sqrt{3}\beta_1^{3/2}}] [1 - \frac{(3 - \frac{\lambda^2}{3})^{2}\pi^{3/2}}{2\sqrt{3}\beta_2^{3/2}}] - \frac{\lambda^2}{9} \\
\]

Using the FOC equation 10a and 10b, the maximized value of \(V_i\) can be written as
\[
V_i^* = \frac{\beta_i^*}{3^{1/2}}. \\
\]
Maximized net value of the firm is
\[
V_{i,\text{net}} = V_i^* - w_i = \frac{(\beta_i^*)^2}{3^{1/2}} - \frac{(\beta_i^*)^2}{2(\pi)^{3/2}(3 - \frac{\lambda^2}{3})^2} - U. \\
\]
$U$ is the reservation utility which is positive. This implies that

$$\frac{(\beta_i^*)^2}{3^2} > \frac{(\beta_i^*)^2}{2(\pi)^{\frac{3}{2}}(3 - \frac{\lambda^2}{3})^2}$$

leading to

$$\frac{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}{2\sqrt{3\beta_i^*}} > \frac{3}{4}$$

$V_{i,\text{net}}^*$ is the maximized net value of firm $i$, maximized with respect to $\beta_i$. Hence,

$$\frac{dV_{i,\text{net}}^*}{d\beta_i} < 0$$

which leads to

$$\frac{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}{2\sqrt{3\beta_i^*}} < 1.$$ 

Hence we get the upper and lower limits

$$\frac{3}{4} < \frac{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}{2\sqrt{3\beta_i^*}} < 1,$$

$$-\frac{\lambda^2}{9} < [1 - \frac{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}{2\sqrt{3\beta_i^*}} - \frac{\lambda^2}{9}] < 1 - \frac{3}{4} - \frac{\lambda^2}{9}.$$

Sufficient condition for $[1 - \frac{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}{2\sqrt{3\beta_i^*}} - \frac{\lambda^2}{9}] < 0$ is $\frac{3}{2} < \lambda$.

$$-\frac{\lambda^2}{9} < D = [1 - \frac{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}{2\sqrt{3\beta_i^*}}][1 - \frac{(3 - \frac{\lambda^2}{3})^{\frac{3}{2}}}{2\sqrt{3\beta_i^*}}] - \frac{\lambda^2}{9} < 1 - \frac{16 - \lambda^2}{9}$$

Sufficient condition for $D$ to be negative is $\frac{3}{4} < \lambda$.

If $\frac{3}{2} < \lambda$, $\frac{d\beta_2}{dd} > 0$.\footnote{We only consider positive value of $\lambda$}

So $\frac{d\beta_2}{dd} > 0$ when $\frac{3}{2} < \lambda$. Further, when $\frac{3}{2} < \lambda$, $\frac{d\beta_1}{dd} < 0$. So as long as $\frac{3}{2} < \lambda$

$$\frac{d\beta_2}{dd} - \frac{d\beta_1}{dd} > 0$$

Hence we show that as long as $\frac{3}{2} < \lambda$,

$$\frac{d\beta_2}{dd} > 0$$
We note that \( \frac{3}{2} < \lambda \) is a sufficient condition for these to hold, not necessary conditions. There can be other ranges of \( \lambda \) when these two inequalities may hold.

### 4.7.3 Appendix A3

\[
\frac{d\beta_2}{dd} - \frac{d\beta_1}{dd} > 0.
\]

The sufficient condition for \( \frac{dq_2}{dd} > 0 \) is that \( \frac{3}{2} < \lambda \). We should also note that this is a sufficient condition for \( \frac{dq_2}{dd} > 0 \) but not a necessary condition.
Proceeding in the same way for $q_1$, we get,

$$
\frac{dq_1}{dd} = \frac{\frac{de_1}{dd} + \frac{\lambda}{3}c - \frac{\lambda}{3} \frac{de_2}{dd}}{(3 - \frac{\lambda^2}{3})} \\
= \frac{c^{\frac{2}{3}}}{(3 - \frac{\lambda^2}{3})} \frac{(3 - \frac{\lambda^2}{3})^{\frac{1}{2}}}{12 \beta_1^1 \beta_2^1} z^{\frac{1}{2}}
$$

The sufficient condition for $\frac{dq_1}{dd} < 0$ is that $\frac{3}{2} < \lambda$. We should also note that this is a sufficient condition for $\frac{dq_1}{dd} > 0$ but not a necessary condition.

So when $\frac{3}{2} < \lambda$,

$$
\frac{dq_2}{dd} > 0, \\
\frac{dq_2}{dd} < 0, \\
\frac{dq_2}{dd} - \frac{dq_1}{dd} > 0.
$$

QED.

4.7.4 Appendix A4

The various data definitions are as follows:

Sales is Data 12 from COMPUTSTAT. Sales Growth is defined as Sales in year $t$ minus Sales in year $t-1$ divided by Sales at year $t-1$. Proxy for firm aggressiveness is defined as Sales growth of a firm minus the median sales growth of that industry. This is called industry adjusted sales growth. So if a firm is more aggressive in terms of sales than its peers in the industry, this proxy variable should be positive. If the firm is lagging behind others in the same industry, then this variable should come as negative. Industry is defined as the NAICS code. NAICS code better describes industry compared to SIC code. Profitability is defined as the sum of data18, Income before extraordinary items, and data14, Depreciation and Ammortization, divided by total assets, data6. All these data are from Compustat database. Investment is defined as the ratio of data172 by data6, total assets. data 172 is net income (loss). Size is log of assets, log(data6). Leverage is defined as the ratio of data9, long term debt to total assets data6.

We get executive compensation data from ExecuComp. Short Run Bonus is defined as Bonus divided by total current compensation, TDC1 of ExecuComp. Percentage of shares owned by executives is defined as shrown divided by shrsout divided by 10. shrsout is the common shares outstanding. shrown
is the shares owned by the executive. Following Aggarwal and Samwick (1999), CEO compensation is composed of three components: flow compensation, the change in the value of stock holding and the change in the value of stock options. Flow compensation is easily calculated as TDC1, which is available from ExecuComp. TDC1 is composed of salary, bonus, total value of stock options, long term incentive payouts, other annual compensation and all other, as is defined in ExecuComp manual. The change in the value of stock holding is defined as the percentage of stocks held by the CEO at the beginning of the fiscal year multiplied by shareholder dollar return. Total return to shareholders are reported in ExecuComp in percentages. The dollar return is defined as the percentage total return multiplied by the market value of the firm at the beginning of the fiscal year. Once we have the dollar return to shareholder, we can calculate the change in the value of stock holding. The change in the value of stock options is a bit difficult to calculate. We calculate the value of old options as the sum of INMONEX and INMONUN. INMONEX is the value of the unexercised exercisable options. INMONUN is the value of unexercised unexercisable options. The new options are defined as BLK-VALU, which the value of new options granted in ExecuComp. Total option value is the sum of old options and new options. Change in the option value is the value of the option in year t minus the value of the option in year t-1. The total value of CEO’s compensation package is defined as the sum of the flow compensation, the change in the value of stock holding and the change in the value of stock options. The variance of preceding five years stock returns is termed as variance and is used a proxy for stock’s risk. We calculate CEO tenure using BECAMECEO from ExecuComp, which gives us the date an individual has become the CEO. CEO tenure acts a proxy for her abilities when we run pay performance sensitivity regressions.
4.8 Tables

Table 4.1: Descriptive Statistics of mean of sales growth based on two financial constraint criteria and inferences based on t test of the difference of the mean of sales growth

<table>
<thead>
<tr>
<th></th>
<th>Div Payment</th>
<th>LR Credit Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Growth</td>
<td>5.956</td>
<td>10.672</td>
</tr>
<tr>
<td></td>
<td>21.269</td>
<td>17.967</td>
</tr>
<tr>
<td></td>
<td>-17.18***</td>
<td>-5.29***</td>
</tr>
<tr>
<td>Sales Growth Industry Adjusted</td>
<td>-2.133</td>
<td>0.289</td>
</tr>
<tr>
<td></td>
<td>6.649</td>
<td>5.119</td>
</tr>
<tr>
<td></td>
<td>-10.82***</td>
<td>-3.78***</td>
</tr>
</tbody>
</table>

N 2659 2267

Sales is data12 and assets is data6. Sales is normalised by assets data6. Sales is normalised by assets data6. Sales growth in year t is sales in year t minus sales in year t-1 divided by assets in year t-1. Sales growth industry adjusted is the sales growth of the firm minus the median sales growth of the firms in the industry, where industry is defined by the three digit NAICS code. Every entry has three values. The first value is the mean value for the financially unconstrained firms and the second value is that of the financially constrained firm. We conduct a t test to test the difference in the means of sales growth across financially unconstrained and financially constrained firms. The t test assumes inequality of variance between the two samples. T test based on the assumption of equality of variance also gives the similar results. The third value is the t value of that t test. T statistic is the mean sales growth of financially unconstrained minus the mean sales growth of financially constrained firm divided by standard error and adjusted for degrees of freedom. For example, the first entry says that mean sales growth for financially unconstrained firms is 5.956 and mean sales growth for financially constrained firms is 21.269 where dividend payment is the financial constraint measure. The t value of the t test that tests the difference between the mean sales growth of unconstrained and constrained is -14.14 and it is significant at 1 percent significance level. Note that * correspond to significant at 10 percent, ** correspond to significant at 5 percent and *** correspond to significant at 1 percent.
Table 4.2: Fixed Effect Regression of Sales Growth on financial constraint. Dependent Variable is Sales growth with respect to industry.

<table>
<thead>
<tr>
<th></th>
<th>Div Payment</th>
<th>LR Credit Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag Sales Growth Industry Adjusted</td>
<td>-0.079 ***</td>
<td>-0.083 ***</td>
</tr>
<tr>
<td></td>
<td>0.021</td>
<td>0.021</td>
</tr>
<tr>
<td>Size</td>
<td>-0.0004</td>
<td>0.0005</td>
</tr>
<tr>
<td></td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Profitability</td>
<td>-0.107</td>
<td>-0.130</td>
</tr>
<tr>
<td></td>
<td>0.232</td>
<td>0.231</td>
</tr>
<tr>
<td>Lag Profitability</td>
<td>-0.648 ***</td>
<td>-0.637 ***</td>
</tr>
<tr>
<td></td>
<td>0.201</td>
<td>0.201</td>
</tr>
<tr>
<td>Investment</td>
<td>0.667 ***</td>
<td>0.678 ***</td>
</tr>
<tr>
<td></td>
<td>0.196</td>
<td>0.196</td>
</tr>
<tr>
<td>Lag Investment</td>
<td>0.237</td>
<td>0.251</td>
</tr>
<tr>
<td></td>
<td>0.168</td>
<td>0.168</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.071</td>
<td>0.061</td>
</tr>
<tr>
<td></td>
<td>0.046</td>
<td>0.046</td>
</tr>
<tr>
<td>Lag Leverage</td>
<td>-0.069</td>
<td>-0.068</td>
</tr>
<tr>
<td></td>
<td>0.044</td>
<td>0.044</td>
</tr>
<tr>
<td>Cash Capital</td>
<td>0.007 **</td>
<td>0.007 **</td>
</tr>
<tr>
<td></td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>FC dummy</td>
<td>10.457 ***</td>
<td>1.687</td>
</tr>
<tr>
<td></td>
<td>3.601</td>
<td>1.927</td>
</tr>
<tr>
<td>N</td>
<td>2231</td>
<td>2231</td>
</tr>
<tr>
<td>R Square</td>
<td>0.446</td>
<td>0.449</td>
</tr>
</tbody>
</table>

Sales is data12 and assets is data6. Sales is normalised by assets data6. Sales growth in year t is sales in year t minus sales in year t-1 divided by assets in year t-1. Sales growth industry adjusted is the sales growth of the firm minus the median sales growth of the firms in the industry, where industry is defined by the three digit NAICS code. Note that * correspond to significant at 10 percent, ** correspond to significant at 5 percent and *** correspond to significant at 1 percent.
Table 4.3: Fixed Effect Regression of Sales Growth on Managerial Compensation. Instrumental Variable approach

<table>
<thead>
<tr>
<th>L Sales Growth</th>
<th>-0.036**</th>
<th>-0.032*</th>
<th>-0.042**</th>
<th>-0.114***</th>
<th>-0.035**</th>
<th>-0.087***</th>
<th>-0.103***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indus</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
<td>0.019</td>
<td>0.017</td>
<td>0.018</td>
<td>0.019</td>
</tr>
<tr>
<td>Size</td>
<td>-0.0004</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
<td>0.003**</td>
</tr>
<tr>
<td>Profitability</td>
<td>-0.190</td>
<td>-0.198</td>
<td>-0.116</td>
<td>-0.131</td>
<td>-0.077</td>
<td>-0.171</td>
<td>-0.082</td>
</tr>
<tr>
<td>L profitability</td>
<td>-0.605***</td>
<td>-0.627***</td>
<td>-0.779***</td>
<td>-0.656***</td>
<td>-0.547***</td>
<td>-0.711***</td>
<td>-0.373**</td>
</tr>
<tr>
<td></td>
<td>0.159</td>
<td>0.159</td>
<td>0.168</td>
<td>0.178</td>
<td>0.157</td>
<td>0.172</td>
<td>0.170</td>
</tr>
<tr>
<td>Investment</td>
<td>0.688***</td>
<td>0.568***</td>
<td>0.614***</td>
<td>0.569***</td>
<td>0.607***</td>
<td>0.589***</td>
<td>0.573***</td>
</tr>
<tr>
<td>L investment</td>
<td>0.209</td>
<td>0.306**</td>
<td>0.461**</td>
<td>0.546***</td>
<td>0.177</td>
<td>0.538***</td>
<td>0.221</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.090***</td>
<td>0.102***</td>
<td>0.107***</td>
<td>0.104***</td>
<td>0.108***</td>
<td>0.096***</td>
<td>0.110***</td>
</tr>
<tr>
<td>L leverage</td>
<td>-0.052</td>
<td>-0.079**</td>
<td>-0.048</td>
<td>-0.029</td>
<td>-0.058*</td>
<td>-0.032</td>
<td>-0.052</td>
</tr>
<tr>
<td>SR Bonus</td>
<td>0.034</td>
<td>0.034</td>
<td>0.035</td>
<td>0.037</td>
<td>0.034</td>
<td>0.035</td>
<td>0.033</td>
</tr>
<tr>
<td>Stock owned by CEO</td>
<td>0.295***</td>
<td>0.036</td>
<td>0.725**</td>
<td>0.335</td>
<td>0.060**</td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td>Tot comp</td>
<td>0.034</td>
<td>0.034</td>
<td>0.035</td>
<td>0.037</td>
<td>0.034</td>
<td>0.035</td>
<td>0.033</td>
</tr>
<tr>
<td>Flow comp</td>
<td>0.034</td>
<td>0.034</td>
<td>0.035</td>
<td>0.037</td>
<td>0.034</td>
<td>0.035</td>
<td>0.033</td>
</tr>
<tr>
<td>Ch of stock holding</td>
<td>0.034</td>
<td>0.034</td>
<td>0.035</td>
<td>0.037</td>
<td>0.034</td>
<td>0.035</td>
<td>0.033</td>
</tr>
<tr>
<td>Ch of stock option</td>
<td>0.034</td>
<td>0.034</td>
<td>0.035</td>
<td>0.037</td>
<td>0.034</td>
<td>0.035</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Dependent variable is industry adjusted sales growth, which is \((sales_t - sales_{t-1})/sales_{t-1}\). Every entry has two values. Instrument of a variable is the lag of the variable. The top value is the coefficient of the regression coefficient and the bottom one is the corresponding standard error. The data definition is from Kaplan and Zingales 1997. Note that * correspond to significant at 10 percent, ** correspond to significant at 5 percent and *** correspond to significant at 1 percent.
Table 4.4: Regression of Sales Growth on Managerial Compensation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient (Top)</th>
<th>Coefficient (Bottom)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L Sales Growth</td>
<td>-0.034**</td>
<td>-0.036***</td>
</tr>
<tr>
<td>Indus</td>
<td>0.017</td>
<td>0.0005</td>
</tr>
<tr>
<td>Size</td>
<td>0.0003</td>
<td>0.0005</td>
</tr>
<tr>
<td>Profitability</td>
<td>-0.171</td>
<td>-0.135</td>
</tr>
<tr>
<td>Lprofitability</td>
<td>-0.601***</td>
<td>-0.564***</td>
</tr>
<tr>
<td>Investment</td>
<td>0.616***</td>
<td>0.634***</td>
</tr>
<tr>
<td>Linvestment</td>
<td>0.233*</td>
<td>0.179</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.105***</td>
<td>0.089**</td>
</tr>
<tr>
<td>L leverage</td>
<td>-0.075**</td>
<td>-0.041</td>
</tr>
<tr>
<td>SR Bonus</td>
<td>0.111***</td>
<td>0.025</td>
</tr>
<tr>
<td>L SR Bonus</td>
<td>0.062**</td>
<td>0.025</td>
</tr>
<tr>
<td>L stock owned</td>
<td>0.200**</td>
<td>0.092</td>
</tr>
<tr>
<td>by CEO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L tot comp</td>
<td></td>
<td>0.023***</td>
</tr>
<tr>
<td>L flow comp</td>
<td></td>
<td>0.009</td>
</tr>
<tr>
<td>L ch stock</td>
<td></td>
<td>-0.209***</td>
</tr>
<tr>
<td>hold</td>
<td></td>
<td>0.076</td>
</tr>
<tr>
<td>L ch stock</td>
<td></td>
<td>0.074**</td>
</tr>
<tr>
<td>opt</td>
<td></td>
<td>0.035</td>
</tr>
<tr>
<td>N</td>
<td>3507</td>
<td>3501</td>
</tr>
<tr>
<td>R Square</td>
<td>0.385</td>
<td>0.378</td>
</tr>
</tbody>
</table>

Dependent variable is industry adjusted sales growth, which is \((sales_t - sales_{t-1})/sales_{t-1}\). The top value is the coefficient of the regression coefficient and the bottom one is the corresponding standard error. The data definition is from Kaplan and Zingales 1997. Note that * correspond to significant at 10 percent, ** correspond to significant at 5 percent and *** correspond to significant at 1 percent.
### Table 4.5: Fixed Effect Regression of Managerial Compensation on Sales Growth

<table>
<thead>
<tr>
<th></th>
<th>Tot Comp</th>
<th>Fl Comp</th>
<th>Ch Stock Hold</th>
<th>Ch Stock Opt</th>
<th>Stock Own</th>
<th>SR Bonus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ret</td>
<td>0.353***</td>
<td>0.008</td>
<td>0.055***</td>
<td>0.285***</td>
<td>-0.0002</td>
<td>0.060***</td>
</tr>
<tr>
<td>Ret*tenure</td>
<td>-0.0001</td>
<td>-0.0001</td>
<td>-0.001***</td>
<td>0.0007***</td>
<td>0.0000</td>
<td>-0.0002*</td>
</tr>
<tr>
<td>Ret*var</td>
<td>1.843***</td>
<td>-0.007</td>
<td>0.775***</td>
<td>1.064***</td>
<td>0.0027</td>
<td>0.033</td>
</tr>
<tr>
<td>Ret*size</td>
<td>0.166</td>
<td>0.209</td>
<td>0.049</td>
<td>0.114</td>
<td>0.0175</td>
<td>0.083</td>
</tr>
<tr>
<td>Sales growth</td>
<td>-0.0139</td>
<td>0.021</td>
<td>-0.023</td>
<td>0.015</td>
<td>0.004</td>
<td>0.156***</td>
</tr>
<tr>
<td>Adjusted</td>
<td>0.048</td>
<td>0.058</td>
<td>0.014</td>
<td>0.032</td>
<td>0.005</td>
<td>0.023</td>
</tr>
<tr>
<td>N</td>
<td>2407</td>
<td>2672</td>
<td>2565</td>
<td>2516</td>
<td>2565</td>
<td>2679</td>
</tr>
<tr>
<td>R Square</td>
<td>0.558</td>
<td>0.127</td>
<td>0.342</td>
<td>0.586</td>
<td>0.684</td>
<td>0.473</td>
</tr>
</tbody>
</table>

Sales Growth is \((sales_t - sales_{t-1}) / sales_{t-1}\). Sales growth in is industry adjusted sales growth. Ret is dollar return to share holder which is defined as the total market value of equity at the beginning of the year multiplied by the stock return including distributions over the year. Variance is the preceding 60 months variance of the monthly return of the stock. We get the monthly returns from CRSP. Using today’s date, we calculate the variance of monthly returns for the preceding 60 months. Variance is the measure of risk. Size is the log of assets. Tenure of the manager is calculated. Execucomp gives us the date the person became CEO. We subtract that date from today’s date to calculate the tenure of CEO. Every entry has two values. Total compensation is divided into three components, flow compensation, change in value of stock holding (ch of stock holding) and change in value of stock option (ch of stock option). Short run bonus is defined as the ratio of bonus to flow compensation. The top value is the coefficient of the regression coefficient and the bottom one is the corresponding standard error. Note that * correspond to significant at 10 percent, ** correspond to significant at 5 percent and *** correspond to significant at 1 percent.
Table 4.6: Fixed Effect Regression of Managerial Compensation on Financial Constraint. Financial Constraint is based on Dividend Payment

<table>
<thead>
<tr>
<th></th>
<th>Tot Comp</th>
<th>Fl Comp</th>
<th>Ch Stock Hold</th>
<th>Ch Stock Opt</th>
<th>Stock Own</th>
<th>SR Bonus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ret</td>
<td>0.293***</td>
<td>0.005</td>
<td>0.046***</td>
<td>0.237***</td>
<td>-0.0000</td>
<td>0.014*</td>
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<td>0.021</td>
<td>0.037</td>
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<td>Ret*var</td>
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<td>1.408***</td>
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<td>Ret*size</td>
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<td>-0.000</td>
<td>-0.0000***</td>
<td>-0.0000***</td>
<td>0.000</td>
<td>-0.0000*</td>
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<tr>
<td>FC Dummy</td>
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<td>-0.994</td>
<td>-0.176</td>
<td>18.945***</td>
<td>2.68**</td>
<td>7.37**</td>
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<td>9.247</td>
<td>14.025</td>
<td>3.102</td>
<td>6.466</td>
<td>1.102</td>
<td>3.110</td>
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|          | 1595     | 1775    | 1707          | 1665         | 1707      | 1775     |
| R Square | 0.735    | 0.158   | 0.3878        | 0.721        | 0.650     | 0.424    |

Sales Growth is \((sales_t - sales_{t-1})/sales_{t-1}\). Sales growth in is industry adjusted sales growth. ret is dollar return to share holder which is defined as the total market value of equity at the beginning of the year multiplied by the stock return including distributions over the year. Variance is the preceding 60 months variance of the monthly return of the stock. We get the monthly returns from CRSP. Using today's date, we calculate the variance of monthly returns for the preceding 60 months. Variance is the measure of risk. size is the log of assets. tenure of the manager is calculated. Execucomp gives us the date the person became CEO. We subtract that date from today's date to calculate the tenure of CEO. Every entry has two values. Total compensation is divided into three components, flow compensation, change in value of stock holding (ch of stock holding) and change in value of stock option (ch of stock option). Short run bonus is defined as the ratio of bonus to flow compensation. The top value is the coefficient of the regression coefficient and the bottom one is the corresponding standard error. Note that * correspond to significant at 10 percent, ** correspond to significant at 5 percent and *** correspond to significant at 1 percent.
Table 4.7: Fixed Effect Regression of Managerial Compensation on Financial Constraint. Financial Constraint is based on Long Run Credit rating of the firm

<table>
<thead>
<tr>
<th></th>
<th>Tot Comp</th>
<th>Fl Comp</th>
<th>Ch Stock Hold</th>
<th>Ch Stock Opt</th>
<th>Stock Own</th>
<th>SR Bonus</th>
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<td>0.059***</td>
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<td>0.938***</td>
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<td>0.0000</td>
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<tr>
<td>FC Dummy</td>
<td>18.134***</td>
<td>4.76</td>
<td>0.955</td>
<td>11.820***</td>
<td>-0.266</td>
<td>5.046***</td>
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<tr>
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<td>7.027</td>
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<td>1.9717</td>
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<td>1533</td>
<td>1475</td>
<td>1437</td>
<td>1475</td>
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<tr>
<td>R Square</td>
<td>0.564</td>
<td>0.123</td>
<td>0.360</td>
<td>0.604</td>
<td>0.670</td>
<td>0.379</td>
</tr>
</tbody>
</table>

Sales Growth is \((sales_t - sales_{t-1})/sales_{t-1}\). Sales growth in is industry adjusted sales growth. ret is dollar return to share holder which is defined as the total market value of equity at the beginning of the year multiplied by the stock return including distributions over the year. Variance is the preceding 60 months variance of the monthly return of the stock. We get the monthly returns from CRSP. Using today’s date, we calculate the variance of monthly returns for the preceding 60 months. Variance is the measure of risk. size is the log of assets. tenure of the manager is calculated. Execucomp gives us the date the person became CEO. We subtract that date from today’s date to calculate the tenure of CEO. Every entry has two values. Total compensation is divided into three components, flow compensation, change in value of stock holding \((ch of stock holding)\) and change in value of stock option \((ch of stock option)\). Short run bonus is defined as the ratio of bonus to flow compensation. The top value is the coefficient of the regression coefficient and the bottom one is the corresponding standard error. Note that * correspond to significant at 10 percent, ** correspond to significant at 5 percent and *** correspond to significant at 1 percent.
Chapter 5

Conclusion

This thesis investigates product market capital market interaction. Equity holders are protected by limited liability. The traditional argument is that debt financing leads to more aggressive product market strategies. The intuition behind this is that debt is issued first and then output decisions are made. The manager of a firm caters to the equity holders. After the firm has issued debt, the manager is free to choose output. The manager may adopt risky product market strategies thereby shifting wealth from the debt holders to the equity holders. In the second chapter, we discuss that this logic does not hold if either debt is endogenous or switching state of nature is endogenous or both debt and switching state of nature are endogenous. The benefits of acting aggressively are the increase in revenue in the good states of nature. The costs of acting aggressively are the revenue loss during the bad states of nature. When debt and/or switching state is endogenous, the range of bad states of nature is endogenous. More aggressive behavior leads to a bigger range of bad states of nature and a bigger loss to the equity holders. The manager evaluates both the benefits and costs before deciding whether to produce more aggressively. When the demand uncertainty is low, the costs of acting aggressively outweigh the benefits of acting aggressively and the firm acts less aggressively in the product market.

In the third chapter, both debt and equity financing are considered as sources of external financing. The loss to the equity holders is the cost imposed by liquidation of the firm. It is assumed that in case of bankruptcy, the firm is liquidated and the assets of the firm are auctioned off. The proceeds go to the debt holders and the equity holders do not receive anything. In this scenario, a financially constrained firm, which uses both debt and equity financing, shall produce less than what it would have produced if it were not financially constrained. Using comparative statics, a relation between the rate of return on equity and the equilibrium output is developed.
Chapter 4 deals with managerial compensation. It is shown that financially constrained firms may be producing more than the financially unconstrained because of the managerial compensation structure. A financially constrained firm by definition has a higher cost of capital and higher production cost. One of the ways to counter the higher production cost is to induce the manager to provide higher effort. The managerial compensation of a financially constrained firm is such that the manager has higher incentive to perform. The pay performance sensitivity is higher for the financially constrained firm. It is reported that managerial compensation is an explanatory variable of more aggressive product market strategies of a financially constrained firm.

Output decisions and financing decisions are dependent on each other. Instead of modeling the financing decisions and output decisions separately, both these decisions should be modeled simultaneously to develop a better understanding of the firm. This thesis shall help us better understand the interaction of product markets and capital markets.
Bibliography


